The Careful Physicist's Guide to Modified Gravity in the UV

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Abstract

In this thesis we investigate scalar and scalar-tensor field theories and their relation to low energy modifications of Einstein's General Relativity (GR), as well as their mathematical validity and self-consistency. We begin by outlining the cosmological constant problem, and how large quantum corrections lead to unnatural space-time curvatures. Going into depth, we present a rearrangement of GR which emphasises the global structure within the Einstein equations. Preceding in this manner, we examine global modifications of GR which semi-classically act to insulate the highly Ultra-Violet (UV) sensitive loop corrections to the vacuum energy from the curvature of space-time. We explore the consequences of a manifestly local variant of this model, studying the UV sensitivity to place bounds on resulting cosmological profiles, the effect of phase transitions on fine-tuning, and its compatibility with inflation.

Taking a different approach, we examine other local modifications of GR, which introduce new degrees of freedom. We summarise the need for screening mechanisms which are an important feature of any local modification of GR, in order to remain within constraints imposed by observation. Presenting a high energy extension of a massive Galileon theory, we investigate if it exhibits so-called Vainshtein screening; Vainshtein screened theories are commonly incompatible with Wilsonian UV completion. Using this theory as a toy model, we examine what can go wrong when integrating out a heavy field, as well as what kind of role mass term deformations might play in a wider class of theories, and at what scale Vainshtein screening potentially breaks down.

List of papers

This thesis presents material from the following papers:

'Cosmological consequences of Omnia Sequestra'

B. Coltman, Y. Li, and A. Padilla

JCAP 06, 017 (2019)

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'Massive Galileons and Vainshtein Screening'

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Convention & notation

The following conventions, notations and definitions will be applied throughout:

Unless they appear explicitly, we will take $c = \hbar = 1$ via Natural units. For example, in these units we write the (reduced) Planck mass as $M_{\mathrm{Pl}} = (8\pi G_{\mathrm{N}})^{-\frac{1}{2}} \approx 10^{18}\mathrm{GeV}$ where G_{N} is Newton's gravitational constant.

We will use the mostly-positive signature for the metric $(-,+,+,+)$. For example, we express the Minkowski metric as $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$.

Greek indices $\mu, \nu, \ldots = (0,1,2,3)$ are space-time indices, we denote a partial derivative as ∂_μ and a covariant derivative as ∇_μ. The operator $\Box$ is the 4-Laplacian given by $\nabla_\mu \nabla^\mu$.

The space-time average of a scalar quantity Q looks like,

$$\langle Q \rangle = \frac{\int \mathrm{d}^4x \sqrt{-g}\, Q}{\int \mathrm{d}^4x \sqrt{-g}}$$

where the integrals are over the entire 4-volume of a space-time.

We denote symmetrisation of a tensor over enclosed indices as,

$$S_{(\mu\nu)} = \frac{1}{2}(S_{\mu\nu} + S_{\nu\mu}).$$

Contents

Chapter 1

Introduction

1.1 General Relativity

Gravity is the weakest of the four fundamental forces we observe in Nature. The force due to gravity between two electrons is approximately 43 orders of magnitude smaller than the size of the corresponding force due to electromagnetism. Similar statements hold for the comparison between the relative strengths of gravity and the strong and weak forces. In spite of this, gravity is often the only interaction relevant on cosmological scales. As a result, the search for a deeper understanding of the observable universe is intimately related with the pursuit of gravitational knowledge. Likewise, developments in cosmology can strongly influence our approach to gravity.

Einstein's theory of General Relativity (GR), as presented in 1915, is our best picture of gravity to date. Amending Newtonian predictions with relativistic corrections, GR solved several of the observational problems present at the time, including the bending of light by the sun, gravitational red-shift, and the perihelion precession of Mercury. Additionally, it resolved the inconsistencies present between Newtonian gravity and special relativity, such as locality and causality, by elevating space-time to a dynamical construction viewed through a geometrical lens. Both solar system and earthbound observational testing of gravity continually affirms GR as a remarkably accurate theory of gravity, see e.g. [1]–[6] and references therein. Most recently, advances in experimental physics have allowed multiple detections of gravitational waves, predicted to exist by GR a century ago [7].

The Einstein-Hilbert action, with a bare cosmological constant term $\Lambda_{\rm c}$, is given by,

$$S_{\rm GR} = \int {\rm d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \Lambda_{\rm c}\right) \tag{1.1.1}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, with $g_{\mu\nu}$ as the metric tensor and $R_{\mu\nu}$ as the Ricci tensor, $g = \det g_{\mu\nu}$, and $M_{\rm Pl}$ is the Planck mass. The Ricci tensor is defined in terms of the Riemann curvature tensor as $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$, with the Riemann tensor given by,

$$R^{\alpha}{}_{\beta\gamma\delta} = \partial_{\gamma}\Gamma^{\alpha}{}_{\delta\beta} - \partial_{\delta}\Gamma^{\alpha}{}_{\gamma\beta} + \Gamma^{\alpha}{}_{\gamma\sigma}\Gamma^{\sigma}{}_{\delta\beta} - \Gamma^{\alpha}{}_{\delta\sigma}\Gamma^{\sigma}{}_{\gamma\beta} \tag{1.1.2}$$

with Christoffel symbols $\Gamma^{\lambda}{}_{\mu\nu}$ defined as,

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_{\mu}g_{\nu\sigma} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\mu\nu}\right). \tag{1.1.3}$$

The metric $g_{\mu\nu}$ describes the geometry of the system via these various curvature tensors, and through the line element (or space-time interval) describing the distance between two infinitesimally separated points in space-time as,

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}. \tag{1.1.4}$$

We couple a sector of matter fields to gravity minimally, that is to say purely through the covariant volume element $\mathrm{d}^4x\sqrt{-g}$ and any covariant derivatives, with the total action a linear combination of the gravitational and matter sector actions,

$$S = S_{\rm GR} + S_{\rm m}[g_{\mu\nu}, \Psi] \tag{1.1.5}$$

where Ψ denotes the matter fields. After variation with respect to $g_{\mu\nu}$ the ten independent field equations obtained, known as the Einstein equations, are given by,

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu} - \Lambda_{\rm c}g_{\mu\nu} \tag{1.1.6}$$

where the Einstein tensor $G_{\mu\nu}$ is defined by $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, and the energy-momentum tensor of the matter sector $T_{\mu\nu}$ is defined as,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}}. \tag{1.1.7}$$

After some rearranging, the total cosmological constant that appears in the Einstein equations, i.e. the coefficient of the metric term, is a sum of the bare value $\Lambda_{\rm c}$ from the Einstein Hilbert action, and the matter sector vacuum energy $V_{\rm vac}$ which contributes to the energy-momentum tensor as $T_{\mu\nu\,\rm vac} = -g_{\mu\nu}V_{\rm vac}$. Here $V_{\rm vac}$ is the sum of all matter loop diagrams with no external legs; we shall see why it takes this form in chapter 2. The Einstein equations treat each part identically, thus observations can only constrain the combination $\Lambda_{\rm t} = \Lambda_{\rm c} + V_{\rm vac}$.

General relativity has had many experimental successes, which cement its place as our current most accurate gravitational theory. Predictions made by GR such as the precession of the orbit of mercury and the bending of light by massive objects, amongst others, have been verified by experiments to a large degree of precision. However, GR is thought to be an insufficient description of phenomena at small distance scales as quantum mechanical considerations become important or even dominant, and a yet-to-be-discovered quantum theory of gravity is required. Despite this expected breakdown of the theory, it was widely thought that it would be an excellent approximation all the way to cosmological scales. Recently this expectation has come under scrutiny with the discovery of the accelerated expansion of the universe, a consequence of so-called 'dark energy', which earned the Nobel Prize in physics in 2011. Experimental results, such as observations of Type 1a supernovae [8], [9] and precision measurements of the Cosmic Microwave Background (CMB) [10]–[13], require us to accept a tiny non-zero cosmological constant that is not chosen *a priori* by GR. Whilst this is not a problem within the classical theory, once one tries to treat GR as a quantum theory within an EFT framework issues arise regarding naturalness and fine-tuning. This is known as the cosmological constant problem, which will be discussed thoroughly in chapter 2. Additionally, observational evidence strongly suggest that within a GR framework there must exist a large amount of 'dark matter' in order to explain several galactic and cosmological phenomena, such as anomalous behaviour of galactic rotation curves [14]–[16], gravitational lensing measurements [17]–[20], and large scale structure such as galaxy clusters [21]–[25]. These issues introduce the possibility that we may be required to modify our theory of gravity in order to adequately describe our Universe on the largest distance scales.

1.2 Dark Energy

Since the formulation of GR, advancements in observational cosmology provide us with the tools to study gravity on huge distances comparable to the Hubble scale $H_0^{-1} \sim 10^{26}$m. One facet of this research is the increasingly large amount of evidence supporting the aforementioned accelerated expansion of the universe. Initially, observations of Type 1a supernovae appearing dimmer than expected, from a non-relativistic particle dominated universe, hinted at late time expansion acceleration [8], [9]. Numerous additional studies, including precision Cosmic Microwave Background (CMB) measurements [10]–[13], established concretely that the energy density of non-relativistic matter is not the dominant component of the total energy of the universe in the current cosmological epoch, and that an alternative phenomenon, commonly referred to as dark energy, was causing the cosmological acceleration.

There exists an easy solution to the nature of dark energy; a period of late time acceleration can be simply explained by the presence of the cosmological constant $\Lambda_{\rm t} = \Lambda_{\rm c} + V_{\rm vac}$. As the energy density of a cosmological constant term does not dilute upon cosmological expansion, as opposed to any other type of energy density satisfying null energy conditions, it will naturally become the dominant energy contribution at late times. This results in an asymptotically de-Sitter cosmological profile [26], with a curvature directly related to the size of the cosmological constant. In addition, all current data is consistent with a cosmological constant explanation of dark energy, with magnitude $\Lambda_{\rm obs} \sim ({\rm meV})^4$ [13].

However, in reality the situation is not so simple. The total cosmological constant $\Lambda_{\rm t}$ receives radiative corrections through $V_{\rm vac}$ from vacuum fluctuations of each massive particle coupled to gravity. In the standard model of particle physics these fluctuations are extremely sensitive to ultra violet (UV) physics, and the value of $V_{\rm vac}$ is vastly larger than $\Lambda_{\rm obs}$, even when only considering the contribution from the electron. Heavier particles provide corrections that exacerbate the situation, adding of the order of their mass to the fourth power. In order to even approach the diagnosis of a small $\Lambda_{\rm t}$, we must repeatedly fine tune $\Lambda_{\rm c}$ every time our description of the matter sector changes, for instance when changing how many orders of the loops we calculate V_{vac} to, or modifying the effective action's Wilsonian cut-off. This repeated need to fine tune a classical parameter of the action is the cosmological constant problem [27]–[33], and is the subject of a large proportion of this thesis.

A common modification to GR is the introduction of one or more scalar fields into the action. Known as scalar-tensor theories, due to the presence of both the metric and scalar degrees of freedom, they are often used in current attempts to modify gravity in the infrared (IR), in pursuit of a resolution to the cosmological constant problem or a potential source of dynamical dark energy. Scalar fields automatically satisfy the isotropy required from a cosmological standpoint since they do not select a preferred direction, making them good candidates for modifying gravity. Additionally, most models of modified gravity reduce to scalar-tensor theories when appropriate limits are taken, i.e. when considering the departure from GR and how this affects the dynamics. For example, scalar fields usually occur in theories with extra dimensions and can describe such things as the position of a hyper-surface (known as a brane) in an extra-dimensional space, or the size of compactified dimensions in string theory [34]–[38].

1.3 Screening mechanisms

The main problem with modifying gravity to achieve cosmological acceleration is not in actualising suitable behaviour at late times, but rather concurring with tests of gravity on local scales. Any local modification of GR that provides altered long distance behaviour necessitates the introduction of new degrees of freedom acting as force mediators. Commonly, modifications to GR use light scalar fields coupled to the matter sector, as these are often the simplest to work with.

Any deviations from GR resulting from a new force mediated by a varying scalar field must be suppressed by $O(10^{-5})$ within the solar system for the modification to not be invalidated by current observational evidence [1]. Let us illustrate this with a toy model of a massive scalar field coupled to matter linearly through the trace of the energy-momentum tensor, in the limit $M_{\rm Pl} \to \infty$, to obtain the action,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{M_{\rm Pl}}\phi T\right) \tag{1.3.1}$$

where λ is a dimensionless coupling constant and ϕ is a massive scalar field which couples to the trace of the energy-momentum tensor. If $\lambda \sim O(1)$, ϕ is coupled to matter with gravitational strength. The dynamics of the scalar field are understood via the equation of motion,

$$\phi - m^2\phi + \frac{\lambda}{M_{\rm Pl}}T = 0. \tag{1.3.2}$$

Modelling the stress-energy as approximating a non-relativistic astronomical point source, we assume it to be static and spherically symmetric with mass M such that,

$$T_{\mu\nu} = M\delta^{\mu}{}_{0}\delta^{\nu}{}_{0}\delta^3(\mathbf{x}) \tag{1.3.3}$$

so that $T = -M\delta^3(\mathbf{x})$. In this case the scalar field solution is a static, spherically symmetric profile given by (e.g. [39], [40]),

$$\phi(r) = -\frac{\lambda M}{4\pi M_{\rm Pl}}\frac{e^{-mr}}{r} \tag{1.3.4}$$

where we assume asymptotic vanishing of ϕ. As expected for a massive scalar field, we recover the standard potential with a Yukawa suppression. The scalar field acts on the matter sector it is coupled to with force per unit mass of the form $F_\phi = -\frac{\lambda}{M_{\rm Pl}}\nabla\phi$ (see e.g. [40]), giving the full expression,

$$F_\phi = -\frac{\lambda^2 M}{4\pi M_{\rm Pl}^2}\frac{e^{-mr}}{r^2}(1+mr) \tag{1.3.5}$$

where F_ϕ is the magnitude of a radial force. Contrasting with the magnitude of the Newtonian force $F_{\rm N}$ enacted by the source, we find,

$$\left|\frac{F_\phi}{F_{\rm N}}\right| = 2\lambda^2 e^{-mr}(1+mr). \tag{1.3.6}$$

We are free to use the Newtonian approximation here as we are comparing the forces on the scale of the solar system, at which distances this simplification is legitimate and illustrative. Let us consider what parameter values we require to have an interesting field theory that satisfies experimental bounds. Generically we would naturally expect $\lambda \sim O(1)$, and m must be chosen as small relative to the Hubble scale to permit the scalar force to contribute at cosmological distances. Under these choices it seems that this theory provides a force with magnitude comparable to Newtonian gravity, and is therefore unable to meet experimental tests, which require this additional force to be suppressed. To circumvent these issues we usually employ some sort of *screening mechanism*.

A screening mechanism is a feature of a theory that dynamically suppresses the force of additional fields at scales where they are required to be small to meet experimental bounds, while permitting a non-negligible effect elsewhere. In the context of modified GR, we would require suppression at solar system scales, while desiring an effect similar to dark energy at cosmological distances. Our primary screening method of interest is known as the Vainshtein mechanism, which we consider below.

1.3.1 The Vainshtein Mechanism

Vainshtein screening is a mechanism that depends on a non-linear kinetic sector in the action. It was originally introduced in connection with massive gravity, specifically Fierz-Pauli theory [41], where efforts to make the graviton massive led to contradictions with solar system tests, even in the limit of vanishing graviton mass. This is usually a cause for concern, as at least at these distances one would anticipate a massive gravity theory in the massless limit would match GR predictions. This discrepancy between GR and Fierz-Pauli theory is commonly known as the vDVZ discontinuity [42], [43]. This dissonance between the two theories is due to the presence of an additional scalar degree of freedom, originating as the longitudinal mode of the massive graviton, which even in the limit of vanishing mass remains coupled to the trace of the stress-energy tensor. More rigorously, the Fierz-Pauli action is,

$$S_{\mathrm{FP}} = \int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{GR,lin}} - \frac{m^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\mathrm{Pl}}} \right) \tag{1.3.7}$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\mathcal{L}_{\mathrm{GR,lin}}$ is the linearised GR Lagrangian and m is the graviton mass. First used in the context of Abelian gauge theories, and applied to massive gravity in [44], the Stükelberg trick allows us to extract the new degrees of freedom by recovering the gauge symmetry that is broken by the graviton mass. This is done by adding new fields and gauge symmetries to make the degrees of freedom explicit whilst preserving their number, so that we can make sense of the $m \to 0$ limit. To recover the linearised diffeomorphism symmetry we transform (1.3.7) by,

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + 2\partial_\mu \partial_\nu \phi \tag{1.3.8}$$

where we have introduced the vector field A_μ and the scalar field ϕ. This transformation is structurally identical to the gauge symmetry we are trying to recover and so $\mathcal{L}_{\mathrm{GR,lin}}$ is invariant. After integration by parts and the assumption of energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ the matter coupling also remains invariant. However the mass term is transformed, resulting in,

$$\begin{aligned} S = \int \mathrm{d}^4x \bigg(& \mathcal{L}_{\mathrm{GR,lin}} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} \\ & -2m^2(h_{\mu\nu}\partial^\mu A^\nu - h\partial_\mu A^\mu) - 2m^2(h_{\mu\nu}\partial^\mu\partial^\nu\phi - h\partial^2\phi) + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\mathrm{Pl}}} \bigg) \end{aligned} \tag{1.3.9}$$

where we have defined $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the Fierz-Pauli structure of the mass term allows the evasion of a ghostly degree of freedom by sending its mass to infinity and decoupling it from the theory. This action now allows two gauge transformations that it is invariant under, given by,

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad A_\mu \to A_\mu - \xi_\mu \tag{1.3.10}$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \quad \phi \to \phi - \Lambda \tag{1.3.11}$$

and we can see that the linearised diffeomorphism symmetry is now recovered and encapsulated within (1.3.10). Now that we have restored the gauge symmetry of linearised GR we are able to take the limit $m \to 0$, preceded by rescaling $A_\mu \to A_\mu/m$ and $\phi \to \phi/m^2$. We obtain,

$$S = \int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{GR,lin}} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 2h_{\mu\nu}\partial^\mu\partial^\nu\phi + 2h\partial^2\phi + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\mathrm{Pl}}} \right). \tag{1.3.12}$$

Diagonalising the action by transforming $h_{\mu\nu} \to h_{\mu\nu} + \phi\, \eta_{\mu\nu}$ to eliminate the kinetic mixing between $h_{\mu\nu}$ and ϕ, we show explicitly the final action,

$$S = \int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{GR,lin}} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 + \frac{\phi T}{\sqrt{6}M_{\mathrm{Pl}}} + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\mathrm{Pl}}} \right) \tag{1.3.13}$$

where there are now the standard two degrees of freedom contained within the metric, and the extra degrees of freedom present in massive gravity are now incorporated within $F_{\mu\nu}$ and ϕ. It is now apparent that ϕ couples to T with gravitational strength similarly to (1.3.1), providing an extra force in tandem with that produced by the metric, and so (1.3.7) differs from linearised GR even in the massless limit; this is the vDVZ discontinuity.

Vainshtein was the first to notice that, in a non-linear gravity theory that reduces to linearised massive gravity, there will be a scale at which the non-linear interactions become comparable to the linear interactions [45], [46], now designated the Vainshtein scale. At distances shorter than this scale non-linear terms either contribute to or even dominate the dynamics, causing a departure from the linearised approximation. For example, DGP gravity has a Vainshtein radius of $r_{\rm V} = (GM/m^4)^{\frac{1}{5}}$ [38], [47], where m is the graviton mass and the M is the mass of a spherically symmetric non-relativistic source. In the massless limit $m \to 0$, the linear theory is invalidated for all space and therefore any resulting predictions become untrustworthy, and we consequently avoid the vDVZ discontinuity.

If instead deviations from Newton's potential are computed in the non-linear regime, the scalar force F_ϕ is found to be of strength,

$$\frac{F_\phi}{F_{\rm N}} \sim \left(\frac{r}{r_{\rm V}}\right)^{\frac{3}{2}} \tag{1.3.14}$$

for $r \ll r_{\rm V}$ [38]. Clearly the non-linear interaction terms suppress the scalar force, screening F_ϕ in the vicinity of the source. Although as we have presented here Vainshtein screening originated in the context of massive gravity, it is in general more widely applicable, and appears in a variety of theories containing non-linear interactions.

To explore the Vainshtein mechanism in more depth, let us consider the action,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 + \frac{c}{\mu^4}(\partial\phi)^4 + \frac{\lambda}{M_{\rm Pl}}\phi T\right) \tag{1.3.15}$$

where λ and c are dimensionless coupling coefficients of $O(1)$ and μ . $M_{\rm Pl}$ is the cut-off of the theory. It is worth noting that the coupling of the scalar field to matter does not break the shift symmetry that the derivative terms in ϕ clearly have, since $\partial_\mu T^\mu{}_\nu = 0$ means we can write,

$$\frac{\lambda}{M_{\rm Pl}}\phi T = \frac{\lambda}{M_{\rm Pl}}\phi\,\partial_\mu\left(x^\nu T^\mu{}_\nu\right) \tag{1.3.16}$$

which under the shift symmetry transformation is a total derivative.

Once again taking the source to be a static spherically symmetric point source of mass M, by dimensional analysis we can conjecture that far from the source the non-linear term will be subdominant. Under this hypothesis, we linearise the equation of motion to find,

$$\phi_{\rm lin}(r) = -\frac{\lambda M}{4\pi M_{\rm Pl}}\frac{1}{r} \tag{1.3.17}$$

which as expected is (1.3.4) in the limit $m \to 0$. Having approximated the theory, we can then ask when the non-linear terms we have neglected will start to become important, or equivalently the radius at which,

$$\left.\frac{(\partial\phi)^2}{\mu^4}\right|_{\phi_{\rm lin}} \sim O(1). \tag{1.3.18}$$

This radius is known as the Vainshtein radius [45], [46], and in this scenario is given by,

$$r_{\rm V} \sim \frac{1}{\mu}\sqrt{\frac{\lambda M}{4\pi M_{\rm Pl}}} \sim \frac{1}{\mu}\sqrt{\frac{M}{M_{\rm Pl}}}. \tag{1.3.19}$$

Solving the equation of motion far inside the Vainshtein radius where the non-linear term dominates, the solution is of the form,

$$\phi_{\rm nl} = -\left(\frac{27\lambda}{16\pi}\right)^{\frac{1}{3}} c^{-\frac{1}{3}}\mu^{\frac{4}{3}}\left(\frac{M}{M_{\rm Pl}}\right)^{\frac{1}{3}} r^{\frac{1}{3}} + \text{const.} \tag{1.3.20}$$

Comparing the force on matter due to the scalar field F_ϕ with a similarly acting Newtonian force $F_{\rm N}$, we see that,

$$\left|\frac{F_\phi}{F_{\rm N}}\right| \sim \left(\frac{r}{r_{\rm V}}\right)^{\frac{4}{3}}. \tag{1.3.21}$$

It is important to realise that the solution for the scalar field can only be matched continuously across the two regimes if $c < 0$, otherwise we obtain a branch-cut singularity [48]. Comparing equation (1.3.21) to (1.3.6) we see a significant difference in behaviour, i.e. when $r \ll r_{\mathrm{V}}$ we get a relative suppression of the scalar force on matter, which increases as we approach the source.

To explain the restrictions on c more rigorously, we can look at perturbations of the theory about a given background solution, and how those perturbations couple to the matter sector. Choosing a background solution $\overline{\psi}$ and a perturbation ψ about $\overline{\psi}$ such that $\phi = \overline{\psi} + \psi$, we expand the action to obtain,

$$S_{\mathrm{pert}} = \int \mathrm{d}^4x \left(-\frac{1}{2} Z^{\mu\nu}[\overline{\psi}] \partial_\mu \psi \partial_\nu \psi + \frac{\lambda}{M_{\mathrm{Pl}}} \psi \delta T \right) \tag{1.3.22}$$

where we have dropped terms of higher than quadratic order, and,

$$Z^{\mu\nu}[\overline{\psi}] = \eta^{\mu\nu} \left(1 - \frac{4c}{\mu^4} (\partial \overline{\psi})^2 \right) - \frac{8c}{\mu^4} \partial^\mu \overline{\psi} \partial^\nu \overline{\psi}. \tag{1.3.23}$$

We can clearly see that as the non-linear terms begin to dominate Z diverges from η and becomes large. Under a schematic canonical normalisation the fluctuations couple to the matter sector with an effective coupling of $\frac{\lambda}{M_{\mathrm{Pl}}\sqrt{Z}}$, clearly indicating a dynamical decoupling of the scalar field and the matter sector as non-linear terms become large, and hence a reduction of the scalar force acting on matter particles.

To examine this closer, we specify the static spherically symmetric ansatz for the background solution $\overline{\psi} = \overline{\psi}(r)$. Under this assumption, we find,

$$Z^{00} = -1 + \frac{4c}{\Lambda^4} \overline{\psi}'^2 \tag{1.3.24}$$

$$Z^{0i} = 0 \tag{1.3.25}$$

$$Z^{ij} = \delta^{ij} \left(1 - \frac{4c}{\Lambda^4} \overline{\psi}'^2 \right) - \frac{8c}{\Lambda^4} \frac{x^i x^j}{r^2} \overline{\psi}'^2 \tag{1.3.26}$$

where $\overline{\psi}'$ is taken to mean $\frac{\mathrm{d}\overline{\psi}}{\mathrm{d}r}$. Physical considerations constrict us to solutions for which $\lim_{r\to\infty} \overline{\psi}' = 0$, at which point $Z^{\mu\nu}$ becomes the Minkowski metric, and therefore has one negative and three positive eigenvalues. In order to maintain this structure for any r, we must necessarily demand $c < 0$ [48], [49]. If this is not the case, there will be a point at which the fluctuation ψ becomes infinitely strongly coupled. This is an example of disallowed Vainshtein Screening, despite the action having a non-linear operator of the correct form, a situation that has been known to occur in other theories [48].

It is worth noting that different assumptions about the background solution will necessarily inform the bounds on c. For example, if we instead take a homogeneous and isotropic background $\overline{\psi} = \overline{\psi}(t)$, we find that now the requirement is $c > 0$. We would then have a theory manifesting a sort of 'cosmological Vainshtein' effect. We may conclude from this analysis that, in at least this theory if not others, the sign of c is vitally important to whether the Vainshtein mechanism is viable.

This Vainshtein validity interacts with fundamental constraints on coupling constants of leading order irrelevant operators, as discussed in [50]. To summarise, for $2 \to 2$ tree level scattering amplitudes, where s, t, u are the standard Mandelstam variables, [50] concludes that in the forward limit ($t \to 0$) the s^2 term must have a strictly positive coefficient if the UV completion is to be Lorentz invariant with analytic S-matrix. In other words, this positivity condition must be met if the theory is to UV-extend to a local QFT.

If we consider the form of the $2 \to 2$ scattering amplitude $\mathcal{A}(s,t)$ in more detail, we can see where this bound originates from. The amplitude $\mathcal{A}(s,t)$ has a pole at $s = 0$ and cuts on the real axis above some $|s_\star| < \infty$. Using Cauchy's integral formula around a closed curve γ about $s = 0$ gives,

$$\frac{\partial^2}{\partial s^2}\mathcal{A}(s,t)\bigg|_{s=0} = \frac{1}{i\pi}\oint_\gamma \mathrm{d}s \frac{\mathcal{A}(s,t)}{s^3}. \tag{1.3.27}$$

Deforming the contour appropriately and taking the forward limit $t \to 0$, we obtain by symmetry,

$$\frac{\partial^2}{\partial s^2}\mathcal{A}(s,0)\bigg|_{s=0} = \frac{4}{\pi}\oint_{s_\star}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\mathcal{A}(s,0)}{s^3} \geq 0 \tag{1.3.28}$$

where the inequality is given by the optical theorem. The contribution from the contour at infinity will vanish as long as the amplitude is at least s^2 to leading order. This is true in massive theories [50]–[52], however the situation is somewhat less clear for massless theories [50]. The full details of this calculation can be found in [39].

In our example (1.3.15) the $2 \to 2$ scattering amplitude is given by [39],

$$\mathcal{A}(s,t) = \frac{2c}{\Lambda^4}\left(s^2 + t^2 + u^2\right) = \frac{4c}{\Lambda^4}\left(s^2 + t^2 + st\right) \tag{1.3.29}$$

where shift symmetry protects a massless ϕ, and so $s + t + u = 0$. It is now apparent that the positivity bound requires $c > 0$ in order for a standard Wilsonian UV completion to be possible. This restriction directly conflicts with our requirement $c < 0$ for Vainshtein screening scalar forces on static backgrounds, for our example (1.3.15). This strongly implies that applying the Vainshtein mechanism to (1.3.15) to suppress scalar forces on solar system scales results in a theory that cannot be the IR limit of a local, Lorentz invariant UV completion.

Later in this thesis we will examine so-called galileon theories, in particular Wess-Zumino galileons. Positivity conditions affect these theories with even greater strength. The $2 \to 2$ scattering amplitude for galileons has leading order term s^3; with the s^2 coefficient fixed at zero, the theory violates the positivity condition for all parameter choices, and so [50] suggests it does not admit a local, Lorentz invariant UV completion. This disconnect between positivity bounds and the parameter choices necessary for Vainshtein screening is one of the main problems for theories exhibiting the mechanism.

1.4 Effective Field Theories

Effective field theories (EFTs) are a natural step when considering the UV completion of any low energy theory. EFTs are constructed according to the guiding principle that every operator that is invariant under a certain set of symmetries, chosen *a priori*, should be included in the action. The form this EFT takes is therefore substantially determined by the selected symmetries. Sometimes however, there are too many candidate operators that satisfy all symmetries to write down or deal with in a meaningful way. In this case a common practice is to power count in terms of some suppression scale, and truncate at some point along the series expansion of this parameter, with some justification as to why the remaining terms stay small in magnitude. This latter point is very important, as discarding terms that can become large in some regime will produce a completely fictitious theory that makes misleading predictions, as we will see in chapter 4. The expansion parameter has another name in the context of EFTs, where it is known as the cut-off, i.e. the scale at which our effective description breaks down. This is a key feature of EFTs, and what distinguishes them from standard field theories.

