

1 **Probabilistic examination of the change in eigenfrequencies of an offshore wind turbine under**
2 **progressive scour incorporating soil spatial variability**

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16

17 **Abstract**

18 The trend for development in the offshore wind sector is towards larger turbines in deeper water. This
19 results in higher wind and wave loads on these dynamically sensitive structures. Monopiles are the
20 preferred foundation solution for offshore wind structures and have a typical expected design life of
21 20 years. These foundations have strict serviceability tolerances (e.g. mudline rotation of less than
22 0.25° during operation). Accurate determination of the system frequency is critical in order to ensure
23 satisfactory performance over the design life, yet determination of the system stiffness and in
24 particular the operational soil stiffness remains a significant challenge. Offshore site investigations
25 typically focus on the determination of the soil conditions using Cone Penetration Test (CPT) data.
26 This test gives large volumes of high quality data on the soil conditions at the test location, which can
27 be correlated to soil strength and stiffness parameters and used directly in pile capacity models.
28 However, a combination of factors including; parameter transformation, natural variability, the
29 relatively small volume of the overall sea bed tested and operational effects such as the potential for
30 scour development during turbine operation lead to large uncertainties in the soil stiffness values used
31 in design. In this paper, the effects of scour erosion around unprotected foundations on the design
32 system frequencies of an offshore wind turbine is investigated numerically. To account for the
33 uncertainty in soil-structure interaction stiffness for a given offshore site, a stochastic ground model is
34 developed using the data resulting from CPTs as inputs. Results indicate that the greater the depth of
35 scour, the less certain a frequency-based SHM technique would be in accurately assessing scour
36 magnitude based solely on first natural frequency measurements. However, using Receiver Operating
37 Characteristic (ROC) curve analysis, the chance of detecting the *presence* of scour from the output
38 frequencies is improved significantly and even modest scour depths of 0.25 pile diameters can be
39 detected.

40

41 **Keywords:** Scour; Offshore Wind; Uncertainty; Frequency; SHM; Spatial Variability

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44 1.0 Introduction

45 In order to meet increasing energy demands, reduce reliance on non-renewable sources and increase
46 energy security, alternative and renewable energies are in high demand. In Europe, offshore wind
47 turbines provided over 11,000 MW of grid-connected capacity at the end of 2015 [1], supplying 1.5%
48 of the electricity consumption demand of the European Union (EU) [2]. The offshore wind industry is
49 continually undergoing rapid development, tending towards larger capacity turbines, ever-increasing
50 hub heights and locations further offshore in deeper waters. This rapid development is a challenge for
51 foundation design, and design methods are constantly evolving [3,4].

52

53 To date, over 80% of offshore turbines are founded on monopiles, followed by 9% founded on
54 Gravity Base Foundations (GBFs) and approximately 5% on jacket structures [1]. Other solutions
55 include Tripods, Tripiles, floating solutions as well as experimental foundation concepts. Monopiles
56 are by far the preferred foundation solution with typical diameters (D) ranging from 4-6m and larger
57 with typical penetrations (L) of 20-30m [5,6]. As turbines grow larger and water depths increase
58 monopiles with diameters of up to 10m are being planned, leading to increased foundation costs.
59 Foundations typically account for 30% of the cost of the entire system [7]. Larger monopiles will lead
60 to increased lateral hydrodynamic loads, impacting the monopile in the horizontal direction, resulting
61 in significant shear forces and overturning moments. Foundations typically resist this loading
62 through flexural action and rigid rotation and the ultimate capacity is governed by soil strength
63 properties or the structural properties of the pile. Rigid monopiles with slenderness ratios, length
64 normalised by diameter (L/D), of 5 and below have more uncertain lateral load-resistance
65 characteristics as the design methods for offshore piles were originally developed for long, slender
66 piles [8–11] which flex under an applied lateral load.

67

68 Scour erosion around monopiles reduces the lateral load bearing capacity as well as the soil-structure
69 interface stiffness [12] and can result in significant changes to load effects at the mudline level. Scour
70 occurs when the near bed shear stresses applied by hydraulic action exceed the threshold shear stress
71 at which sediment commences movement and occurs as a result of the obstruction caused by the
72 monopile changing the waterflow characteristics locally [13]. It is a complicated mechanism and is
73 environment dependent. In rivers, scour generally occurs under steady current conditions whereas in
74 the marine environment, it occurs due to current, tides and waves [14]. The combined action of
75 current and waves typically gives rise to lower ultimate scour depths than under current only
76 conditions [10,15], however the interaction is complex and uncertain.

77

78 Scour alters the dynamic characteristics of structures, a phenomena that has led to significant research
79 interest related to the performance of river bridges [16–23]. From this research there is consensus that

80 scour reduces the foundation stiffness for bridges. In the offshore environment however, there is
81 uncertainty about the effects of scour on the strength and stiffness properties of soil and the combined
82 effects of load cycling and pore pressure accumulation [6]. In marine conditions, combined effects
83 from currents and waves lead to variations in the equilibrium scour depth, with erosion and
84 backfilling both occurring. Unlike in rivers under live-bed erosion conditions [24], where the
85 deposited material typically has lower strength and stiffness properties, the wave action can densify
86 this material potentially restoring stiffness to pre-scour levels or higher [25]. The uncertainty
87 associated with the effects of scour on the strain dependant stiffness behaviour of the remaining soil,
88 cyclic load response, bearing capacity and other factors potentially makes scour occurrence a critical
89 safety issue.

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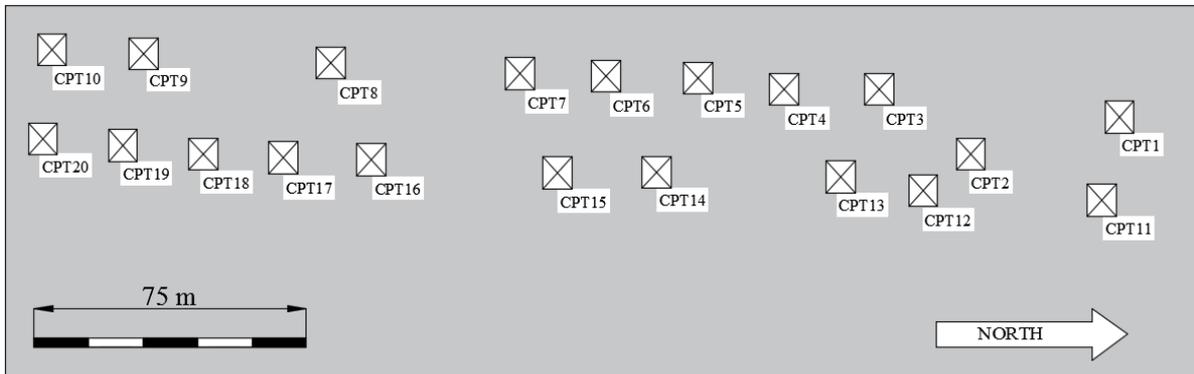
91 The analysis in this paper builds on the study presented by Prendergast et al. [26], which examined the
92 effect of scour on the natural system frequency of an offshore turbine under three idealised soil
93 profiles. The model is expanded in the present work to investigate the effect of spatial variability in
94 soil properties derived from measured Cone Penetration Test (CPT) data on the system frequencies of
95 a typical turbine under scoured conditions. A sample of twenty deep CPT profiles measured in a
96 reclaimed area of Rotterdam Harbour were used to generate 50,000 hypothetical spatially-correlated
97 CPT profiles for the statistical analyses. A Monte-Carlo analysis was performed to derive the likely
98 system frequencies for a typical offshore turbine considering a range of scour depths. The variation in
99 frequency from the spatial uncertainty of the ground conditions and with scour is investigated, with a
100 view to understanding whether the magnitude of the changes could be detected within a SHM
101 framework.

102

103 **2.0 Test Site**

104 The ground model developed in this paper is based on data from the Port of Rotterdam, Netherlands.
105 The site was originally located offshore in the North sea, but was reclaimed by the Dutch Authorities
106 in the 1970s [27]. The site consists of predominately Holocene era sands to a depth of approximately
107 25m below existing ground level (egl) with bulk unit weights ranging from 18.5 to 20.5 kN m⁻³. The
108 relative density (D_r) is approximately 50%. Some modest clay to clayey silt lenses of varying
109 thickness are found in between primarily close to ground level, with a maximum thickness of
110 approximately 1m to 1.5m. The bulk unit weight of the clay layers is in the range of 15 to 18 kN m⁻³.
111 Some medium coarse Pleistocene sands are found at a depth of 24 to 25 m below egl [27], with a bulk
112 unit weight between 19 and 20 kN m⁻³, a D_r of 80% and φ' between 35° and 37°. The perched water
113 table elevation ranges from a minimum of 3.5m below egl to a maximum of 1m below egl. Twenty
114 CPT q_c profiles were measured at the site and corrected to the ordnance datum (NAP). The relative
115 locations of these CPT profiles are shown in Fig. 1. Fig. 2 shows the CPT q_c profiles measured, with

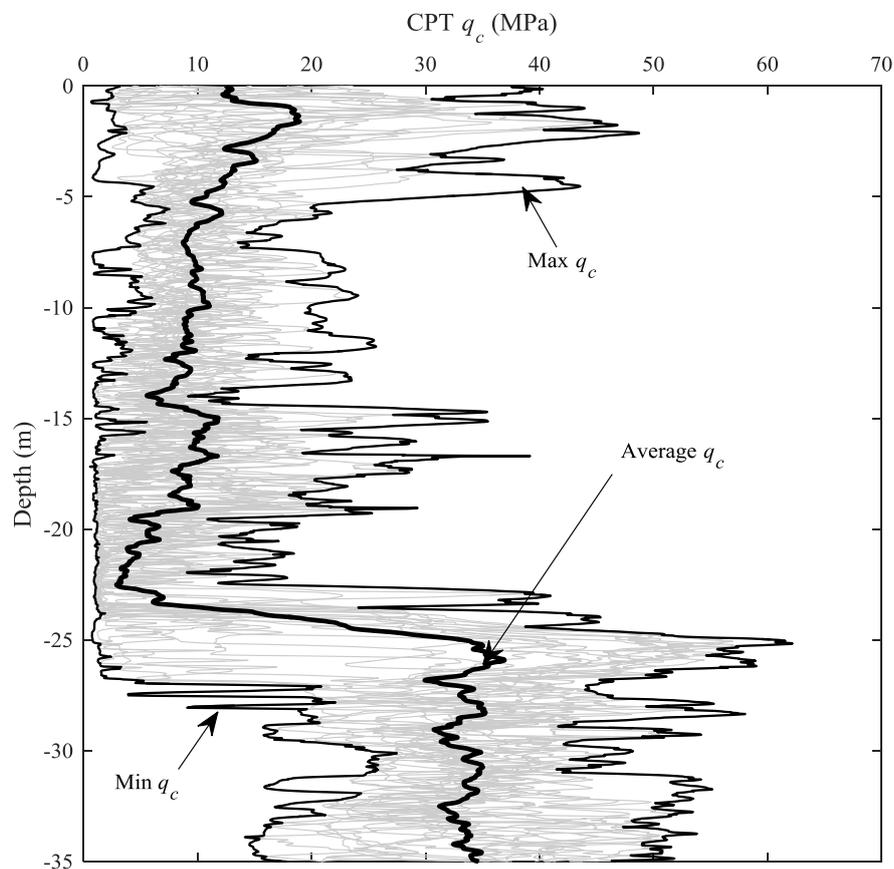
116 the average and maximum/minimum envelopes also shown. As is evident, there are two distinct layers
 117 present in the profiles, transitioning at approximately 23 – 25 m below ground level.



