THE EFFECTS OF REFRACTION ON ENGINEERING SURVEY MEASUREMENTS

by

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To my parents and Beth.

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SYMBOL	DESCRIPTION	UNITS
A	E.D.M. additive constant (Chapter Two)	m
A	Area exposed to wind (Appendix B)	m^2
a	Acceleration due to gravity	$m.s^{-2}$
a	Principle radius of Airy's spheroid	m
a_h	Horizontal acceleration due to gravity	$m.s^{-2}$
a_n	Normal component of acceleration due to gravity	$m.s^{-2}$
a_{RS}	Component of horizontal acceleration along line RS	$m.s^{-2}$
a_v	Vertical component of acceleration due to gravity	$m.s^{-2}$
a_x	x component of horizontal acceleration due to gravity	$m.s^{-2}$
a_y	y component of horizontal acceleration due to gravity	$m.s^{-2}$
C_d	Drag coefficient	
c	Velocity of electromagnetic waves in vacuo	$m.s^{-1}$
D	Distance	m
D_1	Two wavelength measured distance	m
D_2	Two wavelength measured distance	m
D_a	E.D.M. distance measurement in ambient conditions	m
D_c	Chord distance	m
D_r	Reduced (station to station) slope distance	m
D_R	Spheroid distance	m
D_s	E.D.M. distance measurement in standard conditions	m
d	Diameter of sphere	m

d	Difference between theodolite and target heights	m
d'	Difference between E.D.M. and prism height	m
e	Partial water vapour pressure	mb
e	Eccentricity of Airy's spheroid	
e_s	Saturated water vapour pressure	mb
F_D	Drag force on the balloon	N
f	Modulation frequency	Hz
G	Gravitational constant	$m^3.kg^{-1}.s^{-2}$
g	Acceleration due to the earth's gravity	$m.s^{-2}$
g_{45}	Standard acceleration due to the earth's gravity	$m.s^{-2}$
g_m	Mean gravity between two stations	$m.s^{-2}$
H	Height of a rock column	m
h	Height above ground level	m
h'	Mean height above mean sea level	m
h_1	Height of station A above mean sea level	m
h_2	Height of station B above mean sea level	m
h_e	E.D.M. instrument height above survey station	m
h_m	Theodolite target height above survey station	m
h_p	E.D.M. prism height above survey station	m
h_t	Theodolite height above survey station	m
i	Angle of incidence	o ' "
L	Difference in level	m
L_s	Height above mean sea level	m
m	Mass	kg
N	Integer number	
n	Refractive index	
n_1	Two wavelength ambient refractive index 1	

n_2	Two wavelength ambient refractive index 2	
n_a	Ambient refractive index	
n_g	Group refractive index	
n_o	Absolute refractive index	
n_s	Standard refractive index	
P	Atmospheric pressure	mb
P	Plan distance	m
P_o	Atmospheric pressure at mean sea level	mb
P_1	Atmospheric pressure at height, h_1	mb
P_2	Atmospheric pressure at height, h_2	mb
R	Local radius of the spheroid (earth)	m
R_e	Reynold's number	
r	Angle of refraction	o ' "
r	Radius	m
T	Ambient air dry bulb temperature	K
T'	Ambient air dry bulb temperature	^oC
T_m	Mean ambient dry bulb temperature	K
T_w	Ambient wet bulb temperature	K
t	Time	s
V	Wind velocity	$m.s^{-1}$
v	Velocity of electromagnetic waves in a medium	$m.s^{-1}$
v_g	Group velocity of a wave	$m.s^{-1}$
x	Mantissa of cartesian coordinates	m
y	Ordinate of cartesian coordinates	m
α	Angle (for geometry)	o ' "
α	Coefficient of expansion of air	K^{-1}

β	Constant in two wavelength technique	
β	Angle (for geometry)	o ' "
δ	Small change in a function	
$\delta\theta$	Reduction correction to vertical angle measurements	o ' "
$\delta\theta'$	Angle subtended between E.D.M. line and station/station line	o ' "
ζ	Deviation of the vertical	o ' "
ζ_A	Deviation of vertical at A in azimuth of B	o ' "
ζ_B	Deviation of vertical at B in azimuth of A	o ' "
ζ_R	Deviation of vertical at R in azimuth of RS	o ' "
ζ_S	Deviation of vertical at S in azimuth of SR	o ' "
θ	Angle subtended at centre of spheroid	o ' " or radians
θ_A	Upward corrected (refraction only) vertical angle	o ' "
θ_a	Upward measured vertical angle	o ' "
θ_B	Downward corrected (refraction only) vertical angle	o ' "
θ_b	Downward measured vertical angle	o ' "
θ_C	Mean corrected (curvature and refraction) vertical angle	o ' "
θ_{CA}	Upward corrected (curvature and refraction) vertical angle	o ' "
θ_{CB}	Downward corrected (curvature and refraction) vertical angle	o ' "
θ_R	Reduced vertical angle	o ' "
θ_r	Station to station reduced vertical angle	o ' "
λ	Wavelength	μm
λ_a	Ambient modulation wavelength	μm
λ_s	Standard modulation wavelength	μm
μ	Dynamic viscosity	$N.s.m^{-2}$

ρ	Air density	$Kg.m^{-3}$
σ	Radius of curvature of ray path	m
σ	Wave number	μm^{-1}
τ	Tether angle	o ' "
Φ	Latitude	o ' "
ϕ	Phase angle	radians
ψ	Angle subtended by ray path at centre of curvature	radians
Ω_a	Angle of refraction at A	o ' "
Ω_b	Angle of refraction at B	o ' "

INTRODUCTION

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In August 1974 the International Association of Geodesy at a symposium entitled, 'Terrestrial Electromagnetic Distance Measurements and Atmospheric Effects on Angular Measurements' resolved to encourage research into the effects of refraction on survey networks [Curl, 1975]*.

At a recent Science Research Council round table meeting on research in engineering survey [Ashkenazi, 1975] it was stated that

> "Research is required, in co-operation with atmospheric physicists, for a better understanding of the refractive properties of the atmosphere, in so far as they affect,
>
> (i) horizontal angles,
> (ii) vertical angles,
> (iii) distance (E.D.M. measurements).
>
> It is essential to devise improved mathematical models which best represent the effect of the atmosphere on the measurements made and to develop instruments and observational procedures for obtaining the values of the unknown physical parameters." (i.e. temperature, pressure and humidity.)

These statements have been substantiated by conversations between the author and instrument designers, manufacturers and operators. Not only has interest been shown in the research reported in this thesis, but

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* References are listed alphabetically on page 306.

the practical necessity for this type of work has been asserted if both the precision and accuracy of engineering survey measurements are to improve.

There have been many improvements in both survey instruments and techniques in recent years. The introduction of electromagnetic distance measuring (E.D.M.) instruments has had a major influence on engineering survey methods and has established standards of accuracy that would previously have been inconceivable.

E.D.M. can be categorised into two groups; the first are those which are capable of measuring distances up to three kilometres, these instruments usually use visible or infra red light and are frequently used in engineering survey measurements. The second type of E.D.M. are those which use microwaves or laser light and have measuring ranges of the order of one hundred kilometres. This thesis deals exclusively with short range E.D.M., with visible light or infra red carrier waves, and statements made with reference to E.D.M. concern this type of instrument only, and need not necessarily apply to microwave instruments.

The most accurate E.D.M. commercially available at the present time, the Kern Mekometer ME3000, has a specified accuracy of ± 0.2 mm ± 1 mm/km, the instrument has a maximum operating range of 3000 m. This statement implies that a distance of 1000 m can be measured to an accuracy of ± 1.2 mm; in practice such accuracy is very difficult to achieve. Although the phase measurement accuracy of an E.D.M. may be considerably less than 1 mm, there are many other factors, both instrumental and non-instrumental, which have an adverse effect on accurate distance measurement.

These sources of error are considered in detail in this thesis.

The greatest non-instrumental source of error in E.D.M. is that caused by refraction of the electromagnetic waves as they pass through the atmosphere between the instrument and the reflector prisms. To enable a full understanding of refraction errors, Chapter One outlines the theory of refraction from first principles and gives the methods by which the refractive index of air can be calculated.

Before the effects of refraction on E.D.M. can be investigated it is necessary to consider and quantify the other sources of error. These are discussed in Chapter Two and include errors in the E.D.M. modulation frequency, the additive constant and the measurement of phase angle. The methods of applying an atmospheric correction to an E.D.M. measurement are given in sections 2.6. and 2.7. The last section in Chapter Two outlines the two wavelength E.D.M. technique which enables the accurate measurement of distance without the necessity for atmospheric corrections. The limitations of this method for applications in engineering survey are stated.

The mean air temperature, pressure and humidity along the ray path between E.D.M. instrument and prism are required for the refraction correction of distance measurements. To ensure accurate meteorological measurements carefully calibrated barometers and thermometers should be used. Chapter Three discusses the types of meteorological equipment used and the calibration procedures adopted by the author during the experimental work undertaken in connection with this thesis. The problems associated with the derivation of mean atmospheric parameters are considered in Chapter Four, which also describes experimental work carried out in the Vale of Llangollen, North Wales. These experiments were designed to evaluate the errors incurred by assuming that the means of end station pressures, temperatures and humidities are representative of the air along the E.D.M. wave path. Particular attention has been given to the derivation of mean air temperature which is the most important parameter affecting the atmospheric refraction correction of E.D.M. measurements.

The plan distance and the difference in level between two survey stations are normally required in engineering survey. Consequently the corrected E.D.M. slope distance must be reduced to give these dimensions, using either the results of a spirit levelling, or observed vertical angles. The accuracy of vertical angle measurement is considered in detail in Chapter Five. Refractive bending of the line of sight between a theodolite and a target can introduce significant errors in the measured vertical angle, particularly if the distance between the stations is large. Practical investigations into the effects of refraction on vertical angle measurement have been made by the author on test lines in Llangollen, and at Risley in Derbyshire. The results of these experiments give information concerning the optimum weather conditions and time of day for measurements, and also an indication of the attainable accuracy of vertical angle measurement if the greatest care is taken in the measuring and correcting procedures.

The effect of the curvature of the earth on vertical angle measurements is considered, and the method of correction for this source of error is given in section 5.3.

When working to the very highest orders of accuracy, the deviation of the vertical at a survey station can have a significant effect on the measurement of vertical angles. A method of calculating the deviation of the vertical at a survey station, due to the surrounding topography, has been developed by the author and is illustrated by example in section 5.6.

The combination of accurate measurements of vertical angles and E.D.M. slope distances, to give plan distances and differences in level, is discussed in Chapter Six. The necessity to reduce all distances to a common reference plane, before they can be used in the calculation of station co-ordinates, is emphasized.

Results of distance and angle measurements, taken between stations in the Llangollen survey network, have been used to illustrate the high standards of accuracy that can be achieved with careful calibration of equipment, good observational procedure and rigorously applied correction methods.

Published works relating directly to the subject in the text are referenced with the authors name and the year of publication, for example [Author, 1977]. References are listed in alphabetical order on page 306. Where more than one paper by the same author is quoted the references appear in chronological order. A Bibliography of further published work directly relating to the subjects considered in this thesis, but not referenced is given on page 312.

Wherever possible the conventional notation of variables has been adopted in the text. Each symbol is explained when it is first introduced, after which it is used without qualification. A full nomenclature is given on page xii.

CHAPTER ONE

THE REFRACTIVE INDEX OF AIR

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1.1. First Principles

When light, or any other electromagnetic waves, pass through the boundary between two mediums of different density the waves are refracted, that is the velocity of the wave and its direction of propagation are changed. When a light wave passes from a less dense to a more dense medium it is retarded and the wave is bent towards the normal of the density boundary (Figure 1.1.). The converse is true if the waves travel in the opposite direction.

