Essays on Lobbying
and
Club Formation

Priyanka Joshi

School of Economics
University of Nottingham

Thesis submitted to the University of Nottingham for the degree of Doctor of Philosophy.

August 2019

Abstract

Chapter 1 of this doctoral thesis studies that whether micro-targeting (MT) is an effective tool as a lobbying strategy to influence policy and/or election outcomes. Interest groups (IGs) have a membership base who can be sent group-specific messages (MT) on behalf of the political candidate. The IG demands policy favours in exchange for facilitating MT. We investigate how exchange of policy favours for MT influences the election outcome, and hence, policy outcomes. It is shown that IGs who are opposed to the candidate's ideal policy could effectively use MT to change policy outcomes to their advantage. But IGs who have same ideal policy as the candidate's do not find MT effective. However, interestingly, their presence in the political system itself can affect the influence other IGs exert.

Chapter 2 studies a dynamic game of club formation where individuals' utilities depend only on the size of the club they belong to. Clubs are formed and dissolved over time with given rules. The objective is to uncover a 'fear of exclusion' phenomenon caused by long-run dynamics. We find that individuals might form sustainable sub-optimal sized clubs when they fear exclusion in the infinite period game, but this does not happen in finite horizon game where all possible optimal sized clubs form in the stable state.

Keywords: Micro-targeting, lobbying, influence, ideological candidates, private commitment, non-cooperative games, club good, fear of exclusion, stable club structure, infinite horizon, optimal club

JEL Classification Numbers: C72,C73,D72,D71,D82

Acknowledgements

I would like to acknowledge all the support and help I received during the course of my PhD years. First and foremost, I am very grateful to my supervisors Prof. Daniel Seidmann and Prof. Vincent Anesi for guiding and supporting me through this research project. Thank you both for the insights and knowledge you provided without which the completion of this thesis would have not been possible. I would also like to thank Dr. Matthias Dahm for helpful comments and suggestions. I am grateful to Prof. Siddhartha Bandyopadhyay and Dr. Silvia Sonderegger for agreeing to thoroughly examine my thesis.

I thank my family for the support and love they gave me throughout my PhD journey. I would also like to thank my colleagues in the school of Economics. It was great to share the office with such talented and passionate people, and I much appreciate the healthy working environment in B52. I also thank the administrative staff of the School of Economics for their support. I am also very thankful to Soumitro for giving me all the love and support: thank you for being there through my ups and downs. Finally, I acknowledge the financial support received from the Economic and Social Research Council and the School of Economics, without which I could not have pursued this academic path.

Contents

List of Tables

List of Lemmas, Propositions, Observations and Corollaries

Chapter 1

Chapter 2

Introduction

Micro-targeting (MT) means identifying voters based on their preferences and opinions so they can be segmented for content targeting in order to get their support. In contemporary politics, interest groups (IGs) have increasingly been paying attention to MT techniques in furthering their agendas. The literature on lobbying has addressed many different aspects of lobbying strategies in influencing the policy-making process but an analysis of how MT is used by IGs in exerting influence is largely missing in the literature. In Chapter 1, we address this question and investigate the mechanism behind how MT by the IGs affects policy/election outcomes.

Interest groups can potentially influence the policy making process or policy outcomes in their favour by using different lobbying tactics. We study how and under what circumstances IGs have influence on policy outcomes when they can use MT as a lobbying strategy, i.e. the IGs can send group specific messages to a subset of voters. In the absence of IGs, the political candidate does not have means to privately commit a policy to a group of voters, who might agree with her after observing such a policy commitment. Recognizing this, IGs can get policy favours from the candidate in exchange for facilitating the candidate's private commitment. We identify conditions in which MT is influential, in the sense of leading to a different outcome (policy and/or election) in the presence of IGs. First, unlike-minded IGs (IGs who are opposed to candidate's fixed ideal policy stand) can directly influence policy outcomes. Second, like-minded IGs (IGs who agree with the candidate's ideals) do not have direct influence. However, their presence in the political system and the the possibility that they can influence the candidate has an effect on the influence of other IGs. The analysis fully characterizes the set of influential MT equilibria under three different cases: 1. Alike case: two political candidates have same ideal policy on both policy issues; 2. Semi-alike case: two political candidates have the same ideal policy on one of the policy issues; and 3. Polar case: two political candidates have a different ideal policy on each policy issue. Our 1st Chapter concludes with some key insights on the role of MT as a lobbying strategy in influencing policy outcome.

In Chapter 2, we explore a dynamic model of sequential club formation in which identical individuals form and dissolve clubs over an infinite number of periods. An individual's per-period payoff only depends on the number of members in the club to which she belongs, and is common across individuals, so there is a unique optimal sized club. Up to one new club can be formed in each period. We consider whether individuals would form sub-optimal sized clubs in every equilibrium of the club formation game. We also analyse a benchmark, finite-horizon game. We characterize Markov Perfect equilibria of these games, and following Acemoglu et al. (2012), we define stable club structures, i.e. club structures that never change once they are formed. We show that in an infinite horizon game, stable club structures need not have optimal sized clubs. In fact, if individuals are patient enough then there exists a set of stable club structures in which none of the clubs are of optimal size. By contrast, in finite horizon games, all stable club structures have as many optimal sized clubs as possible. This provides us two interesting insights. First, if individuals are forward-looking then they might exhibit fear of exclusion in infinite horizon games. Second, individuals do not form optimal sized clubs if doing so leads to unwanted subsequent changes. A specific class of equilibria is analysed to examine such behaviour.

Chapter 1

Influence under Micro-targeting

1.1 Introduction

Political micro-targeting (MT), in a broad sense, means identifying the subset of voters so that voters can be delivered tailored messages based on their preferences and opinions. There is ample evidence that politics, to some extent, is driven by MT, which has become more effective and intensive over the years. In the US, the political intermediaries have an immense amount of detailed data on voters (Bennett (2016)). Also, companies like CampaignGrid and Cambridge Analytica offer online MT services to politicians that allow the political parties to target voters with ads on platforms like Facebook, LinkedIn etc.[1] Former US president Bill Clinton effectively targeted a particular section of the society by appearing on non-traditional platforms like daytime talk show and MTV (Suggett (2016)). But Barack Obama took this approach to a new level during his 2012 election. Use of social media was the defining feature of his 2008 campaign; data science and micro-targeting stood out most in his 2012 campaign (Young (2012)).

The use of MT in politics has been around for a long time. However, the topic only became popular after the 2012 US presidential elections and more so after the unexpected outcome of the 2016 US presidential election. MT seems very important for political parties and their campaigns, as it allows them to communicate to swing voters alone. However, political parties usually do not have the resources, infrastructure, or valuable data on voters to micro-target directly. This is where the intermediaries can come in and provide their services, either for monetary rewards or to further their political agendas.

[1] https://campaigngriddirect.com/ and https://cambridgeanalytica.org/

What we are interested in looking at is how political motives of interest groups (IGs) affect policies when these lobbying organisations can micro-target voters in exchange for policy favours?

In addition to communication/advertising agencies and consultancies, some IGs are also working towards building issue-specific databases. Since the political parties need data on voters, this could potentially help these lobbying organisations to get policy favours and influence the policy making process. For instance, the National Rifle Association (NRA), a pro-gun lobbying organisation in the US, has built a massive database on gun owners. The NRA has been very successful in pushing for its pro-gun agenda, even though the majority of US citizens support strict gun laws.[2] This prompts us to study the role of MT in the context of lobbying. In this chapter, we study how and which IGs can influence electoral outcomes when they help political parties to make use of sophisticated methods such as micro-targeting. We show that MT is an ineffective tool for IGs that have the same policy preference as the the political candidate; but interestingly, their presence in the political system itself could influence the electoral outcome. In contrast, IGs who promote policies which are different from a candidate's ideal political position could potentially use MT to influence the election and policy outcomes in their favour. The underlying mechanism behind these results is that the candidate is able to send group-specific messages with the help of an IG, in return for which the candidate implements the IG's ideal policy.

There is a fair amount of evidence that information on voters' preferences is used to target specific voters.[3] This information is very useful when the candidates want to micro-target voters. In recent times, Facebook and Cambridge Analytica have combined data mining and data analysis on more than 71 million people and used it for strategic communication. It is claimed that strategic MT, based on this data, was first used in the British EU referendum and then in the 2016 US presidential election.[4] Even though these organisations are mainly businesses which want to sell this information

[2] Source Opensecret.org: 73% of the candidates the NRA directly supported at the federal level won in the 2016 US election. A recent poll conducted by Gallup shows that sixty-one percent of Americans favour stricter laws on the sale of firearms.

[3] There are many private enterprises in the US such as Catalist, Aristotle, and Camelot, which gather data on individual voters, compile this data with other publicly and commercially available data, and sell them for monetary gains to political or non-political campaigns.

[4] Source: The Guardian (https://www.theguardian.com/news/2018/apr/10/cambridge-analytica-and-facebook-face-class-action-lawsuit

for monetary rewards, there are other groups or organisations which make use of this information to exert influence on candidates. A review of the NRA legislation news page shows that it has had 230 victories at the state level from 2004 to 2013. In 2012, the NRA-ILA (NRA's institute for legislative actions) was very influential in promoting the legislation in Florida that would punish doctors who asked their patients about their gun possession (Watkins (2013)).[5] Many argue that the power of the NRA comes from the money it spends on lobbying; but the NRA spends much less than and, arguably, is more influential than lobbying organisations like the National Association of Realtors and the Koch Brothers.[6] The NRA has a huge membership and a powerful activist base. These members are very loyal to the organization and vote one way or the other based on the NRA's view of a particular candidate. Adam Winkler (Winkler (2013)) argues that the real source of the NRA's influence comes from its members. The NRA simply grades the candidates based on their view on gun laws and reports these grades to its members. Thus, what seems more important for the NRA's success is the ability of the NRA to tell its members which candidates are in favour of guns. The NRA has also been collecting data on the the former, current and prospective non-member gun owners (Friess (2013)). The grandeur of this secret database explains, in part, the enduring influence of the NRA.

This is somewhat in line with evidence that money does not play a major role in politics (Azari (2015)). In 1999, the US members of Congress and their staff claimed that there are only 12 and 4 lobbying groups in a list of 25 top most powerful IGs for campaign contributions and lobbying expenses respectively.[7] Kenny et al. (2004) studied the extent of influence the NRA had on the 1994 and 1996 contested House races. They investigated the methods the NRA used to exert influence on the election outcome and found, in addition to campaign contributions, that their membership base played a big role in helping Republican challengers (who were sympathetic with the NRA's policy position) win the election in 1994, but not in 1996 because of the politically unfavourable environment for the NRA.

[5]Source: nraila.org

[6]According to Center for Responsive Politics, the NRA spent \$4.1 million and \$5.1 million on lobbying in 2018 and 2017 respectively, whereas National Association of Realtors spent \$53.7 million and \$54.5 million on lobbying in the same years.

[7]Source for the power ranking: Birndaum (1999). Source for SIGs' money: Center for Responsive Politics (http://www.opensecrets.org/).

In light of these observations, it is natural to ask that how IGs use MT to influence policy makers. There is extensive research on IGs' influence on the policy making process, but the literature barely addresses MT. IGs are traditionally seen to influence policy in two ways: first, IG may provide campaign contributions in exchange (implicit or explicit) for policy favours (Grossman and Helpman (1994); Hillman and Riley (1989)); second, IGs can also provide policy relevant information and thereby influence legislators' beliefs about the consequences of different policies (Milgrom and Roberts (1986); Austen-Smith and Wright (1992)). IGs may also engage in indirect lobbying, such as grass roots lobbying or issue advocacy advertising (Yu (2005); Kollman (1998); Bombardini and Trebbi (2011)). While studying these lobbying tactics is undoubtedly important in understanding IGs' influence, studying MT as a lobbying strategy would shed some light on how MT is being used by IGs to their advantage.

In the literature of lobbying, influence is considered from different perspectives such as influence on the content of a bill or influence on the prospects of a given bill being enacted into law. In this Chapter, influence simply means an IG's ability to change the electoral and/or policy outcome in its favour, which would not be possible without IG's lobbying efforts. The idea is that IGs have information on the policy preferences of some of the voters. A candidate might want to use this information in order to send group-specific messages to win the election; so the IGs are in a position to ask for policy favours in exchange for this information. Influence might come about either because of the IG's direct lobbying effort or indirectly because of the IG's existence in the political system. We therefore further distinguish between direct and indirect influence. Specifically, an IG can have direct influence when the IG is in direct contact with the candidate; and an IG may have indirect influence because of its presence among other IGs. This is defined formally in Section 1.3.

We present a model of lobbying that does not include the traditional channels through which lobbying distorts policy making in favour of IGs. Instead, the IGs can use MT as a tool to lobby political candidates. Since we want to study the impact MT has on policy outcomes when the IGs have political agendas, we assume that information on some voters' preferences is private to the IGs. The analysis characterizes necessary and sufficient conditions for MT to be influential, i.e. to change election and/or policy outcomes compared to an alternative setting in which lobbying is not allowed. This is because MT has the potential to make the candidate win the election by sending group-specific messages that are favourable to the candidate's political position.

We develop the argument using a simple model, in which there is a political candidate, say the challenger, who has an ideal political position on two policy issues and is running for election against a (passive) incumbent. There are IGs who attempt to influence the electoral outcome by lobbying the challenger. In order to avoid complexity, we assume that IGs can only lobby one of the candidates, who we call the 'challenger'. The challenger wants to win the election in order to implement her ideal policy or a policy which is close to her ideal. To this end, the challenger needs to get more than half of the total votes. Voters also have preferences over the two policy issues, and are grouped based on their policy preferences. There are four IGs, who are single issue minded, i.e. they only care about one policy issue and either agree with the challenger or are opposed to the challenger's ideal policy. The IGs can identify the group of voters who have the same policy preferences as the IG; we call these voters members of that IG. We model lobbying as an exchange of favours: an IG, advocating on behalf of different policies, demands policy favours (offers the challenger either to promise a policy in the IG's favour if the IG is opposed to the challenger's ideal or to stand by her ideal if the IG agrees with the challenger's ideal); and in exchange, the IG privately reveals the challenger's commitment to its members. Thus, in our set-up, a policy promise to an IG serves as a (private) commitment device. The members, when voting, compare the challenger's private commitment and the incumbent's ideal policies. Voters know the challenger's ideal policy but do not necessarily know which policy she would implement if she wins the election in the presence of IGs.

In the absence of IGs, the challenger publicly commits (to all voters) to a policy which she would implement if she wins the election. We assume that it does not cost anything for the challenger to make such a public commitment, but that acquiring information on the preferences of the voters is time consuming and requires expertise. Moreover, the IGs do not have information on the preferences of each voter. In our framework, influential MT alters the policy outcome as compared to the policy outcome in the absence of IGs. A necessary condition for MT to be influential is that the challenger loses the election in the absence of lobbying if she were to commit to her ideal policy or second best policy.[8]

[8]The second best policy is the one where she changes her ideal policy on the issue which she cares

The challenger must have enough incentive to change her policy stance on an issue, in response to an offer from one of the IGs, who could help the challenger to win the election by MT. If the challenger can already win with her best policies then IGs have nothing better to offer to the challenger; so there would be no influence. Influence also requires the two candidates to be sufficiently opposed so that the challenger would be willing to compromise on her ideal policy when an IG approaches her with a deal. The analysis identifies conditions under which such behaviour takes place.

There are two features of the model which are essential to our analysis. The first feature is that there is random access: not all IGs have the opportunity to offer a deal to the challenger. Access is randomly allocated to only one of the IGs. There is a consensus view that access is not ordinarily given, where access is valuable as it offers IGs the opportunity to influence the policymakers. Providing campaign contribution is one way to gain the attention of the candidates and thereby access. In this Chapter, we want to study the role of MT in influencing the policy-making process; so, to remove any other source of influence, we do not treat contributions as a way to get access. The second feature is that an IG which gains access can only offer a deal to the challenger on the policy issue the IG cares about. This seems realistic: the NRA lobbies candidates on gun policy, rather than on other policy issues, such as abortion.

The analysis considers how influential MT varies in cases where the two candidates differ in their ideologies. The three cases are: 1) Polar case: the two candidates are completely opposed to each other; 2) Semi-alike case: the two candidates are partially opposed; and 3) Alike case: the two candidates are exactly like each other. In the polar case, MT is influential when IGs who are opposed to the challenger's ideal policy get access and the challenger accepts their offer. Here, lobbying by these IGs makes the challenger better off by making him win the election at the cost of compromising on one of the issues: the challenger cannot implement her ideal policy but her implemented policy is better for her than the policy that the incumbent would implement. In the semi-alike case, MT is influential in altering the policy outcome if the opposed IG on the issue where the challenger and the incumbent agree gets access, and the challenger puts more weight on the issue on which she disagrees with the incumbent. In the alike case, there is no influence.[9] In all the cases, a set of necessary and sufficient conditions

less about, and implements her ideal policy on the other issue.

[9]The challenger and the incumbent then have the same policy preferences. Therefore, it is equally

is identified such that MT is influential in changing the policy and/or election outcomes. Of particular interest is the necessary condition in the semi-alike case, which requires the challenger to promise that she would implement her ideal policy to the IG who agrees with the challenger's ideal, but disagrees with the incumbent's ideal policy. This is required so that the challenger does not lose support of the voters whose preferences are completely aligned to her but doubt her in the presence of IGs. In the semi-alike case, lobbying is more extreme in the sense that it leads to a policy outcome that is completely different from what it would be without IGs. In contrast, in the polar case, influential MT leads to a policy outcome which only partially differs from the policy outcome in the no-lobbying case: there is at least one issue on which the same the policy is implemented with or without MT.

The remainder of the paper is organised as follow. Section 1.2 reviews the most relevant literature. Section 1.3 presents the model and a benchmark case. Section 1.4 presents the main results, which are discussed in Section 1.5. Section 1.6 concludes. All proofs are in the appendix.

1.2 Related Literature

In this section, we discuss the literature which is most relevant to this chapter. In the lobbying literature, campaign contributions and/or provision of policy-relevant information have been the subject of most of the scholarly articles and research (see the surveys by Schlozman and Tierney (1986) and Grossman and Helpman (2001)). A growing number of papers investigate and examine the role of campaign contributions on IGs' influence on policy outcomes (Austen-Smith (1987); Grossman and Helpman (1994, 1996); Baron (1994)). Contributions are treated either as a bribe for policy favours or a way to get access to politicians. Influence through contributions is generally seen as detrimental, as it lead to policy outcomes that do not represent constituents' welfare, but rather favours IGs (Grossman and Helpman (2001)). On the other hand, influence through information is seen as welfare enhancing because competition among IGs reveals relevant information, resulting in better informed policy makers (Austen-Smith and Wright

good for the challenger if the incumbent wins the election, because the incumbent then implements her ideal policy, which is also the challenger's ideal policy. The challenger would therefore never accept an offer which requires her to deviate from her ideal policy; otherwise she could always deviate to rejecting and commit to her ideal policy.

(1992)). Bennedsen and Feldmann (2006) studied an environment in which the IGs make use of both campaign contributions and policy-relevant information to influence policy makers. These studies have covered many aspect of lobbying, but none of them feature MT in their formal models. This chapter investigates a form of lobbying which can be viewed as an intermediate between campaign contributions and informational lobbying. Instead of contributions, IG could deliver votes by facilitating private commitments in exchange for policy favours.

Several authors have used electoral competition models to address the question of IGs' influence on policy-makers where IGs compete for influence (Bernheim and Whinston (1986); Besley and Coate (2001); Dixit et al. (1997); Grossman and Helpman (1994, 1996)). In most of these studies, lobbying is modelled as a "menu-auction", where lobbyists offer contributions which are conditioned on the chosen policy; and the promises of payments and policies defined in the scheme are binding. An implicit assumption of the menu-auction model is that all interest groups play a role in the policy making process. We find this assumption unrealistic for two reasons. First, casual observation suggest that not all IGs have access to politicians to get policy favours in exchange for contributions or any kind of relevant information. IGs' years of experience in politics make them known and relevant in the political system, and that is what puts them at the front of the line to access the candidates. In our model, we propose an alternative setting in which access is random: so not all IGs have access to the candidates. Second, empirical evidence suggest that many IGs co-exist but some of them are just dormant and do not actively engage in lobbying (Wright (1996)), although their presence might be affecting some aspect of the policy-making. In our model, we are able to capture the (indirect) influence of such IGs.

Schelling (2006) explained that commitment must be communicated for it to have its intended impact. He argued that commitment serves no purpose if it is not communicated to its targeted audience. Commitments are made to change expectations of others' behaviour. In our model, there is a commitment which could be communicated by an IG to a specific audience. Private commitment would influence behaviour in the intended manner only if it is communicated to the targeted audience and not communicated to others (no leaks). We adopt a model by explicitly incorporating a mechanism by which different private commitments can be made to different set of voters.

Perhaps, the closest paper in the literature to ours is Grossman and Helpman (1999), which investigated the extent to which IGs get policy favours as a result of endorsements which convey information to voters who share policy concerns. In their model, there are two policy issues: fixed and pliable. Voters know the policy position of the party on both policy issues and their own preferences regarding the fixed policy issue, but are uncertain about their preference on the pliable policy issue. Some of these voters are members of the organised interest group and share the same interest with respect to the pliable policy issue. The IG can convey information to its members by endorsing a particular party. This information tells members where their interest might lie. Based on this information, they update their beliefs and vote accordingly. Grossman and Helpman find that these endorsements are ineffective when all voters hear them and the interests of non-members are diametrically opposed to those of members. But if the endorsements could be conveyed privately to the group members or if the interests of the members and non-members are not perfectly complementary then the parties would find it worthwhile to compete for endorsements; so the policy positions they take might favour interest groups.

There are several differences between their work and ours, in particular the way interactions and preferences are modelled. The main difference is that we have a political candidate who has policy preferences and is indirectly office motivated because of competition with the incumbent: the only way she could implement a policy closer to her ideal policy is by winning office. This is not the case in Grossman and Helpman, where the parties do not have any particular preference over the policy outcome, and choose platforms to attract endorsement. In addition, we allow multiple IGs to co-exist, even though they do not compete directly. This is important because it allows us to observe the pure effect of endorsement in isolation and to avoid any other factor that might be driving the results. In Grossman and Helpman, only a fraction of voters are members of the single IG, whereas our model assigns each voter to one of the IGs; and voters are therefore able to assess the credibility of the message. Finally, voters in our model know their own policy preferences, but may be unsure of the policy which the challenger would implement. In Grossman and Helpman, voters can observe the policy which each party announces, but do not know how one of the policies affects them.

In the literature on political endorsements, McKelvey and Ordeshook (1985) was one of the first studies to recognise the importance of groups' endorsements as an effective medium of communication with imperfectly informed voters. In their model, the candidates do not know voters' preferences, and a large subset of voters do not know the policy stand of the two candidates. One source of information for these voters is through polls which take place before the election. In addition, voters can also acquire information through endorsements; they allow voters to observe whether a candidate's policy position is further to the left or the right.

In contrast to the paper by McKelvey and Ordeshook, the endorsement process was modelled explicitly by Grofman and Norrander (1990). In their model, voters are unclear about the policy position of the two candidates, where the candidates have a fixed policy stance on the real line. There are two informed endorsers whose policy preferences are common knowledge. Voters consider these two endorsers and take cues from them when reaching a voting decision. It is assumed that each endorser supports a candidate if her position is not too far from her own ideal point. Voters update their beliefs about the candidates' policy positions after observing these (non-strategic) endorsements.

Lupia (1992) and Cameron and Jung (1995) analysed how endorsements affect voting behaviour in the context of referendum voting. In both papers, there is a status quo and an "agenda setter" who can propose to alter the status quo. The initiative proposed by the agenda setter will replace the status quo only if it secures majority of the votes. The agenda setter has her own preferences over the policy outcomes, and formulates a proposal to maximise her total benefits. Voters do not know how the proposed initiative affects their utilities (e.g. they do not understand the pros and cons of the referendum initiative), but they know the utility function of the agenda setter. In both papers, there is an endorser who could disseminate information about the initiative. In Lupia (1992), the endorser behaves non-strategically by informing voters whether the alternative is to the left or the right of the status quo. In contrast, Cameron and Jung (1995) introduced a strategic endorser who also has policy preferences and decides whether or not to support the proposal. The endorser's preferences are common knowledge, and the voters update their beliefs based on the information implicit in the endorser's decision.

Our analysis also builds on the theoretical literature on electoral competition, which assumes that communication is public. For example, Laslier (2006) studied ambiguity in electoral competition, where political communication is equivalent to mass communication. Laslier proposed a formal model to explain why parties choose to remain ambiguous in their policy platforms even when voters dislike ambiguity. It is assumed that it is impossible for the parties to target a group of voters. Given the technologies and availability of voters' information to micro-target voters in the modern election campaigns, we take a different view: that a candidate can target specific voters with the help of IGs; so communication does not necessarily have to be public.

Schipper and Woo (2019) studied an environment where the political candidates have information on the preferences of voters and use MT to target a subset of voters. There are two candidates, and each candidate has a fixed ideological policy stance in a multi-dimensional policy space. The candidates compete for voters who are unaware of all political issues and are unsure about the exact policy position of the candidates. Each candidate can send a private message to a voter: where the message consist of a subset of issues and the same information about policy positions on those issues. They found that the election outcome is the same as if voters have full awareness of issues and complete information on political positions. These results may break down, among other reasons, when the candidates are unable to use micro-targeting. The main difference between their paper and ours is that voters in our model are fully aware of the candidates' ideal policy on both issues and that we allow IGs to privately reveal information about a politician's commitment, which is itself partially informative to voters who do not receive such information. The purpose of MT in our Chapter is to inform voters as compared to persuade voters in their paper.

Prummer (2019) studied a model of micro-targeting in which the two political candidates compete to win the election and choose targeting strategies to convince voters. The different advertising media outlets are allowed to have different audiences and thus it creates incentives for candidates to choose the one which maximises the probability of winning, taking into account the preferences of targeted voters. It was shown in the study that polarisation between the candidates increases when the fragmentation in the media outlet increases. We study the micro-targeting in the context of lobbying and find that micro-targeting not always leads to polarisation: polarisation between the candidate can increase or decrease depending on the ideologies of the candidates.

1.3 Model

There are 2 sets of players in the game: a challenger and voters. The challenger (c) is running for election where if the challenger wins the election then she has to implement policy on two issues, indexed by $i = 1, 2$. We denote a policy by $p = (p_1, p_2)$ on two issues: the economy (p_1), and gun control (p_2) where $p_1 \in \{L, R\}$ and $p_2 \in \{A, P\}$ denote the policies on issue 1 and issue 2 respectively. Policy $p_1 = L$ and $p_1 = R$ corresponds to left type and right type policies, respectively, on the economy. Policy $p_2 = P$ and $p_2 = A$ corresponds to pro-gun and anti-gun policies, respectively, on gun control. There is also an incumbent i (a passive player), who top-ranks some policy pair denoted p_i. The incumbent is elected if the challenger loses the election and always implements p_i, which is common knowledge. The challenger commits to policy; but needs to communicate to voters.

The population of size 1 is partitioned into groups of voters based on policy preferences, $j = (L, P), (L, A), (R, P), (R, A)$. We denote the group of voters with the generic element, $j = (j_1, j_2)$ where group j constitutes fraction n_j of the electorate and j_1 and j_2 are the ideal policies of group-j voters on policy issues 1 and 2. Thus, the name of the group and its ideal policy are both given by (j_1, j_2). For notational convenience, we write (j_1, j_2) as $j_1 j_2$, so for instance the group referred to as RA has ideal policy (R, A). Voters in each group are further divided into two subgroups: those who care more about the economy, and those who care more about guns. This essentially means that there are eight groups of voters. The payoff from each policy is 1 if the policy implemented and the voter's ideal policy match, and 0 otherwise. Hence, voters prefer the implemented policy and their ideal policy to coincide. Given policy p, the utility of the group $j = j_1 j_2$ voters is:

$$W_j(p_1, p_2, \alpha) = \begin{cases} 1 + \alpha, & \text{if } p_1 = j_1, p_2 = j_2 \\ 1, & \text{if } p_1 = j_1, p_2 \neq j_2 \\ \alpha, & \text{if } p_1 \neq j_1, p_2 = j_2 \\ 0, & \text{if } p_1 \neq j_1, p_2 \neq j_2 \end{cases} \tag{1.1}$$

where $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$ represents importance of issue p_2 relative to issue p_1, with $\underline{\alpha} < 1 < \overline{\alpha}$. Voters in group j with $\alpha = \overline{\alpha}$ are the voters who care more about p_2 (or guns) and constitutes a proportion δ of the voters in group j. Heterogeneity within each group accounts for the fact that salience of an issue might be different for voters with the same policy preferences. All voters vote sincerely: casting their vote in favour of the challenger if they expect to earn more when the challenger wins, and voting for each candidate with equal probability when indifferent. As there are only two candidates, sincere voting is a weakly dominated strategies. We assume that if both candidates get same number of votes then because of the incumbency bias the incumbent wins the election.

The challenger's ideal policy is given by p_c, which is common knowledge and α_c is the weight she assigns to guns (p_2): she has the same policy preferences as one of the voter groups. Election of the challenger requires a majority of the votes; the incumbent wins otherwise and implements her ideal policy. The best scenario for the challenger would be if she could win the election and implement her ideal policy. That would only happen if the challenger gets a majority of votes after publicly committing to her ideal policy. But if that were not the case, the challenger might not want to reveal the policy she would choose, if elected, to all the constituents. Revealing a policy commitment to only a subset of voters could therefore be useful. However, the challenger only knows the distribution of the voters' preferences (n_j and δ), but cannot identify a given voter's preferences. This information can be provided by IGs, who may require a policy commitment from the challenger in exchange for information on voters' preferences.

Special interest groups (IGs). There are many IGs both in the US and the EU. It is generally the case that each IG cares only about certain issues. For instance, pro-gun IGs in the US will try to influence politicians on second amendment rights, but do not care about tariff rates on textiles. We capture this phenomenon by assuming that each IG only cares about one policy dimension. However, there are typically competing IGs on each issue. Given the binary nature of our model, we assume that there are four IGs, two opposing IGs on each issue. The interest groups are denoted by $l = L, R, A, P$, where L and R are the IGs who only care about the economy, and strictly prefer policy L and R respectively. P and A are the IGs who only care about guns, and strictly prefer P and A respectively. We also define *like-minded* and *unlike-minded* IGs as follows: *like-minded* IGs are the ones whose ideal policies coincide with the challenger's ideal policy; the other IGs are *unlike-minded*.

Each IG has private information on the identity of some voters. The IG can identify the group of voters who care more about the same issue as the IG is concerned with and have the same policy preference on that issue. Such voters are called the members of that IG. For instance, the members of IG L are the voters in group LP and LA for whom $\alpha = \underline{\alpha}$. The policy preference of the members of any particular IG is known to all, but only their IG has their contact information and therefore, the members can only be contacted through the IG. Thus, if the challenger wants to contact these members to solely reveal her commitment to them, she has to convince the IG to do so. However, the IG has a policy preference and might only be willing to communicate the challenger's policy commitment in exchange for a policy promise from the challenger. In this way, the IGs are in a position to deliver voters by convincing the challenger to take a particular policy stand. This is how the IGs could make use of their membership base to deliver votes. There is no cost in either lobbying the challenger or maintaining the membership base and not all IGs have access to the challenger.

We assume that only one of the IGs is randomly selected by Nature, and that Nature's choice is privately revealed only to the chosen IG and the challenger. The selected IG then makes an offer (to the challenger), which prescribes its ideal policy (on the policy the IG cares about). In exchange, the IG would endorse the challenger if the challenger accepts its offer, i.e. it would inform its members about the agreed policy. If the challenger accepts an offer then she is only committed to implement a policy on one issue, and can choose to implement any policy on the other issue. We assume that the challenger implements her ideal policy on that other issue. For example, if $p_c = (R, P)$ and IG L gets access, then IG L's offer is to implement L on the economy; so the challenger implements (L, P) if she accepts L's offer and wins. In exchange, the IG endorses the challenger, i.e. the IG would inform its members that the challenger has accepted the offer and would implement (L, P) if she wins. Let p_l be the policy the challenger implements when IG l gets access and the challenger accept its offer (or when l endorses the challenger).

The challenger can also *publicly* broadcast her commitment. After one of the IGs gets access to the challenger and makes an offer, the challenger chooses whether to accept the offer or not. If the challenger accepts the offer then the IG endorses the challenger, i.e. IG informs its members about the challenger's commitment. If the challenger rejects the offer then she publicly commits to one of the policy pairs. The challenger's strategy specifies : i) whether she accepts or rejects the offer she receives from each IG; and ii) which policy pair to publicly commit to after rejecting an offer. We will explain later why the challenger publicly commits to the same policy pair for each offer she might reject in an equilibrium.

If l gets access and its offer is accepted then it informs its members that it endorses the challenger, in which case members know that the challenger would implement p_l if she wins. If L gets access and its offer is rejected then all voters see public commitment from the challenger. IG l inform its members that it did not get access if it did not get access, in which case members know that some other IG must have endorsed the challenger if they do not see public commitment. We assume that IGs cannot lie or hide any information about the endorsement. Voters cannot directly observe which IG got access but the distribution (prior probability with which the Nature assign access to the IGs) is common knowledge.

The game described above is played once. There are two features of the model that are important for the results: first, that the IGs cannot lobby the incumbent. This aids tractability, and also allows us to focus on the pure effect of MT on electoral/policy outcomes. We could add competition between the two political candidate if we allowed the incumbent to be lobbied as well. This competition might be one of the contributing factors in the IG's influence. Therefore, it would be difficult to separate the influence of MT from IGs' influence due to competition; Second, IGs only offer a deal on the policy dimension which they care about. This is not only realistic, but also aids tractability. This essentially means that the challenger can either commit to both issues (publicly) or to one issue (via an IG).

Payoffs. Given policy p, an IG earns 1 if its ideal policy is implemented and 0 otherwise, while a voter in group j earns $W_j(p, \alpha)$. The challenger belonging to group j earns $W_j(p, \alpha_c)$ if she wins the election and implements p, and $W_j(p_i, \alpha)$ if she loses where p_i is the policy implemented by the incumbent. The challenger might compromise on her ideal policy in order to win and implement a policy which is closer to her ideal policy; so she may be willing to accept an IG's offer.

Timing. In stage 0, Nature chooses IG l with probability $\pi_l \geq 0$ where $0 \leq \sum \pi_l \leq 1$. Nature's choice is privately revealed to the challenger and the chosen IG. In stage 1, the selected IG makes a private offer, p_l to the challenger. In stage 2, the challenger decides whether to accept or reject the offer. If she rejects then the challenger publicly announces her policy commitment. If she accepts, she promises to implement the offered policy conditional on winning and the IG then endorses the challenger and informs its members of the endorsement. In stage 3, voters form their beliefs after receiving any information about the endorsement and vote either for or against the challenger. The winner is announced and policy is implemented.

Strategies and Equilibrium. Since there is no lobbying cost involved, we assume that the IGs always makes an offer to the challenger when they get access. In that sense, IGs do not take any decision. We also know that the incumbent cannot be lobbied and is a passive player in our model. The challenger decides whether to accept or reject the offered deal from the chosen IG. Let $\gamma = (\gamma_L, \gamma_R, \gamma_A, \gamma_P)$ denote the challenger's *acceptance strategy*, where $\gamma_l \in \{reject, accept\}$. $\gamma_l = accept$ is when the challenger accepts the offer from IG l and $\gamma_l = reject$ when the challenger rejects the offer and publicly commits to one of the four policy choices. Let q denote the challenger's *public announcement strategy*. If one policy is optimal when the challenger rejects the offer from any one of the IGs, then the same policy must be optimal when she rejects an offer from any other IG.[10] If the challenger rejects, then we assume that she publicly commits to her ideal policy p_c if she is indifferent between announcing any of the policy pairs.

