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Aspects of de Sitter Vacua in Theoretical Cosmology

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To my parents, family, supervisors and friends.

Abstract

The work presented in this thesis is based on research conducted at the Particle Theory Group, School of Physics and Astronomy, University of Nottingham, United Kingdom. In this thesis we explore de Sitter vacua in theoretical cosmology. The thesis has two parts. In Part I we consider different string-inspired models which are related to the cosmological constant problem. In Part II we study the de Sitter vacua solutions in one string cosmology model, namely, $O(d,d)$ invariant cosmology.

In Part I we study three aspects of de Sitter vacua in string-inspired models. First of all, in chapter 2 we review the related concepts and tools we need in this part, namely, string landscape, swampland conjecture and bubble nucleation. In chapter 3, we consider a wide class of $4D$ effective field theories in which gravity is coupled to multiple four-forms and their dual scalar fields, with membrane sources charged under the corresponding three-form potentials. We propose a "probabilistic" argument to explain the small cosmological constant instead of the dominant anthropic principle. Chapter 3 is based on original work in collaboration with Antonio Padilla and Francisco G. Pedro [1]. In chapter 4, we explore in detail whether or not different models are compatible with the membrane weak gravity conjecture. We find that these models can be generalised to the DBI action. Chapter 4 is based on original work in collaboration with Antonio Padilla and Francisco G. Pedro [2]. In chapter 5, we generalise the results in chapter 3 and 4 to D-dimensions including the Gauss-Bonnet term. We find that the Euclidean action of the bounce, B, describing the decay of a de Sitter vacuum, is proportional to $k_+^{-(D-2)}$, which has a pole as $k_+^2 \to 0$ where k_+^2 is the curvature of the parent vacuum. We find a new decay channel, describing up-tunneling from anti-de Sitter into de Sitter. Chapter 5 is based on original work done by the author [3].

In Part II we explore the de Sitter vacua solutions in $O(d,d)$ invariant cosmology. We briefly review the basic concepts of $O(d,d)$ invariant cosmology in chapter 6. Then in chapter 7 we perform a thorough analysis of de Sitter solutions in $O(d,d)$ invariant cosmologies. We elucidate the nature of the instability in the string frame vacuum. For the Einstein frame, we find that the de Sitter solutions cannot be eternal. Then we extend our analysis to include Bianchi I universes. We show that we can extend the theory to the anisotropic case so that de-Sitter solutions can exist. All results in chapter 7 are presented in this paper [4], which is done in collaboration with Antonio Padilla, Paul Saffin and Robert Smith.

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List of Publications

1. The cosmological constant is probably still zero,
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4. On de Sitter vacua in $O(d,d)$ invariant cosmology,
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   Phys.Rev.D 110 (2024) 063522, arXiv:2404.15401, [Chapter 7]

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except where specifically indicated in the text. No part of this thesis has previously been submitted for a degree or other qualification at this or any other university.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

There are four fundamental interactions in nature: the strong force, the weak force, the electromagnetic force and the gravitational force. At long distances gravity is the weakest force. For example, if we consider the gravitational force between two electrons, we can find that gravity is approximately 10^{-42} in units of the corresponding electromagnetic force. However, gravity plays the most important role in cosmology since gravity is the only relevant interaction for the whole universe among four fundamental interactions. Therefore, in order to give a consistent description of the universe, we need to understand gravity very well.

1.1 General relativity

In 1915 Einstein proposed the theory of general relativity (GR) [5, 6], which is our best understanding of the gravitational force at low energies so far. In order to solve the inconsistencies between Newtonian gravity and special relativity, Einstein introduced a revolutionary idea. In GR the gravitational force has been replaced by a geometric description of a dynamical spacetime. Since then a lot of observations have been done and verified the results of GR, such as gravitational red-shift, the bending of light by the Sun and the perihelion precession of Mercury. More details can be found in [5, 6]. In recent years, one of the most important predictions in GR, gravitational waves, have been observed [7] and the leaders of the research team won the Nobel prize in physics in 2017.

The action of GR, the Einstein-Hilbert action, is given by

$$S_{GR} = \int d^4x\sqrt{-g}(\frac{1}{16\pi G}R - \Lambda), \tag{1.1}$$

where $g_{\mu\nu}$ is the metric tensor, $g = \det g_{\mu\nu}$, $R = g_{\mu\nu}R^{\mu\nu}$ is the Ricci scalar and $R_{\mu\nu}$ is the Ricci tensor. G is the Newton's constant. Λ is the cosmological constant. We have set $c = 1$. Furthermore, if we couple matter fields to

gravity, the total action of the whole system is

$$S = S_{GR} + S_m(g^{\mu\nu}, \Phi) \tag{1.2}$$

where Φ denotes all fields. The resulting field equations after varying the action with respect to $g_{\mu\nu}$ are the following

$$G_{\mu\nu} + 8\pi G\Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.3}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{1.4}$$

is the Einstein tensor and

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} \tag{1.5}$$

is the energy-momentum tensor. The Einstein equation is a set of nonlinear partial differential equations, therefore $g_{\mu\nu}$ is very hard to solve exactly. More details of GR can be found in famous textbooks [8–10].

If we only consider the range of the Solar System, the cosmological constant Λ can be neglected. However, if we apply the Einstein equation to cosmology, the contribution from Λ will play an important role. In theoretical physics how to explain the small value of the cosmological constant is a long-standing problem, which is the so-called "the cosmological constant (CC) problem". We will discuss the CC problem in section 1.2.

If the cosmological constant Λ is neglected, our universe should be shrinking or decelerating because of the attraction of gravity. However, one of the most crucial findings in cosmology in recent years is that our universe has an accelerating expansion. The first series of observations of this kind came from [11, 12]. The leaders of these teams won the Nobel prize in physics in 2011.

The reason for the accelerating expansion of the universe is still unclear.

The driving force of the accelerating expansion is often called the "dark energy", which is consistent with a vacuum energy, or cosmological constant [11, 12]. Therefore, in principle, if the origin of the cosmological constant can be understood, then the origin of the accelerating expansion of universe can be revealed.[1]

At present it has been realized that GR is just the low energy effective field theory of gravity, although it is still the best theory of gravity we have. It breaks down at high energies where we expect it to be replaced by a quantum theory of gravity. In Einstein's eyes, GR was constructed by the principle of equivalence and general coordinate invariance. Thanks to Riemannian geometry, gravity has disappeared in GR and has been replaced by the geometry of spacetime. However, in modern quantum field theory we now understand that GR is an inevitable consequence of a few basic principles without assuming a priori the equivalence principle or general coordinate invariance. We can construct GR as a low energy description of interacting massless spin-2 particle using locality and Lorentz invariance [13–17].

However, GR is not a UV complete theory. Here are the reasons. First, when we quantize GR in the traditional QFT way (perturbatively), infinite quantities that cannot be absorbed by a finite number of counterterms, which means GR is non-renormalizable. Second, the gravitational coupling constant in GR is Newton's constant G, which has negative mass dimension: $[G] = (\text{mass})^{-2}$. This implies that the theory becomes strongly coupled at high energies (or small distances), where loop corrections blow up. This breakdown hints at the incompleteness of GR in the UV – it cannot be extended to arbitrarily small scales without modification. Third, the expansion of GR around flat spacetime (or any background) leads to terms that grow uncontrollably at high energy. The perturbation series does not converge or even make sense beyond a certain scale – usually around the Planck scale ($\sim 10^{19}$GeV). Fourth, GR predicts singularities (e.g., inside black holes, at the Big Bang) where curvature

[1] That is only true if the data remains consistent with $\omega = p/\rho = -1$.

becomes infinite. These are signals that the classical theory breaks down – something quantum needs to resolve them [18, 19]. Therefore, we need a UV complete gravitational theory. In Section 1.3, we will briefly review the most promising unified theory - string theory.

In fact, the other three fundamental interactions can be described by Yang-Mills theory, or gauge field theory, which is the basic principle of interactions between massless spin-1 particles. Theoretical physicists have attempted to derive the value of the cosmological constant from quantum field theory by summing the contributions of vacuum energy from quantum fields. For example, if we add up the vacuum contributions from QED (electromagnetic field), QCD (gluon field, quark condensates), Higgs field, Electroweak gauge bosons (W and Z) and gravitational zero-point fluctuations (in speculative models), we get something on the order of $\rho_{\text{vac}}^{\text{theory}} \sim 10^{60}$ to $10^{120} \times \rho_{\text{vac}}^{\text{obs}}$, where $\rho_{\text{vac}}^{\text{theory}}$ is the theoretical result of vacuum energy density and $\rho_{\text{vac}}^{\text{obs}}$ is the observational result of vacuum energy density [20, 21]. The observational value of Λ is much smaller than the theoretical result, which is one of the biggest challenges in modern theoretical physics. The main topic of this thesis is trying to understand CC problem better.

1.2 The cosmological constant problem

The cosmological constant problem has haunted theoretical physics ever since Pauli calculated the effect of the zero-point energy of the electron on the curvature of spacetime, declaring that the universe would "not even reach to the moon" [22]. The situation has not really improved in the century that followed. When we apply standard quantum field theory methods, radiative corrections to the vacuum energy are extremely sensitive to ultra-violet physics, scaling like the fourth power of the cut-off. Vacuum energy gravitates just like a cosmological constant presenting a major issue for cosmology. The observed value of the cosmological constant lies sixty orders of magnitude below the expected

value predicted from vacuum energy calculations with a TeV cut-off, set by the scale of modern collider experiments. If we push the cut-off all the way up to the Planck scale, the discrepancy extends to 120 orders of magnitude. The problem has at least three aspects [23]:

1. Why the cosmological constant is so small?
2. Why the cosmological constant is positive?
3. How to explain the coincidence problem?[2]

We will focus on the first question. It is hard enough! Now let us review the background.

When we write down the field equation for GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8\pi G\Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.6}$$

we realize one important thing: the cosmological constant Λ was not fixed by the structure of GR. In other words, Λ can be an arbitrary constant. In order to yield a static cosmological solution, Einstein introduced the cosmological constant Λ. However, the cosmological solution is still non-static, which is the "greatest blunder" of Einstein [20]. The cosmological constant Λ returns through the back door. The quantum fluctuations in the vacuum of quantum field theory contribute to the expectation value of the stress tensor in a way which is exactly like a cosmological constant. So the cosmological constant becomes a real problem [20].

In quantum field theory, the nature of vacuum is much more complicated than in classical theory. Let us take an example. When we consider the ground state of a harmonic oscillator, every mode of each field can contribute a zero-point energy to the energy density of the vacuum. In the description of path integral, this energy arises from the so-called "loops", or virtual particle-antiparticle pairs. If the vacuum is assumed to be Lorentz invariant, then the corresponding energy-momentum tensor must also be Lorentz invariant.

[2]The coincidence problem basically asks why are the energy densities of dark energy and matter of the same order right now, even though they evolve very differently over time?

The only Lorentz invariant rank 2 tensor is the Minkowski metric. By general covariance, this gets promoted to the full metric $g_{\mu\nu}$ in curved spacetime. Therefore, the corresponding energy-momentum-stress tensor should be proportional to the metric, namely, [8–10]

$$< T_{\mu\nu} >= -\rho_{vac} g_{\mu\nu}, \tag{1.7}$$

where ρ_{vac} is the density of vacuum energy.

Although it is located on the right-hand side of Einstein's equation, vacuum energy takes the form of a cosmological constant, meaning we could incorporate it and redefine Λ as follows:

$$\Lambda = \Lambda_{Ein} + \rho_{vac}, \tag{1.8}$$

where Λ_{Ein} is the bare cosmological constant appearing in Einstein's equation. With observational data, the upper bound of cosmological constant is $|\Lambda| \leq 3 \times 10^{-122} l_p^2$, where l_p is the Planck length [20].

In this section we will discuss different aspects of the cosmological constant problem. We start by arguing that the CC problem can be traced back to two rigorous foundations of modern physics, i.e., quantum field theory (QFT) and GR. Then we explain how the vacuum energy contributions to the cosmological constant are calculated in QFT and semiclassical gravity theory. We then explain why we must repeatedly fine tune a classical parameter in GR to match observations of a small cosmological constant. We refer the reader to the number of reviews on the cosmological constant problem for more details [20, 21, 23–25].

1.2.1 Two rigorous foundations

The cosmological constant problem arises from the intersection of two fundamental pillars of modern physics: quantum field theory (QFT) and general

relativity (GR).

In QFT, the vacuum is not empty but has energy. This vacuum energy influences physical processes through loop corrections to tree-level scattering amplitudes. Consider, for example, quantum electrodynamics (QED). A specific case is the Coulomb interaction between an electron e^- and a positron e^+, which occurs in Bhabha scattering $e^-e^+ \to e^-e^+$. In QED, the Coulomb potential is mediated by the exchange of a virtual photon, as described by the QED Lagrangian:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m_e)\psi - e\bar{\psi}\gamma^\mu A_\mu\psi, \tag{1.9}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of vector potential A_μ, ψ is the relevant fermion, γ^μ are the gamma matrices and e is the electric charge of relevant fermion. In the non-relativistic limit, the Coulomb potential arising from tree diagram is given by

$$V(r) = -\frac{e^2}{4\pi r}, \tag{1.10}$$

which describes the attractive interaction between two opposite charges. However, the story does not end there. We can calculate a 1-loop correction to the tree-level Coulomb potential, a process known as vacuum polarization. If $e \ll 1$, the 1-loop contribution is much smaller than the tree-level term. The non-relativistic potential, which includes both the tree-level and 1-loop contributions, is referred to as the Uehling potential [26, 27]:

$$V(r) = -\frac{e^2}{4\pi r}\left(1 + \frac{e^2}{6\pi^2}\int_1^\infty dx e^{-2m_e r x}\frac{2x^2+1}{2x^4}\sqrt{x^2-1}\right), \tag{1.11}$$

where the x is the parameter often appears in Feynman integral. The integral in (1.11) is hard to calculate, but we can do it for the following two limits. When the distance scale exceeds the Compton wavelength of the electron,

namely, $r \gg 1/m_e$, the Uehling potential becomes [26, 27]:

$$V(r) = -\frac{e^2}{4\pi r}\left[1 + \frac{e^2}{16\pi^{3/2}}\frac{1}{(m_e r)^{3/2}}e^{-2m_e r}\right]. \tag{1.12}$$

If the distance scale is smaller than the Compton wavelength of the electron, namely, $r \ll 1/m_e$, the Uehling potential is [26, 27]

$$V(r) = -\frac{e^2}{4\pi r}\left[1 + \frac{e^2}{6\pi^2}\left(\log\frac{1}{m_e r} - \frac{5}{6} - \gamma_E\right)\right], \tag{1.13}$$

where $\gamma_E \approx 0.577$ is the Euler-Mascheroni constant. The two limits are easier to analyze, so we just consider these two limits. This correction to the tree-level potential energy arises entirely from the interactions between the virtual photon and the vacuum fluctuations of the virtual electron-positron pair. This serves as an example of how vacuum fluctuations influence physical processes [28].

In 1947, Willis Lamb observed a small splitting between the energy levels of the hydrogen atom, which would be degenerate in both Dirac theory and tree-level QED. This phenomenon, known as the "Lamb shift", led physicists to recognize the crucial role of loop corrections in accurately describing physical processes [28–32].

After discussing the first foundation — vacuum fluctuations in QFT — let us now turn to the second foundation: the principle of equivalence. This principle asserts that all forms of matter and energy experience gravity with the same strength. As the fundamental principle of GR, the equivalence principle suggests that the vacuum energy fluctuations contributing to the Lamb shift should also be affected by gravity [28]. In fact, loop corrections influence not only the inertial energy of an atom but also its gravitational energy. However, since the gravitational force is much weaker than the electromagnetic force, gravitational energy is often neglected in QED. In fact, this small change in the gravitational mass due to Lamb shift has been measured and more details

can be found in [33].

1.2.2 Fine tuning

The cosmological constant problem arises as an unwanted consequence of two foundational pillars of twentieth-century physics: QFT and GR. As discussed in the previous section, the vacuum possesses energy. In QFT, the standard approach to calculating vacuum energy involves computing the vacuum loop diagrams for each particle species [25]. To determine the vacuum energy, we employ a semi-classical framework, where the field theory sector is treated quantum mechanically, while the gravitational degrees of freedom are treated classically. Typically, the action is expanded around Minkowski spacetime at the outset [28]. We compute the loop corrections in the limit $M_{pl} \to \infty$. After calculating the vacuum energy contributions to a given order, we restore the finite value of M_{pl}. This step enables gravity to classically probe the resulting vacuum energy in accordance with gauge invariance. In GR, the symmetry under diffeomorphisms (general coordinate transformations) plays the role of a gauge symmetry. That is:

1. The gravitational field (described by the metric tensor $g_{\mu\nu}$) is not uniquely defined until we fix a coordinate system;

2. Diffeomorphism invariance ensures that all coordinate systems are physically equivalent – just as gauge invariance ensures that all gauge choices are physically equivalent.

Diffeomorphism invariance ensures that vacuum energy gravitates. Specifically, the action term $-V_{vac}\int d^4x$ in flat space gets promoted to

$$-V_{vac}\int d^4x\sqrt{-g}. \tag{1.14}$$

in curved space in order for the action to remain invariance under diffeomorphisms [25]. This means that the vacuum energy couples to metric.

When calculating the loop corrections to the cosmological constant, we en-

counter divergent integrals that require regularization. A widely used approach is dimensional regularization, which involves working in $D = 4 - \epsilon$ dimensions. Consider a scalar field of mass m_ϕ and quartic self-interaction $\frac{\lambda}{4!}\phi^4$. In one loop, the only vacuum diagram is the free theory bubble, a single loop with no interaction vertices (the first term of Figure 1.1). This comes from the Gaussian integral in the path integral. The corresponding one-loop contribution is given by

$$V_{vac}^{\phi,1-loop} = \frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\log(k^2 - m_\phi^2) \tag{1.15}$$

namely,

$$V_{vac}^{\phi,1-loop} = -\frac{m_\phi^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \log\left(\frac{\mu^2}{m_\phi^2}\right) + \text{finite}\right], \tag{1.16}$$

where μ is the arbitrary mass scale introduced in dimensional regularization [25]. The divergence as $\epsilon \to 0$ requires to add the following counter term which depends on an arbitrary subtraction scale, M,

$$V_c^{\phi,1-loop} = \frac{m_\phi^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \log\left(\frac{\mu^2}{M^2}\right)\right]. \tag{1.17}$$

Combining the two we obtain the renormalised vacuum energy at 1-loop [25, 28]

$$V_{ren}^{\phi,1-loop} \sim \frac{m_\phi^4}{(8\pi)^2}\left[\log\left(\frac{m_\phi^2}{M^2}\right) - \text{finite}\right]. \tag{1.18}$$

Since $V_{ren}^{\phi,1-loop}$ explicitly depends on the arbitrary scale M, we do not obtain a definitive result for the renormalized vacuum energy. As a result, we are unable to predict the exact contribution of the vacuum energy from massive scalar particles to the cosmological constant in effective field theory.

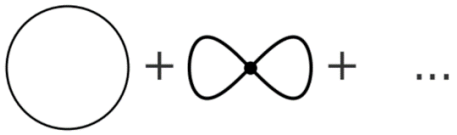

Figure 1.1: *1-loop and 2-loop vacuum diagrams in $\lambda\phi^4$ theory.*

Since we are unable to predict the exact value of the cosmological constant,

it is necessary to measure it and adjust the finite part of Λ_c to align with observational data. For instance, if the massive scalar field is the Higgs boson with a mass of 126 GeV, we can choose M in such a way that the logarithm cancels the "finite" part with an accuracy of 1 part in 10^{60} [28].

We now proceed to examine the two-loop correction to the vacuum energy from a massive scalar field. The two-loop correction corresponds to the second term of Figure 1.1. The two-loop correction to the vacuum energy is

$$V_{vac}^{\phi,2-loop} = \frac{i\lambda}{8} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2}, \tag{1.19}$$

namely, scales as

$$V_{vac}^{\phi,2-loop} \sim \lambda m_\phi^4. \tag{1.20}$$

In 2-loop diagram, there is an interaction vertex $\frac{\lambda}{4!}\phi^4$, but 1-loop diagram does not have the vertex. Therefore, (1.20) depends on λ but (1.16) does not. In perturbative theories that do not have precisely adjusted couplings, this means that the 2-loop correction is not much smaller than the 1-loop contribution. As a result, the cancellation we established at the 1-loop level is entirely disrupted, and we must re-adjust the finite contributions in the counterterms with a similar level of precision [25]. Likewise, the contributions from the 3-loop, 4-loop, and higher-order corrections must also be re-tuned. Consequently, we must re-adjust the bare cosmological constant at each order in loop perturbation theory [28]. This demonstrates that the cosmological constant is highly sensitive to high-energy physics, and its extremely small observed value appears to be highly unnatural [28]. This fact that we cannot predict the magnitude of the cosmological constant is an inevitable consequence of effective field theory. In QED we cannot predict the values of the electron mass or charge as well. However, in QED we do not need to fine tune repeatedly since there is no radiative instability [28].

Here we make a comment on the differences between the electron mass and the cosmological constant. The electron mass and the cosmological constant

behave very differently when it comes to quantum loop corrections. The electron mass receives relatively mild, logarithmic corrections, and importantly, it is protected by a symmetry: chiral symmetry. If the electron mass were zero at tree level, quantum corrections would not generate a mass because chiral symmetry would prevent it. This makes the electron mass technically natural – small values are stable under quantum corrections without needing fine-tuning. In contrast, the cosmological constant receives extremely large, quartically divergent corrections from quantum loops. Every quantum field contributes to the vacuum energy, and these contributions grow rapidly with the energy cutoff. Unlike the electron mass, the cosmological constant is not protected by any known symmetry — setting it to zero doesn't enhance the symmetry of the theory. As a result, keeping the cosmological constant small requires an extraordinary degree of fine-tuning to cancel these large quantum contributions. So, the key distinction is that the electron mass is stable due to symmetry, while the cosmological constant is not, and this lack of protection leads to one of the biggest fine-tuning problems in theoretical physics [21, 27, 34, 35].

It is useful to mention another important problem in physics – the Higgs mass problem: why the physical Higgs mass (~ 125GeV) is much smaller than the Planck scale ($\sim 10^{19}$GeV)? Despite that they are very different, they can be connected. For example, both are about why certain constants of nature are so small, despite large quantum corrections. Both involve vacuum sensitivity to UV physics (Planck scale). In string theory/landscape/multiverse approaches, both problems might have anthropic explanations: Higgs mass might scan across the landscape and only values leading to atoms exist, while the cosmological constant might scan and only small Λ allows structure formation and human existence [36].

An alternative yet equivalent way to describe the cosmological constant problem does not involve altering the loop order in perturbation theory. This approach utilizes the Wilson action, demonstrating that the vacuum energy

becomes unstable when the Wilsonian cut-off is modified [25]. The fundamental concept of the Wilson action is to integrate out the heavy degrees of freedom that lie above a certain mass scale, denoted by μ. We start by assuming the presence of a microscopic action $S[\phi_i]$ constructed from fields ϕ_i, which is applicable at high energy scales. The low and high energy modes decouple, allowing each energy scale to be understood independently. By using the Wilsonian action approach, we can focus on the physics at low energy scales without needing to understand the details at high energy scales. This principle of decoupling is commonly observed in physics; for example, when studying atomic physics, we do not need to consider the behavior of quarks and gluons. However, if decoupling does not apply, it becomes much more challenging to study the system [28].

The degree of freedom ϕ can be decomposed as $\phi = \phi_l + \phi_h$, where the heavy modes ϕ_h correspond to energies greater than μ, and the light modes ϕ_l correspond to energies less than μ. The Wilson effective action, $S_{eff}[\phi_l]$ is obtained by integrating out the heavy modes in the path integral

$$\exp\left(iS_{eff}[\phi_l]\right) = \int D\phi_h \exp\left(iS[\phi_l, \phi_h]\right), \tag{1.21}$$

where $S_{eff}[\phi_l, \phi_h]$ represents the complete microscopic action that includes both the heavy and light degrees of freedom [25]. The vacuum energy is obtained from the effective action $S_{eff}[\phi_l]$, which is valid up to the scale μ and is anticipated to be of order μ^4. This allows us to determine the cosmological counterterm needed to reconcile the renormalized cosmological constant with low-energy observations [25].

Now, let us consider the scenario when the cut-off is shifted to a new scale $\hat{\mu} < \mu$. This entails integrating out the degrees of freedom with energies between $\hat{\mu}$ and μ. The resulting effective action $\hat{S}_{eff}$ generates a vacuum energy of the form $\hat{\mu}^4$, which needs to be canceled by the cosmological counterterm. However, the cosmological counterterm was originally fixed to ensure cancella-

tion in the effective field theory with the higher cut-off μ. Now, it needs to be readjusted. This is not the intended behavior of effective actions; we should not change the low-energy counterterm simply because additional high-energy modes have been integrated out. Thus, we encounter again a high sensitivity to the theoretical details in the UV.

In principle, there is nothing inherently wrong with the vacuum energy's dependence on the cutoff. The actual issue lies in the fact that this dependence follows a power law rather than a logarithmic form[3], requiring us to continuously fine-tune Λ_c to match observations as the effective description of the field theory evolves. This is what makes the vacuum energy particularly problematic, in contrast to many parameters in the Standard Model that undergo logarithmic renormalization [28].

No discussion of the cosmological constant would be comprehensive without addressing the anthropic principle. In fact, there exists a "solution" to the cosmological constant problem that does not require stabilizing the vacuum energy corrections against large radiative corrections, nor does it involve stopping unstable contributions from causing curvature [28]. The anthropic interpretation of the cosmological constant suggests that naturalness is irrelevant, and that the ability to describe nature is rooted in large-scale fine-tuning. It posits that the vacuum in which we make observations is simply the one that allows for our existence, while many other vacua with vastly different values of the cosmological constant exist, where no species could survive [28]. In chapter 2, we will mention anthropic principle again when we discuss string landscape.

[3] A power-law dependence means that a quantity grows like some power of a variable. This causes huge contributions from high-energy (UV) modes. However, logarithmic dependence is much more manageable and common in renormalizable and well-behaved quantum field theories. If vacuum energy had logarithmic dependence instead, it would naturally be much smaller, and closer to what we observe. Unfortunately, nature chose power law dependence, which leads to the cosmological constant problem.

1.2.3 Weinberg's no-go theorem

One might wonder if it is possible to introduce new fields into the matter sector that could dynamically address this issue by adjusting themselves to counterbalance the large contribution of the vacuum energy to the cosmological constant. However, Weinberg's no-go theorem [21] asserts that this is not feasible without merely transferring the fine-tuning of the bare cosmological constant into the potential energy of the self-regulating field, all while preserving the mass hierarchy of the matter sector [28]. Further details of the no-go theorem can be found in [25].

We begin by considering the following field content: a spacetime metric $g_{\mu\nu}$ and self-adjusting matter fields φ_i, with the tensor structure omitted. The dynamics is governed by a general Lagrangian density $\mathcal{L}[g_{\mu\nu}, \varphi_i]$. Additionally, we assume that the vacuum is translationally invariant, implying that on-shell, $g_{\mu\nu} = \eta_{\mu\nu}, \varphi_i =$ constant. Although Weinberg's argument is more broadly applicable to self-adjusting fields of any tensor rank [28], for simplicity we focus on scalar fields, with the index i running from 1 to N. The action for this theory is

$$S = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \varphi_i]. \tag{1.22}$$

In $\mathcal{L}[g, \varphi_i]$ we permit any interactions between the scalars φ_i and the metric $g_{\mu\nu}$. Since the system is translationally invariant, the metric and matter fields are constant, then according to the Euler-Lagrangian equation, the field equations are given by

$$\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\varphi_i} = 0. \tag{1.23}$$

To prevent these solutions from being finely tuned, Weinberg assumed that the trace of the gravitational field equations is connected to the scalar field equations of motion by

$$2g^{\mu\nu}\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} + \sum_i f_i(\varphi_i)\frac{\partial\mathcal{L}}{\partial\varphi_i} = 0, \tag{1.24}$$

where $f_i(\varphi_i)$ are arbitrary functions of the scalar fields. Eq.(1.24) is a linear combination of (1.23) and is a generalization of (1.23). In principle, we can consider more general generalization of (1.23), but we will not discuss here. Considering that the variation of the action (1.22) is

$$\delta S = \int d^4x \left[\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} \delta g^{\mu\nu} + \sum_i \frac{\partial \mathcal{L}}{\partial \varphi_i} \delta \varphi_i \right], \tag{1.25}$$

if the action remains invariant under the infinitesimal transformations $\delta g^{\mu\nu} = 2\alpha g^{\mu\nu}$ and $\delta\varphi_i = \alpha f_i(\varphi_i)$, then the relation (1.24) is ensured to hold with constant fields [28]. Let us define a new field $\tilde{\varphi}_i = \tilde{\varphi}_i(\varphi_i)$. Then we have

$$\delta\tilde{\varphi}_i = \sum_k \frac{\partial \tilde{\varphi}_i}{\partial \varphi_k} \delta\varphi_k = \sum_k \frac{\partial \tilde{\varphi}_i}{\partial \varphi_k} \alpha f_k(\varphi_k). \tag{1.26}$$

Furthermore, we set

$$\delta\tilde{\varphi}_1 = \alpha \sum_k \frac{\partial \tilde{\varphi}_1}{\partial \varphi_k} f_k(\varphi_k) = -\alpha, \tag{1.27}$$

and

$$\delta\tilde{\varphi}_j = \alpha \sum_k \frac{\partial \tilde{\varphi}_j}{\partial \varphi_k} f_k(\varphi_k) = 0, \tag{1.28}$$

then the field redefinitions $\varphi_i \to \tilde{\varphi}_i$ enable us to express the transformations as:

$$\delta g^{\mu\nu} = 2\alpha g^{\mu\nu}, \qquad \delta\tilde{\varphi}_1 = -\alpha, \qquad \delta\tilde{\varphi}_j = 0, \tag{1.29}$$

where j is from 2 to N. This is equivalent to a conformal transformation. Therefore, if the action consists of the following components, it will remain invariant: $\hat{g}_{\mu\nu} = e^{2\tilde{\varphi}_1} g_{\mu\nu}$ and $\tilde{\varphi}_j$. Gauge invariance necessitates that the on-shell action be [28]

$$S = \int d^4x \sqrt{-g} e^{4\tilde{\varphi}_1} V(\tilde{\varphi}_j), \tag{1.30}$$

and the gravity equation $\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = 0$ generates

$$e^{4\tilde{\varphi}_1} V(\tilde{\varphi}_j) = 0. \tag{1.31}$$

Therefore, we must either choose $V(\tilde{\varphi}_j)$ which involves fine-tuning, or take $e^{4\tilde{\varphi}_1} \to 0$. The physical masses in the field theory are all on the scale of $m^2_{phy} = e^{2\tilde{\varphi}_1} m^2$, meaning the second option involves choosing the conformal limit. However, because our universe is not conformally invariant, this option is ruled out by observational data [25, 28].

This no-go theorem is quite general, and any attempts to resolve the cosmological constant problem should provide a solid explanation of how the theorem is circumvented. One option is to break translational invariance in the background by permitting the fields to have time-dependent solutions. For instance, see [37–39].

1.2.4 Dynamical dark energy

For completeness of our discussion, we need to mention dynamical dark energy as an alternative to the cosmological constant to explain the late-time accelerated expansion. The simplest explanation is the cosmological constant (Λ) in Einstein's equations – essentially a constant energy density that fills the space homogeneously. The cosmological constant implies dark energy is static – it does not change with time or space. However, dynamical dark energy proposes that dark energy's properties evolve over time, meaning its energy density or pressure changes [40].

Dynamical dark energy is often described using scalar fields – like the "quintessence" model. Quintessence is a slowly-rolling scalar field with a potential $V(\phi)$, similar to how the inflaton field works during cosmic inflation. The field's energy density and pressure can vary over time. The equation of state $\omega = p/\rho$ is not exactly -1, and can evolve with time. The action for quintessence is

given by

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right], \tag{1.32}$$

where $(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $V(\phi)$ is the potential of the field. In a flat FRW spacetime the variation of the action (1.32) with respect to ϕ gives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{1.33}$$

The energy density and pressure density of the scalar field are

$$\rho = \frac{1}{2}\dot{\phi} + V(\phi), \tag{1.34}$$

$$p = \frac{1}{2}\dot{\phi} - V(\phi). \tag{1.35}$$

Therefore, the equation of state $\omega = p/\rho$ is not exactly -1. More details can be found in [40].

Other dynamical dark energy models include phantom energy, K-essence, chameleon fields, or holographic dark energy, etc. Dynamical dark energy could explain tensions in cosmological measurements (e.g. the Hubble constant discrepancy). A time-evolving dark energy might leave signatures in the cosmic microwave background, structure formation, or supernova observations. It connects with ideas from string theory, modified gravity, or extra dimensions [40]. The findings of DESI 2024 VI provide significant insights into the nature of dark energy and its role in the expansion of the universe. The data indicate a preference for dynamical dark energy over the cosmological constant Λ. However, the results are not yet conclusive [41, 42].

1.3 Brief review of string theory

String theory is the most promising quantum gravity theory at present. Bousso and Polchinski proposed a toy model of "string landscape" [43]. Since we will use some concepts related to string theory and string landscape in the following

sections of this thesis, we briefly review string theory in this section.

1.3.1 World-volume actions, closed strings and open strings

In quantum field theory, point particles evolve in time, sweeping out one-dimensional world lines in spacetime while in string theory, strings evolve in time, sweeping out two-dimensional surfaces in spacetime. The two-dimensional surfaces are called "world sheets" in string theory. This difference between quantum field theory and string theory has two important implications. First, string interactions are naturally independent of short-range singularities, so there is no UV divergence of string theory amplitudes. The string scale $\frac{1}{l_s}$ acts as a UV cut-off. Second, the structure of interactions in string theory can be uniquely determined by a free theory. In other words, we cannot choose interactions arbitrarily in string theory. However, in quantum field theory, we can have arbitrary interactions as long as they satisfy the symmetries of the model [18].

Furthermore, Polchinski found that there exist extend objects in string theory, i.e., p-branes, objects with p spatial dimensions and a tension (or energy density) T_p [44]. p-branes appear in string theory in the form of non-perturbative excitations. The classical motion of a p-brane extremizes the $(p+1)$-dimensional volume V that it sweeps out in space-time. The action of a p-brane action can be given by $S_p = -T_p V$. In particular, if $p = 1$, the string is the so-called "fundamental string" and V is the area of the string world sheet. The corresponding action is the famous Nambu–Goto action,

$$S_\sigma = -\frac{T}{2}\int \sqrt{-h}h^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu d\sigma d\tau \tag{1.36}$$

where $h_{\alpha\beta}(\sigma,\tau)$ is world-sheet metric, $h = \det h_{\alpha\beta}$, and $h^{\alpha\beta}$ is the inverse of $h_{\alpha\beta}$.

The $X^\mu(\sigma,\tau)$ describes the space-time embedding of the string world sheet. The Nambu-Goto action can be used to describe the behavior of bosonic string

theory. Things get even more interesting if we consider the effects of Quantum Mechanics. As a result, there is no conformal anomaly when space-time dimension is $D = 26$, which is the critical dimension of bosonic string theory. Similarly, if we consider the effect of supersymmetry in string theory, which is called superstring theory, then the critical dimension is $D = 10$ [18, 19, 45]. The $X^\mu(\sigma, \tau)$ has two parameters τ and σ. The parameter τ is the world-sheet time coordinate and σ parametrizes the string at a given world-sheet time. There are two basic configurations of strings in string theory: closed strings and open strings. For a closed string, which is topologically a circle, one should impose periodic conditions on the spatial parameter σ, namely, $X^\mu(\sigma, \tau)$ should satisfy $X^\mu(\sigma, \tau) = X^\mu(\sigma + \pi, \tau)$, while for an open string, which is topologically a line interval, each end can satisfy either Neumann or Dirichlet boundary conditions [18, 19, 45].

One important physical object is a D-brane. (D stands for Dirichlet condition.) If all open string boundary conditions were Neumann boundary conditions, then the endpoints of the string could be anywhere in spacetime. However, the Dirichlet condition specifies the spacetime hypersurface where the string's endpoints lie [18].

1.3.2 Superstrings

There are no fermions in bosonic string theory, which is inconsistent with nature. In order to include fermions in string theory, we need to generalize bosonic string theory. Supersymmetry is a natural way to establish the connection between bosons and fermions, so we can try to embed supersymmetry into string theory. The result is the so-called superstring theory. One of the most important results in the first string revolution is that there are only five consistent superstring theories: type IIA theory, type IIB theory, type I theory and two heterotic string theories: $SO(32)$ and $E_8 \times E_8$. Here "consistent" means anomaly free. Let us briefly review them.

There are two modes in string theory: left-shifting mode and right-shifting mode. When we use both modes at the same time, the supersymmetry associated with left-shifting and right-shifting can have the same handedness or opposite handedness. These two possibilities can give two different superstring theories: the type IIB and type IIA superstring theories, respectively. Furthermore, if we mod out the left-right symmetry, a procedure called orientifold projection, then we can deduce a third possibility, namely, type I superstring theory. The resulting strings are unoriented. The gauge group of type I theory is $SO(32)$.

If we use the formalism of the 26-dimensional bosonic string for the left-movers and the formalism of the 10-dimensional superstring for the right-movers, we can get two "heterotic" superstring theories. Their Lie algebras are $SO(32)$ and $E_8 \times E_8$, respectively. The mismatch in space-time dimensions may sound strange, but it is actually exactly what is needed. The extra 16 left-moving dimensions must describe a torus with very special properties, which can provide a consistent string theory [18].

As we said in section 1.3.1, the critical dimensions of all five superstring theories are $D = 10$. In $10D$, type I theory and two heterotic string theories have $\mathcal{N} = 1$ supersymmetry. The minimal spinor in ten dimensions has 16 real components, so these theories have 16 conserved supercharges. However, type IIA and type IIB string theories have $\mathcal{N} = 2$ supersymmetry, in other words, they have 32 supercharges. More details can be found in [18, 19, 45, 46].

1.3.3 The second string revolution

All the above results have been known since the mid-1980s, but there were at least two problems which could not be solved in the first string revolution. The first problem is that if string theory is the so-called unified theory, then there should be only one type of string theory instead of five distinct string theories. The second problem is that all the above results are perturbative, therefore

understanding the non-perturbative effects in string theories is a challenging task. After about ten years, these two problems could be answered better. All these achievements constitute the second string revolution. The key word of this revolution is "duality".

T-duality

String theory has many surprising properties. One of them is "T-duality". T-duality means that two different geometries for the extra dimensions are physically equivalent in many cases! Take a simplest example. When we interchange the radius R of a circle with radius l_s^2/R, where l_s is the fundamental string length scale, we can find that the mass spectrum of string theory is invariant. In other words, they are physically equivalent.

T-duality can relate two different theories. For example, it relates two type II and two heterotic theories. Type IIA and IIB theories (also two heterotic theories) should be considered a single theory because they represent two ends of a geometric continuum that varies with the radius of the circular dimension [18]. Another example is the equivalence between type IIA superstring theory compactified on a Calabi-Yau manifold and type IIB superstring theory compactified on a "mirror" Calabi-Yau manifold [18, 19, 45, 46].

S-duality

In the second string revolution, another important duality was discovered, the S-duality, which relates the string coupling constant g_s to $1/g_s$. The string coupling constant g_s is given by the vacuum expectation value of $\exp(\phi)$, where ϕ is the dilaton. For example, weakly coupled $SO(32)$ heterotic theory is equivalent to strongly coupled type I theory. Another example is that type IIB theory is dual to itself, therefore duality becomes a symmetry [18, 19, 45, 46].

D-branes

When we study non-perturbative string theory, we can discover that superstring theory contains various p-branes, where p is the spatial dimensions. When $g_s \to 0$, p-branes ($p > 2$) become infinitely heavy, thus they cannot appear in perturbative string theory. However, when g_s is not very small, p-branes have dynamical effects.

The type I and II superstring theories contain different p-branes called D-branes, whose tension is proportional $1/g_s$. The D denotes Dirichlet boundary condition, which means that the branes are objects on which fundamental strings can terminate. The Yang–Mills fields can be viewed as the massless modes of open strings attached to the D-branes. In fact,the standard model residing on D-branes has many very interesting implications. For instance, one possible reason we live in $4D$ space-time is that we are living on $D3$-branes [18, 19, 45, 46]. Model-building along these directions are often called the "brane-world" scenario, which is the main topic of Part I.

M theory and F theory

In the mid-1990s, the five different ten-dimensional superstring theories can be related by a web of dualities have been realized. The type IIA superstring theory and the $E_8 \times E_8$ heterotic string theory exhibit an eleventh dimension at strong coupling and thus we can obtain an 11-dimensional limit, a theory called *M theory*. In the decompactification limit, this $11D$ theory does not contain any strings, so *M theory* is not a string theory [18]. Edward Witten introduced the term M-theory, which refers to the "mysterious" or "magical" $11D$ quantum theory whose leading low-energy effective action is 11-dimensional supergravity. The description of M-theory in terms of an effective action is not fundamental, so string theorists are searching for alternative formulations. Although M-theory is not yet fully formulated, it is believed that it should exist [18, 19, 45, 46].

As we mentioned, the type IIA and the $E_8 \times E_8$ theories can be regarded as arising from M-theory. Then one may ask a natural question, namely, if the other three kinds of superstring theories can be derived in a similar way. The answer is yes. Cumrun Vafa introduced *F theory*, which is a $12D$ quantum theory. The nonperturbative $SL(2, \mathbf{Z})$ symmetry of type IIB string theory plays an important role in constructing F-theory. We will not discuss it further since it is not related to the core of the thesis. More details of M-theory and F-theory can be found in textbooks, such as [18, 19].

1.3.4 String cosmology

Since string theories, M-theory or F-theory are viewed as unified theories of nature, they should explain the realistic phenomena in our universe. String cosmology is an exciting research area. It has introduced new insights into cosmology, such as string landscape and dS conjecture [43, 47, 48], which will be discussed in Chapter 2. In order to describe the $4D$ universe, we need to do compactification from $10D$, $11D$ or $12D$ to $4D$.

One of the most important problems in string cosmology is the cosmological constant problem. We want to get a de Sitter vacuum solution with a positive small cosmological constant Λ from superstring theories, M-theory or F-theory to match observations. de Sitter space is a maximally symmetric Lorentzian manifold with constant positive scalar curvature. For example, in type IIB theory after carrying out a flux compactification we can find that the minimum value of the potential is always negative[4], which corresponds to AdS vacua. To get a positive minimum value of the potential, we should carry out an uplifting, such as in the KKLT senario, which introduces anti-$D3$ branes to

[4]Here we do not consider the contributions from racetrack. Racetrack models are a way to stabilize moduli using multiple non-perturbative contributions to the superpotential, typically from gaugino condensation or D-brane instantons. In $\mathcal{N} = 1$ supergravity, the superpotential in a racetrack model looks like $W = Ae^{-aT} + Be^{-bT}$, where T is a modulus field, A and B are constants dependdent on complex structure moduli or fluxes, a and b are are positive constants determined by the gauge group of the hidden sector [49]. By considering the contributions of raectrack, we can obtain supersymmetric Minkowski vacua [49].

uplift the potential [47]. Due to the issue of backreaction of the D-branes, the calculation is not considered reliable in some eras [50]. In fact, Cumrun Vafa and his coauthors wonder if string theories can get $4D$ dS vacua. They proposed a conjecture – dS conjecture, which claims that all string theories cannot obtain dS vacua in $4D$ [48]. If the conjecture holds true, then these vacua do not exist [51–54]. However, only low energy supergravity theories are usually considered, therefore it is natural to suspect that whether we can obtain dS vacua from non-perturbative aspects of string theory, such as M-theory or F-theory. Some results have been obtained, such as [55–57], but these results are still controversial.

