Unpacking the ‘Service Economy’: Innovation, Imitation and Prospects for Productivity Growth

University of Nottingham
School of Economics

Selim Elbadri
20348391
9022 words

2022

Abstract

It is a well-documented fact that the peak of the manufacturing hump that developing countries face today are lower and take place earlier in the development process compared to those experienced by the early industrialisers. Standard models of structural transformation would predict that these countries are under threat of an equilibrium characterised by a dominant services sector with low productivity growth. This paper explores whether it is possible for countries to avoid this equilibrium; more specifically if the services sector can be a source of productivity growth for developing countries. Using data from the 10-sector Groningen Growth & Development Centre, this paper begins by documenting three facts on services productivity growth. First, there is significant heterogeneity in countries' abilities to grow their services productivity. Second, most of this growth (or lack thereof) is driven by the so-called within channel. Third, services productivity growth exhibits conditional convergence. Next, this paper presents a theoretical model to rationalise these facts. In the model, the social planner faces a standard consumption-investment trade-off as in the Ramsey model. However, the planner also faces a static decision whereby physical investments must be allocated to either imitating the frontier or innovation. This step endogenises productivity growth in services. After solving for the transitional dynamics and the balanced growth path, this paper performs quantitative analyses to explore the role of different variables in determining the trajectory of productivity growth in services and concludes by presenting alleys for future research.

1 Introduction

The Kuznets facts posit that over the development process, countries' share of agriculture value added and employment declines with income at the expense of services. The manufacturing sector takes a different path, experiencing a hump whereby it rises at first, peaks and declines as the country develops [Herrendorf et al., 2014]. It is a well-documented fact that the peak of the manufacturing hump that developing countries face today are lower and take place earlier in the development process compared to those experienced by the early industrialisers [Rodrik, 2016]. This fact, known in the literature as 'premature deindustrialisation', is the point of departure of this paper.

The reallocation of production factors to services early on in the development process can have significant implications for late industrialisers, including the rise of the 'service economy'. The canonical model of structural transformation posits that productivity growth in services is the slowest of the three sectors [Ngai and Pissarides, 2007]. Thus, developing countries are under threat of facing a combination of a dominant services sector with slow productivity growth, possibly leading to a divergence in cross-country income differences with advanced countries. The manufacturing sector has developed a reputation for being a 'growth escalator' because of its ability to generate jobs for unskilled labour and absorb the latest technologies relatively easily from the world technology frontier. Are late industrialisers set to miss out on this 'growth escalator' or can services fill this gap? Do these developing countries specialise in the same industries within services (e.g. banking & finance vs retail) or does the specialisation depend on the height of the hump? Can developing countries avoid a combination of a large and dominant services sector with low productivity growth implied by standard models of structural change? How can countries sustainably grow their services productivity?

Some research has attempted to address whether services can drive growth in developing countries. Ghani & O'Connell, for example, argue that latecomers can benefit from the rise of the 'service economy' to drive their development processes through catch-up provided countries have sound physical infrastructure [Ghani and O'Connell, 2016]. Work by Baccini et al. finds that services do have a strong relationship with economic development, but this is only true for specific industries within services (particularly the high-skill services) [Baccini et al., 2021]. Some policymakers have shared this enthusiasm on the potential for the 'service economy' to drive growth in developing countries [Nayyar et al., 2021]. The evidence is nevertheless inconclusive thus far.

This paper aims to shed light on these questions. I proceed in the following steps. To start with, I document three facts on services labour productivity growth using data from the 10-sector Groningen Growth & Development Centre (GGDC) spanning from 1975 to 2009. First, there is significant heterogeneity in countries' abilities to grow their services labour productivity. Most advanced economies have seen their productivity levels grow between 0 and 2% in the last few decades. Most developing countries, meanwhile, either managed to grow their labour productivities in services fairly quickly (most notably China, India and Egypt) or saw their productivity levels decline (most notably Brazil, Tanzania and Mexico) over the same time period. Performing the Macmillan and Rodrik [McMillan et al., 2014] productivity growth decomposition on the services sector leads to the second fact documented here which is that services productivity growth (or the lack thereof) is mostly an artefact of the so-called 'within' channel which corresponds to productivity growth within different industries inside services as opposed to the ‘between’ channel which involves the reallocation of labour to more productive services industries. Unpacking the sources of ‘within’ channel growth, I find evidence of conditional convergence in the services sector across countries which means that, holding constant the determinants of steady-state level of productivity in services (that is, time-invariant factors captured by country-level fixed effects), countries further away from the frontier tend to grow their services productivity faster. This finding formulates the third fact and suggests that, while convergence may exist, this ultimately depends on factors that vary from country to country.

Next, I build a theoretical framework that proposes a mechanism to rationalise these facts. The model is set in continuous time and there exists a sole factor of production – technology – which is owned by the household and used to produce output. The social planner makes two decisions in the interest of maximising social welfare which is defined as the net present value of household utility. The first decision involves choosing the optimal paths of consumption and investment exactly as in the standard neoclassical growth model. This decision results from an intertemporal optimisation problem and investment here is used to grow the technology stock and, by extension, output. The second decision involves a static optimisation problem where the planner must choose the share of this physical investment allocated to both imitation and innovation. The planner does this by comparing the marginal gains from innovation which is determined by the arrival rate of successfully innovation and the gains from imitating which decline as the stock of technology approaches the frontier technology level. With some simplifying assumptions, the model admits a balanced growth path where technology and output grow at a constant rate once the technology stock eventually reaches the frontier level. Once I solve for the balanced growth path, I analyse the properties of the steady state equilibrium consumption and investment levels in this economy.

In the last section of the paper, I discuss and perform some numerical exercises using the theoretical framework to showcase the impact of different variables on the trajectory of services productivity growth. First, I discuss how to solve for the transitional dynamics computationally. Next, I parameterise the model. In the third subsection, I perform some comparative statics to show how different parameter values shift the trajectory for services productivity growth. I show that growth efficiency and the initial productivity level in services relative to the frontier are two important determinants of services productivity growth - especially for countries whose services productivity is behind the frontier.

The remainder of this study is organised as follows. Section 2 presents a brief overview of the literature on structural transformation and long-run growth. Section 3 presents the three stylised facts that I document, each of which is contained in a subsection. Section 4 presents the theoretical framework that rationalises the facts while Section 5 brings the model to the computer, exploring some numerical exercises of interest. Section 6 concludes the study, laying out some alleys for future doctoral research in the topic.

2 Related Literature

This paper first and foremost speaks to the literature on structural transformation. The seminal contribution of Kuznets [Kuznets, 1957] in documenting long-term empirical patterns of sectoral change accompanying economic growth must therefore be acknowledged. Since then, a vast body of literature attempting to provide theoretical explanations for the Kuznets facts started to emerge, begining with Baumol [Baumol, 1967]. More recent efforts in this line of thought can be split into two broad categories of explanations. The first considers homothetic preferences and sector-biased productivity growth as the source of structural change – a class of models led by Ngai & Pissarides [Ngai and Pissarides, 2007]. The other, meanwhile, pins down structural transformation through non-homothetic preferences and equalised growth rates across sectors, a class of models that can be traced back to Kongsamut, Rebelo & Xie [Kongsamut et al., 2001]. Both explanations derive results that are also consistent with the Kaldor facts, namely that GDP per capita grows at a constant rate and that the capital and labour shares do not change over time.

While sectoral shifts in value-added and employment appear to be taking place in today's developing countries, Rodrik [Rodrik, 2016] shows that today's developing countries' manufacturing sector peaks are lower and taking place at earlier levels of the development process (lower levels of per-capita income) than today's advanced countries did back when they were transforming their respective economies. Several papers have provided models designed to replicate the pattern of premature deindustrialisation, most notably Fujiwara & Matsuyama [Fujiwara and Matsuyama, 2020] and Huneeus & Rogerson [Huneeus and Rogerson, 2020]. While both offer competing mechanisms, a necessary condition in both frameworks is a relatively slow growth rate in services. One of the immediate implications of premature deindustrialisation is that industrialisation opportunities are scarcer for these developing countries. Today's developing world is at risk of facing an equilibrium of growing services industries with slow productivity growth, potentially exacerbating cross-country income differences. Unpacking the sources and challenges of productivity growth in services will therefore be a question of interest for academics and policymakers alike going forward.

Several studies have zoomed in on the services sector. Early efforts include the works of Stigler [Stigler et al., 1956] and Fuchs [Fuchs et al., 1968]. More recently, literature on the topic has expanded, particularly on the intersection between the 'service economy' and human capital. Buera & Kaboski's [Buera et al., 2022] paper is one that has served as an inspiration for this line of thought. They examine the rise of the service economy (particularly skill-intensive service industries) in the US by modelling trade-offs between home and market production for high-skill labour in the context of longer-term productivity gains. The paper by Fang & Herrendorf [Fang and Herrendorf, 2021] is another example, though their work focuses exclusively on China to explain the barriers of growth in high-skill intensive services. While informative, one of the main implications of these papers is that economies should rely on the reallocation of labour to high-skill services as a driver of productivity growth in the sector. One of the facts this paper establishes however, is that the importance of the reallocation of labour to more productive industries inside services in driving services productivity growth is dwarfed by the impact of 'within' channel which captures gains in TFP inside different industries that make up services.

In this paper, I take a different approach from the current literature and model productivity growth in services as a function of a decision to imitate or innovate. This is in line with some of the evidence presented in this paper which is that services exhibit conditional convergence, implying benefits from being further away from the frontier. The decision of whether to imitate or innovate depends on different factors such as how far a country is from the frontier and the innovation rate among other variables. This paper is therefore loosely related to some other strands of literature such as that of cross-country income convergence and more generally on endogenous growth theory. For the former, papers by Barro and Sala-i-Martin [Barro et al., 1991] [Barro and Sala-i Martin, 1992] were influential in developing empirical specifications to test for (both unconditional and conditional) convergence. This is in addition to the paper by Rodrik [Rodrik, 2013] which tests for the presence of (unconditional) convergence in the manufacturing sector across countries. The empirical section below uses this class of reduced form models but applies them specifically to the services sector and its various industries to test for convergence.

The literature on endogenous growth theory is also relevant for this paper. One of the main contributions of this paper is that it endogenises productivity growth in services, partly through the option to innovate. This paper is therefore related to the benchmark growth model presented by Aghion & Howitt [Aghion and Howitt, 1990] which endogenises the decision to dedicate resources to innovation. My model nevertheless differs in that I add an option to imitate as well. This is broadly in line with the findings of Ghan & O'Connell which is that the 'service revolution' is creating opportunities for catch-up in developing countries. Given this tradeoff, the model presented below is in the spirit of the class of models treated in Acemoglu [Acemoglu, 2009]. Abstracting from the interest in human capital, the productivity process in the model environment below loosely follows the setup presented by Vandenbussche et al. [Vandenbussche et al., 2006].