At the time of voting, voters might have different information on what the challenger would implement if she won: some voters know the history of the IG's and challenger's move and some do not know. This information (conditional on the history) is denoted by I_j^k where $I_j^k = 1$ if voters k in group j receive information from their representative IG that their IG endorsed the challenger and $I_j^k = 0$ if voters k in group j receive no information. Let $\beta_l = pr(l \mid I_j^k)$ denote the posterior belief of voter k belonging to group j that the IG l got access to the challenger. Voters update their belief about which IG got access to the challenger, according to Bayes' rule. A voter k in group j calculates his expected payoff and votes for the challenger if and only if

[10] The policy the challenger publicly commits to, in case of rejection, depends only on the proportion of votes.

$$\sum_l \beta_l(W_j(p_l;\alpha)) > W_j(p_i;\alpha) \tag{1.2}$$

We call the game described above the MT game. The equilibrium concept is Perfect Bayesian Equilibrium (PBE) in pure strategies. Loosely speaking, an equilibrium consists of strategies γ^* and q^*, and beliefs $\beta^*(\cdot)$ such that: 1) at every decision stage, each agent takes an action that maximises its expected payoff given its belief and the others' behaviour, and 2) beliefs are derived using Bayes' rule whenever possible.

1.3.1 Influence

We have described a model in which IGs are present, and where lobbying involves extracting a policy promise from the challenger in exchange for the IG conveying her commitment to its members (micro-targeting). We first analyse how the introduction of micro-targeting influences the *outcome* of the election, where an *outcome* is a pair of a winner of the election and the policy implemented. Let $O = (w, p)$ be a generic outcome of the MT game, where $w \in \{c, i\}$ is the winner of the election. For every strategy profile σ of the MT game, $O(\sigma) = (O_L(\sigma), O_R(\sigma), O_A(\sigma), O_P(\sigma))$ is the outcome vector of the game under σ where $O_l(\sigma)$ is the outcome conditional on IG l getting access in PBE σ. Throughout the discussion, we use this definition of an *outcome* to measure the influence of lobbying, and focus on determining conditions under which this measure is different in the presence of IGs.

In order to define influence, we study the benchmark case of the MT game in which $\pi_l = 0$ $(\forall l)$, i.e. in which MT is not feasible. Thus, the challenger cannot send group-specific messages. We refer to this special case of the MT game as the no-lobbying game; otherwise whenever we refer to MT game, it is with regard to the game where $\pi_l > 0$ for all l. The working of the no-lobbying game is fairly simple. Since none of the IGs have access to exert influence, the challenger simply decides to publicly commit to one of the four possible policy choices. Given the challenger's public announcement, the voters then decide whether to vote for the challenger or the incumbent by comparing the payoff from the challenger's committed policy and the incumbent's ideal policy. After voting, the winner is announced and her policy is implemented.

Direct influence

Let O^{NL} be an outcome of the no-lobbying game. Then, $O^{NL}(\sigma^{NL}) = \{O_l^{NL}(\sigma^{NL})\}$, where $O_l^{NL}(\sigma^{NL}) = O^{NL}(\sigma^{NL}) : \forall l$. We say that the MT is directly *influential* if $O_l(\sigma) \neq O^{NL}$ for at least one l for some PBE σ of the MT game where $\pi_l > 0$ $(\forall l)$, and that MT is otherwise not directly *influential.* We say that IG l has direct influence when $O_l(\sigma) \neq O^{NL}$ for IG l. The outcome is different when at least one of the winner or the implemented policy is different in the MT game where $\pi_l > 0$ $(\forall l)$ as compared to the no-lobbying game. To study the implication of micro-targeting, we will compare the outcomes of no-lobbying and MT games in Section 1.4. We further examine how the presence of any particular IG affects the direct influence of other IGs. Such influence is called *indirect influence.*

Indirect influence

Voting behaviour depends, among other things, on the access probabilities of IGs, and therefore altering the access probabilities might change voting behaviour, which in turn might affect direct influence. To measure indirect influence, we compare cases in which all the IGs have a positive probability of getting access ($\pi_l > 0$ for all l) with cases in which $\pi_l = 0$ for some (exactly one) IG l. We call the latter case the modified game. We assume that altering the access probability of one IG does not affect the access probabilities of other IGs: so in the modified game, no one gets access when Nature chooses l with probability 0. Given the challenger's ideal policy, her strategies, and the proportion n_j, we just change the access probability of IG l to 0 and analyse how that changes the direct influence of other IGs. We define $O^{-l}(.)$ as the outcome of the modified game in which $\pi_l = 0$ for some (exactly one) l. Formally, let $O_j^{-l}(\sigma^{-l})$ be the outcome of the MT game under σ^{-l} and $\pi_l = 0$ when $j \neq l$ gets access. Then, we say that l has indirect influence if $O_j^{-l}(\sigma^{-l}) \neq O_j(\sigma)$ for at least one $j \neq l$ for some PBE σ of the MT game where $\pi_l > 0$ (all l) and some PBE σ^{-l} of the modified game where $\pi_l = 0$ for some (exactly one) l.

To find indirect influence of IG l on the direct influence of other IGs $j \neq l$, we consider the equilibrium of the MT game where direct influence from each IG is possible. Formally this requires that the challenger accepts offers from all IGs. The reason for this is as follows. Let's look at the indirect influence of IG l on direct influence of other

IGs $j \neq l$. If the challenger rejects an offer from an IG $j \neq l$ then there would be no direct influence from IG j, irrespective of the presence of IG l: presence of l would not have any effect on the direct influence of IG j.[11] Thus, to find indirect influence of IG l on other IGs, the challenger must accept offers from all IGs $j \neq l$. Next, the challenger must also accept the offer from IG l: IG l's presence only matters if l can secure a deal with the challenger when it can get access. If the challenger rejected l's offer when l could get access then voters know that l cannot secure a deal with the challenger, and their vote therefore does not depend on the access probability of l: they expect to see public commitment when l gets access, so l's presence would have no indirect influence. Thus, the challenger must accept the offer from IG l for it to have any indirect influence. *Then the challenger must accept from all IGs for IG l to have any indirect influence.*

1.4 Influential Micro-targeting

In this section, we identify conditions under which micro-targeting is influential. Throughout, the incumbent's ideal policy is fixed at $p_i = (R, A)$. We look at different challengers with different ideal policies as compared to the incumbent's: a) The *semi-alike* case: the challenger differs from the incumbent on only one policy issue e.g. $p_c = (R, P)$ (we focus on this semi-alike case wlog.); b) The *polar* case: the challenger differs from the incumbent on both policy issues, i.e. $p_c = (L, P)$; and c) The *alike* case: the challenger shares policy with the incumbent on both policy issues, i.e. $p_c = (R, A)$. To find direct influence, we assume that each IG has positive probability of getting access ($\pi_l > 0$).

We investigate the influence of MT. We start by discussing direct and indirect influence in the three different cases: 1.4.1) alike, 1.4.2) semi-alike, and 1.4.3) polar. We then interpret and discuss the main results in section 1.5. We seek to characterize the PBEs of the no-lobbying and the MT games. In the no-lobbying game, the challenger publicly commits to one of the four policy pairs. In the MT game, the challenger has to make two decisions: she first decides whether to accept or reject the offer she receives from each IG, and then decides which policy to publicly commit to if she rejects an offer. Note that the challenger does not have to commit to her ideal policy if she rejects an offer.

[11] Note that the challenger's strategy is the same when we compare outcomes of the MT and modified games.

Claim 1.1. *If the challenger rejects an offer from IG l then $O_l = O^{NL}$.*

This is very obvious. The challenger has to publicly commit if she rejects an offer. Then, if she commits to policy p in the no-lobbying game then she would commit to the same policy in the MT game after rejecting an offer. The outcome then would be same in both the no-lobbying and MT games.

1.4.1 Alike case: $p_c = (R, A)$

We first analyse the case in which the challenger and the incumbent are alike. It is easy to see that MT is not influential in this case. The challenger and the incumbent's interests are completely aligned to each other, and therefore, MT by IGs would not change the election/policy outcome. Irrespective of whether the challenger's ideal policy is popular or not, she can always get her ideal policy implemented, either by winning if her ideal policy is popular or by losing otherwise in which case the incumbent wins and implements (R, A). Note that the voter votes for either candidate with equal probability if she is indifferent between voting for the challenger and the incumbent, in which case incumbent wins. In fact, the challenger is indifferent between winning after publicly committing to her ideal policy (R, A) and losing. Thus, the challenger publicly commits to her ideal policy and loses the election, so the outcome in the no-lobbying game is the incumbent wins and implements (R, A).

In the MT game, the challenger would never accept offer from any unlike-minded IG if she were to win after such a private commitment. Compromising on any issue by accepting an offer from an unlike-minded IG is not rational for her because she could profitably deviate to lose the election. Thus, the challenger accepts their offer only if she were to lose, otherwise she rejects and publicly commits to her ideal policy. In any case, the challenger loses the election: the incumbents wins and implements (R, A). Hence, there is no direct influence from unlike-minded IGs. Voters anticipate this and know that irrespective of the IGs presence, the challenger would always implement her ideal policy (R, A) if she wins. Thus, when like minded IGs endorse the challenger, all voters vote as if the challenger publicly committed to her ideal policy: so, the outcome remains the same. Hence, like minded IGs also have no direct influence.

The same reasoning applies to indirect influence. The presence of any IG does not change the fact that the challenger would always implement her ideal policy: so voters' votes do not depend on the access probabilities of IGs. Hence, none of the IGs have indirect influence. The idea is that the challenger does not have an opposition whom voters can choose to vote for if they do not share challenger's ideals. Thus, MT by IGs cannot affect the election/policy outcome, precisely because the challenger does not want to change the outcome.

1.4.2 Semi-alike case: $p_c = (R, P)$.

We now analyse the case in which the challenger and the incumbent are semi-alike: $p_c = (R, P)$ and $p_i = (R, A)$. Lemma 1.1 below identifies necessary conditions for MT to be influential in the semi-alike case.

Lemma 1.1. *If $p_c = (R, P)$ then MT is only influential if $n_{LP} + n_{RP} < 1/2$.*

The proof is in the Appendix. The premise means that the challenger must lose the election if she publicly committed to her ideal policy: she would then receive proportion $n_{LP} + n_{RP}$ of votes.[12] Consequently, the challenger may be motivated to accept offers from one or both unlike-minded IGs. If $n_{LP} + n_{RP} > 1/2$ then there is no equilibrium in which the challenger accepts any offer from the unlike-minded IGs. This is true for a very obvious reason. If $n_{LP} + n_{RP} > 1/2$, the challenger can publicly commit to her ideal policy (R, P) and get the highest possible payoff, i.e. $1 + \alpha_c$. Accepting an offer from an unlike-minded IG means implementing a policy which is different from (R, P), earning a lower payoff. The premise implies that if the above condition did not hold, the challenger would reject all offers or only accept the offer from some like-minded IG(s) in every equilibrium.

Observation 1.1. *The like-minded IGs have no direct influence.*

This happens for two reasons: 1. The hate (for the challenger) of the group of voters who are completely opposed to the challenger's ideal policy, i.e voters with policy preference (L, A). These voters are the members of unlike-minded IGs L and A and would never vote for the challenger unless their representative IG endorses the challenger.[13]

[12] Refer to Table 1.2 in the appendix to see the proportion of votes the challenger gets for each of the four public policy commitment.

[13] In which case some might vote for the challenger and some vote for either candidate with equal probability depending on which IG they belong to. The promise the IG L gets is (L, P). The members of L who see the endorsement know that the challenger implements their ideal policy on the economy and the incumbent implements her ideal policy on guns. Since they care more about the economy, they

Thus, these voters do not vote for the challenger when they neither see public commitment from the challenger nor an endorsement from their IG and expect the challenger to accept like-minded IGs' offers in equilibrium. 2. The loyalty (to the incumbent) of the group of voters who completely agree with the incumbent's ideal policy, i.e voters with policy preference (R, A). Again, these voters vote for the incumbent since her ideal policy gets them the highest payoff. Similarly, these voters also vote for the incumbent when they neither see public commitment from the challenger nor an endorsement from their IG and expect the challenger to accept an offer from any other IG. When like-minded IGs endorse the challenger, the voters in groups (L, A) and (R, A) vote for the incumbent and the challenger can only get votes from the voters in groups (R, P) and (L, P). Thus, the maximum vote the challenger can get when like-minded IGs endorse the challenger is $n_{LP} + n_{RP}$.

From Lemma 1.1, we know that in any influential equilibria, we have $n_{LP}+n_{RP} < 1/2$ and therefore, in any influential equilibrium, whenever like-minded IGs endorse the challenger, the challenger loses the election. Hence, like-minded IGs have no direct influence.

Influence might depend on whether the challenger cares more about the economy or guns. Therefore, we have two cases to consider: the semi-alike case in which the challenger cares more about the economy, and the semi-alike case in which the challenger cares more about guns.

Semi-alike case in which the challenger cares more about guns. The challenger cares more about the issue where she has a different ideal policy from the incumbent: $p_c = (R, P)$ and $\alpha_c > 1$.

We first specify the equilibrium outcome of the no-lobbying game. The challenger's public commitment depends on the share of votes she gets from each commitment and whether it constitutes a majority of votes. Since $\alpha_c > 1$, the challenger's policy preference order is: her ideal policy is (R, P); her second best policy is (L, P); her third best policy is (R, A); and the least preferred policy is (L, A). The idea is that the challenger would publicly commit to her ideal policy if that attracted enough votes to win. If not,

vote for the challenger when they receive information about policy from their representative IG L. The promise that IG A gets is (R, A), which is same as the incumbent's ideal. Then, the members of A vote for either candidate with equal probability when they get a policy promise through their representative IG.

she commits to her second best policy if it is popular among the voters. If that is not the case, then she commits to her third best policy (R, A) in which case she loses and gets her third best payoff. Note that her reservation utility is 1, which she can always get by publicly committing to the incumbent's ideal policy. Thus, any policy which gives her less than 1 will never be implemented. This is formally given below:

Outcome in no-lobbying game when $p_c = (R, P)$ *and* $\alpha_c > 1$. The outcome is:

$$O^{NL} = \begin{cases} (c, (R, P)) & \text{if } n_{LP} + n_{RP} > 1/2 \\ (c, (L, P)) & \text{if } n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2 > n_{LP} + n_{RP} \\ (i, (R, A)) & \text{otherwise} \end{cases}$$

The proof is in the Appendix. The following Lemmas identify necessary conditions for MT to be influential.

Lemma 1.2. *Let* $p_c = (R, P)$ *and* $\alpha_c > 1$*. MT is directly influential only if* $n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} < 1/2$*.*

The proof is in the Appendix. Lemma 1.2 is a direct consequence of Lemma 1.1. Since the challenger cares more about guns, (L, P) is the challenger's second best policy. The premise then means that the challenger's second best policy must be unpopular among voters. We already know from Lemma 1.1 and Observation 1.1 that, in any influential equilibrium, the challenger's ideal policy is not popular and that like-minded IGs have no direct influence. Hence, only unlike-minded IGs L and A can have direct influence. For this to happen, the challenger must accept one or both offers from unlike-minded IGs, changing the outcome.[14]

Suppose by way of contradiction that $n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2$, i.e. the challenger were to win if she publicly committed to (L, P). Then the outcome of the no-lobbying game is that the challenger wins and implements (L, P). We show that in the MT game the equilibrium outcome when L endorses the challenger is the same as O^{NL}. Now, the equilibrium in which the challenger accepts L's offer exists only if the challenger were to win after accepting L's offer.[15] Thus, the equilibrium outcome when L endorses

[14] When the challenger rejects any offer, the outcome is same as in the no-lobbying game.

[15] If the challenger loses after accepting L's offer then the incumbent wins and implements (R, A), in which case she earns 1. If that is the case then the challenger could profitably deviate to reject L's offer, and publicly commit to (L, P) because $\alpha_c > 1$. Note that the challenger in both cases (no-lobbying and L's endorsement) commits to (L, P), but she could lose when L endorses the challenger because she

the challenger is $O_L = O^{NL} = (c,(L,P))$. Therefore, L has no direct influence; and only unlike-minded IG A could be influential. But then the challenger would not accept A's offer because if she accepted she would have to implement the incumbent's ideal policy, whereas she could do better by rejecting, announcing her second best policy, and winning. Thus, A is not influential either. As a consequence, MT can only be influential if the challenger's second best policy is also unpopular among the voters: $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$ is the proportion of votes the challenger gets if she publicly commits to (L,P).

Lemma 1.3. *Let $p_c = (R,P)$ and $\alpha_c > 1$. Unlike-minded IG L can only have direct influence if the challenger accepts the offer from like-minded IG P.*

The proof is in the Appendix. We know from Observation 1.1 that like-minded IGs do not have direct influence. Therefore, only unlike-minded IGs L and A can have direct influence. We also know that if the challenger rejects an offer from an IG then that IG cannot have direct influence. Thus, unlike-minded IGs can exert direct influence only if the challenger accepts their offers.

Interestingly, L can only have direct influence if the challenger accepts the offer from like-minded IG P in equilibrium. P does not have direct influence but its presence and the possibility that its offer would be accepted affect the direct influence of other unlike-minded IGs. When L endorses the challenger, it convinces its members in group LA to vote for the challenger instead of for the incumbent. Members of L in group LP vote for the challenger anyway since they dislike the incumbent's ideal policy.[16] All the members of L then vote for the challenger if L's offer is accepted. We know from Observation 1.1 that all members of P and A vote for the challenger and the incumbent respectively, when they are uninformed.[17] If the members of R with ideal policy (R,P) do not vote for the challenger then the challenger gets proportion $n_{LP}+\delta n_{RP}+(1-\delta)n_{LA}$ of votes, which is less than half by Lemma 1.2. Thus, L cannot have direct influence. The only way the challenger could win is if she can get some votes from the members

privately commits to (L,P) when L endorses the challenger, whereas in no-lobbying game she publicly commits to (L,P).

[16]Note that the voters with ideal policy (L,A) least prefer the challenger's ideal policy and vote for the incumbent unless they a get policy commitment through their representative IG.

[17]Members of P with ideal policy (L,P) vote for the challenger because they the hate incumbent's ideal policy and those with ideal policy (R,P) also vote for challenger because they care more about guns and only the challenger implements their ideal policy on guns. Members of A with ideal policy (L,A) vote for the incumbent because they hate the challenger's ideal policy and those with ideal policy (R,A) also vote for the incumbent because her ideal policy is same as their ideal policy.

of IG R in group RP[18], which could only happen if the challenger accepted P's offer in the equilibrium. This creates enough incentives for a member of IG R in group RP to vote for the challenger after updating their beliefs in the event that his IG did not get access. These are the groups of voters whose preferences are completely aligned with the challenger. But they put more weight on the issue on which they share policy with the incumbent. That is why their vote depends on what they expect the challenger to do, which in turn determines the policy the challenger would implement.

Proposition 1.1 below identifies the regions of the parameter space in which both unlike-minded IGs have direct influence. For each of the parameter configurations satisfying the conditions in the Proposition, equilibrium behaviour satisfies the necessary condition of Lemma 1.3 for both unlike-minded IGs to have direct influence.

Proposition 1.1. *Let $p_c = (R, P)$ and $\alpha_c > 1$. Both unlike-minded IGs L and A could have direct influence*

P1.1 L has direct influence iff $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ *and* $n_{LP} + (1-\delta)n_{LA} + n_{RP} > 1/2$

P1.2 A has direct influence iff $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ *and* $\frac{1}{2}\delta(n_{LA} + n_{RA}) + n_{RP} + n_{LP} > 1/2$ *or* $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$ *and* $\frac{1}{2}\delta(n_{LA} + n_{RA}) + \delta n_{RP} + n_{LP} > 1/2$

The proof is in the Appendix. To see how unlike-minded IGs have direct influence, we need to first look at the equilibrium outcome in the no-lobbying game. Lemmas 1.1 and 1.2 must hold if MT is directly influential. If Lemmas 1.1 and 1.2 hold then the outcome in the no-lobbying game is that the incumbent wins and implements her ideal policy: $O^{NL} = (c, (R, A))$.

Whenever micro-targeting is influential, lobbying by IGs L and A changes the outcome. This requires that on getting access, L and A's offers are both accepted and the challenger wins the election. As discussed above in Lemma 1.3, this also requires that the challenger accepts P's offer in equilibrium for L to have direct influence.

When L gets a policy commitment from the challenger,[19] it informs its members about the endorsement. The members of IG L then know for sure which policy the challenger would implement, whereas the members of other IGs do not have this information. As a result, all the members of L vote for the challenger when they see that their IG endorses the challenger. The members of IG P, when unsure of the challenger's

[18]The members of IG R in group RA vote for the incumbent because their ideal policy is completely aligned with the incumbent's.

[19]The challenger implements policy (L, P) if she accepts L's offer and wins.

policy commitment, vote for the challenger if they expect the challenger to accept the offer from at least one of L or R. The challenger implements her ideal (pro-gun) policy on guns when L or R get access. All members of P are pro-gun and care more about guns, whereas the incumbent is anti-gun. The exact opposite is true for the members of IG A, who do not vote for the challenger if they expect the challenger to accept an offer from L or R. This gives the challenger a vote share of $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$, which is less than half by Lemma 1.2. The challenger can get extra votes from the some members of IG R.[20] For these voters, the challenger's policy is better than the incumbent's if P endorsed the challenger; and vice-versa if L endorsed the challenger. Condition P1.1 guarantees that those members of R have a large enough incentive to vote for the challenger. When $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$, the voters in group R do not vote for the challenger and thus the challenger gets exactly the same number of votes as she would get by publicly announcing (L, P). Hence, condition P1.1 is necessary for the challenger to get extra votes. This condition basically means that P's probability of access is large enough compared to L's. How much larger it need be depends on the value of $\underline{\alpha}$.

Condition P1.2 guarantees that the challenger wins the election when unlike-minded IGs L or A endorses the challenger. The terms in condition P1.2 are the respective share of votes the challenger gets when L or A endorses her. The challenger needs more than half of the total votes to win the election. Hence, condition P1.2 is also necessary.

To summarise, the equilibrium outcome of the game without IGs is:

- The incumbent wins the election and implements her ideal policy

$$O^{NL} = (i, (R, A)) \tag{1.3}$$

The equilibrium outcome of the game with IGs is:

- The challenger accepts L's offer.
  - Since condition P1.1 is satisfied, the challenger gets share $n_{LP} + (1-\delta)n_{LA} + n_{RP}$ of the total votes.
  - Since condition P1.2 is satisfied, the challenger wins and implements policy (L, P). Thus, we have $O_L = (c, (L, P))$: the outcome when L endorses the

[20] Some members of IG R who belong to group RA vote for the incumbent unless they get assurance from their representative IG, in which case they vote for either candidate with equal probability. Other members of R who belong to group RP might vote for the challenger.

challenger.

- The challenger accepts A's offer.
  - Since condition P1.1 is satisfied, the challenger gets $\frac{1}{2}\delta(n_{LA}+n_{RA})+n_{RP}+n_{LP}$ of the total votes.
  - Since condition P1.2 is satisfied, the challenger wins and implements policy (R, A). Thus, we have $O_A = (c, (R, A))$: the outcome when A endorses the challenger.
- The challenger loses when R or P get access. Thus, we have $O_R = O_P = (i, (R, A))$: the outcome when R or P endorses the challenger.

Let O^{MT} be the outcome vector in the MT game: $O^{MT} = (O_L, O_R, O_A, O_P)$. There is direct influence because $O^{NL} \neq O_l$ for $l = \{L, A\}$.

We now discuss indirect influence when the challenger in the semi-alike challenger cares more about guns.

Proposition 1.2. *Let $p_c = (R, P)$, $\alpha_c > 1$, and $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$. Only like-minded IG P has indirect influence. IG P's presence affects the:*
2.1. Direct influence of IG L iff $n_{LP} + (1-\delta)n_{LA} + n_{RP} > 1/2$
2.2. Direct influence of IG A iff $\frac{1}{2}[\delta(n_{LA} + n_{RA})] + n_{LP} + n_{RP} > 1/2 > \frac{1}{2}[\delta(n_{LA} + n_{RA})] + n_{LP} + \delta n_{RP}$

The proof is in the Appendix. The presence of IG P affects the direct influence of unlike-minded IG L in that IG L does not have direct influence if $\pi_P = 0$. The presence of P may also affect the direct influence of unlike-minded IG A. The purpose of MT is to privately promise a policy to a subset of voters to get their votes, and let the other voters be uninformed. Some of these uninformed voters might vote for the challenger but would have voted against the challenger if they knew which IG endorsed the challenger. The uninformed voters' behaviour depends on their expectation about play: in particular, whether other IGs are able to exert direct influence and whether other IGs are able to get access to the challenger. Thus, the direct influence of a particular IG might depend on how uninformed voters vote, which in turn might depend, among other things, on the access probability of other IGs. In this case, P's presence is important for these uninformed voters to vote for the challenger. We show that it is only one group of voters whose vote depends on the access probability of IG P.

Voters in group LP always vote for the challenger as long as one of IGs L, R, or P have a positive probability of getting access: they are anti-incumbent and vote for the challenger if they believe that the challenger implements something different from (R, A) with positive probability. Voters in group RA always vote for the incumbent unless the challenger publicly commits to (R, A) or only A has positive probability of getting access.[21] Voters in group LA always vote for the incumbent: voters in this group who care more about the economy belong to IG L and any of the policies the challenger implements when their IG did not get access is either worse than (R, A) or is (R, A); voters who care more about guns belong to A and their second best policy is (R, A).[22] Voters in group RP who belong to group P vote for the challenger when their IG did not get access: any of the policies the challenger implements is either better than or the same as the incumbent's ideal policy. Now, the crucial group of voters in group RP are the voters who are members of IG R who share the challenger's ideal policy but the incumbent's ideal policy is their second best policy. Their vote is crucial for the challenger to win. They would vote for the challenger when their IG did not get access if they believed that the challenger is likely to implement a policy closer to their ideal. When their IG did not get access, they know that the challenger's implemented policy may be worse than the incumbent's (if L endorses the challenger and the challenger wins). But they are willing to take that risk only in the presence of IG P, which gives them more confidence to vote for the challenger. However, if P cannot get access, they know with certainty that the challenger's implemented policy would be worse than the incumbent's.

It is interesting to see that even though P does not have direct influence, the mere fact that it might get access (and extract a policy commitment) can influence the policy and/or election outcomes. This is equivalent to the necessary condition in Lemma 1.3, which requires that the challenger must accept the offer from like-minded IG P for unlike-minded IG L to have direct influence. IG P's indirect influence turns out to favour IG P in the sense that there is some chance that P's ideal policy is implemented if P is active and could possibly get a policy commitment from the challenger when

[21] The challenger would implement (R, A) only if either A endorses the challenger or if the challenger publicly commits to RA. The challenger implements (L, P) if L endorses the challenger, (R, P) if R or P endorses the challenger.

[22] Note that the challenger never implements (L, A).

pro-gun policy is not popular. That happens when L has direct influence, in which case the challenger implements her ideal policy on guns.

We now consider cases in which $\frac{\pi_L}{\pi_P}$ is large.

Proposition 1.2a. *Let $p_c = (R, P)$, $\alpha_c > 1$ and $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. Only like-minded IG L has indirect influence. L's presence affects the direct influence of IG A iff $\frac{1}{2}[\delta(n_{LA}+n_{RA})]+n_{LP}+n_{RP} > 1/2 > \frac{1}{2}[\delta(n_{LA}+n_{RA})]+n_{LP}+\delta n_{RP}$.*

Given, $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$, Proposition 1.1 implies that unlike-minded IG L does not have direct influence: only unlike-minded IG A could have direct influence. Suppose that unlike-minded A has direct influence in the presence of all IGs. In that case, the presence of unlike-minded IG L does not affect A's direct influence. In the absence of L, the challenger (after accepting A's offer) gets more votes than what she gets in the presence of L. Thus, if the challenger wins (after accepting A's offer) in the presence of L, she will definitely win in the absence of L. Therefore, L could only have indirect influence if A did not have direct influence when $\pi_L > 0$: A could have direct influence if $\pi_L = 0$. This would happen if the challenger's vote share (after accepting A's offer) were less than a half when $\pi_L > 0$ and more than a half when $\pi_L = 0$. Hence, the condition in the premise must be satisfied.

Again, the critical set of voters are the members of IG R with ideal policy (R, P). Their vote is based on their expectation about the challenger's decision and the access probabilities of other IGs. In this case, these voters do not vote for the challenger when $\pi_L > 0$ because $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$, which essentially mean that L's access probability is higher than R's and their weight on guns $\underline{\alpha}$ is too low to risk voting for the challenger. But if L cannot get access then they know that whoever endorsed the challenger, the implemented policy would either be better than or the same as the incumbent's. Hence, they vote for the challenger.

Note that the indirect influence of A does not bring a change in the policy outcome. If A has direct influence, policy (R, A) will be implemented; and the same policy will be implemented otherwise.

The challenger in the semi-alike challenger who cares more about the economy: the policy on which the challenger and the incumbent agree. Specifically, $p_c = (R, P)$ and $\alpha_c < 1$.

Outcome in the no-lobbying game when $p_c = (R, P)$ and $\alpha_c < 1$. The outcome is:

$$O^{NL} = \begin{cases} (c, (R, P)) & \text{if } n_{LP} + n_{RP} > 1/2 \\ (i, (R, A)) & \text{otherwise} \end{cases}$$

The proof is in the Appendix.

Proposition 1.3. *Let $p_c = (R, P)$ and $\alpha_c < 1$. Unlike-minded IG L does not have direct influence.*

The proof is in the Appendix. Given Lemma 1.1, the outcome in the no-lobbying game is that the incumbent wins and implements her ideal policy, $O^{NL} = (i, (R, A))$.

The challenger would only accept the offer from unlike-minded IG L if she were to otherwise lose the election. If she wins by accepting L's offer, she gets a payoff of α_c and therefore she could profitably deviate to publicly commit to (R, P) or (R, A), in which case she loses and gets a payoff of 1. The challenger can always get her second highest payoff by losing the election if her ideal policy is not popular among the voters. Thus, unlike-minded IG L does not have direct influence. Compromising on the issue which she cares about more and agree with the incumbent is not rational.

Proposition 1.4. *Let $p_c = (R, P)$ and $\alpha_c < 1$. Unlike-minded IG A could then have direct influence if the challenger accepts the offer from A and the offer from at least one of L or R.*

The proof is in the Appendix. If the challenger accepts A's offer then she would commit to implement (R, A), i.e. IG A's ideal policy on guns and her own ideal policy on the economy. Note that the challenger is indifferent between accepting and rejecting A's offer. If she accepts A's offer then she gets a payoff of 1 irrespective of whether she wins or loses. If she rejects A's offer then the outcome is same as in the the no-lobbying game: her payoff is 1.

Since the challenger is indifferent, she might accept A's offer in some instances. If she wins after accepting A's offer then the outcome would be $(c, (R, A)$, giving A direct influence. In the Appendix, we prove that the challenger might win after accepting A's offer. The policy implemented would remain the same in the MT and the no-lobbying games. The only difference is that in the presence of IGs, the challenger might win.

In some sense, the direct influence of IG A has no significance. Provided that the challenger cares more about the economy and her ideal policy is unpopular, IG A anticipates that her ideal policy is always implemented, regardless of who wins.

Indirect influence. When the challenger cares more about the economy, only unlike-minded IG A could have direct influence. Then, for indirect influence, we see how the presence of IGs P, L and R affects A's direct influence.

Proposition 1.5. *Let* $p_c = (R, P)$*,* $\alpha_c < 1$ *and* $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$*. None of the IGs except for P have indirect influence. P affects A's direct influence iff* $\frac{1}{2}[\delta(n_{LA}+n_{RA})]+n_{LP}+n_{RP} > 1/2 > \frac{1}{2}[\delta(n_{LA}+n_{RA})] + n_{LP} + \delta n_{RP}$*.*

The argument and the reasoning are the same as in Proposition 1.2a. But in this case, the challenger puts more weight on the economy; so the unlike-minded IG L has no direct influence irrespective of the presence of any IG. Therefore, the presence of any IG does not affect L's direct influence. However, P could affect the direct influence of unlike-minded IG A in the same manner as discussed in Proposition 1.2a. There is a group of voters who vote for the challenger only in the presence of P, given $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$. If P cannot get access, they do not vote for the challenger in the absence of endorsement from their IG. Then if A had direct influence and losing these voters meant losing majority then P would have indirect influence.

Note that the indirect influence of P only affects who wins the election and not the policy outcome. If the challenger's ideal policy is not popular then the incumbent's ideal policy would be implemented in both the no-lobbying game and MT game because like-minded IGs cannot have direct influence and (R, A) is the challenger's second best policy.[23]

Proposition 1.5a. *Let* $p_c = (R, P)$*,* $\alpha_c < 1$ *and* $\frac{\alpha}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$*. None of the IGs except for L have indirect influence. L affects A's direct influence iff* $\frac{1}{2}[\delta(n_{LA}+n_{RA})]+n_{LP}+n_{RP} > 1/2 > \frac{1}{2}[\delta(n_{LA}+n_{RA})] + n_{LP} + \delta n_{RP}$

Given $\frac{\alpha}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$, members of R with policy preference (R, P) do not vote for the challenger in the presence of all IGs. But they vote for the challenger in the absence of L. Thus, if $\pi_L = 0$ then they vote for the challenger $\frac{\alpha}{1-\underline{\alpha}} < 0$: so, if A did not have direct

[23]The only case in which the challenger can implement something better than the incumbent's ideal is when the like-minded IGs have direct influence.

influence in the presence of all IGs and gaining these voters meant forming majority in the absence of L then L would have indirect influence.

1.4.3 Polar case: $p_c = (L, P)$

We now investigate the polar case. The challenger and the incumbent top-rank different policies on both issues. The worst outcome for the challenger is if the incumbent wins the election, in which case the challenger's least preferred policy would be implemented. Thus, she would be willing to compromise on any single issue if that results in a win, as she could then implement a policy which differs from the incumbent's ideal. Therefore, we expect to find more influence in the polar case, compared to the semi-alike case. In other words, we expect to see less restrictive conditions for an influential MT equilibrium to exist. Lemma 1.4 below identifies a necessary condition for an influential MT.

Lemma 1.4. *Let $p_c = (L, P)$. MT is influential only if $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2$.*

The proof is in the Appendix. Following the same argument as in the semi-alike case, the premise ensures that the challenger's ideal policy is unpopular among the voters, and the challenger has enough incentives to accept offers from unlike-minded IGs. The condition implies that if the challenger were to publicly commit to her ideal policy (L, P), she would lose the election. If her ideal policy is popular among the voters, the challenger would never accept offers from any of the unlike-minded IGs and would only accept offers from like-minded IGs if she were to win. In the no-lobbying game, the challenger publicly commits to her ideal policy because she wins and gets the highest payoff. The share of votes she gets in the presence of IGs would be same as in the no-lobbying game: thus, the outcomes in both the games would be same. Hence, no IGs have direct influence if $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2$.

Observation 1.2. *The like-minded IGs have no direct influence.*

The argument is similar to Observation 1.1. We start by analysing two cases. First, consider the case where the challenger's ideal policy is popular among voters, i.e. $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2$. We know from Lemma 1.4 that in that case none of the IGs would have direct influence. Second, consider the case where the challenger's ideal policy is not popular among the voters; i.e $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2$. We then just need to show that the challenger can get a maximum of $n_{LP} + \delta n_{RP} + (1-\delta) n_{LA}$ of the total votes when like-minded IGs L or P endorses the challenger.[24]

Observe that $n_{LP}+\delta n_{RP}+(1-\delta)n_{LA}$ is the challenger's vote share if all the members of like-minded IGs vote for the challenger.[25] Therefore, when L or P endorses the challenger, the challenger must get votes from some members of unlike-minded IGs R and A to win, else there is no direct influence. What happens is that none of the members of unlike-minded IGs vote for the challenger when they are uninformed about the challenger's policy commitment. This happens for two reasons: 1. The strong support of some of the voters, who are members of unlike-minded IGs, for the incumbent. These are the members whose policy preferences match perfectly with the incumbent. They never vote for the challenger even if their IG endorses the challenger because the challenger only changes her policy position partially, and 2. The strong intensity of preference of some of the voters, who are members of unlike-minded IGs, for the incumbent's ideal. These members of an unlike-minded IG care more intensely about the policy, which is perfectly aligned with the incumbent's policy; and the challenger implements these policies only when the unlike-minded IGs endorse the challenger. But when unlike-minded IGs do not get access, a member of unlike-minded IGs knows that the challenger would not implement his ideal policy on the issue he cares about more. Thus, these voters do not vote for the challenger when the like-minded IG endorses the challenger.

As in the semi-alike case, we need to consider cases of the challenger who cares more about the economy and the challenger who cares more about guns. We discuss them one by one.