1.4 Overview of the thesis

In this thesis we explore aspects of de Sitter vacua in theoretical cosmology. The thesis has two parts. In Part I we will consider different string-inspired models which are related to the cosmological constant problem. In Part II we will study de Sitter vacua solutions in one string cosmology model, namely, $O(d, d)$ invariant cosmology.

In Part I – String-inspired Models, we will explore three aspects of de Sitter vacua in string-inspired models. First of all, in chapter 2 we review the related concepts and tools we need in this part, namely, string landscape, swampland conjecture and bubble nucleation. Then in chapter 3, we will consider a wide class of $4D$ effective field theories in which gravity is coupled to multiple four-forms and their dual scalar fields, with membrane sources charged under the corresponding three-form potentials. We propose a "probabilistic" argument to explain the small cosmological constant instead of the dominant anthropic principle. Furthermore, we will explore in detail whether or not different models are compatible with the membrane weak gravity conjecture in chapter 4. We can find that these models can be generalized to DBI action. In chapter 5, we will generalize the results of vacuum transitions in chapter 3 and 4 to

D-dimension including the Gauss-Bonnet term. It can be found that the Euclidean action of the bounce, B, describing the decay of a de Sitter vacuum, is proportional to $k_+^{-(D-2)}$, which has a pole as $k_+^2 \to 0$ where k_+^2 is the curvature of the parent vacuum. We can find a new decay channel, describing up-tunneling from anti-de Sitter into de Sitter.

Furthermore, in Part II – de Sitter Vacua in String Cosmology, we will explore the de Sitter vacua solutions in $O(d,d)$ invariant cosmology. We briefly review the basic concepts of $O(d,d)$ invariant cosmology in chapter 6. Then in chapter 7 we will perform a thorough analysis of de Sitter solutions in $O(d,d)$ invariant cosmologies. We elucidate the nature of the instability in the string frame vacuum. For the Einstein frame, we will find that the de Sitter solutions cannot be eternal. Then we can extend our analysis to include Bianchi I universes. We show that we can extend the theory to the anisotropic case so that de-Sitter solutions can exist.

Part I

String-inspired Models

Chapter 2

String landscape, swampland conjecture and bubble nucleation

In Part I, we will use some concepts of string landscape, swampland conjecture and bubble nucleation. Therefore, first of all we briefly review these concepts in chapter 2. This chapter does not contain new results.

2.1 Landscape of the cosmological constant

String compactification generates a non-zero superpotential that depends on the moduli of the supergravity models, leading to moduli stabilization. Additionally, string theories can produce an immense number of vacua, with the count jumping from 10^8 to 10^{600} [58]. For instance, type IIB theory can produce approximately 10^{500} vacua [58]. A small subset of these vacua is expected to feature broken supersymmetry and a small positive cosmological constant. If this is the case, it suggests that the fundamental parameters of our physics are not intrinsic but are drawn from a vast set of solutions—referred to as the **string landscape** [58]. The current vacuum could be selected through the "anthropic principle" [43]. The anthropic principle suggests that the current vacuum is selected through anthropic reasoning: if the vacuum curvature were excessively large and positive, structure formation would be impossible due to rapid exponential expansion; if it were too large and negative, the universe would have begun contracting before stars, planets, and complex life could emerge [1]. In contemporary theoretical cosmology, the anthropic approach has become the leading explanation for addressing the cosmological constant problem [1].

The flux landscape is not merely a collection of distinct theories; it also exhibits a dynamical nature that enables transitions between these theories. This dynamic process is known as **bubble nucleation**. The surface of these bubbles are domain walls composed of high-dimensional charged objects. This crucial feature will persist in the *string landscape* [58].

The cosmological constant problem has inspired numerous intriguing ideas in theoretical physics. In certain approaches, the cosmological "constant" is

treated as a dynamic variable. Several mechanisms exist by which the cosmological constant can become a dynamical variable. One such mechanism involves a four-form field strength. In this scenario, the field strength is required to be constant, meaning it has no local dynamics. However, field strengths can contribute positively to the energy density, potentially canceling out a cosmological constant that arises from other fields, particularly if the latter is negative [43]. In Section 2.1, we will discuss three key frameworks for addressing the cosmological constant problem: the Abbott framework, the Brown-Teitelboim framework, and the Bousso-Polchinski framework.

2.1.1 The Abbott set-up

This set-up introduces a scalar field whose vacuum energy compensates for the initial cosmological constant. The compensating sector involves small mass scales but will not make unnatural adjustments to parameters [59]. The model consists of a scalar field, B, coupled to a gauge theory. The couplings of the scalar field, are constrained by the symmetry [59]

$$B \to B + \text{constant}. \tag{2.1}$$

The symmetry of (2.1) is softly broken in two ways to achieve a non-trivial potential for the B field. Firstly, B is coupled to a gauge theory through the term

$$L_{int} = (\alpha_{ph}/4\pi)(B/f_B)\epsilon^{\mu\nu\alpha\beta}\text{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right), \tag{2.2}$$

where the parameter f_B is a large mass (perhaps of order m_{pl}) associated with the complete high-energy theory and $F_{\mu\nu}$ is the gauge field strength. We explain why (2.1) is a symmetry of (2.2). The term in $\epsilon^{\mu\nu\alpha\beta}\text{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) = 4\text{tr}(F \wedge F)$. This is a total derivative: in $4D$ spacetime, this object can be written as the divergence of a current (the Chern-Simons current). That is $\epsilon^{\mu\nu\alpha\beta}\text{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) = \partial_\mu K^\mu$. This means that $\int d^4x\epsilon^{\mu\nu\alpha\beta}\text{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right)$ is a topo-

logical invariant: its value depends only on the global properties of the gauge field configuration, not on the local gauge field values. Suppose the shift (2.1), then the second term of (2.2) is independent of B and just adds a constant piece to the Lagrangian. It is a total derivative. It does not affect the classical equations of motion. Therefore, (2.1) is a symmetry of (2.2).

It has been known that the coupling (2.2) leads to a non-perturbative correction to the potential for the B field of the form [59]

$$V_1(B) = -\Lambda_{ph}^4 \cos\left(B/f_B\right). \tag{2.3}$$

The gauge coupling of the B field breaks the symmetry (2.1) but still maintains the symmetries

$$B \to B + 2\pi f_B \qquad \text{and} \qquad B \to -B. \tag{2.4}$$

We should point out that the symmetry $B \to -B$ is not a subset of the symmetry in (2.1). By introducing a linear term into the potential for B, these symmetries are once again softly broken

$$V_2(B) = \epsilon B/f_B. \tag{2.5}$$

Assuming the parameter ϵ is less than Λ_{ph}^4, but otherwise arbitrary. Due to its violation of symmetry, all radiative corrections for ϵ must be proportional to ϵ. Therefore there is no need for fine-tuning to maintain its small value [59].

When this compensating sector is added to the standard model of particle physics, the total vacuum energy is given by the following equation

$$V = \epsilon B/f_B - \Lambda_{ph}^4 \cos\left(B/f_B\right) + V_0, \tag{2.6}$$

where V_0 represents the vacuum energy density of all the fields except for B.

For $\epsilon \ll \Lambda_{ph}^4$ the potential has local minimum at

$$B \approx 2\pi N f_B. \tag{2.7}$$

for integer N with energy densities

$$V_N \approx 2\pi N\epsilon - \Lambda_{ph}^4 \cos\left(B/f_B\right) + V_0. \tag{2.8}$$

Since ϵ is less than $(10^{-34}eV)^4$, it is always possible to find values of N for which (2.8) can give a sufficiently small vacuum energy [59].

We can assume that at some initial time we are in a de Sitter spacetime, namely, the universe is in a state with a large, positive cosmological constant initially. Due to the linear term of (2.6), the local minima of the potential are unstable. Therefore, as time passes, the B field will descend down the linear slope stepping from one local minimum to the next, continually decreasing the energy density of vacuum [59]. Finally, by descending down the potential, the B field reduces the vacuum energy density to an acceptable small value,

$$V_N < m_{pl}^2 \Lambda_{ph}^2 \leq (0.001\text{eV})^4. \tag{2.9}$$

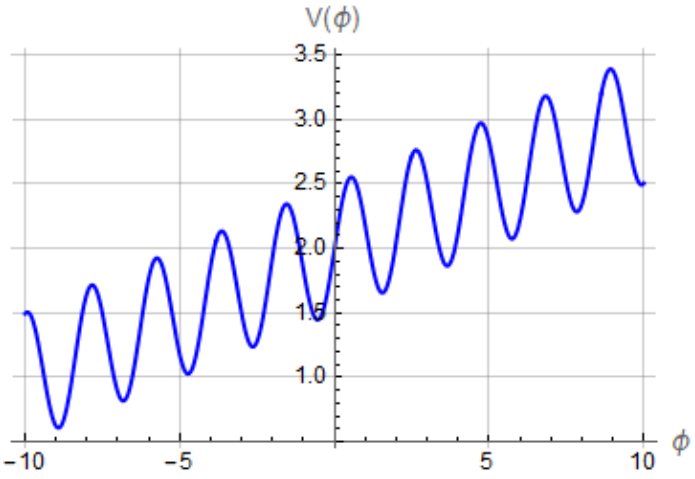

Figure 2.1: *Schematic diagram of the Abott set-up. The horizontal axis is the scalar field ϕ. The vertical axis is the corresponding potential $V(\phi)$.*

It can be shown that once we get to states with $V_N \ll m_{pl}^2 \Lambda_{ph}^2$ the tunneling

process is highly suppressed and further downward progress is extremely slow. For $V_N \ll m_{pl}^2 \Lambda_{ph}^2$ the tunneling rate per unit volume is given by

$$\Gamma/V \approx \Lambda_{ph}^4 \exp\left(-\frac{3}{8} m_{pl}^4 / V_N\right). \tag{2.10}$$

Therefore, we obtain states with acceptable small values of the cosmological constant, ultimately resulting in an extremely long lifetime of the universe [59].

However, the mechanism of the Abbott set-up will cause the "empty universe problem". The "empty universe problem" is that the universe undergoes long periods of accelerated expansion, diluting away all matter and leaving us with a cold and empty universe. The empty universe problem spoils the mechanism for slowly descending through adjacent vacua in a "washboard" potential for a scalar field, such as the Abbott set-up. Therefore, it is hard to incorporate the Abbott set-up into a realistic cosmology, so we should consider other mechanisms.

2.1.2 The Brown-Teitelboim set-up

Now we turn to the Brown-Teitelboim framework. Brown and Teitelboim introduced an intriguing toy model in which a landscape of different vacua is generated by the vacuum expectation value of a four-form field strength. The vacua are explored through the nucleation of membranes charged under the corresponding three-form field [60, 61]. In section 2.1.2, we will first review the fundamentals of four-form field strengths, which are essential to this thesis, and then explore the key concepts of the Brown-Teitelboim setup.

Four-form quantization

The action for gravity, including a bare vacuum energy Λ and the kinetic term for the four-form field, is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4^2} R - \Lambda_{bare} - \frac{Z}{2 \cdot 4!} F_4^2 \right) + S_{branes}, \tag{2.11}$$

where $\kappa_4 = 1/M_{pl}$, A_3 is 3-form and $F_4 = dA_3$ represents the corresponding 4-form flux. Additionally, Z is a normalization constant in the kinetic term.

We need to include certain boundary terms in this action, even though they do not influence the equations of motion. Typically, the boundary term is provided by the Gibbons-Hawking term

$$S_{GH} = \frac{1}{\kappa_4^2} \int_\Sigma d^3x \sqrt{h} K, \tag{2.12}$$

where h is the induced metric and K the extrinsic curvature. When we vary the Einstein-Hilbert action, we can find that the boundary terms do not vanish, even if you fix the metric on the boundary. This happens because R contains second derivatives of the metric. Varying it introduces total derivative terms that, via Stokes' theorem, become boundary integrals. The action is not well-posed unless we cancel those unwanted boundary terms. This Gibbons-Hawking term precisely cancels the problematic boundary variation from R, making the variational principle well-defined [9].

Moreover, we need to add the Duncan-Jensen (DJ) boundary term for the four-form field

$$S_{DJ} = \frac{Z}{3!} \int d^4x \partial_\mu (\sqrt{-g} F^{\mu\nu\rho\lambda} A_{\nu\rho\lambda}). \tag{2.13}$$

The DJ boundary term is introduced to fix the action when the scalar field couples to gravity non-minimally [61].

When we discuss boundary conditions, in principle, we need to express them for all fields in the system, and they can differ for different fields. For the metric, we assume Dirichlet boundary conditions, which requires us to include the Gibbons-Hawking boundary term. This means that the metric is assumed to be fixed on the boundary, but not its normal derivative. For the 3-form field, we assume Neumann boundary conditions. So it is the opposite. The 3-form is not necessarily fixed on the boundary, but its normal derivative is.

If we disregard the brane sources, the equation of motion for the four-form

is given by $\partial_\mu(\sqrt{-g}F^{\mu\nu\rho\lambda}) = 0$, which yields the following solution:

$$F^{\mu\nu\rho\lambda} = c\epsilon^{\mu\nu\rho\lambda}, \tag{2.14}$$

where c is any constant and $\epsilon^{\mu\nu\rho\lambda}$ is the totally antisymmetric Levi-Civita tensor.

As a result, we can obtain the energy density

$$\Lambda = \Lambda_{bare} - \frac{Z}{48}F_4^2 = \Lambda_{bare} + \frac{Zc^2}{2}. \tag{2.15}$$

In $4D$ theory, only the cosmological constant Λ is an observable quantity, while Λ_{bare} and the four-form F cannot be detected independently. To account for the observed small value of the cosmological constant, Λ_{bare} must nearly cancel out the contribution from the four-form field [43].

The constant c can assume any real value, meaning that the bare cosmological constant Λ_{bare} can be canceled with arbitrary precision [62]. However, we will assume that c is quantized. The rationale behind this is as follows.

Firstly, if gravitational instantons exist on the Euclidean manifold X, we can anticipate that the integral of the Euclidean four-forms over X will be quantized [43],

$$\int_X F_4 = \frac{2\pi n}{e}, \qquad n \in \mathbf{Z}. \tag{2.16}$$

This is the extended form of the Dirac quantization condition [63–66]. The derivation of this generalized condition can be found in [19]. It originates from the quantum mechanics of membranes, interacting with the potential energy A_3, and is expressed by the following equation:

$$S = e\int_W A_3. \tag{2.17}$$

Here, e is the membrane charge and W represents the world-volume. The membrane amplitudes must be single-valued, which implies the quantization

condition given in (2.16).

Furthermore, when considering membranes, we can infer that c is a local constant. The value of constant c will change across a membrane,

$$\Delta c = \frac{e}{Z}. \tag{2.18}$$

The overall variation in c caused by the nucleation of any number of membranes is an integer multiple of e/Z [43], so we obtain:

$$c = \frac{en}{Z}, \qquad n \in \mathbf{Z}, \tag{2.19}$$

namely, the value of c is quantized.

The Brown-Teitelboim set-up

After reviewing the fundamentals of four-forms, we now turn to the key concepts of the Brown-Teitelboim framework. Consider a space-time with $D = d + 1$ dimensions, where a totally antisymmetric tensor field with d indices is coupled to its natural source, a $d - 1$ (spatial) dimensional membrane. Just as an electric field can generate particle pairs, an antisymmetric tensor field can produce membranes. When this system is coupled to gravity and a cosmological constant term Λ_{bare} is included, representing the combined effects of various physical processes like spontaneous symmetry breaking, it contributes to the cosmological constant. The total cosmological constant Λ is then the sum of Λ_{bare} and a contribution from the antisymmetric tensor field [61].

Let $x^\mu = z^\mu(\xi)$ describe the d-dimensional history of a membrane in $D = d+1$ space-time dimensions, where ξ^a (with $a = 0, 1, ..., d-1$) are the coordinates of the membrane. The induced metric is given by $g^d_{ab} = g_{\mu\nu} z^\mu_{,a} z^\mu_{,b}$. Furthermore, let $A_{\mu_1 ... \mu_d}$ represent the antisymmetric tensor field, with its field strength

defined as $F_{\mu_1\ldots\mu_D} = \frac{1}{d!}\partial_{[\mu_1}A_{\mu_2\ldots\mu_D]}$. Then the action is given by

$$\begin{aligned} S = &- m\int d^d\xi\sqrt{-g^d} + \frac{e}{d!}\int d^d\xi A_{\mu_1\ldots\mu_d}\frac{\partial z^{\mu_1}}{\partial\xi^{a_1}}\ldots\frac{\partial z^{\mu_d}}{\partial\xi^{a^d}}\epsilon^{a_1\ldots a_d} \\ &- \frac{1}{2D!}\times\int d^Dx\sqrt{-g}F_{\mu_1\ldots\mu_D}F^{\mu_1\ldots\mu_D} \\ &+ \frac{1}{d!}\int d^Dx\partial_{\mu_1}\left(\sqrt{-g}F^{\mu_1\ldots\mu_D}A_{\mu_2\ldots\mu_D}\right) + S^{grav}(\Lambda_{bare}). \end{aligned} \tag{2.20}$$

Here, m represents the mass per unit $(d-1)$-dimensional volume of the membrane, and e is the coupling constant between the membrane and the antisymmetric tensor field. Additionally, $S^{grav}(\Lambda_{bare})$ is the gravitational action, which includes the explicit contribution Λ_{bare} to the cosmological constant, assumed to be negative. The total derivative term in (2.20) is a topologically invariant term [60, 61].

The key findings of [60, 61] are as follows. The quantum creation of closed membranes by totally antisymmetric tensor and gravitational fields is examined in arbitrary space-time dimensions. This creation event is described through instanton tunneling. As membranes are generated, the energy density associated with the antisymmetric tensor field decreases, which in turn lowers the effective value of the cosmological constant. For a wide range of parameters and initial conditions, once the cosmological constant approaches zero, the process naturally halts, even if the remaining energy in the antisymmetric tensor field is large. Some of the resulting instantons are interpreted as representing topology changes, where open spaces spontaneously compactify. However, the quantum probability for these processes has vanished [60, 61].

The problem with the Brown and Teitelboim proposal is that the descent is too slow. In order for the landscape to be sufficiently dense to readily accommodate vacua with energy densities around the dark energy scale, we must assume that the membrane charge is extremely small in units of a Planckian cut-off. Descent from high scale vacua with energy densities close to the cut off to the low density vacuum we see today proceeds via a large number of intermediate steps, slowly descending from one cosmological constant to an-

other. The time scales are such that the universe undergoes long periods of accelerated expansion, diluting away all matter and leaving us with a cold and empty universe [2].

2.1.3 The Bousso-Polchinski set-up

In the Brown-Teitelboim set-up there is only one four-form, so if we want to obtain the observed value of the cosmological constant, we need to do fine-tunning for the membrane charge. In order to get a dense landscape of vacua and avoid the extremely small membrane charge, Bousso and Polchinski introduced a model which has multiple four-forms [43].

Multiple four-forms

String flux compactifications can produce several four-form fluxes. When we consider Calabi-Yau compactification we have several ways to obtain four-form fluxes, such as, a seven-form wrapping on a 3-cycle can give a four-form, or an eight-form wrapping on a 4-cycle can give a four-form. In this section we consider how to use multiple four-forms to neutralize the cosmological constant Λ. If there are J fluxes, with

$$\Lambda = \Lambda_{bare} + \frac{1}{2}\Sigma_{i=1}^{J} n_i^2 q_i^2, \tag{2.21}$$

then the cosmological constant problem becomes that if there is a set of n_i which can satisfy the following condition

$$2|\Lambda_{bare}| < \Sigma_{i=1}^{J} n_i^2 q_i^2 < 2(|\Lambda_{bare}| + \Delta\Lambda), \tag{2.22}$$

where $\Delta\Lambda$ is the observational bound, which is about 10^{-120} in Planck units. Consider a sphere of radius $r = |2\Lambda_{bare}|^{1/2}$, whose center is at $n_i = 0$. The allowed values of the four-form energy density are given by the radius-squared of points in the grid, whose dimension is J. The spacing in each direction i

is q_i. One should consider a thin shell whose width encodes the width of the observational bound,

$$\Delta r = |2\Lambda_{bare}|^{-1/2}\Delta\Lambda. \tag{2.23}$$

In order to estimate the density of vacua, we need that the shell should at least have one grid point. In other words, if there exists a degeneracy D, it should have D grid points at least.[1] Therefore, the volume of the shell should be greater than the volume per D grid points, namely, $V_{shell} \geq D\Pi_{i=1}^{J} q_i$. The volume of shell V_{shell} is $\omega_{J-1} r^{J-1}\Delta r$, where the area of a unit sphere is $\omega_{J-1} = 2\pi^{J/2}\Gamma(J/2)$. Therefore we have

$$\Pi_{i=1}^{J} q_i \leq \frac{\omega_{J-1}}{D}|2\Lambda_{bare}|^{J/2-1}\Delta\Lambda, \tag{2.24}$$

or

$$\frac{D}{\omega_{J-1}}\Pi_{i=1}^{J}\frac{q_i}{|2\Lambda_{bare}|^{1/2}} \leq \frac{\Delta\Lambda}{|2\Lambda_{bare}|}. \tag{2.25}$$

The density of vacuum energies is dependent on where we are in the landscape. Near Minkowski spacetime, the density is

$$\Delta\Lambda_{min} \approx \frac{D\Pi_{i=1}^{J} q_i}{\omega_{J-1}|2\Lambda_{bare}|^{J/2-1}}. \tag{2.26}$$

From (2.26), we can find that if there exist more than two four-form fields then the membrane charge q_i need not be very small.

We can give an illustrative example of that. If we take $J \approx 100$, $|\Lambda_{bare}| \sim M_{pl}^4$ and $q_i \sim q \sim 0.02 M_{pl}^2$, we can get the desired observational bound, namely, the smallest possible separation in curvature δk^2 is less than H_0^2, where $H_0 \sim 10^{-60} M_{pl}$ is the current Hubble scale [43].

[1] Eq.(2.21) has a symmetry $q_i \to -q_i$ for any i. Therefore, if there are $\mathcal{N}$ four-form fluxes, the degeneracy D should be $D \sim 2^{\mathcal{N}}$. In other words, D points give the same cosmological constant. As a result, the shell should at least have $2^{\mathcal{N}}$ grid points.

M-theory compactification

As mentioned earlier, string compactifications can lead to four-form fluxes. In this subsection, we provide a brief overview of how the Bousso-Polchinski model can be derived from an M-theory compactification [43].

For instance, consider compactifying M-theory on a 7-dimensional manifold K. The associated bosonic action in 11-dimensional spacetime is given by

$$S = 2\pi M_{11}^9 \int d^{11}X\sqrt{-g_{11}}\left(R - \frac{1}{2\cdot 4!}F_4^2\right) + S_{branes}. \tag{2.27}$$

In M theory there are two important branes: M2-brane and M5-brane. The tension and charge of M2-brane is $2\pi M_{11}^3$, while the tension and charge of M5-brane is $2\pi M_{11}^6$.

The M5-brane couples to A_6, whose action is given by

$$2\pi M_{11}^6 \int_W A_6, \tag{2.28}$$

where W represents the volume of M5-brane and $dA_6 = F_7 = *_{11}F_4$. The corresponding generalized version of Dirac quantization condition is

$$2\pi M_{11}^6 \int_K F_7 = 2\pi n,\ n \in \mathbf{Z}. \tag{2.29}$$

Let us examine the $4D$ theory. In this case, the $11D$ F_4 reduces to a $4D$ F_4, and the corresponding bosonic action in $4D$ is given by

$$S = V_7 2\pi M_{11}^9 \int d^4x\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_4^2\right) + S_{branes}, \tag{2.30}$$

where V_7 is the volume of K. Moreover, (2.29) will become

$$F_0 = \frac{n}{M_{11}^6 V_7},\ n \in \mathbf{Z}. \tag{2.31}$$

By comparing (2.30) and (2.31), we can obtain

$$(2\kappa_4^2)^{-1} = Z = 2\pi M_{11}^9 V_7. \tag{2.32}$$

Here, Z represents the normalization constant in the kinetic term. Further details can be found in [43]. However, it is important to note that this is a toy model, and [43] does not address the proper stabilization of moduli.

Avoid empty universe problem

Bousso and Polchinski showed how the empty universe problem could be avoided by extending the Brown and Teitelboim mechanism to a large number of four-forms [43]. This allows for a sufficiently dense landscape without fine tuning the membrane charges to very small values. The empty universe problem can then be avoided as follows. In the moments before the jump, the underlying vacuum energy is large and the inflaton field is displaced to large values thanks to quantum diffusion. Immediately after the jump, the minimum of the inflaton potential and the underlying vacuum energy can be small but the inflaton itself is still displaced. The universe undergoes a period of slow roll inflation until the inflaton rolls down its potential and begins to oscillate about the minimum, allowing the universe to reheat. The empty universe problem can be avoided in a similar way with just two species of four-form by assuming that the corresponding membranes have an (almost) irrational charge ratio [67–69]. Note that Garriga and Vilenkin have argued that the empty universe problem is a fallacy, at least in the context of eternal inflation [70]. In chapter 3 we will discuss the empty universe problem as well.

This type of descent from high to low scale vacua is able to explain how we can arrive at the current vacuum, but it does not explain why. Bousso and Polchinski argue that the current low curvature vacuum is selected on anthropic grounds, although the validity of this claim has been criticised in [71]. Alternatively, we might look for ways to slow down the descent, or even

stop it altogether, whenever the curvature approaches zero. In [72], the authors consider the Brown-Teitelboim model, but take into account membrane stacks and degeneracy factors. They claim that the tunneling rates depend on the ambient de Sitter temperature, being enhanced at high scales and switching off at low scales. However, Garriga and Vilenkin argue the opposite: that these enhancement factors are independent of the ambient de Sitter temperature and do not switch off at low curvature [70].

2.1.4 Summary and discussion

Now let us summarize the main differences between the three models: the Abott model, the Brown-Teitelboim model and the Bousso-Polchinski model and the issues with each. The Abott model introduces a scalar field coupled to gravity. The field can tunnel between minima in a potential, each with a different vacuum energy. The process slows as Λ gets smaller. However, the Abott model can cause empty universe problem. Brown and Teitelboim introduced a toy model in which a landscape of different vacua is generated by the vacuum expectation value of a four-form field strength. The vacua are explored through the nucleation of membranes charged under the corresponding three-form field. The problem with the Brown and Teitelboim proposal is that the descent is too slow and the membrane charge is extremely small. This model can cause the empty universe problem as well. In order to obtain a dense landscape of vacua and avoid the extremely small membrane charge, Bousso and Polchinski introduced a model that has multiple four-forms. The Bousso-Polchinski set-up can avoid the empty universe problem, but they introduced anthropic principle to select our current vacuum. Anthropic principle is unnatural in theory. We would like to avoid this fine-tuning mechanism. In Chapter 3, we will propose a "probablistic" explanation for the extremely small value of the cosmological constant.

We briefly comment on the relationship between the three models and loop

corrections discussed in Chapter 1. The full cosmological constant is from renormalized vacuum energy and model-dependent corrections, where model-dependent corrections correspond to the four form corrections in the Brown-Teitelboim set-up and the Bousso-Polchinski set-up or the potential pieces in Abbott set-up. The renormalized vacuum is radiatively unstable in the usual sense. However, the value of full cosmological constant is selected on anthropic grounds. As long as we have a landscape of solutions with sufficient density and the ability to jump between solutions in some way, we can anthropically select the vacua based on the full cosmological constant value. The radiatively unstable part is hidden from view by model-dependent corrections [43, 59–61].

2.2 Swampland and weak gravity conjecture

In recent years, it has become increasingly clear that not everything is possible within the framework of quantum gravity . In other words, not all quantum field theory (QFT) models are compatible with quantum gravity theory. The aim of the swampland program is to identify the constraints that an effective field theory (EFT) must adhere to in order to be consistent with a quantum gravity theory, such as string theory [73–81]. Quantum EFTs that appear consistent (anomaly-free) but cannot be incorporated into a UV-complete theory of quantum gravity are classified as being in the **swampland**. Conversely, EFTs that can be embedded in a UV-complete quantum gravity theory are considered part of the **landscape**. Although the concept of the swampland is not exclusive to string theory, the related swampland conjectures are often inspired by string theory.

The swampland program includes at least five different but related aspects [73–81]:

1. No global symmetry conjecture:

If a theory which couples gravity has finite number of states, then the theory has no exact global symmetries [73–81].

2. Completeness of spectrum:

If a theory that couples gravity also possesses a gauge symmetry, then the theory must include states corresponding to all possible charges, in accordance with the Dirac quantization condition under the gauge symmetry [73–81].

3. Distance conjectures:

- Consider a theory that couples gravity with a moduli space $\mathcal{M}$, where the expectation values of some field ϕ^i have no potential. In this case, for any point P in $\mathcal{M}$, there always exists another point Q in $\mathcal{M}$ such that the geodesic distance between P and Q, denoted $d(P,Q)$, is infinite [73–81].

- An infinite tower of states exists, with the corresponding mass scale M between P and Q being

$$M(Q) \sim M(P)e^{-\alpha d(P,Q)}, \tag{2.33}$$

  where α is some positive constant and $d(P,Q)$ is the corresponding geodesic distance [73–81].

4. de Sitter conjectures:

At every point in the field space, the effective scalar potential satisfies one of the following two inequalities

$$|\nabla V| \geq \frac{c_1}{M_{pl}} V, \quad \text{or} \quad \min_{i,j}(\nabla_i \nabla_j V) \leq -c_2 V, \tag{2.34}$$

where c_1 and c_2 are two positive constants of order one in Planck units and $\min_{i,j}(\nabla_i \nabla_j V)$ is the minimum eigenvalue of the Hessian in an orthonormal basis [73–81].

5. Weak gravity conjecture:

Let us explain the weak gravity conjecture in four dimensions. Consider a theory coupled to gravity that includes a $U(1)$ gauge symmetry with gauge

coupling g; the action is then given by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{4g^2}F^2 + ...\right] + S_{branes}. \tag{2.35}$$

There are two versions of weak gravity conjecture, namely, electric WGC and magnetic WGC. There ought to be at least one species of particle satisfies the two weak gravity conjectures both. For 1-form field, we have [73–81]:

- (electric WGC) In the theory, there is a particle with m and charge q satisfying the inequality

$$m \leq \sqrt{2}gqM_{pl}, \tag{2.36}$$

  where g represents the gauge coupling.

- (magnetic WGC) The EFT cutoff M_{UV} is constrained from above by the gauge coupling [73–81]

$$M_{UV} \leq gM_{pl}. \tag{2.37}$$

Moreover, for p-form field we should consider membrane weak gravity conjecture since p-form field couples to membranes. The electric and magnetic weak gravity conjecture for membranes are given by:

- (electric WGC for membranes) The membrane tension τ should satisfy

$$\tau < M_{pl}gq. \tag{2.38}$$

- (magnetic WGC for membranes) The UV cutoff scale M_{UV} of the effective theory is constrained by the following inequality

$$M_{UV}^3 < qM_{pl}. \tag{2.39}$$

In chapter 4 we will mainly consider the constraints of weak gravity conjecture

for membranes on different EFT models.

2.3 Bubble nucleation – gravitational effects on vacuum decay

In classical field theory it is possible to have two stable homogeneous ground states, only one of which is an absolute energy minimum. However, in quantum field theory the ground state of higher energy is a false vacuum, rendered unstable by barrier penetration. There is a semi-classical theory of vacuum decay [82]. In [82] Coleman and De Luccia extended this theory to include gravitation. Now let us review the result of this paper briefly.

The action in [82] is

$$S = \int d^4x\sqrt{g}\left[\frac{R}{16\pi G_N} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{2.40}$$

where R is the Riemann curvature, ϕ is a scalar field and $V(\phi)$ is the potential of scalar field ϕ. To compute the rate at which membranes are nucleated, mediating transitions between vacua, we analytically continue to Euclidean signature, setting

$$t \to -it_E, \qquad S \to iS_E, \tag{2.41}$$

where S_E is the Euclidean action. The Euclidean action is

$$S_E = -\int d^4x_E\sqrt{g}\left[\frac{R}{16\pi G_N} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{2.42}$$

Since we need to consider the vacuum decay of the theory, the potential $V(\phi)$ should have at least two vacua: a higher vacuum and a lower vacuum. The potential can decay from the higher vacuum (false vacuum) to lower vacuum (true vacuum) [82]. Fig.2.1 is a schematic diagram of potential energy $V(\phi)$.

We will look for an $O(4)$ invariant metric

$$ds^2 = dr^2 + \rho^2(r)d\Omega_3^2, \tag{2.43}$$

where $d\Omega_3^2$ is the element of distance on a unit three-sphere,

$$d\Omega_3^2 = d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\theta d\phi^2. \tag{2.44}$$

and ϕ obeys the field equations

$$\phi'' + \frac{3\rho'}{\rho}\phi' = V'(\phi), \tag{2.45}$$

$$\phi'^2 = 1 + \frac{8}{3}\pi G_N \rho^2 \left(\frac{1}{2}\phi'^2 - V(\phi)\right), \tag{2.46}$$

where $'$ denotes the derivative with respect to r [83].

We now seek the bounce solution of (2.45) and (2.46) and plug into the Euclidean action (2.40), giving

$$S_E[\phi, g] = 2\pi^2 \int_0^\infty dr \left[\rho^3 \left(\frac{\phi'^2}{2} + V(\phi)\right) + \frac{3}{8\pi G_N}(\rho^2\rho'' + \rho\rho'^2 - \rho)\right]. \tag{2.47}$$

The bounce solution interpolates between the true vacuum $\phi(0) = \phi_B$ and the false vacuum $\phi(\infty) = \phi_A$ [84]. There is a symmetry between the two vacua so the bounce solution is the same for going from $A \to B$ or $B \to A$ (See Fig.2.1). The only difference between up-tunneling and down-tunneling is that the background action which is subtracted to get the tunneling amplitude is different. This process is called the "detailed balance" [84]. The tunneling rate for the down-tunneling is given by

$$\Gamma \sim e^{-\Delta S_E/\hbar}, \tag{2.48}$$

where $\Delta S_E = S_E(\text{instanton}) - S_E(\text{parent})$. Here $S_E(\text{instanton})$ is the Euclidean action evaluated on the bubble configurations, interpolating between

the false vacuum and true vacuum. In contrast, S_E(parent) is the Euclidean action evaluated on the complete false vacuum, with no bubbles. More details of bubble nucleation can be found in review [83].

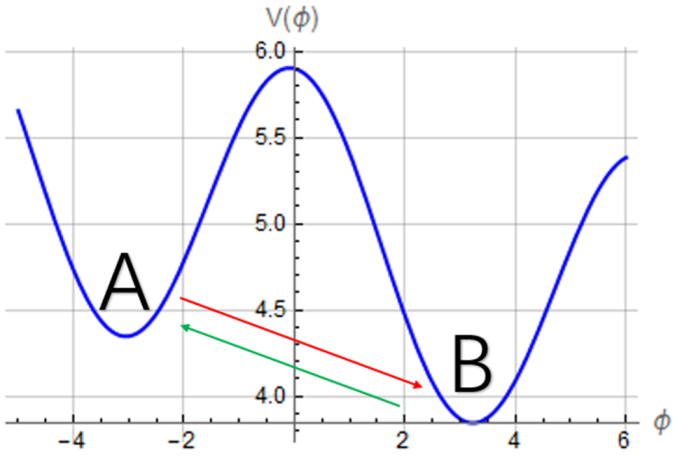

Figure 2.2: *Tunneling between vacua. The horizontal axis represents the scalar field ϕ. The vertical axis depicts the corresponding potential $V(\phi)$. Tunneling between vacua. The horizontal axis is the value of the scalar field ϕ. The vertical axis is the corresponding value of potential $V(\phi)$. There are two vacua A and B. The red arrow denotes the tunneling from A to B. The green arrow denotes the tunneling from B to A.*

In the following, we will explore the string landscape of different string-inspired models and explore if they can satisfy weak gravity conjecture in chapter 3, 4 and 5. We also calculate the rates of bubble nucleation, in particular the decay rates from dS_+ to Min_-. Furthermore, we will propose a probabilistic argument for the cosmological constant problem.

Chapter 3

The cosmological constant is probably still zero

In chapter 3, we will consider a wide class of four-dimensional effective field theories in which gravity is coupled to multiple four-forms and their dual scalar fields, with membrane sources charged under the corresponding three-form potentials. Four-form flux, quantised in units of the membrane charges, generically generates a landscape of vacua with a range of values for the cosmological constant that is scanned through membrane nucleation. We list various ways in which the landscape can be made sufficiently dense to be compatible with observations of the current vacuum without running into the empty universe problem. Further, we establish the general criteria required to ensure the absolute stability of the Minkowski vacuum under membrane nucleation and the longevity of those vacua that are parametrically close by. This selects the current vacuum on probabilistic grounds and can even be applied in the classic model of Bousso and Polchinski, albeit with some mild violation of the membrane weak gravity conjecture. We present other models where the membrane weak gravity conjecture is not violated but where the same probabilistic methods can be used to tackle the cosmological constant problem.

3.1 Introduction

Recently, Kaloper [67] and later, Kaloper and Westphal [69] have constructed a simple dynamical model in which the cosmological constant is relaxed to near zero through the nucleation of membranes, charged under a pair of three-forms. Membrane nucleation triggers a jump in the effective cosmological constant that is controlled by the corresponding charges. Crucially, these charges are not assumed to be exponentially small. Instead, they are assumed to have an irrational ratio, which guarantees a dense landscape of vacua, including those whose cosmological constant is close to zero. Given a more modest upper bound on the charge (which may be in some tension with the weak gravity conjecture [85, 86]) transitions between vacua are rapid, at least until the system finds a vacuum with a small positive cosmological constant, at

which point, the vacuum is very long lived. This latter result has its roots in Hawking's famous solution to the cosmological constant problem in which he computes the Euclidean action for a landscape of vacua and argues that the most probable configuration is the one with the smallest absolute curvature [62]. Thanks to the irrational charge ratio in [67, 69] the transition from a very high energy de Sitter vacuum to near Minkowski can occur quickly via a single membrane nucleation and the empty universe problem is said to be avoided.

The simple set-up presented in [69] is a clever generalisation of Henneaux and Teitelboim's covariant formulation of unimodular gravity [87], with a pair of four-form field strengths that enter only through a bilinear mixing with a dual scalar field, and charged membranes (for a review of unimodular gravity, see [88]). Although the model is phenomenologically interesting, its embedding within fundamental theory is likely to prove challenging. However, in this chapter we will show that key aspects of the relaxation mechanism apply far more generally, increasing the likelihood that it could apply in some of the four-dimensional effective theories obtained from string theory compactifications. Remarkably, this is true of the famous Bousso-Polchinski set-up [43], offering a probabilistic explanation for the small observed value of the cosmological constant instead of an anthropic one.

We consider a very general set-up involving Einstein-Hilbert gravity coupled to three-forms fields and their dual scalars, with membrane sources charged under the three-forms. This includes the Kaloper-Westphal model [69] as a special case but also other effective theories including those expected to appear in compactifications of string theory, such as the Bousso-Polchinski set-up [43]. A key feature of the entirely family of models is that in vacuum, the scalars and the three-forms always gravitate like a cosmological constant, even in the presence of a non-trivial flux. The flux of the four-field strength is quantised, giving rise to a landscape of vacua. To have any hope of solving the cosmological constant problem, this landscape must be sufficiently dense in and around Minkowski space in order to admit vacua that match observations.

Tunneling between vacua is achieved through membrane nucleation. This is described by instantons, solutions to the Euclidean equations of motion with vacua of different vacuum energies, separated by charged membranes. The geometry of the instantons depends on the curvature of the corresponding vacua and the membrane tensions. Many configurations are physically irrelevant because the membranes would have to have negative tension or because the corresponding tunneling rate is infinitely suppressed. As in [67, 69], those instanton solutions that are physically relevant can be dissected even further. In our general setting, this dissection depends on the value of a parameter $X = 4M_{pl}^4\Delta k^2/T^2$, where Δk^2 is the jump in vacuum curvature and T is the membrane tension. We consider several example theories where the bounds on this parameter can be related to bounds on other microscopic parameters such as the membrane charges. This is important since the value of X not only controls which instanton geometries are allowed but also the corresponding tunneling rates. In particular, when $|X| < 1$, any near Minkowski vacua become very long lived and robust against the nucleation of anti-de Sitter bubbles. Thus, if you find yourself in a near Minkowski vacuum, chances are you will stay there.

Of course, by itself, this is not enough to solve the cosmological constant problem. One must show that we avoid the so-called empty universe problem. This occurs when the universe descends to the current vacuum incrementally, via a slow cascade of de Sitter vacua, the curvature jumping by a small amount each time, forever diluting away any matter excitations. The density of the landscape and the abundance of dangerous de Sitter vacua render this a real possibility in generic theories of this type. However, through a detailed study of the kinematics and an analysis of tunneling rates we show that the empty universe problem can often be avoided: one is able to transition quickly from a high scale de Sitter vacuum to near Minkowski via a single transition. All of this suggests a wide-ranging mechanism for solving the cosmological constant problem, inspired by, but extending far beyond, the original proposal of [67,

69].

The rest of this chapter is organised as follows: in section 3.2, we present the generalised set-up along with the corresponding field equations and junction conditions. Since they will play an important role in the evaluation of tunneling rates later in the chapter, we dwell a little on the choice of boundary conditions and corresponding boundary terms. In section 3.3, we present the landscape of Lorentzian vacuum solutions. In section 3.4, we Wick rotate to Euclidean signature and solve the Euclidean field equations to find the corresponding instanton solutions and compute the corresponding transition rates. In section 3.5, we demonstrate the role of the parameter X in controlling the stability of near Minkowski vacua, protecting them from decay into anti de Sitter. We also discuss conditions to avoid the empty universe problem. In section 3.6, we run through four different models, including Brown-Teitelboim, Bousso-Polchinski, Kaloper-Westphal, and a fourth model including a single three-form (with standard kinetic term) and its dual scalar. In the latter three cases, we show how the parameters of the theory can be chosen to successfully implement the mechanism for solving the cosmological constant problem originally proposed in [67]. In section 3.7, we conclude.

3.2 The generalised set-up

First, in Table 3.1 we list the properties of the three models discussed in Chapter 2. None of the three models can avoid the empty universe problem and the use of the anthropic principle at the same time. This suggests the need for a more general set-up that can address the cosmological constant problem without using either.

Then we begin with a general four-dimensional effective theory on a manifold, $\mathcal{M}$, with a dynamical metric $g_{\mu\nu}$ and a family of three-form fields, A^i,

Model	Empty universe problem	Anthropic principle
The Abott set-up	+	−
The Brown-Teitelboim set-up	+	−
The Bousso-Polchinski set-up	−	+

Table 3.1: The table summarizes the key features of the Abbott, Brown-Teitelboim, and Bousso-Polchinski models. In the second column, a "+" denotes that the corresponding model leads to the empty universe problem, while a "−" indicates that it does not. In the third column, a "+" signifies that the model requires an anthropic explanation to address the cosmological constant problem, whereas a "−" means that such an explanation is not necessary.

and dual scalars ϕ_i,

$$S = \int_{\mathcal{M}} d^4x\sqrt{|g|}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\omega^{ij}(\phi)\nabla_\mu\phi_i\nabla^\mu\phi_j - V(\phi)\right] + \int_{\mathcal{M}}\left[-\frac{1}{2}Z_{ij}(\phi)F^i\wedge\star F^j + \sigma_i(\phi)F^i\right] + S_{\text{boundary}} + S_{\text{membranes}}, \quad (3.1)$$

where R is the Ricci scalar. The four-form field strengths are given in terms of the three-form fields $F^i = dA^i$ and $\star$ denotes the Hodge star operator on the manifold $\mathcal{M}$. In components, the three-form is written as $A^i = \frac{1}{3!}A^i_{\mu\nu\alpha}dx^\mu \wedge dx^\nu \wedge dx^\alpha$ and the corresponding field strength as $F^i = \frac{1}{4!}F^i_{\mu\nu\alpha\beta}dx^\mu \wedge dx^\nu \wedge dx^\alpha \wedge dx^\beta$ where $F^i_{\mu\nu\alpha\beta} = 4\partial_{[\mu}A^i_{\nu\alpha\beta]}$. When acting on the four-form, the Hodge star operator yields $\star F^i = \epsilon_{\mu\nu\alpha\beta}F^{i\mu\nu\alpha\beta}$, where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor on the manifold. The three models discussed in Chapter 2 are the special cases of the generlised set-up. For example, in the Brown-Teitelboim set-up, there are no scalars and a single four-form, corresponding to the case where $i, j = 1$ only and $\omega^{ij} = \sigma_i = 0, Z_{ij} = 1, V = V_{QFT}$, where V_{QFT} is the renormalised vacuum energy. In the Bousso-Polchinski set-up, we have $\omega^{ij} = \sigma_i = 0, Z_{ij} = \delta_{ij}, V = V_{QFT}$, with i, j running from 1 to $\mathcal{N}$, where $\mathcal{N} \sim \mathcal{O}(100)$ is the number of four-form fluxes. In the Abott set-up, there are no four-form fluxes, but $\omega^{ij} = 1$ and V is given by (2.6).