3 Three Facts on Productivity Growth in Services

This section establishes three stylised facts on service sector productivity growth. First, there is significant heterogeneity in countries' ability to grow their services labour productivity. Second, services labour productivity growth is driven by the 'within' channel which reflects growth in productivity inside different services industries. The 'between' channel – which reflects changes in labour allocation within services – plays a far smaller role. Third, inside the 'within' channel, most industries exhibit conditional convergence with the frontier.

3.1 Data

The key data source for this section is the well-known 10-sector Groningen Growth & Development Centre (GGDC, 2014 release). The database offers a unique sectoral breakdown of aggregate data on an annual basis spanning back at least 45 years for a total of 42 countries. Data on sectoral-level employment and both nominal and real value-added is available for these countries. The sectoral breakdown follows International Standard Industrial Classification (ISIC) Rev. 3.1. As such, the services sector is defined as the aggregation of trade, transport, business, government and personal services. In the interest of maximising the number of countries with full time series in the sample, I focus specifically on the years between 1975 and 2009, leaving me with a total of 1,470 country-year observations. For the remainder of this paper, services productivity is defined as:

$$A_{(i)ct} = \frac{RVA_{(i)ct}}{EMP_{(i)ct}} \tag{1}$$

where $RVA_{(i)ct}$ and $EMP_{(i)ct}$ are real value added and employment in services in for country c in year t. It follows that the additional subscript i reflects a within-sector disaggregation of services into the aforementioned industries. The level of development of a country is proxied by real GDP per capita which is calculated using the Penn World Tables 10.0[1].

3.2 Facts

3.2.1 Fact 1: Significant Heterogeneity in Productivity Growth in Services

The first stylised fact to highlight is that there is significant heterogeneity in service sector productivity growth. Table 1 splits the 42 countries into three groups based on how quickly they grew labour productivity in services between 1975 and 2009. The 'high' group is composed of countries with mean growth in services labour productivity above 2.0% while the 'moderate' group is the set of countries with mean growth between 0 and 2.0%. The 'contraction' group, meanwhile, is the group of countries that saw a mean decline in services labour productivity growth. Looking at both the 'high' and 'contraction' groups, one would find it difficult to immediately draw out similarities within the groups. For example, we observe a geographical mix within groups, with no specific continent particularly dominating either group. This is also true for the 'moderate' group, though most advanced countries are placed here. Indeed, these countries are seeing slow growth in their economies partly because they have dominant services sectors. Some exceptions in the 'moderate' group are Ghana, Thailand and Colombia, all of which are developing countries.

Such heterogeneity in experiences already hints at some of the implications of 'premature deindustrialisation'. As the peak in manufacturing employment begins to fade, the employment share of the services sector will start to grow in line with the so-called 'Kuznets facts'. Less than a third of the countries in the database have seen average growth in services productivity exceed 2.0%, meaning that the threat of having a dominant services sector with stagnant or declining productivity is certainly conceivable for developing countries.

3.2.2 Fact 2: Services Productivity Growth is Driven by the 'Within' Channel

Naturally, one would ask how countries can avoid this combination by maintaining average growth above 2.0%. One would think that countries that were able to reallocate their factors of production (particularly labour) to industries within services that are more productive are those that managed to grow their productivities the fastest in the last few decades. Decomposing the sources of services labour productivity growth would clarify whether this is actually the case. To do so, I follow the decomposition method suggested by Macmillan & Rodrik [McMillan et al., 2014], applying it specifically to the services sector as opposed to the entire economy:

$$\Delta A_{ct} = \sum_{i=n} \theta_{ict-k} \Delta A_{ict} + \sum_{i=n} A_{ict} \Delta \theta_{ict} \tag{2}$$

The Δ represents the change operator, while A_{ct} is labour productivity in services for country c at time t. θ_{ict} is the employment share of industry i (inside services) over the entire sector's employment

---

[1] Real GDP per capita is calculated by dividing output-side real GDP at chained PPPs by the population

High		**Moderate**		**Contraction**	
Country	Average Services Productivity Growth	Country	Average Services Productivity Growth	Country	Average Services Productivity Growth
CHN	4.9	JPN	1.9	ARG	-0.2
BWA	4.1	GBR	1.6	BRA	-0.8
TWN	3.9	SWE	1.5	CRI	-1.0
ZMB	3.7	GHA	1.1	VEN	-1.2
MYS	3.6	THA	1.0	MEX	-1.5
IND	3.5	USA	1.0	SEN	-1.8
EGY	3.5	FRA	0.9	KEN	-1.9
NGA	3.0	KOR	0.9	BOL	-1.9
HKG	2.4	CHL	0.9	PER	-2.0
SGP	2.2	DNK	0.8	TZA	-2.1
IDN	2.2	PHL	0.7	MWI	-2.8
MUS	2.0	ETH	0.6		
		MOR	0.5		
		NLD	0.4		
		ZAF	0.1		
		ITA	0.0		
		COL	0.0		
		ESP	0.0		

Table 1: Mean Services Labour Productivity Growth by Group (1975-2009)

Notes. Average services productivity growth is the mean year-on-year labour productivity growth of the services sector between 1975 and 2009 for each country. The 'high' group refers to countries with average services productivity growth above 2.0%. The 'moderate' group refers to countries with an average growth rate between 0 and 2.0%. The 'contraction' group refers to the group of countries that saw an average decline in productivity.

while A_{ict} is the industry-level labour productivity. t will be 2009 while $t-k$ will be 1975. Combined, (2) represents a statistical decomposition in services labour productivity into the two main forces that drive sectoral productivity growth – the 'within' and 'between' forces. The first term in (2) reflects the weighted sum of the 'within' force which aggregates the increases in labour productivity inside a given services industry over time. The second term, meanwhile, reflects the 'between' factor which captures the change in employment shares or labour reallocation within services over time.

The statistical decomposition above allows one to assess the drivers of services productivity growth across time. Performing this exercise leads to the second stylised fact of this paper, which is that productivity growth in services (or lack thereof) is primarily driven by the 'within' as opposed to the 'between' component.

Figure 1 plots the share of the increase in service sector labour productivity between 1975 and 2009 driven by the 'within' and 'between' shares. The takeaway from this is that virtually all the countries included (with the exceptions of Argentina and Morocco) rely on the 'within' channel for labour productivity growth as opposed to the 'between' channel. Surprisingly, this result appears to be robust to the different groups, meaning that even countries that saw contractions did so because

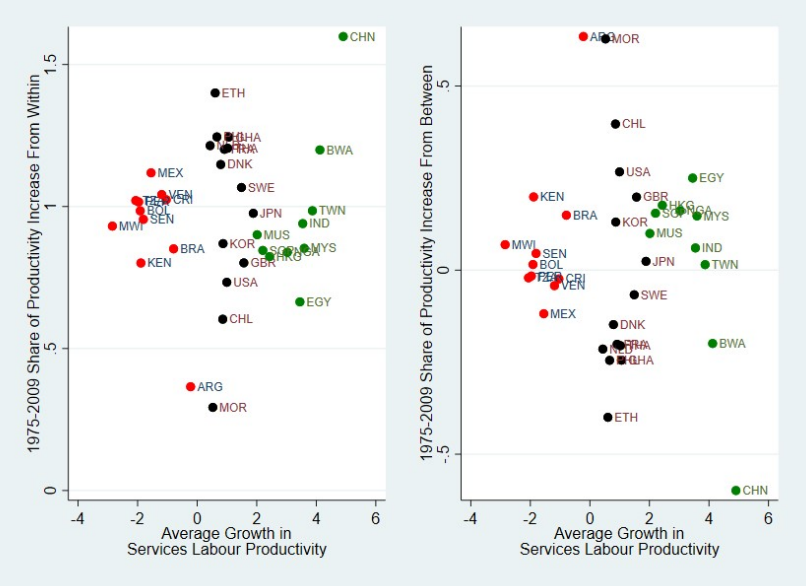

Figure 1: 1975-2009 Share of Productivity Increase Driven by Within and Between by Group
Notes. This figure plots the results of the statistical decomposition of services labour productivity growth between 1975 and 2009. The y-axis the share of the increase in services labour productivity between 1975 and 2009 driven by the 'within' (LHS) and 'between' components (RHS). The x-axis is mean year-on-year growth in services labour productivity between 1975 and 2009. Red countries are the contraction group, black countries are the moderate group and green countries are the high group. Colombia, Zambia, India, Italy, South Africa and Spain are excluded due to excessively large positive and negative shares. The interaction term is not included because it plays a minor role in this exercise.

of declines in the 'within' industry productivity. One can even argue that these results suggest that, for many countries that experienced growth in services labour productivity between 1975 and 2009 like Ethopia, Ghana and some European countries like Sweden and Denmark, their respective growth rates were dragged down by the 'between' component. This can be interpreted as the reallocation of labour within services from more productive industries to less productive ones. This is particularly true for China.

Table 2 presents some of the statistics on the decomposition, namely the mean, median and standard deviations of the share of the services labour productivity increase between 1975 and 2009 driven by the 'between' and 'within' forces as well as the interaction term. It confirms that, on average, the 'within' component drives over nine-tenths of the services labour productivity increase between the two periods. This is true for all three groups, reinforcing the observation in Figure 1 that this trend is robust across all countries regardless of how well they performed in those three and a half decades. Another thing to note is that the interaction term is virtually non-existent. This measures the part of the productivity increase not explained by either one of the two forces which is captured by the difference between the implied productivity increase calculated from the decomposition and the actual observed data. The standard deviations, while fairly sizable, suggest that these conclusions hold for most of the countries.

	Contraction	Moderate	High
Mean			
Within	0.92	1.00	0.96
Between	0.08	-0.01	0.03
Interaction	0.00	0.01	0.01
Median			
Within	0.98	1.11	0.88
Between	0.02	-0.11	0.12
Interaction	0.00	0.00	0.00
Standard Deviation			
Within	0.20	0.31	0.26
Between	0.20	0.30	0.25
Interaction	0.00	0.02	0.03

Table 2: Statistics on the Decomposition of Productivity Increase (1975-2009)

Notes. This table presents the mean, median and standard deviations of the share of the services labour productivity increase between 1975 and 2009 driven by the 'between' and 'within' forces as well as the interaction term. The interaction term is the difference between the productivity increase implied from the decomposition and the actual productivity increase observed in the data. Colombia, Zambia, India, Italy, South Africa and Spain are excluded due to excessively large positive and negative shares.