General Relativity, in the form set forth by Einstein, is fundamentally a classical theory. This distinguishes it from every other force in the Standard Model which is described quantum mechanically. The common saying is that General Relativity and Quantum Mechanics are incompatible because of this, but it is possible to express GR as an effective theory of gravity, where quantisation is permitted in an approach akin to Yang-Mills theories [53]. By expressing GR as an EFT we are able to make predictions, but with the understanding that they only remain valid below the cut-off. Using this formalism for GR permits us to make gravitational quantum predictions up to the Planck scale [47], [53], whereupon loop perturbation theory breaks down and we judge our effective theory to be no longer legitimate. Most notably, it is possible to calculate the correction to the non-relativistic gravitational potential at 1-loop as [54],

$$\frac{\delta V(r)}{V(r)} = \frac{41}{10\pi}\frac{G\hbar}{r^2} \tag{1.4.1}$$

where $V(r) = -\frac{Gm_1m_2}{r}$ is the Newtonian potential, and the presence of $\hbar$ highlights the quantum character of this correction. This being said, we would like to push the cut-off higher than the Planck scale, and potentially find some sort of UV completion of GR, and eventually unify this new theory with the rest of the standard model to find a Grand Unified Theory describing all forces in nature. There are many avenues of research pursuing this, including String Theory [55]–[57], but the rest of this thesis will have little to say about this lofty goal.

1.5 Vacuum Decay

As previously discussed, scalar fields are commonly used in theoretical physics, from inflationary model building [58]–[62] to the Higgs mechanism [63]–[65] which gives standard model particles mass. In these scenarios it is possible for the potential of the scalar field to possess a false minimum. For example, the experimentally verified value of the Higgs mass ($\approx$ 125GeV) suggests that radiative corrections to the Higgs potential cause it to possess such a minimum [66]. The first analysis of these metastable fields was conducted by Voloshin, Kobzarev, and Okun [67], and this work was later progressed by Coleman [68].

Coleman describes in his seminal work an analogy to vacuum decay of phase transitions from the field of statistical mechanics, wherein a superheated fluid in a liquid phase (the false minimum) forms bubbles of vapour via thermal fluctuations. If the bubble is too small then it is energetically favourable for it to shrink to nothing, as it takes more energy to form the bubble wall than is gained from the transition inside the bubble. Vice versa, if the bubble is large enough then it is favourable for the bubble to grow, and so it does so until it has converted the entire fluid to a vapour phase. The corresponding picture for vacuum decay is represented by a replacement of thermal fluctuations with their quantum counterparts. Once a bubble of true vacuum is formed that is large enough, Lorentz invariance informs us that it will start to grow at the speed of light [69], spreading through the universe changing false vacuum into true vacuum.

The probability for such a bubble to form per unit 4-volume is such that vacuum decay occurs on cosmological timescales. Such cosmological phenomena can be utilised in hybrid inflation, combining a slow-roll effect with the aforementioned phase transitions [70], as well as in the field of cosmic strings [71].

As will be shown below, perturbation theory will be inadequate to study non-perturbative phenomena such as bubble formation. Instead, Coleman describes a semi-classical formulation involving a Wick rotation to a Euclidean action, which solves the Euclidean field equations to approximate bubble formation [68].

We present a summary of Coleman's work, preceded by an illustrative example of a perturbative calculation of barrier penetration in non-relativistic quantum mechanics.

1.5.1 The WKB Method

The WKB approximation allows one to solve linear differential equations with varying coefficients, under certain approximations. The method comes in use when performing semiclassical calculations in quantum mechanics. The usual procedure, demonstrated below, involves an exponential ansatz for an approximation to the equation, under the assumption of a slowly varying phase.

We reduce the one-dimensional, time-independent Schrödinger equation given by,

$$-\frac{\hbar}{2m}\frac{\mathrm{d}^2\Psi(x)}{\mathrm{d}x^2} + V(x)\Psi(x) = E\Psi(x) \tag{1.5.1}$$

to the form,

$$i\Phi''(x) - (\Phi'(x))^2 + (k(x))^2 = 0 \tag{1.5.2}$$

by making the substitution $\Psi(x) = Ae^{i\Phi(x)}$, where $k(x) = \hbar^{-1}\sqrt{2m(E - V(x))}$, A is some normalisation and $\Phi(x)$ is a varying phase that for a flat potential is linear in x as $\Phi(x) = \pm\kappa x$. For slowly varying $V(x)$, we approximate $\Phi''(x) \approx 0$, which gives the solution,

$$\Psi_0(x) = e^{\pm i\left(\int k(x)\,\mathrm{d}x + C_0\right)} \tag{1.5.3}$$

or to first order, $\Phi_0''(x) = k'(x)$, from which we obtain,

$$\Psi_1(x) = e^{\pm i\left(\int \sqrt{(k(x))^2 \pm ik'(x)}\,\mathrm{d}x + C_1\right)}. \tag{1.5.4}$$

We note this is to first order the same solution that we would've obtained had we considered $\Psi(x) = \rho(x)e^{i\phi(x)}$ with ρ and ϕ both real functions.

Let us provide an example. Taking a top hat potential for $V(x)$,

$$V(x) = \begin{cases} V & 0 \leq x \leq L \\ 0 & \text{otherwise} \end{cases}$$

where we choose $V > E$, and a basic approximation $\Psi(x) = \Psi_0(x) = \Psi_0(0)e^{\pm i\left(\int_0^x k(x)\,\mathrm{d}x\right)}$, we attempt to compute the tunnelling probability T, given by the ratio of the transmitted intensity $|\Psi(L)|^2$ to the incident intensity $|\Psi(0)|^2$,

$$T(L,E) = \frac{|\Psi(L)|^2}{|\Psi(0)|^2} = e^{\pm 2i\left(\int_0^L \hbar^{-1}\sqrt{2m(E-V(x))}\,\mathrm{d}x\right)} = e^{-2\hbar^{-1}\sqrt{2m(V-E)}L} \tag{1.5.5}$$

where L is the barrier width, E is the particle energy, and the sign of the exponent has been resolved such that the probability is less than 1. This is the probability of a particle tunnelling through the potential barrier.

1.5.2 Instantons

We have seen from the WKB approximation that the amplitude of transmission through a potential barrier is of the form,

$$|T(E)| = e^{-2\hbar^{-1}\left(\int_{x_1}^{x_2}\sqrt{2m(V(x)-E)}\,\mathrm{d}x\right)} \tag{1.5.6}$$

This equation vanishes more rapidly than any power of $\hbar$ in the classical limit $\hbar \to 0$, and therefore cannot hope to be captured by perturbation theory. In standard quantum mechanics this phenomenon is captured by the WKB approximation. We must look to an equivalent non-perturbative description or method in QFT in order to fully capture effects such as this.

In the following section we will discuss such a non-perturbative process, which aims to capture tunnelling from a local minimum of the potential to a global minimum. In the context of QFT this is known as false vacuum decay. We will illustrate the approach by considering a problem in quantum mechanics, to which we already know the answer to by the standard means, but which will illuminate the workings of the method. We will proceed to a field theory generalisation, before lastly considering gravity as well, and what effect space-time curvature has on false vacuum decay. This outline will roughly follow that given in Coleman's papers on the subject [68], [72], [73], with an emphasis on our particular area of interest, but for brevity and clarity we omit some technical details which can be found in said works.

There exist two equivalent descriptions of quantum mechanics, expressed via states and operators or the path integral formulation. Our primary tool for evaluating the non-perturbative questions in quantum mechanics we wish to answer will be the main equation that connects the two, describing in each interpretation the transition amplitude between position eigenstates, $|x_\mathrm{i}\rangle$ at $t = -\frac{t_0}{2}$ and $|x_\mathrm{f}\rangle$ at $t = \frac{t_0}{2}$, which takes the form,

$$\int \mathcal{D}x\, e^{-iS[x(t)]} = \langle x_\mathrm{f}|e^{-iHt_0}|x_\mathrm{i}\rangle \tag{1.5.7}$$

where H is the Hamiltonian of a system, $S[x(t)]$ is the corresponding action evaluated on paths between points $(x_\mathrm{i}, -t_0/2)$ and $(x_\mathrm{f}, t_0/2)$, and $\mathcal{D}x$ is a normalised measure for the integration over trajectories satisfying these boundary conditions.

The boundary conditions are important, and we set them based on the problem we wish to consider. Inserting a complete set of energy eigenstates, and identifying $\langle x|n\rangle = \psi_n(x)$, the right hand side gives us,

$$\langle x_{\rm f}|e^{-iHt_0}|x_{\rm i}\rangle = \Sigma_n e^{-iE_n t_0}\psi_n(x_{\rm f})\psi_n^*(x_{\rm i}). \tag{1.5.8}$$

We can now see that if we perform a Wick rotation, $\tau = it$, then the oscillating exponentials become exponentially decreasing instead, and as τ becomes large only the terms with the lowest energy E_0 will survive, which helpfully picks out the ground state of the theory,

$$\lim_{\tau_0(=it_0)\to\infty} \Sigma_n e^{-iE_n t_0}\psi_n(x_{\rm f})\psi_n^*(x_{\rm i}) \approx e^{-E_0\tau_0}\psi_0(x_{\rm f})\psi_0^*(x_{\rm i}). \tag{1.5.9}$$

If we can now match with a corresponding expression on the left hand side of (1.5.7), it will be possible to read off the value of E_0 and the overlap of the two position states. Although in this specific example there are simpler ways to find these quantities, Coleman's method has the advantage that it is immediately generalisable to non-perturbative QFT in a way that 'simpler' methods are not.

Let us proceed to treat the path integral in the same way and simplify the left hand side. Firstly, the transformation from real to imaginary time means that the relevant quantity is now the Euclidean action,

$$S = \int_{-t_0/2}^{t_0/2}\left(\frac{1}{2}\dot{x}^2 - V(x)\right){\rm d}t \longrightarrow_{\tau=it} iS_{\rm E} = i\int_{-\tau_0/2}^{\tau_0/2}\left(\frac{1}{2}\dot{x}^2 + V(x)\right){\rm d}\tau. \tag{1.5.10}$$

Expressing the path integral in terms of the Euclidean action, we obtain an exponential weight for each path; the Euclidean version of (1.5.7) is,

$$\int \mathcal{D}x\, e^{-S_{\rm E}[x(\tau)]} = \langle x_{\rm f}|e^{-H\tau_0}|x_{\rm i}\rangle. \tag{1.5.11}$$

It is now easy to see that the dominant contribution originates from the trajectory with the lowest value for $S_{\rm E}$, i.e. the solution to the Euclidean equations of motion. The Euclidean path integral then behaves as,

$$\int \mathcal{D}x\, e^{-S_{\rm E}[x(\tau)]} \propto e^{-S_{\rm E}[x_{\rm cl}]} \tag{1.5.12}$$

where $x_{\rm cl}(\tau)$ is a solution of $\delta S_{\rm E} = 0$, which in this case is the equation of motion $\ddot{x} - V'(x) = 0$. We now draw some parallels between this system and a classical particle moving in a potential: Firstly that this equation describes a particle moving in the inverted potential "$-V(x)$", and secondly the conserved energy of such a system looks like,

$$E = \frac{1}{2}\dot{x}^2 - V(x) = \text{const.} \tag{1.5.13}$$

Now if the potential has but a single minimum, which without loss of generality we will put at $x = 0$, the only path that satisfies the Euclidean equation of motion while keeping the action finite is the path that sits at the minimum for all τ. Treating (1.5.12) a little more carefully we could consider fluctuations around $x_{\rm cl}(\tau)$, and by integrating the resulting gaussians we would obtain,

$$\sqrt{\frac{\omega}{\pi}} e^{-\frac{\omega\tau_0}{2}} = \langle 0|e^{-H\tau_0}|0\rangle \longrightarrow_{\tau_0\to\infty} |\langle x=0|n=0\rangle|^2 e^{-E_0\tau_0} \tag{1.5.14}$$

where $x_{\rm i}$ and $x_{\rm f}$ have been set to zero by our boundary conditions, hence we are considering the state $|0\rangle$. We see that by inserting a complete set of states and comparing the exponents and coefficients of each side of the equation in the limit $\tau_0 \to \infty$ we obtain the standard results for a single-minimum potential, namely the ground state energy $E_0 = \frac{1}{2}\omega$ and the probability of the particle being localised at the minimum in the ground state $\sqrt{\frac{\omega}{\pi}}$.

To illustrate the workings of this method further we consider a non-trivial example of a potential with two minima. In principle we could treat this generally, however as it is an example it will be sufficient for our purposes to take the potential,

$$V(x) = \lambda(x^2 - \eta^2)^2. \tag{1.5.15}$$

We now have a non-trivial ground state, on account of the double minima at $x = \pm\eta$. As opposed to the simple choice of boundary conditions $x_{\rm i} = x_{\rm f}$ above, we can now choose our start and end positions to be $\pm\eta$. Let us cut to the chase and choose $x_{\rm i} = -\eta$ and $x_{\rm f} = \eta$. Classically this process is disallowed, but there is no such barrier once we consider the inverted potential; in the limit $\tau \to \infty$ there exists a solution where the particle reaches the top of either hill at $\pm\infty$, and makes the transition between the two at some finite time. The particle must then reach the hill with zero energy, and so we can set E to zero in (1.5.13), which allows us to simplify the Euclidean action as,

$$S_0 = \int_{-\infty}^{\infty} \left(\frac{1}{2}\dot{x}_{\rm cl}^2 + V(x_{\rm cl}) \right) \mathrm{d}\tau = \int_{-\infty}^{\infty} \dot{x}_{\rm cl}^2 \, \mathrm{d}\tau = \int_{-\eta}^{\eta} \dot{x}_{\rm cl} \, \mathrm{d}x = \int_{-\eta}^{\eta} \sqrt{2V(x_{\rm cl})} \, \mathrm{d}x \tag{1.5.16}$$

and so (1.5.11) becomes,

$$\lim_{\tau\to\infty} \langle \eta | e^{-H\tau} | -\eta \rangle \approx e^{-S_0[x_{\rm cl}]} = \exp\left(-\frac{\omega^3}{12\lambda} \right) \tag{1.5.17}$$

where ω is defined by $\omega^2 = V''(\pm\eta) = 8\lambda\eta^2$. Once again, treating (1.5.17) more rigorously with integration of perturbations around $x_{\rm cl}$, we obtain an expression which by matching allows us to obtain the lowest lying energy states.

1.5.3 Unstable Vacua

Having explored this instanton method in quantum mechanics, and found it reproduces known results accurately, we turn to generalising it to QFT, and investigate what happens once we have an infinite amount of degrees of freedom. For clarity and brevity we will only consider the case we will ultimately be interested in, namely a real scalar field theory ϕ with a double-minimum potential where only one of them is a global energy minimum. Let us label the global minimum (or "true vacuum") as at $\phi = \phi_{\rm T}$, while the local minimum (or "false vacuum") is at $\phi = \phi_{\rm F}$.

We choose our initial conditions such that the field is in the false vacuum state $\phi = \phi_{\rm F}$ over all space. Again, this state is classically stable due to the potential barrier between the false and the true vacuum, but may decay in a quantum system. Naively one might expect that all the degrees of freedom of a QFT should tunnel simultaneously, which results in a zero probability, and the conclusion that tunnelling is forbidden. Examining the situation in more detail leads to a different result however.

Similar to the previously described thermal phase transition, or nucleation process, in statistical physics, there is a non-zero probability that some number of degrees of freedom within a finite spherical region simultaneously transition from $\phi_{\rm F}$

to ϕ_{T}. More rigorously, the gain in energy from the transition is proportional to the difference in energy between the two minima, and to the volume of the sphere, which scales as R^3, where R is the sphere radius. There is also an energy cost in creating the bubble wall, which behaves like R^2. Balancing the two, there exists a critical radius R_{c}, where if $R > R_{\mathrm{c}}$ at the point of formation the sphere will grow until it converts the whole space to the true vacuum. Having qualitatively described tunnelling in QFT, we proceed to cement this description with a more technical discussion.

It should be noted that the quantity we want to compute is the decay rate per unit time, Γ, per unit volume. This is because in any given volume Γ must be proportional to said volume, on account of the bubble of true vacuum being equally likely to form anywhere within the space. We have demonstrated that the important quantity in determining tunnelling probability is the exponential of the Euclidean action on the classical solution to the equations of motion with the appropriate boundary conditions.

A key simplification we will make is that the behaviour of phase transitions is dominated by $O(4)$ invariant solutions, and therefore we will limit our considerations to this scenario, where ϕ is a function that only depends on the four-dimensional euclidean distance $\rho = \sqrt{\tau^2 + \mathbf{x}^2}$. This was shown to hold true in the case of a single scalar field in [74].

Within this simplification the Euclidean action becomes,

$$S_{\mathrm{E}} = 2\pi^2 \int \mathrm{d}\rho \, \rho^3 \left(\frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}\rho} \right)^2 + V(\phi) \right) \tag{1.5.18}$$

and the equations of motion simplify to,

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}V}{\mathrm{d}\phi}. \tag{1.5.19}$$

For the sake of simplicity we will add a constant to the potential such that the energy of the false vacuum is equal to zero, $V(\phi_{\mathrm{F}}) = 0$. This will not be allowed once we wish to consider gravity, as the potential energy at the minimum will determine curvature, but we will accept the leeway while we can.

Remembering that the equation of motion must be accompanied by boundary conditions, we assume that initially the field sits in the false vacuum for all space,

$$\lim_{\tau \to -\infty} \phi(\tau, \mathbf{x}) = \phi_{\mathrm{F}} \tag{1.5.20}$$

where we have now taken τ_0 to infinity, and that ϕ should take the false vacuum value at large distances,

$$\lim_{|\mathbf{x}| \to \infty} \phi(\tau, \mathbf{x}) = \phi_{\mathrm{F}} \tag{1.5.21}$$

in order for the action to be finite. Recognising that the action is invariant under $\tau \to -\tau$, these boundary conditions can be expressed as one in terms of the radial distance,

$$\lim_{\rho \to \infty} \phi(\rho) = \phi_{\mathrm{F}}. \tag{1.5.22}$$

That the boundary conditions combine to an $O(4)$-invariant form gives us some more faith in our assumption.

Noting that the action exhibits time translation symmetry, we can assume without loss of generality that the tunnelling occurs at $\tau = 0$, and that the bubble comes into existence at rest, hence,

$$\left. \frac{\partial}{\partial \tau} \phi(\tau, \mathbf{x}) \right|_{\tau=0} = 0 \tag{1.5.23}$$

and we find a final boundary condition for (1.5.19) of the form,

$$\left. \frac{\mathrm{d}\phi(\rho)}{\mathrm{d}\rho} \right|_{\rho=0} = 0 \tag{1.5.24}$$

so as to avoid a $\rho = 0$ singularity.

We see we have simplified the problem of tunnelling in QFT, a system with an infinite amount of degrees of freedom, into the study of a single partial differential equation. We can make an analogy to the situation we have already described in the previous section: a classical particle in a potential '$-V$', although this time with an additional friction-like force whose 'friction coefficient' is inversely proportional to the 'time' ρ. The boundary conditions tell us that the particle starts at rest at $\phi \approx \phi_{\mathrm{T}}$, then travels down the potential to arrive at the local maximum $\phi = \phi_{\mathrm{F}}$ at $\rho = \infty$. If the particle starts too far from ϕ_{T} the frictional force means it would fall short of reaching ϕ_{F}. If however the particle begins too close to ϕ_{T} then it will

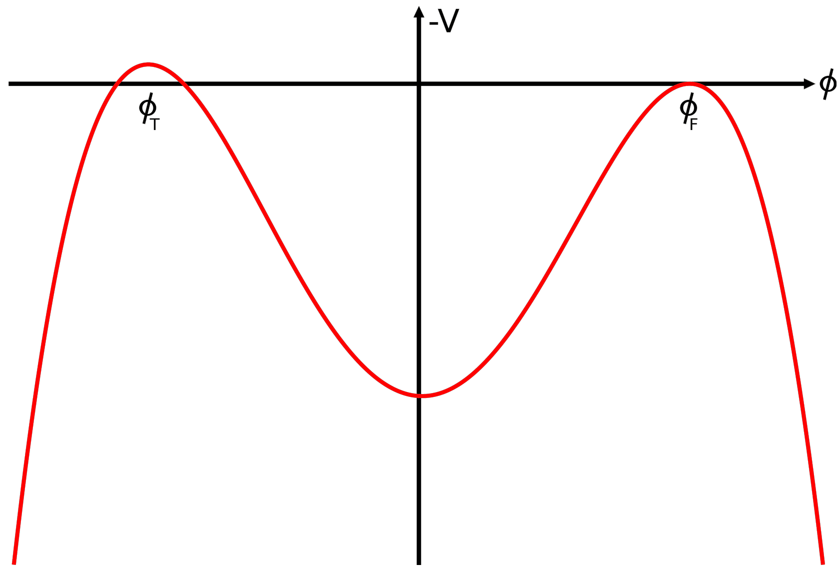

Figure 1.5.1: A schematic of the type of potential we wish to consider, showing two local maxima within the thin wall approximation. The field starts close to $\phi_{\rm T}$ and remains there until $\rho_{\rm b}$ at which point it quickly traverses the gap to $\phi_{\rm F}$, where it stays.

overshoot $\phi_{\rm F}$ and run to infinity; this is helped by the frictional force becoming smaller the longer the particle stays at the top of the hill. A continuity argument suggests there will be an initial position that fulfils our boundary conditions.

It is easy to see that (1.5.19) becomes easier to solve once the frictional term is negligible. To this end we make the assumption that the difference in energy between the global and local maxima $\epsilon = V(\phi_{\rm T}) - V(\phi_{\rm F}) > 0$ is small. The path taken by the particle now looks like this; the particle spends time at $\phi \approx \phi_{\rm T}$ at rest while the friction force is becoming smaller due to increasing ρ, until it becomes negligible and the particle quickly traverses the gap between the two vacua, remaining at $\phi \approx \phi_{\rm F}$ for the rest of the motion, reaching it at infinite time. This is known as a thin-wall limit, because it results in the transition between the two maxima happening at $\rho \approx$ const, i.e. the 'wall' between the two vacua is small.

In this limit we can split the integral (1.5.18) into three regimes. Firstly, for $\rho < \rho_{\rm b}$, (the bubble radius), we have $\phi \approx \phi_{\rm T}$ and any derivative terms are negligible,

$$S_{\rm b} = 2\pi^2 \int_0^{\rho_{\rm b}} {\rm d}\rho\, \rho^3 V(\phi_{\rm T}) = -\frac{1}{2}\pi^2 \rho_{\rm b}^4 \epsilon. \tag{1.5.25}$$

Secondly we evaluate the integral in the transition between vacua. As stated, in a thin-wall limit the conversion to false vacuum happens at $\rho \approx \rho_{\rm b}$. Here the bubble wall takes the form of a 3-dimensional surface that divides the two regions of vacuum, with some surface tension σ. Close enough to the bubble wall we can neglect the small difference in the energy of the two vacua, and generalising (1.5.16) to our QFT scenario we obtain,

$$S_{\rm w} = 2\pi^2 \int_{\rho_{\rm b}-\delta}^{\rho_{\rm b}+\delta} {\rm d}\rho\, \rho^3 \left(\frac{1}{2} \left(\frac{{\rm d}\phi}{{\rm d}\rho} \right)^2 + V(\phi) \right) = 2\pi^2 \rho_{\rm b}^3 \int_{\phi_{\rm F}}^{\phi_{\rm T}} \sqrt{2V(\phi)}\, {\rm d}\phi = 2\pi^2 \rho_{\rm b}^3 \sigma. \tag{1.5.26}$$

Finally, outside of the bubble ϕ sits at the false vacuum; a negligible derivative and $V(\phi_{\rm F}) = 0$ gives a zero action.

Combining these terms, the total action is,

$$S_{\rm E} = S_{\rm b} + S_{\rm w} = -\frac{1}{2}\pi^2 \rho_{\rm b}^4 \epsilon + 2\pi^2 \rho_{\rm b}^3 \sigma \tag{1.5.27}$$

The last thing to do is to find the value for $\rho_{\rm b}$ in terms of our thin-wall limit parameter ϵ. We know we want to extremise the action, hence by varying $S_{\rm E}$ with respect to the $\rho_{\rm b}$ we can find,

$$\frac{\partial S_{\rm E}}{\partial \rho_{\rm b}} = 0 \implies \rho_{\rm b} = \frac{3\sigma}{\epsilon} \tag{1.5.28}$$

giving a final expression for the Euclidean action in the thin-wall limit as,

$$S_{\rm E} = \frac{27\pi^2 \sigma^4}{2\epsilon^3}. \tag{1.5.29}$$

Once we have determined the field configuration after the tunnelling event the field evolves according to the Lorentzian equation of motion, and by matching at $\tau = t = 0$ the field evolves according to the analytic continuation of the tunnelling solution to real time. Wick rotating ρ,

$$\rho^2 = \tau^2 + \mathbf{x}^2 = \mathbf{x}^2 - t^2 \tag{1.5.30}$$

we see that surfaces of constant Euclidean distance relate to three dimensional surfaces of constant distance expanding with real time t. After tunnelling, the bubble wall follows the hyperboloid $\mathbf{x}^2 - t^2 = \rho_{\rm b}^2$, and hence the bubble of true vacuum rapidly accelerates to an almost light-speed expansion.

1.5.4 Inclusion of Gravity

As will become clear in the later parts of this thesis, we wish to account for the effects of gravitation on vacuum decay. A hasty analysis might tell us that vacuum decay itself occurs at scales of negligible gravitational effect, however the growth of the bubble must necessarily stray into gravitational territory, so let us try to estimate at what distances this happens. Vacuum decay releases energy proportional to the bubble volume, which therefore dictates the behaviour of its Schwarzschild radius; with the growth of the bubble there must be a point at which it is comparable with the radius of the bubble itself. A sphere of radius l and energy density ϵ has a Schwarzschild radius of the form $2G_{\mathrm{N}}(\frac{4\pi l^3}{3})\epsilon$. The two radii are equal at,

$$l = \left(\frac{8\pi G_{\mathrm{N}}\epsilon}{3}\right)^{-\frac{1}{2}} = M_{\mathrm{Pl}}\sqrt{\frac{3}{\epsilon}}. \tag{1.5.31}$$

For example, $\epsilon \sim (1\mathrm{GeV})^4$ results in $l \sim 1\mathrm{km}$. Certainly we wish to consider larger energy densities than this, and therefore smaller values of l. It is apparent that gravity is non-negligible well before we arrive at any kind of cosmological distance, and gravity must be included in our understanding of tunnelling if we are to obtain accurate conclusions.

A subtlety that we were able to ignore in the previous section when we set $V(\phi_{\mathrm{F}}) = 0$ raises its head once we introduce gravity. Up to this point there has been no absolute zero of energy density, and we have been able to add a constant to V as we please. This is no longer the case once gravity is included, as we can see from the gravitational action,

$$S = \int \sqrt{-g}\left(\frac{1}{2}\left(\partial_\mu\phi\right)^2 - V(\phi) - \frac{1}{2}M_{\mathrm{Pl}}^2 R\right) \tag{1.5.32}$$

where R is the Ricci scalar. Now, shifting V by a constant introduces a term proportional to $\sqrt{-g}$ to the integrand, i.e. a cosmological constant. Vice versa, once the vacuum has decayed from false to true vacuum the cosmological constant will change within that volume. We must therefore determine the absolute zero of energy density by giving its initial value as an additional boundary condition.

We will undergo the remainder of this section following the approach taken by Coleman and de Luccia [73]; we consider there to be two useful examples based on the experimental declaration that the observed value of the cosmological constant is approximately zero in natural units. These two examples will therefore be: 1) $V(\phi_{\mathrm{T}})$ is zero, and we reside after the decay of the false vacuum. 2) $V(\phi_{\mathrm{F}})$ is zero, and we currently reside in the false vacuum. In general this method would work for any values of the cosmological constant, but it will be helpful to be aware of these two special cases.

As before we wish to solve the Euclidean equations of motion, only now for the action (1.5.32), with corresponding boundary conditions. In principle, this gives us an additional ten degrees of freedom to find, originating from the metric tensor. However, it is reasonable to assume that the introduction of gravity doesn't break the symmetry of the original problem; in our semi-classical approach we are considering classical gravity, in which the vacuum solutions are exactly those that allow us to retain $O(4)$ invariance. It should be noted that while this is merely a supposition, some attempts to consider solutions that are not $O(4)$ invariant have been made, and so far they suggest that our assumption is accurate [75].

The most general $O(4)$ invariant Euclidean metric is of the form,

$$\mathrm{d}s^2 = \mathrm{d}\xi^2 + \rho(\xi)^2 \mathrm{d}\Omega_3^2 \tag{1.5.33}$$

where $\mathrm{d}\Omega_3^2$ is the element of distance on a unit three-sphere, ρ gives the radius of curvature of each three-sphere, and ξ is a radial coordinate which is orthogonal to all angular coordinates on the sphere. Once again rotational invariance has allowed us to considerably simplify our problem; instead of considering ten functions of four variables we can work with a single function of one variable.

Denoting differentiation with respect to ξ with a prime from hereon out, the Einstein equations for this metric are given by,

$$\rho'^2 = 1 + \frac{1}{3}\frac{\rho^2}{M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\phi'^2 - V\right) \tag{1.5.34}$$

with all others resulting in trivial expressions. The equation of motion for ϕ, assuming ξ dependence only, is given by,

$$\phi'' + 3\frac{\rho'}{\rho}\phi' = \frac{\mathrm{d}V}{\mathrm{d}\phi} \tag{1.5.35}$$

where we can see compared to the corresponding equation without gravity, (1.5.19), only the coefficient of ϕ' has changed. We can again use the thin-wall approximation to neglect this term [73], and we once again obtain the simplified equation $\phi'' = \frac{\mathrm{d}V}{\mathrm{d}\phi}$ that we did in the scalar scenario. We now wish to compute the Euclidean action for this theory in the thin-wall limit. As we have said, we cannot simply set $V(\phi_{\mathrm{F}}) = 0$ due to the metric coupling, but we do need to perform a background subtraction in order to obtain a finite decay rate. The probability will now be related to the difference between the Euclidean actions for the solution to the equations of motion, and for a field sitting in the false vacuum,

$$B = S_{\mathrm{E}}[\phi_{\mathrm{b}}] - S_{\mathrm{E}}[\phi_{\mathrm{F}}]. \tag{1.5.36}$$

The other difference when compared to the purely scalar calculation is the addition of the Einstein-Hilbert term to S_{E}. The Ricci scalar for our $O(4)$ invariant metric (1.5.33) is,

$$R = \frac{6}{\rho^2}\left(1 - \rho\rho'' - \rho'^2\right). \tag{1.5.37}$$

Combining with (1.5.34), we substitute into the Euclidean version of (1.5.32), and integrate by parts to arrive at the expression,

$$S_{\mathrm{E}} = 4\pi^2 \int \mathrm{d}\xi \left(\rho^3 V - 3M_{\mathrm{Pl}}^2 \rho\right). \tag{1.5.38}$$

Following the same approach as the scalar case, we split the integration into three regions. Outside of the bubble $\phi = \phi_{\mathrm{F}}$, which exactly matches our background subtraction, hence $B_{\mathrm{outside}} = 0$. For the bubble wall $\rho \approx \rho_{\mathrm{b}}$, hence using (1.5.38) we get,

$$B_{\mathrm{wall}} = 4\pi^2\rho_{\mathrm{b}}^3 \int \mathrm{d}\xi \left(V(\phi) - V(\phi_{\mathrm{F}})\right) = 2\pi^2\rho_{\mathrm{b}}^3\sigma. \tag{1.5.39}$$

Lastly, inside of the bubble $\phi = \phi_{\mathrm{T}}$. From (1.5.34) we obtain the relation,

$$\mathrm{d}\xi = \frac{\mathrm{d}\rho}{\sqrt{1 - \frac{1}{3}M_{\mathrm{Pl}}^{-2}\rho^2 V}} \tag{1.5.40}$$

which we can use in (1.5.38) to integrate from $\rho = 0$ to $\rho = \rho_{\rm b}$, and get,

$$B_{\rm inside} = \frac{12\pi^2 M_{\rm Pl}^4}{V(\phi_{\rm T})}\left(\left(1 - \frac{1}{3}\frac{\rho_{\rm b}^2 V(\phi_{\rm T})}{M_{\rm Pl}^2}\right)^{\frac{3}{2}} - 1\right) - (\phi_{\rm T} \to \phi_{\rm F}) \tag{1.5.41}$$

where the first term is from $S_{\rm E\,inside}[\phi_{\rm T}]$ and the second from the background subtraction in terms of $\phi_{\rm F}$.