In 1620 Snell established a simple mathematical relationship for the degree of refractive bending at a density boundary [Nelkon and Parker, 1968]. He stated that for two mediums of specific but different density (Figure 1.1.) the ratio of the sines of the angle of incidence (i) and the angle of refraction (r) was constant. This constant is the refractive index (n).

$$n = \frac{\sin i}{\sin r} \qquad \ldots 1.1.$$

The refractive index has a specific value for the boundary between any two regions of different density, it is dependent on the direction of the incident wave. The refractive index of a material or medium is that when light enters the medium from a vacuum.

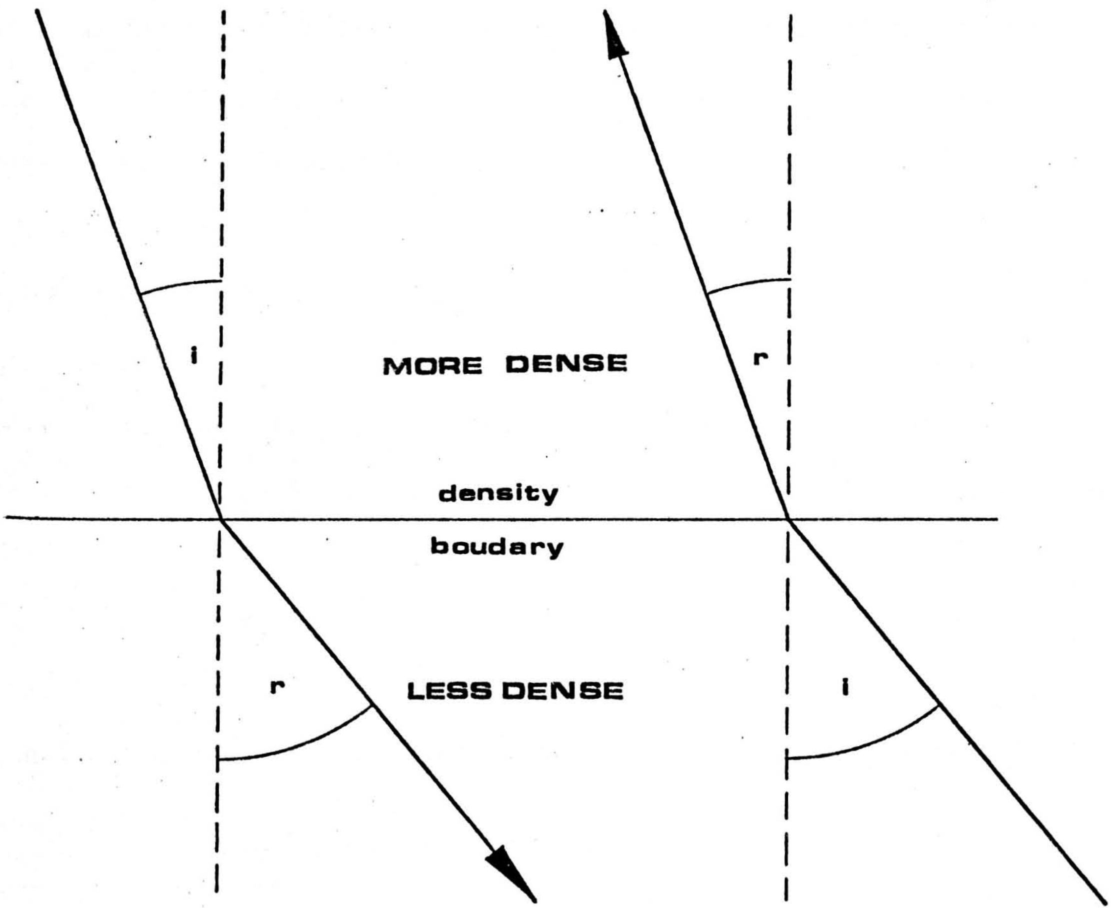


Figure 1.1. Snell's Law of Refraction

$$n = \frac{\sin i}{\sin r}$$

Using Huyghen's [Nelkon and Parker, 1968] wave theory of light it is possible to relate the refrrctive index of the boundary between two density regions to the velocity of light in the regions. Consider Figure 1.2. in which a beam of light, between AB and LM, is incident on the density boundary between region one and region two. The wavefront BM is first incident on the density boundary at B, it then takes point M a further time, t, to reach the boundary. In so doing the wavefront travels a distance MN where

$$MN = v_1 t$$

where v_1 is the velocity of light in region 1. In the same time, t, the wave AB has been refracted and has travelled to C in region 2, where

$$BC = v_2 t$$

v_2 being the velocity of light in region 2. Now Snell states that

$$n = \frac{\sin i}{\sin r}$$

but referring to the geometry of Figure 1.2.

$$n = \frac{\sin NBM}{\sin BNC} = \frac{NM/BN}{BC/BN} = \frac{NM}{BC} = \frac{v_1 t}{v_2 t}$$

so refractive index,

$$n = \frac{v_1}{v_2}$$

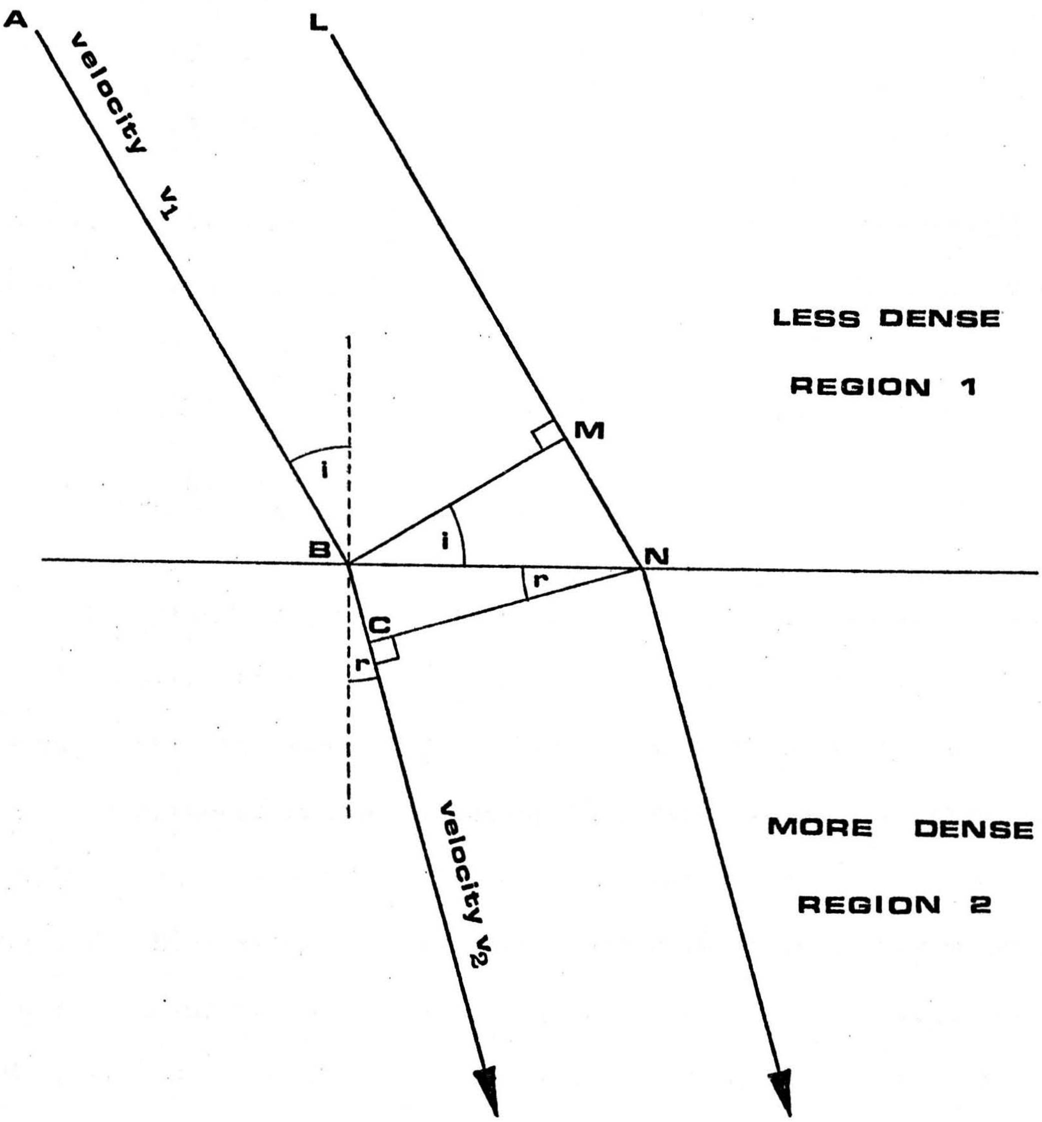


Figure 1.2. The Relationship between Refractive Index and Velocity

This expression gives the refractive index between two density regions. The absolute refractive index of a specific medium in which the velocity of light is v, is given by

$$n_o = \frac{c}{v} \qquad \ldots\ 1.2.$$

where c is the velocity of light in a vacuum. The vacuum velocity of light was accepted by the International Association of Geodesy [Anon, 1958] as

$$c = 299\ 792\ 500 \qquad m.s^{-1}$$

No medium has one specific refractive index; the index depends on the type of light passing through the medium. Electromagnetic waves including light are classified by their wavelength as shown in Figure 1.3. Most survey methods, including theodolite, tacheometer and level work, use light in the visible spectrum which is detectable by the human eye. However, E.D.M. instruments use both visible light (e.g. Mekometer ME3000, Geodimeter 6) and infra red radiation which is invisible (e.g. Tellurometer MA100, Wild DI3S). All the present short range E.D.M.'s for engineering survey are either light or infra red instruments. Radio and Microwave equipment is manufactured but is normally used for long distance measurement, in consequence the calculation of refractive indices for microwaves is not covered in this thesis.

1.2. The Dispersion Formula

In 1939 Barrell and Sears [1939] outlined their experimental

WAVELENGTH µm		E.D.M. EQUIPMENT	OTHER SURVEY EQUIPMENT
	γ RAYS		
10^{-5}	X RAYS		
10^{-2}	ULTRA VIOLET		
0.4	VISIBLE SPECTRUM	0.48 µm KERN ME3000 0.55 µm GEODIMETER 6	THEODOLITES LEVELS TACHEOMETERS ALL OTHER EQUIPMENT
0.8	INFRA RED	0.875 µm WILD DI10 0.875 µm WILD DI3S 0.875 µm KERN DM500 0.900 µm KERN DM1000 0.910 µm HEWLETT PACKARD 3800B 0.910 µm GEODIMETER 12 0.910 µm ZEIS ELDI 2 0.930 µm TELLUROMETER MA100	
10^{2}	MICROWAVES	29 mm TELLUROMETER MRA3 29 mm SIAL MD60	
10^{6}	RADIOWAVES		

Figure 1.3. The Electromagnetic Spectrum

(Not to Scale)

work and stated the following relationship between the refractive index of air, at standard temperature and pressure ($0^{o}C$, 1013.25 mb), and the wavelength of the refracted electromagnetic wave. They gave the absolute refractive index, n_o, as

$$(n_o - 1) \times 10^6 = 287.604 + \frac{1.6288}{\lambda^2} + \frac{0.0136}{\lambda^4} \qquad \ldots 1.3.$$

where λ is the wavelength of the electromagnetic wave in micrometres (μm). For E.D.M. instruments the carrier wavelength should always be used in equation 1.3. The generally accepted value of absolute refractive index for sunlight or white light in air is

$$n_o = 1.000\ 293$$

This value should be used in all refraction calculations involving optical sighting, including theodolite angle measurements.

When light is not homogeneous, as is the case with the visible light of the Geodimeter 6 and all other light and infra red wave E.D.M. instruments, it is necessary to use the group refractive index (n_g) instead of the absolute refractive index (n_o).