Exploring the data further, it is clear that the relative importance of the 'between' and 'within' forces in driving productivity growth change over the development process, but do so differently for the different groups. Figure 2 plots these relationships by country groups. The 'moderate' and 'high' group countries both see the relative importance of the 'within' channel decline vis-a-vis the 'between' channel as they get richer. This is broadly consistent with the findings of Buera et al. [Buera et al., 2022] and in turn means that as countries develop, labour is reallocated to more productive industries within services. This trend appears to not hold for the 'contraction' group, though in all cases the 'within' channel seems to still maintain dominance regardless of where in the development process a country is.

It is clear from the decomposition above that the fundamental driver of productivity growth in services is the 'within' as opposed to the 'between' channel. Figure 3 which plots average labour productivity growth in services against the employment share difference presents more evidence against the importance of the labour reallocation story. The main insight from the figure is that the flow of labour into or out of services as a whole is not related to a country's success in growing labour productivity in the sector. Both Zambia and Hong Kong, for instance, recorded two of the highest growth rates in services labour productivity in the sample over the years, though the former did so with a net decline in services share of employment while the latter saw the employment share of services rise by over 40 percentage points. Similarly, the magnitude of the flow of labour into the sector does not seem to affect the lack of productivity growth in the 'contraction' group with Brazil and Bolivia, for instance, seeing fairly sizable increases in services employment share compared to Venezuela and Senegal who have seen comparatively smaller increases. The 'moderate' group also

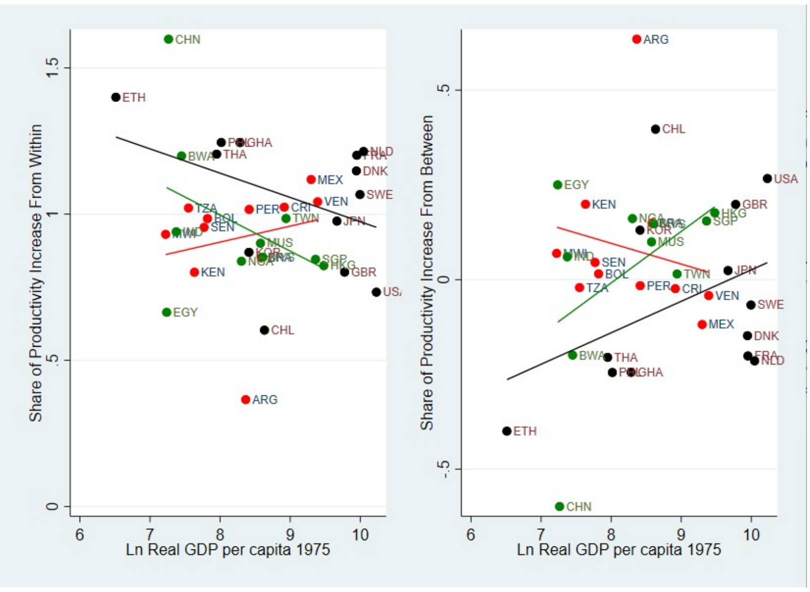

Figure 2: 1975-2009 Within and Between Shares and Real GDP per capita by Group

Notes. This figure plots the share of the productivity increase between 1975 and 2009 on the y-axis against the natural logarithm of real GDP per capita in 1975 on the x-axis by country groups. Red countries are the contraction group, black countries are the moderate group and green countries are the high group. The lines of best fit are group-specific. Colombia, Zambia, India, Italy, South Africa and Spain are excluded due to excessively large positive and negative shares.

follows the same trend.

3.2.3 Fact 3: The 'Within' Channel's Effect is Partly Driven by Conditional Convergence

The main takeaways thus far are that there is significant heterogeneity in countries' successes in growing their services labour productivity and that the 'within' channel is the dominant driver of productivity growth. What remains is pinning down how exactly the 'within' channel operates and drives labour productivity growth in services. Here it is important to distinguish between the two main sources of 'within' sector growth, which are convergence and country-sector-specific innovation. The former would suggest that growth, particularly in developing countries, is underpinned by knowledge diffusion and imitation of advanced countries' technologies used in services. The latter, on the other hand, would suggest that labour productivity growth is driven by country-specific trends, namely innovation within sectors that are specific to different countries. To test for the presence of convergence, I borrow the following regression model from the cross-country growth empirics literature:

$$g_{(i)ct,t+1} = \alpha + \beta ln\_initial\_A_{(i)c} + \chi_t + \epsilon_{(i)ct} \tag{3}$$

where $g_{(i)ct,t+1}$ is year-on-year growth in services labour productivity in country c between years

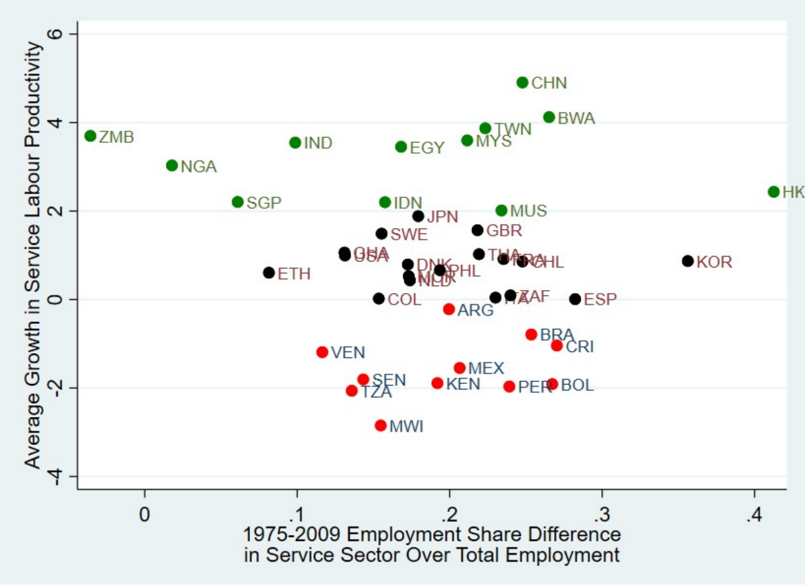

Figure 3: 1975-2009 Effect Of Reallocation of Labour on Service Productivity Growth by Group
Notes. This x-axis is the change in the employment share in services over total employment in the economy between 1975 and 2009. The y-axis is the mean year-on-year growth in services labour productivity over the same time period. Red countries are the contraction group, black countries are the moderate group and green countries are the high group. All countries are included.

t and $t+1$. As before, the subscript i is present if this regression is run for the different services industries. The explanatory variable is $ln\_initial\_A_{(i)c}$ which represents the natural logarithm of the initial productivity level in country c. The coefficient of interest is therefore the corresponding β, which is referred to as β-convergence in the cross-country growth literature. As a partial remedy for potential cross-section dependence, I include χ_t which represents year fixed effects that absorb changes in global annual trends in the services sector and its industries. Finally, $\epsilon_{(i)ct}$ is the error term that is specific to each industry, country and year. The essence of this model is to test whether there exists a link between initial labour productivity levels and future growth rates. The presence of convergence would imply a negative and statistically significant coefficient on the the initial productivity level as countries with lower initial productivity should imply higher growth later on.

While in theory (3) would represent the appropriate test for convergence, it generates a problem of statistical power. Due to fairly limited number of countries in the database (small N), there is limited variation in the explanatory variable because it measures initial labour productivity levels which do not change over time. The solution chosen here is to create a new variable, $ln[\frac{A_{(i)ct}}{Frontier\_A_{(i)t}}]$, which is ratio of a country's labour productivity level and the frontier labour productivity level of that year for services or a given industry. The frontier here – which is normalised to 1 – is defined as the median labour productivity level of the top 5 highest labour productivities in a given year which is equivalent to the third highest labour productivity in the year. The regression estimated is therefore:

$$g_{(i)ct,t+1} = \alpha + \beta ln[\frac{A_{(i)ct}}{Frontier\_A_{(i)t}}] + \chi_t + \epsilon_{(i)ct} \tag{4}$$

One important note around interpretation is that (4) purposefully excludes country fixed effects because it tests for *unconditional* convergence. In other words, it tests if the gap between two countries' labour productivity would decline (i.e. converge) regardless of these countries' structural characteristics and environment.

	(1) Services	(2) Trade	(3) Transport	(4) Business	(5) Government	(6) Personal
Ln Services Distance	-0.04					
	(0.05)					
Ln Trade Distance		-0.04				
		(0.09)				
Ln Transport Distance			-0.08			
			(0.07)			
Ln Business Distance				-0.16		
				(0.11)		
Ln Government Distance					-0.42***	
					(0.07)	
Ln Personal Distance						-0.15
						(0.09)
N	1394	1394	1394	1394	1088	1326
R^2	0.046	0.043	0.046	0.033	0.051	0.034

Table 3: Test for Unconditional Convergence

Notes. This table presents the standard OLS cross-section regression results of equation (4). All columns include year fixed effects. Column (1) regresses mean year-on-year growth rate of services labour productivity on the natural logarithm of the ratio of the distance to the frontier. Columns (2)-(6) run the same regressions but for specific industries within services. No country-level fixed effects are included. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 3 presents the regression results of equation (4), where the first column is the result for services while columns (2)-(6) are for the breakdown of services by industry. The first observation is that the coefficent for services, being negative and close to zero, is statistically insignificant. This suggests that there is no evidence of unconditional convergence in services which is indeed consistent with Rodrik [Rodrik, 2013] who finds that unconditional convergence is only evident in manufacturing.

Second, there is no evidence of unconditional convergence in four of the five services industries. The only industry that does show unconditional convergence is government services, and this is statistically significant at the 1.0% level. Overall however, Table 3 shows insufficient evidence for one to argue that unconditional convergence is the main driver of 'within' productivity growth in services.

We can also run a test for conditional convergence which simply adds country fixed effects to equation (4) such that the estimated model becomes:

$$g_{(i)ct,t+1} = \alpha + \beta ln[\frac{A_{(i)ct}}{Frontier\_A_{(i)t}}] + \chi_t + \gamma_c + \epsilon_{(i)ct} \tag{5}$$

where γ_c are the country fixed effects. Examining the conditional distribution of the β-convergence coefficient is deemed more appropriate in the cross-country growth empirics literature [Barro et al., 1991] as it tests for convergence conditioning on countries' characteristics. Including country fixed effects would hold constant variations in countries' structural characteristics such as institutions and environment, hence making the test one of conditional convergence. The results of equation (5) are presented in Table 4. Once the country fixed effects are included, the beta coefficients for services as a whole and four of the five industries are both negative and statistically significant. The only exception is government services where evidence of unconditional convergence emerged in Table 3.

