Pricing Futures and Real Options with a Liquidity Factor: Theory and Evidence

by

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Abstract

Liquidity is one of the most intensively topics researched in financial economics for the last decade. Against this backdrop, this thesis attempts to address issue of liquidity in derivative markets and derivative models.

It begins with the provision of empirical evidence that liquidity risk can serve as an additional risk factor to market risk factor in pricing the commodity futures and it also outlines the vital role played by liquidity in futures prices of commodity co-movement and co-integration. Empirical evidence yields strong support on futures pricing model building, where factors should include both market risk and liquidity risk.

On above basis, this thesis builds two-factor futures pricing model by taking liquidity risk into account. I have also used 20-year oil futures market data to empirically justify liquidity-adjusted futures pricing model compared with traditional future pricing model without liquidity factor. I utilize mean pricing error (MPE) and root mean squared error (RMSE) to estimate errors for both models and I also adopt T-test for statistical significance justifications. For most years, liquidity adjusted futures pricing model performs better than the traditional model with results being statistically significant.

More importantly, liquidity adjusted futures pricing model can predict spot prices and futures prices simultaneously, which means only one model can be applied in both spot price predictions and futures price predictions based purely on historical market information. Existing models either predict futures prices by using spot prices (e.g. Black, 1976) or use futures prices to predict spot prices (e.g. Reichsfeld and Roache, 2011). As a result, my model has a great degree of prediction power with its prediction errors being less than 3%, which is relatively small.

Therefore, it is arguable, that liquidity risk plays a key role in commodity futures markets and illiquidity of those assets could prove influential on firms' daily operations. I also build an intrinsic nexus between real options theory and real asset illiquidity to accommodate this issue. Study of the new real options

model reveals effects of real asset illiquidity towards investment threshold and flexibility values, namely, exercise boundary and real options values, which is complementary to existing real options and corporate finance literatures. Instead of constructing free boundary line, which shows effects of time and asset price, the model presents a three-dimensional free surface, which indicates not only effects of time and asset price, but also that of asset illiquidity.

The new model contributes to two types of existing literatures. The first type focuses on effects of real asset illiquidity (mainly physical asset) on corporate investment and cost of capital. Illiquidity of existing physical assets will decrease corporate investment and increase cost of capital (Gan, 2007; Flor and Hirth, 2013, and Ortiz-Molina and Phillips, 2014). In addition to physical asset illiquidity, I distinguish physical asset from (expected) inventory asset within real asset category. The new model shows that the inventory asset illiquidity would also shape the corporate investment behaviors.

Additionally, the model also relates to literatures that document investment booming during unfavourable market conditions. I argue real asset illiquidity could engender the suboptimal exercise of real options. Simulation results of the new model illustrate that investment threshold becomes lower as waiting value and flexibility get eroded by asset illiquidity. Because of lower exercising boundaries, firms have higher a probability to exercise real options, but at lower values, which results in suboptimal exercise of real options. The suboptimal exercise of the real options due to the asset illiquidity might provide an interpretation for the investment booming during unfavourable market conditions. More importantly, I argue that the suboptimal exercise of real options might undermine firm value and thus firms shall be more prudent to invest when the environment is unfriendly.

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Capital Asset Pricing Model, CAPM 10

Controlled Autoregressive Moving Average Model, CARAM 37

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Chapter 1

Introduction

1.1 Motivations of the Thesis

The astounding 2007-2009 financial crisis has shocked the entire financial world. It is argued one cause of the 2007-2009 financial crisis was the drying up of market with liquidity. Liquidity risk is a highly-concerned issue in asset pricing and financial market research. Early notable work tackling liquidity issue would be Amihud and Mendelson (1986). They denote liquidity risk premium as the 'illiquidity premium' and suggest asset pricing model shall take liquidity risk into account. Later, Acharya and Pedersen (2005) and Liu (2006) try to incorporate liquidity risk into asset pricing model. More recently, further work on this issue includes Cornett et al. (2011), Calvo (2012), and Brunnermeier and Sannikov (2014) among others.

Liquidity measures the level of ease in trading. Specifically, it reveals the degree at which asset can be traded fast, and at low cost. As such, liquidity in real terms signifies trading frictions in financial markets, which suggests that real financial market is neither perfect nor frictionless. Acharya and Pedersen (2005) regard liquidity as one of the important trading frictions in the market. Empirically, commodity futures and real assets do have liquidity risk (Marshall et al., 2012; Gavazza, 2010). Therefore, including liquidity can put asset pricing model into more accurate and more realistic context. From this standpoint, the futures pricing model based on Black (1973) and real options theory based on Myers (1977) that relies on the frictionless market assumptions may need to be updated.

Plenty of studies have identified the issue of perfect market assumption but they mainly focus on the stock market. Research later conducted discovered assumption was unrealistic in real financial markets and tried to relax perfect market assumption when pricing asset and derivatives. Figlewski (1989), for instance, examines inconsistency of the perfect market assumption and options trade in practice. He concludes effects of market imperfections were larger, than what researchers may expect. Given the situation, he maintains the effects will lead to inaccurate options value because of weakness in assumption being so weak that theoretical value of options will depart from actual price. Dumas and Luciano (1991) highlight transaction cost was one of pronounced frictions in financial market. More importantly, they argue that any presence of market frictions will change the nature of the optimisation problem but they use it to study options pricing model. Dai, Zhong and Kwok (2011) examine optimal arbitrage strategies in stock index futures with position limits and transaction costs. However, little attention has been paid in futures pricing and real options valuation field. The vast majority of existing futures pricing models and real options models have ignored liquidity risk when valuing futures and real options. Main state variable of existing futures and real options study is asset price and one-dimensional real options model has been developed.

1.1.1 Futures and Options Pricing Model with Asset Liquidity

Asset pricing model mostly considers risk factors involved in asset prices and returns. For futures pricing, risk premia also plays an important role in influencing future prices. It is arguable that deviations of futures prices from expected future spot prices are outcomes from risk premia of futures. Hedging and diversification benefits will also affect futures risk premia (De Roon et al., 2000). Liquidity risk is a type of pivotal risk premiums in futures market and liquidity in futures markets indicated by open interest and trading volume, and it has been noticed in literatures for the futures market (Bessembinder and Seguin, 1993). Evidence also indicates the existence of liquidity risk in commodity futures market. Marshall et al. (2012) research reveals 24 commodity futures showing positive liquidity risk under different liquidity proxies. Marshall et al. (2013) demonstrates distinct and common liquidity factor in commodity market and close relationship of liquidity factor with commodity price. Even though liquidity risk has been demonstrated in the futures markets, only liquidity risk in stock markets are substantially investigated (Shleifer and Vishny, 1992; DeAngelo, DeAngelo and Wruck, 2002; Pastor and Stambaugh, 2003; Officer, 2007; Bakke, Jens and Whited, 2012) as well as liquidity effects in terms of trading costs and bid-ask spread (Schwert, 1997; Bessembinder, Maxwell and Venkataraman, 2006). However, futures pricing model, which includes liquidity factor is scarce. As a result, incorporating liquidity as a risk factor into futures pricing model is complementary to existing literatures.

Most of futures markets and real asset markets are not perfect in reality. Therefore, I relax perfect market assumption through introducing liquidity risk into futures and options pricing model. This study's most important contribution is to introduce liquidity risk into futures and options pricing model. It argues that liquidity risk is a quintessential part of futures and options pricing model and introducing this specific extension would constitute an important innovation and contribution to existing works on the topic in question. There are a range of reasons why liquidity risk is so indispensable to futures and options pricing model.

Firstly, introducing liquidity risk factor into futures and options pricing model can improve the model accuracy since the risk factor shall be reflected in the asset pricing model. Holmstrom and Tirole (2000) also agree different assets have different liquidity premiums, which could also be true for different commodity futures and different real options whose underlying assets are commodity futures. Like Amihud and Mendelson (1991a) declare investment with less liquid asset shall provide higher expected returns to compensate investors who bear the additional liquidity risk. Brennan and Subrahmanyam (1996) suggest relatively illiquid security has a higher expected return compared with more liquid security. Eleswarapu (1997) empirically examines illiquidity premium in NASDAQ market and find strong evidence for supporting illiquidity premium in financial market. Brennan et al. (1998) also confirm the role of liquidity risk in determining expected asset return. O'Hara (2003) suggests market liquidity may enter into asset price formation. Allen and Gale (2004) claim commodity market liquidity to significantly influence commodity prices. Underestimating total risk may lead to biased low risk compensation and risk measures and therefore overestimate value of futures and options (Jarrow, 2005). Roll et al. (2007) assert liquidity in the financial market could play a key role in influencing price movements.

Datar et al. (1998) and Jacoby et al. (2000) affirm a positive relationship between expected return and asset liquidity. Therefore, those illiquid commodity futures could have additional liquidity risk because relatively illiquid assets require additional expected return, which results in decrease of futures prices since higher expected return will raise the discount rate. Rather than assuming market to be perfectly-liquid like existing studies, including liquidity risk can differentiate futures and options with various underlying assets whose liquidity risk may be different. Therefore, including liquidity risk can have more accurate risk reflections of futures markets as well as real asset market, and also distinguish illiquid assets from more liquid ones. The classical model neglects liquidity risk may overestimate the asset values and thus I expect my model to have lower estimated values. Since classical model overestimates market price, my lower estimated values will be closer to actual market price and project values. The new model thereby improves accuracy of value estimations for both futures and options.

Moreover, liquidity can be considered as a risk factor because it can vary over time (Hasbrouck and Seppi, 2001). Chordia et al. (2000) find time-varying feature is commonality for asset liquidity. Taking crude oil as an example, when oil market is active, liquidity of oil is high. However, when there is extreme market condition, such as 2008 oil bubble, market liquidity will drop dramatically (Tokic, 2011). Under such conditions, there is little liquidity provided in the market with oil companies incurring substantial losses in that period. One of the most important reasons for collapsing of oil price is lack of liquidity (Tokic, 2012). Liquidity risk is an important component of market risk in real financial market and shall not be neglected (Bangia et al., 1999). Therefore, it would be a worthwhile endeavor to develop a liquidity-adjusted derivative pricing model in order to furnish an up-to-date investigation into liquidity effects on derivative prices.

1.1.2 The Linkage between Real Options Theory and Asset Liquidity

Since real asset markets are not perfect and have presented liquidity risk, there is huge demand for relaxing the perfect market assumption in real options valuation. However, relaxing perfect market assumption is considered a big challenge for real options pricing and several researchers appealed for research in this field (Contreras et al., 2010). Lander and Pinches (1998) demonstrate relaxing market completeness and no-arbitrage assumption is one of the biggest challenges of real options valuation. Hubalek and Schachermayer (2001) note real options limitations by assuming perfect market and no arbitrage conditions. Triantis (2005) remarks perfect market assumption as first gap between real options theory and practice and appeals for more realistic assumptions in order to make model more practical and accurate.

By extending the traditional real options model, I introduce liquidity factor (measures the illiquidity of the inventory asset) into traditional real options model because real asset liquidity could reflect the market imperfections. For example, the inventory asset of oil project is crude oil, and most of crude oil is traded in futures market, which has liquidity risk. Gavazza (2011) maintains that this kind of liquidity can be considered as cost arising from market and this cost will affect investment behavior. Since real asset is closely related to real options value, real asset liquidity might also play an important role in real options theory. Consequently, my study is motivated by the linkage between real asset liquidity and real options and the link is twofold. The first linkage is real asset liquidity relating to project cash flows and thereby project value and free boundary for project launch. Real asset here mainly refers to inventory assets produced by projects and expected to be sold for generating cash flows, such as crude oil from oilfield project, and copper from copper-mine developing project.

In financial literature, project value and real inventory asset values are highly correlated. In copious classical works of real options, they are perfectly linked (Dixit and Pindyck, 1994). Many real options literatures thereby directly used project output as underlying asset of real options. For instance, Brennan and Schwartz (1985) apply real option framework to value copper mines and they use copper as underlying asset in their model. Paddock et al. (1988) adopt real options method to evaluate offshore oil fields and use oil as underlying asset. Grenadier (1996) uses real options method to evaluate real estate value by employing housing price as underlying asset price. Another example of real options underlying asset could be the aircraft and many scholars try to extend real options theory into aircraft valuation with the aircraft being considered as underlying asset under framework (Stonier, 1999 and Gibsona and Morrell, 2004).

In practice, real options theory is a risk management tool helping managers evaluate project and make investment decisions accordingly. When firms evaluate real investment opportunity, their main concern is future cash flows that project can generate. Potential risk of future cash flows could be a driving force in the investment decisions. I identify the main source of cash flow risk comes from inventory asset illiquidity. Such as, when a firm launches project and asset production sale from project generates cash flows for project. For instance, oil project extracting crude oil from oilfield and firms selling extracted oil in oil market. Oil market liquidity thereby dominates future cash flows since one of the most important reasons for oil price collapsing has been lack of liquidity (Tokic, 2012). Inventory assets produced are more difficult to sell when inventory asset market becomes less liquid leading to potential uncertainty facing cash flows. At such times, firms either wait for a longer period to sell asset, or sell it at relatively lower price. Thus, cash flow risk could be reflected from asset liquidity risk.

Potential cash flow risk links to firms cash holdings has been an intensively researched topic in recent times (Duchin, 2010; Fresard, 2010; Palazzo, 2012; Harford et al., 2014; Hugonnier et al., 2015). Studies such as Kim et al. (1998) and Harford (1999) suggest an increase of cash flow risk will eventually raise firms cash holdings. They confirm cash holdings are related positively with future cash flow volatility. Bouvard (2014) maintains it is costly for entrepreneurs to finance externally because of information asymmetry and adverse selection in capital market. Meanwhile, internal financing resolves adverse selection problem and grasps investment opportunities. As the pecking order theory states, asymmetric information makes external funding more costly (Myers, 1984; Myers and Majluf, 1984). As such, it is reasonable to presume entrepreneurs are financially constrained and prefer financing internally.

External financing being extremely costly and risky, vast majority of firms prefer to fund investment opportunity from internal financing, leading internal capital market to play a crucial role in resource allocation (Stein, 1997; Fee et al., 2009). Likewise, Bhagat et al. (2005) maintain most of the corporate investment to be funded by internally-generated cash flows. Fazzari et al. (1988) demonstrate that investment spending largely depends on internal fund availability. Cash flows from project revenues ranks among most important sources of internal fund. As a result, I identify risk from this source of internal fund, which might impact corporate investment behaviors. Liquidity adjusted real options model takes this effect into account and shows potential cash flow risk from asset illiquidity relates to suboptimal exercise of real options.

Second aspect of the nexus between real asset liquidity and real options relies on real assets illiquidity reducing firms operating flexibility. In other words, the second linkage is real asset illiquidity relating to project flexibility and thereby real options values. Asset liquidity held by a firm reflects its general liquidity level. As demonstrated by Flor and Hirth (2013) and Ortiz-Molina and Phillips (2014), real asset with higher liquidity can provide more flexibility and illiquid assets may provide less. Holding liquid assets provides firm with not only greater flexibility but also reduces liquidity demand of firms (Jones and Ostroy, 1984). Morellec (2001) notes operating flexibility associated with asset liquidity leading to liquid assets holdings increasing firm values. Flexibility will be significantly reduced when assets become more illiquid. It is clear from this perspective that real asset liquidity is a nexus of real options theory as operating flexibility remains the vital concern of real options theory. For instance, Mardones (1993) reveals operating flexibility can add value to project and can be measured by contingent claim analysis due to its option-like features. Kogut and Kulatilaka (1994) claim operating flexibility such as production shifting is identical as owning an option and Zhang (2005) gives return implications of flexibility under real options framework. Since real assets illiquidity will affect operating flexibility and real options theory is the best way to evaluate such flexibilities, it shall be considered in the real options model.

This study is also motivated by substantial liquidity effects of real assets. Centralized-traded assets such as futures have liquidity risk. Real assets are often traded in decentralized markets such as Over-the-Counter (OTC) markets within the industry. Without facilitation of the centralized trading system, the OTC markets trading structure makes liquidity effects more noticeable. Without central trading coordination system, investors cannot buy or sell asset instantaneously in the OTC markets. They have to trade asset through search and bargaining process, which underlines liquidity effects (Jankowitsch et al., 2011). Since liquidity effects are noticeably important and dramatic in OTC markets, liquidity premium will be much higher (Ang et al., 2013). It is therefore rather crucial to investigate liquidity effects in such fairly-illiquid markets.

Liquidity adjusted real options model can help investors and managers to evaluate the investment decisions more precisely when they face asset liquidity uncertainty. Under real options framework, the underlying asset is project output, such as copper, oil, and real estate, which directly relates to project value and I denote as inventory asset. Therefore, the price at which output asset is sold will have considerable impact on project value. As Fisher et al. (2003) demonstrate, real estate market liquidity will affect property investment values. Consequently, investors and managers shall consider asset liquidity as an important factor for future investment decisions. In order to help investors and managers make optimal investment decisions by considering liquidity effects, I make the practical contribution that I derive from liquidity adjusted real options model.

To conclude, the traditional model is based on unrealistic assumptions; futures and options may be overpriced oftentimes (Kairys and Valerio, 1997). Theoretically, the new model improves accuracy and applicability of classical futures options pricing model by relaxing unrealistic assumptions and reflecting real market conditions and risks. As such, I have built a more applicable and practical framework. In practice, my model provides more precise results compared to classical model. From modelling perspective, based on existing liquidity adjusted European options pricing model, I first establish liquidity adjusted futures pricing model, validating it by actual market data followed by enabling of investors to value American options with liquidity risk, and of managers to evaluate real projects by considering liquidity risk. I also take current market liquidity level into the model, which makes it closer to actual market conditions and reflects promptly changes in market conditions. By establishing real options model with asset illiquidity, I show asset illiquidity effects towards investment thresholds. More importantly, I can quantify reduction in flexibilities amount by asset illiquidity in terms of real options values. The effect, in turn, downgrades investment threshold, leading to suboptimal exercise of real options. I argue that firm values would be impaired by the suboptimal exercise of the real options since firms acquire less project values than expected.

1.2 Contributions and Structure of the Thesis

I have found motivations of building liquidity-adjusted futures pricing model and also liquidity-adjusted real options model from existing literatures. Since the thesis focuses on the liquidity issue, my first research question would be to find out the role played by liquidity in the futures market, taking the commodity futures market as an example. More specifically, like the liquidity-augmented Capital Asset Pricing Model (CAPM), can liquidity risk serve as an additional risk factor to market risk factor in pricing commodity futures?

Consequently, Chapter 3 settles those research questions and confirms liquidity risk can serve as additional risk factor to the market risk factor in pricing the commodity futures. Demonstration of the conclusion in Chapter 3 is twofold. I first run regression of commodity futures return with market CRB index and store the residuals. This is followed by the identification of statistically significant close relations between liquidity risk and residual part not explained by market risk factor. I also run commodity futures returns regression with market CRB index by adding liquidity risk as another independent variable and discover that by controlling market risk factor, liquidity risk also presented a statistically significant coefficient in explaining commodity futures returns. I conclude liquidity risk can serve as an additional risk factor to market risk factor in pricing commodity futures. Based on empirical findings, I claim futures pricing model shall be two-factor model, where one factor is market risk and the other is liquidity risk.

By identifying two risk factors, how would I incorporate liquidity factor to develop a new futures pricing model? By building such model, will liquidity-adjusted futures pricing model empirically perform better than traditional futures pricing model without liquidity factor? The main contribution of Chapter 4 is to deal with those research questions by building up a liquidity adjusted futures pricing model based on Chapter 3. The newly-developed model considers both market risk factor and liquidity risk factor. In addition, Chapter 4 adopts 20-year oil futures market data to empirically justify liquidity adjusted futures pricing model compared with traditional future pricing model without liquidity factor. I use mean pricing error (MPE) and root mean squared error (RMSE) to estimate errors for both models and also adopt T-test for statistical significance justifications. For most years, liquidity adjusted futures pricing model performs better than traditional model with results being statistically significant.

Foremost, liquidity adjusted futures pricing model can predict spot prices and futures prices simultaneously, which means only one model can be applied in both spot price predictions and futures price predictions purely based on historical market information. The existing models either predict futures prices by using spot prices (e.g. Black, 1976) or uses futures prices to predict spot prices (e.g. Reichsfeld and Roache, 2011). As a result, my model has great degree of prediction power. Moreover, my model testifies short-term spot and futures market integrations since short-term futures prices could be a sound approximation of short-term spot prices. On this basis, my model adopts a two-step forecasting method. Firstly, I estimate spot prices and use estimated prices as inputs to predict futures prices. The market has different views on futures with different maturities since implied discount factors for futures with different maturities are different. In order to obtain more precise results, I choose different discount factors for different maturity groups. Lastly, prediction errors of my model are less than three per cent, which are relatively small.

Therefore, it is arguable that liquidity risk plays a vital role in commodity futures market and assets in these markets are real assets. What will be the effects of real asset illiquidity in real investment and real project values? Will the illiquidity of those assets impact on values of the project and thus value of the firms, which are actually producing them? In Chapter 5, I build up a two-factor real options model with real asset illiquidity to address those research questions, and I distinguish those assets as inventory assets since they are expected to be sold.

The model in Chapter 5 builds an intrinsic connection between real options theory and inventory asset illiquidity. The model study reveals effects of inventory asset illiquidity towards investment threshold and flexibility values, namely, exercise boundary and real options values, which are complementary to existing real options and corporate finance literatures. Instead of constructing free boundary line, which shows effects of time and asset price, the model presents three-dimensional 'free surface', which indicates not only effects of time and asset price, but also that of inventory asset illiquidity. The results can aid managers to have a deeper understanding of the role of real asset illiquidity in corporate finance and risk management.

The study of the model unveils the effects of inventory asset illiquidity towards investment threshold and flexibility values, namely, the exercise boundary and the real options values, which are complementary to the existing real options and corporate finance literatures. Instead of constructing a free boundary line which shows the effects of time and asset price, the model presents a three-dimensional "free surface" which indicates not only the effects of time and asset price, but also the effects of inventory asset illiquidity. The results can aid managers to have a deeper understanding of the role of real asset illiquidity in corporate finance and risk management.

In fact, the new model contributes to two types of existing literatures. The first type focuses on effects of real asset illiquidity (mainly physical asset) on corporate investment and cost of capital. Illiquidity of existing physical assets will decrease corporate investment and increase cost of capital (see Gan, 2007; Flor and Hirth, 2013, and Ortiz-Molina and Phillips, 2014). In addition to physical asset illiquidity, I distinguish physical asset from (expected) inventory asset within real asset category. To be specific, inventory asset from project, such as crude oil and metal, is waited for sale once developed, and plays a vital role in future cash flow generation. Model shows illiquidity of inventory assets would also influence corporate investment decisions.

More importantly, I can utilize the model to specifically quantify the amount of flexibilities in terms of real options values been reduced by inventory asset illiquidity. The model shows flexibilities have been significantly reduced by inventory asset illiquidity when real options are at/in the money, but the effects is considerably less when options are out of the money. It is arguable that asset illiquidity primarily serves a negative factor in the model. When real options are out of money, asset liquidity is unable to lift the options values getting them to be in the money. On other hand, when real options are in the money, asset illiquidity might shift options values downwards. Additionally, asset liquidity has little impact on real options when maturity time is near. Since time remaining is short, options will be exercised so long as they are in the money. Therefore, asset liquidity has little impact, otherwise the 'in the money' options will be abandoned, which is irrational for managers.

According to the new model, illiquidity of inventory assets would lessen project waiting value leading to shrinkage of real options value. This effect, in turn, reduces investment threshold, which might result in suboptimal exercise of real options during unfavorable market conditions. I also argue that inventory assets illiquidity could result in cash flow uncertainty and thus affect firms internal funds. Therefore, internal fund shortfall will restrict firm's investment spending thereby influencing corporate investment behavior. As a result, understanding effects of inventory asset illiquidity and cash flow uncertainty can enhance firm's liquidity management and project management.

Additionally, the model also relates to literatures documenting investment boom during unfavorable market conditions such as declining market demand and economic recessions where underlying asset tends to illiquid. Kahn (1985) gives evidence that investment boomed after 1981-82 recession. Ghemawat (1993) maintains firms could overemphasise on financial risk on investment during recession. Grenadier (1996) shows it may be rational to invest when prices decline in imperfectly competitive markets. He notes during the decline in demand of real estate, which indicated market liquidity of the real estate was low, there was significant increase in new building development. As they have argued that the reason why the investment might boom during the unfavourable market conditions might be twofold. Firstly, the costs such as labor cost might be lower during the unfavourable market conditions. More importantly, there is a first-mover advantage for those firms that invest in advance. There might be a time lag between building and selling the property. My thesis perceives these investment behaviors are results of suboptimal exercise of real options, which has been mentioned in Boyle and Guthrie (2003). They argue that potential future cash shortfall will result in the suboptimal exercise of real options. I further maintain the risk of potential future cash shortfall comes from inventory asset illiquidity and I testify the threshold of investment becomes lower since waiting value and flexibility are eroded by asset illiquidity. As a result of lower exercising boundaries, firms have a higher probability to exercise real options, but at lower value, which results in suboptimal exercise of real options and the suboptimal exercise might be harmful to the firm values.

Importantly, real options theory with asset illiquidity sheds light on corporate liquidity management where liquidity management becomes more relevant in current risk management context. Financial literatures agree firms can acquire flexibility through liquidity management (Denis, 2011). Gamba and Triantis (2013) developed an integrated risk management system, which comprises liquidity management, derivatives hedging, and operating flexibility. They reveal liquidity management playing a vital role in risk management and likely to be more important than hedging with derivatives. As a result, I seek to enhance operating flexibility and firm values through liquidity management. As a result, the real options model with asset liquidity is not only can help managers to understand the effects of asset illiquidity, but also enhance liquidity and project management of the firm. Bates et al. (2009) demonstrated that significant increment in cash flow risk for firms has resulted in high cash holdings for US firms. Cash holdings can serve as cushions to buffer when firms encounter cash shortfalls. Hirth and Uhrig-Homburg (2010) also suggest firms prefer to hold liquid funds. As a result, projects future cash flows play important roles in corporate finance in terms of firms cash holdings. I demonstrate firms with high liquid projects can be more patient on exercising real options.

According to new model, when asset market is liquid, then firms need not need be anxious about project discontinuations because of cash shortfalls. Instead, firms are more willing to hold those projects with liquid asset just like holding liquid funds. Firms know those assets can be sold quickly at fair prices and those assets thereby can act as buffering cushions for other illiquid projects. As argued by Ozkan and Ozkan (2004), conversion cost for liquid assets into cash is much lower as compared with other assets. Firms with sufficient liquid assets can easily obtain internal funds than other firms. Consequently, when asset is liquid, threshold for exercising real options on project will be higher.

From the modelling perspective, compared with the model developed by Feng et al. (2014), I developed a liquidity adjusted futures pricing model and real options models whereas they only adopt the framework for European option pricing. The existing literatures do not have the liquidity adjusted futures pricing model or liquidity adjusted real options model. For the liquidity adjusted real options model, most real options are American typed and American options are quite different from European options. To be specific, the key difference between European options and American options is the exercise time, where American options allow early exercise. The early exercise premium becomes a crucial part of the American options values. Options traders generally agree that American options are more valuable than European options because of the early exercise potential (Jorion and Stoughton, 1989). Carr et al. (1992) present the American options price as the sum of its early exercise premium with its intrinsic value. Detemple and Osakwe (2000) demonstrated that the early exercise premium has its own value which shall be included in American options values and it will be higher for those more volatile options. Moreover, the convexity of the early exercise premium has paramount implications for the hedging behavior of American options. Furthermore, it may not be irrational to exercise American call options early when there are market frictions, but it will be rational when market frictions present with an equilibrium model (Poteshman and Serbin, 2003). In this liquidity-adjusted model, the asset liquidity could be considered as a market friction and also the model is imposed by a market-clearing equilibrium condition. It should be reasonable to have massive early exercised options and then extending European options pricing to American options pricing will be quite essential.

More importantly, the ignorance of the early exercised options will impair the powerful implications of options pricing theory toward the real projects. Like Luehrman (1998) argued that the implementation of real projects are more complicated. Situations such as regulation changes, potential loss of market share and movement of asset prices might cause early exercise of real options. Grenadier and Weiss (1997) also contend that firms hold real options should choose to exercise options whenever optimal. Sarkar (2000) points out that the project that can be views as a series of real options to invest and thus the options shall be exercised when it brings higher payoffs to the firm. Consequently, the early exercise of American options has significant effects on both options pricing theory and real options theory and including early exercise policy of options will play a pushing role in improving both theories. Technically, European options pricing is a pure valuation problem whereas American options pricing involves optimal stopping, which makes the valuation process more complicated (Jacka, 1991). As a result, American options pricing is still a challenging practical problem, and thereby extending liquidity adjusted European options valuation model to liquidity adjusted American options pricing model will also be a contribution (Haugh and Kogan, 2004). Based on liquidity adjusted futures pricing model, I further derived the American options pricing model as well as the liquidity adjusted real options model since the real options model mainly concerns about the optimal timing of the project development and optimal stopping of developed projects (Sarkar, 2000).The derivation of the liquidity adjusted futures pricing model and real options model could also be considered as one of contributions of this thesis.

This thesis comprises six chapters following the introduction in Chapter 1. Chapter 2 provides an overview of the related literatures and gives more pronounced theoretical foundations of the thesis. Chapter 3 conducts a series of empirical analysis on the liquidity effects in the commodity futures markets. Chapter 4 develops a liquidity adjusted futures pricing model and I validate the model by using the crude oil futures data. The newly-developed model turns out to be much more accurate than the traditional futures pricing model. Chapter 5 builds a real options model with asset illiquidity and the model shows the influence of asset illiquidity on the option exercise boundaries and real options values. Base on the new model, I argue that the asset illiquidity could lead to the suboptimal exercise of real options. Chapter 6 concludes the thesis.

Chapter 2

Literature Review

2.1 Liquidity Risk in Futures Market and Existing Futures Pricing Models

Liquidity risk has been demonstrated to exist in the futures markets, especially commodity futures markets. Literatures show that all commodity futures exhibit positive liquidity risk. Marshall et al. (2012) research 24 commodity futures and all commodity futures show positive liquidity risk under different liquidity proxies. Moreover, Marshall et al. (2013) demonstrate that there is a distinct and common liquidity factor in the commodity market and the liquidity factor is closely related to the commodity price. Although the liquidity risk presents in the commodity futures market, the futures pricing model that includes the liquidity risk is scarce. Black (1976) derives the futures and forward pricing model with cost of carry. One seminal model in the futures pricing is developed by Gibson and Schwartz (1990). They introduce the convenience yield into the futures pricing model and propose a two-factor model, one is the spot price and the other is the instantaneous convenience yield. They empirically testify the two-factor model by adopting the oil futures data and they conclude that the two-factor model is more accurate than the original one factor model.

After the seminal model of Gibson and Schwartz (1990), there are many follow up works. Schwartz (1997) sets a three-factor model, where the third factor is the instantaneous interest rate. Hilliard and Reis (1998) investigate the impact of stochastic convenience yield, stochastic interest rate and jump effects on the futures prices under the three-factor model framework. They discover that there are huge differences between futures prices estimated from a deterministic convenience yield model and a stochastic convenience yield model. Cortazar and Schwartz (2003) also create a three-factor model, where the third factor they consider is the long-term spot price return. They find the model is well fitted with oil and copper futures market data. Casassus and Collin-Dufresne (2005) use the same three-factor model as Schwartz (1997) and they allow the convenience yield as a function of spot commodity prices. They utilize the function to explain the mean reversion in spot prices.

Afterwards, Trolle and Schwartz (2009) incorporate the stochastic volatility factor when pricing commodity derivatives and decompose the stochastic volatility into spanned and unspanned volatility factors. They suggest that both factors will affect the spot commodity price volatility and cost of carry. Nakajima and Ohashi (2012) extend the Gibson and Schwartz two-factor model by adding the linear relations among commodity prices. Most of the existing futures pricing models consider the stochastic convenience yield and stochastic volatility factors. More recently, Feng et al. (2014) develop a European options pricing model that includes stochastic liquidity risk. They add a series of liquidity related parameters into the original Black- Scholes model.

2.2 Introduction to the Real Options Theory

The futures pricing model is quite similar as the American-type options pricing model and the widely applied American-type options model is the real options model. Myers (1977) cites the idea real options in his paper and he applies the options pricing theory into the real investment strategies. Dixit and Pindyck (1994) define this approach as the investment opportunity valuation method that focuses on the options-like features of the investment opportunity and it considers the values created by uncertainty and flexibility of the investment. Although the real options method is used to evaluate the investments, it is still in an options valuation model, which indicates that the calculation of NPV will be based on the options theory that has been developed for the financial markets. The only difference is that the underlying assets of the basic options are the financial assets such as stock or bond. In contrast, the underlying assets of the real options are the real assets such as investment opportunities or project outputs. Therefore, options pricing models and theories are the foundations of the real options theory.

2.2.1 The Applications of Real Options Theory

As the real options method has been proposed, it was applied widely in many areas of business. The first area that the real options theory applies is the real estate industry. Timan (1985) firstly applies the real options theory to land price valuation. Quigg (1993) uses the empirical real estate transaction prices in Seattle to test the real options theory and she argued that holding an undeveloped land was like holding an American call options. She adopts 2,700 land transaction prices in Seattle to test the explanatory power of the real options model empirically. Moreover, Grenadier (1996) uses real options theory to study the development timing of the real estate and he develops an option exercise strategies in real estate development. In addition, Cunningham (2006) and Cunningham (2007) test the relationship between the investment and the uncertainty within the real estate industry under the real options framework. His findings prove that investors consider real options when exploring the future real estate and making investment decisions.

The second area of real options application might be the commodity related projects, especially in the metal and energy sectors. Brennan and Schwartz (1985) value the copper mining with options-like approach to improve the basic NPV method. McDonald and Siegel (1985) utilize the real options to evaluate the risky projects with shutdown options and they mainly focus on the projects that produce commodity. Paddock et al. (1988) argue that oil firms own the developed oil field can choose when to extract and produce, which is known as the optimal timing for extracting and can use real options to solve. Morck et al. (1989) apply real options theory to investigate the optimal timing of harvest the forest resource. Tufano (1998) introduces the real options theory into the gold mining industry.

Other areas like pharmaceutical projects and information technology (IT) projects with Research and Development (R&D) were also studied by the real options theory. Mitchell and Hamilton (1988) introduce the real options theory into the R&D area. Nichols (1994) demonstrates the underestimate of the R&D project value by using NPV method, especially in the pharmaceutical industry. He maintained that it was because the NPV method cannot estimate the future cashflow accurately and more importantly, the volatility of the cashflow. It has been argued that many pharmaceutical companies started to use real options method to evaluate projects such as Merck Company. Weeds (2002) applies real options theory to evaluate the strategic delay in competing R&D projects. Boer (2000) uses real options theory to focus on the fast-growing industries, such as information technology.

2.2.2 The Classifications of Different Real Options

A large number of scholars have been devoted to studying the real options theory and the fundamental categorization of real options has been specified by Trigeorgis (1998) based on their pay-off functions. The first real option is the option to defer, which concerns the optimal time to start the investment. Tourinho (1979) gives the edged idea of the option defer. Later on, McDonald and Siegel (1986) examine the value of options to defer and developed a real options model to evaluate the projects. Paddock et al. (1988) apply the option to defer to the oilfield evaluation and they argue that at the oilfield exploration stage, oilfields are potential to develop like owning an option. As a result, oil firms need to wait until the market conditions are favored and then they can develop the oilfield. Therefore, the option to defer is to locate the optimal timing for firms to develop the oilfield. To develop the option to defer further, Ingersoll and Ross

(1992) emphasize the effects of interest rate in the option to defer and investigate the relationship between interest rate and investment. Cortazar and Schwartz (1998) also consider the value of undeveloped oilfield by comprising the timing option value where the option value is presented in the optimal investment time. More recently, Folta et al. (2006) empirically examine the option to defer and they underline the interactive effects between uncertainty and irreversibility on the likelihood of entry into a new industry. Abid and Kaffel (2009) develop a methodology for valuing the option to defer and they focus on determining the appropriate stochastic processes for modeling relevant risk factors.

The second type of real option is the option to abandon, Myers and Majd (1990) claim the existence of option to abandon in the real projects. Berger et al. (1996) confirm that the value of option to abandon will be taken into account when investors price the firm. More recently, Huang et al. (2006) apply the option to abandon model to the evaluation of Build-Operate-Transfer infrastructure projects.

Another popular type of real option is the time-to-build and This type of real options are especially relevant for projects with multiple stages, like real estate constructions and energy generation. Majd and Pindyck (1987) notice the options value for the sequential construction. Carr (1988) values staged investment as compounded options and considered the effects of time to build in real investment. Pacheco-de-Almeida and Zemsky (2003) scrutinize the effects of the "time-to-build" in the strategic investment.

The "option on regime switching" is also an important kind of real options and this kind of option mainly applies to the energy generation, especially electricity. Margrabe (1978), first value this kind of option in his paper. Kulatilaka (1988) studies the flexibilities embedded in the Flexible Manufacturing System (FMS) and he identifies various benefits of FMS from the flexibilities that it can change the production modes. More recently, Chiu et al. (2015) apply the real options under regime switching to study fashionable and perishable products.

Another type of real option needs to mention is the "option to change operating scale". This kind of option mainly concerns the expansion or the reduction of the project production. McDonald and Siegel (1985), Brenann and Schwartz (1985) both mention the flexibility in the operating scale. Moreover, Pindyck (1988) develops the model of capacity choice with real options. Further, Bollen (1999) combines the product life cycle with the change of project's capacity. Recently, Boomsma et al. (2012) use the real options approach to assess investment timing and capacity choice of the renewable energy projects.

The final type of real option is the growth option. Myers (1977) refers the growth option as the value of growth opportunities stems from the firm's options to make future investments. Following work by Kester (1984) and Chung and Charolnwong, (1991) further studied the growth option. Kulatilaka and Perotti (1998) combine the growth options with imperfect competition. More recently, Dcamps and Villeneuve (2007) investigate the interactions and trade-off between paying out dividends and exercising growth options.

2.2.3 The Risk Factors Involved in Real Options Theory

The real options model initially only has one risk factor, the underlying asset price, whose movements are treated as the only source of uncertainty and are modeled as the stochastic process. For instance, Paddock et al. (1988) map the options theory into the oil field development and they use a stochastic process to model the oil price and the stochastic process commonly used is the Geometric Brownian Motion (GBM). They first value the developed oil reserve from the market equilibrium perspective because the oil produced from the project can be sold directly in the market. Then, they value the unexploited reserves by a replicating strategy, which is proposed by McDonald and Siegel (1984) based on Merton (1973). They reproduce the undeveloped oil reserve as a portfolio of producing developed oil reserve with riskless bond. The replicating strategy method has been generalized by Dixit and Pindyck (1994).

On the other hand, other than model the underlying asset price as the GBM, Schwartz and Smith (2000) develop a two-factor model, which combines features of mean-reversion process and GBM. They effectively decompose the oil price into two parts, namely, the short-term variation and long run equilibrium level of oil price. In the combined stochastic process, the short-term variation follows a mean-reversion process whereas the long-term dynamics follow a GBM process. Moreover, Askari and Krichene (2008) demonstrate that the oil price is excessively sensitive to the unexpected new such as earthquake or strategic actions by Organization of the Petroleum Exporting Countries (OPEC). As a result, the jump component is introduced to the stochastic process when the asset price is sensitive to the shocks from the environment such as Hilliard and Reis (1998). Aguerrevere (2009) introduces demand shocks into the real options model. Furthermore, Grenadier and Malenko (2010) separate the temporary shocks from permanent shocks by adopting a Bayesian approach to real options.

In addition to the underlying asset price, abundant studies look at other risk factors and extend the classical one-state variable model into a two- or three- state variables model. For commodity related valuation, stochastic convenience yield will always be considered as the second state variable. For instance, Gibson and Schwartz (1990) present a two-state variable real options model, which includes stochastic asset price and stochastic convenience yields. They prove that the two-state variable model is more accurate than one-state variable model for valuing oil contingent claims. Miltersen and Schwartz (1998) establish a three-state variable model for pricing commodity futures options. The three state variables in their models are stochastic asset price, stochastic convenience yields and stochastic interest rates and their model is also under BlackScholes economy. On their basis, Hilliard and Reis (1998) add jump component into Miltersen and Schwartz (1998)s three-state variable model. Detemple and Tian (2002) value a series of American options under different stochastic processes and the two-dimensional model would be preferred. In addition, they also argue that the hardest problem for valuing American options is to tackle the mutildimensional case. They model American options by including stochastic interest rate and stochastic volatility.

Additionally, stochastic volatility will be a pivotal component as the volatility can affect the option value dramatically. As a consequence, stochastic volatility is constantly included into the real options models. Patel and Sing (2000) use

Dixit and Pindycks (1994) model to find the implied volatilities from commercial property in the UK. They find that the implied volatilities vary from 5.06% to 36.43%. So, they vindicate that stochastic volatility shall be included in the real options model when evaluate commercial property. Bond and Hwang (2003) also maintain the same argument, especially for real estate valuation. It is arguable that the two or three dimensional options pricing model generally outperforms the one dimensional model. Furthermore, Trolle and Schwartz (2009) discover that the stochastic volatility factor will be essential for pricing commodity options. Also, Henriques and Sadorsky (2011) have developed a model for testing the effects of changes in oil price volatility on oil companies strategic investments from the real options literature. The empirical results have shown there was a U shape relationship between oil price volatility on oil companies strategic investments.

Other possible risk factors may be also presented in the real options model, such as geological and technical uncertainties. Lund (2000) has valued the option on developed oilfield under two types of uncertainties. One is market risk and the other type of risk is reservoir risk, namely, the uncertainty of the volume contained in the reservoir. They used probability distribution to estimate the reservoir. The uncertainty of the reservoir volume was measured by the variance of the reservoir implied in the distribution. Additionally, Cortazar et al. (2001) evaluate the option value for three different phases under both market risk and geological-technical risk. They summarize all geological and technical factors into one vector G. They model the vector G by using zero-drift GBM and assume it has no correlation with oil price.