The action is also equipped with boundary terms which depend on the choice of boundary conditions. These are integrals over the boundary, $\partial\mathcal{M}$, which we take to be a co-dimension one surface described by the embedding $x^\mu =$

$X^\mu(\xi^a)$. The induced metric and the pullback of the three-forms on the boundary are given respectively by $\gamma_{ab} = g_{\mu\nu}X^\mu_{,a}X^\nu_{,b}$ and $\alpha^i = \frac{1}{3!}A^i_{\mu\nu\alpha}X^\mu_{,a}X^\nu_{,b}X^\alpha_{,c}d\xi^a \wedge d\xi^b \wedge d\xi^c$, where $X^\mu_{,a} = \partial X^\mu/\partial\xi^a$ are the boundary tangent vectors. We also define the extrinsic curvature of the boundary, $K_{ab} = \frac{1}{2}\mathcal{L}_n\gamma_{ab}$, as the Lie derivative of the induced metric with respect to the outward pointing unit normal, n^μ. If $K = \gamma^{ab}K_{ab}$ is the trace of the extrinsic curvature on the boundary, the boundary action is given by

$$S_{\text{boundary}} = M_{pl}^2\int_{\partial\mathcal{M}} d^3\xi\sqrt{|\gamma|}K - \int_{\partial\mathcal{M}}\mu p^i\phi_i + \lambda\chi_i\alpha^i \tag{3.2}$$

where we define "conjugate momenta",

$$p^i = -d^3\xi\sqrt{|\gamma|}\omega^{ij}n^\mu\nabla_\mu\phi_j, \quad \chi_i = \sigma_i - Z_{ij}(\star F^j). \tag{3.3}$$

The extrinsic curvature piece ensures that the action can be extremised under metric variations with Dirichlet boundary conditions, $\delta\gamma_{ab} = 0$ [89]. Meanwhile, the parameter λ allows us to interpolate between Dirichlet ($\lambda = 0$) and Neumann ($\lambda = 1$) boundary conditions on the three-forms, while the parameter μ allows us to interpolate between Dirichlet ($\mu = 0$) and Neumann ($\mu = 1$) boundary conditions on the scalars. To see this, note that variation of the action with respect to all fields yields a boundary contribution [89, 90],

$$\begin{aligned}\delta S_{boundary} = &-\frac{M_{pl}^2}{2}\int_{\partial\mathcal{M}} d^3\xi\sqrt{|\gamma|}(K^{ab} - K\gamma^{ab})\delta\gamma_{ab} \\ &+\int_{\partial\mathcal{M}}(1-\lambda)\chi_i\delta\alpha^i - \lambda\delta\chi_i\alpha^i + (1-\mu)p^i\delta\phi_i - \mu\delta p^i\phi_i\end{aligned} \tag{3.4}$$

that is required to vanish under the appropriate choice of boundary conditions. Finally we consider the membrane contributions. We can include contributions from membranes and anti-membranes, Σ_I, charged under any of the three-

forms, such that

$$S_{\text{membranes}} = -\sum_I \left\{ \eta_I^i q_i \int_{\Sigma_I} \alpha_I^i + \tau_i \int_{\Sigma_I} d^3\xi \sqrt{|\gamma_I|} \right\}. \tag{3.5}$$

Membranes charged under A^i carry a fundamental charge $\pm q_i$ depending on whether they are branes or antibranes and tension τ_i. In the action (3.5), $\eta_I^i = 0, \pm 1$ depending on whether the membrane Σ_I carries positive ($\eta_I^i = 1$), negative ($\eta_I^i = -1$) or vanishing charge ($\eta_I^i = 0$) under A^i [1]. The induced metric and the pullback of the three-forms on Σ_I are given in a similar way to the boundary, by $\gamma_{Iab} = g_{\mu\nu} X_{I\,,a}^{\mu} X_{I\,,b}^{\nu}$ and $\alpha_I^i = \frac{1}{3!} A^i_{\mu\nu\alpha} X_{I\,,a}^{\mu} X_{I\,,b}^{\nu} X_{I\,,c}^{\alpha} d\xi^a \wedge d\xi^b \wedge d\xi^c$, where $X_{I\,,a}^{\mu} = \partial X_I{}^{\mu}/\partial \xi^a$ are the tangent vectors on Σ_I. Throughout this chapter we will restrict attention to timelike membranes so that their unit normal, n_I^{μ} is spacelike.

Away from the membranes, the field equations resulting from variation of the action (3.1) with respect to the metric, three-forms and scalars are given by

$$M_{pl}^2 G^{\mu\nu} = T_F^{\mu\nu} + T_{\phi}^{\mu\nu} \tag{3.6}$$

$$\nabla_{\mu} \chi_i = 0 \tag{3.7}$$

$$\nabla_{\mu} \left(\omega^{kj} \nabla^{\mu} \phi_j \right) = V^{,k} + \frac{1}{2} \omega^{ij\,,k} \nabla_{\mu} \phi_i \nabla^{\mu} \phi_j - \left[\frac{1}{2} Z_{ij}{}^{,k} (\star F^i)(\star F^j) - \sigma_i{}^{,k} (\star F^i) \right] \tag{3.8}$$

where $V^{,k} = \partial V/\partial \phi_k$ etc. The energy momentum tensors for the three-forms, the scalar and the membranes are

$$T_F^{\mu\nu} = \frac{Z_{ij}}{3!} \left(F^{i\mu\alpha\beta\gamma} F^{j\nu}{}_{\alpha\beta\gamma} - \frac{1}{8} g^{\mu\nu} F^i_{\alpha\beta\gamma\delta} F^{j\alpha\beta\gamma\delta} \right) \tag{3.9}$$

$$T_{\phi}^{\mu\nu} = \omega^{ij} \left(\nabla^{\mu} \phi_i \nabla^{\nu} \phi_j - \frac{1}{2} g^{\mu\nu} \nabla_{\alpha} \phi_i \nabla^{\alpha} \phi_j \right) - V g^{\mu\nu}. \tag{3.10}$$

At the membranes, we can derive junction conditions which determine how

[1] In what follows we will assume that any given membrane, I, is charged under just one of the three forms, namely $A^{i_\star}$ for some particular value of $i_\star$. This means that $\eta_I^{i_\star} = \pm 1$ and $\eta_I^{i \neq i_\star} = 0$.

the conjugate momenta of the system jump across the membranes. The momentum conjugate to γ_{Iab} is proportional to the following combination of the extrinsic curvature $\pi_I^{ab} = M_{pl}^2(K_I^{ab} - K^I\gamma_I^{ab})$. As we pass over Σ_I this jumps in accordance with the Israel junction conditions [91]

$$M_{pl}^2\Delta\pi_I^{ab} = -|\eta_I^i|\tau_i\gamma_I^{ab}. \tag{3.11}$$

The momentum conjugate to the three-forms also jumps

$$\Delta\chi_i = -\eta_I^i q_i \qquad \text{(no sum over } i\text{)} \tag{3.12}$$

while the scalar momenta remains continuous $\Delta p^i = 0$. The metric and the three-form potentials are assumed to be continuous at Σ_I in order for the membrane action to be well defined. Note that the junction condition (3.12) is consistent with the momentum χ_i quantised in units of the fundamental membrane charge, $\chi_i = -N_i q_i$ (no sum) where $N_i \in \mathbb{Z}$.

3.3 Vacua

We take vacua to be real Lorentzian solutions with constant scalars, four-forms of constant flux $\star F^i = c^i$, and a maximally symmetric metric with constant curvature k^2, corresponding to de Sitter ($k^2 > 0$), Minkowski ($k^2 = 0$) or anti de Sitter ($k^2 < 0$) spacetime. Away from any membranes, the field equations imply that

$$3M_{pl}^2k^2 = V + \frac{1}{2}Z_{ij}c^ic^j \tag{3.13}$$

$$V^{,k} = \frac{1}{2}Z_{ij}{}^{,k}c^ic^j - \sigma_i{}^{,k}c^i \tag{3.14}$$

while the conjugate momentum,

$$\chi_i = \sigma_i - Z_{ij}c^j \tag{3.15}$$

is locally constant. At membranes, we get a jump in χ_i according to (3.12), generically triggering a jump in the spacetime curvature. This means we have a landscape of possible vacua with different cosmological constants, scanned through membrane nucleation.

In order to be compatible with observations, this landscape must include vacua whose curvature is of order the Hubble scale today, H_0^2. This suggests a dense landscape with vacua separated by no more than this very low scale. In the original Brown-Teitelboim scenario [60, 61], this required the membrane charge to be exponentially small in Planck units. That condition was lifted in Bousso-Polchinski [43] where a sufficiently dense landscape follows from a family of $\mathcal{O}(100)$ three-form fields coupled to membranes with incommensurate charges. In Kaloper-Westphal [67, 69] the dense landscape follows from an irrational ratio for the fundamental membranes charges.

Of course, the density of the landscape raises the possibility that transitions between vacua of similar curvature are possible. If this is the case, we may encounter an empty universe problem where the universe arrives at a low scale vacuum via a slow cascade through higher-curvature vacua, cooling the universe down through many efolds of exponential expansion. Although the empty universe problem is known to be an issue for the original Brown-Teitelboim scenario, we will show how it is avoided in the other cases we consider.

Membrane nucleation, necessary for scanning the landscape of vacua, is a quantum process. When we compute transition rates we do so between eigenstates of constant χ_i. This suggests a path integral formalism equipped with Neumann boundary conditions, fixing χ_i in both the in-state and in the out-state [90]. We will compute these transition rates in the next section. Corresponding formulae for Dirichlet boundary conditions, fixing A^i in in and out-states, are presented in Appendix B.

3.4 Nucleation rates

To compute the rate at which membranes are nucleated, mediating transitions between vacua, we analytically continue to Euclidean signature, setting

$$t \to -it_E, \qquad S \to iS_E \tag{3.16}$$

where S_E is the Euclidean action. Following [60, 61], we assume that scalars are unchanged under this Wick rotation, including the dual of the four-forms $\star F \to \star F$. It follows that the three-forms should be analytically continued as $A \to iA$ [2]. The resulting Euclidean action now takes the form

$$\begin{aligned} S_E = &- \int_{\mathcal{M}} d^4x_E \sqrt{|g|} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2}\omega^{ij}(\phi)\nabla_\mu\phi_i\nabla^\mu\phi_j - V(\phi) \right] \\ &+ \int_{\mathcal{M}} \left[-\frac{1}{2} Z_{ij}(\phi) F^i \wedge \star F^j + \sigma_i(\phi) F^i \right] + S^E_{\text{boundary}} + S^E_{\text{membranes}}, \end{aligned} \tag{3.17}$$

where the boundary terms are chosen to be consistent with Neumann boundary conditons on the three-form fields ($\lambda = 1$). Since we will only be considering bounce configurations that transition between two vacua, we assume that there is a single membrane, Σ, which is charged under A^i for some particular choice of $i = i_*$. The membrane charges under A^i are given by $Q_i = \delta_{ii_*} Q_{i_*}$ where $Q_{i_*} = \pm q_{i_*}$ and the membrane tension is $T = \tau_{i_*}$. As a result, the (Euclidean) membrane action is given by

$$S^E_{\text{membranes}} = -Q_{i_*} \int_\Sigma \alpha_\Sigma^{i_*} + T \int_\Sigma d^3\xi_E \sqrt{|\gamma_\Sigma|}\,. \tag{3.18}$$

We consider $O(4)$ symmetric Euclidean field configurations, with metric

$$ds^2 = dr^2 + \rho(r)^2 d\Omega_3 \tag{3.19}$$

[2] If η and $\eta_E = i\eta$ are the volume forms for Lorentzian and Euclidean signature respectively, then $F = -(\star F)\eta$ and $F_E = (\star F)_E \eta_E$. Setting $\star F = (\star F)_E$, as desired, we infer that $F = iF_E$. This is consistent with our choice of $A = iA_E$.

where $d\Omega_3 = h_{ij}d\xi^i d\xi^j$ is the metric on a unit 3-sphere, Euclidean three-form potentials

$$A^i = A^i(r)\sqrt{|h|}d^3\xi \tag{3.20}$$

and scalars $\phi_i = \phi_i(r)$. The radial coordinate is assumed to run from $r_{\min}$ to $r_{\max}$ (to be determined) and the membrane lies at $r = 0$. We label the interior geometry, where $r_{\min} \leq r < 0$, by $\mathcal{M}_-$ and the exterior, where $0 < r < r_{\max}$, by $\mathcal{M}_+$.

With this ansatz, the field equations away from the membrane simplify to the following

$$\begin{aligned}
3M_{pl}^2\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2\right] &= V - \frac{1}{2}\omega^{ij}\phi_i'\phi_j' + \frac{1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^6} && (3.21)\\
M_{pl}^2\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2 - 2\frac{\rho''}{\rho}\right] &= V + \frac{1}{2}\omega^{ij}\phi_i'\phi_j' + \frac{1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^6} && (3.22)\\
\chi_i' &= 0 && (3.23)\\
\frac{1}{\rho^3}\left(\rho^3\omega^{kj}\phi_j'\right)' &= V^{,k} - \frac{1}{2}Z_{ij}{}^{,k}\frac{A^{i\prime}A^{j\prime}}{\rho^6} + \frac{A^{i\prime}}{\rho^3}\sigma_i{}^{,k} && (3.24)
\end{aligned}$$

where $'$ denotes the derivative with respect to r and $\chi_i = \sigma_i - Z_{ij}\frac{A^{j\prime}}{\rho^3}$.

Considering (3.21) and (3.22), we can find that if we set $A^{i'} = c^i\rho^3$, then for constant scalars ϕ_i we have

$$3M_{pl}^2\left[\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2}\right] - (V + \frac{1}{2}Z_{ij}c^ic^j) = 0, \tag{3.25}$$

and

$$3M_{pl}^2\left[\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2} - 2\frac{\rho''}{\rho^2}\right] - 3(V + \frac{1}{2}Z_{ij}c^ic^j) = 0. \tag{3.26}$$

Let

$$V + \frac{1}{2}Z_{ij}c^ic^j = 3k^2M_{pl}^2, \tag{3.27}$$

(3.25) and (3.38) become

$$\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2} - k^2 = 0, \tag{3.28}$$

and

$$\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2} - 2\frac{\rho''}{\rho^2} - 3k^2 = 0. \tag{3.29}$$

Furthermore, letting $\rho = \frac{1}{\alpha}\sin\theta$ (α is a constant) and insert in (3.28), we can find that

$$\rho = \frac{1}{k}\sin k(r_0 + \epsilon r), \tag{3.30}$$

where $\epsilon = \pm 1$. The solution (3.30) also satisfies (3.29). In fact, we can find that for $A^{i'} = c^i\rho^3$ and constant scalars, (3.23) and (3.24) can be satisfied automatically.

Therefore, assuming all scalars are locally constant and satisfy the constraints listed from (3.13) to (3.15), this system is solved by

$$\rho = \frac{\sin(k(\epsilon r + r_0)}{k}, \qquad \epsilon = \pm 1, \tag{3.31}$$

where the curvature k^2 matches the curvature of the Lorentzian solution (3.13), and three-form potentials

$$A^i(r) = A^i(0) + c^i \int_0^r d\tilde{r}\rho(\tilde{r})^3. \tag{3.32}$$

Note that the solution for ρ extends to $k^2 \leq 0$ by analytic continuation. The parameter r_0 is an integration constant, setting the radius of the 3-sphere at the membrane.

For $k^2 > 0$, the geometry is that of a section of a 4-sphere and it turns out that ρ is invariant under $\epsilon \to -\epsilon$, $r_0 \to \pi/k - r_0$. This allows us to set $\epsilon = +1$, WLOG, while also assuming $r_0 \in [0, \pi/k]$. The poles of the 4-sphere are located at $r_{\min} = -r_0$ and $r_{\max} = \pi/k - r_0$.

For $k^2 = 0$ and $k^2 < 0$, the geometry is that of a section of a four-dimensional Euclidean plane or hyperboloid respectively. Since ρ is assumed to be non-negative at the membrane, we take $r_0 \geq 0$. Note that we can no longer fix ϵ WLOG, and must consider each sign separately. For $\epsilon = +1$, $r_{\min} =$

$-r_0$ corresponding to the point where the 3-spheres shrink to zero size, while $r_{\max} = \infty$ corresponding to where they diverge. For $\epsilon = -1$, the reverse is true: $r_{\min} = -\infty$ corresponding to the point where the 3-spheres diverge, while $r_{\max} = r_0$ corresponding to where they shrink to zero size.

At the membrane, the induced metric $ds^2_\Sigma = \rho^2(0)d\Omega_3$ and the pullback of the three-forms $\alpha^i_\Sigma = A^i(0)\sqrt{|h|}d^3\xi$ are well defined. Note that for the induced metric this gives us a continuity constraint

$$\Delta\rho(0) = \Delta\left[\frac{\sin kr_0}{k}\right] = 0\,, \tag{3.33}$$

where we introduce the notation $\Delta x = x^+ - x^-$ and $\langle x\rangle = \frac{1}{2}(x^+ + x^-)$ corresponding respectively to the difference and average of some quantity x defined on either side of the membrane. We also have the following junction conditions on the conjugate momenta,

$$6M^2_{pl}\Delta\left[\frac{\rho'(0)}{\rho(0)}\right] = -3T \tag{3.34}$$

$$\Delta\chi_i = -\delta_{ii_*}Q_{i_*}. \tag{3.35}$$

Physically realistic membranes always carry non-negative tension, resulting in the following constraint on the allowed configurations

$$\Delta\left[\epsilon\cos kr_0\right] \leq 0\,. \tag{3.36}$$

The instanton solutions can also be classified according to the value of

$$X = \frac{4M^4_{pl}\Delta k^2}{T^2} \tag{3.37}$$

with $X > 0$ in downward transitions and $X < 0$ in upward transitions. More importantly, however, it turns out that some configurations are kinematically allowed only when $|X| \leq 1$ with the remainder allowed when $|X| \geq 1$. An analogous result given explicitly in terms of membrane charges was shown

to be true in [67, 69]. Here we show how it is generalised to a much wider class of models which could, in principle, include the Bousso-Polchinski set-up [43]. Indeed, we can always relate X to the charge of the membrane and the curvature of the parent vacuum using equations (3.13) to (3.15), along with the junction condition (3.35), although the relationship is not always a simple one[3].

To see how X relates to the geometry of the instanton we use the junction conditions,

$$\Delta\rho(0) = 0, \qquad \Delta[\rho'(0)] = -\frac{T}{2M_{pl}^2}\rho(0) \tag{3.38}$$

along with the expression $\rho'^2 = 1 - k^2\rho^2$ to prove the following useful relations

$$\langle\rho'(0)\rangle = \frac{M_{pl}^2}{T}\rho(0)\Delta k^2 \tag{3.39}$$

and

$$\rho(0) = \frac{\frac{T}{M_{pl}^2}}{\sqrt{(\Delta k^2)^2 + \frac{T^2}{M_{pl}^4}\langle k^2\rangle + \left(\frac{T^2}{4M_{pl}^4}\right)^2}}. \tag{3.40}$$

We now take the ratio of (3.38) and (3.39) to see that

$$X = -\frac{2\langle\rho'(0)\rangle}{\Delta[\rho'(0)]} = \frac{1+z}{1-z}, \qquad z = \frac{(\epsilon\cos kr_0)_+}{(\epsilon\cos kr_0)_-}. \tag{3.41}$$

The value of X now corresponds to a kinematic constraint on the geometry. As an illustrative example, consider an instanton describing a $\mathrm{dS}_+ \to \mathrm{dS}_-$ transition with $(kr_0)_+ \geq \frac{\pi}{2} \geq (kr_0)_-$ and, of course, $\epsilon_\pm = 1$. Since $(\cos kr_0)_+ \geq 0 \geq (\cos kr_0)_-$ it immediately follows that this solution has $T \geq 0$ and $z \leq 0$, or in other words, $-1 \leq X \leq 0$.

A summary of these classifications is presented for all potential instanton

[3]In principle, this can be done as follows for a generic model. First, we use (3.14) and (3.15) to express the fluxes c^i and the scalars ϕ_i in terms of χ_i. Plugging this into (3.13) we obtain an expression for the bulk cosmological constant of the form $k^2 = \mathcal{K}(\chi_i)$. For example, for tunneling from near Minkowski into AdS, we then have χ_i^+ fixed such that $k_+^2 = \mathcal{K}(\chi_i^+) \approx 0$, with $\chi_i^- = \chi_i^+ + Q_i$ fixed by the junction conditions (3.35). Finally we have that $X^2 = 4M_{pl}^4\Delta k^2/T^2 = 4M_{pl}^4[\mathcal{K}(\chi_i^+) - \mathcal{K}(\chi_i^+ + Q_i)]/T^2$

<table>
<tr><th></th><th colspan="2">dS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = -1$</th></tr>
<tr><td>dS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_+ \geq \frac{\pi}{2} \geq (kr_0)_-$
allowed for $|X| \leq 1$
$\frac{\pi}{2} \geq (kr_0)_+ \geq (kr_0)_-$
allowed for $X \geq 1$</td><td>$(kr_0)_+ \geq (kr_0)_- \geq \frac{\pi}{2}$
allowed for $X \leq -1$
$(kr_0)_+ < (kr_0)_-$
negative tension</td><td colspan="2">negative tension</td><td>$(kr_0)_- \geq \frac{\pi}{2}$
kinematically
allowed for $X \leq -1$,
infinitely suppressed</td><td>$\frac{\pi}{2} \geq (kr_0)_-$
kinematically
allowed for $-1 \leq X \leq 0$,
infinitely suppressed</td></tr>
<tr><td>Minkowski/AdS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_+ \geq \frac{\pi}{2}$
allowed for $0 \leq X \leq 1$</td><td>$\frac{\pi}{2} \geq (kr_0)_+$
allowed for $X \geq 1$</td><td>$|k_-| \geq |k_+|$
allowed for $X \geq 1$</td><td>$|k_-| < |k_+|$
negative tension</td><td colspan="2">kinematically allowed for $|X| \leq 1$,
infinitely suppressed</td></tr>
<tr><td>Minkowski/AdS$_-$
$\epsilon_- = -1$</td><td colspan="2">negative tension</td><td colspan="2">negative tension</td><td>$|k_-| > |k_+|$
negative tension</td><td>$|k_-| \leq |k_+|$
kinematically
allowed for $X \leq -1$,
infinitely suppressed</td></tr>
</table>

Table 3.2: Summary of transitions $\mathcal{M}_+ \to \mathcal{M}_-$ where $\mathcal{M}$ is de Sitter, Minkowski or anti de Sitter, as indicated. If a transition is forbidden because it violates the condition of non-negative tension, we mark it accordingly. For the remaining transitions, we mark whether they are kinematically allowed for $|X| \leq 1$ or $|X| \geq 1$ where $X = 4M_{pl}^4 \Delta k^2/T^2$. In some of these cases, the transition may be kinematically allowed but is ruled out because the transition rate is infinitely suppressed. Recall that we can take $\epsilon = +1$ for all dS configurations WLOG, so any examples contrary to this are not applicable. For a given model, equations (3.35) and (3.38) can be used to determine, X and identify precisely where we are on the Baedeker.

geometries in table 3.2. The table also lists those geometries for which the corresponding tunneling rates are infinitely suppressed and so can be ignored. As we will see presently, these correspond to configurations with $k_+^2 \leq 0$ and $\epsilon_+ = -1$. Table 3.2 mirrors the "Baedeker" of [67], although we emphasize again how it applies to a much broader set of models.

In semi-classical theory of vacuum decay, transition rates between vacua $\mathcal{M}_+ \to \mathcal{M}_-$ are given by [82, 92, 93]

$$\frac{\Gamma}{\text{Vol}} \sim e^{-B/\hbar} \tag{3.42}$$

where

$$B = S_E(\text{instanton}) - S_E(\text{parent}). \tag{3.43}$$

Here S_E(instanton) is the Euclidean action evaluated on the bubble configurations described above, interpolating between the vacua $\mathcal{M}_+$ and $\mathcal{M}_-$. In contrast, S_E(parent) is the Euclidean action evaluated on the complete parent vacuum, $\mathcal{M}_+$, with no bubbles.

The tunneling exponent can be computed in all cases after a relatively

lengthy calculation and plenty of heart warming cancellations, giving

$$B = -2M_{pl}^2\Omega_3\Delta\left\{\frac{1}{k^2}\left[\rho'(r)^3\right]_{r_{\min}}^0\right\} + \Omega_3 T\rho(0)^3, \tag{3.44}$$

where $r_{\min}$ denotes the minimal value of the radial coordinate and Ω_3 is the volume of the unit 3-sphere. The derivation of (3.44) can be found in Appendix A. Note that this is quantitatively identical to the tunneling exponent computed in GR for the same geometry, reflecting the fact that the scalars are constants in vacuo, while the three-forms gravitate like a cosmological constant. Whenever we have a parent AdS or Minkowski vacuum with $\epsilon_+ = -1$, there are configurations with non-negative membrane tension whose tunneling exponents contain divergent contributions at $r_{\min}$. These correspond to exotic configurations where an infinite asymptotic space is removed by membrane nucleation. The corresponding tunneling rates are either infinitely suppressed, ruling out the transition, or infinitely enhanced, signalling a catastrophic instability. As we have imposed Neumann boundary conditions on the three-forms, these particular configurations are always infinitely suppressed[4].

As expressed in table 3.2, there are now only three configurations that avoid any problems with negative brane tension or infinitely suppressed tunneling rates. These are the configurations of physical interest corresponding to

- $\mathrm{dS}_+ \to \mathrm{dS}_-$
- $\mathrm{dS}_+ \to$ Minkowski/AdS_-
- Minkowski/$\mathrm{AdS}_+ \to$ Minkowski/AdS_- $(|k_-| \geq |k_+|)$

each with $\epsilon_\pm = 1$ and so $\rho'(r_{\min}) = 1$. To simplify the corresponding formulae for the tunneling rates, we make use of equations (3.38) to (3.40). The tunneling exponents for the three configurations of interest can now be written

[4]For $\mathrm{AdS}_+ \to \mathrm{AdS}_-$ configurations with $\epsilon_\pm = -1$, there are divergent contributions to the tunneling exponent for both vacua. If we assume that the divergence in the proper distance occurs at the same rate in each case, $|r^+_{\min}| \sim |r^-_{\min}| \to \infty$, then the positivity of the membrane tension ensures that $|k|_+ > |k|_-$, and the dominance of the divergent contribution of the + vacuum.

as

$$B = \frac{2M_{pl}^2\Omega_3}{k_+^2 k_-^2}\left\{-\Delta k^2 + \frac{M_{pl}^2}{T}\rho(0)\left[(\Delta k^2)^2 + \frac{T^2}{2M_{pl}^4}\left\langle k^2\right\rangle\right]\right\}. \tag{3.45}$$

If we recall that $X = 4M_{pl}^4\Delta k^2/T^2$ and introduce

$$Y(X) = \sqrt{(X-1)^2 + 16k_+^2 M_{pl}^4/T^2} \tag{3.46}$$

we can rewrite the tunneling exponent in the following simple form

$$B = \frac{4M_{pl}^2\Omega_3}{k_+^2}\left[\frac{1+Y-X}{Y(1+Y+X)}\right]. \tag{3.47}$$

3.5 Cascades, bungees and the stability of the Minkowski vacuum

Armed with a general expression for the tunneling exponent and a detailed understanding of the instanton geometries as shown in table 3.2, we are ready to take a more in-depth look at the tunneling dynamics. In particular, we are interested in what happens if the universe begins in a high scale de Sitter vacuum with curvatures far above the current Hubble scale. Can we descend into the current low scale vacuum in a single jump, avoiding the empty universe problem, or do we need to cascade into it via a series of incremental changes in curvature? Is there a danger we fall deep into an anti-de Sitter vacuum, so that the universe is driven to a apocalyptic crunch? How long is the current vacuum expected to survive before it gives way to these apocalyptic AdS vacua?

Consider tunneling from a parent vacuum with a non-negative curvature, so that $k_+^2 \geq 0$. It follows that $Y \geq |X-1| \geq 0$, and so rewriting the tunneling exponent as

$$B = \frac{4M_{pl}^2\Omega_3}{k_+^2}\left[\frac{Y-(X-1)}{Y(2+Y+X-1)}\right], \tag{3.48}$$

we see that it is everywhere positive[5]. Further, for k_+ fixed, it is monoton-

[5]This contradicts [67] where it is claimed that the bounce action can be negative when the membrane radius is close to the de Sitter radius of the two vacua. This involved neglecting

ically decreasing from $4M_{pl}^2\Omega_3/k_+^2$ as $X \to -\infty$ to zero as $X \to +\infty$[6]. For a given parent de Sitter vacuum, this means that transitions towards smaller values of the curvature are likely to be the most rapid, provided they are kinematically allowed. This raises the possibility of decay deep into anti de Sitter space, which would be incompatible with the current low scale vacuum, with curvature no larger than the current Hubble scale.

Generically we find that descent into AdS is kinematically forbidden for parent vacua with very large de Sitter curvatures. This is because the membrane does not carry enough charge to trigger a sufficiently large jump in the cosmological constant. For parent vacua with curvature less than some critical value, $k_+^2 \leq k_c^2$, descent into AdS is kinematically allowed. This is potentially dangerous if the tunneling rates are too high. One way to avoid this is as follows: first we choose the parameters of the model such that $0 < X < 1$ for all downward transitions from parent vacua below the critical curvature, $k_+^2 \leq k_c^2$. It follows that

$$B > B(X=1) = \frac{2M_{pl}^2\Omega_3}{k_+^2\left(1+\frac{2M_{pl}^2k_+}{T}\right)} > \frac{M_{pl}^2\Omega_3}{k_c^2} = B_c \tag{3.49}$$

where we have also used the fact that $4M_{pl}^4k_+^2/T^2 < 1 + 4M_{pl}^4k_-^2/T^2 \leq 1$ for $k_+^2 \leq k_c^2$ and $k_-^2 \leq 0$. Since we typically expect $k_c \lesssim M_{pl}$, we can choose parameters such that B_c is large, and all vacua in danger of decay into anti de Sitter are long lived. Furthermore, the lifetime of these vacua increases as we reduce the curvature of the parent vacuum, with B containing a pole as $k_+ \to 0$.

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terms that we said to be small compared to a "leading" term, equal to the difference in curvature. However, this "leading" term is just as small as the neglected terms in the relevant limit.

[6]The quickest way to see this is to define $\zeta \in (-\infty, \infty)$ such that $X \equiv \frac{4M_{pl}^4\Delta k^2}{T^2} = 1 + \frac{4M_{pl}^2k_+}{T}\sinh\zeta$. The tunneling exponent then reduces to the following simple form

$$B = \frac{4M_{pl}^2\Omega_3}{k_+^2(1+e^{2\zeta})\left(1+\frac{2M_{pl}^2k_+}{T}e^{\zeta}\right)} \ .$$

This is clearly monotonically decreasing between the two limits stated in the text.

We can see the importance of the proposed bound on X by taking a closer look at transitions from parent vacua near Minkowski space. For these transitions, we have

$$X_{M_+\to AdS_-} \approx -\frac{4M_{pl}^4 k_-^2}{T^2}. \tag{3.50}$$

Note that $X_{M_+\to AdS_-} = |X_{M_+\to AdS_-}|$ since the daughter vacuum is AdS and so $k_-^2 < 0$. The tunneling exponent for the transition goes as

$$B_{M_+\to AdS_-} \sim \frac{2M_{pl}^2\Omega_3}{k_+^2}(1-S(X))+\frac{8M_{pl}^6\Omega_3}{T^2X(X-1)^2}\left[(X-1)^2(1-S(X))+2S(X)\right] \tag{3.51}$$

where $S(X) = \mathrm{sgn}(X-1)$ and we understand that $X = X_{M_+\to AdS_-}$ here and in the remainder of this section. The lifetime of the parent near Minkowski vacuum is clearly determined by the sign of $X-1$. In particular, when $X > 1$, we have

$$B_{M_+\to AdS_-} \sim \frac{16M_{pl}^6\Omega_3}{T^2X(X-1)^2}, \qquad X > 1. \tag{3.52}$$

For tunneling into deep AdS so that X is large, this exponent is suppressed and the transition is quick, rendering the near Minkowski vacuum short lived. In contrast, if $X < 1$, the tunneling exponent goes as

$$B_{M_+\to AdS_-} \sim \frac{4M_{pl}^2\Omega_3}{k_+^2}, \qquad X < 1. \tag{3.53}$$

As the parent vacuum approaches Minkowski, this exponent diverges, suppressing any transition into AdS and ensuring a long lived Minkowski vacuum. This is the key ingredient for addressing the cosmological constant advocated in [67, 69], although we now see how it can apply more generally, as long as the microscopic details of transition from Minkowski into AdS enforce $X_{M_+\to AdS_-} = -\frac{4M_{pl}^2 k_-^2}{T^2} < 1$.

Of course, it is not enough for the near Minkowski vacuum to be long lived. We need to show that we are likely to reach it from a generic initial state corresponding to a high scale de Sitter vacuum. There are two ways in which

this might happen. The first involves a bungee jump, a single transition from a high scale de Sitter vacuum towards a near Minkowski vacuum, $k_+^2 \gg H_0^2 \gtrsim |k_-|^2$. If a bungee jump is kinematically possible, the tunneling exponent goes as

$$B_{dS_+ \to M_-} \sim \frac{4M_{pl}^2 \Omega_3}{k_+^2 \left(1 + \frac{4M_{pl}^4 k_+^2}{T^2}\right)^2}. \tag{3.54}$$

For de Sitter vacua, the larger the value of k_+, the more this exponent is suppressed and the faster the transition. Bungee jumps avoid the empty universe problem [59]. As explained in [43], the inflaton is significantly displaced from its minimum by quantum diffusion before the transition. After the transition, the inflaton is in slow roll and follows the classical evolution towards the minimum, at which point it begins to oscillate and reheating occurs. However, because the vacuum energy is now no larger than the current Hubble scale, there is no rapid cooling.

The second way in which we might finally arrive at the current vacuum is via a cascade, several transitions in which the cosmological constant descends from a high scale in small increments, each with $\Delta k^2 \lesssim H_0^2$. For small incremental transitions where the vacuum energy changes by a small amount, $k_+^2 \to k_-^2 = k_+^2(1-\epsilon)$, for $\epsilon \ll 1$, the tunneling exponent goes as

$$B_{dS_+ \to dS_- \approx dS_+} \sim \frac{4M_{pl}^2 \Omega_3}{k_+^2 \sqrt{1 + \frac{16M_{pl}^4 k_+^2}{T^2}}}. \tag{3.55}$$

Unfortunately, these cascades do run into the empty universe problem [59]. As the cosmological constant slowly descends, quantum diffusion effects are scaled down, allowing the inflaton to settle into its minimum and reheat the universe. However, this happens too soon, before the vacuum energy has settled into its current value. The universe then spends far too long in de Sitter vacua at scales above the current Hubble scale, and any matter produced after reheating is diluted away by exponential expansion. To avoid the empty universe problem, it is clear that bungee jumps with $k_+^2 \gg H_0^2 \gtrsim |k_-|^2$ must be kinematically

allowed. Furthermore, a cascade of incremental transitions with $\Delta k^2 \lesssim H_0^2$, should either be kinematically forbidden, or suppressed relative to a bungee jump. If this is the case, the universe initially in a high scale de Sitter vacuum is expected to find the current vacuum before inflation has ended and reheating has begun. Furthermore, if we can consistently impose a condition $|X| < 1$ for a parent vacuum close to Minkowski, then the current vacuum is long lived, and the catastrophe of descent into AdS is delayed until the far future, or avoided altogether.

3.6 Examples

As we have seen, the key features for addressing the cosmological constant problem proposed in [67, 69] could apply quite generally to a large class of models under consideration. This is because the scalars and the four-form field strengths all gravitate as cosmological constants in the vacuum. Consequently, the tunneling exponent, B, takes a universal form which we can express in terms of the tension of the membrane and the curvature of the two vacua involved in the transition. Things only become model dependent when we ask how the quantities relate to other model parameters, such as the membrane charge. In particular, the condition $|X| < 1$ will mean different things for different models. Kinematic considerations can also vary. We shall now explore a few cases of particular interest.

3.6.1 The Brown-Teitelboim set-up

We begin with the pioneering model of this class, originally proposed by Brown and Teitelboim [60, 61]. As is well known, this model runs into problems with the empty universe and cannot be taken seriously as a solution to the cosmological constant problem[7]. However, we include a discussion of its main

[7] A variant of this setup that circumvents the empty universe problem by allowing for the nucleation of brane stacks was proposed in [72].

features as a warm up to the more interesting models. In the Brown-Teitelboim set-up, there are no scalars and a single four-form, corresponding to the case where $i, j = 1$ only and

$$\omega^{ij} = \sigma_i = 0, \quad Z_{ij} = 1, \quad V = V_{QFT} \tag{3.56}$$

where V_{QFT} is the renormalised vacuum energy. In the absence of scalars, the bulk constraints (3.13) to (3.15) yield

$$3M_{pl}^2 k^2 = V_{QFT} + \frac{1}{2}c^2 \tag{3.57}$$

$$\chi = -c \tag{3.58}$$

from which it follows that

$$k^2 = \frac{V_{QFT} + \frac{1}{2}\chi^2}{3M_{pl}^2} \ . \tag{3.59}$$

Membrane nucleation triggers a jump $\Delta\chi = -Q = -\eta q$, where q is the fundamental membrane charge and $\eta = +1$ for branes and $\eta = -1$ for anti-branes. This is consistent with the quantisation condition $\chi = -Nq$ for $N \in \mathbb{Z}$, so that membrane nucleation gives $\Delta N = \eta$. The spectra of vacua are now characterised by the following curvature

$$k^2(N) = \frac{V_{QFT} + \frac{1}{2}N^2 q^2}{3M_{pl}^2} \ . \tag{3.60}$$

For this to include near Minkowski vacua, we assume $V_{QFT} < 0$, which is easy to engineer in string compactifications. The smallest possible separation in curvature, δk^2, occurs between neighbouring vacua and takes the value

$$\delta k^2 = k^2(N \pm 1) - k^2(N) = \frac{(1 \pm 2N)q^2}{6M_{pl}^2} \ . \tag{3.61}$$

This density varies with where we are in the landscape, through the dependence on N. For a near Minkowski vacuum, $N = N^{\rm Mink} \approx \pm\sqrt{2|V_{QFT}|}/q$ and the separation takes two possible values

$$\delta k^2|_{k^2=0} \approx \frac{q^2 \pm q\sqrt{8|V_{QFT}|}}{6M_{pl}^2} \,. \tag{3.62}$$

To be compatible with observation, we require the absolute value of both to be no larger than the current Hubble scale, H_0^2. Assuming $|V_{QFT}| \gg M_{pl}^2 H_0^2$, this imposes an upper bound on the membrane charge

$$q \lesssim \frac{6M_{pl}^2 H_0^2}{\sqrt{8|V_{QFT}|}} \,. \tag{3.63}$$

Nucleation of a membrane triggers a jump $N^+ \to N^- = N^+ \pm 1$, where we take the $+$ for a brane and the $-$ for an anti-brane. In the Brown-Teitelboim setup, this corresponds to a minimal change in curvature

$$\Delta k^2 = \frac{(1 \mp 2N^-)q^2}{6M_{pl}^2} = \frac{(-1 \mp 2N^+)q^2}{6M_{pl}^2} \,. \tag{3.64}$$

In a general set-up, adjacent vacua in flux space are separated by a curvature, Δk^2, which is generically *not* the same as the smallest jump in curvature, δk^2. Crucially, however, in the Brown-Teitelboim set-up, the simplicity of the model renders the two to be equivalent: $\Delta k^2 = \delta k^2$. By the density assumption, nearest neighbour jumps are now no larger than the current Hubble scale whenever the transitions include the current vacuum. More explicitly, consider the transition from a parent de Sitter vacuum to a near Minkowski vacuum. For this to the arise from the nucleation of a single membrane, the parent vacuum will have curvature

$$k_+^2 = \frac{(1 + 2|N^{\rm Mink}|)q^2}{6M_{pl}^2} \approx \frac{q}{6M_{pl}^2}\sqrt{8|V_{QFT}|} \lesssim H_0^2 \tag{3.65}$$

where we have used the fact that $|N^{\text{Mink}}| \gtrsim |V_{QFT}|/M_{pl}^2 H_0^2 \gg 1$ along with the density constraint (3.63) on the fundamental charge. This demonstrates the fact that bungee jumps to the current vacuum cannot be mediated by single membrane. As is well known, this suggests that the Brown-Teitelboim set-up suffers from an empty universe problem [43]. Tunneling rates in this set-up are easily computed and can be shown to agree with [60, 61]. This can also be obtained from the corresponding analysis in the next section by taking the limit of a single three-form potential.