The evidence presented here suggests that the argument of conditional convergence being the driver of 'within' productivity growth in the services industry should be taken seriously. Indeed, upon observing the country groups in Table 1, it is easy to see that the more industrialised countries – ones closer to the frontier such as Japan, the US, Sweden and Great Britain – see slower services productivity growth compared to the 'high' group which is mostly composed of developing countries. Crucially, the presence of *conditional* convergence captures the difference between the 'high' and 'contraction' groups which are both mainly composed of developing countries. Countries' policies, institutions, environment and geography distinguish between developing countries that grow their services productivity. The next section is concerned with building a model that replicates these three stylised facts presented here.

4 Model

The purpose of this section is to present a theoretical framework that sheds light on the mechanisms driving the facts presented in the empirical section. The planner economy is populated with one representative household endowed with just one factor of production that is, technology, which is used to produce output in services. The planner faces two problems. The first is an intertemporal problem where the planner must maximise lifetime utility. The second problem involves a static optimisation problem where the planner must maximise productivity growth through the allocation of physical investments. Labour does not feature in this model and the population is normalised to unity.

	(1) Services	(2) Trade	(3) Transport	(4) Business	(5) Government	(6) Personal
Ln Services Distance	-2.30*** (0.48)					
Ln Trade Distance		-3.50*** (0.67)				
Ln Transport Distance			-3.12*** (0.66)			
Ln Business Distance				-2.33*** (0.68)		
Ln Government Distance					0.32 (0.50)	
Ln Personal Distance						-2.91*** (0.70)
N	1394	1394	1394	1394	1088	1326
R^2	0.017	0.020	0.016	0.008	0.000	0.013

Table 4: Test for Conditional Convergence

Notes. This table presents the fixed effects panel regression results of equation (5), so all columns include country and time fixed effects. Column (1) regresses mean year-on-year growth rate of services labour productivity on the natural logarithm of the ratio of the distance to the frontier. Columns (2)-(6) run the same regressions but for specific industries within services. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

4.1 Environment

4.1.1 Utility

The social planner's goal is to maximise the household's lifetime utility. The household's utility function u(c) = log(c) is strictly increasing, concave and twice differentiable with u′(c)>0 and u″(c)<0:

$$\max_{\{c_t\}} U = \int_0^\infty e^{-\rho t} \log c_t dt \tag{6}$$

c_t and ρ represent consumption and the subjective discount factor, respectively. The planner maximises lifetime household utility facing the following resource constraint:

$$y_t = c_t + i_t \tag{7}$$

where y_t and i_t represent output in services and investment, respectively. This simple maximisation problem already implies an intertemporal trade-off between consumption and investment observed in the standard neoclassical growth model (NGM). The planner can increase consumption at the expense of investment at time t, implying slower growth in future output.

4.1.2 Technology

The planner makes two decisions in this model. The first involves the choice of consumption which was just introduced. The second corresponds to the choice of how much to innovate and imitate. To see this, first consider the stock of technology which features directly in the production function of y_t, so it is used to produce a final output in services. I assume the following technology:

$$y_t = a_t \tag{8}$$

where the key property is monotonicity. In this model, the stock of technology in the economy is the sum of its first-order lag and a growth function. The technology growth function is simply defined as investment which is formed of two inputs. The first is z_t which is the physical investment into technology decided by the planner. The second is $\Gamma(a, \delta, t)$ which represents the return per unit of investment and depends on whether the planner decides to imitate or innovate:

$$\dot{a}_t = i_t = z_t \Gamma(a, \delta, t) \tag{9}$$

where:

$$\Gamma(\delta, t, a) = \max_{\{\delta_t\}} \gamma \left[\delta_t \left(\bar{a}_t - a_t \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \delta_t) \eta^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \tag{10}$$

and:

$$a_0 > 0 \tag{11}$$

The social planner chooses δ_t which is simply a distribution parameter. It determines the shares of the physical investment allocated to imitation and innovation. The choice is endogenous in that the planner makes the decision simply by comparing the marginal gains between both forms of technology growth. The first term in the square bracket in (10) represents the marginal gains from imitating which is determined by how far the current technology stock is from the frontier technology stock, $\bar{a}_t$. The alternative decision is to innovate where with probability η, the poisson arrival rate of innovation, the planner successfully innovates and is able to increase the technology stock. The elasticity of substitution between imitation and innovation as drivers of technology growth is regulated by σ which for now is assumed greater than 1 (gross substitutability). γ is a growth efficiency parameter that captures the institutional and geographical environment and is therefore time-invariant. This corresponds to factors captured in the country-level fixed effects. Finally, the frontier is assumed to grow at a constant rate $g_{\bar{a}}$:

$$\bar{a}_t = \bar{a}_0 e^{g_{\bar{a}} t} \tag{12}$$

It is important to reiterate that $\Gamma(\delta, a, t)$ captures the channel of productivity growth at the sectoral level that abstracts from the factor reallocation effects and focusing exclusively on improvements in the production technology (within channel). One can already see that this is incorporates the final fact which documents that some countries are able to catch-up with their services sector while others do not. This heterogeneity is dealt with using γ which varies by country. Another thing to note is that $\Gamma(a, \delta, t)$ shrinks as a_t rises because of decreasing marginal returns from imitation. Once the planner starts to innovate, $\Gamma(a, \delta, t)$ is simply just a function of the constant arrival rate of innovation and the growth efficiency parameters. The social planner therefore faces a static optimisation problem of maximising technology growth and then, conditional on this choice, maximises the household's welfare. Note that the no-ponzi condition is also imposed for the intertemporal problem:

$$\lim_{t \to \infty} \left[a_t e^{-\int_0^t r_s ds} \right] \geq 0 \tag{13}$$

The standard interpretation holds, which is that the no-ponzi condition ensures that in the limit, the net present value discounted at rate $r(s)$ of the household's stock of technology, a_t, cannot be negative. This constraint rules out the consumption path that involves accumulating infinite debts.

4.2 Equilibrium

Given the model environment, the following definition of a competitive equilibrium is imposed:

Definition 4.1 (Equilibrium) *A planner equilibrium consists of paths of consumption, technology, allocation to imitation, $[c_t, a_t, \delta_t]_{t=0}^{\infty}$ such that:*

1. *The social planner maximises the net present value of the household's utility by choosing optimal paths of physical investment and consumption $[z_t, c_t]_{t=0}^{\infty}$;*
2. *The social planner maximises technology growth by choosing the path of allocation to imitation $[\delta_t]_{t=0}^{\infty}$ taking the frontier technology, the flow rate of innovation and the current level of technology as given;*
3. *There is market clearing such that (7) binds.*

The social planner makes two decisions, beginning with the choice of δ_t for the optimal allocation of the physical investment. The solution to the static problem is a sequence of the following actions:

$$\delta_t^* = \begin{cases} 1 & \text{if} \quad \bar{a}_0 e^{g_{\bar{a}} t} - a_t > \eta \\ \text{mix} & \text{if} \quad \bar{a}_0 e^{g_{\bar{a}} t} - a_t = \eta \\ 0 & \text{if} \quad \bar{a}_0 e^{g_{\bar{a}} t} - a_t < \eta \end{cases} \tag{14}$$

Since the current stock of technology, the arrival rate of innovation and the frontier technology are all given, the planner simply compares the gains from innovating and imitating. The social planner chooses $\delta_t = 1$ if the current technology stock is far below the frontier to the extent that it outweighs the potential gains from innovating which is represented by arrival rate of innovation η. Conversely, the planner can choose $\delta_t = 0$ if it is too close to the frontier that it has less to gain from imitating the frontier than if the planner allocates the physical investment to innovating. The only exception to this is if the gains from imitating and innovating are exactly equal. While this is a rare scenario, one can think about this as a country whose productivity in services is high, but not quite at the frontier. It can potentially imitate, but it can also innovate because with some probability it might be successful. In this case, the planner would opt for a mixed strategy. Any share of δ_t would maximise technology growth so the planner can just randomise.

Before proceeding with the equilibrium paths of c_t and i_t, I introduce the following assumption for the remainder of the paper.

Assumption 1 *Perfect substitutability between innovation and imitation:*

$$\sigma = \infty \tag{15}$$

Assumption 1 is stronger than assuming gross substitutability between imitation and innovation. It is imposed for analytical tractability and does not greatly compromise the intuition behind the model. This assumption can be relaxed in future research[2].

4.2.1 Transitional Dynamics

The transitional dynamics in this model refer to the case where the planner's optimal allocation of the physical investment is to imitation. The equilibrium consumption and investment functions will naturally differ in this case compared to the case where the technology stock has reached the frontier and the planner prefers to innovate ($\delta^* = 0$), in which case the economy enters a balanced growth path and a steady state can be computed using normalised values for consumption and physical investment.

Proposition 1 *The equilibrium paths of consumption, investment and physical investment* $[c_t, i_t, z_t]_{t=0}^{\infty}$ *for the case where the planner prefers to imitate* ($\delta^* = 1$) *evolve according to the following equations:*

1.
$$\frac{\dot{c}_t}{c_t} = -\rho + \frac{\bar{a}_t g_{\bar{a}-(\bar{a}_t - a_t)\gamma z_t}}{(\bar{a}_t - a_t)} - \frac{\dot{\mu}_t}{\mu_t} \tag{16}$$

2.
$$\dot{a}_t = i_t = z_t \gamma (\bar{a}_t - a_t) \tag{17}$$

3.
$$z_t = \frac{a_t - c_t}{\gamma(\bar{a}_t - a_t)} \tag{18}$$

---

[2] See Section 6.4 for a discussion on this assumption.

where $-\frac{\dot{\mu}_t}{\mu_t} = \frac{1}{\gamma(\bar{a}_t - a_t)} + \frac{a_t}{\gamma(\bar{a}_t - a_t)^2} - \frac{c_t}{\gamma(\bar{a}_t - a_t)^2}$

Proof 1 *See Appendix (Proof 1)* ■

(16) is the standard Euler equation which dictates the equilibrium growth in consumption in the economy. μ_t, the co-state variable in this optimal control problem, represents the marginal utility coming from an additional unit increase in technology. Meanwhile, (17) reflects the optimal choice of the static problem where the planner prefers to imitate. As such, growth in productivity will be a function of the physical investment and distance to the frontier. The further away the domestic productivity is from the frontier, the faster productivity growth is. The optimal choice of physical investment is then retrieved from the resource constraint. (16), (17) and (18) dictate how consumption, investment and physical investment evolve when the planner finds it optimal to allocate all physical investments to imitation.