However, the existing real options literatures has paid little attention to the liquidity risk that is presented in both commodity and asset markets. The early works such as McDonald and Siegel (1986), Dixit and Pindyck (1994) both assume that the underlying project values only depend on the asset prices. Later on, Cortazar et al. (2001) value the real options for oil related projects under both market risk and geological-technical risk. Boomsma et al. (2012) use the real options approach to assess investment timing and capacity choice of the renewable energy projects using both spot and futures prices.

2.3 Black- Scholes Options Pricing Model and the Perfect Market Assumption

Although the liquidity risk recently has been recognized to be presented in the commodity futures markets, the previous derivative pricing literature usually postulates that the market is perfect and market frictions do not exist. Based on this theoretical foundation, Black and Scholes developed the remarkable model for options pricing in 1973, which became the widely used pricing model in the financial market. Their model works well under the perfect market assumption.

The perfect market assumption quintessentially means two things. First, the market is frictionless, which indicates there are no costs and frictions on the asset trading. The other side is the market is perfectly competitive, which means any trader can buy and sell any amount of securities without influencing the price (Cetin et al., 2004). The perfect competitive market also rests on the assumption that all financial instruments such as common stock have perfect substitutes (Loderer et al., 1991). The perfect market condition could have an inference, that is, the demand of the asset is independent of its price and thus there will be no liquidity risk for asset pricing. It is because the demand curve of an asset in the perfect financial market will be kept flat by arbitraging between substitutes of the asset (Wurgler and Zhuravskaya, 2002). All assets related arbitrage opportunities will be arbitraged away as soon as they appear in the market. Otherwise, arbitrageurs can gain infinite wealth from the market by taking the arbitrage opportunity (Scholes, 1972). Since the demand curve is flat, it makes the asset demand elasticity perfect and infinite. Therefore, liquidity risk becomes irrelevant for pricing the asset under perfect market assumption.

However, many followed researches found the assumption was unrealistic in the real financial markets and tried to relax the perfect market assumption when pricing asset and derivatives. For instance, Merton (1976) addresses the problem of discontinuity of the underlying asset price. Figlewski (1989) examines the inconsistency of the perfect market assumption and the options trade in practice. He concludes the effects of market imperfections were much larger than the researchers may expect. Given the condition, he maintains the effects will lead to an inaccurate options value because the assumption is so weak that the theoretical value of options will depart from the actual price. Dumas and Luciano (1991) highlight the transaction cost was one of the pronounced frictions in the financial market. More importantly, they argue that any presence of market frictions will change the nature of the optimization problem and they use it to study the options pricing model. Longstaff (1995a) elucidates that the models with market frictions perform much better than models without market frictions in the real financial markets. Liquidity cost is treated as one of the important trading frictions in the market and thus it is a vital ingredient composing the market with frictions (Acharya and Pedersen, 2005). Therefore, including the liquidity cost will help asset pricing model to be more accurate and more realistic.

As declaimed by Wurgler and Zhuravskaya (2002), in the perfect market, the demand curve of stock will be kept flat by arbitrage between the perfect substitutes of stocks. However, they question the function of arbitrage and they reveal the fact that most stocks do not have perfect substitutes and arbitrage usually fails to correct the mispriced stock as it is expected to. As a result, the demand curve of stock tends to be declining rather than be flat. Likewise, many other authors such as Wayne and Partch (1985), Shleifer (1986) also affirm that the demand curve of common stock to be declining. This fact, in turn, challenges the demand irrelevance argument for valuing financial instruments and the trading of financial instruments may also have an impact on their price. For instance, Amin et al. (2002) argue that the options price could be influenced by the demand and supply of the options under the imperfect market. Bates (2003) underlines that the empirical features of options prices cannot be seized by the parametric implementations under the perfect market assumption. He appeals that a new way of approaching the options pricing was necessary in order to capture the actions of market participants. Bollen and Whaley (2004) contend that the change in options demand will lead to options price changes as well. Garleanu et al. (2009) justify the importance of the demand-pressure of the options prices. As the options market is not completed because of the stochastic volatility and jumps in the underlying assets (Figlewski, 1989), they conclude that the options price is not independent of the demand.

As the perfect market assumption has been relaxed, it is arguable that the options price can deviate from the theoretical values under perfect market conditions (Figlewski and Webb, 1993; Grossman, 1995). Consequently, a new model needs to be developed in order to provide a more accurate and realistic model of options valuation in the real financial market. Since the perfect market assumption has been relaxed, not only demand and supply will become relevant, but also there will be liquidity risk (Jarrow and Protter, 2007). So, the liquidity adjusted options pricing model could capture the features in the real financial markets and provide more accurate options values.

2.4 The Imperfections of the Oil Market

As liquidity adjusted derivative pricing model could capture the features in the real financial markets and provide more accurate options values, the real options theory will also need to incorporate the liquidity factor. Since financial market imperfections have dramatic influence on the financial options pricing, the real asset market imperfections will also have big impacts on the real options valuation model. Grullon et al. (2012) argue that the industry that has intensive real options application is the natural resource industry, particularly the oil and gas firms. Moreover, the oil market is also an important commodity futures market. Therefore, I take the natural resource market as an example to demonstrate the imperfections of the real market. This demonstration underpins the idea of liquidity adjusted future pricing model and liquidity adjusted real options model because the existence of liquidity risk reflects the crucial part of market imperfections.

Resemble to the options pricing theory, the jump, stochastic volatility and existence of arbitrage opportunities are the three main aspects for discrediting the perfect market assumption (Romano and Touzi, 1997; Dritschel and Protter, 1999). The oil market is not a perfect market mainly in these three dimensions.

Firstly, stochastic volatility of oil price has been adequately evidenced in the literature (Vo, 2009; Larsson and Nossman, 2011). Agnolucci (2009) disagrees with the traditional implied volatility method and he demonstrates that the GARCH model transcends the implied volatility method for forecasting the oil price volatility. Sadorsky (2006) asserts that GARCH (1, 1) model performed better than more complex models such as state space. Furthermore in the literature, many comparisons for the models of oil price volatility were undertaken. Day and Lewis (1993) compare forecasts of oil volatility by employing GARCH(1,1), Exponential GARCH(1,1), implied volatility and historical volatility methods. They find that the GARCH model performs better in out-of-sample test and adds large part of price information to the implied volatility model. Yaziz et al. (2011) study the West Texas Intermediate (WTI) crude oil price by applying the GARCH family models. They conclude that the ARIMA (1, 2, 1) and GARCH (1, 1) are two appropriate models for the crude oil price volatility. Through their further studies and other measures, they compared ARIMA (1, 2, 1) and GARCH (1, 1) models. They find that GARCH (1, 1) model will be better for describing the daily crude oil prices as it can capture the volatility of the non-constant of the conditional variance. As a consequence, the stochastic volatility exists in the oil market. Moreover, Costa and Suslick (2006) demonstrate that the stochastic volatility also exists in the oil projects, especially offshore oil projects and the volatility is much higher for the oil projects than oil price. Chordia et al. (2001) elucidate that volatility will influence liquidity. So, change of volatility will also cause liquidity change over time, which makes liquidity more volatile. As a result, it will be quite essential for the real options model to include liquidity as a source of uncertainty.

Secondly, jumps are often occurring in the oil market and used to model the oil price shock. This is also a source of imperfect market as the price of the underlying asset is not continuing. Hamilton (2003) concludes that the oil price shock will create huge uncertainties for the oil-related assets and has critical implications for firms. Meade (2010) also argues that the model which can properly capture the properties of oil price should be able to include the jump. The jump is mainly utilized to reflect the political and economic instability of the leading oil-producing regions like the Middle East and Africa. Miller and Zhang (1996) develop a GBM model of oil price with jumps. They testify the positive jumps always occurred in peacetime when there is a war and the price tends to revert back after the war ends. They point out that the presence of jumps would increase the oil price volatility, which made the oil price more volatile. Abid and Kaffel (2009) confirm that the GBM model with jump performs better than other stochastic models for oil price by using simulation. Larsson and Nossman (2011) test a number of models and also have the same conclusion that the stochastic model with jumps strongly outperforms other models regarding the oil price. As a result, jumps commonly exist in the oil market and verify the market imperfections. Jumps could also enlarge the liquidity spread, which may lead to liquidity change over time (Kagraoka, 2005). Therefore, liquidity risk may play an important role in reflecting the price discontinuity as a friction in the real asset market.

Finally, the arbitrage opportunities exist in the oil market as well as the demand for oil is not perfectly elastic to the oil price. Crowder and Hamed (1993) and Girma and Paulson (1999) testify the arbitrage opportunities existence in the oil market. Alizadeh and Nomikos (2004) find that the freight price is not related to the oil price, which resists the law of cost of carry relationship. This finding in turn verifies the existence of arbitrage opportunities in the oil market. The demand for oil is not perfectly elastic to the oil price, which has been investigated by the Copper (2003). He studies oil demand elasticity in 23 countries, where demand in most countries behaved negatively against the oil price. The increase of oil price will erode the oil demand. Other papers such as Griffin and Schulman (2005) and Moore (2011) also give the similar results. The presence of arbitrage opportunities may induce arbitrage activities. The arbitrage activities would have two contradictory effects on market liquidity empirically. Choi et al. (2009)

illuminate that arbitrage activities will improve the market liquidity. On the other hand, Roll et al. (2007) substantiate that arbitrage activities may deteriorate market liquidity. The conclusion thereby could be the arbitrage activities have substantial effects on the market liquidity.

To sum up, the oil market is not a perfect market in three dimensions and it might be irrational to value the oil-related real options under the perfect market assumption. Similarly, other commodity markets are not perfect either. Since the market is not perfect, there will be frictions in the real asset markets. As a result, liquidity becomes relevant for real asset related project valuation. Firstly, liquidity factor can be considered as one of the most important trading frictions in the real markets (Brockman et al., 2009; Gavazza, 2009). Secondly, because of the arbitrage opportunities existence, perfect demand elasticity no longer holds, which makes liquidity relevant (Jarrow and Protter, 2007). Finally, the imperfections of the market such as jump will drive the fluctuation of market liquidity and then liquidity becomes a type of risk and uncertainty in the real asset market.

Since the underlying asset of real options is mostly the real assets such as commodity and I will mainly focus on the commodity market. The liquidity risk indeed exists in the commodity market and copious studies attempted to measure it and attest its existence empirically. For instance, one of the widespread measures of liquidity risk is the bid-ask spread (Amihud and Mendelson, 1986). The bid ask spread in the commodity market has been documented in many studies (Hirshleifer, 1988; Wang and Yau, 2000; Bryant and Haigh, 2004; Tse and Zabotina, 2004). Marshall et al. (2011) measure the commodity market liquidity risk in a number of ways and give the actual liquidity cost of trading the commodity. Moreover, Marshall et al. (2013) demonstrate that there is a distinct liquidity factor in the commodity market and the liquidity factor is related to the commodity price. They also assert that the neglect of the liquidity risk in the commodity market will lead to huge losses of companies and funds.

Furthermore, liquidity not only exists in the futures market, but also in the options market. Cao and Wei (2010) verify liquidity existence in the options market. Based on their study, Chou et al. (2011) testify the impact of liquidity on options price and they concluded that liquidity has a big impact on the options price and options price volatility. Further, their conclusion lined with Cetin et al. (2006) that every option has an intrinsic and significant liquidity cost. As a result, neglecting the liquidity cost of the options will lead to inaccurate results. Since liquidity is so crucial and it has influences on the options price and volatility, including liquidity risk can make the real options model more applicable and reliable. Brenner et al. (2001) test the liquidity effects on options empirically and they reject the hypothesis that liquidity has no effects on the options prices. Pastor and Stambaugh (2003) also affirm that the liquidity risk was an important element for asset pricing and the asset price is quite sensitive to liquidity. Consequently, including the liquidity risk as a proxy to reflect the imperfect market conditions in both futures pricing model and real options model will be excessively necessary.

2.5 Asset Liquidity and Derivative Pricing

It is clear that the underlying asset of commodity futures is real assets such as oil and metals. On the other hand, the value of the project is the underlying of the real options model and the project value depends on the future cash flows it can create, while the future cash flows come from the sale of the outputs such as commodity, which are real assets. Therefore, the value of the project and the real asset values are highly correlated. In copious classical works of real options, they are perfectly linked (Dixit and Pindyck, 1994). Many real options literatures thereby directly used the output of the project as the underlying asset. For instance, Brennan and Schwartz (1985) apply the real options framework to value copper mines and they used copper as the underlying asset in their model. Paddock et al. (1988) used the real options method to evaluate the offshore oil fields and they use the oil as the underlying asset. Grenadier (1996) uses the real options method to evaluate the real estate value by using the housing price as the underlying asset price. Another example of the real options underlying asset could be the aircraft and many scholars tried to extend the real options theory into the aircraft valuation and the aircraft is considered as the underlying asset (Stonier, 1999; Mun, 2002; Gibsona and Morrell, 2004).

Accordingly, it is arguable that the underlying assets of commodity futures and real options are basically real assets. This thesis aims to build up a liquidity adjusted futures pricing model and a liquidity adjusted real options model. As a result, the overview of the asset liquidity measurement would be imperative. Unlike the centralized-traded financial assets such as stocks, one of the common features of those real assets is that they are mainly traded in the decentralized markets such as Over-the-Counter (OTC) markets, which are influential markets in the modern economy (Childs et al., 2001). Without the facilitation of the centralized trading system, the trading structure of the OTC markets makes the liquidity effects even more noticeable and also investors cannot buy or sell the asset instantaneously in the OTC markets. In turn, they have to trade the asset through a searching and bargaining process, which underlines the liquidity effects (Jankowitsch et al., 2011). Since the liquidity effects are noticeably important and dramatic in the OTC markets, the liquidity premium will be much higher (Ang et al., 2013). Hence, it is rather important and interesting to research liquidity effects in the markets that have more visible liquidity risk. Further, OTC markets are an important part of the modern economy, especially for real asset trading. For example, the trading amount for real assets such as commodities in OTC markets is extraordinarily large. Like Bank for International Settlements (BIS) has reported that the total amount of commodity contracts traded in the OTC markets was 3,197 billion dollars in 2011 and 2,994 billion dollars in 2012 (BIS, 2013). Duffie et al. (2005) also corroborate that plenty of real asset trading within the OTC markets and the OTC markets played an important role in asset trading.

There are many theories trying to explain and model the liquidity effects in the financial markets. A large number of literatures have assigned the liquidity effects to the information asymmetry and used the information-based theory to explain asset liquidity (Glosten and Milgrom, 1985; Easley et al., 1996; Koski and Michaely, 2000; Lester et al., 2012). Dufour and Engle (2000) also agree that the trade will affect the asset price because it contains tremendous information. Further, plentiful scholars also confirm the close relationship between liquidity and information, such as Diamond and Verrecchia (1991), Hasbrouck and Seppi (2001). They maintain that trades generally reflect the information of price expectations in the near future.

However, other scholars disagree with this view and they demonstrate an ambiguous relationship between the information asymmetry and the asset liquidity (O'Hara, 2003; Garleanu and Pedersen, 2004; Vayanos and Wang, 2012). Furthermore, Biais (1993) compares the fragmented markets with centralized markets, and he finds, surprisingly, the expected bid-ask spread is much smaller in fragmented markets and traders prefer to trade in the fragmented markets. Based on his findings, it is clear that information transparency in the centralized market does not improve the asset liquidity. Therefore, the information-based theory cannot explain the liquidity well enough.

On the other hand, many existing literatures have linked the asset liquidity with search theory (Hirshleifer, 1968; Vayanos and Wang, 2007; Lagos and Rocheteau, 2009). The basic idea of search theory is that trading will take place when traders are successfully matched through a searching and bargaining process (Vayanos and Wang, 2007). The search theory can be applied in both centralized and decentralized markets since the successful trading in each market based on the successful matching. Compared with centralized market, it will be more difficult to find the trading partner in the decentralized market. Consequently, the search cost will be more considerable in the OTC markets (Duffie et al., 2005). Moreover, Duffie et al. (2007) suggest that the searching and bargaining abilities of investors are quite important for trading in the OTC markets. Similarly, Lagos et al. (2011) point out that the search and bargaining is particularly relevant to the OTC markets. They argue that in the OTC markets, investors need to find the counterparty to process the trade, which involves searching and the trading price is set through bilateral negotiation. As a result, searching theory is particularly relevant in modeling the liquidity effects.

Since the asset liquidity effects are related to searching and bargaining and search theory has been used to explain the liquidity within financial markets, it is necessary to identify the intrinsic link between the liquidity effects and the search theory. Vayanos and Wang (2007) propose a new idea that they use the standard search model developed by Diamond (1982) to address asset liquidity relies on the fact that it will take time for traders to find counterparties inside the decentralized markets. They argue that more liquid assets involve more buyers and sellers, which leads to higher trading volume and less trading time, and high trading volume and short trading time imply less search cost. It will be much easier for traders to find the counterparties in the markets with more buyers and sellers and the trading will be more likely to be successful. According to Krainer (2001), liquidity can be defined as how fast the asset can be traded at the prevailing market price. Therefore, the time of trade becomes an important dimension of the liquidity measure, which has also been mentioned in a number of literatures such as Kyle (1985) and Hallin et al. (2011).

Therefore, this conclusion leads to our first dimension of the asset liquidity: the time dimension, which can be measured as the trading speed. Furthermore, the time dimension of asset liquidity is associated with the number of traders in the specified market. The asset with more traders will be traded faster and tend to be more liquid. As a result, the number of potential buyers could be served as one measure of the real asset liquidity from the time dimensions (Benmelech and Bergman, 2009). Likewise, Gavazza (2011) defines the number of potential buyers of a particular aircraft as the liquidity of that aircraft. He maintains that this kind of liquidity can be considered as a cost arising from the market and this cost will affect the investment behavior. According to Ortiz-Molina and Phillips (2014), they use three kinds of asset illiquidity measurements. Similar to Gavazza (2011), they also adopt the number of potential buyers of a firm's asset as the first measure of asset illiquidity. The second measure they use is the average book leverage net of cash of the rivals within the same industry. The final measure is the value of M&A activity in the firm's industry scaled by industry assets with a minus sign, and Sibilkov (2009) also adopts the similar measure. Specifically, the Time-on-the-Market (TOM) is a popular measure of real estate liquidity, which is also from the time side of asset liquidity. Caplin and Leahy (2011) agree that the market tightness is directly connected with housing liquidity and the liquidity of real estate can be defined as TOM. Since liquidity can be considered as a trading friction in the imperfect market, it can be broadly defined as the ease of trading, which involves finding the trading partners (Weill, 2008). Lagos and Rocheteau (2009) maintain that the trading delays are an exemplary characteristic of OTC markets. Since less trading time implies less searching cost and higher trading volume, the trading time can be used as a proxy of liquidity in the decentralized markets, which, in turn, can be measured by searching time (Vayanos and Wang, 2007). Similarly, Lippman and McCall (1986) point out that the time that an asset has been exchanged for money is the most important dimension of the asset liquidity and this idea also consistent with other liquidity definitions.

Another type of conceptual measures of liquidity could be trading delay in the OTC market, which is also focused on the time dimension. As Lagos and Rocheteau (2009) maintain that trading delays are one of the important aspects of market liquidity and correlated with trading volume. Pagano (1989) also highlights the close relationship between asset liquidity and trading volume. Karpoff (1987) also discovers that trading volume will increase with the growing number of active traders. According to Vayanos and Wang (2007), the emergence of the imperfectly liquid asset of the other group depends on the demand condition within the market, especially when there is a supply surplus. To be specific, there are μ_b^i active buyers and μ_s^i active sellers in the market for the real asset type i. The measure of market thickness thereby is $MktTk = \frac{\mu_b^i}{\mu_b^i + \mu_s^i}$. They also assume that sellers seek the buyers to make the trade as a Poisson process with arrival rate λ. Therefore, the seller meets the buyer at the rate $\lambda \mu_b^i$. In their paper, the liquidity is measured by the expected time for sale as $1/\lambda \mu_b^i$. Since the expected time measure is connected with active buyers and the active buyers is also related to the trading volume. It will be justifiable to assume that trading volume can be a proxy for indicating the expected time of sale. The higher the trading volume is, the fast the asset can be sold.

The early work of liquidity analysis, such as Garbade (1982), Kyle (1985) and

Harris (1990), points out three main dimensions of liquidity: spread, depth and resiliency. Likewise, Hallin et al. (2011) claim that trading time, transaction cost and trading volume are three crucial dimensions of liquidity concept. Moreover, Liu (2006) identifies four aspects of liquidity, which are trading amount, trading speed, trading cost, and price impact. For commodity markets, the liquidity measures like Amihud measure, Roll measure, as well as trading speed and trading volume, can be applied well since the trading is traceable in those markets.

Moreover, the output of the real projects are usually real assets, which could also be connected with the transaction cost and price discount dimension. As mentioned before, the eventual purpose of investing in a project is to sell the project output and get future cash flows and investment return if the project is assumed to be alive. The output needs to be traded in the market as an asset and thereby the asset price is the key issue shall be concerned. The asset price dispersion is a common phenomenon in the financial markets (LeRoy and LaCivita, 1981). Jankowitsch et al. (2011) also mention that the same asset can be traded in different prices at nearly the same time in the OTC markets. They attributed these dispersions to the trading friction in terms of inventory and search costs and the trading friction, which can be also interpreted as the liquidity effect. They also argued that in a highly liquid market, the dispersion effects are imperceptible and investor will identify those effects as transaction costs and take them into account when making investment decisions. Vayanos and Wang (2007) also show that liquidity can be translated into asset price differentials through search costs. Duffie et al. (2005) assign trading frictions to searching and bargaining process in the OTC markets and prove their significant effects on asset prices. Accordingly, the search theory is not only linked with liquidity effects, but can also consider the impact of asset liquidity, in terms of searching time, on asset prices. As mentioned in Rocheteau and Wright (2013), the asset liquidity has impacts on asset trading and asset price, and the search theory can explain them fairly well.

Although the asset liquidity effect turns out to be rather important in asset trading and asset pricing, the futures pricing model and real options model literatures do not show enough concerns about it. Like Williams (1995) criticizes that a large number of literatures on real options ignored the part of costly searching when they apply the options pricing theory to value real assets. More recently, Duffie et al. (2007) argue that asset pricing in the market with searching frictions should develop a model to reflect this friction.

The recent developments of futures pricing model and real options work have not identified this issue specifically. Thijssen (2010) adopts real options method into game theory to examine the first-mover advantage. Zhu and Lian (2012) develop a futures pricing model for VIX futures with jumps in both the asset price and volatility stochastic processes. Hwang et al. (2013) apply the real options theory to analyze the relationship between higher education and unemployment rates. Bouvard (2014) finds the real options theory is useful in explaining adverse selection effects when companies try to finance in the capital market. Benth et al. (2014) build an electricity futures pricing model by adopting the stable Controlled Autoregressive Moving Average Model (CARAM) to establish the electricity spot price process. In addition, Hackbarth and Johnson (2015) examine the effects of changes in productivity and production technology on firms' equity risk and the expected return under the real options framework. Therefore, adding the liquidity factor to the futures pricing model and real options model might be supplementary to the existing literatures.

Therefore, I aim to build a liquidity adjusted futures pricing model along with a liquidity adjusted real options model. Through building up the model, I try to learn how futures prices and real options values respond to the change of asset liquidity and how sensitive they are. How asset liquidity might impact on the real options exercise boundaries and compare the exercise boundaries for different scenarios. Based on the comparison of exercise boundaries, I further analyze how the investment decisions have been influenced by the asset liquidity.

Chapter 3

Liquidity Effects on Prices and Returns Co-movement and Co-integration in Commodity Futures Markets

3.1 Introduction

3.1.1 Research Background and Related Literatures

Commodity price behaviors, especially commodity futures price behaviors, have attracted large academic attentions. It stems from the fact that commodity price movements not only influence countries trading balances, but more importantly, have implications towards their fiscal and monetary policies. Since the commodity is traded continuously in the financial market, the price movement can provide useful insights of the economic situation (Cody and Mills, 1991). For instance, if commodity prices rise too fast, then, the economy might face the risk of inflation acceleration. If policy makers notice the risk, they might adopt restrictive monetary policies and inflation might be controlled in advance. In fact, trading activities and investor behaviors in the futures market play important roles in affecting futures prices as well as price volatility (Chatrath, Ramchander, and Song, 1996). The trading activities were demonstrated to be closely related to the market liquidity in the stock market (Chordia, Roll and Subrahmanyam, 2001). On the other side, market liquidity tends to be influential for individual asset returns in the stock market (Avramov, Chordia and Goyal, 2006). More importantly, market liquidity might have important regulatory implications for financial markets (Bloomfield, O'Hara and Saar, 2015). As a result, I aim to study the liquidity effects in the commodity futures markets and to deliver the implications for both price behaviors and market regulations and policies in this chapter.

On the other hand, existing literatures mainly focus on the integral relations be between commodity prices and economic variables. Cody and Mills (1991) illustrate the close relationship between commodity prices and economic indicators such as the consumer price index (CPI). Their empirical results conclude that commodity prices can deliver early information of the current state of the economy. They affirm that use commodity prices as indicators to formulate monetary policy could improve the economic performance. Clarida et al. (1998) demonstrate that commodity prices are firmly connected with inflation, interest rates and outputs. Stock and Watsonl (2003) argue that commodity prices can serve as predictors of inflation and output growth. Bernanke, the former chairman of the Federal Reserve, (2008) suggests that commodity price movements exhibit tremendous influence on monetary policy. He demands a better understanding of the factors that drive commodity prices. Insufficient understanding of commodity price impetuses will result in misleading monetary policy and encountering investment losses. Hong and Yogo (2012) contend that inflation and exchange rates could be forecasted by analyzing commodity futures prices. Chinn and Coibion (2014) also point out that understanding the movement and changes of commodity prices could be helpful in near future policy formulation. Moreover, Bhar and Hamori (2008) present the empirical results to show that commodity prices would be informative in formulating monetary policy. They argue that commodities are the primary source of industrial inputs.

Therefore, commodity price movements have direct impact on the price level and understanding commodity price movements becomes important. Because the close relationship between commodity prices and the inflation level, the commodity price level can be considered as intermediate indicators for monetary policy (Garner, 1989). The monetary authorities adjust monetary policies in advance according to the commodity price movements. Garner (1989) also maintains that the rocketing of commodity prices has a signal effect in economic overheating and high inflation level. Awokuse and Yang (2003) adopt U.S. data to empirically prove that the commodity prices not only influence the consumer price index but also the industrial production index. Similarly, Gospodinov and Ng (2013) provide statistical evidence that commodity prices are robust indicators of inflation rate. As a result, understanding of commodity prices, especially the determinants of commodity price movements will provide insights of the economic state and convey helpful information on formulating monetary policies.

Since commodity prices can provide implications towards monetary policy and economic state, the commodity price co-movement may give even richer information. The existence of the commodity price co-movement has been well documented. Pindyck and Rotemberg (1990) develop the excess co-movement hypothesis and examine the commodity returns co-movement. They demonstrate the commodity returns co-movement by using the data from 1960 to 1985 for seven commodities (wheat, cotton, copper, gold, crude oil, lumber, and cocoa). Ai et al. (2006) display strong evidence of the agricultural commodity prices co-movement by adopting the commodity data of wheat, barley, corn, oats and soybeans commodities from 1957 to 2002. Natanelov et al. (2011) suggest that the rise of crude oil price should be responsible for the increasing agricultural commodity prices. Byrne et al. (2013) emphasize the necessity of understanding the commodity price co-movement. More importantly, they argue that the co-movement plays an informational role in social welfare for commodity importers. As a result, it will also be useful for China since China is a large oil importer worldwide (Zhang et al., 2013). Casassus et al. (2013) support the argument that commodity prices not only rely on commodity characteristics such as return and convenience yield, but also on the fundamentals of other related commodities, like production relationship. Daskalaki et al. (2014) try to find out the common factors in determining cross-sectional commodity futures return and they conclude that none of the employed factors can explain the cross-section commodity future returns. Since commodity prices have monumental effects on economy, the co-movement of commodity prices will have even larger effects.

In order to understand the commodity price movements, plentiful researchers scrutinize the impact of macroeconomic variables on commodity prices. Svensson (2008) argues that the increase of real interest rates will decrease the future values of commodities due to the rise of the discount rate. Likewise, Akram (2009) uses quarterly data from 1990 to 2007 to empirically examine the impact of the real interest rate change on commodity prices. He attributes the increasing commodity prices to the low real interest rates. Vansteenkiste (2009) uses 32 commodities from 1957 to 2007 for the empirical test and finds that the global demand, exchange rate and real interest rates play significant roles in determining the commodity prices. Byrne et al. (2013) also provide evidence that real interest rate is a determinant of commodity prices.

Nevertheless, most existing literatures focus on the macroeconomic variables explanation and little work has been dedicated to the commodity market conditions. In the financial asset pricing area, the impact of liquidity risk on asset price has received substantial attention recently. A range of liquidity adjusted asset pricing models have been developed, especially Capital Asset Pricing Model (CAPM) with liquidity. Liquidity risk may not be recognized as part of the market risk but as an augmentation of the market risk (Sadka, 2006 and Liu, 2006). In addition to the financial asset, plentiful studies pay attention to the impact of liquidity risk on derivative pricing more recently, such as Chou et al. (2011), Bongaerts et al. (2011) and Feng et al. (2014). Furthermore, a large number of literatures demonstrate the existence of liquidity commonality in both stock and commodity markets (Chordia et al., 2000; Korajczyk and Sadka, 2008; Marshall et al., 2013; Frino et al., 2014). So I investigate the relationship between liquidity risk and the residual part of the market risk and I find that the liquidity risk can explain most part of the market risk residuals.

3.1.2 Research Purposes and Research Methods

Since the existing literatures have not explored the causal explanatory variables of commodity prices movements on daily basis other than the macroeconomic variables on low frequency basis, such as monthly data or quarterly data. The main target of this chapter is to determine that the market liquidity could serve as the microeconomic variables for explaining the daily price co-movement and co-integration. Furthermore, I also show that liquidity shocks may contribute to the excessive returns co-movement after I control the market risk, which is the common risk factor.

Moreover, I identify the causal relationship between commodity liquidity commonality and the commodity price movements by adopting the Granger test. The Granger test was developed by Granger (1969) to study the causality between two time series. This approach has also been widely applied in the financial field. For instance, Hiemstra and Jones (1994) use the linear and non-linear Granger test to examine the relationship between aggregate stock price and trading volume. They conclude that there is a significant non-linear Granger causality relationship between aggregate stock price and trading volume. Moreover, Bhar and Hamori (2008) apply the approach to investigate the causal relations among the Reuters-CRB index (represent for commodity prices), the consumer price index (CPI) and the industrial production index (IP). They discover that CRB is the cause of both CPI and IP, however, CPI and IP cannot well explain CRB. Fernandez (2014) uses the Granger test to examine the causality between four U.S. price indices and 31 commodity series and he argues that the causality is strong from individual commodities to price indices. Other macroeconomic variable such as exchange rate also has been investigated by adopting Granger causality analysis (Sadorsky, 2000). In addition, the causality within the commodity prices are also well studied, especially the relationship between oil price and other agricultural commodity prices (Nazlioglu, 2011; Nazlioglu and Soytas, 2012).

In order to scrutinize the commodity prices behaviors, I also conduct the co-integration test, which is also widely applied in the commodity futures market studies. The co-integration test has been emphasized in the financial literatures such as Engle and Granger (1987) and Brenner and Kroner (1995) because it produces useful information about market price trends and for financial model implementation. Zhang and Wei (2010) show a significant co-integration relationship between the crude oil price and gold price and they imply a long-term equilibrium interaction for the two markets. Nazlioglu and Soytas (2012) demonstrate the co-integration relationship between the crude oil price and twenty four world agricultural commodity prices. On their basis, I adopt a wider co-integration relationship test, including crude oil price, gold price and agricultural commodity prices. The puzzle is that the commodity markets are supposed to be co-integrated. However, after confirming the Granger relations among those markets, the results from co-integration tests tend to be non-stationary, which indicates the non-cointegrated relations. Surprisingly, when I control the liquidity variables from the residual part, the residuals of co-integration regression become stationary. These results are complementary to those arguments that liquidity contains most of the market noise from the commodity futures markets perspective.

Therefore, I argue that after controlling the liquidity variables all markets exhibit co-integration relationships because liquidity is an inference variable. The liquidity in commodity markets contains abundant information for a number of reasons. Firstly, market liquidity can impact the behaviors of commodity traders (see Bertsimas and Lo, 1998; Huberman and Stanzl, 2005). Secondly, liquidity is closely related to the demand and supply of the risky assets (Obizhaeva and Wang, 2013). According to Sockin and Xiong (2015), the supply shock can be viewed as noise information in the commodity markets. The demand/supply of the asset and the flow of investment to the commodity markets can be reflected in the market liquidity. As a result, liquidity can serve as a proxy for market noise information. So I conclude that crude oil price, gold price, copper price and agricultural commodity prices are all co-integrated without the liquidity factor inference. More importantly, the liquidity factor contains a large number of market noise information, which is aligned with the results presented by Greene and Smart (1999) and is complementary to the results presented by Sockin and Xiong (2015). On the other side, it can be also argued that the liquidity variables are co-integrated with commodity futures prices, which also implies the vital role that liquidity plays in the commodity futures market.

In order to investigate the effects of liquidity shocks on residual movements of futures returns after controlling the market return, I introduce the liquidity innovations as the liquidity shock. Liquidity innovations come from the residual of autoregression AR (1) model for liquidity dynamics. Based on the findings, it is justifiable to regard the liquidity shock as a risk factor in the two factor regression model, in addition to the market risk.

3.1.3 Research Contributions

In this chapter, I elaborate the role of liquidity in the commodity futures market from three main perspectives. Firstly, the commodity market liquidity is a determinant of commodity prices co-movement in different commodity futures markets. Motivated by the first finding, I further investigate the liquidity explanatory power. I find that the liquidity risk, which I define it as the liquidity innovation, is closely related to the residual risk part that is not explained by the market risk. Since the liquidity risk is linked with the residual information, I argue that the market liquidity incorporates most of the noise information in the commodity futures markets. Likewise, Hu et al. (2013) point out the informational role of market noise in the U.S. bond market and they suggest that the noise could be served as a measure of overall market illiquidity. Then I connect the informational role of liquidity with the co-integration tests of commodity futures. The co-integration tests of the selected commodity futures markets turns out to be non-stationary. Considering the informational role of market liquidity, I then control the liquidity variables for the co-integration tests, and thereby the residuals of the co-integration regression become stationary. The stationary residuals imply the long-run co-integration relationship between five commodity futures markets.

Based on my findings, I build the linkage between macro-liquidity and micro-liquidity through commodity futures markets. Chordia et al. (2005) establish the link between macro-liquidity and micro-liquidity in the bond and stock markets and Belke et al. (2010) give implications of global liquidity towards the commodity prices. Correspondingly, I declare that the macro-liquidity level and commodity futures market liquidity can be connected. In the first place, Browne and Cronin (2010) claim that the commodity prices vary proportionally to the money supply, which can be defined as the macro-liquidity, in the long-run. Then, I demonstrate that the commodity prices movement is affected by the commodity futures market liquidity, which can be defined as the micro-liquidity. Therefore, the linkage between macro-liquidity and micro-liquidity has been established through commodity futures markets. The macro-liquidity is closely related to the monetary policy formulation and the micro-liquidity is correlated with the trading actives in the futures market. Thus, the trading activities in the futures market might deliver implications to the policy makers.

As a result, in this chapter, my results can help researchers to learn the properties of futures market liquidity and give implications towards monetary policy. The chapter is organized as follows. In section 3.2, I describe the data and suitable liquidity measure for commodity markets. In section 3.3, I investigate the commodity futures prices co-movement, the liquidity commonality among commodity futures markets and test the relationship between the two. In section 3.4, I discuss the liquidity effects on futures markets co-integrations. In section 3.5, I demonstrate that the liquidity is firmly connected with the risk part that is not explained by the market risk and I analyze the role of liquidity shocks in the commodity futures markets. In section 3.6, I give the research implications and make the conclusions.

3.2 Data and Methodology

According to Marshall et al. (2013), there are five families of commodities, namely, energy (such as crude oil, (oil thereafter)), agricultural (such as corn), livestock (such as live cattle), precious metals (such as gold) and industrial metal (such as copper). I pick up one commodity from each family (as indicated in the brackets) to construct a cross-sectional commodity portfolio. The representative commodities are the actively traded commodities. The trading volume of crude oil, corn, copper, live cattle and gold is the highest of each section (Kowalski, 2014). The commodity prices are the spot prices, which are the nearest maturities of the particular commodity. For commodity liquidity measure, Marshall et al. (2012) test a large number of liquidity proxies for 19 commodities. They find that the Amihud liquidity proxy has the maximal correction ratio among all proxies and they strongly recommend researchers to use this proxy when modeling commodity liquidity. As a result, I will use the proxy mentioned in Amihud (2002) and it takes the form:

$$Amihud_t = \frac{|R_t|}{Vol_t}$$

where R_t is the asset return at time t and Vol_t is the asset trading volume at time t.

Intuitively, when trading volume is high, the amount of liquidity measure is small and the asset is denoted to be more liquid. It is clear that the Amihud measure extracts the information from the trading volume. I use five commodity prices and the Amihud liquidity measure as the two main variables.For the data description, the superscript 'ca' represents live cattle commodity futures, 'co' represents corn commodity futures, 'c' represents copper commodity futures, 'o' represents oil commodity futures, 'g' represents gold commodity futures and 'ML_t' represents market liquidity indicator. 'L' stands for the Amihud measure of liquidity, 'P' stands for the commodity price and 'r' stands for the return of commodities. The data provider is Thomson Datastream and the data period is from 1st Jan, 2005 to 31st Dec, 2013, which is the maximal available data for the common period. I normalize the commodity prices by taking the natural log. All data are collected on daily basis.

3.3 Price Co-movements and Liquidity Commonality

3.3.1 Commodity Futures Price Co-movements

Firstly, I plot all five commodity prices and returns in Fig 3.1 and Fig 3.2. From Fig 3.1, it is clear that the commodity prices co-move with each other and exhibit a common moving trend. For instance, all commodity prices display a downward trend during the period of late 2008 to early 2009. Another period from middle 2010 to early 2011, all commodity prices display an upward trend. As a result, the co-movement exists among the commodities. In addition, high and low returns of different commodities occur at similar dates. It is also clear from Fig 3.2 that the returns exhibit a similar co-movement. Then, I conduct a series of correlation matrices of the variables for the five commodities. It is clear that the correlation between commodity prices is positively correlated and all the correlation coefficients are larger than 0.5 (see Table 3.1). The p-value of the strong cross-sectional positive relationship between commodity prices is also presented to be statistically significant. The cross-sectional correlations might intimate that all the spot commodity prices move in the same direction and have positive impacts on each other, which can explain the commodity price co-movements. It is arguable that the prices of different commodities are firmly and positively correlated and the correlation may result in price co-movement. In this section, I will determine the cause of price co-movement and I will discuss the return co-movement in section 3.5.

3.3.2 Liquidity Commonality in Commodity Futures Markets

The commodity futures prices has been demonstrated to co-move over the sample period. Next, the commodity futures liquidities also present a common trend, which I denote as the commodity futures liquidities commonality. The correlation

	$\ln P_t^{ca}$	$\ln P_t^{co}$	$\ln P_t^{c}$	$\ln P_t^{o}$
$\ln P_t^{co}$	0.7452			
	(0.00)			
$\ln P_t^{c}$	0.5458	0.6706		
	(0.00)	(0.00)		
$\ln P_t^{o}$	0.6615	0.7065	0.7825	
	(0.00)	(0.00)	(0.00)	
$\ln P_t^{g}$	0.7624	0.6529	0.8792	0.6371
	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.1: Spot commodity prices correlation matrix with p-value

matrix for the five commodity liquidities is presented in Table 3.2. For the spot Amihud measure correlation matrix in Table 3.2, all the commodity liquidities are positively correlated and all the coefficients are statistically significant. It is thereby arguable that the spot commodity liquidity has a common part.

	L_t^{ca}	L_t^{co}	L_t^{c}	L_t^{o}
L_t^{co}	0.5004			
	(0.00)			
L_t^{c}	0.5217	0.4088		
	(0.00)	(0.00)		
L_t^{o}	0.5172	0.2507	0.8565	
	(0.00)	(0.00)	(0.00)	
L_t^{g}	0.4342	0.4454	0.5793	0.5080
	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.2: Spot commodity liquidities correlation matrix with p-value

Furthermore, it is arguable that the liquidity commonality is correlated with market liquidity, which can be measured through equally weighted average liquidity (Chordia et al., 2000). Enlightened by Alquist and Coibion (2014), who decompose the productivity shocks into common and idiosyncratic parts and each part has its own impact on the price movement. More importantly, Chinn and Coibion (2014) assert that liquidity mainly varies in a systematic way. I therefore propose that the commodity liquidity could have two components, one is the market liquidity component and the other is the idiosyncratic component. The market liquidity component is the main reason why mainly varies in a systematic manner. The commodity liquidity thereby can be decomposed as:

$$L_{c,t} = \lambda * ML_t + (1 - \lambda) * IL_t$$

, where $L_{c,t}$ is the commodity liquidity and λ is the weights of the two components, ML_t is the market liquidity and IL_t, is the idiosyncratic part of commodity liquidity. Since all the correlations are positive, the coefficient λ should be positive and thus, $0 \leq \lambda \leq 1$.This is one possible way to explain why the commodity prices are connected with liquidities of other commodities. When the liquidity measurement varies, it has a synthetical effect, by representing the variation of both market liquidity and idiosyncratic liquidity.

3.3.3 The Linkage between Price Co-movements, Micro Liquidity and Macro Liquidity

Since the commodity prices tend to co-move and the liquidity from five commodity markets also exhibit commonality, I testify that the liquidity commonality can serve as an explanatory variable for the commodity prices co-movements. Firstly, I test whether the individual commodity futures prices are fully integrated with CRB market index. The results turn to be negative, where the residuals for all five commodity futures prices are not stationary (see Table 3.3). Then, I adopt the correlation test, which is similar to Leybourne et al. (1994)s paper, by setting commodity liquidity as the determinant variable instead of macroeconomic variable and regress the commodity price with all the commodity liquidity measured by Amihud as well as the CRB market index. In order to deal with the endogeneity, I utilize the lag of liquidity as the independent variable. The regression equations and results are shown in Table 3.4.

