Stochastic Petri net-based modelling of the durability of renderings

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Abstract:

In this study, a methodology to model and predict the life-cycle performance of building façades based on Stochastic Petri Nets is proposed. The proposed model evaluates the performance of rendered façades over time, evaluating the uncertainty of the future performance of these coatings. The performance of rendered façades is evaluated based on a discrete qualitative scale composed of five condition levels, established according to the physical and visual degradation of these elements. In this study, the deterioration is modelled considering that the transition times between these condition states can be modelled as a random variable with different distributions. For that purpose, a Stochastic Petri Nets model is used, as a formal framework to describe this problem. The model's validation is based on probabilistic indicators of performance, computed using Monte-Carlo simulation and the probability distribution parameters leading to better fit are defined as those maximizing the likelihood, computed using Genetic Algorithm. In this study, a sample of 99 rendered façades, located in Portugal, is analysed, and the degradation condition of each case study is evaluated through *in-situ* visual inspections. The model proposed allows evaluating: i) the transition rate between degradation conditions; ii) the probability of belonging to a given degradation condition over time; and iii) the mean time of permanence in each degradation condition. The use of Petri Nets shows to be more accurate than a more traditional approach based on Markov Chains, but

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also allows developing future research to consider different environmental conditions, maintenance actions or inspections, amongst other aspects of life-cycle analysis of existing assets.

Keywords: Petri nets; rendered façades; genetic algorithms; degradation.

1. Introduction

According to Jensen and Rozenberg (2012), the net theory can be seen as “a system theory that aims at understanding systems whose structure and behaviour are determined by a combinatorial nature of their states and changes”. The first proposal of nets of places and transitions, proposed by C. A. Petri (Petri, 1962), allows developing a non-idealizing methodology to concurrency and information flow, in organizational systems (Genrich and Lautenbach, 1981). Petri nets are considered a mathematical and graphical tool for the formal description of systems whose dynamics are characterized as being concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic, mutual exclusive, and conflicting, which are typical features of distributed environments (Murata, 1989). Therefore, Petri nets allow capturing the static and the dynamic nature of a real system, thus characterizing the rate of transition between states or conditions (Marsan et al., 1994).

Due to their characteristics, Petri nets have been successfully applied in different fields of knowledge, namely in robotics (Al-Ahmari, 2016), in the optimization of manufacturing systems (Chen et al., 2014; Uzam et al., 2015), business process management (van der Aalst, 2002), human computer interaction (Tang et al., 2008), among others. Petri nets are not widely used in the construction industry, and particularly in building asset modelling. Nevertheless, there are various works (Li, 1998; Cheng et al., 2011; Molinero and Núñez, 2011; Cheng et al., 2013; Rinke et al., 2017) that use Petri nets to manage resources, to estimate equipment availability and scheduling of tasks on the site-work during the building design process. On the other hand, recent work has been published on the use of Petri Nets to model the deterioration of other civil engineering infrastructures (Andrews, 2013; Rama and Andrews, 2013; Le and Andrews, 2015; Le and Andrews, 2016; Leigh and Dunnett, 2016; Yianni et al., 2017; Zhang et al., 2017). In the last decades, various authors proposed several extensions and adaptations of ordinary Petri nets; all of them based on the basic Petri net formalism, but presenting very different characteristics and assumptions, in order to adapt themselves to the phenomena under analysis. Consequently, there is a reasonable expertise in the application of Petri nets to different application domains, thus allowing transferring knowledge and methodologies from one field to another (Girault and Valk, 2002).

This study intends to evaluate the suitability and advantages of the use of Stochastic Petri Nets (SPN) as deterioration models in building asset management. The main advantages of SPN are their graphical representation, allowing a better and more intuitive understanding of the modelling principles, and their versatility, allowing the modelling complex stochastic processes. In the particular case of deterioration modelling, and compared to the more traditional Markov Chains, SPN allow the seamless use of different probabilistic distributions. Furthermore, their versatility allow modelling, in a common framework, multiple aspects of asset management, including deterioration, maintenance, inspection, and decision-making. In this study, a model to predict the life-cycle performance of building façades based on stochastic Petri nets is proposed. To analyse the degradation condition of rendered façades over time, a set of Petri net models considering different probabilistic distributions are used to estimate the transitions times between condition levels. Since there are no closed form expressions for the probability distribution of the condition state at a certain time, Monte Carlo simulation is used to compute the likelihood of each model. However, the errors introduced by Monte Carlo simulation require the use of gradient-independent optimization methods, like Genetic Algorithms, to identify the optimal parameters of the probability distributions.

The sample analysed in this study comprises 99 renderings, located in Portugal, for which degradation condition was evaluated through *in situ* visual inspections. The classification system adopted in this study to evaluate the deterioration state of rendered façades is a discrete qualitative scale divided in five condition levels, proposed by Gaspar and de Brito (2008, 2011), ranging between “no visible degradation” (condition A) and “generalised degradation” (condition E), which requires an immediate rehabilitation or maintenance action.

In the first part of this study, a traditional method, based on Markov chains is applied, in order to define a benchmark model. The benchmark model and the Petri net model with transition times exponentially distributed are used to validate the methodology proposed. The comparison of the models is possible since the stochastic Petri net with transitions exponentially distributed is equivalent to a finite Markov chain. After that, a set of probabilistic distributions are used to analyse the degradation condition of rendered façades over time. The information obtained from the Petri net models allows the identification of the degradation rate of rendered façades, characterizing the pattern that characterizes the loss of performance of these claddings over time. This information is crucial to identify the future need for interventions, optimizing the maintenance needs, and thus avoiding unnecessary cost associated with urgent interventions.

The outline of this paper is as follows: Section 2 provides a literature review concerning the classification system and modelling techniques used to model the evolution of the degradation in rendered façades; Section 3 introduces the concept of Petri nets, as well as the procedure used to predict the life-cycle performance of renderings. Finally, the discussion of the results is presented in Section 4 and conclusions are drawn in Section 5.