1.3. Group Velocity and Group Refractive Index

Non homogeneous light is a combination or a group, of light waves which have slightly different wavelengths and, due to dispersion, different velocities in the same medium. The group velocity is the mean velocity of the waves in a group. Consider a group of two such waves represented on Figure 1.4., they have wavelengths of λ and $\lambda + \delta\lambda$ and velocities of v and $v + \delta v$ respectively.

If the waves are in coincidence at A1 and A2 at time $t = 0$, the next time the waves will coincide will be when points B1 and B2 are coincident after a time lapse of δt where,

$$\delta t = \frac{\delta\lambda}{\delta v} \qquad \ldots 1.4.$$

This is best appreciated by considering the relative velocity of the waves. In the time δt the point of coincidence will have 'slipped back' one wavelength of the shorter wave, which means that the wave group has travelled a distance $v\delta t - \lambda$ which gives the group velocity v_g,

$$v_g = \frac{\text{distance travelled by group}}{\text{time taken}} = \frac{v\,\delta t}{\delta t} - \frac{\lambda}{\delta t}$$

and substituting δt from equation 1.4. we obtain,

$$v_g = v - \lambda\frac{dv}{d\lambda} \qquad \ldots 1.5.$$

which is the final expression for the group velocity of waves in a medium.

Equation 1.2. gave the refractive index of a medium as

$$n = \frac{c}{v} \qquad \ldots 1.6.$$

from which it follows that the refractive index of a group of waves in

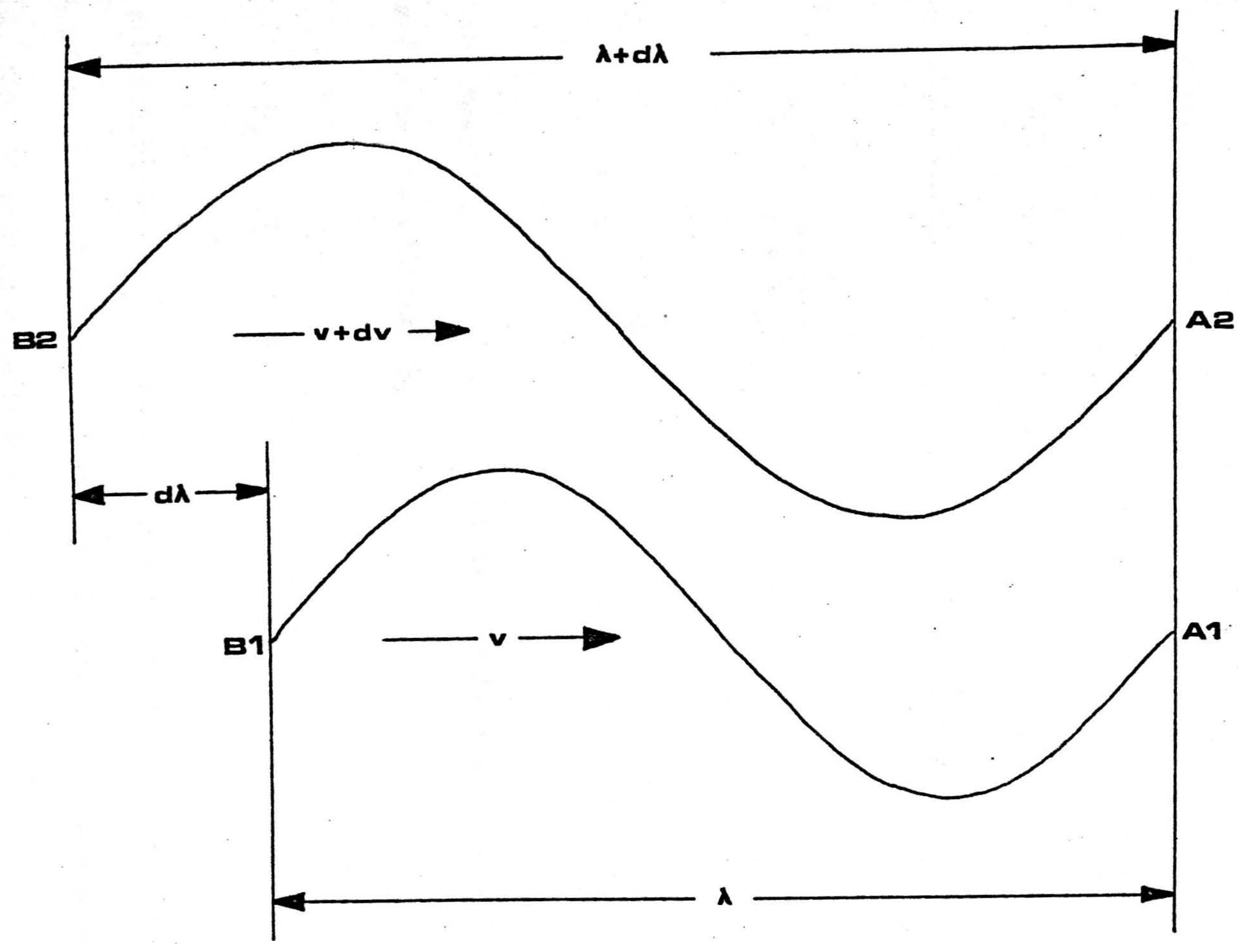


Figure 1.4. Group Velocity of Electromagnetic Waves

non homogeneous light can be defined by

$$\text{group refractive index, } n_g = \frac{c}{v_g} \qquad \dots 1.7.$$

To obtain a relationship between refractive index and group refractive index, equations 1.5. and 1.6. are combined to give

$$v_g = \frac{c}{n} + \lambda \frac{c}{n^2} \frac{dn}{d\lambda}$$

which, substituting for v_g from equation 1.7., gives

$$n_g = \frac{n}{1 + \frac{\lambda}{n}\frac{dn}{d\lambda}}$$

or

$$n_g = n - \lambda \frac{dn}{d\lambda} \qquad \dots 1.8.$$

For the specific case of air at standard temperature and pressure this equation can be written as,

$$n_g = n_o - \lambda \frac{dn_o}{d\lambda} \qquad \dots 1.9.$$

By substitution of n_o from equation 1.3. the following expression for the group refractive index of air at standard temperature and pressure (0^oC, 1013.25 mb) is obtained

$$(n_g - 1) \times 10^6 = 287.604 + \frac{3(1.6288)}{\lambda^2} + \frac{5(0.0136)}{\lambda^4} \qquad \dots 1.10.$$

The wavelength in this equation should be in units of micrometres (μm), the group refractive index is dimensionless.

1.4. Ambient Refractive Index

The ambient refractive index of air at specific temperature, pressure and humidity can be calculated if the group refractive index of the transmitted light is known. The National Physical Laboratory [Anon, 1960] recommended the following formula be accepted for the calculation of the ambient refractive index (n_a) for waves in the visible spectrum, it is also applicable to near visible infra red radiation (e.g. MA100, $\lambda = 0.93\ \mu m$). The formula is the result of experimental work by Barrell and Sears [1939],

$$n_a = 1 + \frac{n_g - 1}{\alpha T} \cdot \frac{P}{1013.25} - \frac{4.12e \times 10^{-8}}{\alpha T} \qquad \ldots 1.11.$$

where α = coefficient of expansion of air = $3.661 \times 10^{-3}\ K^{-1}$

P = ambient atmospheric pressure, mb (1 mb = $10^2\ N.m^{-2}$)

T = ambient atmospheric dry bulb temperature, K

e = partial water vapour pressure, mb

The partial water vapour pressure, a measure of humidity, is calculated from the difference between wet and dry bulb hygrometer readings [Hinsley and Parczewski, 1956]. Assuming a typical value of the psychrometric constant we have,

$$e = e_s - 0.659 \times 10^{-3} \cdot P(T - T_w) \qquad \ldots 1.12.$$

where e_s = saturated water vapour pressure at wet bulb temperature, mb,

and T_w = ambient atmospheric wet bulb temperature, K.

The value of e_s may be obtained from psychometric tables [Barenburg, 1955] using the pressure and the dry bulb temperature, or it can be calculated from the following expression [McPherson, 1971]

$$e_s = \frac{\exp(60.009 - 6814.6/T_w)}{T_w^{5.106}} \cdot 10^{-2} \qquad \ldots 1.13.$$

where T_w is the wet bulb temperature in degrees Kelvin.

The method of calculating ambient refractive index using the Barrell and Sears formulae is summarised in Figure 1.5. With polychromatic light, such as sunlight, which has the complete spectrum of wavelengths, the absolute refractive index is used in place of the group refractive index in equation 1.11. The group refractive index must be calculated for monochromatic waves, which are those with nearly constant wavelengths.

An interesting aspect of the work by Barrell and Sears [1939] concerns the earlier work of Tilton [1933] who suggested that variations in the refractive index of air were related to sun spot activity. Barrell and Sears found no evidence in their experimental results to support this theory.

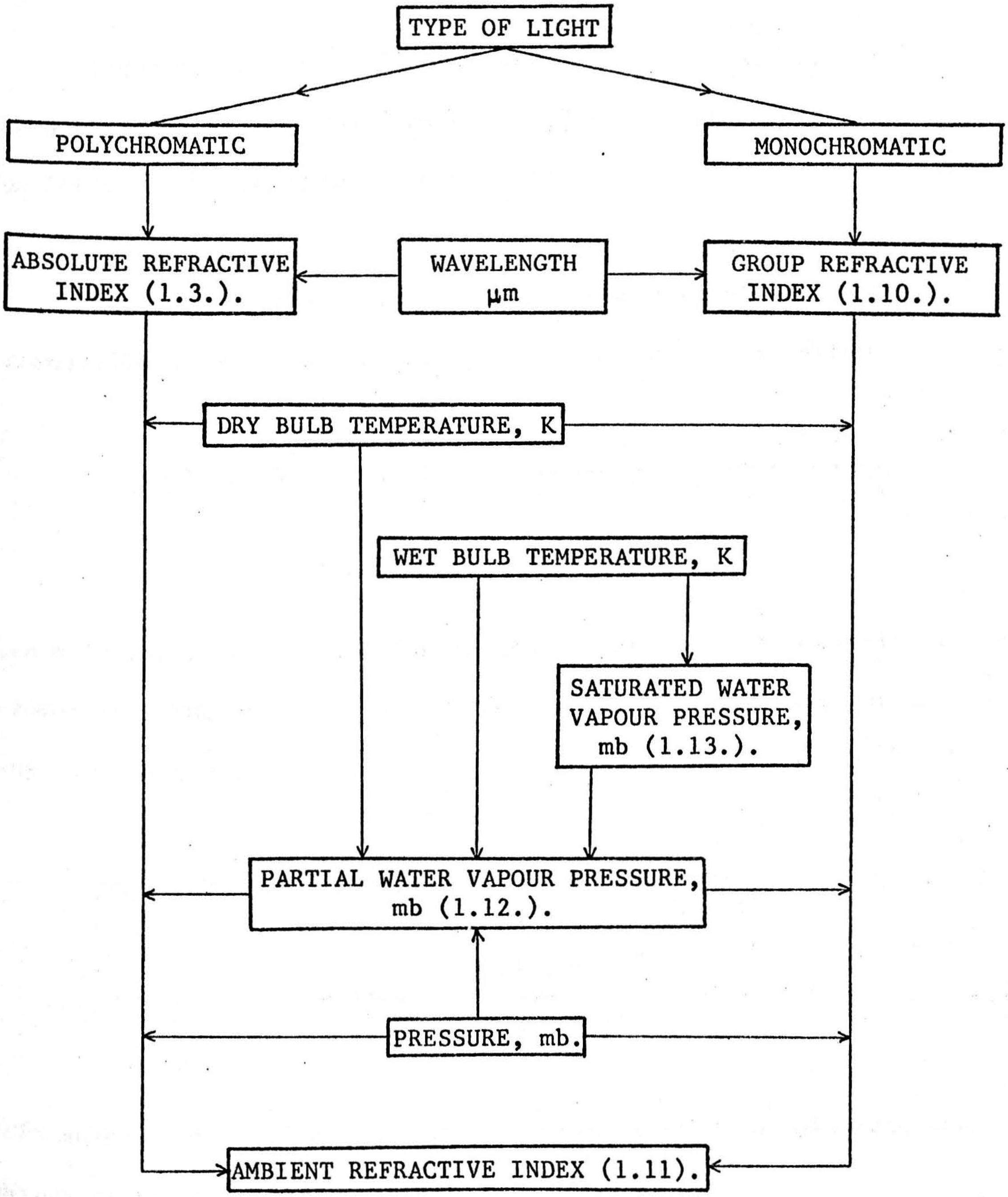


Figure 1.5. Flowchart for the Calculation of the Ambient Refractive Index of Air

Figures in brackets refer to equation numbers in text

1.5. The Edlén Formulae

Alternatives to the Barrell and Sears formulae 1.3., 1.10. and 1.11. are the Edlén formulae [Edlén, 1953] which were later improved after further investigations [Edlén, 1965].