Challenger in the polar case who cares more about guns. Specifically, $p_c = (L, P)$ and $\alpha_c > 1$: so the challenger's policy preference order is: her ideal policy is (L, P); her second best policy is (R, P); her third best policy is (L, A); and her least preferred policy is (R, A). Below is the equilibrium outcome of the no-lobbying game.

Outcome in no-lobbying case when $p_c = (L, P)$ and $\alpha_c > 1$. The outcome is:

[24] Note that if the challenger rejects any offer then the outcome is same as in the no-lobbying game. Thus, there is no change in the outcome when IG l gets access and the challenger rejects l's offer.

[25] The members of L are voters in group LA and LP with $\alpha = \underline{\alpha}$. The members of P are voters in group RP and LP with $\alpha = \overline{\alpha}$. δ proportion of the voters have $\alpha = \overline{\alpha}$ and $1-\delta$ proportion of the voters have $\alpha = \underline{\alpha}$. If all the members of L and P vote for the challenger, the challenger gets a vote share of $(1-\delta)(n_{LA} + n_{LP}) + \delta(n_{RP} + n_{LP})$.

$$O^{NL} = \begin{cases} (c,(L,P)), & \text{if } n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2 \\ (c,(R,P)), & \text{if } n_{LP} + n_{RP} > 1/2 > n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} \\ (c,(L,A)), & \text{if } n_{LP} + n_{LA} > 1/2 \quad \text{and} \quad max\{n_{LP} + n_{RP}, n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}\} < 1/2 \\ (i,(R,A)), & \text{otherwise} \end{cases}$$

The proof is in the Appendix. The following Lemma identifies a necessary condition for MT to be influential in the polar case when the challenger cares more about guns.

Lemma 1.5. *Let $p_c = (L, P)$ and $\alpha_c > 1$. MT is influential only if $n_{LP} + n_{RP} < 1/2$.*

The proof is in the Appendix. For the same reasons mentioned in the semi-alike case, the challenger's second best policy must be unpopular among the voters. Suppose by way of contradiction that the challenger's second best policy is popular. The challenger would then reject offers from L, P and A. If she accepted A's offer she would be committed to implement her third best policy, whereas she could do better by rejecting and publicly committing to her second best policy. If she accepts offers from L and P then she loses, because we know from Observation 1.2 that like-minded IGs do not have direct influence. Thus, she could deviate by publicly committing to her second best policy. She would accept R's offer only if she were to win, otherwise she has a profitable deviation to publicly committing to (R, P).

In the no-lobbying game, the challenger commits to her second best policy and receives her second highest payoff. In the MT game, the outcome would remain the same. Hence, there is no direct influence if $n_{LP} + n_{RP} > 1/2$.

Direct influence of unlike-minded IGs R and A. From Lemmas 1.4 and 1.5, we know the challenger's ideal and second best policies must be unpopular among the voters for MT to be influential. Given this, we consider two cases: 1. When the challenger's third best policy (L, A) is popular i.e. $n_{LP} + n_{LA} > 1/2$, and 2. When (L, A) is unpopular, i.e. $n_{LP} + n_{LA} < 1/2$: $n_{LP} + n_{LA}$ is the share of votes the challenger gets if she publicly commits to (L, A).

Case 1: Influential MT when the challenger would lose in the no-lobbying game. If the challenger cannot win with her ideal policy or her second best policy then she could win with her third best policy (L, A), which is better than incumbent's ideal.

She publicly commits to policy (L, A) if $n_{LA} + n_{LA} > 1/2$; otherwise she is indifferent between announcing any of the policies because in any case she loses. Then in case 1, the outcome in the no-lobbying game is that the incumbent wins and implements (R, A), i.e. $O^{NL} = (i, (R, A))$.

In case 1, there could possibly exist four influential equilibria in which both the unlike-minded IGs have direct influence: the challenger accepts from both the unlike-minded IGs and either accepts or rejects from like-minded IGs. In contrast to the semi-alike case, both the unlike-minded IGs can have direct influence even if the challenger rejects offers from both like-minded IGs. Proposition 1.6 describes the equilibrium in which the challenger rejects offers from both like-minded IGs. The Supplementary Appendix describes the other three influential equilibria in which both unlike-minded IGs have direct influence, where the analysis is the same but the conditions for influence differ among different equilibria.

Proposition 1.6. *Let $p_c = (L, P)$, $n_{LP} + n_{LA} < 1/2$ and $\alpha_c > 1$. If the challenger rejects offers from both the like-minded IGs then R and A have direct influence if and only if each of the following conditions hold:*
(P6.1) $\underline{\alpha} < \frac{\pi_A}{\pi_R} < 1$ *or* $\overline{\alpha} > \frac{\pi_A}{\pi_R} > 1$ *and,*
(P6.2) $min\{n_{LP} + n_{RP} + (1 - \delta)n_{LA}, n_{LP} + n_{LA} + \delta n_{RP}\} > 1/2$.

The proof is in the Appendix. Lemmas 1.4 and 1.5, and given condition $n_{LP}+n_{LA} < 1/2$ imply that the challenger loses if she publicly commits to (L, P), (R, P) or (L, A). Then, the outcome in the no-lobbying game is that the incumbent wins and implements (R, A). Since the challenger loses in the no-lobbying game, there exists an equilibrium in which the challenger accepts offers from both R and A.[26] If she accepts R's [resp.A's] offer, she: either wins, in which case she would implement (R, P) [resp.(L, A)] and has no profitable deviation to public commitment by Lemma 1.3 [resp.Lemma 1.4]; or she loses, in which case she cannot profitably deviate (to a public announcement) as she loses anyway.

Condition P6.1 guarantees that the challenger gets more votes than she would get with any public commitment when she accepts offers from R and A.[27] Now we see how different groups of voters vote. The voters in group LP vote for the challenger unless

[26]The challenger must accept offers from R and A in order for them to exert direct influence.
[27]She loses if she publicly commits to any policy.

they see a public commitment of (R, A) because their preferences are perfectly aligned with the challenger: they are anti-incumbent.[28] The opposite holds true for the voters in group RA. The preferences of voters in group LA and RP are partially aligned with each of the candidates. They either belong to like-minded IG or unlike-minded IG.[29] Their vote depends on whether their IG endorses the challenger and on their beliefs when their IG did not get access. Note that the challenger is committed to implement (L, A) [resp.(R, P)] if she accepts A's [resp.R's] offer and publicly commit if L or P gets access. Thus, if the challenger does not publicly commit, it must be that she would either implement (L, A) or (R, P).

The voters in group LA who are members of A, vote as follows: they vote for the challenger if their IG endorses the challenger and vote for the incumbent if their IG did not get access and they see no public commitment because their second best policy (R, A) is better than (R, P). The voters in group LA who are members of L vote for the challenger if their IG did not get access and they see no public commitment if they believe that the challenger is more likely to implement their ideal policy ($\pi_A > \pi_R$) or the payoff they derive from the policy they share with the incumbent is low enough, i.e. $\underline{\alpha} < \frac{\pi_A}{\pi_R} < 1$.

The voters in group RP who are members of R vote as follows: they vote for the challenger if their IG endorses the challenger; and for the incumbent if their IG did not get access and they see no public commitment because their second best policy (R, A) is better than (L, A). The voters in group RP, who are members of P, vote for the challenger if their IG did not get access and they see no public commitment if they believe that the challenger is more likely to implement their ideal policy ($\pi_A < \pi_R$) or the payoff they derive from the policy they share with the challenger is high enough, i.e. $\overline{\alpha} > \frac{\pi_A}{\pi_R} > 1$.

We have established that voters in group LP and RA vote for the challenger and the incumbent respectively. The voters in group RP and LA who are members of unlike-minded IGs only vote for the challenger if their IG endorses the challenger; and those who are members of like-minded IGs vote for the challenger if and only if the conditions mentioned above are satisfied.

[28]The challenger only implements (R, A) when she publicly commits to it.

[29]The voters in group LA who care more about the economy belong to L; others belong to A. The voters in group RP who care more about the economy belong to R; others belong to P.

If R gets access then the voters in group LA who are members of L are crucial for R's direct influence. If these voters do not vote for the challenger, the maximum vote share the challenger can get (when she accepts R's offer) is $n_{LP} + n_{RP}$, which is less than a half by Lemma 1.5. For A to exert direct influence, the crucial set of voters are in group RP who are members of R. If these voters do not vote for the challenger, the maximum vote share the challenger can get (when she accepts A's offer) is $n_{LP} + n_{LA}$, which is less than a half by the given condition in the premise. Hence, condition P6.1, which is necessary for these group of voters to vote for the challenger.

A [resp. R] would only have direct influence if the challenger wins after accepting its offer. When the conditions in P6.1 are satisfied, the challenger gets more votes than she could get with a public commitment; so, the challenger wins if the vote share the challenger gets after accepting A's [resp. R's] offer is more than 1/2. Hence, condition P6.2 must be satisfied: where $n_{LP} + n_{RP} + (1 - \delta)n_{LA}$ is the vote share the challenger gets when R endorses the challenger, and $n_{LP} + n_{LA} + \delta n_{RP} > 1/2$ is her vote share when A gets endorses the challenger.

Case 2: Influential MT when the challenger wins in the no-lobbying game. We now analyse the case where the challenger wins when she publicly announces policy (L, A) , i.e. $n_{LA} + n_{LP} > 1/2$. In the no-lobbying game, the challenger announces (L, A) and wins the election: $O^{NL} = (c, (L, A))$.

Proposition 1.7. *Let $p_c = (L, P)$, $n_{LP} + n_{LA} > 1/2$ and $\alpha_c > 1$. There is a unique influential equilibrium in which the challenger rejects the offers from both like-minded IGs and accepts the offers from both unlike-minded IGs. In such an equilibrium only unlike-minded IG R has direct influence iff $\frac{\pi_A}{\pi_R} > \underline{\alpha}$ or $\frac{\pi_A}{\pi_R} > 1$ and $n_{LP} + n_{RP} + (1 - \delta)n_{LA}$.*

We know from Observation 1.2 that like-minded IGs have no direct influence; so the challenger loses when she accepts an offer from like-minded IGs. If she accepts offers from like-minded IGs then the incumbent wins and implements (R, A), which is the challenger's least preferred policy. Given $n_{LP} + n_{LA} > 1/2$, the challenger will never accept offers from like-minded IGs because she loses and gets a payoff of 0. But she could do better by deviating to publicly commit to (L, A) because she wins, which gives her a payoff of 1. Hence, the challenger rejects the offers from both the like-minded IGs.

The challenger must accept offers from at least two IGs for any IGs to have direct influence. If the challenger rejects offers from more than two IGs then voters can correctly infer what policy the challenger implements if they neither see an endorsement from their IG nor the public commitment from the challenger. Suppose the challenger accepts an offer from l, and rejects offers from all other IGs $l' \neq l$. In such an equilibrium, when l gets access members of l expect their IG to endorse the challenger and members of other IGs expect to see neither an endorsement from their IG nor a public commitment. Thus, all voters know that it is l who got access when l gets access. If the challenger were to win after agreeing to implement l's ideal policy then she would also win by publicly committing to it. Then, l would have no direct influence. Thus, the challenger must accept offers from at least two IGs. In this case, the challenger rejects the offers from both the like-minded IGs; so unlike-minded IGs can only have direct influence if the challenger accepts offers from both R and A.

Such an influential equilibrium would exist if and only if the challenger wins after accepting A's and R's offers; otherwise she could profitably deviate to publicly committing to (L, A); so the challenger wins after accepting R's and A's offer in such an equilibrium. When the challenger accepts A's offer, she wins and commits to (L, A). Thus, A does not have direct influence because the challenger wins and implements (L, A) even without the help of A; so, only IG R can have direct influence. When the challenger accepts R's offer, she wins and implements (R, P). Thus, R has direct influence: $O_R = (c, (R, P)) \neq O^{NL}$.

Observation 1.3. *Suppose unlike-minded IG A does not have direct influence. Then, unlike-minded IG R has direct influence only if the challenger accepts an offer from at least one of A or P.*

If unlike minded IG on guns has no direct influence then R cannot have direct influence if the challenger rejects offers from both IGs on guns. The maximum vote share the challenger can get (when R endorses the challenger) is $n_{LP} + n_{RP}$ if the challenger rejects offers from both the IGs on guns. See how voters vote when R endorses the challenger. The voters in group RA vote for the incumbent because their ideal policy is perfectly aligned with the incumbent. The voters in group LP vote for the challenger because their ideal policy is perfectly aligned with the challenger. The voters in group LA neither see an endorsement from their IG nor see a public announcement when R endorses the challenger. The voters in LA who are members of A vote for the incumbent because their preferences are partially aligned with both the challenger and the incumbent, but they put more weight on the issue which they share with the incumbent.[30] The voters in group LA who are members of L also vote for the incumbent; they know that the challenger only accepts offers from L and R, so it must be R who endorsed the challenger in which case the challenger implements (R, P), their least preferred policy. Thus, the challenger loses votes from all the voters in groups RA and LA. The voters in group RP who are members of R see that their IG endorses the challenger, so the challenger would implement (R, P) and therefore vote for the challenger. The voters in group RP who are members of P vote for the challenger because any of the policies the challenger implements without a public announcement is better than the incumbent's ideal policy.[31] The challenger only gets the votes from the voters in groups RP and LP. This is $n_{LP} + n_{RP}$ of the total votes, which is less than a half by Lemma 1.5.

What is important in this argument is how members of L vote: members of L with policy preference (L, P) vote for the challenger whereas the members with ideal policy (L, A) do not vote for the challenger because they know that R must have endorsed the challenger since the challenger rejects offers from both IGs on guns. However, if the challenger accepts an offer from one or both of the IGs on guns then these members cannot know for sure, when their IGs did not get access, whether the challenger would implement their least preferred policy. If A endorsed the challenger then she would implement their ideal policy or if P endorsed the challenger then she would implement their second best (L, P). Then there is some chance that the challenger would implement a policy better than the incumbent's ideal. Thus, they vote for the challenger (when their IG did not get access) if the challenger accepts offers from one or both of the IGs on guns and A is more likely to get access than R or the weight they put on the economy is high enough to compensate for the risk (if R endorsed the challenger then she wins). *Same argument applies to the direct influence of unlike-minded IG A. If unlike-minded IG R does not have direct influence then unlike-minded IG A has direct influence only if the challenger accepts the offer from at least one of L or R.*

[30] The challenger only implements (L, A) when A endorse the challenger.

[31] When L endorses the challenger, the challenger implements (L, P), and when R endorses the challenger, the challenger implements (R, P).

We now discuss indirect influence when the challenger in the polar case cares more about guns. Remember that to find indirect influence, we only consider the case where the challenger accepts offers from all IGs. Now, if the challenger's third best policy is popular among the voters then the challenger would reject offers from the like-minded IGs and publicly commit to (L, A). Thus, to find indirect influence we must impose the condition that the challenger's third best policy is unpopular, i.e. $n_{LP} + n_{RP} < 1/2$.

Proposition 1.8. *Let $p_c = (L, P)$, $\pi_A > \pi_R$ and $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$. All IGs except for like-minded IG P can have indirect influence:*

8.1. L affects A's direct influence iff $n_{LP} + n_{LA} + \delta n_{RP} > 1/2$ and $\overline{\alpha} < \frac{\pi_A}{\pi_R}$.

8.2. L affects R's direct influence iff $\overline{\alpha} < \frac{\pi_A}{\pi_R}$. and $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2 > n_{LP} + (1-\delta)n_{RP} + (1-\delta)n_{LA}$.

8.3. R affects A's direct influence iff $n_{LP} + n_{LA} + \delta n_{RP} > 1/2$ and $\overline{\alpha} < \frac{\pi_L+\pi_A}{\pi_L}$.

8.4. A affects R's direct influence iff $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$ and $\underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$.

The reason we observe indirect influence from more IGs in the polar case as compared to the semi-alike case is that there are two groups of voters whose vote depends (when their IG did not get access and they see no public commitment) on which issue they care about more (value of α) and their posterior beliefs, which in turn depend on the access probabilities of the IGs. These voters are: members of L with policy preference (L, A); and members of P with policy preference (R, P). By contrast in the semi-alike case, this is true for only one group of voters.

Any particular IG could then have indirect influence if as a result of its access probability becoming 0, the outcome changes as a result of the change in the voting behaviour of these groups of voters. Given the conditions in the premise, both of these groups of voters vote for the challenger if their IG did not get access when $\pi_l > 0$ for all l. Since A is more likely to get access than R ($\pi_A > \pi_R$), members of L with policy preference (L, A) vote for the challenger. If $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$ then the members of R with policy preference (R, P) put high enough weight on guns, on which only the challenger could implement their ideal policy because the incumbent is anti-gun.

We first explain why P cannot have indirect influence. Suppose P cannot get access to the challenger, i.e. $\pi_P = 0$. The absence of P does not change the vote of its own members. The members of L with ideal policy (L, A) vote for the challenger because A

is still more likely to get access than R and these voters care more about the economy: the challenger is more likely to implement their ideal policy (L, A). Hence, P's presence does not affect their voting behaviour.

On the other hand, L's presence could affect the direct influence of R and A. Members of R with policy preference (R, P) care more about guns. They know that the challenger would implement their ideal policy if R endorsed the challenger, their second best policy if L endorsed the challenger and implement their worst policy if A endorsed the challenger. In the absence of L, they know that one of A or R endorsed the challenger; and since A is more likely to get access, there is greater chance that the challenger would implement their worst policy. If $\overline{\alpha}$ is not high enough to compensate for the risk then these voters would vote for the incumbent. The votes of these voters are important for the challenger (after accepting A's offer) to win. Thus, in the absence of L, these voters do not vote for the challenger if $\overline{\alpha} < \frac{\pi_A}{\pi_L}$. If that is the case then the challenger loses when A endorses the challenger and A would have no direct influence. Then, L has indirect influence if A had direct influence in the presence of all IGs and in the absence of L these voters vote against the challenger. Similarly, the presence of L would affect R's direct influence if the vote share the challenger gets (after accepting R's offer) exceeded 1/2 in the presence of all IGs and the vote share she gets is less than 1/2 in the absence of L.

Note that none of the IGs' presence affects the direct influence of like-minded IGs, who can never have direct influence. To have direct influence, they would have to get the votes of the members of unlike-minded IGs. As we discussed above, only (some) members of like-minded IGs L and P alter their voting behaviour. The voting behaviour of members of the unlike-minded IGs does not change. This implies that R [resp. A] could only affect the direct influence of A [resp. R].

The presence of R does not alter the voting behaviour of the members of L with ideal policy (L, A) as these voters vote for the challenger as long as A is more likely to get access than R. However, R's presence could affect the voting behaviour of the members of P with ideal policy (R, P): these voters' vote is important for A's direct influence. Note that their ideal policy is implemented only if R endorses the challenger; otherwise the challenger either implements their second best policy (L, P) or their worst policy (L, A). The incumbent implements their third best policy. Thus, in the absence of R they vote for the incumbent if $\overline{\alpha}$ is not high enough to compensate for the risk if A endorses the challenger in which case the challenger implements their worst policy. Thus, R has indirect influence if A had direct influence before and in the absence of R, the challenger does not get enough votes and loses; so A loses its direct influence.

Similarly, the presence of A does not alter the voting behaviour of the members of P with ideal policy (R, P).[32] However, A's presence could affect the voting behaviour of the members of L with ideal policy (L, A): these voters' vote is important for R's direct influence. Note that their ideal policy is implemented only if A endorses the challenger; otherwise the challenger either implements their second best policy (L, P) or their worst policy (R, P). The incumbent implements their third best policy. Thus, they vote for the incumbent if they care enough about guns, where only the incumbent implements their ideal policy, i.e $\underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$. Then, A has indirect influence if R had direct influence when $\pi_l > 0$ for all l, but when $\pi_A = 0$, the challenger does not get enough vote and loses; so A loses its direct influence.

Proposition 1.8a. *Let $p_c = (L, P)$, $\pi_A > \pi_R$ and $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} > \overline{\alpha}$. None of the IGs except for A have indirect influence. A affects the direct influence of R iff either of the following conditions hold:*

8a.1. $n_{LP} + (1-\delta)(n_{LA} + n_{RP}) > 1/2$ and $\underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$; or

8a.2. $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2 > n_{LP} + (1-\delta)(n_{LA} + n_{RP})$ and $\underline{\alpha} < \frac{\pi_P}{\pi_P+\pi_R}$

Condition $\pi_A > \pi_R$ implies that when they are uninformed about the challenger policy commitment, the members of L with ideal policy (L, A) would vote for the challenger; and $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} > \overline{\alpha}$ implies that the members of P with ideal policy (R, P) would vote for the incumbent. The presence of P would not affect the voting behaviour of its own members or the members of L with policy preference (L, A).[33]

L could have indirect influence if its presence could alter the voting behaviour of the members of P with ideal policy (R, P). In the presence of all IGs, the members of P vote for the incumbent because their payoff from the incumbent's ideal policy is greater than their expected payoff from voting for the challenger, i.e. $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} > \overline{\alpha}$. If these voters do not vote for the challenger in the presence of L, they would definitely not vote for

[32]Those members of P vote for the challenger if $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$. Since $\overline{\alpha} > 1$, $\frac{\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$ if A's probability of access becomes 0.

[33]They vote for the challenger as long as A is more likely to get access than R. The access probability of P does not change this condition.

the challenger in the absence of L. The weight they put on guns is not high enough and A is more likely to get access than R. This remains true for them in the absence of L.[34] The presence of L does not change their vote. Hence, L has no indirect influence.

The presence of R does not change the voting behaviour of any of the members of L or P. The members of L vote for the challenger as long as A is more likely than R to get access. This becomes certain if R cannot get access. The members of P with ideal policy (R, P) have more reasons to vote for the incumbent in the absence of R because their ideal would never be implemented, and their weight on guns is low. Thus, the presence of R does not change the voting behaviour of any of these voters. Hence, R does not have direct influence.

The presence of A, however, affects the voting behaviour of these voters. In the absence of A, members of P would vote for the challenger since any policy the challenger would implement is better than the incumbent's. The challenger would either implement (L, P) if L endorses the challenger, or (R, P) if R endorses the challenger. Members of P care more about guns and hence any of these polices is better than (R, A). In the absence of A, the members of L might or might not vote for the challenger. If the weight they put on guns is high enough they vote for the incumbent; otherwise they vote for the challenger. The vote of these members is crucial for R's direct influence; R does not have direct influence if these voters do not vote for the challenger. Then A has indirect influence if R had direct influence in the presence of all IGs and now in the absence of A, these voters do not vote for the challenger. Hence, condition 8a.1 must be satisfied. Now, suppose R did not have influence before, when the members of L voted for the challenger and members of P voted for the incumbent. Then, A has indirect influence if, in the absence of A, members of L vote for the challenger, i.e. $\underline{\alpha}$ is low enough. The challenger's vote share would be larger than before because the members of P now vote for the challenger. If this vote share exceeds 1/2 whereas the old vote share did not, then R has no direct influence in the presence of all IGs, but has direct influence in the absence of A. Hence condition 8a.2 must be satisfied.

Proposition 1.8b. *Let* $p_c = (L, P)$, $\pi_R > \pi_A$, *and* $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} > \underline{\alpha}$. *All IGs except for L can have indirect influence:*

8b.1. P affects A's direct influence iff $n_{LA} + n_{LP} + \delta n_{RP} > 1/2 > n_{LP} + \delta n_{LA} + \delta n_{RP}$

[34] $\pi_A > \pi_R \Rightarrow \frac{\pi_A}{\pi_R} > \frac{\pi_A + \pi_L}{\pi_R + \pi_L}$.

and $\underline{\alpha} > \frac{\pi_A}{\pi_R}$.

8b.2. P affects R's direct influence iff $\underline{\alpha} > \frac{\pi_A}{\pi_R}$ *and* $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$

8b.3. R affects A's direct influence iff $n_{LP} + n_{LA} + (1-\delta)n_{RP} > 1/2$ *and* $\overline{\alpha} < \frac{\pi_L + \pi_A}{\pi_L}$.

8b.4. A affects R's direct influence iff $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$ *and* $\underline{\alpha} > \frac{\pi_P}{\pi_P + \pi_R}$

The argument is similar to the one in Proposition 1.8a. The group of voters whose vote depends on the access probabilities of other IGs are: members of L with ideal policy (L, A); and members of P with ideal policy (R, P). L's presence does not change the vote of its own members; so the vote of members of L with ideal policy (L, A) does not change. Members of R with ideal policy (R, P) vote for the challenger as long as $\overline{\alpha} > \frac{\pi_A + \pi_L}{\pi_R + \pi_L}$: since $\pi_R > \pi_A$, they vote for the challenger irrespective of π_L. Thus, L has no direct influence.

P's presence could affect the direct influence of A and R if the members of L with ideal policy (L, A) do not vote for the challenger in the absence of P. These voters vote for the challenger as long as $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} > \underline{\alpha}$. This condition might not hold in the absence of P. If these voters do not vote for the challenger and the new vote share (after losing these voters) is less than 1/2 then the challenger loses, and hence A and R lose their direct influence. Thus, conditions 8b.1 and 8b.2 must be satisfied.

R's presence could affect the direct influence of A if the members of P with ideal policy (R, P) do not vote for the challenger in the absence of R. These voters vote for the challenger as long as $\overline{\alpha} > \frac{\pi_A + \pi_L}{\pi_R + \pi_L}$. This condition might not hold in the absence of R. If these voters do not vote for the challenger and the new vote share (after losing these voters) is less than 1/2 then the challenger loses and hence A loses its direct influence. Thus, the condition 8b.3 must be satisfied.

A's presence could affect the direct influence of R if the members of L with ideal policy (L, A) do not vote for the challenger in the absence of R. These voters vote for the challenger as long as $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} > \underline{\alpha}$. This condition might not hold in the absence of R. If these voters do not vote for the challenger and the new vote share (after losing these voters) is less than 1/2 then the challenger loses and hence R loses its direct influence. Thus, the conditions 8b.4 must be satisfied.

Proposition 1.8c. *Let* $p_c = (L, P)$, $\pi_R > \pi_A$ *and* $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} < \underline{\alpha}$. *None of the IGs except for R have indirect influence. R affects the direct influence of A iff either of the following* *conditions hold:*

8c.1. $n_{LP} + \delta(n_{LA} + n_{RP}) > 1/2$ *and* $\overline{\alpha} < \frac{\pi_A + \pi_L}{\pi_L}$*; or*

8c.2. $n_{LP} + \delta(n_{LA} + n_{RP}) < 1/2 < n_{LP} + n_{LA} + \delta n_{RP}$ *and* $\overline{\alpha} > \frac{\pi_A + \pi_L}{\pi_L}$.

Condition $\pi_R > \pi_A$ implies that when they are uninformed about the challenger's commitment, the members of P with ideal policy (R, P) would vote for the challenger; and condition $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} < \underline{\alpha}$ implies that the members of L with ideal policy (L, A) would vote for the incumbent. The presence of L would not affect the voting behaviour of its own members or the members of P with ideal policy (R, P).[35]

P could have indirect influence if its presence could alter the voting behaviour of the members of L with ideal policy (L, A). In the presence of all IGs, members of L vote for the incumbent because their payoff from the incumbent's ideal policy is greater than their expected payoff from voting for the challenger, i.e. $\frac{\pi_A + \pi_P}{\pi_R + \pi_P} < \underline{\alpha}$. If these voters do not vote for the challenger in the presence of P then they would definitely not vote for the challenger in the absence of P. The value they have for guns is not high enough and R is more likely to get access than A. This remains true for them in the absence of P.[36] Hence, P has no indirect influence.

The presence of A does not change the voting behaviour of any of the members in L and P. Members of P vote for the challenger as long as R is more likely than A to get access. This becomes certain if A cannot get access. The members of L with ideal policy (L, A) certainly vote for the incumbent in the absence of A because their ideal policy would never be implemented by the challenger, and their weight on guns is low. Hence, A does not have direct influence.

The presence of R, however, affects the voting behaviour of these voters. In the absence of R, members of L with ideal policy (L, A) would vote for the challenger since any policy the challenger would implement is better than the incumbent's. The challenger would either implement (L, A) if A endorses the challenger, or (L, P) if P endorses the challenger. Members of L care more about the economy, so any of these polices is better than (R, A). In the absence of R, members of P might or might not vote for the challenger. If the weight they put on guns is not high enough then they vote for the incumbent; otherwise they vote for the challenger. The vote of these members is

[35] They vote for the challenger as long as R is more likely to get access than A. The access probability of P does not change this condition.

[36] $\pi_R > \pi_A \Rightarrow \frac{\pi_A}{\pi_R} < \frac{\pi_A + \pi_P}{\pi_R + \pi_P}$.

crucial for A's direct influence. Then, R has indirect influence if A had direct influence in the presence of all IGs and now in the absence of R, these voters do not vote for the challenger. Hence, condition 8c.1 must be satisfied. Now, suppose A did not have direct influence in the presence of all IGs, when members of P voted for the challenger and members of L voted for the incumbent. Then R has indirect influence if, in the absence of R, members of L vote for the challenger, i.e. $\underline{\alpha}$ is low enough. The challenger's vote share would be larger than before because all members of L now vote for her. If this vote share exceeds 1/2 whereas the old vote share did not, then A had no direct influence in the presence of all IGs, but has direct influence in the absence of R. Hence condition 8c.2 must be satisfied.

Challenger in the polar case who cares more about the economy. Specifically, $p_c = (L, P)$ and $\alpha_c < 1$. Here, we have analogous arguments as for the case where the challenger cares more about guns. We have the same claims to make where we observe direct influence from both of unlike-minded IGs and indirect influence, same as in Proposition 1.8, 1.8a, 1.8b and 1.8c. Thus, there is no difference in terms of finding influence in the polar case irrespective of whether the challenger cares more about the economy or guns.

As expected, we see that both unlike-minded IGs have direct influence when the challenger would definitely lose in the no-lobbying game. We also observe influence from one of the unlike-minded IGs when the challenger wins with her third favourite policy in the no-lobbying game. This is different from the semi-alike case where we observe influence only when the challenger definitely loses in the no-lobbying game. The reason why α_c does not change the result in the polar case is that the challenger and the incumbent are completely opposed to each other; so the challenger accepts any deal which gives her the opportunity to win. Irrespective of whether the challenger cares more about the economy or guns, the incumbent's ideal policy remains her least favourite policy, and therefore, this does not change the influence.

1.5 Discussion

We now summarize the results above and discuss the intuition.

Result 1. *Like-minded IGs have no direct influence.*

We identify two reasons underlying an IG's influence: (i) how the different groups of voters are informed about the challenger's private commitment to the IG, and (ii) different policy preferences of voters between and within groups. Suppose the challenger is rightist and pro-gun and the incumbent is rightist and anti-gun. Now direct influence implies that the IG convinces the challenger to change her policy stand on a particular issue in favour of the IG (if the challenger and the IG have opposite ideal policies). In return for this the IG disseminates that information to only a group of voters, who would (probably) be favourable to the challenger after receiving such information. But if the challenger and the IG have the same ideal policy then it does not make any sense for the IG to make a deal with the challenger to change her policy stand. Thus, according to that reasoning, like-minded IGs cannot have direct influence. Direct influence could also mean that, on getting access, the IG convinces the challenger to maintain her stand on her ideal policy and the challenger agrees to it, and wins where she could not win without such a commitment. However, this commitment would not get the challenger enough votes to win because the challenger must get some votes from the anti-gun supporters who are leftist and care more the economy but they do not vote for the challenger unless they get a private commitment. The point is that like-minded IGs cannot deliver any more votes than the challenger could get anyway.

To some extent, this result reflects how influence actually comes about in the policy making process or in the elections when interest groups try to persuade candidates to act in their favour. If we look at how the NRA behaves in regards to the policy stand of any particular candidate, one thing that can be seen is that the NRA becomes very aggressive if the candidate is anti-gun. It then threatens the candidate with a loss of votes unless the candidate changes her policy stance on guns. But if the candidate is already in line with the NRA's agenda, the NRA endorses this candidate and delivers the votes (from its members), but cannot provide more votes to the candidate to win the election because non-members are all anti-gun voters and some pro-gun voters who care more about the other issue. All non-members who are anti-gun vote for the incumbent and all non-members who are pro-gun vote for the challenger. But as the challenger could get these votes with a public announcement, she does not need the NRA's help with micro-targeting.

It is known from the model that the members of like-minded IGs are sympathetic (partially or fully) to the challenger's ideal policy. So when the like-minded IG endorses the challenger, the members get assurance and definitely vote for the challenger. But a significant majority of them still vote for the challenger even if their IG does not endorse the challenger. Members of unlike-minded IGs resent (partially or fully) the challenger's ideal policy and would need an assurance from their IG to vote for the challenger.

In the above example where the challenger is rightist and pro-gun and the incumbent is rightist and anti-gun, suppose the challenger would lose if she publicly committed to her ideal policy. The challenger gets the vote of all pro-gun voters: they vote for the challenger if they are rightist because the challenger's policy is also their ideal policy; and if they are leftist, they vote for the challenger because they hate the incumbent's ideal policy. If the pro-gun IG endorses the challenger, the maximum number of votes the challenger can get is from all the pro-gun voters, which she gets anyway. She would want votes from the anti-gun voters who do not see an endorsement from their IG. But anti-gun voters who are rightist always vote for the incumbent because she implements their ideal policy and anti-gun voters who are leftist also vote for the incumbent if it is likely enough that the challenger has a deal with the like-minded IG because they hate the challenger's ideal policy. Thus, the challenger cannot get any votes from anti-gun voters.

Result 2. *Let $p_c = (R, P)$ and $p_i = (R, A)$. Like-minded IG P has indirect influence if the challenger cares more about guns.*

The idea behind MT is that the challenger gets some votes by committing to some policy without letting the other voters know about it. Since the challenger does not choose an IG but rather access is random, the uninformed voters do not draw an adverse inference from non-access. As a result, the challenger can accept an IG's offer and get votes from its members without losing the votes of non-members, who either partially or fully share the challenger's ideal policy, and believe that the challenger is likely to implement a better policy than the incumbent. Thus, an IG's influence depends on how the uninformed voters vote, which in turn depends on whether the challenger accepts offers from other IGs. That is how the presence of one IG and the challenger's private commitment to an IG affects the direct influence of other IGs. For example, consider the same challenger and incumbent in Result 1. If voters believe that the challenger rejects L's offer then they know that the challenger will never implement policy (L, P)

and they update beliefs about the policy the challenger would implement accordingly. However, their vote might change if they expected the challenger to accept L's offer, in which case the challenger might implement (L, P).

In Proposition 1.2, we showed that some voters do not change their votes as long as the challenger accepts the offers from all IGs. Voters who are rightist and anti-gun always vote for the incumbent because they share the ideal policy with her on both issues. Voters who are leftist and pro-gun always vote for the challenger because they hate the incumbent's ideal policy. Voters who are leftist and anti-gun always vote for the incumbent because they hate the challenger's ideal policy. Voters who are rightist and pro-gun and care more about guns always vote for the challenger because they know that any policy the challenger would implement is either better than or the same as the incumbent's ideal. But those who care more about the economy either vote for the challenger or the incumbent depending on their beliefs about the access probabilities and the weight they put on guns. These swing voters are the members of R and their votes depend on the access probabilities of the IGs L, P and A. If their IG did not endorse the challenger then they know one of L, A or P must have endorsed the challenger. If P endorsed the challenger then they would vote for the challenger; if L endorsed the challenger then they vote for incumbent; and if A endorsed the challenger then they are indifferent. They vote for the challenger if relative to the weight they put on guns, π_P is high enough as compared to π_L. If $\pi_L = 0$ or $\pi_A = 0$, then they vote for the challenger: so L and A do not have indirect influence. If $\pi_P = 0$, they vote for for the incumbent: so P has indirect influence.

Relating this result back to the NRA example, we can see how the presence of the NRA is important for the group of voters who are pro-gun but do not belong to the NRA because they care more about the economy. If the NRA could not get access then the challenger would lose votes from the voters who are pro-gun but care more about the economy. If the challenger did not have a deal with the NRA then there is a high chance that she deviates on the economy. But if the NRA is present and could endorse a candidate then these voters may vote for the challenger. If the NRA endorses the challenger then they believe that there is some chance that the challenger would stand by her ideal policy on guns and therefore are willing to take a chance to vote for the challenger who might or might not deviate from her ideal policy on the economy.