Let us now consider the two examples outlined at the start of this section. The first case is for an absolute zero vacuum energy defined by the true vacuum, $V(\phi_{\rm T}) = 0$, which in the thin-wall approximation gives $V(\phi_{\rm F}) = \epsilon$. This is equivalent to a tunnelling scenario from a de Sitter into a Minkowski space-time. Simplifying (1.5.41), our full expression is now given by,

$$B = B_{\rm wall} + B_{\rm inside} = 2\pi^2\rho_{\rm b}^3\sigma - 6\pi^2 M_{\rm Pl}^2\rho_{\rm b}^2 - \frac{12\pi^2 M_{\rm Pl}^4}{\epsilon}\left(\left(1 - \frac{1}{3}\frac{\rho_{\rm b}^2\epsilon}{M_{\rm Pl}^2}\right)^{\frac{3}{2}} - 1\right). \tag{1.5.42}$$

Varying with respect to $\rho_{\rm b}$, and demanding that the actual bubble radius is a stationary point, we find,

$$\rho_{\rm b} = \frac{12\sigma}{4\epsilon + 3M_{\rm Pl}^{-2}\sigma^2} = \frac{\rho_0}{1 + (\rho_0/2l)^2} \tag{1.5.43}$$

where ρ_0 is the value of the bubble radius in the purely scalar case, given by (1.5.28), and $l = M_{\rm Pl}\sqrt{\frac{3}{\epsilon}}$, as in (1.5.31). This results in a simplified version of the total tunnelling exponent (1.5.42) of the form,

$$B = \frac{B_0}{\left(1 + (\rho_0/2l)^2\right)^2} \tag{1.5.44}$$

where B_0 is the Euclidean action in the purely scalar case (1.5.29); for a decay from de Sitter into Minkowski space-time gravity increases the decay probability by diminishing the action.

The second case is tunnelling from a Minkowski space-time, $V(\phi_{\rm F}) = 0$, to an Anti de Sitter space-time, $V(\phi_{\rm T}) = -\epsilon$, where now the absolute zero of vacuum energy is set by the false vacuum, and the true vacuum has an associated negative vacuum energy. Following the same procedure, we arrive at similar expressions for the bubble radius,

$$\rho_{\rm b} = \frac{\rho_0}{1 - (\rho_0/2l)^2} \tag{1.5.45}$$

and tunnelling exponent,

$$B = \frac{B_0}{\left(1 - (\rho_0/2l)^2\right)^2} \tag{1.5.46}$$

where the denominator now carries a sign change when compared with the equations for case 1. We see that for this case gravity will decrease the decay probability, and increase the radius of the materialised bubble. In fact, for $\rho_0 = 2l$, gravity has the effect of completely stabilising the false Minkowskian vacuum, by requiring the bubble to form with infinite size. This phenomenon has a simple explanation in terms of energy requirements: Energy conservation demands a materialised bubble has zero energy, the balance of a negative energy volume term and a positive energy surface term. In the purely scalar case it is always possible to make a bubble satisfy this regardless of the value of ϵ; for a big enough bubble the volume/surface ratio will automatically fulfil our requirements. In contrast, when we include gravitation, the negative energy density of the bubble curves the space-time so as to reduce the volume/surface ratio. We therefore reach the requirement of infinite bubble radius at a finite value of ϵ, past which no bubble will have zero energy.

Chapter 2

The Cosmological Constant Problem

2.1 Vacuum Energy

In the previous chapter we have stated that we expect there to be corrections to the cosmological constant coming from vacuum energy matter loops. Let us now explain why we expect them to contribute to the energy momentum tensor with form $T_{\mu\nu\,\mathrm{vac}} = -g_{\mu\nu}V_{\mathrm{vac}}$, and estimate their magnitude.

In order to estimate the vacuum energy in GR coupled to matter, we utilise a semi-classical method, treating the field theory sector quantum mechanically but the gravitational degrees of freedom classically. We initially expand the action around a Minkowski background with canonically normalised fluctuations of the metric. We then decouple gravity by sending $M_{\mathrm{Pl}} \to \infty$, and calculate loop corrections in this limit; we now have a theory on a Minkowski background where standard quantum field theory is applicable. After calculating the vacuum energy corrections to a selected order in perturbation theory, we return M_{Pl} to a finite value and let gravity interact classically with the resulting vacuum corrections as imposed by the gauge invariance.

It should be made clear that this is an approximation to the full prescription. Although pure graviton loops do not contribute, once the graviton is coupled to matter we will see similar large radiative corrections. However, the matter loops already present enough of a conundrum when comparing our perturbation theory result to experimental observation, as will become apparent, therefore as a first attempt this semi-classical method will be sufficient. Of course the corrections from virtual gravitons are expected to be non-negligible relative to the experimentally observed value of the cosmological constant [76], even with suppression from the Planck mass, and so in a full solution to the cosmological constant problem these would need to be considered. We will examine these contributions later within an EFT approach.

Let us take an example action to illustrate the calculation,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4\right) \tag{2.1.1}$$

The 1-loop correction to the cosmological constant then takes the form,

$$S = -\frac{1}{2}\int \mathrm{d}^4x \int \frac{\mathrm{d}^4p}{(2\pi)^4}\log\left(p^2 + m_\phi^2\right). \tag{2.1.2}$$

In simplifying this equation divergent momentum integrals arise, which require some form of regularisation. Here we choose to use dimensional regularisation working in $d = 4 - \epsilon$ dimensions (closely following [31]). We obtain a result at the 1-loop level for our example of [33],

$$V_{\mathrm{vac}}^{\phi,\text{1-loop}} = -\frac{m_\phi^4}{(8\pi)^2}\left(\frac{1}{2\epsilon} + \log\left(\frac{\mu^2}{m_\phi^2}\right) + \text{finite}\right) \tag{2.1.3}$$

where μ is an arbitrary mass scale introduced during the regularisation procedure, which by dimensional analysis must be inserted. Any finite contributions to the loop correction are therefore also arbitrary, as μ can always be redefined to absorb them. We can clearly see that, in the limit $\epsilon \to 0$, (2.1.3) is divergent. As is standard QFT procedure, we cancel the divergence by adding a counter term Λ_{c}, of the form,

$$\Lambda_{\mathrm{c}}^{\phi,\text{1-loop}} = \frac{m_\phi^4}{(8\pi)^2}\left(\frac{1}{2\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + \text{finite}\right) \tag{2.1.4}$$

where M is the scale of renormalisation. We can recognise $\Lambda_{\mathrm{c}}^{\phi,\text{1-loop}}$ as the bare cosmological constant that was present in our gravitational action, since it appears in exactly the same form, hence the notation choice. The resulting renormalised vacuum energy is the sum of the two and is given by,

$$\Lambda_{\mathrm{ren}}^{\phi,\text{1-loop}} = \frac{m_\phi^4}{(8\pi)^2}\left(\log\left(\frac{m_\phi^2}{M^2}\right) + \text{finite}\right). \tag{2.1.5}$$

We can once again see that the finite part of the loop correction is arbitrary since we can always redefine M to absorb it. This is an example of an important principle in the study of EFTs: it is impossible to predict *a priori* the exact size of the loop corrections to the cosmological constant, or indeed any relevant operator, as the result will always contain an arbitrary mass scale. Instead, we have to perform a measurement, and modify the finite contribution to Λ_c accordingly in order to match theory with observation.

2.2 Fine Tuning and Re-Fine Tuning

As an example, let's assume our scalar field is the Higgs boson, with a mass of $m_\phi = 126\text{GeV}$ [64], [65]. Observations give an upper bound of $(\text{meV})^4$ on the total cosmological constant, so the counterterm must cancel vacuum energy contributions at the 1-loop level to an accuracy of 1 part in 10^{60}.

Having shown that at the 1-loop level the counterterm needs to be fine-tuned to match observation, we now calculate the 2-loop correction. The exact form of this calculation is not important, and from dimensional analysis we see it is of the form,

$$V_{\text{vac}}^{\phi,\text{2-loop}} \sim \lambda m_\phi^4 \tag{2.2.1}$$

where the λ originates from the 4-point vertex in this interaction. It is important to note that this is an additional correction on top of our result for 1-loop. Again taking the Higgs boson to be our example scalar, with $\lambda \sim 0.1$, this is another large correction relative to the observed cosmological constant value . However, the bare cosmological constant counterterm has already been fixed such that Λ_{ren} matches observations at 1-loop. We now must re-tune it to cancel the 2-loop contributions as well, as a similar level of accuracy. This pattern repeats and we must continue to re-tune at 3-loops, 4-loops and beyond. At every order the correction to the vacuum energy from the scalar field is not perturbatively suppressed relative to the previous loop correction like we might expect, and remains well above the observed value for the cosmological constant. This is known as radiative instability, and is a symptom of the highly UV sensitive nature of the cosmological constant; we denote its small observed value as being unnatural.

It is worth noting that the same result, that the cosmological constant is radiatively unstable, can be obtained from a Wilsonian viewpoint [77], [78]. This involves integrating out high energy modes above some cut-off, leaving a lower energy EFT. There one finds that the correction to the vacuum energy is highly sensitive to the cut-off of the effective theory, and that if one changes the cut-off, again re-fine tuning is required. This is an equivalent argument to that discussed above, and produces the same understanding of the nature of the cosmological constant. For a full discussion of this approach see [29], [79].

For comparison to the cosmological constant, the calculation of loop corrections to the electron mass results in a series of terms, where each loop order contributes a correction that is suppressed relative to the previous order. Additionally, these contributions have a logarithmic form as opposed to a power law, making them much more robust to changes in UV physics [80]. This means we can set the bare mass of the electron in our EFT such that we obtain the correct observed value, and calculating to additional loop orders will not cause an appreciable difference. This occurs because massless fermions are protected by their chiral symmetry, which prevents a mass term from being generated by loops. If we were to introduce a small symmetry breaking term, i.e. a mass term, to obtain the theory of a massive fermion, corrections will now be generated that affect the mass term. However, these will be proportional to the bare mass and not the mass of some heavy particle from the UV theory, therefore keeping the corrections small at any loop order. We will use a similar argument to this in chapter 4.

This contrast between the cosmological constant and our example of a massive fermion shines a light on the real issue of the cosmological constant problem - the power law dependence of the quantum corrections on the cut-off. There is nothing intrinsically wrong with cut-off dependent corrections to the vacuum energy. However, a power law dependence rather than logarithmic means we must repeatedly fine tune Λ_c to agree with observations when our description of the EFT changes, i.e. when we change the cut-off or calculate to additional loop orders. This is the real issue with the vacuum energy, as opposed to most parameters in the standard model which undergo logarithmic re-normalisation. The exception to this is the Higgs mass, which also receives large radiative corrections due to not being protected, by symmetry or otherwise. This is known as the hierarchy problem, and we generously leave the solution to someone else's thesis.

2.3 Global Structure of General Relativity

A key distinguishing feature of the cosmological constant as a form of energy density is the fact that it does not dilute under cosmological expansion. In order to comprehend the consequences of this, we examine the cosmological constant more rigorously by introducing the space-time average, denoted by angled brackets. This average applied to a scalar quantity Q looks like,

$$\langle Q \rangle = \frac{\int \mathrm{d}^4x\sqrt{-g}\, Q}{\int \mathrm{d}^4x\sqrt{-g}} \tag{2.3.1}$$

where the integrals are over the entire 4-volume of space-time. It is then easy to see that the cosmological constant is the unique contribution to the energy-momentum tensor that is equal to its space-time averaged value at all points in time of cosmological evolution,

$$\Lambda = \langle \Lambda \rangle = \frac{\int \mathrm{d}^4x\sqrt{-g}\, \Lambda}{\int \mathrm{d}^4x\sqrt{-g}}. \tag{2.3.2}$$

Equivalently, Λ is a zero momentum, infinite wavelength source.

With this new perspective on the cosmological constant, we will re-examine Einstein's equations (1.1.6). Let us initially decompose them into an equation containing pure trace terms, and a remaining set of traceless equations,

$$M_{\mathrm{Pl}}^2 R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T^{\alpha}{}_{\alpha}\, g_{\mu\nu} \tag{2.3.3}$$

$$M_{\mathrm{Pl}}^2 R = 4\Lambda_{\mathrm{c}} - T^{\alpha}{}_{\alpha} \tag{2.3.4}$$

with $T^{\alpha}{}_{\alpha} = g^{\mu\nu} T_{\mu\nu}$ as the trace of the energy-momentum tensor. This way of writing the GR equations of motion is equivalent to (1.1.6), and in fact we can recover the original form from these new equations. We see that the cosmological constant only affects curvature via the trace equation. Likewise, the loop corrections to the total cosmological constant of the form $T_{\mu\nu} = -g_{\mu\nu} V_{\mathrm{vac}}$ are removed in the right hand side of equation (2.3.3), by virtue of cancellation. Let us further decompose these equations by taking the space-time average of the trace equation,

$$M_{\rm Pl}^2 \langle R \rangle = 4\Lambda_{\rm c} - \langle T^{\alpha}{}_{\alpha} \rangle \tag{2.3.5}$$

which has been simplified by utilising $\Lambda_{\rm c} = \langle \Lambda_{\rm c} \rangle$. Taking the difference between this equation and (2.3.4), we obtain the full set of equations,

$$M_{\rm Pl}^2 R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T^{\alpha}{}_{\alpha}\, g_{\mu\nu} \tag{2.3.6}$$

$$M_{\rm Pl}^2 (R - \langle R \rangle) = \langle T^{\alpha}{}_{\alpha} \rangle - T^{\alpha}{}_{\alpha} \tag{2.3.7}$$

$$M_{\rm Pl}^2 \langle R \rangle = 4\Lambda_{\rm c} - \langle T^{\alpha}{}_{\alpha} \rangle \tag{2.3.8}$$

which fully characterise the behaviour of GR, over the entire space-time. We see that the cosmological constant term now only appears in the globally averaged equation (2.3.8).

As we are concerned with the cosmological constant and how it appears in these rearranged GR equations, we shall separate the vacuum energy from the energy originating from localised sources by defining,

$$T_{\mu\nu} = \tau_{\mu\nu} - V_{\rm vac} g_{\mu\nu} \tag{2.3.9}$$

where $\tau_{\mu\nu}$ encapsulates all localised sources. This leads to a final set of equations,

$$M_{\rm Pl}^2 R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \tau_{\mu\nu} - \frac{1}{4} \tau^{\alpha}{}_{\alpha}\, g_{\mu\nu} \tag{2.3.10}$$

$$M_{\rm Pl}^2 (R - \langle R \rangle) = \langle \tau^{\alpha}{}_{\alpha} \rangle - \tau^{\alpha}{}_{\alpha} \tag{2.3.11}$$

$$M_{\rm Pl}^2 \langle R \rangle = 4(\Lambda_{\rm c} + V_{\rm vac}) - \langle \tau^{\alpha}{}_{\alpha} \rangle. \tag{2.3.12}$$

Let us reiterate that we are simply rearranging the Einstein equations to show manifestly how the cosmological constant sources curvature, which is that its only direct effect is through $\langle R \rangle$. Its effect on terms describing local curvature is indirect, and summarised by equations (2.3.10), (2.3.11). We may designate (2.3.12) as a global equation, since it singularly involves quantities that depend on the entirety of the space-time. We again see an obligation for repeated fine tuning of $\Lambda_{\rm c}$ to oppose the radiatively unstable V_{vac}; this is necessary to prevent the curvature term $\langle R \rangle$ from being sourced by radiatively unstable corrections to the total cosmological constant. This equation therefore suggests that perhaps a solution to the cosmological constant problem lies in a global modification of gravity.

Additionally, there exists a supplementary reason why we may want to modify gravity on a global scale to solve the cosmological constant problem. As we have seen when combining two separate contributions to produce the total cosmological constant, the only distinguishing feature of the cosmological constant is that it does not evolve i.e. to no big surprise, it is constant. As a consequence of this, any gravity candidate can only identify a true cosmological constant by having knowledge of the future dynamics of the system. If this were not true, how would it distinguish between vacuum energy and for instance a scalar field in extreme slow roll until asymptotically late times. This means that we may only consider separating the vacuum energy from the full energy-momentum tensor if we have knowledge of the entire space-time. If we wish to decouple exclusively the vacuum energy from sourcing curvature, we must make a global modification to GR. This argument involving causality was first explored in [81].

Chapter 3

Sequestering & Vacuum Decay

3.1 Sequestering

The following chapter features heavily the sequestering theories, whose first iteration was proposed by Kaloper and Padilla in 2013 as a potential solution to the cosmological problem. We will showcase the original theory [82]–[84] and see how it impedes vacuum energy-like contributions to the cosmological constant from sourcing curvature, while making sure all other types of source gravitate as in GR. This automatically implies sequestering conforms to all experimental bounds imposed by tests of gravity on solar system scales without the need for any type of screening to occur. We will proceed to present more recent versions of the sequester, which deal with a formulation of the original proposal as a local theory, and subsequently the treatment of virtual gravitons. Both of these amendments serve to improve the theory from the perspective of UV completion.

3.1.1 Global Sequestering

We have seen previously the manner in which radiatively unstable contributions to the cosmological constant originating from the matter sector of the standard model source curvature. More precisely, we highlighted that it is necessary to probe the entirety of the cosmological timeline in order to separate the vacuum energy from the full energy-momentum tensor $T_{\mu\nu}$. Following this, we would like to modify gravity such that only vacuum energy is impeded from sourcing curvature, and all other forms of energy-momentum behave as in General Relativity. We saw that in order to implement this we must make changes to gravity on a *global* scale by modifying the global element present in the GR field equations (2.3.12). Sequestering satisfies precisely these criteria. At the level of the action, sequestering and GR differ by the introduction of global variables Λ_c and λ. Global variables are defined here by being unvarying in space-time, but allowed to vary in the action. They act to enforce global constraints rather than supplying any new local degrees of freedom. It shall become clear that these global constraints are a key component of vacuum energy sequestering.

The original version of Sequestering is represented by the *Einstein frame* action [82],

$$S = \sigma\left(\frac{\Lambda_\mathrm{c}}{\lambda^4\mu^4}\right) + \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_\mathrm{Pl}^2}{2}R - \Lambda_\mathrm{c} - \lambda^4\mathcal{L}_\mathrm{m}(\lambda^{-2}g^{\mu\nu},\Psi)\right) \tag{3.1.1}$$

where we introduce a smooth function σ which resides explicitly outside the integral as a global contribution to the modified GR action. The mass scale μ is included to make the argument of the function dimensionless, and its value can be fixed by a redefinition of σ. Any standard model matter that is minimally coupled to the rescaled metric $\lambda^2 g_{\mu\nu}$ is represented by Ψ.

Having introduced (3.1.1) in the form in which it was first proposed, we will immediately transform this action into the *Jordan frame*, in order to make better comparisons with the later versions of Sequestering shown in this chapter. Under the rescalings,

$$g_{\mu\nu} \to \widetilde{g}_{\mu\nu} = \lambda^2 g_{\mu\nu} \tag{3.1.2}$$

$$\Lambda_\mathrm{c} \to \widetilde{\Lambda}_\mathrm{c} = \frac{\Lambda_\mathrm{c}}{\lambda^4} \tag{3.1.3}$$

the action reads,

$$S = \sigma\left(\frac{\widetilde{\Lambda}_\mathrm{c}}{\mu^4}\right) + \int \mathrm{d}^4x\sqrt{-\widetilde{g}}\left(\frac{M_\mathrm{Pl}^2}{2\lambda^2}\widetilde{R} - \widetilde{\Lambda}_\mathrm{c} - \mathcal{L}_\mathrm{m}(\widetilde{g}^{\mu\nu},\Psi)\right) \tag{3.1.4}$$

where $\widetilde{R}$ is the Ricci scalar defined with respect to the conformally rescaled metric $\widetilde{g}_{\mu\nu}$. The full equations of motion are obtained by varying over the degrees of freedom given by $\widetilde{g}_{\mu\nu}$, $\widetilde{\Lambda}_\mathrm{c}$ and λ. Varying in $\widetilde{g}_{\mu\nu}$ gives,

$$M_\mathrm{Pl}^2\widetilde{G}_{\mu\nu} = \lambda^2\widetilde{T}_{\mu\nu} - \lambda^2\widetilde{\Lambda}_\mathrm{c}\widetilde{g}_{\mu\nu} \tag{3.1.5}$$

where $\widetilde{T}_{\mu\nu}$ is the energy-momentum tensor related to $\widetilde{g}_{\mu\nu}$. Converting $\widetilde{\Lambda}_{\mathrm{c}}$ and $\widetilde{g}_{\mu\nu}$ back to their Einstein frame counterparts, and using $\lambda^2\widetilde{T}_{\mu\nu} = T_{\mu\nu}$ and $\widetilde{G}_{\mu\nu} = G_{\mu\nu}$, if we recognise Λ_{c} as the bare cosmological constant of the Einstein-Hilbert action, then we obtain exactly the GR equations of motion (1.1.6). In contrast to GR, Sequestering also has two additional constraint equations, obtained by variation over the global parameters of the theory. Varying over Λ_{c} provides an equation of motion constraining the 4-volume of the entire space-time in terms of global variables, of the form,

$$\int \mathrm{d}^4x\sqrt{-\widetilde{g}} = \frac{\sigma'}{\mu^4} \tag{3.1.6}$$

where σ' is the differential of σ by its argument. We therefore constrain σ to the set of differentiable functions, in order to have a well defined variational principle. We now see why σ is an integral part of the sequestering formalism, as without it we would obtain a zero space-time volume. It also clear that as μ and σ' are assumed to be finite, this theory does not allow an infinite Jordan frame space-time volume.

To obtain the twelfth and last equation of motion of Global Sequestering, variation over λ provides an expression constraining the integral of the Ricci scalar over the entire space-time to depend on the global variables as,

$$\frac{M_{\mathrm{Pl}}^2}{\lambda^3}\int \mathrm{d}^4x\sqrt{-\widetilde{g}}\,\widetilde{R} = 0. \tag{3.1.7}$$

Lastly, we take the ratio of (3.1.7) and (3.1.6), giving a single constraint $\langle\widetilde{R}\rangle = 0$. For clarity, we wish to examine what modifications the introduction of these two constraint equations have made to the gravitational dynamics. To this purpose, let us replace the global variable $\widetilde{\Lambda}_{\mathrm{c}}$ from (3.1.5), to obtain modified Einstein equations. Taking the trace of (3.1.5) we get,

$$-M_{\mathrm{Pl}}^2\widetilde{R} = \lambda^2\widetilde{T}^{\alpha}{}_{\alpha} - 4\lambda^2\widetilde{\Lambda}_{\mathrm{c}} \tag{3.1.8}$$

which, after integrating over all space-time, in combination with (3.1.7), yields,

$$4\widetilde{\Lambda}_{\mathrm{c}}\int \mathrm{d}^4x\sqrt{-\widetilde{g}} = \int \mathrm{d}^4x\sqrt{-\widetilde{g}}\,\widetilde{T}^{\alpha}{}_{\alpha} \tag{3.1.9}$$

or alternatively, with the help of (3.1.6),

$$4\widetilde{\Lambda}_{\mathrm{c}}\frac{\sigma'}{\mu^4} = \int \mathrm{d}^4x\sqrt{-\widetilde{g}}\,\widetilde{T}^{\alpha}{}_{\alpha} \tag{3.1.10}$$

which tells us the integral of the Jordan frame traced energy-momentum tensor, or energy-momentum scalar, is wholly constrained by global variables.

We can also now see that by rearranging (3.1.9) we obtain,

$$4\widetilde{\Lambda}_{\mathrm{c}} = \langle \widetilde{T}^{\alpha}{}_{\alpha} \rangle. \tag{3.1.11}$$

The global variable equations of motion have enabled us to make a global constraint of the space-time average of the energy-momentum scalar as a function of the bare cosmological constant. Alternatively, we can write the bare cosmological constant wholly as a function of the space-time average of the energy-momentum scalar, in contrast to GR where Λ_{c} is a free parameter. This on-shell constraint equation is key to the sequestering of vacuum energy.

Substituting this equation back into (3.1.5) eliminates the global variables and produces our final modified Einstein equations, given by,

$$M_{\mathrm{Pl}}^2 \widetilde{G}_{\mu\nu} = \lambda^2 \widetilde{T}_{\mu\nu} - \frac{1}{4}\lambda^2 \langle \widetilde{T}^{\alpha}{}_{\alpha} \rangle \widetilde{g}_{\mu\nu}. \tag{3.1.12}$$

Having successfully obtained field equations describing gravitational dynamics in the Global Sequestering theory, we now switch back to Einstein frame for the remainder of the section in order to make a concrete comparison with GR. The field equations (3.1.12) become,

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} \langle T^{\alpha}{}_{\alpha} \rangle g_{\mu\nu}. \tag{3.1.13}$$

Let us separate the energy-momentum tensor as $T_{\mu\nu} = \tau_{\mu\nu} - V_{\mathrm{vac}}\, g_{\mu\nu}$, to manifest the cosmological constant contribution, so that we may readily track the quantum corrections. Then we obtain,

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = \tau_{\mu\nu} - \frac{1}{4} \langle \tau^{\alpha}{}_{\alpha} \rangle g_{\mu\nu}. \tag{3.1.14}$$

It is now apparent that the vacuum energy is unable to contribute to (3.1.14) and is therefore unable to source curvature. The cancellation of vacuum energy-like contributions between the two terms on the right hand side is active for all perturbative loop orders and likewise for any Wilsonian cut-off. The sequestering constraint (3.1.11) demands that the bare cosmological constant Λ_{c} compensates for all quantum corrections originating from V_{vac} to the total cosmological constant Λ_{t} without the need for fine tuning. The repeated fine tuning that we saw was necessary in GR, and was the root of the cosmological constant problem, is completely taken care of by the global constraint equation in Sequestering.

Returning to examine (3.1.13), we see that curvature is now solely determined by the local contributions contained in $\tau_{\mu\nu}$, which contribute exactly as in GR, as well as the residual cosmological constant term $\frac{1}{4}\langle\tau^{\alpha}{}_{\alpha}\rangle g_{\mu\nu}$. This residual vacuum energy-like contribution $\frac{1}{4}\langle\tau^{\alpha}{}_{\alpha}\rangle$ is radiatively stable, and takes the form of a space-time average of the energy-momentum representing local matter excitations. As in any EFT its value should be fixed by experimental observations, which give $\Lambda_{\rm obs}$. (meV)^4.

Having arrived at modified Einstein equations for the Global Sequestering theory, we remind ourselves of (2.3.8), a global equation which we claimed would be important in solving the cosmological constant problem. Let us attempt to connect our Sequestering theory to equations (2.3.7-2.3.8) by splitting the field equations we have just obtained in a similar fashion, and manifest the global character of this gravitational modification. We again decompose the equations of motion (3.1.5) into a trace equation and the traceless degrees of freedom. Repeating the procedure we underwent for GR, we subtract the trace equation from its space-time average. Unsurprisingly, this produces precisely the Einstein equations of GR, but with the additional global constraint of Sequestering. After this process, the field equation decomposition for Global Sequestering is,

$$M_{\rm Pl}^2 R^{\mu}{}_{\nu} - \frac{1}{4}R\delta^{\mu}{}_{\nu} = \tau^{\mu}{}_{\nu} - \frac{1}{4}\tau^{\alpha}{}_{\alpha}\delta^{\mu}{}_{\nu} \tag{3.1.15}$$

$$M_{\rm Pl}^2(R - \langle R\rangle) = \langle\tau^{\alpha}{}_{\alpha}\rangle - \tau^{\alpha}{}_{\alpha} \tag{3.1.16}$$

$$M_{\rm Pl}^2\langle R\rangle = 4(\Lambda_{\rm c} + V_{\rm vac}) - \langle\tau^{\alpha}{}_{\alpha}\rangle \tag{3.1.17}$$

$$4(\Lambda_{\rm c} + V_{\rm vac}) = \langle\tau^{\alpha}{}_{\alpha}\rangle \tag{3.1.18}$$

and the final equation (3.1.18) makes transparent the exact cancellation of any vacuum energy contributions.

In conclusion, we have seen that imposing global constraints on the Einstein equations can completely decouple radiatively unstable quantum corrections to the cosmological constant from sourcing classical gravity. The mechanism behind Global

Sequestering is the introduction of global variables to the base Einstein-Hilbert action with matter coupling, as well as a function containing these global parameters outside of the action space-time integral.

3.1.2 Local Sequestering

In order to thoroughly discuss the local version of the sequestering theory, it is helpful to first present a brief overview of unimodular gravity, and the gauge invariant version of Henneaux and Teitelboim [85]. Unimodular gravity attempts to solve the cosmological constant problem but ultimately fails [86]. The theory itself differs from general relativity only in that it enforces the additional constraint $\sqrt{-g} = 1$. This has the effect of a vanishing metric determinant variation,

$$\frac{\delta}{\delta g_{\mu\nu}}\sqrt{-g} = 0. \tag{3.1.19}$$

Under this constraint the GR action (1.1.1) gives,

$$M_{\rm Pl}^2 R_{\mu\nu} - \frac{1}{4}R g_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu} \tag{3.1.20}$$

which are the Einstein equations in a traceless form. The cosmological constant is not present in these equations since it appears in the action as $\sqrt{-g}\Lambda_{\rm c}$, and likewise radiative corrections $V_{\rm vac}$ also disappear. It may appear as if the cosmological constant problem has been solved, but alas as we shall see this is not the case [87]. Using the Bianchi identity on the divergence of (3.1.20) yields,

$$\nabla_\mu (M_{\rm Pl}^2 R + T) = 0 \tag{3.1.21}$$

which when integrated gives,

$$M_{\rm Pl}^2 R + T = 4\Lambda_{\rm c}. \tag{3.1.22}$$

It is apparent that this is merely the trace of the Einstein equations and $\Lambda_{\rm c}$ has reentered our equations in the form of an integration constant. The combination of (3.1.20) and (3.1.22) yields the full GR field equations,

$$M_{\rm Pl}^2 G_{\mu\nu} = T_{\mu\nu} - \Lambda_{\rm c} g_{\mu\nu} \tag{3.1.23}$$

and the return of the cosmological constant problem. The main effect of having imposed (3.1.19) is that $\Lambda_{\rm c}$ is now an integration constant as opposed to a constant present in the action. So instead of the repeated fine-tuning we were forced to do in GR we must now constantly change the value of our integration constant in reaction to the quantum corrections from the matter sector, i.e. the cosmological constant problem is alive and well.