2. Literature review

The façades can be seen as the skin of the building, i.e. they can be considered the first layer of protection against the deterioration agents (Silva et al., 2015), thus being the element more prone to degradation. According to Flores-Colen and de Brito (2010) the claddings' degradation level can influence the quality of the urban environment, since it affects the architectural appearance of buildings, which has a considerable effect on the physical comfort of inhabitants of larger cities (Korjenic et al., 2016). Rendered façades are the most common type of cladding in Portugal (Census, 2001). In the present context of societies aiming at achieving a more sustainable use of resources, it is increasingly important to define rational maintenance strategies so as to avoid unnecessary costs (Wang and Xie, 2002; Arain and Pheng, 2006; Wong and Li, 2009). For that purpose, it is essential to develop new and versatile tools to support the decision-making process regarding the instant in which maintenance actions must be performed, knowing the degree of uncertainty associated with the estimates (Frangopol, 2011). To achieve this, the present work focuses on the use of probabilistic based methods for modelling performance, including Stochastic Petri Nets and Markov Chains.

The definition of maintenance strategies is, in general, related with the users' demands, i.e. more demanding users may demand a high level of performance, requiring that the cladding be replaced as soon as it starts to deteriorate; on the other hand, some users may accept a lower level of performance, thus minimizing the maintenance costs (Shohet et al., 1999). Consequently, the definition of maintenance strategies requires the condition assessment of rendered façades and the knowledge of their expected service life. According to Hertlein (1999), condition-based maintenance by inspection planning can be a useful tool to reduce the life cycle costs, achieving a more rational and efficient way to manage maintenance budgets (Flores et al., 2011).

In the last decades, different studies (Shohet et al., 2002; Shohet and Paciuk, 2004; Gaspar and de Brito, 2008; Paulo et al., 2014; Paulo et al., 2016) propose visual and physical scales to characterize the type, extension and severity of defects observed in rendered façades. Gaspar and de Brito (2008) and Silva et al. (2014) proposed a discrete scale to evaluate the degradation condition of rendered façades (Table 1).

This qualitative scale, based on the evaluation of the physical and visual degradation of rendered façades analysed during a comprehensive fieldwork, can be associated with a quantitative index that portrays the global performance of the façades. This numerical index, initially proposed by Gaspar and de Brito (2008, 2011), expresses the global degradation of façade coatings through the ratio between the degraded area weighted as a function of its condition and a reference area, equivalent to the whole and having the maximum degradation level possible - equation (1).

$$S_w = \frac{\Sigma\left(A_n \times k_n \times k_{a,n}\right)}{A \times k} \quad \textbf{(1)}$$

Where S_w is the degradation severity of the coating, expressed as a percentage; k_n is the multiplying factor of anomaly n, as a function of their degradation level, within the range $K = \{0, 1, 2, 3, 4\}$; $k_{a,n}$ is a weighting factor corresponding to the relative weight of the anomaly detected ($k_{a,n}$ Є R+); $k_{a,n} = 1$ by default; A_n is the area of coating affected by an anomaly n; A is the façade area; and k is the multiplying factor corresponding to the highest degradation level of a coating of area A.

In this study, the anomalies that occur in rendered façades are grouped in three categories: stains; cracking; and detachment. The coefficient $k_{a,n}$ allows establishing a relative weight between these groups of anomalies, based on the cost of repair of each anomaly, its aesthetic impact, the influence on the renderings' service life, the fulfilment of performance requirements (e.g. watertightness) and its propensity to generate new anomalies. In this study, the following $k_{a,n}$ values are adopted for the different groups of anomalies: 1.0 for cracking; 1.5 for detachment; and 0.25 for stains in condition B and 0.67 for stains in more serious condition levels (C, D and E).

Figure 1 shows the correlation between the condition of some façades inspected and the numerical index, illustrating the visual conditions of rendering in each degradation condition.

2.1 Application of Markov chains to model the evolution of the degradation of rendered façades

Markov chains are widely used by researchers in several fields of civil engineering (Wang and Zaniewski, 1996; Hawk and Small, 1998; Thompson et al., 1998; McDulling, 2006; Ortiz-García et al., 2006). Particularly, continuous-time Markov chains are commonly used in modelling the deterioration of civil engineering assets (Kallen and van Noortwijk, 2006). This modelling technique is considered an intuitive, simple and computationally inexpensive stochastic process, since analytical solutions exist and the memoryless property allows estimating the future performance only based on the current performance, becoming particularly relevant when limited information is available.

Silva et al. (2015) used continuous-time Markov chains to evaluate the degradation process of external render over time, based on the visual inspections of characteristics and condition of façades located in Portugal. In this work, it is assumed that the progression of damage is continuous and, over a small time interval, the condition of the façade can only remain constant or deteriorate to the next condition state. The intensity matrix defines the rate of transitions between states (Kalbfleisch and Lawless, 1985) as:

$$\mathbf{Q} = \begin{bmatrix} -q_{1,2} & q_{1,2} & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & -q_{n-1,n} & q_{n-1,n} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad \textbf{(2)}$$

The estimation of the optimal intensity matrix, leading to the best fit between the model and the observed condition, was based on the concept of maximum likelihood described by Kalbfleisch and Lawless (1985). Likelihood is defined as the predicted probability of occurrence of the observed transitions:

$$L(\mathbf{Q}) = \prod_{i=1}^{m} \prod_{j=1}^{k} p_{ij} \quad \textbf{(3)}$$

Where i is the condition level in the initial instant, j is the condition level in the final instant, m is the number of elements, k is the number of intervals between inspections, and p_{ij} is the probability of transition from condition level i to condition level j, (i, j) entry of the transition probability matrix, **P**, given by:

$$\mathbf{P}(\Delta t) = e^{\mathbf{Q} \cdot \Delta t} \quad \textbf{(4)}$$

Where Δt is the time interval between inspections.