4.2.2 Balanced Growth Path

Thus far it has been assumed that γ is large enough such that the technology stock eventually catches up to the frontier. Provided that this is the case, the planner eventually switches from being an imitator to being an innovator ($\delta^* = 0$). The growth dynamics thereafter are important because they define the limiting case.

Proposition 2 *The equilibrium paths of consumption, investment and physical investment* $[c_t, i_t, z_t]_{t=0}^{\infty}$ *for the case where the planner prefers to innovate* ($\delta^* = 0$) *evolve according to the following equations:*

1.
$$\frac{\dot{c}_t}{c_t} = -\frac{\dot{\mu}_t}{\mu_t} - \rho \tag{19}$$

2.
$$\dot{a}_t = i_t = z_t \eta \gamma \tag{20}$$

3.
$$z_t = \frac{a_t - c_t}{\eta \gamma} \tag{21}$$

where $-\frac{\dot{\mu}_t}{\mu_t} = \frac{1}{\gamma \eta}$.

Proof 2 *See Appendix (Proof 2)* ■

Proposition 2 presents the equilibrium paths of consumption, investment and physical investment once the planner decides to innovate. The key thing to note is that a higher arrival rate of innovation and growth efficiency stimulate faster growth in productivity. To ensure that the planner switches just once (the t at which this will happen will be referred to as τ for the remainder of this paper), a new simplifying assumption will be introduced for the rest of the paper.

Assumption 2 *The growth rate of the frontier technology is equal to the growth rate of technology* *from innovation:*

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{\bar{a}}_t}{\bar{a}_t} = g_{\bar{a}} \tag{22}$$

Assumption 2 is imposed because it guarantees a unique point in time at which the planner switches from being an imitator to being an innovator provided its growth efficiency allows it to reach the frontier. When $t < \tau$, the planner prefers to imitate until the stock of technology eventually converges to the frontier. Once $t \geq \tau$, the planner begins to innovate. The growth rate of output from innovation matches the growth rate of the frontier technology stock. This is useful because it limits the transitional dynamics and prevents cases where the planner crosses time τ multiple times. Assumption 2 can be extended to the following equality by virtue of (8) and (9):

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{\bar{a}}_t}{\bar{a}_t} = \frac{\dot{y}_t}{y_t} = \frac{\eta\gamma z_t}{a_t} = g_{\bar{a}} \tag{23}$$

Assumption 2 and (23) both imply that there exists a balanced growth path whereby both output growth and the technology-output ratio are constant from τ onward. This model is therefore consistent with the Kaldor facts. Having established that there exists a balanced growth path, one can now begin to think about the steady state.

4.2.3 Steady State

Given that the growth rate of output is constant once the planner switches to being an innovator, it would be useful to normalise the variables of interest, namely consumption and investment, to work with stationary values. Letting $\tilde{c}_t$ represent consumption per efficiency units (i.e. consumption divided by the frontier technology stock which is the same as the domestic technology stock), the Euler equation for normalised consumption can be expressed as:

$$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\gamma\eta} - \rho - g_{\bar{a}} \tag{24}$$

One implication of a steady state is that consumption per efficiency units must not grow over time. This prompts the introduction of a final assumption for the remainder of this paper.

Assumption 3 *The growth rate of the frontier, $g_{\bar{a}}$, is defined as:*

$$g_{\bar{a}} = \frac{1}{\gamma\eta} - \rho \tag{25}$$

such that $\gamma = \bar{\gamma}$.

Assumption 3 is important for guaranteeing the existence of a steady state. Assumption 2 shows that output grows at a rate $g_{\bar{a}}$. Similarly, Assumption 3 suggests that consumption grows at that same rate. Thus, investment must grow at the same rate $g_{\bar{a}}$ by virtue of the resource constraint (7).

A simplification that was made for this to be the case is that the growth efficiency of the frontier is the same as the growth efficiency of the domestic economy[3].

Having solved for the balanced growth path and ensured a steady state where by definition normalised variables do not grow, what remains is computing the steady-state levels of normalised output, consumption, investment and physical investment where the normalisation involves dividing these four variables by $\bar{a}_t$. Note that in the steady-state, the production function (8) guarantees that output in efficiency units is normalised to unity. Proposition 3 takes care of this task for normalised consumption, investment and physical investment.

Proposition 3 *The steady state levels of normalised consumption, investment and physical investment can be expressed as:*

1.
$$\tilde{c}^* = \frac{\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{26}$$

2.
$$\tilde{i}^* = 1 - \frac{\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{27}$$

3.
$$\tilde{z}^* = \frac{1}{\eta\gamma} - \frac{1}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{28}$$

where $d = \frac{1}{\gamma[\bar{a}_t - a_t]}\left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right]$.

Proof 3 *See Appendix (Proof 3)* ■

5 Quantitative Analysis

The optimal consumption and physical investment functions that dictate the equilibrium conditions of the model have been recovered. The model admits a balanced growth path that has been solved for and the variables of interest have been normalised to study the steady-state equilibrium. This section performs some numerical exercises that would benefit from bringing the model to the computer.

5.1 Solving for c_0

The remaining task in order to fully solve the model is to retrieve c_0 – the initial consumption level. It can be shown that the model is saddle path stable and reaching this stable path requires choosing the appropriate c_0. The value of c_0 that is of interest is the one that, having fixed the initial domestic technology stock, would imply that the equilibrium consumption and investment functions would bring the economy to the steady state characterised above. Not only is this step important to get the solutions above in closed form, this step is required in order to fully solve for the transitional

---

[3] Section 6.2 discusses this assumption further

dynamics of the model. Once having solved for the transitional dynamics, one can experiment with different model parameters and their implications on the pace of catch-up in services productivity and the steady-state level of consumption.

This task is a numerical exercise that involves the following steps. First, set model parameters namely growth efficiency, the arrival rate of innovation, the discount factor and the frontier growth rate (γ, η, ρ, $g_{\bar{a}}$) as these parameters govern either the optimal consumption and investment functions or the frontier level of technology stock at any given time[4]. These parameters must be set such that Assumption 3 holds. The initial domestic and frontier technology stocks whose laws of motion are dictated by (9) and (12) respectively must also be chosen. The only condition is that the former must be smaller than the latter such that the domestic economy needs to catch up with the frontier.

The second step involves guessing c_0 and creating a time series for the optimal paths of consumption and technology stock using (16) and (17). The first guess can be the midpoint between 0 and the initial domestic technology stock acknowledging that the resource constraint is binding. Between the initial output and steady-state output, (14) says that the planner will choose to imitate until the economy's technology stock reaches the frontier level[5]. The point thereafter is when the planner switches to innovating, the economy begins to grow at the rate of the balanced growth path and the normalised values of consumption and investment hit their steady-state levels. Thus, the task is to guess a c_0 such that given the optimal consumption and investment paths and the parameter values above allows the economy to eventually reach the steady-state normalised consumption and output levels. Having guessed a c_0, a time series for the domestic and frontier technology stocks and consumption is built through forward iterations.

The time series end at the point in time when the domestic technology stock reaches the frontier level. At that point, the normalised output ($a_t/\bar{a}_t$) is 1 so the steady-state output is reached. Counting the number of time iterations it takes until this happens gives the value of τ which is the point at which the planner switches to becoming an innovator. With τ, one can proceed with the third step which is calculating the steady-state level of consumption and comparing it with the current normalised consumption level implied by the optimal consumption path. If they are not the same, then the initial guess for c_0 must be updated. If normalised consumption is less than the frontier at time τ, then c_0 should be higher and vice versa. The correct value of c_0 is the one that given the fixed initial technology stock allows the economy to reach the steady-state through these optimality conditions.

5.2 Parameterisation

Here I present and interpret some parameter values that can be used for shooting sequences of consumption and investment. These numbers will be used as a baseline for the next section which performs some comparative statics using the optimal consumption paths above.

ρ represents the discount factor which governs the rate of discounting future consumption. Set at

---

[4] See next subsection for a detailed discussion on parameterisation.

[5] A simplifying assumption has been made here which is that as soon as soon as the planner switches from being an imitator to being an innovator, the technology stock and output are both set to the frontier technology stock and output level.

Parameter	Value	Description
ρ	0.03	discount factor
γ	3.00	growth efficiency
η	0.001	arrival rate of innovation
$\mathrm{g}_{\bar{a}}$	0.05	frontier growth rate
a_0	0.50	initial domestic technology stock
$\bar{a}_0$	3.00	initial frontier technology stock

Table 5: Chosen Parameter Values

Notes. This table presents the choice of parameter values used for the numerical exercise that aims to recover c_0.

0.03 – a standard value in the literature – this implies that the same level of consumption in the next period brings 3.0 percentage points less utility than the current period. The next parameter is γ which represents the growth efficiency in the model. This is an important parameter that will govern the rate at which a country reduces its gap with the frontier per unit of investment. A higher γ implies a higher growth efficiency meaning that a country would be able to grow faster. This parameter is akin to the country-level fixed effects in Section 3 and so does not change over time. Countries with the high growth efficiency occupy the 'high' group while countries with low (or even negative) growth efficiencies define the 'low' group. The interpretation for the latter would be countries that are far away from the frontier labour productivity and any physical investments into the technology stock drive the country even further away from the frontier. Two countries with the same initial capital stock and therefore distance to the frontier may reach the frontier at different rates depending on growth efficiency. For now, I set it at 3.00 but this number will be subject to changes in the next subsection to explore the implications of different subsections.

Another important parameter is η – the arrival rate of innovation. It is set at 0.001, and the correct interpretation for this is that innovations are expected to be successful 0.1% of the time. Another way to consider this is that for every 100 research & development projects, 0.1 are successful. In the transitional dynamics, the arrival rate of innovation is important because it partly determines τ. A relatively higher η leaving all else constant means that τ declines because the planner would prefer to switch to being an innovator at an earlier stage. η nevertheless plays a bigger role in the innovation phase where the economy enters a balanced growth path. Finally, the initial domestic and frontier technology stocks are set at 0.50 and and 3.00 respectively. The only condition for those was that the domestic stock must be less than the frontier technology stock in order to allow for catch-up. Finally, the per-period growth rate of the frontier technology stock, $g_{\bar{a}}$, is set at 5.0%.