The Edlén dispersion formula for absolute refractive index (n_o) at standardised conditions of 15°C and 1013.25 mb is as follows

$$(n_o - 1) \times 10^8 = 8342.13 + \frac{2406030}{(130 - \sigma^2)} + \frac{15997}{(38.9 - \sigma^2)} \qquad \ldots 1.14.$$

where σ is the wavenumber, which is the reciprocal of the wavelength in micrometers. The group refractive index is calculated from equation 1.14. using 1.9. to give,

$$(n_g - 1) \times 10^8 = 8342.13 + 2406030 \frac{(130 + \sigma^2)}{(130 - \sigma^2)^2} + 15997 \frac{(38.9 + \sigma^2)}{(38.9 - \sigma^2)^2} \qquad \ldots 1.15.$$

Edlén suggests the following formula should be used to calculate the ambient refractive index of air

$$(n_a - 1) = \frac{(n_g - 1)P}{960.937} \cdot \frac{1 + 0.75P\,(0.817 - 0.0133\,T') \times 10^{-6}}{1 + \alpha T'} \qquad \ldots 1.16.$$

where α = 3.661×10^{-3}

P = ambient atmospheric pressure, mb.

T' = ambient air dry bulb temperature, ^{o}C.

Figure 1.6. compares results from the Barrell and Sears, and the Edlén formulae. Considering the measuring accuracy of modern E.D.M. instruments (± 1 ppm) there is no significant difference between the two methods. More sophisticated and accurate instrumentation, such as two wavelength E.D.M., should be corrected with the Edlén formulae to maintain high accuracy.

The atmospheric correction of E.D.M. measurements and the relevance of the accurate determination of the refractive index of air is covered in detail in Chapter Two.

METHOD	REFRACTIVE INDEX		
	GROUP T = 15°C, P = 1013.25 mb.	GROUP T = 0°C, P = 1013.25 mb.	AMBIENT T = 20°C, P = 950 mb.
Barrell and Sears (1939)	(1.000 288 7) ←	1.00 304 5	1.00 266 0
Edlén (1965)	1.000 288 6	-	1.000 265 9

Figure 1.6. Comparison of Barrell and Sears with Edlén Formulae

$\lambda = 0.55\ \mu m.$

CHAPTER TWO

THE EFFECTS OF REFRACTION ON E.D.M. MEASUREMENTS

CHAPTER TWO

THE EFFECTS OF REFRACTION ON E.D.M. MEASUREMENTS

2.1. Introduction

There are many factors which affect the accuracy of E.D.M. measurements; of these, errors due to refraction are the most difficult to quantify. Inadequate methods of obtaining representative mean atmospheric conditions may be the limiting factor affecting the development of high accuracy engineering survey measurements with E.D.M. It is only possible to investigate the effects of refraction when all the other errors affecting E.D.M. have been considered, quantified and, if possible, corrected for.

Chapter Two derives the basic E.D.M. equation from first principles. The effects and methods of correcting for possible errors on the distance measurement are discussed. Operational sources of error, including E.D.M. warm-up time, are considered. The theory of refractive index corrections to E.D.M. is explained and the methods of applying the corrections are compared. Section 2.9. outlines the principle of the two wavelength system of E.D.M. measurement, and explains why, at the present time, such a technique is not a solution to the atmospheric correction problem in engineering survey measurements.

2.2. Fundamental E.D.M. Theory

Modern short range E.D.M. instruments transmit a carrier wave of visible or infra red radiation which is returned to the instrument by a retrodirective prism at a remote survey station. Superimposed on the carrier wave is a modulation wave, the wavelength of which is the basic measuring unit of the E.D.M. technique. The modulation wave is used as a 'yardstick'; by counting the number of times it is placed along the line to be measured the distance is obtained. Because the waves travel from the E.D.M. to the prisms and back to the E.D.M. the path length is twice the distance to be measured.

The path length of an E.D.M. wave will be an integral number of wavelengths plus a fractional part of a wavelength. Considering Figure 2.1., for example, the path length (2D) is measured as four complete wavelengths plus a fractional part (1/x) of a wave. More generally,

$$2D = N\lambda + \frac{\lambda}{x} \qquad \ldots\ 2.1.$$

where N is an integer. To give the measured distance, D, as

$$D = N\frac{\lambda}{2} + \frac{\lambda}{2x} \qquad \ldots\ 2.2.$$

This is comparable to N complete revolutions of a vector OA (Figure 2.1.) plus a fractional part of a revolution $\phi/2\pi$, or

$$D = N\frac{\lambda}{2} + \frac{\phi}{2\pi}\frac{\lambda}{2} \qquad \ldots\ 2.3.$$

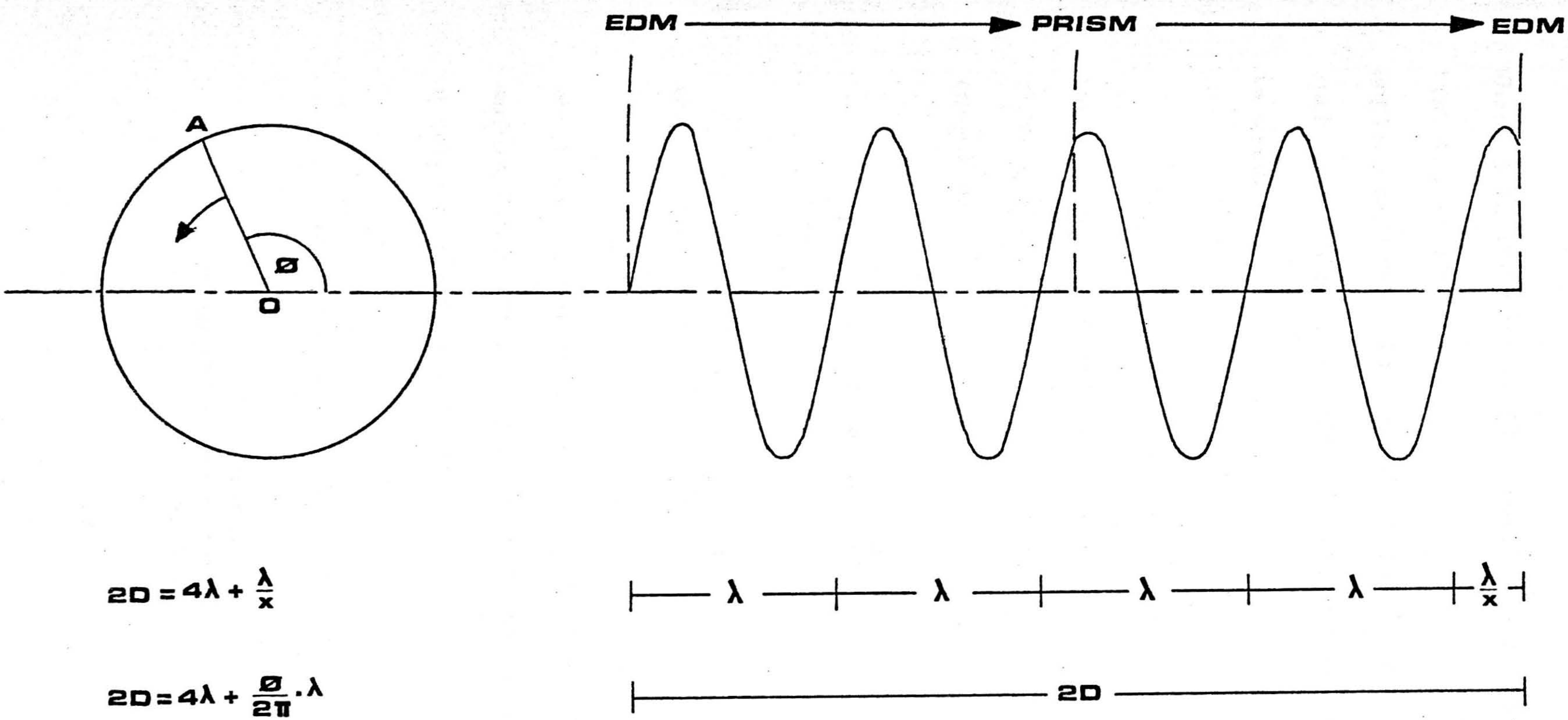


Figure 2.1. Diagram of the E.D.M. Measuring Principle

Where ϕ is the phase angle, which indicates the phase shift between transmitted and received waves. In most E.D.M. instruments phase angle measurement is electronic but in some high accuracy instruments, such as the Kern Mekometer ME3000, mechanical-optical phase measurement is used.

To determine the number of wavelengths along the wave path the distance is measured with two electromagnetic waves of slightly different frequency and wavelength, thus giving two simultaneous measuring equations,

$$D = N_1 \frac{\lambda_1}{2} + \frac{\phi_1}{2\pi} \frac{\lambda_1}{2} \qquad \ldots\ 2.4.$$

and

$$D = N_2 \frac{\lambda_2}{2} + \frac{\phi_2}{2\pi} \frac{\lambda_2}{2} \qquad \ldots\ 2.5.$$

which can be solved for N_1 or N_2 to give the required distance D. Most modern E.D.M. instruments solve these equations internally and give a digital readout of the distance.

Now considering the basic wave equation,

$$\text{wavelength} = \frac{\text{velocity}}{\text{frequency}}$$

or

$$\lambda = \frac{v}{f} \qquad \ldots\ 2.6.$$

and the velocity definition of refractive index, from equation 1.2.

$$n = \frac{c}{v}$$

an expression for the wavelength is obtained,

$$\lambda = \frac{c}{fn} \qquad \ldots\ 2.7.$$

Since the wave passes through the air between the instrument and reflector survey stations the mean ambient refractive index (n_a) should be used in equation 2.7., giving

$$\lambda = \frac{c}{fn_a} \qquad \ldots\ 2.8.$$

This expression can be substituted into equation 2.3. to give

$$D = N \cdot \frac{c}{2fn_a} + \frac{\phi}{2\pi} \cdot \frac{c}{2fn_a} + A \qquad \ldots\ 2.9.$$

where A is the additive constant for the E.D.M. instrument. This takes account of any eccentricity between the electro-optical centre of the instrument, or prism, and their geometrical centres which are plumbed vertically over the survey stations.

There are four factors in equation 2.9. which govern the accuracy that can be obtained with the E.D.M. method of distance measurement, they are

(a) Modulation frequency (f).

(b) Additive constant (A).

(c) Phase angle (ϕ).

(d) Ambient refractive index (n_a).

The first three factors are connected with the internal mechanisms of an E.D.M. instrument, although the additive constant is sometimes combined for both E.D.M. and prisms. However, the ambient refractive index is related to both the instrument specifications and, more importantly, the atmospheric conditions along the line being measured.