	${\varepsilon_t}^{ca}$	${\varepsilon_t}^{co}$	${\varepsilon_t}^{c}$	${\varepsilon_t}^{o}$	${\varepsilon_t}^{g}$
DF	-0.54	-1.81	-1.11	-1.31	-1.03
	(0.88)	(0.38)	(0.71)	(0.62)	(0.74)

Table 3.3: Dicky-Fuller test for five commodity futures with CRB market index

$$\ln P_t^{ca} = \alpha_{11} L_{t-1}^{ca} + \alpha_{12} L_{t-1}^{c} + \alpha_{13} L_{t-1}^{co} + \alpha_{14} L_{t-1}^{o} + \alpha_{15} L_{t-1}^{g} + \alpha_{16} \ln P_t^{CRB} + \varepsilon_t^{ca}$$
$$\ln P_t^{c} = \alpha_{21} L_{t-1}^{ca} + \alpha_{22} L_{t-1}^{c} + \alpha_{23} L_{t-1}^{co} + \alpha_{24} L_{t-1}^{o} + \alpha_{25} L_{t-1}^{g} + \alpha_{26} + \ln P_t^{CRB} + \varepsilon_t^{c}$$
$$\ln P_t^{co} = \alpha_{31} L_{t-1}^{ca} + \alpha_{32} L_{t-1}^{c} + \alpha_{33} L_{t-1}^{co} + \alpha_{34} L_{t-1}^{o} + \alpha_{35} L_{t-1}^{g} + \alpha_{36} \ln P_t^{CRB} + \varepsilon_t^{co}$$
$$\ln P_t^{o} = \alpha_{41} L_{t-1}^{ca} + \alpha_{42} L_{t-1}^{c} + \alpha_{43} L_{t-1}^{co} + \alpha_{44} L_{t-1}^{o} + \alpha_{45} L_{t-1}^{g} + \alpha_{46} \ln P_t^{CRB} + \varepsilon_t^{o}$$
$$\ln P_t^{g} = \alpha_{51} L_{t-1}^{ca} + \alpha_{52} L_{t-1}^{c} + \alpha_{53} L_{t-1}^{co} + \alpha_{54} L_{t-1}^{o} + \alpha_{55} L_{t-1}^{g} + \alpha_{56} \ln P_t^{CRB} + \varepsilon_t^{g} \tag{3.1}$$

	$\ln P_t^{ca}$	$\ln P_t^{c}$	$\ln P_t^{co}$	$\ln P_t^{o}$	$\ln P_t^{g}$
L_{t-1}^{ca}	-3.04***	-0.84***	-2.62***	-1.91 ***	-3.75***
	(0.02)	(0.37)	(0.48)	(0.21)	(0.49)
L_{t-1}^{c}	-0.05***	-0.01	0.42***	-0.07***	-0.32***
	(0.02)	(0.03)	(0.04)	(0.01)	(0.04)
L_{t-1}^{co}	-0.68***	-0.75***	-4.01***	-1.31***	-5.21***
	(0.11)	(0.17)	(0.22)	(0.09)	(0.23)
L_{t-1}^{o}	-17.27***	-277.67***	-123.91***	-154.32***	-246.51***
	(9.61)	(15.11)	(19.91)	(8.43)	(20.16)
L_{t-1}^{g}	-4.47***	-0.95***	-22.6***	-5.51**	-20.39***
	(0.32)	(0.49)	(0.65)	(0.27)	(0.65)
$\ln P_t^{CRB}$	0.13***	0.77***	1.06***	1.04***	-0.25
	(0.02)	(0.03)	(0.05)	(0.02)	(0.46)

Table 3.4: Regression results for spot prices and lagged spot liquidity

where *,**,*** indicate statistical significance at 10%,5% and 1% levels, respectively.

	ε_t^{ca}	ε_t^{co}	ε_t^{c}	ε_t^{o}	ε_t^{g}
DF	-1.85	-3.61	-3.57	-4.96	-3.67
	(0.35)	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.5: Dicky-Fuller test for five commodity futures with CRB market index by controlling liquidity variables

From Table 3.4, it is observable that most coefficients are statistically significant. As a result, not only the liquidity of the commodity itself can explain the its price variation, but also liquidity of other commodities from different families can explain even after controlling the market index variance. More importantly, most the signs of the coefficients are negative, which illuminate that liquidity might drive the commodity prices to the same direction. The cross-sectional explanatory power provides cognition that there might be a common driving force.

	$\ln P_t^{ca}$	$\ln P_t^{c}$	$\ln P_t^{co}$	$\ln P_t^{o}$	$\ln P_t^{g}$
ML_{t-1}	-1.31***	-2.72***	-2.99***	-2.42 ***	-3.61***
	(0.40)	(0.07)	(0.11)	(0.06)	(0.11)
$\ln P_t^{CRB}$	0.75***	0.77***	0.99***	1.07 ***	-0.23***
	(0.03)	(0.06)	(0.03)	(0.02)	(0.06)

Table 3.6: Regression results for commodity price and Amihud market liquidity measure lag with CRB market index

where *,**,*** indicate statistical significance at 10%,5% and 1% levels, respectively.

Then, I test whether the residual of the regression after controlling the liquidity variables tend to be stationary compared with previous equation. From Table 3.5, it is clear that expect the residuals of cattle futures price, all other residuals become stationary. Compared with Table 3.3, it can be shown that liquidity may include the information that is involved in the residual part, but not incorporated in the market price index.

$$
\begin{aligned}
\ln P_t^{ca} &= \alpha_1 + \beta_1 ML_{t-1} + \varepsilon_t^{ca} \\
\ln P_t^{c} &= \alpha_2 + \beta_2 ML_{t-1} + \varepsilon_t^{c} \\
\ln P_t^{co} &= \alpha_3 + \beta_3 ML_{t-1} + \varepsilon_t^{co} \\
\ln P_t^{o} &= \alpha_4 + \beta_4 ML_{t-1} + \varepsilon_t^{o} \\
\ln P_t^{g} &= \alpha_5 + \beta_5 ML_{t-1} + \varepsilon_t^{g}
\end{aligned} \tag{3.2}
$$

	ε_t^{ca}	ε_t^{co}	ε_t^{c}	ε_t^{o}	ε_t^{g}
DF	-1.85	-3.61	-3.57	-4.96	-3.67
	(0.35)	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.7: Dicky-Fuller test for five commodity futures price co-integration test with market liquidity and CRB market index

It is noticeable that all the sign of liquidity coefficients are all negative and coefficients are statistically significant from Table 3.6. Moreover, from Table 3.7, the residuals of the regression all exhibit stationarity. According to Leybourne et al. (1994), if the sign of the first derivative regarding the same variable is the same in the regression model, then, the variable drives the price to the same direction. Namely, the first derivative equations are

$$sign(\frac{d\ln P_t^{ca}}{dML_{t-1}}) = sign(\frac{d\ln P_t^{c}}{dML_{t-1}}) = sign(\frac{d\ln P_t^{co}}{dML_{t-1}}) = sign(\frac{d\ln P_t^{o}}{dML_{t-1}}) = sign(\frac{d\ln P_t^{g}}{dML_{t-1}})$$

As a result, all the commodity liquidities drive the commodity prices to the same direction, which may result in the commodity price co-movement. As a consequence, I believe that the common factor lying in the commodity liquidity may also drive all commodity prices to the same direction. The driving force can result in commodity price co-movement. Consequently, the commodity liquidity commonality could be an impetus of commodity price co-movement since it influences the all the commodity prices negatively. When the commodities become illiquid, they are difficult to sell.

Therefore, the Amihud measured liquidity induces all commodity prices negatively. The Amihud measured liquidity mainly bases on the trading volume, which acts as the denominator in the formula. Since the Amihud measured liquidity has a negative impact on the commodity prices, the trading volume should have a positive relation with commodity prices. The conclusion is consistent with existing financial literatures.

Since the negative relationship between commodity price and market liquidity has been confirmed, I further test the causality between the two variables by adopting the Granger test. The results for Granger test has been presented in Table 3.5.The results indicate that the commodity market liquidity in general can be used to predict the commodity futures prices except for cattle and corn price. So I argue that the commodity market liquidity is not that useful in predicting futures prices where the prediction powers for cattle and corn are not significant. As a consequence, commodity liquidity commonality might be helpful in predicting commodity futures price co-movements for most non-agricultural commodity markets. The movement of commodities could give implications on the monetary policy formulation and economic development.

It is clear that the micro liquidity can explain the variance of futures prices. On the other hand, Browne and Cronin (2010) find that the macro liquidity is

Table 3.8: Granger test results for causality from spot market liquidity toward spot commodity prices with F-statistical values.

	$\ln P_t^{ca}$	$\ln P_t^{c}$	$\ln P_t^{co}$	$\ln P_t^{o}$	$\ln P_t^{g}$
ML_t(Lags)	2	3	2	4	1
F-Value	1.54	9.35***	1.34	2.39**	2.26*
P-Value	(0.21)	(0.00)	(0.26)	(0.05)	(0.08)

H_0: Spot market liquidity does not have Granger-cause toward commodity spot prices and *,**,*** indicate statistical significance at 10%,5% and 1% levels, respectively.

also a determinant of commodity futures price. As a result, there may be links between macro liquidity and micro liquidity. As Cochrane (1989) shows that the money supply growth is strictly related with interest rate, I use a daily annualized interest rate as a proxy of macro liquidity to investigate its connections with micro liquidity. The correlation matrix presented in Table 3.9 has confirmed the relations. All micro liquidities are positively and significantly correlated with the interest rates. It gives us the idea that the interest rate can influence the commodity futures prices through the market liquidity channel. The linkage between macro-liquidity and micro-liquidity can also be formulated through commodity futures markets. Furthermore, it is arguable that a higher interest rate indicates tighter money supply and also leads to less liquidity in the market.

	L_t^{ca}	L_t^{co}	L_t^{c}	L_t^{o}	L_t^{g}
IR_t	0.52	0.53	0.39	0.74	0.77
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.9: Correlation matrix of interest rate and micro liquidity with p-value

3.4 Liquidity Effects on Commodity Prices Co-integration

Since the prices of commodity futures tend to co-move and the liquidities of five markets exhibit the commonality, it is arguable that the five commodity prices tend to be co-integrated. The evidence could be also found in the literatures. For instance, Zhang and Wei (2010) find the co-integration relationship between gold and oil futures markets. Moreover, Corradi et al. (2000) propose that co-movement and co-integration are interchangeable concepts. As a consequence, it is reasonable to claim that there is a co-integration relationship among different commodity futures markets. So I implement a series of co-integration tests to testify the co-integration relationship among the commodity futures markets.

The co-integration test has been developed by Granger (1986), Engle and Granger (1987). The series must be differenced d times to achieve stationarity then the series is known as I(d). According to Engle and Granger (1987), the residuals of the regression of the two series are stationary, then there may be a co-integration relationship between the two series. It is also true for multiple series (Engsted et al., 1997). As a result, I use the regression model to obtain the residuals for each market and then I use Dicky-Fuller Test to testify whether the residuals of the five futures markets are stationary. If they are stationary, I can argue that the five different commodity futures markets are co-integrated. The results are presented in Table 3.10. I found that only oil market and copper market are co-integrated with other markets. The rest of the commodity markets seem to be not co-integrated with other markets.

The results may result from the issue presented in Granger (1986). Granger (1986) argued that if the pair of two price series from a jointly efficient, speculative market, they cannot be co-integrated. The way that two prices are co-integrated, is that one series can help to predict the other, which is not valid under the efficient market assumption. Therefore, according to his argument, gold and silver prices cannot be co-integrated. So the prerequisite of co-integration is that there must be Granger causality in at least one direction for the two series, as one variable can help forecast the other. As a result, I conduct a series of Granger tests to see whether there is Granger causality for the three commodity prices that display the negative results in the co-integration tests. From Table 3.11, it is clear that gold prices indeed have Granger prediction power for cattle and corn futures prices. However, it has mostly insignificant results for the predictions of gold from other commodities. On the other hand, cattle and corn prices do not have Granger causality for each other, but they exhibit Granger causality with other futures prices. As a result, I thereby redo the two co-integration tests for cattle and corn and I do not redo the gold case since it has the most insignificant results. I remove cattle price from the corn price regression equation as the independent variable and remove corn price from the corn price regression equation as the independent variable. Then, I test whether the residuals of new regression equations become stationary. The new test results are presented in Table 3.12. It is surprising that the residuals are not stationary still. It is subtle that even they have Granger causality for each other, however, they are not integrated. I propose that there should be a factor that interferes the residuals and makes residuals non-stationary. I try to find the interference factor and filter it from the regression residuals such that the residuals can become stationary.

$$\ln P_t^i = \alpha_i + \sum_{j=1, j \neq i}^{5} \beta_i \ln P_t^j + \varepsilon_{i,t} \tag{3.3}$$

where i=1, 2 , 3, 4, 5, representing ca, co, g, o, c, correspondingly.

	${\varepsilon_t}^{ca}$	${\varepsilon_t}^{co}$	${\varepsilon_t}^{c}$	${\varepsilon_t}^{o}$	${\varepsilon_t}^{g}$
DF	-1.53	-1.98	-2.39	-3.02	-2.62
	(0.52)	(0.29)	(0.14)	(0.03)	(0.09)

Table 3.10: Dicky-Fuller test for five commodity futures markets co-integration with p-value

	$\ln {P_t}^{ca}$	$\ln {P_t}^{co}$	$\ln {P_t}^{g}$
$\ln {P_t}^{ca}$	-	1.06	1.21
$\ln {P_t}^{co}$	1.29	-	1.64
$\ln {P_t}^{c}$	2.09*	2.13*	1.86
$\ln {P_t}^{o}$	3.38***	2.97**	1.35
$\ln {P_t}^{g}$	2.99*	2.8**	-

Table 3.11: Granger test results of F-statistic with p-value for cattle, corn and cold

where *,**,*** indicate statistical significance at 10%,5% and 1% levels, respectively.

$$\ln P_t^i = \alpha_i + \sum_{j=1, j\neq ca, j\neq co}^{5} \beta_i \ln P_t^j + \varepsilon_{i,t} \tag{3.4}$$

where i=1, 2, representing ca, co, correspondingly.

	${\varepsilon_t}^{ca}$	${\varepsilon_t}^{co}$	${\varepsilon_t}^{c}$	${\varepsilon_t}^{o}$	${\varepsilon_t}^{g}$
DF	-1.73	-2.17	-	-	-
	(0.41)	(0.21)			

Table 3.12: Dicky-Fuller test for cattle and corn commodity futures markets with p-value

Since the liquidity level contributes to the commodity co-movements, it is arguable that liquidity can be the interference factor. As a result, I control all the relevant liquidity variables from the regression equation to see whether the residuals can become stationary. Table 3.13 compares the two sets of results. The first set of result does not take liquidity into account and the residuals are nonstationary. On the other hand, the second set of result takes liquidities as the control variables and then residuals become stationary. I can maintain that most of the nonstationary part of the residuals stem from the commodity futures liquidity. After I extract the liquidity information from the residuals, they become more stationary and thus markets are co-integrated.

$$\begin{aligned}
\ln P_t^i &= \alpha_i' + \sum_{j=1, j\neq ca, j\neq co}^{5} \beta_i \ln P_t^j + \sum_{j=1, j\neq ca, j\neq co}^{5} \gamma_i L_t^j + \varepsilon_{i,t}' \\
\ln P_t^g &= \alpha_g' + \sum_{j=1, j\neq g}^{5} \beta_g \ln P_t^j + \sum_{j=1, j\neq g}^{5} \gamma_g L_t^j + \varepsilon_{g,t}'
\end{aligned}$$

where i=1, 2, representing ca, co, correspondingly.

In summary, gold futures movement cannot be predicted by other commodity futures prices. Cattle and corn have Granger causality from oil, gold and copper futures but they are not co-integrated with each other. Oil and copper futures have co-integration relationship with other commodity futures. By controlling the liquidity variables, I found that the gold, cattle and corn futures prices are co-integrated with other commodity prices and with liquidity level as well.

	$\underline{\varepsilon_t}^{ca}$	$\underline{\varepsilon_t}^{co}$	$\underline{\varepsilon_t}^{c}$	$\underline{\varepsilon_t}^{o}$	$\underline{\varepsilon_t}^{g}$
DF [1]	-1.73	-2.17	-2.39	-3.02	-2.62
	(0.41)	(0.21)	(0.14)	(0.03)	(0.09)
DF[2]	-3.33	-3.36	-3.24	-	-
	(0.01)	(0.01)	(0.02)		

Table 3.13: Dicky-Fuller test for five commodity futures markets co-integration with p-value

[1] Dicky-Fuller test results without controlling the liquidity variables

[2] Dicky-Fuller test results with controlling the liquidity variables

3.5 Liquidity Risk Impact on Commodity Futures Returns

In section 3, I present the co-movement of commodity prices are correlated with liquidity levels. Further in this section, I test the co-movement between excess commodity futures returns and liquidity innovations. I define the excess commodity futures returns as the residuals of commodity futures returns that are not explained by the market returns and I define the liquidity risk as the liquidity innovations. Furthermore, the co-movement relationship between liquidity risk and asset returns has been demonstrated by Pastor and Stambaugh (2003) in the stock markets. I extend their findings to commodity futures markets and I testify the co-movement between liquidity innovations and excess commodity futures returns. I first test the co-movement among futures returns themselves. From the return correlation matrix, all the commodity returns seem to be positively correlated with each other (see Table 3.14). More importantly, the p-values of return indicate that all the correlation coefficients are statistically significant as well. Consistent with section 3, futures returns also exhibit co-movements.

Then, I investigate the co-movement between liquidity innovations and excess commodity futures returns. I try to empirically testify whether the liquidity risk can abundantly explain the residual part that the market risk cannot explain. If the liquidity risk relates to the residual part isolated from the market risk, it probably comprises most of the noise information in the market. It is also

	r_t^{ca}	r_t^{c}	r_t^{co}	r_t^{o}
r_t^{co}	0.1576			
	(0.00)			
r_t^{c}	0.1580	0.2254		
	(0.00)	(0.00)		
r_t^{o}	0.1585	0.2999	0.4507	
	(0.00)	(0.00)	(0.00)	
r_t^{g}	0.1162	0.3553	0.2267	0.2275
	(0.00)	(0.00)	(0.00)	(0.00)

Table 3.14: Spot commodity returns correlation matrix with p-value

consistent with the findings that after controlling the liquidity variable from the co-integration tests, the residual part becomes stationary. I adopt the CRB Index as the proxy for the commodity market returns. The CRB Index is composed with 28 commodities, including energy, agriculture and metal, which could be a solid representation of overall commodity prices. I first run the regression of the returns of five commodity families with the returns of CRB Index and take the residuals. For the regression equations: where λ_t^i is the residuals that is not explained by the market risk Then, I try to measure the liquidity innovations in the commodity futures markets. A more recent paper by Feng et al. (2014) has valued the options with liquidity risk. The following two stochastic equations are identified in Feng et al. (2014) for asset prices and asset liquidity respectively.

$$\begin{aligned} dS_t/S_t &= rdt + \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2}dW_t^S + \rho\beta L_t dW_t^L \\ dL_t &= \alpha(\theta - L_t)dt + \xi dW_t^L \end{aligned} \tag{3.5}$$

where σ is individual asset price volatility, ρ is correlation between individual asset return and market liquidity, β is sensitivity of asset price to market liquidity, α is the mean reversion speed of market liquidity, θ is the long-run mean of market liquidity and ξ is the volatility of market liquidity.

From those two equations, it can be seen that the volatility part has been decomposed into two parts. One is related to the market risk dW_t^s and the other is related to the liquidity risk, dW_t^L. It may indicate that the variance of commodity futures prices that are not explained by the market risk could relate to the liquidity risk. In this section, I will test whether the residuals that not explained by the market risk can be explained by the liquidity risk. Since the liquidity risk part is measured by the residual of the liquidity stochastic equation, which is $\xi dW_t^{L,Q} = dL_t - \alpha(\theta - L_t)dt$, I can empirically approximate the liquidity risk through following regressions:

$$L_t^i - L_{t-1}^i = \alpha_1^i + \alpha_2^i L_{t-1}^i + \alpha_3^i (L_{t-1}^i - L_{t-2}^i) + \delta_t^i \tag{3.6}$$

${\delta_t}^i$ is the residuals that is not explained by the liquidity autoregression, which can be noted as the liquidity shocks. For the return regression equation, I save the residual parts of each commodity family, denoting the residuals as ${\lambda_t}^i$. Then, I run the autoregression in equation (3.6) and take the residuals as the liquidity risk, denoted as ${\delta_t}^i$. The ${\delta_t}^i$ is the residual part of the autoregression process, which is the innovation part of the liquidity and it is also known as the liquidity shock. The residual part captures the unexpected part of the process, which is the liquidity shock part and the liquidity shock in the market mainly represents the liquidity risk. It is consistent with the existing literature that the liquidity risk is measured as the volatility of the liquidity shocks (Pastor and Stambaugh, 2003; Sadka, 2006). Then, I construct the correlation matrix between ${\lambda_t}^i$ and ${\delta_t}^i$. I illustrate that the liquidity risk is firmly correlated with the residuals that are not explained by the market risk and the results are presented in Table 3.15. All of the commodity liquidity risk is significantly related to the market residuals that are not explained by the market index return. Then, I reveal relationship between ${\delta_t}^i$ and individual commodity return r_t^i by controlling the market index return. The regression results are shown in Table 3.16. All commodity futures returns are significantly related with the market index return and the liquidity risks (the ${\delta_t}^i$). The liquidity risk can be a explanatory factor for commodity futures returns after controlling the market index return. The liquidity risk can explain the commodity futures returns' variances for which market index return fails to explain. Moreover, the ${\delta_t}^i$ seems to have significantly negative relationship with all commodity futures returns, which implies the negative impact of liquidity risks on the commodity futures returns .

	λ_t^{ca}	λ_t^{co}	λ_t^{c}	λ_t^{o}	λ_t^{g}
δ_t^{ca}	-0.06	-	-	-	-
	(0.00)				
δ_t^{co}	-	-0.05	-	-	-
		(0.04)			
δ_t^{c}	-	-	-0.07	-	-
			(0.00)		
δ_t^{o}	-	-	-	-0.16	-
				(0.00)	
δ_t^{g}	-	-	-	-	-0.07
					(0.00)

Table 3.15: Liquidity risk and market residual correlation matrix with p-value

	r_t^{ca}	r_t^{co}	r_t^{c}	r_t^{o}	r_t^{g}
r_t^{CRB}	0.14***	1.19***	0.92***	1.51***	0.38***
	(0.017)	(0.028)	(0.034)	(0.021)	(0.022)
δ_t^{i}	-0.04***	-0.06***	-0.01**	-8.41***	-0.17***
	(0.018)	(0.003)	(0.033)	(1.241)	(0.058)

Table 3.16: Regression results for commodity futures returns and liquidity risk

where *,**,*** indicate statistical significance at 10%,5% and 1% levels, respectively.

$$r_t^i = \alpha_1^i + \alpha_2^i r_t^{CRB} + \alpha_3^i \delta_t^i + \varepsilon_t \tag{3.7}$$

Then, I construct a correlation matrix for liquidity innovations from different futures markets. I found that all correlations exhibit positive relations and over a half of the correlations are statistically significant (see Table 3.15). Since the liquidity dynamics have been shown in section 3.3 being a driving factor for commodity futures prices co-movement, it is arguable that the liquidity innovations can serve as the deterministic factor for return co-movement. When there is a liquidity shock in one futures market, the shock can spread over to other markets and all futures market can be affected. As a result, the positive correlations among liquidity innovations can be an influential factor for the return co-movement, as shown in Table 3.16.

The crucial role of liquidity on futures prices and futures returns relates to liquidity measures. The liquidity measure I use is mainly based on the trading volume in the commodity markets. The research of trading volume in the literatures points out that trading volume plays an informative role in the financial market. For example, Blume et al. (1994) testify the potential usefulness of trading volume in the financial market. They assert that trading volume can offer investors with additional information that the market price cannot offer. In addition, Lee and Swaminathan (2000) claim that trading volume will be helpful in predicting cross-sectional stock returns and closely related to price momentum.

The trading volume associates with asset price in three dimensions. Firstly, trading volume is positively correlated with price change (see Karpoff, 1987). One reason for the phenomena might be the disposition effect which causes investors who hold a security to be less willing to sell after a price decline than a price rise. Odean (1998) shows that stocks with gains are sold by individual investors at twice the rate of stocks with losses. Secondly, the trading volume is correlated with transaction costs. Like Karpoff (1987) points out that trading volume will increase with the growing number of active traders. When the trading volume is higher, the Amihud measure is lower and the market is more active. Since sellers are more likely to sell the asset, the cost for the asset trading would be reduced, which illustrates that the trading cost will diminish with the trading volume. Admati and Pfleiderer (1988) develop a model that states the negative relationship between trading volume and transaction cost. Finally, there are several studies focusing on the relationship between trading volume and asset price volatility in the future market. Bessembinder and Seguin (1993) discover that trading volume has strong effects on price volatility, especially when there is a volume shock, the price volatility will be significantly influenced. Similarly, Wang and Yau (2000) also verify the positive relationship between trading volume and asset price volatility in the future market. As a result, when trading volume is high, the price will be more fluctuating.

3.6 Implications and Conclusions

The main results of this chapter can be summarized as follows. First of all, I have found that there exists significant price co-movement among five different commodity families. My results are based on daily data while the existing literatures studied low frequency data, such as monthly data or quarterly data. I also show that market liquidity is a driving factor for the daily price co-movement. I also find that liquidity effects can penetrate across different commodity classes. In other words, the price movements are not only driven by its own liquidity level, but also the liquidity levels from different commodity families. Furthermore, I established the link between the micro-liquidity (i.e. the trading liquidity) with the macro-liquidity (i.e. the money supply) through commodity price co-movements.

I also investigate the liquidity effects on commodity price co-integration. I find that the prices of oil and copper have co-integration relationship with other commodity futures prices, but cattle and corn prices are not co-integrated with other futures prices. I show that the liquidity effects have contributed the non-existence of co-integration of cattle and corn prices. After incorporating the liquidity factors, I reestablished the co-integration relationship of gold, cattle and corn prices with other commodity prices and their corresponding liquidity levels, which makes the residuals stationary. Finally, I study the impact of liquidity risk on futures returns. I find that futures returns are also positively correlated. I show that such correlated movements are not only driven by the market return, but also by positively correlated liquidity innovations.

This chapter also has research implications towards monetary policy. The market liquidity is a reflection of trading activities and liquidity conditions at a micro level. By defining the market liquidity as the micro-liquidity level, I construct the connection between macro-liquidity and micro-liquidity via commodity futures markets. The trading activities in the futures market can be the reflection of market conditions and the market liquidity can deliver fruitful information to the policy makers. More importantly, market liquidity is a representation of the monetary policy implementation, the expected policy results can be observed in the futures market through relevant trading activities. Furthermore, commodity prices convey implications on macroeconomic variables such as inflation. Beckerman and Jenkinson (1986) and Garner (1989) agree that commodity price will have strong effects on inflation. Clarida et al. (1998) argue that plenty of countries use targeted inflation rate as one of the most important indicators on formulating monetary policy. Because the co-movement of prices has a magnified much larger effect on inflation rate change than an uncorrelated price movement, the commodity co-movement has a significant impact on other key macroeconomic variables such as GDP, inflation, interest rate exchange rate trading balance and so on. Thus, the movement of commodity price has an indirect influence on monetary policy via inflation rate. As a result, the commodity liquidity commonality which drives the commodity price co-movement can be viewed as a market signal of commodity price moving trend and thereby give indications to the monetary policy makers.

The monetary policy tools such as interest rate correlated with money growth, which is known as aggregated liquidity. The commodity prices are firmly correlated with monetary policy as mentioned before. The futures market liquidity is also connected with commodity futures prices. As a consequence, the futures market liquidity, which I denote as the micro liquidity and the aggregated liquidity, which I denote as macro liquidity are connected through the commodity futures prices. Thus, policy makers can mitigate the inflation risk by influencing the commodity prices. The commodity liquidity can be used to predict the price moving trend and provide policy maker suggestions on monetary policy formulation in advance.

Since the monetary policy is mainly concerned about money supply, I shall start with the famous equation of exchange, MV = Py, where M is the money supply; V is the velocity of money – that is, the speed at which money circulates; P is the price level; and y is the real GDP (Snyder, 1924). According to Browne and Cronin (2010), they decompose the total price level P into two parts: $P = \omega P_s + (1 - \omega) P_c$, where P_s represents the consumer goods, P_c represents the commodity and ω is coefficient constant for $0 \leq \omega \leq 1$. I plug the decomposition into the equation of exchange, which gives$MV = [\omega P_s + (1-\omega)P_c] * y$. Therefore, the money supply has the relation with commodity price: $M = \frac{y}{V}(1-\omega)P_c + \frac{y}{V}\omega P_s$. Holding other variables constant, the money supply can be viewed as a linear function of the commodity price. Since the liquidity commonality has a negative impact on commodity price moving trend, the implications to monetary policies are clear. When the market becomes active and liquid, the commodity prices may have a common upward trend in the future. As a result, the policy makers shall implement the relatively tight monetary policies to control the coming inflation. On the other hand, when the market becomes inactive and illiquid, the commodity prices may have a common downward trend in the future, in which case, the policy maker shall provide liquidity by lowering the interest rate.

In addition, the demand for an asset can be reflected in the asset liquidity. When the demand of an asset is high, the asset becomes more liquid and the trading volume increases. Like Pindyck and Rotemberg (1990) document that a rise in demand of commodities such as crude oil will have a direct impact on the commodity prices. The rise in commodity demand not only increases the current commodity prices but also the expectation of future commodity prices. It is well known that trading volume and price changes are positively correlated in the stock market (Karpoff, 1987). The Amihud liquidity measure takes trading volume into account and thereby the commodity futures market liquidity will be a vital factor for understanding the commodity futures markets and prices. I find that the commodity futures market liquidity is not only a determinant of the commodity prices co-movement but also contains a large number of information about the trading activities. Since the liquidity factor contains rich information, it is firmly correlated with the residual risk that is not explained by the market risk and therefore sheds lights on the wider understanding of the commodity futures markets.

To sum up, commodity price co-movements are important indicators for macroeconomic conditions, such as inflation. As a result, policy makers who wish to regulate the economic environment would have a deep understanding of the commodity prices and the market liquidity. More importantly, from this chapter, I have empirically demonstrated that the liquidity factor can serve as an additional factor in pricing commodity futures for market risk factor. As a result, in the next chapter, I will develop a two-factor future pricing model, which incorporates both market risk factor and liquidity factor. Afterwards, I will compare the newly developed model with traditional benchmark model to see whether the model will be more accurate.

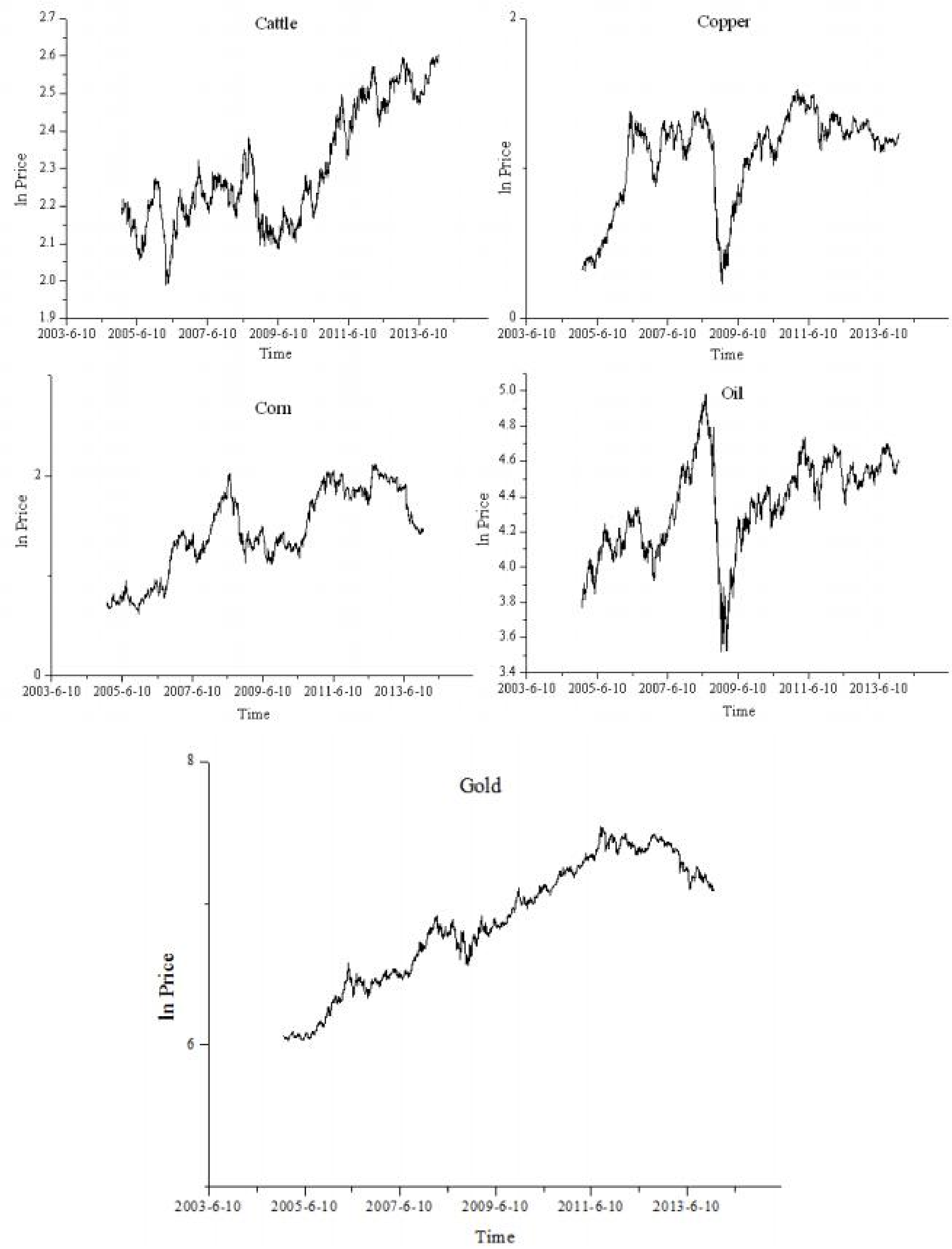


Figure 3.1: Natural log of spot commodity daily prices movements

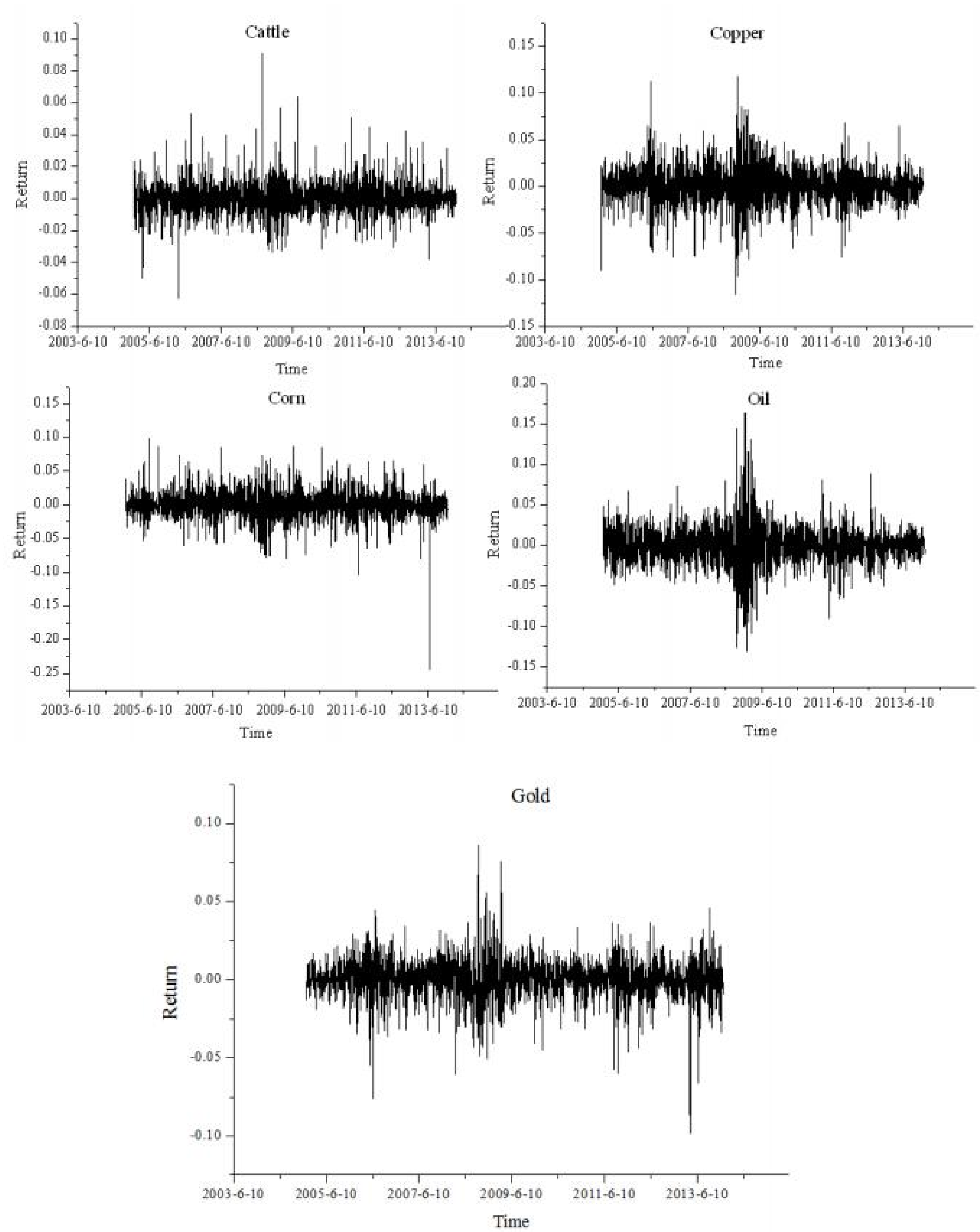


Figure 3.2: Daily spot returns of five commodity markets

Chapter 4

Pricing Futures with a Liquidity Factor: Theory and Evidence

4.1 Introduction

The impact of liquidity risk factor on asset price has received substantial attention in the financial research field. A number of liquidity-adjusted asset pricing models have been developed, including the Capital Asset Pricing Model (CAPM) with liquidity, such as, Acharya and Pedersen (2005) and Liu (2006). In addition to financial assets, a number of recent studies have paid attention to the impact of liquidity risk factor on derivative pricing, including those of Chou et al. (2011), Bongaerts et al. (2011) and Feng et al. (2014). In this chapter, I develop a liquidity-adjusted commodity futures pricing model. This model is applicable not only for valuing real commodity futures with liquidity factor but also for its ability to provide insights on pricing American options and real options whose underlying assets have liquidity risk.

One cause of the 2007-2009 financial crisis was the fact that the market was dried up with liquidity. Since the financial crisis, liquidity risk has become a focus of financial research as a result, with many papers taking on the problem, including those of Cornett et al. (2011) and Calvo (2012). Brunnermeier and Sannikov (2014) contend that market illiquidity is a driving force of market endogenous risk. Liquidity risk could therefore be recognized as a component of market risk, a linkage previously illustrated by a number of derivative pricing models that have included liquidity risk. For example, Bongaerts et al. (2011) incorporate liquidity risk in their pricing model of credit default swaps. Most recently, Feng et al. (2014) develop a model to value European options considering the impact of liquidity.

Based on previous research on the derivative pricing models with liquidity, this chapter develops a liquidity adjusted futures pricing model and tends to make four main contributions to the financial research area. First, based on the existing literature, I have developed a futures pricing model that considers the liquidity factor. The model builds on the liquidity adjusted European option pricing model of Feng et al. (2014). However, Feng et al. (2014), only utilize the estimated liquidity parameters as inputs to improve the standard Black-Scholes option pricing model. As a consequence, their model is only capable of valuing European option with stochastic liquidity, but the model cannot be adopted to value futures and contingent claims as well as American options. On the other hand, my model can be utilized to value futures and can be extended easily to value American options. In my model, the current market liquidity level is taken as an input and thereby the current market liquidity level can be reflected in the asset price under the model framework. Thus, my model contributes to the theoretical side of the literatures on asset pricing with liquidity.

More importantly, as mentioned in Bongaerts et al. (2011), the empirical work on liquidity and derivative pricing is quite limited, especially in the commodity futures market. Two recent papers put effort on the empirical work on liquidity and derivative pricing. Li and Zhang (2011) compare the price of warrants with identical options by using data from the Hong Kong market during 2002-2007. They demonstrate that the price difference between two derivatives reflects the liquidity premium of warrants over options. Mancini et al. (2013) systematically study the liquidity risk in the foreign exchange market and they show the strong liquidity effects in the foreign exchange market. This chapter also conducts an empirical analysis by comparing two estimated values from two models. The empirical results show that the liquidity-adjusted model has lower values compared with nonliquidity-adjusted model adjusted with convenience yield and the estimated values are much closer to the actual market values. As a result, this chapter contributes to the empirical literature by illustrating that I shall include the liquidity factor when I value the commodity futures and other derivatives with liquidity factor.

Third, the liquidity adjusted futures pricing model can predict spot prices and futures prices simultaneously, which means the very one model can be applied in both spot price predictions and futures price predictions purely based on historical market information. The existing models either predict futures prices by using spot prices (e.g. Black, 1976) or use futures prices to predict spot prices (e.g. Reichsfeld and Roache, 2011). As a result, my model has a great degree of prediction power. Moreover, my model testifies the short-term spot and futures market integrations since the short-term futures prices could be a sound approximation of short-term spot prices. On this basis, the newly developed model adopts a two-step forecasting method. The first step is to estimate the spot prices and then I use the estimated prices as inputs to predict futures prices. In order to obtain more precise results, I choose different discount factors for different maturity groups. Not surprisingly, the prediction errors of my model are less than 1.6%, which are relatively small.

Last but not least, the futures markets have different views on futures with different maturities since the implied discount factors in my model for futures with different maturities are different. In the light of my model, researchers can learn the term structure effects due to the limited maturity of the futures contract and the mean-reverting nature of the stochastic liquidity process. One reason for the term structure effects may be because maturity has strong effects on trading volume (Grammatikos and Saunders, 1986). The main liquidity measure mainly used in this chapter is the Amihud measure and it extracts information from the trading volume. As a consequence, there could be interaction effects between maturity and liquidity.