5.3 Comparative Statics

The goal of this paper was to identify the sources and challenges facing countries in growing their services productivity. This subsection uses the theoretical framework to explore some comparative statics, examining the role of different parameters on the trajectory of productivity growth. I focus on the role of the growth efficiency and the initial domestic technology stock on the trajectory of productivity (and output) in services.

5.3.1 The Role of Growth Efficiency

The growth efficiency parameter, γ plays a fairly large role in determining the pace of catch-up in the transitional dynamics of the model. Figure 4 experiments with different values of γ and explores their implications on the trajectory of growth in output.

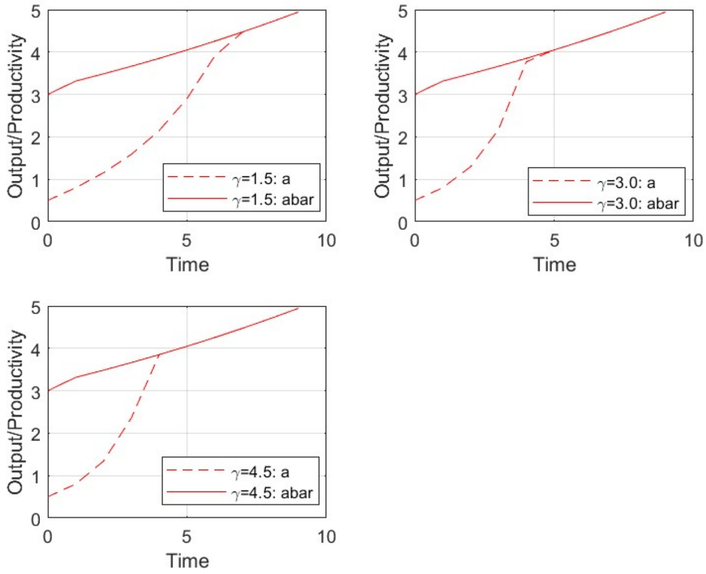

Figure 4: The Effect of Growth Efficiency on the Pace of Catch-Up

Notes. abar refers to the evolution of the frontier technology stock while a refers to the evolution of domestic technology stock in all panels. The difference between the different panels is the growth efficiency parameter. The top-left builds a sequence for a using a growth efficiency of 1.5, the top-right with 3.0 and the bottom-left with 4.5.

Two observations are in order. First, it is clear that countries with a higher growth efficiency parameter are able to catch-up quicker with the frontier. The top-right panel shows that a growth efficiency of 1.5 means full convergence takes place after 7 periods. Meanwhile, a growth efficiency of 3.0 means 5 periods are required for full convergence while a growth efficiency of 4.5 means 4 periods are needed. The second observation to note is that once the technology stock is close enough to the frontier – where close enough implies that the difference is less than η, the arrival rate of innovation – the growth rate shrinks because the planner switches from being an imitator to being an innovator and hence kickstarts the balanced growth path. This is especially clear in the top-right panel where the economy is too close to the frontier and therefore experiences a kink in the trajectory of output and productivity growth.

5.3.2 The Role of Initial Domestic Technology Stock

Another important factor in determining the pace of catch-up is the initial technology stock. This essentially determines the distance in the productivity in services between the frontier and the domestic economy. Here, I experiment with three different values for the initial domestic technology stock leaving all variables constant. The results are presented in Figure 5.

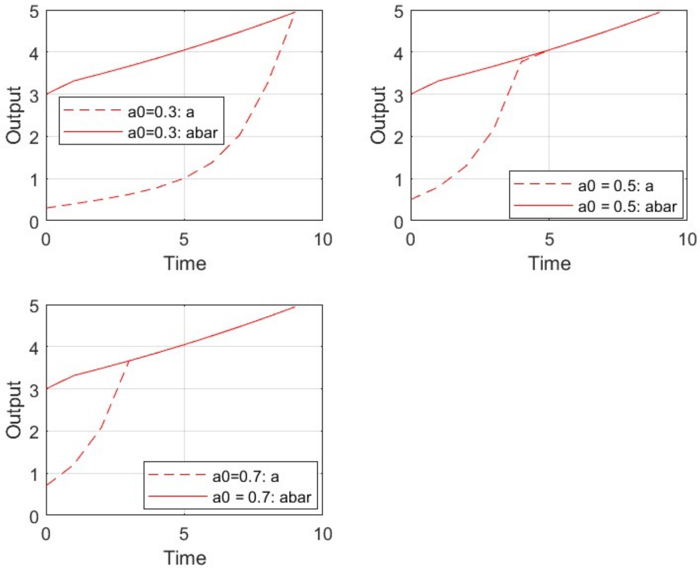

Figure 5: The Effect of The Initial Domestic Technology Stock on the Pace of Catch-Up
Notes. abar refers to the evolution of the frontier technology stock while a refers to the evolution of domestic technology stock in all panels. The difference between the different panels is the initial domestic technology stock. The top-left builds a sequence for a using an initial domestic technology stock of 0.3, the top-right with 0.5 and the bottom-left with 0.7.

As with the growth efficiency parameter, it is clear that the rate of convergence is rising with the initial domestic productivity level. The top-left panel in Figure 5 shows that an initial domestic technology stock of 0.3 means it takes 9 periods to fully converge with the frontier. An initial stock of 0.7, meanwhile, implies that it takes just 3 periods to converge. One aspect to emphasise is that the initial consumption level, c_0, has been left unchanged at 0.2 in all three panels. This can be thought of as maintaining a subsistence level of consumption at the start. The results in Figure 5 are therefore intuitive. At a low initial technology stock and high initial consumption level, the resource constraint guarantees that only very little space is left for investment which grows the technology stock. This is irrespective of the current distance to the frontier.. As such, output is noticeably flatter in the beginning when the initial technology stock is low relative to the case of a high initial stock.

6 Conclusion: What's Next?

Motivated by the presence of 'premature deindustrialisation', this paper has examined some of the implications of the late industrialisers' increasing reliance on the 'service economy' on these countries' long-run growth trajectories. I started by documenting three facts on productivity growth in services. First, there exists significant heterogeneity in countries' abilities to grow their services labour productivity growth in the last few decades. Second, most of this productivity growth stemmed from the 'within' channel – involving increases in labour productivity across the different services industries as opposed to the reallocation of labour to more productive services industries. Third, the 'within' channel's effect is partly underpinned by catch-up conditional on developing countries sharing key structural characteristics with the frontier economies.

I proceed by building a theoretical model to rationalise the facts and propose a mechanism driving the empirical section. In the model, the economy reaches the frontier of services productivity by allocating all of the physical investments to imitation. The time it takes to reach the frontier is a function of the economy's growth efficiency, and a growth efficiency that is too low would imply that the economy may never catch-up to the frontier. The model predicts fast growth in the initial phase until the marginal gains from imitating decline due to proximity to the frontier. Having arrived to the frontier, the social planner reaches a steady state level of normalised consumption and physical investment. Along the balanced growth path, the planner decides it is optimal to innovate and the frontier and domestic technology stock grow at the same pace – a pace dictated by the arrival rate of innovation.

Then, I present the steps to solve for the initial consumption level before parameterising the model and performing some numerical exercises using this theoretical framework. I show how the trajectory of productivity growth depends on the growth efficiency parameter γ and the initial capital stock a_0. The results in this paper are in fact indicative that the rise of the 'service economy' can offer a new development path for late industrialisers – though this is by no means guaranteed. While having unpacked some of the dynamics that characterise the 'service economy', I believe this paper leaves many questions for future research. I therefore dedicate the remainder of this section to presenting some of the questions that will inform my research agenda going forward.

6.1 What is Special About the 'Service Economy'?

This is a big question and has not been answered in this paper. Besides the use of data on services labour productivity, the theoretical framework abstracted away from any specifics that may characterise and distinguish the services sector from the rest of the economy. For example, whether and how the services production function may potentially differ from its manufacturing counterpart is still unknown to the best of my knowledge. Shedding light on this question will help explain the role (if any) of factor endowments in explaining the variation observed in countries' abilities to grow the services sector (Fact 1). Some other questions that should be considered for future research is whether imitation is harder in services than in manufacturing, and what the complementarities are between both sectors. One potential result that the empirical facts might be picking up is that productivity growth is driven by improvements in manufacturing inputs' quality, and this could be explaining the importance of the 'within' channel for driving productivity growth. Further work should be done to rule out these potential explanations.

6.2 What is γ?

The role of the growth efficiency parameter γ is very large in this model. Apart from determining the pace of the balanced growth, it affects the pace of convergence for countries lagging behind.

With a low enough γ, some countries may not be able to catch-up with the frontier. γ was modelled as the theoretical counterpart to the country fixed effects in Section 3 and must therefore capture time-invariant characteristics of the different countries. Interpreting the results in Section 3 suggests that holding constant these structural features, countries' productivity growth in services do indeed converge with the frontier. One must therefore ask: what exactly does γ capture? It is natural to suspect that these characteristics will have something to do with the classic debate of geography vs institutions. Even if these factors do not directly affect the trajectory of services productivity growth, it is likely that they have an impact on the skill endowment of countries, for example. Given the importance of γ in determining the trajectory of the services productivity growth, it is important to dig deeper into its drivers in order to fully unpack the rise of the 'service economy'.

On a more technical note, the model's structure is such that the frontier and converging countries must share the same γ in the balanced growth path. This assumption, while crucial in this iteration of the model, should be relaxed in the future. One potential solution is endogenising γ such that it grows over time to match the frontier country's, though the theoretical counterpart of the fixed effects would need to be incorporated in some other way. The structure presented in the paper is unrealistic and should therefore be modified to allow for greater flexibility on the value of γ.

6.3 How Can We Increase η?

Sections 4 and 5 showed that the arrival rate of innovation η is key in explaining why the frontier and countries that catch-up to it grow slowly as η determines the rate of balanced growth. This is not very far from the conclusions coming from the endogenous growth literature [Aghion and Howitt, 1990]. The question thus becomes: How can countries improve the arrival rate of innovation? What is the role of policy in this? A dominant services sector with a low arrival rate of innovation threaten to drive a divergence in cross-country income differences and severely slow down the development process for today's poorer countries. Raising the innovation rate should become a priority for governments that want to avoid this equilibrium. This applies mostly to countries at the frontier, including the advanced economies, as well as countries that successfully manage to catch-up with that frontier. Some potential policy interventions that should be studied include R&D subsidies and subsidised credit access to allow for greater investments in services technology.