118

119

Fig. 1 Cone Penetration Test (CPT) spatial layout at Rotterdam Harbour



120

121

Fig. 2 CPT q_c profiles with maximum and minimum envelopes

122 3.0 Stochastic ground model

123 Soil is a naturally heterogeneous material, understanding how it varies is essential to the development
 124 of accurate mechanics based ground models, which can encapsulate and subsequently represent soil
 125 physical properties. Traditionally variability within soil was accounted for by subdividing the soil into

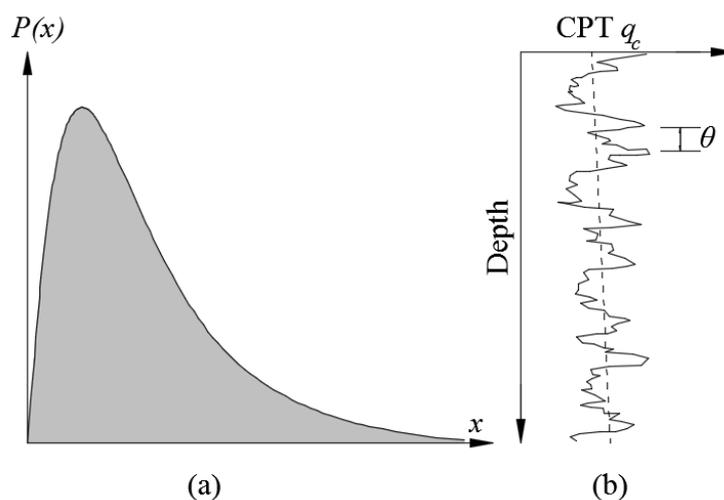
126 a number of discrete layers, with each layer having a different set of deterministic soil parameters to
 127 describe the soil properties within that layer [28,29]. Naturally, given the significant uncertainty
 128 present in such an approach, conservative values have to be chosen.

129

130 In an effort to eradicate such gross oversimplifications, probabilistic techniques have come to the fore
 131 for geotechnical applications [30–34]. Such approaches utilise all of the available data from a soil
 132 layer in the form of a probability distribution. While the majority of structural engineering problems
 133 can be modelled using a simple random variable approach, the stratified nature and heterogenic
 134 composition of soil demands a more complex stochastic approach [35]. To account for this soil is
 135 frequently modelled using a number of layered non-homogeneous random fields (2D or 3D) or
 136 processes (1D) [36–39]. These random fields or processes model the scope of a given property's
 137 variance and define how it varies temporally and/or spatially.

138

139 For variables that can be described using normal and log-normal distributions (See Fig. 3(a)) the
 140 random process of a soil property can be described in terms of three variables, namely mean, standard
 141 deviation and a third term describing the spatial variability, in this case the scale of fluctuation (θ), see
 142 Fig. 3(b). The scale of fluctuation is the distance over which soil properties are significantly correlated
 143 [40,41]. While the mean and standard deviation are easy to obtain from a given dataset, determining
 144 the scale of fluctuation is somewhat more complicated. The general procedure adopted in this paper,
 145 to generate spatially correlated CPT tip resistance (q_c) profiles in the vertical direction, is outlined
 146 below. As this is a complex field of study in its own right, interested readers are directed to [42,43]
 147 for a more in depth discussion and alternative methods for investigating spatial variability. Only a
 148 fundamental overview is provided herein for the present application.



149

150 Fig. 3 (a) Initial lognormal distribution defined by mean and standard deviation at a given depth, (b)
 151 Scale of fluctuation (θ) adjusts the general shape of distribution to account for spatial variability

152 To determine the CPT spatial correlation structure in the vertical direction it is necessary to first
 153 remove any underlying trend from the data. Typically, only first order trends (for example the strength
 154 increase with depth typically seen in normally consolidated soil deposits) are considered as higher
 155 order trends may result in overfitting and their use would demand further justification. By redefining
 156 the mean and standard deviation such that they are functions of depth (See Fig. 3(b)), the mean trend
 157 can be removed from the dataset using a curve fitting approach, thus isolating any variability. This
 158 variability can then be fitted to a spatial correlation structure. Following the removal of any
 159 discernible trend, the soil property (in this case q_c) for a normal distribution can be described by
 160 Eq.(1).

$$161 \quad q_c = \mu + \sigma \mathbf{G} \quad [1]$$

162 where μ is the mean value described at a depth z using Eq. (2), σ is the standard deviation at the same
 163 depth and \mathbf{G} is a matrix containing n spatially correlated normal random processes of zero mean and
 164 unit variance which account for the vertical spatial correlation structure.

$$165 \quad \mu(z) = a_i + b_i z \quad [2]$$

166 where a_i is the value of the mean trend at the beginning of the i th layer, b_i is the slope of that trend at
 167 the same layer and z is the depth into the stratum.

168 When the linear depth trend of each q_c profile in the dataset is removed, the standard deviation of the
 169 detrended tip resistances is calculated. Normalised detrended tip resistances are then obtained by
 170 dividing the individual detrended CPTs by their respective standard deviations. This approach
 171 produces normal random processes with a mean of zero and a standard deviation of 1. These normal
 172 random processes can be used to estimate the spatial correlation structure $\hat{\rho}(\tau_j)$ of the CPTs with
 173 depth, see Eq. (3).

$$174 \quad \hat{\rho}(\tau_j) = \frac{1}{\sigma^2(n-j)} \sum_{i=1}^{n-j} (X_i - \mu)(X_{i+j} - \mu) \quad [3]$$

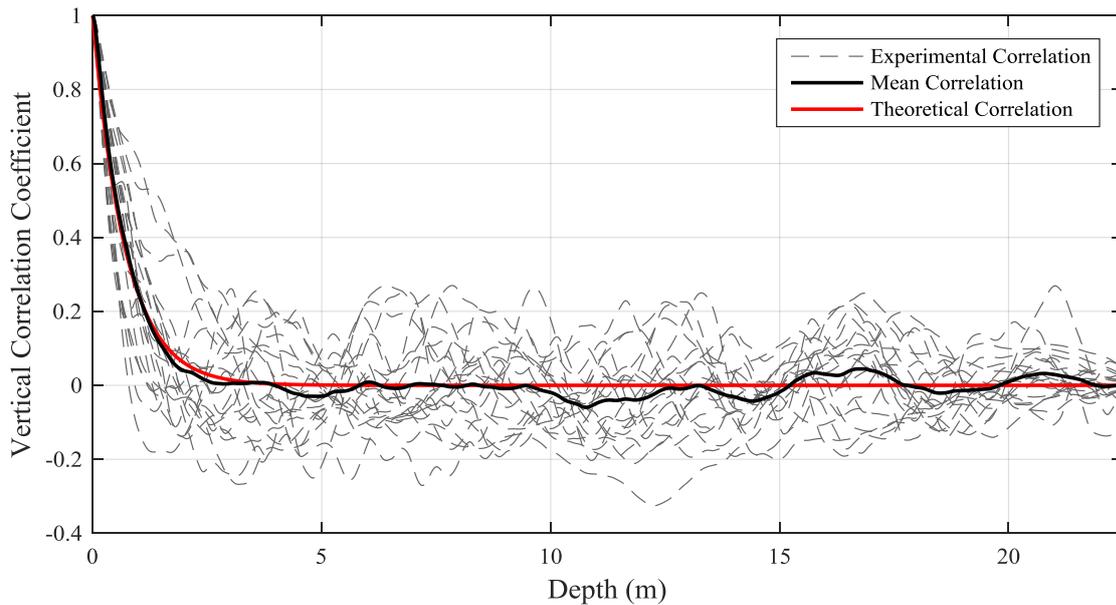
175 where $j = 0, 1, \dots, n-1$ with n being the number of data points, $\tau_j = j\Delta\tau$ is the lag distance between
 176 the two points in question where $\Delta\tau$ is the distance between two adjacent points, μ is the estimated
 177 mean, σ is the standard deviation and X is the random soil property. A Markov correlation function
 178 [41,44] was used to approximate the spatial correlation structure, see Eq. (4). The Markov function,
 179 which assumes that the correlation between two points decreases exponentially with distance was then
 180 fitted to the estimated correlation structure obtained from Eq. (3). This was accomplished by

187 minimising the scale of fluctuation, θ , until the difference between $\hat{\rho}(\tau)$ and $\rho(\tau)$ was negligible, see
 188 Fig. 4. A vertical scale of fluctuation of 1.424 m with a 95% confidence interval of {1.403 m, 1.445
 189 m} was determined for the 23m deep sand layer (from 0 to 23m in Fig. 2) in the Port of Rotterdam
 190 and 1.771 m with a 95% confidence interval of {1.735 m, 1.807 m} for the bottom layer .

191

192

$$\rho(\tau_j) = \exp\left(\frac{-2|\tau_j|}{\theta}\right) \quad [4]$$



193

194

195 Fig. 4 Estimated vertical correlation structure from 20 CPTs and fitted theoretical correlation function
 196 using a 1.424m scale of fluctuation for top layer.

197

198 The resulting correlation matrix ρ is positive definite and can be decomposed into upper \mathbf{L}^T and
 199 lower \mathbf{L} triangular forms using Cholesky Decomposition, see Eq. (5).

200

$$\rho = \mathbf{L}\mathbf{L}^T \quad [5]$$

202

203 The correlated matrix of normalised random processes, \mathbf{G} , is then obtained by multiplying the lower
 204 triangular matrix with \mathbf{U} , a vector of 50,000 independent normal random numbers with zero mean and
 205 unit standard deviation per depth increment, see Eq. (6).

$$\mathbf{G} = \mathbf{L}\mathbf{U} \quad [6]$$

207

208 If a normal distribution is required \mathbf{G} can be inserted directly into Eq. (1), however in this paper, a
 209 lognormal distribution was used, as it demonstrated a better fit than the normal distribution and
 210 prevented negative tip resistance values from being generated, as these are physically inadmissible
 211 [45,46]. Note, a bounded normal distribution or a Beta distribution could also have been used. To
 212 generate a lognormally distributed random processes, the mean and standard deviation first need to be
 213 transformed into the lognormal domain, see Eqs. (7) & (8).