2.3. Modulation Frequency

The modulation frequency is the critical design parameter of an E.D.M. instrument. The frequency controls the modulation wavelength which is the basic measuring unit of the E.D.M. system. Any error in the modulation frequency will cause a proportional error in distance measurement, this error is usually expressed in parts per million (ppm). The higher the frequency of an E.D.M., the shorter the modulation wavelength and the greater is the possible distance measuring accuracy.

The E.D.M. frequency is modulated by an oscillator. The characteristics of this oscillator, and hence the modulation frequency, depend on the ambient temperature of the environment in which it operates. There are two methods currently in use to overcome this variation, firstly the oscillator can be heated to a steady temperature in a thermostatically controlled oven, the temperature of which is outside the range of normal atmospheric conditions. The second method involves integration of the oscillator into a stabilized network of components, which are arranged within the network so that the temperature dependent effects of each part combine to form a temperature independent stabilized network.

The frequency generated by crystals in an E.D.M. instrument changes slowly with time due to ageing. Ageing does not normally produce a significant error in the modulation frequency in less than about a year. Frequency ageing is typically 1 ppm per year.

It is advisable to check the modulation frequency of first order E.D.M. instruments before any high accuracy measurements are made. This checks that the temperature compensating devices are functioning correctly and also quantifies any frequency drift that may have occurred due to ageing. As frequency errors are proportional to the distance being measured, frequency checking is not necessary before the measurement of short lines, unless these form part of a traverse in which the proportional errors are cumulative. Frequency checks are also unnecessary for the less accurate second order E.D.M. instruments, some of which have built in safeguards that stop the measuring procedure if the frequency drift is outside specifications.

The nominal modulation frequencies for E.D.M. instruments are usually quoted in the operators handbook. Any small deviations from these values will cause errors in distance measurement which should be corrected. The correction for frequency error is equal to

$$\frac{\text{nominal frequency - actual frequency}}{\text{nominal frequency}} \times 10^6 \text{ ppm}$$

Large frequency errors will of course necessitate maintenance from the manufacturer.

A suitable system for checking modulation frequencies has been assembled and used by the author. A Hewlett-Packard 5382A frequency counter was used in conjunction with an Advance Instruments off-air frequency standard OFS2B as shown in Plate 2.1. The frequency counter has a range from 10 Hz to 225 MHz with an eight digit display. The accuracy of the counter alone is inadequate for checking E.D.M. frequencies; it uses a 10 MHz crystal as a standard which ages with time and is also temperature dependent [Anon, 1974,a]. To alleviate these problems the off-air-standard is used, this instrument receives a standard radio transmission from Droitwich at 200 kHz and provides standard frequency outputs of 1 MHz and 10 MHz, which are accurate to better than two parts in one hundred million with a one second counter gate time. The accuracy can be increased ten fold if the gate time is increased by the same magnitude. The standard is connected to the external input of the frequency counter, thus by-passing the counter's crystal, and forming a stable and temperature independent frequency reference. This system provides a simple, economical and very accurate method of testing E.D.M. frequencies. The frequency counting equipment should be switched on to 'soak' for at least ten hours before use. Figure 2.2. shows in diagrammatic form the arrangement of the frequency checking apparatus, the feed-through device acts as a filter to stop double counting of the frequency.

Frequency checks are best made in the clean environment of a laboratory. Although frequency can be checked in the field this is not normally necessary because frequency changes are usually gradual. Serious

PLATE 2.1.

Checking the modulation frequencies of
a Tellurometer MA 100.


GATE TIME
ATTENUATOR
INPUT
LINE ON (IN)
5382A 225MHz FREQUENCY COUNTER
Outputs
1MHz
10MHz
Supply
Off On
Lock
Meter
Supply
AUTOMATIC
MANUAL

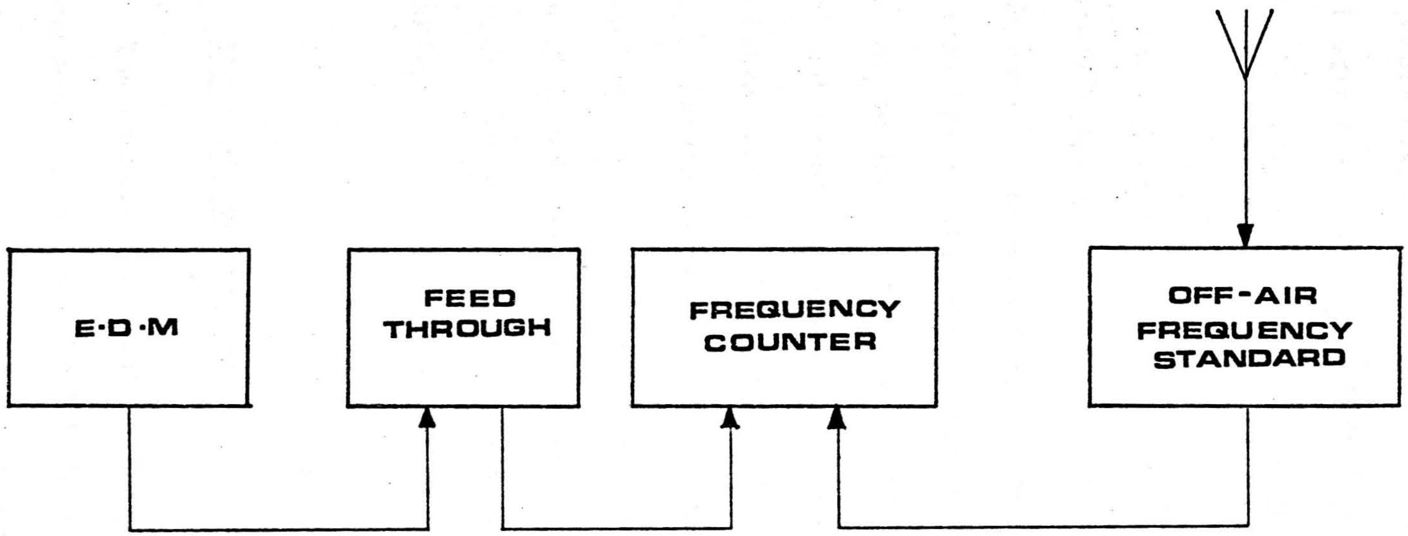


Figure 2.2. Diagrammatic Representation of Frequency Counting Equipment

frequency malfunctions would manifest themselves in the observed distance measurements. Instruments such as the Tellurometer MA100 have frequency output terminals as a standard fitting. However, with the Geodimeter 6 a simple search coil has to be inserted through the bulb holder opening and held next to the Kerr cell.

The crystals which generate the modulation frequency of an E.D.M. take a finite time from being switched on to reaching their correct operating value. Consequently, E.D.M. instruments should be allowed a warm up time, particularly if a long distance is to be measured. Warm up times can be determined by monitoring the E.D.M. frequency from the time the instrument is turned on. Figure 2.3. shows a warm up curve for a Tellurometer MA100 from which it can be seen that a minimum of ten minutes should be allowed if accuracies of better than 1 ppm are to be achieved in distance measurements. Warm up times will depend on the type of E.D.M. instrument in use. Some second order short range instruments need no warm up time and can operate within specifications immediately.

2.4. The Additive Constant

The additive constant represents the difference between the true length of a line and the E.D.M. measured distance after all other corrections have been applied. The additive constant is equal to the eccentricity of the optical and physical centres of both E.D.M. instrument and prisms. The claimed accuracy of any E.D.M. measurement cannot be better than that with which the additive constant is known, unless differential measuring techniques are used. The additive constant is not

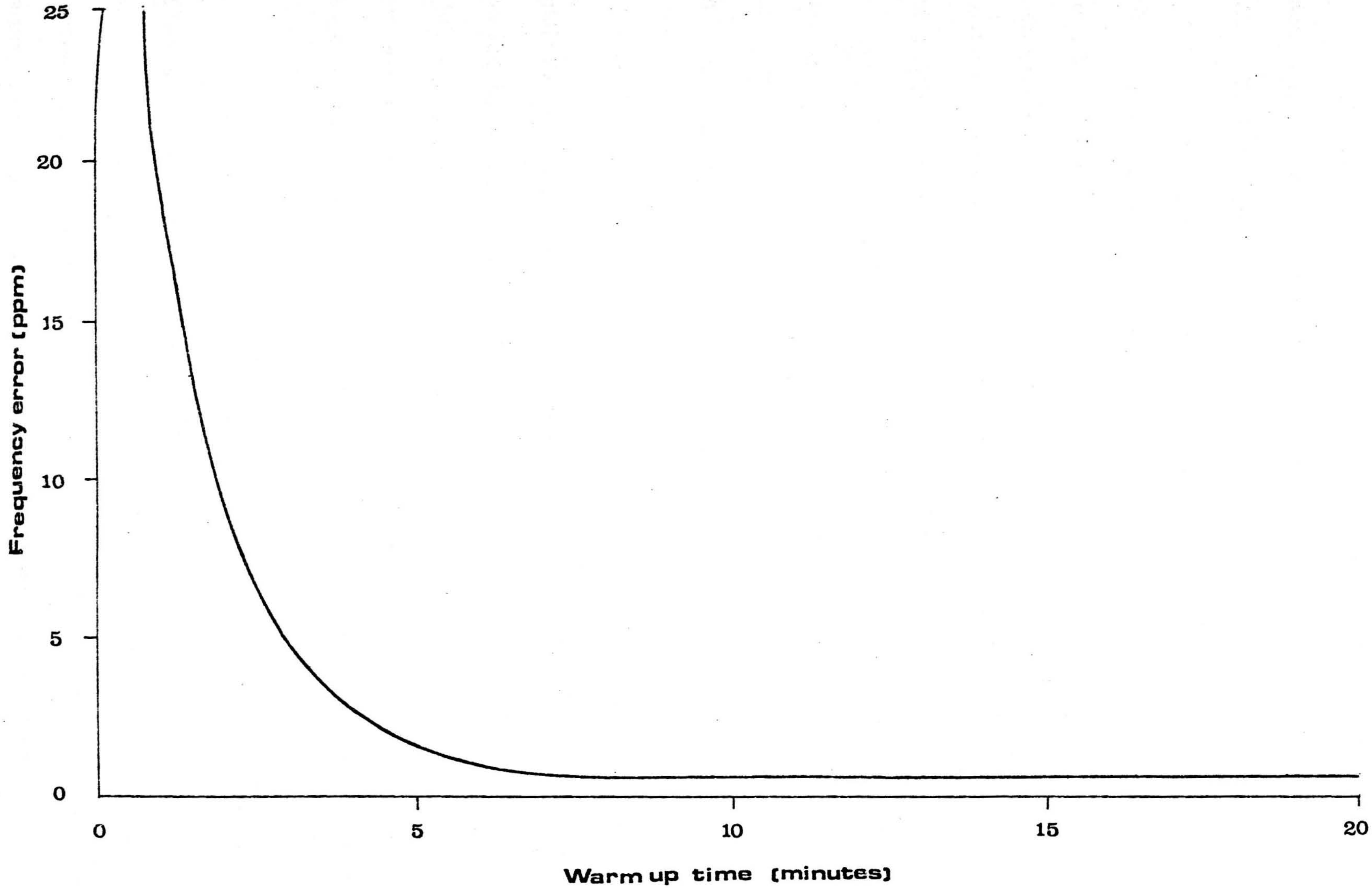


Figure 2.3. Frequency Warm Up Curve for Tellurometer MA100 No.366

proportional to the distance being measured.

The additive constant can be determined on a baseline of known length, or preferably on a multi station baseline [Schwendener, 1972]. Such a line has been established, with twelve stations spaced over 942 m, along a disused railway track at Quorn [Hodges and Curl, 1976] (Appendix A). An ideal baseline is one designed such that if all the combinations of distances are measured the fractional excess of each measurement is different in length. The range of these lengths should be evenly distributed over a complete modulation wavelength of the E.D.M. instrument being tested. A good mean additive constant can be obtained by measuring all the combinations of distances in such a baseline [Ashkenazi and Dodson, 1975]. Alternatively, the three point method [Hodges, 1975] is a quick, but less accurate method of determining the additive constant when no known baseline is available.