In sum, pro-gun voters who care more about the economy and are uninformed prefer to vote for the incumbent in the absence of the NRA, the result of which is that the challenger does not get enough votes to win, and the outcome is that the incumbent wins and implements the anti-gun policy, which is bad for the NRA. Thus, the NRA's endorsement cannot deliver more voters but can definitely take away some important votes.

Result 3. *When the challenger cares more about guns, the like-minded IG on guns, i.e. IG P, has indirect influence in the semi-alike case whereas this is not necessarily the case for any of the IGs who might have indirect influence in the polar case.*

Observation 1.3 implies that in the polar case, the challenger does not have to accept offer from a like-minded IG for the unlike-minded IGs to have direct influence. This is not true in the semi-alike case, where the challenger has to accept the offer from like-minded IG P for L to have direct influence (cf. Proposition 1.2). Moreover, in the polar case, if both unlike-minded IGs are directly influential then the challenger does not have to accept offers from like-minded IGs for the unlike-minded IGs to have direct influence. The intuition behind this is that in the polar case, like-minded IGs' interests are completely misaligned with the incumbent's ideals; so they want the challenger to win the election and to implement a policy closer to their ideal policy. Some members of like-minded IGs then unconditionally support the challenger: they share the challenger's interests.

In the polar case, the challenger need not accept offers from like-minded IGs to get the votes of their members, because some members of both like-minded IGs share the challenger's interests on both issues (disagree with incumbent's ideal on both issues), and therefore always support the challenger unless she publicly commits to the incumbent's ideal policy. Thus, in the polar case, when the challenger rejects offers from like-minded IGs, she does not fear losing her most supportive voters. But these are not the only voters the challenger is concerned about losing. There are some members of like-minded IGs who could vote for the incumbent. These voters partially agree with the challenger: depending on the agreed policy, they may vote for the challenger (if there is a successful deal between the challenger and the unlike-minded IG on the issue where they do not agree with the challenger) or for the incumbent (if there is a successful deal between the challenger and the unlike-minded IG on the issue on which they agree with the challenger). For these voters, voting for the challenger would either give them the highest or the lowest payoff. If they value the highest payoff enough to compensate for the loss when the challenger implements their least favourite policy then they vote for the challenger. But if these voters are not willing to take that risk, they would not vote for the challenger even if the challenger guarantees that she would implement her ideal policy (by accepting the offer from the like-minded IG) because the challenger's ideal is not their favourite policy. Thus, their voting decision does not depend on whether the challenger accepts the offer from like-minded IGs.

By contrast, in the semi-alike case, only one like-minded IG has members who share the challenger's interests on both issues (disagrees with the incumbent's ideal policy on both issues). In fact, the other like-minded IG has members who share the incumbent's interests on both issues; in the polar case, none of the members of like-minded IGs share the incumbent's interests on both. Therefore, in the semi-alike case, the challenger has to make sure to get their votes, and to do so she has to accept an offer from the other like-minded IG because this ensure those members that there is some chance that the challenger would stand by her ideal policy.

Although, the IGs are passive players, the preferences of their members are very important for the challenger to decide whether to accept or reject an offer, which in turn determines the impact they can have on the outcome. In essence, what is happening in the polar case is that the challenger does not fear losing the votes of members of like-minded IGs by not accepting their offer. The members of unlike-minded IGs only vote for the challenger if they get direct assurance via their IG; so the challenger does not get more votes from these members if she accepts offers from like-minded IGs. Some members of like-minded IGs have policy preference such that their ideal policy is implemented when one of the unlike-minded IGs endorses the challenger. For these voters, the presence of that unlike-minded IG is more important than the presence of like-minded IG. Thus, the presence of a like-minded IGs is not mandatory for the extra votes the challenger needs to get to win.

In essence, in the semi-alike case, members of R who agree with the challenger's ideal policy on both issues care more about the economy, on which the incumbent implements their ideal. These members only vote for the challenger (when their IG does not get access) if they know that there is some chance that the challenger would not deviate from her ideal policy (when P endorses the challenger). But P cannot endorse the challenger if it cannot get access. In that case, these voters vote for the incumbent: there is a high chance that the challenger implements L on the economy on which they prefer R and also they care more about the economy. Thus, if P cannot get access, the challenger loses these voters' vote when L endorses the challenger and hence L loses its direct influence if it had direct influence when P could endorse the challenger. Therefore, unlike minded IG P has indirect influence in the semi-alike case.

Result 4. *More IGs have direct influence in the polar case than in the semi-alike case. Direct influence alters policy implemented on both issues in the semi-alike case but only on one of the issues in the polar case.*

In the semi-alike case, only IG L's direct influence changes policy (as compared to no-lobbying). But in the polar case, direct influence of both IGs R and A brings about a change in policy. When the challenger and the incumbent are polar and the challenger would lose without IGs, the challenger can do better by moving closer to the incumbent's ideal policy. By doing so, the challenger could win and implement a policy closer, but more desirable than the incumbent's ideal. If R [resp. A] has direct influence then the outcome is (R, P) [resp. (L, A)]; whereas the policy outcome without IGs is (R, A). In the semi-alike case, if the challenger cares more about the economy then there is no change in policy. By contrast in the polar case, irrespective of whether the challenger cares more about the economy or guns, direct influence is possible which alters policy outcomes. MT leads to a change in policy more often in the polar case than in the semi-alike case: in the polar case, direct influence of both unlike-minded IGs brings a policy change whereas this is true for only one unlike minded IG in the semi-alike when the challenger cares more about guns. However, whenever there is a change in policy because of MT, it is more extreme (a change in policy on both issues) in the semi-alike case than in the polar case.

In the semi-alike case, if the challenger deviates from her ideal policy on guns then she promises to implement the same policy as the incumbent. Thus, she does not gain anything from this because she cares only about the policy implemented. But if the challenger deviates on the economy then her implemented policy would entirely differ from the incumbent's, and therefore is motivated to privately commit if she cares more about guns and is able to win after such a private commitment. Thus, if the incumbent were to win in the absence of IGs, the change in the policy outcome as a result of MT is extreme in the sense that the policy outcome would differ on both issues. If L has direct influence then the outcome is (L, P) when L endorses the challenger; the outcome without IGs is (R, A).

1.6 Conclusion

In this chapter, we argue that IGs could effectively use MT as a lobbying tool to influence electoral outcomes. The IGs' ability to use MT can help to explain why and how some IGs have undue influence on relevant policy issues, even though most voters oppose their agendas. Politicians can privately promise policies through an IG by providing policy favours in order to appease a small group of voters on an issue which is of primary importance to them. Private commitment of the political candidate exploits the uncertainty of the voters who do not know who lobbied the candidate. This creates an opportunity for the political candidate to win by partially compromising on the issue the IG is concerned about.

To capture this idea, we have described a model of lobbying in which the challenger has preferences over two policy issues: the economy and guns. The challenger has to decide whether to accept an IG's offer which is favourable to the IG, in exchange for the IG micro-targeting in favour of the challenger. The challenger is policy motivated in that by winning the election, the challenger implements a policy closer to her ideal. Voters know which policy the challenger would implement if she agreed with an IG, but they may not know which IG endorsed the challenger unless they are privately informed about the challenger's policy commitment. Because of the voters' uncertainty, the challenger could make a deal with the IG, in order to increase her vote share. In this setting, the challenger's decision to accept an offer and the voters' decision depend, among other things, on how much voters and the challenger care about the issue.

We find that IGs could potentially use MT as a way to exert influence on politicians who are ideologically motivated. In our framework, the IGs can facilitate private disclosure of information to a subset of voters. MT essentially implies that different information is provided to different group of voters. There may, nevertheless, be some spillover of information between groups. The question then is how MT can be effective if voters share their information with other voters? Note that it is reasonable to think that voters with opinions and preferences would most likely share their views with those who have similar ideologies. This phenomenon is quite evident in the viral campaigns on social media like Facebook, Whatsapp, Twitter etc. There is no reason why voters would willingly look for someone with opposing interests to share their views with.

Our model has several predictions. First, we find that like-minded IGs do not have direct influence. Second, like-minded IGs can have indirect influence. Like-minded IGs have members who are already sympathetic to the challenger's ideals; so these like-minded IGs cannot provide any extra help to the politician. But what they can do is to induce the challenger to commit to her ideal policy as she would otherwise lose votes of the group of voters who partially or fully share challenger's ideal and belong to a like minded IG but whose IG could not get access. In deciding how to vote under uncertainty about what policy the challenger would implement, these voters rely on, among other factors, whether the challenger stand by her ideal policy, which is enforced when the like-minded IGs endorses the challenger. Thus, a like-minded IG's endorsement might turn out to be an important factor for some members of other like-minded IGs, instead of for its own members. This is precisely why like-minded IGs could be indirectly influential by taking away votes of the members of the other like-minded IG. Third, like-minded IGs are less directly influential in the polar case than in the semi-alike case. Fourth, direct influence is more extreme in the semi-alike case than in the polar case in the sense of changing policy outcomes on both issues. Our results are robust to positive rents from office where the challenger would be willing to compromise on one or both policy issues if that results in a win and rents from office are high enough. Positive rents would not make any difference in the analysis of influence in the polar case, as the challenger is willing to compromise on both issues in any case. In the semi-alike case, the challenger would be willing to accept an offer on the issue which she cares about more if rents are high enough. Thus, allowing for positive rents does not provide any more insights.

In the case of gun control, although a majority of the voters favour stricter regulations, a minority opposes them with greater intensity; and those who support strict regulations might not care intensely about guns. As a result, a pro-gun policy might come into effect if the candidate has the means to send group-specific messages. This observation is reflected in our semi-alike case where the challenger is rightist and pro-gun and the incumbent is rightist and anti-gun, and a majority of voters favour strict gun regulations, i.e. in the absence of IG, the voters elect a candidate who is anti-gun ($n_{LP} + n_{RP} < 1/2$ or $n_{LA} + n_{RA} > 1/2$). We showed in Proposition 1.1 that with the help of private commitment (MT), the challenger could win the election and implement a pro-gun policy on guns if she could get the votes of those who are anti-gun but do not care intensely about guns without losing votes from the minority who favour lenient gun regulations. Since the challenger cares about winning and MT is effective in winning the elections, IGs can get policy favours because they can facilitate private commitment.

Our findings can help to understand why some policies are more likely to be implemented when IGs can lobby policy-makers by using MT to influence their decision. For example, the U.S. Congress is reluctant to introduce stricter regulation on gun control. There have been long obstructions to the Manchin-Toomey amendment to have stricter background checks for gun buyers in the wake of the Newtown massacre. On April 17 2013, it failed to get enough votes to pass in the U.S. Senate, even though the majority of Americans favoured these regulations according to contemporary opinion polls. On the same day, former U.S. President Obama urged voters to put more pressure on their representatives to pass gun control regulations :"Ultimately, you outnumber those who argued the other way. But they're better organized. They're better financed. They've been at it longer. And they make sure to stay focused on this one issue during election time. And that's the reason why you can have something that 90 percent of Americans support and you can't get it through the Senate or the House of Representatives".[37]

Our analysis suggests that MT is effective, i.e. policy outcomes may diverge from what the majority of the electorate want when they cannot be micro-targeted. This is because citizens have intense preferences on different issues and can be provided targeted information to get their support. The main attraction of MT is that the politician can get support of a subset of voters by privately committing to them through an IG

[37] Before the defeat of gun-control amendments in April 2013, it was noted that "large majorities back expanding background checks to cover all purchases. (. . .) And yet, as Newtown disappeared further in the political rearview mirror, the same politics that had turned guns into a dormant issue on the national political stage for much of the 1990s and 2000s began to take hold. Senate Democrats up for re-election in Republican-leaning states in 2014–think Montana, North Carolina, Alaska, Arkansas and Louisiana–were loathe to vote on things like the assault weapons ban." ("Newtown didn't change the politics of guns," Washington Post, March 22, 2013)

without necessarily having to lose vote from the voters who (partially or fully) share the politician's ideal policy. Since the challenger only compromise on one issue, they are very likely to vote for the challenger if they believe that the challenger is less likely to deviate on an issue they share with her. Therefore, some of these voters do not change their vote when lobbying is introduced. As a consequence of this different voting behaviour of voters, some IGs have direct influence but some do not. Unlike-minded IGs are effective in directly influencing policy outcomes because they have members who only vote for a particular candidate if their IG endorses the candidate. But like-minded IGs are not effective in directly influencing policy outcomes because they have members who more or less vote the same as they do without lobbying. The analysis provided also shows the extent of change in the policy outcomes. MT can lead to a change in policy on both issues when candidates are semi-alike, but is less harmful (change in policy on only one issue) when the candidates are polar.

Our paper calls for more theoretical and empirical research on this topic to understand how influential MT is. We have measured influence by comparing the outcome in the presence of IGs to the outcome without IGs and made claims about whether MT is effective in changing the election outcome. It would be interesting to study MT in the context of social welfare, i.e. to explore whether MT is detrimental or welfare enhancing. Another avenue for future research is to explore the effectiveness of MT as compared to other lobbying strategies, such as campaign contributions. The NRA's financial contributions are nominal in the lobbying business; but the role that its contributions play is still unknown.

The model could be extended to cover cases where the incumbent can also be lobbied. In this extension, IGs might have an active role to decide who to lobby. This might provide us some insights on why some interest groups are more likely to lobby a particular party or candidate: for instance why the NRA almost always endorses Republican candidates? In this setting it would be difficult to keep track of voters' beliefs about the policy of the candidate who is not lobbied. However, the analysis could be simplified by assuming that when candidates are not lobbied, they stand by their public commitment, as in the no-lobbying game. It seems that, on the one hand, we would expect to see more influence because IGs have more opportunities to push their agendas; but on the other hand, competition between the candidates might have negative effects on their influence. It would be interesting to see how these two mechanisms interact. Another interesting extension would to be allow for more than one IG to get access and endorse the candidate, say two opposing IGs get access. In this setting, competition between the IGs might lead to better informed voters. Although answers to these interesting questions would provide further insights on the use of MT in influencing policy-making process, the question remains of how to model them. One attraction of our model is that we do not have to address these questions in the analysis we are interested in.

Table 1.1: Voters' payoff

Voters	**Payoff**			
	(R,A)	(R,P)	(L,P)	(L,A)
RP with $\alpha=\overline{\alpha}$	1	$1+\overline{\alpha}$	$\overline{\alpha}$	0
RP with $\alpha=\underline{\alpha}$	1	$1+\underline{\alpha}$	$\underline{\alpha}$	0
LP with $\alpha=\overline{\alpha}$	0	$\overline{\alpha}$	$1+\overline{\alpha}$	1
LP with $\alpha=\underline{\alpha}$	0	$\underline{\alpha}$	$1+\underline{\alpha}$	1
LA with $\alpha=\overline{\alpha}$	$\overline{\alpha}$	0	1	$1+\overline{\alpha}$
LA with $\alpha=\underline{\alpha}$	$\underline{\alpha}$	0	1	$1+\underline{\alpha}$
RA with $\alpha=\overline{\alpha}$	$1+\overline{\alpha}$	1	0	$\overline{\alpha}$
RA with $\alpha=\underline{\alpha}$	$1+\underline{\alpha}$	1	0	$\underline{\alpha}$

Table 1.2: Voters' vote in no-lobbying game

Voters' group (Proportion of voters)	**Public commitment**			
	(R,P)	(L,P)	(L,A)	(R,A)
RP with $\alpha=\overline{\alpha}$ (δn_{RP})	c	c	i	1/2
RP with $\alpha=\underline{\alpha}$ $((1-\delta)n_{RP})$	c	i	i	1/2
LP with $\alpha=\overline{\alpha}$ (δn_{LP})	c	c	c	1/2
LP with $\alpha=\underline{\alpha}$ $((1-\delta)n_{LP})$	c	c	c	1/2
LA with $\alpha=\overline{\alpha}$ (δn_{LA})	i	i	c	1/2
LA with $\alpha=\underline{\alpha}$ $((1-\delta)n_{LA})$	i	c	c	1/2
RA with $\alpha=\overline{\alpha}$ (δn_{RA})	i	i	i	1/2
RA with $\alpha=\underline{\alpha}$ $((1-\delta)n_{RA})$	i	i	i	1/2
Total share of vote	$n_{LP}+n_{RP}$	$n_{LP}+\delta n_{RP}+(1-\delta)n_{LA}$	$n_{LP}+n_{LA}$	1/2

Appendix

Proof of no-lobbying game.

Proof. Table 1.1 shows the payoff voters in each group get for different implemented policies. For instance, in row 3 and column 2, voters with ideal policy (R,P) who care more about guns get a payoff of 1 if policy (R,A) is implemented. Table 1.2 shows how voters vote for each of the four possible public policy commitment by the challenger. Column 1 has the the eight groups of voters with their proportion in the total population given in the brackets. Column 2 shows whether voters in a particular group vote for the challenger or the incumbent when the challenger announces policy (R,P), so on and so forth for policies (L,P), (L,A), and (R,A) in column 3,4 and 5: where we write c if they vote for the challenger, i otherwise. The last row specifies the challenger's vote share for each public policy commitment.

The outcome in no-lobbying game:

Outcome when $p_c=(R,P)$ **and** $\alpha_c>1$.

The challenger's order of policy preference is

$$(R, P)[1 + \alpha_c] \succ (L, P)[\alpha_c] \succ (R, A)[1] \succ (L, A)[0]$$

where the payoff from the respective policy is given in the square brackets.

The challenger would commit to her ideal policy if her ideal policy is popular; the share of the vote she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + n_{RP} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} = (c, (R, P)) \quad \text{if} \quad n_{LP} + n_{RP} > 1/2$$

The challenger commits to her second best policy if her ideal policy is unpopular but her second best policy is popular; the vote share she gets by committing to (R, P) is less than half and the vote share she gets by committing to (L, P) is more than half, i.e. $n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2 > n_{LP} + n_{RP}$. The outcome then is that the challenger wins and implements her second best policy. Formally,

$$O^{NL} = (c, (L, P)) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2 > n_{LP} + n_{RP}$$

If the conditions above do not hold then the challenger announces her third best policy, which is also incumbent's ideal policy. She loses because she gets half of the total votes. The challenger would never commit to her least preferred policy unless she were to lose. This happens because by implementing (L, A), she gets a payoff 0, but she can profitably deviate to announce (R, A), in which case her payoff is 1. In any case the outcome is incumbent wins and implements her ideal policy.[38]

$$O^{NL} = (i, (R, A)) \quad \text{otherwise}$$

Outcome when $p_c = (R, P)$ and $\alpha_c < 1$.

The challenger's order of policy preference is

$$(R, P)[1 + \alpha_c] \succ (R, A)[1] \succ (L, P)[\alpha_c] \succ (L, A)[0]$$

[38]Remember that if both the candidates commit to the same policy then the incumbent wins because of the incumbency bias.

where the payoff from each policy is given in the square brackets.

The challenger commits to her ideal policy if her ideal policy is popular; the share of vote she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + n_{RP} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} = (c, (R, P)) \quad \text{if} \quad n_{LP} + n_{RP} > 1/2$$

The challenger commits to her second best policy (R, A) if her ideal policy is unpopular. She loses because she gets half of the total votes. The challenger would never commit to policy (L, P) or (L, A) unless she were to lose. She can always deviate to publicly commit to (R, A) and get a higher payoff. The outcome is

$$O^{NL} = (i, (R, A)) \quad \text{otherwise}$$

Outcome when $p_c = (L, P)$ and $\alpha_c > 1$.

The challenger's order of policy preference is

$$(L, P)[1 + \alpha_c] \succ (R, P)[\alpha_c] \succ (L, A)[1] \succ (R, A)[0]$$

The challenger commits to her ideal policy if her ideal policy is popular; the share of vote she gets by committing to (L, P) is more than a half, i.e. $n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} = (c, (L, P)) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} > 1/2$$

The challenger commits to her second best policy if her ideal policy is unpopular but her second best policy is popular; the vote share she gets by committing to (L, P) is less than a half and the vote share she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} < 1/2 < n_{LP} + n_{RP}$. The outcome then is that the challenger wins and implements her second best policy. Formally,

$$O^{NL} = (c, (R, P)) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1 - \delta) n_{LA} < 1/2 < n_{LP} + n_{RP}$$

The challenger commits to her third best policy (L, A) if her first two best policies are unpopular but her third best policy is popular. The outcome then is that the challenger wins and implements (L, A). Formally,

$$O^{NL} = (c, (L, A)) \quad \text{if} \quad n_{LP}+n_{LA} > 1/2 \quad \text{and} \quad max\{n_{LP}+n_{RP}, n_{LP}+\delta n_{RP}+(1-\delta)n_{LA}\} < 1/2$$

If none of the conditions above hold, i.e if the challenger cannot win with any of her best three policies then the incumbent wins and implements her ideal policy. Formally,

$$O^{NL} = (i, (R, A)) \quad \text{otherwise}$$

□

Proof of Lemma 1.1

Proof. The proof is by contradiction. Suppose that $n_{LP}+n_{RP} > 1/2$. Then, the outcome in no-lobbying game is that the challenger wins and policy (R, P) is implemented, i.e. $O^{NL} = (c, (R, P))$. Let U_{max} denote the utility of the challenger when the challenger wins and implements her ideal policy. Thus, she earns $U_{max} = 1 + \alpha_c$ in the no-lobbying game.

In the MT game, if $n_{LP}+n_{RP} > 1/2$, then the challenger must reject offers from both unlike-minded IGs. No other strategy profile in which the challenger accepts offers from one or both unlike-minded IGs survives in the equilibrium. She commits to implement (L, P) if L endorses her and (R, A) if A endorses her. She could either win or lose the election after accepting offers from unlike-minded IGs. If she wins she implements a policy worse than her ideal or she could lose in which case the incumbent wins and implement a policy worse than the challenger's ideal. In any case, the challenger's payoff will be less than U_{max}; so, she could profitably deviate to publicly commit to her ideal policy, thereby earning U_{max}. The challenger would accept offers from like-minded IGs only if she were to win, otherwise she could profitably deviate to publicly announce her ideal policy. In any case, the challenger wins and implements her ideal policy; so $O_R = O_P = O_{NL}$.

We know from Claim 1 that if the challenger rejects an offer from l then $O_l = O^{NL}$.

If $n_{LP} + n_{RP} > 1/2$, then the challenger rejects offers from both unlike-minded IG. Therefore, we have $O_L = O_A = O_R = O_P = O^{NL}$.

□

Proof of Lemma 1.2

Proof. The proof is by contradiction. Suppose that $n_{LP} + (1 - \delta)n_{LA} + \delta n_{RP} > 1/2$. Then, by Lemma 1.1 the outcome in the no-lobbying game is that the challenger wins and implements her second best policy: $O^{NL} = (c, (L, P))$.

In MT game, there exists only two equilibria if $n_{LP}+(1-\delta)n_{LA}+\delta n_{RP} > 1/2$: one in which the challenger rejects all offers and publicly commits to (L, P); and one in which the challenger accepts the offer from IG L and rejects all other IGs' offers and publicly commit to (L, P). From Observation 1.1 and Lemma 1.1, we know that the challenger loses when like-minded IGs endorse her in which case policy (R, A) is implemented, earning 1. But, she could reject offers from like-minded IGs and publicly commit to (L, P), earning α_c. Since $\alpha_c > 1$, she rejects offers from like-minded IGs. If she accepts A's offer, she implements (R, A) if she wins, (R, A) is implemented otherwise. In any case, she earns 1. But, she could reject A's offer and publicly commit to (L, P), earning α_c. Thus, the challenger rejects from A, R and P and publicly commit to (L, P). The challenger accepts from L only if she were to win, otherwise she could profitably deviate to publicly committing to (L, P). The challenger promises to implement (L, P) if she accepts L's offer.

In both the equilibria, $O_l = (c, (L, P) = O^{NL}$ for each l. Thus, $n_{LP} + (1 - \delta)n_{LA} + \delta n_{RP} < 1/2$ in any equilibrium where an IG has direct influence. □

Proof of Lemma 1.3

Proof. Assume by the way of contradiction that the challenger rejects P's offer. Here, we want to show that L has no direct influence when L endorses the challenger. Now consider an equilibrium where the challenger accepts the offer from L. Note that by Lemmas 1.1 and 1.2, the challenger is indifferent between publicly announcing (R, P), (L, P) and (R, A) because she loses anyway. She strictly prefer to lose than to implement her least preferred policy (L, A). In case of indifference, we assume that she publicly commits to her ideal policy: so, $O^{NL} = (i, (R, A))$.

Now we show what proportion of total votes the challenger gets when L endorses the challenger. Members of L see the endorsement from their IG and therefore know that the challenger would implement (L, P) if she wins. When L endorses the challenger, the members of IGs R, A and P neither receive any endorsement nor see a public announcement. Then,

Members of L vote for the challenger. Members with ideal policy (L, P) vote for the challenger because $1 + \underline{\alpha} > 0$ and members with ideal policy (L, A) also vote for the challenger because $1 > \underline{\alpha}$.

Members of R vote for the incumbent. Members of R know that either case 1) L or A endorsed the challenger if the challenger's strategy prescribes her to accept A's offer; or case 2) L endorsed the challenger if the challenger rejects A's offer. Payoff of the members with ideal policy (R, P) if they vote for the challenger is case 1) $\frac{\pi_L \underline{\alpha} + \pi_A 1}{\pi_A + \pi_L}$ or case 2) $\underline{\alpha}$. In either case, their payoff is less than 1 (payoff if they vote for the incumbent). Payoff of the members with ideal policy (R, A) if they vote for the challenger is case 1) $\frac{\pi_L 0 + \pi_A (1 + \underline{\alpha}}{)} \pi_A + \pi_L$ or case 2) 0. In either case, their payoff is less than $1 + \underline{\alpha}$ (payoff if they vote for the incumbent).

Members of A vote for the incumbent. Members of A know that either case 1) L or R endorsed the challenger if the challenger's strategy prescribes her to accept R's offer; or case 2) L endorsed the challenger if the challenger rejects A's offer. Payoff of the members with ideal policy (L, A) if they vote for the challenger is case 1) $\frac{\pi_L 1 + \pi_R 0}{\pi_L + \pi_R}$ or case 2) 1. In either case, their payoff is less than $\overline{\alpha}$ (payoff if they vote for the incumbent). Payoff of the members with ideal policy (R, A) if they vote for the challenger is case 1) $\frac{\pi_L 0 + \pi_R 1}{\pi_A + \pi_R}$ or case 2) 0. In either case, their payoff is less than $1 + \overline{\alpha}$ (payoff if they vote for the incumbent).

Members of P vote for the challenger. Members of P know that either case 1) L or R or A endorsed the challenger if the challenger's strategy prescribes her to accept both R's and A's offer; or case 2) L or R endorsed the challenger if the challenger's strategy prescribes her to accept R's offer and rejects A's offer; otherwise case 3) L or A endorsed the challenger if the challenger's strategy prescribes her to accept A's offer and rejects R's offer. Payoff of the members with ideal policy (L, P) if they vote for the challenger is case 1) $\frac{\pi_L (1 + \overline{\alpha}) + \pi_A 0 + \pi_R \overline{\alpha}}{\pi_A + \pi_L + \pi_R}$ or case 2) $\frac{\pi_L (1 + \overline{\alpha}) + \pi_R \overline{\alpha}}{\pi_L + \pi_R}$ or case 3) $\frac{\pi_L (1 + \overline{\alpha}) + \pi_A 0}{\pi_A + \pi_L}$. In any case, their payoff is more than 0 (payoff if they vote for the incumbent). The payoff of the members with ideal policy (R, P) if they vote for challenger is case 1) $\frac{\pi_L\overline{\alpha}+\pi_A 1+\pi_R(1+\overline{\alpha})}{\pi_A+\pi_L+\pi_R}$ or case 2) $\frac{\pi_L\overline{\alpha}+\pi_R(1+\overline{\alpha})}{\pi_L+\pi_R}$ or case 3) $\frac{\pi_L\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L}$. In any case, their payoff is more than 1 (payoff if they vote for the incumbent).

Thus, only members of L and P vote for the challenger. Then, the vote share the challenger gets when L endorses the challenger is $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$, which is less than a half by Lemma 1.2. Thus, the challenger loses the election: so, $O_L = (i, (R, A)) = O^{NL}$. Hence L has no direct influence.

□

Proof of Proposition 1.1

Proof. (***Necessity***)

We first establish the necessity of condition P1.1. Assume by way of contradiction that $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. Now, direct influence from IG l is only possible if the challenger accepts offer from l. We know from Lemma 1.3 that in the MT game, the challenger must accept offer from P for L to have direct; so for unlike-minded IGs L and A to have direct influence, the challenger must accept offers from L, A and P. In such an equilibrium, the challenger either accepts offer from R or rejects. But, our premise holds irrespective of whether the challenger accepts or rejects R's offer.

When L endorses the challenger, members of L know that their IG endorsed the challenger so they know that the challenger would implement (L, P) if she wins; members of other IGs neither see an endorsement from their IG nor see a public commitment from the challenger. Then, the proportion of votes the challenger receives when L endorses her is calculated as follows:

1) **Members of L vote for the challenger.** Members in group LP vote for the challenger: the challenger implements (L, P) which earns them $1 + \underline{\alpha}$ and the incumbent implements (R, A) which earns them 0. Members in group LA vote for the challenger: the challenger implements (L, P) which earns them 1 and the incumbent implements (R, A) which earns them $\underline{\alpha}$

2) **Members of P vote for the challenger**

i) Members in group (R, P) vote for the challenger because $\frac{\pi_L(\overline{\alpha})+\pi_R(1+\overline{\alpha})+\pi_A(1)}{\pi_L+\pi_A+\pi_R} > 1$, where the left hand side is the expected payoff from voting for the challenger and the right hand side is the payoff from voting for the incumbent.

ii) Members in group (L, P) vote for the challenger because $\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 0}{\pi_L+\pi_A+\pi_R} > 0$

3) **Members of A vote for the incumbent**

i) Members in group (R, A) vote for the incumbent because $\frac{\pi_R+\pi_P}{\pi_L+\pi_P+\pi_R} < 1 + \overline{\alpha}$

ii) Members in group (L, A) vote for the incumbent because $\frac{\pi_L}{\pi_L+\pi_P+\pi_R} < \overline{\alpha}$

4) **Members of R vote as follows:**

i) Members in group (R, A) vote for the incumbent because $\frac{\pi_A(1+\underline{\alpha})+\pi_P}{\pi_L+\pi_P+\pi_A} < 1 + \underline{\alpha}$

ii) *Members in group (R, P) vote for the challenger if and only if* $\frac{\underline{\alpha}(\pi_L+\pi_P)+\pi_A+\pi_P}{\pi_L+\pi_P+\pi_A} > 1 \Rightarrow \frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$

Thus, the challenger gets $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$ of the total votes if $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. We know from Lemma 1.2 that $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP} < 1/2$, which means challenger loses and L has no direct influence. Thus, direct influence from L requires $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$

Second, we establish the necessity of condition P1.2. When condition P1.1 holds, the challenger gets $n_{LP} + (1-\delta)n_{LA} + n_{RP}$ when L endorses her and gets $\frac{1}{2}\delta(n_{LA} + n_{RA}) + n_{RP} + n_{LP}$ when A endorsed her. Condition P1.2 ensures that the proportion of voters the challenger gets when L or A endorses her is greater than a half. The challenger otherwise loses, and there is no direct influence.

(***Sufficiency***)

For sufficiency, we show that the strategies described above are equilibrium strategies and that L and A have direct influence in such an equilibrium.

If conditions P1.1 and P1.2 are satisfied, then there exists an equilibrium in which the challenger accepts offers from L, A and P and wins when L or A endorses the challenger.

The challenger cannot profitably deviate to reject L's offer. Accepting L's offer gives the challenger a payoff of α_c. If the challenger rejects L's offer, the outcome is that the incumbent wins and implements (R, A), which gives the challenger a payoff of 1. Since $\alpha_c > 1$, the challenger cannot profitably deviate to rejecting L's offer.

Similarly, the challenger cannot profitably deviate to rejecting A's offer. The challenger gets a vote share of $\frac{1}{2}\delta(n_{LA}+n_{RA})+n_{RP}+n_{LP}$ if she accepts A's offer. Condition P1.2 implies that the challenger wins the election and gets a payoff of 1 if she accepts A's offer. Rejecting A's offer results in losing the election, giving her a payoff of 1. There is, therefore, no profitable deviation from accepting A's offer.

The challenger has no profitable deviation to rejecting P's offer. Note that P is like-

Table 1.3: The challenger's equilibrium strategy in Proposition 1.2a

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(R, A)
P	Accepts	(R, P)

minded and like-minded IGs have no direct influence. The outcome when P endorses the challenger is $(i, (R, A))$, giving her a payoff of 1. If she deviates to reject P's offer, she loses the election and gets a payoff of 1. Thus, there is no profitable deviation from accepting R's offer.

Now, it is easy to see that in such an equilibrium, both L and A have direct influence.

□

Proof of Proposition 1.2

Proof. Remember that to find indirect influence we only consider the equilibrium in which the challenger accepts offers from all IGs. For that we first look at the outcome of such an equilibrium.

Table 1.3 describes an equilibrium in which the challenger accepts all offers where column 3 has policies the challenger implements after accepting offer from each IG.

Given the challenger's strategy profile in Table 1.3, Table 1.4 describes how voters vote in the MT game. Column 4 specifies whether voters vote for the challenger (c) or the incumbent (i) when their IG endorses the challenger, where the payoff from voting for the challenger is given in the brackets. The payoff from voting for the incumbent is given in column 5. Column 6 specifies whether voters vote for the challenger or the incumbent when they neither see public commitment nor do they see endorsement from their IG; (expected) payoff from voting for the challenger is given in brackets. When payoffs are the same, the voters vote for each candidate with equal probability, which is written as 1/2. Given $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$, members of R with ideal policy (R, P) vote for the challenger.

Lemmas 1.1 and 1.2 imply that the challenger would only accept offer from unlike-minded IGs if her ideal policy and second best policy are unpopular. Thus, this equilibrium would only exist if Lemmas 1 and 2 hold. Then, the outcome in MT game is:

Table 1.4: Voters' vote for strategy profile given in Table 1.3

IGs	α	**Members**	**Vote (Endorsement)**	**Incumbent**	**Vote (no endorsement)**
L	$\alpha = \underline{\alpha}$	LP $(n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{(\pi_R+\pi_P)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_P+\pi_R})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$\mathbf{i}(\frac{(\pi_P+\pi_R)0+\pi_A\underline{\alpha}}{\pi_A+\pi_P+\pi_R})$
R	$\alpha = \underline{\alpha}$	RP $(n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{c}(\frac{\pi_L\underline{\alpha}+\pi_A 1+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_L+\pi_P})$
		RA $(n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\frac{\pi_L 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_L+\pi_P})$
A	$\alpha = \overline{\alpha}$	LA (δn_{LA})	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 1+(\pi_R+\pi_P)0}{\pi_L+\pi_P+\pi_R})$
		RA (δn_{RA})	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 0+(\pi_R+\pi_P)1}{\pi_L+\pi_P+\pi_R})$
P	$\alpha = \overline{\alpha}$	LP (δn_{LP})	$\mathbf{c}(\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_L 1+\pi_A 0+\pi_R\underline{\alpha}}{\pi_A+\pi_L+\pi_R})$
		RP (δn_{RP})	$\mathbf{c}(1+\overline{\alpha})$	1	$\mathbf{c}(\frac{\pi_L\overline{\alpha}+\pi_A 1+\pi_R(1+\underline{\alpha})}{\pi_A+\pi_L+\pi_R})$

- $$O_L = \begin{cases} (c,(L,P)) & \text{if } n_{LP}+n_{RP}+(1-\delta)n_{LA} > 1/2 \\ O^{NL}, & \text{otherwise} \end{cases}$$

- $O_R = O^{NL}$

- $$O_A = \begin{cases} (c,(R,A)) & \text{if } n_{LP}+n_{RP}+\frac{1}{2}[\delta(n_{LA}+n_{RA})] > 1/2 \\ O^{NL}, & \text{otherwise} \end{cases}$$

- $O_P = O^{NL}$

Any IG would have indirect influence if its absence would alter voting behavior. Since voters know which policy is implemented when their IG endorses the challenger, their vote does not change in column 4 when we change access probabilities. Therefore, we just need to check whether voting changes in column 6.

We first prove that L, R and A cannot have indirect influence.