$$214 \quad \sigma_{\ln} = \sqrt{\ln(1 + \sigma^2)} \quad [7]$$

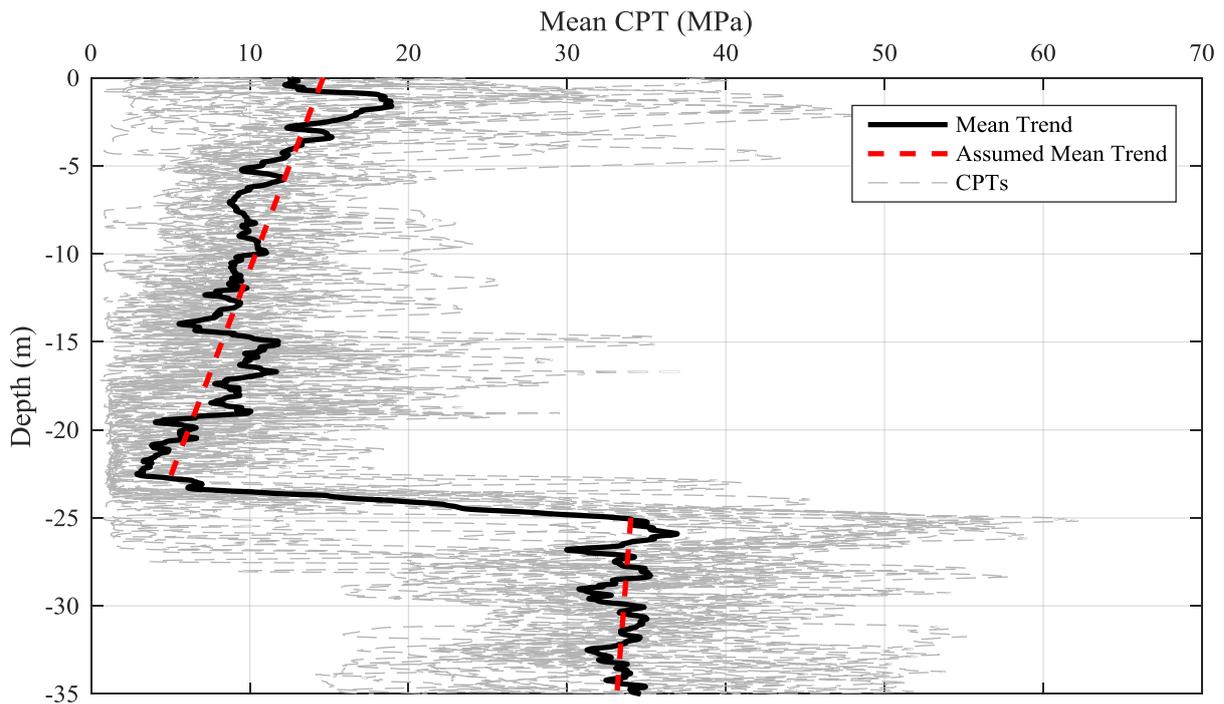
$$215 \quad \mu_{\ln} = \ln \mu - \frac{1}{2} \sigma_{\ln}^2 \quad [8]$$

216 The spatially correlated lognormally distributed random process is then obtained using Eq. (9).

$$217 \quad \mathbf{q}_c = \exp(\mu_{\ln} + \sigma_{\ln} \mathbf{G}) \quad [9]$$

218 The calculations in this paper were carried out using 50,000 random spatially correlated CPT profiles
 219 generated using the methodology presented in Eqs. (4)-(9). The mean profiles used to generate these
 220 CPTs is shown in Fig. 5 (Rotterdam Harbour CPT data), with the standard deviation calculated per
 221 CPT and averaged over each layer. The previously evaluated vertical scale of fluctuation of 1.424 m
 222 (for the top layer) and 1.771 (for the bottom layer) was used.

223



224

225 Fig. 5 The 20 CPT profiles with assumed mean trend for determining spatial variation and
 226 subsequently generating a random process model for the soil layers

227

228 4.0 Wind turbine model

229 A numerical model of a wind turbine was developed using 1-D finite elements (FE). The properties of
 230 the model were initially derived based on the recommendations in Sørensen and Ibsen [13], who state
 231 that monopiles supporting wind turbines have typical diameters of $D=4-5\text{m}$, wall thicknesses of 50-
 232 120mm and penetrations of $L=15-30\text{m}$. They currently support 2-5MW turbines in 10-25m water
 233 depths. The model used in the present study consists of a 6m diameter monopile [26], with an overall
 234 length of 75m (water depth of 30m) and an embedded length of 30m ($L/D = 5$). The embedded length
 235 was derived using the Critical Pile Length Criterion, described in Arany et al. [3]. The pile supports a
 236 70m high tower and nacelle assembly, see Fig. 6. A pile wall thickness of 0.08 m was adopted as the
 237 cross-sectional properties of the monopile were tailored to the required design protocols (see section
 238 4.2). The primary geometric and material properties adopted are outlined in Table 1.

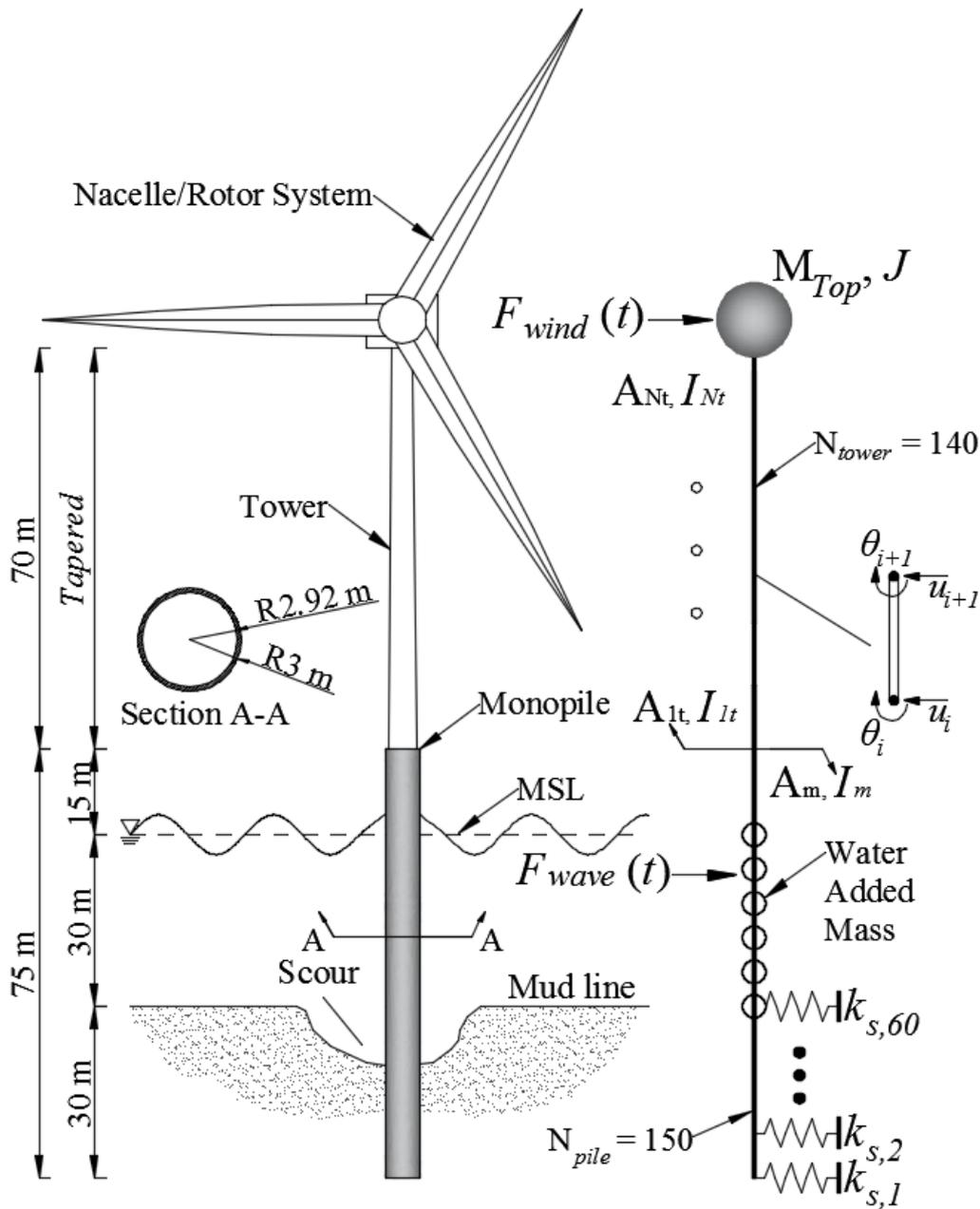


Fig. 6 Wind turbine model schematic.

4.1 Structural modelling

The monopile and tower were formulated numerically using four degree of freedom (4-DOF) Euler-Bernoulli beam elements, the elemental stiffness \mathbf{K}_i and mass \mathbf{M}_i matrices are available in Kwon and Bang [47]. Each element is 0.5m in length. Table 1 provides the primary material and geometrical information. The mass of soil within the monopile is treated as an added mass, by increasing the effective cross-sectional area of the elements below the mudline. A bulk unit weight of 20 kN m^{-3} is assumed for the internal soil. For the portion of the pile under water, hydrodynamic (external) and entrapped (internal) water added mass is incorporated using Eq.(10).

$$m_w = C_a \rho_w \frac{\pi D^2}{4} H \quad [10]$$

where m_w is the added mass acting over the entire water column height, H , ρ_w is the density of sea water (1025 kg m^{-3}) and C_a is the coefficient of added mass multiplying the area of fluid displaced by the monopile. A value of $C_a=2$ is adopted, 1 for the external mass [48] and 1 for the internal mass [49]. Water added mass is formulated using an effective cross-sectional area for the elements below the water-line. All submerged elements are formulated using buoyant densities ($\rho-\rho_w$). Relative changes in added mass due to tidal action are not considered in the present study.

The tower is assumed to taper from a diameter of 5m at the base to 3.5m at the top so the cross-sectional area, A and moment of inertia, I vary along its length, as indicated in Table 1. The nacelle and rotor/blades system is modelled as a lumped mass at the top of the tower, formulated by adding a lumped mass matrix to the final beam element at the tower top, shown in Eq.(11). Eccentricities due to the offset of the nacelle mass from the vertical, gyroscopic motion of the blades and aerodynamic damping are not considered in this study.

$$\mathbf{M}_N = \mathbf{M}_i + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & M_{nacelle} & 0 \\ 0 & 0 & 0 & J \end{bmatrix} \quad [11]$$

where $M_{nacelle}$ is the mass of the nacelle (kg) taken as 230,000 kg [50] and J is the rotational inertia in the fore-aft direction (kg m^2) taken as $3.5 \times 10^7 \text{ kg m}^2$. The soil dynamic stiffness is incorporated using Winkler spring elements [26,51–54], with linear stiffness. It is assumed that the soil springs have a null mass matrix.

The discrete spring stiffnesses are derived from the stochastic soil model, see section 4.2 for a discussion on the derivation of soil spring stiffness (geotechnical) for the present study. The various local elemental matrices are assembled into global ($n \times n$) mass and stiffness matrices [47] for the full system and the undamped natural frequencies and mode shapes are obtained by solving the eigenproblem shown in Eq.(12).

$$([\mathbf{M}^{-1}\mathbf{K}] - \lambda[\mathbf{I}])\{\mathbf{A}\} = \{\mathbf{0}\} \quad [12]$$

where $[\mathbf{I}]$ is the identity matrix, $[\mathbf{M}^{-1}\mathbf{K}] - \lambda[\mathbf{I}]$ is the characteristic matrix, $\lambda = \omega_n^2$ are the eigenvalues and $\{\mathbf{A}\}$ the associated eigenvectors. The eigenvalues and eigenvectors (natural frequencies and mode shapes) are obtained by solving the characteristic equation. In total the model is

276 formulated using 140 elements for the tower (height = 70m), 150 elements for the monopile (length =
 277 75m) and 60 springs for the un-scoured soil profile (depth of embedment of 30m). A water depth of
 278 30m with 15m freeboard is assumed [26]. The scour process is modelled in the numerical model as
 279 the iterative removal of springs starting at the top (removing the apportioned spring stiffness from the
 280 assembled global stiffness matrix), corresponding to an increase in scour depth equating to the FE
 281 length discretisation, L .