The model I build is designed to value futures contracts, especially for commodity futures since liquidity risk plays an important role in the futures markets. Like Marshall et al. (2011) research 24 commodity futures and all commodity futures show positive liquidity risk under different liquidity proxies. Moreover, Marshall et al. (2013) demonstrate that there is a distinct and common liquidity factor in the commodity market and the liquidity factor is closely related to the commodity price.

Assets with different liquidity risk will be priced differently. Less liquid assets demand additional risk premium, a phenomenon Amihud and Mendelson (1986) referred to as illiquidity premium. Amihud and Mendelson (1991) declare that investment with less liquid assets will provide a higher expected return in order to compensate investors who bear the additional liquidity risk. Brennan and Subrahmanyam (1996) suggest the relatively illiquid security has a higher expected return compared with more liquid security. Eleswarapu (1997) empirically examines the illiquidity premium in the NASDAQ market, finding strong evidence supporting the illiquidity premium in the financial market. Brennan et al. (1998) also confirm the role of liquidity risk in determining the expected asset return. Brenner et al. (2001) empirically distinguish liquid options from illiquid ones and demonstrate that they have different values. As a result, the total risk of the asset will be biased low when excluding liquidity risk factor. A lower risk measure will thereby require a lower expected return, which, in turn, will cause the asset price to be biased high. Thus, I expect that the pricing model without liquidity factor will have a higher estimated price than the model including liquidity factor.

Although liquidity risk is present in the commodity futures market, the futures pricing model that includes liquidity factor is scarce. Black (1976) derives the futures and forward pricing model with cost of carry. One seminal model in futures pricing is developed by Gibson and Schwartz (1990). They introduce the concept of convenience yield into the futures pricing model and propose a two-factor model based on both spot price and instantaneous convenience yield. They subject the two-factor model to a series of empirical tests using oil futures data, concluding that the two-factor model is more accurate than the original one-factor model.

There have been many follow-up works on the seminal model put forward by Gibson and Schwartz (1990). Schwartz (1997) outlines a three-factor model wherein the third factor is the instantaneous interest rate. Hilliard and Reis (1998) investigate the impact of stochastic convenience yield, stochastic interest rate and jump effects on the futures prices under the three-factor model framework. They discover that there are huge differences between futures prices estimated from a deterministic convenience yield model and a stochastic convenience yield model. Cortazar and Schwartz (2003) also create a three-factor model, wherein the third factor they consider is the long-term spot price return. They find that the model is well fitted with oil and copper futures market data. Casassus and Collin-Dufresne (2005) use the same three-factor model as Schwartz (1997), allowing for convenience yield as a function of spot commodity prices. They then utilize this function to explain the mean reversion in spot prices.

More recently, Trolle and Schwartz (2009) incorporate a stochastic volatility factor when pricing commodity derivatives and decompose the stochastic volatility into spanned and unspanned volatility factors. They suggest that both factors will affect the spot commodity price volatility and cost of carry. Nakajima and Ohashi (2012) extend the Gibson and Schwartz two-factor model by adding linear relations among commodity prices. Most of the existing futures pricing models consider stochastic convenience yield and stochastic volatility factors. Conversely, this chapter proposes a futures pricing model that considers stochastic liquidity factor in the commodity market. Most recently, Feng et al. (2014) promote a European option pricing model that includes stochastic liquidity factors. They add a series of liquidity-related parameters into the original Black-Scholes model. On this basis, I build a model that takes the current market liquidity level into account and adjusts risk accordingly. As a result, my model will have lower estimated prices than the original one-factor model, making the results more accurate.

The remainder of this chapter is structured as follows. In section 4.2, I derive the one-factor benchmark model and my two-factor liquidity-adjusted futures pricing model. In section 4.3, I give the empirical results on performance of the two models. In section 4.4, I further elaborate the research implications of my model and finally, I make the conclusion of this chapter.

4.2 Model Setup

4.2.1 One-Factor Benchmark Model

I firstly introduce the one-factor benchmark model without liquidity factor. I establish a filtered probability space (Ω, F, $\{F_t\}_{t\geq 0}$), P) with ($0 \leq t \leq T$) for a fixed time, T, where the T can be considered as the lifetime of the futures contract and P is the probability measure either statistical or empirical. Specifically,Ω is the set of all possible outcomes of the stochastic economy within the time horizon and F is the sigma algebra on Ω (Longstaff and Schwartz, 2001). All of the stochastic processes involved in this study are assumed to be $\{F_t\}_{t\geq 0}$) adapted. Secondly, I postulate the asset under one-factor model is perfect liquid and has no liquidity consideration. The price of the real asset is S_t, and the natural log of the price is X_t, thereby, $X_t = \ln(S_t)$. The interest rate, r, which can be considered as the risk-free discount rate, is accumulated in the money account B_t. The real assets with perfect liquidity are assumed to follow a pure geometric Brownian Motion (GBM):

$$\frac{dS_t}{S_t} = u_s dt + \sigma_s dW_t^{S'P} \tag{4.1}$$

where $W_t^{S'P}$is a Wiener process under P measure and perfect market condition. The GBM process also requires that the asset return has a log-normal stationary distribution. From the property of log-normal stationary distribution, I have the futures price:

$$F = S * exp(r * \tau), for S_t = S \tag{4.2}$$

where τ=T-t, which is the time to maturity. It should be also clear that the equation (4.2) is the solution to the standard Black-Scholes Partial Differentiation

Equation (PDE) for the GBM process:

$$\frac{\partial F}{\partial \tau} = rS\frac{\partial F}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 F}{\partial S^2} \tag{4.3}$$

The equation (4.3) serves as the benchmark model of this paper since it values futures price without liquidity. In addition, many studies have documented the concept of "convenience yield " such as Gibson and Schwartz (1990) and Casassus and Collin-Dufresne (2005). Some scholars may argue that the inclusion of convenience yield in the benchmark model will therefore reduce the modeling errors by making the theoretical prices strictly lower. As a result, I explore my model performance comparing with the convenience yield adjusted benchmark model. The method of estimating convenience yield I use is developed by Heaney (2002). He proposes that the convenience yield is the difference between two trading strategies, where $\delta = TS\left(S_{it}, T\right) - TS\left(F_{itT}, T\right)$ and he defines the trading strategies as:

$$\begin{aligned} TS(S_{it},T) &= \ln\left\{\left[2+\tfrac{\sigma_s{}^2(T-t)}{2}\right]N\left[\tfrac{\sqrt{\sigma_s{}^2(T-t)}}{2}\right]+\sqrt{\tfrac{\sigma_s{}^2(T-t)}{2\pi}}\exp\left[-\tfrac{\sigma_s{}^2(T-t)}{8}\right]\right\} \\ TS(F_{itT},T) &= \ln\left\{\left[2+\tfrac{\sigma_F{}^2(T-t)}{2}\right]N\left[\tfrac{\sqrt{\sigma_F{}^2(T-t)}}{2}\right]+\sqrt{\tfrac{\sigma_F{}^2(T-t)}{2\pi}}\exp\left[-\tfrac{\sigma_F{}^2(T-t)}{8}\right]\right\} \end{aligned} \tag{4.4}$$

where δ is the convenience yield, σ_s is the spot asset volatility and σ_F is the futures asset volatility.

Based on those arguments, I use the benchmark model with convenience yield adjustment to justify my model's performance. So the Stochastic Differential Equation (SDE) becomes:

$$\frac{dS_t}{S_t} = \left(u_s - \delta\right)dt + \sigma_s dW_t^{S'P} \tag{4.5}$$

Then, the correspond PDE becomes:

$$\frac{\partial F}{\partial \tau} = (r-\delta)S\frac{\partial F}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 F}{\partial S^2} \tag{4.6}$$

Therefore, the benchmark model becomes F=S*exp((r-δ)τ), which is the original

Black model adjusted with convenience yield and the solution to equation (4.6).

4.2.2 Liquidity-adjusted Futures Pricing Model

In contrast, I introduce liquidity factor into the futures pricing model, presuming that commodity trading in the market will face liquidity risk. Liquidity factor is modeled as a stochastic process. According to Feng et al. (2014), asset liquidity (L_t) follows a mean-reversion stochastic process:

$$dL_t = \alpha(\theta - L_t)dt + \xi dW_t^{L,P} \tag{4.7}$$

where α is the mean reversion speed of the asset liquidity depending on the market condition and θ is the equilibrium level of asset liquidity and ξ is the volatility of the asset liquidity.

In addition, another important measure must be introduced: the sensitivity of the asset price to the level of market liquidity, as denoted by β. Price sensitivity to liquidity has been mentioned in many existing studies, including those of Kyle (1985), Vives (1995) and Ozsoylev and Werner (2011). Different assets have different βs, as defined by Feng et al. (2014) as an asset specific measure. There is normally one particular real asset traded in the commodity market. For example, the oil market only trades oil, and therefore, β may not be as useful in such markets. Because price is sensitive to liquidity, there should be a liquidity discount factor γ_t, which captures the influence of liquidity on asset price. It should be a function of market liquidity L_t and price sensitivity β, with $\beta \geq 0$. According to Feng et al. (2014), I have:

$$d\gamma_t/\gamma_t = (-\beta L_t + \frac{1}{2}\beta^2 {L_t}^2)dt - \beta L_t dW_t^{\gamma,P} \tag{4.8}$$

For simplicity, as Brunetti and Caldarera (2006) prove, I assume that there is always a price that can clear the entire market. Under such market-clearing conditions, the illiquid asset price will follow:

$$dS_t/S_t = (\mu + \beta L_t + \frac{1}{2}\beta^2 {L_t}^2)dt + \beta L_t dW_t^{\gamma,P} + \sigma dW_t^{S',P} \tag{4.9}$$

For the purpose of futures pricing, I first disaggregate the Brownian motion of the imperfect liquid asset price into two parts:

$$W_t^{\gamma,P} = \rho W_t^{L,P} + \sqrt{1-\rho^2} W_t^{u,P} \tag{4.10}$$

where

$$dW_t^{L,P} dW_t^{u,P} = 0$$
$$W_t^{S,P} = \int_0^t \frac{\sigma}{\sqrt{\sigma^2+(1-\rho^2)\beta^2 L_t^2}} dW_t^{S',P} + \int_0^t \frac{\beta L_t \sqrt{1-\rho^2}}{\sqrt{\sigma^2+(1-\rho^2)\beta^2 L_t^2}} dW_t^{u,P}$$

Therefore, using Levys theorem,$W_t^{S,P}$ is a Brownian motion under the measure of P; therefore, the asset price will be:

$$dS_t/S_t = (\mu+\beta L_t+\frac{1}{2}\beta^2 {L_t}^2)dt+\sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} dW_t^{S,P} + \rho\beta L_t dW_t^{L,P} \tag{4.11}$$

where $dW_t^{L,P} dW_t^{S,P} = 0$

As Bingham and Kiesel (1998) conclude, all possible martingale measures can be categorized by so-called Girsanov densities. Under the Girsanov densities, I will find the appropriate risk-neutral martingale measures for futures pricing valuation with illiquidity. According to Girsanov theory, the equivalent martingale measures can be represented by a Girsanov derivative:

$$M(T) = \frac{dQ}{dP}$$
$$M_t = \exp(-\int_0^t \lambda_1(s) dW_t^{S,P} - \int_0^t \lambda_2(s) dW_t^{L,P} - \frac{1}{2}\int_0^t \lambda_1^2(s)ds - \frac{1}{2}\int_0^t \lambda_2^2(s)ds)$$

As a result, the Brownian motions are changed under the new measure Q:

$$dW_t^{L,Q} = dW_t^{L,P} + \lambda_1 dt$$
$$dW_t^{S,Q} = dW_t^{S,P} + \lambda_2 dt$$

where λ_1 and λ_2 should satisfy:

$$\mu + \beta L_t + \frac{1}{2}\beta^2 {L_t}^2 - r = \lambda_1 \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} + \lambda_2 \rho \beta L_t \tag{4.12}$$

Thus, the new SDEs become:

$$dS_t/S_t = rdt + \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} dW_t^{S,Q} + \rho\beta L_t dW_t^{L,Q} \tag{4.13}$$

$$dL_t = \tilde{\alpha}(\tilde{\theta} - L_t)dt + \xi dW_t^{L,Q} \tag{4.14}$$

where

$$\tilde{\alpha} = \alpha + w$$
$$\tilde{\theta} = \tfrac{\alpha\theta}{\alpha + w}$$
$$dW_t^{L,Q} dW_t^{S,Q} = 0$$

and w is constant.

It can be seen from the SDEs that there are six parameters to be determined, namely, individual asset price volatility σ, correlation between individual asset return and market liquidity ρ, sensitivity of asset price to market liquidity β, the mean reversion speed of market liquidity $\tilde{\alpha}$, the long-run mean of market liquidity $\tilde{\theta}$ and the volatility of market liquidity ξ. Based on derived stochastic processes, I can obtain the liquidity-adjusted futures pricing model. I apply the Feynman-Kac formula for the two-dimensional derivation, adopting F (S, L, τ) to represent the value of futures contracts when S_t=S and L_t=L. By using matrix multiplication and the Feynman-Kac formula, we develop PDE for the function F (S, L, τ):

$$\frac{\partial F}{\partial \tau} = rS\frac{\partial F}{\partial S} + \tilde{\alpha}(\tilde{\theta} - L)\frac{\partial F}{\partial L} + \frac{1}{2}S^2(\sigma^2 + \beta^2 L^2)\frac{\partial^2 F}{\partial S^2} + \frac{1}{2}\xi^2 \frac{\partial^2 F}{\partial L^2} + \rho\xi\beta L S\frac{\partial^2 F}{\partial S \partial L} \tag{4.15}$$

where τ=T-t is the time to maturity T. In comparison with the traditional PDE, the PDE I acquire is similar in that it is only a two-dimensional extension of the traditional version with asset liquidity. They also share the same boundary conditions. This PDE is subject to both initial and boundary conditions. The

PDE of my model has an initial condition, which is

$$F(S, L, 0) = S. \tag{4.16}$$

The PDE is also subject to the following boundary conditions:

$$F(S, 0, \tau) = S * exp(r\tau) \tag{4.17}$$

$$F(S, 1, \tau) = S * exp(r\tau) * d \tag{4.18}$$

$$F(0, L, \tau) = 0 \tag{4.19}$$

$$F(S_{max}, L, \tau) = S_{max} * exp(r\tau) \tag{4.20}$$

The first boundary condition holds that when the market is perfectly liquid, futures prices equal the classical model value under perfect market assumptions. The second boundary condition holds that when the market is significantly illiquid, the theoretical futures prices must be discounted for illiquidity, where d represents an implied discount factor from the market. At its default value, the discount rate is set at 10%, indicating that when the market is frozen, assets can only be sold at 10% of their market values. The third boundary condition states that when spot prices are zero, theoretical futures prices will be zero as well. The final boundary condition holds that when the spot prices are at their maximum value, the theoretical futures prices will be at maximal as well.

4.3 Data and Model Performance

4.3.1 Data Description

In this section, I will empirically demonstrate the new model improves the pricing accuracy compared with the classical model. I adopt the market data of oil futures to validate the accuracy of my model. The oil futures data I use comes from the oil spot and futures prices with oil trading volume in New York Mercantile Exchange. I use 20-year oil data as the sample, which is from January 1995 to December

2014. To be specific, it is the settlement price of light crude oil in NYMEX. The data provider is Thomson Datastream.

Since I include an additional liquidity variable, it will inevitably increase the number of parameters within the model. The increasing number of parameters will then result in better in-sample fitting of the future prices and potentially improves the results. As a consequence, I will test the out-of-sample performance in valuing oil futures contracts. The estimation method I use is the rolling over method. I use last year data to estimate the parameters for the next year and I plug in the parameters in the model and obtain the results for four groups of futures prices. For instance, I use the market data during 1994 to estimate the six parameters. Then, I plug in the six parameters with spot prices in 1995 into my model to estimate the four different groups of futures prices in 1995 and compare the results with the benchmark models results. Followed Gibson and Schwartz (1990), I adopt the nearest future market data as the proxy for spot market for the input of the spot price and the liquidity level. Then, I use the model to predict the oil futures values of different maturity groups over the same period, namely, 3-month futures, 4-month futures, 5-month futures and 6-month futures in the oil market. Although there may be other dominated economic variables that influence the oil futures prices such as inflation and interest rates, as mentioned in Chapter 3, this thesis mainly focuses on the factors from the futures market, namely, market price and market liquidity. The angle this thesis takes is from microeconomic view and market structural perspective.

In order to show the improvement and validation of my model, I use the classical model as the benchmark model and compare the results from the new-developed model and the benchmark model. The benchmark model is the classical model developed by Black (1976), which prices the futures and forward contracts with cost of carry. It is the precursor of the Black-Scholes options pricing model. I use two methods in Gibson and Schwartz (1990) and Harvey (1991) to exhibit the performance of the two models. The first method is called mean pricing error (MPE) in dollars, which is measured as:$MPE = \frac{1}{N}\sum_{n=1}^{N}\left|\hat{F}_n - F_n\right|$. The other method is root mean squared error (RMSE) in dollars, namely,$RMSE =$

$\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\hat{F}_n - F_n\right)^2}$. In the two methods, N is the number of total observations; F_n is the actual future price observed in the market and $\hat{F}_n$ is the theoretical future price estimated from the model.

The methodology I follow is developed by Gibson and Schwartz (1990) who also propose a two-factor model and use estimation method of joint stochastic process. Similarly, followed their study, I apply the new futures pricing model to value oil futures. I estimate the parameters of the joint stochastic process followed by the spot price with liquidity.

4.3.2 Parameter Estimations

Plenty studies have documented a large number of measures for liquidity such as bid-ask spread and trading volume. Marshall et al. (2011) test a large number of liquidity proxies for 19 commodities. They find that the Amihud liquidity proxy has the maximal correction ratio among all proxies and they strongly recommend researchers to use this proxy when modeling commodity liquidity. As a result, I will use the proxy mentioned in Amihud (2002) to measure asset liquidity and it takes the form:

$$Amihud_t = \frac{|R_t|}{Vol_t} \tag{4.21}$$

where R_t is the asset return at time t and Vol_t is the asset trading volume at time t. Intuitively, when trading volume is high, the expected trading time is short and the asset is more liquid. I scale asset liquidity level from 0 (perfect liquid) to 1 (maximal illiquid) and I will adopt Amihud measure as the main liquidity measure of the thesis.

According to Feng et al. (2014), there are six parameters to be determined: long-run mean, mean reversion speed and volatility of the market liquidity, the volatility of asset price, sensitivity of individual asset price towards market liquidity and the correlation between asset return and market liquidity. Regarding the volatility of asset return, they will take the usual form $\sigma = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}\left(R_t - E[R_t]\right)^2}$, which are also known as the sample standard deviation (Poon and Granger, 2003). Since the empirical return volatility contains components of fundamental return volatility and volatility in liquidity discounts, I decompose the empirical return volatility as $\sigma^2 = \sigma_s^2 + \beta^2 L_t^2$, where σ is the empirical return volatility. Moreover, β measures the individual sensitivity of the asset volatility against the market liquidity and β can be set equal to 1 since in my model, there is only one common asset trading in the market.

After collecting the data, I then testify the mean-reverting property of the market liquidity by regressing the Amihud measure in AR (1) process since the discretization of the mean-reverting process is AR (1) process. It takes the general form: $L_t = \varphi_1 + \varphi_2 L_{t-1} + \varepsilon_t$ and if the absolute value of coefficient φ_2 is less than 1, then the process will be a mean-reverting process. The commonly used unit root test is the Augmented Dickey-Fuller (ADF) test. If the time series does not have a unit root, then the process is probably mean-reverting since the process without a unit root is considered as a stationary process (Wu, 2000; Narayan and Prasad, 2007). The results for Amihud and Roll measure with one and two lags in the oil market are performed in STATA and both results are strongly significant to reject the null hypothesis. From the results, I can see that the oil liquidity measure has no unit root and is a mean-reverting process, which supports my argument that asset liquidity is mean-reverting. In addition, I presume that investors are risk-neutral and thereby I set w=0.

Since the process is mean-reverting, it has the long-run mean:$\theta = \frac{\varphi_1}{1-\varphi_2}$. Then, the mean reversion speed can be estimated from the following stochastic process (Balvers et al., 2000):$L_t - L_{t-1} = \varphi_3 + \varphi_4(\theta - L_{t-1}) + \varepsilon_t$, where φ_4 is the mean reversion speed and $0 < \varphi_4 < 1$. φ_1 andφ_3 are positive constants and ε_t is the noise term with unconditional mean of zero. Since the model $L_t = \varphi_1 + \varphi_2 L_{t-1} + \varepsilon_t$ is equivalent to $L_t - L_{t-1} = \varphi_1 + (\varphi_2 - 1)L_{t-1} + \varepsilon_t$. Combined the two regression models, I find that the $\varphi_4 = 1 - \varphi_2$.

Moreover, the volatility of liquidity is usually measured as the volatility of the liquidity shocks (Pastor and Stambaugh, 2003; Sadka, 2006). The shock of liquidity is usually estimated by the residuals of the liquidity autoregression process. As a result, I estimate the volatility as follows. Firstly I discretize the mean-reverting process of liquidity as $L_t - L_{t-1} = \tilde{\alpha}(\widetilde{\theta} - L_{t-1}) + \varepsilon_t$ and I set dt=1. Then, the residual can be known as $\varepsilon_t = L_t - L_{t-1} - \tilde{\alpha}(\widetilde{\theta} - L_{t-1})$.

Then, I plug in the parameters and find the liquidity volatility $\xi = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(\varepsilon_t - E[\varepsilon_t])^2}$. Finally, the correlation between asset return and asset liquidity volatility, which can be estimated as:$\rho = \frac{Cov(R_t, \varepsilon_t)}{\xi\sigma}$. It is observable that the six parameters represent different aspects of the market situation or asset specific features. The ρ measures the co-movement between asset returns and asset liquidity, whereas the β measures the contribution of asset liquidity in the asset volatility. Furthermore, σ_s and ξ represents the volatility of asset return and asset liquidity respectively. Since I model the liquidity process as the mean-reversion process, θ and α presents for long-run mean and the mean-reversion speed respectively. Therefore, it might be essential to have all six parameters in the model to reflect the detailed market and asset information in the model.

In addition, I use annual 6-month T-Bill rate as the risk free rate. As I obtain the parameters of the two stochastic processes along with the risk free rate, I can use the new model to estimate the oil futures values. In order to analyze the pricing errors more precisely, I follow Gibson and Schwartz (1990) to separate futures contracts into three groups according to different maturities. Then, after obtaining the solution of liquidity adjusted PDE for options pricing, I can plug in these parameters into my model and calculate the final results. Table 4.1 summarizes the 20 year results for the main parameters and since the Amihud measure is too small, usually less 10E-8, so I enlarge the whole sample by 10,000,000 times to obtain the regression results for the parameters.

4.3.3 Robustness Check

In order to make the model performance more robust, I adopt another measure of liquidity. Because liquidity cannot be observed directly from the market, and thus liquidity must be measured by proxies, I adopt another widely used liquidity measure: the bid-ask spread. I use the effective spread estimator developed by Roll (1984). The measure is also broadly used in a number of financial papers such as Goyenko et al. (2009), Holden (2009) and Corwin and Schultz (2012). The proxy utilizes the auto-covariance of the price changes as an effective measure of the bid-ask spread and I use on a daily basis. Roll (1984) assumes that the stocks have fundamental values, denoted as V_t at time t and V_t follows:

$$V_t = V_{t-1} + \varepsilon_t \tag{4.22}$$

Next, he denotes S_t as the last observed trade price on day t and presumes that S_t follows:

$$S_t = V_t + \frac{1}{2}EQ_t \tag{4.23}$$

where E is the effective spread and Qt is a buy/sell indicator for the last trade that equals +1 for a buy and -1 for a sell. He further assumes that Qt is equally likely to be +1 or -1 and Qt is also serially uncorrelated, and independent of ε_t. Then he takes the first difference of equation 4.23 and plugs in the result from equation (4.22), which yields

$$\Delta S_t = \frac{1}{2}E\Delta Q_t + \varepsilon_t \tag{4.24}$$

As a result,$Cov(\Delta S_t, \Delta S_{t-1}) = -\frac{1}{4}E^2$ or equivalently, $spread = 2\sqrt{-\text{cov}(\Delta S_t, \Delta S_{t-1})}$. Because when the auto-covariance is positive, the formula is undefined. I therefore use a modified version of the Roll estimator (Goyenko et al., 2009):

$$spread = \begin{cases} 2\sqrt{-\text{cov}(\Delta S_t, \Delta S_{t-1})}, \text{cov}(\Delta S_t, \Delta S_{t-1}) \leq 0 \\ 0, \text{cov}(\Delta S_t, \Delta S_{t-1}) > 0 \end{cases} \tag{4.25}$$

Because the oil liquidity is relatively stable and thus, I use the averaged spread measure as the estimator of the oil liquidity over the whole episode.

4.3.4 Model Performance

To make the model comparison, I outline the two PDEs for the two models. The PDE for classical non-liquidity (NLA) but convenience yield adjusted model of oil future is equation (4.6) with the solution F=S*exp(r-δ)τ, where F is the future contract value, S is the spot price and τ is the time to maturity. On the other hand, the PDE for liquidity-adjusted (LA) model of oil future is equation (4.15) with initial condition equation (4.16) and boundary conditions equations (4.17)-(4.20).

Since the LA model is a two dimensional PDE, it will be appropriate to use numerical methods to approximate the values. The method I use for numerical approximation is the finite different method, which is also mentioned in Hull (2012) and has been adopted in many studies (see Brennan and Schwartz, 1978; Courtadon, G., 1982; Wu and Kwok, 1997). I replace the partial differentiation terms by discretization form and I discretize two SDEs, namely, equation (4.13) and(4.14), and also the PDE from equation (4.15) to make them available for discrete data application. Then, I plug the parameters in and the PDE solution F(S, L, τ) is obtained on a cubic grid of F(S, L, τ) (see Figure 4.11 as an example). On any particular day, I will get the spot price S, the liquidity level L and the maturity τ. Next, I plug in the spot price, liquidity level and maturity as inputs to interpolate the futures prices in the solution surface, which gives us the theoretical values for different maturities predicted by my model.

Thus, I am able to compare the theoretical values from two models with the observed future market values and see which model is more accurate. I find that my model effectively reduces the pricing errors compared to the traditional futures pricing model for both evaluation criteria, namely, MPE and RMSE. As mentioned early, the benchmark model does not take liquidity factor into account, which results in the overestimate of the market price. On the other side, my model is liquidity adjusted and thus has lower estimated values than the benchmark model. Figure 1a and 1b show the daily errors comparing both LA model and NLA model by using Roll and Amihud measure respectively for the sample year of 2014, where error is defined as $Error = \left|\hat{F}_t - F_t\right|$ From the figure, it is salient that the red line (representing errors for LA model) is generally below the black line (representing errors for NLA model). It is aligned with my previous theoretical predictions that my model will have lower predicted values but closer to the actual market prices.

As I use the rolling-over out-of-sample comparing strategy, I compare the results using both MPE and RMSE with Amihud and Roll measures for four different maturity groups and so I obtain 16 tables in total. The rolling-over strategy is to use the previous year data to get the model parameter estimations and to use the fitted model to compare the theoretical futures prices. Under the MPE comparing groups, I also conduct the T-test between the two model results to affirm whether the differences are significant.

There are several conclusions that I can draw from Tables 4.2-4.17. First of all, the averaged improvement rate is around 30% and thus the out-of-sample performance of the LA model will surpass the benchmark for most cases. Here the improvement rate is defined as the absolute value of the averaged difference between the pricing errors of LA and NLA models divided by those of benchmark model, namely, the NLA model. Moreover, almost all individual improvement rates are positive except the 2008 case. The reason might attribute to the financial crisis influence on the oil market. From Figure 4.3, I can see that the oil price experienced a sharp decline during 2008. This sharp decline may be resulted from the financial crisis and made the oil price exceedingly volatile. This unusually high volatility of oil price may result in the LA model working less advantageous since the model applies the volatility from the previous year. Most other improvement rates are positive and all the p-values of T-test are significant. As a result, it is arguable that the errors of the LA model are statistically lower than the benchmark model. Furthermore, the standard deviation of errors are consistently smaller in LA model than NLA model, which indicates that the pricing errors of the LA model are more stable.

4.4 Model and Research Implications

4.4.1 Model Extension

In addition, many researchers have studied "convenience yield " for futures pricing model and they argue that the convenience yield plays an important role in the commodity futures markets. Therefore, I extend my model discounting by both convenience yield and liquidity factor and try to see whether the model performance is improved compared with my previous model. So, the new SDEs with convenience yield become:

$$dS_t/S_t = (r-\delta)dt + \sqrt{{\sigma_s}^2 + (1-\rho^2)\beta^2 L_t^2}\,dW_t^{S,Q} + \rho\beta L_t dW_t^{L,Q} \tag{4.26}$$

$$dL_t = \tilde{\alpha}(\tilde{\theta} - L_t)dt + \xi dW_t^{L,Q} \tag{4.27}$$

where

$$\tilde{\alpha} = \alpha + w$$
$$\tilde{\theta} = \tfrac{\alpha\theta}{\alpha+w}$$
$$dW_t^{L,Q} dW_t^{S,Q} = 0$$

where δ is the convenience yield and w is constant.Based on derived stochastic processes, I can obtain the liquidity-adjusted futures pricing model. I apply the Feynman-Kac formula for the two-dimensional derivation, adopting F (S, L, τ) to represent the value of futures contracts when S_t=S and L_t=L. By using matrix multiplication and the Feynman-Kac formula, I develop PDE for the function F (S, L, τ):

$$\frac{\partial F}{\partial \tau} = (r-\delta)S\frac{\partial F}{\partial S} + \tilde{\alpha}(\tilde{\theta}-L)\frac{\partial F}{\partial L} + \frac{1}{2}S^2(\sigma^2+\beta^2L^2)\frac{\partial^2 F}{\partial S^2} + \frac{1}{2}\xi^2\frac{\partial^2 F}{\partial L^2} + \rho\xi\beta LS\frac{\partial^2 F}{\partial S\partial L} \tag{4.28}$$

In comparison with the traditional PDE, the PDE I acquire is similar in that it is only a two-dimensional extension of the traditional version with asset liquidity. They also share the same boundary conditions. This PDE is subject to both initial and boundary conditions. The PDE of my model has an initial condition, which is

$$F(S, L, 0) = S. \tag{4.29}$$

The PDE is also subject to the following boundary conditions:

$$F(S, 0, \tau) = S * exp(r - \delta)\tau \tag{4.30}$$

$$F(S, 1, \tau) = S * exp(r - \delta)\tau * d \tag{4.31}$$

$$F(0, L, \tau) = 0 \tag{4.32}$$

$$F(S_{max}, L, \tau) = S_{max} * exp(r - \delta)\tau \tag{4.33}$$

The first boundary condition states the fact that when the market is perfect liquid, the futures prices equal to the classical model value under perfect market assumption discounted by convenience yield. The second boundary condition states the fact that when the market is significantly illiquid, the theoretical futures prices must be discounted for the illiquidity and the boundary is also subject to the convenience yield. The third boundary condition states that when the spot prices are zero, the theoretical futures prices shall be zero as well. The final boundary condition states that when the spot prices are at maximal, the theoretical futures prices shall be at maximal as well by taking convenience yield into account.

Then, I compare my new PDE model with the benchmark model with convenience yield. Since the convenience yield and interest rate vary across time, I take 6-month oil futures data as an example since it has the lowest improvement rate among the four products and I use the methodology I previously adopted to acquire the model values. The results are presented in Table 4.18 for Amihud measure and Table 4.19 for Roll measure. It appears that the new model in general outperforms the benchmark model. The overall improvement rate is about 33%, which is highly above the previous rate of 23% for the Amihud case. On the other hand, The overall improvement rate of Roll measure is about 25%, which is slightly above the previous rate of 22% Therefore, the model adjusts with both convenience yield and liquidity factor performs better than the previous model in general. Nevertheless, comparing the new results with my previous results, namely, Table 4.18 and Table 4.14, Table 4.19 and Table 4.15, I find that in some years, the improvement rates become even negative. The reason is that, in these years, the liquidity adjusted model has already adjust the price strictly low, when I further adjust with convenience yield, the theoretical values become even lower and thereby the accuracy declines. Therefore, in some years, the model will over adjust the theoretical values if I consider liquidity factor and convenience yield simultaneously. As a result, it may not be suitable for all years to discount with both liquidity factor and convenience yield simultaneously and the model performance highly depends on the market conditions if I consider two factors.

4.4.2 Implied Liquidity Discount Factor and Model Prediction Tests

Implied volatility is the volatility which makes the model price to be equal to the market price of options. The options-implied volatility from BlackScholes model has been intensively researched. Many studies agree that the options-implied volatility is an efficient tool to forecast future volatility (see Sheikh 1989; Christensen and Prabhala, 1998). More recently, Yu et al. (2010) discover that the implied volatility outperforms either historical volatility or a GARCH-type volatility forecast for predicting future volatility in both the OTC options market and exchange-traded stock index options market.

Likewise, my model has an implied discount factor, which suggests that when the market becomes significantly illiquid, what would be the discount for the asset values. Similar to the role of implied volatility in option prices, the implied discount factor makes the difference between model price and the market price of futures to be minimal. This discount factor can provide helpful insights of the liquidity factor impact on the asset values. More importantly, I can adopt the discount factor to forecast future market price by adjusting the discount factor. I pick up the discount factor (d in equation 4.18) such that the errors between theoretical values and actual market prices are minimal in the sense that the mean error is minimum in the previous year. Then, I can adjust the implied discount factor to improve the accuracy of the model. Since I can adjust the discount rate (d in equation 4.18) to make the model more accurate, I can use the implied discount factor to forecast the future market prices based on my model. In order to forecast the futures price, I shall have the liquidity level, time to maturity and spot price level as main inputs. Because liquidity level usually does not fluctuate vigorously, it would be acceptable to use the past month moving average liquidity level as the input. However, the spot price moves more forcefully, so I need to estimate the spot price first. Previously, I use the current spot price as an input to forecast futures prices on that day. Now I try to estimate the spot price first, then I use the estimated values to forecast futures prices.

It is alleged that the spot and futures prices tend to converge as the time elapses (Errera and Brown, 2002). Garbade and Silber (1983) point out that the commodity spot and futures markets co-integrate fairly well in one or two days. Therefore, the futures with maturity of the next day will be a sound approximation for spot price on the next day. So I use my model to estimate the price of futures with maturity of next day based on todays market information. Then, I use the estimated futures price as the input as the next days spot price to predict longer maturity futures prices on the next day. I use this roll over strategy to predict the spot price one day ahead and use the estimated spot price to forecast the futures prices with different maturities. So the first step is to determine the implied discount factor based on the historical market data.

Futures with different maturities could have different discount rates and I adopt different discount rates according to Table 4.19 and Table 4.20. From those two tables, It can be seen that the discount rate for 3-month is 0.50 because it has the lowest MPE and relative low RMSE. For the spot price estimation, I adopt discount rate of 0.80 since the spot market is more liquid than the 3-month futures market. Moreover, it would be clear that the discount rates for 4-month will be 0.10 and for 5-month and 6-month will be 0.01 since they all have the lowest errors estimated from historical market data. As the discount rates have been determined, I shall first estimate the spot price. The sample period I use to determine the discount rate is from January 1, 2013 to December 31, 2013. I shall testify the accuracy of my model in an out-of-sample method and thus I choose the sample period from January 1, 2014 and to March 31, 2014. The market price in January 1 will be used as an input to estimate the next days spot price and the time to maturity is set at 1/260. I define the daily error as the difference between the estimated price and the actual market price over the market price:

$$Error = \frac{\left|\hat{F}_t - F_t\right|}{F_t}$$

In order to forecast futures prices, I need the spot price first and I use my model to estimate the spot price for the next day. For example, I use the price \$98.42 on January 1, 2014 as the input to estimate the spot price on January 2, 2014, which is \$96.10. By using the predicted spot prices for next day, I can predict futures price for the next day through the same model. All the predicted prices are based on the market information on the previous day. Figure 4.4 compares the actual market price and the model estimated prices and Figure 4.5 shows the daily prediction errors in percentage. My model seems doing a good job on predicting spot prices in the near future since the averaged percentage prediction error is about 0.8% and it is smaller than the actual daily volatility, which is about 2.2%. Based on the historical oil market data, the change of actual spot return can range from -16.9% to 23.5%. In contrast, my model prediction only ranges from -3.07% to 3.07%, which is well below the observed data. Since the predictions of spot price are generally accurate, I utilize the estimated values as inputs of daily spot prices to forecast the futures price with different maturities. Figure 4.6 presents the market prices observed and theoretical prices estimated. As mentioned before, I adopt different discount rates for different maturity groups and I calculate the difference between estimated prices and market prices and present in Figure 4.7. The averaged percentage prediction errors for 3-month, 4-month 5-month and 6-month are 1.13%, 1.51%, 1.25% and 1.22% respectively. It is apparent that all errors are less than 1.6%, which can be considered as an accurate model. Since the prediction errors of my model are less than 1.6%, which are relatively small. Therefore, the predictions of my model can be considered as being accurate.

The source of the prediction power of my model may come from two aspects. As French (1986) maintains that the shocks of demand and supply on the underlying asset will impact the future spot price changes. The shock of demand and supply will impact on the trading volume as well. Since the Amihud liquidity measure extracts information from the trading volume, my model takes the shocks on demand and supply into account. On the other hand, Foster (1995) argues that the trading volume statistics can improve forecasts of futures price movements and therefore my model can have a better prediction of futures prices. Liquidity is a variable that can react in advance, which is before the price movement.

4.4.3 Theoretical Analysis and Solution Surface for the LA Futures Pricing Model

The newly-developed model has revealed the fact that liquidity factor matters for futures price and the model is useful for pricing futures. As a result, I conduct a further theoretical analysis for the model. I use computer programs to obtain the solution surfaces of the new PDE and apply them to scrutinize the relationships among asset price, asset liquidity and time to maturity. Since the model has three main variables, namely, spot price, asset liquidity and time to maturity, I fix one of the variables and show the two dimensional graphs between the other two variables. To maintain the consistency, I also adopt Amihud measure as liquidity proxy in graph plotting. The first graph in Figure 4.11 is obtained by setting the market is perfect liquid. Under the situation, I notice two issues. Firstly, the futures price is a linear function of spot price, which is consistent with the generalized formula: $F = S * exp(r\tau)$. More importantly, the formula is also the solution to the classical PDE without liquidity factor: $\frac{\partial F}{\partial \tau} = rS\frac{\partial F}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 F}{\partial S^2}$. Secondly, the time to maturity will not change the linear relation of futures prices and spot prices. The conclusion is in line with findings in Amihud and Mendelson

(1991b). They argue that maturity mostly represents for asset liquidity. As a result, in the market with full liquidity, the impact of maturity on asset price will be limited. In comparison, for the market has liquidity frictions in the second graph in Figure 4.11 (where I set liquidity equals a half), the time to maturity does affect the futures values. I observe a non-linear relationship between futures and spot prices for a given maturity. The futures prices are adjusted downward from the full liquidity futures prices. The degree of such adjustment depends on the time to maturity. The longer the time to maturity is, the lower the futures values are adjusted. The peak futures values (red regions) centralize in the regions where time to maturity is short.

Finally, I fix the spot price at \$100, which is half of the spot price range, to investigate the relationship between futures values and asset liquidity and the solution surface is presented in the third graph in Figure 4.11. It is clear that the lowest futures values (blue regions) mostly distribute in the regions where asset is most illiquid and the highest futures values (red regions) mostly distribute in the regions where the asset is most liquid. So, suppose that I also fix time to maturity, then, the cross-section of the surface is a smoothing curve that represents the relation between asset liquidity and futures prices. When the asset is liquid, the futures price is high, whereas when the asset is illiquid, the futures price is low.

4.4.4 Liquidity and Maturity Term Structure Effects

Based on the theoretical PDE solutions I obtained, I investigate a coupling effect of illiquidity discounting and futures maturities. In order to exhibit the discount factor more visibly, I define an adjustment ratio, which is $AdjustmentRatio = \frac{F(S,L,\tau)}{Se^{(r-\delta)\tau}}$.This ratio represents the difference between theoretical values implied by my model and theoretical values implied by Blacks classical model. The Figure 4.8 shows the adjustment ratio against the asset liquidity. It is visible that when the market is maximal liquid (i.e. liquidity=0), the theoretical values from two models are almost identical (i.e. adjustment ratio=1). On the other side, when the market becomes more illiquid, the difference between two models enlarges. When the market is perfect illiquid, the asset value with liquidity factor is around 10% of the classical model, which is the discount factor 10%. The downward sloping curve is consistent with the fact that the asset bears higher liquidity risk will require a higher return, which will have a lower market price. Furthermore, the black line presents the 3-month futures prices and the blue line presents the 9-month futures prices. It is clear that the blue line is steeper than the black line, which suggests that the liquidity effects are more noticeable for the long maturity futures. This is in accord with the existing empirical evidence from bond market. It is alleged that the liquidity premium will increase with the maturity (Amihud and Mendelson, 1991b; Fontaine and Garcia, 2012). The asset with longer maturity tends to have larger liquidity effects.

In order to present the relationship more clearly, I plot the adjustment ratio against the maturity (0 represents now and 1 represents a year) in Figure 4.9. It is visible that the adjustment ratio is a downward sloping and it gradually decreases as the maturity becomes longer. This also indicates the liquidity is related to the maturity and long maturity assets generally have larger liquidity effects and will be discounted more than short maturity assets. When the assets are more illiquid, future prices are adjusted downward in greater degree compared to standard no arbitrage future prices. The Figure 4.10 shows that the adjustment ratio against the price with a fixed maturity and the curve is generally flat. The liquidity adjustment ratios seem independent of spot prices for a fixed maturity. To sum up, the adjustment ratio depends on the future maturities and liquidity level. Futures with longer maturity might imply deeper adjustment for a fixed spot liquidity level. Additionally, futures with more illiquid spot trading may also have deeper adjustment for a fixed spot price.