Let $\pi_L = 0$. Then, voting does not change. Thus, L has no indirect influence.

- It does not affect the payoff (in column 6) of members of L.
- For members of R, setting $\pi_L = 0$
  - Members in group RP do not change their vote because $\frac{\pi_A 1+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_P} > 1$
  - Members in group RA do not change their vote because $\frac{\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_P} < 1+\underline{\alpha}$
- For members of A, setting $\pi_L = 0$
  - Members in group LA do not change their vote because $\frac{(\pi_R+\pi_P)0}{\pi_P+\pi_R} < \overline{\alpha}$
  - Members in group RA do not change their vote because $\frac{(\pi_R+\pi_P)1}{\pi_P+\pi_R} < 1+\underline{\alpha}$
- For members of P, setting $\pi_L = 0$

- Members in group LP do not change their vote because $\frac{\pi_A 0+\pi_R \underline{\alpha}}{\pi_A+\pi_R} > 0$
- Members in group RP do not change their vote because $\frac{\pi_A 1+\pi_R(1+\underline{\alpha})}{\pi_A+\pi_R} > 1$

Let $\pi_R = 0$. Then, voting does not change. Thus, R has no indirect influence.

- It does not affect the payoff (in column 6) of members of R.
- For members of L, setting $\pi_R = 0$
  - Members in group LP do not change their because $\frac{\pi_P \underline{\alpha}+\pi_A 0}{\pi_A+\pi_P} > 0$
  - Members in group LA do not change their because $\frac{\pi_P 0+\pi_A \underline{\alpha}}{\pi_A+\pi_P} < \underline{\alpha}$
- For members of A, setting $\pi_R = 0$
  - Members in group LA do not change their because $\frac{\pi_L 1+(\pi_P)0}{\pi_L+\pi_P+\pi_R} < \overline{\alpha}$
  - Members in group RA do not change their because $\frac{\pi_L 0+(\pi_P)1}{\pi_L+\pi_P} < 1+\underline{\alpha}$
- For members of P, setting $\pi_R = 0$
  - Members in group LP do not change their because $\frac{\pi_L 1+\pi_A 0}{\pi_A+\pi_L} > 0$
  - Members in group RP do not change their because $\frac{\pi_A 1+\pi_R(1+\underline{\alpha})}{\pi_A+\pi_R} > 1$

Let $\pi_A = 0$. Then, voting does not change. Thus, A has no indirect influence.

- It does not affect the payoff (in column 6) of members of A.
- For members of L, setting $\pi_A = 0$
  - Members in group LP do not change their vote because $\frac{(\pi_R+\pi_P)\underline{\alpha}}{\pi_P+\pi_R} > 0$
  - Members in group LA, do not change their vote because $\frac{(\pi_P+\pi_R)0}{\pi_P+\pi_R} < \underline{\alpha}$
- For members of R, setting $\pi_A = 0$
  - Members in group RP do not change their vote because $\frac{\pi_L \underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_L+\pi_P} > 1$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+\pi_P 1}{\pi_L+\pi_P} < 1+\underline{\alpha}$
- For members of P, setting $\pi_A = 0$
  - Members in group LP do not change their vote because $\frac{\pi_L 1+\pi_R \underline{\alpha}}{\pi_L+\pi_R} > 0$
  - Members in group RP do not change their vote because $\frac{\pi_L \overline{\alpha}+\pi_R(1+\underline{\alpha})}{\pi_L+\pi_R} > 1$

We now prove that P could have indirect influence.

If $\pi_P = 0$, then the members of IG R with policy preference (R, P) change their vote from the challenger to the incumbent.

- It does not affect the payoff (in column 6) of members of P.
- For members of L, setting $\pi_P = 0$
  - Members in group LP do not change their vote because $\frac{(\pi_R)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_R} > 0$
  - Members in group LA do not change their vote because $\frac{(\pi_R)0+\pi_A\underline{\alpha}}{\pi_A+\pi_R} < \underline{\alpha}$
- For members of R, setting $\pi_P = 0$
  - Members in group RP, *vote for incumbent* because $\frac{\pi_L\underline{\alpha}+\pi_A 1}{\pi_A+\pi_L} < 1$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_L} < 1+\underline{\alpha}$
- For members of A, setting $\pi_P = 0$
  - Members in group LA do not change their vote because $\frac{\pi_L 1+(\pi_R)0}{\pi_L+\pi_R} < \overline{\alpha}$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+(\pi_R)1}{\pi_L+\pi_R} < 1+\overline{\alpha}$

If $\pi_P = 0$ and L endorses the challenger, the challenger loses vote of the members of R who belong to group RP. The new vote share the challenger gets is $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$. This is less than half by Lemma 1.2. The challenger loses and policy (R, A) is implemented: so, $O_L^P = O^{NL}$ where O_L^{-P} be the outcome when $\pi_P = 0$ and L endorses the challenger in the modified game. The outcome does not change if the challenger were to loses when $\pi_P > 0$; it changes only when the challenger wins when $\pi_p > 0$ and loses when $\pi_P = 0$. Hence, the condition in 2.1 must be satisfied. Then, $O_L = (c, (L, P)) \neq O_L^{-P} = (i, (R, A)$ if $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$.

If $\pi_P = 0$ and A endorses the challenger, the challenger loses the vote of the members of R who belong to group RP. If $\pi_P = 0$ then vote share the challenger gets is $\frac{1}{2}[\delta(n_{LA} + n_{RA})] + n_{LP} + \delta n_{RP}$, which is less than the vote share she gets when $\pi_P > 0$. If she loses when $\pi_P > 0$, then she also loses when $\pi_P = 0$. Therefore, the outcome changes only when the challenger wins when $\pi_P > 0$ and loses when $\pi_P = 0$. Hence, the condition in 2.2 must be satisfied. □

Proof of Proposition 1.2a

Proof. To prove Proposition 1.2a, we can use Table 1.4 where the only difference come from the condition $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$: in Proposition 1.2, members of R with ideal policy (R, P) vote for the challenger because $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$; whereas in Proposition 1.2a, these same members vote for the incumbent because $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. Same as in Proposition 1.2, we need to show whether changing probability of one of the IGs changes voters' vote when they see no endorsement and no public commitment, i.e. their vote in Column 6 of Table 1.4. The above mentioned condition does not affect payoff and hence vote of the members of L, A and P. We know that to find indirect influence of an IG, we need to show whether the access probabilty of l changes the vote of members of other IGs $j \neq l$: changing access probability of l would not affect the payoff of its own members when they see no endorsement and no public commitment.

R **does not have indirect influence.** We know from Proposition 1.2a that setting $\pi_R = 0$ does not change the vote of the members of L, A and P and we also know that the payoff of the members of L, A, and P is the same in Proposition 1.2a and Proposition 1.2. Thus, R has no direct influence.

A **does not have indirect influence.** We know from Proposition 1.2 that setting $\pi_A = 0$ does not change vote of the members of L and P. We also know that the payoff of members of L and P do not change because their payoff is independent of condition mentioned above. Therefore, we just need to check whether setting $\pi_A = 0$ changes the vote of members of R. Note that now given the condition $\frac{\alpha}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$, all the members of R vote for the incumbent.

For members R, setting $\pi_A = 0$

- Members with ideal policy (R, P) do not change their vote because $\frac{\pi_L\underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_L+\pi_P} < 1$
- Members with ideal policy (R, A) do not change their vote because $\frac{\pi_L 0+\pi_P 1}{\pi_L+\pi_P} < 1+\underline{\alpha}$

P **does not have indirect influence.** We know from Proposition 1.2 that setting $\pi_P = 0$ does not change the vote of the members of L and A. We also know that the payoff of members of L and A do not change because their payoff is independent of condition mentioned above. So, we just need to check whether setting $\pi_A = 0$ changes the vote of members of R. Note that all the members of R vote for the incumbent when $\pi_l > 0$ for all l.

For members R, setting $\pi_R = 0$

- Members with ideal policy (R, P) do not change their vote because $\frac{\pi_L\underline{\alpha}+\pi_A 1}{\pi_A+\pi_L} < 1$
- Members with ideal policy (R, A) do not change their vote because $\frac{\pi_L 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_L} < 1+\underline{\alpha}$

We now prove that L could have indirect influence

L **has indirect influence.** We know from Proposition 1.2 that setting $\pi_L = 0$ does not change the vote of the members of A and P. We also know that the payoff of members of A and P do not change because their payoff is independent of condition mentioned above. So, we just need to check whether setting $\pi_L = 0$ changes the vote of members of R. Note that now all the members of R vote for the incumbent.

For members R, setting $\pi_L = 0$

- Members with ideal policy (R, P) *now vote for the challenger* because $\frac{\pi_A 1+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_P} > 1$
- Members with ideal policy (R, A) do not change their vote because $\frac{\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_P} < 1+\underline{\alpha}$

If $\pi_L = 0$ and A endorses the challenger, the challenger gets extra votes from the members of R with ideal policy (R, P). The new vote share the challenger gets is $\frac{1}{2}[\delta(n_{LA} + n_{RA})] + n_{LP} + n_{RP}$ and the old share vote share was $\frac{1}{2}[\delta(n_{LA} + n_{RA})] + n_{LP} + \delta n_{RP}$. If the old share was less than a half and new share is more than a half, then the challenger loses (when A endorses the challenger) when $\pi_L > 0$, but wins when $\pi_L = 0$. Let O_A^{-L} be the outcome when $\pi_L = 0$ and A endorses the challenger. Then, $O_A = (i, (R, A)) \neq O_A^{-L} = (c, (R, A)$ if $\frac{1}{2}[\delta(n_{LA} + n_{RA}] + n_{LP} + n_{RP} > 1/2 > n_{LP} + n_{RP} + (1 - \delta)n_{LA}$. The outcome changes only when the challenger loses when $\pi_L > 0$ and wins when $\pi_L = 0$. Hence, the condition in $2a$ must be satisfied. □

Proof of Proposition 1.3

Proof. Unlike-minded IG L would only have direct influence if it endorses the challenger and the challenger wins. Suppose by way of contradiction that the challenger accepts L's offer and wins in some equilibrium. We know by Lemma 1.1 that $O^{NL} = (i, (R, A))$.

If the challenger accepts L's offer and wins, she gets a payoff of α_c. The challenger can profitably deviate to rejecting L's offer and publicly committing to (R, P) or (R, A) and lose the election, thereby earning 1. The outcome is the same as in the no-lobbying game, where the challenger earns 1. Since $\alpha_c < 1$, the challenger can profitably deviate to reject L's offer. Hence, there is no equilibrium in which the challenger accepts L's offer and wins. Thus, L has no direct influence. □

Proof of Proposition 1.4

Table 1.5: The challenger's equilibrium strategy in Proposition 1.4

IGs	Challenger's decision	Policy implemented
L	Rejects	(R, P)
R	Rejects	(R, P)
A	Accepts	(R, A)
P	Accepts	(R, P)

Proof. The two necessary conditions are obvious. If the challenger rejects A's offer, there can be no direct influence because the outcome is same as the no-lobbying game. Second, the challenger has to accept the offer from at least one other IG. Suppose by way of contradiction that there is an equilibrium in which the challenger rejects offers from all other IGs and accept only A's offer. Then it will be a fully revealing equilibrium. Consider what happens when A endorses the challenger. The members of A receive information that the challenger would implement (R, A). Thus, the members of A vote for each candidate with equal probability.[39]. Members of other IGs see no endorsement and no public announcement. This happens on the equilibrium path only if IG A endorsed the challenger because the challenger rejects offer from all the other IGs. Voters know in that event that A must have endorsed the challenger an would implement (R, A). Thus, they all votes vote for each candidate with equal probability. The challenger gets half of the total votes and loses. Hence, A has no direct influence.

Second, A can only have direct influence if the challenger accepts offer from at least one of L or R. Suppose by way of contradiction that the challenger rejects the offers from both L and R: the challenger's strategy profile is given in Table 1.5.

We want to show that A has no direct influence for the strategy profile given in Table 1.5. We now look at the share of votes the challenger gets what when A endorses the challenger for the strategy profile given in Table 1.5. Table 1.6 specifies voters' payoff when A endorses the challenger. Column 4 and 5 respectively specify voters' payoff from voting for the challenger and for the incumbent. Voters vote for the challenger iff payoff in column 4 is greater than payoff in column 5. If the two payoffs are equal, the voters vote for each candidate with equal probability, for which we write 1/2. Column 6 specifies whether voters vote for the challenger (c) or the incumbent (c) by comparing payoffs in columns 4 and 5.

In this equilibrium, the challenger only accepts offers from A and P. When A

[39]The challenger implements (R, A) when she accepts A's offer.

Table 1.6: Voters' vote when A endorses the challenger

IGs	α	Members	Payoff challenger	Payoff incumbent	Vote for
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$\frac{\pi_P \underline{\alpha} + \pi_A 0}{\pi_A + \pi_P}$	0	c
		$LA(n_{LA}(1-\delta)$	$\frac{\pi_A \underline{\alpha} + \pi_P 0}{\pi_A + \pi_P}$	$\underline{\alpha}$	i
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$\frac{\pi_A 1 + \pi_P (1+\underline{\alpha})}{\pi_A + \pi_P}$	1	c
		$RA\ (n_{RA}(1-\delta)$	$\frac{\pi_A (1+\underline{\alpha}) + \pi_P 1}{\pi_A + \pi_P}$	$1+\underline{\alpha}$	i
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\overline{\alpha}$	$\overline{\alpha}$	1/2
		$RA\ (\delta n_{RA})$	$1+\overline{\alpha}$	$1+\overline{\alpha}$	1/2
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	0	0	1/2
		$LP\ (\delta n_{LP})$	1	1	1/2

endorses the challenger, the members of A see that their IG endorsed the challenger, so the challenger would implement (R, A). Members of other IG neither see endorsement from their IGs nor see a public announcement. On the equilibrium path, members of P neither see endorsement nor see public announcement if A endorsed the challenger. Thus, members of P know which policy the challenger would implement when A endorsed the challenger. Members of L and R, know that either A or P endorsed the challenger when their IG did not get access and they see no public announcement.

The vote share the challenger gets when A endorses the challenger is $\frac{1}{2}\delta + (1 - \delta)(n_{LP} + n_{RP})$. Lemma 1.1 implies that $n_{LP} + n_{RP} < 1/2$ in any influential equilibrium. This implies that $n_{LP} + n_{RP} < 1/2$; so, $\frac{1}{2}\delta + (1-\delta)(n_{LP} + n_{RP}) < 1/2$.

Thus, the challenger loses when A endorses the challenger. Hence, A has no direct influence. $\square$

Proof of Proposition 1.5

Proof. Note that the only difference between Propositions 1.5 and 1.2 is in α_c: $\alpha_c > 1$ in Proposition 1.2. But the condition $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ is the same in both the Propositions. Now, for indirect we consider the equilibrium of the MT game in which the challenger accepts all offers. Then we can use Tables 1.3 and 1.4 as it is because α_c does not affect voters' vote. We also know that given $\alpha_c < 1$, the challenger must lose when L endorses the challenger; otherwise the challenger could profitably deviate to rejecting L's offer; so, L does not have direct influence.

Since only α_c changes, which does not change voters behaviour, we can make same arguments for the indirect influence of L, R and A. However, for P, the argument before was that the presence of P affects the direct influence of L: L does not have direct influence when $\pi_P = 0$. But now, L does not have direct influence, irrespective of the presence of P because the challenger must lose when L endorses the challenger. Thus, P does not affect L' direct influence. But by the same argument as in Proposition 1.2, P can affect direct influence of IG A. $\square$

Proof of Proposition 1.5a See proof of Proposition 1.2a.

Proof of Lemma 1.4

Proof. The proof is by contradiction. Suppose $n_{LP} + \delta n_{RP} + (1 - \delta)n_{LA} > 1/2$. Thus, the outcome in no-lobbying game is that the challenger wins and policy (L, P) is implemented, i.e. $O^{NL} = (c, (L, P))$. Let U_{max} denote the utility of the challenger when she wins and implements her ideal policy. Thus, she $U_{max} = 1 + \alpha_c$ in the no-lobbying game.

In MT game, if $n_{LP}+\delta n_{RP}+(1-\delta)n_{LA} > 1/2$ then challenger must reject offer from both unlike-minded IGs, otherwise the challenger would have to promise to implement a policy different from her ideal but then she could deviate to reject and publicly commit to her ideal policy. Thus, unlike-minded IGs do not have direct influence. The challenger implements her ideal policy when like-minded IGs endorse the challenger. She would accept offers from like-minded IGs only if she were to win, otherwise she can deviate to publicly commit to her ideal policy. Thus, if like-minded IGs gets access, she either accepts their offers and win or she rejects and publicly commits to her ideal policy and wins. Thus, like-minded IGs do not have direct influence either. $\square$

Proof of Lemma 1.5

Proof. $n_{LP}+n_{RP}$ is the proportion of votes the challenger gets if she publicly commit to policy (R, P), which is the challenger's second best policy. Lemma 1.4 implies that the challenger loses the election if she publicly commits to her ideal policy in the no-lobbying game. Now, assume by way of contradiction that $n_{LP}+n_{RP} > 1/2$. Then, the outcome in the no-lobbying game is that the challenger wins and policy (R, P) is implemented; $o^{NL} = (c, (R, P))$. Thus, she earns α_c.

In the MT game, the challenger rejects the offer from unlike-minded IG A, otherwise she would have to promise to implement (L, A), which is her third best policy; but then she could do better by rejecting and publicly committing to (R, P). Thus, A has no direct influence. If she accepts R's offer, she would have to commit to implement (R, P).

Table 1.7: The challenger's equilibrium strategy in Proposition 1.6

IGs	Challenger's decision	Policy implemented
L	Rejects	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Rejects	(L, P)

She would only accept R's offer if she were to win. If she accepts R's offer and loses then incumbent wins, earning 0. But, she could then deviate to publicly announce her second best policy, earning α_c. In any case, the outcome is the same. Thus, R has no direct influence. From Observation 1.2, we know that like-minded IGs never have direct influence. Hence, MT is not directly influential. □

Proof of Proposition 1.6

Proof. (***Necessity***)

We first establish the necessity of conditions P6.1 and P6.2. Given that $n_{LP}+n_{LA} < 1/2$ and that the challenger rejects from both like-minded IGs, unlike minded IGs could only have direct influence if the challenger accepts their offers. Table 1.7 specifies the equilibrium decision of the challenger and policies she would implement for each of the offers she accepts. Note that $n_{LP}+n_{LA} < 1/2$; so by Lemmas 1.4 and 1.5, the challenger loses if she publicly commits to any policy. In case of indifference, the challenger commits to her ideal policy: $O^{NL} = (i, (R, A))$.

Table 1.8 specifies how voters vote. Column 4 specifies whether they vote for the challenger (c) or incumbent (i) when they see endorsement from their IG. Column 5 specifies their payoffs if they vote for the incumbent. Column 6 specifies whether they vote for the challenger (c) or incumbent (i) when they neither see public commitment nor endorsement. Note that the challenger only accepts offers from unlike-minded IGs R and A so, members of L and P do not see endorsement on the equilibrium path.[40]

By looking at Table 1.8, we can see that: members of L with policy preference (L, A) vote for the challenger iff $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_R} > \underline{\alpha} \Rightarrow \frac{\pi_A}{\pi_R} > \underline{\alpha}$; members of P with policy preference (R, P) vote for the challenger iff $\frac{\pi_R(1+\overline{\alpha})+\pi_A 0)}{\pi_A+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A}{\pi_R}$.

Note that the voters in at least one of these groups vote for the challenger. If $\pi_A > \pi_R$, then the members of L vote for the challenger since $1 > \underline{\alpha}$. If $\pi_A < \pi_R$, then

[40]But they can only see endorsement off the equilibrium path if their IGs got access in which case they know that the challenger would implement her ideal policy.

Table 1.8: Voters' vote for strategy profile given in Table 1.7

IGs	α	Members	Vote (Endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{\pi_R \underline{\alpha} + \pi_A 1}{\pi_A + \pi_R})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$(\frac{\pi_R 0 + \pi_A (1+\underline{\alpha})}{\pi_A + \pi_R})$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{i}\ (0)$
		$RA\ (n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\underline{\alpha})$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(0)$
		$RA\ (\delta n_{RA})$	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(1)$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$\mathbf{c}(1+\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_A 1 + \pi_R \underline{\alpha}}{\pi_A + \pi_R})$
		$RP\ (\delta n_{RP})$	$\mathbf{c}(\overline{\alpha})$	1	$(\frac{\pi_R (1+\overline{\alpha}) + \pi_A 0)}{\pi_A + \pi_R})$

the members of P vote for the challenger since $1 < \overline{\alpha}$.

When R endorses the challenger, only members of R see endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$, if members of L do not vote for the challenger. This vote share is less than half by Lemma 1.5. Thus, the members of L must vote for the challenger for R to have direct influence. This requires either $\frac{\pi_A}{\pi_R} > 1$ or $1 > \frac{\pi_A}{\pi_R} > \underline{\alpha}$.

When A endorses the challenger, only members of A receive an endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{LA}$, if members of P do not vote for the challenger. This vote share is less than half the given condition in the premise. Thus, the members of P must vote for the challenger for A to have direct influence. This requires either $\frac{\pi_A}{\pi_R} < 1$ or $1 < \frac{\pi_A}{\pi_R} < \overline{\alpha}$. These two conditions gives condition in P6.1.

When condition P6.1 is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} + (1-\delta)n_{LA}$ or $n_{LP} + n_{LA} + \delta n_{RP}$ when R or A endorses the challenger respectively. The challenger wins if these vote shares form majority. Otherwise, the challenger loses and the IGs do not have direct influence. Hence, condition P6.2 must be satisfied.

(***Sufficiency***)

For sufficiency, we prove that the challenger does not have profitable deviation from the prescribed strategy profile.

The challenger rejects offers from both like-minded IGs L and P and loses, earning 0. The challenger cannot profitably deviate to (accepting) L's offer. If she accepts L offer, she gets votes from members of L and P. This gives her a total vote share of $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 1.4. Thus, the challenger loses and earns 0. If she accepts P's offer, she gets votes from members of L and P. This gives her a total vote share of $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half

Table 1.9: The challenger equilibrium strategy in Proposition 1.7

IGs	**Challenger's decision**	**Policy implemented**
L	Rejects	(L, A)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Rejects	(L, A)

by Lemma 1.4. Thus, the challenger loses and earns 0.

The challenger accepts offers from unlike-minded IGs R and A. If conditions P6.1 and P6.2 are satisfied, the challenger wins and implements (R, P) or (L, A) when R or A endorses the challenger respectively. Implementing (R, P) gives her a payoff of α_c and implementing (L, A) gives her a payoff of 1. The challenger cannot profitably deviate (to rejecting) R's or A's offer. If she rejects, she publicly commits to (L, P) and thus loses the election and gets a payoff of 0. □

Proof of Proposition 1.7

Proof. First, we show the uniqueness. Given $n_{LA} + n_{LP} > 1/2$, $O^{NL} = (c, (L, A))$, so the challenger earns 1 in the no-lobbying game. We know from Observation 1.1 that like-minded IGs have no direct influence. If the challenger accepts like-minded IGs offer then she loses and gets a payoff of 0. But she could profitably deviate to reject their offers and publicly commit to (L, A). Thus, the challenger rejects offers from both like-minded IGs and publicly commit to (L, A). We also know that for any IG to have direct influence, the challenger must accept offers from at least two IGs. Since the challenger rejects offers from both like-minded IGs, the challenger must accept offers from both unlike-minded IGs for their direct influence.

(***Necessity***)

Next we show the necessary conditions for such an equilibrium to exist. Table 1.9 describes the equilibrium in which the challenger rejects offers from both like-minded IGs and accepts offers from both unlike-minded IGs. The challenger publicly commits to (L, A) in the no-lobbying game and when she rejects an offer in MT games. For such an equilibrium to exist, the challenger must win the election after accepting offers from unlike-minded IGs R and A, otherwise she could profitably deviate to publicly committing to (L, A): so, the challenger must get enough votes when R or A endorses the challenger.

When R endorses the challenger, members of R get to know that their IG endorsed the challenger and that the challenger would implement (R, P); members of other IG do not see endorsement or public commitment. In such an equilibrium, only R and A can endorse the challenger: so when R endorses the challenger, members of A know that R must have endorsed the challenger and members of L and P know that one of R or A must have endorsed the challenger. Then, the proportion of votes the challenger receives when R endorses the challenger is given below.

1) **Members of R with ideal policy (R, P) vote for the challenger because $1 + \overline{\alpha} > 1$, and members with ideal policy (R, A) vote for the incumbent because $1 + \overline{\alpha} > 1$.**

2) **Members of L vote as follows:**

i) Members with ideal policy (L, P) vote for the challenger because $\frac{\pi_A 1 + \pi_R \alpha}{\pi_A + \pi_R} > 0$.

ii) *Members with ideal policy (L, A) vote for the challenger iff* $\frac{\pi_R 0 + \pi_A (1 + \underline{\alpha})}{\pi_A + \pi_R} > \underline{\alpha} \Rightarrow \frac{\pi_A}{\pi_R} > \underline{\alpha}$.

3) **Members of A vote for the incumbent**

i) Members with ideal policy (R, A) vote for the incumbent because $1 < \overline{\alpha}$.

ii) Members with ideal policy (L, A) vote for the incumbent because $0 < \overline{\alpha}$.

4) **Members of P vote as follows:**

i) Members with ideal policy (R, A) vote for the challenger because $\frac{\pi_R \overline{\alpha} + \pi_A 1}{\pi_A + \pi_R} > 0$.

ii) *Members with ideal policy (R, P) vote for the challenger if and only if* $\frac{\pi_R(1+\overline{\alpha}) + \pi_A 0}{\pi_A + \pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A}{\pi_R}$

Thus, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$ if the members of L with ideal policy (L, A) do not vote for the challenger. We know from Lemma 1.5 that $n_{LP} + n_{RP} < 1/2$, which means the challenger loses and we know that the challenger must not lose the election (when R endorses her) for this equilibrium to exist. Thus, direct influence from R requires $\frac{\pi_A}{\pi_R} > \underline{\alpha}$ or $\frac{\pi_A}{\pi_R} > 1$. If this condition is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} + (1 - \delta) n_{LA}$ when R endorses the challenger. For R to have direct influence, the challenger must win, meaning that this vote share must form a majority. Hence, the conditions must be satisfied.

We know that the challenger must win the election when R or A endorses the challenger. Then, A has no direct influence because the challenger wins either way and implements the same policy. The outcome when R endorses the challenger is $O_R = (c, R, P)$.

Table 1.10: The challenger equilibrium strategy in Proposition 1.8a,b,c,d

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Accepts	(L, P)

Thus, only R has direct influence.

(***Sufficiency***)

The challenger cannot profitably deviate to rejecting R's offer (to publicly committing to (L, A)). She wins when R endorses the challenger and gets a payoff of α_c. Rejecting R's offer and publicly committing to (L, A) would give her a payoff of 1. Since $\alpha_c > 1$, there is no profitable deviation. Similarly, there is no profitable deviation for the challenger to reject A's offer. When A endorses the challenger, she wins and implements (L, A), which earns her 1. If she deviates, she would still earn a payoff of 1.

The challenger has no profitable deviation to accept L's or P's offer. If she reject L's offer, she gets a payoff of 1 by publicly committing to (L, A). The maximum vote she can get if she accepts L or P's offer is $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 1.4, earning 0. $\square$

Arguments and Tables we use to prove Propositions 8, 8a, 8b, 8c. Since we want to find indirect influence, we look at the equilibrium of the MT game in which the challenger accepts offers from all IGs. We know that such an equilibrium would only exist only if $n_{LP} + n_{LA} < 1/2$; otherwise the challenger can profitably deviate to reject like-minded IGs' offer and publicly commit to (L, A). Table 1.10 specifies the challenger equilibrium strategy and the policy she implements if she wins after an IG endorses her.

Given the challenger's equilibrium strategy, we specify in Table 11 the payoff voters get if they vote for the challenger after they see endorsement from their IG (column 4), payoff they get if they vote for the incumbent (column 5), and (expected) payoff by voting for the challenger if they see no endorsement and no public commitment (column 6).

Vote share when L endorses the challenger. When L endorses the challenger, members of L see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

Table 1.11: Voters' vote for the strategy profile given in Table 1.10

IGs	α	Members	Vote (Endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$1+\underline{\alpha}$	0	$\frac{\pi_A 1+\pi_R \underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_R+\pi_P}$
		$LA(n_{LA}(1-\delta)$	1	$\underline{\alpha}$	$\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P}$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$1+\underline{\alpha}$	1	$\frac{(\pi_L+\pi_P)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_P}$
		$RA\ (n_{RA}(1-\delta)$	1	$1+\underline{\alpha}$	$\frac{(\pi_L+\pi_P)+0+\pi_A\underline{\alpha}}{\pi_A+\pi_L+\pi_P}$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\overline{\alpha}$	$\overline{\alpha}$	$\frac{(\pi_L+\pi_P)1+\pi_R 0}{\pi_R+\pi_L+\pi_P}$
		$RA\ (\delta n_{RA})$	$1+\overline{\alpha}$	$1+\overline{\alpha}$	$\frac{(\pi_P+\pi_L)0+\pi_R 1}{\pi_R+\pi_L}$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$1+\overline{\alpha}$	0	$\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L+\pi_R}$
		$RP\ (\delta n_{RP})$	$\overline{\alpha}$	1	$\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R}$

- Members of L vote for the challenger (column 4 > column 5).
- Members of R and A vote for the incumbent (column 5 > column 6).
- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy (R, P) vote for the challenger iff $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$*

Vote share when P endorses the challenger. When P endorses the challenger, members of P see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of P vote for the challenger (column 4 > column 5).
- Members of R and A vote for the incumbent (column 5 > column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy (L, A) vote for the challenger iff $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$*

Vote share when R endorses the challenger. When R endorses the challenger, members of R see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of R
  - with ideal policy (R, P) vote for the challenger (column 4 > column 5).
  - with ideal policy (R, A) vote for the incumbent (column 4 < column 5).
- Members of A vote for the incumbent (column 5 > column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).

- *With ideal policy (L, A) vote for the challenger iff* $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$

- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy (R, P) vote for the challenger iff* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$

Vote share when A endorses the challenger. When A endorses the challenger, members of A see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of A
  - with ideal policy (L, A) vote for the challenger (column 4 > column 5).
  - with ideal policy (R, A) vote for the incumbent (column 4 < column 5).
- Members of R vote for the incumbent (column 5 > column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy (L, A) vote for the challenger iff* $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$
- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy (R, P) vote for the challenger iff* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$

Proof of Proposition 1.8

Proof. Conditions $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$ and $\pi_A > \pi_R$ in the proposition implies that that the members of P with ideal policy (R, P) and members of L with ideal policy (L, A) vote for the challenger if they see no endorsement and no public commitment.

We know from Observation 1.2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (i, (R, A))$.

When R endorses the challenger, the vote share of the challenger is $n_{LP} + n_{RP} + (1 - \delta)n_{LA}$. The outcome then is $O_R = (c, (R, P))$ if $n_{LP} + n_{RP} + (1 - \delta)n_{LA} > 1/2$, O_{NL} otherwise. When A endorses the challenger, the vote share of the challenger is $n_{LP} + \delta n_{RP} + n_{LA}$. The outcome then is $O_A = (c, (L, A))$ if $n_{LP} + \delta n_{RP} + n_{LA} > 1/2$, O_{NL} otherwise.

We first prove L's indirect influence. L might have indirect if setting $\pi_L = 0$ changes votes when voters neither see endorsement nor a public commitment. Setting $\pi_L = 0$ does not change the vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, their payoff in column 5 is still greater than their expected payoff in column 6. Members of P with ideal policy (L, P) do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of P with ideal policy (R, P) might change their vote: setting $\pi_L = 0$, their payoff in column 5 is greater than their expected payoff in column 6 if $\overline{\alpha} < \frac{\pi_A}{\pi_R}$, in which case they vote for the incumbent.* Then, when A endorses the challenger when $\pi_L = 0$ and $\overline{\alpha} < \frac{\pi_A}{\pi_R}$, she loses votes of the members of R with ideal policy (R, P) and therefore loses the election because $n_{LP} + n_{LA} < 1/2$. If the challenger won when A endorsed her when $\pi_l > 0$ for all l and loses when $\pi_L = 0$ then the outcome O_A changes. Hence, condition 8.1 must be satisfied. Similarly, if the challenger won when R endorsed her when $\pi_L = 0$, and loses the challenger loses if losing these voters (members of R with ideal policy (R, P)) means losing majority then the outcome changes. Hence, condition 8.2 must be satisfied.

Next, we prove R's indirect influence. Setting $\pi_R = 0$ does not change vote of the members of R: their payoff in column 6 is independent of π_R. Members of IGs L and A do not change their: setting $\pi_R = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of L in column 6 is still greater than payoff in column 5. Members of P with ideal policy (L, P) do not change their vote: setting $\pi_R = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of P with ideal policy (R, P) might change their vote: setting $\pi_R = 0$, their payoff in column 5 is greater than their expected payoff in column 6, if $\overline{\alpha} < \frac{\pi_A + \pi_L}{\pi_L}$, in which case they vote for the incumbent.* So, when A endorses the challenger when $\pi_R = 0$, she loses votes of the members of R with ideal policy (R, P) and therefore loses the election because $n_{LP} + n_{LA} < 1/2$. If the challenger won when A endorsed her when $\pi_l > 0$ for all l and loses when $\pi_L = 0$ then the outcome O_A changes. Hence,condition 8.3 must be satisfied.

Next, we prove A's indirect influence. Setting $\pi_A = 0$ does not change vote of the members of A: their payoff in column 6 is independent of π_A. Members of IGs L and P do not change their vote: setting $\pi_A = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of P in column 6 is still greater than payoff in column 5. Members of L with ideal policy (L, P) do not change their vote: setting $\pi_A = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of L with ideal policy (L, A) might change their vote: setting $\pi_A = 0$, their payoff in column 5 is greater than their expected payoff in column 6, if $\underline{\alpha} > \frac{\pi_P}{\pi_P + \pi_R}$, in which they vote for the incumbent.* So, when R endorses the challenger when $\pi_A = 0$, she loses votes of the members of L with ideal policy (L, A) and therefore loses the election because $n_{LP} + n_{RP} < 1/2$ by Lemma 1.5. If the challenger won when R endorsed her when $\pi_l > 0$ for all l and loses when $\pi_A = 0$ then the outcome O_R changes. Hence, condition 8.4.

Next, we prove why P cannot have indirect influence. Setting $\pi_P = 0$ does not change vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their vote: setting $\pi_P = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of L do not change their vote: setting $\pi_P = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. So setting $\pi_P = 0$ does not change any vote. Hence P has no indirect influence. □

Proof of Proposition 1.8a.

Proof. Conditions $\frac{\pi_A + \pi_L}{\pi_R + \pi_L} > \overline{\alpha}$ and $\pi_A > \pi_R$ in the proposition imply that the members of P with ideal policy (R, P) vote for the incumbent and members of L with ideal policy (L, A) vote for the challenger if they see no endorsement and no public commitment.

We know from Observation 1.2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (i, (R, A))$.

When R endorses the challenger, the vote share of the challenger is $n_{LP} + (1 - \delta)(n_{RP} + n_{LA})$. The outcome then is $O_R = (c, (R, P))$ if $n_{LP} + (1-\delta)(n_{RP} + n_{LA}) > 1/2$, O_{NL} otherwise.

When A endorses the challenger, the vote share of the challenger is $n_{LP} + n_{LA}$, which is less than a half so, the the outcome O_{NL}.

We first prove that why L, P and R cannot have direct influence. Let $\pi_L = 0$. Setting $\pi_L = 0$ does not change vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of P do not change their vote: setting $\pi_L = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (R, P) still vote for the incumbent because $\pi_A > \pi_R \Rightarrow \frac{\pi_A}{\pi_R} > \frac{\pi_A + \pi_L}{\pi_R + \pi_L}$.

Let $\pi_P = 0$. Setting $\pi_P = 0$ does not change vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their vote: setting $\pi_P = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of L do not change their vote: setting $\pi_P = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (L, A) still vote for the challenger because $\frac{\pi_A}{\pi_R} > \underline{\alpha}$.