282 Table 1 Model Properties

Tower/Nacelle Properties:	Value:	Monopile Properties:	Value:
Tower length (m)	70	Monopile length (m)	75
Material	Steel	Embedded length (m)	30
Density (kg m ⁻³)	7850	Material	Steel
Young's modulus (GPa)	210	Density (kg m ⁻³)	7850
Tower diameter (m)	5 – 3.5	Young's modulus (GPa)	200
Tower wall thickness (m)	0.045	Monopile diameter (m)	6
X-sectional area (A_{it}) (m ²)	0.7005-0.4884	Monopile wall thickness (m)	0.08
Moment of inertia (I_{it}) (m ⁴)	2.15-0.7289	X-sectional area (A_m) (m ²)	1.4879
Nacelle/Rotor mass (M_{Top}) (kg)	230,000	Moment of inertia (I_m) (m ⁴)	6.5192
Nacelle rotational inertia fore-aft direction (J) (kg m ²)	3.5×10^7	Mass of power unit at interface level ($M_{Transition}$) (kg)	27,000

283

284 4.2 Geotechnical modelling

285 Arany et al. [3] present a step-by-step monopile design procedure covering the Ultimate Limit State
 286 (ULS), Serviceability Limit State (SLS), Fatigue limit State (FLS), Target Natural Frequency (TNF)
 287 and Installation Criteria. The purpose of the present paper is to highlight how geotechnical uncertainty
 288 and spatial variability in soil strength combined with scour erosion can affect a wind turbine's system
 289 frequencies. Therefore, the TNF design is the most important to ensure design compliance for the
 290 given ground conditions. Once the TNF is evaluated, basic SLS checks are undertaken to ensure
 291 compliance against the wind and wave environment. Section 4.2.1 discusses the basis for simple wind
 292 and wave loading calculations, section 4.2.2 describes how the soil-structure interaction for the scour

293 modelling is incorporated, section 4.2.3 presents an overview of the TNF analysis and section 4.2.4
 294 presents the compliance checks for the SLS for the derived model properties. Note, only the minimum
 295 design checks are conducted in this paper, a full design should consider ULS, SLS, FLS, TNF,
 296 driveability and buckling, among others.

297 **4.2.1 Load basis for pile design**

298 For compliance checking of the monopile in SLS, a baseline load estimation is required. Note, the
 299 loading for SLS is assumed as that applied under normal turbine operating conditions. Extreme loads
 300 for ULS calculations are not considered, as their effect on serviceability is assumed negligible since
 301 they will not occur very often. Interested readers are directed to Arany et al. [3] for a more in-depth
 302 load calculation basis. The recommendations of Corciulo et al. [49] are adopted herein, which
 303 describes a simplified wind/wave loading regime. The assumptions are that wind and wave thrusts,
 304 F_{wind} and F_{wave} (i) depend on wind velocity and system geometry, (ii) depend on the application of
 305 empirical aero- and hydrodynamic factors and (iii) are co-directional. Also, the effect of rotor
 306 revolution on wind speed is neglected. Wind thrust can be calculated according to Eq.(13).

$$307 \quad F_{wind} = \frac{1}{2} A_R C_T \rho_{air} V_{wind}^2 \quad [13]$$

308 where A_R is the swept area of the rotor (m^2), $\rho_{air} = 1.2 \text{ kg m}^{-3}$, V_{wind} is the wind speed ($m \text{ s}^{-1}$) and $C_T =$
 309 0.688 (empirical wind thrust coefficient). By postulating a sustained wind field, an equilibrium sea
 310 state is assumed. A Pierson-Moskowitz wave spectrum [55] is postulated to quantify the wave energy
 311 S associated with each frequency f , see Eq.(14).

$$312 \quad S(f) = \frac{\alpha g^2}{(2\pi f)^5} \exp \left[-\beta \left(\frac{g}{2\pi f V_{wind}^{19.5m}} \right)^4 \right] \quad [14]$$

313 where $\alpha=0.0081$ and $\beta=0.74$ are empirical constants, $g=9.81 \text{ m s}^{-2}$, $V_{wind}^{19.5m}$ = wind speed at 19.5m
 314 above sea level. Wind speeds can be extrapolated from a reference measurement using a power law
 315 formulation [56]. Wave frequency f_s (at maximum spectral amplitude) and wave height H_s (distance
 316 between crest and trough) are shown in Eq.(15).

$$317 \quad f_s^4 = \frac{4\beta}{5} \left(\frac{g}{2\pi V_{wind}^{19.5m}} \right)^4 \quad [15a]$$

$$318 \quad H_s = 2 \sqrt{\frac{\alpha}{\beta}} \frac{(V_{wind}^{19.5m})^2}{g} \quad [15b]$$

319 The mono-harmonic sea state defined by f_s and H_s can be transformed to a hydrodynamic thrust F_{wave}
 320 using the Morison equation [57], with drag and inertial force components as shown in Eq.(16).

$$321 \quad F_{wave}^{DRAG} = \rho_w g \frac{C_d D}{8} H_s^2 \left(\frac{1}{2} + \frac{kH}{\sinh 2kH} \right) \quad [16a]$$

$$322 \quad F_{wave}^{INERTIA} = \rho_w g \frac{C_m \pi D^2}{8} H_s \tanh kH \quad [16b]$$

323 The overturning moments with respect to the mudline are shown in Eq.(17).

$$324 \quad M_{wave}^{DRAG} = \rho_w g \frac{C_d D}{8} H_s^2 \left[\frac{H}{2} + \frac{2(kH)^2 + 1 - \cosh 2kH}{4k \sinh 2kH} \right] \quad [17a]$$

$$325 \quad M_{wave}^{INERTIA} = \rho_w g \frac{C_m \pi D^2}{8} H_s H \left[\tanh kH + \frac{1}{kH} \left(\frac{1}{\cosh kH - 1} \right) \right] \quad [17b]$$

326 C_d and C_m , the drag and inertia coefficients are taken as 0.65 and 1.6 respectively. H is the height of
 327 the water column (m), D is the monopile diameter (m), ρ_w is the seawater density (1025 kg m^{-3}) and k
 328 is the wave number, related to the wave length (λ_w) by $k = 2\pi / \lambda_w$ [58]. k can be obtained from the
 329 dispersion relation [3], shown in Eq.(18).

$$330 \quad \omega^2 = gk \tanh(kH) \quad [18]$$

331 where $\omega = 2\pi f_s$. Eq.(18) is an implicit equation, therefore solutions must be found numerically.
 332 However, an explicit approximation may be obtained by Eq.(19) [59].

$$333 \quad k = \frac{\omega^2}{g \left[\tanh \left\{ \left(\frac{2\pi \sqrt{\frac{H}{g}}}{T} \right)^{\frac{3}{2}} \right\} \right]^{\frac{2}{3}}} \quad [19]$$

334 where $T = 2\pi / \omega$. The drag and inertial components of the wave thrust will be out of phase,
 335 therefore resultant mudline forces and moments are calculated using the Square Root Sum of
 336 Squares (SRSS)[58], see Eq.(20).

$$337 \quad F_{wave} = \sqrt{\left(F_{wave}^{INERTIA} \right)^2 + \left(F_{wave}^{DRAG} \right)^2} \quad [20a]$$

338
$$M_{wave} = \sqrt{(M_{wave}^{INERTIA})^2 + (M_{wave}^{DRAG})^2} \quad [20b]$$

339 It is assumed that the sustained wind speed under normal turbine operating conditions in combination
 340 with associated wave loading is the critical load scenario for SLS design compliance in this paper.
 341 The wind turbine modelled is a 3.6MW turbine [50], with nominal power production at a wind speed
 342 of 12 m s⁻¹. The derived unfactored wind and wave loads used in this paper are show in Table 2.

343 Table 2 loading for SLS compliance

V_{wind}	$V_{wind}^{19.5m}$	F_{wind}	Lever Arm	F_{wave}	M_{wind}	M_{wave}	Mudline F	Mudline M
m s ⁻¹	m s ⁻¹	kN	m	kN	kNm	kNm	kN	kNm
12	10.05	670	115	480	77050	15980	1155	93225

344

345 **4.2.2 Soil-Structure Interaction**

346 The soil-structure interaction between the monopile and the surrounding soil is incorporated via the
 347 Winkler hypothesis [51], using an array of discrete, mutually independent, 1-D spring elements. For
 348 the purpose of sizing the monopile, the design spring stiffness was derived from the average CPT q_c
 349 profile in Fig. 2. Linear springs were used for the small-strain TNF analysis, and non-linear springs
 350 were derived for the SLS check.

351 When the system parameters were adequately sized, the effect of spatial variability and geotechnical
 352 uncertainty on system natural frequencies for various scour depths was assessed. For this analysis,
 353 small-strain linear springs were developed from the randomly generated, spatially correlated ground
 354 profile.

355 The process of calculating individual spring moduli is discussed herein. A hypothetical CPT profile
 356 developed using the stochastic ground model is converted to small-strain linear springs distributed
 357 along the monopile shaft. Each CPT profile is discretised into 0.5m depth increments (to correspond
 358 to the discretisation in the FE model) and each increment is transformed to the small-strain shear
 359 modulus (G_0) using Eq.(21). In the absence of laboratory or geophysical measurements of G_0 [53,60],
 360 correlations between G_0 and q_c developed by Lunne et al.[61] and Schnaid et al.[62] have been shown
 361 to provide reasonable estimates of the small strain stiffness response when the stress history, age and
 362 degree of cementation is considered. By taking the average q_c profile for the site (Fig. 2) and assuming
 363 a 30m embedded monopile the relationship proposed by Schnaid et al. [62] suggest a value of $n = 6$ is
 364 appropriate for this deposit. This is within the expected range for dense sands.