6.4 Imitation & Innovation – Complements or Substitutes?

One important assumption that has been made in this paper, especially in the theoretical framework, is that imitation and innovation are perfect substitutes (Assumption 1). While logical, the premise of this assumption should be rigorously explored. One can think of a scenario where imitation and innovation are complementary, implying that more imitation increases the rate of innovation and vice versa. If this is truly the case, not only may this change the main results of the model, it can suggest that countries that are unable to grow their services productivity are facing frictions that are constraining firms' abilities to perform these two activities and grow their productivity in services. The role of financial frictions is one example that comes to mind. Financial frictions could be behind firms' in services abilities to grow their productivity through the 'within' channel. This question also relates to subsection 6.1 in the sense that services may differ from manufacturing in the extent to which imitation and innovation are complements or substitutes.

7 Appendix

7.1 Proof 1

Set-up the present-value Hamiltonian:

$$\hat{H} = e^{-\rho t}\log c_t + \mu_t \left[(a_t - c_t)\frac{1}{\Gamma(\delta, a, t)} \right] \tag{29}$$

Take the first order conditions of $\hat{H}$ with respect to c_t, μ_t and a_t:

$$\hat{H}_c : \frac{e^{-\rho t}}{c_t} = \frac{\mu_t}{\Gamma(\delta, t, a_t)} \tag{30}$$

$$\hat{H}_a : \frac{\frac{\mu_t}{\Gamma(\delta,t,a_t)} + (a_t - c_t)\,\delta_t \mu_t}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \Rightarrow \frac{\mu_t}{\Gamma(\delta, t, a_t)} + \frac{\mu_t \delta_t (a_t - c_t)}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \tag{31}$$

$$\hat{H}_\mu : \quad \dot{a}_t = (a_t - c_t)\frac{1}{\Gamma(\delta, t, a_t)} \tag{32}$$

and the TVC:

$$\lim_{t \to \infty} \mu_t a_t = 0 \tag{33}$$

Take $\hat{H}_c$ and manipulate:

$$c_t = \frac{\Gamma(\delta, t, a)e^{-\rho t}}{\mu_t} \tag{34}$$

We are dealing with the case where the planner prefers to imitate, so $\delta^* = 1$ and (31) turns into:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, a, t)} + \frac{\mu_t}{\Gamma^2(\delta, a, t)}(a_t - c_t) \tag{35}$$

Then, divide both sides by μ_t:

$$-\frac{\dot{\mu}_t}{\mu_t} = \frac{1}{\Gamma(\delta, a, t)} + \frac{1}{\Gamma^2(\delta, a, t)}(a_t - c_t) \tag{36}$$

Acknowledging that $\Gamma(\delta, a, t) = \gamma(\bar{a}_t - a_t)$, take the time derivative of (34), first by taking logs:

$$-\rho t \log e + \log \gamma + \log(\bar{a}_t - a_t) - \log \mu_t = \log c_t \tag{37}$$

and then the time derivative of this:

$$\frac{\dot{c}_t}{c_t} = -\rho + \frac{\dot{\bar{a}}_t - \dot{a}_t}{\bar{a}_t - a_t} - \frac{\dot{\mu}_t}{\mu_t} \tag{38}$$

Substitute (36) into (38) to get the consumption Euler equation when $\delta^* = 1$.

To get the equilibrium law of motion for investment for imitation, substitute $\delta^* = 1$ into (10) and substitute (10) into (9) to get:

$$\dot{a}_t = i_t = z_t \gamma (\bar{a}_t - a_t) \tag{39}$$

Then, the path of z_t is simply derived by substituting (39) into the resource constraint and solving for z_t:

$$z_t = \frac{a_t - c_t}{\gamma(\bar{a}_t - a_t)} \tag{40}$$

7.2 Proof 2

Start with the same steps as in Proof 1. Set-up the present-value Hamiltonian:

$$\hat{H} = e^{-\rho t} \log c_t + \mu_t \left[(a_t - c_t) \frac{1}{\Gamma(\delta, a, t)} \right] \tag{41}$$

Take the first order conditions of $\hat{H}$ with respect to c_t, μ_t and a_t:

$$\hat{H}_c : \frac{e^{-\rho t}}{c_t} = \frac{\mu_t}{\Gamma(\delta, t, a_t)} \tag{42}$$

$$\hat{H}_a : \frac{\frac{\mu_t}{\Gamma(\delta,t,a_t)} + (a_t - c_t)\, \delta_t \mu_t}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \Rightarrow \frac{\mu_t}{\Gamma(\delta, t, a_t)} + \frac{\mu_t \delta_t\, (a_t - c_t)}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \tag{43}$$

$$\hat{H}_\mu : \quad \dot{a}_t = (a_t - c_t) \frac{1}{\Gamma(\delta, t, a_t)} \tag{44}$$

and the TVC:

$$\lim_{t \to \infty} \mu_t a_t = 0 \tag{45}$$

Take $\hat{H}_c$ and manipulate:

$$c_t = \frac{\Gamma(\delta, t, a)e^{-\rho t}}{\mu_t} \tag{46}$$

We are dealing with the case where the planner prefers to innovate, so $\delta^* = 0$. As a result, (43) turns into:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, a, t)} \tag{47}$$

Divide (47) by μ_t:

$$-\frac{\dot{\mu}_t}{\mu_t} = \frac{1}{\Gamma(\delta, a, t)} \tag{48}$$

Next, take the time derivative of (46) now acknowledging that $\Gamma(\delta, a, t) = \gamma\eta$:

$$-\rho t \log e + \log \gamma + \log \eta - \log \mu_t = \log c_t \tag{49}$$

The time derivative is:

$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{\dot{\mu}_t}{\mu_t} \tag{50}$$

Substituting (48) into (50) gives the consumption Euler equation under innovation.

To get the equilibrium law of motion for investment for innovation, substitute $\delta^* = 0$ into (10) and substitute (10) into (9) to get:

$$\dot{a}_t = i_t = z_t \gamma \eta \tag{51}$$

Then, the path of z_t is simply derived by substituting (51) into the resource constraint and solving for z_t:

$$z_t = \frac{a_t - c_t}{\eta\gamma} \tag{52}$$

7.3 Proof 3

To get the closed form values of $\tilde{c}^*$, one must first retrieve the closed form solution for c_t under both imitation and innovation. Set-up the present-value Hamiltonian:

$$\hat{H} = e^{-\rho t} \log c_t + \mu_t \left[(a_t - c_t) \frac{1}{\Gamma(\delta, a, t)} \right] \tag{53}$$

Take the first order conditions of $\hat{H}$ with respect to c_t, μ_t and a_t:

$$\hat{H}_c : \frac{e^{-\rho t}}{c_t} = \frac{\mu_t}{\Gamma(\delta, t, a_t)} \tag{54}$$

$$\hat{H}_a : \frac{\frac{\mu_t}{\Gamma(\delta,t,a_t)} + (a_t - c_t)\,\delta_t \mu_t}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \Rightarrow \frac{\mu_t}{\Gamma(\delta, t, a_t)} + \frac{\mu_t \delta_t\,(a_t - c_t)}{\Gamma^2(\delta, t, a_t)} = -\dot{\mu} \tag{55}$$

$$\hat{H}_\mu : \quad \dot{a}_t = (a_t - c_t)\,\frac{1}{\Gamma(\delta, t, a_t)} \tag{56}$$

and the TVC:

$$\lim_{t\to\infty} \mu_t a_t = 0 \tag{57}$$

Take $\hat{H}_c$ and manipulate:

$$c_t = \frac{\Gamma(\delta, t, a) e^{-\rho t}}{\mu_t} \Rightarrow \frac{e^{-\rho t}}{\mu_t} = \frac{c_t}{\Gamma(\delta, t, a)} \tag{58}$$

This expression is not in closed form because μ_t is unknown which means c_t cannot be pinned down. To find μ_t, take $\hat{H}_a$ and recognise that it is a first-order ordinary differential equation that needs to be solved:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, a, t)} + \frac{\mu_t \delta_t}{\Gamma^2(\delta, a, t)}\,(a_t - c_t) \tag{59}$$

Multiplying out:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, a, t)} + \frac{\mu_t \delta_t a_t}{\Gamma^2(\delta, a, t)} - \frac{\mu_t \delta_t c_t}{\Gamma^2(\delta, a, t)} \tag{60}$$

Note that (60) is not a homogeneous differential equation. To see this, substitute (58) in the last term on the RHS:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, a, t)} + \frac{\mu_t \delta_t a_t}{\Gamma^2(\delta, a, t)} - \frac{\delta_t}{\Gamma(\delta, a, t)} e^{-\rho t} \tag{61}$$

(61) is a differential equation that needs to be solved in order to pin down c_t and by extension z_t and i_t. First, note that the solution depends on the value of δ_t. Since δ_t is binary, there will be two values for μ_t. Consider the first scenario where $\delta_t = 1$:

$$-\dot{\mu}_t = \frac{\mu_t}{\Gamma(\delta, t, a)} + \frac{\mu_t a_t}{\Gamma^2(\delta, t, a)} - \frac{e^{-\rho t}}{\Gamma(\delta, t, a)} \tag{62}$$

Consider also the following three equations that follow from the model environment:

$$\Gamma(\delta, t, a) = \gamma[\bar{a}_t - a_t] \tag{63}$$

$$\dot{a}_t = \gamma[\bar{a}_t - a_t] z_t \tag{64}$$

$$g_t = e^{\rho t} \mu_t \tag{65}$$

Now take (62) and multiply both sides by $e^{\rho t}$:

$$-\dot{\mu}_t e^{\rho t} = \frac{e^{\rho t}\mu_t}{\Gamma(\delta, a, t)} + \frac{\mu_t a_t e^{\rho t}}{\Gamma^2(\delta, a, t)} - \frac{1}{\Gamma(\delta, a, t)} \tag{66}$$

Using (65):

$$-\dot{g}_t = g_t \frac{1}{\Gamma(\delta, t, a)} + \frac{g_t a_t}{\Gamma^2(\delta, a, t)} - \frac{1}{\Gamma(\delta, a, t)} \tag{67}$$

With some re-arranging:

$$-\dot{g}_t = \underbrace{g_t \left[\frac{1}{\Gamma(\delta, a, t)} + \frac{a_t}{\Gamma^2(\delta, a, t)}\right]}_{\text{homogeneous}} - \underbrace{\frac{1}{\Gamma(\delta, a, t)}}_{\text{particular}} \tag{68}$$