Let $\pi_R = 0$. Setting $\pi_R = 0$ does not change vote of the members of R: their payoff in column 6 is independent of π_R. Members of IGs A do not change their vote: setting $\pi_R = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6. Members of L do not change their vote: setting $\pi_R = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (L, A) still vote for the challenger because $\frac{\pi_A + \pi_P}{\pi_P} > \underline{\alpha}$. Members of P do not change their vote: setting $\pi_R = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (R, P) still vote for the incumbent because $\overline{\alpha} < \frac{\pi_A + \pi_L}{\pi_R + \pi_L} < \frac{\pi_A + \pi_L}{\pi_L}$.

Next, we prove A's indirect influence. Let $\pi_A = 0$. Setting $\pi_A = 0$ does not change vote of the members of A: their payoff in column 6 is independent of π_A. Members of IGs R do not change their vote: setting $\pi_A = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6. Member of L and P with ideal policy (L, P) do not change their vote: setting $\pi_A = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. Members of L with ideal policy (L, A) might change their vote: setting $\pi_A = 0$, these members vote for the incumbent if $\frac{\pi_P}{\pi_P + \pi_R} < \underline{\alpha}$. *Members of P with ideal policy (R, P) certainly change their vote: setting $\pi_A = 0$, their payoff in column 6 is greater than their expected payoff in column 5 because $\overline{\alpha} > \frac{\pi_L}{\pi_R + \pi_L}$, in which they vote for the challenger.* So when R endorses the challenger and $\pi_A = 0$, the challenger gains more votes from members of R with ideal policy (R, P) and might lose votes from members of L with ideal policy (L, A). If $\overline{\alpha} > \frac{\pi_L}{\pi_R + \pi_L}$ then the new vote share of the challenger gets when R endorses her is $n_{RP} + n_{LP}$, which is less than a half by Lemma 1.5. We know that the outcome $O_R = (c, (R, P))$ when $\pi_l > 0$ for all l if $n_{LP} + (1 - \delta)(n_{RP} + n_{LA}) > 1/2$. The outcome when $\pi_A = 0$ is $O_R = O_{NL}$ (the challenger loses) if $\frac{\pi_P}{\pi_P + \pi_R} < \underline{\alpha}$. Then, there is change in outcome. Hence, condition 8a.1 must be satisfied. We know that the outcome is $O_R = O_{NL}$ when $\pi_l > 0$ for all l if $n_{LP} + (1 - \delta)(n_{RP} + n_{LA}) < 1/2$. The outcome when $\pi_A = 0$ is $O_R = (c, (R, P))$ (the challenger wins) if $\frac{\pi_P}{\pi_P + \pi_R} > \underline{\alpha}$ and $n_{LP} + n_{RP} + (1 - \delta)n_{LA} > 1/2$ (new vote share when the challenger gets more votes but does not lose vote). Then, there is change in outcome. Hence, condition 8a.2 must be satisfied. □

Proof of Proposition 1.8b

Proof. Conditions $\frac{\pi_A + \pi_P}{\pi_P + \pi_R} > \underline{\alpha}$ and $\pi_R > \pi_A$ in the proposition implies that that the members of P with ideal policy (R, P) and members of L with ideal policy (L, A) vote for the challenger if they see no endorsement and no public commitment.

We know from Observation 1.2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (i, (R, A))$.

When R endorses the challenger, the vote share of the challenger is $n_{LP} + n_{RP} + (1 - \delta)n_{LA}$. The outcome then is $O_R = (c, (R, P))$ if $n_{LP} + n_{RP} + (1 - \delta)n_{LA} > 1/2$, O_{NL} otherwise.

When A endorses the challenger, the vote share of the challenger is $n_{LP} + \delta n_{RP} + n_{LA}$. The outcome then is $O_A = (c, (L, A))$ if $n_{LP} + \delta n_{RP} + n_{LA} > 1/2$, O_{NL} otherwise.

We first prove P's indirect influence. P might have indirect if setting $\pi_P = 0$ changes votes when voters neither see endorsement nor a public commitment. Setting $\pi_P = 0$ does not change the vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their: setting $\pi_P = 0$, their payoff in column 5 is still greater than their expected payoff in column 6. Members of L with ideal policy (L, P) do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of L with ideal policy (L, A) might change their vote: setting $\pi_P = 0$, their payoff in column 5 is greater than their expected payoff in column 6 if $\underline{\alpha} > \frac{\pi_A}{\pi_R}$, in which case they vote for the incumbent.* So, when R endorses the challenger when $\pi_P = 0$ and $\underline{\alpha} > \frac{\pi_A}{\pi_R}$, she loses votes of the members of L with ideal policy (L, A) and therefore loses the election because $n_{LP} + n_{LA} < 1/2$. If the challenger won when R endorsed her when $\pi_l > 0$ for all l and loses when $\pi_P = 0$ then the outcome O_R changes. Hence, condition 8b.1 must be satisfied. Similarly, if the challenger won when A endorsed her when $\pi_L = 0$,the challenger loses if losing these voters (members of L with ideal policy (L, A)) means losing majority then the outcome changes. Hence, condition 8b.2 must be satisfied.

Next, we prove R's indirect influence. Setting $\pi_R = 0$ does not change vote of the members of R: their payoff in column 6 is independent of π_R. Members of IGs L and A do not change their: setting $\pi_R = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of L in column 6 is still greater than payoff in column 5. Members of P with ideal policy (L, P) do not change their vote: setting $\pi_R = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of P with ideal policy (R, P) might change their vote: setting $\pi_R = 0$, their payoff in column 5 is greater than their expected payoff in column 6, if $\overline{\alpha} < \frac{\pi_A + \pi_L}{\pi_L}$, in which case they vote for the incumbent.* So, when A endorses the challenger when $\pi_R = 0$, she loses votes of the members of R with ideal policy (R, P) and therefore loses the election because $n_{LP} + n_{LA} < 1/2$. If the challenger won when A endorsed her when $\pi_l > 0$ for all l and loses when $\pi_L = 0$ then the outcome O_A changes. Hence, condition 8b.3 must be satisfied.

Next, we prove A's indirect influence. Setting $\pi_A = 0$ does not change vote of the members of A: their payoff in column 6 is independent of π_A. Members of IGs L and P do not change their vote: setting $\pi_A = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of P in column 6 is still greater than payoff in column 5. Members of L with ideal policy (L, P) do not change their vote: setting $\pi_A = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of L with ideal policy (L, A) might change their vote: setting $\pi_A = 0$, their payoff in column 5 is greater than their expected payoff in column 6, if $\underline{\alpha} > \frac{\pi_P}{\pi_P + \pi_R}$, in which they vote for the incumbent.* So, when R endorses the challenger when $\pi_A = 0$, she loses votes of the members of L with ideal policy (L, A) and therefore loses the election because $n_{LP} + n_{RP} < 1/2$ by Lemma 1.5. If the challenger won when R endorsed her when $\pi_l > 0$ for all l and loses when $\pi_A = 0$ then the outcome O_R changes. Hence, condition 8b.4.

Next, we prove why L cannot have indirect influence. Setting $\pi_L = 0$ does not change vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of P do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. So setting $\pi_L = 0$ does not change any vote. Hence L has no indirect influence. $\square$

Proof of Proposition 1.8c

Proof. Conditions $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} < \underline{\alpha}$ and $\pi_R > \pi_A$ in the proposition imply that the members of L with ideal policy (L, A) vote for the incumbent and members of P with ideal policy (R, P) vote for the challenger if they see no endorsement and no public commitment.

We know from Observation 1.2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (i, (R, A))$.

When A endorses the challenger, the vote share of the challenger is $n_{LP} + \delta(n_{RP} + n_{LA})$. The outcome then is $O_R = (c, (R, P))$ if $n_{LP} + \delta(n_{RP} + n_{LA}) > 1/2$, O_{NL} otherwise.

When R endorses the challenger, the vote share of the challenger is $n_{LP}+n_{RP}$, which is less than a half by Lemma 1.5 so, the the outcome O_{NL}.

We first prove that why L, P and A cannot have direct influence. Let $\pi_L = 0$. Setting $\pi_L = 0$ does not change vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of P do not change their vote: setting $\pi_L = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (R, P) still vote for the incumbent because $\overline{\alpha} > \frac{\pi_A}{\pi_R}$.

Let $\pi_P = 0$. Setting $\pi_P = 0$ does not change vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their vote: setting $\pi_P = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of L do not change their vote: setting $\pi_P = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (L, A) still vote for the challenger because $\pi_R > \pi_A \Rightarrow \frac{\pi_A}{\pi_R} < \frac{\pi_A+\pi_P}{\pi_R+\pi_P}$.

Let $\pi_A = 0$. Setting $\pi_A = 0$ does not change vote of the members of A: their payoff in column 6 is independent of π_A. Members of IGs R do not change their vote: setting $\pi_A = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6. Members of P do not change their vote: setting $\pi_A = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (R, A) still vote for the challenger because $\frac{\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$. Members of L do not change their vote: setting $\pi_A = 0$, payoff of members with ideal (L, P) in column 6 is still greater than in column 5; members with ideal policy (L, A) still vote for the incumbent because $\underline{\alpha} > \frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \frac{\pi_P}{\pi_P+\pi_R}$.

Next, we prove R's indirect influence. Let $\pi_R = 0$. Setting $\pi_R = 0$ does not change vote of the members of R: their payoff in column 6 is independent of π_R. Members of IGs A do not change their vote: setting $\pi_R = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6. Member of L and P with ideal policy (L, P) do not change their vote: setting $\pi_A = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. *Members of R with ideal policy (R, P) might change their vote: setting $\pi_R = 0$, these members vote for the incumbent if $\frac{\pi_A+\pi_L}{\pi_L} > \overline{\alpha}$. Members of L with ideal policy (L, A) certainly change their vote: setting $\pi_R = 0$, their payoff in column 6 is greater than their expected payoff in column 5 because $\underline{\alpha} < \frac{\pi_A+\pi_P}{\pi_P}$, in which they vote for the challenger.* So when A endorses the challenger and $\pi_R = 0$, the challenger gains more votes from members of L with ideal policy (L, A) and might lose votes from members of R with ideal policy (R, P). If $\frac{\pi_A+\pi_L}{\pi_L} > \overline{\alpha}$ then the new vote share of the challenger gets when R endorses her is $n_{LA} + n_{LP}$, which is less than a half. We know that the outcome $O_A = (c, (L, A))$ when $\pi_l > 0$ for all l if $n_{LP} + \delta(n_{RP} + n_{LA}) > 1/2$. The outcome when $\pi_A = 0$ is $O_R = O_{NL}$ (the challenger loses) if $\frac{\pi_A+\pi_L}{\pi_L} > \overline{\alpha}$. Then, there is change in outcome. Hence, condition 8c.1 must be satisfied. We know that the outcome is $O_A = O_{NL}$ when $\pi_l > 0$ for all l if $n_{LP} + \delta(n_{RP} + n_{LA}) < 1/2$. The outcome when $\pi_R = 0$ is $O_A = (c, (L, A))$ (the challenger wins) if $\frac{\pi_A+\pi_L}{\pi_L} < \overline{\alpha}$ and $n_{LP} + \delta n_{RP} + n_{LA} > 1/2$ (new vote share when the challenger gets more votes but does not lose vote). Then, there is change in outcome. Hence, condition 8c.2 must be satisfied. $\square$

Table 1.12: The challenger's equilibrium strategy in Proposition 1.6a

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Rejects	(L, P)

Supplementary appendix

Proposition 1.6a. *Let $p_c = (L, P)$, $n_{LP} + n_{LA} < 1/2$ and $\alpha_c > 1$. If the challenger rejects the offer from like-minded IG P and accept offer from like-minded IG L then unlike-minded IGs R and A have direct influence if and only if each of the following conditions hold:*

(P6a.1) $\underline{\alpha} < \frac{\pi_A}{\pi_R} < 1$ or $\pi_A > \pi_R$ and $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L} > 1$ and,

(P6a.2) $min\{n_{LP} + n_{RP} + (1-\delta)n_{LA}, n_{LP} + n_{LA} + \delta n_{RP}\} > 1/2$.

Proof of Proposition 1.6a

(***Necessity***)

Proof. Table 1.12 specifies the equilibrium decision of the challenger and policies she would implement for each of the offers she accepts. Note that $O^{NL} = (i, (R, A))$. Table 1.13 specifies how voters vote.

Looking at Table 1.13 we can see that: members of L with policy preference (L, A) vote for the challenger iff $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_R} > \underline{\alpha} \Rightarrow \frac{\pi_A}{\pi_R} > \underline{\alpha}$; members of P with policy preference (R, P) vote for the challenger iff $\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$. Note that the voters in at least one of these groups vote for the challenger. If $\pi_A > \pi_R$, then the members of L vote for the challenger. If $\pi_A < \pi_R$, then the members of P vote for the challenger.

When R endorses the challenger, only members of R see endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$, if members of L do not vote for the challenger. This vote share is less than half by Lemma 1.5. Thus, the members of L must vote for the challenger for R to have direct influence. This requires either $\frac{\pi_A}{\pi_R} > 1$ or $1 > \frac{\pi_A}{\pi_R} > \underline{\alpha}$.

When A endorses the challenger, only members of A receive an endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{LA}$, if members of P do not vote

Table 1.13: Voters' vote for strategy profile given in Table 1.12

IGs	α	**Members**	**Vote (Endorsement)**	**Incumbent**	**Vote (no endorsement)**
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{\pi_R\underline{\alpha}+\pi_A 1}{\pi_A+\pi_R})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$(\frac{\pi_R 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_R})$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{i}\ (\frac{\pi_L\underline{\alpha}+\pi_A 0}{\pi_A+\pi_L})$
		$RA\ (n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\frac{\pi_L\ 0+\pi_A\underline{\alpha}}{\pi_A+\pi_L})$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 1+\pi_R 0}{\pi_R+\pi_L})$
		$RA\ (\delta n_{RA})$	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 0+\pi_R 1}{\pi_R+\pi_L})$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$\mathbf{c}(1+\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L+\pi_R})$
		$RP\ (\delta n_{RP})$	$\mathbf{c}(\overline{\alpha})$	1	$(\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R})$

for the challenger. This vote share is less than half the given condition in the premise. Thus, the members of P must vote for the challenger for A to have direct influence. This requires either $\frac{\pi_A}{\pi_R} < 1$ or $1 < \frac{\pi_A}{\pi_R}$ and $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$. These two conditions gives condition in P6a.1.

When condition P6a.1 is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} + (1-\delta)n_{LA}$ or $n_{LP} + n_{LA} + \delta n_{RP}$ when R or A endorses the challenger respectively. The challenger wins if these vote shares form majority. Otherwise, the challenger loses and the IGs do not have direct influence. Hence, condition P6a.2 must be satisfied.

(***Sufficiency***)

For sufficiency, we prove that the challenger does not have profitable deviation from the prescribed strategy profile. The challenger rejects the offers from IG P and loses, earning 0. The challenger cannot profitably deviate to (accepting) P's offer. If she accepts P offer, she gets votes from members of L and P. This gives her a total vote share of $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 1.4. Thus, the challenger loses and earns 0. The challenger accepts offers from unlike-minded IGs R and A and unlike-minded IG L. If conditions P6a.1 and P6a.2 are satisfied, the challenger wins and implements (R, P) or (L, A) when R or A endorses the challenger respectively. The challenger loses when L endorses the challenger, earning 0. Implementing (R, P) gives her a payoff of α_c and implementing (L, A) gives her a payoff of 1. The challenger cannot profitably deviate (to rejecting) R's or A's or L's offer. If she rejects, she publicly commits to (L, P) and thus loses the election and gets a payoff of 0. $\square$

Proposition 1.6b. *Let $p_c = (L, P)$, $n_{LP} + n_{LA} < 1/2$ and $\alpha_c > 1$. If the challenger accepts offers from both like-minded IGs then unlike-minded IGs R and A have direct influence if and only if each of the following conditions hold:*

Table 1.14: The challenger's equilibrium strategy in Proposition 1.6b

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Accepts	(L, P)

(P6b.1) $\frac{\pi_A}{\pi_R} > 1$ *and* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$ *or* $\frac{\pi_A}{\pi_R} < 1$ *and* $\underline{\alpha} < \frac{\pi_A+\pi_P}{\pi_R+\pi_P}$ *and,*

(P6b.2) $min\{n_{LP} + n_{RP} + (1-\delta)n_{LA}, n_{LP} + n_{LA} + \delta n_{RP}\} > 1/2.$

Proof of Proposition 1.6b

Proof. (***Necessity***)

Table 1.14 specifies the equilibrium decision of the challenger and policies she would implement for each of the offers she accepts. Note that $O^{NL} = (i, (R, A))$. Table 1.15 specifies how voters vote.

Looking at Table 1.15 we can see that: members of L with policy preference (L, A) vote for the challenger iff $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P} > \underline{\alpha} \Rightarrow \frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$; members of P with policy preference (R, P) vote for the challenger iff $\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$. Note that the voters in at least one of these groups vote for the challenger. If $\pi_A > \pi_R$, then the members of L vote for the challenger. If $\pi_A < \pi_R$, then the members of P vote for the challenger.

When R endorses the challenger, only members of R see endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$, if members of L do not vote for the challenger. This vote share is less than half by Lemma 1.5. Thus, the members of L must vote for the challenger for R to have direct influence. This requires either $\frac{\pi_A}{\pi_R} > 1$ or $1 > \frac{\pi_A}{\pi_R}$ and $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$.

When A endorses the challenger, only members of A receive an endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{LA}$, if members of P do not vote for the challenger. This vote share is less than half the given condition in the premise. Thus, the members of P must vote for the challenger for A to have direct influence. This requires either $\frac{\pi_A}{\pi_R} < 1$ or $1 < \frac{\pi_A}{\pi_R}$ and $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$. These two conditions gives condition in P6b.1.

When condition P6b.1 is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} +$

Table 1.15: Voters' vote for strategy profile given in Table 1.14

IGs	α	Members	Vote (Endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	LP $(n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{C{=}c}(\frac{\pi_A 1+\pi_R\underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_R+\pi_P})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$(\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P})$
R	$\alpha = \underline{\alpha} < 1$	RP $(n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{i}$ $(\frac{(\pi_L+\pi_P)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_P})$
		RA $(n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\frac{(\pi_L+\pi_P)+0+\pi_A\underline{\alpha}}{\pi_A+\pi_L+\pi_P})$
A	$\alpha = \overline{\alpha} > 1$	LA (δn_{LA})	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(\frac{(\pi_L+\pi_P)1+\pi_R 0}{\pi_R+\pi_L+\pi_P})$
		RA (δn_{RA})	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(\frac{(\pi_P+\pi_L)0+\pi_R 1}{\pi_R+\pi_L})$
P	$\alpha = \overline{\alpha} > 1$	LP (δn_{LP})	$\mathbf{c}(1+\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L+\pi_R})$
		RP (δn_{RP})	$\mathbf{c}(\overline{\alpha})$	1	$(\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R})$

$(1-\delta)n_{LA}$ or $n_{LP}+n_{LA}+\delta n_{RP}$ when R or A endorses the challenger respectively. The challenger wins if these vote shares form majority. Otherwise, the challenger loses and the IGs do not have direct influence. Hence, condition P6b.2 must be satisfied.

(***Sufficiency***)

For sufficiency, we prove that the challenger does not have profitable deviation from the prescribed strategy profile. The challenger accepts offers from unlike-minded IGs R and A and unlike-minded IG L. If conditions P6b.1 and P6b.2 are satisfied, the challenger wins and implements (R, P) or (L, A) when R or A endorse the challenger respectively. The challenger loses when L or P endorses the challenger, earning 0. Implementing (R, P) gives her a payoff of α_c and implementing (L, A) gives her a payoff of 1. The challenger cannot profitably deviate (to rejecting) R's or A's or L's or P's offer. If she rejects, she publicly commits to (L, P) and thus loses the election and gets a payoff of 0. □

Proposition 1.6c. *Let $p_c = (L, P)$, $n_{LP} + n_{LA} < 1/2$ and $\alpha_c > 1$. If the challenger rejects the offer from like-minded IG L and accept offer from like-minded IG P then unlike-minded IGs R and A have direct influence if and only if each of the following conditions hold:*

(P6c.1) $\overline{\alpha} > \frac{\pi_A}{\pi_R} > 1$ or $\pi_A < \pi_R$ and $\underline{\alpha} < \frac{\pi_A+\pi_P}{\pi_R+\pi_P} > 1$ and,

(P6c.2) $min\{n_{LP} + n_{RP} + (1-\delta)n_{LA}, n_{LP} + n_{LA} + \delta n_{RP}\} > 1/2$. □

Proof of Proposition 1.6c

Proof. (***Necessity***)

Table 1.16 specifies the equilibrium decision of the challenger and policies she would implement for each of the offers she accepts. Note that $O^{NL} = (i, (R, A))$. Table 1.17 specifies how voters vote.

Table 1.16: The challenger's equilibrium strategy in Proposition 1.6c

IGs	Challenger's decision	Policy implemented
L	Rejects	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Accepts	(L, P)

Looking at Table 1.17 we can see that: members of L with policy preference (L, A) vote for the challenger iff $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P} > \underline{\alpha} \Rightarrow \frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha}$; members of P with policy preference (R, P) vote for the challenger iff $\frac{\pi_R(1+\overline{\alpha})+\pi_A 0}{\pi_A+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A}{\pi_R}$. Note that the voters in at least one of these groups vote for the challenger. If $\pi_A > \pi_R$, then the members of L vote for the challenger. If $\pi_A < \pi_R$, then the members of P vote for the challenger.

When R endorses the challenger, only members of R see endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$, if members of L do not vote for the challenger. This vote share is less than half by Lemma 1.5. Thus, the members of L must vote for the challenger for R to have direct influence. This requires either $\frac{\pi_A}{\pi_R} > 1$ or $1 > \frac{\pi_A}{\pi_R}$ and $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha}$.

When A endorses the challenger, only members of A receive an endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{LA}$, if members of P do not vote for the challenger. This vote share is less than half the given condition in the premise. Thus, the members of P must vote for the challenger for A to have direct influence. This requires either $\frac{\pi_A}{\pi_R} < 1$ or $1 < \frac{\pi_A}{\pi_R} < \overline{\alpha}$. These two conditions gives condition in P6c.1.

When condition P6c.1 is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} + (1-\delta)n_{LA}$ or $n_{LP} + n_{LA} + \delta n_{RP}$ when R or A endorses the challenger respectively. The challenger wins if these vote shares form majority. Otherwise, the challenger loses and the IGs do not have direct influence. Hence, condition P6c.2.

(***Sufficiency***)

For sufficiency, we prove that the challenger does not have profitable deviation from the prescribed strategy profile. The challenger rejects offers from IG L and loses, earning 0. The challenger cannot profitably deviate to (accepting) L's offer. If she accepts L offer, she gets votes from members of L and P. This gives her a total vote share of

Table 1.17: Voters' vote for strategy profile given in Table 1.16

IGs	α	Members	Vote (Endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{\pi_A 1+\pi_R \underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_R+\pi_P})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$(\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P})$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{i}\ (\frac{\pi_A 0+\pi_P \underline{\alpha}}{\pi_A+\pi_P})$
		$RA\ (n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\frac{\pi_P 0+\pi_A \underline{\alpha}}{\pi_A+\pi_P})$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(\frac{\pi_R 0+\pi_P 1}{\pi_R+\pi_P})$
		$RA\ (\delta n_{RA})$	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(\frac{\pi_R 1+\pi_P 0}{\pi_R+\pi_P})$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$\mathbf{c}(1+\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_R \overline{\alpha}+\pi_A 1}{\pi_A+\pi_R})$
		$RP\ (\delta n_{RP})$	$\mathbf{c}(\overline{\alpha})$	1	$(\frac{\pi_R(1+\overline{\alpha})+\pi_A 0}{\pi_A+\pi_R})$

$n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 1.4. Thus, the challenger loses and earns 0. The challenger accepts offers from unlike-minded IGs R and A and unlike-minded IG P. If conditions P6c.1 and P6c.2 are satisfied, the challenger wins and implements (R, P) or (L, A) when R or A endorses the challenger respectively. The challenger loses when P endorses the challenger, earning 0. Implementing (R, P) gives her a payoff of α_c and implementing (L, A) gives her a payoff of 1. The challenger cannot profitably deviate (to rejecting) R's or A's or P's offer. If she rejects, she publicly commits to (L, P) and thus loses the election and gets a payoff of 0. $\square$

Chapter 2

Fear of Exclusion and Dynamics of Club Formation

2.1 Introduction

The study of club goods has spawned a huge literature in economics going back to at least Tiebout (1956) and Buchanan (1965). Club formation is an integral part of our economic, political, and social pursuits and its significance is evident from the several studies attempting to model it. The study of clubs is important for a number of reasons: it can provide insights for jurisdictional designs as the theory of clubs can capture decisions about provision, membership size, and partitioning of population into clubs; it can be applied in fields such as treaty formation, the internet, custom unions etc; it can reveal how membership size is derived endogenously. Several extensions and refinements of Buchanan's theory of club goods have been modelled and analysed, but the analysis of membership size in a dynamic context is largely missing in the club literature.[1]

In contrast to much of the prior literature which has mainly focused on static incentives, we aim to capture dynamic considerations with both finite and infinite horizons. In this spirit, we investigate a dynamic model of club formation where individuals have preferences which depend solely on the size (number of members) of the club they belong to. The decision making process on the part of the individuals involves proposals and responses (to proposals) to form a club of a particular size. A club forms if all potential members agree unanimously. To secure membership in a club, it may seem obvious from

[1]These refinements include different forms of the congestion functions, hedonic games (the composition of members), different types of mechanism for exclusion, optimality of clubs, financing of clubs, etc.

the outset that individuals would always prefer to form or remain members of a club which provides them the highest benefit, i.e. an optimal sized club. But uncertainty about the future in a dynamic setting may instil fear among individuals about sustainability of optimal sized clubs. In short, fear of exclusion from a sustainable popular sub-optimal club can induce individuals not to form optimal sized clubs or to break their existing optimal sized clubs. In this chapter, we study the dynamic decision making process of club formation which may explain and uncover an endogenous "fear of exclusion" phenomenon that is caused by long-run interactions, and can be supported by a simple class of stationary equilibria.

Club theory, as developed by Buchanan (1965), views club goods as public goods that are excludable and partially rivalrous: there is excludability in the sense that the club goods are restricted only to the members of the club; and there is rivalry because of the crowding effect. If the cost of provision is shared equally among members then increasing the size of the club reduces the cost. This, in turn, increases the utility of each member but only until reaching a point where congestion may set in. Therefore, individuals' preferences will incorporate a trade-off between cost reduction and crowding as size increases. We can think of many diverse examples of clubs goods. For instance, trade unions can be thought of as clubs. The union's objective is to negotiate terms for better wages while simultaneously securing employment. Increasing the size of the membership may result in compromise in the goals of high wages to secure employment. The consequence is that members have preferences over the size of the club.

The early literature on club theory mainly aimed at examining the welfare aspect of club formation (optimal provision of the club good and optimal size) in a static setting. However, it is evident that players can join or leave clubs repeatedly. The future implications of such decisions may be particularly important if the agents in the economy are far-sighted and patient enough. For instance, suppose a group of individuals decide to form a club X that gives them the current desired benefits. But (some) members of club X may nonetheless choose to form a different club in the future which could potentially hurt (some) other members of X. If so, the members may rationally avoid joining such a club. Decision makers would consider these future changes induced by their current decisions and this will affect their current decisions. Previous studies of endogenous club formation have mainly captured the one-stage game approach (the game ends after the stable state is reached) but largely ignored the dynamics in the decision making mentioned above. Konishi et al. (1997) studied a coalition formation game with free mobility of players where the population partitions itself into clubs, but the game ends as soon as the stable club structure is reached. Stiglitz (1977) analysed club formation with a median voting rule by assuming that the current changes will not lead to future changes. Klevorick and Kramer (1973) also considered a median voter rule in a one period game with single peaked preferences over the decision variable. Layard (1990) studied a bargaining model of wages for a democratic trade union where the median voter's choice of wage is bargained with the firm. Under the assumption of zero discounting, the equilibrium level of wages and employment was analysed. An important distinction between these papers and ours is that the clubs' decisions (not just membership) are agreed at the formation stage through negotiations.

This chapter takes a step forward by developing a model of dynamic club formation in which players can form or leave clubs over time. We study two versions of this framework: one with a finite horizon and one with a infinite horizon. Specifically, we suppose that players can form clubs at each time period by agreeing unanimously. For example, if there is high congestion, some members might (if they get an opportunity) form a new club of smaller size. The question then is whether we can support as, an equilibrium, players forming sub-optimal sized clubs when they can possibly form optimal sized clubs. To do so we compare the benchmark game of the club formation process where the process must stop by some time period (finite period game) as compared to when there is no deadline (infinite period game).[2] Note that finite horizon game is only a *benchmark* against which to compare the impact of long-run dynamics.

We define a non-cooperative game of coalition formation in which each player's strategy is either to propose to form a new club or respond (say yes or no) to the proposal offered to her. The order in which players propose is given by an exogenous protocol. Each player only cares about the size of her club, rather than the identity of its members.[3] Utility increases initially with size, and then deceases because of a crowding effect. There is, therefore, a unique optimal club size which maximises its members' utility. Since all individuals are identical and have the same preferences over the size of the club, they would all like to be members of an optimal sized club. The objective of this exercise is to obtain equilibria where fear of exclusion (FOE) arises endogenously. A class of FOE equilibria are the ones in which agents refrain from forming optimal sized clubs. Observe that it is well-known from folk theorems that repeated interactions with

[2]We assume that players are aware of the horizon of their economic environment.

[3]The size of the club can appear directly in the players' utility function through the cost sharing of the club good or indirectly through decisions taken by a club of a given size.

rational and patient enough players can allow for many SPE outcomes: so, of course, it is easy to obtain FOE equilibria with SPE. But, in fact, we can obtain FOE equilibria even with refined SPEs, that is with a class of Markov perfect equilibria (MPEs) to which standard folk theorems do not apply. To this end, we concentrate on a very specific class of stationary Markov perfect equilibria (SMPEs). In the infinite-horizon game, we consider those equilibria in which clubs of only two sizes prevail in the long-run: a good (bigger size) club whose members receive a higher utility; and a bad (smaller size) club whose members receive a low utility. We establish the existence of SMPEs of interest of such a game by construction.

Under the assumption that players are patient enough, we prove the existence and characterize of a very specific class of (dynamically) stable club structures. The properties we consider when providing such a characterization are: a) existence of stable club structures, and b) formation of optimal sized clubs in stable club structures. We explore the different kinds of behaviour individuals might exhibit, depending on whether they are in finite horizon or infinite horizon game.

Our main conclusions are derived from comparing the results of finite and infinite games given in Propositions 1, 2 and, 3. If the players are patient enough, we show that club structures with no optimal sized clubs may be stable in an infinite horizon game. By contrast, this never occurs in the finite horizon game: using backward induction, we find an unique equilibrium outcome in which as many clubs of optimal size as possible form in the stable club structure. The assumption of a high discount factor is essential and enables us to get the desired results; but it is also empirically relevant. A high discount factor is natural when club formation and dissolution take little time.

Why may we observe stable club structures which have no optimal sized clubs in the infinite horizon game? Despite the dynamic nature of the game, we can point out two straightforward reasons that dictate the equilibrium behaviour which leads to such club structures: 1) shared expectations that optimal sized clubs are not sustainable, and 2) fear of exclusion. Players might believe, in equilibrium, that optimal sized clubs would not survive for long. They might then fear exclusion from a club of sub-optimal size which provides a larger return than the other sustainable club. In other words, they fear exclusion from their best available option *among clubs that are sustainable in the future*. Fear of exclusion manifests itself in players' behaviour in such a way that it compels disloyalty to optimal sized clubs and other non-stable clubs. By contrast, these arguments do not apply in finite period game. When a rational individual sees a deadline, she can use backward induction and correctly anticipate the outcome at the end: once the game ends, club structure cannot be amended any more. Even though membership commitment is not feasible in our set-up, players' ability to see the end outcome guarantees non-betrayal. Since all players are aware that the horizon is finite, this guarantee is mutually understood among the players. Thus, by the very nature of a finite horizon, certainty about future outcomes offsets the limitations of the lack of commitment. In contrast, when the horizon is infinite, players' inability to commit plays a crucial role: players cannot commit to remain loyal in the optimal sized clubs when they expect that such clubs are not sustainable. This instils a fear of exclusion among players to be left out of the best (among the surviving) clubs.

The rest of the chapter is organised as follows. Section 2.2 reviews the relevant literature. We present our model in Section 2.3, and provide results for finite and infinite period games in Sections 2.4 and 2.5, respectively. In Section 2.6, we discuss the results and provide some intuitions. Section 2.7 concludes.

2.2 Related literature

Buchanan (1965) was one of the first studies that looked at the welfare analysis of the in-between case of pure public and private goods. Buchanan developed a general theory of clubs to address the question of how the size of the club influences the provision of public goods. For predecessors of club theory such as Tiebout (1956), Wiseman (1957), Olson (1965) among others, it was not clear how the provision of shared good and membership interacted. The reason for this was that the provision was exogenous, which is in contrast to Buchanan clubs, where membership size is an endogenous choice which depends on the provision decision. Several models and extensions have built on this concept of Buchanan clubs: earlier papers include Pauly (1970), Wooders (1978) and Shubik and Wooders (1982, 1983); more recent papers include Page and Wooders (2007), Banerjee et al. (2001) and Bogomolnaia and Jackson (2002).

This chapter is closely related to the models which allow for congestion effects. Konishi et al. (1997) analysed a non-cooperative game where individuals have the same congestion function. They proved existence for the general case and showed that existence may fail without the assumption of a common congestion function. Hollard (2000) studied a similar model with an anonymous congestion function which allowed for externalities on non-members. Holzman and Law-Yone (1997) looked at a more specific case of congestion games, which reflects the negative effects of the congestion. Similarly, Milchtaich (1996) studied a class of non-cooperative games in which the utility of a player derived from using a specific strategy depends only on the total numbers of players who are employing that same specific strategy, and the utility decreases with that numbers in a way which is defined for that particular player. While these studies prove existence and find stable Nash equilibrium strategy choices, the static nature of these models cannot explain how an equilibrium is reached, if at all. By contrast we follow Acemoglu et al. (2012) by allowing for repeated interactions, and characterize dynamically stable states.

Arnold and Wooders (2015) studied a dynamic club formation game where the mobility of players is modelled explicitly, and players are myopic. In their model, there is no restriction on how clubs are formed and dissolved, in the sense that players simultaneously choose locations, and individuals who choose the same location form a club. The result is that the existing members of a club cannot restrict further entry. By contrast, we define a protocol and allow players to restrict entry by not proposing or by rejecting proposals to expand the club. This enables us to find an equilibrium in which the maximum feasible number of optimal sized clubs form when the horizon is finite. In Arnold and Wooders (2015), such an equilibrium fails because of the indivisibility problem: when the population size is not an integer multiple of the optimal size club, the remaining individuals who are not members of optimal sized club prevent the process from settling down. In further contrast, the players in our model are far-sighted and therefore our formulation allows us to investigate equilibria in which players rationally avoid forming optimal sized clubs.

Roberts (2015) studied the dynamic formation of a single club which N players wish to join. Decision makers are the members of the club, and the decision involves changing the size of the club. The voting is by majority rule, and individuals are ordered according to single-crossing preferences. One limitation of the paper is that the order in which the individuals join or leave a club is determined by an exogenous system of seniority. In contrast, we suppose that all members of a proposed club vote, and the club only form if all members agree. Moreover, all individuals have the same preferences over club size.

Barbera et al. (2001) studied a dynamic game of club formation where any member of the club can vote to include a non-member/s unilaterally. The utility of the players depends on the stream of the members included in the club; and players have (binary) preferences over the other players. Non-members become active only after they are included in the club. They presented conditions for the existence of a pure strategy perfect equilibrium for finite horizon games. In addition to finite horizon game, we also explore the infinite horizon case, simplifying preferences by assuming that players only care about the size of the club they belong to, and share preferences over the size of the club. This enables us to compare equilibria in the finite and infinite horizon games.

This chapter also relates to the literature on non-cooperative coalition formation. Seidmann and Winter (1998) studied endogenous bargaining games where the agreements can be reversed after they have been implemented. They consider superadditive games and focus on equilibria in which the grand coalition forms in one step or a number of steps. They show that if players are allowed to renegotiate then the process results in the grand coalition, but this might not happen otherwise.