3.6.2 The Bousso-Polchinski set-up

We now switch to a set-up with no scalars and a large number of four-forms as originally proposed by Bousso and Polchinski [43] as a way to overcome the limitations of [60, 61]. This corresponds to the case where

$$\omega^{ij} = \sigma_i = 0, \quad Z_{ij} = \delta_{ij}, \quad V = V_{QFT} \tag{3.66}$$

with i, j running from 1 to $\mathcal{N}$. In the absence of scalars, the bulk constraints (3.13) to (3.15) yield

$$3M_{pl}^2 k^2 = V_{QFT} + \frac{1}{2}\delta_{ij} c^i c^j \tag{3.67}$$

$$\chi_i = -\delta_{ij} c^j \tag{3.68}$$

from which it follows that

$$k^2 = \frac{V_{QFT} + \frac{1}{2}\delta^{ij}\chi_i\chi_j}{3M_{pl}^2} \ . \tag{3.69}$$

Recall the quantisation condition $\chi_i = -N_i q_i$ (no sum), for some $N_i \in \mathbb{Z}$. Assuming nucleation of a brane or anti-brane of type i_*, this is consistent with the membrane junction condition (3.35), $\Delta\chi_i = -\delta_{ii_*} Q_{i_*}$, with $Q_{i_*} = \pm q_{i_*}$ (no sum), or equivalently, $\Delta N_i = \pm\delta_{ii_*}$. The spectra of vacua is therefore

characterised by curvature

$$k^2 = \frac{V_{QFT} + \frac{1}{2}\sum_{i=1}^{\mathcal{N}} N_i^2 q_i^2}{3M_{pl}^2} \ . \tag{3.70}$$

For this to include near Minkowski vacua, we must once again assume that $V_{QFT} < 0$. The vacua span an $\mathcal{N}$ dimensional grid with spacing q_i. Surfaces of constant vacuum curvature correspond to spheres centred at $N_i = 0$. Consider two such surfaces of curvature $k^2 = \frac{V_{QFT}+\frac{1}{2}r^2}{3M_{pl}^2}$ and $k'^2 = \frac{V_{QFT}+\frac{1}{2}r'^2}{3M_{pl}^2}$. The corresponding spheres form the boundary of a shell of volume $V_{\text{shell}} \approx \frac{1}{2}\Omega_{\mathcal{N}-1} r^{\mathcal{N}-2}\delta r^2$ where $\delta r^2 = r'^2 - r^2$ and $\Omega_{\mathcal{N}-1} = 2\pi^{\mathcal{N}/2}/\Gamma(\mathcal{N}/2)$ is the volume of a unit $(\mathcal{N}-1)$-sphere. To estimate the density of vacua, we require that this shell should contain at least one grid point (or if there is degeneracy D, it should contain D grid points), setting a bound $V_{\text{shell}} \gtrsim D\prod_{i=1}^{\mathcal{N}} q_i$. This now translates into vacua with a typical curvature separation of [43]

$$\delta k^2 \approx \frac{D\prod_{i=1}^{\mathcal{N}} q_i}{3M_{pl}^2\Omega_{\mathcal{N}-1} r^{\mathcal{N}-2}}, \qquad r = \sqrt{6M_{pl}^2 k^2 - 2V_{QFT}} \ . \tag{3.71}$$

For this model, we assume the q_i are incommensurate so that $D \sim 2^{\mathcal{N}}$, arising from the vacuum degeneracy $N_i \to -N_i$. As in the Brown-Teitelboim set-up, the density of vacuum energies depends on where you are in the landscape. Near Minkowski, the density is

$$\delta k^2|_{k^2=0} \approx \frac{2^{\mathcal{N}}\prod_{i=1}^{\mathcal{N}} q_i}{3M_{pl}^2\Omega_{\mathcal{N}-1}(2|V_{QFT}|)^{\mathcal{N}/2-1}} \ . \tag{3.72}$$

If we assume that there are no hierarchies in the membrane charges, $q_i \sim \mathcal{O}(q)$ for all i, whilst remaining incommensurate, it turns out that

$$\delta k^2|_{k^2=0} \sim \frac{2|V_{QFT}|}{3M_{pl}^2}\sqrt{\frac{\pi}{\mathcal{N}}}\left(\frac{\mathcal{N}q^2}{e\pi|V_{QFT}|}\right)^{\mathcal{N}/2} \tag{3.73}$$

Provided $\mathcal{N}q^2 < e\pi|V_{QFT}|$, by taking $\mathcal{N}$ sufficently large, we can achieve a density well within the desired observational bound $\delta k^2|_{k^2=0} \lesssim H_0^2$ where $H_0 \sim$

$10^{-60}M_{pl}$ is the current Hubble scale. For example, if $|V_{QFT}| \sim M_{pl}^4$ and $q \sim 0.02M_{pl}^2$ then we can achieve the desired density of vacuum energies for $\mathcal{N} \approx 100$.

Nucleation of a membrane of type i_* triggers a jump in one of the flux integers, $N_{i_*}^+ \to N_{i_*}^- = N_{i_*}^+ \pm 1$, while leaving the remaining $N_{i\neq i_*}$ unchanged. This translates into a jump in vacuum curvature

$$\Delta k^2 = \frac{(1 \mp 2N_{i_*}^-)q_{i_*}^2}{6M_{pl}^2} = \frac{(-1 \mp 2N_{i_*}^+)q_{i_*}^2}{6M_{pl}^2}. \tag{3.74}$$

As in the previous example, this is sensitive to where we are in the landscape through the dependence on $N_{i_*}^{\pm}$, a consequence of the curvature having quadratic four-form dependence.

Let us consider descent from a parent de Sitter vacuum with curvature $k_+^2 \geq 0$ and compute the important quantity, $X = 4M_{pl}^4\Delta k^2/T^2$. To do this, we first relate the curvature to the integer fluxes using the relation

$$\sum_{i=1}^{\mathcal{N}}(N_i^+)^2q_i^2 = r_+^2, \quad r_+ = \sqrt{6M_{pl}^2k_+^2 - 2V_{QFT}}. \tag{3.75}$$

Since $\Delta k^2 \geq 0$, it now follows that

$$X = \frac{2M_{pl}^2q_{i_*}^2}{3\tau_{i_*}^2}(2|N_{i_*}^+| - 1) \sim \frac{2M_{pl}^2q^2}{3\tau^2}(2|N_{i_*}^+| - 1) \tag{3.76}$$

where we have used the fact there are no hierarchies, $q_i \sim \mathcal{O}(q)$, $\tau_i \sim \mathcal{O}(\tau)$ for all i. Further, equation (3.75) now implies that $|\vec{N}|^2 \sim r_+^2/q^2$, where $|\vec{N}|^2 = \sum_{i=1}^{\mathcal{N}} N_i^2 \geq |N_{i_*}|^2$ and so

$$X \lesssim \frac{4M_{pl}^2qr_+}{3\tau^2} \equiv X_{\max}(k_+)\,. \tag{3.77}$$

Dangerous transitions into AdS are kinematically forbidden if $X_{\max}(k_+) < X_0(k_+)$, where $X_0(k_+) = 4M_{pl}^4k_+^2/\tau^2$ represents the critical value of X for which the daughter vacuum is approximately Minkowski. For the largest values of

$k_+^2 \gg |V_{QFT}|/3M_{pl}^2$, it is clear that $r_+ \approx \sqrt{6}M_{pl}k_+$ and so

$$X_{\max} \approx \frac{4\sqrt{6}M_{pl}^3 q k_+}{3\tau^2} < \frac{4M_{pl}^2 k_+}{3\tau^2}\sqrt{\frac{6e\pi|V_{QFT}|}{\mathcal{N}}} \ll X_0(k_+) \tag{3.78}$$

where we have used the density condition $\mathcal{N}q^2 < e\pi|V_{QFT}|$. It is, therefore, kinematically impossible for parent vacua with $k_+^2 \gg |V_{QFT}|/3M_{pl}^2$ to decay directly into AdS.

What about parent vacua with $k_+^2 \lesssim |V_{QFT}|/3M_{pl}^2$? These have $r_+ \sim \sqrt{2|V_{QFT}|}$ and so

$$X_{\max} \sim \frac{2M_{pl}^2 q}{3\tau^2}\sqrt{8|V_{QFT}|}. \tag{3.79}$$

For sufficiently small $k_+^2 < k_c^2 = q\sqrt{8|V_{QFT}|}/6M_{pl}^2$, we can have $X_{\max} > X_0(k_+)$ and so decay into AdS is now kinematically possible.

The potential for decay into AdS might be troubling at first glance. However, as we hinted in the previous section, we can impose the condition

$$\frac{2M_{pl}^2 q}{3\tau^2}\sqrt{8|V_{QFT}|} < 1 \tag{3.80}$$

ensuring that $0 < X < 1$ whenever the parent vacuum has $k_+^2 < k_c^2$ and is therefore vulnerable to this dangerous decay. However, the lifetime of these vacua can be made large by ensuring that $B_c \gg 1$, where B_c is defined by equation (3.49). Inputting the value of k_c we have just derived for the Bousso-Polchinski set-up, we find that

$$B_c \gtrsim \frac{6M_{pl}^4\Omega_3}{q\sqrt{8|V_{QFT}|}} \tag{3.81}$$

For our canonical example, with $|V_{QFT}| \sim M_{pl}^4$, $q \sim 0.02M_{pl}^2$ and $\mathcal{N} \approx 100$, we have $B_c \gtrsim 2000$, and so vacuum decay into AdS is heavily suppressed in all instances. Furthermore, the condition (3.80) ensures that $|X_{M_+ \to AdS_-}| < 1$, guaranteeing the presence of a pole in the transition rate as the parent vacuum approaches Minkowski, rendering it arbitrarily long-lived.

All of this suggests that the current vacuum can be selected using probabilistic arguments, similar to those presented in [67, 69]. This is a significant departure from the anthropic arguments usually presented for the Bousso-Polchinski set-up [43]. There is a small price to pay for this success: the condition (3.80) is in violation of the weak gravity conjecture for membranes [86], which requires

$$\tau_i \lesssim g_i M_{pl} q_i \tag{3.82}$$

where g_i is the strength of the three-form coupling. To see this, note that we have two inequalities on the charge,

$$\sqrt{\frac{e\pi|V_{QFT}|}{\mathcal{N}}} > q > \frac{2M_{pl}^2 q^2}{3\tau^2}\sqrt{8|V_{QFT}|} \tag{3.83}$$

and so

$$\frac{M_{pl}q}{\tau} < \left(\frac{9e\pi}{32\mathcal{N}}\right)^{1/4} < 1 \tag{3.84}$$

in violation of (3.82).

If we assume sub-Planckian membrane tension consistent with a well-defined effective theory and the maximum charge, $q_{\max}$, tolerated by the density constraint $\delta k^2 \lesssim H_0^2$ (where δk^2 is given by equation (3.73)), the stability condition (3.80) is also incompatible with an arbitrarily large underlying vacuum energy $|V_{QFT}|$. To see this, we note that

$$q_{\max} = \sqrt{\frac{e\pi|V_{QFT}|}{\mathcal{N}}}\left(\frac{3M_{pl}^2H_0^2}{2|V_{QFT}|}\sqrt{\frac{\mathcal{N}}{\pi}}\right)^{\frac{1}{\mathcal{N}}} \approx \sqrt{\frac{e\pi|V_{QFT}|}{\mathcal{N}}}\epsilon(\mathcal{N}), \tag{3.85}$$

where, in the last step, we have taken a limit of large $\mathcal{N}$ and defined $\epsilon(\mathcal{N}) = \left(\frac{H_0^2}{M_{pl}^2}\right)^{\frac{1}{\mathcal{N}}} < 1$. We include the $\epsilon(\mathcal{N})$ correction to the large $\mathcal{N}$ limit to allow for the fact there is a large hierarchy between H_0 and M_{pl}. For $q \sim q_{\max}$, the stability condition (3.80) now implies that

$$\frac{2|V_{QFT}|}{3M_{pl}^4}\sqrt{\frac{8e\pi}{\mathcal{N}}}\left(\frac{3M_{pl}^2H_0^2}{2|V_{QFT}|}\sqrt{\frac{\mathcal{N}}{\pi}}\right)^{\frac{1}{\mathcal{N}}} \lesssim \frac{\tau^2}{M_{pl}^6} < 1 \tag{3.86}$$

where we have also used the condition of sub-Planckian membrane tensions. This can be used to place a limit on the scale of the QFT contribution to the cosmological constant. For large $\mathcal{N}$, this bound on $|V_{QFT}|$ takes on a particularly simple form

$$|V_{QFT}| \lesssim \frac{3}{2\epsilon(\mathcal{N})}\sqrt{\frac{\mathcal{N}}{8e\pi}} \times \frac{\tau^2}{M_{pl}^2}. \tag{3.87}$$

This is quadratically sensitive to the brane tension and not especially restrictive as can be seen from the numerical examples in Table 3.3. Note, in particular, the case where $\mathcal{N} = 100$, corresponding to the benchmark point in [43]. Here we can have Planckian $|V_{QFT}|$ with the current vacuum being both natural and quantum mechanically stable if the brane tensions, $\tau \sim 0.2M_{pl}^3$ and the charges, $q \leq 0.02M_{pl}^2$. The last thing we need to check is already well known:

$\mathcal{N}$	Upper bound on $\lvert V_{QFT}\rvert$	Upper bound on $q_{\max}$
10	$10^{12} \times \frac{\tau^2}{M_{pl}^2}$	$10^{-7} \times \frac{\tau}{M_{pl}}$
60	$10^{2} \times \frac{\tau^2}{M_{pl}^2}$	$10^{-2} \times \frac{\tau}{M_{pl}}$
100	$10 \times \frac{\tau^2}{M_{pl}^2}$	$10^{-1} \times \frac{\tau}{M_{pl}}$

Table 3.3: Maximum scale of QFT contribution to the cosmological constnat (in units of τ^2/M_{pl}^2) compatible with the quantum mechanical stability of low scale de Sitter space. We also show the corresponding upper bound on the brane charges (in units of τ/M_{pl}). The quoted numbers are order of magnitude estimates.

is it possible to transition from a high scale de Sitter vacuum to the current vacuum in a single jump? Near Minkowski daughter vacua satisfy

$$\sum_{i=1}^{\mathcal{N}} (N_i^{\text{Mink}})^2 q_i^2 \approx 2|V_{QFT}|. \tag{3.88}$$

If this is the daughter vacuum generated by the nucleation of a brane of type i_*, then the parent vacuum has curvature

$$k_+^2 = \frac{\sum_{i=1}^{\mathcal{N}}(1 + 2|N_{i_*}^{\text{Mink}}|)q_{i_*}^2}{6M_{pl}^2} \sim \frac{q\sqrt{8|V_{QFT}|}}{6M_{pl}^2} = k_c^2 \tag{3.89}$$

For physically allowed configurations with positive brane tension this will be positive, corresponding to a *high* scale de Sitter vacuum provided q is not tuned to exponentially small values. In particular, for our canonical parameter values, $|V_{QFT}| \sim M_{pl}^4$, $q \sim 0.02 M_{pl}^2$ and $\mathcal{N} \approx 100$, we have a bungee jump from vacua of curvature $k_+^2 \sim 0.01 M_{pl}^2$ to the current vacuum. This enables us to avoid the empty universe problem in the Bousso-Polchinski set-up, as is well known.

To summarise, if we are prepared to accept some violation of the membrane weak gravity conjecture, we can select the near Minkowski vacuum using probabilistic methods even for the Bousso-Polchinski set-up. This offers a tantalising alternative to the standard anthropic arguments used to recover a vacuum consistent with observation.

3.6.3 The Kaloper-Westphal set-up

We now switch to the original motivation for our work: the Kaloper-Westphal set-up [69]. This corresponds to the case where there are just two three-forms (so i, j runs from 1 to 2) and

$$\omega^{ij} = Z_{ij} = 0, \quad V = V_{QFT} + M_{pl}^2(\phi_1 + \phi_2), \quad \sigma_i = -2\phi_i. \tag{3.90}$$

Crucially, the membrane charges are assumed to have an irrational ratio, $q_1/q_2 = w \notin \mathbb{Q}$. The bulk constraints on the vacua (3.13) to (3.15) now give

$$3M_{pl}^2 k^2 = V_{QFT} + M_{pl}^2(\phi_1 + \phi_2) \tag{3.91}$$

$$M_{pl}^2 = 2c^k \tag{3.92}$$

$$\chi_i = -2\phi_i \tag{3.93}$$

from which it follows that

$$k^2 = \frac{V_{QFT} - \frac{M_{pl}^2}{2}(\chi_1 + \chi_2)}{3M_{pl}^2}. \tag{3.94}$$

Recall that the nucleation of the membrane of type i_* triggers a jump $\Delta\chi_i = -\delta_{ii_*}Q_{i_*}$, where $Q_{i_*} = \pm q_{i_*}$, consistent with the quantisation condition on $\chi_i = -N_i q_i$ (no sum) for some $N_i \in \mathbb{Z}$. The spectra of vacua is characterised by the curvature

$$k^2 = \frac{V_{QFT} + \frac{M_{pl}^2}{2}(N_1 q_1 + N_2 q_2)}{3M_{pl}^2}. \tag{3.95}$$

The irrational ratio in membrane charge ensures that this landscape of vacua is dense and passes arbitrarily close to the Minkowski vacuum. The nucleation of our membrane triggers a consistent jump $N_{i_*}^+ \to N_{i_*}^- = N_{i_*}^+ \pm 1$ leaving $N_{i\neq i_*}$ unchanged. This gives rise to a jump in the vacuum curvature

$$\Delta k^2 = \pm\frac{q_{i_*}}{6} \tag{3.96}$$

giving

$$X = \pm\frac{2M_{pl}^4 q_{i_*}}{3\tau_{i_*}^2}. \tag{3.97}$$

If we start in a parent de Sitter vacuum with curvature k_+^2, transitions to AdS are only possible for $k_+^2 < k_c^2 = q/6$, where $q = \max\{q_1, q_2\}$. In [69], the authors impose the condition,

$$\frac{2M_{pl}^4 q_i}{3\tau_i^2} < 1, \qquad i = 1, 2 \tag{3.98}$$

which is equivalent to $|X| < 1$ in all cases. In particular, as we have seen in the previous section, this type of condition ensures that those vacua vulnerable to decay into anti de Sitter can be long lived. More precisely, when $k_+^2 < k_c^2 = q/6$, we recall that the tunneling exponent $B \geq B_c$, where B_c is computed using (3.49),

$$B_c \sim \frac{6M_{pl}^2\Omega_3}{q}. \tag{3.99}$$

As long q is sub-Planckian, this remains large. Indeed, for $q_{\max} \sim 0.1M_{pl}^2$, the low scale de Sitter vacua can be made very long lived, with $B_c \gtrsim 1000$. Since $|X| < 1$, we also have absolute stability of the Minkowski vacuum thanks to

the presence of the pole in the tunneling exponent as $k_+ \to 0$. Note that unlike in the Bousso-Polchinski setup, the condition (3.98) is independent of $|V_{QFT}|$ or, in other words, the quantum mechanical stability of the Minkowski vacuum can be guaranteed whatever the depth of the bare AdS vacuum.

The Kaloper-Westphal model also avoids the empty universe problem. This is because we can bungee jump to the current vacuum from a parent de Sitter vacuum with curvature $k_+^2 \sim q/6$, where q is not constrained to be exponentially small in Planck units. Assuming sub-Planckian brane tension, we might worry that the one condition we do have (3.98) is at odds with the weak gravity conjecture for membranes (3.82). However, in the absence of kinetic terms for the three-forms in the Kaloper-Westphal set-up, it is not immediately clear how one should implement the weak gravity condition in the first place. Of course, the absence of the kinetic terms would itself present a challenge to string theory model builders.

3.6.4 More fun with irrationals

The key ingredients of the Kaloper-Westphal set-up can be realised in a simple model of a single axion coupled to a four-form. In particular, we consider an action of the form

$$S = \int_{\mathcal{M}} d^4x\sqrt{|g|}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + \int_{\mathcal{M}}\left[-\frac{1}{2}F\wedge\star F - \mu\phi F\right] + S_{\text{boundary}} + S_{\text{membranes}} \quad (3.100)$$

with $V(\phi) = V_{QFT} - m^4\cos(\phi/f)$. In terms of our generalised model described earlier, this corresponds to the case where $i, j = 1$ only and $\omega^{ij} = 1, Z_{ij} =$

$1, \sigma_i = -\mu\phi$. The bulk constraints (3.13) to (3.15) imply that

$$3M_{pl}^2 k^2 = V_{QFT} - m^4 \cos(\phi/f) + \frac{1}{2}c^2 \tag{3.101}$$

$$\frac{m^4}{f}\sin(\phi/f) = \mu c \tag{3.102}$$

$$\chi = -\mu\phi - c. \tag{3.103}$$

It follows that

$$k^2 = \frac{V_{QFT} - m^4\cos(\bar{\phi}(\chi)/f) + \frac{m^8}{2\mu^2 f^2}\sin^2(\bar{\phi}(\chi)/f)}{3M_{pl}^2}\,, \tag{3.104}$$

where $\bar{\phi}$ solves the equation

$$\chi = -\mu\bar{\phi} - \frac{m^4}{\mu f}\sin(\bar{\phi}/f)\,. \tag{3.105}$$

Nucleation of the membrane leads to a jump $\Delta\chi = -Q = -\eta q$, where q is the fundamental membrane charge and $\eta = +1$ for branes and $\eta = -1$ for anti-branes. This is consistent with the quantisation condition on $\chi = -Nq$ for some $N \in \mathbb{Z}$, with the membrane nucleation giving $\Delta N = \eta$. We can rewrite (3.105) as

$$N = \frac{\mu}{q}\bar{\phi}(N) + \frac{m^4}{q\mu f}\sin(\bar{\phi}(N)/f)\,, \tag{3.106}$$

with the vacuum spectrum characterised by curvature

$$k^2(N) = \frac{V_{QFT} - m^4\cos(\bar{\phi}(N)/f) + \frac{m^8}{2\mu^2 f^2}\sin^2(\bar{\phi}(N)/f)}{3M_{pl}^2}\,. \tag{3.107}$$

If the cosine potential were generated by non-perturbative corrections to the axion ϕ, and nothing else, we would expect the decay constant and therefore the periodicity of the potential, to be set by the fundamental membrane charge. This corresponds to the critical choice, $f_{\text{crit}} = q/2\pi\mu$. The problem is that in such a set-up, the vacuum curvature (3.107) is single valued, making it of no use in addressing the cosmological constant problem. To see this note that

if $f = f_{\text{crit}}$, then (3.105) is invariant under $N \to N + n, \bar{\phi} \to \bar{\phi} + 2\pi n f_{\text{crit}}$. It follows that nucleation of a membrane simply triggers a shift in $\bar{\phi}$ of size $\pm 2\pi f_{\text{crit}}$, leaving the curvature unchanged.

Inspired by the Kaloper-Westphal model, we consider instead the case where $f = w f_{\text{crit}}$ for some $w \notin \mathbb{Q}$. This could have arisen from integrating out the heavier of two sectors in a doubled system with an irrational ratio of membrane charges, reinforcing the connection to Kaloper and Westphal's original set-up[8]. The curvature spectrum is now dense over a finite range of values. To see this, we assume a rational approximation for $w \approx p_1/p_2$, accurate to order ϵ,

$$w = \frac{p_1}{p_2}(1 + \epsilon) \tag{3.108}$$

for coprimes p_1 and p_2. If $\phi(N)$ solves (3.106), then we can show that

$$\phi(N + p_1) \approx \bar{\phi}(N) + 2\pi f p_2 - \frac{2\pi f p_1 \epsilon}{\frac{2\pi m^4}{q\mu f}\cos(\bar{\phi}(N)/f) + w}\,. \tag{3.109}$$

We now compare the vacuum curvature for the vacuum with flux number N and that with number $N + p_1$,

$$\delta k^2 = k^2(N+p_1) - k^2(N) \approx -\left[\frac{2\pi p_1 m^4 \sin(\bar{\phi}(N)/f)(1 + \frac{m^4}{\mu^2 f^2}\cos(\bar{\phi}(N)/f))}{3M_{pl}^2\left(\frac{2\pi m^4}{q\mu f}\cos(\bar{\phi}(N)/f) + w\right)}\right]\epsilon. \tag{3.110}$$

Since we can make ϵ arbitrarily small but non-zero with better and better choices of p_1 and p_2, it is clear that the distance between vacua can be made arbitrarily small and that the spectrum is dense. It turns out that the curvature lies in a range $k^2 \in [k^2_{\text{min}}, k^2_{\text{max}}]$, where

$$k^2_{\text{min}} = \frac{1}{3M_{pl}^2}\left(V_{QFT} - m^4\right) \tag{3.111}$$

[8]Here we outline how quantized charges and irrationals could come from string theory. Charge quantization appears naturally from the quantization of motion in extra dimensions, such as Kaluza-Klein mechanism (compact extra dimensions). In addition, charges carried by strings and branes are quantized because they are sourced by topologically quantized fluxes. Irrational numbers in string theory can be from geometry of compact space, moduli fields, integration over manifolds, dualities, etc [18, 36, 45].

and

$$k^2_{\rm max} = \frac{1}{3M_{pl}^2}\left[V_{QFT} + m^4 + \frac{1}{2}m^4\left(\frac{m^2}{\mu f} - \frac{\mu f}{m^2}\right)^2 \theta\left(\frac{m^2}{\mu f} - 1\right)\right] \tag{3.112}$$

where

$$\theta\left(\frac{m^2}{\mu f} - 1\right) = \begin{cases} 1 & m^2 \geq \mu f \\ 0 & m^2 < \mu f \end{cases}. \tag{3.113}$$

We now require that V_{QFT}, μf and m^4 are such that the range of curvature passes from large and positive, $k^2_{\rm max} \gg H_0^2 > 0$, to large and negative values, $k^2_{\rm min} \ll -H_0^2 < 0$, with respect to the current Hubble scale. Once this is guaranteed, the density of the spectrum ensures that vacua exist whose curvature are sufficiently close to observations.

Requiring the curvature of vacua to scan either side of the observed value directly constrains the size of the QFT contribution to the vacuum energy. In particular, we find that

$$3M_{pl}^2H_0^2 - m^4 - \frac{1}{2}m^4\left(\frac{m^2}{\mu f} - \frac{\mu f}{m^2}\right)^2\theta \ll V_{QFT} \ll -3M_{pl}^2H_0^2 + m^4. \tag{3.114}$$

Assuming $m^4 \gg M_{pl}^2H_0^2$, we can satisfy this just by imposing an upper bound on the scale $|V_{QFT}| \lesssim m^4$. Typically we expect $m^4 \sim \Lambda_{\rm UV}^4 e^{-S}$, where $\Lambda_{\rm UV}$ is some UV scale[9] and $S > 1$ is the action for the instanton that generates the potential. Although we cannot extend the scale of the underlying vacuum energy to arbitrarily large values in this model, we can easily accommodate scales just a few orders of magnitude below the Planck scale.

As ever, nucleation of a membrane triggers a jump in one of the flux integers, $N^+ \to N^- = N^+ \pm 1$, translating into a jump in vacuum curvature

$$\Delta k^2 = -\frac{m^4\Delta[\cos(\bar{\phi}/f)]}{3M_{pl}^2}\left[1 + \frac{m^4}{\mu^2 f^2}\langle\cos(\bar{\phi}/f\rangle)\right] \tag{3.115}$$

[9]In QCD, this is set by the QCD scale, or more precisely, by the scale of the quark condensates and the quark masses. In string compactifications it is typically a few orders of magnitude below the Planck scale, being suppressed by the volume of the Calabi-Yau.

where

$$\Delta[\cos(\bar{\phi}/f)] = \cos(\bar{\phi}(N^+)/f) - \cos(\bar{\phi}(N^+ \pm 1)/f) \tag{3.116}$$

and

$$\langle \cos(\bar{\phi}/f) \rangle = \frac{1}{2}\left[\cos(\bar{\phi}(N^+)/f) + \cos(\bar{\phi}(N^+ \pm 1)/f)\right] . \tag{3.117}$$

This depends implicitly on where we are in the landscape through the dependence on N^+. Descent from parent de Sitter vacua with curvature $k_+^2 > 0$ is characterised by the value of $X = 4M_{pl}^4 \Delta k^2/T^2 > 0$. If τ is the fundamental membrane tension, we can use the fact that $\Delta[\cos(\bar{\phi}/f)] \geq -2$ and $\langle \cos(\bar{\phi}/f) \rangle \leq 1$ to show that

$$X \leq \frac{8M_{pl}^2 m^4}{3\tau^2}\left[1 + \frac{m^4}{\mu^2 f^2}\right] = X_{\max} . \tag{3.118}$$

As in our analysis for the Bousso-Polchinski set-up, dangerous transitions into AdS are kinematically forbidden if $X_{\max} < X_0(k_+) = 4M_{pl}^2 k_+^2/\tau^2$. This introduces a critical curvature $k_c^2 = \frac{2}{3}\frac{m^4}{M_{pl}^2}\left[1 + \frac{m^4}{\mu^2 f^2}\right]$ beyond which we cannnot descened into AdS through a single membrane nucleation. For $k_+^2 \lesssim k_c^2$, descent into AdS may be possible. However, if we impose the modest constraint

$$\frac{8M_{pl}^2 m^4}{3\tau^2}\left[1 + \frac{m^4}{\mu^2 f^2}\right] < 1 \tag{3.119}$$

we are guaranteed to have $0 < X < 1$ for all downward transitions. This ensures that the lifetime of these vacua is determined by a tunneling exponent $B \geq B_c$ where

$$B_c = \frac{3M_{pl}^4 \Omega_3}{2m^4}\left[1 + \frac{m^4}{\mu^2 f^2}\right]^{-1} . \tag{3.120}$$

By choosing parameters so that this is large we can ensure that the metastable vacua are very long-lived. Furthermore, the condition (3.119) also ensures that $|X_{M_+ \to AdS_-}| < 1$. As we saw previously, this guarantees the presence of a pole in the transition rate as the parent vacuum approaches Minkowski, rendering

it arbitrarily long-lived.

Interestingly, this set-up seems to avoid any issues with the membrane weak gravity conjecture. To see this, note that $f = wf_{\rm crit} = \frac{wq}{2\pi\mu}$, and so we can rewrite the condition (3.119) as

$$\frac{8M_{pl}^2 m^4}{3\tau^2}\left[1+\frac{4\pi^2 m^4}{w^2 q^2}\right] < 1 \tag{3.121}$$

which does not indicate any violation of (3.82). The last thing we need to do is to check that we can indeed transition to a near Minkowski vacuum in a single bungee jump and avoid the empty universe problem. Using equation (3.107), we see that the flux integer for the near Minkowski vacuum satisfies

$$\cos\left[\bar{\phi}\left(N^{\rm Mink}\right)/f\right] = -\frac{\mu^2 f^2}{m^4}\left(1 \pm \upsilon\right) \tag{3.122}$$

where

$$\upsilon = \frac{1}{\mu f}\sqrt{2\left(V_{QFT}+\frac{1}{2}\mu^2 f^2 + \frac{1}{2}\frac{m^8}{\mu^2 f^2}\right)} \geq \sqrt{6}\frac{M_{pl}k_{\rm max}}{\mu f} \tag{3.123}$$

where in the inequality we have used the formulae for $k_{\rm max}$ for both $\mu f \leq m^2$ and $\mu f > m^2$ given in equation (3.112). If Minkowski is the daughter vacuum, it is generated by membrane nucleation in a parent de Sitter vacuum of curvature

$$k_+^2 = \frac{m^4}{3M_{pl}^2}|\Delta[\cos(\bar{\phi}/f)]|\left(\upsilon - \frac{m^4}{2\mu^2 f^2}|\Delta[\cos(\bar{\phi}/f)]|\right) \tag{3.124}$$

where now

$$\Delta[\cos(\bar{\phi}/f)] = \cos(\bar{\phi}(N^{\rm Mink} \mp 1)/f) - \cos(\bar{\phi}(N^{\rm Mink})/f)\ . \tag{3.125}$$

Let us now assume that $\upsilon \gg m^4/M_{pl}\mu f > H_0$ and $\omega = f/f_{\rm crit} > 1$. As we will show, these are a set of sufficient conditions that allow us to avoid the empty

universe problem. First up, note that $\omega > 1$ implies $|\Delta[\cos(\bar{\phi}/f)]| \sim \mathcal{O}(1)$[10]. For $\upsilon \gg \frac{m^4}{\mu^2 f^2}$ and $|\Delta[\cos(\bar{\phi}/f)]| \sim \mathcal{O}(1)$ the curvature of the parent vacuum is bounded from below as follows

$$k_+^2 \sim \frac{m^4 \upsilon}{3M_{pl}^2} \geq \frac{m^4 k_{\max}}{M_{pl}\mu f}\,, \tag{3.127}$$

where in the second inequality we have neglected $\mathcal{O}(1)$ factors and used equation (3.123). As mentioned before, the absence of finely tuned cancellations between the various contributions to the curvature requires $k_{\max} \gg H_0$. If we further assume that the axion parameters satisfy the inequality $m^4/M_{pl}\mu f > H_0$, as stated above, we find that

$$k_+^2 \gg \frac{m^4 H_0}{M_{pl}\mu f} \gg H_0^2. \tag{3.128}$$

Thus, the curvature of the parent vacuum vastly exceeds that of its offspring, the current vacuum. It follows that we can bungee jump from a high scale vacuum to the current vacuum with a single bubble nucleation and the empty universe problem is avoided.

We conclude that this model of a single four-form scalar pair addresses the cosmological constant problem in much the same way as the Kaloper-Westphal model. However, there are some differences. For example, in the Kaloper-Westphal model, the quantum mechanical stability of Minkowski is independent of $|V_{QFT}|$, whereas here we find it can only be achieved for $|V_{QFT}| < m^4$. This is not a significant limitation since we can easily imagine m lying just below the Planck scale. This new model also contains certain features that are appealing from the perspective of fundamental theory, including a stan-

[10] To show that $|\Delta[\cos(\bar{\phi}/f)]| \sim \mathcal{O}(1)$ for $\omega > 1$, consider the case where $|\Delta[\cos(\bar{\phi}/f)]| \ll 1$. It follows that $\bar{\phi}(N^{\text{Mink}} \mp 1) \approx \sigma\bar{\phi}(N^{\text{Mink}}) + 2n\pi f$ for some $n \in \mathbb{Z}$ and $\sigma = \pm 1$, which is consistent with the condition on the flux integers (3.106) if and only if

$$\mp 1 \approx n\omega + (\sigma - 1)N^{\text{Mink}} \tag{3.126}$$

For $\sigma = 1$, this implies $\omega \approx 1/|n|$, contradicting the requirement that $\omega > 1$. For $\sigma = -1$, the condition (3.126) depends explicitly on N^{Mink} and is destabilised by radiative corrections to V_{QFT}. We therefore ignore this possibility.

dard kinetic term for the four-form without any violation of the weak gravity conjecture for membranes.

Of course, the form of the axion potential may be further constrained by the *axion* weak gravity conjecture [94]. Naively, this would correspond to the condition $fS < M_{pl}$ which is not especially restrictive here. However, we think a more careful analysis in the context of a particular UV completion of the model would be worthwhile.

Finally, it is worth noting that besides the vacuum transitions mediated by the nucleation of fundamental membranes charged under the four-form, the present model also allows for transitions between local minima while holding the four-forms fixed. These correspond to the well known Coleman-de Luccia instantons [82], where local vacua are separated by domain walls, and provide an extra channel for the relaxation of the cosmological constant.

3.7 Conclusions

Within the standard framework of quantum field theory, radiative corrections to the vacuum energy are large, scaling like the fourth power of the cut-off. If we assume naturalness, this scale should determine the overall size of the renormalized vacuum energy. Corrections to the cosmological constant from other sectors are now required to bring this result in line with observations, bringing the corresponding vacuum curvature down to the present Hubble scale. In an effective four-dimensional description of the universe in which gravity is coupled to four-form field strengths, four-form flux provides a correction to the cosmological constant. Nucleation of membranes charged under the corresponding three-form potential, then allows the flux to jump by a single quantised unit, changing the value of the cosmological constant. This yields a landscape of possible vacua labelled by the quantised flux, scanned through quantum tunneling. To make contact with observation the landscape should be sufficiently dense, with the curvature of vacua separated by no more than

the current Hubble scale. Furthermore, to avoid becoming lost in a cold and empty universe, we must be able to reach the current vacuum from a high scale de Sitter vacuum in a single nucleation event.

In Bousso and Polchinski's seminal paper [43], there are a large number of four-forms, realising the required density of the landscape without running into the empty universe problem. However, this is not enough to address the cosmological constant problem. We also need to ask how the observed vacuum is selected from the landscape of possibilities. Bousso and Polchinski argue that the current vacuum should be selected using anthropic arguments: if the vacuum curvature were too large and positive, structure could not have formed due to the rapid exponential expansion; if it were too large and negative, the universe would have started contracting before stars, planets and complex life had ever come into existence. Anthropic arguments have also been used in the context of the electroweak hierarchy problem where, like in the cosmological constant case, naturalness is under pressure. For example, a coupling between the Higgs-squared and one [95, 96] or more four-forms [97] yields a landscape of electroweak vacua, from which the current vacuum is selected through anthropics.

In the recent history of string cosmology, anthropic ideas have indubitably become the dominant explanation for the small value of the cosmological constant seen in nature. We now seek to challenge that assertion, showing how the current vacuum can often be selected on *probabilistic* grounds for a wide class of string-like models of four-forms coupled to scalar fields, including Bousso and Polchinski's original set-up. With this new perspective, our world exists independently of mankind or some other complex species. It is born from probability.

The main idea was inspired by the recent proposal of Kaloper [67] and Kaloper and Westphal [69], although we have shown how it can be readily extended. In the wide class of models under consideration, we have identified a parameter that controls the lifetime of low scale vacua, similar to the current

vacuum we observe. If this parameter can be suitably bounded, a pole appears in the corresponding bounce actions as the curvature of the parent vacuum goes to zero. This guarantees the quantum stability of the Minkowski vacuum, and the longevity of those vacua close to Minkowski in Planck units. In the Kaloper-Westphal model, this bound corresponds to an absolute bound on the membrane charge in terms of its tension. In the Bousso-Polchinski model, the bound is also sensitive to the scale of the renormalised vacuum energy. However, the sensitivity is not too severe: one can easily accommodate a Planckian energy density for the QFT vacuum, with the membrane tension and charge lying just an order of magnitude or two below the Planck scale. That said, in both the Kaloper-Westphal and the Bousso-Polchinski set-ups, these probabilistic arguments seem to require a mild violation of the membrane weak gravity conjecture. With this in mind, we have presented a new model where the same probabilistic methods are used but where there is no such violation.

This new approach offers some hope for addressing the cosmological constant problem using probabilistic methods. Notwithstanding some concerns associated with various swampland considerations, this could be relevant to a wide class of four-dimensional effective field theories whose underlying structures are compatible with string compactifications.

Chapter 4

The cosmological constant and the weak gravity conjecture

In chapter 4, we examine the descent via membrane nucleation through a landscape of vacua where the cosmological constant is given by a combination of four-form fluxes. It has been shown in chapter 3 that this descent can be slowed exponentially for very low curvature vacua close to Minkowski space in a wide class of models satisfying certain parametric conditions, providing a possible solution to the cosmological constant problem. We explore in detail whether or not those parametric conditions are compatible with the membrane weak gravity conjecture. Whilst it is true that there is often tension, we show that this is not always the case and present an explicit model where Minkowski space is absolutely stable and the weak gravity conjecture is satisfied. This corresponds to an extension of the Bousso-Polchinski model into a generalised DBI action for four-forms. We also clarify how the landscape should be populated in a consistent model.

4.1 Introduction

In a universe with a landscape of different vacua, as one might expect to occur in string theory, we can imagine a scenario in which the cosmological constant starts out large but descends to smaller values via some process, until it reaches the small value we see today. Now we briefly summarize the background we used in chapter 3.

A mechanism for halting the descent is also presented in [67–69]. Indeed, in [69], the authors consider a generalisation of Henneaux and Teitelboim's covariant formulation of unimodular gravity [87] with two species of four-form. Transitions between vacua go through via membrane nucleation. By making some relatively mild assumptions on the membrane charge and tensions, the nucleation rate slows down exponentially quickly, halting the decay of the vacuum at vanishing curvature.

In chapter 3, this mechanism was shown to extend to a much larger class of models, including the classic framework of Bousso and Polchinski. This

meant there was a way to halt the descent between vacua at vanishingly low curvature in a wide class of effective field theories, providing an interesting alternative to anthropic selection. However, in this work it was also noted that the corresponding condition on the membrane charges and tensions was at odds with the membrane version of the weak gravity conjecture (WGC) [85, 86], especially in the case of the Bousso-Polchinski model.

Therefore in chapter 4, we will further explore the dynamics of this wide class of models, paying close attention to descent between vacua and the role played by the WGC. A related analysis was recently performed in [98], although our more general approach allows us to clarify some important points. In particular, there is certainly a tension between the WGC and the halting conditions for a canonical theory quadratic in the flux, as already pointed out in chapter 3. However, if a higher power of the flux dominates the dynamics, these tensions can be eased by assuming a hierarchy between the renormalised vacuum energy and the cut-off scale, as one could easily obtain in the presence of supersymmetry. Of course, the dominance of a single higher power is not consistent with a well defined effective theory at weak coupling. This motivates us to study the dynamics at strong coupling, taking into account a full tower of operators. We first consider the generic case using naive dimensional analysis [99, 100] and once again find tension between the WGC and the halting conditions. However, we also explore an elegant extension of the Bousso-Polchinski model based on a generalised form of the DBI action [101, 102]. This has the property that it behaves like a Bousso-Polchinski set-up at smaller values of the four-form flux, although at large values it behaves more like the linear four-form proposal presented in [69]. As we will show explicitly, this generalised model can be taken to be compatible with the WGC *without* spoiling the halting of vacuum descent at very low curvatures.

Our analysis of the dynamics also helps us to better understand how very low curvature vacua are populated. In particular, following [98], we might consider two distinct phases in the membrane nucleation rate: the boiling phase, where

the rate is high (in other words, the tunneling bounce $B < 1$) and the braking phase, where the rate is low (in other words, the tunneling bounce $B > 1$). Having arrived at a very low curvature vacuum with further descent almost halted, we can ask how we got here. Did the universe start out in a high curvature vacuum and boil rapidly down to the current low curvature vacuum? Or did we reach the current vacuum having already entered the braking phase before the last transition? Under a very mild set of assumptions, we will show that it must be the latter. This now suggests the following set-up: the universe starts out in a high curvature vacuum and boils down to a bubbling soup of vacua of intermediate curvature. Within those bubbles of intermediate curvature we are now in the braking phase and future transitions are slow. Nevertheless, given enough time, we do reach the low curvature vacua like ours, which are even more stable than their ancestors. These low curvature vacua are by far the longest lived. The rest of this chapter is organised as follows.

In section 4.2, we study specific models, including those for which the cosmological constant is a homogeneous polynomial of the four-form flux and the generalised DBI set-up, with a view to understanding the constraints imposed by both the WGC and the halting condition. In section 4.3 we consider the dynamics of vacuum decay in a general setting focussing on parametric behaviour of the rate of decay. In section 4.4, we discuss our findings and summarize some remaining concerns.

4.2 Implications of WGC on specific models

We still begin with a generalised theory of four-forms on a manifold, $\mathcal{M}$, with a dynamical metric $g_{\mu\nu}$ described by the following action,

$$S = \int_{\mathcal{M}} d^4x \sqrt{|g|} \left[\frac{M_{pl}^2}{2} R + f(\star F_i) \right] + S_{\text{boundary}} + S_{\text{membranes}}, \tag{4.1}$$

where we have a family of three-form fields, $A^i = \frac{1}{3!}A^i_{\mu\nu\alpha}dx^\mu \wedge dx^\nu \wedge dx^\alpha$, with corresponding field strength $F^i = dA^i$ and $\star$ denotes the Hodge star operator on the manifold. R is the Ricci scalar on the manifold while the function f is assumed to admit an expansion of the form

$$f(\star F_i) = \sum_{n\geq 2} \frac{a_{i_1\dots i_n}}{n! M_{\rm UV}^{2n-4}} (\star F^{i_1}) \dots (\star F^{i_n}), \tag{4.2}$$

where $M_{\rm UV} \lesssim M_{pl}$ is the cut-off of the EFT and the dimensionless coefficients $a_{i_1\dots i_n}$ are naturally $\mathcal{O}(1)$.

For the ith membrane species to be part of the effective field theory, we require

$$q_i \lesssim M_{\rm UV}^2, \qquad \tau_i \lesssim M_{\rm UV}^3. \tag{4.3}$$

Furthermore, there ought to be at least one species for which we also satisfy the constraints coming from the WGC [85, 86]

$$\tau_i < M_{pl} q_i \qquad \text{electric WGC} \tag{4.4}$$

$$M_{\rm UV}^3 < q_i M_{pl} \qquad \text{magnetic WGC .} \tag{4.5}$$

To make contact with chapter 3, we switch to a dual formulation, introducing a Lagrange multlplier ϕ_i fixing $F^i = dA^i$ on shell, via a term $\int \phi_i (F^i - dA^i)$. Integrating out the scalars, $\star F_i$, yields

$$S = \int_{\mathcal{M}} d^4x \sqrt{|g|} \left[\frac{M_{pl}^2}{2} R - L_f(\phi_i) \right] - \int \phi_i dA^i + S_{\rm boundary} + S_{\rm membranes}, \tag{4.6}$$

where L_f is the Legendre transform of f. This action now falls into the general class studied in chapter 3, with $Z_{ij} = \omega^{ij} = 0$, $\sigma_i = -\phi_i$ and $V(\phi) = L_f(\phi)$.