4.5 Conclusion

The liquidity factor has been a focus of financial research after the financial crisis. The liquidity factor does matter for the derivative prices, especially for futures pricing. The existing futures pricing model ignores the liquidity factor. As a consequence, I develop a liquidity-adjusted futures pricing model and I argue that the model without liquidity factor may overestimate the futures prices. I empirically compare the LA model and NLA model by adopting the oil futures market data. The LA model transcends the NLA model because it provides lower but more precise theoretical values than NLA model compared with actual market futures data. So I conclude that liquidity factor shall be incorporated in the derivative pricing models such as futures pricing model and option pricing model. My model is practical and can shed light on the development of new commodity trading strategies by considering liquidity effects.

The main results of this chapter can be summarized as follows. Firstly, my liquidity adjusted models are more accurate than standard models without liquidity effects. My model has about 30% of the improvement rate in the sense of error reductions. The results of my new model are more stable than the benchmark model. My results are robust in comparing performance tests using two different types of liquidity measurement methods: Amihud and Roll measures. Secondly, I introduce the notion of implied discount rate for the liquidity effect into my model. By using the implied discount rate, my model can be applied in predicting both spot price and futures price simultaneously. More importantly, the forecast errors of my model for predicting the next day futures prices for different maturities are less than 1.6%.

Finally, I also carry out a theoretical study on liquidity effects on future prices in couple with maturity effects. I find that when the assets are more illiquid, futures prices are adjusted downward in greater degree compared to standard no-arbitrage futures prices. Such adjustment seems independent of spot prices for a fixed maturity. However, the illiquidity adjustment depends on the futures maturities. Longer maturity implies deeper adjustment for a fixed spot liquidity level. Therefore, spot illiquidity has bigger impacts on long maturity futures than short maturity products. Since the liquidity adjusted futures pricing model is proved to more accurate empirically, it is legitimate to consider the extension of the liquidity adjusted models, such as liquidity adjusted American options model and real options model. As a result, I will develop a liquidity adjusted real options model in the next chapter and further investigate the liquidity effects of real assets in the real options model.

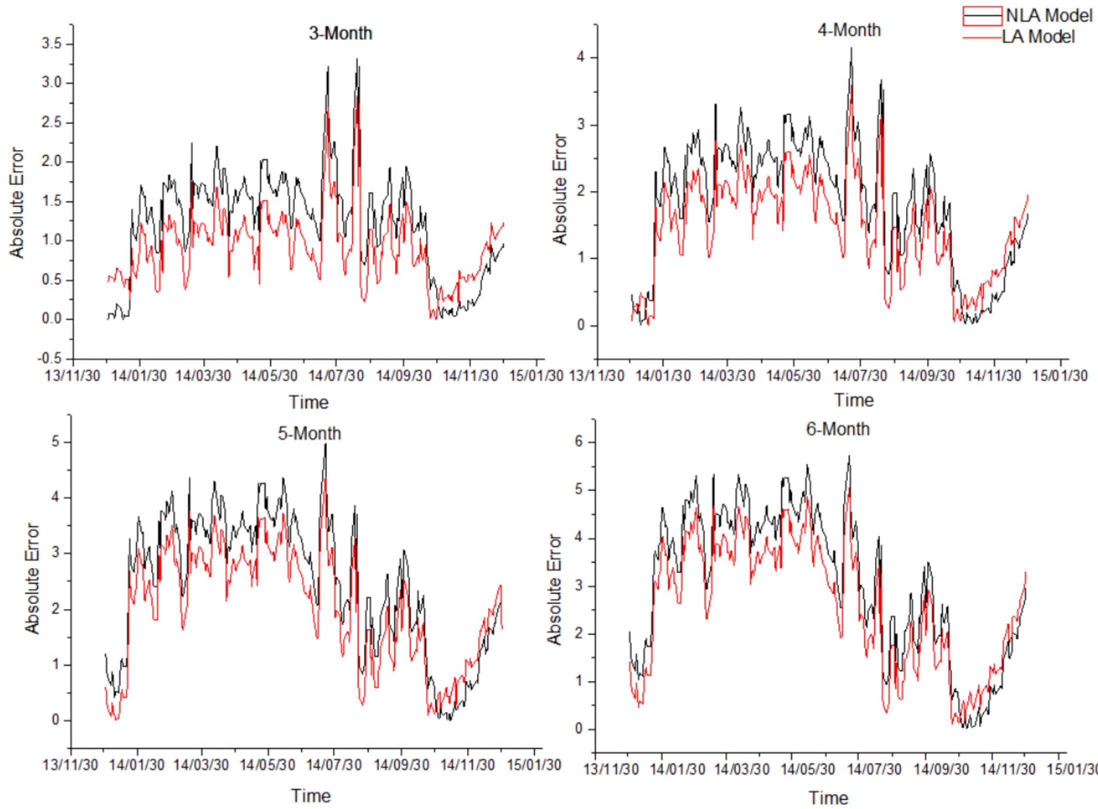


Figure 4.1: The daily absolute error comparison for two models. The error is defined as where F_t is the market price and F_t is the theoretical prices. The liquidity measure is Roll measure and the sample period is January 1, 2014, to December 31, 2014, with liquidity discount rate =0.10

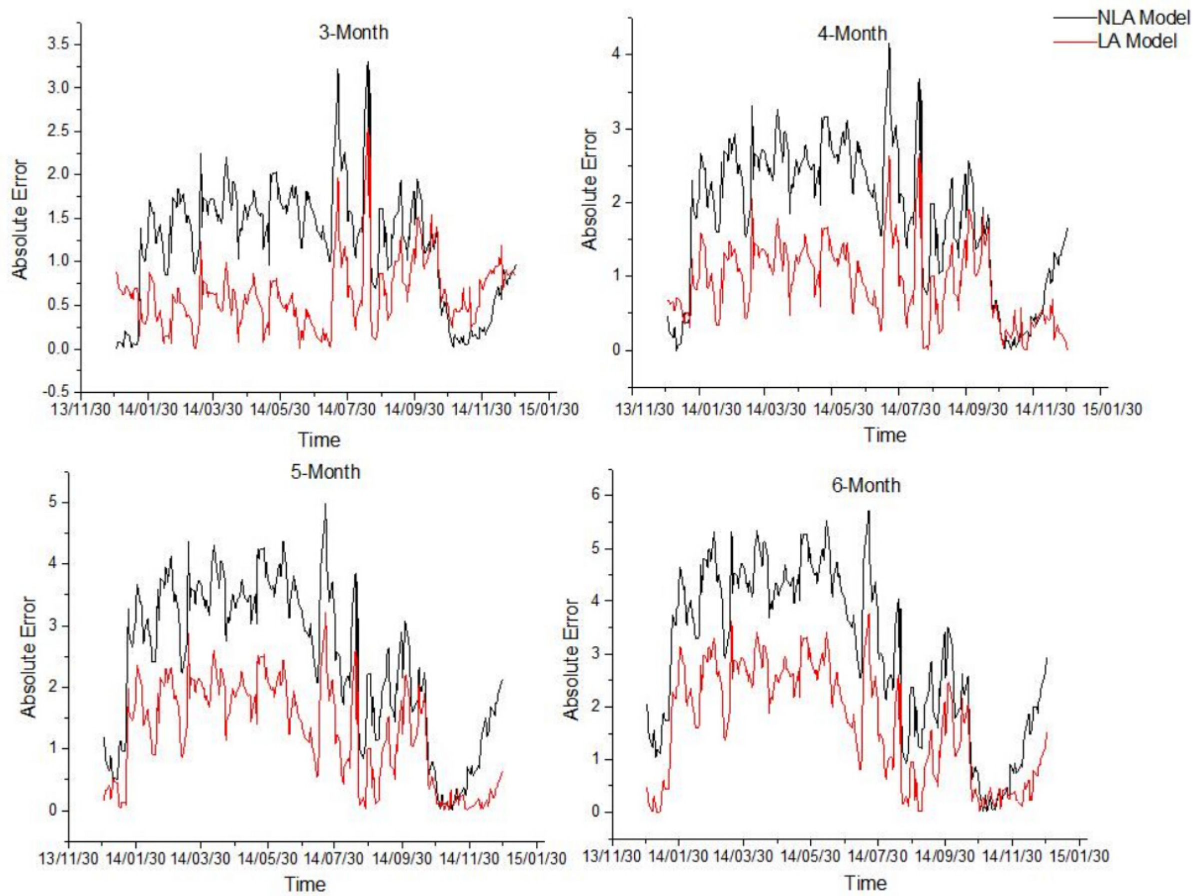


Figure 4.2: The daily absolute error comparison for two models

Figure 4.3: Twenty-year oil price movement

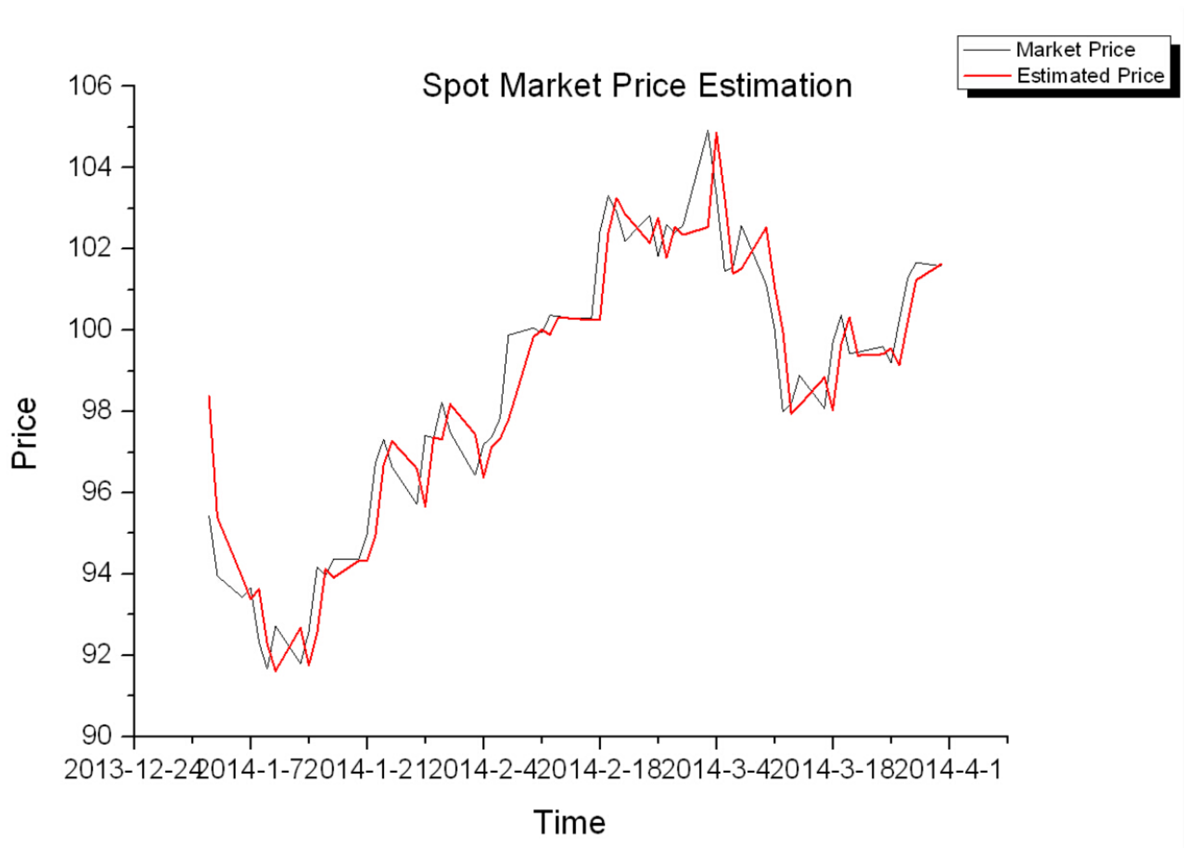


Figure 4.4: Comparison of spot market price and estimated price

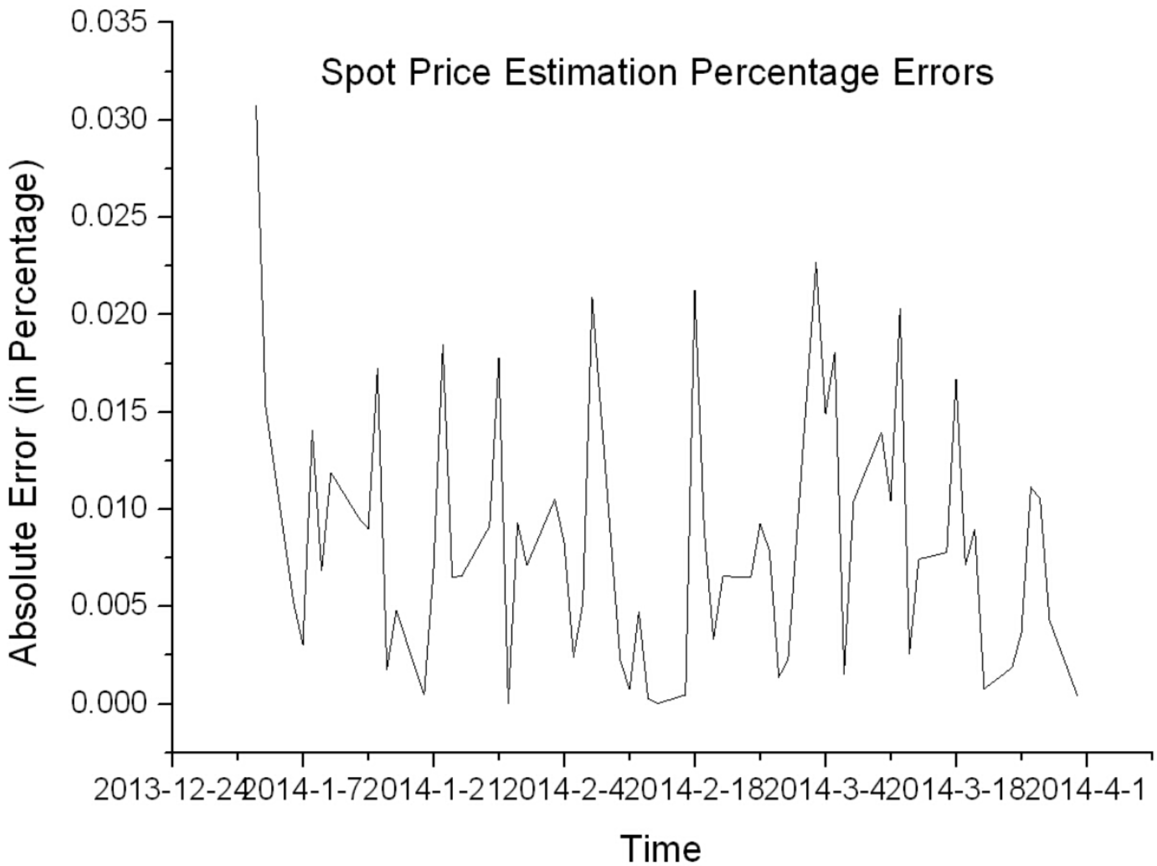


Figure 4.5: Daily errors for spot market price estimation

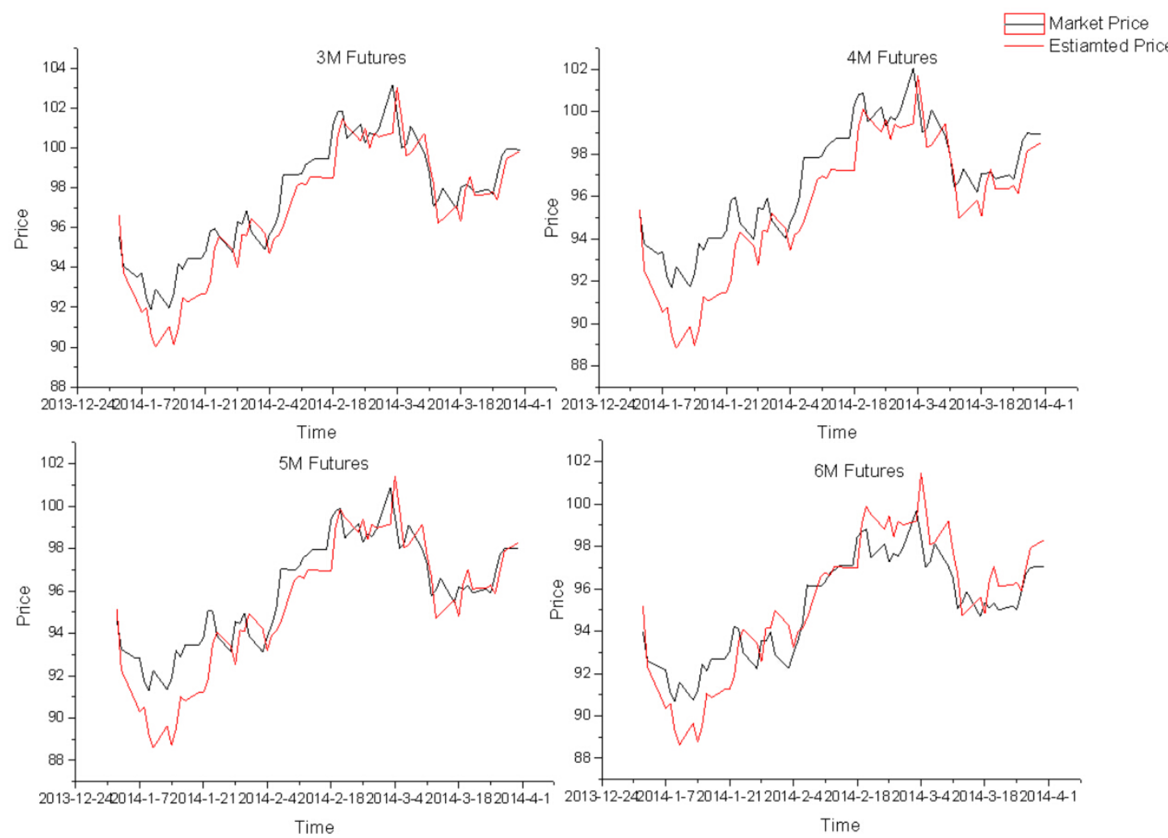


Figure 4.6: Comparison for 3-month, 4-month, 5-month and 6-month futures market prices and model estimated prices

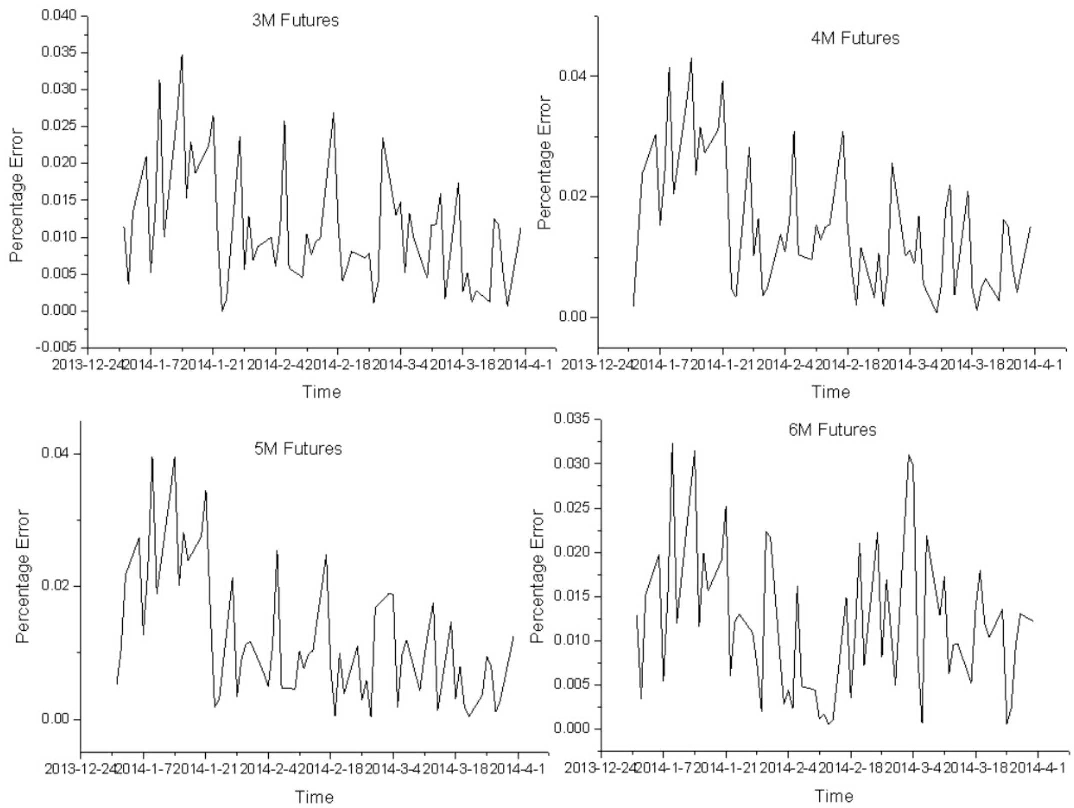


Figure 4.7: Percentage errors for 3-month, 4-month, 5-month and 6-month futures market price estimation

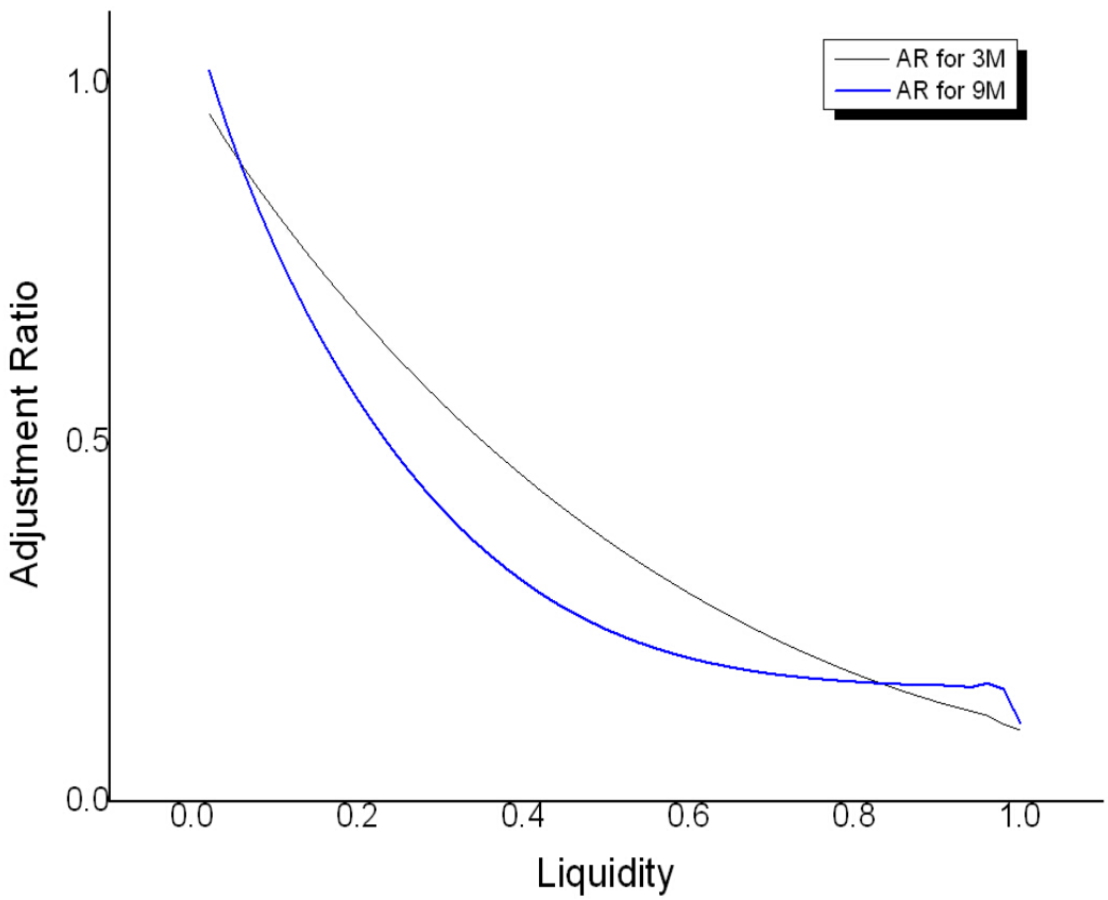


Figure 4.8: The plot of liquidity against adjustment ratio

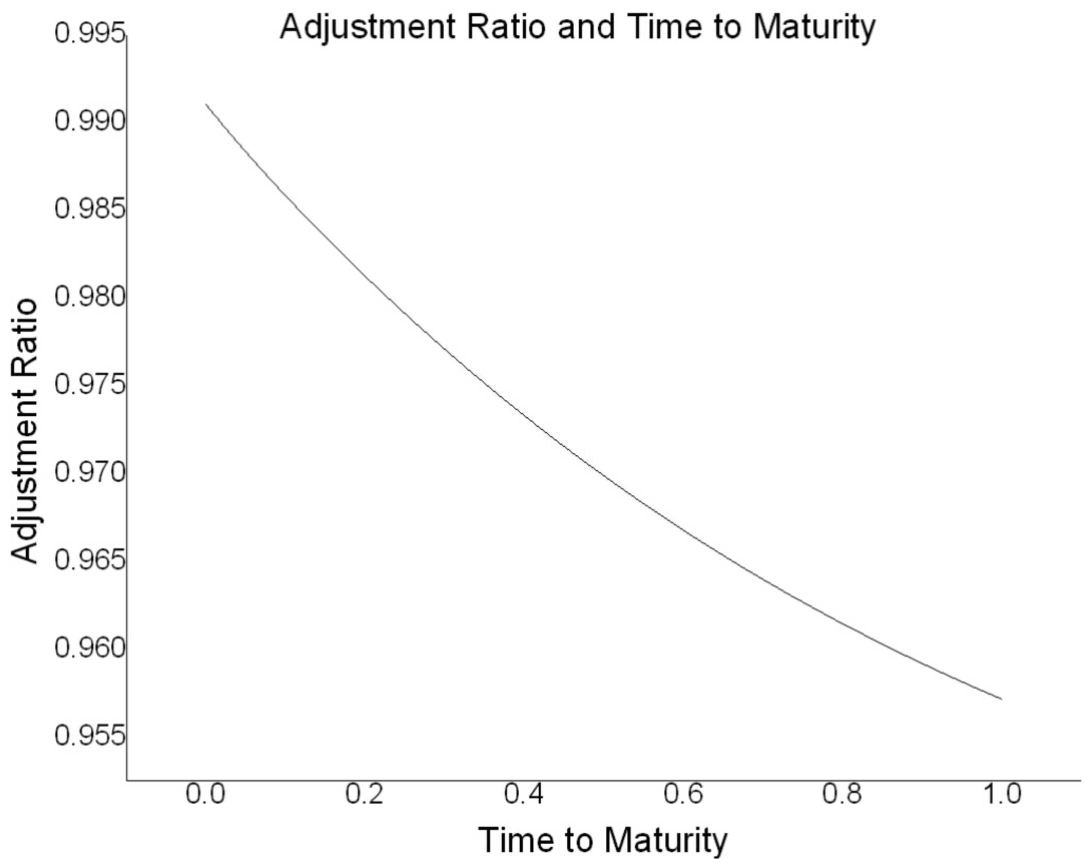


Figure 4.9: The plot of time to maturity against adjustment ratio

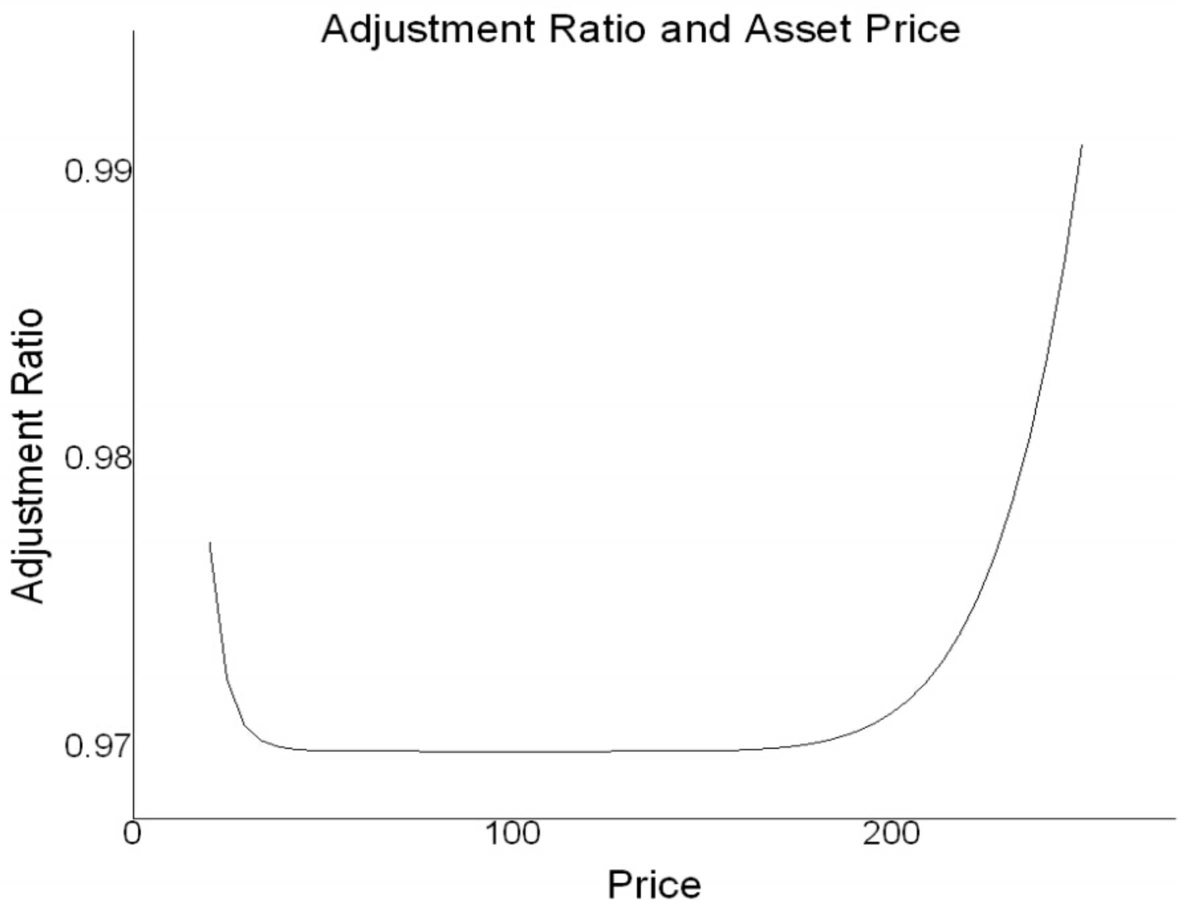
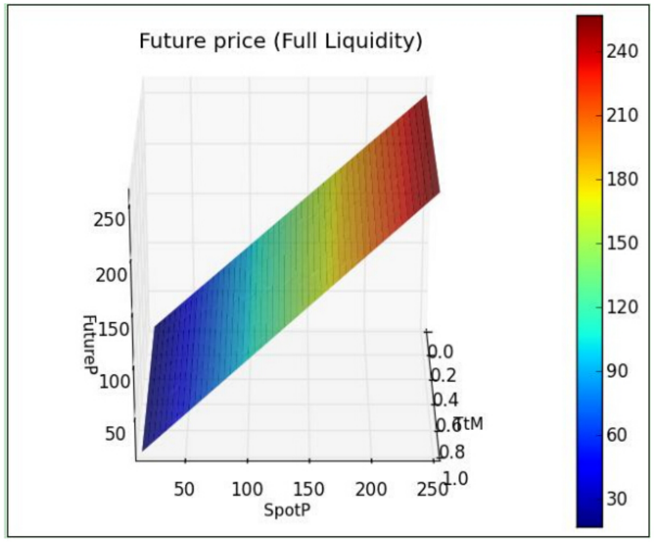
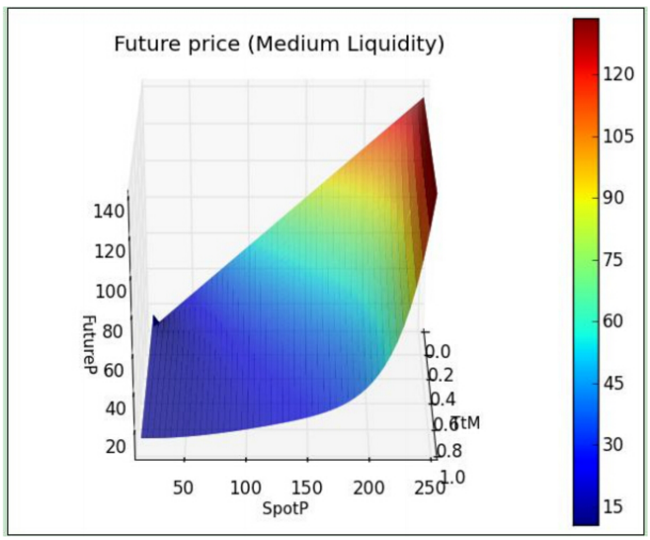


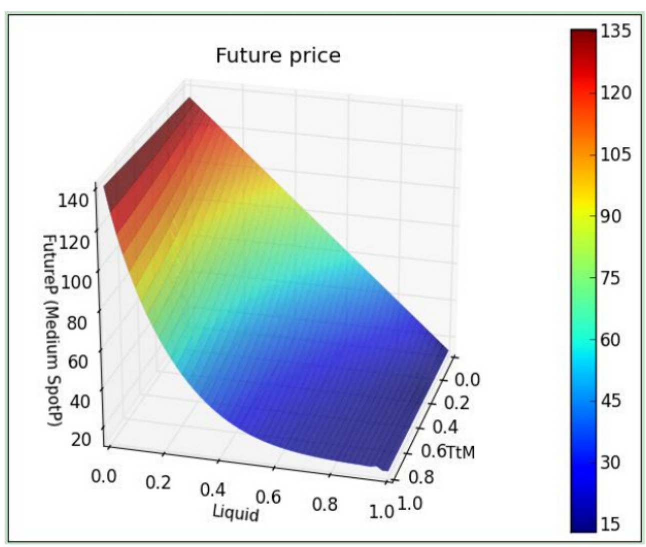
Figure 4.10: The plot of spot price against adjustment ratio



The perfect liquid case



The medium liquidity case



The medium spot price case

Figure 4.11: PDE surface presentation

Year	r	σ	ξ_{Ami}	ξ_{Roll}	Amihud	Roll
1994	0.0464	0.28	0.01	0.05	0.077	0.009
1995	0.0556	0.20	0.01	0.03	0.056	0.004
1996	0.0508	0.37	0.01	0.09	0.088	0.011
1997	0.0518	0.28	0.01	0.07	0.067	0.014
1998	0.0483	0.46	0.01	0.07	0.11	0.013
1999	0.0475	0.35	0.01	0.08	0.078	0.012
2000	0.059	0.42	0.01	0.16	0.066	0.033
2001	0.0334	0.42	0.01	0.1	0.076	0.018
2002	0.0168	0.34	0.01	0.09	0.045	0.03
2003	0.0105	0.38	0.01	0.14	0.033	0.04
2004	0.0158	0.35	0.01	0.15	0.02	0.05
2005	0.0339	0.31	0.01	0.19	0.012	0.06
2006	0.0481	0.27	0.01	0.19	0.008	0.04
2007	0.0444	0.30	0.01	0.18	0.005	0.06
2008	0.0162	0.61	0.01	0.23	0.007	0.07
2009	0.0028	0.53	0.03	0.23	0.01	0.05
2010	0.002	0.27	0.01	0.19	0.002	0.03
2011	0.001	0.34	0.03	0.23	0.003	0.04
2012	0.0013	0.25	0.08	0.2	0.02	0.07
2013	0.0009	0.18	0.01	0.17	0.002	0.04

Table 4.1: Twenty-year annual parameters estimations. r is the risk free rate; σ is the asset return volatility; ξ is the volatility of liquidity; Amihud and roll are two measures of liquidity.

Year	LA Model		NLA Model		P-value of T-test	Improvement Rate (%)
	Mean	Std Dev	Mean	Std Dev		
1995	0.15	0.13	0.52	0.26	0	71.15
1996	0.51	0.63	1.02	0.96	0	50.00
1997	0.32	0.35	0.48	0.59	0	33.33
1998	0.49	0.39	0.77	0.48	0	36.36
1999	0.44	0.54	0.81	0.81	0	45.68
2000	0.99	0.67	2.49	0.97	0	60.24
2001	0.53	0.51	0.54	0.61	0	1.85
2002	0.57	0.44	0.58	0.46	0	1.72
2003	0.69	0.67	1.33	0.9	0	48.12
2004	0.55	0.42	0.89	0.54	0	38.20
2005	0.84	0.56	0.88	0.57	0	4.55
2006	1.27	0.65	1.31	0.66	0	3.05
2007	1.28	0.89	1.8	0.93	0	28.89
2008	1.11	1.21	1.36	1.56	0	18.38
2009	1.21	1.63	2.82	2.24	0	57.09
2010	0.98	0.67	1.59	1.12	0	38.36
2011	0.92	1.02	1.19	1.36	0	22.69
2012	0.59	0.23	0.77	0.45	0	23.38
2013	0.95	0.47	1.15	1.1	0	17.39
2014	0.63	0.38	1.19	0.61	0	47.06
Average	0.75	0.62	1.17	0.86		32.38

Table 4.2: Twenty-year model MPEs with T-test for 3-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		P-value	Improvement
	Mean	Std Dev	Mean	Std Dev	of T-test	Rate (%)
1995	0.15	0.12	0.52	0.26	0	71.15
1996	0.59	0.54	1.02	0.96	0	42.16
1997	0.31	0.33	0.48	0.59	0	35.42
1998	0.48	0.39	0.77	0.48	0	37.66
1999	0.42	0.27	0.81	0.81	0	48.15
2000	0.57	0.45	2.49	0.97	0	77.11
2001	0.53	0.52	0.54	0.61	0	1.85
2002	0.58	0.45	0.58	0.46	0	0.00
2003	0.69	0.69	1.33	0.9	0	48.12
2004	0.65	0.42	0.89	0.54	0	26.97
2005	0.88	0.58	0.88	0.57	0	0.00
2006	1.25	0.64	1.31	0.66	0	4.58
2007	1.13	0.79	1.8	0.93	0	37.22
2008	1.03	1.18	1.36	1.56	0	24.26
2009	1.78	1.71	2.82	2.24	0	36.88
2010	0.77	0.98	1.59	1.12	0	51.57
2011	1.09	0.82	1.19	1.36	0	8.40
2012	0.36	0.3	0.77	0.45	0	53.25
2013	0.99	0.45	1.15	1.1	0	13.91
2014	0.94	0.52	1.19	0.61	0	21.01
Average	0.76	0.61	1.17	0.86		31.98

Table 4.3: Twenty-year model MPEs with T-test for 3-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.21	0.04	0.82	0.45	74.39
1996	0.81	1.7	1.63	6.01	50.31
1997	0.47	0.49	0.56	1.28	16.07
1998	0.63	0.79	0.72	1.27	12.50
1999	0.69	0.99	1.15	2.35	40.00
2000	1.19	1.79	1.68	5.29	29.17
2001	0.74	1.25	0.78	2.23	5.13
2002	0.72	0.83	0.74	0.87	2.70
2003	0.96	1.61	1.61	3.3	40.37
2004	0.69	0.61	1.04	1.08	33.65
2005	1.01	1.25	1.06	1.52	4.72
2006	1.44	2.09	1.46	2.13	1.37
2007	1.58	3.33	2.04	3.76	22.55
2008	1.63	10.08	2.07	14.2	21.26
2009	2.02	11.49	3.6	23.7	43.89
2010	1.18	2.5	1.95	6.73	39.49
2011	1.37	4.92	1.81	8.46	24.31
2012	0.74	0.33	0.81	0.53	8.64
2013	1.06	1.07	1.6	3.82	33.75
2014	0.73	0.74	1.36	1.04	46.32
Average	0.99	2.40	1.42	4.50	27.53

Table 4.4: Twenty-year model RMSEs for 3-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.19	0.06	0.82	0.45	76.83
1996	0.8	1.28	1.63	2.01	50.92
1997	0.46	0.45	0.56	1.28	17.86
1998	0.62	0.78	0.72	1.27	13.89
1999	0.49	0.36	1.15	2.35	57.39
2000	0.72	0.69	1.68	5.29	57.14
2001	0.77	1.27	0.78	2.23	1.28
2002	0.74	0.87	0.74	0.87	0.00
2003	0.98	1.68	1.61	3.3	39.13
2004	0.77	0.69	1.04	1.08	25.96
2005	1.05	1.28	1.06	1.52	0.94
2006	1.41	2.12	1.46	2.13	3.42
2007	1.38	2.25	2.04	3.76	32.35
2008	1.56	10.49	2.07	14.2	24.64
2009	2.46	13.12	3.6	23.7	31.67
2010	1.24	4.27	1.95	6.73	36.41
2011	1.46	3.79	1.81	8.46	19.34
2012	0.46	0.31	0.81	0.53	43.21
2013	1.09	0.98	1.6	3.82	31.88
2014	1.08	1.13	1.36	1.04	20.59
Average	0.99	2.39	1.42	4.30	29.24

Table 4.5: Twenty-year model RMSEs for 3-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model		NLA Model		P-value of T-test	Improvement Rate (%)
	Mean	Std Dev	Mean	Std Dev		
1995	0.25	0.15	0.76	0.33	0	67.11
1996	0.81	0.9	1.65	1.01	0	50.91
1997	0.45	0.54	0.64	0.76	0	29.69
1998	0.7	0.48	0.76	0.55	0	7.89
1999	0.78	0.79	1.16	1.06	0	32.76
2000	1.64	0.92	2.15	1.13	0	23.72
2001	0.67	0.7	0.72	0.77	0	6.94
2002	0.76	0.52	0.85	0.62	0.1	10.59
2003	1.1	1.04	1.93	1.13	0	43.01
2004	0.76	0.45	1.36	0.73	0	44.12
2005	0.99	0.65	1.05	0.72	0	5.71
2006	1.65	0.83	1.69	0.85	0	2.37
2007	1.82	1.14	2.46	2.43	0.01	26.02
2008	2.1	1.96	1.88	1.93	0	(11.70)
2009	1.85	2.02	3.75	2.69	0	50.67
2010	1.19	1.2	2.19	1.33	0	45.66
2011	0.9	0.91	1.61	1.73	0	44.10
2012	0.62	0.32	1.17	0.37	0	47.01
2013	1.33	1.12	1.9	1.75	0	30.00
2014	0.92	0.52	1.84	0.95	0	50.00
Average	1.06	0.86	1.58	1.14		30.33