The solution to the differential equation takes the following form:

$$g_t = g_t^h + g_t^p \tag{69}$$

Focus on the homogeneous part first and substitute in (63):

$$-\dot{g}_t^h = g_t^h \left[\frac{1}{\gamma\left(\bar{a}_t - a_t\right)} + \frac{a_t}{\left[\gamma\left(\bar{a}_t - a_t\right)\right]^2}\right] \tag{70}$$

$$-\dot{g}_t^h = g_t^h \underbrace{\frac{1}{\gamma[\bar{a}_t - a_t]}\left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right]}_{d} \tag{71}$$

Next, take g_t^h on the RHS, move to the other side, recognise that the LHS is $\frac{dln g_t^h}{dt}$, integrate and take the exponential of both sides to get:

$$g_t^h = g_0^h e^{-dt} \tag{72}$$

The initial equation was:

$$-\dot{g}_t = \underbrace{g_t \left[\frac{1}{\Gamma(\delta, a, t)} + \frac{a_t}{\Gamma^2(\delta, a, t)}\right]}_{\text{homogeneous}} - \underbrace{\frac{1}{\Gamma(\delta, a, t)}}_{\text{particular}} \tag{73}$$

substitute in (69) and (72):

$$-\dot{g} = [g_t^h + g_t^p] \frac{1}{\gamma[\bar{a}_t - a_t]} \left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right] - \frac{1}{\gamma[\bar{a}_t - a_t]} \tag{74}$$

$$-\dot{\mu^h}_t - \dot{\mu^p}_t = [g_0^h e^{-dt} + g_t^p] \frac{1}{\gamma[\bar{a}_t - a_t]} \left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right] - \frac{1}{\gamma[\bar{a}_t - a_t]} \tag{75}$$

Now we need to solve for g_t^p. Since it does not change with time:

$$0 = g_t^p \frac{1}{\gamma[\bar{a}_t - a_t]} \left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right] - \frac{1}{\Gamma(\delta, a, t)} \tag{76}$$

$$0 = g_t^p d - \frac{1}{\Gamma(\delta, a, t)} \tag{77}$$

$$d = \frac{1}{\gamma[\bar{a}_t - a_t]} \left[1 + \frac{a_t}{\gamma[\bar{a}_t - a_t]}\right] \tag{78}$$

Now solve for g_p:

$$g_t^p = \frac{1}{\Gamma(\delta, a, t) d} \tag{79}$$

which means:

$$g_t = g_0 e^{-dt} + \frac{1}{\Gamma(\delta, a, t) d} \tag{80}$$

Divide both sides by $e^{\rho t}$ to get:

$$\mu_t = \mu_0 e^{-(\rho+d)t} + \frac{e^{-\rho t}}{\Gamma(\delta, a, t)d} \tag{81}$$

and by definition $\mu_0 = 1/c_0$ which means:

$$\mu_t^{Imitation} = \frac{e^{-(\rho+d)t}}{c_0} + \frac{e^{-\rho t}}{\Gamma(\delta, a, t)d} \tag{82}$$

This was all done to get μ_t during the imitation phase. This will be needed to get μ_t in the innovation phase. To do so, we need to repeat these steps to get μ_t during innovation phase. During innovation $\delta = 0$ which means $\hat{H}_a$ is:

$$\hat{H}_a : \frac{\mu_t}{\Gamma(\delta, t, a)} = -\dot{\mu} \tag{83}$$

where $\Gamma(\delta, t, a)$ is in fact just $\gamma\eta$:

$$\hat{H}_a : \frac{\mu_t}{\gamma\eta} = -\dot{\mu} \tag{84}$$

Solving the differential equation:

$$\frac{-\dot{\mu}_t}{\mu_t} = \frac{1}{\gamma\eta} = \tilde{d} \tag{85}$$

$$\mu_t = K e^{-\tilde{d}t} \tag{86}$$

where K is just a constant. Importantly, this satisfies the TVC because it tends to 0 as t goes to infinity. Next, assume K is such that μ_t is the same as in the imitation case when switching to innovation which happens at time τ:

$$\mu_t^{Innovation} = [\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta, a, t)d}] e^{-\tilde{d}t} \tag{87}$$

(82) and (87) are the μ_t under imitation and innovation. Now they can be substituted for μ_t in $\hat{H}_c$ to obtain c_t under both imitation and innovation. From $\hat{H}_c$, we know:

$$c_t = \frac{\Gamma(\delta, t, a)e^{-\rho t}}{\mu_t} \tag{88}$$

Thus, we have two equations, one for $t \geq \tau$ and one for $t < \tau$:

$$\boxed{c_t = \Gamma(\delta, t, a)\frac{e^{-\rho t}}{\mu_t} = \begin{cases} e^{-\rho t}[\gamma\left(\bar{a}_t - a_t\right)]\left[\frac{e^{-(\rho+d)t}}{c_0} + \frac{e^{-\rho t}}{\Gamma(\delta,a,t)d}\right]^{-1} & t < \tau \\ e^{-\rho t}\eta\gamma\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]e^{-\widetilde{d}t}\right)^{-1} & t \geq \tau \end{cases}} \tag{89}$$

Armed with Assumption 3, one can solve for the steady-state level of consumption efficiency units. First, divide both sides by $\bar{a}_t$ (which is the same as a_t once the planner switches to innovation):

$$\tilde{c_t} = \frac{e^{-\rho t}\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]e^{-\widetilde{d}t}\right)a_t} = \frac{e^{-\rho t}\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]e^{-\widetilde{d}t}\right)\bar{a}_t} \tag{90}$$

Substituting (12) in the denominator produces:

$$\tilde{c_t} = \frac{e^{-\rho t}\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]e^{-\widetilde{d}t}\right)\bar{a}_0 e^{g_{\bar{a}}t}} \tag{91}$$

Invoking Assumption 3 means (91) is reduced simply to the steady state level of efficiency consumption units in closed form:

$$\tilde{c}^* = \frac{\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{92}$$

As with consumption, one can divide output y_t by a_t which from thereafter will be denoted as $\tilde{y}_t$. By virtue of constant returns to scale in the production function, one can write $\tilde{y}^* = 1$ for every $t > \tau$. The implication of this is that because of the initial resource constraint (7), steady state investment efficiency units is simply the difference between 1 and $\tilde{c}_t$ which is just:

$$\tilde{i}^* = 1 - \frac{\eta\gamma}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{93}$$

A closed form expression for the steady-state level of z_t can also be expressed using (9), (10) and (93). Dividing both sides by $\Gamma(\delta, t, a)$ which is simply $\gamma\eta$ in the steady state yields the closed form value of $\tilde{z}^*$ in the steady state:

$$\tilde{z}^* = \frac{1}{\eta\gamma} - \frac{1}{\left([\frac{e^{-(\rho+d)\tau}}{c_0} + \frac{e^{-\rho\tau}}{\Gamma(\delta,a,t)d}]\right)\bar{a}_0} \tag{94}$$

Expressions (92), (93) and (94) represent the steady-state levels of consumption, investment and physical investment efficiency units in this model in closed form – all of which do not grow when $t > \tau$.

References

[Acemoglu, 2009] Acemoglu, D. (2009). *Introduction to Modern Economic Growth.* Princeton University Press.

[Aghion and Howitt, 1990] Aghion, P. and Howitt, P. (1990). A model of growth through creative destruction.

[Baccini et al., 2021] Baccini, L., Fiorini, M., Hoekman, B. M., and Sanfilippo, M. (2021). *Services and Economic Development in Africa.* International Growth Centre.

[Barro and Sala-i Martin, 1992] Barro, R. J. and Sala-i Martin, X. (1992). Convergence. *Journal of political Economy*, 100(2):223–251.

[Barro et al., 1991] Barro, R. J., Sala-i Martin, X., Blanchard, O. J., and Hall, R. E. (1991). Convergence across states and regions. *Brookings papers on economic activity*, pages 107–182.

[Baumol, 1967] Baumol, W. J. (1967). Macroeconomics of unbalanced growth: the anatomy of urban crisis. *The American economic review*, 57(3):415–426.

[Buera et al., 2022] Buera, F. J., Kaboski, J. P., Rogerson, R., and Vizcaino, J. I. (2022). Skill-biased structural change. *The Review of Economic Studies*, 89(2):592–625.

[Fang and Herrendorf, 2021] Fang, L. and Herrendorf, B. (2021). High-skilled services and development in china. *Journal of Development Economics*, 151:102671.

[Fuchs et al., 1968] Fuchs, V. R. et al. (1968). The service economy. *NBER Books.*

[Fujiwara and Matsuyama, 2020] Fujiwara, I. and Matsuyama, K. (2020). A technology-gap model of premature deindustrialization. *Available at SSRN 3753930.*

[Ghani and O'Connell, 2016] Ghani, E. and O'Connell, S. D. (2016). Can services be a growth escalator in low-income countries? *Revue deconomie du developpement*, 24(2):143–173.

[Herrendorf et al., 2014] Herrendorf, B., Rogerson, R., and Valentinyi, A. (2014). Growth and structural transformation. *Handbook of economic growth*, 2:855–941.

[Huneeus and Rogerson, 2020] Huneeus, F. and Rogerson, R. (2020). Heterogeneous paths of industrialization. Technical report, National Bureau of Economic Research.

[Kongsamut et al., 2001] Kongsamut, P., Rebelo, S., and Xie, D. (2001). Beyond balanced growth. *The Review of Economic Studies*, 68(4):869–882.

[Kuznets, 1957] Kuznets, S. (1957). Quantitative aspects of the economic growth of nations: Ii. industrial distribution of national product and labor force. *Economic development and cultural change*, 5(S4):1–111.

[McMillan et al., 2014] McMillan, M., Rodrik, D., and Verduzco-Gallo, Í. (2014). Globalization, structural change, and productivity growth, with an update on africa. *World development*, 63:11–32.

[Nayyar et al., 2021] Nayyar, G., Hallward-Driemeier, M., and Davies, E. (2021). *At Your Service?: The Promise of Services-Led Development.* World Bank Publications.

[Ngai and Pissarides, 2007] Ngai, L. R. and Pissarides, C. A. (2007). Structural change in a multi-sector model of growth. *American economic review*, 97(1):429–443.

[Rodrik, 2013] Rodrik, D. (2013). Unconditional convergence in manufacturing. *The quarterly journal of economics*, 128(1):165–204.

[Rodrik, 2016] Rodrik, D. (2016). Premature deindustrialization. *Journal of economic growth*, 21(1):1–33.

[Stigler et al., 1956] Stigler, G. J. et al. (1956). Trends in employment in the service industries. *NBER Books*.

[Vandenbussche et al., 2006] Vandenbussche, J., Aghion, P., and Meghir, C. (2006). Growth, distance to frontier and composition of human capital. *Journal of economic growth*, 11(2):97–127.