Let us now explore the impact of the WGC on the set-up described in the previous section. To this end, it will be convenient to express the spacetime curvature in terms of the dimensionless flux $Q_i = N_i \delta_i$ for $\delta_i = q_i / M_{\rm UV}^2$, via an

expression of the form

$$3M_{pl}^2 k^2 = V_{\rm QFT} + M_{\rm UV}^4 \kappa(Q), \tag{4.7}$$

where $\kappa(Q) = L_f(M_{\rm UV}^2 Q)/M_{\rm UV}^4$ is a dimensionless function of the flux. Note that if the membranes of type i are inside the EFT, we have $\delta_i < 1$, as per equation (4.3). Now suppose we nucleate such a membrane, so that $N_i \to N_i \pm 1$ and $N_j \to N_j$ for $j \neq i$. The process induces a corresponding change in k^2,

$$\Delta k^2 = \frac{M_{\rm UV}^4}{3M_{pl}^2} \Delta\kappa_i \tag{4.8}$$

where we define

$$\begin{aligned} \Delta\kappa_i &\equiv \kappa(\ldots, Q_{i-1}, Q_i, Q_{i+1}, \ldots) - \kappa(\ldots, Q_{i-1}, Q_i \pm \delta_i, Q_{i+1}, \ldots) \\ &= -\sum_{n=1}^{\infty} \frac{(\pm\delta_i)^n}{n!} \frac{\partial^n}{\partial Q_i^n} \kappa(Q) \ , \end{aligned} \tag{4.9}$$

and in the last equality we Taylor expanded $\kappa(\ldots, Q_{i-1}, Q_i \pm \delta_i, Q_{i+1}, \ldots)$. As we described in chapter 3, the nucleation rate is controlled by the value of a parameter

$$X_i = \frac{4M_{pl}^4 \Delta k^2}{\tau_i^2} = \frac{4M_{pl}^2 M_{\rm UV}^4}{3\tau_i^2} \Delta\kappa_i \ . \tag{4.10}$$

For a parent vacuum with $k_+^2 \approx 0$, we must choose the flux such that $\kappa \approx -\frac{V_{\rm QFT}}{M_{\rm UV}^4}$. If we further assume that $|N_i| \gg 1$ when $k^2 \approx 0$, it follows that $|Q_i| \gg \delta_i$ where we can truncate the Taylor expansion to leading order in (4.9) to show that

$$\left| X_i^{M_+ \to AdS_-} \right| \approx \frac{4}{3} \left(\frac{q_i M_{pl}}{\tau_i} \right)^2 \frac{M_{\rm UV}^2}{q_i} \left| \frac{\partial\kappa}{\partial Q_i} \right|_{k^2=0} \ . \tag{4.11}$$

For membrane nucleation to be halted at very low curvature, we require $\left|X_i^{M_+\to AdS_-}\right| < 1$, or equivalently

$$\left|\frac{\partial\kappa}{\partial Q_i}\right|_{k^2=0} < \frac{3}{4}\left(\frac{\tau_i}{q_i M_{pl}}\right)^2 \frac{q_i}{M_{\rm UV}^2} \tag{4.12}$$

for *all* species. If the membranes of type i are part of the EFT (4.3) and satisfy the WGC (4.4), it is clear that the corresponding first partial derivative of κ should not be too large near the $k^2 \approx 0$ vacuum. We will now explore the implications of this for several models.

4.2.1 Homogeneous polynomial models

In the Kaloper-Westphal (KW) model [69], the cosmological constant is linear in the flux, $\kappa = a^i Q_i$. This represents a multi-field generalisation of Hennueaux and Teitelboim's covariant formulation of unimodular gravity (with branes). It cannot be obtained from a generic action of the form (4.1), since the Legendre transform cannot be inverted in the linear case. Even so, it is instructive to explore how the WGC may be avoided even when the halting condition (4.12) holds for near Minkowski vacua. In the KW model, the q_i have irrational ratios, so we can always find a solution to $k^2 \approx 0$ to any desired level of accuracy as long as we choose sufficiently large $|N_i|$. Descent between vacua is halted at very low curvatures provided

$$|a^i| < \frac{3}{4}\left(\frac{\tau_i}{q_i M_{pl}}\right)^2 \frac{q_i}{M_{\rm UV}^2} \tag{4.13}$$

for all values of i. Clearly, if there are any species satisfying WGC (4.4) along with the EFT constraint (4.3), the corresponding value of a^i should be tuned to be suitably small.

When the cosmological constant is quadratic in flux as in the Bousso-Polchinski model difficulties can arise, as was first noted in [1] and emphasized in [98]. To see this note that $\kappa(Q) = \frac{1}{2}\sum_i Q_i^2$ where the coefficients are fixed by canonical

normalisation $a_{ij} = \delta_{ij}$. The halting condition (4.12) now requires

$$|Q_i| < \frac{3}{4}\left(\frac{\tau_i}{q_i M_{pl}}\right)^2 \frac{q_i}{M_{\rm UV}^2} \implies |N_i| < \frac{3}{4}\left(\frac{\tau_i}{q_i M_{pl}}\right)^2 \tag{4.14}$$

for all values of i. The flux numbers are expected to be large in a neighbourhood of the Minkowski vacuum in order to achieve the necessary cancellation of the presumably large bare vacuum energy. This is clearly incompatible with any species satisfying the WGC (4.4).

What is happening here? The point is that in order to halt the descent between vacua at very low curvatures the flux dependent function $\kappa(Q)$ is required to be flat along any direction satisfying the WGC, as per the inequality (4.12). In the case of the Bousso Polchinski model, the function $\kappa(Q)$ is just a sum of squares, being flatter near their minimum at $Q_i = 0$. However, in the generic case where $V_{\rm QFT} \gg M_{pl}^2 H_0^2$, this is not a low curvature vacuum.

We can generalise this result to the case where $\kappa(Q)$ is a homogeneous function of any degree $n > 1$. Indeed, in this case, the Euler identity implies that $\sum_i Q_i \frac{\partial \kappa}{\partial Q_i} = n\kappa$, from which we infer

$$\frac{\partial \kappa}{\partial Q_i} = \frac{1}{n-1}\sum_j Q_j \frac{\partial^2 \kappa}{\partial Q_i \partial Q_j}. \tag{4.15}$$

Plugging this result back into the inequality (4.12), we see that the halting condition requires

$$\left|\sum_j N_j \frac{q_j}{q_i}\frac{\partial^2 \kappa}{\partial Q_i \partial Q_j}\right|_{k^2=0} < \frac{3}{4}\left(\frac{\tau_i}{q_i M_{pl}}\right)^2 (n-1) \tag{4.16}$$

for all values of i. Recall that the $|N_i|$ are generically expected to be large on the Minkowski vacuum. Assuming the absence of any charge hierarchies $q_i \sim \mathcal{O}(q)$, there are two ways in which the inequality (4.16) might be satisfied alongside the WGC (4.4): either there are miraculous cancellations between terms or certain components of the Hessian turn out to be extremely small.

Note that the inequality (4.16) must hold even if there are order one corrections to the underlying vacuum energy. In a generic scenario, this is likely to spoil any miraculous cancellations, so we are led to assume fine tuning of the Hessian. Indeed, if we suppose that the first species with $i = 1$ satisfies the WGC, we must have that

$$\left| \frac{\partial^2 \kappa}{\partial Q_1 \partial Q_j} \right|_{k^2=0} \ll \frac{3}{4}(n-1) \tag{4.17}$$

for all j. If more species satisfy the WGC, then more components of the Hessian will need to be tuned.

For a quadratic model with $n = 2$, this constraint on the Hessian requires the corresponding couplings to be unnaturally small. This is consistent with our previous result for the Bousso-Polchinski model where we saw that the halting condition for low curvature vacua was incompatible with the weak gravity conjecture [1]. However, for higher powers, $n > 2$, the Hessian can be rendered small by taking the Q_i to be small in the corresponding limit. For a suitable choice of membranes, this may still be possible even for large N_i.

To see this explicitly, let us consider the case where κ is dominated by a sum of monomials of degree n,

$$\kappa(Q) = \sum_i \frac{c_n^i}{n!} Q_i^n \tag{4.18}$$

where $n \geq 1$ and c_n^i are constants, assumed to be order one in a natural scenario. Recall that the nucleation of the membrane of type i triggers a change in flux $N_i \to N_i \pm 1$ and $N_{j\neq i} \to N_{j\neq i}$. The corresponding value of X_i controlling the rate of the transition is given by

$$|X_i| \approx \frac{4}{3} n \left(\frac{q_i M_{pl}}{\tau_i} \right)^2 \frac{M_{\rm UV}^2}{q_i} \frac{|c_n^i|}{n!} |Q_i|^{n-1} \tag{4.19}$$

where we have assumed that the flux number is large $|N_i| > 1$. In the absence of any hierarchies, $q_i \sim \mathcal{O}(q)$, $\tau_i \sim \mathcal{O}(\tau)$ and $c_n^i \sim \mathcal{O}(c_n)$. Near the Minkowski vacuum, we have $\kappa \approx -\frac{V_{\rm QFT}}{M_{\rm UV}^4}$, and since we expect each $|Q_i| \sim |n!\kappa/c_n|^{\frac{1}{n}}$ we

infer that

$$|N_i| \sim \frac{M_{\rm UV}^2}{q} \left| \frac{V_{\rm QFT} n!}{M_{\rm UV}^4 c_n} \right|^{\frac{1}{n}} \tag{4.20}$$

and

$$\left| X_i^{M_+ \to AdS_-} \right| \sim \frac{4}{3} \left(\frac{q M_{pl}}{\tau} \right)^2 \frac{M_{\rm UV}^2}{q} \left| \frac{V_{\rm QFT}}{M_{\rm UV}^4} \right|^{1-\frac{1}{n}} \left(\frac{n^n}{n!} \right)^{\frac{1}{n}} |c_n|^{\frac{1}{n}} \ . \tag{4.21}$$

Recall that generically we expect $|N_i| > 1$. For the descent between vacua to halt at very low curvatures, we also require $\left| X_i^{M_+ \to AdS_-} \right| < 1$ for each species. From equation (4.21), we see that this is only possible if there are one or more of the following:

- a violation of the WGC, $\frac{q M_{pl}}{\tau} < 1$
- the membrane charges are no longer consistent with the EFT, $\frac{M_{\rm UV}^2}{q} < 1$
- suppression of the couplings, $|c_n| < 1$
- suppression of the bare cosmological constant: $\frac{|V_{QFT}|}{M_{UV}^4} < 1$

None of these scenarios are ideal but perhaps the least worrying is the suppression of the bare cosmological constant. Indeed, if the scale of supersymmetry breaking lies many orders of magnitude below the cut-off, this is exactly what we would expect to find.

Note that we can recast the large flux condition $|N_i| > 1$ and the halting condition, $\left| X_i^{M_+ \to AdS_-} \right| < 1$, as the following constraint on the couplings

$$|c_n| < \min\{\lambda_1, \lambda_2\} \tag{4.22}$$

where

$$\lambda_1 = \left(\frac{M_{\rm UV}^2}{q} \right)^n \frac{|V_{\rm QFT}|}{M_{\rm UV}^4} n!, \qquad \lambda_2 = \left(\frac{3}{4} \left(\frac{\tau}{q M_{pl}} \right)^2 \frac{q}{M_{\rm UV}^2} \right)^n \left(\frac{|V_{\rm QFT}|}{M_{\rm UV}^4} \right)^{1-n} \frac{n!}{n^n} \tag{4.23}$$

If we now demand that both $\lambda_1 > 1$ and $\lambda_2 > 1$, as required to avoid fine

tuning of the couplings, we find that

$$\frac{3}{4}\left(\frac{\tau}{qM_{pl}}\right)^2 > \frac{1}{(n-1)!}\left(\frac{q}{M_{\rm UV}^2}\right)^{n-2} \tag{4.24}$$

For $n = 2$, it is clear that this is not consistent with the weak gravity conjecture, as we have already seen. However, for $n > 2$, we can be more optimistic. Indeed, as an example, consider the case where $n = 10$, $|V_{\rm QFT}| \sim 10^{-6}M_{pl}^4$, $q \sim 10^{-2}M_{pl}^2$, $\tau \sim 10^{-3}M_{pl}^3$, $|c| \sim 1$ and $M_{\rm UV} \sim M_{pl}$, so that

$$|N_i| \sim 114, \qquad \left|X_i^{M_+ \to AdS_-}\right| \sim 0.1 \tag{4.25}$$

In this scenario, the membrane charges and tensions are chosen to be small but are consistent with the EFT constraint (4.3) and the WGC (4.4). Similarly, the bare cosmological constant is many orders of magnitude below the cut-off, as one might expect in a supersymmetric scenario. Nevertheless, for order one couplings in $\kappa(Q)$, it seems we can have scenarios in which membrane nucleation is halted at very low curvatures, without any violation of our EFT constraints or the WGC.

Of course, in a consistent weakly coupled scenario we would not expect higher order operators to dominate over the quadratic terms. Given that we have already seen that canonical quadratic terms are incompatible with the halting condition if the WGC and EFT constraints are satisfied, it follows that we can only exploit the higher order operators by going to strong coupling. Further, in such a scenario, we would expect a whole tower of higher order operators to be present beyond the homogeneous polynomials considered here. What effect do they have on the interplay between the WGC and the longevity of low curvature vacua? We shall now explore this question in two different ways: first, with a generic tower of EFT operators whose coefficients are set using naive dimensional analysis (NDA) [99, 100]; and second, in the special case of a generalised DBI action for four-forms, where the couplings are motivated

by the dynamics of a spacetime filling 3-brane [101, 102].

4.2.2 A generic tower of higher order operators

Consider a generic tower of EFT operators whose coefficients are set using naive dimensional analysis [99, 100]. This allows us to be a little more precise about the form of the dimensionless coefficients in the dual potential in equation (4.6). In particular, we expect that [103, 104]

$$L_f(\phi) = \sum_{n\geq 2} \frac{b^{i_1\ldots i_n}}{(M_{\rm UV}^2/4\pi)^{n-2} n!} \phi_{i_1} \ldots \phi_{i_n} \tag{4.26}$$

where $b^{ij} = \delta^{ij}$ and the $b^{i_1\ldots i_n} \sim \mathcal{O}(1)$. In terms of the dimensionless potential, this gives

$$\kappa(Q) = \sum_{n\geq 2} \frac{b^{i_1\ldots i_n}}{n!} (4\pi)^{n-2} Q_{i_1} \ldots Q_{i_n} \tag{4.27}$$

In the weakly coupled regime, $|Q_i| < \frac{1}{4\pi}$, the leading order quadratic terms dominate and we recover the negative results of the previous section. However, the EFT is valid even in the strongly coupled regime where $\frac{1}{4\pi} < |Q_i| < 1$, allowing us to go beyond the leading order quadratic approximation. To see what happens then, consider the case where $|Q_i| \sim \mathcal{O}(Q)$ in the near Minkowski limit, where the scale Q is assumed to lie in the interval, $\frac{1}{4\pi} < |Q| < 1$. Recall that the lower limit means we are at strong coupling and cannot truncate the expansion in the Q_i, while the upper limit ensures that the EFT remains valid. We then find that[1]

$$\left| \frac{\partial \kappa}{\partial Q_i} \right|_{k^2=0} \sim 4\pi |\kappa|_{k^2=0} \sim 4\pi \frac{|V_{\rm QFT}|}{M_{\rm UV}^4} \tag{4.28}$$

[1]For example, this result is easily understood for the case where the dimensionless potential scales as $\kappa \sim e^{4\pi Q}/(4\pi^2)$ in the strong coupling regime.

Near the Minkowski vacuum $|\kappa|_{k^2=0} \approx \frac{|V_{\rm QFT}|}{M_{\rm UV}^4}$ and so we expect $Q \sim \frac{1}{4\pi}\ln(4\pi|V_{\rm QFT}|/M_{\rm UV}^4)$. We then infer that the flux numbers scale as

$$|N_i| \sim \frac{M_{\rm UV}^2}{q}Q \sim \frac{M_{\rm UV}^2}{4\pi q}\ln(4\pi|V_{\rm QFT}|/M_{\rm UV}^4), \tag{4.29}$$

while

$$\left|X_i^{M_+\to AdS_-}\right| \sim \frac{16\pi}{3}\left(\frac{qM_{pl}}{\tau}\right)^2\left(\frac{M_{\rm UV}^2}{q}\right)\frac{|V_{\rm QFT}|}{M_{\rm UV}^4}, \tag{4.30}$$

where we have, once again, assumed the absence of any hierarchies between species, so that $q_i \sim \mathcal{O}(q)$ and $\tau_i \sim \mathcal{O}(\tau)$. If the WGC is to be respected and the membranes stay inside the EFT, we might hope to satisfy the halting condition, $\left|X_i^{M_+\to AdS_-}\right| < 1$, by taking the bare vacuum energy to be well below the cut-off, as we did in the previous section. The problem is that we expect the scale Q to be in the strong coupling window, $\frac{1}{4\pi} < |Q| < 1$, constraining $|V_{\rm QFT}| \gtrsim M_{\rm UV}^4 e/4\pi$. It follows that

$$\left|X_i^{M_+\to AdS_-}\right| \gtrsim \frac{e}{3}\left(\frac{qM_{pl}}{\tau}\right)^2\left(\frac{M_{\rm UV}^2}{q}\right) \sim \frac{e}{3}\left(\frac{qM_{pl}}{\tau}\right)^2\frac{|N_i|}{Q} > \frac{e}{3}\left(\frac{qM_{pl}}{\tau}\right)^2|N_i|.$$

Given that flux numbers are always assumed to be large, there is no way this can be less than unity without violating the WGC. This suggests that generic strong coupling scenarios do not help alleviate tensions between the WGC and our ability to halt the descent between vacua at very low curvatures. Of course, there may be some theories where the desired flattening of $\kappa(Q)$ is achieved at strong coupling thanks to elegant cancellations between terms, perhaps motivated by some enhanced symmetry. We will consider exactly such a scenario in the next section.

4.2.3 Generalised DBI

We now consider the generalised DBI action for three-form fields [101] (see also [102] for a more transparent description of the action) with multiple species. First we discuss the motivation for using the generalised DBI action. The gen-

eralised DBI action arises in string theory as the effective action for the dynamics of D-branes. It resums an infinite set of higher-derivative corrections and remains well-behaved at high energies. Thus, it provides a non-perturbative, string-theoretically motivated effective action for brane dynamics in warped geometries [101, 102].

For the generalised DBI action, (4.1) is given by

$$f(\star F_i) = \sum_i \Lambda_i^4 \left(1 - \sqrt{1 - \frac{(\star F^i)^2}{\Lambda_i^4}}\right). \tag{4.31}$$

This can be seen as the action for a spacetime filling 3-brane with tension Λ_i^4 on whose world volume the F_i are defined. The square root structure of (4.31) is special and we can expect it to have a symmetry. It will be very interesting to explore the symmetry of (4.31) in the future. The Legendre transform is easily computed to give

$$L_f(\phi) = \sum_i \Lambda_i^4 \left(\sqrt{1 + \frac{\phi_i^2}{\Lambda_i^4}} - 1\right) \tag{4.32}$$

with a corresponding dimensionless potential

$$\kappa(Q) = \sum_i \lambda_i^4 \left(\sqrt{1 + \frac{Q_i^2}{\lambda_i^4}} - 1\right), \qquad \lambda_i = \frac{\Lambda_i}{M_{\rm UV}}. \tag{4.33}$$

To avoid falling into the trap of quadratic domination and problems with the WGC, let us assume that when $k^2 \approx 0$, we have $|Q_i| \gtrsim \lambda_i^2$. In this limit, we recover the linear behaviour familiar from the KW model, albeit from a more well motivated initial set-up

$$\kappa(Q) \approx \sum_i \lambda_i^2 |Q_i| \ . \tag{4.34}$$

Since this is always positive, it is clear that we must have $V_{\rm QFT} < 0$ in order to have a spectrum of vacua that includes very low curvatures. For near Minkowski vacua, let us further assume that $|Q_i| \sim \mathcal{O}(Q)$ for some scale Q

and $\lambda_i \sim \mathcal{O}(\lambda)$ for some scale λ, so that $Q \sim |V_{\rm QFT}|/M_{\rm UV}^4 \lambda^2 \mathcal{N}$ where $\mathcal{N}$ is the number of species. It also follows that $|\partial\kappa/\partial Q_i|_{k^2=0} \approx \lambda_i^2$. For the EFT constraints (4.3) and the WGC (4.4) to be satisfied alongside the halting condition (4.12), we know that we must have a flat potential, and so $\lambda_i^2 < 1$, or equivalently, the underlying brane tensions lying below the cut-off $\Lambda_i < M_{\rm UV}$. This is certainly what we would expect from a consistent EFT.

We now consider the density of vacua at low curvature for the generalised DBI model. If we have just two species of three-form field, we can adopt the KW strategy and assume an irrational charge ratio. This guarantees a dense set of vacua even when the membrane charges are not especially small, allowing us to avoid the empty universe problem. Alternatively, we can consider the generalised DBI framework as a UV completion of the Bousso-Polchinski model,[2] with a large number of fields. To read off the density of vacua at large $\mathcal{N}$ and large $Q_i = N_i q_i / M_{\rm UV}^2 \gg \lambda_i^2$, we note that the curvature

$$k^2 \approx \frac{V_{\rm QFT} + \sum_i \Lambda_i^2 q_i |N_i|}{3M_{pl}^2}. \tag{4.35}$$

This means the vacua span an $\mathcal{N}$ dimensional grid with spacing $\Lambda_i^2 q_i$. Surfaces of constant vacuum curvature correspond to the boundaries of cross-polytopes, as opposed to spheres in the case of the original Bousso-Polchinski model. Consider two such surfaces of curvature $k^2 = \frac{V_{\rm QFT}+r}{3M_{pl}^2}$ and $k'^2 = \frac{V_{\rm QFT}+r'}{3M_{pl}^2}$, where $r' > r > 0$. The corresponding polytopes differ by a shell of volume $V_{\rm shell} = \frac{2^{\mathcal{N}}}{\mathcal{N}!}(r'^{\mathcal{N}} - r^{\mathcal{N}}) \approx \frac{2^{\mathcal{N}} r^{\mathcal{N}-1}}{(\mathcal{N}-1)!}(r'-r)$. To ensure that this shell contains at least one grid point, we require that $V_{\rm shell} \gtrsim D \prod_{i=1}^{\mathcal{N}} \Lambda_i^2 q_i$, where D is the degeneracy factor. This implies that vacua are separated by a curvature

$$\delta k^2 \approx \frac{(\mathcal{N}-1)! D \prod_{i=1}^{\mathcal{N}} \Lambda_i^2 q_i}{3M_{pl}^2 2^{\mathcal{N}} r^{\mathcal{N}-1}}, \qquad r = 3M_{pl}^2 k^2 - V_{\rm QFT} \tag{4.36}$$

[2]The Bousso-Polchinski model is an effective model with simplified potential terms – it ignores backreaction, brane dynamics, and quantum corrections. The generalised DBI action arises from string theory and includes backreaction, brane dynamics, and quantum corrections. Therefore, the generalised DBI action provides a UV-complete embedding of the Bousso-Polchinski model [43, 105].

Assuming the $\Lambda_i^2 q_i$ are incommensurate, the degeneracy $D = 2^{\mathcal{N}}$ due to the $N_i \to -N_i$ symmetry. The density of vacua near Minkowski space is therefore given by

$$\delta k^2|_{k^2=0} \approx \frac{(\mathcal{N}-1)! \prod_{i=1}^{\mathcal{N}} \Lambda_i^2 q_i}{3M_{pl}^2 |V_{\rm QFT}|^{\mathcal{N}-1}}, \tag{4.37}$$

In the absence of hierarchies in membrane charge and DBI tension, we assume that $q_i \sim \mathcal{O}(q)$ and $\Lambda_i \sim \mathcal{O}(\Lambda)$, so that

$$\delta k^2|_{k^2=0} \sim \frac{(\mathcal{N}-1)!(\Lambda^2 q)^{\mathcal{N}}}{3M_{pl}^2 |V_{\rm QFT}|^{\mathcal{N}-1}} \sim \frac{|V_{\rm QFT}|}{3M_{pl}^2 \mathcal{N}} \left(\frac{\Lambda^2 q}{|V_{\rm QFT}|}\right)^{\mathcal{N}} \mathcal{N}! \tag{4.38}$$

Using Stirling's approximation $\mathcal{N}! \approx \sqrt{2\pi\mathcal{N}} \left(\frac{\mathcal{N}}{e}\right)^{\mathcal{N}}$ for large $\mathcal{N}$ we have that

$$\delta k^2|_{k^2=0} \sim \frac{|V_{\rm QFT}|}{3M_{pl}^2} \sqrt{\frac{2\pi}{\mathcal{N}}} \left(\frac{\mathcal{N}\Lambda^2 q}{|V_{\rm QFT}| e}\right)^{\mathcal{N}}.$$

Requiring $\delta k^2|_{k^2=0} \lesssim H_0^2$ only imposes a constraint

$$\sqrt{\frac{2\pi}{\mathcal{N}}} \left(\frac{\mathcal{N}\Lambda^2 q}{|V_{\rm QFT}| e}\right)^{\mathcal{N}} \lesssim \frac{3M_{pl}^2 H_0^2}{|V_{\rm QFT}|}\ . \tag{4.39}$$

From our analysis of the linear model and equation (4.13), descent between vacua is halted at low curvature provided

$$\lambda_i^2 < \frac{3}{4} \left(\frac{\tau_i}{q_i M_{pl}}\right)^2 \frac{q_i}{M_{\rm UV}^2}. \tag{4.40}$$

In the absence of hierarchies this translates into the following constraint on the tensions $\tau_i \sim \mathcal{O}(\tau)$,

$$\tau \gtrsim 2M_{pl}\Lambda \sqrt{\frac{q}{3}}\ . \tag{4.41}$$

Finally, recall that the EFT constraints (4.3), the electric WGC (4.4) and the magnetic WGC (4.5) require that

$$\tau \lesssim M_{pl} q, \qquad \frac{M_{\rm UV}}{M_{pl}} \lesssim \frac{q}{M_{\rm UV}^2} \lesssim 1\ . \tag{4.42}$$

All constraints can be satisfied if we take, for example, $|V_{\rm QFT}| \sim M_{pl}^4$, $\mathcal{N} \approx 108$, $\tau \sim 0.1 M_{pl}^3$, $q \sim 0.2 M_{pl}^2$, $\Lambda \sim 0.1 M_{pl}$ and $M_{\rm UV} \sim 0.5 M_{pl}$. Note that this implies that $Q \sim |V_{\rm QFT}|/M_{\rm UV}^4 \lambda^2 \mathcal{N} \sim 3.7 \gg \lambda^2 \sim 0.04$, which confirms that we are indeed in the asymptotic regime of the DBI action.

This suggests that the generalised DBI set-up provides an elegant completion of the Bousso-Polchinski model, capable of halting the descent between vacua at low curvatures without running into any problems with the WGC or the consistency of the EFT. The model recovers the dynamics of a linear model in the asymptotic regime, with the required suppression of the effective coupling understood in terms of the underlying brane tensions lying inside the cut-off.

We should point out that the generalised DBI set-up is not the only one which halts the descent and satisfies the membrane WGC simultaneously. We will explore other possible models in the future.

4.2.4 Comment

We need to make a comment here. All the scenarios we discussed are landscape scenarios. The landscape scenarios are notoriously difficult to test with experiment because they predict too many possibilities, and we do not currently have a guiding principle or observable data to narrow them down [23]. We will explore this problem in the future.

4.3 de Sitter decay dynamics

As we have seen, membrane nucleation allows for descent from high scale de Sitter vacua to the low scale vacuum we see today, and perhaps even beyond. The dynamics of this descent is controlled by the tunneling exponent given in equation (3.48). Our goal here is to investigate how this changes as we move through the landscape, with a view to better understanding how the landscape is populated and how we arrive at the current vacuum. In fact, this section explains the results in chapter 3 more carefully.

The first thing to note is that for a parent de Sitter or Minkowski vacuum, $k_+^2 \geq 0$, the tunneling exponent is always positive and monotonically decreasing with X, as already emphasized in [1]. To delve a little deeper, we note that the behaviour of the tunneling exponent depends both on the curvature of the parent vacuum through the dimensionless combination $k_+^2 M_{pl}^4/T^2$ and on the magnitude of the curvature jumps between successive vacua through $X = 4M_{pl}^4 \Delta k^2/T^2$. Therefore curvature and its jumps between vacua are classified as large or small with respect to the reference scale T^2/M_{pl}^4. This observation prompts us to analyse equation (3.48) in the asymptotic limits $k_+^2 \gg T^2/M_{pl}^4$ and $k_+^2 \ll T^2/M_{pl}^4$. We focus on the case of vacuum descent, so that $\Delta k^2 > 0$ and by association, $X > 0$.

For $k_+^2 \gg T^2/M_{pl}^4$ we have that for fixed $X > 0$

$$B \approx \frac{4M_{pl}^6 \Omega_3}{T^2} \left[\frac{1}{4} \left(\frac{T^2}{k_+^2 M_{pl}^4} \right)^{3/2} - \frac{X}{8} \left(\frac{T^2}{k_+^2 M_{pl}^4} \right)^2 + \mathcal{O}\left(\frac{T^2}{k_+^2 M_{pl}^4} \right)^{5/2} \right] . \quad (4.43)$$

This approximation is valid in the early stages of vacuum descent when we are in a high scale de Sitter vacuum. At later stages, when the vacuum curvature has fallen to smaller values there are two different scenarios depending on the size of the jump. In particular, when $k_+^2 \ll T^2/M_{pl}^4$, we find that

$$B \approx \begin{cases} \frac{4M_{pl}^6 \Omega_3}{T^2} \left[\frac{4}{X(X-1)^2} + 16 \frac{1-4X}{X^2(X-1)^4} \frac{k_+^2 M_{pl}^4}{T^2} + \mathcal{O}\left(\frac{k_+^2 M_{pl}^4}{T^2} \right)^2 \right] & \text{for } X > 1 \\ \frac{4M_{pl}^6 \Omega_3}{T^2} \left[\frac{T^2}{k_+^2 M_{pl}^4} + 4 \frac{X-2}{(X-1)^2} + 16 \frac{6-4X+X^2}{(X-1)^4} \frac{k_+^2 M_{pl}^4}{T^2} + \mathcal{O}\left(\frac{k_+^2 M_{pl}^4}{T^2} \right)^2 \right] & \text{for } 0 < X < 1. \end{cases} \quad (4.44)$$

de Sitter decay via flux discharge can now be understood as a model dependent trajectory in the $(k_+^2, \Delta k^2)$ plane, starting at some presumably large curvature k_+^2 and moving in the direction in which it decreases. The changing value of the tunneling exponent is shown in figure 4.1 as a function of $k_+^2 M_{pl}^4/T^2$ and $X = 4M_{pl}^4 \Delta k^2/T^2$. In the limit where $k_+^2 M_{pl}^4/T^2 \to \infty$ we see from equation (4.43) that $B \to 0$ indicating that very high scale vacua are short lived. This

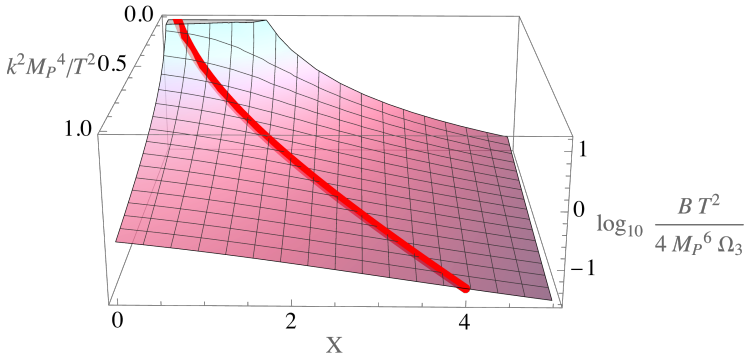

Figure 4.1: *The logarithm of the tunneling exponent,* $\log_{10}(BT^2/4M_{pl}^6\Omega_3)$ *as a function of* $k_+^2 M_{pl}^4/T^2$ *and* X*. Note that as the curvature of the parent vacuum decreases, the value of* B *increases.*

rapid decay of high scale vacua with $B < 1$ was recently dubbed the "boiling phase" of vacuum descent [98]. As the figure shows, as the curvature of the parent vacuum falls, the tunneling exponent increases and the discharge of the cosmological constant begins to slow down. Eventually we enter the "braking phase" of vacuum descent with $B > 1$, where the discharge rate becomes exponentially slow [98].

Once the curvature drops below the tension of the membranes, $k_+^2 \ll T^2/M_{pl}^6$, we see from equation (4.44) that the decay dynamics branches out depending on the size of the jumps in curvature Δk^2. Indeed, as the curvature of the parent vacuum tends towards zero, we find that there are two possible asymptotic limits, with the rate of transition from a near Minkowski vacuum ($k_+^2 \to 0$) to anti de Sitter ($k_-^2 < 0$) controlled by the following tunneling exponent,

$$B_{M_+ \to AdS_-} \sim \begin{cases} \frac{16 M_{pl}^6 \Omega_3}{T^2 X (X-1)^2} & \text{for } X > 1 \\ \frac{4 M_{pl}^6 \Omega_3}{k_+^2 M_{pl}^4} & \text{for } 0 < X < 1 \ , \end{cases} \tag{4.45}$$

where now $X = -4M_{pl}^4 k_-^2/T^2$. For $X > 1$, the tunneling exponent reaches a finite upper limit, and the corresponding rate of transition settles down to a finite value depending on the precise value of X. Note that the closer X is to unity from above, the slower the rate of transition. We might even say that there is a pole at $X = 1^+$ that could be used to halt any further discharge

of the cosmological constant in the Minkowski limit. However, the precise value of X depends on the depth of the would-be daughter vacuum and the brane tension. In any given model, these quantities are exposed to radiative corrections and any attempt to tune X to be close to unity from above in the appropriate limit would not be natural.

In contrast, for $0 < X < 1$ we see the presence of the Minkowski pole in the tunneling exponent, as $k_+^2 \to 0$, indicating that the rate of transition tends towards zero for a range of values of X. This is the halting condition that prevents any further discharge of the cosmological constant whenever the vacuum curvature vanishes. By association, low scale de Sitter vacua are extremely long lived. It follows that in order to achieve a reliable halting mechanism as the parent vacuum approaches Minkowski, we need a model that gives $0 < X < 1$ in the appropriate limit.

Having established that the discharge of the cosmological constant slows down exponentially at very low curvature provided $0 < X < 1$, we now ask how we arrive at such a vacuum. In particular, is it possible to "boil" directly from a high scale de Sitter vacuum to an exponentially long lived near Minkowski vacuum? Or do we always have pass through an intermediate scale vacuum, with the final transition in the so-called braking regime?

To answer this, consider a transition from a high scale parent vacuum with curvature $k_+^2 > 0$ to a Minkowski daughter vacuum, with $k_-^2 = 0$. This has $X_{dS_+ \to M_-} = 4M_{pl}^4 k_+^2/T^2$, corresponding to the red line in figure 4.1, where the tunneling exponent is given by

$$B_{dS_+ \to M_-} = \frac{16 M_{pl}^6 \Omega_3}{T^2 X (X+1)^2} \ . \tag{4.46}$$

Let us suppose that this transition is mediated by a membrane of type i, with charge $\pm q_i$ and tension, τ_i. Following a similar reasoning to the discussion at the beginning of section 4.2, it follows that the value of X is given approxi-

mately by

$$X_i^{dS_+\to M_-} \approx \frac{4M_{pl}^2 M_{\rm UV}^4}{3\tau_i^2}\left(\frac{\pm q_i}{M_{\rm UV}^2}\right)\frac{\partial\kappa}{\partial Q_i}\Big|_{k^2=0} \tag{4.47}$$

where we recall that $M_{\rm UV}$ is the cut-off and $\kappa(Q)$ is the dimensionless function of the flux defined through equation (4.7). As it happens, the halting condition for Minkowski vacua imposes a flatness condition for κ along all directions in a neighbourhood of $k^2 = 0$, as per equation (4.12). Assuming the flux function is differentiable in this neighbourhood of the Minkowski vacuum, we can use this to bound the value of X describing our transition from de Sitter to Minkowski, giving

$$\left|X_i^{dS_+\to M_-}\right| \approx \left|X_i^{M_+\to AdS_-}\right| < 1 \ . \tag{4.48}$$

This must be true for all species. In other words, tunneling from a high scale de Sitter vacuum to a very long lived near Minkowski vacuum must have $0 < X_{dS_+\to M_-} < 1$ and so

$$B_{dS_+\to M_-} > \frac{4M_{pl}^6\Omega_3}{T^2} \ . \tag{4.49}$$

Since the membranes are assumed to be part of the EFT, we have from (4.3) that $T < M_{\rm UV}^3 \lesssim M_{pl}^3$. It immediately follows that $B_{dS_+\to M_-} > 1$, or in other words, transitions to exponentially long lived vacua of very low curvature can only occur in the braking phase. If we assume that we have found ourselves in the current vacuum with any further descent almost halted, we must conclude that we did not get here by boiling.

We now arrive at the following picture of our journey through the landscape: the universe starts out in some high scale de Sitter vacuum close to the cut-off and rapidly boils down to some intermediate scale vacua, with curvature still far in excess of the current vacuum. Further descent is much slower. Nevertheless, provided we can ensure $0 < X < 1$, once a vacuum of very low curvature is nucleated, it lives for a very very long time, far longer than the vacua at intermediate scales.

As a final comment, we note that in most models there are multiple decay channels corresponding to the nucleation of different membrane species with different values of X. However, for fixed values of the curvature in the parent vacuum, the tunneling exponent can be shown to be a monotonically decreasing function of X in $X > 0$, since

$$\frac{\partial B}{\partial X}\Big|_{k_+^2 \text{ fixed}} = -\frac{4M_{pl}^2 k_+^4 B^2 (X+1+2Y)}{\Omega_3 T^2 Y (X-1+Y)^2} \tag{4.50}$$

where we have also used the fact that $Y > 0$. This monotonicity implies that largest possible values of X will tend to dominate the transitions. Indeed, consider a low curvature vacuum which can, in principle, decay via instantons with both $X < 1$ and with $X > 1$. The latter will dominate since $B(X < 1) > B(X > 1)$ for all $k_+^2 > 0$, leading to a landscape with no preference for small values of the cosmological constant. Therefore, in order to exploit the halting condition, preventing the decay of Minkowski vacua, it must be that all decay channels satisfy $X < 1$, with possible consequences for the WGC across *all* species for some models.

4.4 Discussion

In this chapter we have performed a deep dive into the dynamics of models with three-form fields exhibiting discharge of the cosmological constant via membrane nucleation. For suitable choices of parameters, these models can often admit a halting mechanism that stops any further discharge whenever we reach a Minkowski vacuum, presenting a possible solution to the cosmological constant problem. By continuity of the tunneling rates, it also follows that very low curvature vacua are exponentially long lived in the same parametric regime. As was first noticed in Chapter 3, these parametric choices tend to run into tension with the membrane WGC.

In [98] it was argued that parametric violation of the WGC, in taking the

membrane tension to infinity (while holding the charge fixed) stabilises de Sitter space. Whilst this is certainly true, we note that the existence of charged membranes in the EFT ($\tau < M_{\rm UV}^3$) is sufficient to trigger the decay of de Sitter, independently of their charge-to-tension ratio, rendering de Sitter space unstable regardless of the membrane WGC.

In contrast, the status of Minkowski space is a little more subtle. As we have seen, the instability of Minkowski space can, in some instances, be connected to membranes satisfying the WGC. For example, in a canonical quadratic model, like the one proposed by Bousso and Polchinski [43], a Minkowski vacuum could only be stable if all species of membrane violate the WGC. Said another way, if just one species of membrane should satisfy the bound set by the membrane WGC, a Minkowski vacuum would eventually decay. This reminds us of the inevitable decay of charged extremal black holes whenever there is at least one particle whose charge exceeds its mass in Planck units [85].

Whilst it is tempting to use the WGC to draw analogies between the stability of Minkowski and/or de Sitter and the stability of extremal black holes, we urge caution in doing so. Indeed, following [106], we can identify the cosmological constant with conserved charges coming from the global part of the three-form gauge symmetries. More precisely, the conserved charges correspond to the four-form fluxes, $N_i q_i$. In the usual form of the WGC, decay of black hole charge is required to avoid the existence of stable charged remnants. In analogy, we might say that a spacetime can always decay away its charge, so that the only stable solution is the one with vanishing charge. In the Bousso-Polchinski set-up, this corresponds to the anti de Sitter vacuum with vanishing flux and curvature $k^2 = V_{\rm QFT}/3M_{pl}^2 < 0$.

We have also shown that tension between the WGC and stability of Minkowski space is not inevitable and can actually be avoided in some special cases. In particular, we presented an elegant extension of the Bousso-Polchinski model of multiple three-form species, given by a four-form generalisation of the DBI action. At small values of the four-form flux, the leading order quadratic terms

dominate, yielding the dynamics of the original Bousso-Polchinski model. However, at larger values of the flux, relevant for transitions to and from low curvature vacua, the DBI structure approximates a linear behaviour, albeit in absolute value. In this limit, the theory admits a halting mechanism that picks out the vacuum with the lowest absolute curvature from the landscape, even when we choose natural values for the couplings without any violation of the WGC. Aside from being well motivated, the generalised DBI model also has the property that the effective potential of the flux is bounded from below, even though it approximates a linear regime at large values of the flux. This is not the case for the original linear flux model [69] where the potential can become more and more negative for as long as the EFT remains valid.

Beyond these considerations of the WGC, we have also taken a closer look at how we journey through the landscape of four-form flux vacua, in a relatively model independent way. In particular, our analysis of the dynamics of de Sitter decay demonstrated that the decay rate slows down as the curvature of the parent vacuum decreases. Decay from high scale de Sitter vacua is rapid, said to be in the so-called boiling phase, whereas decay from low scale de Sitter vacua is much slower, said to be in the so-called braking phase [98]. The transition from boiling to braking occurs when the tunneling exponent passes from $B < 1$ to $B > 1$. With this perspective, how did we arrive at the current low scale vacuum? The answer depends on whether or not our vacuum is exponentially long lived. If it is, and the halting conditions hold, $|X| < 1$, we find that the descent from high scale de Sitter to the current vacuum will not occur through boiling alone - at least one transition must occur in the braking phase.

Whilst the idea of boiling and braking is a useful one to help visualise the vacuum descent, it does not play a significant role in understanding why we are in this particular vacuum amongst the vast number present in the landscape. Of course, we know that anti de Sitter vacua crunch on a time scale inversely proportional to their curvature. For de Sitter vacua, what really matters is

the fact that the decay rate slows down exponentially quickly as we descend through the de Sitter part of landscape, until it comes to a halt for a Minkowski vacuum. In practice, this means de Sitter vacua of least curvature are the longest lived.

This mechanism takes us on a quantum journey through the landscape, leading us towards the vacuum of least absolute curvature. In a continuous landscape of vacua where a Minkowski vacuum is guaranteed, we are led to that vacuum, but in a discrete but dense landscape, we are led to the vacuum closest to Minkowski. The task of the model builder is to find a landscape where that near Minkowski vacuum has a curvature given by the current Hubble scale. This is achieved in the Bousso-Polchinski model (and its DBI extension) with relatively mild tunings of fundamental parameters and including of order a hundred species.