Table 4.6: Twenty-Year Model MPEs with T-test for 4-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		P-value	Improvement
	Mean	Std Dev	Mean	Std Dev	of T-test	Rate(%)
1995	0.33	0.2	0.76	0.33	0	56.58
1996	0.64	0.8	1.65	1.01	0	61.21
1997	0.43	0.52	0.64	0.76	0	32.81
1998	0.71	0.48	0.76	0.55	0	6.58
1999	0.53	0.62	1.16	1.06	0	54.31
2000	0.81	0.66	2.15	1.13	0	62.33
2001	0.61	0.68	0.72	0.77	0	15.28
2002	0.63	0.49	0.85	0.62	0.1	25.88
2003	1.07	1.03	1.93	1.13	0	44.56
2004	0.88	0.65	1.36	0.73	0	35.29
2005	0.96	0.63	1.05	0.72	0	8.57
2006	1.68	0.87	1.69	0.85	0	0.59
2007	1.69	1	2.46	2.43	0.01	31.30
2008	2.01	1.97	1.88	1.93	0	(6.91)
2009	2.71	1.98	3.75	2.69	0	27.73
2010	1.23	1.26	2.19	1.33	0	43.84
2011	1.14	1.11	1.61	1.73	0	29.19
2012	0.65	0.4	1.17	0.37	0	44.44
2013	1.37	1	1.9	1.75	0	27.89
2014	0.73	0.49	1.84	0.95	0	60.33
Average	1.04	0.84	1.58	1.14		33.09

Table 4.7: Twenty-year model MPEs with T-test for 4-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.22	0.08	0.88	0.7	75.00
1996	1.15	3.13	2.11	7.12	45.50
1997	0.7	1.1	0.76	2.18	7.89
1998	0.85	1.24	0.87	1.73	2.30
1999	1.11	2.23	1.12	3.96	0.89
2000	1.88	3.55	2.33	7.58	19.31
2001	0.96	2.15	1.05	2.47	8.57
2002	0.93	0.96	1.06	1.75	12.26
2003	1.52	3.68	2.24	5.9	32.14
2004	0.89	0.85	1.54	2.03	42.21
2005	1.19	1.68	1.25	2.1	4.80
2006	1.85	3.39	1.89	3.45	2.12
2007	2.18	5.64	2.75	6.08	20.73
2008	2.88	18.64	2.61	18.54	(10.34)
2009	2.74	17.79	4.6	34.5	40.43
2010	1.28	4.23	2.56	9.62	50.00
2011	1.85	8.75	2.36	13.79	21.61
2012	0.73	0.5	1.21	0.79	39.67
2013	1.74	4.32	2.59	9.49	32.82
2014	1.05	1.12	2.06	3.25	49.03
Average	1.39	4.25	1.89	6.85	24.85

Table 4.8: Twenty-year model RMSEs for 4-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.39	0.18	0.88	0.7	55.68
1996	1.03	2.71	2.11	7.12	51.18
1997	0.68	1.03	0.76	2.18	10.53
1998	0.86	1.24	0.87	1.73	1.15
1999	0.82	1.37	1.12	3.96	26.79
2000	1.04	1.67	2.33	7.58	55.36
2001	0.92	2.09	1.05	2.47	12.38
2002	0.81	0.85	1.06	1.75	23.58
2003	1.48	3.57	2.24	5.9	33.93
2004	1.09	1.59	1.54	2.03	29.22
2005	1.21	1.69	1.25	2.1	3.20
2006	1.87	3.42	1.89	3.45	1.06
2007	1.96	3.83	2.75	6.08	28.73
2008	2.82	18.59	2.61	18.54	(8.05)
2009	3.35	19.8	4.6	34.5	27.17
2010	1.76	6.69	2.56	9.62	31.25
2011	1.59	5.8	2.36	13.79	32.63
2012	0.77	0.62	1.21	0.79	36.36
2013	1.7	3.7	2.59	9.49	34.36
2014	0.88	1.06	2.06	3.25	57.28
Average	1.35	4.08	1.89	6.85	27.19

Table 4.9: Twenty-year model RMSEs for 4-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models

Year	LA Model		NLA Model		P-value	Improvement
	Mean	Std Dev	Mean	Std Dev	of T-test	Rate(%)
1995	0.32	0.23	0.99	0.36	0	67.68
1996	1.19	0.96	1.82	1.08	0	34.62
1997	0.58	0.73	0.63	0.89	0	7.94
1998	0.88	0.55	1.05	0.61	0	16.19
1999	1.12	1.03	1.39	1.19	0	19.42
2000	2.28	1.09	3.15	1.27	0	27.62
2001	0.89	0.86	0.96	0.91	0	7.29
2002	0.69	0.61	1.13	0.82	0	38.94
2003	1.65	1.27	2.18	1.33	0	24.31
2004	1.08	0.63	1.87	0.86	0	42.25
2005	1.09	0.68	1.15	0.78	0	5.22
2006	1.87	0.97	1.92	0.99	0	2.60
2007	2.29	1.38	3.02	1.5	0	24.17
2008	2.67	2.25	2.33	2.21	0	(14.59)
2009	2.59	2.32	4.43	3	0	41.53
2010	0.93	1.14	2.67	1.47	0	65.17
2011	1.48	1.64	1.93	2	0	23.32
2012	0.46	0.32	1.51	0.42	0	69.54
2013	1.87	1.74	2.75	2.33	0	32.00
2014	1.25	0.81	2.38	1.22	0	47.48
Average	1.36	1.06	1.96	1.26		29.13

Table 4.10: Twenty-year model MPEs with T-test for 5-month oil futures (Amihud Measure) This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		P-value	Improvement
	Mean	Std Dev	Mean	Std Dev	of T-test	Rate(%)
1995	0.52	0.29	0.99	0.36	0	47.47
1996	1.08	0.89	1.82	1.08	0	40.66
1997	0.56	0.69	0.63	0.89	0	11.11
1998	0.89	0.55	1.05	0.61	0	15.24
1999	0.88	0.91	1.39	1.19	0	36.69
2000	1.31	0.93	3.15	1.27	0	58.41
2001	0.85	0.82	0.96	0.91	0	11.46
2002	0.67	0.59	1.13	0.82	0	40.71
2003	1.53	1.25	2.18	1.33	0	29.82
2004	0.8	0.65	1.87	0.86	0	57.22
2005	1.05	0.67	1.15	0.78	0	8.70
2006	1.83	0.95	1.92	0.99	0	4.69
2007	2.2	1.13	3.02	1.5	0	27.15
2008	2.57	2.26	2.33	2.21	0	(10.30)
2009	3.56	2.23	4.43	3	0	19.64
2010	1.72	1.38	2.67	1.47	0	35.58
2011	1.21	1.35	1.93	2	0	37.31
2012	0.97	0.45	1.51	0.42	0	35.76
2013	1.79	1.6	2.75	2.33	0	34.91
2014	0.7	0.53	2.38	1.22	0	70.59
Average	1.33	1.01	1.96	1.26		30.64

Table 4.11: Twenty-year model MPEs with T-test for 5-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate(%)
1995	0.34	0.22	0.95	0.86	64.21
1996	1.5	4.11	2.65	8.11	43.40
1997	0.93	1.95	1.02	3.15	8.82
1998	1.04	1.66	1.05	2.09	0.95
1999	1.53	3.83	1.72	5.09	11.05
2000	2.51	5.46	3.81	9.7	34.12
2001	1.24	3.16	1.32	3.46	6.06
2002	0.92	1.37	1.4	3	34.29
2003	2.08	6.12	2.83	8.71	26.50
2004	1.25	1.48	2.05	2.97	39.02
2005	1.28	1.84	1.35	1.98	5.19
2006	2.12	4.58	2.17	4.66	2.30
2007	2.69	8.9	3.36	9.87	19.94
2008	3.49	22.88	3.12	22.92	(11.86)
2009	3.48	24.51	5.41	43.9	35.67
2010	1.48	5.71	3.06	11.97	51.63
2011	2.2	12.04	2.77	18.19	20.58
2012	0.56	0.37	1.57	1.31	64.33
2013	2.55	9.71	3.6	17.27	29.17
2014	1.48	2.05	2.65	5.61	44.15
Average	1.73	6.10	2.39	9.24	26.48

Table 4.12: Twenty-year model RMSEs for 5-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.39	0.18	0.88	0.7	55.68
1996	1.03	2.71	2.11	7.12	51.18
1997	0.68	1.03	0.76	2.18	10.53
1998	0.86	1.24	0.87	1.73	1.15
1999	0.82	1.37	1.12	3.96	26.79
2000	1.04	1.67	2.33	7.58	55.36
2001	0.92	2.09	1.05	2.47	12.38
2002	0.81	0.85	1.06	1.75	23.58
2003	1.48	3.57	2.24	5.9	33.93
2004	1.09	1.59	1.54	2.03	29.22
2005	1.21	1.69	1.25	2.1	3.20
2006	1.87	3.42	1.89	3.45	1.06
2007	1.96	3.83	2.75	6.08	28.73
2008	2.82	18.59	2.61	18.54	(8.05)
2009	3.35	19.8	4.6	34.5	27.17
2010	1.76	6.69	2.56	9.62	31.25
2011	1.59	5.8	2.36	13.79	32.63
2012	0.77	0.62	1.21	0.79	36.36
2013	1.7	3.7	2.59	9.49	34.36
2014	0.88	1.06	2.06	3.25	57.28
Average	1.35	4.08	1.89	6.85	27.19

Table 4.13: Twenty-year model RMSEs for 5-month oil futures (Roll Measure) This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model		NLA Model		P-value of T-test	Improvement Rate (%)
	Mean	Std Dev	Mean	Std Dev		
1995	0.37	0.27	0.54	0.55	0	31.48
1996	1.49	0.96	2.94	1.08	0	49.32
1997	0.69	0.86	0.55	0.45	0	(25.45)
1998	0.97	0.63	0.85	0.53	0	(14.12)
1999	1.33	1.08	1.48	0.81	0	10.14
2000	2.7	1.22	2.86	0.71	0	5.59
2001	1.05	1.01	1.1	0.98	0	4.55
2002	0.74	0.61	1.32	0.9	0	43.94
2003	2.02	1.43	2.63	0.93	0	23.19
2004	1.41	0.79	2.42	0.77	0	41.74
2005	1.16	0.68	1.21	0.78	0	4.13
2006	1.99	1.14	2.05	1.14	0	2.93
2007	2.69	1.59	3.5	1.72	0	23.14
2008	3.23	2.51	2.73	2.5	0	(18.32)
2009	3.21	2.48	5.28	3.14	0	39.20
2010	1.07	1.31	3.11	1.56	0	65.59
2011	1.6	1.63	2.15	2.14	0	25.58
2012	0.43	0.29	1.76	0.53	0	75.57
2013	2.49	2.31	3.67	2.84	0	32.15
2014	1.66	1.06	2.92	1.51	0	43.15
Average	1.62	1.19	2.25	1.28		23.18

Table 4.14: Twenty-year model MPEs with T-test for 6-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		P-value	Improvement
	Mean	Std Dev	Mean	Std Dev	of T-test	Rate(%)
1995	0.68	0.36	0.54	0.55	0	(25.93)
1996	1.49	0.96	2.94	1.08	0	49.32
1997	0.65	0.83	0.55	0.45	0	(18.18)
1998	0.97	0.61	0.85	0.53	0	(14.12)
1999	1.19	1.07	1.48	0.81	0	19.59
2000	1.74	1.11	2.86	0.71	0	39.16
2001	1.01	0.95	1.1	0.98	0	8.18
2002	0.78	0.59	1.32	0.9	0	40.91
2003	1.81	1.4	2.63	0.93	0	31.18
2004	0.81	0.62	2.42	0.77	0	66.53
2005	1.12	0.63	1.21	0.78	0	7.44
2006	1.93	1.11	2.05	1.14	0	5.85
2007	2.65	1.24	3.5	1.72	0	24.29
2008	3.13	2.52	2.73	2.5	0	(14.65)
2009	4.23	2.39	5.28	3.14	0	19.89
2010	2.13	1.47	3.11	1.56	0	31.51
2011	1.26	1.35	2.15	2.14	0	41.40
2012	1.19	0.48	1.76	0.53	0	32.39
2013	2.32	2.13	3.67	2.84	0	36.78
2014	0.95	0.54	2.92	1.51	0	67.47
Average	1.60	1.12	2.25	1.28		22.45

Table 4.15: Twenty-year model MPEs with T-test for 6-month oil futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Mean Pricing Errors (MPE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate(%)
1995	0.46	0.4	0.54	0.33	14.81
1996	1.87	4.81	3.13	8.31	40.26
1997	1.1	2.84	1.56	2.47	29.49
1998	1.14	2.01	1.36	0.03	16.18
1999	1.79	4.9	1.49	0.11	(20.13)
2000	2.96	7.23	2.86	0.14	(3.50)
2001	1.47	4.11	1.53	4.34	3.92
2002	0.96	1.44	1.59	3.28	39.62
2003	2.47	8.11	3.2	10.9	22.81
2004	1.62	2.2	2.49	4.16	34.94
2005	1.34	1.93	1.41	2.05	4.96
2006	2.29	5.67	2.35	5.79	2.55
2007	3.11	9.76	3.9	13.68	20.26
2008	4.08	27.17	3.63	27.45	(12.40)
2009	4.05	30	6.1	52.1	33.61
2010	1.69	7	3.47	14.04	51.30
2011	2.28	12.3	3.03	21.08	24.75
2012	0.52	0.34	1.83	1.88	71.58
2013	3.4	16.9	4.64	26.83	26.72
2014	1.97	3.57	3.28	8.56	39.94
Average	2.03	7.63	2.67	10.38	22.08

Table 4.16: Twenty-year model RMSEs for 6-month oil futures (Amihud Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model		NLA Model		Improvement
	Mean	Std Dev	Mean	Std Dev	Rate (%)
1995	0.77	0.61	0.54	0.33	(42.59)
1996	1.78	4.81	3.13	8.31	43.13
1997	1.06	2.65	1.56	2.47	32.05
1998	1.15	2.02	1.36	0.03	15.44
1999	1.6	4.16	1.49	0.11	(7.38)
2000	2.06	4.75	2.86	0.14	27.97
2001	1.43	4.06	1.53	4.34	6.54
2002	0.98	1.53	1.59	3.28	38.36
2003	2.29	7.42	3.2	10.9	28.44
2004	1.02	1.4	2.49	4.16	59.04
2005	1.3	1.92	1.41	2.05	7.80
2006	2.24	5.62	2.35	5.79	4.68
2007	2.91	6.81	3.9	13.68	25.38
2008	4.01	27.15	3.63	27.45	(10.47)
2009	4.85	32.6	6.1	52.1	20.49
2010	2.58	10.38	3.47	14.04	25.65
2011	1.84	8.43	3.03	21.08	39.27
2012	1.28	1.17	1.83	1.88	30.05
2013	3.15	14.59	4.64	26.83	32.11
2014	1.09	1.11	3.28	8.56	66.77
Average	1.97	7.16	2.67	10.38	22.14

Table 4.17: Twenty-year model RMSEs for 6-month oil Futures (Roll Measure). This table represents the results comparison for the Liquidity-adjusted (LA) model and Nonliquidity-adjusted (NLA) model. It compares the means and volatilities of Root Mean Squared Error (RMSE) for both models.

Year	LA Model with Convenience Yield		NLA Model		P-value of T-test	Improvement Rate (%)
	Mean	Std Dev	Mean	Std Dev		
1995	0.34	0.47	0.54	0.55	0	37.04
1996	1.28	0.95	2.94	1.08	0	56.46
1997	0.35	0.43	0.55	0.45	0	36.36
1998	0.67	0.46	0.85	0.53	0	21.18
1999	0.31	0.78	1.48	0.81	0	79.05
2000	1.66	0.68	2.86	0.71	0	41.96
2001	0.92	0.86	1.1	0.98	0	16.36
2002	0.72	0.6	1.32	0.9	0	45.45
2003	2.14	1.44	2.63	0.93	0	18.63
2004	1.38	0.74	2.42	0.77	0	42.98
2005	1.22	0.68	1.21	0.78	0	(0.83)
2006	2.06	1.14	2.05	1.14	0	(0.49)
2007	3.47	1.61	3.5	1.72	0	0.86
2008	3.22	2.52	2.73	2.5	0	(17.95)
2009	3.2	2.43	5.28	3.14	0	39.39
2010	1.04	1.3	3.11	1.56	0	66.56
2011	1.53	1.63	2.15	2.14	0	28.84
2012	0.41	0.26	1.76	0.53	0	76.70
2013	2.45	2.23	3.67	2.84	0	33.24
2014	1.63	1.02	2.92	1.51	0	44.18
Average	1.42	1.11	2.25	1.28		33.30

Table 4.18: Twenty-year model MPEs with T-test for 6-month oil Futures (Amihud Measure). This table comprares new Liquidity-adjusted (LA) model that considers both liquidity factor and convenience yield with Nonliquidity-adjusted (NLA) model. This table presents the means and volatilities of Mean Pricing Errors (MPE) for both models and it is comparable with Table 4.14.

Year	LA Model with Convenience Yield		NLA Model		P-value of T-test	Improvement Rate (%)
	Mean	Std Dev	Mean	Std Dev		
1995	0.47	0.36	0.54	0.55	0	12.96
1996	1.54	1.06	2.94	1.08	0	47.62
1997	0.48	0.88	0.55	0.45	0	12.73
1998	0.76	0.61	0.85	0.53	0	10.59
1999	1.21	1.05	1.48	0.81	0	18.24
2000	1.78	1.11	2.86	0.71	0	37.76
2001	1.03	0.95	1.1	0.98	0	6.36
2002	0.81	0.44	1.32	0.9	0	38.64
2003	1.75	1.39	2.63	0.93	0	33.46
2004	1.02	0.64	2.42	0.77	0	57.85
2005	1.24	0.65	1.21	0.78	0	(2.48)
2006	2.09	1.12	2.05	1.14	0	(1.95)
2007	3.02	1.55	3.5	1.72	0	13.71
2008	3.11	2.54	2.73	2.5	0	(13.92)
2009	4.25	2.33	5.28	3.14	0	19.51
2010	2.11	1.48	3.11	1.56	0	32.15
2011	1.33	1.45	2.15	2.14	0	38.14
2012	1.23	0.51	1.76	0.53	0	30.11
2013	2.02	2.14	3.67	2.84	0	44.96
2014	1.05	0.66	2.92	1.51	0	64.04
Average	1.62	1.15	2.25	1.28		25.02

Table 4.19: Twenty-year model MPEs with T-test for 6-month oil futures (Roll Measure). This table comprares new Liquidity-adjusted (LA) model that considers both liquidity factor and convenience yield with Nonliquidity-adjusted (NLA) model. This table presents the means and volatilities of Mean Pricing Errors (MPE) for both models and it is comparable with Table 4.15.

	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
3-Month	0.99	0.98	0.95	0.94	0.92	0.90	0.89	0.87	0.84
4-Month	1.35	1.34	1.33	1.3	1.34	1.33	1.34	1.34	1.37
5-Month	1.84	1.85	1.87	1.88	1.89	1.91	1.93	1.97	1.97
6-Month	2.45	2.47	2.49	2.52	2.54	2.57	2.59	2.65	2.65

Table 4.20: MPE for different discount factors and different maturities. The first row represents different discount factors and the first column represents different matutirities

	0.01	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
3-Month	1.09	1.07	1.06	1.06	1.05	1.05	1.06	1.07	1.08
4-Month	1.73	1.74	1.76	1.78	1.80	1.82	1.86	1.91	
5-Month	2.50	2.52	2.55	2.57	2.60	2.63	2.66	2.72	2.72
6-Month	3.34	3.37	3.40	3.43	3.46	3.49	3.53	3.59	3.60

Table 4.21: RMSE for different discount factors and different maturities. The first row represents different discount factors and the first column represents different matutirities

Chapter 5

Extending Real Options Theory with Real Asset Illiquidity

5.1 Introduction

5.1.1 Related Literatures and Motivations of the Chapter

The real options theory links option pricing theory with corporate investment decisions. From the corporate investment decisions perspective, cash plays a crucial role in the investment decision making process. Stulz (1990) maintains that when cash flow is too low, managers are forced to make little investment. Likewise, Lamont (1997) argues that the decrease in cash will decrease the investment, holding the profitability of investment fixed. However, on the other hand, Almeida, Campello and Weisbach (2004) point out that holding cash is costly and is detrimental for investment. Bates et al. (2009) declare that the average cash-to-assets ratio of U.S. firms has significantly risen from 1980 to 2006 and U.S. firms tend to hold much more cash today. One reason they maintain is that the firm's cash flows turn to be more uncertain and firms are financially constrained. In fact, the topic of cash holdings and financing constraints on investment has been intensively studied (see Stulz, 1990; Whited, 2006; Denis and Sibilkov, 2010; Denis, 2011).

In fact, cash represents the liquidity within firms and the liquidity effects on option values have seen significant growth in interest amongst academics and practitioners for the last few decades. Brenner, Eldor, and Hauser (2001) examine the liquidity effects currency options prices and their test affirms the liquidity effects on options prices. Chou et al. (2011) also test the liquidity effects of options and they find that every option has an intrinsic liquidity cost and higher option liquidity level means a higher implied volatility level, which is coined in the argument of illiquidity premium by Amihud and Mendelson (1986). Furthermore, given that options are contingent assets, it is arguable that the liquidity of the underlying asset can be also involved in pricing of options. As a result, several studies, such as Frey (1998), consolidate the liquidity effects into the option pricing model. Therefore, I argue that the liquidity factor is crucial for options pricing model and then build the real options pricing model by incorporating the stochastic liquidity factor.

Therefore, this chapter is mainly motivated by the liquidity effects on the real options value and the effects are exhibited through linkages between real options theory and the asset illiquidity. The first linkage is that the real asset illiquidity relates to the cash flows the project and thereby the project value and the free boundary for launching the project. The first linkage is that the real asset illiquidity relates to the cash flows of the project and thereby the project value and the free boundary for launching the project. The real asset here mainly refers to the inventory assets that produced by the projects and expected to be sold to generate cash flows, such as crude oil from oilfield project and copper from copper mining project.

In the financial literature, the value of the project and the real asset values are also highly correlated. In copious classical works of real options, they are perfectly linked (Dixit and Pindyck, 1994). Many real options literatures thereby directly used the output of the project as the underlying asset of the real options. For instance, Brennan and Schwartz (1985) apply the real option framework to value copper mines and they use copper as the underlying asset in their model. Paddock et al. (1988) adopt the real options method to evaluate the offshore oil fields and they use the oil as the underlying asset. Grenadier (1996) uses the real options method to evaluate the real estate value by employing the housing price as the underlying asset price. Another example of the real options underlying asset could be the aircraft and many scholars tried to extend the real options theory into the aircraft valuation and the aircraft is considered as the underlying asset under the framework (see Stonier, 1999 and Gibsona and Morrell, 2004).

Practically, the real options theory is a risk management tool that helps managers to evaluate the project and make investment decisions accordingly. When firms evaluate a real investment opportunity, they are mainly concerned about the future cash flows that the project can generate. The potential risk of future cash flows could be a driving force in the investment decisions. I identify that the main source of cash flow risk comes from the inventory asset illiquidity. Intuitively, when a firm launches a project and the sale of the inventory asset generates the cash flows for the project. For instance, the oil project extracts the crude oil from the oilfield and oil firms sell the extracted oil in the oil market to obtain the cash flows. The liquidity of the crude oil thereby dominates the future cash flows since one of the most important reasons for the oil price collapsing is lacking of liquidity (Tokic, 2012). When the market of the inventory asset becomes less liquid, then the inventory assets produced are more difficult to sell and thus the cash flows face potential uncertainty. In that case, firms either wait for a longer period to sell the asset or they sell the asset at a relatively lower price, which introduces the cash flow risk. Thus, cash flow risk might be embedded in asset liquidity risk.

Moreover, the potential cash flow risk links to the cash holdings of the firm, which is an intensively studied topic more recently (see Duchin, 2010; Fresard, 2010; Palazzo, 2012; Harford et al., 2014; Hugonnier et al., 2015). Many studies suggest that the increase of cash flow risk will eventually raise the cash holdings of the firm. Like Kim et al. (1998) and Harford (1999) confirm that cash holdings are positively related to the future cash flow volatility. Cash holding is the fundamental component of the internal funding resource and internal financing resolves the adverse selection problem and grasps the investment opportunities since it is costly for entrepreneurs to finance externally. As the pecking order theory states, asymmetric information makes the external funding more costly (Myers, 1984; Myers and Majluf, 1984; Bouvard, 2014). As a result, it is reasonable to presume entrepreneurs prefer to finance internally and they tend to be financially constrained. Since most firms prefer to fund the investment opportunity from internal financing, then the internal capital market plays a crucial role in resource allocation (Stein, 1997; Fee et al., 2009). Likewise, Bhagat et al. (2005) also maintain that most of the corporate investment is funded by the internally-generated cash flows. Fazzari et al. (1988) demonstrate that the investment spending largely depends on the availability of the internal fund. The cash flow from the project revenues is one of the most important sources of internal fund. As a result, asset illiquidity creates the cash flow uncertainty and this uncertainty influences the internal funding of the firm, which might impact the corporate investment behaviors.

In fact, cash holdings are valuable to firms, especially for the firms that are financially constrained. Faulkender and Wang (2006) show the evidence that cash holdings are more valuable for constrained firms than for unconstrained firms. Brown and Petersen (2011) display that financially constrained rely heavily on the cash holdings to smooth their R&D projects. Denis and Sibilkov (2010) argue that cash holdings are connected with investment levels for constrained firms. They also suggest that higher cash holdings allow constrained firms to undertake value-increasing projects. Moreover, cash holdings can also enhance the firm values for constrained firms, where the effect is stronger than unconstrained firms. Hence, cash holdings are valuable to financially constrained firms and the cash flows uncertainty will impact on the cash holdings. Since the inventory asset illiquidity might affect the cash flows, it might have impact on corporate investment decisions. Therefore, the real options theory shall incorporate the asset illiquidity factor because it is a theory for corporate investment decisions.

The second aspect of nexus between real asset liquidity and real options relies on the fact that the real assets illiquidity will reduce the operating flexibility of the firm. In other words, the second linkage is that the real asset illiquidity relates to the flexibility of the project and thereby the real options values. The liquidity of the assets that a firm holds reflects the generally liquidity level of the firm. As demonstrated by Flor and Hirth (2013) and Ortiz-Molina and Phillips (2014), real asset with higher liquidity can provide more flexibility to the firm and the illiquid assets provide less flexibility to the firm. Holding liquid assets not only gives great flexibility to the firm but also reduces the demand for liquidity (Jones and Ostroy, 1984). Almeida et al. (2011) demonstrate that the firms will prefer short payback period projects with liquid assets and the liquid assets can enhance the firm's flexbility. Furthermore, Morellec (2001) notices that the operating flexibility associated with the asset liquidity and thus liquid assets holdings can increase firm values. On the other hand, when the asset becomes more illiquid, the flexibility will be significantly reduced. From this perspective, it is clear that the real asset liquidity is closely related to the real options theory since the operating flexibility is a vital concern of the real options theory. For instance, Mardones (1993) reveals the fact that the operating flexibility can add value to a project and can be measured by contingent claim analysis due to its option-like features. Likewise, Kogut and Kulatilaka (1994) claim that the operating flexibility such as production shifting is identical as owning an option and Zhang (2005) gives the return implications of flexibility under the real options framework. Since the real assets illiquidity will affect the operating flexibility and the real options theory is the best way to evaluate such flexibilities, it shall be considered in the real options model.

This study is also motivated by the substantial liquidity effects of the real assets. The centralized-traded assets such as futures have liquidity risk. Other real assets like real estate are often traded in decentralized markets such as Over-the-Counter (OTC) markets. Without the facilitation of the centralized trading system, the trading structure of the OTC markets makes the liquidity effects even more noticeable. Without the central trading coordination system, investors cannot buy or sell the asset instantaneously in the OTC markets. In turn, they have to trade the asset through a searching and bargaining process, which underlines the liquidity effects (Jankowitsch et al., 2011). Since the liquidity effects are noticeably important and dramatic in the real asset markets, the liquidity premium will be much higher (Ang et al., 2013). Hence, it is rather crucial to investigate the liquidity effects in such fairly illiquid markets.

Nevertheless, the oversight of asset illiquidity factor in existing real options model might provide less accurate results for option values (for example, Dias and Nunes, 2011; Adkins and Paxson, 2016). In real options theory, other factors have been considered such as time preference (Grenadier and Wang, 2007) and utility (Wang and Miao, 2011). More importantly, since the real assets illiquidity might impact on the firm values, can managers cope with the issue to improve the situation and thus increase the firm values by making better investment decisions and liquidity management. Furthermore, like Cleary (1999) and Kaplan and Zingales (2000) investigate the sensitivity of investment regarding the cash flow of the firm and they find that the sensitivity is not monotonic in the degree of liquidity constraints.

Therefore, this chapter will focus on the real options model with the asset illiquidity factor. The effects of financial asset liquidity in the financial markets have been studied in many financial models, such as the Capital Asset Pricing Model (CAPM) with liquidity (Acharya and Pedersen, 2005 and Liu, 2006). More recently, rather than the financial asset liquidity, but the real asset illiquidity issue becomes a focus of financial researches, such as Flor and Hirth (2013) and Ortiz-Molina and Phillips (2014). In the chapter, I argue that the inventory asset illiquidity not only influences the firm value through the channel cost of capital, but also the channel of real project values and real options values. The higher asset liquidity may increase the value of project value and lower the level of the cash holdings, which can induce higher firm values since cash holdings are costly. Therefore, I provide an intrinsic link between real asset illiquidity and real options theory. More importantly, I build a real options model that takes the real asset illiquidity into account and delivers practical implications to real investment decision problems. The developed real options framework with finite option rights might help the managers to manage the cash legitimately and enhance firm values especially for those financially constrained firms.

5.1.2 Contributions and Structure of the Chapter

The first contribution of this chapter is that I build a two-factor real options model which incorporates the inventory asset illiquidity. This model builds an intrinsic connection between real options theory and inventory asset illiquidity. The study of the model reveals the effects of inventory asset illiquidity towards investment threshold and flexibility values, namely, the exercise boundary and the real options values, which is complementary to the existing real options and corporate finance literatures. Instead of constructing a free boundary line which shows the effects of time and asset price, the model presents a three-dimensional "free surface" which indicates not only the effects of time and asset price, but also the effects of inventory asset illiquidity. The results can aid managers to have a deeper understanding of the role of real asset illiquidity in corporate finance and risk management.

Moreover, the new model relates to two categories of existing literatures. The first class of literature focuses on the effects of real asset illiquidity (mainly physical asset) on the corporate investment and cost of capital. The illiquidity of existing physical assets will decrease the corporate investment and increase cost of capital (see Gan, 2007; Flor and Hirth, 2013 and Ortiz-Molina and Phillips, 2014). In addition to the physical asset illiquidity, I distinguish the physical asset from (expected) inventory asset within the real asset category. To be specific, the inventory asset from the project, such as crude oil and metal, is waited for sale once being developed, which plays a vital role in future cash flow generation. The model shows that the illiquidity of inventory assets would also influence the corporate investment decisions.

More importantly, I can utilize the model to specifically quantify the amount of flexibilities in terms of real options values that has been reduced by the inventory asset illiquidity. The model shows that the flexibilities have been significantly reduced by the inventory asset illiquidity when the real options are at/in the money, but the effect is considerably less when the options are out of the money. It is arguable that asset illiquidity primarily serves a negative factor in the model.

When the real options are out of the money, asset liquidity is unable to lift the options values bringing them to be in the money. On the other hand, when the real options are in the money, asset illiquidity might shift the options values downwards. Additionally, asset liquidity has little impact on real options when the time is near maturity. Since the time remaining is short, the options will be exercised so long as they are in the money. Therefore, the asset liquidity has little impact when the options are near maturity. In the money options shall be exercised not abandoned when the remaining time is short.

According to the new model, the illiquidity of the inventory assets would lessen the waiting value of the project and thus shrink the value of real options. This effect, in turn, reduces the threshold of the investment, which might result in the suboptimal exercise of real options during the unfavorable market conditions. Moreover, I argue that the inventory assets illiquidity might result in cash flow uncertainty and thus affect the internal funds of the firm. Therefore, the shortfall of the internal fund will restrict the investment spending of the firm and thus impact on the corporate investment behavior. As a result, understanding the effects of inventory asset illiquidity and cash flow uncertainty can enhance the firm's liquidity management and project management.

Additionally, the model also relates to the literatures that document the investment booming during unfavorable market conditions such as declining market demand and economic recessions where the underlying asset tends to illiquid. For instance, Kahn (1985) gives evidence that the investment after the 1981-82 recession boomed. Ghemawat (1993) maintains that firms might overemphasize on the financial risk on the investment during the recession time. Grenadier (1996) show that it may be rational to invest when prices decline in imperfectly competitive markets. He noticed that when there was a decline in demand of real estate, which indicated that the market liquidity of the real estate was low, but there was a significant increase in new building development. I argue that these investment behaviors are resulted from suboptimal exercise of the real options, which has been mentioned in Boyle and Guthrie (2003). They argue that the potential future cash shortfall will result in the suboptimal exercise of the real options. I further maintain that the risk of potential future cash shortfall comes from the inventory asset illiquidity and I testify that the threshold of the investment becomes lower since the waiting value and the flexibility are eroded by the asset illiquidity. Therefore, because of the lower exercising boundaries, firms have a higher probability to exercise the real options, but at a lower value, which results in the suboptimal exercise of the real options. The suboptimal exercise of the real options might have negative impact on the firm values. Hence, firms should be prudent to make investment decisions when the market of the asset presents illiquidity.

More importantly, the real options theory with asset illiquidity sheds light on the corporate liquidity management where liquidity management becomes more relevant in current risk management context. Recent financial literatures agree that firms can acquire flexibility through the liquidity management (Denis, 2011). More recently, Gamba and Triantis (2013) develop an integrated risk management system, which comprises liquidity management, derivatives hedging and operating flexibility. They show that the liquidity management plays a vital role in risk management and will be more important than hedging with derivatives. As a result, I seek to enhance the operating flexibility and firm values through the liquidity management. Therefore, the real options model with asset illiquidity is not only can help firms to manage liquidity risk, but also could reinforce the flexibilites of firms. As demonstrated by Bates et al. (2009) that the significantly increased cash flow risk for firms has resulted in high cash holdings for the U.S. firms. Cash holdings can serve as cushions to buffer when firms encounter cash shortfalls. Hirth and Uhrig-Homburg (2010) also suggest that firms prefer to hold liquid funds. As a result, future cash flows of the project act an important part in corporate finance in terms of cash holdings of the firm. I demonstrate that firms with high liquid projects can be more patient on exercising the real options and hold those projects as the buffering cushions.

Furthermore, when the market for the asset is liquid, firms are more willing to hold those projects with liquid asset just like holding liquid funds. Firms know that those assets can be sold quickly at a fair price and those assets thereby can act as buffering cushions for other illiquid projects. Like argued by Ozkan and Ozkan (2004), the conversion cost for liquid assets into cash is much lower as compared with other assets. Firms with sufficient liquid assets can easily obtain internal funds than other firms. As a consequence, when the asset is liquid, the threshold for exercising the real options on that project will be higher.

Finally, the liquidity adjusted real options model provides an inherent connection between real options theory and the prospect theory. Similar to the real options theory, the prospect theory is also a theoretical framework explaining the decision making under conditions of risk (Kahneman and Tversky, 1979). The prospect theory states that when the conditions are prosperous, individuals tend to be risk averse. On the other side, when the conditions are unfavorable, individuals become risk seeking. The simulated results from the liquidity adjusted real options model share the similar idea. When the market is liquid, managers shall be risk averse and the thresholds of exercising options are much higher. On the other hand, when market turns to be illiquid, managers might be risk seeking and the thresholds of exercising options are much lower. In other words, the real options are more likely to be suboptimally exercised.

The remainder of the chapter is organized as follows. The section 5.2 gives a theoretical foundation of the model setup and section 5.3 shows the basic results obtained from the model. Section 5.4 and 5.5 conduct a series of sensitivity analyses regarding the impacts of parameters in liquidity process on the exercising surface and the real options values respectively.

5.2 Optimal Investment Timing Model

5.2.1 Benchmark Real Options Model and Basic Model Setup

I establish a filtered probability space (Ω, F, $\{F_t\}_{t\geq 0}$), P) with $(0 \leq t \leq T)$ for a fixed time, T, where the T can be considered as the lifetime of the project development contract and P is the probability measure either statistical or empirical.

Specifically,Ω is the set of all possible outcomes of the stochastic economy within the time horizon and F is the sigma algebra on Ω (Longstaff and Schwartz, 2001). All of the stochastic processes involved in this study are assumed to be $\{F_t\}_{t\geq 0})$ adapted. The benchmark model I choose is the one-factor traditional real options model. The traditional real options framework was firstly developed by McDonald and Siegel (1986) and this type of real options is to choose the optimal time for the investment, which is also known as the optimal timing. Under the framework, the market is perfect and there is no liquidity issues for real assets. Taking the undeveloped oilfield as an example and I denote B_t as the number of barrels of oils in the developed reserve and V_t is the value per barrel of oil and R_t is the instantaneous return on the developed reserve and ω is the fraction of oils produced each year. If I assume the production follows an exponential decline, which is $dB_t = -\omega B_t dt$. Therefore, the instantaneous return on the developed reserve can be written as $R_t dt = \omega B_t \Pi_t dt + d(B_t V_t)$, where Π_t is the after-tax profit from oil reserve operation. If the instantaneous return on the developed reserve is assumed to follow the Geometric Brownian Motion (GBM):

$$\frac{R_t dt}{B_t V_t} = \mu_V dt + \sigma_V dW_t^V \tag{5.1}$$

From equation (5.1), it is clear to deduce the unit value of developed reserve V, as

$$\frac{dV}{V} = (\mu_V - y)dt + \sigma_V dW_t^V \tag{5.2}$$

where y is the convenience yield, or the payout rate of each unit asset.

The concept of convenience yield was firstly proposed by Kaldor (1939). It refers to the price spread of the spot commodity price and the price of future delivery under the condition that the spot price is above the future delivery. It illustrates the benefits of having the physical commodities over the future contract. I set τ=T-t as the time to maturity of the project development, where the development option expires at time T. The value of the real options on this investment opportunity is postulated as a function of V, denoted as C(V, τ). According to equation (5.2), I can have the following Partial Differential Equation

(PDE):

$$\frac{\partial C}{\partial \tau} = (\mu_V - y)V\frac{\partial C}{\partial V} + \frac{1}{2}V^2{\sigma_V}^2\frac{\partial^2 G}{\partial V^2} - rC \tag{5.3}$$

If the development cost per unit is denoted as I, the equation (5.3) can be solved under the following conditions: The initial condition of the project is:

$$C(V, 0) = max(V - I, 0) \tag{5.4}$$

The PDE shall also satisfy the following boundary conditions:

$$C(0, \tau) = 0 = 0 \tag{5.5}$$

$$C(V^*(\tau), \tau) = V^*(\tau) - I \tag{5.6}$$

$$C_v(V^*(\tau), \tau) = 1 \tag{5.7}$$

$$C(V, \tau) = V - I \quad for V \geq V^* \tag{5.8}$$

The equation (5.5) indicates that when the real asset value is 0, the option is worthless. The equation (5.6) and (5.7) are usual conditions for value-matching and smooth-pasting. Equation (5.8) is the exercise boundary.

5.2.2 Liquidity Adjusted Real Options Model

On the other hand, I develop a liquidity adjusted real options model for optimal investment timing based on Chapter 4. I adopt the liquidity measure for the financial asset since the financial markets and the commodity markets are similar. The commonly used liquidity measure such as trading volume, quoted spread and Amihud measure can be applied and for consistency purpose, I choose the Amihud measure. The liquidity adjusted real options model can be based on Feng et al. (2014)'s paper, there will be a traceable trading market and market liquidity therefore clearly defined.