Chatterjee et al. (1993) analysed a game of coalitional bargaining with n players where players can transfer utility. The sequence of proposers and respondents is determined by a given protocol. They consider superadditive games and mainly investigate the effeciecny (no-delay and formation of grand coalition) properties of SSPEs. They show that inefficiently small coalitions may arise in equilibrium and/or agreement may be delayed. However, the sequence in which the players move significantly affects the efficiency of the equilibrium. As opposed to them, the players in our model cannot transfer utility and our efficiency results do not depend on the order of moves.

Hyndman and Ray (2007) studied a coalition formation game in which agreements are binding until all affected parties agree to renegotiate it. Allowing for history-dependent strategies, they showed that for characteristic function games, efficiency is achieved on every equilibrium path. Ray and Vohra (1999) studied a coalition formation game where players have utilities which depends on the whole coalition structure. They formulated the analysis after defining a partition function, and assuming that players can write binding agreements. They provided sufficient conditions for no-delay equilibria, and used these finding in the context of Cournot oligopoly. In contrast, members of a club in our model cannot write binding contracts; in other words, they cannot formally provide commitment to members within or outside the club they belong to.

Acemoglu et al. (2012) studied a dynamic decision making problem in a general framework where the individuals are sufficiently forward looking. They proved existence and characterized dynamically stable states, which are functions of the initial states. A state, in their set-up, corresponds to an economic and/or political system, and individuals can take decisions over time to change the status quo. They provided useful insights into why a particular state might be dynamically stable. We follow their approach in the context of club formation with respect to club size, and exploit their design to analyse a very specific class of dynamically stable club structures.

2.3 The Model

In this section, we introduce the model of club formation. There are n identical individuals denoted by $N \equiv \{1, \ldots, n\}$ with $n \geq 3$. The players are indexed by i. A *club* S is a non-empty subset of players. We write $s \in N$ as the size of club S or the number of members of S. A *club structure*, π is partition of N. Let Π be the set of all club structures. For any non-empty subset of K of N, the set of partitions of K is denoted by Π_K, with typical element π_K.

The focus of this chapter is on the size of the club; so, we assume that the each individual's preferences do not depend on the identity of the members of her club, but only on the size of the club. An individual who belongs to club S gains a per-period payoff of $v(s)$.[4] For each $i \in N$ and each $\pi \in \Pi$, define $v_i(\pi)$ as $v_i(\pi) = v(s)$ where $i \in S \in \pi$ and $s = |S|$. We assume that v is strictly concave and single-peaked. This implies that there exists an optimal club size for each individual.

Let P be the set of permutations of N and $\mathcal{N} \equiv 2^N \setminus \{\emptyset\}$. A *rule of order* $\rho \in \Delta(P)$ is an exogenous protocol, where ρ determines the order of moves of proposers in the sequential game of club formation. Suppose that Nature chooses the sequence of proposers $(\iota_1, \ldots, \iota_n)$. Every agent is chosen as a proposer exactly once. Thus, there are n proposers in every period.

In each period $t = 1, 2..., T$ (where T may be infinite) the sequential game of club formation proceeds as follows. The first proposer, ι_1, starts the game by proposing to form a club $S \ni \iota_1$ or passes. If ι_1 proposes S to which she belongs then all $i \in S$ simultaneously decide whether to accept or reject ι_1's proposal. If all members accept, the members of S form a coalition and each member then incurs a positive cost of forming a club, denoted $\varepsilon > 0$, in that period and the period ends. Note that the cost of forming a new club could, in principle, depend on the size of the club but, here, we only introduce this cost as a tie breaking device. Therefore, we assume this cost to be given and fixed. If any $i \in S$ rejects the proposal or the proposer passes then the next proposer, ι_2, makes a proposal $S' \ni \iota_2$ or passes. The stage game then proceeds as

[4] Buchanan (1965) assumed that there are n identical individuals, with utility represented by $U(x, s, G)$, where x is the private good consumed, s is the number of club members, and G is the provision of the club good. Utility increases in x and G, and decreases in s (congestion from overcrowding).

before, until a new club has been formed or the last proposer has proposed.[5] The next period then starts and the game proceeds with the first proposer in the order selected by Nature.

All agents seek to maximise their average discounted per-period payoff, and share a common discount factor $\delta \in [0, 1)$. In each period, only one club is formed, if at all. In any period t, if all the proposers pass or if all the proposals are unsuccessful then no club is formed in that period.

Thus, at the end of each period t, we obtain a coalition structure π^t, which contains: (i) a new club (possible empty) that has been formed in the current period; (ii) clubs in π^{t-1} that have not been affected by the moves in the current period (none of their members have successfully proposed or agreed to form another club); and (iii) the broken clubs (one or more of whose members has left the club, to join the new club). We assume that members of the broken club remain in their old club after some of the members have left.

Period t starts with the club structure obtained from the last period, $t - 1$. In any period t, the club structure from the last period, π^{t-1}, can be altered at most once. Thus, when a player i gets to move, the club structure from the last period is intact and she has the opportunity to change it, either as a proposer or as a respondent. Hence, the current coalition structure for an active player i in period t is π^{t-1}.

A history at any stage of the game is a complete list of all proposals, acceptances and rejections that took place in all the previous periods and stages. A player is called *active* at history h^t at date t if it is her turn to move after history h^t. We assume that all players possess perfect recall and that the game is of perfect information. At any stage, each player observes and recalls everything that has previously transpired in the game, which we call a complete history. We focus on subgame perfect equilibria in stage-undominated strategies, i.e., those in which, at any stage, no player uses a weakly dominated strategy. We will refer to such strategy profiles, simply, as equilibria.

Optimality or efficiency. In our analysis of sequential club formation, we define optimality as follows: optimal club structures are the ones which have all possible optimal-sized clubs; on the other extreme, sub-optimal club structures are the ones in which none of the clubs are of optimal size. In the literature of club goods or public goods, it is not clear whether efficiency or Pareto-optimality should be viewed from the point of social welfare or the welfare of the representative member of a club (See Helpman and Hillman

[5]The period ends when either a club is formed or the last proposer has proposed.

(1977), Ng (1973), and Ng (1978)). As a first step in this chapter, we refrain from the complete welfare analysis of equilibrium club structure and follow the approach of Buchanan clubs to find optimal club structure, so we use the word 'optimal' even though we are not using it in the sense of welfare analysis.[6] In some instances, we will comment on whether an equilibrium club structure is Pareto-optimal.

2.4 Benchmark: Short-run interactions

In this section we study the game with finite number of periods, i.e. $1 \leq T < \infty$. The main purpose of this section is to characterize equilibria of the finite game, so as to compare equilibrium outcomes in the finite and infinite games. We start by studying the special case of $T = 1$ period.

We know that v is strictly concave and let $s^* = min\{n, \arg\max_{s \in \mathbb{N}} v(s)\}$ be the optimal club size. Throughout the chapter we assume that $v(s) > v(s') \Rightarrow v(s) - \varepsilon > v(s')$. This assumption, plus the concavity of v imply that

$$v(s^*) - \varepsilon > v(s) \quad \forall s \neq s^* \tag{2.1}$$

where $\varepsilon > 0$.

Proposition 2.1. *Let k^* be implicitly defined by $k^* s^* \leq n \leq (k^* + 1)s^*$. If $T = 1$ then in any subgame perfect equilibrium, the following holds: (i) no club of size-s^* breaks up; and (ii) if at the start of the period there are fewer than k^* clubs of size-s^* then an additional club of size-s^* forms.*

Proof. Let π be the initial club structure and $\overline{A}$ be the set of agents who are members of a size-s^* club at the the start of the game; that is

$$\overline{A} \equiv \{i \in N : \exists S \subseteq N \text{ such that } i \in S \in \pi \ \& \ s = s^*\} \ .$$

Part (i) To prove part i, it suffices to show that whenever a proposer offers to form a club which includes member/s of $\overline{A}$, the proposal is rejected. To see this, consider the proposer, ι_n, who proposes last. Suppose the last proposer offers to form a club which includes member/s of $\overline{A}$ or in other words, she proposes to break club/s of size-s^*. The utility of the agents in $\overline{A}$ is $v(s^*)$ and thus the best offer the proposer can offer is a club

[6] In deriving the Pareto-optimality conditions, Buchanan equilibrium clubs maximise the benefits of a representative club member rather than maximising the total net benefit of the whole population.

of size-s^*. If a respondent in $\overline{A}$ accepts the offer, she gets a payoff of $v(s^*) - \varepsilon$, whereas, if she rejects then she receives a payoff of $v(s^*)$. Thus, each respondent $i \in \overline{A}$ rejects the offer from ι_n to form a club of size-s^* because $v(s^*) > v(s^*) - \varepsilon$. As a result, the last proposer is unsuccessful in breaking size-s^* club/s.

Consider now the second to last proposer, ι_{n-1} and suppose that she offers to break a club of size-s^*. Anticipating that the club of size-s^* will remain intact if she rejects proposal from ι_{n-1}, a respondent $i \in \overline{A}$ rejects the offer and the proposal is unsuccessful for the same reason as above. Proceeding along the same lines, we can say that a proposer ι_i is unsuccessful in breaking size-s^* club if all the proposers in the order from ι_i onwards till the last proposer, ι_n, are unsuccessful in their attempt to break club/s of size-s^*. We proved that ι_n is unsuccessful in breaking size-s^* club and therefore by inductive reasoning, no proposer is successful in forming a club which includes members of $\overline{A}$, proving part (i).

Part (ii) Let m be the number of clubs of size-s^* at the start of the game. Note that only one club is formed in every period and therefore we just need to prove that whenever a club is formed, it must be of size-s^* and that a club forms if $m < k^*$. If $m < k^*$ then the number of remaining players, $s^*(k^* - m)$, are a multiple of s^* who are outside $\overline{A}$. Now, consider the last proposer ι_n.

• If $\iota_n \in \overline{A}$ then no new club forms. From part (i) we know that the proposer either passes or proposes an unsuccessful club.

• If $\iota_n \notin \overline{A}$ then she successfully forms a size-s^* club. Given condition (2.1), all the respondents in $N \setminus \overline{A}$ would accept a proposal to form the size-s^* club. If ι_n successfully offers the members of $N \setminus \overline{A}$ to form a size-s^* club then she gets a payoff of $v(s^*) - \varepsilon$.[7] If she passes or offers an unsuccessful proposal then she gets a payoff of $v(s) < v(s^*) - \varepsilon$, and, therefore, she forms a size-s^* club in every equilibrium.

Consider now, the penultimate proposer.

• If $\iota_{n-1} \in \overline{A}$ then no new club forms. From part (i) we know that such a proposer would either pass or offer an unsuccessful proposal.

• If $\iota_{n-1} \notin \overline{A}$ then a club of size-s^* forms, either now or at some later stage. If ι_{n-1} anticipates that she would not otherwise be a part a size-s^* club which forms at some later stage then she successfully proposes to the members of $N \setminus \overline{A}$ to form a club of size-s^*. She is indifferent between forming a size-s^* club now and passing if she anticipates

[7]If ι_n offers a proposal to a member/s in $\overline{A}$ then she is unsuccessful. Thus, the only way she can form a new club is by offering the proposal to the agents in $N \setminus \overline{A}$. Also, she would offer to form a size-s^* club because it gives her the highest payoff.

that she would be included in the club of size-s^* at some later stage. In either case, a size-s^* club forms.

Proceeding recursively, any proposer who does not belong to the optimal sized club would:

1. Successfully propose to form a new club of size-s^* if she anticipates that she would not otherwise be part of club of size-s^* which forms at some later stage or

2. Passes or successfully proposes to form a new club of size-s^* if she anticipates joining a size-s^* club at some later period if she passes. In either case, a club of size-s^* forms.

As $m < k^*$, some individual, and therefore some proposer must not be in $\overline{A}$: for every protocol ρ. Consequently, a club of size-s^* must form, proving part (ii). $\square$

Proposition 2.1 shows that, in a single period game, an optimal sized club forms if it is possible to form one, i.e. if s^* or more than s^* individuals do not belong to a club of size-s^* at the start of the period. It also shows that members of optimal sized club never break their clubs (pass if they are proposer or reject any proposal if they are respondents). This implies that if k^* optimal sized clubs already exist at the start of the game then k^* clubs would remain intact by the end of the game. Note that size-$(n - s^*k^*)$ is the next best club after size-s^* club since v is strictly concave. Then, Proposition 1 also implies that if k^* optimal sized clubs exist at the start of the game then the individuals who are not in optimal sized club form a club of size-$(n - s^*k^*)$ if they already do not belong to size-$(n - s^*k^*)$ club; otherwise every proposer passes. Individuals know that they will get payoff only once and therefore they strive to get the highest payoff, i.e to form or join an optimal sized club as soon as they get an opportunity. If they are already in optimal sized club then they can anticipate that none of the members of their club would break their club because optimal sized club provide them the highest payoff since forming new clubs incurs a cost. We now consider the more general case of $T > 1$ finite.

Proposition 2.2. *If T is finite then the following holds in each period: (i) no club of size-s^* breaks up; and (ii) if at the start of the period there are fewer than k^* clubs of size-s^* then an additional club of size-s^* forms.*

Proof. We start by proving **Part (i)**. *In the last period T, no club of size-s^* breaks up.* The last period is equivalent to the one period game. Hence, the same argument applies to the last period as to the one period game.

In period $t = T - 1$, no club of size-s^* breaks up. We begin with the last proposer. We show that last proposer, ι_n, is unsuccessful in breaking any size-s^* club, either by forming a club which includes member/s who are in the optimal sized club or by successfully forming a new club when she is already a member of an optimal-sized club. Suppose that she proposes to form a new club of size-s^* to a member of an optimal-sized club. This respondent anticipates that her club will remain in place in the next (last) period. If she accepts the offer then she gets a total discounted payoff of no more than $v(s) - \varepsilon + \delta v(s)$ and she gets a payoff of $v(s^*) + \delta v(s^*)$ if she rejects. The respondent then rejects the offer because $v(s^*) + \delta v(s^*) > v(s) - \varepsilon + \delta v(s)$. Thus, the last proposer in the last period is unsuccessful in forming a new club which includes any member of an optimal-sized club. Finally, suppose that $\iota_n \in \overline{A}$. In equilibrium, she cannot break her current club: anticipating that her club remains intact in the next period, ι_n gets a payoff of $v(s^*) + \delta v(s^*)$ if she passes or offers an unsuccessful proposal. This is the maximum payoff she can get. If she successfully forms a new club, she incurs a cost of ε. Thus, the last proposer in period $T-1$ cannot break any size-s^* club.

Any respondent in a size-s^* club, would reject any offer to form a new club if she anticipates that her club will never be dissolved in all the later stages and periods, i.e. if all the proposers in later stages and periods are unsuccessful in breaking any size-s^* club. In other words, a proposer in period t is unsuccessful in breaking any size-s^* club in that period if all subsequent proposers in that and in later periods fail to break up any size-s^* club. We know that no proposer in the last period is successful in breaking a size-s^* club and that the last proposer in the second last period is unsuccessful. Hence by inductive reasoning, no proposer in any period is successful in breaking any optimal-sized club, proving part (i).

We now prove **Part (ii)**.

Consider a period t in which there are m^t clubs of size-s^* and that $m^t < k^*$ and $\overline{A}$ is a set of agents who belong to a size-s^* club in that period. Since $m^t < k^*$, $s^*(k^* - m^t) \geq s^*$ individuals are not in $\overline{A}$.

Now, consider a proposer k in period t.

• If $\iota_k \in \overline{A}$, she passes or offers an unsuccessful proposal. To see this note that:

1 If she passes or offers an unsuccessful proposal, she gets $v(s^*) + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s^*)$

2 If she successfully forms a new club, say of size-$\overline{s}$, she gets a payoff of $v(\overline{s}) - \varepsilon + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s)$

We know that $v(s^*) \geq v(\overline{s})$ and that $\sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s^*) \geq \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s)$. Therefore, $\iota_k \in \overline{A}$ passes or offers an unsuccessful proposal because $v(s^*) + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s^*) >$

$v(\overline{s}) - \varepsilon + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s)$.[8]

Consequently, any new club must be offered by a proposer who does not belong to $\overline{A}$.

• If $\iota_k \notin \overline{A}$ then a new club of size-s^* forms, either now or at some later stage of the current period.

Suppose that $k = n$ and that the proposer belongs to a club of size $s_k \neq s^*$. The proposer gets a payoff of $v(s_k) + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s)$ if she passes or offers an unsuccessful proposal. The proposer can successfully offer to the agents in $N \setminus \overline{A}$.[9] Since s^* is the unique maximum and $s^*(k^* - m^t) \geq s^*$, the proposer offers to individuals in $N \setminus \overline{A}$ to form a club of size s^*. Condition (2.1) implies that all respondents accept. The proposer gets a payoff of $v(s^*) - \varepsilon + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s^*)$ if she offers a club of size-s^* to individuals in $N \setminus \overline{A}$. Thus, the proposer forms a new club of size-s^* because $v(s^*) - \varepsilon + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s^*) > v(s_k) + \sum_{\tau=t+1}^{T} \delta^{\tau-t} v(s)$.

Suppose $k \neq n$ and that the proposer belongs to a club of size-$s_k \neq s^*$. She successfully forms a club of size-s^* if she anticipates that she would not otherwise be part of size-s^* club at some later stage. She is indifferent between forming a club now and passing if she anticipates joining a size-s^* club at some later stage. In either case, a club of size-s^* forms in the period t.

In sum, a club of size-s^* forms in period t if $\iota_k \notin \overline{A}$. As $m^t < k^*$, some individual, and therefore some proposer must not be in $\overline{A}$: for every protocol ρ. Consequently, a club of size-s^* must form, proving part (ii). □

Proposition 2.2 implies that in every period an optimal sized club would form if it if possible to form one and that no exiting optimal sized club would break until the end of the game. Individuals can anticipate that optimal sized clubs remain intact until the very last period if they form or join an optimal sized club. Then, from there onwards they can get the highest payoff in every period. Thus, if there are enough periods to form all optimal sized clubs then will be k^* clubs of size-s^* in the stable club structure.

Corollary 2.1. *Let m^* (possibly 0) be the number of size-s^* clubs at the start of period 1. Then, at the end of period $t = (k^* - m^*)$ there exist k^* clubs of size-s^*.*

This is a direct consequence of Proposition 2.2. We know from Proposition 2.2 that

[8] Since there is discounting, the agents do not wait to become part of size-s^* club till the next period and they take the opportunity to be part of a size-s^* club either by forming one or by agreeing to the proposer who wants to form one.

[9] From part (i) we know that no club of size-s^* breaks and therefore individuals who belong to $\overline{A}$ reject any offer.

a club of size-s^* never breaks. Then, after all the optimal-sized clubs are formed, all the proposers who are members of a size-s^* club would pass when given the opportunity to propose, and all the respondents who are members of a size-s^* club reject any offer to form a new club. Thus, all clubs at the end of period $k^* - m^*$ are optimal sized, and there are some left over individuals who cannot form an optimal-sized club because there are not enough individuals left to form one.

The analysis of the finite-horizon game implies that the outcome (number of clubs and club sizes) of all possible equilibria is unique. Depending on the protocol and the initial club structure, some individuals will end up in an optimal sized club and those who did not get the opportunity to become member of an optimal sized club would end up in a sub-optimal sized club. Agents anticipate that at the end of the game, there will only be two types of clubs: one club of size-$(n - m^*s^*)$ and rest of the clubs would be of size-s^*. The finite horizon allows members of the optimal sized club to implicitly "commit" not to break their club if agents are aware that the process of club formation ends at some point. Note that outcome in the finite game is also Pareto optimal.

The club structure at the end of period $k^* - m^*$ would have k^* clubs of size s^*. In the next period, players who are not in a size-s^* club would form a club among themselves, i.e. a club of size-$(n - k^*s^* < s^*)$, if they already do not belong to a club of size-$(n - k^*s^*)$: a club of size-$(n - k^*s^*)$ is the next best club after a size-s^* club. Then, the club structure at the end of period $k^* - m^* + 1$ would have one club of size-$(n - k^*s^*)$ and k^* clubs of size-s^*. This club structure would never change because, as we mentioned, size-s^* clubs never break, and size-$(n - k^*s^*)$ is the best club for its members if they cannot become members of a size-s^* club. Thus, all the possible stable club structures of the finite period game have the maximum possible number of clubs of optimal size.

2.5 Long-run interactions: $T = \infty$

In this section, we analyse how clubs of inefficient size may form in the infinite horizon game. We saw in Section 2.4 that, in the case of short-term interactions, all the possible optimal-sized clubs form after a finite number of periods and never dissolve. Once all the optimal-sized clubs have been formed, the rest of the agents form the next best club and remain in that club forever. As a result, the equilibrium stable club structure does not change once all the optimal-sized clubs and next best club have been formed. Thus, optimality (stable optimal club structure) is achieved in the case of finite periods. The objective of this section is to show that this may not hold if there is no deadline. In particular, fear of exclusion may appear endogenously in an equilibrium. If agents are far-sighted then fear of exclusion from a relatively better club can make agents behave differently as compared to the case of finite periods. A change in behaviour, in turn, might change the club structure obtained, since membership in a club is itself influenced by the behaviour of agents.

2.5.1 Equilibrium and strategies

Note that games with infinitely repeated interactions and patient players may have many equilibrium outcomes (cf. the folk theorem). It is therefore easy to find an equilibrium in which clubs of inefficient sizes form . Following the lead of Acemoglu et al. (2012), we therefore use the Markovian solution concept. Specifically, a player's strategy can, generally, depend on the complete history, i.e. everything that has transpired in all stages of all previous periods; but in this section we focus on Markovian strategies (cf. Maskin and Tirole (2001)). We will therefore shed unneeded generality and only provide a formal definition for such strategies.

The definition of Markov strategies begins with the set of payoff-relevant states, which in this case are of two types: proposer states, and respondent states. A *proposer state* is a pair of a club structure $\pi \in \Pi$ and a list of remaining proposers $(\iota_\ell, \ldots, \iota_n) \in N^\ell$, for some $\ell \in \{1, \ldots, n\}$. (We identify ι_ℓ as the identity of the proposer, whose turn it is to make a proposal). A *responder state* is a pair of a proposer state $(\pi, ((\iota_\ell, \ldots, \iota_n))$ and a proposal to form a club $S \in 2^N$.

For a player who is a proposer and active at history h^t, the only payoff-relevant variables of the history are the current coalition structure π, and the remaining sequence of proposers; for a respondent, the payoff-relevant variables are the current coalition structure, the remaining sequence of proposers, and the proposal just made to her, S. (We interpret $S = \emptyset$ as passing). The sets of proposer and responder states are denoted by K^p and K^r respectively, with generic element κ.

Let K_i^p be the set of proposer states in which it is player i's turn to make a proposal i.e., $K_i^p \equiv \big\{\big(\pi, (\iota_\ell, \ldots, \iota_n)\big) \in K^p \colon \iota_\ell = i\big\}$; and let K_i^r be the set of responder states in which i has to respond to a proposal i.e., $K_i^r \equiv \big\{\big(\pi, (\iota_\ell, \ldots, \iota_n), S\big) \in K^r \colon S \ni i\big\}$. A *(stationary) Markov strategy* for player $i \in N$ is a pair $\sigma_i = (\alpha_i, \beta_i)$ of a proposer strategy and a responder strategy, where $\alpha_i \colon K_i^p \to 2^N$, and $\beta_i \colon K_i^r \to \{\text{yes}, \text{no}\}$. Note that strategy profile σ_i is stationary in the sense that it only depends on history via payoff-relevant states.

A stationary Markov perfect equilibrium (SMPE) is a stage-undominated subgame perfect equilibrium in which all players use stationary Markov strategies. In particular, we are interested in finding pure strategy SMPEs. Henceforth, any reference to 'equilibria' is to the pure strategy SMPEs.

2.5.2 (Sub-optimal) dynamic stable club structure

Now we characterise the equilibria of this game. To this end we will use the approach of Acemoglu et al. (2012) to define *dynamically stable club structures.*

Definition 2.1. *We say that a club structure* $\pi \in \Pi$ *is dynamically stable if there exists a threshold* $\bar{\delta} \in (0,1)$ *such that for all* $\delta \in (\bar{\delta}, 1)$*, there exists a SMPE in which the following holds: (i)* π *is formed after a finite number of periods with positive probability on the equilibrium path; and (ii) whenever* π *is formed (on or off the path), it remains in place in all future periods.*

In other words, π is dynamically stable if it does not change once it has been formed in some period. Our objective in this section is to show that there exists a dynamically stable club structure π which only consists of clubs of sub-optimal size, i.e. where none of the clubs are of size-s^*. We now state the main result of this section.

Now, we state Proposition 2.3 and provide a proof and intuition for it. We will use the following notation to prove the Proposition. For any integer $\bar{s} < n$, let $m(\bar{s})$ be the maximum number of clubs of size-$\bar{s}$ that can possibly be formed, i.e. $m(\bar{s}) \in \mathbb{N}$ is implicitly defined by $m(\bar{s})\bar{s} \leq n \leq (m(\bar{s})+1)\bar{s}$.

Proposition 2.3. *For any* $\bar{s} \in \mathbb{N}$ *that satisfies* $m(\bar{s})\bar{s} < n$ *and* $v(n - m(\bar{s})\bar{s}) < v(\bar{s})$*, there is a threshold* $\bar{\delta} < 1$ *such that, if* $\delta > \bar{\delta}$ *then there exists an equilibrium in which the following holds in each period: (i) no club of size-*$\bar{s}$ *ever breaks up; and (ii) if at the start of any period there are fewer than* $m(\bar{s})$ *clubs of size-*$\bar{s}$ *then an additional club of size-*$\bar{s}$ *forms.*

For notational ease, let $m(\bar{s}) \equiv \bar{m}$. From Proposition 2.3 we show that the dynamic stable structures that exhibit FOE can be obtained with stationary strategies. In these stable club structures, members of size-$(n - \bar{m}\bar{s})$ are excluded from size-$\bar{s}$ clubs. We proved through Propositions 2.1 and 2.2 that every possible stable club structure of the finite horizon game has as many optimal sized clubs as possible. Proposition 2.3 shows that the result of the finite horizon game might not hold in the infinite horizon game where it is possible to generate equilibria in which none of the clubs in the stable club structure are of optimal size. We provide the intuition for this result in Section 6 and discuss why this result cannot be replicated in the finite horizon game.

Proof. The proof of Proposition 2.3 is constructive and proceeds in four steps. In Step 1, we construct a function W_i for each player i, and establish some properties which will be useful in the next steps. Step 2 defines a stationary Markov strategy σ. Step 3 defines the continuation values; and finally, in Step 4, we show that σ is an SPE, thus completing the proof of the Proposition. Note that $\bar{s} \leq s^*$ is consistent with the premise and some $\bar{s} > s^*$ may also satisfy the premise.

Step 1: Preliminaries. We begin with the construction of n real functions $W_1, \ldots, W_n$. The domain of each W_i is the union of the set of proposer states K^p and the set of coalition structures Π: $\mathbf{K} \equiv K^p \cup \Pi$.

To define W_i, $i \in N$, we need to establish some additional notation. First define the order $\lhd_i$ on $N \setminus \{i\}$ as follows: if $i = 1$ or $i = n$, then $\lhd_i =<$, where $<$ is equivalent to strictly less than; otherwise

$$i + 1 \lhd_i \ldots \lhd_i n \lhd_i 1 \lhd_i 2 \lhd_i i - 1 \ .$$

Proposer i proposes to other agents to form a club as defined by the order $\lhd_i$. Now for each club structure $\pi \in \Pi$, let $\bar{A}(\pi)$ be the set of agents who are members of club of size-$\bar{s}$, that is

$$\bar{A}(\pi) \equiv \{i \in N \colon i \in S \text{ for some } S \in \pi \text{ such that } s = \bar{s}\} \ .$$

For each $i \in N$ and $\pi \in \Pi$, we define $S_i(\pi) \in 2^N$ as the coalition comprising the first $\bar{s} - 1$ agents (according to order $\lhd_i$) in $N \setminus \bar{A}(\pi)$ if $n - \left|\bar{A}(\pi)\right| \geq \bar{s}$, and as $N \setminus \bar{A}(\pi)$ otherwise.

We are now in a position to define $W_i(\mathbf{k})$ for every $\mathbf{k} \in \mathbf{K}$. To this end, consider the following path of the game that begins with proposer state $\mathbf{k} = \big(\pi, (\iota_\ell, \ldots, \iota_n)\big)$, i.e., player ι_ℓ is about to move in a proposal stage at which the current club structure is π and the list of remaining proposers is $(\iota_\ell, \ldots, \iota_n)$:

1. In any proposal stage at which she is called upon to propose, player i behaves as follows:

    (a) If i belongs to $\bar{A}(\pi)$, or if the current club structure comprises $\bar{m}$ size-$\bar{s}$ clubs and one size-$(n - \bar{m}\bar{s})$ club, then she passes.

(b) Otherwise she proposes to form club $S_i(\pi)$.

2. All proposals made (on this path) are successful.

It is readily checked that on this path, the game reaches a dynamically stable club structure after a finite number of rounds, which comprises $\bar{m}$ size-$\bar{s}$ clubs and one club of size-$(n - \bar{m}\bar{s})$. Let $W_i(\mathbf{k})$ be the expected payoff to player i resulting from this path (where expectations are taken over the distributions of proposer orders in each period).

To complete the definition of W_i, we must define the values it takes when Nature has not yet selected the sequence of proposers and $\mathbf{k}$ contains some coalition structure $\pi \in \Pi$. In this case, we set $W_i(\pi) \equiv \mathbb{E}_\rho\big[W_i\big(\pi, (\iota_1, \ldots, \iota_n)\big)\big]$, where the expectation is taken over the set of proposer lists $(\iota_1, \ldots, \iota_n)$ which Nature may select at the start of any period using probability distribution ρ.

Lemma 2.1. *For each $i \in N$, let $W_i \colon \mathbf{K} \to \mathbb{R}$ be defined as above. Then there exists $\bar{\delta} \in (0,1)$ such that the following holds for all $\delta > \bar{\delta}$:*
(i) $\bar{W} \equiv \max\big\{W_i(\pi) \colon i \notin \bar{A}(\pi)\big\} < v(\bar{s}) - \varepsilon$;
(ii) $(1-\delta)v(s^*) + \delta W_i(\pi) < v(\bar{s}) - \varepsilon$, for all $i \in N$ and $\pi \in \Pi$ such that $i \notin \bar{A}(\pi)$;
(iii) $W_i(\mathbf{k}) \leq v(\bar{s})$, for all $\mathbf{k} \in \mathbf{K}$ and $i \in N$.

Proof. (i) To establish the first part of the Lemma, it suffices to show that $W_i(\pi) < v(\bar{s}) - \varepsilon$, for all $i \in N$ and $\pi \in \Pi$ such that $i \notin \bar{A}(\pi)$. As there is only a finite number of pairs (i, π) such that $i \notin \bar{A}(\pi)$, this indeed guarantees that $\bar{W} < v(\bar{s}) - \varepsilon$.

Let $i \in N$ and $\pi \in \Pi$ be such that $i \notin \bar{A}(\pi)$, and suppose first that $\big|\bar{A}(\pi)\big| = \bar{m}\bar{s}$. By definition of W_i, this implies that

$$W_i(\pi) = \begin{cases} v(n - \bar{m}\bar{s}) & \text{if } N \setminus \bar{A}(\pi) \in \pi \ , \\ v(n - \bar{m}\bar{s}) - \varepsilon & \text{otherwise;} \end{cases}$$

so that $W_i(\pi) < v(\bar{s}) - \varepsilon$ because $v(n - \bar{m}\bar{s}) < v(\bar{s})$, where ε is the cost of forming a new club[10]. Indeed, if $N \setminus \bar{A}(\pi) \notin \pi$, then all proposers in $\bar{A}(\pi)$ pass, and the next proposer outside $\bar{A}(\pi)$ successfully forms the coalition $N \setminus \bar{A}(\pi)$, which contains i.

Now suppose that $\big|\bar{A}(\pi)\big| < \bar{m}\bar{s}$. It follows that the next proposer who is not in $\bar{A}(\pi)$, say j, forms the size-$\bar{s}$ club $S_j(\pi)$. If i is a member of $S_j(\pi)$ then she receives $v(\bar{s}) - \varepsilon < v(\bar{s})$. If i is not a member of $S_j(\pi)$ then it follows from the definition of the

[10] Remember the assumption that $v(s) > v(s') \Rightarrow v(s) - \varepsilon > v(s')$

path in Step 1 that she will either end up in a size-$\bar{s}$ club or in a size-$(n-\bar{m}\bar{s})$ club after a (random) finite number, say τ, of periods. As her stage payoff is bounded above by $v(s^*)$ and $\tau \leq \bar{m}+1$, i's expected payoff (conditional on $i \notin S_j(\pi)$) is itself bounded above by

$$(1-\delta)\sum_{s=1}^{\bar{m}} \delta^{s-1} v(s^*) + \delta^{\bar{m}}(1-\delta)\sum_{s=1}^{\infty} \delta^{s-1}\big(\mathbb{E}\big[v(\tilde{s})\big]-\varepsilon\big) = (1-\delta^{\bar{m}})v(s^*) + \delta^{\bar{m}}\big(\mathbb{E}\big[v(\tilde{s})\big]-\varepsilon\big) \ ,$$

where $\tilde{s} \in \{\bar{s}, n-\bar{m}\bar{s}\}$ is the random variable describing the size of i's club from period $\bar{m}+1$ onward, and the expectation is computed from the distribution of proposer orders. As $v(n-\bar{m}\bar{s}) < v(\bar{s})$ and the probability of i ending up in the $(n-\bar{m}\bar{s})$-sized club is positive, $\mathbb{E}\big[v(\tilde{s})\big] < v(\bar{s})$. Moreover, as $\lim_{\delta\to 1}(1-\delta^{\bar{m}})v(s^*) + \delta^{\bar{m}}\big(\mathbb{E}\big[v(\tilde{s})\big]-\varepsilon\big) = \mathbb{E}\big[v(\tilde{s})\big]-\varepsilon$, there exists $\bar{\delta}_1 \in (0,1)$ such that $(1-\delta^{\bar{m}})v(s^*) + \delta^{\bar{m}}\big(\mathbb{E}\big[v(\tilde{s})\big]-\varepsilon\big) < v(\bar{s})-\varepsilon$, for all $\delta \in (\bar{\delta}_1, 1)$. This in turn implies that $W_i(\pi) < v(\bar{s})-\varepsilon$. Henceforth, we assume that $\delta > \bar{\delta}_1$.

(ii) Let $i \in N$ and $\pi \in \Pi$ be such that $i \notin \bar{A}(\pi)$. By definition of $\bar{W}$, we have

$$\lim_{\delta\to 1}\big[(1-\delta)v(s^*) + \delta W_i(\pi)\big] \leq \lim_{\delta\to 1}\big[(1-\delta)v(s^*) + \delta\bar{W}\big] = \bar{W} < v(\bar{s})-\varepsilon \ ,$$

where the last inequality follows from part *(i)* and $\delta > \delta_1$. Hence, there exists $\bar{\delta}_2(i,\pi) \in (0,1)$ such that $(1-\delta)v(s^*) + \delta W_i(\pi) < v(\bar{s})-\varepsilon$ whenever $\delta > \bar{\delta}_2(i,\pi)$. We obtain the result for all $i \in N$ and $\pi \in \Pi$ be such that $i \notin \bar{A}(\pi)$ by imposing $\delta > \bar{\delta}_2 \equiv \max\big\{\bar{\delta}_2(i,\pi) \colon i \notin \bar{A}(\pi)\big\}$. Henceforth, we assume that $\delta > \bar{\delta}_2$.

(iii) Let $i \in N$ and $\mathbf{k} \in \mathbf{K}$ and let π_S denote the club structure obtained at the end of the period if S forms. Suppose first that $i \in \bar{A}(\pi)$. Then, by definition $W_i(\mathbf{k}) = v(\bar{s})$. Suppose now that $i \notin \bar{A}(\pi)$ and $|\bar{A}(\pi)| < \bar{m}\bar{s}$. It follows that the next proposer (not in $\bar{A}(\pi)$), say j, forms the size-$\bar{s}$ club $S_j(\pi)$. If i is a member of $S_j(\pi)$, then she receives $v(\bar{s})-\varepsilon < v(\bar{s})$. Otherwise, her payoff is $(1-\delta)v(s_0) + \delta W_i(\pi_{S_j(\pi)}) < v(\bar{s})$, where s_0 is the size of her current club and the inequality follows from from part *(ii)*.