This suggests a new perspective on the cosmological constant problem as an alternative to the standard anthropic ideas usually employed in the presence of a landscape of vacua. However, although we can explain how we were driven towards a vacuum of low absolute curvature, we cannot explain how it is so young, at least in cosmological terms. Indeed, if the low curvature vacuum is so stable and long-lived, and matter is diluted over time, how come we find ourselves in an era where the density of matter is comparable to that of the constant vacuum energy? This is the so-called coincidence problem [107, 108]. This is usually addressed with some new late time physics. For example, there could be a mechanism that forces the universe to end not long after dark energy begins to dominate, as was proposed in [109–112].

As a solution to the cosmological constant problem, the halting mechanism for membrane nucleation is undoubtedly interesting and one that can be incorporated into a wide class of models with multiple three-form fields coupled to scalars. Although this is often in tension with the WGC, that is not exclusively the case: there are elegant models such as the generalised DBI set up, where the halting mechanism exists for a family of membranes satisfying the

WGC. Nevertheless, there are still some interesting and important problems that must be addressed, not least how it is aligned with a solution to the *why now?* problem.

Chapter 5

Vacuum transitions with the Gauss-Bonnet term in D dimensions

In chapter 3 and 4, we proposed a probabilistic argument to explain the reason why the cosmological constant is very small in $4D$. We can ask a question: can the behavior of the tunneling exponent B can be generalized to D-dimension? Moreover, in higher dimensional theory motivated by string theory, the Gauss-Bonnet term plays an important role [113]. Therefore, in this chapter, we generalize our result in $4D$ to arbitrary D dimensions including the Gauss-Bonnet term. As a result, we have two main results. We find that the Euclidean action of the bounce, B, describing the decay of a de Sitter vacuum, is proportional to $k_+^{-(D-2)}$, which has a pole as $k_+^2 \to 0$ where k_+^2 is the curvature of the parent vacuum. This result is similar to the result in $4D$. The other result is that we find a new decay channel, describing up-tunneling from anti-de Sitter into de Sitter. The meaning of this new decay channel in the string landscape should be explored in the future.

5.1 Introduction

The famous Bousso-Polchinski set-up [43] is one of the examples of the $4D$ EFT. In current string cosmology, anthropic principle has become the dominant explanation for the small value of cosmological constant. We proposed a "probabilistic" argument for the cosmological constant to challenge the anthropic principle in chapter 3. One crucial thing we need to compute is the tunneling exponent B,

$$B = S_E(\text{instanton}) - S_E(\text{parent}), \tag{5.1}$$

where S_E(instanton) is the Euclidean action of instanton and S_E(parent) is the Euclidean action of parent vacuum. We will explain more details about B in the following.

The cosmological constant problem is of course a $4D$ problem. We have explored the transition between vacua in $4D$ [1, 2]. If we want to consider

the properties of string landscape in higher dimensions, we can ask if exist transitions between vacua in higher dimensions. The physics in higher dimensions can affect the physics in $4D$ [45]. For example, by seeing how gravity behaves in other D, we can understand which features are unique to $4D$ (e.g., Schwarzschild black holes), and which are universal – this helps sharpen our understanding of gravitational theory and its limits. Furthermore, string theory requires higher dimensions ($10D$ for superstrings). To recover $4D$ physics, we compactify the extra dimensions. The shape and size of these compact spaces (such as Calabi-Yau manifolds) determine particle types, forces, and couplings in $4D$. $4D$ physics arises as an effective theory from higher-dimensional models, such as string theory, can give a potential explanation for the origin of the Standard Model and its parameters [18, 19, 45].

Moreover, in higher dimensional theory motivated by string theory the Gauss-Bonnet term plays an important role [113]. It is usually considered that the Gauss-Bonnet term has no dynamical effect in $4D$ since it is a topological term in $4D$. However, in [114] the authors found that in the limit of strong coupling, the Gauss-Bonnet term in $4D$ gives rise to non-trivial contributions to gravitational dynamics, while preserving the number of graviton degrees of freedom and being free from Ostrogradsky instability. Besides, the Gauss-Bonnet term also plays an important role in other aspects of physics. On the one hand, the Gauss-Bonnet term is a part of the Lovelock gravity family – the most general metric theories of gravity with second-order field equations. Including the Gauss-Bonnet term leads to modified gravity theories can avoid ghosts and have richer cosmological and black hole solutions [115]. On the other hand, the Gauss-Bonnet term is part of α' expansion in string theory, where higher-order curvature terms appear due to quantum corrections. Its presence helps cancel anomalies and modify gravitational interactions at high energies [116]. Therefore, we can ask if the Gauss-Bonnet term could play a role in solving the cosmological constant problem.

So in the present chapter we will study transition between vacua, or in other

words, the tunneling rates of vacuum decay, in higher dimensions including the Gauss-Bonnet term. According to the stability analysis of vacua in Gauss-Bonnet gravity, the associated coupling constant of the Gauss-Bonnet term, θ, should be positive, otherwise the whole theory will be unstable [117]. In the following calculation, we assume that θ is positive. We should point out that there exist two possibilities if we require positive membrane tension. We can find a new decay branch $AdS_+ \to dS_-$ in one of the possibilities. This new branch does not exist in GR and our previous model [1]. We have not figured out the meaning of the new decay branch in string landscape so far. It should be studied in the future.

The rest of this chapter is organised as follows: in section 5.2, we present the generalised set-up and the landscape of Lorentzian vacuum solutions. In section 5.3, we Wick rotate to Euclidean signature and solve the Euclidean field equations to find the corresponding instanton solutions and compute the corresponding transition rates. We demonstrate the role of the parameter X in controlling the stability of near Minkowski vacua, protecting them from decay into anti-de Sitter. We find that if we consider the effect of the Gauss-Bonnet term, there exists a new decay branch, AdS_+ to dS_-, which does not exist in GR. In section 5.4, we conclude and discuss our results.

5.2 The generalised set-up and vacua

5.2.1 The generalised set-up

We start with a general D-dimensional effective field theory on a manifold, $\mathcal{M}$, with a dynamical metric $g_{\mu\nu}$ and a family of $(D-1)$-form fields, A^i, and dual scalars ϕ_i,

$$S = \int_{\mathcal{M}} d^D x \sqrt{|g|} \left[\frac{1}{2} M_{pl}^{D-2} R - \frac{1}{2} \omega^{ij}(\phi) \nabla_\mu \phi_i \nabla^\mu \phi_j - V(\phi) + \theta(\phi) R_{GB} \right] + \int_{\mathcal{M}} \left[-\frac{1}{2} Z_{ij}(\phi) F^i \wedge \star F^j + \sigma_i(\phi) F^i \right] + S_{\text{boundary}} + S_{\text{membranes}}, \quad (5.2)$$

where D is the dimension of spacetime and R the Ricci scalar. θ is a linear Gauss-Bonnet coupling parameter, which is a function of scalar fields and $R_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the associated Gauss-Bonnet term. The D-form field strengths F^i are given in terms of the $(D-1)$-form fields $F^i = dA^i$ and $\star$ denotes the Hodge star operator on the manifold $\mathcal{M}$.

The action is also equipped with boundary terms which depend on the choice of boundary conditions. These are integrals over the boundary, Σ, which we take to be a co-dimension one surface described by the embedding $x^\mu = X^\mu(\xi^a)$. The induced metric and the pullback of the $(D-1)$-forms on the boundary are given respectively by $\gamma_{ab} = g_{\mu\nu}X^\mu_{,a}X^\nu_{,b}$ and $\alpha^i = \frac{1}{(D-1)!}A^i_{\mu_1\mu_2\ldots\mu_{D-1}}X^{\mu_1}_{,i_1}X^{\mu_2}_{,i_2}\ldots X^{\mu_{D-1}}_{,i_{D-1}}d\xi^{i_1}\wedge d\xi^{i_2}\ldots\wedge d\xi^{i_{D-1}}$, where $X^\mu_{,a} = \partial X^\mu/\partial\xi^a$ are the boundary tangent vectors. For an action of the form (5.2), the analogue of the Gibbons-Hawking boundary term, is given by [118–120],

$$\int_\Sigma d^{D-1}x\sqrt{|\gamma|}\left[M_{pl}^{D-2}K + 4\theta(J - 2\hat{G}^{ij}K_{ij})\right] - \int_\Sigma \mu p^i\phi_i + \lambda\chi_i\alpha^i, \tag{5.3}$$

where we define "conjugate momenta",

$$p^i = -d^{D-1}\xi\sqrt{|\gamma|}\omega^{ij}n^\mu\nabla_\mu\phi_j, \quad \chi_i = \sigma_i - Z_{ij}(\star F^j). \tag{5.4}$$

In eq.(5.3), γ_{ij} is the induced metric on the spacetime boundary, Σ, with corresponding Einstein tensor $\hat{G}^{ij}$. The extrinsic curvature, $K_{ij} = \frac{1}{2}\mathcal{L}_n\gamma_{ij}$, is defined in terms of the Lie derivative of the induced metric with respect to the *outward* pointing normal, n^a, and $K = \gamma^{ij}K_{ij}$ is its trace. Finally, we can define [119, 120]

$$J_{ij} = \frac{1}{3}\left[(K_{kl}K^{kl} - K^2)K_{ij} + 2KK_{ik}K^k_{\ j} - 2K_{ik}K^{kl}K_{lj}\right] \tag{5.5}$$

along with its trace $J = \gamma^{ij}J_{ij}$. The extrinsic curvature piece ensures that the action can be extremised under metric variations with Dirichlet boundary conditions, $\delta\gamma_{ab} = 0$ [89]. Meanwhile, the parameter λ allows us to interpolate

between Dirichlet ($\lambda = 0$) and Neumann ($\lambda = 1$) boundary conditions on the $(D-1)$-forms, while the parameter μ allows us to interpolate between Dirichlet ($\mu = 0$) and Neumann ($\mu = 1$) boundary conditions on the scalars [1].

Moreover, we consider the membrane contributions. We can include contributions from membranes and anti-membranes, Σ_I, charged under any of the $(D-1)$-forms, such that

$$S_{\text{membranes}} = -\sum_I \left\{ \eta_I^i q_i \int_{\Sigma_I} \alpha_I^i + \tau_i \int_{\Sigma_I} d^{D-1}\xi \sqrt{|\gamma_I|} \right\}. \tag{5.6}$$

Membranes charged under A^i carry a fundamental charge $\pm q_i$ depending on whether they are branes or antibranes and tension τ_i. In the action (5.6), $\eta_I^i = 0, \pm 1$ depending on whether the membrane Σ_I carries positive ($\eta_I^i = 1$), negative ($\eta_I^i = -1$) or vanishing charge ($\eta_I^i = 0$) under A^i. The pullback and the induced metric of the three-forms on Σ_I are given in a similar way to the boundary, by $\alpha_I^i = \frac{1}{(D-1)!} A^i_{\mu_1\mu_2\ldots\mu_{D-1}} X^{\mu_1}_{I,i_1} X^{\mu_2}_{I,i_2} \ldots X^{\mu_{D-1}}_{I,i_{D-1}} d\xi^{i_1} \wedge d\xi^{i_2} \ldots \wedge d\xi^{i_{D-1}}$, where $X_{I,a}^{\mu} = \partial X_I{}^{\mu}/\partial \xi^a$ are the tangent vectors on Σ_I. In the current chapter we only consider the timelike membranes so that their unit normal n_I^μ is spacelike.

5.2.2 Vacua

We take vacua to be real Lorentzian solutions with constant scalars, D-forms of constant flux $\star F^i = c^i$, and a maximally symmetric metric with constant curvature k^2, corresponding to dS ($k^2 > 0$), Minkowski ($k^2 = 0$) or AdS ($k^2 < 0$) spacetime. Since the scalars are constant, then the Gauss-Bonnet coupling parameter θ is a constant as well. The conjugate momentum,

$$\chi_i = \sigma_i - Z_{ij} c^j \tag{5.7}$$

is locally constant. The equations of motion away from membranes will be given in (5.39), (5.40) and (5.41). At membranes, we get a jump in χ_i,

$$\Delta\chi_i = -\eta_I^i q_i \qquad \text{(no sum over } i\text{)}, \tag{5.8}$$

generically triggering a jump in the spacetime curvature. Thus we have a landscape of possible vacua with different cosmological constants, scanned through membrane nucleation. Membrane nucleation, which is a quantum process, is necessary for scanning the landscape of vacua. When we compute transition rates we do so between eigenstates of constant χ_i. This suggests a path integral formalism equipped with Neumann boundary conditions, fixing χ_i in both the in-state and in the out-state [90]. We will compute these transition rates in the next section.

5.3 Nucleation rates

Transitions between vacua go through via membrane nucleation. In order to compute the nucleation rate at which transitions occur, we need to do analytical continuation to Euclidean signature

$$t \to -it_E, \qquad \star F^i \to \star F^i, \qquad A^i \to iA^i, \qquad S \to iS_E, \tag{5.9}$$

where S_E is the Euclidean action

$$\begin{aligned} S_E = &-\int_{\mathcal{M}} d^D x_E \sqrt{|g|}\left[\frac{1}{2}M_{pl}^{D-2}R - \frac{1}{2}\omega^{ij}(\phi)\nabla_\mu\phi_i\nabla^\mu\phi_j - V(\phi) + \theta R_{GB}\right] \\ &+\int_{\mathcal{M}}\left[-\frac{1}{2}Z_{ij}(\phi)F^i\wedge\star F^j + \sigma_i(\phi)F^i\right] + S^E_{\text{boundary}} + S^E_{\text{membranes}}, \end{aligned} \tag{5.10}$$

Moreover, we should solve for the instanton solution interpolating between the parent vacuum, $\mathcal{M}_+$, with curvature k_+^2 and the daughter vacuum, $\mathcal{M}_-$, with curvature k_-^2.

The boundary terms have been chosen to be consistent with Neumann boundary conditons on the $(D-1)$-form fields ($\lambda = 1$). We will only consider bounce configurations that transition between two vacua, therefore we assume that there is a single membrane, Σ, which is charged under A^i for some particular choice of $i = i_*$. The membrane tension is $T = \tau_{i_*}$ and the membrane charges under A^i are given by $Q_i = \delta_{ii_*} Q_{i_*}$, where $Q_{i_*} = \pm q_{i_*}$. After the Wick rotation, the Euclidean action of membrane is given by

$$S^E_{\text{membranes}} = -Q_{i_*} \int_\Sigma \alpha^{i_*}_\Sigma + T \int_\Sigma d^{D-1}\xi_E \sqrt{|\gamma_\Sigma|}\,. \tag{5.11}$$

We consider $O(D)$ symmetric Euclidean field configurations, with metric

$$ds^2 = N^2(r)dr^2 + \rho(r)^2 d\Omega_{D-1}, \tag{5.12}$$

where $d\Omega_{D-1} = h_{ij}d\xi^i d\xi^j$ is the metric on a unit $(D-1)$-sphere, Euclidean $(D-1)$-form potentials

$$A^i = A^i(r)\sqrt{|h|}d^{D-1}\xi \tag{5.13}$$

and scalars $\phi_i = \phi_i(r)$. We will set $N(r) = 1$ in the following calculation. The membrane is assumed to lie at $r = 0$ and radial coordinate runs from $r_{\min} < 0$ to $r_{\max} > 0$. Note that upon Wick rotation back to the Lorentzian signature, the instanton solution corresponds to a bubble of daughter vacuum in the parent spacetime.

We consider $O(D)$ symmetric Euclidean field configurations with metric (5.12). We denote r as the radial coordinate and i's as the other coordinate components. Then we can obtain the corresponding components of Riemann tensor

$$R^i{}_{jkl} = \left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2\right](\delta^i{}_j\delta_{kl} - \delta^i{}_l\delta_{jk}), \tag{5.14}$$

$$R^i{}_{rrj} = \frac{\rho''}{\rho}\delta^i{}_j, \tag{5.15}$$

and other components are vanishing. Then by contraction we have the components of Ricci tensor in D-dimension:

$$R^i{}_k = \left((D-2)\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2\right] - \frac{\rho''}{\rho}\right)\delta^i{}_k, \tag{5.16}$$

$$R^i{}_r = 0, \tag{5.17}$$

$$R^r{}_r = -(D-1)\frac{\rho''}{\rho}. \tag{5.18}$$

As a result, we can get the Ricci scalar in D-dimension:

$$R = (D-1)(D-2)\frac{1}{\rho^2} - (D-1)(D-2)\left(\frac{\rho'}{\rho}\right)^2 - 2(D-1)\frac{\rho''}{\rho}. \tag{5.19}$$

Finally, the associated Gauss-Bonnet term is given by

$$\begin{aligned} R_{GB} \equiv & R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} \\ = & (D-1)(D-2)(D-3)(D-4)\frac{1}{\rho^4} + (D-1)(D-2)(D-3)(D-4)\frac{\rho'^4}{\rho^4} \\ & - 2(D-1)(D-2)(D-3)(D-4)\frac{\rho'^2}{\rho^4} - 4(D-1)(D-2)(D-3)\frac{\rho''}{\rho^3} \\ & + 4(D-1)(D-2)(D-3)\frac{\rho'^2\rho''}{\rho^3}. \end{aligned} \tag{5.20}$$

From (5.20) we can find that the Gauss-Bonnet term is zero if $D \leq 4$.

Then we calculate the boundary term in D-dimension. The extrinsic curvature K_{ij} of metric (5.12) is

$$K_{ij} = \frac{\rho'}{\rho}\gamma_{ij}, \tag{5.21}$$

where $\gamma_{ij} = \rho^2(r)h_{ij}$ and its trace is

$$K \equiv \gamma^{ij}K_{ij} = (D-1)\frac{\rho'}{\rho}. \tag{5.22}$$

Then according to (5.5), then we have

$$J_{ij} = -\frac{1}{3}(D-2)(D-3)\frac{\rho'^3}{\rho^3}\gamma_{ij}, \tag{5.23}$$

and its trace is

$$J \equiv \gamma^{ij}J_{ij} = -\frac{1}{3}(D-1)(D-2)(D-3)\frac{\rho'^3}{\rho^3}. \tag{5.24}$$

The corresponding Ricci tensor $\hat{G}_{ij}$ on the spacetime, Σ, is given by

$$\hat{G}_{ij} \equiv \hat{R}_{ij} - \frac{1}{2}\gamma_{ij}\hat{R} = (D-2)\frac{1}{\rho^2}\delta_{ij} - \frac{1}{2}\gamma_{ij}(D-1)(D-2)\frac{1}{\rho^2}. \tag{5.25}$$

Then we can obtain that

$$J - 2\hat{G}^{ij}K_{ij} = (D-1)(D-2)(D-3)\left(\frac{\rho'}{\rho^3} - \frac{1}{3}\frac{\rho'^3}{\rho^3}\right). \tag{5.26}$$

Finally, the first term of the boundary term (5.3), the Gibbons-Hawking term, is given by

$$\int_\Sigma d^{D-1}x\sqrt{|\gamma|}M_{pl}^{D-2}K = \int_\Sigma d^{D-1}xM_{pl}^{D-2}(D-1)\frac{\rho'}{\rho}, \tag{5.27}$$

and the second term of the boundary term (5.3), the Myers term, is given by

$$\int_\Sigma d^{D-1}x\sqrt{|\gamma|}4\theta(J-2\hat{G}^{ij}K_{ij}) = 4\theta(D-1)(D-2)(D-3)\int_\Sigma d^{D-1}x\sqrt{|\gamma|}\left(\frac{\rho'}{\rho^3} - \frac{1}{3}\frac{\rho'^3}{\rho^3}\right). \tag{5.28}$$

According to the bulk action of (5.2), we can obtain the field equations away from membrane for N and ρ for constant scalars ϕ_i.

First, for N we have

$$\frac{\delta_N S_{bulk}}{\Omega_{D-1}} = \left(\int_{0^+}^{r_{max}} dr + \int_{r_{min}}^{0^-} dr\right)\delta N E_N, \tag{5.29}$$

where

$$E_N = \rho^{D-1}\left(\frac{1}{2}(D-1)(D-2)M_{pl}^{D-2}\left[\frac{1}{\rho^2}-\left(\frac{\rho_N}{\rho}\right)^2\right] + \theta(D-1)(D-2)(D-3)(D-4)\frac{1}{\rho^4}(1-\rho_N^2)^2 - V\right) - \frac{1}{2}Z_{ij}\frac{A_N^i A_N^j}{\rho^{D-1}}, \tag{5.30}$$

$$\rho_N \equiv \frac{1}{N}\rho' = \frac{1}{N}\frac{d}{dr}\rho, \tag{5.31}$$

and

$$A_N^i \equiv \frac{1}{N}A^{i'} = \frac{1}{N}\frac{d}{dr}A^i. \tag{5.32}$$

Set $E_N = 0$ and $N = 1$, we obtain the field equation for N away from membrane for constant scalars (5.39).

Second, for ρ we have

$$\frac{\delta_\rho S_{bulk}}{\Omega_{D-1}} = \left(\int_{0^+}^{r_{max}} dr + \int_{r_{min}}^{0^-} dr\right)\delta\rho E_\rho, \tag{5.33}$$

where

$$E_\rho = NM_{pl}^{D-2}\rho^{D-2}E_{\rho 1} + N\rho^{D-2}E_{\rho 2} + NE_{\rho 3}, \tag{5.34}$$

$$E_{\rho 1} = \frac{1}{2}(D-1)(D-2)(D-3)\frac{1}{\rho^2} - (D-1)(D-2)\frac{\rho_{NN}}{\rho} - \frac{1}{2}(D-1)(D-2)(D-3)\frac{\rho_N^2}{\rho^2}, \tag{5.35}$$

$$E_{\rho 2} = (D-1)V - \frac{(D-1)}{2}Z_{ij}\frac{A_N^i A_N^j}{\rho^{2D-2}}, \tag{5.36}$$

$$E_{\rho 3} = \theta(D-1)(D-2)(D-3)(D-4)(D-5)\rho^{D-6}(1-\rho_N^2)^2 + 4\theta(D-1)(D-2)(D-3)(D-4)\rho_N^2\rho_{NN}\rho^{D-5} - 4\theta(D-1)(D-2)(D-3)(D-4)\rho^{D-5}\rho_{NN}, \tag{5.37}$$

and

$$\rho_{NN} \equiv \frac{1}{N}\frac{d}{dr}\rho_N = \frac{1}{N}\frac{d}{dr}\left(\frac{1}{N}\frac{d}{dr}\rho\right). \tag{5.38}$$

Set $E_\rho = 0$ and $N = 1$, we obtain the field equation for ρ away from membrane

for constant scalars (5.40).

Therefore, the resulting field equations away from the membrane for constant scalars ϕ_i are the following (we have set $N(r) = 1$):

For N:

$$\begin{aligned} 0 =& \frac{1}{2}(D-1)(D-2)M_{pl}^{D-2}\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2\right] - V - \frac{1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^{2D-2}} \\ &+ \theta(D-1)(D-2)(D-3)(D-4)\frac{1}{\rho^4}(1-\rho'^2)^2 \end{aligned} \tag{5.39}$$

For ρ:

$$\begin{aligned} 0 =& \frac{1}{2}(D-1)(D-2)M_{pl}^{D-2}\left[\frac{D-3}{\rho^2} - (D-3)\left(\frac{\rho'}{\rho}\right)^2 - 2\frac{\rho''}{\rho}\right] \\ &- (D-1)V - \frac{D-1}{2}Z_{ij}\frac{A^{i\prime}A^{j\prime}}{\rho^{2D-2}} \\ &+ \theta(D-1)(D-2)(D-3)(D-4)(D-5)\frac{1}{\rho^4}(1-\rho'^2)^2 \\ &+ 4\theta(D-1)(D-2)(D-3)(D-4)\frac{\rho'^2}{\rho^3}\rho'' \\ &- 4\theta(D-1)(D-2)(D-3)(D-4)\frac{\rho''}{\rho^3} \end{aligned} \tag{5.40}$$

Moreover, for A^i, we still have:

$$\chi_i' = 0 \tag{5.41}$$

where $\chi_i = \sigma_i - Z_{ij}\frac{A^{j\prime}}{\rho^{D-1}}$ and $'$ denotes the derivative with respect to r. The derivation of (5.39) and (5.40) can be found in Appendix F.

Since we have assumed that all scalars are locally constant and satisfy the constraints listed in (5.40) and (5.41), this system is solved by

$$\rho = \frac{\sin(k(\epsilon r + r_0))}{k}, \qquad \epsilon = \pm 1 \tag{5.42}$$

where k^2 is the curvature of spacetime, and $(D-1)$-form potentials

$$A^i(r) = A^i(0) + c^i \int_0^r d\tilde{r}\rho(\tilde{r})^{D-1}. \tag{5.43}$$

Note that the solution for ρ extends to $k^2 \leq 0$ by analytic continuation. The parameter r_0 is an integration constant, setting the radius of the $(D-1)$-sphere at the membrane.

For $k^2 > 0$, the geometry is that of a section of a D-sphere and we can check that under $\epsilon \to -\epsilon$, $r_0 \to \pi/k - r_0$, ρ is invariant. Therefore, without loss of generality, we can always set $\epsilon = +1$, while also assuming $r_0 \in [0, \pi/k]$. The poles of the D-sphere are located at $r_{\max} = \frac{\pi}{k} - r_0$ and $r_{\min} = -r_0$.

For $k^2 = 0$ and $k^2 < 0$, the geometries are different. The geometry for $k^2 = 0$ is that of a section of a D-dimensional Euclidean plane. While the geometry for $k^2 < 0$ is that of a section of a D-dimensional hyperboloid. We can take $r_0 \geq 0$ since the ρ at the membrane is non-negative. Unlike the case of $k^2 > 0$, for these two cases we cannot always set ϵ without loss of generality and must consider each case separately. For $\epsilon = -1$, $r_{\max} = r_0$ corresponding to where the $(D-1)$-spheres shrink to zero size, while $r_{\min} = -\infty$ corresponding to the point where they diverge. For $\epsilon = +1$, the result is reversed: $r_{\max} = \infty$ corresponding to where the $(D-1)$-spheres diverge, while $r_{\min} = -r_0$ corresponding to the point where they shrink to zero size.

We have a continuity constraint at the membrane,

$$\Delta\rho(0) = \Delta\left[\frac{\sin kr_0}{k}\right] = 0\,, \tag{5.44}$$

where we introduce two notations $\Delta x = x^+ - x^-$ and $\langle x \rangle = \frac{1}{2}(x^+ + x^-)$, which correspond to the difference and average of some quantity x defined on either side of the membrane, respectively.

We also have the following junction conditions

$$\begin{aligned} T =& -(D-2)M_{pl}^{D-2}\Delta\left[\frac{\rho'(0)}{\rho(0)}\right]+\frac{4}{3}\theta(D-2)(D-3)(D-4)\Delta\left[\frac{\rho'^3(0)}{\rho^3(0)}\right] \\ & -4\theta(D-2)(D-3)(D-4)\Delta\left[\frac{\rho'(0)}{\rho^3(0)}\right], \end{aligned} \tag{5.45}$$

$$\Delta\chi_i = -\delta_{ii_*}Q_{i_*}. \tag{5.46}$$

According to eq.(5.45), the Gauss-Bonnet term has no contribution when $D \leq 4$.

Physically realistic membranes always carry non-negative tension, resulting in the following constraint on the allowed configurations

$$A\Delta\rho'(0) > 0, \tag{5.47}$$

where

$$\begin{aligned} A =& -M_{pl}^{D-2}\rho^2(0)+\frac{8}{3}\theta(D-3)(D-4)\langle\rho'^2(0)\rangle \\ & -\frac{1}{3}\theta(D-3)(D-4)\Delta\rho'^2(0)-4\theta(D-3)(D-4). \end{aligned} \tag{5.48}$$

If θ is zero and $D = 4$, then we have

$$-M_{pl}^2\rho^2(0)\Delta\rho'(0) > 0, \tag{5.49}$$

namely,

$$\Delta\left[\epsilon\cos kr_0\right] \leq 0\,, \tag{5.50}$$

which is the result we have obtained in [1].

Eq. (5.48) has two possibilities when $D \geq 5$. The first possibility is $A < 0$. Then we have $\Delta\rho'(0) < 0$, namely,

$$\Delta\left[\epsilon\cos kr_0\right] \leq 0\,. \tag{5.51}$$

<table>
<tr><th></th><th colspan="2">dS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = -1$</th></tr>
<tr><td>dS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_+ \geq \frac{\pi}{2} \geq (kr_0)_-$
allowed for $|X| \leq 1$
$\frac{\pi}{2} \geq (kr_0)_+ \geq (kr_0)_-$
allowed for $X \geq 1$</td><td>$(kr_0)_+ \geq (kr_0)_- \geq \frac{\pi}{2}$
allowed for $X \leq -1$
$(kr_0)_+ < (kr_0)_-$
negative tension</td><td colspan="2">negative tension</td><td>$(kr_0)_- \geq \frac{\pi}{2}$
kinematically
allowed for $X \leq -1$,
infinitely suppressed</td><td>$\frac{\pi}{2} \geq (kr_0)_-$
kinematically
allowed for $-1 \leq X \leq 0$,
infinitely suppressed</td></tr>
<tr><td>Minkowski/AdS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_+ \geq \frac{\pi}{2}$
allowed for $0 \leq X \leq 1$</td><td>$\frac{\pi}{2} \geq (kr_0)_+$
allowed for $X \geq 1$</td><td>$|k_-| \geq |k_+|$
allowed for $X \geq 1$</td><td>$|k_-| < |k_+|$
negative tension</td><td colspan="2">kinematically allowed for $|X| \leq 1$,
infinitely suppressed</td></tr>
<tr><td>Minkowski/AdS$_-$
$\epsilon_- = -1$</td><td colspan="2">negative tension</td><td colspan="2">negative tension</td><td>$|k_-| > |k_+|$
negative tension</td><td>$|k_-| \leq |k_+|$
kinematically
allowed for $X \leq -1$,
infinitely suppressed</td></tr>
</table>

Table 5.1: Summary of transitions $\mathcal{M}_+ \to \mathcal{M}_-$ where $\mathcal{M}$ is de Sitter, Minkowski or anti de Sitter, are listed. The transition is forbidden if a transition has negative tension and we mark it accordingly. We mark whether they are kinematically allowed for $|X| \geq 1$ or $|X| \leq 1$ for the remaining transitions. In some of these cases, we rule out some transitions although they are kinematically allowed since the transition rate is infinitely suppressed. We can take $\epsilon = +1$ for all dS configurations without loss of generality, so any examples contrary to this are not applicable. While for AdS and Minkowski configurations, we need to consider the two cases $\epsilon = +1$ and $\epsilon = -1$, respectively. This table is given in chapter 3, namely, [1].

Then this possibility has the same allowed configurations as we obtained in chapter 3, namely, [1]. For this possibility, we have only three configurations that avoid any problems with negative brane tension or infinitely suppressed tunneling rates. These are the configurations of physical interest corresponding to

- dS$_+ \to$ dS$_-$
- dS$_+ \to$ Minkowski/AdS$_-$
- Minkowski/AdS$_+ \to$ Minkowski/AdS$_-$ ($|k_-| \geq |k_+|$)

each with $\epsilon_\pm = 1$ and so $\rho'(r_{\min}) = 1$. Table 5.1 summaries all possible transformations for this possibility [1].

The second possibility is $A > 0$. Then we have $\Delta\rho'(0) > 0$, namely,

$$\Delta\left[\epsilon \cos kr_0\right] \geq 0\,. \tag{5.52}$$

This possibility has some new allowed configurations. For the second possibility, we also have three configurations that avoid any problems with negative

brane tension or infinitely suppressed tunneling rates as well. These are the configurations of physical interest corresponding to

- $\mathrm{dS}_+ \to \mathrm{dS}_-$
- $\mathrm{Minkowski/AdS}_+ \to \mathrm{dS}_-$
- $\mathrm{Minkowski/AdS}_+ \to \mathrm{Minkowski/AdS}_-$ $(|k_+| \geq |k_-|)$

where the second configuration, $\mathrm{Minkowski/AdS}_+ \to \mathrm{dS}_-$, is a new configuration, which is not possible in GR and the model in chapter 3 [1]. Table 5.2 summaries all possible transformations for this possibility.

It is not hard to get positive A. For example, if we take $\theta = 0.8 M_{pl}^{D-2}$, $r_0 = 0.5$, $|k_+| = 0.8$ and $\cos(k_- r_0) = 0.9$, then we can have $A > 0$. We define $\theta_{eff} = \theta(D-3)(D-4)$. Since the stability of Gauss-Bonnet gravity requires θ to be positive [117], considering $A > 0$, we can have the sufficient condition which θ should satisfy

$$\theta_{eff} > \frac{M_{pl}^{D-2} \frac{1}{|k_+^2|} [\cosh^2(|k_+| r_0) - 1]}{\Sigma}, \tag{5.53}$$

where

$$\Sigma = \frac{1}{3}\cosh^2(|k_+| r_0) + \frac{1}{3}\cos^2(k_- r_0) + \frac{1}{3}\cosh(|k_+| r_0)\cos(k_- r_0) - 4. \tag{5.54}$$

We should emphasize that the Gauss-Bonnet term can be treated perturbatively. According to (5.48), even if θ is very small, we can still choose the proper parameters, such as membrane tension T and r_0, to make A positive. As a result, the up-tunneling channel can exist. In other words, as long as the Gauss-Bonnet term exists, we can find the up-tunneling channel.

The tunneling rates between two vacua $\mathcal{M}_+ \to \mathcal{M}_-$ in semi-classical theory

<table>
<tr><th></th><th colspan="2">dS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = +1$</th><th colspan="2">Minkowski/AdS$_+$
$\epsilon_+ = -1$</th></tr>
<tr><td>dS$_-$
$\epsilon_- = +1$</td><td>$(kr_0)_- \geq \frac{\pi}{2} \geq (kr_0)_+$
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$\frac{\pi}{2} \geq (kr_0)_- \geq (kr_0)_+$
allowed for $X \leq -1$</td><td>$(kr_0)_- \geq (kr_0)_+ \geq \frac{\pi}{2}$
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$(kr_0)_+ > (kr_0)_-$
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kinetically
allowed for $X \leq -1$</td><td>$(kr_0)_- > \frac{\pi}{2}$
kinetically
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$\epsilon_- = +1$</td><td colspan="2">negative tension</td><td>$|k_+| \geq |k_-|$
kinetically
allowed for $X < -1$</td><td>$|k_+| < |k_-|$
negative tension</td><td colspan="2">negative tension</td></tr>
<tr><td>Minkowski/AdS$_-$
$\epsilon_- = -1$</td><td>$(kr_0)_+ \leq \frac{\pi}{2}$
kinetically
allowed for $0 \leq X \leq 1$
infinitely suppressed</td><td>$(kr_0)_+ \geq \frac{\pi}{2}$
kinetically
allowed for $X \geq 1$
infinitely suppressed</td><td colspan="2">kinematically
allowed for $|X| \leq 1$,
infinitely suppressed</td><td>$|k_+| \leq |k_-|$
kinetically
allowed for $X \leq -1$
infinitely suppressed</td><td>$|k_+| > |k_-|$
negative tension</td></tr>
</table>

Table 5.2: Summary of transitions $\mathcal{M}_+ \to \mathcal{M}_-$ when $A > 0$.

of vacuum decay are given by the following formula

$$\frac{\Gamma}{\text{Vol}} \sim e^{-B/\hbar}, \tag{5.55}$$

where the tunneling exponent B is

$$B = S_E(\text{instanton}) - S_E(\text{parent}) \tag{5.56}$$

and Γ is the transition (or tunneling) rate. Here $S_E(\text{instanton})$ is the Euclidean action evaluated on the bubble configurations described above, interpolating between the two vacua $\mathcal{M}_+$ and $\mathcal{M}_-$. While $S_E(\text{parent})$ is the Euclidean action evaluated on the complete parent vacuum, $\mathcal{M}_+$, which has no bubbles.

After a relatively lengthy calculation and plenty of heart warming cancellations, the tunneling exponent B can be computed in all cases:

$$\begin{aligned}\frac{B}{\Omega_{D-1}} = \Delta\Bigg[\int_{r_{\min}}^{0} dr\, \frac{(D-1)(D-2)}{2} M_{pl}^{D-2} \frac{1}{k^{D-3}} \left(1+\cos^2 k(r_0+r)\right) \sin^{D-3} k(r_0+r) \\ + \theta(D-1)(D-2)(D-3)(D-4) \frac{1}{k^{D-5}} \sin^{D-5} k(r_0+r) \\ \times \left(1 - \frac{1}{3}\cos^4 k(r_0+r) + 2\cos^2 k(r_0+r)\right) \\ + \theta(D-1)(D-2)(D-3)(D-4) \frac{1}{k^{D-5}} \sin^{D-1} k(r_0+r) \\ - \frac{(D-1)(D-2)}{2} M_{pl}^{D-2} \frac{1}{k^{D-3}} \sin^{D-1} k(r_0+r)\Bigg] + T\rho(0)^{D-1},\end{aligned} \tag{5.57}$$

where $r_{\min}$ denotes the minimal value of the radial coordinate and Ω_{D-1} is the volume of the unit $(D-1)$-sphere. In (5.57), we have used (5.42) for dS vacua.

Of course, for AdS vacua, ρ should be replaced by

$$\rho_{AdS} = \frac{1}{|k|}\sinh(|k|(\epsilon r + r_0)), \quad \epsilon = \pm 1. \tag{5.58}$$

As we did in chapter 3, we now define a parameter X, which is given by

$$X = -\frac{2\langle\rho'(0)\rangle}{\Delta[\rho'(0)]} \equiv \frac{1+z}{1-z}, \qquad z \equiv \frac{(\epsilon\cos kr_0)_+}{(\epsilon\cos kr_0)_-}. \tag{5.59}$$

The value of X now corresponds to a kinematic constraint on the geometry. When $z < 1$, $X \in (-1, +\infty)$; when $z > 1$, $X \in (-\infty, -1)$. Since the values of X in these two parts are not continuous, we can expect that the behaviour near $X = 1$ should be very important.

According to the junction conditions (5.45) and (5.46), X can also be rewritten in terms of Δk^2 and T^2. The expression is complicated and we do not use it in the present chapter, thus we do not list the formula for X. In particular, if $\theta = 0$, we can get

$$X = \frac{(D-2)^2 M_{pl}^{2(D-2)}\Delta k^2}{T^2} \tag{5.60}$$

with $X > 0$ in downward transitions and $X < 0$ in upward transitions. Furthermore, if $D = 4$, we have $X = \frac{4M_{pl}^4\Delta k^2}{T^2}$, which is consistent with the X we have obtained in [1]. More importantly, however, it turns out that some configurations are kinematically allowed only when $|X| \leq 1$ with the remainder allowed when $|X| \geq 1$.

When we consider the down-tunneling from near Minkowski space to anti-de Sitter space, which k_+ goes to zero while k_- is finite, according to (5.57), the tunneling exponent for $Min_+ \to AdS_-$ goes as

$$B_{M_+\to AdS_-} \sim \frac{1}{k_+^{D-2}}, \qquad X \approx 1, \tag{5.61}$$

where k_+^2 is the curvature of parent vacuum. As the parent vacuum approaches Minkowski, this exponent diverges, suppressing any transition into AdS and

ensuring a long lived Minkowski vacuum. In fact, according to (5.57), the Gauss-Bonnet term does not affect the behaviour of pole. In other words, if $X \approx 1$, in the absence of the Gauss-Bonnet term, $B_{M_+ \to AdS_-}$ still has the same behavior. We can give an intuitive understanding for this result. According to (5.2), the GR term is proportional to curvature R (or k^2) while the Gauss-Bonnet term is proportional to R^2 (or k^4). Therefore, when k goes to zero, the GR term should play the dominant role in tunneling exponent and the GB term cannot affect the behaviour near $k \approx 0$.

We have a comment on $X = 1$. If $X = 1$, even if there is no Gauss-Bonnet term, according to (5.60), the precise value X is dependent on the membrane tension and the depth of the daughter vacuum. In any given model, these quantities are exposed to radiative corrections. We need to tune the value of X to be close to unity, but this is not natural. If we include the contribution from the Gauss-Bonnet term, we can expect that more quantities should be tuned. Therefore, we will not accept $X = 1$ limit.

There is an argument in [121] that up-tunneling is impossible from Minkowski or anti-dS space. The argument is the following. Every instanton describes not one but two transitions. If we calculate the rate for one process, then we should calculate the rate for the reverse process as well. In other words, this means that the instanton not only describes "down-tunneling" from A to B, but also describes "up-tunneling" from B to A [121]. These processes are described by the same instanton both, and the rate to down-tunneling and up-tunneling are given by

$$\Gamma_{A\to B} \sim e^{-\Delta S_E/\hbar}, \quad \Delta S_E = S_E(\text{instanton}) - S_E(A), \tag{5.62}$$

and

$$\Gamma_{B\to A} \sim e^{-\Delta S_E/\hbar}, \quad \Delta S_E = S_E(\text{instanton}) - S_E(B). \tag{5.63}$$

The ratio of (5.62) and (5.63) is given by $e^{S_E(A)-S_E(B)}$, the ratio of the exponentials of the entropies. The essential feature of de Sitter space which allows

up-tunneling is that the system has finite size in the sense that it has a finite horizon volume and a finite entropy, namely, $S_E(B)$ is finite. However, when sizes of system are infinite, entropy differences between two states are infinite, therefore based on principle of detailed balance the up-tunneling is impossible. Anti-de Sitter space and Minkowski space have infinite horizon volumes and infinite entropies, therefore in [121] the authors claimed that up-tunneling is impossible from anti-de Sitter space or Minkowski space. So the readers may ask the question: do our results contradict the argument in [121] ?

Here is our explanation. As we have seen in Table 5.2, we have a new decay branch $AdS_+ \to dS_-$, when $\epsilon = \pm 1$. In fact, the ratio of (5.62) and (5.63) is given by $e^{S_E(dS)-S_E(AdS)}$. Since $S_E(dS)$ is finite while $S_E(AdS)$ is infinite, the difference between these two action is infinite. However, this does not mean that $B_{AdS_+\to dS_-}$ needs to be infinite. In GR and other previous models [1, 121], the decay branch $B_{AdS_+\to dS_-}$ does not exist because of the negative membrane tension. However, as we have pointed out, when we include the contribution of the Gauss-Bonnet term, there exists a new decay branch $B_{AdS_+\to dS_-}$, when $\epsilon = \pm 1$, which has positive membrane tension. Hence there is no contradiction between our result and the argument of [121].

5.4 Discussions

The cosmological constant problem is a long-standing problem in theoretical physics. In the current string cosmology, anthropic principle is the dominant explanation for the small value of the cosmological constant of our universe. We challenged the dominant assertion in our previous paper [1]. For a family of string-inspired models of four-form fluxes coupled to scalar fields, including Bousso and Polchinski's set-up, we can show how the current vacuum can be selected on "probabilistic" grounds. In other words, our universe is born from probability. Hence the existence of our world is independent of mankind and other biological species.

The main idea of our chapter 3 was inspired by the model constructed by Kaloper [67] and Kaloper and Westphal [69], though we have shown how to generalize this special model. We proposed an important parameter X which controls the lifetime of low scale vacua, similar to the current vacuum we are living. When the parent vacuum goes to Minkowski spacetime, we found that a pole can appear in the corresponding bounce if X is suitably bounded. This can guarantee the stability of the Minkowski vacuum and hence the longevity of those vacua near Minkowski in Planck units.

In chapter 3, we only considered the cosmological constant problem in $4D$. If we are interested in the properties of string landscape in higher dimensions, we can ask that if the tunneling exponent B has a similar behavior in arbitrary D-dimension as in $4D$. Moreover, the Gauss-Bonnet term appears often in string theory and play an important role in gravitational theory [113]. So we can consider transition between vacua in D-dimension including the Gauss-Bonnet term. According to higher dimensional theory motivated by string theory, the associated coupling constant of the Gauss-Bonnet term θ should be positive, otherwise the whole theory will be unstable [117]. In the previous calculation, we assume that θ is a positive constant since ϕ_i are locally constants. We find that the tunneling bounce B of $\mathrm{Min}_+ \to \mathrm{AdS}_-$ is proportional to $k_+^{-(D-2)}$, which has a pole at k_+^2 and is similar to the result in $4D$. This means that the Gauss-Bonnet term does not play a major role near the pole. Furthermore, we should point out that when we include the Gauss-Bonnet term, there exist two possibilities if we require positive membrane tension. As a result, we can find a new decay branch $AdS_+ \to dS_-$, $\epsilon_\pm = 1$. This new branch does not exist in GR and our previous model [1].