The optimization of the intensity matrix, **Q**, was performed using the active set algorithm implemented in MATLAB®. The aim of the optimization algorithm is to find the parameters of the intensity matrix, **Q**, which maximize the fitness function, $\log V = \max\left(\sum \sum \log p_{ij}\right)$, while keeping all terms of matrix **Q** positive, $q_{i-1,i} > 0$ where $i \in \{2, \dots, n\}$. This optimization algorithm is a reasonable tool for problems that use analytical expressions. In situations where analytical expressions are not available, the numerical estimation of the functions can lead to convergence problems and lack of robustness of the solution.

3. Petri nets

3.1 Conventional Petri nets

The concept of Petri nets was originally present by Carl A. Petri, who in his doctoral thesis developed a new model of information flow and control in systems (Petri, 1962). Petri nets are a graphical and mathematical modelling tool, suitable for characterizing concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic systems (Murata, 1989).

An ordinary Petri net is considered a directed, weighted, bipartite graph with an initial state called the initial marking, M_0 (Murata, 1989; Schneeweiss, 2004). It is called a bipartite graph because nodes are divided into two different types: places, usually represented by circles, and transitions, usually represented by rectangles. Both nodes are linked by directed arcs, from places to transitions (input arcs) and from transitions to places (output arcs) (Peterson, 1977; Murata, 1989; Schneeweiss, 2004). The third element of a Petri nets are tokens, usually represented by black dots, which represent the elements in the system (Peterson, 1977; Murata, 1989). Figure 2 shows a simple Petri net model. Transition T_1 has two input places (P_1 and P_2) and one output place (P_3). The arcs that connect the input places to the transition and the transition to the output place represent the pre- and post-conditions of the transition, respectively. When all input places are occupied by a token the transition is said to be enabled. At this point, the transition fires, the tokens are removed from the input places, and new tokens are created in the output places. In this example, transition T_1 is not enabled because the pre-conditions of the transition are not complied with, i.e. there is a token in place P_1, but no token at place P_2. Once tokens exist in both P_1 and P_2, transition T_1 will fire, tokens from places P_1 and P_2 will be removed and a token will be placed in place P_3.

In the context of this study, places represent resources or conditions while transitions can represent actions or events that cause the system to change (Murata, 1989). Tokens are stored in places representing the present state of the system and transitions allow the tokens to move between two places modelling, in this way, the dynamic behaviour of the system.

When analysing a PN, conflicts might occur when two or more transitions are enabled from a common place and the firing of one transition disables the other transitions (Bowden, 2000). In the literature, there are several proposals for solving conflicts, either deterministically, for example through the introduction of a priority transition by the user, or probabilistically, by assigning probabilistic properties to the conflicting transitions (David and Alla, 2010; Wang, 2012). However, in timed Petri nets, the most common way to solve conflicts is through firing times, assuming that the transition with the shortest delay will fire first (Murata, 1989).

3.2 Stochastic Petri net

In the original definition of Petri nets, the concept of time is not explicitly included (Murata, 1989). However, many applications are time dependent and the introduction of time delays has to be considered. The notion of time in Petri nets was initially introduced by Ramamoorthy and Ho (1980) and Zuberek (1980). In these two works, deterministic time intervals are used for each transition, creating a delay between the instant the transition becomes enabled and the firing instant. Molly (1982) introduced the concept of stochastic Petri net by assigning an exponentially distributed firing rate to each transition for continuous time systems. After that, several classes of stochastic Petri nets have been proposed for performance and reliability analysis of systems, the more relevant of which are: the generalized stochastic Petri net (Marsan et al., 1984), the extended stochastic Petri net (Dugan et al., 1984), and the deterministic and stochastic Petri net (Marsan and Chiola, 1987).

The model employed in this work considers Petri nets with transitions times defined as a random variable, as proposed by Molly (1982). However, the results obtained showed that the exponential distribution for the firing times, proposed by Molly (1982), were not always adequate. To overcome this limitation, the proposal of Dugan et al. (1984), allowing any probability distribution to be used to model the firing rate was used.

Mathematically, the theory behind the stochastic Petri net is the same as the Petri net; their mode of operation is identical, applying the same firing rules. The only difference is the random time interval between the transition becoming enabled and firing.

3.3 Deterioration Petri net model

Deterioration can be modelled with Petri nets by considering that each place is a condition state of the classification system adapted, tokens indicate the current condition of an element, and timed transitions define the movement between condition states (Le, 2014, Yianni et al., 2016, 2017). In this work, a five condition levels Petri net scheme is defined. Since maintenance actions are not considered, the condition level of the infrastructure deteriorates continuously over time until it reaches the worst condition level defined in the performance scale.

The time dependent nature of the problem is included by defining timed transitions. The time specified by each transition represents the sojourn time in the condition level, i.e., the time that infrastructure spends in condition level i before moving to condition level $i+1$. The timed transitions are modeled by probability distributions.

3.4 Parameter estimation

The probability distribution that best describes the deterioration process of an infrastructure is that resulting in higher probabilities of occurrence of observed transitions.

In order to identify the parameters of the probability distribution that provide a best fit, parameter estimation is required. The parameters of the probability distribution are fitted to historical data through the maximum likelihood method proposed by Kalbfleisch and Lawless (1985) and shown in equation (3). To simplify the computations and improve robustness, the logarithm of the likelihood is maximized.

3.4.1 Monte Carlo simulation

The probability of occurrence of the observed transition, p_{ij}, is estimated by Monte Carlo simulation. This is a helpful approach to compute numerical approximations in situations where it is not feasible to obtain analytical solutions and can be used to consider the propagation of uncertainties during the lifetime of the infrastructure. This method allows generating random sojourn times to each condition level from the inverse CDF (cumulative distribution function) of probability distribution.