To see this in more detail, we can repackage the unimodular constraint into the action via a Lagrange multiplier $\lambda(x)$ in the form,

$$S_{\rm unimodular} = S_{\rm GR} + \int \mathrm{d}^4x \lambda(x)(\sqrt{-g} - 1). \tag{3.1.24}$$

This action explicitly breaks gauge invariance, which tells us that the constraint $\sqrt{-g} = 1$ is merely a local choice of gauge. In a full diffeomorphism invariant theory it is always possible to select a reference frame that locally obeys the unimodular constraint. Fixing a gauge in this way cannot give us insight into the cosmological constant problem.

Hence, let us consider Henneaux and Teitelboim's gauge invariant unimodular gravity [85]. To implement a diffeomorphism invariant construction, it is possible to use a different measure in the action. The usual covariant four volume is a 4-form, which written in full is given by,

$$\int \mathrm{d}^4x\sqrt{-g} = \frac{1}{4!}\int \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho}\epsilon^{\mu\nu\lambda\rho}\mathrm{d}^4x = \frac{1}{4!}\int \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\lambda \wedge \mathrm{d}x^\rho \tag{3.1.25}$$

where $\epsilon_{\mu\nu\lambda\rho}$ is the Levi-Civita symbol, which behaves as a tensor density under coordinate transformations. However, although $\sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho}$ is the 4-form that appears in the Einstein-Hilbert action, in general any 4-form $F_{\mu\nu\lambda\rho}$ could be used as a measure and the action would be diffeomorphism invariant. Henneaux and Teitelboim used this to modify unimodular gravity into the form,

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R - \int \Lambda_{\rm c}(x)\left(\sqrt{-g}\,\mathrm{d}^4x - \frac{1}{4!}F_{\mu\nu\lambda\rho}\,\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\lambda \wedge \mathrm{d}x^\rho\right) \tag{3.1.26}$$

where the 4-form $F_{\mu\nu\lambda\rho}$ is defined as the exterior derivative of a 3-form $A_{\nu\lambda\rho}$, as given by,

$$F_{\mu\nu\lambda\rho} = 4\partial_{[\mu}A_{\nu\lambda\rho]}. \tag{3.1.27}$$

It is worth noting that $F_{\mu\nu\lambda\rho}$ is completely independent of the metric off-shell; it transforms correctly due to its totally anti-symmetric nature, resulting in its absence from the modified Einstein equations. Varying $\Lambda_{\mathrm{c}}(x)$ generates a constraint equation for $\sqrt{-g}$ and varying $A_{\nu\lambda\rho}$ gives us $\partial_\mu \Lambda_{\mathrm{c}}(x) = 0$. Solving this equation yields an integration constant that is exactly the cosmological constant present in the GR action, and is the fixed on-shell value of our Lagrange multiplier. More generally, it is possible to have gauge invariant unimodular gravity with a more complicated $\Lambda_{\mathrm{c}}(x)$ in the topological sector, such as,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \Lambda_{\mathrm{c}}(x)\right) + \frac{1}{4!}\int \sigma\left(\frac{\Lambda_{\mathrm{c}}(x)}{\mu^4}\right) F_{\mu\nu\lambda\rho}\,\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \wedge \mathrm{d}x^\lambda \wedge \mathrm{d}x^\rho \tag{3.1.28}$$

where μ is a mass scale necessary for dimensional consistency and σ is some smooth function. Continuing along our line of inquiry, in keeping with our analysis of the global sequester, let us introduce a matter sector with the standard minimal coupling to the metric and show that unimodular gravity once again falls short of solving the cosmological constant problem. The $g^{\mu\nu}$ field equation is given by,

$$M_{\mathrm{Pl}}^2 G^\mu{}_\nu = T^\mu{}_\nu - \Lambda_{\mathrm{c}}(x)\delta^\mu{}_\nu \tag{3.1.29}$$

which we can identify as the GR field equations where the cosmological constant has been promoted to a dynamical scalar field. As mentioned, varying $\Lambda_{\mathrm{c}}(x)$ leads to,

$$\frac{\sigma'}{\mu^4}F_{\mu\nu\lambda\rho} = \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho} \tag{3.1.30}$$

and the variation of $A_{\nu\lambda\rho}$ now yields,

$$\frac{\sigma'}{\mu^4}\partial_\mu \Lambda_{\mathrm{c}}(x) = 0 \tag{3.1.31}$$

which again constrains $\Lambda_{\mathrm{c}}(x)$ on-shell to be a constant. Substituting this constant into (3.1.29) results in the Einstein equations with the added constraint of (3.1.30) on-shell. As a result of the rigidity of $\Lambda_{\mathrm{c}}(x)$ we can integrate (3.1.30) to obtain a constraint on the space-time volume of the form,

$$\frac{1}{4!}\frac{\sigma'}{\mu^4}\int F_4 = \int \mathrm{d}^4x\sqrt{-g} \tag{3.1.32}$$

However, as opposed to the global sequester, this constraint equation is unable to inform us about $\langle R\rangle$ or $\langle T^\alpha{}_\alpha\rangle$. In short, it has nothing to say about the global equation of GR (2.3.12) which we have previously identified as a significant aspect of any theory hoping to address the cosmological constant problem.

For the sake of clarity we shall once again decompose the field equations, similarly to our analysis of general relativity and the global sequester. The resulting equations are given by,

$$M_{\mathrm{Pl}}^2 R^\mu{}_\nu - \frac{1}{4}R\delta^\mu{}_\nu = \tau^\mu{}_\nu - \frac{1}{4}\tau^\alpha{}_\alpha\delta^\mu{}_\nu \tag{3.1.33}$$

$$M_{\mathrm{Pl}}^2(R - \langle R\rangle) = \langle\tau^\alpha{}_\alpha\rangle - \tau^\alpha{}_\alpha \tag{3.1.34}$$

$$M_{\mathrm{Pl}}^2\langle R\rangle = 4(\Lambda_{\mathrm{c}} + V_{\mathrm{vac}}) - \langle\tau^\alpha{}_\alpha\rangle \tag{3.1.35}$$

$$\star F - \langle\star F\rangle = 0, \qquad \langle\star F\rangle = \frac{\mu^4}{\sigma'} \tag{3.1.36}$$

where we have separated the energy-momentum content into $T^\mu{}_\nu = \tau^\mu{}_\nu - V_{\mathrm{vac}}\,\delta^\mu{}_\nu$ and $\star$ denotes the Hodge dual of a form, which for our purposes is given by,

$$\star F = \frac{1}{4!}\frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho}\,. \tag{3.1.37}$$

This presentation of the field equations shows explicitly that the defining constraint of unimodular gravity is insufficient, since the constraint equations (3.1.36) are totally decoupled from the Einstein equations represented by (3.1.33)–(3.1.35). Once again it is necessary to continually re-tune Λ_{c} in order to address the radiative instability of V_{vac}.

In conclusion, we have demonstrated that Λ_{c}, a global parameter in the original sequester, can emerge as an integration constant from on-shell field equations. Additionally, we have shown how via the introduction of a new covariant measure it is possible to produce global constraints from a theory comprised solely of local fields. We have illustrated how these mechanisms are insufficient for addressing the cosmological constant problem in the arena of unimodular gravity. We proceed to present how these methods can instead be applied to a theory that produces local field equations resembling the design of the global sequester, and thus connect to the cosmological constant problem.

We recall that the global sequestering theory in the Jordan frame is given by,

$$S = \int \mathrm{d}^4x \sqrt{-\widetilde{g}} \left(\frac{M_{\mathrm{Pl}}^2}{2\lambda^2} \widetilde{R} - \widetilde{\Lambda}_{\mathrm{c}} - \mathcal{L}_{\mathrm{m}}(\widetilde{g}^{\mu\nu}, \Phi) \right) + \sigma \left(\frac{\widetilde{\Lambda}_{\mathrm{c}}}{\mu^4} \right) \tag{3.1.38}$$

where the equation of motion of the global parameter λ constrained the integral of the Ricci scalar R over all space-time. We now wish to produce a corresponding constraint in a theory constructed from only local fields. In our examination of unimodular gravity $\widetilde{\Lambda}_{\mathrm{c}}$ was replaced by a scalar field, forced to be rigid on-shell by the variation of a 3-form, with its off-shell fluctuations providing a global constraint. Let us continue in a similar manner by promoting the remaining global parameter λ to a scalar field as,

$$\frac{M_{\mathrm{Pl}}^2}{\lambda^2} \to \kappa^2(x) \tag{3.1.39}$$

where we aim for $\kappa^2(x)$ to be similarly fixed rigid on-shell, whilst allowing an off-shell variation that produces a constraint on $\langle R \rangle$. In order to realise this we implement a second 4-form, supplementary to that discussed in the Henneaux and Teitelboim formulation of unimodular gravity, which is coupled to $\kappa^2(x)$ whilst being independent of the metric off-shell, and that leaves the gauge invariance of the theory intact. These requirements are satisfied by the theory given by [88],

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{\kappa^2(x)}{2} R - \Lambda_{\mathrm{c}}(x) \right) + S_{\mathrm{m}} \left[g^{\mu\nu}, \Psi \right] + \frac{1}{4!} \int \mathrm{d}^4x \, \epsilon^{\mu\nu\lambda\rho} \left(\sigma \left(\frac{\Lambda_{\mathrm{c}}(x)}{\mu^4} \right) F_{\mu\nu\lambda\rho} + \hat{\sigma} \left(\frac{\kappa^2(x)}{M_{\mathrm{Pl}}^2} \right) \hat{F}_{\mu\nu\lambda\rho} \right) \tag{3.1.40}$$

where σ and $\hat{\sigma}$ are smooth functions independent of each other, both $F_{\mu\nu\lambda\rho}$ and $\hat{F}_{\mu\nu\lambda\rho}$ are defined as exterior derivatives of 3-forms in the same way, and μ . M_{Pl} where M_{Pl} and μ are the gravitational and field theory cutoffs. The function σ is still restricted in the same way that it cannot be a purely logarithmic function. This is the full, manifestly local, Sequestering theory.

We now have in total five fields to vary over. As in previous sections, it is important to note that the following is carried out in a semi-classical picture, where gravity is treated classically and we ask how the vacuum energy influences classical curvature if fine tuning is absent.

Firstly, variation with respect to $g^{\mu\nu}$ gives,

$$\kappa^2(x)G^\mu{}_\nu = (\nabla^\mu\nabla_\nu - \delta^\mu{}_\nu\nabla^2)\kappa^2(x) + T^\mu{}_\nu - \delta^\mu{}_\nu\Lambda_c(x). \tag{3.1.41}$$

The two scalar fields $\Lambda_c(x)$ and $\kappa^2(x)$ yield equations of motion of the form,

$$\frac{\sigma'}{\mu^4}F_{\mu\nu\lambda\rho} = \sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho} \tag{3.1.42}$$

$$\frac{\hat{\sigma}'}{M_{\rm Pl}^2}\hat{F}_{\mu\nu\lambda\rho} = -\frac{1}{2}\sqrt{-g}R\,\epsilon_{\mu\nu\lambda\rho} \tag{3.1.43}$$

from which we can determine that neither σ nor $\hat{\sigma}$ are allowed to be linear functions, so as to not completely constrain either 4-form in terms of the space-time geometry. Lastly, variation of the 3-forms restricts the scalars on-shell to be constant,

$$\frac{\sigma'}{\mu^4}\partial_\alpha\Lambda_c(x) = 0 \tag{3.1.44}$$

$$\frac{\hat{\sigma}'}{M_{\rm Pl}^2}\partial_\alpha\kappa^2(x) = 0. \tag{3.1.45}$$

This removes the possibility of new long range fifth forces and as a consequence there is no need to use any kind of screening mechanism in order to match with solar system tests of gravity, as we would with other theories of modified gravity; this is a direct consequence of the modification only effecting the global structure of gravity. This system of equations boils down to GR with additional global constraints on $\langle R\rangle$, in comparison to the original sequestering model which sets $\langle R\rangle = 0$; the advantages of having a non-zero constraint will become apparent shortly.

Integrating the scalar equations and substituting the resulting integration constants, which we shall denote by Λ_c and κ^2, back into (3.1.41), the gravity equations become,

$$\kappa^2 G^\mu{}_\nu = T^\mu{}_\nu - \delta^\mu{}_\nu\Lambda_c \tag{3.1.46}$$

where $\kappa \sim 10^{18}$ GeV is now the bare Planck mass. Fixing κ so as to match observations generates a hierarchy between itself and the scales present in the matter sector, but it will still be a radiatively stable quantity against matter loop corrections as long as μ . $M_{\rm Pl}$. Integrating the constraints (3.1.42) and (3.1.43) gives,

$$\frac{1}{4!}\frac{\sigma'}{\mu^4}\int F_4 = \int \mathrm{d}^4x\sqrt{-g} \tag{3.1.47}$$

$$\frac{1}{4!}\frac{\hat{\sigma}'}{M_{\rm Pl}^2}\int \hat{F}_4 = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}R. \tag{3.1.48}$$

Upon taking the ratio of the two equations, we obtain,

$$\langle R\rangle = -2\frac{\hat{\sigma}'}{\sigma'}\frac{\mu^4}{M_{\rm Pl}^2}\frac{\int \hat{F}_4}{\int F_4}. \tag{3.1.49}$$

The space-time average of the Ricci scalar is now constrained by the ratio of the 4-form fluxes. In order to make the proceeding discussion clearer, we will write this equation as $\kappa^2\langle R\rangle = 4\Delta\Lambda$ with $\Delta\Lambda$ taking its definition from (3.1.49).

Let us go back to the gravity equations (3.1.46) and make explicit the $\langle R\rangle$ dependence by taking the trace and space-time average, giving us,

$$4\Lambda_{\rm c} = \langle T^\alpha{}_\alpha\rangle + \kappa^2\langle R\rangle \tag{3.1.50}$$

where we have used the fact that $\Lambda_{\rm c} = \langle\Lambda_{\rm c}\rangle$ and $\kappa^2 = \langle\kappa^2\rangle$. Now by using (3.1.49) we can replace $\langle R\rangle$ in (3.1.50) by the ratio of the integrated 4-forms, expressed in terms of $\Delta\Lambda$. Finally we can substitute for $\Lambda_{\rm c}$, resulting in a set of gravitational field equations of the form,

$$\kappa^2 G^\mu{}_\nu = T^\mu{}_\nu - \frac{1}{4}\langle T^\alpha{}_\alpha\rangle\delta^\mu{}_\nu - \Delta\Lambda\delta^\mu{}_\nu \tag{3.1.51}$$

which are the same as the equations we obtained from the global sequestering model, but for the addition of $\Delta\Lambda$. Once again splitting the energy-momentum tensor into global vacuum energy and local matter contributions as $T^\mu{}_\nu = \tau^\mu{}_\nu - V_{\rm vac}\delta^\mu{}_\nu$, it is apparent that the radiatively unstable vacuum energy contributions to the cosmological constant do not appear in these modified Einstein equations, and are therefore unable to source curvature. The only sources allowed to affect curvature are local matter excitations and the residual cosmological constant. This is similar to the original Sequestering model, but with a new addition to the previous residual cosmological constant $\frac{1}{4}\langle\tau^\alpha{}_\alpha\rangle$ of the form $\Delta\Lambda$. These two terms are both radiatively stable and are therefore allowed to be set by observation without issue. In contrast to the global Sequester, this residual cosmological constant can now be a Dark Energy candidate and responsible for observed late time cosmological acceleration as we no longer need be restricted to a finite space-time volume.

If this residual cosmological constant is the only source of acceleration then it will result in an infinite space-time volume, in which case matter fields satisfying null energy conditions will fulfill $\langle \tau^\alpha{}_\alpha \rangle = 0$, due to the denominator of the space-time average diverging while the numerator dilutes as the universe expands. We are left with only a $\Delta\Lambda$ contribution, which can now be fixed by observation to be $\Delta\Lambda \sim (\text{meV})^4$. We know $\Delta\Lambda$ must be radiatively stable, as it is comprised of 4-form fluxes and the ratio of the derivatives of σ and $\hat{\sigma}$. The integrals of F_4 and $\hat{F}_4$ are infrared quantities and so are not sensitive to variation of the cut-off μ. The functions σ and $\hat{\sigma}$ will receive corrections from matter loops on account of their Λ_c and κ dependencies, but if we restrict ourselves to sufficiently smooth functions (i.e. $\sigma(O(1)z) \sim O(1)\sigma(z)$) then these matter loops can only ever contribute corrections of $O(1)$ as they are suppressed by the cut-offs as Λ_c/μ^4 and κ^2/M_Pl^2.

Finally, in order to highlight the significance of the global constraints, we present the decomposition of the field equations for local sequestering, which take the form,

$$\kappa^2 R^\mu{}_\nu - \frac{1}{4} R\delta^\mu{}_\nu = \tau^\mu{}_\nu - \frac{1}{4}\tau^\alpha{}_\alpha \delta^\mu{}_\nu \tag{3.1.52}$$

$$\kappa^2 (R - \langle R \rangle) = \langle \tau^\alpha{}_\alpha \rangle - \tau^\alpha{}_\alpha \tag{3.1.53}$$

$$\kappa^2 \langle R \rangle + \langle \tau^\alpha{}_\alpha \rangle = 4(\Lambda_\text{c} + V_\text{vac}) \tag{3.1.54}$$

$$\star F - \langle \star F \rangle = 0, \ \langle \star F \rangle = \frac{\mu^4}{\sigma'} \tag{3.1.55}$$

$$\star \hat{F} - \langle \star \hat{F} \rangle = \frac{M_\text{Pl}^2}{2\kappa^2 \sigma'} (\tau^\alpha{}_\alpha - \langle \tau^\alpha{}_\alpha \rangle) \tag{3.1.56}$$

$$\Delta\Lambda = \frac{1}{4}\kappa^2 \langle R \rangle = -\frac{\kappa^2 \hat{\sigma}'}{2M_\text{Pl}^2} \langle \star \hat{F} \rangle. \tag{3.1.57}$$

The equations (3.1.52)–(3.1.54) correspond exactly to the Einstein equations and they confirm that this version of sequestering is able to locally reproduce GR. The next equations, labelled (3.1.55), are identified as the unimodular gravity constraint equations. It should be obvious that these would appear in our decomposition, as local sequestering (3.1.40) reduces to the generalisation of gauge invariant unimodular gravity (3.1.28) in the limit $\hat{\sigma} = 0$. Lastly, the modifications to unimodular gravity, and hence GR, are represented by (3.1.56) and (3.1.57). As requested they interact with the global GR equation by constraining the space-time average of the Ricci scalar. Equation (3.1.56) demonstrates the radiative stability of $\hat{F}_4$, due to the cancellation of vacuum energy corrections and the fact that $\kappa^2\sigma'/M_{\rm Pl}^2$ as previously discussed only shifts by $O(1)$. Likewise, equation (3.1.57) demonstrates that the space-time averaged Ricci scalar is made radiatively stable by virtue of the $\Delta\Lambda$ constraint. With the left hand side of (3.1.54) radiatively stable, $\Lambda_{\rm c}$ is forced to absorb radiatively unstable contributions originating from $V_{\rm vac}$. Finally, it is worth mentioning that it is the 4-forms that permit local sequestering to avoid Weinberg's no-go theorem. This is due to them allowing diffeomorphism invariant non-gravitating measures independent of the metric off-shell.

The local sequester as we have described it does not produce a numerical value for the Ricci scalar space-time average, and as in any EFT it would need to be measured. Any a priori insight would necessarily come from a more complete understanding of the UV physics with regards to the source of the four-forms and their fluxes.

In summary, presenting the material of [82]–[84], we have built upon our analysis of the cosmological constant problem and the global description of the vacuum energy within GR from chapter 2. We have proceeded to show how the problematic vacuum energy contributions from the matter sector can be completely decoupled from the curvature by implementing a modification to GR involving global constraints. The original sequester achieves this via global variables, whose equations of motion demand that the cosmological constant cancel the radiative corrections without a need for fine tuning. We then demonstrated how this mechanism could be implemented within a local theory, by introducing a topological sector that leads to similar global constraints, as shown and built upon in [88]–[90]. This again leaves finite wavelength sources untouched, and so replicates GR locally, automatically passing solar system tests. Now we turn our attention to the problem of including the virtual graviton in our analysis of the cosmological constant problem, which up until now we had put to one side. It is an important consideration, as it is found to be essential to the complete sequestration of the radiatively unstable vacuum energy [91]. We will also study the cosmological consequences of such a model in greater detail.

3.2 Omnia Sequestra

So far we have a mechanism for protecting the vacuum energy from matter loop corrections. We have achieved this by utilising constraints at the largest wavelengths, as well as new gauge symmetries preventing additional local degrees of freedom. Curvature is then sourced by a residual radiatively stable vacuum energy, with a value that must be measured, as is the case for other parameters in EFTs e.g. the electron mass. For a full portfolio of work surrounding the local sequester, see [82]–[84], [88]–[90], [92]–[94].

We now aim to modify the mechanism to accommodate vacuum energy loops that include virtual gravitons. It will become apparent that higher dimensional operators are useful as conjugate variables to constraint parameters that sequester said loops from the gravitational field equations. Since this modification is focused on vacuum energy loops as opposed to the locality of the theory, we truncate our analysis by concentrating on the global sector from the start. We therefore integrate out the 3-forms from (3.1.40), which forces the scalars κ and Λ to be space-time constants. The global sector of the local sequestering theory is now described by the action,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{\kappa^2}{2}R - \Lambda - \mathcal{L}_{\mathrm{m}}(g^{\mu\nu}, \Psi)\right) + \sigma\left(\frac{\Lambda}{\mu^4}\right)c + \hat{\sigma}\left(\frac{\kappa^2}{M_{\mathrm{Pl}}^2}\right)\hat{c} \tag{3.2.1}$$

where c and $\hat{c}$ are the fluxes of 3-forms A and $\hat{A}$ respectively, with constraints originating from the variation of the 'rigid' scalars κ^2 and Λ. The field equations are given by,

$$\kappa^2 G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \Lambda\delta^{\mu}{}_{\nu}, \quad \frac{\sigma'}{\mu^4}c = \int \sqrt{-g}\,\mathrm{d}^4x, \quad \frac{\hat{\sigma}'}{M_{\mathrm{Pl}}^2}\hat{c} = -\frac{1}{2}\int R\sqrt{-g}\,\mathrm{d}^4x. \tag{3.2.2}$$

Tracing and averaging over space-time as in the previous section, we once again obtain (3.1.51). The vacuum energy cancellation that occurs as a consequence of (3.1.51) is a result of two approximate symmetries of the local sequestering action

[82]–[84]. There is an approximate shift symmetry $\mathcal{L}_{\rm m} \to \mathcal{L}_{\rm m} + \nu^4$, $\Lambda \to \Lambda - \nu^4$, for constant ν, which is promoted to an exact symmetry in the limit $c/\mu^4 \to 0$ as the topological sector is suppressed.

There also exists an approximate symmetry in κ^2. To observe it, let us consider metric and κ^2 fluctuations about a Minkowski space-time with negligible cosmological constant. Setting $\kappa^2 = M_{\rm Pl}^2(1 + \phi/M_{\rm Pl})$ and $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$, the theory is invariant under the transformation $\phi \to \phi + \hat{\nu}$ in the limit $M_{\rm Pl} \to \infty, \hat{c}/M_{\rm Pl}^2 \to 0$. However, this symmetry is broken at finite $M_{\rm Pl}$, hence graviton loops will not be cancelled in (3.1.40).

We can alternatively see this by examining vacuum energy loops that include both matter and gravitons. Let us give an outline of the calculation in a locally Lorentzian frame, treating the background geometry as flat. This adequately describes all the UV contributions. Expanding in $1/\kappa^2$, we obtain,

$$-\left(a_0 M^4 + a_1 \frac{M^6}{\kappa^2} + a_2 \frac{M^8}{\kappa^4} + \dots\right) \int \sqrt{-g}\, \mathrm{d}^4 x \tag{3.2.3}$$

where $a_i \sim O(1)$. The first term in the expansion contains the pure matter (and pure gravity) loop corrections; these are the contributions that are sequestered by (3.1.40). The following terms contain graviton interactions. For $\kappa \sim M_{\rm Pl}$, and a cutoff of $M \sim$ TeV, M^6/κ^2 is thirty orders of magnitude above $\Lambda_{\rm obs}$. The contribution $\sim M^8/\kappa^4$ represents diagrams containing either more, or higher order, graviton interactions. This term is smaller than the proceeding one, but is nevertheless problematic for a high enough cutoff. Regardless, κ^2-dependent corrections to vacuum energy will not be sequestered in (3.1.40), due to different cutoff scaling.

Explicitly, the equivalent of equation (3.1.51) is given by,

$$\kappa^2 G^\mu{}_\nu = -\left(\Delta\Lambda - a_1 \frac{M^6}{2\kappa^2} - a_2 \frac{M^8}{\kappa^4} + \dots\right) \delta^\mu{}_\nu \tag{3.2.4}$$

with $\Delta\Lambda = \kappa^2 \langle R \rangle /4 = -\frac{\mu^4}{2} \frac{\kappa^2 \hat{\sigma}'}{M_{\rm Pl}^2 \sigma'} \frac{\hat{c}}{c}$ as before. Clearly the terms independent of κ from (3.2.3) are absent, and are replaced by a residual, radiatively stable, finite contribution $\Delta\Lambda$. However, the κ-dependent terms are still present. The largest corrections are of the form $\sim \frac{M^6}{2\kappa^2}$, but the following contributions are potentially also well above the dark energy scale, hence demanding an additional mechanism to sequester them.

We now proceed to show that such a mechanism can be obtained by a simple modification of the local sequestering theory (3.1.40). Local sequestering makes three main modifications to GR: (i) promote the constants κ^2, Λ into local fields; (ii) fix them to be rigid via 4-form gauge symmetries; (iii) use the field equations of κ^2 and Λ to fix the counterterms and decouple radiatively unstable contributions from the metric. These radiative instabilities instead only couple to the physically unobservable local 4-form fluctuations. The key constraint then originates from the κ equation of motion, which in the global limit fixes the space-time average of R to be a radiatively stable ratio of 4-form fluxes, dubbed $\Delta\Lambda$. This is the condition that results in the cutoff-dominated terms cancelling in the $\Lambda - \langle T^{\alpha}{}_{\alpha} \rangle/4$ portion of the field equations.

It is clear that if we are able to retain all of the main points of the local sequester, whilst somehow not promoting κ^2 to a scalar field, then we will simultaneously deal with all erroneous terms appearing in (3.2.4). It is verifiable that a similar set of conditions would be met by the vanishing of any space-time averaged curvature invariant not purely comprised of scale invariant quantities. The Einstein equations in vacua then imply this invariant would be polynomial in $\Lambda - \langle T^{\alpha}{}_{\alpha} \rangle/4$. Constraining it via the field equations in terms of a radiatively stable quantity would then produce a similar construction to that given by (3.1.40). Additionally, as previously stated, if the constraint is not as a consequence of M_{Pl} variability then vacuum energy corrections involving M_{Pl} will automatically cancel from the residual cosmological constant.

There are many candidate invariants that fill our requirements in four dimensions, but the Gauss-Bonnet term, $R_{\text{GB}} = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$, is in some sense a 'minimal modification'. As the Gauss-Bonnet invariant is a total derivative it only affects the topological sector, leaving finite wavelength behaviour unchanged. It is also not scale invariant, and so it successfully constrains the counterterms required to sequester all large vacuum energy loop corrections.

More rigorously, our candidate action is of the form,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \Lambda(x) + \theta(x)R_{\rm GB} - \mathcal{L}_{\rm m}\left(g^{\mu\nu},\Psi\right)\right) + \frac{1}{4!}\int \mathrm{d}^4x\,\epsilon^{\mu\nu\lambda\rho}\left(\sigma\left(\frac{\Lambda}{\mu^4}\right)F_{\mu\nu\lambda\rho} + \hat{\sigma}(\theta)\hat{F}_{\mu\nu\lambda\rho}\right) \quad (3.2.5)$$

where we have replaced the promotion of the Planck mass with that of the Gauss-Bonnet coupling $\theta(x)$. We shall call this theory Omnia Sequestra (OS) due to its ability to sequester all [95], as we will go on to show.

As before, we work 'in the action' concentrating on the global sector by once again integrating out the 3-forms, which leaves us with an effective action of,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \Lambda + \theta R_{\rm GB} - \mathcal{L}_{\rm m}\left(g^{\mu\nu},\Psi\right)\right) + \sigma\left(\frac{\Lambda}{\mu^4}\right)c + \hat{\sigma}(\theta)\hat{c}. \quad (3.2.6)$$

The two scalars are now fixed to be stiff, and have no local off-shell fluctuations. The parameters c and $\hat{c}$ are the boundary fluxes of the two 3-forms. The equations of motion become,

$$M_{\rm Pl}^2 G^\mu{}_\nu = T^\mu{}_\nu - \Lambda\delta^\mu{}_\nu, \quad \frac{\sigma'}{\mu^4}c = \int\sqrt{-g}\,\mathrm{d}^4x, \quad \hat{\sigma}'\hat{c} = -\int R_{\rm GB}\sqrt{-g}\,\mathrm{d}^4x. \quad (3.2.7)$$

It is possible to express the Gauss-Bonnet invariant as $R_{\rm GB} = -2\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right)^2 + W_{\mu\nu\alpha\beta}^2 + \frac{1}{6}R^2$, where the Weyl tensor $W_{\mu\nu\alpha\beta}$ vanishes in vacuum. Undergoing the same procedures as the previous section of tracing, averaging and substituting, we arrive at,

$$M_{\rm Pl}^2 G^\mu{}_\nu = T^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle T^\alpha{}_\alpha\rangle - \Delta\Lambda\delta^\mu{}_\nu \quad (3.2.8)$$

where $\Delta\Lambda$ is now given by,

$$\Delta\Lambda^2 = \frac{3M_{\rm Pl}^4}{8}\left(\langle R_{\rm GB}\rangle - \langle W_{\mu\nu\alpha\beta}^2\rangle + \frac{2}{M_{\rm Pl}^4}\langle(T_{\mu\nu} - \frac{1}{4}Tg_{\mu\nu})^2\rangle - \frac{1}{6M_{\rm Pl}^4}\left(\langle T^2\rangle - \langle T\rangle^2\right)\right) \quad (3.2.9)$$

and with the space-time average of the Gauss-Bonnet term expressed as the ratio of the two global constraints,

$$\langle R_{\rm GB}\rangle = \frac{\int R_{\rm GB}\sqrt{-g}\,\mathrm{d}^4x}{\int\sqrt{-g}\,\mathrm{d}^4x} = -\mu^4\frac{\hat{\sigma}'\hat{c}}{\sigma' c}. \quad (3.2.10)$$

Naturally, the full analysis of (3.2.5) is more complex, however the global limit of that analysis is fully represented by the field equations given by (3.2.6).