Suppose now that $i \notin \bar{A}(\pi)$ and $\big|\bar{A}(\pi)\big| = \bar{m}\bar{s}$. By definition of W_i, this implies that

$$W_i(\pi) = \begin{cases} v(n-\bar{m}\bar{s}) & \text{if } N \setminus \bar{A}(\pi) \in \pi \ , \\ v(n-\bar{m}\bar{s})-\varepsilon & \text{otherwise;} \end{cases}$$

Setting $\bar{\delta} \equiv \max\{\bar{\delta}_1, \bar{\delta}_2\}$, we obtain the Lemma. □

Step 2: Construction of the strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$. For each player $i \in N$, strategy σ_i prescribes her the following behaviour. At any proposer state $\big(\pi, (\iota_\ell, \ldots, \iota_n)\big) \in K^p$:

(a) If i belongs to $\bar{A}(\pi)$, or if the current club structure comprises $\bar{m}$ size-$\bar{s}$ clubs and one size-$(n - \bar{m}\bar{s})$ club, then she passes.

(b) Otherwise she proposes club $S_i(\pi)$.

Note that the club structure at the start of the period can be altered at most once per period. For every club structure $\pi \in \Pi$ and any proposal (offered club) $S \in 2^N \setminus \{\emptyset\}$, let π_S denote the coalition structure obtained at the end of the period if S forms. At any responder state $\big(\pi, (\iota_\ell, \ldots, \iota_n), S\big) \in K^r$:

(c) If i belongs to $\bar{A}(\pi)$, then she rejects proposal S.

(d) If i does not belong to $\bar{A}(\pi)$, and either $|S| = \bar{s}$ or $\big[|\bar{A}(\pi)| = \bar{m}\bar{s}\ \&\ S = N \setminus \bar{A}(\pi)\big]$, then she accepts proposal S.

(e) Otherwise, she accepts the proposal if and only if:

$$(1-\delta)v\big(|S|\big) + \delta W_i(\pi_S) > \begin{cases} (1-\delta)v(s_0) + \delta W_i(\pi) & \text{if } \ell = 1 \ , \\ W_i\big(\pi, (\iota_2, \ldots, \iota_\ell)\big) & \text{if } \ell \geq 2 \ , \end{cases}$$

where s_0 is the size of the club i belongs to in π.

Note that σ defined above depends on K^r or K^p which only contain the payoff-relevant part of the history; so σ is Markovian.

Step 3: Continuation values. We now define the continuation values from play according to the strategy profile define above. Let $V_i^\sigma(\kappa)$ be the continuation values of player i from play that begins in state κ (according to σ).[11]

From conditions (a) and (c) in the definition of σ, agents in $\bar{A}(\pi)$ always pass when they are proposers and reject when they are responders. As a consequence, no existing club of size-$\bar{s}$ will ever break up. Note that (a) and (b) in Step 2 correspond to (1) in the path defined in Step 1. Similarly, (c) and (d) in Step 2 correspond to (2) in the path defined in Step 1. We know from Step 1 that, for any $\mathbf{k} \in \mathbf{K}$, the payoff of player i is $W_i(\mathbf{k})$ if she plays according to (a)-(d). Hence, conditions (a)-(d) imply

[11]Note that the sets of proposer and responder states are denoted by K^p and K^r respectively, with generic element κ.

that $V_i^\sigma(\kappa) = W_i(\mathbf{k})$ for $i \in N$, where $\mathbf{K} \equiv K^p \cup \Pi$ and κ is either a proposer state or respondent state.

Step 4: Verification that σ is an SPE. Let $\bar{\delta}$ be defined as in Lemma 2.1 and, from now on, assume that $\delta \geq \bar{\delta}$. We saw in Step 2 that σ is a stationary Markov strategy profile. To complete the proof, therefore, it remains to establish that σ is a subgame perfect equilibrium. By the One-shot Deviation Principle, it suffices to check that there is no state at which an agent has a profitable deviation from the prescribed strategy profile.

One-shot deviations at proposer states. We begin with proposer states. Take an arbitrary state $\mathbf{k} = (\pi, (j_1, \ldots, j_l))$ and let i be the agent whose turn it is to propose.

(a) We show that there is no profitable deviation from (a), as defined in Step 2.

a.1 Consider first the case when $i \in \bar{A}(\pi)$. Strategy σ prescribes her to pass. If she does so then, from (c), we know that her current club will never be dissolved. Hence, her total discounted payoff from playing according to σ is $v(\bar{s})$. By deviating from σ, either (i) she makes an unsuccessful proposal, in which case her payoff is $v(\bar{s})$, or (ii) she forms another club of size-$\bar{s}$, so her payoff is $(1-\delta)(v(\bar{s}) - \varepsilon) + \delta v(\bar{s})$, or (iii) she successfully proposes S with $s \neq \bar{s}$. In cases (i) and (ii), she does not have a profitable deviation. In case (iii), she dissolves her size-$\bar{s}$ to form a club of a different size. The payoff she obtains by doing so is bounded above by

$$(1-\delta)(v(s^*) - \varepsilon) + \delta \bar{W} \leq v(\bar{s})$$

where the inequality follows from our assumption that $\delta \geq \bar{\delta}$ and Lemma 2.1(ii). Hence, proposer i cannot profitably deviate from σ.

a.2 We now consider a case where π is such that there are $\bar{m}$ clubs of size-$\bar{s}$ and one size-$(n - \bar{m}\bar{s})$ club. Strategy σ prescribes i to pass. If she passes then her payoff is $v(n - \bar{m}\bar{s})$. If she deviates, she can only make a successful proposal to agents who are not in $\bar{A}(\pi)$, since all agents in $\bar{A}(\pi)$ reject all future proposals. So, by deviating, i can only be in clubs of size $s' \leq n - \bar{m}\bar{s}$. Since $v(s') \leq v(n - \bar{m}\bar{s})$, there is no profitable deviation.

(b) We show that there is no profitable deviation from (b) defined in Step 2. We consider two cases: when $|\bar{A}(\pi)| < \bar{m}\bar{s}$ and when $|\bar{A}(\pi)| = \bar{m}\bar{s}$.

b.1 If $|\bar{A}(\pi)| < \bar{m}\bar{s}$ then $S_i(\pi)$ is the first $\bar{s} - 1$ successors of i in $N \setminus \bar{A}(\pi)$. If she offers $S_i(\pi)$ then, from (d), we know that she successfully forms a club of size-$\bar{s}$; so her payoff is $v(\bar{s}) - \varepsilon$. By deviating from σ, either (i) she successfully forms another club S with $s \neq \bar{s}$, or (ii) she makes an unsuccessful proposal or passes. In case (i), we know that her payoff is $v(\bar{s}) - \varepsilon$. In case (ii), her payoff is

$$\begin{array}{ll} (1-\delta)v(s_0) + \delta W_i(\pi) & \text{if } \ell = 1 \ , \\ W_i\big(\pi, (\iota_2, \ldots, \iota_n)\big) & \text{if } \ell \geq 2 \ , \end{array}$$

where s_0 is the size of her current club. If $\ell = 1$, then we know from Lemma 1(ii) that her payoff is bounded above by $(1-\delta)v\,(s^*) + \delta\bar{W} \leq v(\bar{s}) - \varepsilon$. If $\ell \geq 2$, then from Lemma 2.1(i), we know that her payoff is bounded above by $\bar{W} \leq v(\bar{s}) - \varepsilon$. Hence, i has no profitable deviation.

b.2 Let $|\bar{A}(\pi)| = \bar{m}\bar{s}$ such that $i \in S$ and $s < n - \bar{m}\bar{s}$. Then, if she moves according to σ, she successfully forms $S_i(\pi)$ such that $i \in S_i(\pi)$ and $s_i(\pi) = n - \bar{m}\bar{s}$. Her payoff then is $v(n - \bar{m}\bar{s}) - \varepsilon$. If she deviates, she can only make a successful proposal to agents who are not in $\bar{A}(\pi)$, since all agents of $\bar{A}(\pi)$ reject all the future proposals. So, by deviating, i can only be in clubs of size $s' \leq n - \bar{m}\bar{s}$. Since $v(s') - \varepsilon \leq v(n - \bar{m}\bar{s}) - \varepsilon$, she has no profitable deviation.

One-shot deviations at responder states. We now turn to responder states.

(c) We show that there is no profitable deviation from (c) defined in Step 2. Let $i \in \bar{A}(\pi)$. Strategy σ prescribes her to reject any offer. If she rejects, we know from the equilibrium construction that her current club will never dissolve. Thus, her payoff is $v(\bar{s})$. If she rejects, she gets $V_i^\sigma(\kappa)$. From Step 3 we know that $W_i(\mathbf{k}) = V_i^\sigma(\kappa)$. We also know from Lemma 2.1(iii) that $v_i(\bar{s}) \geq W_i(\mathbf{k}) = V_i^\sigma(\mathbf{k})$ for all $\mathbf{k} \in \mathbf{K}$. Hence, i cannot profitably deviate.

(d) We show that there is no profitable deviation from (d) defined in Step 2.

(d.1) Suppose that $i \notin \bar{A}(\pi)$ and that i is offered $S \ni i$ with $s = \bar{s}$. Strategy σ prescribes her to accept the offer. If she does so then her payoff is $v(\bar{s}) - \varepsilon$. By deviating to reject the offer, her payoff is

$$\begin{array}{ll} (1-\delta)v(s_0) + \delta W_i(\pi) & \text{if } \ell = 1 \ , \\ W_i\big(\pi, (\iota_2, \ldots, \iota_n)\big) & \text{if } \ell \geq 2 \ , \end{array}$$

If $\ell = 1$ then her payoff is bounded above by $(1-\delta)v\,(s^*) + \delta\bar{W} \leq v\,(\bar{s}) - \varepsilon$. If $\ell \geq 2$, then from Lemma 2.1(i), we know that her payoff is bounded above by $\bar{W} < v(\bar{s}) - \varepsilon$.

Hence, i cannot profitably deviate.

d.2 Suppose that $i \notin \bar{A}(\pi)$, $|\bar{A}(\pi)| = \bar{m}\bar{s}$ and that i is offered $S \ni i$ such that $s = n - \bar{m}\bar{s}$. Strategy σ prescribes her to accept the offer. Then, from the same argument as in (b.2), she has no profitable deviation.

(e) Strategy σ prescribes player i to accept the offer S iff the expression on the LHS is strictly greater than the expression on the RHS of the inequality. Note that the expression on the LHS is the payoff player i gets if she accepts offer S, and the expressions on the RHS are the payoffs she gets if she rejects offer S for $\ell = 1$ and $\ell \geq 2$. Thus, i cannot profitably deviate from accepting offer S if the expression on the LHS is strictly greater than the expression on the RHS. □

Corollary 2.2. *Let $\bar{s} \in \mathbb{N}$ that satisfies $m(\bar{s})\bar{s} < n$ and $v(n - m(\bar{s})\bar{s}) < v(\bar{s})$, then any club structure that comprises $\bar{m}$ clubs of size-$\bar{s}$ and one club of size-$(n - \bar{m}\bar{s})$ is dynamically stable*

This is a direct consequence of Proposition 2.3. Note that there is a multiplicity of equilibria and hence there are also other equilibria which have properties different from the one defined above. If $\bar{s} < s^*$ then the premise in Proposition 2.3 is satisfied without any condition. If $\bar{s} > s^*$, then we need to impose the condition that $v(n - \bar{m}\bar{s}) < v(\bar{s})$ or $n - \bar{m}\bar{s} < 2s^* - \bar{s}$ for the condition in the premise to be satisfied.

The following example illustrates Definition 2.1 and demonstrate the mechanism behind our equilibrium construction. This example provides some intuition for the general result in Proposition 2.3.

Example 1. Let $N = \{1, 2, ..., 8\}$ and $v(s) = 10s - s^2$ for all $i \in N$. Hence, the optimal club is of size 5. Take for example a club structure $\pi = \{S_1, S_2, S_3\}$ such that $s_1 = s_2 = 3$ and $s_3 = 2$. Let $\bar{A}$ be the set of players who are in a club of size 3 and let $X = [1, 2, 3, 4, 5, 6, 7, 8]$ be a ternary relation. If $\delta \geq 8/9$ then the following strategy profile forms a pure strategy SMPE in which π is a *dynamically stable club structure*:

- If proposer i belongs to a club of size 3 or π has been formed then she always passes, and otherwise proposes to the next two agents in $X \setminus \bar{A}$ to form a club of size 3 if $8 - |\bar{A}| \geq 3$, otherwise proposes to next agent in $X \setminus \bar{A}$ to form a club of size 2.

- Let s_0 be the size of the current club of respondent i and s be the size of the club she has been offered. If $s_0 = 3$ then i rejects any offer. If $s_0 \neq 3$, and either $s = 3$

or $|\bar{A}| = 6$ and $s = 2$ then she accepts the offer. Otherwise, she accepts the offer iff $(1 - \delta)v(s) + 205\delta/10 > (1 - \delta)v(s_0) + 205\delta/10$

The intuition is as follows. Even though the result would hold for any club structure, assume that the club structure at the start of the game is such that there exists one club of size 3 and one club of size 5. It is readily checked that the strategy profile above leads to dynamically stable club structure π at the end of period 2: one club of size 3 forms in the first period and one club of size 2 forms in the second period, and never changed thereafter.

Note that every player can end up in one of the two clubs in the long run and remains in that club forever: a "good club" (club of size-3) in which she receives 21 in every period, and a "bad club" (club of size-2) in which she receives 16 in every period. A club structure at the start of any period is either a stable club structure π or would ultimately lead to a stable club structure π. In the former case, player i's expected discounted payoff is 21 if i is in a good club, and 16 otherwise. In the latter case, player i receives $(1 - \delta)v(s)$ in the current period and $9/10 \times 21 + 1/10 \times 16 = \delta 205/10$ in the next period (i belongs to a good club with probability 9/10). Her expected payoff is therefore $(1 - \delta)v(s) + (\delta 205/10)$, which is less than 21 for all values of s (recall that $\delta \geq 8/9$). The agents in good clubs would never want to leave their current club because of the fear of ending up in a bad club at some later stages of the game. (There is a positive probability that a member of a good club club ends up in a bad club if she decides to break her current (good) club now to form an optimal sized club).

Thus, every agent i wants to minimise the chance of ending up in a bad club. In respondent stages, this includes rejecting any proposal if respondent i already belongs to a good club: even if the proposal is to form an optimal sized club, the respondent knows that optimal sized clubs are not sustainable and fears ending up in a bad club. It also includes accepting any proposal $S \ni i$ such that $s = 3$ when i is not already in a good club. Any attempt by the members of a bad club to form a club which includes members of a good club would be unsuccessful; and any proposal to form a good club when enough agents are not in a good club is successful. In the proposal stage, it is therefore optimal for player i to pass if she already is in a good club and/or the current club structure is π, and otherwise to propose a good club if enough agents are outside a good club, and to propose a club with the rest of the agents otherwise.

This example illustrates why the result in the infinite horizon game is different from that in the finite period game. In particular, it explains how and why there might be situations where none of the clubs which form are of optimal size in equilibrium: any deviation to propose to form a optimal club would be either unsuccessful or is not profitable, as the club structure would revert back to (sub-optimal) dynamically stable club structure π. The role of the discount factor is crucial to get this result. If the discount factor is small then the agents do not regard future payoff highly enough. Therefore, they would form an optimal sized club as soon as they have an opportunity because only the current period payoff really would matter; and forming optimal sized club would earn them the highest payoff. In conclusion, the above mentioned strategy profile fails to survive in equilibrium if agents' discount factors are not high enough.

2.6 Implications

The analysis of dynamic coalition formation in infinite periods contrasts starkly with the analysis of finite periods in Section 2.4. In the finite period game, we saw that optimal sized clubs form as long as there are enough agents left to form one; and that once an optimal club forms, it never dissolves. The idea is that the game ends at some point and players, in the last period, form the optimal sized club because they only care about the payoff in that period. If they are already in an optimal sized club then they choose to remain in that club. Thus, players can infer what would happen in the last period: if all optimal sized clubs have already been formed then these clubs remain intact, otherwise one optimal sized club forms. Knowing what happens in the last period, the agents in an optimal sized club can guarantee their fellow members not to dissolve their current club in the penultimate period. Applying this reasoning, players use backward induction to anticipate that once an optimal sized club forms, it does not dissolve till the end of the game. However, if the players are unsure about the sustainability of optimal sized clubs then they cannot always guarantee not to dissolve an optimal sized club, as they do not know what happens eventually.

The uncertainty mentioned above about the future leads to a natural lack of *commitment*: if players, for some reason, are pessimistic about the stability of optimal sized clubs then they would not commit to stay in the optimal sized club, as it might hurt them in future (if they end up in the worse club). Note that commitment here does not mean that players can write binding contracts; this is mutually understood among the members of a club based on their beliefs and time horizon. We know that the ideal scenario for each player is if she could become a member of an optimal sized club and stay in it forever. Thus, if players were able to commit by writing binding agreements, then there would be no difference between stable states of finite and infinite horizon games. Players would then form optimal sized clubs as soon as they can and enforce the commitment (to not to dissolve their club) by writing contracts.[12] The resulting (unique) outcome is that all optimal sized clubs form at some time even in the infinite horizon game and never dissolve, given that players can write binding contracts. Thus, for the results in Section 2.5 to hold, it is important that players do not have the means to formally commit.

In Proposition 2.3, we prove the existence of and characterize a (dynamically) stable club structure in which none of the clubs need be of optimal size. This characterization relies on the observation that sufficiently forward looking agents do not support a change which might ultimately lead to a situation in which they are worse off. Consider an equilibrium which prescribes clubs of size $\bar{s} < s^*$. Agents who can make changes in the current period to get a higher return by forming an optimal sized club cannot guarantee that they will remain in that club. This is because they may believe (based on what other agents do) that the optimal sized club is not sustainable in the future. Note that the agents who are currently in a bad club have high incentives to change the clubs: they know that if they are in a bad club and the stable state in reached, they will remain in the bad club forever. Those who immediately gain by forming an optimal sized club now cannot refrain from taking decisions later that would hurt some of their fellow agents who made it possible to form an optimal sized club. For instance, suppose some of the agents in the bad club are offered membership in an optimal club. As, optimal sized clubs are not sustainable, members of such an optimal sized club cannot commit not to dissolve the optimal club.

Based on players' pessimistic views and their response to such views, a steady state will be reached at some point in time in which only clubs of two different sizes exist. Players strictly prefer to be included in the larger club, and therefore take advantage of an opportunity to join such a club, as they fear being excluded from such a club in the future. The presence of an inferior club in the stable state is a requisite condition for individuals to exhibit fear of exclusion. If this is not the case, players do not fear being excluded from sustainable (same size) clubs in the stable state: knowing that everyone would eventually end up in same size club, they could profitably deviate in the current period from forming sub-optimal sized to optimal sized clubs. Thus, in the absence of inferior clubs in the stable state, the strategy profile which prescribes players to form a

[12]Commitments can be enforced by imposing cost on those who break the agreement.

sub-optimal sized club cannot survive in equilibrium. The presence of an inferior club and hence fear of exclusion not only sustains sub-optimal sized clubs but may also lead the proposers and responders to form clubs of sub-optimal size out of fear that the current club structure will be replaced by a new one, and that they may be left out of the better club.

Fear of exclusion (from a better sustainable club) is the main reason why players cannot assure that they will remain in the optimal sized club. This leads to two intuitive results. First, the stability of a club structure turns on whether there are enough players excluded from better clubs to jointly form such a club. It does not depend on whether players would prefer to be members of a club that is not in the structure. For instance, in Example 1, members of A and B can form a new club of size 5; but this club structure is not stable when the players fear exclusion. Second, a dynamically stable club structure can be inefficient, i.e. there might be another club structure whose payoff dominates the payoffs in the dynamically stable club structure. Again in Example 1, a club structure with one club of size 5 and one club of size 3 Pareto dominates the club structure with two clubs of size 3 and one club of size 2.

2.7 Conclusion

This chapter has examined a process of club formation in a dynamic setting when individuals are far-sighted. An important feature of the dynamic club formation process is that given the rules that govern the formation and dissolution of clubs, the current decision makers have an opportunity to take a decision to their advantage that affects their and others' choices in the future. This implies that dynamic club formation must recognize that current decision-makers make choices knowing that their decision will have an impact on their future choices, and, therefore, their current decision might depend on what they know or believe about future outcomes.

We developed a framework to study this problem of dynamic club formation when individuals are only concerned about the size of the club they belong to. All players are identical and have the same preference over club size. Assuming that per-period payoff functions of the players are strictly concave and single-peaked, there exists an optimal sized club which maximises the utilities of its players. In each period, players get an opportunity to either propose to form a new club of a particular size or respond to such proposals. A club forms when all the potential members agree. The game is played T times where T could be infinite. In addition to other players' equilibrium strategies, a player's decision to form a new club might depend on whether the player knows that the process goes on for a finite or an infinite number of periods. Players are far-sighted; and in each period they take a decision to maximise their expected average discounted per-period payoffs.

Agents are aware whether they are in a finite or a infinite horizon game. In both finite and infinite horizon game, agents are uncertain which club they will end up in the stable state, which depends on the exogenous protocol. But in the finite horizon game, agents are certain that optimal sized clubs do not break until the end of the game: so if an agent join an optimal sized club then she will remain in it until the game ends. By contrast in the infinite horizon game, agents are not sure whether optimal sized clubs will survive in the long run. If agents believe that optimal sized clubs are not sustainable then they do not form optimal sized and aim to become part of larger and sustainable sub-optimal sized club as soon as possible. In this case, agents are certain that if optimal sized club forms, it must eventually break. Uncertainty then kicks in when the optimal sized club breaks and they do not know which club they will belong to in the stable club structure.

We use the same approach as Acemoglu et al. (2012) in finding stable club structures, i.e. club structures which never change after they have been formed. In this chapter, we investigate stable club structures of the finite horizon and the infinite horizon games. The idea revolves around whether we can generate an equilibrium in which players have fear of exclusion which induces them to not form optimal sized clubs. In particular, whether and how many optimal sized clubs exists in stable club structures: optimal stable club structures are the ones where all possible clubs are of optimal size and sub-optimal club structures are the ones in which none of the clubs are of optimal size. In the infinite horizon game, because of the multiplicity of equilibria we focus only on a very specific class of equilibria to find stable club structures with interest in the properties defined above about the optimal clubs. We compare why and how the results differ in the two games, if at all. The main aim is to we uncover the underlying phenomenon for such a difference between the strategic behaviour of players in the finite and infinite horizon games that leads to different outcomes.

We have investigated and provided the characterization for stable club structures that have all possible optimal sized clubs and stable club structures that have no optimal sized clubs (sub-optimal club structures). We found that sub-optimal stable club structures can only exist in infinite horizon games. It is interesting to note that sub-optimal club structures cannot be found in the finite horizon game, in which the only difference is that the players can apply backward induction argument to find that optimal sized clubs remain intact until the end once they are formed. As the model of an infinite horizon examined in Section 2.5 shows, players might fear exclusion when they are uncertain about future outcomes. These sub-optimal stable club structures exhibit a lack of commitment on the part of players to stay in optimal sized clubs because they have negative views on the sustainability of optimal sized clubs, and therefore fear exclusion from the best sustainable club. Such optimal sized clubs would therefore break down over time in the infinite horizon game when players have these sort of beliefs. However, in the finite horizon game, players use backward induction to anticipate that in the last period all possible clubs of optimal size would form and therefore they do not fear exclusion; so they know that optimal sized clubs are fool proof. The presence of a club worse than the sustainable sub-optimal club is a requisite for individuals to have fear of exclusion. It is reasonable to think that the population is a not an integer multiple of the size of the sub-optimal club. Thus, at the end, some individuals would be left out of an sub-optimal sized club in the stable club structure. When the sub-optimal sized club is smaller than the optimal sized club, the remaining players left out of sub-optimal sized clubs cannot form an optimal sized club. However when the sub-optimal sized club is larger than the optimal sized club, a simple condition is required to discourage players from deviating.

Club coalition equilibria of the infinite horizon game are based on particularly simple strategies with proposers proposing to form a sub-optimal sized club if they are not already members of such a club and enough players are left, and otherwise they either pass or form a club, which is worse (a club with leftover players). The respondents respond by accepting an offer to form a sub-optimal sized club if they are not already members of one, and otherwise reject if they are in a sub-optimal sized club or accept to form any other club if it is beneficial. When players are sufficiently forward-looking, this equilibrium strategy profile generates a sub-optimal club structure since players fear exclusion and are uncertain about the future.

The theory in this chapter has several interesting extensions. In our study, individuals whose sole aim is to join a club only care bout the size of the club. It would be interesting to look at the hedonic setting in which the individuals have different tastes, and preferences not only over the club size but also who they share the club with. Another interesting extension would be to investigate other possible equilibria (in the infinite horizon case) such as clubs of three different sizes, all different from the optimal size and to examine the behaviour which supports these kinds of club structures. We could also do robustness checks on the equilibria of the infinite period game by proving that there are no joint deviations. We can also introduce a refinement that satisfies forward induction.

Conclusion

Micro-targeting has increasing been used in the political campaign to effectively communicate with a sub-group of voters. On one hand, there are commercial organisations which are consistently collecting date on voters to provide it for a fee, and on the other hand, more interestingly, there are some lobbying organisations which, in addition to their usual lobbying activities, are also engaged in collecting data with the intention to push for their agendas. The first contribution of this thesis is made to the literature of lobbying which studies IGs' influence on the policy outcomes. Chapter 1 of this doctoral thesis, provides a analysis of the influence lobbying organisations (IGs) have on the policy-making process when these IGs use MT as a lobbying tool. The analysis demonstrates that some IGs can influence policy outcomes in their favour when they are the only one who have information on voters' preferences. In particular, IGs who are opposed to the candidate's ideology can influence the candidate to take a policy stand in their favour, provided that the challenger does not care enough about the issue that IG is concerned with. This happens because the IG can send group-specific messages to some voters, which might prove worth for the candidate to win. In exchange for MT on behalf of the candidate, the IG demands policy favours. We also find that MT is not an effective tool to change policy outcomes in its favour if the IG has the same ideology as the candidate. However, their existence in the political system can affect the influence other IGs have on the policy outcomes. We also show that in a two dimensional policy space, MT is less harmful when the two candidate are completely opposed to each other as compared to when they are partially opposed to each other.

The second contribution of this thesis is to the literature of club goods. In Chapter 2, we study a dynamic game of club formation in which individuals' utilities depend only on the size (number of members) of the club they belong. The earlier literature on this topic has mainly analysed this class of club formation game in a static setting. Chapter 2 aims to fill the gap by investigating the club formation game in a dynamic setting. We find that in the infinite horizon game, individuals might refrain from forming clubs that provides the highest per period payoff, but this behaviour is never observed in the finite horizon game. In never ending long term interactions, individuals might have pessimistic views on the sustainability of what is regarded as the best club. In that case, they may form clubs which are not best, but popular and sustainable than the other sustainable clubs. Fear of exclusion is the main factor which compels this kind of behaviour. Since, there is no way to formally commit to remain member of a club, individuals cannot remain loyal not break club if they form best club. However, in the finite horizon game, backward induction naturally lead individuals to remain in best clubs.

Bibliography

Acemoglu, D., G. Egorov, and K. Sonin (2012). Dynamics and stability of constitutions, coalitions, and clubs. *American Economic Review 102*(4), 1446–76.

Arnold, T. and M. Wooders (2015). Dynamic club formation with coordination. *Journal of Dynamics and Games 2*(2164-6066), 341.

Austen-Smith, D. (1987). Interest groups, campaign contributions, and probabilistic voting. *Public Choice 54*(2), 123–139.

Austen-Smith, D. and J. R. Wright (1992). Competitive lobbying for a legislator's vote. *Social Choice and Welfare 9*(3), 229–257.

Azari, J. (2015). The roots of political science frustration. *Mischiefs of Factions, April 20 http://www.mischiefsoffaction.com/2015/04/the-roots-of-political-science.html.*

Banerjee, S., H. Konishi, and T. Sönmez (2001). Core in a simple coalition formation game. *Social Choice and Welfare 18*(1), 135–153.

Barbera, S., M. Maschler, and J. Shalev (2001). Voting for voters: A model of electoral evolution. *Games and Economic Behavior 37*(1), 40 – 78.

Baron, D. P. (1994). Electoral competition with informed and uniformed voters. *American Political Science Review 88*(1), 33–47.

Bennedsen, M. and S. E. Feldmann (2006). Informational lobbying and political contributions. *Journal of Public Economics 90*(4), 631 – 656.

Bennett, C. J. (2016). Voter databases, micro-targeting, and data protection law: can political parties campaign in europe as they do in north america? *International Data Privacy Law 6*(4), 261–275.

Bernheim, B. D. and M. D. Whinston (1986). Menu auctions, resource allocation, and economic influence. *The Quarterly Journal of Economics 101*(1), 1–31.

Besley, T. and S. Coate (2001). Lobbying and welfare in a representative democracy. *The Review of Economic Studies 68*(1), 67–82.

Bogomolnaia, A. and M. O. Jackson (2002). The stability of hedonic coalition structures. *Games and Economic Behavior 38*(2), 201 – 230.

Bombardini, M. and F. Trebbi (2011). Votes or money? Theory and evidence from the US Congress. *Journal of Public Economics 95*(7-8), 587–611.

Buchanan, J. M. (1965). An economic theory of clubs. *Economica 32*(125), 1–14.

Cameron, C. M. and J. P. Jung (1995). Strategic endorsements in hidden offer bargaining. *Mimeo, Stanford University*.

Chatterjee, K., B. Dutta, D. Ray, and K. Sengupta (1993). A noncooperative theory of coalitional bargaining. *The Review of Economic Studies 60*(2), 463–477.

Dixit, A., G. Grossman, and E. Helpman (1997). Common agency and coordination: General theory and application to government policy making. *Journal of Political Economy 105*(4), 752–69.

Friess, S. (2013, August 20). How the NRA built a massive secret database of gun owners. *https://www.buzzfeednews.com/article/stevefriess/how-the-nra-built-a-massive-secret-database-of-gun-owners*.

Grofman, B. and B. Norrander (1990, Mar). Efficient use of reference group cues in a single dimension. *Public Choice 64*(3), 213–227.

Grossman, G. M. and E. Helpman (1994). Protection for sale. *The American Economic Review 84*(4), 833–850.

Grossman, G. M. and E. Helpman (1996). Electoral competition and special interest politics. *The Review of Economic Studies 63*(2), 265–286.

Grossman, G. M. and E. Helpman (1999). Competing for endorsements. *The American Economic Review 89*(3), 501–524.

Grossman, G. M. and E. Helpman (2001). *Special Interest Politics*. MIT Press.

Helpman, E. and A. L. Hillman (1977). Two remarks on optimal club size. *Economica 44*(175), 293–296.

Hillman, A. L. and J. G. Riley (1989). Politically rents and transfers. *Economics and Politics 1*(1), 17–39.

Hollard, G. (2000). On the existence of a pure strategy nash equilibrium in group formation games. *Economics Letters 66*(3), 283 – 287.

Holzman, R. and N. Law-Yone (1997). Strong equilibrium in congestion games. *Games and Economic Behavior 21*(1), 85 – 101.

Hyndman, K. and D. Ray (2007, 10). Coalition Formation with Binding Agreements. *The Review of Economic Studies 74*(4), 1125–1147.

Kenny, C., M. McBurnett, and D. Bordua (2004). The impact of political interests in the 1994 and 1996 congressional elections: The role of the national rifle association. *British Journal of Political Science 34*(2), 331–344.

Klevorick, A. K. and G. H. Kramer (1973). Social choice on pollution management: the genossenschaften. *Journal of Public Economics 2*(2), 101 – 146.

Kollman, K. (1998). *Outside lobbying: Public opinion and interest group strategies.* Princeton University Press.

Konishi, H., S. Weber, and M. L. Breton (1997). Free mobility equilibrium in a local public goods economy with congestion. *Research in Economics 51*(1), 19 – 30.

Laslier, J.-F. (2006). Ambiguity in electoral competition. *Economics of Governance 7*(2), 195–210.

Layard, R. (1990). Lay-offs by seniority and equilibrium employment. *Economics Letters 32*(3), 295 – 298.

Lupia, A. (1992). Busy voters, agenda control, and the power of information. *The American Political Science Review 86*(2), 390–403.

Maskin, E. and J. Tirole (2001). Markov perfect equilibrium: I. observable actions. *Journal of Economic Theory 100*(2), 191 – 219.

McKelvey, R. D. and P. C. Ordeshook (1985). Elections with limited information: A fulfilled expectations model using contemporaneous poll and endorsement data as information sources. *Journal of Economic Theory 36*(1), 55 – 85.

Milchtaich, I. (1996). Congestion games with player-specific payoff functions. *Games and Economic Behavior 13*(1), 111 – 124.

Milgrom, P. and J. Roberts (1986). Relying on the information of interested parties. *The RAND Journal of Economics 17*(1), 18–32.

Ng, Y.-K. (1973). The economic theory of clubs: Pareto optimality conditions. *Economica 40*(159), 291–298.

Ng, Y.-K. (1978). Optimal club size: A reply. *Economica 45*(180), 407–410.

Olson, M. (1965). The logic of collective action. *Cambridge,MA 9*.

Page, F. H. and M. Wooders (2007). Networks and clubs. *Journal of Economic Behavior and Organization 64*(3), 406 – 425.

Pauly, M. V. (1970). Cores and clubs. *Public Choice 9*, 53–65.

Prummer, A. (2019). Micro-targeting and polarization. *Journal of Public Economics*.

Ray, D. and R. Vohra (1999). A theory of endogenous coalition structures. *Games and Economic Behavior 26*(2), 286 – 336.

Roberts, K. (2015). Dynamic voting in clubs. *Research in Economics 69*(3), 320 – 335. Special issue on Political Economy.

Schelling, T. (2006). Strategies of commitment and other essays. *Cambridge, Mass.: Harvard University Press*.

Schipper, B. C. and H. Y. Woo (2019). Political awareness, microtargeting of voters, and negative electoral campaigning. *Quarterly Journal of Political Science 14*(1), 41–88.

Schlozman, K. L. and J. T. Tierney (1986). Organized interests and american democracy. *American Political Science Review 80*(4), 1353–1354.

Seidmann, D. J. and E. Winter (1998, 10). A theory of gradual coalition formation. *The Review of Economic Studies 65*(4), 793–815.

Shubik, M. and M. H. Wooders (1982). Near-markets and market games. *The Economic Studies Quarterly 37*(4), 289–299.

Shubik, M. and M. H. Wooders (1983). Approximate cores of replica games and economies: Part ii: Set-up costs and firm formation in coalition production economies. *Mathematical Social Sciences 6*(3), 285 – 306.

Stiglitz, J. E. (1977). *The Theory of Local Public Goods*, pp. 274–333. London: Palgrave Macmillan UK.

Suggett, P. (2016, November 7). A look at our history of political advertising. *Retrieved from https://www.thebalancecareers.com/a-brief-history-of-political-advertising-in-the-usa-38925*.

Tiebout, C. M. (1956). A pure theory of local expenditures. *Journal of Political Economy 64*(5), 416–424.

Watkins, T. (2013). How the NRA wields its influence. *https://edition.cnn.com/2013/01/09/us/nra-gun-research/*.

Winkler, A. (2013). *Gunfight: The Battle over the Right to Bear Arms in America.* W.W.Norton and Company.

Wiseman, J. (1957). The theory of public utility price-an empty box. *Oxford Economic Papers 9*(1), 56–74.

Wooders, M. (1978). Equilibria, the core, and jurisdiction structures in economies with a local public good. *Journal of Economic Theory 18*(2), 328 – 348.

Wright, J. (1996). *Interest Groups and Congress: Lobbying, Contributions, and Influence.* Allyn and Bacon.

Young, A. (2012, November 20). How data and microtargeting won the 2012 election for obama. *https://www.mediavillage.com/article/how data and microtargeting won the 2012 election for obama antony young mindshare north america/*.

Yu, Z. (2005). Environmental protection: A theory of direct and indirect competition for political influence. *The Review of Economic Studies 72*(1), 269–286.