If only a short baseline is available for the determination of additive constant greater care must be taken to avoid pointing errors, caused by electromagnetic misalignment of the instrument. To minimise this problem E.D.M. instruments should always be pointed on the criteria of maximising return signal rather than relying on optical pointing using telescopic sights. Excessive return signal should always be attenuated.

Great care should be taken to ensure that both instrument and prisms are accurately centred over the survey stations during additive constant determinations and all subsequent measurements. Constrained centring on monuments or pillars is the best method of mounting instruments and prisms. Ideally this method should always be used when the highest order of accuracy is required. However, this is not always possible and tripod mounted measurements may have to be made. In this situation the optical plummets and level bubbles of the tribrachs should be in good adjustment and carefully checked. Regular and routine adjustment of this type of equipment is to be recommended (Plate 2.2.a,b).

2.5. Phase Angle Measurement

The measurement of the phase angle between the transmitted and received waves enables the fractional part of a wavelength, or the fractional excess, to be determined. The accuracy of phase angle measurement governs the internal distance measuring accuracy of an E.D.M. instrument. If two E.D.M. instruments have the same phase angle measurement capabilities, then the instrument with the shorter modulation wavelength will be capable of more accurate distance measurement.

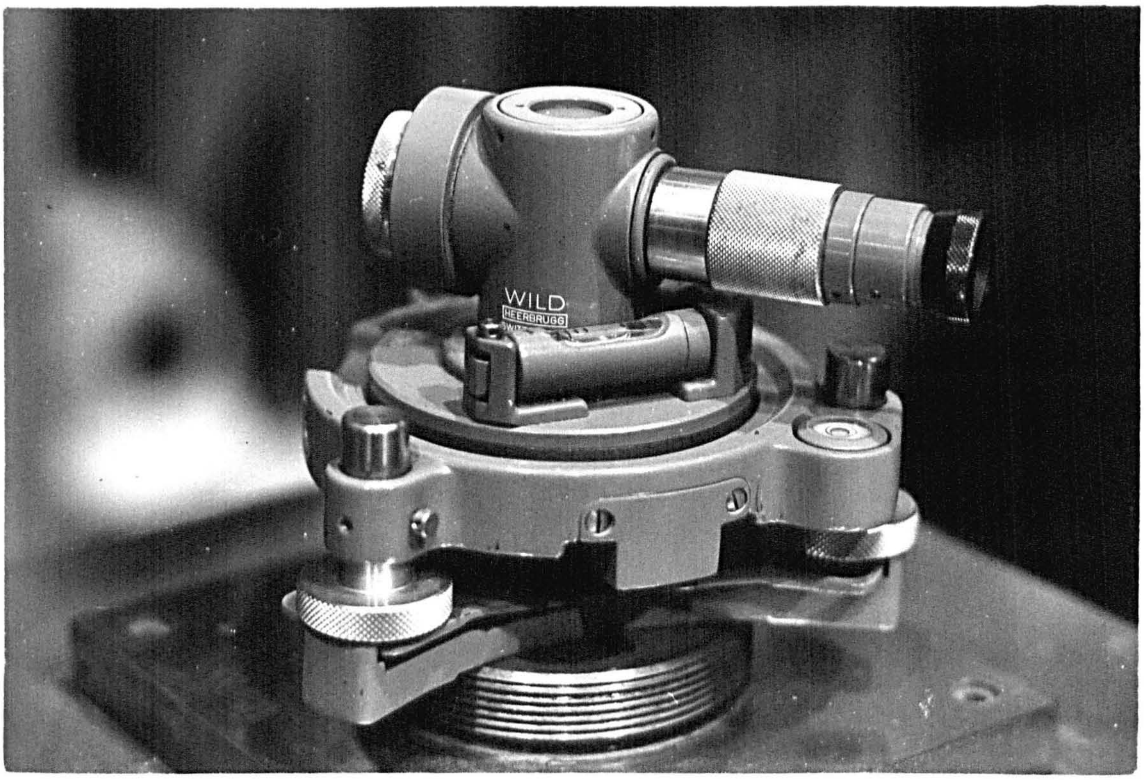
The errors in phase measurement will vary in a cyclic manner depending on the part of the modulated wavelength received by the instrument, they are not proportional to the distance measured. Errors in distance measurement attributable to phase error will be cyclic with a period of one half of a modulation wavelength. These errors are prevalent in electronic phase measurement, mechanical-optical phase measurement is more consistent.

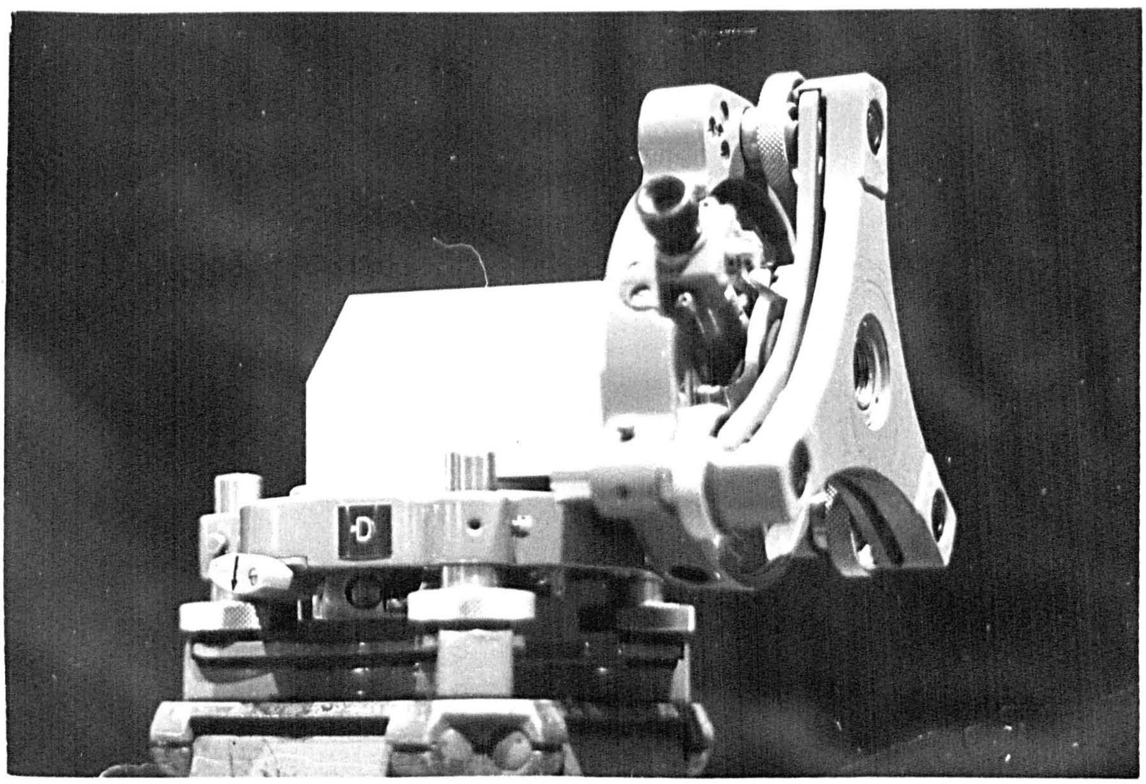
Cyclic errors in phase measurement, or non linearity errors as they are sometimes called, can be determined by taking distance measurements to a series of points a known distance apart in line with the instrument. These distances should be evenly distributed over a

PLATE 2.2.

(a) Checking a tribrach's level bubble.

(b) Checking a tribrach's optical plummet.


WILD
HEERBRUGG



minimum of a half wavelength, however, distribution over a greater range gives further consistency checks. A three metre calibration bar (Plate 2.3.a) has been used to check the phase errors of the E.D.M. used for measurements in connection with this thesis. A prism is mounted on the bar and located by a pin which can be inserted into holes at ten millimetre intervals along the bar. A steel tape is used as a guide to the prism position, but the holes, which are drilled to machine shop accuracy, determine the exact prism position. The calibration bar is mounted on a monument at one end of a 43 metre baseline. Figure 2.4. shows the cyclic error curves of two different Tellurometer MA100 instruments. Instrument number 366 indicates no cyclic error outside the manufacturers claim of accuracy, but number 309 shows large and inconsistent cyclic errors which, in fact, necessitated maintenance by the manufacturing company.

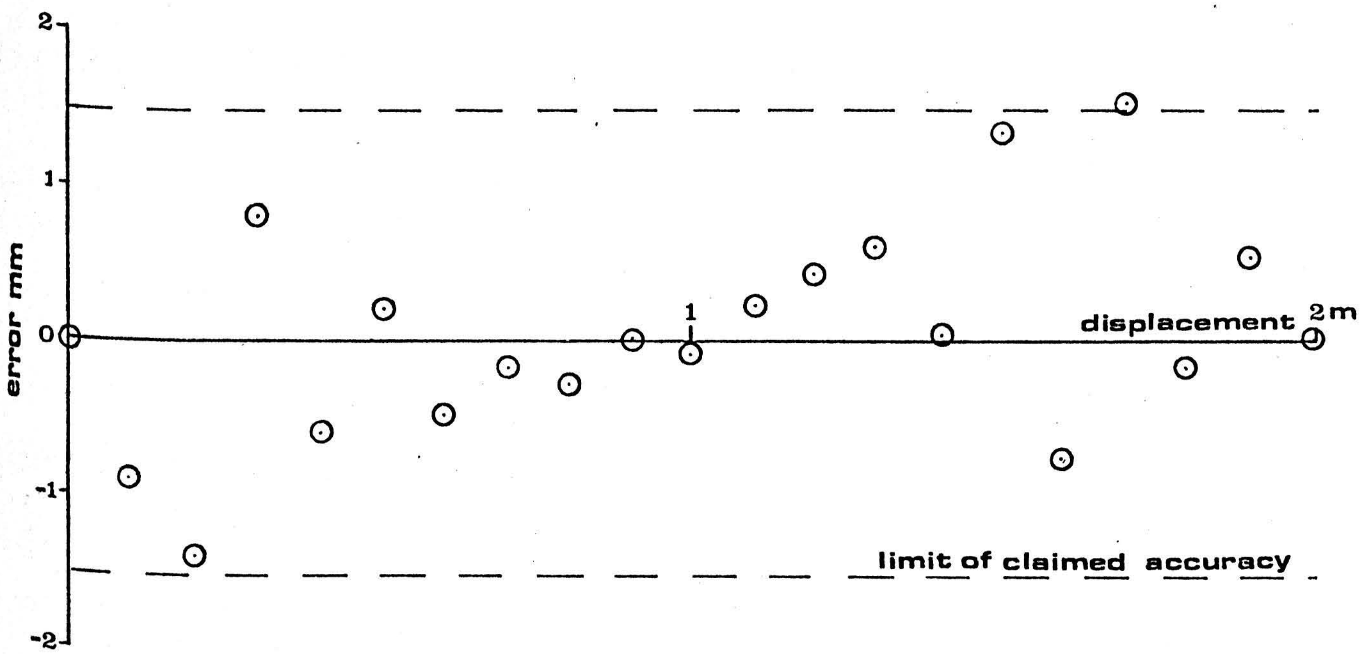
A micrometer capable of providing very accurate prism movement of ± 0.1 mm over a range of 200 mm is useful for studying particular regions of the phase error curves (Plate 2.3.b).

Cyclic error curves can be used to apply a correction to accurate distance measurements. These corrections can be incorporated in the additive constant, which is equivalent to applying a 'variable additive constant'. If phase measurement errors are small then a mean additive constant can be used with confidence.