As we saw previously, the regularised vacuum energy, $\langle \text{vac}|T^\mu{}_\nu|\text{vac}\rangle = -\delta^\mu{}_\nu V_{\text{vac}}$, automatically cancels from the right hand side of (3.2.8). The Weyl tensor gives a vanishing contribution by virtue of scale invariance, and so $\Delta\Lambda$ only receives radiative corrections through $\langle R_{\text{GB}}\rangle$ [96]. These corrections are of the form $\Lambda \to \Lambda + O(M^4)$, and $\theta \to \theta + O(1)\log(M/m)$, where m represents a generic EFT mass scale [97]. Therefore, under the same requirements on σ and $\hat{\sigma}$ as before, $\Delta\Lambda$ is radiatively stable. This cancellation in the resulting equation of motion now accommodates both matter loops, and loops containing virtual gravitons. Additionally, graviton loops are prevented from introducing extra θ dependence in the effective action because the Gauss-Bonnet term is a total derivative. Background curvature effects and their corresponding IR corrections are suppressed by the background curvature scale, and so any extra dependence on the rigid scalars is expected to be negligible. Other curvature contributions to radiative corrections, coming from the renormalisation of (3.2.5), will similarly be sub-leading below the cutoff $M \lesssim M_{\text{Pl}}$.

The improved nature of the global sector (3.2.6) of OS, as presented in (3.2.5), when contrasted with the global sector of the local sequester (3.1.40) is due to the second approximate shift symmetry. This shift symmetry is now represented by $\theta \to \theta + \alpha$, which remains unbroken regardless of whether M_{Pl} is finite, up to the topological terms. In the limit $\hat{c} \to 0$ with M_{Pl} fixed to be finite, the symmetry is restored. This modified approximate shift symmetry, which in the local sequester was unavailable for finite M_{Pl}, forces the cancellation of the large virtual graviton loop contributions to the vacuum energy. That the symmetry is untouched by all but the topological terms restricts any generation of terms involving θ that would interfere with the sequester of the vacuum energy.

In summary, a modification of the local sequester allows for the de-gravitation of cutoff-dominated corrections to the vacuum energy, including diagrams involving virtual gravitons. Using a loop expansion, we consider gravity as an EFT with a sub-Planckian cutoff. This additional capability of the sequester results from an employment of the Gauss-Bonnet topological invariant, promoting the approximate shift symmetry to a bulk invariant independent of the value of M_{Pl}.

3.2.1 Sequestering at the Boundaries

Having obtained an action for the OS theory that deals with both matter and graviton loops (3.2.5), a more complete definition of OS should include boundary conditions and any additional boundary terms required for a well defined variational principle, the analogue of the Gibbons-Hawking term in General Relativity [98]. For the original sequestering theory it was found that a simple Dirichlet boundary condition on the Einstein frame metric was sufficient [89], but as we will see the equivalent for OS is a little more involved. For an action of the form (3.2.5), the analogue of the Gibbons-Hawking boundary term is the Myers boundary term given by [99]–[101],

$$-\int_\Sigma \mathrm{d}^3x\sqrt{|h|}\left[M_{\rm Pl}^2 K + 4\theta(J - 2\hat{G}^{ij}K_{ij})\right] \tag{3.2.11}$$

where h_{ij} is the induced metric on the space-time boundary, Σ, with corresponding Einstein tensor $\hat{G}^{ij}$. The extrinsic curvature, $K_{ij} = -\frac{1}{2}\mathcal{L}_n h_{ij}$, is defined in terms of the Lie derivative of the induced metric with respect to the *outward* pointing normal, n^a, and $K = h^{ij}K_{ij}$ is its trace. Finally, we define,

$$J_{ij} = \frac{1}{3}\left[(K_{kl}K^{kl} - K^2)K_{ij} + 2KK_{ik}K^k{}_j - 2K_{ik}K^{kl}K_{lj}\right] \tag{3.2.12}$$

along with its trace $J = h^{ij}J_{ij}$. The full action is now given by (3.2.5) supplemented with the boundary term (3.2.11). Its variation now yields a boundary contribution of the form [99]–[101],

$$-\frac{1}{2}\int_\Sigma \mathrm{d}^3x\sqrt{|h|}\left[I^{ij}\delta h_{ij} + I^\theta\delta\theta\right] \tag{3.2.13}$$

with,

$$I^{ij} = -M_{\rm Pl}^2(K^{ij} - Kh^{ij}) - 4\theta(3J^{ij} - Jh^{ij} + 2\hat{P}^{iklj}K_{kl}) + \dots \tag{3.2.14}$$

$$I^\theta = 8(J - 2\hat{G}^{ij}K_{ij}) \tag{3.2.15}$$

where $\hat{P}^{iklj}$ is the double dual of the Riemann tensor and the ellipsis denote terms proportional to gradients of θ that will vanish automatically thanks to the bulk equations of motion.

If we were to impose Dirichlet boundary conditions on all fields, the action and variational principle would now be well defined. However, as explained analogously in [89], Dirichlet boundary conditions on either θ or Λ would suppress their off-shell global fluctuations which are crucial to the success of the sequestering mechanism. To preserve the vacuum energy cancellation we must impose Neumann boundary conditions instead,

$$n^a \partial_a \delta\Lambda|_\Sigma = 0, \quad n^a \partial_a \delta\theta|_\Sigma = 0 \tag{3.2.16}$$

Further imposing Dirichlet boundary conditions on the metric would now be problematic. Instead, we seek a boundary condition of the form $\delta h_{ij}|_\Sigma = A_{ij}\delta\theta|_\Sigma$ where A_{ij} is chosen so that,

$$\left(I^{ij}\delta h_{ij} + I^\theta \delta\theta\right)|_\Sigma = 0 \tag{3.2.17}$$

guaranteeing a stationary action on-shell. The task of finding a suitable choice of A_{ij} is simplified for a three dimensional boundary by noting that the double dual of the Riemann tensor, $\hat{P}^{iklj}$, vanishes identically in 3 dimensions. We can also use the Cayley-Hamilton theorem for a 3×3 matrix, applied to K_{ij}, to show that J_{ij} is a pure trace, $J_{ij} = -\frac{2}{3}h_{ij}\det K$. As a result, the final expression for (3.2.14) simplifies considerably, giving $I^{ij} = -M_{\rm Pl}^2(K^{ij} - Kh^{ij})$. In the end, we found a one parameter (z) family of suitable choices for A_{ij},

$$A_{ij} = \frac{1}{M_{\rm Pl}^2}\Bigg[-16\left(\hat{R}_{ij} - \frac{1}{4}\hat{R}h_{ij}\right) - \frac{16}{3}\left(K_{ik}K^k{}_j - KK_{ij} - \frac{1}{4}(K_{kl}K^{kl} - K^2)h_{ij}\right)\Bigg] \\ + z\left[2KK_{ij} + (K_{kl}K^{kl} - K^2)h_{ij}\right]. \tag{3.2.18}$$

We have not been able to establish an intuitive geometric interpretation of this choice, although we note that for $z = 0$, the extrinsic curvature terms appear in combinations familiar to the bulk curvature tensor, via the Gauss-Codazzi equations.

3.3 Cosmological Implications

Before studying the cosmological dynamics in detail, it is convenient to rewrite our effective gravity equation (3.2.8) after explicitly splitting the energy-momentum tensor up into its constant vacuum energy part, $V_{\rm vac}$ and local excitations, described by

$\tau_{\mu\nu}$. To this end we write $T_{\mu\nu} = -V_{\rm vac}\, g_{\mu\nu} + \tau_{\mu\nu}$ so that the vacuum energy drops out altogether and we obtain,

$$M_{\rm Pl}^2 G_{\mu\nu} = \tau_{\mu\nu} - \Lambda_{\rm res}\, g_{\mu\nu} \tag{3.3.1}$$

where we have a residual cosmological constant given by,

$$\Lambda_{\rm res} = \frac{1}{4}\langle\tau\rangle + \Delta\Lambda \tag{3.3.2}$$

with,

$$\Delta\Lambda^2 = \frac{3M_{\rm Pl}^4}{8}\left[\langle R_{GB}\rangle - \langle (W_{\mu\nu\alpha\beta})^2\rangle + \frac{2}{M_{\rm Pl}^4}\left\langle \left(\tau_{\mu\nu} - \frac{1}{4}\tau g_{\mu\nu}\right)^2\right\rangle - \frac{1}{6M_{\rm Pl}^4}\left(\langle\tau^2\rangle - \langle\tau\rangle^2\right)\right] \tag{3.3.3}$$

where $\tau = g^{\mu\nu}\tau_{\mu\nu}$. As emphasised previously, this residual cosmological constant is stable against radiative corrections to the vacuum energy and should now be fixed empirically. Of course, this is the same approach one takes for any relevant operator in effective field theory. For example, the electron mass is radiatively stable thanks to chiral symmetry, but its value cannot be predicted in effective field theory and should be set by measurement.

Let us now focus on a homogeneous and isotropic background, described by the standard cosmological metric,

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}_\kappa^2 \tag{3.3.4}$$

where $a(t)$ is the scale factor at time t, and $d\mathbf{x}_\kappa^2$ is the metric on unit sphere ($\kappa = 1$), plane ($\kappa = 0$) or hyperboloid ($\kappa = -1$). Assuming that the local matter content of the Universe is described by a homogeneous energy density, ρ and pressure, $p = w\rho$, we find that the dynamics is described by a Friedmann equation,

$$H^2 + \frac{\kappa}{a^2} = \frac{\rho + \Lambda_{\rm res}}{3M_{\rm Pl}^2} \tag{3.3.5}$$

where $\Lambda_{\rm res} = -\frac{1}{4}\langle(1-3w)\rho\rangle + \Delta\Lambda$ and,

$$\Delta\Lambda = \pm\sqrt{\frac{1}{2}\langle\rho^2(1+3w)\rangle + \frac{1}{16}\langle\rho(1-3w)\rangle^2 - \frac{3}{8}M_{\rm Pl}^4\mu^4\frac{\hat{\sigma}'}{\sigma'}\frac{\hat{c}}{c}} \tag{3.3.6}$$

Here we have used the fact that the Weyl tensor vanishes on a Friedmann-Robertson-Walker metric (3.3.4), as well as the constraint (3.2.10). Following [83] , we evaluate the space-time averages by assuming that the cosmology takes place over a (regulated) proper time interval $t_{\text{in}} < t < t_{\text{out}}$, with a (regulated) spatial co-moving volume Vol_3. For example, when we explicitly compute the constraint (3.2.10) in this way we obtain,

$$\langle R_{GB} \rangle \stackrel{\text{def}}{=} \frac{\int_{t_{\text{in}}}^{t_{\text{out}}} dt\ a^3 \left[24\frac{\ddot{a}}{a}\left(H^2 + \frac{\kappa}{a^2}\right)\right]}{\int_{t_{\text{in}}}^{t_{\text{out}}} dt\ a^3} = -\mu^4 \frac{\hat{\sigma}'\hat{c}}{\sigma' c}. \tag{3.3.7}$$

The cancellation of the spatial volumes will be generic for all space-time averages computed on this background.

3.3.1 Calculation of historic integrals

Let us now estimate the historic integrals that appear in (3.3.2) and (3.3.3). To do so, we follow [83] and split the cosmological history into intervals (t_i, t_{i+1}), for which the dominant source, ρ, has equations of state w_i and the cosmological evolution has an *effective* equation of state $\bar{w}_i$. Generically we expect $w_i = \bar{w}_i$, although exceptions could include an epoch of curvature domination or domination by the residual cosmological constant, Λ_{res}, as one might expect to see at late times. In this ith interval, we can use the energy conservation equation $\dot{\rho} = -3H(\rho + p)$ and the Friedmann equations (3.3.5) to obtain,

$$H = H_{i+1}\left(\frac{a}{a_{i+1}}\right)^{-\frac{3}{2}(1+\bar{w}_i)}, \ \rho = \rho_{i+1}\left(\frac{a}{a_{i+1}}\right)^{-3(1+w_i)} \tag{3.3.8}$$

where a_j and H_j denote the scale and Hubble factors evaluated at time t_j. Let us define the generic contributions to the integrals in (3.3.6) and evaluate them using (3.3.8). For $n = 0, 1, 2$, we write,

$$I_{n,i} \stackrel{\text{def}}{=} f_{n,i} \int_{t_i}^{t_{i+1}} dt\, a^3 \rho^n \tag{3.3.9}$$

$$= \left(\frac{a^3\rho^n}{H}\right)_{i+1} \frac{f_{n,i}}{g_{n,i}} \left[1 - \left(\frac{a_i}{a_{i+1}}\right)^{g_{n,i}}\right] \tag{3.3.10}$$

$$= \left(\frac{a^3\rho^n}{H}\right)_{i} \frac{f_{n,i}}{g_{n,i}} \left[\left(\frac{a_{i+1}}{a_i}\right)^{g_{n,i}} - 1\right] \tag{3.3.11}$$

where,

$$f_{n,i} = \begin{cases} 1 & n = 0 \\ 1 - 3w_i & n = 1 \\ 1 + 3w_i & n = 2 \end{cases} \tag{3.3.12}$$

and,

$$g_{n,i} = \frac{3}{2}(3 + \bar{w}_i) - 3n(1 + w_i). \tag{3.3.13}$$

Note that for $g_{n,i} = 0$, we understand the formulae for $I_{n,i}$ by taking the limit as $g_{n,i} \to 0$, in which case we obtain logarithms. Let us also define $I_n = \sum_i I_{n,i}$ where the sum is performed over all intervals in the entire cosmic history, so that now we may write,

$$\Lambda_{\text{res}} = -\frac{1}{4}\frac{I_1}{I_0} \pm \sqrt{\frac{1}{2}\frac{I_2}{I_0} + \frac{1}{16}\left(\frac{I_1}{I_0}\right)^2 - \frac{3}{8}M_{\text{Pl}}^4\mu^4\frac{\hat{\sigma}'}{\sigma'}\frac{\hat{c}}{c}}. \tag{3.3.14}$$

Owing to the quadratic nature of the global constraint, our solution comes with two roots. At this stage, we have no compelling reason to pick one root over the other. In higher dimensional Gauss-Bonnet gravity, solutions also split into two branches, and it is the branch that admits a smooth Einstein limit that typically avoids pathological behaviour [102].

Consider first an expanding phase, so that adjacent intervals satisfy $a_{i-1} \ll a_i \ll a_{i+1}$. We obtain the following ratio,

$$\left|\frac{I_{n,i}}{I_{n,i-1}}\right| = \frac{\frac{f_{n,i}}{g_{n,i}}\left[\left(\frac{a_{i+1}}{a_i}\right)^{g_{n,i}} - 1\right]}{\frac{f_{n,i-1}}{g_{n,i-1}}\left[1 - \left(\frac{a_{i-1}}{a_i}\right)^{g_{n,i-1}}\right]}. \tag{3.3.15}$$

Depending on the values for the $g_{n,i}$, there are three possible scenarios[1]:

1. $|I_{n,i}| \gg |I_{n,i-1}|$ e.g. when $g_{n,i} > 0, g_{n,i-1} > 0$

2. $|I_{n,i}| \sim |I_{n,i-1}|$ e.g. when $g_{n,i} < 0, g_{n,i-1} > 0$

3. $|I_{n,i}| \ll |I_{n,i-1}|$ e.g. when $g_{n,i} < 0, g_{n,i-1} < 0$.

[1] When $g_{n,i} > 0, g_{n,i-1} < 0$ we could in principle be in any of the three cases, depending on the relative size of the scale factors.

When case 1 occurs, the later interval dominates thanks to the largeness of a_{i+1}, for appropriate values of $g_{n,i}$. In contrast, when case 3 occurs, the earlier interval dominates thanks to the smallness of a_{i-1}, again, for appropriate values of $g_{n,i}$. One can obtain analogous results in a contracting phase (if there is one). What all this tells us is that the sums I_n are dominated by their extreme infra-red and ultra-violet intervals, where the scale factor is largest and smallest respectively. To develop this further, let us define the infra-red interval as $a_\star < a < a_{\max}$ and the ultra-violet interval as $a_{\min} < a < a_\dagger$, where $a_{\max}$ is the largest scale factor in the cosmic history, and $a_{\min}$ is the smallest. $a_{\min}$ is not taken to be zero, as one might naively expect, but to a regulated finite value consistent with the UV cut-off of the theory. In contrast, we do allow $a_{\max}$ to be infinite, in principle. The precise values of $a_\star$ and $a_\dagger$ are not important in what follows. We may now write[2],

$$I_n \sim I_n^{UV} + I_n^{IR} \tag{3.3.16}$$

where,

$$I_n^{UV} = \left(\frac{a^3\rho^n}{H}\right)_\star \frac{f_{n,UV}}{g_{n,UV}}\left[1-\left(\frac{a_{\min}}{a_\star}\right)^{g_{n,UV}}\right] \tag{3.3.17}$$

$$I_n^{IR} = \left(\frac{a^3\rho^n}{H}\right)_\dagger \frac{f_{n,IR}}{g_{n,IR}}\left[\left(\frac{a_{\max}}{a_\dagger}\right)^{g_{n,IR}}-1\right]. \tag{3.3.18}$$

These terms contain possible divergences as $a_{\min} \to 0$ (for $g_{n,UV} \leq 0$) and $a_{\max} \to \infty$ (for $g_{n,IR} \geq 0$). Of course, what we are really interested in are the ratios I_n/I_0. To this end we note that $g_{0,i} = \frac{3}{2}(3+\bar{w}_i) \in [3,6]$ for an effective equation of state $\bar{w}_i \in [-1,1]$. This range is consistent with sources that satisfy the dominant energy condition. In any event, it follows that there is no divergent UV contribution to I_0, so that we simply have,

$$I_0 \sim I_0^{IR} \sim \left(\frac{a^3}{H}\right)_\dagger \frac{1}{\frac{3}{2}(3+\bar{w}_{IR})}\left(\frac{a_{\max}}{a_\dagger}\right)^{\frac{3}{2}(3+\bar{w}_{IR})}. \tag{3.3.19}$$

[2]In an expanding then contracting Universe, we would get UV and IR contributions from both phases, but we suppress this sum in the interest of brevity.

Now consider the ratios. From the infra-red regime, we have,

$$\begin{aligned}\frac{I_n^{IR}}{I_0} &\sim \frac{3}{2}(3+\bar{w}_{IR})\rho_\dagger^n \frac{f_{n,IR}}{g_{n,IR}} \left(\frac{a_{\max}}{a_\dagger}\right)^{-3n(1+w_{IR})} \\ &\sim \frac{3}{2}(3+\bar{w}_{IR})\rho_{\max}^n \frac{f_{n,IR}}{g_{n,IR}}. \end{aligned} \tag{3.3.20}$$

The matter equation of state satisfies the dominant energy condition with vacuum energy excluded[3], $w_i \in (-1, 1]$, so there is no divergence in the ratio I_n^{IR}/I_0 for $n = 1, 2$. Indeed, we see that this ratio scales like $\rho_{\max}^n$, where $\rho_{\max}$ is the homogeneous energy density associated with localised matter sources at the point where the Universe is at its largest. This contribution vanishes in an infinite Universe thanks to the dilution of such sources.

Now consider the ultra-violet regime. Here we have,

$$\frac{I_n^{UV}}{I_0} \sim -\frac{3}{2}(3+\bar{w}_{IR})\frac{\left(\frac{a^3\rho^n}{H}\right)_\star}{\left(\frac{a^3}{H}\right)_\dagger}\frac{f_{n,UV}}{g_{n,UV}}\frac{\left[\left(\frac{a_{\min}}{a_\star}\right)^{g_{n,UV}}-1\right]}{\left(\frac{a_{\max}}{a_\dagger}\right)^{\frac{3}{2}(3+\bar{w}_{IR})}}. \tag{3.3.21}$$

For $g_{n,UV} < 0$, there is a dangerous *power law* divergence as $a_{\min} \to 0$ in a finite Universe (where $a_{\max}$ is finite). Such a divergence could contaminate the observed cosmological constant, $\Lambda_{\rm res}$, with power law cut-off dependence, in violation of naturalness. Indeed, given the allowed values $w_i \in (-1, 1], \bar{w}_i \in [-1, 1]$, we have that $g_{n,i} \in [3-6n, 6)$ and therefore a potentially dangerous cut-off dependence for $n = 1, 2$. If we choose to identify $\bar{w}_{UV} = w_{UV}$, we can reduce the cut-off scaling to at worst a logarithmic one (for $w_{UV} = 1$) for $n = 1$ [83], although for $n = 2$, power law dependence remains for $w_{UV} \in [-1/3, 1]$.

This unnatural cut-off dependence can be eliminated in an infinite Universe, thanks to the volume suppression as $a_{\max} \to \infty$. This suggests that there is a lower bound on the size of the Universe set by naturalness. Let's have some fun by estimating this, noting first that $\frac{I_n^{UV}}{I_0} \sim \left(\frac{a^3\rho^n}{H}\right)_{\min} / \left(\frac{a^3}{H}\right)_{\max}$. If we take $\rho_{\min} \sim$

[3]The constant underlying vacuum energy gets sequestered. We will deal wth vacuum energy phase transitions in the next section.

$M_{\rm Pl}^2 H_{\rm min}^2$ then we can write,

$$\frac{I_n^{UV}}{I_0} \sim \left(\frac{\mathcal{N}}{a_{\rm max}/a_0}\right)^3 \frac{H_{\rm max}}{H_0}(M_{\rm Pl}^2 H_0^2)^n \tag{3.3.22}$$

$$\lesssim \left(\frac{\mathcal{N}}{a_{\rm max}/a_0}\right)^3 (M_{\rm Pl}^2 H_0^2)^n \tag{3.3.23}$$

where a_0 is the present day scale factor and,

$$\mathcal{N} = \left(\frac{1}{H_0 l_{UV}}\right)^{\frac{1}{3}\left[2n-1-\frac{2}{1+\bar{W}(a_{\rm min},a_0)}\right]} . \tag{3.3.24}$$

Here we have integrated over the cosmic history from the cutoff to the present day, giving,

$$\frac{H_{\rm min}}{H_0} = \left(\frac{a_0}{a_{\rm min}}\right)^{\frac{3}{2}(1+\bar{W}(a_{\rm min},a_0,))} \tag{3.3.25}$$

where,

$$1+\bar{W}(a_{\rm min},a_0) = \frac{\int_{\ln a_{\rm min}}^{\ln a_0} d\ln a \ (1+\bar{w}(\ln a))}{\int_{\ln a_{\rm min}}^{\ln a_0} d\ln a} \tag{3.3.26}$$

and $\bar{w}(\ln a)$ is the effective equation of state when the scale factor has size a. We have also assumed $H_{\rm min} \sim l_{UV}^{-1}$ where l_{UV} is the length scale at which we cut-off the theory (possibly the string length or the Planck length). In any event, provided $a_{\rm max}/a_0 \gtrsim \mathcal{N}$, we are guaranteed that the UV contribution does not exceed the scale set by the critical density today, $I_n^{UV}/I_0 \lesssim (M_{\rm Pl}^2 H_0^2)^n$.

The condition $a_{\rm max}/a_0 \gtrsim \mathcal{N}$ is only required for $n = 1, 2$, and given that $\bar{W} \in [-1, 1]$ our strongest bound comes from $n = 2$ and $\bar{W} = 1$. This yields $a_{\rm max}/a_0 \gtrsim (H_0 l_{UV})^{-2/3}$, which for a Planckian cut-off, is a comforting $a_{\rm max}/a_0 \gtrsim 10^{40}$ or 92 more efolds of expansion! In any event, we trust that the reader has enough time to finish going through the rest of this chapter.

Bringing everything together, we see that the residual cosmological constant receives up to three distinct contributions: the IR part of the historic integrals scaling as $\rho_{\rm max} \lesssim M_{\rm Pl}^2 H_0^2$, the UV part scaling as $\left(\frac{\mathcal{N}}{a_{\rm max}/a_0}\right)^{\frac{3}{n}} \left(\frac{H_{\rm max}}{H_0}\right)^{\frac{1}{n}} M_{\rm Pl}^2 H_0^2 \lesssim M_{\rm Pl}^2 H_0^2$, and the flux contribution scaling as $\Lambda_{\rm flux} = \sqrt{-\frac{3}{8} M_{\rm Pl}^4 \mu^4 \frac{\hat{\sigma}'}{\sigma'}\frac{\hat{c}}{c}}$. The latter can be fixed empirically and assumed to lie below the dark energy scale. In conclusion, then, provided the Universe grows sufficiently large, the residual cosmological constant will not exceed the critical density of the Universe today.

3.3.2 Homogeneous phase transitions

We now consider the effect of a single homogeneous phase transition in the vacuum energy. Such transitions shift the potential by a constant amount $\mathcal{O}(M^4)$, where M is the scale of the transition, with well known examples being the electroweak and the QCD phase transitions. Assuming a rapid transition, we can model this by a step function of size $\Delta V = V_2 - V_1$, at time t_*, so that the energy momentum tensor is given by $T_{\mu\nu} = -V(t) g_{\mu\nu} + \tau_{\mu\nu}$, where,

$$V(t) = \begin{cases} V_1 & t < t_* \\ V_2 & t > t_* \end{cases} \tag{3.3.27}$$

and $\tau_{\mu\nu}$ represents localised sources with equation of state in the range $(-1, 1]$, consistent with the dominant energy condition. In what follows, we will make use of the following shorthand for the space-time volume before transition,

$$\Omega_1 = \text{Vol}_3 \int_{t_{\text{in}}}^{t_*} dt \; a^3 \tag{3.3.28}$$

the space-time volume after,

$$\Omega_2 = \text{Vol}_3 \int_{t_*}^{t_{\text{out}}} dt \; a^3 \tag{3.3.29}$$

and their ratio $\mathcal{I} = \frac{\Omega_2}{\Omega_1}$. We also define the following "before" and "after" averages, respectively,

$$\langle \tau \rangle_1 = \frac{\text{Vol}_3 \int_{t_{\text{in}}}^{t_*} dt \; a^3 \tau}{\Omega_1}, \qquad \langle \tau \rangle_2 = \frac{\text{Vol}_3 \int_{t_*}^{t_{\text{out}}} dt \; a^3 \tau}{\Omega_2}. \tag{3.3.30}$$

Finally we introduce the local excitation of the potential,

$$\delta V = V(t) - \langle V \rangle = \begin{cases} -\Delta V \frac{\mathcal{I}}{(1+\mathcal{I})} & t < t_* \\ \Delta V \frac{1}{(1+\mathcal{I})} & t > t_*. \end{cases} \tag{3.3.31}$$

We are now ready to write down the effective gravity equation in the presence of a homogeneous transition. It is given by $M_{\text{Pl}}^2 G_{\mu\nu} = -\Lambda_{\text{eff}}(t) g_{\mu\nu} + \tau_{\mu\nu}$ where the effective cosmological constant is,

$$\Lambda_{\text{eff}}(t) = \delta V + \Delta\Lambda + \frac{1}{4}\langle \tau \rangle \tag{3.3.32}$$

and,

$$\Delta\Lambda^2 = -\frac{\mathcal{I}}{(1+\mathcal{I})^2}\left[(\Delta V)^2 - \frac{1}{2}\Delta V\left(\langle\tau\rangle_2 - \langle\tau\rangle_1\right)\right] + \frac{3}{4}\left\langle\left(\tau_{\mu\nu} - \frac{1}{4}\tau g_{\mu\nu}\right)^2\right\rangle - \frac{1}{16}\left(\langle\tau^2\rangle - \langle\tau\rangle^2\right) - \frac{3}{8}M_{\rm Pl}^4\mu^4\frac{\hat{\sigma}'}{\sigma'}\frac{\hat{c}}{c}. \quad (3.3.33)$$

For $\Delta V = 0$, this result reduces to (3.3.2) and (3.3.3) for vanishing Weyl tensor, as of course it should. To study the effect of the phase transition, we focus on the ΔV dependent terms in our expression. These introduce some time dependence in the effective cosmological constant, through δV. To develop some intuitive understanding let us first consider very early and very late transitions. For a very early transition, we expect $\mathcal{I} \gg 1$ and so to get some insight we take the limit $\mathcal{I} \to \infty$. In this case, the effective cosmological constant after the transition loses all knowledge of the scale of the jump. Prior to the transition, the effective cosmological constant is strongly sensitive to ΔV. In contrast, for late transitions, modelled intuitively with the limit $\mathcal{I} \to 0$, we have the opposite: no sensitivity to ΔV prior to transition, but strong sensitivity after. Although the details are different, these conclusions are qualitatively the same as for earlier models of sequestering: sequestering works best in the volume that dominates the space-time. This means that we always have late time suppression of the jump for early transitions [83].