The proposed procedure for computing the probability of occurrence of the observed transition, p_{ij}, is illustrated in Figure 4. The procedure depicted is repeated for each transition observed in the historical database. The input for the algorithm includes the information about each observed transition: time interval between observations, Δt, condition level in the initial instant, i, and condition level in the final instant, j. The condition level in the initial instant, i, is used to define the initial marking, M_0, of the Petri net, the time interval between observations, Δt, is the time horizon of the analysis, and the condition level in the final instant, j, is used to compute the probability of occurrence at the end of the procedure. The first transition to fire is identified by checking which transitions are enabled. When more than one transition is enabled, the transition with less time delay is the first to fire. However, since the Petri net defined for the deterioration model is arranged in sequential manner and there is only one token in the Petri net, i.e.

conflicts do not need to be considered. In the next analysis step, the sojourn time in the initial condition level is computed, and the Petri net and time are updated. The process is repeated until Δt is reached. The output of the procedure is the condition index at the time horizon for each sample. Using Monte-Carlo simulation the distribution of the final condition can be computed and the probability of the observed transition occurring can be calculated.

3.4.2 Optimization

The optimization of the parameters of the probability distributions is performed using Genetic Algorithms (GA), which were selected for being widely available, robust and efficient for objective functions computed using Monte-Carlo simulation. In fact, by using only information on the objective function, not requiring the computation of gradients, GA avoid the potential consequences of numerical errors, significantly simplifying the problem (Man et al, 1999; Morcous and Lounis, 2005).

The GA used for optimization of the parameters of the probability distributions is simply depicted in Figure 5. The optimization procedure begins with the definition of optimization variables, objective function, and constraints. The objective function is used to measure the degree of “goodness” of each individual of the population (Man et al, 1999; Morcous and Lounis, 2005). All parameters of the probability distributions are defined as problem parameters, and the Monte-Carlo procedure described above is used to compute the objective function.

In the following step, the initial population is randomly generated. A population is composed by a set of individuals, where each individual is a potential solution of the problem. All individuals of the initial population are evaluated through the objective function, where the best individual is the one with the highest value of the likelihood. At each step of the optimization process, the GA uses the best individual of the current population to create the offspring generation (MatLab, 2016), using the crossover and mutation procedures. The new population generated is then evaluated using the objective function and used as a new parent population. This process is repeated iteratively until a predefined stopping criteria is satisfied.

In this study, the optimization of the parameters of the probability distributions was performed using the GA available in MATLAB® (MatLab, 2016). The parameters used in the GA are the following:

- Size of the population: 50 individuals when the number of optimization variables is less than or equal to 5; and 200 individuals otherwise;
- Stopping criteria: the algorithm stops if the average relative change in the best fitness function value over 50 generations (minimum number of generations) is less than or equal to 10^{-6};
- Mutation procedure was performed using the Gaussian algorithm implemented in MATLAB®.

In the extension of Petri nets proposed by Molly (1982), the stochastic sojourn time is modelled as an exponentially distributed random variable. In this case, a stochastic Petri net is isomorphic to a finite Markov chain.

4. APPLICATION TO RENDERED FAÇADES

The deterioration Petri net model for façades is illustrated in Figure 3. It is composed of five places C_1 to C_5 each representing one of the five discrete states that characterize the degradation condition of external render façades defined in section 2. Level A means there is no visible degradation and Level E indicates the presence of extensive damage in the render façade. The transitions T_1 to T_4 represent the time interval required for the façades to progress to a more deteriorated state.

Since Markov chains are widely used to evaluate the condition level over time and taking into account the isomorphism between Markov chains and stochastic Petri nets, the Petri net model proposed can be validated by comparison with the Markov chains model proposed by Silva et al. (2015). In this manner, the efficiency of the numerical procedure and the optimization algorithm described in section 3.4 can be evaluated.

The data presented by Silva et al. (2015) is therefore used to calibrate both the Markov chain model and the Petri net models. The database is composed of 99 visual inspection records of external render façades located in Portugal. For each façade, only the initial condition level (assuming that at time zero the render is in Level A) and final condition, corresponding to the inspection date, are known.

4.1 Validation of the Petri net model

In Table 2 the optimal transition rates considering a Markov chain model, implemented using analytical expressions, and a Petri Net model with exponentially distributed sojourn times, are compared. The values of the parameters for each condition level are quite similar as expected. The differences obtained are due to sampling errors associated with the Monte-Carlo simulation used in conjunction with the Petri Net model (Figure 6).

Table 3 shows the number of observed and predicted façade in each condition level for both models. The results show that both models are suitable to model the deterioration process of the external façade renders. The biggest relative error is obtained for the Level D (15.3% for Markov chains and 16.5% for Petri nets).

Taking into account the results obtained by Petri nets, it is confirmed that the proposed model is suitable to evaluate the degradation of external façade renders.

4.2 Probabilistic analysis

4.2.1 Two-parameter distributions

When using Petri net models, in addition to the exponential distribution, four distributions were studied: Weibull, Lognormal, Gumbel, and Normal. Table 4 shows the optimal parameters obtained in all probability distribution analysed as the likelihood computed for each set of optimal parameters. All the studied distributions lead to a better likelihood than the exponential distribution.

Table 5 shows the number of observed and predicted façades in each condition level for each probability distribution and Table 6 shows the relative error obtained for each case. The values obtained for the relative error are low and, in all cases, acceptable; the largest errors occur for the exponential distribution. Amongst the alternative distributions, the largest error is associated with the Gumbel distribution in Level A (8.6%). The results in those two tables show that the exponential distribution is the one with the largest mean relative error for all states (7.0%), while the smallest mean relative error for all states is for the lognormal distribution (2.1%). The normal distribution presents a mean relative error for all states of 3.3% (second lower value).

Figure 7 presents the average predicted condition profile of the external render façade over time for each probability distribution analysed. The profiles obtained for the four distributions are showing some differences to the profile obtained for the exponential distribution.

The deterioration curve obtained by exponential distribution does not have inflection points (concave up). The other distribution curves have two inflection points (Figure 7a). In the transition between levels B and C there is an inflection point, where the concavity of the curve changes. The second inflection point occurs between levels C and D. In terms of dispersion of the results (Figure 7b), any of the distributions (Weibull, Lognormal, Gumbel, Normal) has lower dispersion values over the simulated period than the exponential distribution. In fact, the exponential distribution has a mean value equal to the standard deviation. There is no physical reason indicating this occurs for the sojourn times. As a result, the use of Markov chains has limited ability to model the variability of performance, frequently overestimating it.