365
$$G_0 = nq_c \quad [21]$$

366 The small-strain shear modulus G_0 profile can be converted to a profile of the small-strain Young's
 367 modulus according to $E_0 = 2G_0(1 + \nu)$, where ν is the small-strain Poisson ratio. The modulus of
 368 subgrade reaction (K) can then be derived using the procedure outlined in [12,26,63] (originally
 369 developed by Vesic [64]), see Eq. (22).

$$370 \quad K = \frac{E_0}{1 - \nu^2} \left[\frac{E_0 D^4}{E_p I_p} \right]^{1/12} \quad [22]$$

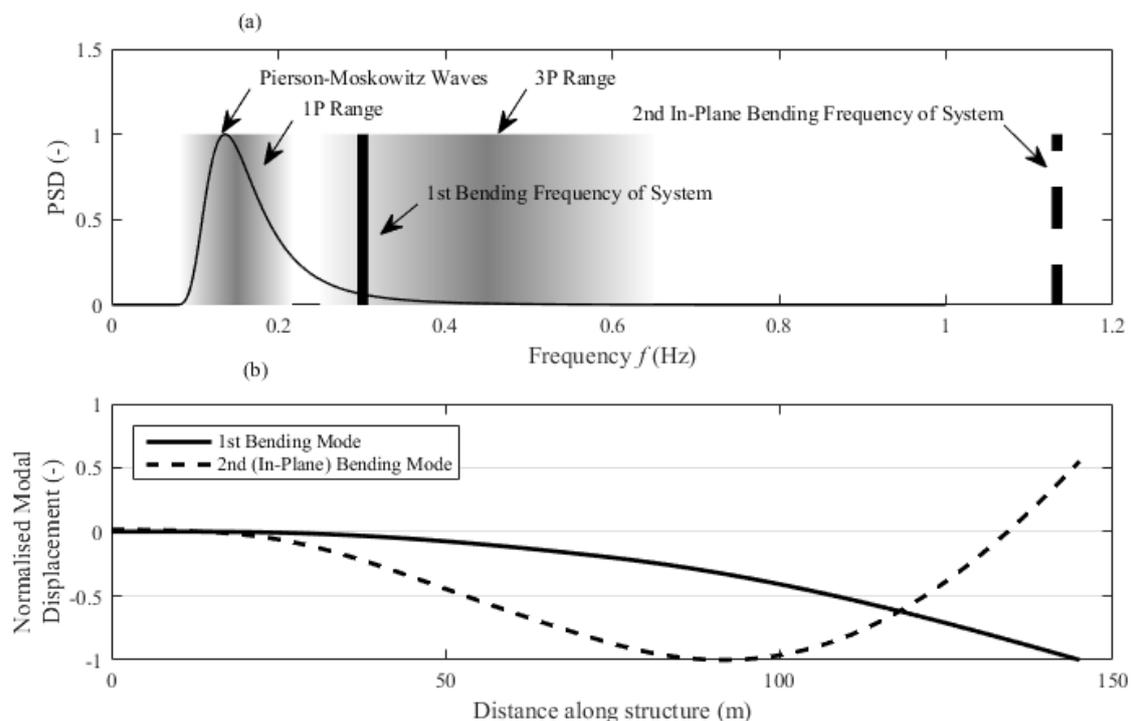
371 where E_p and I_p are the Young's modulus and moment of inertia of the pile, respectively. K is
 372 subsequently converted to individual spring moduli ($k_{s,i}$) by multiplying the K profile at a given depth
 373 by the distance between subsequent springs (L), at each spring depth.

374 Deriving soil-structure interface stiffness using this method has been shown to be accurate in
 375 experimental studies previously conducted. Prendergast and Gavin [53] performed experimental
 376 vibration tests on two piles with varying slenderness ratios (L/D) in dense sand and compared the
 377 results to numerical models developed employing five different modulus of subgrade reaction
 378 formulations. The model employing the Vesic formulation [64], a variation on Eq. (22) provided the
 379 closest approximation of the natural frequencies for both piles tested. Moreover, Prendergast et
 380 al.[12,26] derived soil stiffness profiles using Eq. (22) from shear wave velocity and CPT
 381 measurements and compared experimental results to numerical models at modelling the change in
 382 frequency due to scour. In one study [12], a pile with L/D of 19 was used and in the second [26], a
 383 pile with L/D of 6.5 was used. For both cases, the stiffness derived using Eq. (22) proved accurate at
 384 tracking the frequency changes due to scour imposed on the physical systems. Ashford and
 385 Juirnarongrit [63] performed a study to evaluate the effect of pile diameter on the initial modulus of
 386 subgrade reaction. They derived the subgrade reaction using Eq. (22) and compared numerical models
 387 to experimental piles with diameters of 0.4m, 0.6m, 0.9m and 1.2m. The study concluded that models
 388 employing Eq. (22) were capable of estimating the natural frequencies of each system to within a ratio
 389 of 0.98 to 1.04 times the measured values.

390 **4.2.3 Target Natural Frequency (TNF)**

391 Unlike other large civil structures such as oil and gas platforms, offshore wind turbines are
 392 particularly dynamically sensitive [26]. An over or under prediction in the system frequency can be
 393 detrimental to the stability and fatigue life of these structures in operation. The system is subjected to
 394 periodic loading from a number of sources including wind and wave as well as those arising due to
 395 the operation of the turbine. The frequency generated by the rotational velocity of the rotor is termed
 396 the 1P frequency [3,6,26]. A further loading frequency is generated due to the turbulent interaction
 397 when the blades pass the tower (shadowing effect), termed the $N_b P$ frequency, where N_b is equal to the

398 number of blades on the turbine. Wind loading occurs with typical frequencies lower than the 1P
 399 frequency. Fig. 7(a) shows nominal ranges for the 1P and 3P frequencies of the turbine modelled in
 400 this paper [50], along with the Pierson-Moskowitz wave spectrum. The wind spectrum is omitted.
 401 PSD magnitudes are normalised for illustrative purposes. The 1P and 3P frequency ranges represent
 402 the lowest and highest revolutions per minute (RPM) of the rotor [3,50] (5-13 RPM). For monopile
 403 supported turbines, typical design frequencies reside in the soft-stiff range, between the 1P and 3P
 404 bands. It is first necessary to size the tower assuming it is clamped at the bottom (fully fixed and no
 405 monopile). The clamped first frequency for a soft-stiff founded system should be close to 0.5Hz [65].
 406 Using this threshold, a 70m long tower with a tapering diameter of 5m (base) to 3.5m (top), average
 407 diameter 4.25m, yields a frequency of 0.496 Hz (≈ 0.5 Hz). When connected to the monopile, the
 408 whole system should have a frequency in the range 0.28-0.31 Hz. A 6m diameter monopile with a
 409 wall thickness of 80 mm provides a first natural system frequency of 0.3012 Hz and a second in-plane
 410 bending frequency of 1.1331 Hz, using the design average CPT profile from Fig. 2 and incorporating
 411 water added mass. The second in-plane bending frequency equates to the third mode of vibration, as
 412 the second mode will be out of plane and very close in value to the first frequency for symmetrical
 413 structures, see Fig.7(b) for mode shapes. Note, that the first natural frequency resides in the tail of the
 414 3P band. This is not an issue, however, as the nominal operating RPM will typically be at the upper
 415 end of the range, therefore resonance due to blade shadowing at this low rotational velocity is not
 416 expected (and can be avoided using the control system of the turbine). The following section
 417 describes the serviceability check undertaken to ensure the chosen pile dimensions are compliant with
 418 wind and wave loading.



420 Fig. 7 (a) Frequency bands for present system, (b) First and second in-plane bending mode shapes

421 4.2.4 Serviceability Limit State (SLS)

422 Basic SLS checks are carried out to ensure model compliance with accepted thresholds. The allowable
423 accumulated mudline rotation over the lifetime of a wind turbine founded on a monopile is normally
424 limited to 0.25° rotation, in addition to an initial allowable tilt of 0.25° at the mudline to allow for
425 errors upon installation of the pile [3]. Furthermore, the initial mudline deflection is limited to 0.2m as
426 is the accumulated deflection over the lifetime of the system [3].

427 To perform preliminary checks, a nonlinear p - y analysis was carried out using a finite-difference
428 solver, whereby the pile is modelled using linear beam elements and the soil is modelled as a series of
429 discrete, nonlinear p - y springs. Two approaches are used to generate p - y springs for this study, the
430 American Petroleum Institute (API) method [66] and a CPT-based approach for piles in sand [67].

431 The API design code for laterally loaded piles in sand characterises soil spring p - y relation as a
432 hyperbolic function, as shown in Eq.(23) [8,11]. It was originally derived based on a database of
433 lateral load tests on piles with relatively high slenderness ratios [8].

$$434 \quad p = Ap_u \tanh\left(\frac{kz}{Ap_u} y\right) \quad [23]$$

435 where p_u is the ultimate resistance at depth 'z' below the ground surface (kN m^{-1}), k is the constant
436 coefficient of subgrade reaction (kN m^{-3}), A is an empirical factor accounting for static or cyclic
437 external loading and y is the lateral deflection (m). Numeric values for k are specified in the API
438 design code [66], and depend on the friction angle or density of the soil and vary for saturated and
439 unsaturated conditions. For the analysis in this paper, API springs were generated based on a design
440 friction angle profile, derived using the average CPT profile from Fig. 2. The average CPT profile was
441 converted to a profile of the angle of internal friction using a relation from Kulhawy and Mayne [68],
442 shown in Eq.(24). The design profile was then obtained by discretising this profile into layers, see Fig.
443 8(b).

$$444 \quad \varphi = 17.6 + 11 \log \left[\left(\frac{q_c}{\sigma_{atm}} \right) / \left(\frac{\sigma'_{v0}}{\sigma_{atm}} \right) \right]^{0.5} \quad [24]$$

445 where σ'_{v0} is the effective stress (kN m^{-2}) and σ_{atm} is the atmospheric pressure (taken as 100 kN m^{-2}).
446 The CPT-based approach is based on Suryasentana and Lehane [67], who described a p - y curve
447 derivation technique for laterally loaded piles in sands, which may be more applicable to the rigid pile

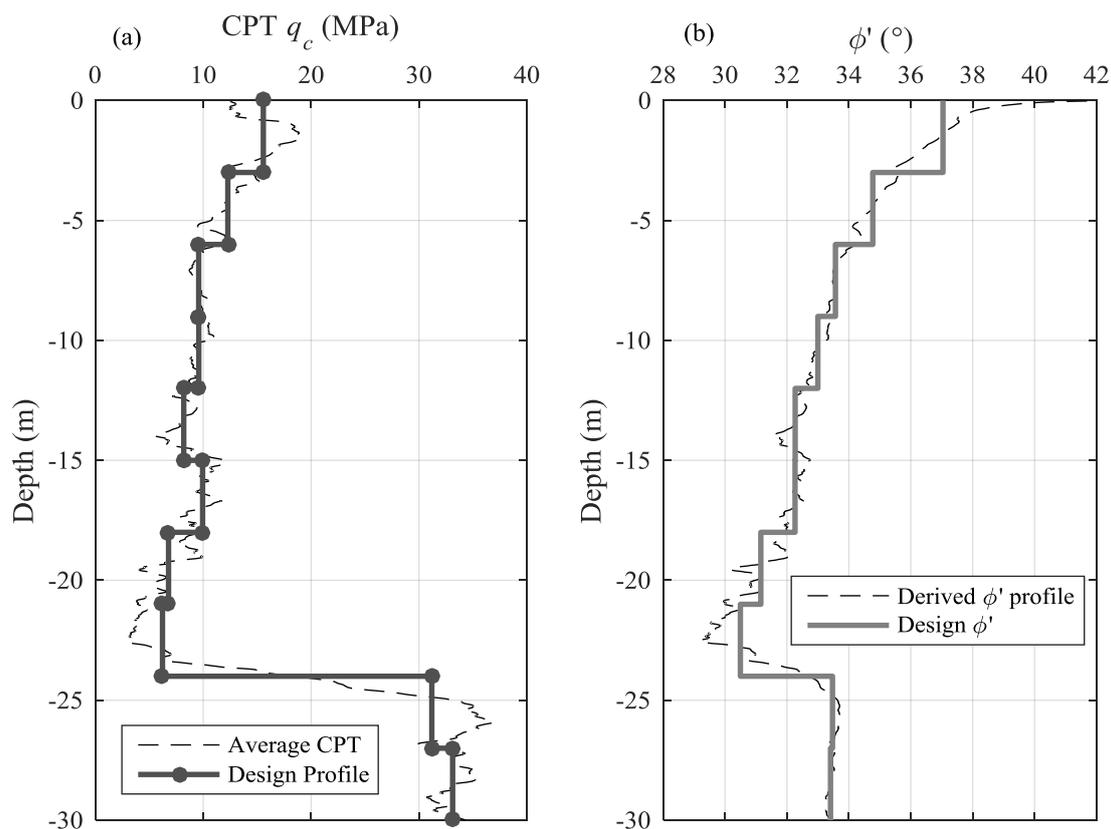
448 geometries used in the offshore wind sector. They propose an exponential relationship for the p - y
 449 curves, shown in Eq. (25).

$$450 \quad \frac{p}{\gamma z D} = 2.4 \left(\frac{q_c}{\gamma z} \right)^{0.67} \left(\frac{z}{D} \right)^{0.75} \left\{ 1 - \exp \left[-6.2 \left(\frac{z}{D} \right)^{-1.2} \left(\frac{y}{D} \right)^{0.89} \right] \right\} \quad [25]$$

451 where p is the soil reaction at a given spring depth (kN m^{-1}), γ is the bulk unit weight of the soil (kN
 452 m^{-3}), z is the depth to the middle of each design layer (m), D is the monopile diameter (m) and y is the
 453 lateral deflection (m).