Let us now estimate the size of this volume ratio and the impact on the effective cosmological constant more carefully. As we saw in the previous section, historic integrals are generically dominated by the period in which the Universe is largest. This corresponds to the latest time during an expanding phase. We shall consider phase transitions occurring in the past, during expansion, consistent with the structure of the Standard Model. The results of the previous section (see equation (3.3.19) and use (3.3.8)) then suggest that,

$$\Omega_1 + \Omega_2 = I_0 = \mathcal{O}(1)\left(\frac{a^3}{H}\right)_{\rm max}, \quad \Omega_1 = \mathcal{O}(1)\left(\frac{a^3}{H}\right)_* \quad (3.3.34)$$

and so,

$$\mathcal{I} = \mathcal{O}(1)\left(\frac{a_{\rm max}}{a_*}\right)^3\frac{H_*}{H_{\rm max}} - 1 \sim \mathcal{O}(1)\left(\frac{a_{\rm max}}{a_*}\right)^3\frac{H_*}{H_{\rm max}} \quad (3.3.35)$$

where we have used the fact that $a_{\rm max} \gg a_*$ and $H_{\rm max} \ll H_*$. Since $\mathcal{I} \gg 1$, we have that,

$$\Delta\Lambda^2 \approx -\left[\frac{(\Delta V)^2}{\mathcal{I}} - \frac{\Delta V}{2\mathcal{I}}\left(\langle\tau\rangle_2 - \langle\tau\rangle_1\right)\right] + \dots \qquad (3.3.36)$$

where $\dots$ denote transition independent terms and,

$$\delta V \approx \begin{cases} -\Delta V & t < t_* \\ \frac{\Delta V}{\mathcal{I}} & t > t_*. \end{cases} \qquad (3.3.37)$$

As anticipated, we get strong dependence on the scale of the jump, prior to the transition. This will yield a short burst of inflation just before the transition occurs. After the transition, it would seem that any dependence on the scale of the jump is heavily suppressed. To see by how much, recall that integrating the cosmic history from the transition to the maximum size, we can show that,

$$\frac{H_*}{H_{\rm max}} = \left(\frac{a_{\rm max}}{a_*}\right)^{\frac{3}{2}(1+\bar{W}(a_*,a_{\rm max}))} \qquad (3.3.38)$$

where $1 + \bar{W}(a_*, a_{\rm max}) = \frac{\int_{\ln a_*}^{\ln a_{\rm max}} d\ln a \ (1+\bar{w}(\ln a))}{\int_{\ln a_*}^{\ln a_{\rm max}} d\ln a}$. It then follows that the contribution to δV after the transition goes as,

$$\delta V_{\rm after} \approx \frac{\Delta V}{\mathcal{I}} = \mathcal{O}(1)\frac{\Delta V}{M_{\rm Pl}^2 H_*^2}\left(\frac{H_{\rm max}}{H_*}\right)^{\frac{1-\bar{W}}{1+\bar{W}}} M_{\rm Pl}^2 H_{\rm max}^2. \qquad (3.3.39)$$

We expect $|\Delta V| = \mathcal{O}(1) M_{\rm Pl}^2 H_*^2$ and so since $\bar{W}(a_*, a_{\rm max}) \in [-1, 1]$, it follows that this contribution is no larger than the critical density at maximum size, or indeed the critical density today, $\delta V_{\rm after}\ .\ M_{\rm Pl}^2 H_{\rm max}^2\ .\ M_{\rm Pl}^2 H_0^2$. This reflects similar conclusions drawn in [83]. In an infinitely old, asymptotically de Sitter Universe, we get exponential suppression since $\bar{W}(a_*, a_{\rm max}) = -1$.

Now consider the jump contributions to $\Delta\Lambda$ as shown in (3.3.36). Similar considerations yield,

$$\frac{(\Delta V)^2}{\mathcal{I}} = \mathcal{O}(1)\left(\frac{\Delta V}{M_{\rm Pl}^2 H_*^2}\right)^2 \left(\frac{H_{\rm max}}{H_*}\right)^{-\frac{1+3\bar{W}}{1+\bar{W}}} M_{\rm Pl}^4 H_{\rm max}^4. \qquad (3.3.40)$$

For the other contribution, we adapt the results of the previous section to estimate the "before" and "after" averages as $\langle\tau\rangle_1 \sim \mathcal{O}(1)\rho_* \gg \langle\tau\rangle_2 \sim \mathcal{O}(1)\rho_{\rm max}$. This then gives the scale,

$$\frac{\Delta V}{2\mathcal{I}}\left(\langle\tau\rangle_2-\langle\tau\rangle_1\right)=\mathcal{O}(1)\frac{\Delta V}{M_{\rm Pl}^2H_*^2}\frac{\rho_*}{M_{\rm Pl}^2H_*^2}\left(\frac{H_{\rm max}}{H_*}\right)^{-\frac{1+3\bar{W}}{1+\bar{W}}}M_{\rm Pl}^4H_{\rm max}^4. \tag{3.3.41}$$

Assuming $|\Delta V|,\rho_*=\mathcal{O}(1)M_{\rm Pl}^2H_*^2$ the result is that the jump contributions to $\Delta\Lambda$ both come in at the scale,

$$[\Delta\Lambda]_{\rm jump}=\mathcal{O}(1)\left(\frac{H_{\rm max}}{H_*}\right)^{-\frac{1+3\bar{W}}{2(1+\bar{W})}}M_{\rm Pl}^2H_{\rm max}^2. \tag{3.3.42}$$

In contrast to $\delta V_{\rm after}$, this contribution has the potential to be enhanced relative to the critical density at maximum size $M_{\rm Pl}^2H_{\rm max}^2$, whenever $\bar{W}\in(-1/3,1]$. This enhancement could easily make $\Delta\Lambda$ larger than the critical density today. Requiring that this is *not* the case imposes the following bound,

$$\bar{W}<-\frac{1}{3}\left(\frac{1-4r}{1-\frac{4}{3}r}\right),\qquad r=\frac{\ln\frac{H_0}{H_{\rm max}}}{\ln\frac{H_*}{H_{\rm max}}} \tag{3.3.43}$$

where we have assumed $r<\frac{3}{4}$. As we have stated previously, in an infinitely old, asymptotically de Sitter Universe, we get $\bar{W}(a_*,a_{\rm max})=-1$ and so there are no dangerously large contributions to $\Delta\Lambda$. But what if the current de Sitter phase is only transient? Let's have more fun and estimate how long this quasi de Sitter stage needs to last in order to ensure there is no dangerous enhancement of $[\Delta\Lambda]_{\rm jump}$. To do this, we crudely model the history of the universe as radiation dominated from a_* until $a_{\rm eq}$, then matter dominated from $a_{\rm eq}$ until $a_{\rm de}$, and finally quasi-de Sitter behaviour from $a_{\rm de}$ until $a_{\rm max}$. We shall not assume that $a_{\rm max}$ is infinite, allowing for the possibility that the quasi de Sitter stage comes to an end close to the maximum size. In any event, we find that,

$$\bar{W}=\frac{\ln\left[\left(\frac{a_{\rm eq}}{a_*}\right)^{\frac{4}{3}}\frac{a_{\rm de}}{a_{\rm eq}}\right]}{\ln\frac{a_{\rm max}}{a_*}}-1. \tag{3.3.44}$$

Assuming r to be small then requiring $\bar{W}<-1/3$, we obtain the following lower bound on the would-be size of the Universe,

$$a_{\rm max}>\frac{\sqrt{a_{\rm eq}a_{\rm de}}}{a_*}a_{\rm de}. \tag{3.3.45}$$

To bring this to life, we note that the QCD phase transition, matter-radiation equality and matter-dark energy equality occur at redshifts of $10^{12}, 3400$ and 0.4 respectively. Setting $a_* \sim a_{QCD}$, our bound then implies $a_{\max}/a_0$ & 10^{10} which is less constraining than our estimate in the previous section. Earlier transitions would suggest a longer future, of course.

3.3.3 Inflation

We have seen in previous sections how a large and old Universe can eliminate potentially large and unnatural contributions to the residual cosmological constant. The standard mechanism for achieving a large Universe is through inflation so it is natural to ask if it can be embedded in a theory of OS. We might be concerned that the inflaton source behaves like a constant vacuum energy to zeroth order in slow roll and will therefore be sequestered. This conclusion is too quick, however. Inflation resembles a (slow) phase transition and, as we have just seen, the corresponding scale is visible in the effective cosmological constant *prior* to the end of the transition. Compatibility with inflation was shown for earlier models of sequestering [83], and we will now show that this is also the case here.

We assume, for simplicity, standard single field inflation (for a review, see [103]), described by a canonical scalar φ with potential $V(\varphi)$, minimally coupled to the metric. During inflation, all other sources of energy-momentum are quickly diluted away, and, during slow roll, we have that the effective Friedmann equation and energy conservation equation are given by,

$$H^2 \approx \frac{V + \Lambda_{\rm res}}{3M_{\rm Pl}^2}, \qquad 3H\dot{\varphi} \approx -V' \tag{3.3.46}$$

where we have also neglected spatial curvature. We now ask whether or not the inflationary contribution to the residual cosmological constant can significantly affect the dynamics. If inflation were to go on like this forever, the answer would be "yes", since the sequestering mechanism would force an exact cancellation between a constant value for V and $\Lambda_{\rm res}$. Of course, inflation must end, and it turns out that its contribution to $\Lambda_{\rm res}$ is nowhere near large enough to compete with the potential.

To see this, let us now estimate the inflationary contribution to Λ_{res}. Again, assuming slow roll, we have that $\tau_{\mu\nu} \approx -V(\varphi)g_{\mu\nu}$. It follows that,

$$\langle \tau \rangle \approx -4\frac{\int_{t_{\text{start}}}^{t_{\text{end}}} dt\ a^3 V(\varphi)}{\int_{t_{\text{in}}}^{t_{\text{out}}} dt\ a^3} \tag{3.3.47}$$

where inflation starts at time $t_{\text{start}} \approx t_{\text{in}}$ and ends at time $t_{\text{end}} \ll t_{\text{out}}$. We can estimate the integrals to give,

$$\langle \tau \rangle = \mathcal{O}(1) V_{\text{inf}} \left(\frac{a_{\text{end}}}{a_{\text{max}}}\right)^3 \frac{H_{\text{max}}}{H_{\text{inf}}} \tag{3.3.48}$$

where $V_{\text{inf}} = M_{\text{Pl}}^2 H_{\text{inf}}^2$ and H_{inf}^2 is the scale of inflation. Since $H_{\text{max}} \ll H_{\text{inf}}$ and $a_{\text{end}} \ll a_{\text{max}}$ we have that $|\langle \tau \rangle|$ is much less than the scale of the potential during inflation V_{inf}. Similarly, we find that,

$$\langle \tau^2 \rangle = \mathcal{O}(1) V_{\text{inf}}^2 \left(\frac{a_{\text{end}}}{a_{\text{max}}}\right)^3 \frac{H_{\text{max}}}{H_{\text{inf}}} \ll V_{\text{inf}}^2 \tag{3.3.49}$$

and $\left\langle \left(\tau_{\mu\nu} - \frac{1}{4}\tau g_{\mu\nu}\right)^2 \right\rangle \approx 0$. Since the flux contribution, Λ_{flux} . $M_{\text{Pl}}^2 H_0^2 \ll V_{\text{inf}}$, we conclude that, $|\Lambda_{\text{res}}| \ll V_{\text{inf}}$, or in other words, inflation in OS goes through as normal.

3.3.4 Geometric consequences of choosing the flux

The boundary fluxes, given by c and $\hat{c}$, are taken to be infra-red geometric quantities, whose values are simply given as fixed boundary conditions in the effective field theory. Nevertheless, it is interesting to explore the consequences of particular choices. For example, in an homogeneous universe, vanishing $\hat{c}$ forces the spatial curvature to be negative, consistent with a spatially open Universe. To see this we simply set $\hat{c} = 0$ in (3.3.7), then solve the integral to give,

$$\kappa|_{\hat{c}=0} = -\frac{[\dot{a}^3]_{t_{\text{in}}}^{t_{\text{out}}}}{3[\dot{a}]_{t_{\text{in}}}^{t_{\text{out}}}}. \tag{3.3.50}$$

The right hand side of this expression is negative for all real choices of $\dot{a}_{\text{in}}$ and $\dot{a}_{\text{out}}$. We emphasise that for generic $\hat{c}$, there are no such well defined constraints on the spatial geometry. Indeed, more generally we have from (3.3.7),

$$\kappa = -\frac{[\dot{a}^3]_{t_{\text{in}}}^{t_{\text{out}}}}{3[\dot{a}]_{t_{\text{in}}}^{t_{\text{out}}}} - \mu^4 \frac{\hat{\sigma}'\hat{c}}{\sigma' c} \frac{\int_{t_{\text{in}}}^{t_{\text{out}}} dt\ a^3}{24[\dot{a}]_{t_{\text{in}}}^{t_{\text{out}}}} \tag{3.3.51}$$

where the second term can take either sign and be as large or small as we like, depending on the choices for the flux and the cosmological dynamics.

3.4 Inhomogeneous Phase Transitions

Transitions in vacuum energy can also occur locally through bubble nucleation. In standard Einstein gravity, the formalism for describing this was pioneered by Coleman and collaborators [68], [72], [73] and adapted to early models of sequestering in [89]. There it was shown that vacuum energy was most efficiently sequestered in regions of space-time of largest volume, favouring near-Minkowski configurations without fine-tuning. We shall now show that similar conclusions can be drawn for OS.

First we assume a potential that interpolates between two minima, separated by a scale ΔV. Tunnelling from one vacuum to the other can occur via spontaneous nucleation of a spherical bubble containing the new vacuum in the interior, then expanding at the speed of light. As we will see, not all configurations are kinematically allowed, at least if we assume a sensible microscopic structure in the bubble wall. Further, for the kinematically allowed configurations, we can estimate the rate of transition per unit volume by computing the so-called bounce solution to the Euclidean field equations.

Let us proceed by first computing the bounce. As usual, we will work in the thin wall approximation [73], and assume that the bounce solution is $O(4)$ invariant [104], [105]. Under these assumptions we can write the metric with the ansatz $ds^2 = dr^2 + \rho^2(r) d\chi^2$ where $d\chi^2 = \gamma_{ij} dx^i dx^j$ is the unit 3-sphere. In a neighbourhood of the bubble wall, we adopt a coordinate system with the wall at $r = 0$, the bubble exterior corresponding to $r > 0$ (which we will call denote $\mathcal{M}_+$), and the interior $r < 0$ (which we will denote $\mathcal{M}_-$). We shall also refer to the exterior as the "old" vacuum, and to the interior as the "new". The rotational invariance allows us to write all fields as functions of the radial coordinate r only. For example, the 3-forms components are now,

$$A_{ijk} = A(r)\sqrt{\gamma}\epsilon_{ijk}, \qquad \hat{A}_{ijk} = \hat{A}(r)\sqrt{\gamma}\epsilon_{ijk}\,. \tag{3.4.1}$$

The computation of the Gauss-Bonnet term gives,

$$R_{\rm GB} = -24\left(\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2}\right)\frac{\rho''}{\rho} \tag{3.4.2}$$

while the Ricci scalar is still,

$$R = 6\left(\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2} - \frac{\rho''}{\rho}\right). \tag{3.4.3}$$

We can now write down the equations of motion. We obtain constant Λ and θ on-shell, while the remaining equations can be written,

$$3M_{\rm Pl}^2\left(\frac{\rho'^2}{\rho^2} - \frac{1}{\rho^2}\right) = -(\Lambda + V(r)) \tag{3.4.4}$$

$$M_{\rm Pl}^2\left(\frac{\rho'^2}{\rho^2} - \frac{1}{\rho^2} + 2\frac{\rho''}{\rho}\right) = -(\Lambda + V(r) + \sigma_{\rm w}\delta(r)) \tag{3.4.5}$$

$$\frac{\sigma'}{\mu^4}A'(r) = \rho^3 \tag{3.4.6}$$

$$\hat{\sigma}'\hat{A}'(r) = 24\left(1-\rho'^2\right)\rho''. \tag{3.4.7}$$

It should be noted that (3.4.4) and (3.4.5) are unchanged from General Relativity (GR), while (3.4.6) is the same as in [89]. The potential,

$$V(r) = \begin{cases} V_+ & r > 0 \\ V_- & r < 0 \end{cases} \tag{3.4.8}$$

is taken to be a step function interpolating between the constant minima, whereas the bubble wall is modelled with a delta-function weighted by a tension $\sigma_{\rm w}$.

Solving away from the bubble wall, we find that,

$$\rho(r) = \frac{1}{q}\sin q(r_0 + \epsilon r) \tag{3.4.9}$$

where $\epsilon = \pm 1$, r_0 is a constant of integration which in principle can differ between interior and exterior, and,

$$q^2 = \frac{\Lambda + V}{3M_{\rm Pl}^2} \tag{3.4.10}$$

represents the local value of the space-time vacuum curvature. Here q^2 can be positive, zero, or negative for a spherical, planar or hyperbolic geometry respectively[4].

[4]Later, when we Wick rotate back to Lorentzian signature, these will correspond to locally de Sitter, Minkowski and anti-de Sitter space-times.

For the planar geometry, we can formally take the limit of (3.4.9) as $q \to 0$, while for the hyperbolic case we analytically continue the formula to imaginary values of q. In all cases, we can rewrite (3.4.2) and (3.4.3) in terms of the local curvature q,

$$R = 12q^2\,, \qquad R_{\rm GB} = 24q^4\,. \tag{3.4.11}$$

Matching conditions across the wall require continuity in 3-sphere radius, ρ, and the 3-form, A, at $r = 0$, or in other words,

$$\left[\frac{1}{q}\sin qr_0\right]_+ = \left[\frac{1}{q}\sin qr_0\right]_-\,, \qquad A(0^+) = A(0^-) \tag{3.4.12}$$

where labels $\pm$ denote evaluation in $\mathcal{M}_\pm$. In contrast, integrating equations (3.4.5) and (3.4.7) across the bubble wall yields the following discontinuities,

$$2M_{\rm Pl}^2\frac{\Delta\rho'(0)}{\rho_0} = -\sigma_{\rm w}\,, \qquad \Delta\hat{A}(0) = \frac{24}{\hat{\sigma}'}\left(\Delta\rho'(0) - \frac{\Delta\left(\rho'(0)^3\right)}{3}\right) \tag{3.4.13}$$

where Δ is defined by $\Delta Q = Q_+ - Q_-$ and $\rho_0 = \rho(0^+) = \rho(0^-)$. The jump in ρ' is just the jump in extrinsic curvature across the bubble wall, familiar from the Israel junction conditions [100], [106]. Less familiar is the jump in $\hat{A}$, which can be rewritten as,

$$\hat{A}(0^+) - \hat{A}(0^-) = -\frac{12}{\hat{\sigma}'}\frac{\rho_0\sigma_{\rm w}}{M_{\rm Pl}^2}\left[1 - \left(\overline{\rho'(0)}\right)^2 - \frac{\rho_0^2\sigma_{\rm w}^2}{48M_{\rm Pl}^4}\right] \tag{3.4.14}$$

where a bar refers to the average of a quantity across the wall, as $\bar{Q} = (Q_+ + Q_-)/2$. The jump in $\hat{A}$ occurs because $\hat{A}$ couples to energy-momentum through the curvature. Tensional thin walls therefore behave as membranes charged under $\hat{A}$, as in [89], although the mapping between the wall tension and the effective 3-form charge is now different. In a physical set-up, we would, of course, expect the bubble wall to have finite thickness, allowing for a smooth but rapid transition in the value of $\hat{A}$.

Requiring that the bubble wall is supported by a sensible microscopic configuration, we require that it carries non-negative tension. Through (3.4.13) this places the usual kinematic constraint on the allowed configurations,

$$\Delta(\epsilon\cos qr_0) \leq 0 \tag{3.4.15}$$

Now let us turn our interest to the tunnelling rates between vacua. In the semi-classical theory of vacuum decay, including gravity, these rates are given by [68], [72], [73],

$$\frac{\Gamma}{V} \sim e^{-B/\hbar}, \tag{3.4.16}$$

where,

$$B = \delta S_{\mathrm{E}} \stackrel{\mathsf{def}}{=} S_{\mathrm{E}}|_{\text{bounce}} - S_{\mathrm{E}}|_{\text{initial vac}} \tag{3.4.17}$$

is the difference in the Euclidean actions for the bounce and the initial vacuum. Splitting B into parts originating from different terms in the action, we can write,

$$B = B_{\mathrm{GR}} - \sigma\delta c - \hat{\sigma}\delta\hat{c} \tag{3.4.18}$$

where $B_{\mathrm{GR}} = -2M_{\mathrm{Pl}}^2\Omega_3\Delta\left[\frac{1}{q^2}[\rho'^3]^0_{r_{\min}}\right] + \sigma_{\mathrm{w}}\Omega_3\rho_0^3$, represents the tunnelling exponent computed in GR for the same geometrical configuration and Ω_3 is the volume of the unit 3-sphere. The flux terms are of the form,

$$\begin{aligned}
\delta c &\stackrel{\mathsf{def}}{=} \int_{\text{bounce}} F_4 - \int_{\text{initial vac}} F_4 \\
&= -\frac{\mu^4}{\sigma'}\Omega_3\Delta\left[\int_{r_{\min}}^0 dr\rho^3\right] \\
&= -\frac{\mu^4}{\sigma'}\Omega_3\Delta\left[-\frac{1}{3q^4}[\rho'(3-\rho'^2)]^0_{r_{\min}}\right]
\end{aligned} \tag{3.4.19}$$

and,

$$\begin{aligned}
\delta\hat{c} &\stackrel{\mathsf{def}}{=} \int_{\text{bounce}} \hat{F}_4 - \int_{\text{initial vac}} \hat{F}_4 \\
&= \frac{24\Omega_3}{\hat{\sigma}'}\Delta\left[\rho'(r_{\min}) - \frac{1}{3}\rho'^3(r_{\min})\right].
\end{aligned} \tag{3.4.20}$$

Note that $\delta\hat{c}$ does not depend on quantities on the brane thanks to an exact cancellation that occurs due to the junction condition on $\hat{A}$. It is also worth highlighting that $r_{\min}$ is *a priori* different for the false vacuum and the bounce solution. Indeed, for the bounce, the radial coordinate $r \in [r^-_{\min}, r^+_{\max}]$, passing from the interior, with $r < 0$, to the exterior, $r > 0$. The precise values of $r_{\max}$ and $r_{\min}$ depend on the sign

	$S_+ - S_-$	$S_+ - H_-$	$H_+ - S_-$	$H_+ - H_-$
$\epsilon_\pm = 1$	$(qr_0)_+ \geq (qr_0)_-$	✓	✗	$\lvert q\rvert_+ \leq \lvert q\rvert_-$
$\epsilon_\pm = -1$	$(qr_0)_+ \leq (qr_0)_-$	✗	✗	✗
$\epsilon_+ = 1, \epsilon_- = -1$	$\overline{qr_0} \in [\pi/2, \pi]$	✗	✗	✗
$\epsilon_+ = -1, \epsilon_- = 1$	$\overline{qr_0} \in [0, \pi/2]$	✓	✗	✗

Table 3.4.1: Summary of allowed configurations. Those marked with a "✓" are allowed while those marked with a "✗" are not. Note that S denotes the sphere, H the hyperboloid. Planar limits can be extracted by taking $q_\pm \to 0$.

of the curvature and the orientation of the bubble [89]:

$$r_{\text{min}} = \begin{cases} -r_0\,, & \epsilon = +1 \\ r_0 - \frac{\pi}{q}\,, & \epsilon = -1,\ q^2 > 0 \\ -\infty\,, & \epsilon = -1,\ q^2 \leq 0 \end{cases} \qquad r_{\text{max}} = \begin{cases} \frac{\pi}{q} - r_0\,, & \epsilon = +1,\ q^2 > 0 \\ \infty\,, & \epsilon = +1,\ q^2 \leq 0 \\ r_0\,, & \epsilon = -1\,. \end{cases} \tag{3.4.21}$$

Similarly, for the initial vacuum, the radial coordinate spans a range $r \in [r^+_{\text{min}}, r^+_{\text{max}}]$, although there is no longer any notion of exterior versus interior.

The contribution from the Gauss-Bonnet term in (3.4.18) is notable by its absence. Because of its topological nature in four dimensions, the bulk Gauss-Bonnet contribution is a total derivative, and is projected into a pure boundary contribution, at r_{max} and r_{min}. These are then cancelled by the Myers boundary term (3.2.11).

In principle, the constraint on the wall tension (3.4.15) does not forbid configurations in which the unbounded part of a Minkowski or AdS space tunnels to a new vacuum. However, these cannot be considered bubble solutions and are inconsistent with a suitable boundary prescription. The complete list of allowed transitions are summarised in table 3.4.1. focusing now on the allowed configurations we note that they all have [89],

$$\rho'(r_{\text{min}}) = 1, \qquad -1 \leq \rho'(0^+) \leq \rho'(0^-) \tag{3.4.22}$$

and so,

$$B_{\rm GR} = 2\Omega_3 M_{\rm Pl}^2 \rho_0^2 \Delta\left[\frac{1}{1+\rho'(0)}\right] \geq 0 \tag{3.4.23}$$

$$-\sigma\delta c = \Omega_3 \frac{\mu^4\rho_0^4}{3}\frac{\sigma}{\sigma'}\Delta\left[\frac{1}{1+\rho'(0)} + \left(\frac{1}{1+\rho'(0)}\right)^2\right] \tag{3.4.24}$$

$$-\hat{\sigma}\delta\hat{c} = 0. \tag{3.4.25}$$

Bringing it all together, we find that the tunnelling rate is given by an exponent,

$$B = 2\Omega_3 M_{\rm Pl}^2\rho_0^2\left(1+\frac{\mu^4\rho_0^2}{6M_{\rm Pl}^2}\frac{\sigma}{\sigma'}\right)\Delta\left[\frac{1}{1+\rho'(0)}\right] + \Omega_3\frac{\mu^4\rho_0^4}{3}\frac{\sigma}{\sigma'}\Delta\left[\left(\frac{1}{1+\rho'(0)}\right)^2\right]. \tag{3.4.26}$$

This suggests that a sufficient condition to avoid infinitely enhanced tunnelling rates, and a catastrophic instability in the theory, is $\frac{\sigma}{\sigma'} > 0$.

We now consider two special cases as in [73]: tunnelling from de Sitter into Minkowski and tunnelling from Minkowski into Anti de Sitter. For tunnelling from de Sitter into Minkowksi ($q^2 \to 0$), we have that $\rho'(0^-) = 1$ and $\rho'(0^+) \in [-1, 1]$, and a tunnelling exponent,

$$B = B_{\rm GR}\left[1 + \frac{\mu^4}{12q^2M_{\rm Pl}^2}\frac{\sigma}{\sigma'}s(8-3s)\right] \tag{3.4.27}$$

where, as in [73], [89], $B_{\rm GR} = \Omega_3\frac{M_{\rm Pl}^2}{q^2}s^2$ and,

$$s = 1-\rho'(0^+) = \frac{\sigma_{\rm w}^2}{2M_{\rm Pl}^4q^2}\left(\frac{1}{1+\sigma_{\rm w}^2/4M_{\rm Pl}^4q^2}\right). \tag{3.4.28}$$

Given the constraint $\frac{\sigma}{\sigma'} > 0$ and the fact that in this case we have $s \in [0, 2]$, we see that the corrections due to OS always suppress this tunnelling event relative to GR.

Now consider the tunnelling from Minkowski into anti de Sitter ($0 \to -|q|^2$). Now we have $\rho'(0^+) = 1$ and $\rho'(0^-) \geq 1$, and a tunnelling exponent,

$$B = B_{\rm GR}\left[1 - \frac{\mu^4}{12|q|^2M_{\rm Pl}^2}\frac{\sigma}{\sigma'}s(8-3s)\right] \tag{3.4.29}$$

where, now, $B_{\rm GR} = \Omega_3\frac{M_{\rm Pl}^2}{|q|^2}s^2$ and,

$$s = 1-\rho'(0^-) = -\frac{\sigma_{\rm w}^2}{2M_{\rm Pl}^4|q|^2}\left(\frac{1}{1-\sigma_{\rm w}^2/4M_{\rm Pl}^4|q|^2}\right). \tag{3.4.30}$$

Transitions for which $|q|^2 < \sigma_{\rm w}^2/4M_{\rm Pl}^4$ are forbidden by energetic considerations [73]. In anti de Sitter the bubble cannot get big enough for the energy stored in the wall to balance the energy stored in the interior. Once again, given the constraint $\frac{\sigma}{\sigma'} > 0$ and the fact that in this case we have $s \leq 0$, we see that OS corrections always suppress this tunnelling event. To sum up, for a consistent theory of OS satisfying the constraint $\frac{\sigma}{\sigma'} > 0$, the allowed inhomogeneous tunnelling events coincide exactly with those in GR, but always occur at a slower rate.

Finally we consider the evolution of the bubble once it has materialised. To see what it does, we simply Wick rotate the bounce solution back to Lorentzian signature. The Lorentzian solutions in our case are geometrically identical to those described in considerable detail, including their global structure, in [89]. It is far too lengthy to repeat here and we refer the reader to [89] for further details. The only difference in the generalised case under consideration here is the mapping between the local curvature and the fluxes.

To find this relation, we note that the integrated versions of (3.4.6) and (3.4.7) are written as,

$$c = \int F_4 = \frac{\mu^4}{\sigma'} \int d^4x\sqrt{-g} = \frac{\mu^4}{\sigma'}(\Omega_+ + \Omega_-) \tag{3.4.31}$$

$$\hat{c} = \int \hat{F}_4 = -\frac{1}{\hat{\sigma}'} \int d^4x\sqrt{-g}R_{\rm GB} = -\frac{24}{\hat{\sigma}'}(q_+^4\Omega_+ + q_-^4\Omega_-) \tag{3.4.32}$$

where Ω_+ is the space-time volume corresponding to the initial vacuum and Ω_- to the new vacuum. In particular, Ω_+ includes the entire spatial volume at all times up until the nucleation of the bubble, and then the exterior spatial volume afterwards. Ω_- is simply the bubble interior.

Taking ratios of the two fluxes, we obtain,

$$\frac{\Lambda_{\rm flux}^2}{9M_{\rm Pl}^4} = \frac{q_+^4}{1+\mathcal{I}^{-1}} + \frac{q_-^4}{1+\mathcal{I}} \tag{3.4.33}$$

where $\mathcal{I} = \frac{\Omega_+}{\Omega_-}$ is ratio of the space-time volumes occupied by each particular vacuum, and we recall that $\Lambda_{\rm flux} = \sqrt{-\frac{3}{8}M_{\rm Pl}^4\mu^4\frac{\hat{\sigma}'}{\sigma'}\frac{\hat{c}}{c}}$. From equation (3.4.10), we also have that,

$$\Delta q^2 \stackrel{\rm def}{=} q_+^2 - q_-^2 = \frac{\Delta V}{3M_{\rm Pl}^2}. \tag{3.4.34}$$

It follows that,

$$q_{\pm}^2 = \frac{1}{6M_{\rm Pl}^2}\left[-\Delta V(\mathcal{R} \mp 1) + \alpha\sqrt{(\Delta V)^2(\mathcal{R}^2-1) + 4\Lambda_{\rm flux}^2}\right] \tag{3.4.35}$$

where $\mathcal{R} = \frac{\mathcal{I}-1}{\mathcal{I}+1}$. Owing to the quadratic nature of the global constraint, our solution comes in two families, parametrised by $\alpha = \pm 1$.

Now, if the old vacuum dominates the space-time volume, then $\mathcal{I} \gg 1$ and so $\mathcal{R} \approx 1$. It then follows that the local curvature in this region, q_+^2, is largely insensitive to the jump in vacuum energy, being given entirely by $\Lambda_{\rm flux}$. In contrast, q_-^2 is highly sensitive to ΔV. The reverse is true when the new vacuum dominates the space-time volume. Then we have $\mathcal{I} \ll 1$ and so $\mathcal{R} \approx -1$: q_+^2 becomes highly sensitive to ΔV, while q_-^2 is given by $\Lambda_{\rm flux}$.