These differences between the degradation curves also have high impact on the probabilistic distribution of the degradation condition level over time (Figure 8a-c). Despite the peaks occurring, approximately, in the same years, their values are quite different.

For level A, the predicted probabilities for all distributions are similar, beginning with probability equal to 1 and decreasing rapidly over time; at year 10 the probability of a render façade being in level A is near zero (Figure 8a). Also, for level B (Figure 8a), the predicted probabilities for all distributions are similar; the maximum probability of a façade belonging to level B occurs between years 3 and 4; after that, the value of the probability decreases rapidly. In level C significant differences can be observed between models (Figure 8b). The maximum probability of belonging to level C is close to 0.50 for the exponential distribution while for the other distributions it varies between 0.70 and 0.80. After the maximum probability is achieved, the slope of the exponential distribution is softer, when compared with the other distributions. For level D (Figure 8b), the exponential distribution has a smoother growth when compared to other distributions, then the peak occurs in all distributions between years 18 and 19 (the maximum probability of belonging to level D is close to 0.40 for the exponential distribution while for the other distributions it varies between 0.70 and 0.80). After that, the slope of the exponential distribution is softer. Finally, as expected, the probability of belonging to level E (Figure 8c) increases over time; however, the increase for the exponential distribution is softer than for the other distributions. At year 40, for the other distributions, the probability of a façade belonging to level E is bigger than 0.95 while, for the exponential distribution, it is closer of 0.80.

In the analysis of the service life and durability of rendered façades, it is assumed that level D corresponds to the end of the service life of rendered façades, beyond which a maintenance action must be performed. Figure 8b shows the probabilistic distribution of the degradation condition D over time. The results reveal that the exponential models and, consequently, the Markov chain models, are less accurate in predicting the behaviour of deteriorated serious conditions, due to the reduced number of samples available. According to the Markov chain model proposed by Silva et al. (2015), the probability of a rendering belonging to level D reaches a peak at 15 years. In this study, using a Petri net model, this peak is between 18 and 19 years. These values seem coherent with physical reality, in agreement with the results present in the literature: i) Shohet et al. (1999) obtained an expected service life for cementitious renders of 20 years; ii) Shohet and Paciuk (2004) estimated a predicted service life of 15 years for a stricter level of demand (with a range of results between 12 and 19 years), and a service life of 23 years (with a range of results between 19 and 27 years) for a lower level of demand; iii) Mayer and Bourke (2005) obtained an estimated service life of 18 years for current renderings; iv) Gaspar and de Brito (2008) estimated a service life of cement-rendered façades of 22 years; v) Silva et al. (2013), using an artificial neural network model, obtained an estimated service life of 22 years with a 16-28 years range, and using a multiple linear regression model, obtained an average estimated service life of 15 years with a range between 12 and 17 years; vi) a comparative analysis of service life prediction methods applied to rendered façades (Silva et al. 2016), led to an average value of the estimated service life of rendered façades ranging between 16 and 22 years.

4.2.2 Three-parameter distributions

The results of the previous section show that the probability distributions with two parameters show a better fit to the historical data when compared with the exponential distribution. In an attempt to examine whether a probability distribution with three parameters is an option to better model the degradation of façades over time, the Weibull 3-parameters distribution was used. The probability density function of this distribution is given by:

$$f(t|\alpha,\beta,\gamma) = \frac{\beta}{\alpha}\left(\frac{t-\gamma}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}} \quad (5)$$

where α, β has the same definition given in Table 4 and $\gamma \in \mathbb{R}$ is the location parameter of the distribution.

The optimal parameters obtained for Weibull 3-parameters distribution and the likelihood obtained for this set of parameters are shown in Table 7. Table 8 shows the number of observed and predicted façades in each condition level.

From the analysis of the results obtained in the two-parameters distribution (Tables 4 and 5) and the three-parameters distribution (Tables 7 and 8), it is found that the Weibull 3-parameters shows a better fit than the two-parameters distribution, both in terms of likelihood and mean relative error. However, the Weibull 3-parameters distribution increases the level of complexity of the analysis (the number of parameters to be optimized increase from 8 to 12).

5. CONCLUSIONS

In this paper, a model to asses and predict the life-cycle performance of building façades based on stochastic Petri nets is proposed. The application of Petri nets to degradation models is a recent research field, but this modelling technique has shown several advantages relative to the more traditional Markov chains. The graphical representation can be used to describe the problem in an intuitive way; PN are more flexible than the Markov chains, allowing the incorporation of a multitude of rules in the model to accurately simulate complex situations and keeping the model size within manageable limits. Moreover, with this modelling technique, transition times are not required to be exponential distributed.

The sojourn time is defined as a random variable for each condition level. The deterioration rates were estimated from available historical data, based on the analysis of the degradation condition of 99 rendered façades, located in Portugal. The Petri net model with transition times exponentially distributed was used to validate the methodology proposed by comparison with a benchmark model based on Markov chains. In order to investigate whether other probability distributions would fit the historical data better than the exponential distribution, five probability distributions were analysed using Petri net models (Weibull 2-parameters, Weibull 3-parameters, Lognormal, Gumbel, and Normal).

From the results of the probabilistic analysis performed with Petri nets model, it was found that the use of distributions with two parameters greatly improves the model's goodness of fit. The likelihood values of the four distributions (Weibull 2-parameters, Lognormal, Gumbel, and Normal) are quite similar and all significantly better than the exponential distribution. Some improvement is obtained when a Weibull 3-parameters distribution in considered, but this is obtained at the expense of a significant increase in the complexity of the model.