Although the Gauss-Bonnet term is a topological term when $D = 2, 3, 4$, we can still consider the $4D$ limit, namely, $D \to 4$ Gauss Bonnet gravity [114]. In the strong coupling limit ($\theta \to \infty$), the Gauss-Bonnet term in $4D$ gives rise to non-trivial contributions to gravitational dynamics, while preserving the number of graviton degrees of freedom and being free from Ostrogradsky

instability [114]. However, so far the up-tunneling in $D \to 4$ Gauss Bonnet gravity has not been found. We need to clarify two points. First, although it is not central to the results of the current chapter, the strong regime limit as $D \to 4$ is still unclear and has been critiqued by several authors, such as [122]. Second, the inclusion of the GB term is not without problems. As demonstrated in [123], the equations are not weakly hyperbolic when the spacetime curvature becomes sufficiently large.

Based on our results, there are at least two speculative pictures. One picture is that AdS vacua decay to near Minkowski vacua ($k_-^2 > 0$) directly. We can expect that there is a pole k_-^{-2} near Minkowski vacua. The other possible picture is that in the first step vacua decay from AdS parent vacua, which can be obtained in string theory, to high dS daughter vacua. And then in the second step high dS vacua decay to near Minkowski vacua ($k_-^2 > 0$). In order to get a very small positive cosmological constant, we need to introduce a mechanism to halt the decay process. The dS vacua cannot be long-lived. In fact it is very hard to get de Sitter vacua from string theory [124]. Many scenarios have been tried, such as KKLT scenario [47]. We should find more terms which can induce $AdS \to dS$ decay channel, which is new and interesting. At present we have not figured out the meaning of the new decay branch in string landscape. All of these should be studied in the future.

Part II

de Sitter Vacua in String Cosmology

Chapter 6

Basics of $O(d, d)$ invariant Cosmology

In Part I, we study de Sitter vacua in string-inspired models. Furthermore, we will study de Sitter vacua in string cosmology in Part II. In particular, we will explore the properties of de Sitter vacua in $O(d,d)$ invariant cosmology in chapter 7. Hence we introduce the basic ideas of $O(d,d)$ invariant cosmology in chapter 6.

6.1 Introduction

String theory is the most promising candidate of quantum gravity theory at present. Since the early days of string theory there have been intriguing ideas of how its unique characteristics could play a role in cosmological scenarios [125, 126]. It is a natural idea that the higher-order corrections of string theory play an important role in resolving the problems in cosmology, for instance, the big-bang singularity [127]. In the 1980s the first few orders of string theory have been obtained [128–130], but the complete computation or classification of these corrections is still out of reach. However, the situations have been improved in recent years. The duality-covariant formulations of string theories, such as double field theory [131–134], have been used to make progress in describing α' corrections [135–141]. One crucial paper which is related to this thesis is written by Hohm and Zwiebach [127]. Now let us review some basic results of this paper.

The analysis of [127] is based on the result by Veneziano and Meissner [142] and Sen [143]. Hohm and Zwiebach considered the classical field theory of the metric $g_{\mu\nu}$, the b-field, and the dilaton ϕ arising from $D = d + 1$ dimensional string theory. This field theory keeps an $O(d,d,\mathbf{R})$ symmetry to all orders in α' if the fields do not depend on the d spatial coordinates. This symmetry contains the scale-factor duality $a \to 1/a$, therefore referred to as "duality". The work of Meissner [144] shows that the duality transformations receive α' corrections in terms of standard fields. Hohm and Zwiebach assumed that duality transformations remain unchanged to all orders in α'. With this assumption

they can classify the duality invariant α' corrections completely. They performed duality-covariant field redefinitions to show that only first-order time derivatives are included in the action, thereby arriving at a minimal set of duality invariant higher-derivative terms to all orders [127].

First we will briefly review the core results of the first part of [127]. We begin with a review of all orders in α' duality invariant cosmology, as originally proposed by Hohm and Zwiebach [127, 145] building on earlier pioneering work [142, 143, 146–148]. Consider the leading order theory describing the universal massless sector of closed strings in $d+1$ dimensions. This corresponds to the well known Neveu-Schwarz action depending on the string frame metric, $\mathbf{g}_{\mu\nu}$, the antisymmetric Kalb-Ramond field, $\mathbf{b}_{\mu\nu}$, and the scalar dilaton ϕ,

$$I_0 = \frac{1}{2l_s^{d-1}} \int \mathrm{d}^{1+d}x\sqrt{-\mathbf{g}}e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} - 2\Lambda\right], \tag{6.1}$$

where the string length $l_s = \sqrt{\alpha'}$, $H_{\mu\nu\rho} = 3\partial_{[\mu}\mathbf{b}_{\nu\rho]}$ is the Kalb-Ramond field strength and R is the Ricci scalar for the string frame metric. The cosmological term, Λ, is present for non-critical strings and scales as l_s^{-2}. For critical strings it vanishes, so we shall neglect it in the following. We make a cosmological ansatz for the metric, b-field and dilaton,

$$\mathbf{g}_{\mu\nu} = \begin{bmatrix} -n^2(t) & 0 \\ 0 & g_{ij}(t) \end{bmatrix}, \tag{6.2}$$

$$\mathbf{b}_{\mu\nu} = \begin{bmatrix} 0 & 0 \\ 0 & b_{ij}(t) \end{bmatrix}, \tag{6.3}$$

$$\phi = \phi(t), \tag{6.4}$$

where $n(t)$ is the lapse and $g_{ij}(t)$ are the components of the spatial metric. This breaks diffeomorphisms down to the subgroup $t \to f(\hat{t}), x^i \to x^i$, with ϕ, g_{ij} and b_{ij} transforming as scalars, and the lapse as a vector, $n \to n\dot{f}$, where

$\dot{f} = \frac{df(\hat{t})}{d\hat{t}}$.[1] In this limit, the corresponding field equations also exhibit an $O(d,d)$ symmetry. Indeed, the full system can be written in terms of $O(d,d)$ covariant objects: a generalised dilaton,

$$\Phi = 2\phi - \ln\sqrt{\det g}, \tag{6.5}$$

and a generalised metric

$$\mathcal{H} = \begin{bmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{bmatrix}. \tag{6.6}$$

The latter transforms as a tensor, $\mathcal{H} \to \Omega^T \mathcal{H} \Omega$, under $\Omega \in O(d,d)$ where

$$\Omega^T \eta \Omega = \eta, \qquad \eta = \begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}. \tag{6.7}$$

In contrast, the generalised dilaton and the lapse function are $O(d,d)$ scalars, transforming trivially as $\Phi \to \Phi$ and $n \to n$.

Since we have assumed that all fields and metric are only dependent on time, then the resulting action can be written in a form that is manifestly invariant under the residual diffeomorphisms as well as the underlying $O(d,d)$ symmetry [146, 148],

$$I_0 = \kappa^2 \int \mathrm{d}t\; n e^{-\Phi} \left[-(\partial_n \Phi)^2 - \frac{1}{8} \mathrm{tr}[(\partial_n S)^2] \right]. \tag{6.8}$$

where $\kappa^2 > 0$ is some dimensionful constant that plays no role in what follows. Note that we have introduced the following notation for the covariant time derivative

$$\partial_n = \frac{1}{n(t)} \frac{\mathrm{d}}{\mathrm{d}t}, \tag{6.9}$$

[1] We consider the metric $ds^2 = n^2(t)dt^2 + \cdots$. If $t = f(\hat{t})$, then $dt = \dot{f} d\hat{t}$, where $\dot{f} = \frac{df(\hat{t})}{d\hat{t}}$. After transformation, n becomes $\hat{n}$. In order to make $n^2 dt^2 = \hat{n}^2 d\hat{t}^2$, we should have $\hat{n} = n\dot{f}$, namely, $n \to \hat{n} = n\dot{f}$.

and further defined

$$S = \eta\mathcal{H} = \begin{bmatrix} bg^{-1} & g - bg^{-1}b \\ g^{-1} & -g^{-1}b \end{bmatrix}. \tag{6.10}$$

Note also that both η and S are involutory matrices, squaring to the identity, $S^2 = \eta^2 = 1$. The action is now seen to be manifestly invariant under $\mathrm{O}(d,d)$ since S transforms as[2]

$$S \to S' = \Omega^{-1} S \Omega, \tag{6.11}$$

We now consider the residual diffeomorphism and $\mathrm{O}(d,d)$ invariant α' corrections to I_0. These terms are constructed out of $\mathrm{O}(d,d)$ scalars such as Φ and its covariant derivatives, $\partial_n\Phi, \partial_n^2\Phi, \ldots$ along with traces of $\mathrm{O}(d,d)$ covariant terms such as $\partial_n S$ and higher covariant derivatives $\partial_n^2 S, \partial_n^3 S, \ldots$. Through a judicious use of equations of motion, field redefinitions, integration by parts, and useful relations of the form

$$\mathrm{tr}(S) = \mathrm{tr}(\partial_n S) = \mathrm{tr}(\partial_n^2 S) = ... = 0, \tag{6.12}$$

$$\mathrm{tr}((\partial_n S)^{2k+1}) = 0 \qquad \text{for} \qquad k = 0, 1, ..., \tag{6.13}$$

$$\mathrm{tr}(S(\partial_n S)^k) = 0 \qquad \text{for} \qquad k = 0, 1, ..., \tag{6.14}$$

we can eliminate all higher order derivatives in the system. Indeed, Holm and Zwiebach [127] showed that the most general action at all order in α' must take the following form

$$I = \kappa^2 \int dt\; n e^{-\Phi} \left[-(\partial_n\Phi)^2 + \sum_{k=1}^{\infty} c_k \alpha'^{k-1} \,\mathrm{tr}[(\partial_n S)^{2k}] + \text{multitrace} \right] \tag{6.15}$$

where $c_1 = -\frac{1}{8}$ to match the leading order Neveu-Schwarz action (6.8). For any particular string theory, there ought to be a tower of dimensionless coefficients

[2] To see this, note that $S = \eta\mathcal{H} \to S' = \eta\Omega^T\mathcal{H}\Omega = \eta\Omega^T\eta S\Omega$, where we have used the fact that $\eta^2 = 1$. It then follows from (6.7) that $\eta\Omega^T\eta = \Omega^{-1}$ and so $S' = \Omega^{-1}S\Omega$, as stated in (6.11). In [127, 145], these transformations are expressed in terms of $h = \Omega^{-1}$ where (6.7) along with the fact that $\eta^2 = 1$ now implies that $h\eta h^T = \eta$.

of higher order operators given by c_k, along with a tower of coefficients for the multitrace operators.

In establishing the above result (6.15), Hohm and Zwiebach repeatedly used field redefinitions iteratively in increasing orders of α'. One striking result is that only first derivatives of the fields appear in the action. Furthermore, we can find that all duality invariant terms with more than one time derivative on S and all terms with one or more derivatives of the dilaton can be redefined away. This is a major, somehow unexpected, simplification [127].

To summarize briefly, it was shown in [127] that field redefinitions can be used to recast the α'^k corrections into the form given in [127, 149]

$$I_k = \alpha'^k \int dtne^{-\Phi} X(\partial_n S), \quad S_k = \frac{1}{2\kappa^2} \int d^d x I_k, \tag{6.16}$$

where X is a completely arbitrary function of $\partial_n S$, with no extra time derivatives and no powers $(\partial_n S)^2$ included.

The proof begins by assuming that the $(k-1)$th order action already has the desired structure. It is then shown that, via field redefinitions at order α'^k, the kth order action can be brought into the form (6.16). A crucial detail is that only the zeroth-order action should be varied under the field redefinitions; the higher order terms only contribute to the action of $\mathcal{O}(\alpha'^{k+1})$. Explicitly, under the transformation [127, 149]

$$\Phi \to \Phi + \alpha'^k \delta\Phi, \quad S \to S + \alpha'^k \delta S \tag{6.17}$$

the variation arising from the second term in

$$I = I_0 + \sum_{p=1}^{k-1} I_p + \alpha'^k \int dtne^{-\Phi} X(\partial_n \Phi, S) + \mathcal{O}(\alpha'^{k+1}), \tag{6.18}$$

only contributes to $\mathcal{O}(\alpha'^{k+1})$. The proof involves showing the following:

1. A factor $\partial_n^2\Phi$ in the action can be replaced by a factor

$$Q_\Phi = \frac{1}{2}(\partial_n\Phi)^2 - \frac{1}{16}\,\mathrm{tr}(\partial_n S)^2, \tag{6.19}$$

which only includes first order derivatives of Φ;

2. A factor $\partial_n^2 S$ in the action can be substituted with a factor

$$Q_S = -S(\partial_n S)^2 + (\partial_n\Phi\partial_n S), \tag{6.20}$$

where (6.20) only contains first order derivatives of S;

3. Any action can be rewritten to include only first-order time derivatives of Φ and S. This is done by integrating by parts the higher-order derivatives until only second-order derivatives are left, and then applying the earlier properties to remove them;

4. Any action at order in α'^k (with $k > 0$) is equivalent to one in which $\partial_n\Phi$ does not appear. This equivalence is established through field redefinitions of Φ;

5. Any term in the correction action that includes a factor of $\mathrm{tr}(\partial_n S)^2$ can be eliminated through an appropriate redefinition of the lapse function $n(t)$.

Consequently, the action involves only first derivatives of the fields. In addition, duality-invariant terms with multiple time derivatives on S, as well as any terms with derivatives of the dilaton, can be eliminated via field redefinitions [127, 149]. For more information, see [127, 149].

In the second part of [127] Hohm and Zwiebach studied this general α'-complete theory for the simplest cosmological ansatz, a Friedmann-Lemaître-Robertson-Walker (FLRW) background. The spatial metric of FLRW metric is given by a time-dependent scale factor times the Euclidean metric. And they set b-field equal to zero. Then the Friedmann equations to all orders in α' can be derived. Hohm and Zwiebach found that we can write the equations compactly in terms of $F(H)$ whose Taylor expansion is determined by the

following equation:

$$F(H) \equiv 4d \sum_{k=1}^{\infty} (-\alpha')^{k-1} 2^{2k-1} c_k H^{2k} \tag{6.21}$$

where H is the Hubble parameter and we can have $c_k \equiv c_{k,0} + 2dc_{k,1} + ...$ in terms of (6.15).

Then the Friedmann equations take the concise form

$$\frac{d}{dt}\left(e^{-\Phi} j(H)\right) = 0, \tag{6.22}$$

$$\ddot{\Phi} + \frac{1}{2} H j(H) = 0, \tag{6.23}$$

$$\dot{\Phi}^2 + k(H) = 0, \tag{6.24}$$

where the functions $f(H)$ and $g(H)$ are defined as

$$j(H) \equiv F'(H), \qquad k(H) \equiv HF'(H) - F(H), \tag{6.25}$$

where $'$ denotes the derivative with respect to H.

Hohm and Zwiebach discussed a general nonperturbative initial-value formulation, and they stated conditions on the function $F(H)$ which the theory permits de Sitter vacua (in string frame) [127]. They considered the dS vacua solutions with constant dilaton ϕ and found that string theory may realize de Sitter vacua if non-perturbative contributions are included. More details can be found in [127].

In chapter 7, we will discuss the dS solutions in $O(d,d)$ invariant cosmology with non-constant dilaton ϕ thoroughly.

Chapter 7

On de Sitter vacua in $O(d,d)$ invariant cosmology

In chapter 7, we perform a thorough analysis of de Sitter solutions in $\mathrm{O}(d,d)$ invariant cosmologies. Starting with a homogeneous and isotropic framework we examine conditions for the existence of such solutions to the vacuum field equations, non-perturbative in α' in both the string frame and the Einstein frame. We elucidate the nature of the instability in the string frame vacuum. For the Einstein frame, we demonstrate that the de Sitter solutions cannot be eternal. We then extend our analysis to include Bianchi I universes where the $\mathrm{O}(d,d)$ symmetry includes scale factor exchange as well as scale factor duality. We show how the theory can be extended to the anisotropic case so that it admits de Sitter solutions, noting the crucial role played by the $\mathrm{O}(d,d)$ symmetry in satisfying any additional constraints.

7.1 Introduction

String theory is a consistent quantum theory of gravity, with a well defined low energy description in terms of higher dimensional supergravities. For this to be relevant to our universe, it must be able to admit a compactification down to four dimensions, where gravity is well described by GR and matter by a suitable extension of the standard model of particle physics. It should also be able to accommodate an accelerated cosmological expansion. Of course, it is still possible that the concerns surrounding de Sitter vacua in perturbative string theory will go away and we will be able to find consistent accelerated cosmological solutions that everyone is happy with. However, if this is not the case, we will need a non-perturbative understanding of the theory to probe the full vacuum structure and establish the existence of de Sitter vacua far from the boundary of moduli space. An important no-go theorem, valid at any order in the string scale, suggests that de Sitter vacua of dimension four or higher are not possible for the classical heterotic string, or indeed classical type II strings in the absence of RR fluxes [150]. To say much more will likely require a third string revolution - a step change in our current approach to

string theory/M theory. We can also consider the possibility that de Sitter is a resonant or excited state of string theory [151–153].

We can try and get some insight into what can happen non-perturbatively by focussing on the effective field theory for classical strings in the homogeneous limit. The Neveu-Schwarz sector describing the metric, the Kalb-Ramond field and the dilaton, are known to possess an $\mathrm{O}(d,d)$ symmetry to leading order [142, 143, 146–148]. This symmetry is expected to be preserved to all orders in the slope parameter α', motivating corrections to the leading order Neveu-Schwarz theory using higher order $\mathrm{O}(d,d)$ invariant operators [144]. Note that $\mathrm{O}(d,d)$ invariance can be made manifest using double field theory, albeit at the expense of manifest diffeomorphism invariance [132, 133, 135, 136].[1] Duality invariant cosmology has been used to tackle the problem of the initial singularity (see eg. [126, 154–157]).

Remarkably, for homogeneous fields, Hohm and Zwiebach have been able to provide a complete classicfication of all higher order terms invariant under $\mathrm{O}(d,d)$ [127, 145, 158]. With field redefinitions, the system is reduced to first order time derivatives allowing them to write down the most general duality invariant theory to all orders in α' for the metric, Kalb-Ramond field and dilaton. To leading order in the expansion in α' the theory coincides with the Neveu-Schwarz sector of the supergravity theories describing the effective theory of classical strings, in the homogeneous limit. In the limit of vanishing Kalb-Ramond field and an isotropic spatially flat metric, the theory exhibits scale factor duality $a \to 1/a$ as a remnant of the underlying $\mathrm{O}(d,d)$ symmetry. The full theory is now tractable at all orders in α', enabling the authors of [127, 145] to identify string frame de Sitter solutions to the vacuum field equations that are *non-perturbative* in α', with a time dependent dilaton. The dynamics of this new class of theories has been studied in a series of subsequent papers

[1] $O(d,d)$ and diffeomorphism symmetry are both real symmetries of string theory. Making $O(d,d)$ manifest requires a formalism that treats regular coordinate x^i and dual coordinate $\tilde{x}_i$ symmetrically. Diffeomorphisms only act on regular coordinates, not the duals. Therefore, we cannot make both symmetries manifest at the same time. In double field theory, we choose to make $O(d,d)$ manifest [132, 133, 135, 136].

[149, 159–170]. In particular, one can extend the system to include $\mathrm{O}(d,d)$ invariant matter fields [148, 149], or find theories that admit de Sitter solutions even in Einstein frame [149, 159, 162]. Instabilities for homogeneous and isotropic perturbations were studied in detail in [160, 162, 165, 166] with subsequent extensions to anisotropic perturbations [167]. Bianchi I universes were studied in [168–170], along with two-dimensional black hole solutions [168] and bouncing cosmologies [169, 170].

In this chapter, we revisit the dynamics of $\mathrm{O}(d,d)$ invariant cosmologies, starting with isotropic and spatially flat metrics before moving on to the anisotropic case. Our focus is on the existence of de Sitter solutions in both the string frame and the Einstein frame, with a time dependent dilaton. Of course, the isotropic case is simpler and much more extensively studied. Within the isotropic framework, string frame de Sitter solutions are known to exist but have been shown to be unstable [160, 165, 166], something we are able to verify using slightly different methods. de Sitter solutions in the Einstein frame have been obtained in the presence of a source [149] and in vacuum [159, 162]. The latter only exist when the theory takes on a very bespoke form at all orders in α'. As we will show, these Einstein frame de Sitter solutions are the only solutions to the vacuum field equations for these particular theories for a finite time period, ensuring their local stability under homogeneous fluctuations. However, the solutions are not eternal in both past and future. This is because the dilaton evolves. After a finite time (towards either the past or future), the solution is kicked into a new dynamical phase where an Einstein frame de Sitter metric is no longer possible. This resonates (!) with the idea of de Sitter as a resonance [151, 152].

These solutions enjoy a natural extension to anisotropic universes thanks to the $\mathrm{O}(d,d)$ symmetry. Here we consider a Bianchi I universe, with a number of different scale factors with corresponding Hubble parameters. In a notable departure from the isotropic framework, the $\mathrm{O}(d,d)$ symmetry now implies an invariance under the exchange of any two of those Hubble parameters, placing

important restrictions on the structure of the underlying theory. Although isotropy is no longer assumed at the level of the theory, we are interested in isotropic solutions: namely, the de Sitter solutions in both string frame and Einstein frame, with a time dependent dilaton. In string frame, de Sitter solutions are possible, provided the anisotropic theory obeys properties analogous to the isotropic case. These solutions are once again unstable. In the Einstein frame, we find conditions for the de Sitter solutions to exist. This mirrors the isotropic case discussed in the previous paragraph, with additional conditions pointing along the anisotropic directions. Remarkably, these extra conditions hold automatically thanks to the $\mathrm{O}(d,d)$ symmetry.

The rest of this chapter is organized as follows. In section 7.2, we review the structure of the underlying theory for an isotropic and spatially flat universe, including sources. When studying vacua, the Einstein frame is arguably the most relevant since the corresponding fluctuations are diagonalised. However, it is often the case that observational probes follow geodesics in the string frame, so it is important to consider both scenarios. In section 7.2.1 we review the derivation and instability of de Sitter solutions in the string frame, providing some added insight into the source of the instability. In section 7.2.2 we demonstrate how de Sitter solutions can also be found in the Einstein frame. As discussed above, these solutions exist for a finite time period, during which time they are stable against homogeneous fluctuations. However, the solutions are not external, and will eventually give way to a new cosmological phase. In section 7.3, we change gears and extend our analysis to include Bianchi I universes. In section 7.3.1 we establish the appropriate conditions for string-frame de Sitter solutions, commenting on the stability. In section 7.3.2 we do the same for de Sitter solutions in the Einstein frame, noting the crucical role played by the $\mathrm{O}(d,d)$ symmetry. In section 7.4, we summarise our conclusions, and extend the results to non-critical dimensions.

7.2 Isotropic cosmologies

In the case of isotropic and spatially flat cosmology, $g_{ij} = a^2(t)\delta_{ij}$, with a vanishing b field, the system simplifies considerably and the action (6.15) can be written in the following compact form [127]:

$$I = \kappa^2 \int \mathrm{d}t\, n e^{-\Phi} \left[-\Phi_n^2 + \frac{1}{\alpha'} f(\sqrt{\alpha'} H) \right], \tag{7.1}$$

where we have introduced the shorthand $\Phi_n = \partial_n \Phi, \Phi_{nn} = \partial_n^2 \Phi, \ldots$ and define the string frame Hubble factor as $H = \partial_n \ln a$. Matching with (6.15) we note that the function f is analytic in a neigbourhood of the origin where it takes the following form

$$f(\sqrt{\alpha'} H) = \sum_{k=1}^{\infty} d_k \alpha'^k H^{2k}, \qquad d_1 = d, \tag{7.2}$$

and the dimensionless coefficients d_k can, in principle, be determined from the c_k and the coefficients of the multitrace operators. Note that f is a manifestly even function in H. This follows from the $\mathrm{O}(d,d)$ symmetry which takes the form of a simple scale factor duality, $a \to 1/a$ and so $H \to -H$, in the homogeneous and isotropic limit.

In the remainder of the chapter, we choose units where $\alpha' = 1$. It will also be convenient to represent the scale factor through its logarithm, $q = \ln a$. The equations of motion for the full system can be obtained from varying the action with respect to the lapse, the dilaton and the scale factor. This yields the equations $E_a = 0$ where $a = n, \Phi, q$ and

$$E_n = \frac{\delta I}{\delta n} = \kappa^2 e^{-\Phi} \left[\Phi_n^2 - L_f(H) \right], \tag{7.3}$$

$$E_\Phi = \frac{\delta I}{\delta \Phi} = \kappa^2 n e^{-\Phi} \left[2\Phi_{nn} - \Phi_n^2 - f(H) \right], \tag{7.4}$$

$$E_q = \frac{\delta I}{\delta q} = -\kappa^2 \frac{\mathrm{d}}{\mathrm{d}t} \left[e^{-\Phi} f'(H) \right]. \tag{7.5}$$

Here, $L_f(H) = H f'(H) - f(H)$ denotes the Legendre transform of f and

$f'(H) = \mathrm{d}f/\mathrm{d}H$. Thanks to the residual diffeomorphisms, these equations are not independent and satisfy the following identity

$$\frac{\mathrm{d}}{\mathrm{d}t}E_n = \Phi_n E_\Phi + H E_q. \tag{7.6}$$

We can include matter in an $\mathrm{O}(d,d)$ invariant way by including a matter action of the form $S_m[\Phi, \mathcal{S}, \chi_I]$, where $\chi_I(t)$ are $\mathrm{O}(d,d)$ invariant matter fields [148, 149]. For a homogeneous and isotropic cosmology, this is equivalent to $S_m[\Phi, q, \chi_I]$. The vacuum equations of motion are now sourced as follows

$$E_n = \bar{\rho}, \quad E_q = -dn\bar{p}, \quad E_\Phi = \frac{1}{2}n\bar{\sigma} \tag{7.7}$$

where the $\mathrm{O}(d,d)$ covariant energy density, pressure and dilaton charge are respectively given by

$$\bar{\rho} = -\frac{\delta S_m}{\delta n}\Big|_{\Phi,q}, \quad \bar{p} = \frac{1}{dn}\frac{\delta S_m}{\delta q}\Big|_{n,\Phi}, \quad \bar{\sigma} = -\frac{2}{n}\frac{\delta S_m}{\delta \Phi}\Big|_{n,q}. \tag{7.8}$$

Note that the $\mathrm{O}(d,d)$ covariant sources differ from their diffeomorphism invariant counterparts by a factor of $\sqrt{-g} = a^d$, namely, $\rho = \bar{\rho}/a^d$, $p = \bar{p}/a^d$, $\sigma = \bar{\sigma}/a^d$. For the pressure, we should also note that p is now defined by holding $\Phi = 2\phi - dq$ fixed when we vary the matter action. This differs from the usual definition of pressure given by

$$p_{\text{standard}} = \frac{1}{dna^d}\frac{\delta S_m}{\delta q}\Big|_{n,\phi},$$

where ϕ is held fixed instead. It is easy enough to show that the two pressures can be related using the dilaton charge, $p_{\text{standard}} = p + \frac{1}{2}\sigma$ [164]. The equations of motion including matter are the following [164]:

$$\Phi_n^2 + HF'(H) - F(H) = 2\kappa^2 e^\Phi \bar{\rho}, \tag{7.9}$$

$$H_n F''(H) - \Phi_n F'(H) = -2d\kappa^2 e^{\Phi} \bar{p}, \tag{7.10}$$

$$2\Phi_{nn} - \Phi_n^2 + F(H) = \kappa^2 e^{\Phi} \bar{\sigma}, \tag{7.11}$$

where $F(H)$ is given by (6.21). The three equations of motion (7.9)-(7.11) can be related via the equation

$$\partial_n \bar{\rho} + dH\bar{p} = \frac{1}{2}\Phi_n \bar{\sigma}. \tag{7.12}$$

Although we have included the matter sources for completeness, in the remainder of this chapter we shall mostly neglect them, since our focus will be on finding consistent solutions to the vacuum field equation. In particular, we will seek conditions for consistent de Sitter solutions for the metric in both the string and the Einstein frame. We refer the reader to [148, 149] for more details on matter couplings.

Let us explain the meaning of string frame and Einstein frame. The string frame and Einstein frame are two different ways of writing the action in string theory or in related gravitational theories (like supergravity). They are related by a conformal transformation – basically a rescaling of the metric – and choosing one frame over the other can make certain calculations or interpretations easier. In the string frame, the action is written in a way that naturally comes from string theory. In this frame, the gravitational field (the metric $g_{\mu\nu}$) couples to the dilaton field ϕ non-minimally. This form comes from the worldsheet formulation of string theory. The string frame is good for describing how strings "see" spacetime because the metric here is the one that appears in the string sigma model [18, 19]. In the Einstein frame, the action is rewritten so that the gravitational part looks like standard general relativity, i.e. the Einstein-Hilbert action. The Einstein frame is generally considered to be the most relevant for observational physics, cosmology, and experiments. It can give us standard general relativity with minimal coupling to matter. The metric in the Einstein frame determines geodesics, redshifts, gravitational

lensing, etc., in a form we can directly compare with astronomical and lab data [8, 9].

7.2.1 dS solutions in the string frame

It was already shown in [127, 145] that string frame de Sitter solutions are possible in this class of α' complete cosmologies for suitable choices of the function $f(H)$. These solutions are necessarily non-perturbative in α', and could not have been found using standard techniques in perturbative string theory. However, it has also been shown that these solutions are unstable under vacuum fluctuations [160, 165, 166]. We will review the derivation of these solutions and add some additional insight into the nature of the instability.

We know that in a neighbourhood of the origin, the function $f(H)$ is even and admits the following Taylor expansion in H^2

$$f(H) = dH^2 + d_2H^4 + \dots \tag{7.13}$$

Although we cannot assume that the function remains analytic for all values of H, we shall assume that it is twice differentiable in order to have well defined field equations. We shall also assume it to be an even function in order to preserve the $\mathrm{O}(d,d)$ symmetry.

Here we are interested in the properties of the function in a neighbourhood of a string frame de Sitter solution with $H = H_c > 0$, for some constant H_c. Whilst it is tempting to motivate this with the current phase of accelerated expansion, it is important to note that we are working in $d+1$ dimensions. For the critical dimension ($d+1 = 10, 26$), we must perform compactification down to four dimensions before making contact with observation. Therefore, we remain agnostic as to the value of H_c.

Setting $E_a = 0$ for $a = n, \Phi, q$, we see that $H = H_c > 0$, for some constant

H_c, if and only if [162]

$$\Phi_n^2 = -f(H_c) = Q^2, \quad f'(H_c) = 0, \tag{7.14}$$

for some real constant Q. This means that $f(H)$ must have an extremum at $H = H_c$. Thanks to the $\mathrm{O}(d,d)$ symmetry, it must also have a extremum at $H = -H_c$. This suggests the following general form

$$f(H) = -Q^2 + \frac{g(H)}{H_c^4}(H^2 - H_c^2)^2 \tag{7.15}$$

for some even function $g(H)$ that admits a Taylor expansion in $H^2 - H_c^2$

$$g(H) = g_0 + g_1(H^2 - H_c^2) + \dots . \tag{7.16}$$

For the form of $f(H)$ given in (7.15) to overlap with the form close to the origin (7.13), we obtain the following constraints

$$g(0) = Q^2, \qquad g'(0) = 0, \qquad g''(0) = 2d + \frac{4Q^2}{H_c^2}. \tag{7.17}$$

It is also important to establish the stability of these de Sitter solutions. Of course, a full analysis of cosmological perturbations requires knowledge of the underlying theory beyond the homogeneous limit under consideration. Although there has been some interesting recent work in this direction [168], it is well known that $\mathrm{O}(d,d)$ invariance in $d+1$ dimensions is at odds (!) with manifest diffeomorphism invariance, as evidenced through double field theory [135, 136] (see also [171] for a related duality observed in cosmological perturbations). Faced with these limitations, we can only consider the stability of solutions under homogeneous perturbations captured by the general action (6.15). Instabilities for homogeneous and isotropic perturbations of string frame de Sitter solutions were identified in [160, 162, 165, 166] with subsequent extensions to anisotropic perturbations [167]. Here we take a subtlely different

approach to those works but draw the same conclusions.

We consider only homogeneous and isotropic perturbations, so that the system of equations $E_i = 0$ remains unchanged. Assuming $f''(H) \neq 0$, these can be written in the following form

$$f'(H) = ce^{\Phi}, \qquad \partial_n H = s\frac{\sqrt{L_f(H)}f'(H)}{f''(H)}, \tag{7.18}$$

where c is a constant of integration. Here $s = \pm 1$ corresponds to the sign of $\dot{\Phi} = d\Phi/dt$, appearing when we take the square root in (7.3). This ability to take either sign was identified as a potential source of error in the stability analysis, as discussed in [166]. Next we will show how the choice of sign plays a crucial role in opening up the unstable channel.

We begin by making a choice of the time co-ordinate by setting the lapse function to unity, $n = 1$, and solve for the Hubble parameter in a neighbourhood of the vacuum, giving

$$H = H_c + \delta H \tag{7.19}$$

where δH satisfies

$$\delta\dot{H} \approx \begin{cases} s\sqrt{H_c f''(H_c)}\delta H^{3/2} & Q = 0 \\ s|Q|\delta H & Q \neq 0 \end{cases} \tag{7.20}$$

For $Q = 0$, this gives the consistent solution

$$\delta H \approx \frac{4}{H_c f''(H_c)(t - t_c)^2} \tag{7.21}$$

with $s = -\mathrm{sgn}(t - t_c)$. We immediately see the unstable channel if and only if the initial time $t_i < t_c$, or equivalently $s = +1$. For $Q \neq 0$, we also get an instability for $s = +1$, as

$$\delta H \approx e^{s|Q|(t-t_c)} \tag{7.22}$$

In all cases, the instability is excited if and only if $\dot{\Phi}(t_i) > 0$. These results are consistent with the instability analysis presented in [162].

7.2.2 dS solutions in Einstein frame

We now turn our attention to the Einstein frame. Einstein frame de Sitter solutions with constant $\dot{\Phi}$ were obtained in [149] in the presence of a source with $\bar{p} = -\bar{\rho}$. Here we shall present an Einstein frame de Sitter metric as solutions to the *vacuum* equations of motion for bespoke choices of the function $f(H)$. Although the solutions are stable, we will also show that the required form of $f(H)$ cannot be smoothly connected to the leading order Neveu-Schwarz theory in a neighbourhood of $H = 0$. This agrees with the conclusions drawn in [159] and [162].

The string frame and the Einstein frame are related by a conformal transformation. Labelling the Einstein frame with the letter "E", we write $\mathbf{g}_{\mu\nu} = \mathcal{A}^2 \mathbf{g}^E_{\mu\nu}$ and so $n = \mathcal{A} n_E$, $a_S = \mathcal{A} a_E$, where $\mathcal{A}$ is the conformal factor to be derived presently. Focussing on the leading order curvature term in the action, we note that

$$\int \mathrm{d}^{1+d}x \sqrt{-\mathbf{g}} e^{-2\phi} R = \int \mathrm{d}^{1+d}x \sqrt{-\mathbf{g}_E} e^{-2\phi} \mathcal{A}^{d-1} \left[R_E + \ldots\right] \tag{7.23}$$

where the ellipsis include derivatives of $\mathcal{A}$ and R_E is the Ricci scalar for the transformed metric. For the latter to coincide with the Einstein frame metric, we must choose $\mathcal{A} = e^{\frac{2\phi}{d-1}}$.

Setting $E_i = 0$ for $i = n, \Phi, q$, we see that the vacuum field equations are equivalent to

$$f'(H) = c e^{\Phi}, \qquad \Phi_n = s\sqrt{L_f(H)} \tag{7.24}$$

where we recall that c is a constant of integration and $s = \pm 1$. We wish to recast these equations in terms of the Hubble parameter defined in the Einstein

frame

$$H_E = \partial_{n_E} q_E \tag{7.25}$$

where $\partial_{n_E} = \frac{1}{n_E}\frac{d}{dt}$ and $q_E = \ln a_E$. From the definition of the generalised dilaton $\Phi = 2\phi - dq$ and the conformal mapping between frames $q - q_E = \ln(n/n_E) = \frac{2\phi}{d-1}$, we can derive the following useful relations

$$\Phi = -q - (d-1)q_E \implies \Phi_n = -H - (d-1)\frac{n_E}{n}H_E \tag{7.26}$$

and

$$\phi = -\frac{(d-1)}{2}(\Phi + dq_E) \implies \frac{n}{n_E} = \frac{e^{-\Phi}}{\mathcal{V}} \tag{7.27}$$

where $\mathcal{V} = a_E^d$ is volume scale factor in Einstein frame.

As first noted in [162], we can inherit an Einstein frame de Sitter vacuum from the string frame de Sitter vacua presented in the previous section, by fixing the dilaton, ϕ, to be constant. This corresponds to the case where $\Phi_n = -dH_c$, with H_c the constant Hubble scale in string frame. These solutions preserve all the de Sitter isometries and would seem to be in contradiction of the classical no-go theorem of [150]. This suggests that the corresponding form of $f(H)$ does not emerge from classical heterotic or type II strings. For this reason, we will focus instead on quasi-de Sitter solutions in Einstein frame, where the dilaton breaks the de Sitter isometries.

After some manipulations, the system of equations (7.24), (7.26) and (7.27) can be expressed as follows

$$e^{\Phi} = \frac{f'(H)}{c}, \qquad \frac{n}{n_E} = \frac{c}{f'(H)\mathcal{V}}, \qquad \partial_{n_E}\mathcal{V} = c\mathcal{F}(H), \qquad \partial_{n_E}H = \frac{c\mathcal{G}(H)}{\mathcal{V}} \tag{7.28}$$

where

$$\mathcal{F}(H) = \frac{d}{1-d}\left[\frac{H + s\sqrt{L}}{f'(H)}\right], \qquad \mathcal{G}(H) = \frac{s\sqrt{L}}{f''(H)} \tag{7.29}$$

and we have assumed $f'(H)f''(H) \neq 0$. Note that the choice $f'(H) \neq 0$ is consistent with a time dependent dilaton. To explore the possibility of a de

Sitter solutions in the Einstein frame, it is convenient to construct

$$\partial_{n_E} H_E = dH_E^2 \left[\frac{\mathcal{V}\partial_{n_E}^2 \mathcal{V}}{(\partial_{n_E}\mathcal{V})^2} - 1 \right] = dH_E^2 \left[\frac{\mathcal{F}'(H)\mathcal{G}(H)}{\mathcal{F}(H)^2} - 1 \right], \qquad c\mathcal{F}(H) \neq 0. \tag{7.30}$$

If we choose the function $f(H)$ such that the right hand of this equation vanishes identically, the Hubble parameter in Einstein frame is guaranteed to be constant, $H_E = H_\star$, for a solution to the vacuum field equations. Thus, the existence of the de Sitter solution in Einstein frame requires that

$$\left(\frac{1}{\mathcal{F}}\right)' + \frac{1}{\mathcal{G}} = 0, \tag{7.31}$$

or in other words, that $f(H)$ satisfies the following differential equation,

$$\frac{d+1}{d-1} sH\sqrt{Hf'-f}f'' + \frac{1}{d-1}(Hf'-f)f'' + \frac{d}{d-1}H^2 f'' + s\sqrt{Hf'-f}f' + \frac{1}{2}Hf'f'' = 0. \tag{7.32}$$

Note further that we can use the third equation in (7.28) to show that the Hubble parameters in the Einstein and string frame are related according to the following formula

$$H_E = \frac{c\mathcal{F}(H)}{d\mathcal{V}}. \tag{7.33}$$

For a suitable choice of $f(H)$ satisfying (7.32), the corresponding constant curvature solution is stable under vacuum perturbations, as it is the *only* solution to (7.30).

Finding a suitable choice of $f(H)$ is not easy as the differential equation (7.32) is challenging to solve in full generality. However, if we assume that $H > 0$, it does admit the following analytic solution when $s = -1$,

$$\bar{f}(H) = (H + H_m)^2 - H_m^2, \tag{7.34}$$

where H_m is a constant [162].

With $f = \bar{f}$ and taking the negative ($s = -1$) root we find that $\mathcal{F} = 0$

and $\mathcal{G} = -H/2$. The vanishing of $\mathcal{F}$ suggests this solution lies outside of the original regime of validility for which (7.32) was derived. Nevertheless, it is interesting to explore the solutions to the system of equations (7.28) in this particular case. It turns out that they imply a constant but vanishing Hubble rate in Einstein frame

$$H_\star = 0, \tag{7.35}$$

consistent with (7.33), along with the following solutions for the string frame Hubble parameter, the dilaton and the lapse,

$$H(t) = H_c \exp\left(-\frac{ct}{2a_0^d}\right), \tag{7.36}$$

$$\Phi(t) = \ln\left[\frac{2}{c}\left(H_c \exp\left(-\frac{ct}{2a_0^d}\right) + H_m\right)\right], \tag{7.37}$$

$$n(t) = \frac{c}{2a_0^d}\left[H_c \exp\left(-\frac{ct}{2a_0^d}\right) + H_m\right]^{-1}, \tag{7.38}$$

where H_c is a dimensionful constant of integration and we have assumed $d \neq 1$ and $c \neq 0$. Clearly the solution runs into a singularity as $H \to -H_m$ although this can be trivially avoided by taking $H_m > 0$.