The computation of the space-time volumes, which ultimately control which regions sequester vacuum energy most efficiently, is a highly non-trivial exercise. The volumes are formally divergent to the infinite past and the infinite future. However the divergence rates can be correlated using the covariant junction conditions. Full details are presented in the appendix of [89], and the results can be carried over to the present case. We do so, however, with an additional word of caution. These ratios were computed using a global time regulator. Other regulators exist and could yield potentially different results due to the so-called measure problem, familiar from eternal inflation [107]. The global time regulator was chosen in [89] because global coordinates cover the entire space-time. We have nothing more to say on this difficult question. Let us simply quote the stated ratios and explore their consequences for the case under consideration here.

For a transition from X to Y, where X, Y are dS (de Sitter), M (Minkowski) or AdS (anti de Sitter), we label the corresponding volume ratio as $\mathcal{I}_{X\to Y}$. From [89], we then have,

$$\mathcal{I}_{dS\to dS} \sim \frac{q_-}{q_+} \tag{3.4.36}$$
$$\mathcal{I}_{dS\to M} = 0 \tag{3.4.37}$$
$$\mathcal{I}_{dS\to AdS} = \infty \tag{3.4.38}$$
$$\mathcal{I}_{M\to AdS} = \infty \tag{3.4.39}$$
$$\mathcal{I}_{AdS\to AdS} = \infty\,. \tag{3.4.40}$$

The consequences of these ratios turn out to be the same as in [89], so we summarise those results. For phenomenologically interesting de Sitter to de Sitter transitions, we can have transitions in either direction. Transitions that lower the curvature ($q_- \ll q_+$) are far more probable and for these we have $\mathcal{I} \ll 1$, ensuring insensitivity to ΔV in the low curvature new vacuum. For the suppressed transitions that raise the curvature ($q_- \gg q_+$), we have $\mathcal{I} \gg 1$, again ensuring insensitivity to ΔV in the low curvature vacuum, although this time it is the old vacuum. More generally, the following behaviour prevails: for a given transition, insensitivity to ΔV is achieved in the vacuum with lowest absolute curvature. The one exception to this rule is transitions from large curvature de Sitter to small curvature anti de Sitter vacua.

This generic behaviour is important. It suggests that vacua with low absolute curvature do not require fine-tuning to achieve their low curvature: the sequestering mechanism will always take care of the required cancellations. We now see how this is common to all sequestering models.

3.5 Discussion

In this chapter, we have explored the cosmological framework of *Omnia Sequestra*, the generalised theory of vacuum energy sequestering with the capacity to enforce cancellation of all radiative corrections to vacuum energy, including both matter and graviton loops [95].

As in older models of sequestering, the cosmological behaviour relies on certain historic integrals, although their structure is different in subtle but important ways. As usual, the historic integrals feed into the residual cosmological constant that we observe through the large scale curvature. In OS, we find that there are potentially dangerous divergences coming from the singular region of space-time. These represent a potential UV instability that could render the observed cosmological constant power law dependent on the UV cut-off of the theory. Such a scenario would mean a violation of naturalness and the theory would do no better than General Relativity. However, it turns out that this behaviour can be tamed in a sufficiently large and old Universe, and eliminated altogether in a Universe that continues for eternity. For a Planckian cut-off, 92 more efolds in expansion will be sufficient. We also find that the scale of residual cosmological constant can be assumed to be bounded above by the scale of the critical density today. This relies on two things: that the Universe grows old enough to tame any cut-off dependence in the historic integrals, and that the flux contribution is not too large.

We also studied the effect of phase transitions through these historic integrals. For homogeneous transitions, we once again encountered potential naturalness problems that mirrored the UV sensitivity problem described in the previous paragraph. More precisely, we find that the residual cosmological constant at late times can become sensitive to jumps in vacuum energy from transitions at early times. Again, these contributions can be tamed as long as the Universe gets sufficiently old and eliminated altogether in an eternal universe. In particular, in a crude historical model, the effect of the QCD phase transition at high redshift would require the Universe to continue for at least 23 more efolds. Again, with this proviso, we found that the late time behaviour became insensitive to the scale of the phase transition.

The role of the 3-form fluxes was also investigated. This is boundary data, assumed to be UV insensitive and taking on values that should be set empirically within the effective field theory. Nevertheless, there are geometric consequences of certain choices. In particular, we showed that for a vanishing flux ratio, the spatial geometry is forced to be that of a hyperboloid.

The formalism for OS was reviewed in some detail in section 3.2, and built upon to include the effect of space-time boundaries. Owing to the non-trivial global dynamics in sequestering models, this extension is non-trivial but was important to allow for a study of inhomogeneous transitions, through the nucleation of a spherical bubble and the bounce computation originally developed for GR by Coleman and De Luccia [73]. Indeed, via a calculation of the bounce, we were able to show that the allowed transitions coincided with those from GR. An important new ingredient, however, was the mapping from the source potential to the local curvature. The local curvature became insensitive to the scale of the transition in the region of space-time that dominated the volume. As in [89], the consequence of this is that generically those vacua with low absolute curvature are the least sensitive to the scale of the transition. This may seem obvious, but it is not. One could have a scenario in which the low curvature is highly sensitive to the transition scale and one has to fine-tune. Indeed, there is one particular scenario where precisely this happens, although it is not generic.

The meaning of tunnelling probabilities in sequestering models may seem unclear at first glance, since the local value of the cosmological constant seems to have knowledge of whether or not tunnelling will occur. Indeed, for a space-time without any bubbles of true vacuum, there is complete cancellation of vacuum energy, whereas if a bubble exists to the future the cancellation is inexact, depending on the ratio of space-time volumes as explained above. However, there is no tension with the probabilistic interpretation of quantum tunnelling. On the one hand, the tunnelling rate per unit volume per unit time is faithfully captured by the bounce, corresponding to a saddle point of the *Euclidean* action. The various space-time configurations that may occur with and without bubble nucleation are all stationary points of the *Lorentzian* action. This is exactly as in General Relativity, the only difference being that the sequestering solutions are also required to satisfy an additional global constraint. Furthermore, as a local observer, we have no way of knowing if the residual cosmological constant we measure contains contributions from inexact cancellations due to future bubble nucleation, or some future fluctuation in the local energy-momentum and its resulting contribution to the space-time average.

Although our analysis has been thorough, some specific questions remain. In particular, we noted that the quadratic nature of Gauss-Bonnet ultimately means that there are multiple roots for the residual cosmological constant. This deserves further investigation: does it lead to problems with well-posedness and branching; is there a physical mechanism for selecting one branch over the other? We have also been unable to attach any extra physical significance to the generalised boundary conditions (3.2.18) we proposed for a well defined variational principle. Establishing this may yield a deeper understanding of the model and how it can be embedded in a more complete theory.

The presiding message is that all sequestering models exhibit similar cosmological behaviour. The phenomenology is consistent with observation, without fine-tuning, and seems to favour Universes that grow old and big. To a large extent, sequestering is best interpreted as a mechanism for cancellation of vacuum energy, rather than a specific model. With this perspective the future focus should really be to better understand how and why it does what it does, at a much deeper level. This depth of understanding should help facilitate the search for the mechanism at a fundamental level, probably as an emergent low energy effect in a UV complete theory. Leading on from this, we spend the remainder of this thesis on the subject of UV completion, in particular with regards to modified gravity theories that are candidates to exhibit Vainshtein screening at low energies.

Chapter 4

Massive Galileons & Vainshtein Screening

This chapter is based upon the candidate UV completion of a massive galileon theory as presented in [108]. We will discuss a generalisation of the theory and explain how it avoids the positivity bounds that would disrupt its UV completion. We proceed to demonstrate its candidacy for Vainshtein screening, and how it fails to retain its screening properties under the process of UV completion, due to the presence of additional EFT operators. Motivated by this analysis, we will then consider similar mass deformations in the context of Wess-Zumino galileons, and ask whether similar EFT operators disrupt screening in this case.

Often in modified gravity model-building, dark energy and the acceleration of the universe can be identified with the dynamics of an ultra-light scalar field which couples to ordinary matter with gravitational strength [38], [39], [109], [110]. If the scalar continued to operate in this way at shorter distances - within the scale of the solar system - it would mediate a fifth fundamental force that so far has not been detected [2], [111]. As introduced in section 1.3, viable models must therefore be able to screen, i.e. suppress, the extra force in environments where it is known to be small. Only a handful of screening mechanisms are known (see eg [45], [112]–[120]), one of which is Vainshtein screening (for a review see [41]). Here, a derivative interaction term dominates close to a matter source, causing a breakdown of the linear theory and suppressing the gradient of the scalar field, thus screening the fifth force within a typically large Vainshtein radius. Vainshtein screening is seen in non-linear massive gravity [121] and Galileon-type models [122]. Theories displaying Vainshtein screening necessarily run into strong coupling at macroscopic scales in order for the derivative interactions to kick in at sufficiently large distances from the source [123]. For this reason, these theories can only be properly understood as effective theories with a limited range of validity. Since the breakdown occurs on macroscopic scales it is important to ask what happens beyond that scale and what impact it has on Vainshtein screening. This question has been studied before [48], [124], where it was argued that a generic ultra-violet (UV) completion of a theory with derivative interactions could introduce further interaction terms that have the potential to destroy the Vainshtein mechanism. However, the difficulty in addressing this question directly has been the absence of a known UV completion of a theory that exhibits Vainshtein screening (indeed, in the case of Galileons [122], it has been argued that a standard Wilsonian UV completion does not exist [50]).

In this chapter, we examine the potential of Vainshtein screening to survive UV completion directly. This has been made possible with the advent of an interacting massive Galileon (IMG) and its UV completion presented in [108]. Motivated by that set-up, we examine a generalised set of IMG theories together with their possible UV completions. We show that Vainshtein screening does occur for each type of interaction provided the Galileon is massive. Armed with an extended description at high energies, we are able to see if screening survives the inclusion of UV corrections. The answer is a resounding no. Through these explicit examples, it becomes clear that a low energy approximation to any UV theory is not automatically trustworthy when pushed into a non-perturbative regime. Such conclusions should not come as a surprise given our understanding of effective field theories in particle physics. Nevertheless, the conclusion is significant in the context of Vainshtein screening, where at least some higher order operators are required to become large by construction. These results cast further doubt on the theoretical viability of Vainshtein screening, even before observational constraints are attempted.

Our approach combines analytic estimates with a careful numerical analysis. A flavour of the numerical results are presented in Fig. 4.0.1 where we plot the ratio of the fifth force to the standard Newtonian force in the vicinity of a spherically symmetric compact source. The dotted lines reveal what happens for a family of

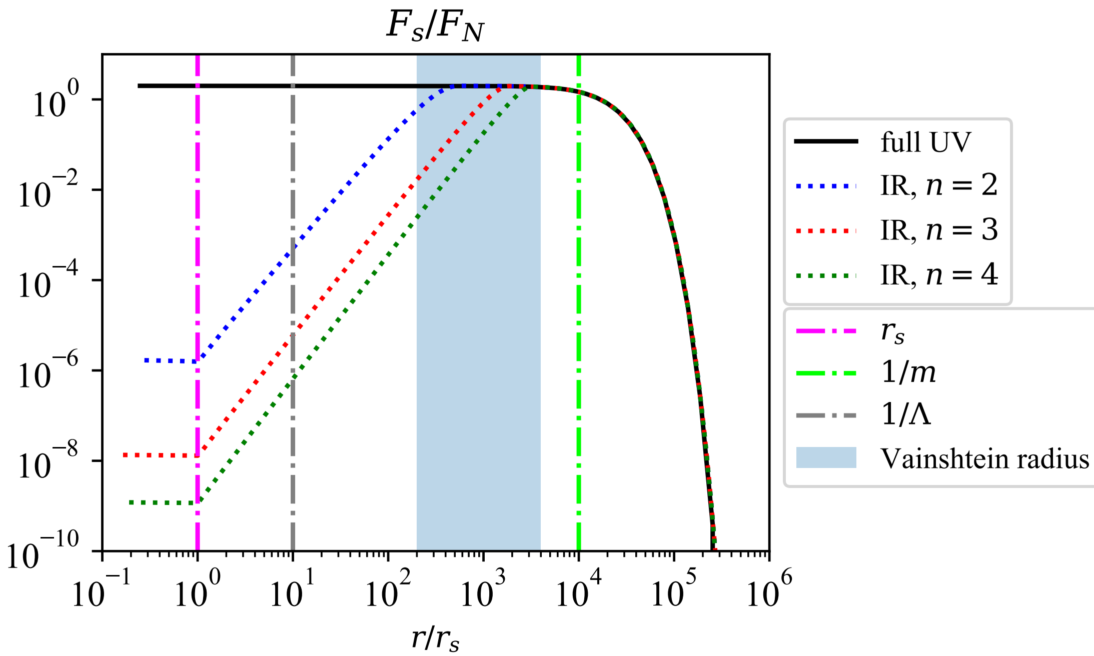

Figure 4.0.1: Ratio of the scalar fifth force to Newtonian force around a compact object, for the IMG theories described in the text (Eq. 4.1.8) (dotted lines) and a possible UV completion for the $n = 3$ case (Eq. 4.2.2) (solid black line). The shaded region indicates the distance scales where the behaviour of the force changes for the different theories. When pushed towards strong coupling, the IMGs show marked suppression of the fifth force around the compact object. However, because this occurs at strong coupling, one really ought to work with a description that extends the theory further into the UV. For the UV complete example shown here (solid black line), we see that there is no longer any suppression of the fifth force. Figure produced by Daniela Saadeh.

massive Galileon theories with particular derivative interactions. In each case, the fifth force is suppressed close to the source as the derivative interaction begins to dominate. The black solid line is the prediction for a UV completion of one of these scenarios. It is easy to see that suppression of the fifth force no longer occurs: the Vainshtein mechanism is completely destroyed by the UV corrections to the theory.

The rest of this chapter is organised as follows: in the next section, we will identify a family of higher order Galileon invariant operators that have the potential to Vainshtein screen, but *only* when the Galileon is massive. We give simple analytic arguments to indicate that screening will take place which are then reinforced by our numerical analysis. In section 4.2 we raise the cut-off of our effective description by integrating in a heavy field. This is done for each family of interactions considered in section 4.1. We present a generic analytic argument for why we expect screening to be spoiled in these UV extended theories. For the special case already identified in [108], the theory in question is UV complete in the limit $M_{\rm Pl} \to \infty$. In section 4.3, we perform a numerical analysis on this UV complete theory and see that screening is destroyed. This allows us to scrutinise the integrating process in detail, and re-examine operators one would usually neglect due to heavy mass suppression. It turns out that a tower of higher order operators can no longer be neglected within a certain macroscopic distance from the source. This is entirely consistent with the generic arguments presented in [48], [124] and reinforces the idea that Vainshtein screening cannot be taken seriously without a much better understanding of the UV effects in any particular model. Our numerical methods and results are presented in section 4.3, with additional details found in a methods paper that accompanies this analytical research [125]. We conclude in section 4.5. In the appendix, we consider adding a mass deformation to so-called Wess-Zumino Galileon theories, and ask whether the screening properties remain intact.

4.1 Interacting Massive Galileons and Vainshtein screening

Galileon theories [122] have been seen to emerge in a variety of interesting cosmological scenarios, from DGP gravity [126] to non-linearly realised massive gravity [127]. Although they contain higher order derivative interactions, the field equations remain at second order, thereby avoiding the Ostrogradski instability [128]. The defining characteristic of a Galileon theory is one that is invariant under a Galilean transformation $\pi \to \pi + b_\mu x^\mu + c$ in flat space, where b_μ and c are constant and we sum repeated indices following the Einstein convention. For the theories with second order field equations defined in [122], the interaction operators shift by a total derivative under the Galileon transformation and for this reason they are sometimes referred to as Wess-Zumino interactions [129]. Of course, we can also include interactions that are *manifestly* invariant under the Galileon transformation, such as $(\partial\partial\pi)^n$, where there are two (or more) derivatives acting on each insertion of the scalar. These are expected to arise anyway as effective field theory (EFT) corrections to the leading order interactions.

The Wess-Zumino interactions are known to facilitate Vainshtein screening [122]. Although, as emphasised earlier, this goes hand in hand with strong coupling and concerns about the validity of our effective description when screening is active [48], [124]. One way to avoid this concern would be to find a UV theory which could reproduce the Vainshtein mechanism at low energies, but generalises to higher energies. Unfortunately, positivity constraints suggest that a standard Wilsonian UV completion of the theory cannot exist [50], [130]. To avoid these bounds, it is necessary to deform the Galileon theory in the infra-red (IR), in order to change the form of the low energy scattering amplitudes. One of the simplest deformations is the inclusion of a mass term. This massive Wess-Zumino galileon theory would take the form,

$$\begin{aligned} \mathcal{L}_{\text{mass gals}} &= \mathcal{L}_{\text{WZ}} - \frac{1}{2}m^2\pi^2 && (4.1.1)\\ &= -\frac{1}{2}(\partial\pi)^2 + \frac{g_3}{\Lambda^3}\mathcal{L}_3 + \frac{g_4}{\Lambda^6}\mathcal{L}_4 + \frac{g_5}{\Lambda^9}\mathcal{L}_5 - \frac{1}{2}m^2\pi^2 && (4.1.2)\end{aligned}$$

where the g_i are dimensionless coupling coefficients, and $\mathcal{L}_i$ are known as the cubic, quartic and quintic Galileons [122], given respectively by,

$$\begin{aligned} \mathcal{L}_3 &= -\frac{1}{2}\partial^2\pi\,(\partial\pi)^2 && (4.1.3)\\ \mathcal{L}_4 &= -\frac{1}{2}\left((\partial^2\pi)^2 - (\partial\partial\pi)^2\right)(\partial\pi)^2 && (4.1.4)\\ \mathcal{L}_5 &= -\frac{1}{2}\left((\partial^2\pi)^3 - 3(\partial^2\pi)(\partial\partial\pi)^2 + 2(\partial\partial\pi)^3\right)(\partial\pi)^2. && (4.1.5)\end{aligned}$$

The tree level $2 \to 2$ scattering amplitude for this theory is then given by,

$$\begin{gathered} \mathcal{A}(s,t) = \mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u + \mathcal{A}_4 \\ \mathcal{A}_X = \frac{9g_3^2X^2(X+4m^2)^2}{4\Lambda^6(m^2+X)}, \quad \mathcal{A}_4 = -6g_4\frac{stu}{\Lambda^6} \end{gathered} \tag{4.1.6}$$

where $X \in \{s,t,u\}$, and $\mathcal{A}_4$ is the amplitude contribution from the 4-point interaction. The 5-point interaction does not contribute to $\mathcal{A}(s,t)$ at tree level. We can see that in the massless limit the quadratic terms vanish, and we necessarily violate the positivity bounds requiring the coefficient of s^2 to be strictly positive [50].

It should be noted that although a mass term breaks the Galileon symmetry, the action continues to respect the Galileon non-renormalisation theorem at low scales [131]. Additionally, loop corrections generated by the addition of a mass term do not violate the Galileon symmetry at any order. We look to be in good shape to have a well-behaved theory that has some hope of being UV completed.

An example of a UV-complete massive Galileon theory was given in [108], where, through the introduction of a single heavy field H, Galileon invariant interactions for the light Galileon field π can be obtained, with the exception of the mass term. Integrating out the heavy field to leading order[1] yields a single field Galileon theory in π at low energies, respecting the same symmetry. Generalising the self interaction term for the heavy field to any integer power $n+1$, the IR theory then contains terms of the form $(\ \ \pi)^n$. Further details on how this is done can be found in section 4.2. The non-linear nature of the derivative interactions opens up the possibility that screening will occur.

With this in mind, we consider the following action assumed to be valid at low energies[2],

$$S[\pi] = \int d^4x \left(-\frac{1}{2} (\partial\pi)^2 - \frac{1}{2} m^2 \pi^2 + \frac{\epsilon}{n+1} \frac{(\ \ \pi)^{n+1}}{\Lambda^{3n-1}} + \frac{\pi T}{M_{\mathrm{Pl}}} \right) \tag{4.1.7}$$

where π is a scalar field with mass m, the integer $n \in \{2, 3, 4 \ldots\}$ and $\epsilon = \pm 1$. One can see that this action is invariant under the Galileon transformation of $\pi \to \pi + b_\mu x^\mu + c$, with the exception of the mass term, verifying that it is indeed a theory of a massive Galileon[3]. As usual, the Galileon is coupled to external sources with gravitational strength through the trace of the energy-momentum tensor T. The

[1] We do this by substituting the equation of motion for H back into the action. We explicitly computed the first-order loop corrections to these results and they do not alter the form of the would-be screening operator.

[2] Note that we employ a different definition of the parameters ϵ and Λ compared to those of the corresponding method paper that accompanies this analytical research [125].

[3] For clarity and brevity, we define a massive Galileon in flat space as a theory for which $\delta\mathcal{L} \propto m^2$ under the Galileon transformation, up to total derivatives.

theory becomes strongly coupled at some scale $\Lambda \ll M_{\rm Pl}$, reflecting its status as an effective theory only valid at large distances. We now ask the following: does this theory exhibit Vainshtein screening close to the source and, if so, how close to the source can we go and still trust its predictions? The latter requires knowledge of the UV completion to be discussed in the next section.

We proceed by varying the action to obtain the equation of motion,

$$\pi - m^2\pi + \epsilon\mathcal{O}_n = \frac{M_{\rm S}}{M_{\rm Pl}}\delta(\mathbf{x}) \tag{4.1.8}$$

where $\mathcal{O}_n \equiv \frac{(\ \pi)^n}{\Lambda^{3n-1}}$, and we have chosen a pressureless delta function source of mass $M_{\rm S}$ with support at $\mathbf{x} = \mathbf{0}$. We shall now look for static, spherically symmetric configurations.

Firstly, considering Eq. (4.1.8) far from the source, we are in the so-called linear regime, and the solution has the form $\pi_{\rm lin} \sim \frac{M_{\rm S}}{M_{\rm Pl}}\frac{e^{-mr}}{r}$. In order to determine at what radius we might expect a breakdown of the linearised theory, and therefore identify a candidate Vainshtein radius, we evaluate $\mathcal{O}_n$ on the linearised solution, and compare it to the other terms in the equation of motion. We find that $\mathcal{O}_n|_{\pi_{\rm lin}} \sim \Lambda^{1-3n}m^{2n} \ \ \pi^n_{\rm lin}$. Assuming that we are well inside the Compton wavelength of π, we can take the approximation $r \ll m^{-1}$, which then simplifies the expression to $\mathcal{O}_n|_{\pi_{\rm lin}} \sim \Lambda^{1-3n}m^{2n}\left(\frac{M_{\rm S}}{M_{\rm Pl}}\right)^n r^{-(n+2)}$.

Comparing with the mass term, the ratio $\mathcal{O}_n|_{\pi_{\rm lin}}/m^2\pi_{\rm lin}$ is given by $\left(\frac{r_v^{(n)}}{r}\right)^{n+1}$, where $r_v^{(n)} \sim (\sigma_{\rm S}\kappa^2)^{\frac{n-1}{n+1}}\Lambda^{-1}$, with $\sigma_{\rm S} \equiv \frac{M_{\rm S}}{M_{\rm Pl}}$ and $\kappa \equiv \frac{m}{\Lambda}$. We see that so long as $\sigma_{\rm S}\kappa^2 \gg 1$, then the linearised theory breaks down at some macroscopic scale $r_v^{(n)} \gg \Lambda^{-1}$. It is worth recognising that, without a mass term, $\kappa = 0$ and there is no screening.

Although we have identified a potential breakdown of the linear theory, we still have not confirmed the existence of screening; we must examine the non-linear regime and determine whether the solution supports a screening mechanism. To this end, we neglect the kinetic and mass terms in (4.1.8), and integrate the equation to obtain $\frac{(\ \pi)^n}{\Lambda^{3n-1}} \sim \frac{M_{\rm S}}{M_{\rm Pl}}\frac{1}{r} + c$ where c is a constant. If the constant is negligible, we integrate to obtain a solution of the form $\pi \sim (\sigma_{\rm S}\Lambda^{3n-1})^{\frac{1}{n}}r^{2-\frac{1}{n}}$. However, if the constant instead dominates, the solution is of the form $\pi \sim (c\Lambda^{3n-1})^{\frac{1}{n}} r^2 + d$. We see that in both cases the scalar force is suppressed at small radii, consistent with screening.

To complete our analysis, we need to show that the two asymptotic solutions, at large and small radii, can be consistently matched onto one another. We have not been able to show this analytically, but our numerical solutions indicate that the two solutions can indeed be matched (see Fig. 4.0.1). This suggests that the family of interacting massive Galileon theories given by equation (4.1.7) will exhibit Vainshtein screening around a heavy source. However, given the importance of the derivative interaction in suppressing the force close to the source, it remains to ask whether or not we really trust this prediction. UV corrections are expected in order to preserve perturbative unitarity and raise the cut-off of the effective theory. What effect do these corrections have on the predictions of the theory close to the source?

4.2 Raising the cut-off eliminates screening

Consider the action,

$$\begin{aligned} S[\pi, H] = \int d^4x \Bigg(-\frac{1}{2}(\partial\pi)^2 - \frac{1}{2}(\partial H)^2 - \frac{1}{2}m^2\pi^2 \\ -\frac{1}{2}M^2H^2 - \alpha H \quad \pi - \frac{\lambda H^{n+1}}{(n+1)!\mu^{n-3}} + \frac{\pi T}{M_{\rm Pl}} \Bigg) \end{aligned} \tag{4.2.1}$$

generalised from [108], where π is the Galileon field, with light mass m, H is some heavy field of mass $M \gg m$, and T is the trace of the energy-momentum tensor of the source, coupling only to the Galileon field. The coupling coefficients λ and α are dimensionless and of order one, although we must impose $\lambda \geq 0$ and $|\alpha| < 1$ to avoid instabilities. The high energy scale μ represents the new cut-off of the theory when $n \geq 4$. For $n \in \{2, 3\}$, the theory is well-defined all the way up to the Planck scale. One can see that in each case the action will transform in the correct way in order to be considered a massive Galileon theory. Variation yields the following field equations,

$$\begin{cases} \quad \pi - m^2\pi - \alpha \quad H = -\frac{T}{M_{\rm Pl}} \\ \quad H - M^2 H - \alpha \quad \pi - \frac{\lambda H^n}{n!\mu^{n-3}} = 0\,. \end{cases} \tag{4.2.2}$$

We assume as boundary conditions that the fields are everywhere regular and asymptoting to the vacuum expectation value.

Once again, we wish to consider $2 \to 2$ tree level scattering to examine the positivity bounds. Using $m \ll M$, we show the example calculation for $n = 3$ from [108], which gives an amplitude of the form,

$$\begin{aligned} \mathcal{A}(s,t) &= \mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u + \mathcal{A}_4 \\ \mathcal{A}_X &= -\frac{\lambda^2\alpha^4}{32\pi^2(1-\alpha^2)^2}\left(\frac{m}{M}\right)^8 \int_0^1 \mathrm{d}x \log\left(\frac{(1-\alpha)^{-1}M + Xx(1-x)}{\mu^2}\right) \\ \mathcal{A}_4 &= -\lambda\alpha^4\left(\frac{m}{M}\right)^8 \end{aligned} \tag{4.2.3}$$

with corresponding positivity bound (obtained in the regime $-4m^2 \leq X \leq 0$),

$$\frac{\lambda^2\alpha^4}{32\pi^2(1+\alpha)^2M^4}\left(\frac{m}{M}\right)^8\left(\frac{1}{15} + \frac{(1-\alpha^2)(4m^2+t)}{70M^2} + O\left(\frac{m^4}{M^4}\right)\right) > 0. \tag{4.2.4}$$

It is clear that no matter what value t takes between $-4m^2$ and 0 we are always able to satisfy the bound.

Proceeding with our attempted connection between IR and UV, we can examine (4.2.1) in the region far outside the Compton wavelength of H, and under the assumption $\quad \ll M^2$ we obtain,

$$H \sim -\frac{\alpha}{M^2} \quad \pi - \frac{\lambda\alpha^n(-1)^n}{n!M^{2(n+1)}}\frac{(\quad\pi)^n}{\mu^{n-3}} + O(\lambda^2)\ . \tag{4.2.5}$$

It should be noted that we have discarded terms of the form $\left(\frac{\quad}{M^2}\right)^j \pi$ in order to write down this expression. While these are legitimate terms under all of our perturbation expansions, they are subdominant in both the linear and non-linear regimes, and only become important at the Compton wavelength of H, at which point one would have to work with the full UV theory anyway.

Using this result, we can write a low energy action as,

$$\widetilde{S} = \int d^4x\left(-\frac{1}{2}(\partial\pi)^2 - \frac{1}{2}m^2\pi^2 + \frac{(-1)^n\lambda\alpha^{n+1}(\quad\pi)^{n+1}}{(n+1)!M^{2(n+1)}\mu^{n-3}} + \frac{\pi T}{M_{\mathrm{Pl}}}\right) \tag{4.2.6}$$

where we have discarded terms of order λ^2 or higher. We then identify this with our IR theory, and see that we must have $M^{2(n+1)}\mu^{n-3} \sim \Lambda^{3n-1}$ and, for n odd, $\epsilon = -1$. If we want the UV theory to be able to describe physics at higher energies reliably, we require it to have a larger strong coupling scale than the corresponding IR theory. For $n \in \{2, 3\}$, the UV theory is renormalisable in the absence of external sources, but for $n \geq 4$ we must restrict ourselves to $\Lambda < \mu$. Writing $\mu = N\Lambda$ for $N > 1$, we see that $M = N^{\frac{3-n}{2(n+1)}}\Lambda$, i.e. the heavy field must be lighter than the strong coupling scale, in keeping with our intuition from Wilsonian UV completions.

We now give analytic arguments to explain why we expect screening to be absent in this extended theory, focusing on the UV complete case with $n = 3$. We start by rewriting the equations of motion as follows:

$$\begin{cases} (\pi - \alpha H) - m^2\pi = \frac{\rho}{M_{\rm Pl}} \\ (H - \alpha\pi) - V'(H) = 0 \end{cases} \tag{4.2.7}$$

where $V'(H) = M^2 H + \frac{\lambda}{3!}H^3$, and for simplicity the source ρ is taken to be a top-hat function of radius $r_{\rm s}$, i.e. $\rho(r) = \bar{\rho}\,\Theta(r_{\rm s} - r)$, so that we may explore the field profiles both inside and outside the source. The main focus here will be on the solution for the Galileon field, π, since this is the one probed directly by matter.