In this study, the degradation condition of rendered façades is described by five condition states, ranging between A (most favourable, without visible degradation) and E (most serious, with generalised degradation). For the analysis of the service life and durability of rendered façades, it is assumed that level D corresponds to the end of the service life of rendered façades, beyond which a maintenance action must be performed. Based on the Petri net model proposed, a rendered façade presents the higher probability of reaching the end of its service (corresponding to level D) between 18 and 19 years. The results obtained are consistent with physical reality and in agreement with the results present in the literature. This study evaluates the loss of performance of rendered façades over time, modelling the probability of transition between degradation conditions through Petri net models. This study demonstrates the validity of this approach to model the degradation of external claddings and, therefore, in future studies, the authors intend to apply a similar methodology to predict the service life of other cladding systems, encompassing the effects of their characteristics in their degradation process (e.g. environmental exposure conditions).

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FIGURE CAPTIONS

Figure 1 - Illustrative example of the degradation conditions of rendered façades (photographs by Gaspar, 2009)

Figure 2 - Example of a Petri net including three places, and one transition

Figure 3 - An example of the Petri net scheme of the deterioration model

Figure 4 - Procedure for computing the probability of occurrence of the observed transition

Figure 5 - Procedure for optimization of the parameters of the probability distributions (adapted from Morcous and Lounis, 2005)

Figure 6 - Comparison of the predicted future condition profile over time for both deterioration models: (a) average condition; (b) standard deviation of condition

Figure 7 - Comparison of the predicted future condition profile over time for all probability distribution analysed: (a) mean condition; (b) standard deviation of condition

Figure 8 - Probabilistic distribution of all degradation condition levels over time: (a) Level A (black) and B (grey); (b) Level C (black) and D (grey); (c) Level E