454 The design profiles used for both methods are shown in Fig. 8. Fig. 8(a) shows the average CPT
 455 profile from Rotterdam Harbour (section 2.0) and the layered averaged profile, used in the lateral load
 456 analysis for the CPT-based p - y approach. Fig. 8(b) shows the derived ϕ' profile and a depth averaged
 457 profile used in the API approach.

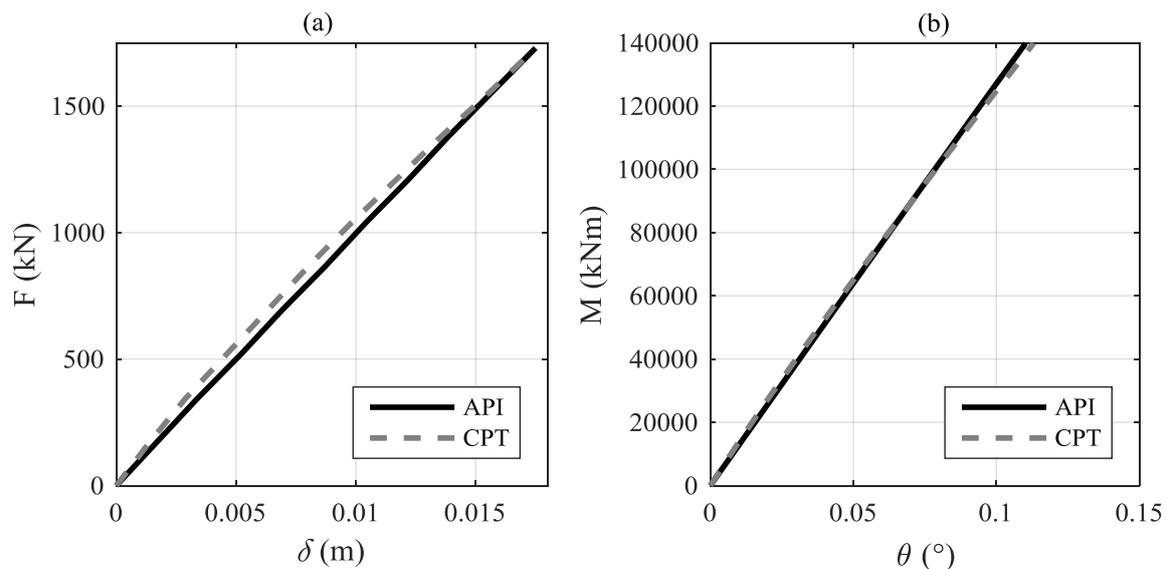
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459

460 Fig. 8 Design profiles for SLS check, (a) Average and design CPT q_c profile, (b) Derived and design
 461 ϕ' profile

462 The analysis is conducted using a finite-difference program that solves for the pile head lateral
 463 displacements and rotations under combined lateral and moment loading. The program operates by
 464 specifying an initial tangent stiffness for each p - y spring, solving for the displacement of the system
 465 under this operating stiffness and iteratively updating the spring stiffness of each spring according to
 466 the relationships specified in Eqs. (23) and (25). The analysis iterates until some predefined tolerance
 467 is achieved. The unfactored lateral load and moment at the mudline are shown in Table 2, as derived
 468 from the wind and wave loading calculations. These loads are factored by 1.5 and applied to the pile
 469 at mudline. The load-displacement and moment-rotation response curves from both p - y approaches
 470 are shown in Fig. 9. The results are broadly in agreement with the API approach predicting a lower
 471 lateral stiffness than the CPT-based approach in the initial stages. This finding is in agreement with
 472 Kallehave and Thilsted [69] who note that the API method can under predict stiffness for rigid piles,
 473 though for the present case this error is minor. As per the limits in Arany et al. [3], a threshold rotation
 474 of 0.25° or pile head displacement of 0.2m is permitted for fundamental SLS checks. Both
 475 displacement and rotation are well within the required bounds for both checks. Note, only
 476 fundamental SLS checks are considered, the plastic accumulation in rotation was not calculated.



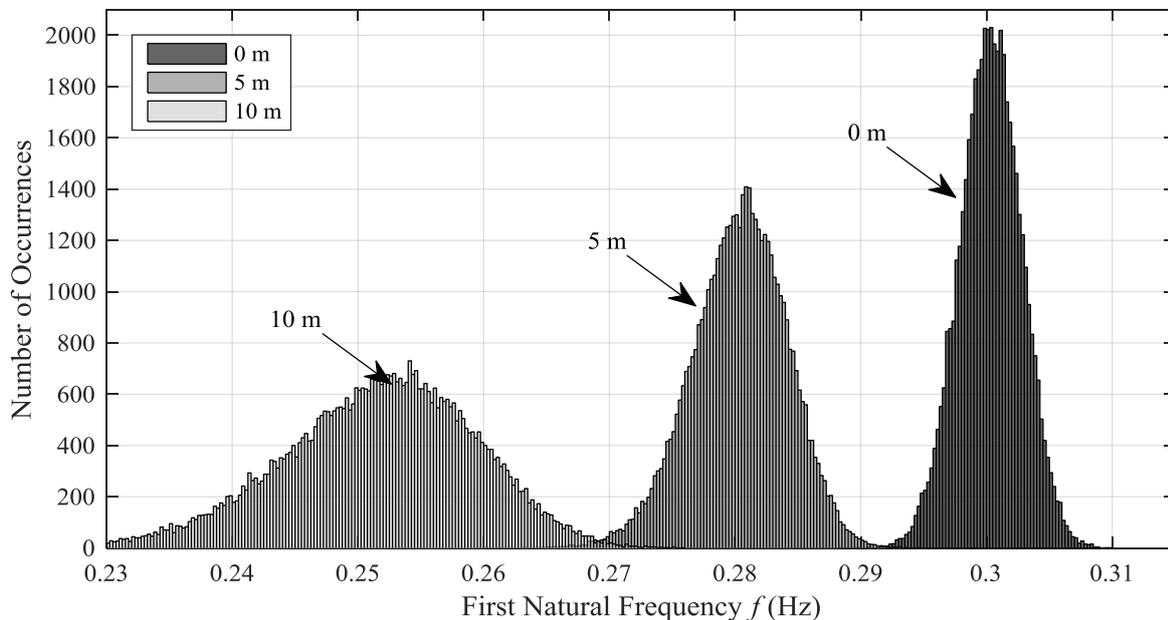
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478 Fig. 9 SLS Checks for API and CPT-based approach, (a) Load-Displacement, (b) Moment-Rotation
 479 responses

480 5.0 Analysis & Results

481 In this paper, the statistical variation in potential frequency changes due to scour incorporating spatial
 482 variability in soil strength is investigated. From the twenty CPT profiles measured at Rotterdam
 483 Harbour, a total of 50,000 hypothetical profiles are generated based on the process outlined in section
 484 3.0. A Monte-Carlo simulation is carried out whereby each hypothetical CPT profile is converted to a
 485 profile of spring coefficients using the procedure discussed previously and then assembled into the

486 global matrices of the turbine structural model using the procedure in section 4.1. The analysis outputs
 487 likely frequency values (first and second in-plane natural system frequency) for each hypothesised
 488 ground profile. The design scour depth for an offshore monopile as recommended by DNV [70] is 1.3
 489 pile diameters (1.3D), though this is based on current-only flow conditions. Physically there is little
 490 merit to this limit as in marine environments, the combined action of currents, tides and waves can
 491 give rise to significantly more complex interactive behaviour [14], where scour temporal variation
 492 could exceed this threshold. In this paper, scour depths ranging from 0m to 10m (1.66D) in discrete
 493 depths of 0.5m are implemented in the model by iteratively removing springs and the likely output
 494 frequencies due to each profile is calculated at each scour depth. Fig. 10 shows a histogram of the
 495 resulting output first natural system frequency values obtained at zero scour, 5m scour and 10m scour
 496 depths.

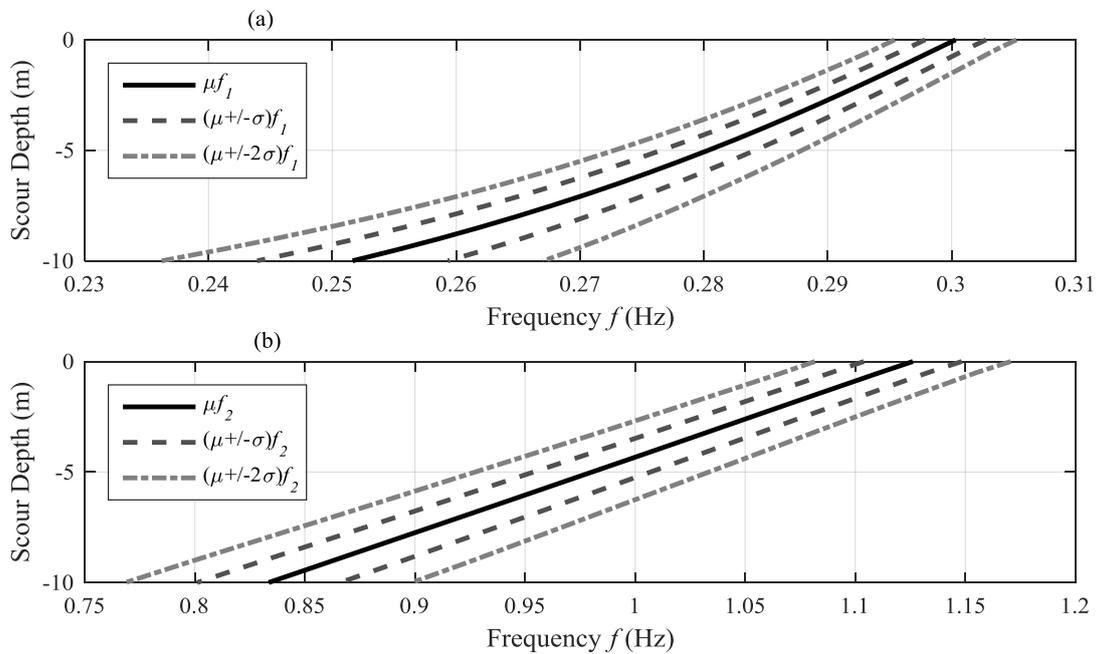


497

498 Fig. 10 Number of occurrence of first natural frequency for 0, 5 and 10 m scour depths

499 It is evident from the results in Fig.10 that the first natural system frequency reduces as the scour
 500 depth increases. A striking feature of the data is that the range (spread) of predicted system
 501 frequencies also increases as the scour progresses. This is a result of the increased flexibility of the
 502 overall system as scour progresses causing a larger relative change in frequency for a given range of
 503 hypothesised ground profiles. This trend is readily observed in Fig. 11(a), which shows the change in
 504 mean first natural frequency plotted against the depth of scour. The mean frequency is obtained at each
 505 scour depth from the distribution of outputs. Also shown in Fig. 11(a) are the envelopes of the change
 506 in frequency with scour at one and two standard deviations away from the mean at each scour depth.
 507 It may be observed that the standard deviation moves further away from the mean profile with
 508 increased scour depth, which mimics the response observed in Fig. 10. Fig.11(b) shows the same