We start by assuming that $\beta \equiv \quad H/V'(H)$ varies slowly. This is consistent with the numerical simulations everywhere away from the source-vacuum transition. In principle the constant value of β could differ from inside to outside the source. The second equation in Eq. (4.2.7) now yields $\quad H = \frac{\alpha}{1-\beta^{-1}} \quad \pi$ and substituting this into the first equation gives,

$$(Z \quad - m^2)\pi = \frac{\rho}{M_{\rm Pl}} \tag{4.2.8}$$

where $Z \equiv 1 - \frac{\alpha^2}{1-\beta^{-1}}$ is assumed to be positive. It is convenient to define effective mass scales $\bar{m}_{\rm in} = m/\sqrt{Z_{\rm in}}$ and $\bar{m}_{\rm out} = m/\sqrt{Z_{\rm out}}$ so that this equation has the regular solution:

$$\pi_{\rm in}(r) = -\frac{\bar{\rho}}{M_{\rm Pl}m^2}\left[1 - \frac{(1 + x_{\rm out})\sinh(\bar{m}_{\rm in}r)}{x_{\rm out}\sinh x_{\rm in} + x_{\rm in}\cosh x_{\rm in}}\frac{r_s}{r}\right] \tag{4.2.9}$$

$$\pi_{\rm out}(r) = -\frac{\bar{\rho}}{M_{\rm Pl}m^2}\left[\frac{e^{x_{\rm out}}(x_{\rm in}\cosh x_{\rm in} - \sinh x_{\rm in})}{x_{\rm out}\sinh x_{\rm in} + x_{\rm in}\cosh x_{\rm in}}\right]\frac{r_s}{r}e^{-\bar{m}_{\rm out}r} \tag{4.2.10}$$

where we define $x_{\rm in} \equiv x/\sqrt{Z_{\rm in}}$ and $x_{\rm out} \equiv x/\sqrt{Z_{\rm out}}$ for $x \equiv mr_s$. Note that the solutions match at the source-vacuum transition, along with their first derivatives.

We will also assume that the source lies deep within the Compton wavelength of the Galileon, so in other words, $x \ll 1$. To examine screening, we compare the exterior solution $\pi_{\rm out}$ with a typical Newtonian potential, $V_N = -\frac{\bar{\rho} r_{\rm s}^3}{6{M_{\rm Pl}}^2 r}$. The ratio,

$$\frac{\pi_{\rm out}/M_{\rm Pl}}{V_N} = \frac{6}{x^2}\left[\frac{e^{x_{\rm out}}(x_{\rm in}\cosh x_{\rm in} - \sinh x_{\rm in})}{x_{\rm out}\sinh x_{\rm in} + x_{\rm in}\cosh x_{\rm in}}\right] e^{-\bar{m}_{\rm out} r} \tag{4.2.11}$$

is suppressed in two cases. The first corresponds to Yukawa suppression in the exterior, with $Z_{\rm out} \ll 1$. Alternatively, if $Z_{\rm out} \gtrsim 1$, suppression can also occur if the scalar decouples in the interior, with $Z_{\rm in} \gg 1$. We shall now demonstrate that these scenarios are incompatible with the required profile for H and so screening is not possible, at least up to the caveat of our approximations.

Recall that $H = \frac{\alpha}{1-\beta^{-1}} \quad \pi = \frac{(1-Z)}{\alpha} \quad \pi$ and so $H = \frac{(1-Z)}{\alpha}\pi + \hat{H}$ where $\hat{H} = 0$. Assuming regularity and continuity of H and its first derivative at the transition, we obtain,

$$H_{\rm in} = \frac{(1 - Z_{\rm in})}{\alpha}\pi_{\rm in} + \frac{(Z_{\rm out} - Z_{\rm in})}{\alpha} x_{\rm out}\pi_{\rm s} \tag{4.2.12}$$

$$H_{\rm out} = \frac{(1 - Z_{\rm out})}{\alpha}\pi_{\rm out} + \frac{(Z_{\rm out} - Z_{\rm in})}{\alpha}(x_{\rm out} + 1)\pi_{\rm s}\frac{r_{\rm s}}{r} \tag{4.2.13}$$

where $\pi_{\rm s} \equiv -\frac{\bar{\rho}}{M_{\rm Pl} m^2}\left[\frac{x_{\rm in}\cosh x_{\rm in} - \sinh x_{\rm in}}{x_{\rm out}\sinh x_{\rm in} + x_{\rm in}\cosh x_{\rm in}}\right]$ is the value of the Galileon at the transition.

For the case of Yukawa suppression for the exterior Galileon, we have $Z_{\rm out} \ll 1$. The Yukawa suppression allows us to neglect $\pi_{\rm out}$ in $H_{\rm out}$. This means that $H_{\rm out}$ scales like a massless field in most of the exterior, and given our definition $\beta \equiv \quad H/V'(H)$, we infer $\beta_{\rm out} \ll 1$. The problem now is that this gives $Z_{\rm out} \approx 1$ in contradiction with the condition for Yukawa suppression.

For the case of suppression through decoupling of the interior Galileon, we have $Z_{\rm in} \gg 1$. It follows that $\pi_{\rm in} \approx -\frac{\bar{\rho}}{6M_{\rm Pl}Z_{\rm in}}\left(\frac{3+x_{\rm out}}{1+x_{\rm out}}{r_{\rm s}}^2 - r^2\right)$ and so $H_{\rm in} \approx -\frac{\bar{\rho}}{M_{\rm Pl}\alpha}$. However, for $Z_{\rm in} \gg 1$ we require $\beta_{\rm in} \approx 1$, and so we now expect $V'(H_{\rm in}) \approx -\frac{\bar{\rho}}{M_{\rm Pl}\alpha}$. This suggests $H_{\rm in} \approx$ constant, in obvious contradiction with $H_{\rm in} \approx -\frac{\bar{\rho}}{M_{\rm Pl}\alpha}$, except in the trivial limit where $\bar{\rho} \to 0$.

In summary then, our heuristic analysis seems to suggest that screening of the Galileon will not be possible when the backreaction of the heavy field is taken into account. Of course, the assumption of constant $H/V'(H)$ was a little crude and the numerics show that this does not hold particularly well near the vacuum-source transition, casting some doubts on our right to apply continuity conditions at this point. For these reasons we do not present our analytics as the main evidence that ultra-violet effects will spoil the Vainshtein effects. We leave that to the numerics.

4.3 Numerical methods and results

Determining the screening property of the UV theory analytically is challenging if one is to avoid some crude assumptions. Likewise, for the IR theory, there is no a-priori guarantee that it is possible to match between the high- and low-density regimes consistently. We therefore address the problem numerically, to obtain the solution to the full equations of motion Eq. (4.1.8), for $n = 2, 3, 4$ and Eq. (4.2.2) for $n = 3$ across all regimes. For this task, we have developed the numerical code φ`enics`[4] [125], building on the `FEniCS` library[132]–[134]. φ`enics` applies the finite element method to the solution of boundary-value problems relevant for screening, and is able to compute the fields' profiles, associated fifth force and high-order operators accurately across the full simulation box, without restricting to the high- and low-density regimes to which analytic understanding is generally confined. The finite element method is well suited for the computation of the high-order operators $(\ \pi)^n$ under study, for which traditionally employed finite-differencing techniques are not sufficient.

For both theories, we compute the field profiles in the presence of a static spherically symmetric compact source of mass $M_{\rm S} = 10^{10} M_{\rm Pl}$ and radius $r_{\rm s} = 10^{47} {M_{\rm Pl}}^{-1}$, following a smoothed top-hat profile:

$$\rho(r) = \frac{M_{\rm S}}{4\pi(-2w^3)\mathrm{Li}_3(-e^{\bar{r}/w})} \frac{1}{\exp\frac{r-\bar{r}}{w} + 1} \tag{4.3.1}$$

where $w = 0.02 r_{\rm s}$, $\mathrm{Li}_3(x)$ is the polylogarithm function of order 3 and $\bar{r}$ is chosen so that 95% of the source mass is included within $r_{\rm s}$. In the limit $w/r_{\rm s} \to 0$, this density profile becomes the step function $\rho(r) = 0.95 \frac{3M_{\rm S}}{4\pi r_{\rm s}^3} \Theta(\bar{t} r_{\rm s} - r)$, where $\bar{t}^{-1} = \sqrt[3]{0.95}$ and

[4] `https://github.com/scaramouche-00/phi-enics`

Θ is the step function.

For the UV theory, we take the masses of the light and heavy fields to be $m = 10^{-51} M_{\rm Pl}$ and $M = 10^{-48} M_{\rm Pl}$, with coupling constants $\alpha = 0.4$ and $\lambda = 0.7$. For the IR theory, we take $\Lambda = 2.07 \times 10^{-48}$ and $\epsilon = -1$. Note that this choice of parameters corresponds to different signs for α in the UV theory for $n = 2, 3, 4$. For both theories, we impose that the fields be regular and asymptoting to the vacuum expectation value, which imposes the boundary conditions $\{\phi(\infty) = H(\infty) = 0; \nabla\phi(0) = \nabla H(0) = 0\}$ and $\{\pi(\infty) = 0; \nabla\pi(0) = 0; \nabla[\nabla^2\pi^n](\infty) = 0\}$. For the IR theory, we supplement these conditions with the requirement $\{\nabla[\nabla^2\pi^n](0) = \text{finite}\}$, which is obtained from the numerical solution to the UV theory ($n = 3$). The latter is applied for consistency with the requirement of UV completion.

We shall now give details of the settings used to solve the UV and IR theories. For both, we use interpolating polynomials of order 5, and the following φ`enics` settings:

- UV theory, $n = 3$: `ArcTanExpMesh` of 150 points spanning a box $r \in [0, 10^{10}] \times r_{\rm s}$, with parameters $k = 8, a = 5 \times 10^{-2}, b = 3 \times 10^{-2}$. Field rescalings: $\mu_\phi = 10^{13} M_{\rm Pl}$, $\mu_H = 10^{12} M_{\rm Pl}$;

- IR theory, $n = 2$: `ArcTanExpMesh` of 400 points, spanning a box $r \in [0, 10^9] \times r_{\rm s}$, with parameters $k = 25, a = 5 \times 10^{-2}, b = 3 \times 10^{-2}$ and declustering at $r_{\rm rm} = 10^3 r_{\rm s}$ with parameters $A_{\rm rm} = 1, k_{\rm rm} = 10$. Field rescaling: $\mu_\pi = 10^{-15} M_{\rm Pl}$.

- IR theory, $n = 3$: `ArcTanExpMesh` of 700 points spanning a box $r \in [0, 10^9] \times r_{\rm s}$, with parameters $k = 25, a = 5 \times 10^{-2}, b = 4 \times 10^{-2}$. Field rescaling: $\mu_\pi = 10^{-15} M_{\rm Pl}$.

- IR theory, $n = 4$: `ArcTanExpMesh` of 600 points spanning a box $r \in [0, 10^9] \times r_{\rm s}$, with parameters $k = 25, a = 5 \times 10^{-2}, b = 3 \times 10^{-2}$. Field rescaling: $\mu_\pi = 10^{-15} M_{\rm Pl}$.

The mesh classes available in φ`enics` are discussed extensively in [125]: they apply a non-linear transformation to a mesh that is initially equally spaced in order to obtain a discretisation that is finer along the source-vacuum transition and coarser everywhere else. All numerical settings reported here are similarly defined in [125] and in the φ`enics` documentation.

In Figure 4.0.1, we show the ratio of the scalar force to the Newtonian gravitational force F_s/F_N for the UV theory ($n = 3$) and the IR theory ($n = 2, 3, 4$), around the compact object in Eq. (4.3.1). When a scalar field couples to matter with a coupling strength $M_{\rm Pl}$, the ratio is equal to 2 if there is no screening. We can see that this is the case for the UV theory, where $F_s/F_N = 2$ for r . $1/m$, (for r & $1/m$ the massive field decays exponentially and the scalar force is correspondingly suppressed). The scenario is radically different for the IR theories, where strong Vainshtein screening is displayed around the source. Here, the scalar force is suppressed compared to the Newtonian force by a factor which can be as large as 10^9, confirming our expectations of Sec. 4.2.

To understand the absence of screening in the UV theory, and its apparent presence in the IR, we consider the neglected higher order terms for $n = 3$. Still under the assumption $\ll M^2$, i.e. far from the Compton wavelength of H, we write down the leading order term for each power of λ, and find them to be of the form:

$$X_j = (-1)^{j+1} \binom{3j}{j} \frac{1}{2j+1} \alpha^{2j+2} \left(\frac{\lambda}{3!}\right)^j (\quad \pi)^{2j+1} M^{-6j-2} \tag{4.3.2}$$

with $j \geq 1$. We might, at first, expect that the terms $j > 1$ are negligible when compared to $X_1 \equiv \mathcal{O}_3$ from Sec. 4.2. However, when evaluated on the full UV solution, we find that actually these terms become important sooner than $\mathcal{O}_3$, and all at roughly the same radius. We check this numerically, and compute the operators X_j for $j = 1, 2, 3, 4$ in the UV theory ($n = 3$): the result is shown in Figure 4.3.1. As expected, the hierarchy of the operators breaks down. We have verified that this numerical result is independent of the specific source profile or theory parameters used. Naïvely, we could consider the radius at which the higher-order operators become important as a new scale at which we might expect the linear theory to break down: however, this is not borne out by the numerical solution. It is therefore clear that the operators we initially neglected, along with $\mathcal{O}_3$, resum to produce an operator that is negligible and unable to provide screening at macroscopic distances.

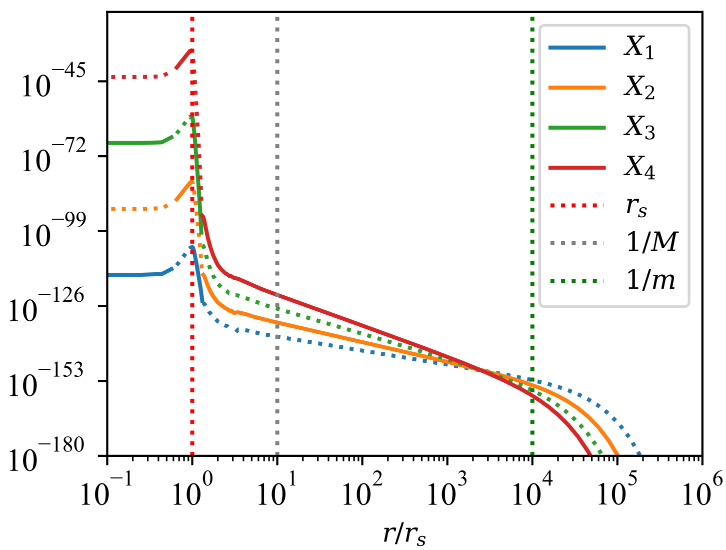

Figure 4.3.1: The operators X_j in Eq. (4.3.2), for the UV theory ($n = 3$); solid (dotted) lines indicate positive (negative) values. The assumption $X_{j>1} \ll \mathcal{O}_3 \equiv \frac{\Box(\Box\pi)^3}{\Lambda^8}$ (for $\Lambda^8 = 6M^8/(\lambda\alpha^4)$) is clearly invalid. Figure produced by Daniela Saadeh.

4.4 Massive Wess-Zumino Galileons

The familiar Wess-Zumino (WZ) Galileons are a popular modified gravity theory, first appearing in the context of DGP gravity [126], [135], giving rise to second-order field equations and Vainshtein screening [122]. They are invariant under the standard Galileon symmetry, up to a total derivative and coupled with the requirement of second-order field equations, this restricts the action to a finite number of operators. However, despite the many desired features WZ Galileons exhibit, they are impeded from a standard Wilsonian UV completion by the existence of positivity bounds, which restrict the form of low energy scattering amplitudes for scalar theories [50], [130]. To avoid this limitation, one may deform the theory at low energies, satisfying the bounds, while attempting to keep all other features of the theory intact.

Having shown in section 4.2 and 4.3 that a mass term acts unexpectedly in our candidate theory, we ask whether this type of deformation is acceptable in the context of WZ Galileons, in particular whether the Vainshtein mechanism is preserved. We consider the theory given in (4.1.1), coupled to matter via the usual $\frac{\pi T}{M_{\rm Pl}}$, and will assume for simplicity that the mass term essentially plays no role in screening - its role here is merely to evade the positivity bounds.

Having posited that simply adding a mass deformation preserves the Vainshtein properties of the theory, while avoiding positivity bounds, we need to consider whether this alteration induces other operators that spoil the screening. Thanks to the Galileon non-renormalisation theorem, neither the mass or the Wess-Zumino couplings receive radiative corrections [135]–[138]. However, higher order EFT corrections are of a more general form, which can be written as,

$$\frac{(m^2\pi^2)^a\partial^{2b}(\partial\partial\pi)^c}{\Lambda^{4a+2b+3c-4}} \tag{4.4.1}$$

where a, b, c are positive integers and we have treated m^2 as a spurion. At the level of the equation of motion, this operator yields a term of the form,

$$\mathcal{O} \sim \frac{m^{2a}\partial^{2(b+c)}\pi^{2a+c-1}}{\Lambda^{4a+2b+3c-4}} \tag{4.4.2}$$

where for the moment we remain agnostic about where the derivatives are operating. Again following the standard procedure, we evaluate the operator on the linearised solution in the static spherically symmetric approximation, for $r \ll m^{-1}$, resulting in,

$$\mathcal{O}\Big|_{\pi_{\rm lin}} \sim \frac{m^{2a}}{\Lambda^{4a+2b+3c-4}} m^{2x} \frac{1}{r}^{2(b+c-x)} \left(\frac{\sigma_{\rm S}}{r}\right)^{2a+c-1} \tag{4.4.3}$$

where $x \in [0, b+c]$ and its value depends on the number of π insertions in $\mathcal{O}$, and $\sigma_{\rm S} \equiv M_{\rm S}/M_{\rm Pl}$ as in Sec. 4.1. Let us now compare this against a standard WZ operator, which looks like,

$$\mathcal{O}_{\rm WZ} \sim \frac{(\partial\partial\pi)^L}{\Lambda^{3(L-1)}} \tag{4.4.4}$$

where $L = 2, 3, 4$. Under the same assumptions, evaluating on the linearised solution gives,

$$\mathcal{O}_{\rm WZ}\Big|_{\pi_{\rm lin}} \sim \left(\frac{\sigma_{\rm S}}{r^3\Lambda^3}\right)^L \Lambda^3 \,. \tag{4.4.5}$$

Comparing the two operators, we obtain a ratio,

$$\frac{\mathcal{O}}{\mathcal{O}_{\rm WZ}}\Big|_{\pi_{\rm lin}} \sim \left(\frac{r_*}{r}\right)^{2a+2b+3c-(1+2x+3L)} \tag{4.4.6}$$

where the radius r_* at which the two operators become of comparable size, is given by,

$$r_* = \frac{1}{\Lambda}\left(\kappa^{2(a+x)}\sigma_{\rm S}^{2a+c-L-1}\right)^{\frac{1}{2a+2b+3c-(1+2x+3L)}} \tag{4.4.7}$$

where $\kappa \equiv m/\Lambda$ as in Sec. 4.1.

We know that screening must be contaminated if $\mathcal{O} \gg \mathcal{O}_{\rm WZ}$ at $r_{\rm V} = \sigma_{\rm S}^{\frac{1}{3}}\Lambda^{-1}$, the Vainshtein radius of the WZ theory, as this would mean that when the WZ terms are supposed to start screening, they would be in fact subdominant to the EFT operators.

Setting $\sigma_{\rm S} \equiv \kappa^{-t}$, we obtain,

$$\left.\frac{\mathcal{O}}{\mathcal{O}_{\rm WZ}}\right|_{\pi_{\rm lin}(r_{\rm V})} \sim \kappa^P \tag{4.4.8}$$

where,

$$P = \frac{2}{3}(b+1)t + \frac{2}{3}a(3-2t) + \frac{2}{3}x(3-t)\ . \tag{4.4.9}$$

If $t \le \frac{3}{2}$ then $P > 0$ and the EFT operators are suppressed relative to the WZ terms. However, if $t > \frac{3}{2}$, then P can be made negative by a sufficiently large choice of a. Incidentally, for the parameter values that correspond to the original mass deformation, P is positive all the way up to $t > 3$, and so we see that EFT corrections are in general more important.

The value of t is essentially dictated by the size of the source, with heavier sources having a larger t. For the Sun, we can estimate $\sigma_{\rm S} \sim 10^{39}$, $m \sim H_0$, $\Lambda \sim (1000\text{km})^{-1}$, which gives $t \sim \frac{39}{20}$. We see that, even for a simple example, EFT terms can spoil the screening of the WZ operators.

There is a loophole in the above discussion. If the Galileon symmetry is only broken by the mass term, then Galileon loops will not generate Galileon breaking operators and we do not obtain arbitrarily high values of a. A similar point was already made in [47]. However, the presence of a Galileon breaking interaction, beyond the original mass term, should be enough to generate a full tower of interactions with high values of a. Such terms might be expected if the breaking of Galileon symmetry is truly inherited from the UV physics and is present in the couplings between light and heavy fields.

4.5 Discussion

In this chapter we have explored a class of UV complete theories of massive Galileons, which at low energy are manifestly Galileon invariant, with the exception of the mass term.

Taking candidate low energy theories, we have shown that operators of the form $(\;\pi)^n$ have the ability to result in Vainshtein screening. This was suggested by our analytic approximations at small and large r. However, to show that the two asymptotic regimes could indeed be connected to one another, we needed to use numerics. It turned out that our asymptotic solutions could match and we did not run into any obstacles involving inconsistent boundary conditions or branch cuts.

Generalising the example action [108] to an arbitrary power of self-interaction for the heavy field H, we have seen that this class of theories exhibits a massive Galileon symmetry in the light field π, and that integrating out H only generates terms that respect the symmetry. However, it turns out operators that would normally be neglected in a naïve analysis of the IR equation of motion, due to being suppressed by large powers of the heavy mass M, play an important role in determining the behaviour of the solution, and in fact become relevant at a larger radius than operators one might have considered leading order. Interestingly, although individually relevant, these additional operators re-sum to produce a negligible effect, giving a free field profile all the way up to the source radius. Although our candidate low energy theory exhibits screening by virtue of a $(\;\pi)^n$ operator, in making contact with the UV we necessarily introduce additional operators that entirely disrupt this effect. It is clear that when integrating out a heavy field, a simple truncation is not always sufficient, and in some cases is catastrophically wrong, forcing a careful consideration of all higher order operators being neglected.

With Ref. [108] having identified a mass term as a potential deformation to avoid positivity bounds in Galileon theories, we investigated its consequences in the context of Wess-Zumino Galileons. At first glance the deformation seems to leave the standard screening picture for this theory unaffected, even when one considers loop corrections. However, if we try to connect it to some UV completion and view it from an EFT standpoint, we must necessarily introduce operators that, for heavy enough sources, can dominate over the standard Wess-Zumino terms. Whether this would ruin the screening enjoyed by the deformationless theory or simply increase the radius at which screening occurs is unclear, but the prior results of this chapter tell us that the former could be more likely than one might naïvely expect.

Chapter 5

Discussion and Future Work

The overarching theme of this thesis has been an analysis of whether a theory can be considered a viable alternative to General Relativity, at the level of both observation and mathematical consistency. The topics that we have examined of screening capability and vacuum decay stability all contribute to a holistic picture of a theory as a sound attempt at modified gravity. The benchmarks that we have discussed are in some sense an important selection of 'base tests' that one must pass in order to approach a plausible model, preceding attempts to match specific data sets.

Chapter 2 presents a detailed discussion of the cosmological constant problem. This issue is a significant hurdle for any attempt to extend gravity into the UV, where generally a semi-classical approach results in a radiatively unstable vacuum energy. We summarise how the standard model matter sector contributes to this instability, which demands constant fine-tuning of the corresponding counterterm.

We proceeded to show that any theory attempting to prohibit the cosmological constant from gravitating should have something to say about the global sector of the GR field equations. This is a natural consequence of its unique feature as the parameter that is fixed over the whole space-time. In chapter 3 we demonstrated a mechanism to achieve this within a well-defined local field theory, firstly addressing solely matter contributions, and then continuing to include virtual graviton effects. These theories are known as sequestering models [82]–[84], [88]–[91], [95], [96], a class of theories that utilise global constraints originating from a topological sector to prevent the sourcing of curvature from the cosmological constant.

There are still however open questions that the sequester must face. For instance, at the very least it is reasonable to expect that kinetic terms for the scalar and 4-form fields may appear in the effective action. These terms will appear once higher order EFT operators are accounted for, and could also be generated by radiative corrections, but this is not a guarantee. This is part of the overarching question of whether the underlying structures of OS are stable with respect to corrections coming from UV considerations. Let us estimate whether the inclusion of kinetic energies invalidates the sequestering mechanism, by examining the following action,

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \Lambda(x) + \theta(x)R_{\mathrm{GB}} - \frac{a_1}{2}(\partial\theta)^2 - \frac{a_2}{2}(\partial\Lambda)^2 - \frac{a_3}{4!}F^2 - \frac{a_4}{4!}\hat{F}^2\right)$$
$$+ \frac{1}{4!}\int \mathrm{d}^4x\,\epsilon^{\mu\nu\lambda\rho}\left(\sigma\left(\frac{\Lambda}{\mu^4}\right)F_{\mu\nu\lambda\rho} + \hat{\sigma}(\theta)\hat{F}_{\mu\nu\lambda\rho}\right) + S_{\mathrm{m}}(g^{\mu\nu}, \Psi) \quad (5.0.1)$$

where the a_i are constants, some of which have dimension, with $F^2 = F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho}$ and similarly for $\hat{F}$ as expected. This becomes the OS action in the limit $a_i = 0$ for all i. Immediately apparent is the presence of terms containing the 4-forms now enter the action minimally coupled to the metric. Consequently, the variation of the 3-forms no longer constrains the scalars to be rigid on-shell. Rather, they set $\star F - \frac{\sigma}{2a_3}$ and $\star\hat{F} - \frac{\hat{\sigma}}{2a_4}$ as constants. We see that the constraint on the space-time averaged Gauss-Bonnet, pivotal to the cancellation of radiatively unstable vacuum energy corrections in OS, now has a dependence on both the non-rigid scalar fields and the metric. Recovering an exact cancellation in a theory including kinetic terms in this way then necessitates some other method of fixing the scalar fields to be rigid on-shell, or else sufficiently suppressing any space-time variation.

On the other hand, it is possible that the introduction of kinetic terms would allow the sequester of the vacuum energy whilst affecting sources of finite wavelength, permitting us to better probe OS via observational experiment. The version of OS presented in chapter 3 is largely phenomenologically equivalent to GR by design, and generally any experiment conducted at low energies will struggle to separate the two. This barrier to observational testing further emphasises the need for a UV completion or extension of the sequester, as this would at least allow us to further examine its mathematical validity, which as we have demonstrated in this thesis is an important component of any model-building endeavour.

Further applications of a sequester-like mechanism, involving a topological sector independent of the metric that preserves gauge invariance, may include novel screening mechanisms. We have demonstrated the importance of such methods in chapter 4, and alternative approaches to the suppression or decoupling of a fifth force would be an exciting prospect.

In the second half of chapter 3 we considered how the OS theory handles phase transitions, and whether any of the results therein are cause for mathematical concern. The tunnelling between maximally symmetric vacua via the nucleation and subsequent evolution of true vacuum bubbles was examined, and the tunnelling rates for such transitions were determined. The generic functions σ and $\hat{\sigma}$ that are present in the OS action were further constrained by the requirements that tunnelling rates should appear qualitatively similar to those of GR, and that importantly instabilities be averted to maintain theoretical validity. Similarly to the local sequester, it was found that a transition from a false de Sitter vacuum to a true de Sitter vacuum with smaller curvature results in the insensitivity of the new vacuum to the size of the jump, and a resulting residual cosmological constant which does not require fine tuning. However, phase transitions are not an infinite wavelength source and so are not fully suppressed by the sequester, but they do not effect our capability to satisfy observational constraints. Nevertheless, this imperfect sequester may allow the theory to be investigated experimentally, for instance [139] suggests that gravitational effects caused by phase transitions could manifest in alterations to the mass-radius correlation of neutron stars. The potential to experimentally distinguish OS from GR, given that they are locally equivalent, is an exciting step on the road to solve the cosmological constant problem.

The work in this chapter lead to a description of OS and its cosmological consequences, but one could instead consider a broader class of sequestering models, as introduced in [95]. The action for this 'generalised sequester' is given by,

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{\kappa^2(x)}{2} R - \Lambda(x) + \theta(x) R_{\mathrm{GB}} - \mathcal{L}_{\mathrm{m}} \left(g^{\mu\nu}, \Psi \right) + \ldots \right) + \frac{1}{4!} \int \mathrm{d}^4x \, \epsilon^{\mu\nu\lambda\rho} \left(\sigma_1 \left(\frac{\Lambda}{\mu^4} \right) F_{1\,\mu\nu\lambda\rho} + \sigma_2 \left(\frac{\kappa^2}{M_{\mathrm{Pl}}^2} \right) F_{2\,\mu\nu\lambda\rho} + \sigma_3(\theta) F_{3\,\mu\nu\lambda\rho} + \ldots \right) \tag{5.0.2}$$

where the '...' signifies additional terms that are compatible with the sequestering mechanism. It would be interesting to undergo a similar analysis of this generalised theory, and identify qualitative and quantitative similarities between sub-classes of these theories. One may also be able to discern observational differences between these theories, which would be essential were relevant experimental data to become available.

In the final section of this thesis, chapter 4, we examined a generalised theory of a massive galileon and its prospective UV completion, proposed to satisfy certain positivity conditions. We considered its potential to match with experimental observation by directly studying the validity of the Vainshtein mechanism applied to the theory and its high energy counterpart, an exercise that has so far been impossible due to the lack of such a pair of theories in the literature. We provide evidence that Vainshtein screening is present in the candidate low energy theories, and proceed to examine the partner theory in the UV, for which we are able to show categorically that Vainshtein screening does not survive the extension. This explicit case study demonstrates that IR approximations of any high energy theory are not automatically valid when moved into a non-perturbative regime. Though this might seem a straightforward conclusion when one considers the full setup within a rigorous EFT context, it is however an important result in a Vainshtein screening circumstance, where large higher order operators are essential for the viability of the mechanism. This direct example simply provides additional evidence of the theoretical invalidity of the Vainshtein mechanism, independent of observational experiment.

Nevertheless, outside of the Vainshtein mechanism and the non-perturbative regime we have the opportunity to study the UV completion of a well-known scalar field theory, albeit with a slight modification, which could potentially provide insights into the high energy extension of other topics in field theory. For instance [140] draws parallels between galileon theories and GR, providing hope that one may learn something about the UV completion of Einstein's theory via this avenue.

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