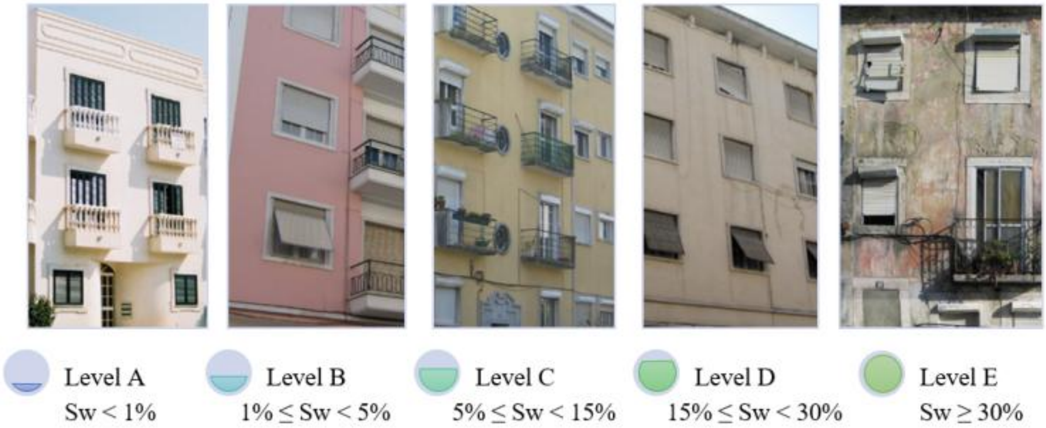


Figure 1

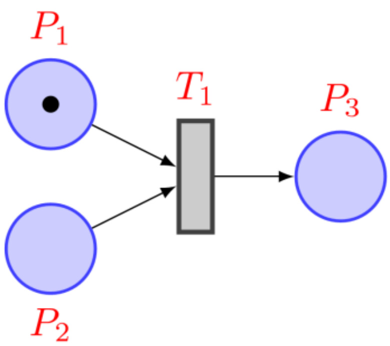


Figure 2

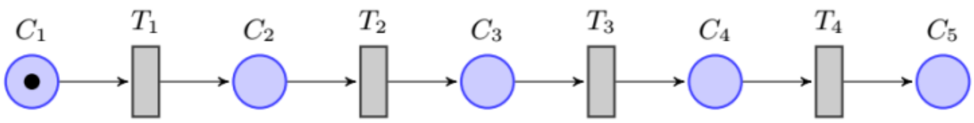


Figure 3

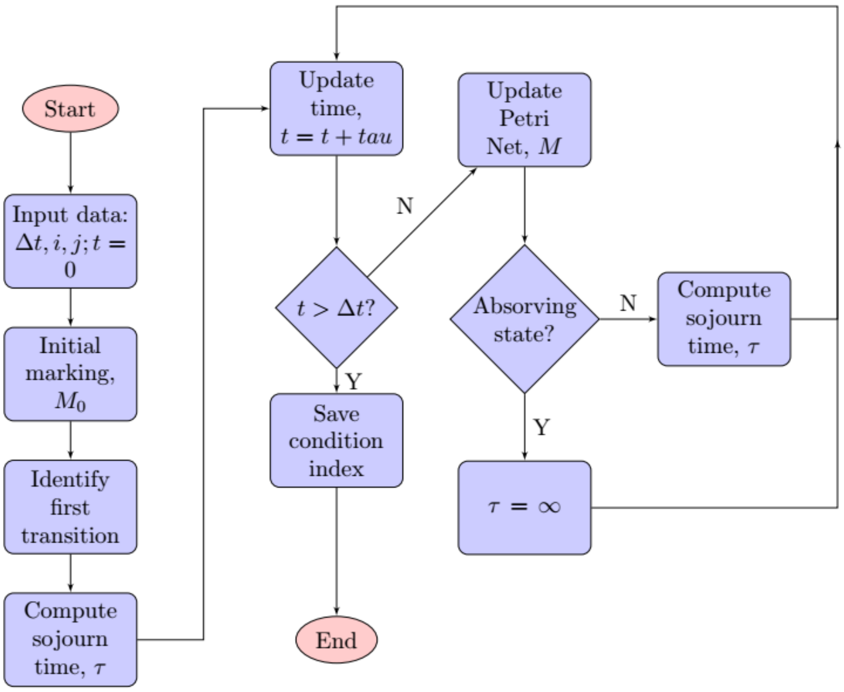


Figure 4

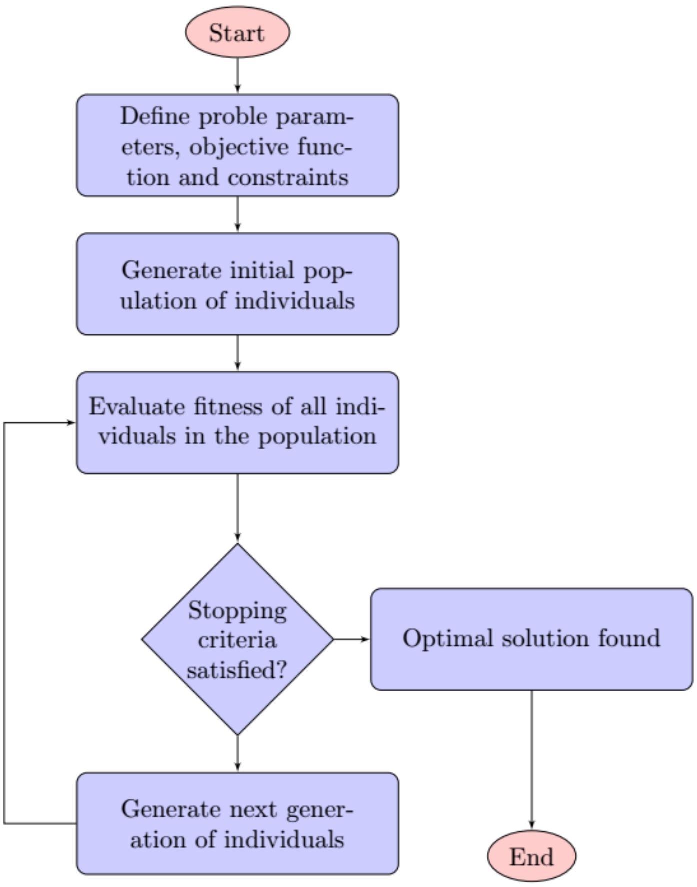
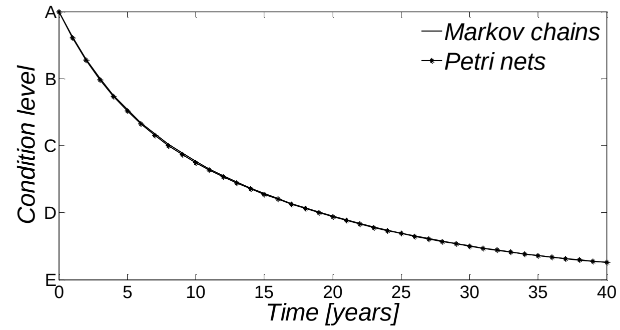
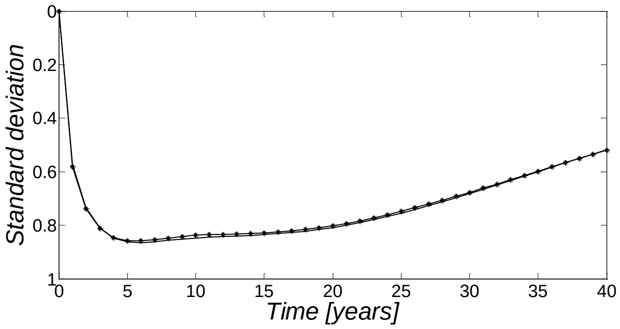


Figure 5

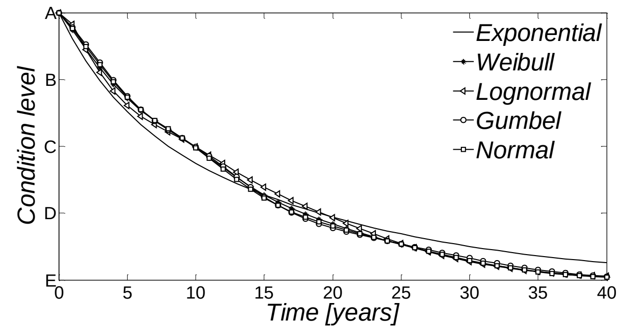


(a)

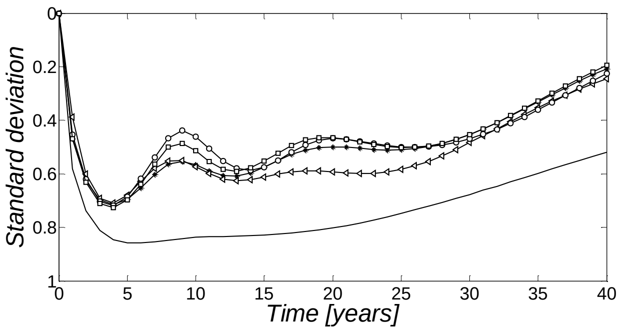


(b)

Figure 6

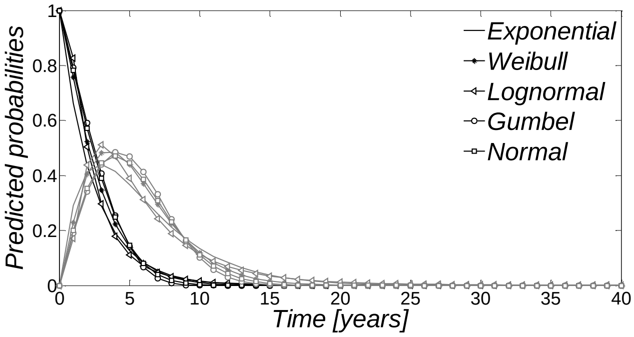


(a)