Therefore, I model the liquidity process as the stochastic process and the rationale is twofold. Firstly, liquidity would fluctuate in the financial markets over time and the fluctuation in liquidity is an impetus of asset price fluctuation in the financial markets (Farmer and Lillo, 2004; Bouchaud et al., 2006). As a result, the stochastic process could capture the dynamics of the liquidity over time. More importantly, as I denote the liquidity shock as the liquidity risk, the liquidity shock contains random effects where the stochastic process can well grasp. The randomness of the liquidity shock in the financial markets comes from the well-informed liquidity traders, who response to new information fast (Gennotte and Leland, 1990). Therefore, the liquidity process is modeled as the stochastic process. Furthermore, I also model the liquidity stochastic process as the mean-reverting process. The mean-reverting behavior of liquidity has been documented in the literatures (Corcuera et al., 2012; Feng et al., 2014). However, some may argue that the liquidity process might persist over the short time interval (Liu et al., 2006). I thereby model the liquidity as a long-run liquidity process, which captures the mean-reverting feature of liquidity behavior in the long-run. Consequently, I model the real asset liquidity (L_t) as the following stochastic process:

$$dL_t = \alpha(\theta - L_t)dt + \xi dW_t^{L,P} \tag{5.9}$$

There are six parameters in the process, namely, individual asset price volatility σ, correlation between individual asset return and market liquidity ρ, sensitivity of asset price to market liquidity β, the mean reversion speed of market liquidity $\tilde{\alpha}$, the long-run mean of market liquidity $\tilde{\theta}$ and the volatility of market liquidity ξ. Thus, the new SDEs become:

$$dV_t/V_t = rdt + \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2}dW_t^{V,Q} + \rho\beta L_t dW_t^{L,Q} \tag{5.10}$$

$$dL_t = \tilde{\alpha}(\tilde{\theta} - L_t)dt + \xi dW_t^{L,Q} \tag{5.11}$$

$$\tilde{\alpha} = \alpha + w$$
$$\tilde{\theta} = \tfrac{\alpha\theta}{\alpha+w}$$
$$dW_t^{L,Q}dW_t^{V,Q} = 0$$

Based on derived stochastic processes, it can generate the liquidity-adjusted real options model. I apply the Feynman-Kac formula for the two-dimensional derivation, adopting G (V, L, τ) to represent the value of real project when V_t=V and L_t=L. By using matrix multiplication and the Feynman-Kac formula, I develop PDE for the function G (V, L, τ):

$$\frac{\partial G}{\partial \tau} = (r-y)V\frac{\partial G}{\partial V}+\tilde{\alpha}(\tilde{\theta}-L)\frac{\partial G}{\partial L}+\frac{1}{2}V^2(\sigma^2+\beta^2L^2)\frac{\partial^2 G}{\partial V^2}+\frac{1}{2}\xi^2\frac{\partial^2 G}{\partial L^2}+\rho\xi\beta LV\frac{\partial^2 G}{\partial V\partial L}-rG \tag{5.12}$$

In comparison with the traditional PDE, the PDE I acquire in Chapter 4 is similar in that it is only a two-dimensional extension of the traditional version with asset liquidity. They also share the same boundary conditions. This PDE is subject to both initial and boundary conditions. The PDE of our model has an initial condition, which is

$$G(V, 0, \tau) = max[V - I, 0] \tag{5.13}$$

The PDE is also subject to the following boundary conditions:

$$G(V, 0, \tau) = C(V, \tau) \tag{5.14}$$

$$G(V, L_{max}, 0) = max[C(V, \tau) * d, V - I] \tag{5.15}$$

$$G(0, L, \tau) = 0 \tag{5.16}$$

$$G(V^*(L, \tau), L, \tau) = V^*(L, \tau) - I \tag{5.17}$$

$$G_v(V^*(L, \tau), L, \tau) = 1 \tag{5.18}$$

$$G(V, L, \tau) = V - I \quad forV \geq V^*(L, \tau) \tag{5.19}$$

The initial condition implies that when the right of the project exploration near expiration date, the project value will be the difference between value and investment outlay. The equation (5.14) indicates that if the market is perfect liquid, then the real option value is the same as the traditional benchmark model. Equation (5.15) introduces an implied discount rate for asset illiquidity. The implied discount rate for the options prices is similar to the idea in Ericsson and Renault (2006), where they argue that if the bondholder needs to sell his position immediately, the realized price would be a fraction of the price in a perfectly liquid market. Likewise, I model this fraction as the implied discount rate d to indicate the discounted option values. It gives the information that if the market is illiquid, near frozen, then the real option value is a fraction of the real option value in the traditional benchmark model, with the discount factor d, when it is below the exercise boundary V-I. Similar to the role of implied volatility in option prices, the implied discount factor makes the difference between model price and the market options price, to be minimal. This discount rate can provide helpful insights of the liquidity factor impact on the real options values. Moreover, the discount rate might vary along with the stochastic liquidity embedded in the market. The equation (5.16) indicates that when the real asset value is 0, the option is worthless. The equation (5.17) and (5.18) are usual conditions for value-matching and smooth-pasting. Equation (5.19) is the exercise boundary for the liquidity adjusted real option model.

5.2.3 Parameter Estimations

According to Dixit and Pindyck (1994), they have developed an empirical research on real options theory application in offshore oil valuation. As a result, I will take their model as one input for my model as shown in equation (5.14), and I will have additional parameters since my model is the two-dimensional model with asset liquidity. For the asset liquidity, I adopt Amihud measure as the asset liquidity measure of this chapter. The additional parameters to be determined are: the long-run mean of asset liquidity, the mean reversion speed of asset liquidity, the volatility of the of asset liquidity, the sensitivity of individual asset price towards asset liquidity and the correlation between asset return and asset liquidity. As mentioned in Chapter 4, the liquidity is a mean-reverting process, so I can adopt the results estimated in Chapter 4 and the data I use is the WTI oil market data for the year 2014. I then estimate the additional parameters in the new model. The mean reversion speed can be estimated from the following stochastic process

(Balvers et al., 2000): $L_t - L_{t-1} = \varphi_3 + \varphi_4(\theta - L_{t-1}) + \varepsilon_t$, where φ_4 is the mean reversion speed and $0 < \varphi_4 < 1$. φ_1 andφ_3 are positive constants and ε_t is the noise term with unconditional mean of zero. Since the model $L_t = \varphi_1 + \varphi_2 L_{t-1} + \varepsilon_t$ is equivalent to $L_t - L_{t-1} = \varphi_1 + (\varphi_2 - 1)L_{t-1} + \varepsilon_t$. Combined the two regression models, I find that the $\varphi_4 = 1 - \varphi_2$. Since the process is mean-reverting, it has the long-run mean: $\theta = \frac{\varphi_1}{1-\varphi_2}$. The shock of liquidity is usually estimated by the residuals of the liquidity autoregression process. As a result, I estimate the volatility as follows. Firstly I discretize the mean-reverting process of liquidity as $L_t - L_{t-1} = \tilde{\alpha}(\widetilde{\theta} - L_{t-1}) + \varepsilon_t$ and I set dt=1. Then, the residual can be known as $\varepsilon_t = L_t - L_{t-1} - \tilde{\alpha}(\widetilde{\theta} - L_{t-1})$. Then, I plug in the parameters and find the liquidity volatility $\xi = \sqrt{\frac{1}{N-1} \sum_{t=1}^{N} (\varepsilon_t - E[\varepsilon_t])^2}$. Finally, the correlation between asset return and asset liquidity volatility, which can be estimated as:$\rho = \frac{Cov(R_t, \varepsilon_t)}{\xi\sigma}$.

5.3 Simulation Result for the Real Options Model with Asset Illiquidity

5.3.1 Simulation Results for Basic Case with Asset Illiquidity

I first discretize equation (5.3) and equation (5.12), making them available for discrete data application. Then, I plug the parameters in and generate an option exercise line for equation (5.3) and an option exercise surface for equation (5.12). The traditional real options theory gives a free boundary which is a line of the exercise of real options. As I add one more state variable with liquidity, the free boundary now becomes a free surface. The free surface has three dimensions, project value, time to maturity and asset liquidity. I set the range of project value from 1 to 3, and set time to maturity from 0 to 3 (i.e. 0 is the maturity date) and asset liquidity from 0 to 0.15, where 0 is the most liquid case and 0.15 is the most illiquid case. For the parameters, I set r=0.0125, y=0.04, σ=0.25, α=0.01, ρ=-0.5, β=1, ξ=0.08, θ=0.01, T=3 and I=1.

From Fig 5.1, it can be seen the free surface of the real options, which has been shaped by the illiquidity effects and the effects will be more pronounced at the time of today. When the market of the real asset is most liquid, the threshold of exercising the real options is about 1.6 and when the market of the real asset is most illiquid, the threshold of exercising the real options is about 1.3. The result is consistent with the cash holding theorem, where firms shall be more patient when they hold liquid assets and they shall be less patient when the market turns to be illiquid. Fig 5.2a produces the similar results in the case of τ=3, where the threshold of exercising is much higher for liquid assets than illiquid assets. Interestingly, when the market is perfect liquid, then the exercise boundary is smooth, while market is perfect illiquid, then the exercise boundary is close to a straight polyline and the waiting value is substantially eroded. Since the waiting benefits are eroded and the threshold is lower, it may provide insights why firms sometimes suboptimally exercise the real options. When the threshold of trigging the project is lower, firms have a higher chance to implement the project. On the other hand, however, the real options value is low and thus the implementation is considered as a suboptimal exercise. As Fig 5.2b presents, when the market is fairly illiquid, the option values are nearly eliminated irrespective of time. When the asset exhibits illiquidity in the market, the longer the managers wait, they have a higher chance of abandoning the real options. It is because that when the asset becomes illiquid, it lowers the waiting value of the project since the asset value can be eroded by the illiquidity. In extreme case, when the market is near frozen, the real options might be exercised immediately whenever the revenue is above the cost. Otherwise, managers are exposed to the risk that the project might be abandoned. As a result, the waiting value of the project will be diminished by the asset illiquidity and thereby the exercising probability increases. It is thereby able to explain the suboptimal exercise of real options and the suboptimal exercise could be detrimental to the firm values.

Specifically, I take a slice of the surface for three lines, with perfect liquid, one-third of the maximum illiquidity and two-thirds of the maximum illiquidity cases respectively (see Fig 5.3a). It is clear that when the market is perfect liquid, the exercising boundary is the highest when the real options are in the money. It is suggested that the waiting benefits are much higher than the waiting cost and the real options are deep in the money. However, the high value of real options yields high exercise boundary for the investment and thus the exercise probability is lower. On the other hand, when the market becomes more illiquid, the exercise boundaries become much lower when the real options are in the money and the exercise probability is much higher but the embedded options are in lower value. It is argued that people are less patient when they have higher waiting cost (Foucault et al., 2005). It is thereby believable that asset illiquidity may increase the waiting costs for managers and thereby increase the chance of suboptimal exercise of the real options. This result aligns with the theory that people are more sensitive to the loss they incurred than the gains they obtained (Tom et al., 2007). Fig 5.3b displays a similar result where the options values are decreasing monotonically with the increase of asset illiquidity. The decreasing of real options values shows a reason why the threshold of exercising real options is lower for the illiquid market conditions. Since the option value is low, the waiting is less valuable and the options are more likely to be exercised.

More specifically, I can quantify the amount of flexibilities in terms of real options values reduced by the inventory asset illiquidity (from completely liquid to completely illiquid). For the options that are in the money, the real option value reduced by asset illiquidity is around 0.07 (i.e. from 0.22 to 0.15). For at the money options, the slope is steeper where the real option value reduced by asset illiquidity is around 0.1 (i.e. from 0.14 to 0.04), which is 10% of the investment cost. For out of the money options, the slope is flat and the reduction amount is comparatively smaller, which is around 0.03 (i.e. from 0.05 to 0.02). It seems that the asset illiquidity has the largest impact on the at the money options. The at the money options are quite sensitive to the illiquidity because the reduced value resulted from asset illiquidity can bring them out of the money.

Fig 5.4a presents the changing slopes of real options values with respect to asset illiquidity. The slope of the perfect liquid case is nearly a straight line where liquidity has little impact on the option values. On the other side, for the most illiquid case, the changing of slope is flat and the line is more concave when the options are out of the money. However, the changing of the slope is steep and the line is more convex when the options are in the money. The value increase of the out of the money options is quite slow before the turning point but much faster after the turning point. There might be a catch up effect for out of the money options. When the asset becomes liquid from illiquid, the project values increase much faster than the liquid assets. In addition, Fig 5.4b presents the changing slope of real options values with respect to project value. When V=0.8, the options are out of the money and the changing slope is nearly a straight line. When options are at/in the money, the line representing the changing slope is flat when market is liquid and becomes steeper when it approaches to the most illiquid region. The asset illiquidity thereby has a larger impact on the at/in the money real options than the out of the money options.

Therefore, from the model, there are two manifest features how asset illiquidity shapes the real options values. Firstly, when the asset is liquid, the real option is more valuable and thereby the exercising boundary is higher and real options are deep in the money. On the other hand, when the asset is illiquid, the real option is less valuable and thereby the waiting value is infinitesimal and the exercising boundary is lower. Secondly, the liquidity effects are more manifest when the options are exercisable (i.e. at/in the money), while asset liquidity has little impact on option values when they are out of the money.

5.3.2 Sensitivity Analysis for Free Surface with Asset Illiquidity

In this section, I further investigate the sensitivity of the real options exercising boundaries regarding the change of the parameters I have set. In particular, I am interested in the effects of the parameters that representing the liquidity nature in the stochastic process and in the PDE. Since the liquidity effects would be more obvious when the market is fairly illiquid, I plot the real options exercising boundaries against different parameters by fixing the liquidity at two-thirds of the maximal liquidity level.

In the basic option pricing theory, asset volatility plays an important role in affecting both call and put values. It is because that increased volatility of the underlying asset will enhance the potential gains for options when costs stay the same, which is also true for the real options theory. Grullon et al. (2012) also ascertain that the firm value is sensitive to the change of underlying volatility and more importantly, firm value sensitivity is higher when firm owns real options. They also find out that the sensitivity of real options values to the underlying volatility plays a significant role in explaining the variation in the relation between returns and contemporaneous changes in volatility. Thus, the changing of the underlying volatility is noticeable for both real options theory and firm values.

However, the impact of underlying volatility variation for real options theory might be complicated. According to Moel and Tufano (2002), the effects of increased asset volatility are twofold. Firstly, the rise of asset volatility will change the distribution of asset prices, which enlarges the existing tail probabilities. Therefore, exercising boundaries are more likely to be touched. When the asset volatility increases, the chance of exercising the real options raises and thus the real option becomes more valuable (McGrath, 1997).

However, on the other hand, the increased volatility also levels up the thresholds. As a result, decision makers tend to be more careful before exercising the options and thus the investment will be delayed. The increased future uncertainty will thereby motivate firms to delay production and investment under the theory of real options (Bredin et al., 2011). From Fig 5.5a, it seems that the second effect dominates the first one. The line with the highest volatility is at the top, which means the project with the highest volatility has the highest threshold. Clearly, the highest volatility levels up the exercise boundaries for managers compared with the other two lines that have a relatively low volatility.

From equation (5.10), it can be seen that there is another volatility term, which is the volatility of the liquidity, namely, the liquidity shock. Liquidity shock also plays a significant role in shaping the exercise boundaries. The top line is the exercise boundary with the lowest liquidity shock and the bottom line is the exercise boundary with the highest liquidity shock (see Fig 5.5b). When there is a low liquidity shock, firms tend to have a higher exercise boundary. When the shock is high, firms tend to lower the boundary because the market downturn with shocks will create more uncertainty to investors (Veronesi, 1999). If the real options are exercised during the high liquidity shock period, then firms may face liquidity constraints. Hence, firms may be forced to externally finance in order to continue the project, which may generate a negative NPV for them if the financing cost is high (Holmstrom and Tirole, 1996). As a result, firms will have a lower exercise boundaries because the real options values are low when the liquidity shock is large. Therefore, when there is a large liquidity shock in the market, it makes waiting more risky and thus reduces the waiting value of current investment if the shock persists, which might reduce the real options values and thus lower the exercising boundaries. It is thereby arguable that the effects of liquidity shock on the exercise boundary are similar to the effects of the asset illiquidity levels.

This theoretical result is consistent with the argument of Boyle and Guthrie (2003). They maintain that the potential risk of a future cash shortfall reduces the value of a firms timing options and leads to suboptimal early exercise of those options. More uncertainty of the firms future cash flow raises the risk of future funding shortfalls, thereby lowering the value of waiting and increasing the value of current investment. When there is a liquidity shock in the market, the expected futures cash flow from the project will be negatively impacted, which may decrease the threshold of exercising those real options. As a result, the line with the highest liquidity shock has the lowest exercise boundaries and vice versa. This feature of our model is also consistent with the empirical finding of Whited (2006).

Consequently, the time length of the liquidity shock persistence in the market creates a risky waiting region for the real options holders and thus the speed of liquidity recovery comes into play. Since the liquidity stochastic process is a mean-reverting process, there is a mean-reversion speed α, which indicates the speed of liquidity recovery when it extensively deviates from its long-run mean.

Since the initial liquidity level is set at a relative illiquid case, then the adjustment made by the stochastic process will toward the more liquid level. From Fig 5.6a, it is observable that the exercising boundary with the highest adjustment speed is at the top while the exercising boundary with the lowest reversion speed is at the bottom. The reason is that if the recovery speed is fast, then the liquidity level will retrieve to the more liquid level much quicker and thus the risky waiting region will be smaller for firms, which makes them more willing to wait. On the other hand, if the recovery speed is slow, then the liquidity level will take much longer time to return to the long-run mean. The waiting value thereby will be significantly lessened and firms thus are less willing to wait.

Another parameter beta, measures the effects that liquidity functions in the process. From Fig 5.6b, it is clear that the exercising boundary with the highest beta is at the top while the exercising boundary with the lowest beta at the bottom. It indicates that when the liquidity has a large impact, then firms tend to wait longer since the threshold of investment will be higher and the exercise probability is lower. The large effects created by asset illiquidity can thereby increase the opportunity of adding value to the project. This effect, in turn, enhances the waiting values of the project and thus levels up the investment threshold.

Finally, the correlation between asset return and asset liquidity also functions in the real options exercising boundaries. From Fig 5.7, when the asset return and asset liquidity is positively correlated, the exercising boundary is higher. When the two factors are positively correlated, it indicates that when the asset turns to be illiquid, the asset generates a higher return. For this reason, firms might be more willing to wait since they can be compensated for the asset illiquidity. On the other hand, when the correlation between asset return and asset liquidity is negative, the exercising boundary is lower. When the asset turns to be illiquid, the asset generates a lower return. The two contemporaneous effects exacerbate the investment situation, which lowers the waiting values. As a result, when the correlation is positive, then the exercising boundary will be higher since it might be more valuable to wait.

5.3.3 Sensitivity Analysis for Real Options Values with Asset Illiquidity

In this section, I further analyze the sensitivity of the real options values regarding the change of the parameters I have set. In particular, I am interested in checking the consistence of the results with previous section and I thereby also plot the real options values against different parameters by fixing the liquidity at two-third of the maximal liquidity level.

The underlying volatility plays a positive role in the real options values since increased volatility of the underlying asset will enhance the potential gains for real options when costs stay the same. From Fig 5.8a, it is observable that the real options are more valuable when the underlying volatility is high and they are less valuable when underlying volatility is low. This result is consistent with previous result where the exercising boundary with the highest underlying volatility is at the top. The valuable real option deserves waiting whereas less valuable options do not.

The liquidity shock has the opposite effects compared with the underlying volatility. The real options with lowest liquidity shock are the most valuable options (see Fig 5.8b). This result is also consistent with previous result where the exercising boundary with the largest liquidity shock is at the bottom. It is because the suffering of market illiquidity weights much heavier than desired effects come from the liquid market. When the market is liquid, firms cannot obtain plenty of benefits since they only sell the asset fast at a reasonable price. On the other side, however, when the market is illiquid, firms will substantially suffer from the potential cash flows uncertainty. As a consequence, firms might prefer the assets with stable liquidity and be willing to wait longer as they are less risky.

The other two parameters mean-reversion speed and liquidity coefficients have infinitesimal effects on the real options values (see Fig 5.9a and Fig 5.9b). From the two figures, separations of real options curves are not so clear for different parameters. The highest liquidity coefficient dominates the other two value curves, suggesting that large liquidity effects will strengthen the real options values. So the waiting value will be reinforced by higher beta, which is consistent with the previous result that highest liquidity coefficient generates highest exercising boundary. Similarly, The highest mean-reversion speed dominates the other two curves, indicating that fast speed of reverting back to the liquid status will build up the real options values. The higher the recovery speed is, the more valuable the real options are, which is also coherent with previous result that higher recovery speed gives a higher exercise boundary.

5.4 Conclusion

To conclude, this chapter builds a three dimension real options model, which takes the asset illiquidity into account. The newly-developed model builds an intrinsic connection between real options theory and inventory asset illiquidity. The study of the model reveals the effects of inventory asset illiquidity towards investment threshold and flexibility values, namely, the exercise boundary and the real options values, which is complementary to the existing real options and corporate finance literatures. The theoretical results can also help managers to have a deeper understanding of the role of asset illiquidity in corporate finance and risk management. The model shows that the inventory asset illiquidity might be one of the reasons why firms suboptimally exercise the real options.

The new model is mainly complementary to two types of existing literatures. The first kind focuses on the effects of real asset illiquidity (mainly physical asset) on the corporate investment. In addition to the physical asset illiquidity, I add the inventory asset to the real asset category and demonstrate the effects of the inventory asset illiquidity. The model shows that the illiquidity of inventory assets would also influence the corporate investment decisions. More importantly, I use the model to specifically quantify the amount of flexibilities in terms of real options values that have been reduced by the inventory asset illiquidity. The model shows that the flexibilities have been significantly reduced by the inventory asset illiquidity when the real options are at/in the money, but the effects are considerably less when the options are out of the money.

In addition, the model also relates to the literatures document the investment booming during unfavorable market conditions such as declining market demand and economy recessions where the underlying asset tends to illiquid. My results show that the asset illiquidity might contribute to the suboptimal exercise of the real options, by demonstrating that when the market is illiquid, the threshold of the investment becomes lower since the waiting value and the flexibility are eroded by the asset illiquidity. The lower real options values will lead to the lower exercise boundaries of the options, which induces the suboptimal exercise of the real options. The results can partially aid the explanation why there might be investment booming when the demand is declining and economy approaches to recession. Since the suboptimal exercise of the real options might be disadvantageous to the firm value, firms shall be more cautious to the investment decisions when the market conditions are unfriendly.

More importantly, the real options theory with asset illiquidity sheds light on the corporate liquidity management. When the asset is liquid, the threshold for exercising the real options on that project will be higher. It is thereby arguable that holding illiquid inventory assets is also toxic for firms' operations and firms tend to exercise those real options with a higher probability. Firms tend to hold liquid inventory assets as the liquidity buffer.

Finally, the new model exhibits two main features how asset illiquidity affects the investment decisions under the real options framework. Firstly, the asset illiquidity has a larger influence when real options are in the money than they are out of the money. Secondly, asset illiquidity has little impact on real options when the time is near maturity. These two effects can aid the understanding of the real asset illiquidity towards the investment decisions and provide helpful insights to managers for both liquidity and project management.

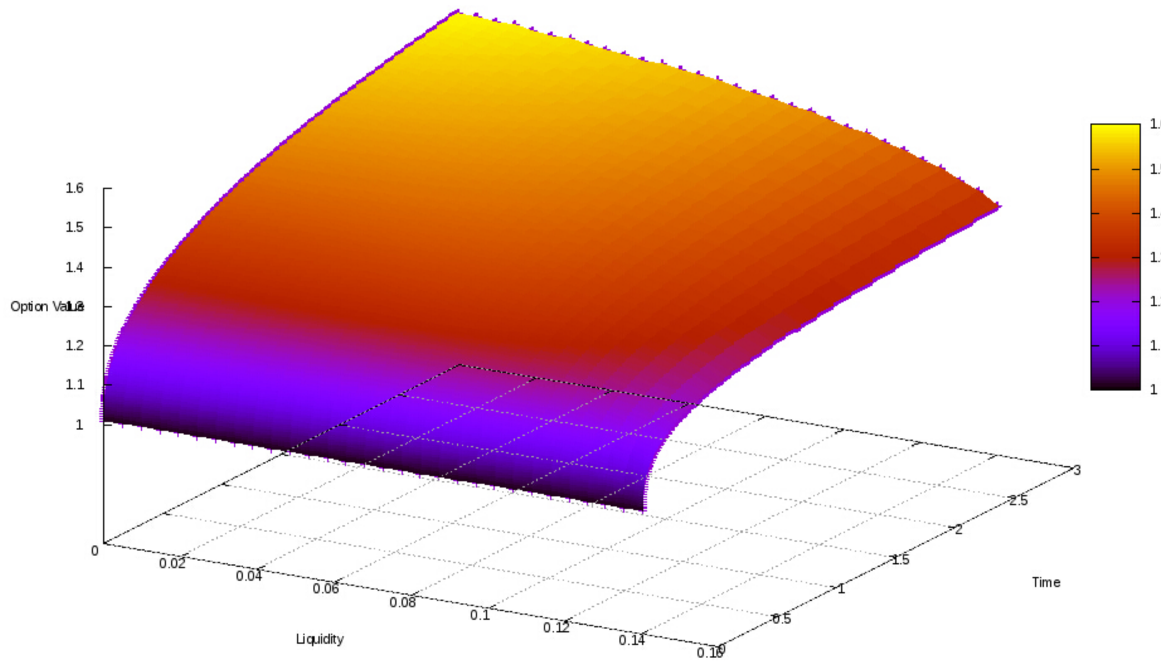


Figure 5.1: The surface of real option exercising with asset liquidity

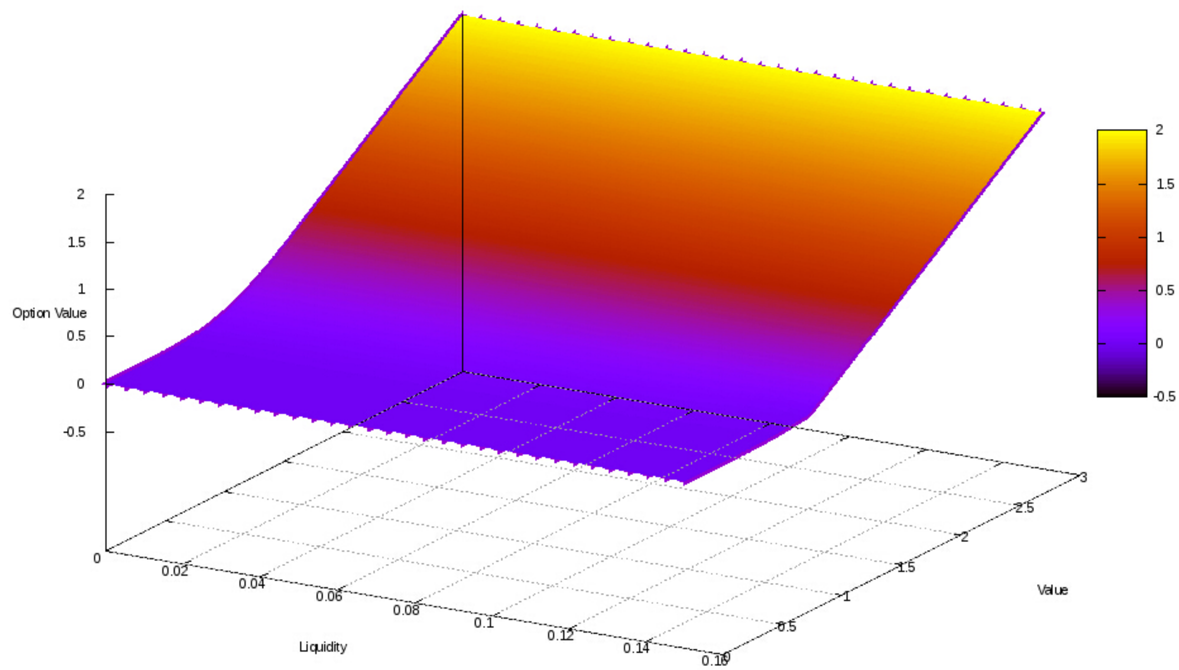


Fig 5.2a: Real options values with asset liquidity when τ=3

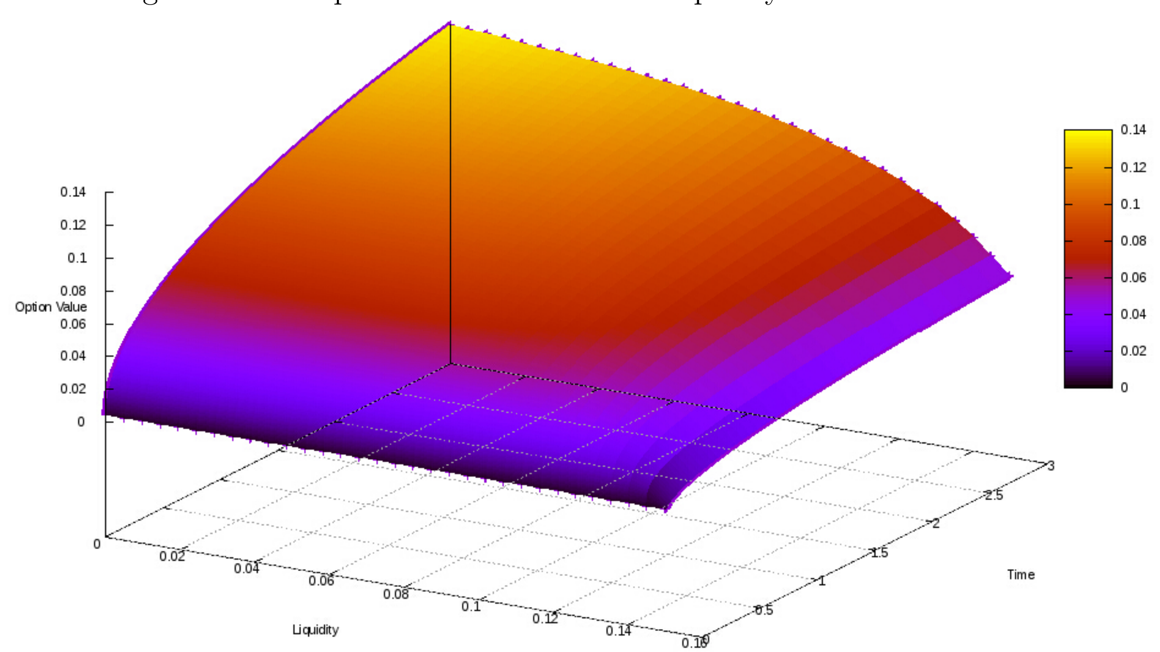


Fig 5.2b: Real options values with asset liquidity when V=1

Figure 5.2: Real options sensitivity

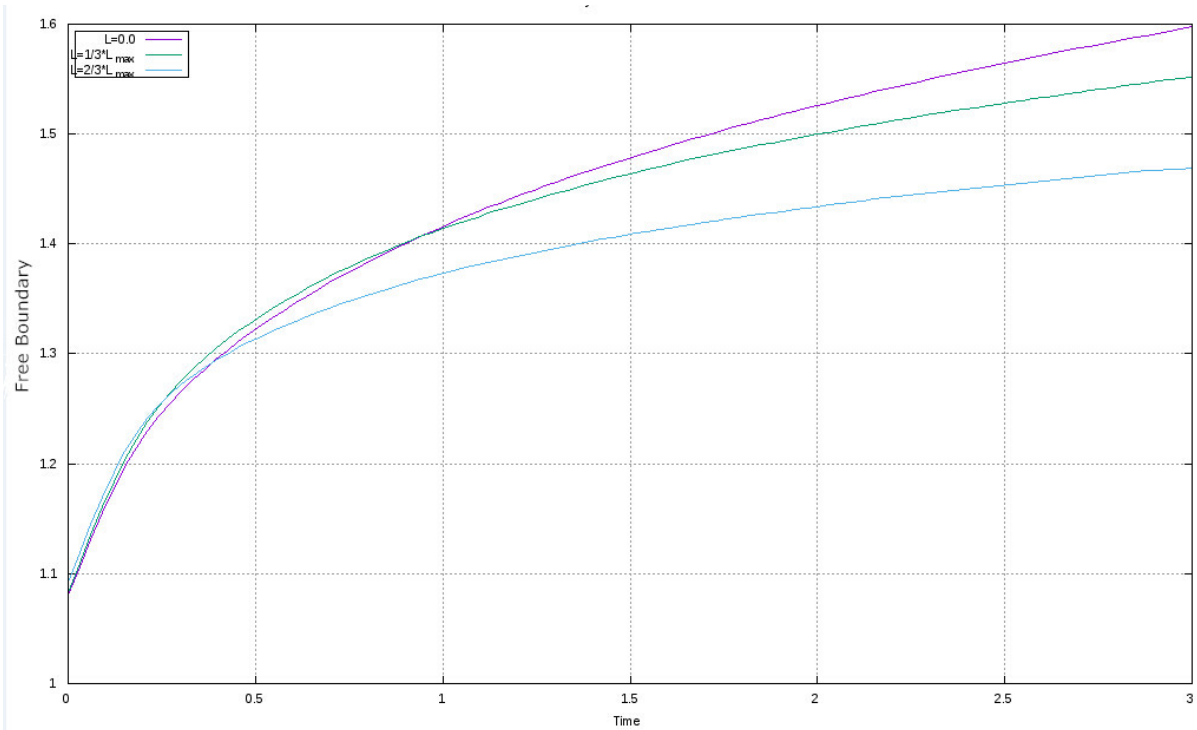


Fig 5.3a: The change of the free boundary at different liquidity levels

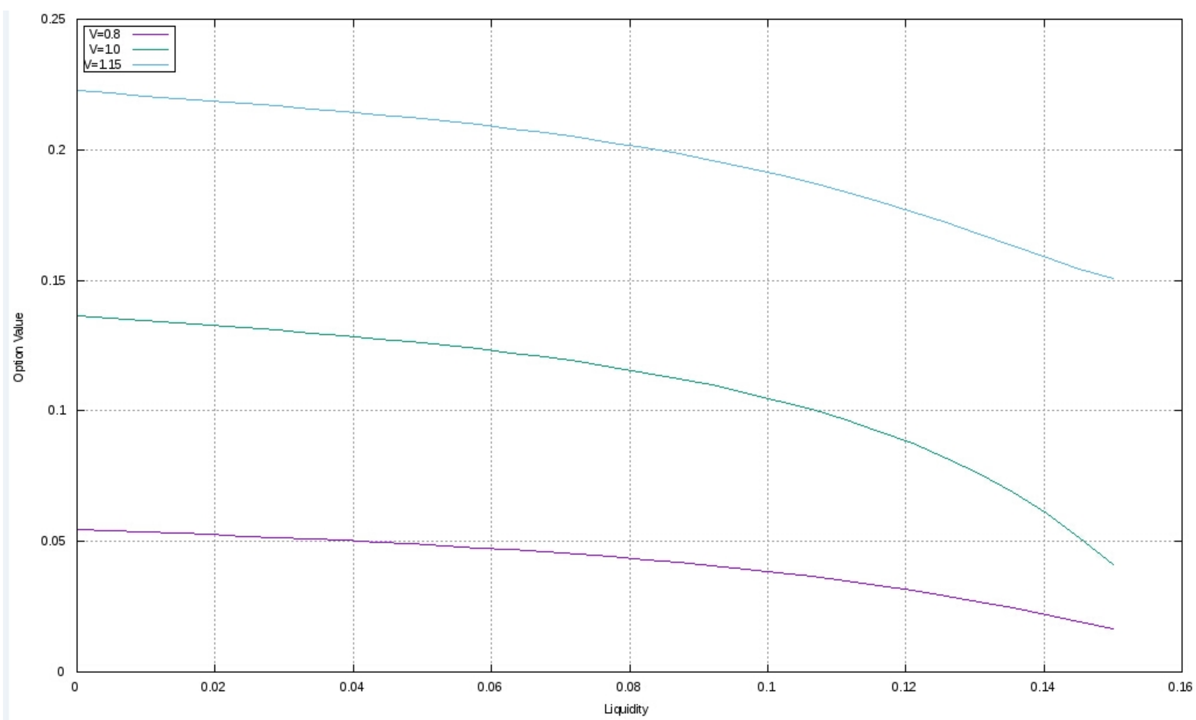


Fig 5.3b: The change of real options values at different project values

Figure 5.3: Two slices of the free surface

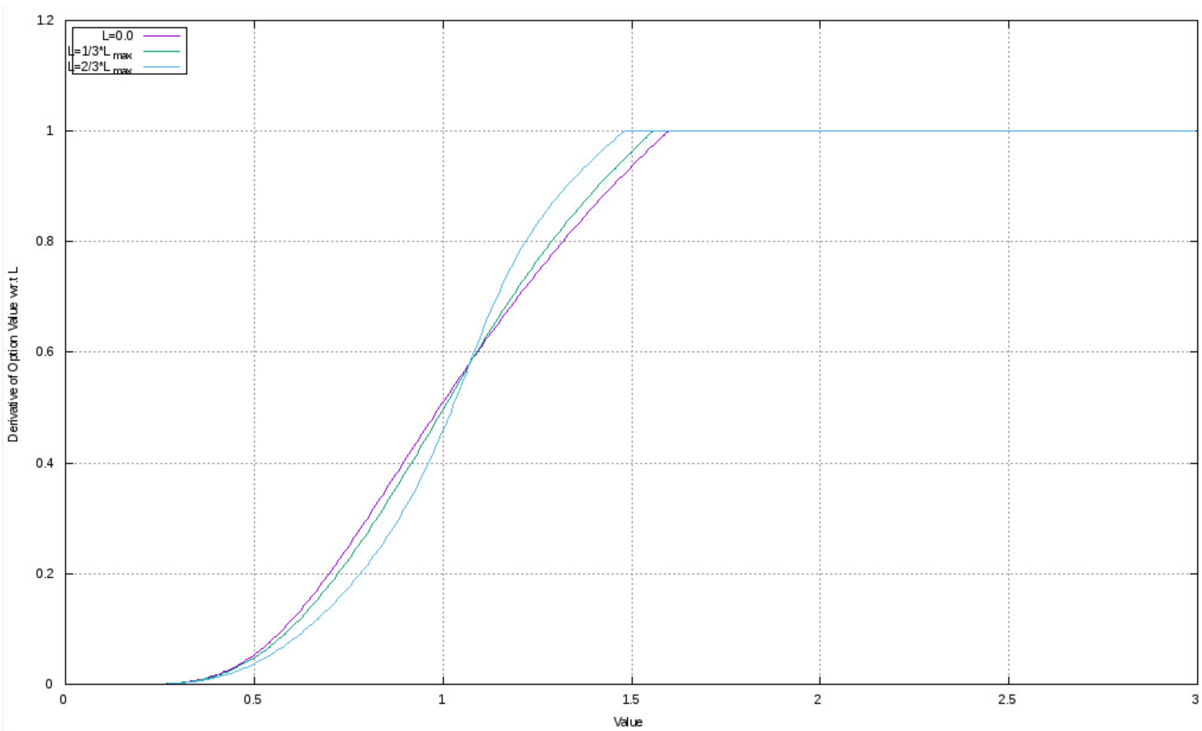


Fig 5.4a: The changing slope of real options values w.r.t liquidity

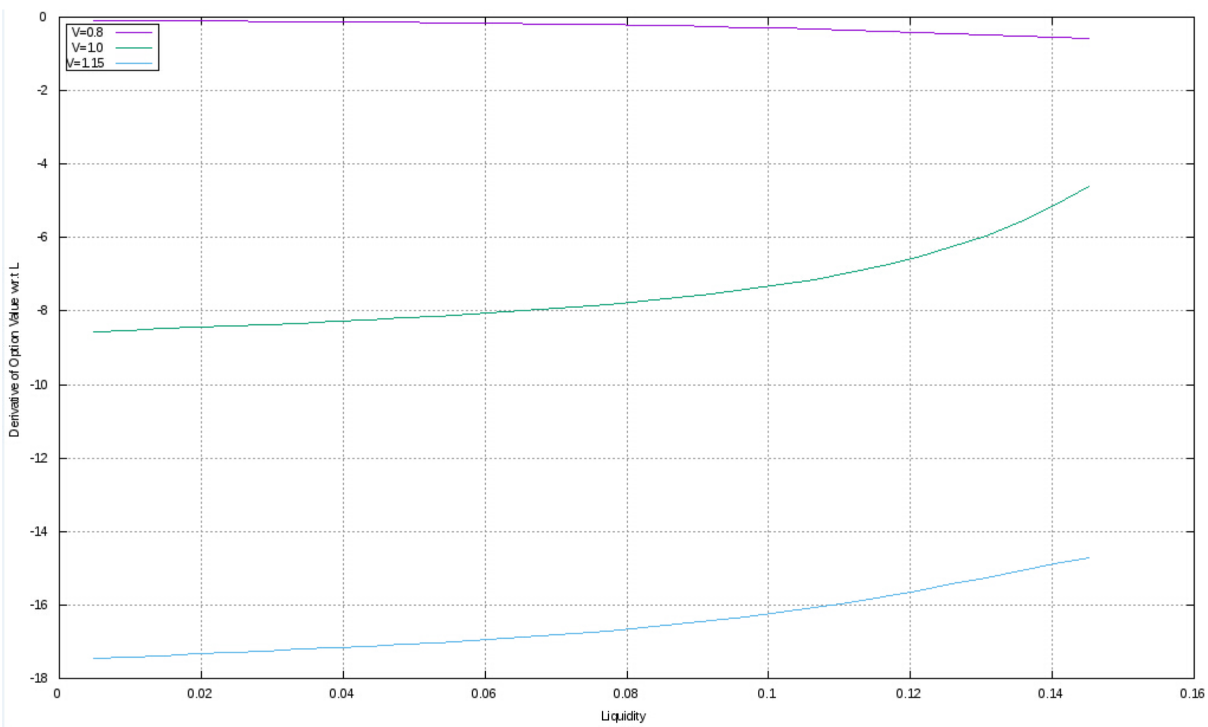


Fig 5.4b: The changing slope of real options values w.r.t project values

Figure 5.4: The changing slope of real options values w.r.t liquidity and project values

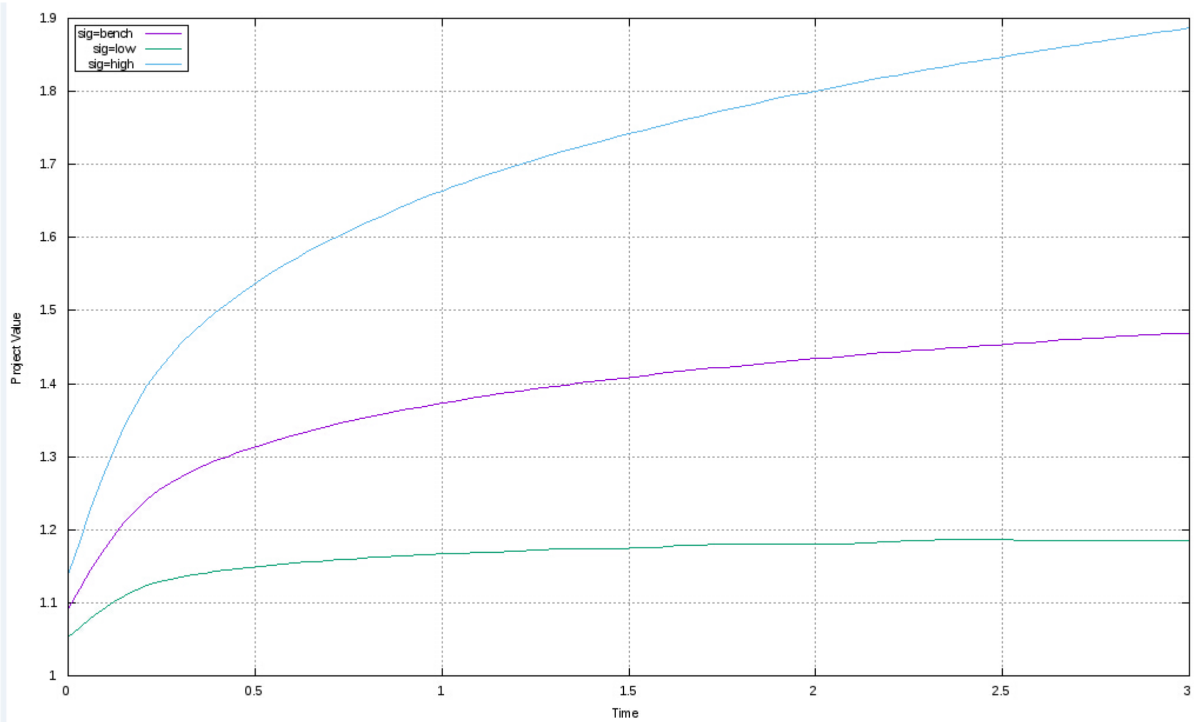


Fig 5.5a: Exercising boundaries with different asset volatilities

(the values of different σ: bench=0.25, high=0.5 and low=0.125)