509 information but for the second in-plane bending frequency. The change in this frequency is more
 510 linear with depth, and the standard deviation still moves away from the mean with increased scour,
 511 though this is less obvious in this case. This indicates that the deeper the scour depth, the less certain
 512 one can be as to the actual depth of scour affecting the system, based solely on frequency
 513 measurements. However, the deeper the scour depth, the more certain one can be that some degree of
 514 scour is affecting the system. For example, if a frequency of 0.28 Hz is measured, this indicates a
 515 scour depth of just over 5m based solely on the mean. However if one considers two standard
 516 deviations either side of the mean frequency, a frequency of 0.28 Hz could indicate a potential scour
 517 depth of anywhere between 3.5m and 7m. For deeper scour, the potential variation is larger. The
 518 likelihood of a given scour depth existing under a detected frequency can be more coherently
 519 visualised by examining the cumulative distribution of the results, see Fig. 12.

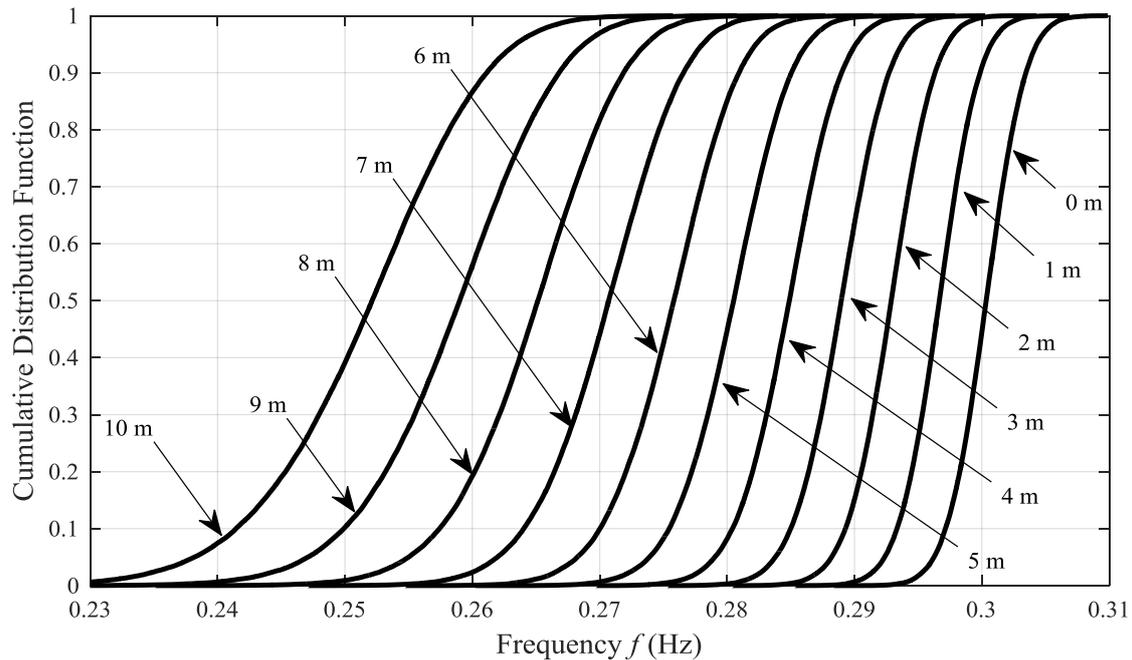


520

521 Fig. 11 Mean and standard deviations of frequencies vs. scour depth (a) first frequency, (b) second in-
 522 plane frequency

523 Fig. 12 shows the Cumulative Distribution Function (CDF) of the first natural frequency results for
 524 scour depths of 0 to 10m in 1m discrete depths (for clarity, the results at each 0.5 m depth increment
 525 are omitted from the figure). The results indicate the probability of scour being a certain depth or less
 526 for a given frequency measurement. For example, if a frequency of 0.28 Hz is measured, this indicates
 527 an almost 0% probability that the depth of scour is 3m or less, an 8% probability of 4m scour or less, a
 528 44% probability of 5m scour or less, an 85% probability of 6m scour or less and almost 97%
 529 probability of 7m scour or less.

530



531

532

Fig. 12 Cumulative Distribution Function (CDF) of frequency with scour

533

534

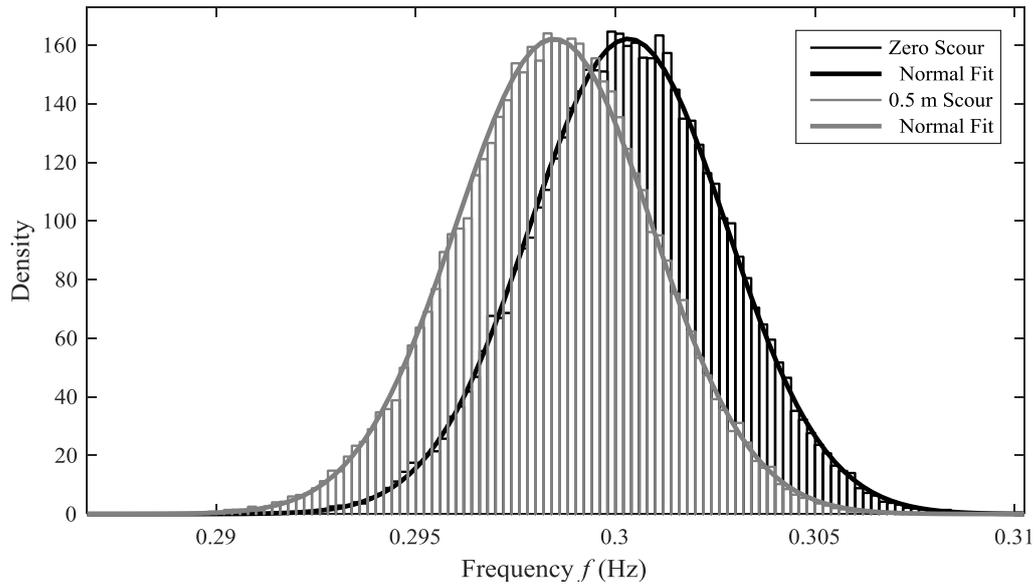
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537

As every scour depth has an associated frequency distribution and the initial shift in mean frequency when scour begins is not substantial, there is a considerable overlap between the “no scour” distribution and the “0.5 m scour” distribution, see Fig. 13. Therefore statistical tests were carried out to ensure that (i) the sampled distributions were not part of the same overall population and (ii) the change in frequency due to the scour was sufficient to ascertain the presence of scour.

538

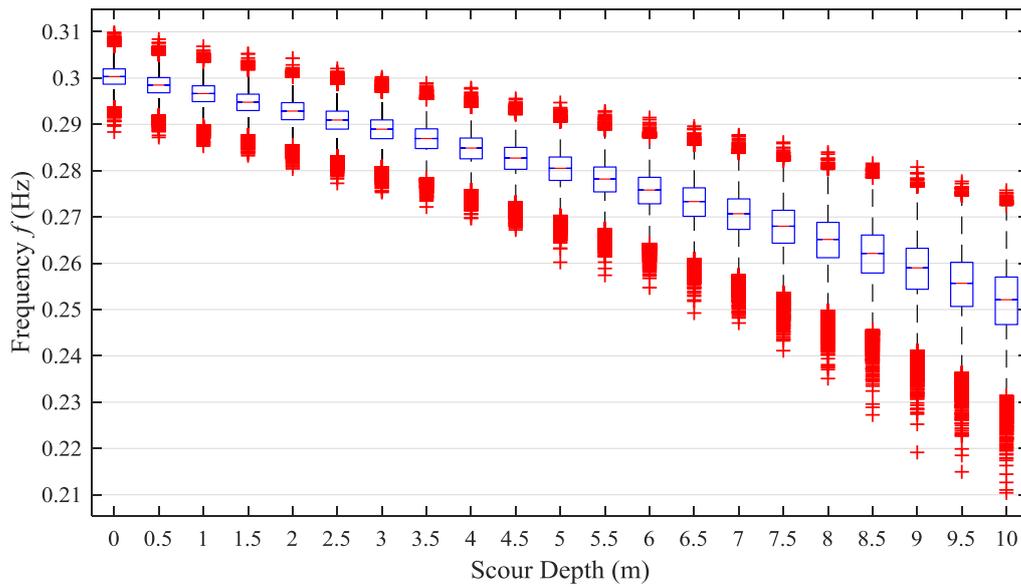


539

540 Fig. 13 Distribution of frequencies for zero scour and 0.5 scour affecting the structure

541 To determine if calculated sampled natural frequency distributions could come from the same
 542 population the Kruskal-Wallis test was used. This test is a non-parametric version of the classical one-
 543 way analysis of variance (ANOVA) approach, and is an extended form of the Mann-Whitney U-test
 544 allowing more than two groups to be tested at any one time. The test orders all the data from low to
 545 high and then utilises this data rank instead of numeric values to compute test statistics. A chi-square
 546 statistic is used along with a probability value (p) which measures its significance. A 5% significance
 547 level was adopted. The test determined that the difference between the median values for every scour
 548 interval (0.5m) was statistically significant and hence that no two distributions could be considered a
 549 subset of one another.

550 Fig. 14 shows the natural frequency plotted against scour depth with median values, interquartile
 551 ranges and outliers. In Fig.14, the red line in the middle of each blue box signifies the median value at
 552 a given scour depth, while the blue box represents the interquartile range. The whisker length
 553 represented by the dotted black line is set as 1.5 times the interquartile range and all outliers are
 554 shown as red crosses at each scour depth.