We have not been able to find an exact solution to (7.32) that gives non-vanishing $\mathcal{F}$ and, correspondingly, $H_\star > 0$. However, numerical solutions are presented in figure 7.1, with the corresponding value of H_E shown in figure 7.2, where we recall that we have set units such that $\alpha' = 1$. We can also

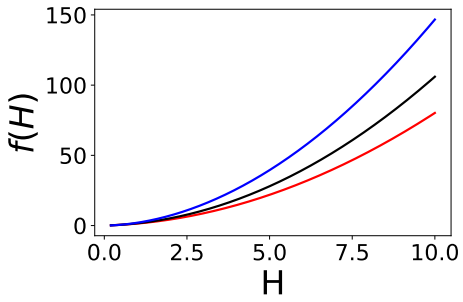

Figure 7.1: *Examples of numerically computed $f(H)$, with the central black curve corresponding to $\bar{f}(H)$.*

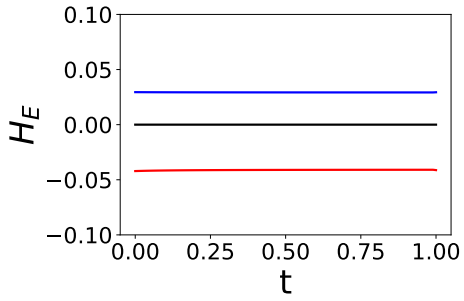

Figure 7.2: *Time evolution of the Einstein-frame Hubble parameter for the $f(H)$ given in Fig. 7.1*

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seek a perturbative solution to (7.32). To this end, we consider

$$f(H) = \bar{f}(H)\,[1 + \epsilon(H)]\,, \tag{7.39}$$

where we assume $|\epsilon| \ll 1$. Plugging this into (7.32) (with $s = -1$) and keeping terms up to linear order in ϵ, we arrive at the linear differential equation

$$(H_m - H)(2H_m - H)H\epsilon''(H) + (2H_m^2 - 3HH_m + 2H^2)\epsilon'(H) - H_m\epsilon(H) = 0 \tag{7.40}$$

with the following solution

$$\epsilon(H) = \frac{C_1 H_m + C_2 H + h_m C_2 \ln\left|\frac{H}{H_m}\right|}{H + 2h_m}, \tag{7.41}$$

where C_1 and C_2 are constants of integration. Note that C_1 can be removed by redefining $H_m \to H_m\,(1 - C_1/2)$ and working to linear order in C_1. Thus we can set $C_1 = 0$ without loss of generality and express (7.41) as a simple function of the ratio $\delta = H/H_m$,

$$\epsilon(H) = C_2\left[\frac{\delta + \ln|\delta|}{\delta + 2}\right]. \tag{7.42}$$

Recall that this expression is valid as long as $|\epsilon(H)| \ll 1$. If we assume $|C_2| \ll 1$ then this condition is violated if and only if δ strays too close to the singularities at $\delta = 0, -2$. More precisely, we have $\epsilon(H) \ll 1$ whenever

$|C_2| \ll 1$, provided

$$\frac{1}{|\delta|} \gg e^{-2/|C_2|} \ , \quad \left|\delta + 2\right| \gg |C_2| \, (2 - \ln 2) \, .$$

We now revisit the field equations (7.28) and (7.31) using the perturbed form of f given by (7.42). To leading order in $|C_2| \ll 1$ (7.31) gives the following expression for Hubble rate in Einstein frame

$$H_E \approx \frac{C_2 c}{4(d-1)\mathcal{V}}. \tag{7.43}$$

If we assume $H_E = H_\star$ for some constant $H_\star$ and set $n_E = 1$, it follows that that we have $\mathcal{V} = a_0^d e^{dH_\star t}$. We can then solve (7.43) explicitly, giving

$$H_\star \approx \frac{1}{dt} W\left(\frac{dC_2 ct}{4(d-1)a_0^d}\right) \approx \frac{C_2 c}{4(d-1)a_0^d} \tag{7.44}$$

where we have used the fact that the Lambert W function $W(x) \approx x$ for small x.[2] We see that we have a constant and *non*-vanishing Hubble rate in the Einstein frame, with the approximation valid as long as $|t| \ll 4(d-1)a_0^d/d|c||C_2|$. Since $H_\star$ is required to be positive in an expanding universe, we assume that $C_2 c > 0$. The remaining fields are dominated by their non-vanishing zeroth order (in ϵ) solutions (7.36)-(7.38). Recall that in a neighbourhood of the origin, perturbative string theory suggests that the function $f(H)$ should admit a Taylor expansion of the form (7.13). We will now show that any solution to the differential equation (7.32) valid for $H > 0$ cannot be analytically continued towards a solution of the form (7.13) in a neighbourhood of $H = 0$.

To see this, we begin by assuming that $f(H)$ admits a Taylor expansions of

[2]The Lambert W function, often written as $W(x)$, is a special function defined as the inverse of the function: $f(\omega) = \omega e^{\omega}$. This means that if $x = \omega e^{\omega}$, then $\omega = W(x)$. For small values of x, we can approximate it using a series expansion: $W(x) = x - x^2 + \frac{3}{2}x^3 - \frac{8}{3}x^4 + \cdots$ [172].

the form (7.13) in a neighbourhood of the origin

$$f(H) = dH^2 + \mathcal{O}(H^4). \tag{7.45}$$

In this neighbourhood, it immediately follows that

$$\frac{\mathcal{F}'(H)\mathcal{G}(H)}{\mathcal{F}(H)^2} = \mathcal{O}(H^2). \tag{7.46}$$

In order to satisfy (7.32), the right hand side of this expression should be equal to unity, which is clearly not possible in a neighbourhood of $H = 0$, regardless of the sign of $s = \pm 1$. Thus, it is not possible to smoothly connect a solution to (7.32) to the desired leading order Neveu-Schwarz theory at low curvatures. However, perhaps this is too demanding. As noted earlier, for the field equations to be well defined, we only need the form of $f(H)$ to be twice differentiable. For any finite N, it is certainly possible to match the first N derivatives of a solution to (7.32) to a function of the form (7.45), at some value of $H = H_{\text{match}} > 0$.

It is, perhaps, optimistic to expect such a matching to emerge from a non-perturbative understanding of string theory. Nevertheless, we could imagine a discontinuity in the higher order derivatives of $f(H)$ to be a remnant of the truncation to the massless sector of the theory. Indeed, if it emerges from string oscillators with string scale masses, we might expect $H_{\text{match}} \sim 1$ (in units of $\alpha' = 1$).

Even if this desired form of $f(H)$ can be justified, it is clear that we can only trust our de Sitter solution in the regime where $H > H_{\text{match}}$. However, recall that H is the Hubble parameter in the *string frame* and this will change over time. If the system evolves into a regime where $H < H_{\text{match}}$, the form of $f(H)$ will no longer admit de Sitter solutions and the Einstein frame de Sitter metric is destabilised.

To avoid this instability, we need to stop H evolving into the unstable region

with $H < H_{\rm match}$ in either time direction. In other words, we need H to have a turning point in its evolution at some $H_{\rm turn} > H_{\rm match}$. At such a turning point, the last equation in (7.28) tells us that $\mathcal{G}(H_{\rm turn}) = 0$. Since $H_{\rm turn} > H_{\rm match}$ by assumption, the differential equation (7.31) ought to be satisfied at $H = H_{\rm turn}$, further implying that $\mathcal{F}(H_{\rm turn}) = 0$ must have a zero at the same point. Since we assume that $f(H)$ is twice differentiable, $\mathcal{F}(H_{\rm turn}) = \mathcal{G}(H_{\rm turn}) = 0$ if and only if $H_{\rm turn} = 0$. This contradicts the underlying assumption that $H_{\rm turn} > H_{\rm match} > 0$. It follows that our de Sitter solution in Einstein frame cannot be eternal.

7.3 Anisotropic cosmologies

In order to gain a deeper insight into the structure of the underlying $\mathrm{O}(d,d)$ invariant theory, at all orders in α', we need to reduce the symmetry of the metric to eliminate any would-be degeneracies. As first step in this direction, we move from an isotropic to an anisotropic framework. Of course, these extensions will be off-shell. On-shell, our goal is still to find solutions that are de Sitter, and therefore isotropic.

For anisotropic cosmologies in $d+1$ spacetime, we consider a string frame line element of the form

$$ds^2 = -n(t)^2 dt^2 + \sum_{i=1}^{d} a_i(t)^2 dx_i^2, \tag{7.47}$$

where $a_i(t)$ is the scale factor along the ith spatial direction. A detailed derivation of the $\mathrm{O}(d,d)$ invariant action requires a careful computation of all the terms in (6.15), including the multitrace operators. Instead of computing that explicitly, we note that the action will take the form

$$I = \kappa^2 \int \mathrm{d}t\; n e^{-\Phi} \left[-\Phi_n^2 + f(H_i) \right], \tag{7.48}$$

where we have assumed units with $\alpha' = 1$. The Hubble parameter along the

ith direction is given by $H_i = \partial_n q_i$ for $q_i = \ln a_i$, while the generalised dilaton is given in terms of the standard dilaton via the following relation

$$\Phi = 2\phi - \sum_i q_i. \tag{7.49}$$

To match with the leading order Neveu-Schwarz action (6.1) at low curvatures, we expect that f admits a Taylor expansion near the origin such that

$$f(H_i) = \sum_i H_i^2 + \mathcal{O}(H^4). \tag{7.50}$$

The $\mathrm{O}(d,d)$ symmetry now includes scale factor duality along each direction, $a_i \to 1/a_i$ and scale factor exchange $a_i \leftrightarrow a_j$ for each i and j. This suggests that the function $f(H_i)$ can also be expressed as

$$f(H_i) = F\left(\hat{e}_i^d(H_j^2)\right), \tag{7.51}$$

where we define the normalised elementary symmetric polynomials

$$\hat{e}_i^d(X_j) = \frac{1}{\binom{d}{i}} \sum_{1 \le j_1 < \ldots j_i \le d} X_{j_1} \ldots X_{j_i} \tag{7.52}$$

for $i = 1, \ldots d$.

Varying the action (7.48) with respect to the lapse, the dilaton and each of the scale factors, we obtain $E_a = 0$ for $a = n, \Phi, q_i$, where

$$E_n = \frac{\delta I}{\delta n} = \kappa^2 e^{-\Phi}\left[\Phi_n^2 - L_f(H_i)\right], \tag{7.53}$$

$$E_\Phi = \frac{\delta I}{\delta \Phi} = \kappa^2 n e^{-\Phi}\left[2\Phi_{nn} - \Phi_n^2 - f(H_i)\right], \tag{7.54}$$

$$E_{q_i} = \frac{\delta I}{\delta q_i} = -\kappa^2 \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{-\Phi} f_i\right], \tag{7.55}$$

where we denote partial derivatives $f_i = \partial f / \partial H_i$ and the Legendre transform $L_f(H_i) = \sum_i H_i f_i - f(H_i)$. It is also convenient to define the average Hubble parameter in string frame as $H = \frac{1}{d}\sum_i H_i$.

7.3.1 dS solutions in the string frame

We now seek solutions to the anisotropic field equations corresponding to the string frame de Sitter vacua. Using similar arguments to the isotropic case in section 7.2.1, we see that there is an isotropic solution to (7.53) to (7.55) with $H_i = H_c > 0$ for all i and some constant H_c if and only if

$$\Phi_n^2 = -f(H_c, \ldots, H_c) = Q^2, \qquad f_i(H_c, \ldots, H_c) = 0, \tag{7.56}$$

where Q is a constant. Given the form of f dictated by the $\mathrm{O}(d,d)$ symmetry (7.51), this suggests that

$$f(H_i) = -Q^2 + \sum_j \frac{g_j(H_l)}{H_c^{4j}} (\hat{e}_j^d(H_l^2) - H_c^{2j})^2, \tag{7.57}$$

where the functions $g_j(H_k)$ take the $\mathrm{O}(d,d)$ symmetric form (7.51) and admit a Taylor expansion in each of $\hat{e}_j^d(H_l^2) - H_c^{2j}$,

$$g_j(H_k) = g_{j0} + \sum_k g_{jk}(\hat{e}_k^d(H_l^2) - H_c^{2k}) + \ldots. \tag{7.58}$$

Close to the origin, the form of f should coincide with (7.50) to recover the leading order Neveu-Schwarz theory at small curvatures. On the isotropic line where $H_i = H$ for all i, the form of f is compatible with the isotropic solutions (7.15), as expected

$$f(H_i)|_{\mathrm{iso}} = -Q^2 + \sum_j \frac{g_j(H, \ldots, H)}{H_c^{4j}} (H^{2j} - H_c^{2j})^2. \tag{7.59}$$

It immediately follows that the corresponding de Sitter solution will once again be unstable to isotropic fluctuations when $\Phi_n(t_i) > 0$.

7.3.2 dS solutions in the Einstein frame

We now switch our attention to the Einstein frame to see if the family of de Sitter solutions found in the isotropic framework [159] carries over to the anisotropic framework. As usual, we perform a conformal transformation to switch to Einstein frame, with $n = \mathcal{A} n_E$ and $a_i = \mathcal{A} a_i^E$ where $\mathcal{A} = e^{\frac{2\phi}{d-1}}$. We therefore have $\ln(n/n_E) = q_i - q_i^E = \frac{2\phi}{d-1}$ for $q_i^E = \ln a_i^E$. This allows us to extract the following useful relation

$$\phi = -\frac{d-1}{2}\left(\Phi + \sum_i q_i^E\right), \tag{7.60}$$

implying

$$\frac{n}{n_E} = \frac{e^{-\Phi}}{\mathcal{V}}, \qquad q_i - q_i^E = -\Phi - \ln \mathcal{V} \tag{7.61}$$

where $\mathcal{V} = \prod_i (a_i^E)^{k_i}$ is the Einstein frame volume scale factor.

In the Einstein frame, we define Hubble parameters $H_i^E = \partial_{n_E} q_i^E$ and the corresponding average $H^E = \frac{1}{d}\sum_i H_i^E$. These are easily related to their string frame counterparts via the relation

$$H_i = \frac{n_E}{n}\left(H_i^E - d H^E\right) - \Phi_n. \tag{7.62}$$

From the field equations (7.53) to (7.55) we can derive expressions for the generalised velocities in Einstein frame

$$\partial_{n_E}\Phi = \frac{e^{-\Phi}}{\mathcal{V}} s\sqrt{L_f}, \qquad H_i^E - H^E = \frac{e^{-\Phi}}{\mathcal{V}}\left(H_i - H\right), \qquad H^E = \frac{e^{-\Phi}}{\mathcal{V}}\frac{\Sigma}{d}\mathcal{F} \tag{7.63}$$

where

$$\mathcal{F} = \frac{d}{1-d}\left(\frac{H + s\sqrt{L_f}}{\Sigma}\right), \qquad \Sigma = \sqrt{\sum_j f_j^2} \tag{7.64}$$

and $s = \pm 1$, depending on the sign of Φ_n. From equation (7.55), we have that

$$e^{-\Phi} f_i = c_i \implies e^{-\Phi}\Sigma = |\underline{c}| \equiv \sqrt{\sum_j c_j^2} \tag{7.65}$$

for some constants c_i. We can now derive the following constraints on the dynamics of the system, analogous to the last two equations in (7.28)

$$\partial_{n_E}\mathcal{V} = |\underline{c}|\mathcal{F}, \qquad \partial_{n_E}H_i = \frac{|\underline{c}|\mathcal{G}_i}{\mathcal{V}} \tag{7.66}$$

where

$$\mathcal{G}_i = \frac{s\sqrt{L_f}\sum_j f^{ij}f_j}{\Sigma} \tag{7.67}$$

and f^{ij} denotes the inverse of the Hessian of the function f, assumed to be well defined. We proceed in a similar way to the isotropic case, and compute the rate of change of the Einstein frame Hubble parameters. We find that

$$\partial_{n_E}H_i^E = d(H^E)^2\left[(1+\xi_i)(\mathcal{O}\ln\mathcal{F}-1)+\mathcal{O}\xi_i\right], \tag{7.68}$$

where

$$\mathcal{O} = \frac{1}{\mathcal{F}}\sum_j \mathcal{G}_j\partial_{H_j}, \tag{7.69}$$

$$\xi_i = \frac{d(H_i-H)}{\Sigma\mathcal{F}}. \tag{7.70}$$

Since $\sum_j \xi_j = 0$, it follows that the rate of change of the average Hubble factor is given by a somewhat simpler formula

$$\partial_{n_E}H^E = d(H^E)^2(\mathcal{O}\ln\mathcal{F}-1), \tag{7.71}$$

which is strongly reminiscent of the corresponding isotropic equation (7.30), as expected. We may alternatively express (7.68) as

$$\partial_{n_E}H_i^E - (1+\xi_i)\partial_{n_E}H^E = d(H^E)^2\mathcal{O}\xi_i. \tag{7.72}$$

Let us now evaluate our system on the isotropic line $H_i = H$, along which we

have $\xi_i = 0$ from (7.70). For consistency, it must follow that

$$(\partial_{n_E} H^E)|_{\rm iso} = d(H^E)^2(\mathcal{O}\ln\mathcal{F} - 1)|_{\rm iso}, \qquad (\mathcal{O}\xi_i)|_{\rm iso} = 0 \tag{7.73}$$

where $|_{\rm iso}$ indicates evaluation on the isotropic line. Requiring the vacuum to be de Sitter in the Einstein frame on the isotropic line, i.e. $H_i^E = H^E = const$, now yields the following constraints

$$\mathcal{O}(\ln\mathcal{F})|_{\rm iso} = 1, \qquad (\mathcal{O}\xi_i)|_{\rm iso} = 0. \tag{7.74}$$

Defining $\mathcal{G} = \sum_i \mathcal{G}_i/d$, we note that

$$\mathcal{O}\left(\ln\mathcal{F}\right) = -\sum_i \mathcal{G}_i \partial_{H_i}\left(\frac{1}{\mathcal{F}}\right), \tag{7.75}$$

$$\mathcal{O}(\xi_i) = \frac{1}{\Sigma\mathcal{F}^2}(\mathcal{G}_i - \mathcal{G}) - \xi_i \mathcal{O}\ln(\Sigma\mathcal{F}), \tag{7.76}$$

and impose (7.74), giving

$$\left[\sum_i \partial_{H_i}\left(\frac{1}{\mathcal{F}}\right) + \frac{1}{\mathcal{G}}\right]_{\rm iso} = 0, \qquad \mathcal{G}_i|_{\rm iso} = \mathcal{G}|_{\rm iso} \neq 0. \tag{7.77}$$

To explore the implications of these results, consider the N dimensional Euclidean space spanned by the Hubble parameters, $(H_1, \ldots, H_d)$. It is convenient to introduce a new set of coordinates (H, v_α), adapted to the isotropic line, defined in terms of the average Hubble parameter $H = \sum_i H_i/d$ and $d-1$ orthogonal directions

$$v_\alpha = \sum_i (H_i - H) n_i^\alpha, \tag{7.78}$$

so that

$$H_i = H + \sum_\alpha n_i^\alpha v_\alpha. \tag{7.79}$$

Here we have introduced an orthonormal set of vectors in Hubble space, n_i^α,

orthogonal to the tangent vector, $u_i = 1/d$, on the isotropic line

$$\sum_i u_i n_i^\alpha = \frac{1}{d}\sum_i n_i^\alpha = 0, \qquad \sum_i n_i^\alpha n_i^\beta = \delta^{\alpha\beta}. \tag{7.80}$$

We can now think of the isotropic line as being parametrized by the average Hubble parameter, H, with each of the remaining coordinates vanishing there, $v_\alpha|_{\rm iso} = 0$. At this point, we recall that the $\mathrm{O}(d,d)$ symmetry implies that $f(H_i)$ is given in terms of the normalised elementary symmetric polynomials, as in (7.51). In a neighbourhood of the isotropic line, we can show that

$$\hat{e}_i^d(H_j^2) = H^{2i} + \frac{i(d+1-2i)}{d(d-1)} H^{2(i-1)} \sum_{\alpha,\beta} \delta^{\alpha\beta} v_\alpha v_\beta + \mathcal{O}(v^3). \tag{7.81}$$

Correspondingly we find that the function f takes the form

$$f(H_i) = f_0(H) + g_0(H) \sum_{\alpha,\beta} \delta^{\alpha\beta} v_\alpha v_\beta + \mathcal{O}(v^3), \tag{7.82}$$

where

$$f_0(H) = F(H^2, \ldots, H^{2d}), \qquad g_0(H) = \sum_{i=1}^{d} \frac{i(d+1-2i)}{d(d-1)} H^{2(i-1)} \frac{\partial F}{\partial \hat{e}_i^d}\bigg|_{\hat{e}_j^d = H^{2j}}. \tag{7.83}$$

For the de Sitter solution to exist, we require the conditions (7.77) to hold. The second of these is equivalent to

$$\Big[\sum u_i \mathcal{G}_i\Big]_{\rm iso} = \mathcal{G}_{\rm iso} \neq 0, \qquad \Big[\sum n_i^\alpha \mathcal{G}_i\Big]_{\rm iso} = 0. \tag{7.84}$$

Making use of the chain rule

$$\frac{\partial}{\partial H} = \sum_i \frac{\partial}{\partial H_i}, \qquad \frac{\partial}{\partial v_\alpha} = \sum_i n_i^{(\alpha)} \frac{\partial}{\partial H_i}, \tag{7.85}$$

and assuming $g_0(H) \neq 0$, we immediately see that the condition (7.84) holds

automatically. Meanwhile, the first condition in (7.77) yields

$$\left[\partial_H\left(\frac{1}{\mathcal{F}}\right)+\frac{1}{\mathcal{G}}\right]_{\rm iso}=0, \tag{7.86}$$

which is equivalent to (7.32) for $f(H)=f_0(H)$. It follows that the Einstein frame de Sitter solutions found in the isotropic framework extend naturally to the anisotropic framework. This is because the underlying $\mathrm{O}(d,d)$ symmetry ensures that deviations from isotropy only enter the theory at $\mathcal{O}(v^2)\sim\mathcal{O}((H_i-H_{j\neq i})^2)$.

7.4 Conclusions

Can we obtain de Sitter solutions in string theory? Although perturbative methods provide some rich phenomenology, they have not been able to uncover de Sitter solutions that are universally accepted as consistent and fully under control. This motivates attempts to probe the non-perturbative structure of string theory. Of course, we do not have the tools to do this in full generality but progress can be made by restricting access to the Neveu-Schwarz fields in the homogeneous limit. This is because they admit an $\mathrm{O}(d,d)$ symmetry that can be extended to higher order operators, extending to all orders in α'.

The underlying structure of these theories is easy to write down whenever we have a homogeneous and isotropic framework, where the $\mathrm{O}(d,d)$ symmetry is equivalent to scale factor dulaity $a\to 1/a$. For a suitable non-linear structure, quasi de Sitter solutions with time dependent dilatons can be found in both the string frame and the Einstein frame. As shown previously [160, 162, 165, 166], homogeneous and isotropic fluctuations about the string frame solutions can trigger an instability, although we have now shown that this is excited if and only if the generalised dilaton is growing at the initial time $\dot{\Phi}(t_i)>0$. The situation in the Einstein frame is even more delicate. Quasi de Sitter solutions can and do exist in the Einstein frame, for particular non-perturbative

extensions of the Neveu-Schwarz theory. However, these extensions do not connect smoothly to the leading order Neveu-Schwarz theory. This means the Einstein frame de Sitter solutions cannot be eternal to both past and future. In other words, a de Sitter solution can survive for a finite period of time, or even a semi-infinite time, but at some point towards either the future or the past, the solution breaks down. The solution enters a new phase of the theory in a neighbourhood of small string frame curvature, where Einstein frame de Sitter solutions are no longer possible. Perhaps this should have been expected, as the breakdown occurs in the perturbative regime where de Sitter solutions are conjectured to be absent [48, 173, 174].

All of these results can be extended to include Bianchi I metrics. The $\mathrm{O}(d,d)$ symmetry now plays a more interesting role than in the isotropic case, where scale factor duality is accompanied by scale factor exchange $a_i \leftrightarrow a_j$. This is crucial, especially for the existence of de Sitter solutions in the Einstein frame where the additional constraints required for existence in an anisotropic setting are satisfied automatically thanks to $\mathrm{O}(d,d)$ symmetry.

A study of anisotropic cosmologies also raises the possibility of some dimensions growing large and others becoming small. In such a scenario, where some dimensions are expanding and some are contracting, it would be interesting to study the existence of de Sitter behaviour along the expanding directions.

Although we have focused our analysis on the case of classical strings in a critical dimension, for which the constant $\Lambda = 0$ in (6.1), the results are easily extended to non-critical dimensions. This is because a non-vanishing $\Lambda \propto D - D_c$ is easily absorbed into a redefinition of the function f that defines theory to all orders in α',

$$f \to f - 2\Lambda. \tag{7.87}$$

This changes the structure of the theory in neighbourhood of the origin, but little else. For the string frame de Sitter solutions in the isotropic framework, this will affect the boundary conditions (7.17), so that we now have $g(0) =$

-2Λ. Similar relations hold in the anisotropic case. The stability analysis is unchanged. For the Einstein frame, we can revisit the question of whether or not the bespoke choice of $f(H)$ can be smoothly connected to the desired form in a neighbourhood of $H = 0$. Focusing on the isotropic case, we set $f(H) = -2\Lambda + dH^2 + \mathcal{O}(H^4)$ and compute

$$\frac{\mathcal{F}'(H)\mathcal{G}(H)}{\mathcal{F}(H)^2} = \frac{d-1}{d} + \mathcal{O}(H). \tag{7.88}$$

Clearly this cannot be set equal to unity, as required for Einstein frame de Sitter solutions with a time dependent dilaton. Thus, we conclude that the situation is qualitatively unchanged from the critical case, $D = D_c$, and one cannot have eternal de Sitter solutions in the Einstein frame. The absence of eternal de Sitter solutions is consistent with the so-called swampland conjectures [48, 173, 174] and of course, the no-go theorem presented in [150].

Chapter 8

Conclusions and Outlook

8.1 Conclusions

One of the most important discoveries in cosmology is that our universe is expanding at an accelerating rate. The driving force is called "dark energy" and is consistent with the vacuum energy or cosmological constant, whose tiny density remains constant over time. When one calculates the contribution of quantum fields to the vacuum energy, a surprising result is that the theoretical result is much larger than the observational result. For example, if we take the cut-off to lie at the TeV scale or beyond, the energy density of the vacuum is then expected to lie at least 60 orders of magnitude higher than the scale of dark energy. How to explain the small cosmological constant (CC) has been a challenging problem in theoretical physics for many years. The arguably most convincing explanation for accelerating expansion is that a positive cosmological constant leads to approximate de Sitter spacetimes [145]. In this thesis, we discuss several different aspects of de Sitter vacua in theoretical cosmology. The thesis has two parts. We consider different string-inspired EFT models which are related to the cosmological constant problem in Part I. Furthermore, we explore the de Sitter vacua solutions in one kind of string cosmology model, namely, $O(d,d)$-invariant cosmology, in Part II.

In Part I we study three aspects of de-Sitter vacua in string-inspired effective field theory models. In Chapter 3, we consider a broad class of $4D$ effective field theories, in which gravity is coupled to multiple four-forms and their dual scalar fields, and membrane sources are charged under corresponding three-form potentials. We list various approaches to obtain sufficiently dense landscapes compatible with current vacuum observations. Furthermore, we introduce a parameter $X = \frac{4M_{pl}^4 \Delta k^2}{T^2}$ and establish the general criteria required to ensure the absolute stability of the Minkowski vacuum. Since the class of models is quite broad and fine-tuning can be avoided mildly, we propose a "probabilistic" argument to explain the cosmological constant, rather than

the dominant anthropic principle [1].

In chapter 3, we establish the parametric conditions for X. In chapter 4 we explore in detail whether or not different models are compatible with the membrane weak gravity conjecture. Although it is true that the models are often incompatible with membrane weak gravity conjecture, we show that this is not always the case and propose an explicit model in which Minkowski space is absolutely stable and the weak gravity conjecture is satisfied. Moreover, we show that this corresponds to an extension of the Bousso-Polchinski model into a generalised DBI action for four-forms [2].

In chapter 5, we generalize the results of vacuum transtions in chapter 3 and 4 to D-dimension including the Gauss-Bonnet term. We find that the Euclidean action of the bounce, B of $Min_+ \to AdS_-$ is proportional to $k_+^{-(D-2)}$, which has a pole as $k_+^2 \to 0$ where k_+^2 is the curvature of the parent vacuum. We find a new decay channel $AdS_+ \to dS_-$ as well. The physical meaning of this new decay channel is still not clear. We will study it in the future [3].

In Part I we studied different aspects of de-Sitter vacua in string-inspired EFT models. It is natural to ask what the properties of de-Sitter vacua in string theory are. In Part II we explore the de-Sitter vacua solutions in one special string cosmology model, namely, $O(d,d)$ invariant cosmology. In chapter 7 we perform an analysis of de Sitter solutions in $O(d,d)$ invariant cosmologies thoroughly. For the homogeneous and isotropic framework we study the conditions for the existence of de Sitter vacuum solutions that are non-perturbative in α' in both the string frame and Einstein frame. We elucidate the nature of the instability in the string frame vacuum. We show that the de Sitter solutions cannot be eternal in the Einstein frame. We then extend the analysis to Bianchi I universes. We show how to extend the theory of homogeneous and isotropic framework to the anisotropic case, allowing de Sitter solutions, and note the crucial role played by the $O(d,d)$ symmetry in satisfying any additional constraints [4].

8.2 Outlook

We end this thesis with a brief discussion for future work on related lines.

8.2.1 Fine-tuning problems

Reducing from ten dimensions to a positively curved (de Sitter) universe with three observed families of chiral fermions, where all moduli are stabilized, remains a challenging objective. We restrict ourselves to the study of intersecting D6-branes models from the perspective of IIA string theory. Specifically, we will address the following two aspects:

Part I. There are at least three fine-tuning problems in theoretical physics: the CC problem, the hierarchy problem and the strong CP problem. However, only few works studied these three problems simultaneously. Therefore, we are considering if the three fine-tuning problems can be "solved" within one stringy model which can get the standard model or at least MSSM [1].

In [175–177], the authors claimed that the following $4D$ potential can explain the strong CP problem

$$V_{EFT}(a) = -\frac{1}{2}X^2 + (ma - \theta_{QCD}\Lambda_{QCD}^2)X + ... \tag{8.1}$$

where X is a four-form flux, a is the QCD axion and Λ_{QCD} is axion decay constant.

When we consider type IIA theory compactified on $\frac{T^6}{Z_2\times Z_2}$ orientifold, we can get $4D$ potential which is rewritten in terms of four-form fluxes. The $4D$ potential has three parts: V_{RR}, V_{NS} and V_{lol}. We noticed that the structure of one term in V_{RR} is similar to (8.1) [178]. Therefore, it suggests that the $\frac{T^6}{Z_2\times Z_2}$ orientifold model in type IIA theory might be used to solve the strong CP problem.

[1] Here, "solved" does not mean that we can really solve these three problems simultaneously. It just means that if we can deal with these three problems within one stringy model.

In [179, 180] the authors claimed that we can obtain stable dS vacua for STU model by uplifting anti-$D6$ brane. (STU model is a special case of type IIA $\frac{T^6}{Z_2\times Z_2}$ orientifold.) Therefore, the cosmologcial constant problem and the strong CP problem could be described in type IIA STU model simultaneously. Furthermore, in [181] the authors considered the mass production of type IIA dS vacua. As an example, they also considered the mechanism of mass production in STU model.

We plan to split this study into a series of steps, following the ideas of [179–181]. First, we will seek supersymmetric Minkowski vacua. The superpotential W of supersymmetric Minkowski vacua satisfy $\partial_i W = 0$ and $W = 0$. Second, we will perform a small downshift ΔW to supersymmetric AdS vacua. Then the total superpotential becomes $W' = W + \Delta W$. The ΔW is so small that the properties of supersymmetric Minkowski vacua can still hold. The downshift is necessary because without it, the vacuum with the observed cosmological constant value, which is uplifted from the Minkowski state with $V = 0$ to the dS state with $V_{dS} = 10^{-120}$, would result in $F^2 \sim 10^{-120}$. This would make the supersymmetry breaking too small. A manageable downshift disentangles supersymmetry breaking from the smallness of the cosmological constant [181]. Third, we will perform an uplift to the dS vacua, using the anti-D6-brane, and then compute the masses of axions, saxions and gravitino [179–181]. Since the gravitino mass can be given by $m_{3/2}^2 = \frac{|V_{AdS}|}{3}$, then if $m_{3/2}$ is about 1 TeV (or 10 TeV), we can "explain" the hierarchy problem in STU model. The extremely small cosmological constant requires that $|V_{AdS}| + V_{\overline{D6}} \approx 0$. Therefore, the cosmological constant problem and the hierarchy problem could be related. In conclusion, the three fine-tuning problems could be "solved" in the type IIA STU model.

One weakness of this project that needs to be acknowledged is that the calculation of uplifting is not reliable. However, ΔW is very small, so the obtained AdS vacua are not deep AdS vacua. Then the calculation of uplifting in this scenario is easier to control than deep AdS vacua. In some sense, this

can be regarded as an indirect reason for the small value of the cosmological constant. Nature gives us an easier problem. Only near Minkowski vacua can the calculations become easier and reliable.

Part II. Intersecting D6-branes in type IIA string theory provide a natural mechanism to realize chiral fermions at D6-brane intersections. Supersymmetric Pati-Salam models arising from intersecting D6-branes on a $\frac{T^6}{Z_2\times Z_2}$ orientifold in type IIA string theory can give a realistic particle model — the Standard Model [182]. So in the future it is natural to ask if we can get the values of some physical quantities from type IIA $\frac{T^6}{Z_2\times Z_2}$ orientifold. For example, in [182], the mass of gravitino has been obtained, so we can use the two relations $m_{3/2}^2 = \frac{|V_{AdS}|}{3}$ and $|V_{AdS}| + V_{\overline{D6}} \approx 0$ to obtain the value of the cosmological constant. In addition, we can use the masses of axions in [182] to constrain the form of (8.1).

One weakness of the calculation in Part II is that in [182] the authors turned off the fluxes. However, in order to do moduli stabilization, we need to turn on the fluxes. Therefore, we need to follow the logic of [182] to get the masses of axions and gravitino with fluxes at first. Then we can obtain the cosmological constant Λ and constrain the form of (8.1).

8.2.2 Gauss-Bonnet term and heterotic string theory

If we use standard Euclidean methods to describe tunneling between vacua, it is well known that up-tunneling from AdS to dS vacua is impossible in GR. However, we recently found that there exists an up-tunneling channel, $AdS_+ \to dS_-$, if we include the contribution from Gauss-Bonnet term [3]. In [183], the authors claimed that the up-tunneling channel, $AdS_+ \to dS_-$ can be used to explain the Hubble tension problem. However, they did not find a concrete example. Therefore, we can explore the following steps:

Dark dimensions and the Hubble tension

We are going to study the model of [3] in dark dimension scenario. Following the logic of [183], we can do compactification for the model of [3] from $5D$ to $4D$ to get a potential V_4 in $4D$. It was recently noted that the Hubble tension problem can be resolved if the cosmological constant parametrizing the dark energy content switches its sign at a critical redshift $z_c \sim 2$ [183]. We can tune the parameter in V_4 to get an up-tunneling channel, $AdS_+ \to dS_-$. Furthermore, if the critical redshift z_c can be tuned to around 2, then we can give a concrete example to solve the Hubble tension problem.

The Gauss-Bonnet term can be obtained from heterotic string theory. In [184], the authors studied the $4D$ Minkowski and dS vacua and moduli stabilisation in heterotic string. There are at least two ways to explore in the future. On the one hand, we can obtain the $4D$ supersymmetric Minkowski vacua and then do the uplift to dS vacua. We can mimic the logic of [181] to get the mass production mechanism of heterotic string theory. Then the hierarchy problem and the cosmological constant problem can be connected in heterotic string theory. On the other hand, we can do dimensional reduction of $10D$ heterotic action to get $5D$ effective action including the Gauss-Bonet term. Similar to the last paragraph, we can do compactification from $5D$ to $4D$ to get the potential V_4. Then following the logic of [183], we can explain the Hubble tension problem. So the connection between the Hubble tension problem and string theory could be built.

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Appendix A

Tunneling rates for Neumann boundary conditions on the three-forms

In semi-classical theory of vacuum decay, transition rates between vacua $\mathcal{M}_+ \to \mathcal{M}_-$ are given by [82, 92, 93]

$$\frac{\Gamma}{\text{Vol}} \sim e^{-B/\hbar} \tag{A.1}$$

where

$$B = S_E(\text{instanton}) - S_E(\text{parent}). \tag{A.2}$$

Here S_E(instanton) is the Euclidean action evaluated on the bubble configurations described above, interpolating between the vacua $\mathcal{M}_+$ and $\mathcal{M}_-$. In contrast, S_E(parent) is the Euclidean action evaluated on the complete parent vacuum, $\mathcal{M}_+$, with no bubbles.

The Euclidean action is

$$S_E = (\int_{0^+}^{r_{max}} dr + \int_{r_{min}}^{0^-} dr)\mathcal{L}_{bulk}(r) + [\mathcal{L}_{bdy}]_{r_{min}}^{r_{max}} + \mathcal{L}_{mem}, \tag{A.3}$$

where the bulk Lagrangian is

$$\mathcal{L}_{bulk} = -\Omega_3\rho^3\left(3M_{pl}^2\left[\frac{1}{\rho^2} + (\frac{\rho'}{\rho})^2\right] - V - \frac{1}{2}Z_{ij}\frac{A^{i'}A^{j'}}{\rho^6} + \sigma_i\frac{A^{i'}}{\rho^3} - \frac{1}{2}\omega^{ij}\phi_i'\phi_j'\right), \tag{A.4}$$

the boundary Lagrangian is

$$\mathcal{L}_{bdy} = \lambda\chi_i A^i - \mu\rho^3\omega^{ij}\phi_i'\phi_j', \tag{A.5}$$

and membrane Lagrangian is

$$\mathcal{L}_{mem} = Q_i A^i(0) + T\rho^3(0) + [\mu\rho^3\omega^{ij}\phi_i'\phi_j']_{0^-}^{0^+}, \tag{A.6}$$

and Q_i is membrane charge of type i and T is membrane tension.

Now for $r > 0$, instanton and parent vacua are both in "+" vacuum, while for $r < 0$ instanton is in "-" vacuum and parent vacuum is still in "+" vac-

uum. The instanton has a membrane while the parent vacuum does not have membrane. So

$$B = \int_{r_{min}^{(-)}}^{0^-} dr\mathcal{L}_{bulk}^{(-)}(r) - \int_{r_{min}^{(+)}}^{0^-} dr\mathcal{L}_{bulk}^{(+)}(r) - [\mathcal{L}_{bdy}^{(-)}(r_{min}^{(-)}) - \mathcal{L}_{bdy}^{(+)}(r_{min}^{(+)})] + \mathcal{L}_{mem}. \tag{A.7}$$

We can compute

$$\int_{r_{min}}^{0^-} dr\mathcal{L}_{bulk}(r) = -\Omega_3 \int_{r_{min}}^{0^-} dr\rho^3 \left(3M_{pl}^2 \left[\frac{1}{\rho^2} + \left(\frac{\rho'}{\rho} \right)^2 \right] - (V + \frac{1}{2} Z_{ij} c^i c^j) + \sigma_i c^i \right), \tag{A.8}$$

where we have assumed that the scalars ϕ_i are constant. Considering (3.13) and (3.15), we have

$$\int_{r_{min}}^{0^-} dr\mathcal{L}_{bulk}(r) = -\Omega_3 \int_{r_{min}}^{0^-} dr\rho^3 \left(3M_{pl}^2 \left[\frac{1}{\rho^2} + \left(\frac{\rho'}{\rho} \right)^2 \right] - 3M_{pl}^2 k^2 + \chi_i c^i \right). \tag{A.9}$$

Now

$$\frac{1}{\rho^2} + \left(\frac{\rho'}{\rho} \right)^2 - k^2 = 2k^2 \frac{\cos^2(k(\epsilon r + r_0))}{\sin^2(k(\epsilon r + r_0))}, \tag{A.10}$$

where $\epsilon = \pm 1$. Thus

$$\int_{r_{min}}^{0^-} dr\mathcal{L}_{bulk}(r) = -\Omega_3 \left(-\frac{6M_{pl}^2}{k^2} \epsilon [\frac{1}{3} \cos^3(k(\epsilon r + r_0))]_{r_{min}}^0 + \chi_i c^i \mathcal{V}_{in} \right), \tag{A.11}$$

where

$$\mathcal{V}_{in} = \int_{r_{min}}^{0} dr\rho^3(r). \tag{A.12}$$

Since $\cos(k(\epsilon r_{min} + r_0)) = \pm 1$ if $\epsilon = \pm 1$, then

$$\int_{r_{min}}^{0} dr\mathcal{L}_{bulk}(r) = -\Omega_3 \left(\frac{2M_{pl}^2}{k^2} [1 - \epsilon \cos^3(kr_0)] + \chi_i c^i \mathcal{V}_{in} \right). \tag{A.13}$$

Moreover, we have

$$\mathcal{L}_{bdy}(r_{min}) = \lambda \chi_i A^i(r_{min}) = \lambda \chi_i (A^i(0) - c^i \mathcal{V}_{in}), \tag{A.14}$$

where we have used (5.43) and

$$\mathcal{L}_{mem} = \Omega_3 \left(Q_i A^i(0) + T\rho^3(0) \right). \tag{A.15}$$

Therefore, the bounce B is given by

$$\begin{aligned} B =& \Omega_3 \Delta \left(\frac{2M_{pl}^2}{k^2} [1 - \epsilon \cos^3(kr_0)] + (1 - \lambda_{r_{min}}) \chi_i c^i \mathcal{V}_{in} + \lambda_{r_{min}} \chi_i A^i(0) \right) \\ &+ Q_i A^i(0) + T\rho^3(0), \end{aligned} \tag{A.16}$$

where $\lambda_{r_{min}}$ denotes the value of λ at r_{min}.

Recall that $\epsilon = 1$ WLOG, then the bounce is

$$\begin{aligned}\frac{B}{\Omega_3} =& 2M_{pl}^2\Delta\left(\frac{1-\cos^3(kr_0)}{k^2}\right) + (1-\lambda_{r_{min}})\Delta(\chi_i c^i \mathcal{V}_{in}) + \lambda_{r_{min}}\chi_i A^i(0) \\ &+ [\lambda_{r_{min}}(\Delta\chi_i) + Q_i]A^i(0) + T\rho^3(0).\end{aligned} \tag{A.17}$$

Considering (3.35),

$$\Delta\chi_i = -\delta_{ii^\star}Q_i, \quad \text{(no sum over } i\text{)}. \tag{A.18}$$

we have

$$\begin{aligned}\frac{B}{\Omega_3} =& 2M_{pl}^2\Delta\left(\frac{1-\cos^3(kr_0)}{k^2}\right) + T\rho^3(0) \\ &+ (1-\lambda_{r_{min}})[\Delta(\chi_i c^i \mathcal{V}_{in}) + Q_i A^i(0)].\end{aligned} \tag{A.19}$$

If we choose the Neumann boundary conditions on the three-forms $\lambda_{r_{min}} = 1$, the bounce B is

$$B = -2M_{pl}^2\Omega_3\Delta\left(\frac{1}{k^2}[\rho'(r)^3]^0_{r_{min}}\right) + \Omega_3 T\rho^3(0), \tag{A.20}$$

Therefore, we can obtain (3.44).

Appendix B

Tunneling rates for general boundary conditions

We can generalize the result of Appendix B to general boundary conditions. For a general choice of boundary condition on the three-forms, the tunneling exponent computed from the geometry of the bounce is given by

$$B = \lambda B_N + (1-\lambda) B_D \tag{B.1}$$

where B_N is the tunneling exponent computed for Neumann boundary conditions ($\lambda = 1$), given by (3.44), and B_D for Dirichlet boundary conditions, given by

$$B_D = B_N - \Omega_3 \sum_i Q_i A^i(0) + \Omega_3 \Delta \left\{ \frac{\sum_i \chi_i c^i}{3k^4} \left[3\rho'(r) - \rho'(r)^3 \right]_{r_{\min}}^0 \right\}. \tag{B.2}$$

Recall that for Neumann boundary conditions ($\lambda = 1$), configurations with $k_+^2 \leq 0$ and $\epsilon_+ = -1$ are always infinitely suppressed; for Dirichlet boundary conditions ($\lambda = 0$), they will also be infinitely suppressed provided $(2V + \sum_i \sigma_i c^i)_+ < 0$. If this is not the case, there will be a catastrophic instability. For configurations of physical interest, the tunneling exponent simplifies. Indeed, B_N is now given by (3.45) for Neumann boundary conditions on the three-forms, and

$$\begin{aligned} B_D =& B_N + \Omega_3 \left[\frac{M_{pl}^2}{3T} \rho(0)^3 \left(v\Delta k^2 - \frac{T^2}{2M_{pl}^4} u \right) - \sum_i Q_i A^i(0) \right] \\ &+ \frac{2\Omega_3}{3k_+^2 k_-^2} \left\{ (u\Delta k^2 - v\langle k^2 \rangle) \left(1 - \frac{M_{pl}^2}{T} \rho(0) \Delta k^2 \right) + \frac{T}{4M_{pl}^2} \rho(0) \left[v\Delta k^2 - 2u\langle k^2 \rangle \right] \right\} \end{aligned} \tag{B.3}$$

for Dirichlet boundary conditions, where $u = -6M_{pl}^2 + \langle (2V + \sum_i \sigma_i c^i)/k^2 \rangle$ and $v = \Delta \left[(2V + \sum_i \sigma_i c^i)/k^2 \right]$.