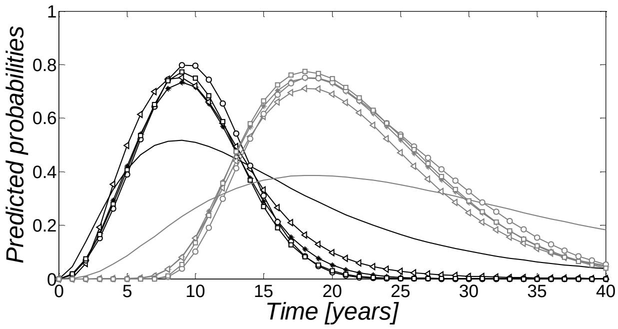


(b)

Figure 7


Exponential
Weibull
Lognormal
Gumbel
Normal
Predicted probabilities
Time [years]
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40

(a)


Predicted probabilities
Time [years]
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0.2
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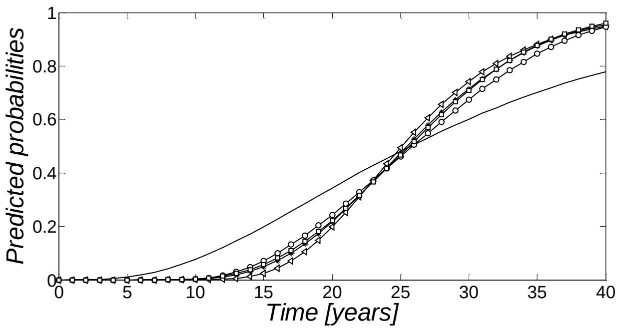
(b)


Predicted probabilities
Time [years]
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40

(c)

Figure 8

TABLE CAPTIONS

Table 1 Description of the degradation conditions of rendered façades

Table 2 - Comparison of the optimal parameters of the Markov chains and Petri net models

Table 3 - Number of observed and predicted coating on each degradation condition for both models

Table 4 - Optimal parameters obtained in all probability distribution analysed

Table 5 - Number of observed and predicted coating in each condition level for each probability distribution

Table 6 - Mean error [%] obtained for each probability distribution

Table 7 - Optimal parameters obtained for Weibull 3-parameters distribution

Table 8 - Number of observed and predicted coating for Weibull 3-parameters distribution

Table 1

Condition level	Description
Condition A	Most favourable condition. Complete mortar surface with no visible degradation, with uniform colour, showing no dirt or detachment
Condition B	Mortar with a non-uniform surface with likelihood of localized voids determined by percussion, but no signs of detachment. Small cracking (0.25 mm to 1.0 mm) in localized areas and changes in the general colour of the surface might exist. Eventual presence of microorganisms.
Condition C	Mortar with localized detachments or perforations, revealing a hollow sound when tapped and detachments only in the socle, with easily visible cracking (1.0 mm to 2.0 mm) and showing dark patches of damp and dirt, often with microorganisms and algae.
Condition D	Mortar with an incomplete surface due to detachments and falling of mortar patches, showing wide or extensive cracking (≥ 2 mm) and very dark patches with probable presence of algae.
Condition E	Most serious condition, requiring an immediate corrective action, associated with incomplete mortar surface due to detachments and falling of mortar patches. Also revealing a wide or extensive cracking (≥ 2 mm), with very dark patches and probable presence of algae.

Table 2

Model	**Optimal parameters**				**Likelihood**
	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\alpha_3}$	$\boldsymbol{\alpha_4}$	
Markov chains[1]	0.4016	0.2819	0.0994	0.0761	82.4245
Petri net (Exponential)	0.4201	0.2743	0.0966	0.0804	82.2582

[1] Data adapted from Silva et al. (2015)

Table3

Degradation condition	**Observed**	**Predicted**		**Error [%]**	
		Markov chains[1]	**Petri net**	**Markov chains**	**Petri net**
Level A	13	12.57	12.17	3.3	6.4
Level B	18	17.77	17.63	1.3	2.1
Level C	31	28.64	28.96	7.6	6.6
Level D	15	17.29	17.47	15.3	16.5
Level E	22	22.74	22.77	3.3	3.5

[1] Data sourced from Silva et al. (2015)

Table 4

	Parameters	Exponential	Weibull	Lognormal	Gumbel	Normal
Parameter 1	α_1	0.4201	2.8616	0.7001	0.6112	0.4811
	α_2	0.2743	3.8199	0.8702	1.5270	1.3919
	α_3	0.0966	7.9483	2.0754	7.8258	7.2940
	α_4	0.0804	14.1976	2.3615	11.4260	11.6725
Parameter 2	β_1	-	1.2149	0.7435	4.2326	3.2519
	β_2	-	1.4040	0.8572	4.4394	3.4303
	β_3	-	6.0816	0.3077	0.4219	0.1330
	β_4	-	2.0100	0.4612	11.7718	7.6393
Likelihood		82.2582	70.4602	70.2610	70.4666	70.1237

Table 5

		Level A	Level B	Level C	Level D	Level E
Observed		13	18	31	15	22
Predicted	Exponential	12.17	17.63	28.96	17.47	22.77
	Weibull	13.58	18.62	29.30	15.49	22.01
	Lognormal	13.09	17.58	32.07	14.67	21.60
	Gumbel	14.12	18.34	29.95	14.53	22.06
	Normal	14.11	18.06	29.38	15.27	22.18

Table 6

	Level A	Level B	Level C	Level D	Level E
Exponential	6.4	2.1	6.6	16.5	3.5
Weibull	4.4	3.5	5.5	3.3	0.0
Lognormal	0.7	2.3	3.4	2.2	1.8
Gumbel	8.6	1.9	3.4	3.2	0.3
Normal	8.5	0.4	5.2	1.8	0.8

Table 7

Parameters	$i = 1$	$i = 2$	$i = 3$	$i = 4$
α_i	1.3998	2.5269	4.5874	1.4221
β_i	0.7026	0.8977	1.8966	0.4718
γ_i	0.8803	0.7551	4.1532	7.7902
Likelihood	69.0345			

Table 8

	Level A	Level B	Level C	Level D	Level E
Observed	13	18	31	15	22
Weibull 3-parameters	12.96	17.68	32.04	14.31	22.01
Mean error [%]	0.3	1.8	3.3	4.6	0.0