555

556

Fig.14 Box plot showing the difference in scour distribution with depth

557

558

559

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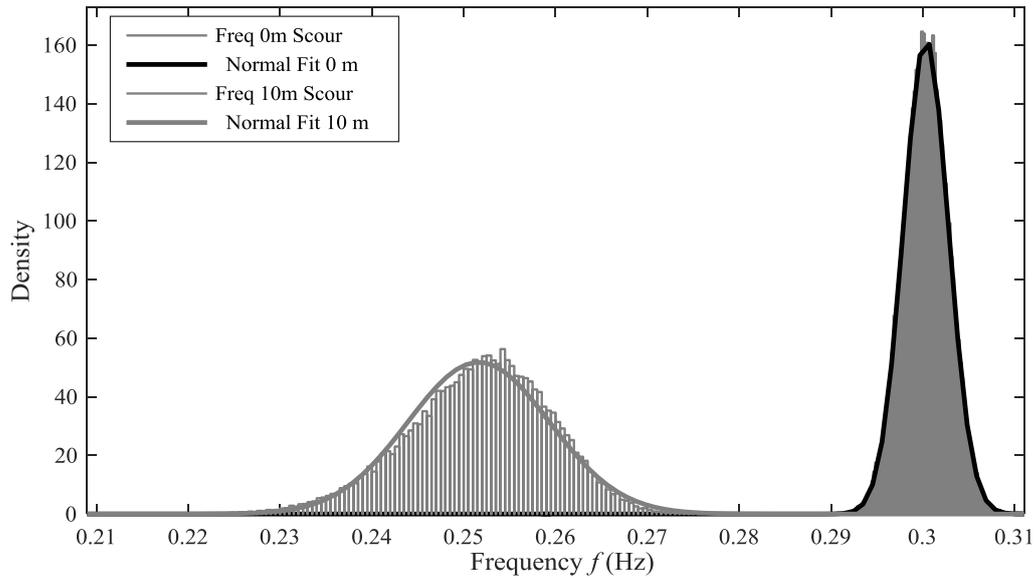
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563

It should be noted that while the computed frequencies were represented quite well by a normal fit for low scour depths, the distribution became less normal with increased scour, see Fig. 15. Fig. 15 shows the fitted normal distribution to the frequency data at 0m scour and 10m scour. The normal fit fits well for the zero scour case but the data is somewhat skewed for the 10m scour case. Strictly speaking the Kruskal-Wallis test assumes all parameters follow the same distribution. However, given that the deviation is small and occurs at a depth where scour is easily detectable through frequency change this can be considered acceptable.

564



565

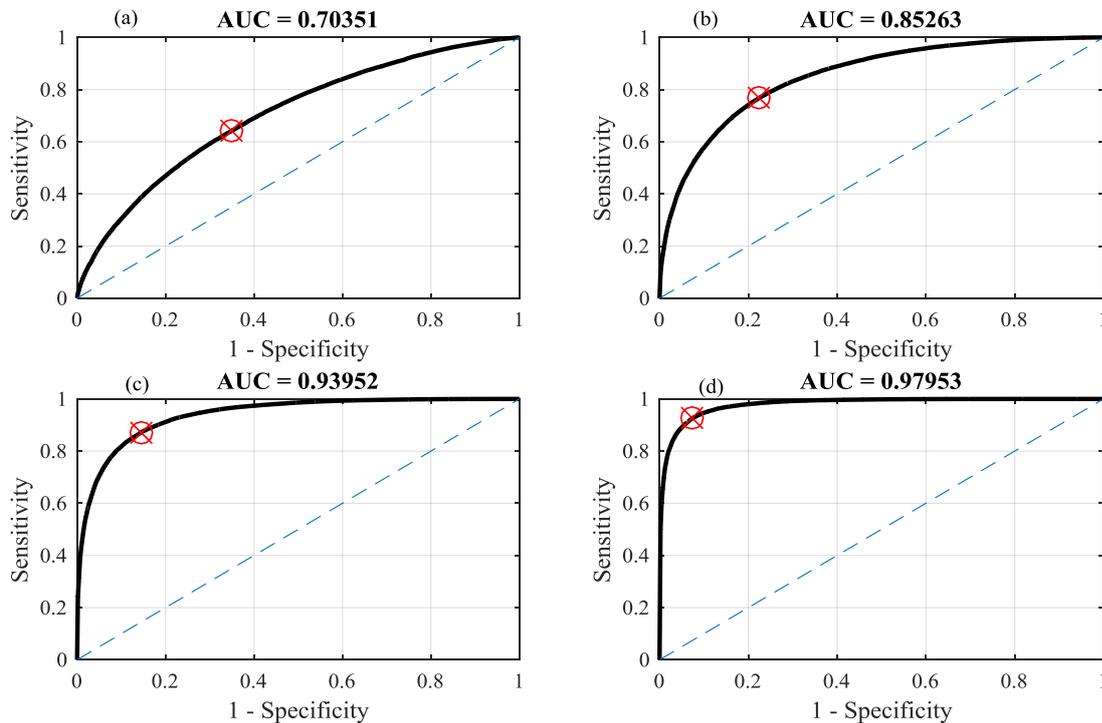
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Fig.15 Frequency profiles become less normal with depth

567 The second test performed was the Receiver Operating Characteristic (ROC) Curve analysis, which is
 568 a test used to illustrate the ability of a system to classify itself between two outcomes as its
 569 discrimination threshold is varied. In the present case the ROC curve represents the diagnosis between
 570 scour and no scour for the overlapping frequency spectrum both outcomes could have. i.e. for any
 571 value where the distributions overlap there is a chance that scour is occurring and is classified as
 572 occurring (true positive), while there is also a chance that scour is occurring and is not detected (false
 573 negative). Similarly there is also a chance that no scour has occurred but is classified as having
 574 occurred (false positive) and finally a chance that no scour has occurred and is predicted as such (true
 575 negative). A ROC curve is therefore the sensitivity of the system expressed in terms of the probability
 576 of false alarm and thus represents the trade-off between a type 1 and type 2 error. Sensitivity is the
 577 probability that the test will indicate scour when it is present and specificity is the probability that the
 578 test will indicate that scour is not present when there is no scour.

579 A ROC curve consists of the true positive rate (Sensitivity) plotted against the false positive rate (1-
 580 Specificity) at different parameter criterion values. Each point on the curve corresponds to a
 581 sensitivity/specificity pair related to a particular decision. The area underneath the curve (AUC)
 582 represents how well one can differentiate between the two distributions in question, the closer AUC is
 583 to 1 the clearer the distinction. If there was no distribution overlap it would be impossible to identify a
 584 value as being from a distribution it is not. In such a case the ROC curve would follow the Sensitivity
 585 axis until it has reached one and would follow the 1-Specificity axis until it too has reached one.
 586 Therefore, the closer the apex of the curve is to the upper left hand corner the easier it is to distinguish
 587 whether a value comes from one distribution or another. The ROC curve analysis as applied to the

588 present case is shown in Fig. 16. The “no scour” natural frequency distribution was compared using
 589 ROC curves to the following scour frequency distributions: (a) 0.5 m, (b) 1 m, (c) 1.5m and (d) 2 m.
 590 The resultant graphs shown in Fig. 16 show that for 0.5 m of scour, curve (a), there is a significant
 591 possibility of scour escaping detection (given the distance of the curve from the top and left-hand axis
 592 and its relatively poor sensitivity and specificity). However, for 1 m of scour, curve (b), this chance
 593 decreases substantially (denoted by the tendency of the apex to move closer to both axes as previously
 594 mentioned and the increase in Positive Predictive Value, PPV, Negative Predictive Value, NPV and
 595 AUC). For scour depths of 1.5 m, curve (c), and 2 m, curve (d), there is an excellent differentiation
 596 between the distributions for zero scour and the relevant scour depth and because of this delineation it
 597 is easy to distinguish whether or not scour exists for these depths and lower. Table 3 displays some
 598 values of interest resulting from this analysis. The PPV is the probability that there is a corresponding
 599 scour hole when scour is indicated, while the NPV is the probability that scour is *not* present when
 600 scour is *not* indicated. As evident, these probabilities increase with scour depth as the separation
 601 between the means of the no scour and scour distributions increases. When combined with the AUC
 602 these values allow us to gauge how reliably the model is predicting, while also informing how likely
 603 one is to predict a false positive (Type 1 error) or a false negative (Type II error).



604

605 Fig. 16 ROC curves examining the ability to distinguish between the no scour frequency distribution
 606 and the frequency distributions corresponding to scour of (a) 0.5 m, (b) 1 m, (c) 1.5 m, (d) 2m.

607

608

609

Table 3 ROC Curve parameters

Scour Depth (m)	AUC	Positive Predictive Value PPV	Negative Predictive Value NPV	Sensitivity	Specificity
0.5	0.704	64.779 %	64.563 %	0.643	0.650
1	0.853	77.421 %	76.996 %	0.768	0.776
1.5	0.940	85.750 %	87.006 %	0.872	0.855
2	0.980	92.639 %	92.579 %	0.926	0.926

610

611 6.0 Concluding remarks

612 In this paper, the effect of spatial variability from CPT profiles and geotechnical uncertainty is
 613 investigated in terms of how it may affect the perceived natural frequencies of a wind turbine system
 614 in the context of using frequency changes due to scour in a SHM framework. Twenty Cone
 615 Penetration Test (CPT) tip resistance q_c profiles, measured at Rotterdam Harbour, were used to
 616 develop a stochastic ground model with spatially correlated strength properties. 50,000 hypothetical
 617 CPT profiles were generated representing likely profiles within the stochastic model and converted
 618 into operational soil-structure interaction stiffnesses for input into a Finite-Element model of a wind
 619 turbine.

620 A numerical model of a wind turbine founded on a monopile embedded in the soil was developed and
 621 eigen-analyses were conducted to calculate the first and second in-plane system natural frequencies of
 622 the turbine under progressive scour. The purpose of the model is to investigate the potential likely
 623 frequency variation that could exist due to likely operating soil stiffness profiles and to observe if the
 624 potential variation in frequency due to scour is larger than these potential variabilities. The question of
 625 how reliably scour can be detected and measured using a SHM regime is investigated.

626 The results indicate that significant variation in frequency at a given scour depth occurs using the
 627 stochastic ground model and as a result, a given frequency measurement gives rise to a relatively large
 628 band of potential scour depths. Moreover, there is increased variation in frequencies at a given scour
 629 level with increased scour progression. This is as a result of the increased bending flexibility of the
 630 system during scour and means it is more difficult to accurately detect the depth of scour as the scour
 631 depth increased. However, the large change in average frequency that occurs as scour progresses
 632 means that it becomes more certain that scour *exists*, even if the actual scour magnitude is more
 633 difficult to quantify. Due to the overlap present in the distributions of output frequencies, the use of
 634 ROC curve analysis to estimate the likelihood of detecting a false positive was investigated. The
 635 results indicate that for a low scour depth of 0.5 m there is significant likelihood of scour not being

636 detected using frequency measurements. However, for deeper scour depths, the existence of scour is
637 much more likely to be observed, even for relatively benign scour depths of 0.25D, in this case.

638 The analysis presented in this paper assumes that the only influence on the frequency of the turbine is
639 the scour affecting the system. It is recognised that other factors will also influence the dynamics of
640 the system such as cyclic loading, large strain soil deformation and soil stiffness degradation, tidal
641 influence on water added mass, corrosion and other damage. The effect of measurement noise may
642 also be a factor. For simplicity these are not considered in the present study and only the influence of
643 scour erosion on the system frequencies is studied. The results in this paper are interesting in the
644 context of the continued development of the offshore wind sector and remote scour monitoring fields
645 and may be important with the development of larger systems in more uncertain design conditions.
646 Future research will focus on the effect of scour on soil damping for offshore wind.

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