2.6. The Ambient Refractive Index

The ambient refractive index of air is a critical factor affecting the accuracy of E.D.M. measurements. The modulation wavelength depends

Tellurometer MA100. no.366



Tellurometer MA100.no.309

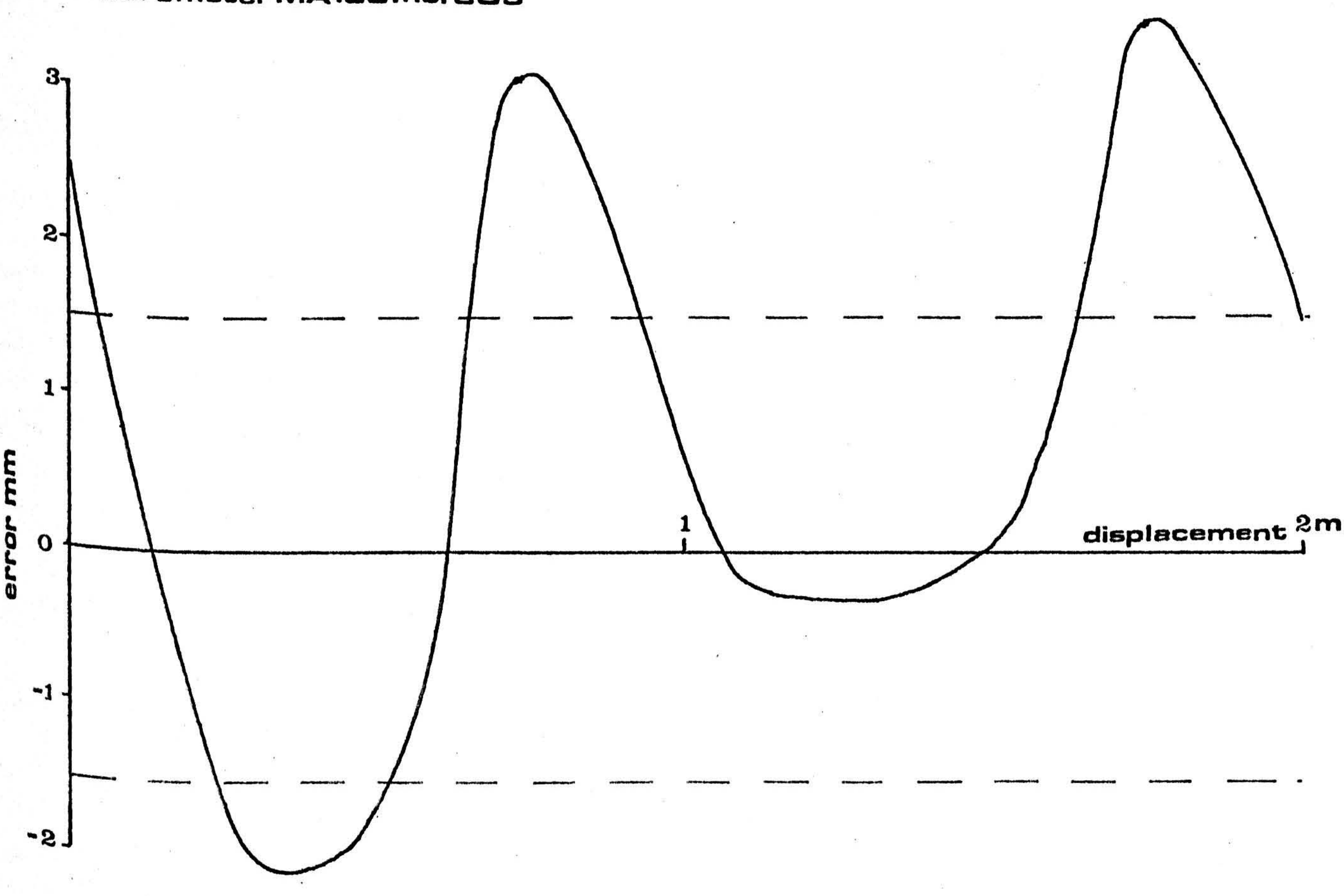
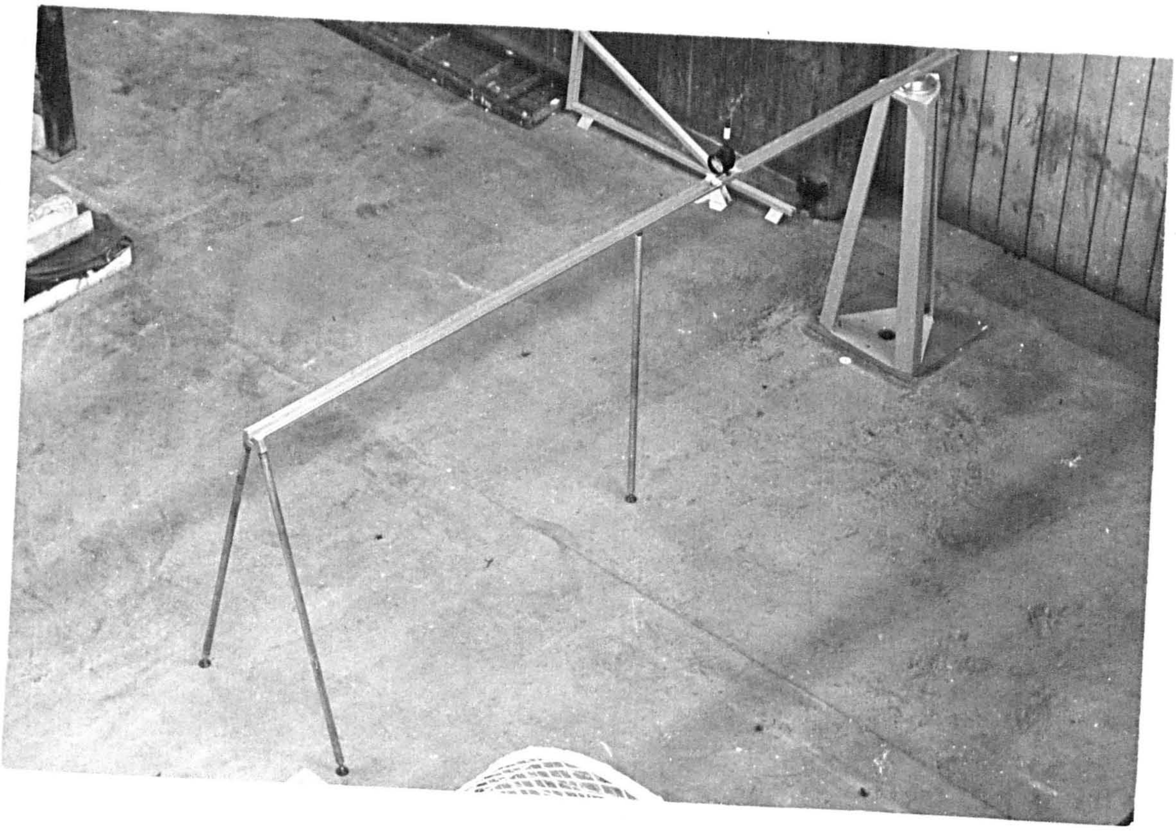


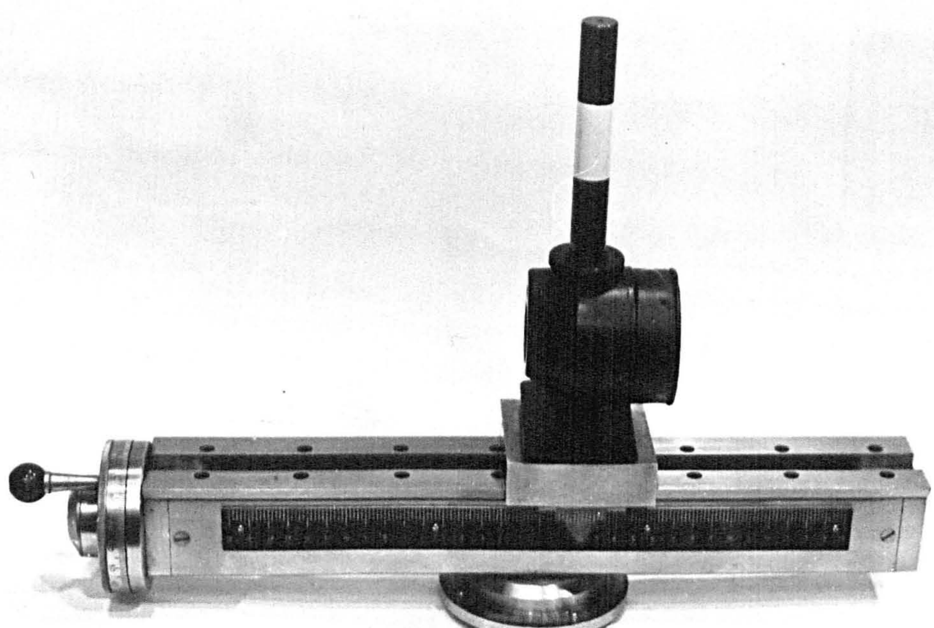
Figure 2.4. Examples of Cyclic Phase Measurement Errors

PLATE 2.3.

(a) The three metre calibration bar.

(b) The micrometer for accurate prism movement.





on the frequency of the modulation wave, a constant for any specific instrument with the exception of the Kern Mekometer, and the velocity of the wave through the air. The wave velocity is not constant, it varies with changes in air density and is defined by the ambient refractive index of the air. It follows that changes in the refractive index of the air will affect the recorded distance. All E.D.M. instruments are designed to make a correct distance measurement through air of a specific refractive index, known as the standard refractive index (n_s). Measurements through air with a different refractive index will require an atmospheric correction. The Mekometer is unique at the present time in that it samples the air at the instrument station and varies the modulation frequency accordingly, thus giving a wave of standard length in all atmospheric conditions. Measurements with the Mekometer will still require atmospheric corrections if the temperature and pressure at the instrument station are not representative of the mean values along the line of sight.

The ambient refractive index of air depends on the air temperature, pressure and humidity (equation 1.11.). For visible light and infra red instruments a change of one ppm in refractive index is caused by a change of $\pm 1^{o}C$ in temperatures or $\pm$ 3 mb of pressure. The effect of humidity on the refractive index of visible and infra red radiation is very small, being less than 0.5 ppm. Figure 2.5. illustrates the effect of atmospheric conditions on refractive index and an E.D.M. measured distance.