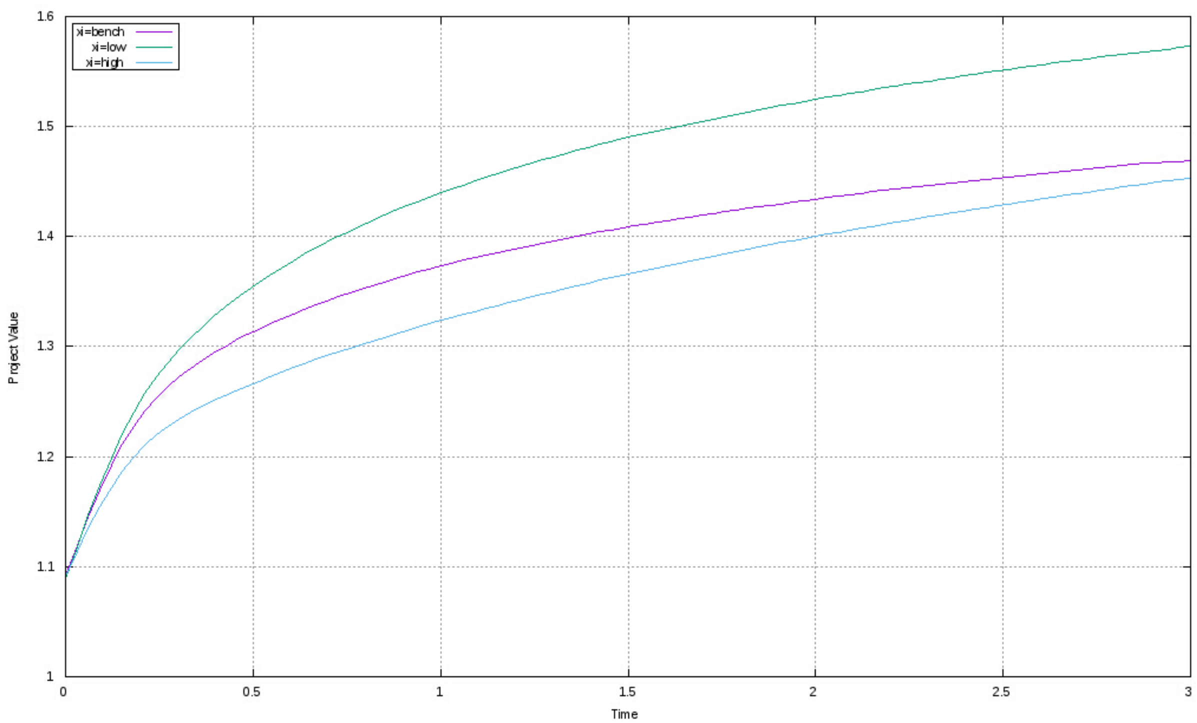


Fig 5.5b: Exercising boundaries with different liquidity shocks

(the values of different ξ: bench=0.08, high=0.16 and low=0.04)

Figure 5.5: The sensitivity of exercising boundaries towards asset volatilities and liquidity shocks

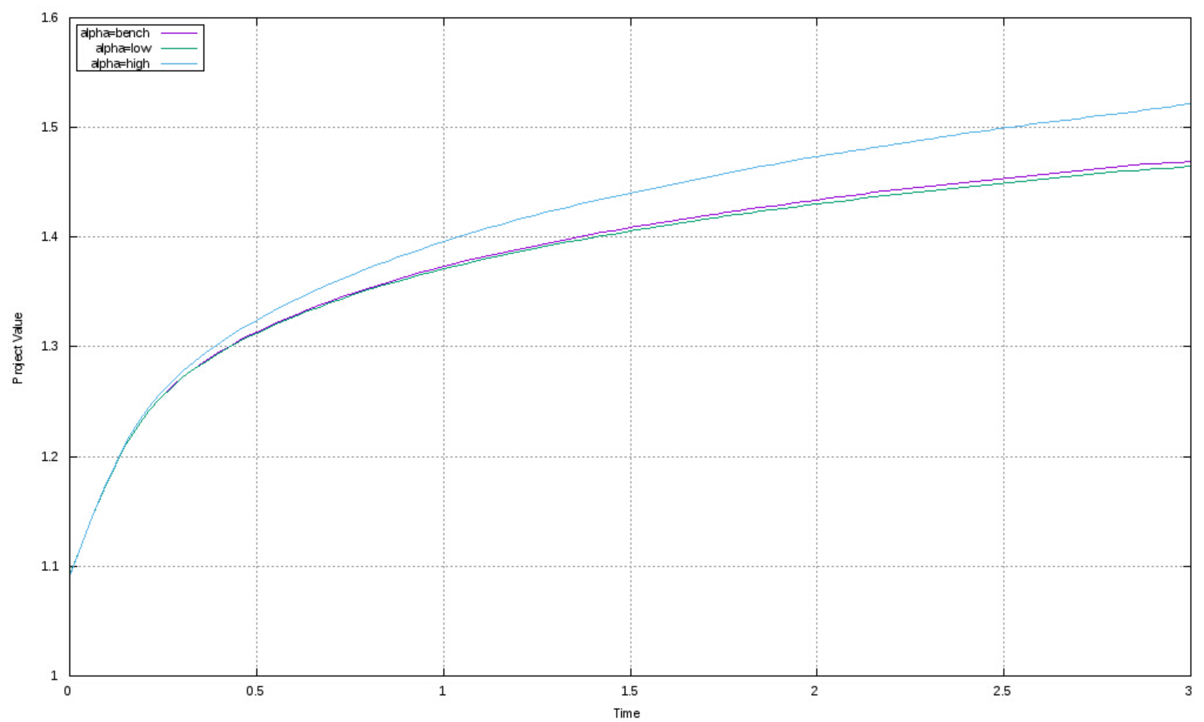


Fig 5.6a: Exercising boundaries with different liquidity mean-revision speeds

(the values of different α: bench=0.05, high=0.5 and low=0.005)

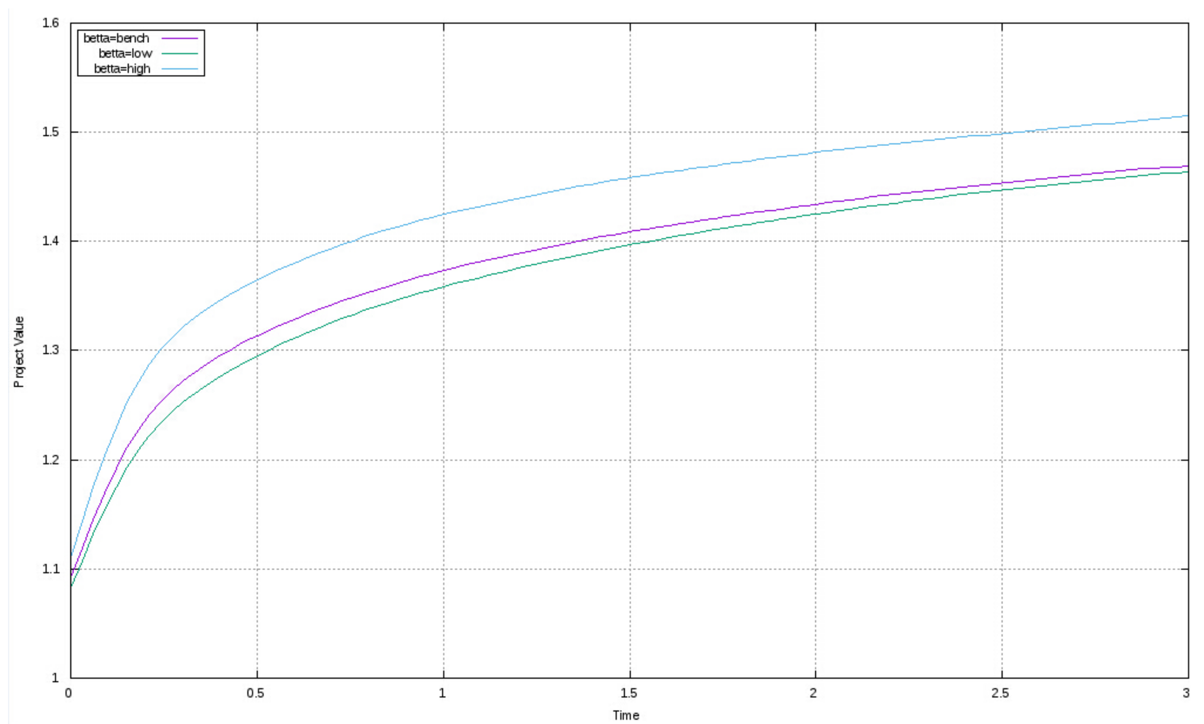


Fig 5.6b: Exercising boundaries with different liquidity coefficients

(the values of different β: bench=1, high=1.75 and low=0.25)

Figure 5.6: The sensitivity of exercising boundaries towards mean-reversion speed and liquidity coefficients

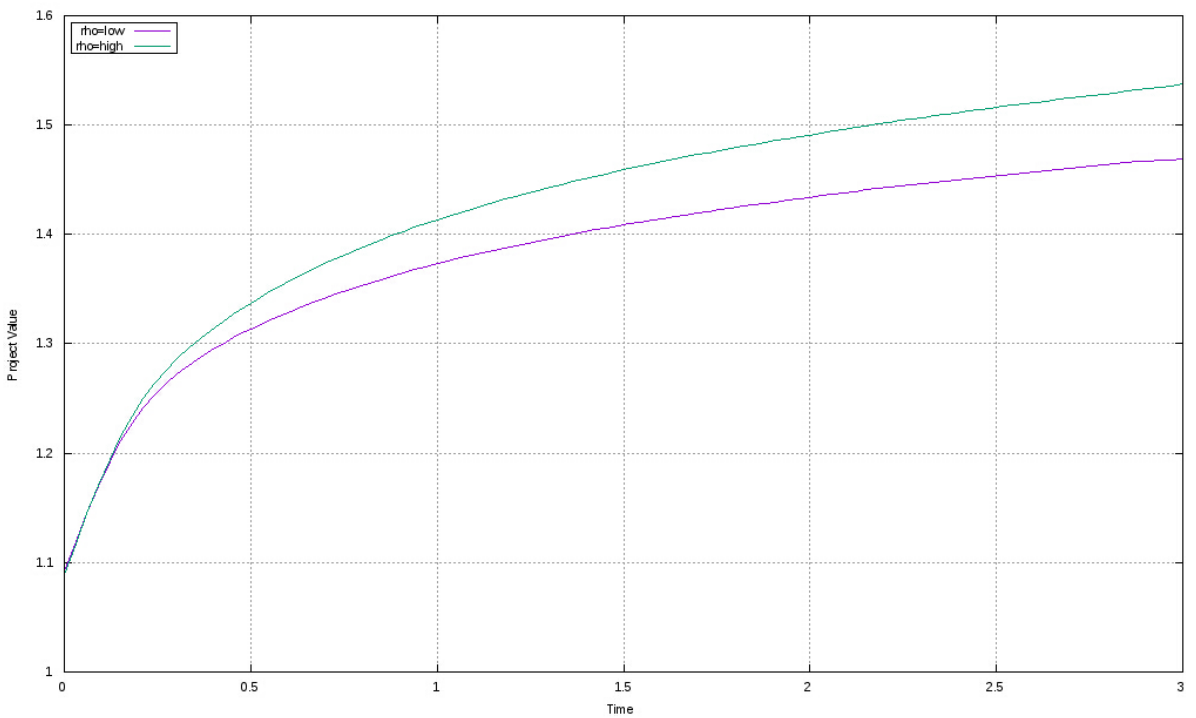


Figure 5.7: Exercising boundaries with different correlations between asset return and asset liquidity (the values of different ρ: high=0.5 and low=-0.5)

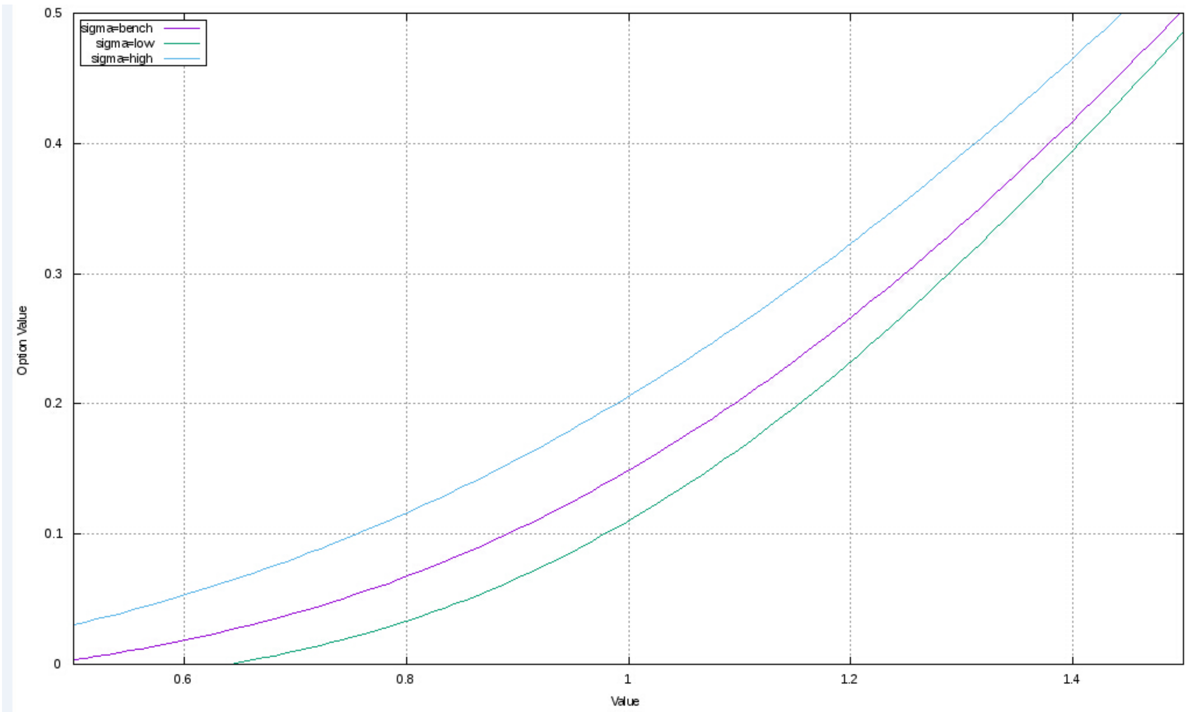


Fig 5.8a: Real options values with different asset volatilities

(the values of different σ: bench=0.25, high=0.5 and low=0.125)

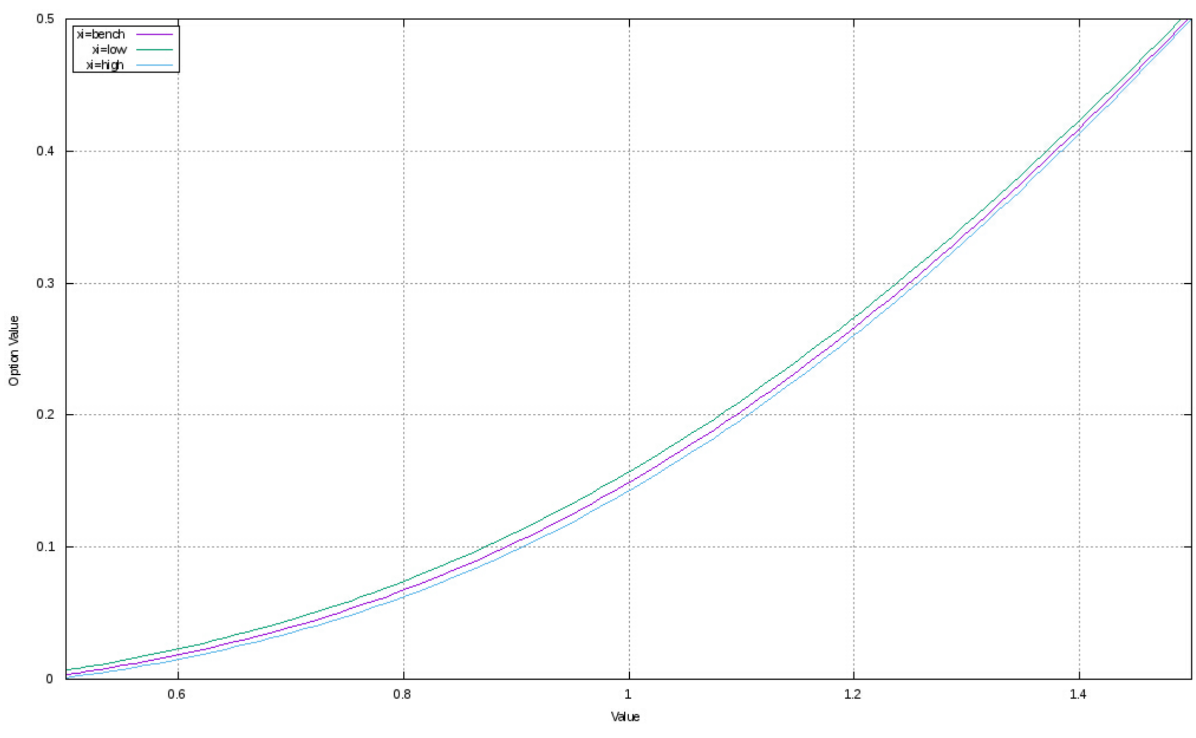


Fig 5.8b: Real options values different liquidity shocks

(the values of different ξ: bench=0.08, high=0.16 and low=0.04)

Figure 5.8: The sensitivity of real options values towards asset volatilities and liquidity shocks

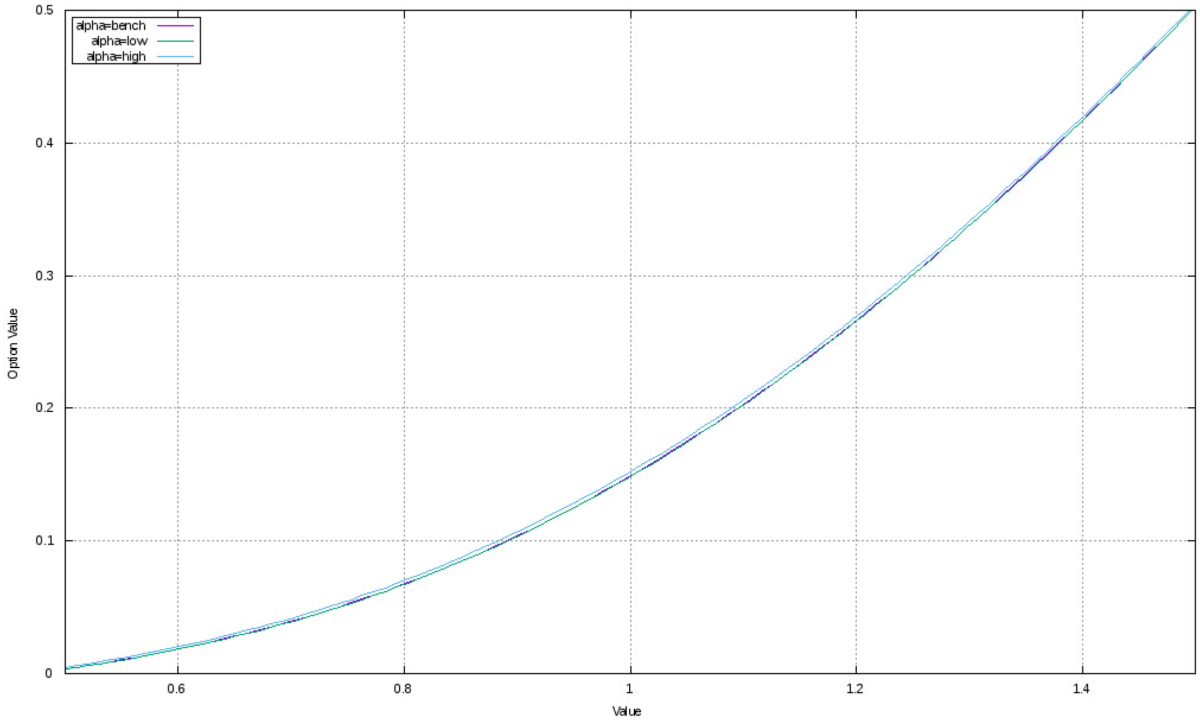


Fig 5.9a: Real options values with different liquidity mean-revision speeds

(the values of different α: bench=0.05, high=0.5 and low=0.005)

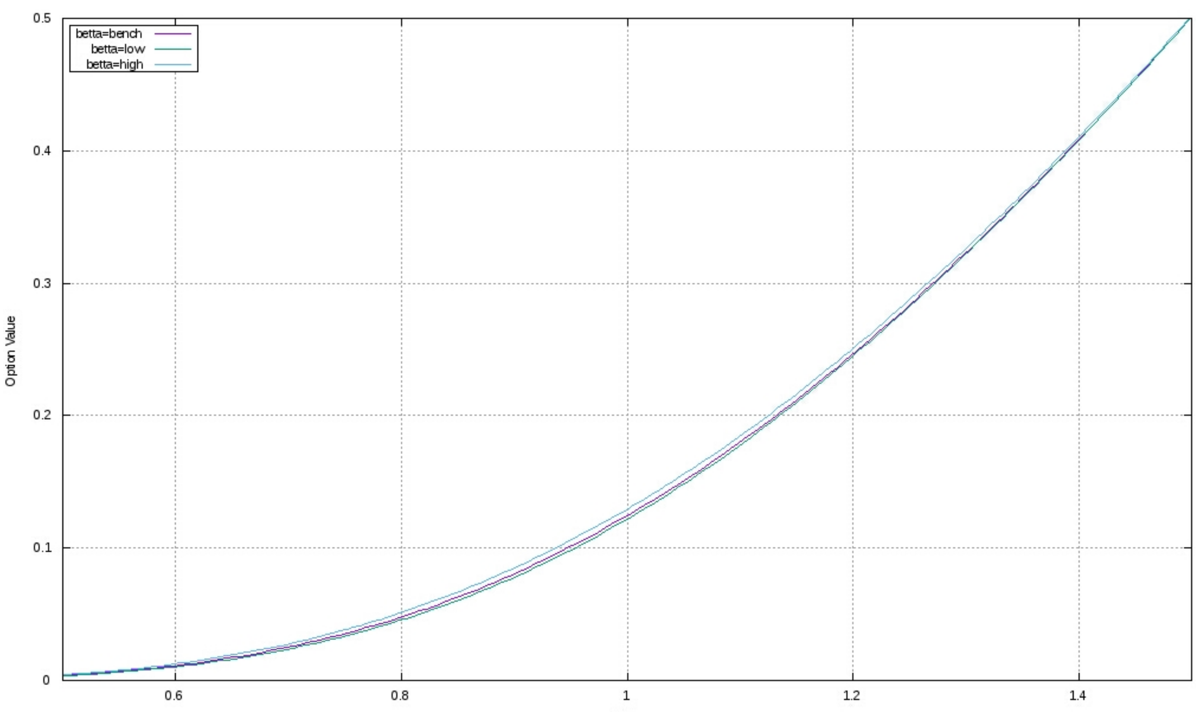


Fig 5.9b: Real options values with different liquidity coefficients

(the values of different β: bench=1, high=1.75 and low=0.25)

Figure 5.9: The sensitivity of real options values towards mean-reversion speed and liquidity coefficients

Chapter 6

Conclusion

Followed by the intensively researched topic of the liquidity risk in the asset pricing and financial market research, the thesis attempts to address the liquidity issue in the derivative markets and pricing models. In particular, this thesis facilitates the understanding of the liquidity in the derivative markets and derivative pricing models mainly from three aspects.

First of all, this thesis conducts a series of regression analysis to examine the liquidity effects in the commodity futures markets and the examination includes five representative futures markets, namely, the agriculture futures markets, the metal futures markets as well as the energy futures markets. Within those markets, this thesis has a number of valuable findings regarding the liquidity effects. Firstly, commodity market liquidity is a determinant of commodity prices comovement in different commodity futures markets. Based on the first finding, I further investigate the liquidity explanatory power in the futures markets and it has been demonstrated that the liquidity risk, which can be defined as the liquidity innovation, is closely related to the residual risk part that is not explained by the market risk.

Since the liquidity risk is linked with the residual information, I argue that the market liquidity incorporates most of the noise information in the commodity futures markets. Then I connect the informational role of liquidity with the cointegration tests of commodity futures. The co-integration tests of the selected commodity futures markets turned out to be non-stationary. By controlling the liquidity variables for the co-integration tests, and then the residuals of the co-integration regression become stationary. The stationary residuals imply the long-run co-integration relationship between five commodity futures markets, where the liquidity can be argued to be integrated with futures prices.

The main result of the empirical analysis is that the liquidity risk is firmly correlated with the residual risk part that the market factor cannot explain. Then I regress the five commodity futures returns on the market returns in addition to the liquidity risk. The results are statistically significant for the liquidity risk in all five markets. So, the liquidity risk can explain the commodity futures returns variances for which market index return is unable to explain. Therefore, it is arguable that the liquidity risk can serve as a complementary factor for explaining the commodity futures returns after controlling the market index return.

Based on this argument and existing literatures, I develop a two-factor futures pricing model by adding liquidity as an additional factor with the market price factor. Supporting by this argument, the liquidity plays an important role in affecting the futures prices. It is thereby the model with liquidity factor should be more accurate than the traditional futures pricing model, which neglects the liquidity effects. For the empirical results comparison, the liquidity adjusted models are more accurate than standard models without liquidity effects. The liquidity adjusted model has lessened the model pricing error by 30%. Moreover, the results of the new model are more stable than the benchmark model since the volatility of the new model is much smaller. The empirical analysis results are robust in performance tests since it adopts two different types of liquidity measurement methods: Amihud and Roll measures. Secondly, the new model introduces the notion of implied discount rate for the liquidity effects. By using the implied discount rate, the new model can be applied in predicting both spot price and futures price simultaneously. More importantly, the forecast errors of the new model for predicting the next day futures prices for different maturities are less than 1.6%.

Furthermore, I also carry out a theoretical study on liquidity effects on future prices in couple with maturity effect. I find that when the assets are more illiquid, futures prices are adjusted downward in greater degree compared to standard no-arbitrage futures prices. Such adjustment seems independent of spot prices for a fixed maturity. However, the illiquidity adjustment depends on the futures maturities. Longer maturity implies deeper adjustment for a fixed spot liquidity level. Therefore, spot illiquidity has bigger impacts on long maturity futures than short maturity products. Since the liquidity adjusted futures pricing model is proved to more accurate empirically, it is legitimate to consider the extension of the liquidity adjusted options pricing models, such as liquidity adjusted American options model and real options model. As a result, this thesis finally develops a two-factor real options model, which considers the liquidity effects on the real investment decision from the firm perspective.

The liquidity adjusted real options model builds a three dimension real options model, which takes the asset illiquidity into account. The newly-developed model builds an intrinsic connection between real options theory and inventory asset illiquidity. The study of the model reveals the effects of inventory asset illiquidity towards investment threshold and flexibility values, namely, the exercise boundary and the real options values, which is complementary to the existing real options and corporate finance literatures. The theoretical results can also help managers to have a deeper understanding of the role of asset liquidity in corporate finance and risk management.

The new model is mainly complementary to two types of existing literatures. The first kind focuses on the effects of real asset illiquidity (mainly physical asset) on the corporate investment. In addition to the physical asset illiquidity, I add the inventory asset to the real asset category and demonstrate the effects of the inventory asset illiquidity. The model shows that the illiquidity of inventory assets would also influence the corporate investment decisions. More importantly, I use the model to specifically quantify the amount of flexibilities in terms of real options values that have been reduced by the inventory asset illiquidity. The model shows that the flexibilities have been significantly reduced by the inventory asset illiquidity when the real options are at/in the money, but the effects are considerably less when the options are out of the money.

In addition, the model also relates to the literatures document the investment booming during unfavorable market when the underlying asset tends to illiquid. My results demonstrate that when the market is illiquid, the threshold of the investment becomes lower since the waiting value and the flexibility are eroded by the asset illiquidity. The lower investment threshold, in turn, yields a higher probability of exercise and the real options are likely to be exercised at a lower value. The results can partially aid the explanation why there might be investment booming when the market is illiquid and the real options are exercised suboptimally.

More importantly, the real options theory with asset illiquidity sheds light on the corporate liquidity and project management. When the asset is liquid, the threshold for exercising the real options on that project will be higher. On the other hand, when the market turns to be illiquid, firms launch the projects in a rapid manner since they neither desire to use their precious cash holdings to rescue the project nor desire to abandon the project when the market conditions get even worse. It is thereby arguable that holding illiquid inventory assets is also toxic for firms' operations and firms might hold liquid inventory assets as the liquidity buffer. Firms thereby have a higher probability of exercising the real options with illiquid assets but the exercise is suboptimal. I argue that the suboptimal exercise of the real options may decrease the firm value and thereby firms shall make judicious decisions for investment when the environment is unfriendly.

Finally, the new model exhibits two main features how asset illiquidity affects the investment decisions under the real options framework. Firstly, the asset illiquidity has a larger influence when real options are in the money than they are out of the money. Secondly, asset illiquidity has little impact on real options when the time is near maturity. These two effects can aid the understanding of the real asset illiquidity towards the investment decisions and provide helpful insights to managers for both liquidity and project management.

Appendix A

Appendix

A.1 *Derivation of the PDEs for Futures Pricing Model and Real Options Model*

For the derivation of the PDE, I assume that there is a liquidity impact factor (γ_t), which reflects the impact of liquidity on the futures prices and is incorporated into the demand function of the underlying asset. This liquidity impact factor is different from the liquidity discount factor (d) I used in the boundary conditions, where the liquidity discount factor (d) captures the discount the seller might offer when the market is close to freeze in order to sell the asset.

The demand function for the futures, D (S_t, γ_t, I_t), is a function of the futures price, the liquidity impact factor γ_t, and the information process I_t:

$$D(S_t, \gamma_t, I_t) = g\left(\frac{I_t^\nu}{S_t\gamma_t}\right)$$

where g is a smooth, strictly increasing function, and ν is a constant and ν >0.

Moreover, the liquidity impact factor, is also a function of the level of liquidity in the market, at, and the sensitivity of the futures price to the level of market illiquidity, β >0:

$$d\gamma_t/\gamma_t = (-\beta L_t + \frac{1}{2}\beta^2 L_t^2)dt - \beta L_t dW_t^{\gamma,P}$$

and the dynamics of the information process, I_t , are:

$$dI_t/I_t = \tilde{\mu}dt + \eta dW_t^{I,P}$$

For simplicity, as Brunetti and Caldarera (2006) prove, I assume that there is always a price that can clear the entire market. Under such market-clearing conditions, the illiquid asset price will follow:

$$dS_t/S_t = (\mu + \beta L_t + \frac{1}{2}\beta^2 {L_t}^2)dt + \beta L_t dW_t^{\gamma,P} + \sigma dW_t^{S',P} \tag{A.1}$$

where

$$\mu = \tilde{\mu}\nu + \frac{1}{2}\nu(\nu - 1)\eta^2$$

and

$$\sigma = \nu\eta$$

For the purpose of futures pricing, I first disaggregate the Brownian motion of the imperfect liquid asset price into two parts:

$$W_t^{\gamma,P} = \rho W_t^{L,P} + \sqrt{1-\rho^2} W_t^{u,P} \tag{A.2}$$

where

$$dW_t^{L,P} dW_t^{u,P} = 0$$
$$W_t^{S,P} = \int_0^t \frac{\sigma}{\sqrt{\sigma^2+(1-\rho^2)\beta^2 L_t^2}} dW_t^{S',P} + \int_0^t \frac{\beta L_t\sqrt{1-\rho^2}}{\sqrt{\sigma^2+(1-\rho^2)\beta^2 L_t^2}} dW_t^{u,P}$$

Therefore, using Levys theorem,$W_t^{S,P}$ is a Brownian motion under the measure of P; therefore, the asset price will be:

$$dS_t/S_t = (\mu+\beta L_t+\frac{1}{2}\beta^2 {L_t}^2)dt+\sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} dW_t^{S,P} + \rho\beta L_t dW_t^{L,P} \tag{A.3}$$

where $dW_t^{L,P} dW_t^{S,P} = 0$

As Bingham and Kiesel (1998) conclude, all possible martingale measures can be categorized by so-called Girsanov densities. Under the Girsanov densities, I will find the appropriate risk-neutral martingale measures for futures pricing valuation with illiquidity. According to Girsanov theory, the equivalent martingale measures can be represented by a Girsanov derivative:

$$
\begin{aligned}
&M(T) = \tfrac{dQ}{dP} \\
&M_t = \exp(-\int_0^t \lambda_1(s) dW_t^{S,P} - \int_0^t \lambda_2(s) dW_t^{L,P} - \tfrac{1}{2}\int_0^t \lambda_1^2(s) ds - \tfrac{1}{2}\int_0^t \lambda_2^2(s) ds)
\end{aligned}
$$

As a result, the Brownian motions are changed under the new measure Q:

$$
\begin{aligned}
dW_t^{L,Q} &= dW_t^{L,P} + \lambda_1 dt \\
dW_t^{S,Q} &= dW_t^{S,P} + \lambda_2 dt
\end{aligned}
$$

where λ_1 and λ_2 should satisfy:

$$
\mu + \beta L_t + \frac{1}{2}\beta^2 {L_t}^2 - r = \lambda_1 \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} + \lambda_2 \rho \beta L_t \tag{A.4}
$$

Thus, the new SDEs become:

$$
dS_t/S_t = rdt + \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} dW_t^{S,Q} + \rho\beta L_t dW_t^{L,Q} \tag{A.5}
$$

$$
dL_t = \tilde{\alpha}(\tilde{\theta} - L_t)dt + \xi dW_t^{L,Q} \tag{A.6}
$$

where

$$
\begin{aligned}
&\tilde{\alpha} = \alpha + w \\
&\tilde{\theta} = \tfrac{\alpha\theta}{\alpha + w} \\
&dW_t^{L,Q} dW_t^{S,Q} = 0
\end{aligned}
$$

and w is constant.

Based on the two stochastic equations, I apply the Feynman-Kac formula for the two-dimensional derivation, adopting F (S, L, τ) to represent the value of futures contracts when S_t=S and L_t=L. By using matrix multiplication and the Feynman-Kac formula, I develop PDE for the function F (S, L, τ):

$$
\frac{\partial F}{\partial \tau} = rS\frac{\partial F}{\partial S} + \tilde{\alpha}(\tilde{\theta} - L)\frac{\partial F}{\partial L} + \frac{1}{2}S^2(\sigma^2 + \beta^2 L^2)\frac{\partial^2 F}{\partial S^2} + \frac{1}{2}\xi^2\frac{\partial^2 F}{\partial L^2} + \rho\xi\beta LS\frac{\partial^2 F}{\partial S \partial L} \tag{A.7}
$$

where τ=T-t is the time to maturity T.

Similarly as above, I can obtain the stochastic equations for real assets, where V stands for the asset values and L stands for the asset illiquidity

$$dV_t/V_t = rdt + \sqrt{\sigma^2 + (1-\rho^2)\beta^2 L_t^2} dW_t^{V,Q} + \rho\beta L_t dW_t^{L,Q} \tag{A.8}$$

$$dL_t = \tilde{\alpha}(\tilde{\theta} - L_t)dt + \xi dW_t^{L,Q} \tag{A.9}$$

$$\tilde{\alpha} = \alpha + w$$
$$\tilde{\theta} = \tfrac{\alpha\theta}{\alpha+w}$$
$$dW_t^{L,Q} dW_t^{V,Q} = 0$$

Based on derived stochastic processes, it can generate the liquidity-adjusted real options model. I apply the Feynman-Kac formula for the two-dimensional derivation, adopting G (V, L, τ) to represent the value of real project when V_t=V and L_t=L. By using matrix multiplication and the Feynman-Kac formula, I develop PDE for the function G (V, L, τ):

$$\frac{\partial G}{\partial \tau} = (r-y)V\frac{\partial G}{\partial V} + \tilde{\alpha}(\tilde{\theta}-L)\frac{\partial G}{\partial L} + \frac{1}{2}V^2(\sigma^2+\beta^2L^2)\frac{\partial^2 G}{\partial V^2} + \frac{1}{2}\xi^2\frac{\partial^2 G}{\partial L^2} + \rho\xi\beta LV\frac{\partial^2 G}{\partial V \partial L} - rG \tag{A.10}$$

In comparison with the traditional PDE, the PDE I acquire in Chapter 4 is similar in that it is only a two-dimensional extension of the traditional version with asset illiquidity. They also share the same boundary conditions. This PDE is subject to both initial and boundary conditions. The PDE of our model has an initial condition, which is

$$G(V, 0, \tau) = max[V - I, 0] \tag{A.11}$$

The PDE is also subject to the following boundary conditions:

$$G(V, 0, \tau) = C(V, \tau) \tag{A.12}$$

$$G(V, L_{max}, 0) = max[C(V, \tau) * d, V - I] \tag{A.13}$$

$$G(0, L, \tau) = 0 \tag{A.14}$$

$$G(V^*(L, \tau), L, \tau) = V^*(L, \tau) - I \tag{A.15}$$

$$G_v(V^*(L, \tau), L, \tau) = 1 \tag{A.16}$$

$$G(V, L, \tau) = V - I \quad forV \geq V^*(L, \tau) \tag{A.17}$$

A.2 *Discretization of the PDEs for Futures Pricing Model and Real Options Model by Adopting Finite Difference Method*

I discretize the PDE model for futures pricing in order to input the real market data, where the real market data is not continuous and is discretized on the daily basis. There are three main dimensions of the PDE model, namely, the time dimension, the futures price dimension and the liquidity dimension. Accordingly, I discretize the time dimension into T tranches, and I discretize the futures price dimension into M tranches and I discretize the liquidity dimension into Q tranches. So I have:

$$\tau = 0, \Delta\tau, 2\Delta\tau ... N\Delta\tau \quad \Delta\tau * N = t_{\max} = T$$
$$S = 0, \Delta S, 2\Delta S ... M\Delta S \quad \Delta S * M = S_{\max} = 250$$
$$L = 0, \Delta L, 2\Delta L ... Q\Delta L \quad \Delta L * Q = L_{\max} = 1$$

Then, I discretize the PDE model as follows:

$$F_{i,j}^k = F(i\Delta\tau, j\Delta S, k\Delta L)$$
$$\frac{\partial F}{\partial \tau} \approx \frac{F_{i,j}^{k+1} - F_{i,j}^k}{\Delta\tau}$$
$$\frac{\partial F}{\partial S} \approx \frac{F_{i+1,j}^k - F_{i-1,j}^k}{2\Delta S}$$
$$\frac{\partial F}{\partial L} \approx \frac{F_{i,j+1}^k - F_{i,j-1}^k}{2\Delta L}$$
$$\frac{\partial^2 F}{\partial S^2} \approx \frac{F_{i+1,j}^k + F_{i-1,j}^k - 2F_{i,j}^k}{\Delta S^2}$$
$$\frac{\partial^2 F}{\partial L^2} \approx \frac{F_{i,j+1}^k + F_{i,j-1}^k - 2F_{i,j}^k}{\Delta L^2}$$
$$\frac{\partial^2 F}{\partial L \partial S} \approx \frac{\frac{F_{i+1,j+1}^k - F_{i-1,j+1}^k}{2\Delta S} - \frac{F_{i+1,j+1}^k - F_{i+1,j-1}^k}{2\Delta S}}{2\Delta L}$$

The discretization is subject to the stability conditions:

$$\Delta\tau \leq \frac{1}{2}\min\{(\Delta S)^2, (\Delta L)^2\}$$

I discretize the PDE model for real options in order to conduct model simulation. There are three main dimensions of the PDE model, namely, the time dimension, the asset value dimension and asset illiquidity dimension. Accordingly, I discretize the time dimension into T tranches, and I discretize the asset value dimension into M tranches and I discretize asset illiquidity dimension into Q tranches. So I have:

$$\tau = 0, \Delta\tau, 2\Delta\tau ... N\Delta\tau \quad \Delta\tau * N = t_{\max} = T$$
$$V = 0, \Delta V, 2\Delta V ... M\Delta V \quad \Delta V * M = V_{\max}$$
$$L = 0, \Delta L, 2\Delta L ... Q\Delta L \quad \Delta L * Q = L_{\max} = 0.1$$

Then, I discretize the PDE for real options model as follows:

$$
\begin{aligned}
&G_{i,j}^{k} = G(i\Delta V, j\Delta L, k\Delta\tau) \\
&\frac{\partial G}{\partial \tau} \approx \frac{G_{i,j}^{k+1} - G_{i,j}^{k}}{\Delta\tau} \\
&\frac{\partial G}{\partial V} \approx \frac{G_{i+1,j}^{k} - G_{i-1,j}^{k}}{2\Delta V} \\
&\frac{\partial G}{\partial L} \approx \frac{G_{i,j+1}^{k} - G_{i,j-1}^{k}}{2\Delta L} \\
&\frac{\partial^2 G}{\partial V^2} \approx \frac{G_{i+1,j}^{k} + G_{i-1,j}^{k} - 2G_{i,j}^{k}}{\Delta V^2} \\
&\frac{\partial^2 G}{\partial L^2} \approx \frac{G_{i,j+1}^{k} + G_{i,j-1}^{k} - 2G_{i,j}^{k}}{\Delta L^2} \\
&\frac{\partial^2 G}{\partial L \partial V} \approx \frac{\frac{G_{i+1,j+1}^{k} - G_{i-1,j+1}^{k}}{2\Delta V} - \frac{G_{i+1,j+1}^{k} - G_{i+1,j-1}^{k}}{2\Delta V}}{2\Delta L}
\end{aligned}
$$

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