To enable accurate atmospheric corrections to E.D.M. distance measurements the mean refractive index along the line of sight from the

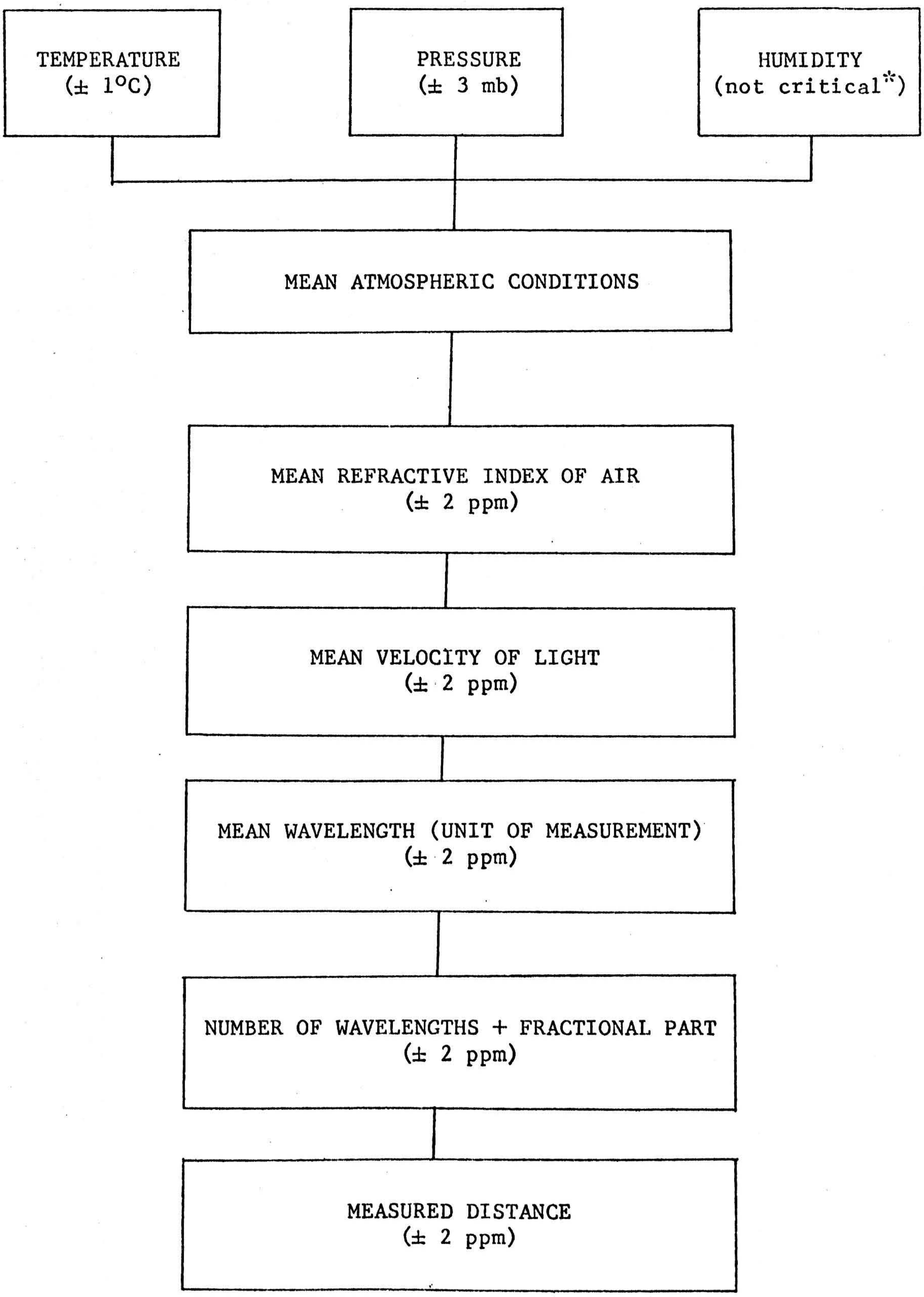


Figure 2.5. The Effects of Errors in Atmospheric Parameters on E.D.M. with Visible and Infra Red Waves

* See Figure 2.6.

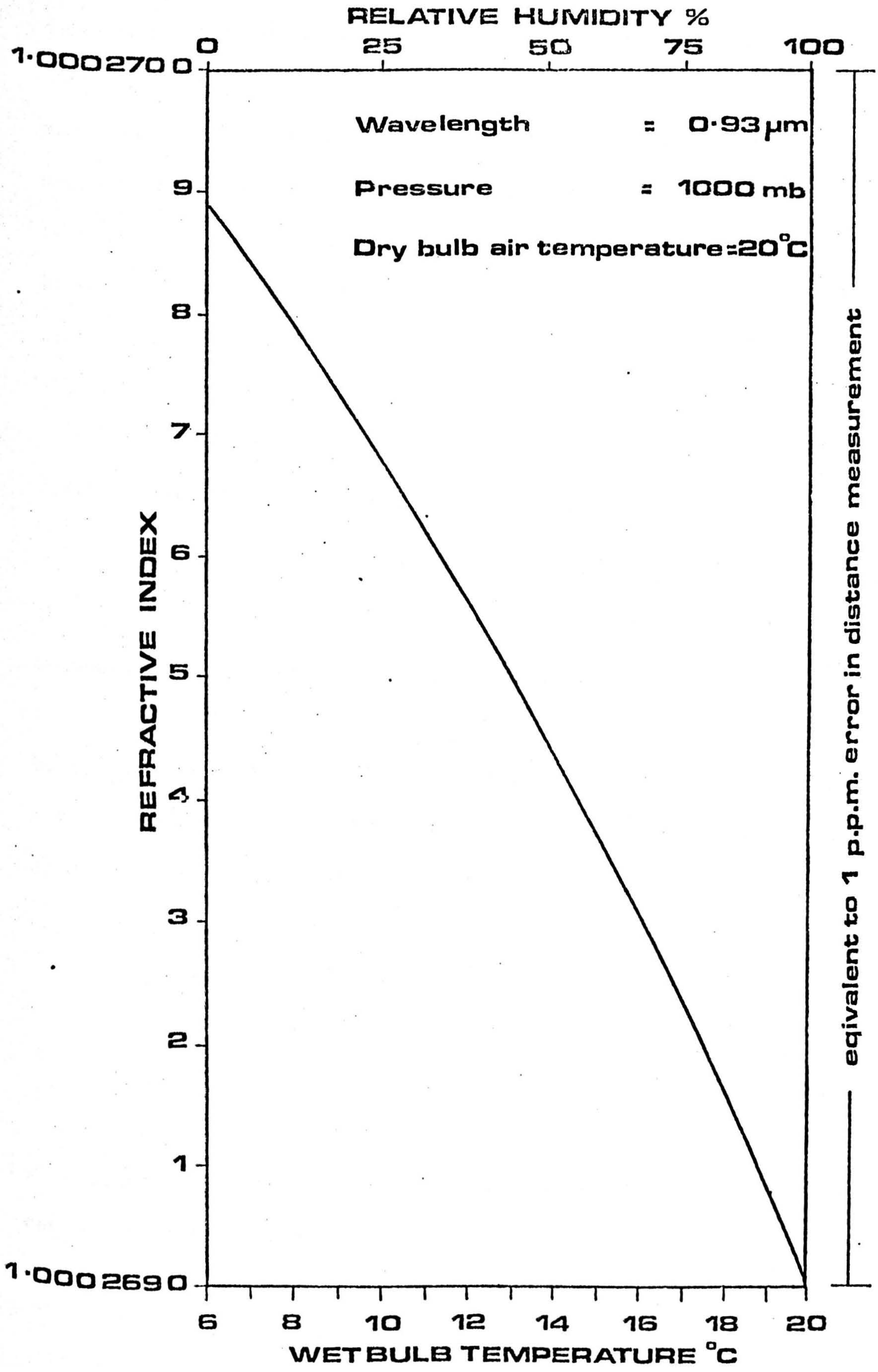


Figure 2.6. The Effect of Humidity on E.D.M. using Visible and Infra Red Waves.

E.D.M. instrument to the prisms must be obtained. This involves the determination of mean air temperature, pressure and to a lesser extent humidity, along the line being measured. Obtaining representative mean atmospheric parameters is the most difficult process in the accurate measurement of a distance by E.D.M. methods, and as such is a major source of error and uncertainty in the corrected distance. The effect of these errors is more significant for longer distance measurements. The derivation of mean atmospheric parameters is covered in detail in Chapter Four.

2.7. Atmospheric Correction of E.D.M. Measurements

Refraction has two effects on electromagnetic waves passing through the atmosphere; firstly, the wave velocity is changed and secondly, the wave path is bent. For short range E.D.M. in engineering survey the influence of refractive bending is negligible, the error caused by ray path curvature on a two kilometre line being less than 0.5 millimetres. However, refractive retardation must be accounted for in all accurate E.D.M. measurements.

Refractive index is defined in Chapter One as

$$n = \frac{c}{v} \qquad \text{from 1.2.}$$

where the velocity of light in a vacuum is a constant (c). This formula can be re-expressed as

$$nv = \text{constant} \qquad \ldots\ 2.11.$$

As the modulation frequency of an E.D.M. is constant (excepting the Mekometer) it follows from the wave equation,

$$v = f\lambda \qquad \text{from 2.6.}$$

that modulation wavelength and wave velocity are directly proportional, so that

$$n\lambda = \text{constant} \qquad \ldots\ 2.12.$$

from equation 2.11. When a distance is measured with an E.D.M. in ambient atmospheric conditions the product of the ambient refractive index (n_a) and the corresponding modulation wavelength (λ_a) will be constant, and will be equal to the product of the standard refractive index (n_s) and the wavelength (λ_s), defined by the instrument manufacturers. Therefore

$$n_a\lambda_a = n_s\lambda_s \qquad \ldots\ 2.13.$$

If the ambient refractive index is greater than the standard, then it follows that the ambient wavelength will be shorter than the standard wavelength.

$$\text{if } n_a > n_s \text{ say, then } \lambda_a < \lambda_s$$

As the true distance to be measured has a constant value a greater number of wavelengths will 'fit into' the distance in ambient rather than standard conditions. The measured distance will therefore be greater in ambient conditions,

$$D_a > D_s$$

The converse is of course true if n_a is less than n_s.

It follows from the previous theory that E.D.M. measured distances are inversely proportional to the mean wavelength in the atmosphere, along the wave path, at the time of measurement.

$$D_a \propto \frac{1}{\lambda_a} \qquad \text{... 2.14.}$$

which combined with equation 2.12. gives

$$\frac{n_a}{D_a} = \text{constant} \qquad \text{... 2.15.}$$

The distance measurement in standard conditions is a special case of a measurement in ambient conditions. Therefore

$$\frac{n_a}{D_a} = \frac{n_s}{D_s} = \text{constant} \qquad \text{... 2.16.}$$

This expression is the basis of atmospheric correction to E.D.M. measurements, the true distance being equivalent to the standard distance.

$$D_s = D_a \frac{n_s}{n_a} \qquad \text{... 2.17.}$$

or $$\text{True distance} = \text{Measured distance} \times \frac{\text{standard refractive index}}{\text{ambient refractive index}}$$

The standard refractive indices of E.D.M. instruments are either stated, or can be derived from information given in the manufacturers' specifications. For example the Tellurometer MA100 manual states, "In all MA100 instruments the assumed refractive index is 1.000 274". The Geodimeter Model 6 handbook specifies, "unit lengths at -4°C and 760 mm Hg", from which the standard refractive index can be calculated, using equation 1.11. to give 1.000 309.

2.8. Example of Atmospheric Correction

The following example illustrates the computational procedure to be followed when correcting an E.D.M. measurement, it combines the theory of Chapters One and Two.

A line of approximately two kilometres was measured with a Geodimeter Model 6. The following data was collected during the measurement.

(a) Data

Mean instrument reading	=	2011.962 6 m	
Mean ambient dry bulb temperature	=	5.05 $^{\circ}$C	(T)
Mean ambient wet bulb temperature	=	4.15 $^{\circ}$C	(T_w)
Mean ambient atmospheric pressure	=	1002.8 mb	(P)

Modulation wavelength of Model 6	=	0.55 μm	(λ)
Additive constant	=	-0.127 m	(A)
Standard refractive index	=	1.000 309 2	(n_s)

(b) Group refractive index (n_g) is calculated using equation 1.10., substituting the modulation wavelength as 0.55 μm, to give,

$$n_g = 1.000\ 304\ 5$$

(c) Saturated water vapour pressure (e_s) is obtained from equation 1.13., remembering that the temperatures in this formula are in degrees Kelvin, to give,

$$e_s = 8.23 \text{ mb}$$

(d) Partial water vapour pressure (e) is calculated from equation 1.12., to give,

$$e = 7.63 \text{ mb}$$

(e) Ambient refractive index (n_a) is calculated from equation 1.11., to give,

$$n_a = 1.000\ 295\ 6$$

It should be noticed that the contribution of the humidity term is only -3×10^{-7}, which will affect the corrected distance by 0.3 ppm.

(f) Additive constant (A) must be applied to the measured distance to give the ambient distance measurement (D_a),

$$D_a = 2011.9626 - 0.127 = 2011.835\,6 \text{ m}$$

(g) Atmospheric correction is made to the ambient distance measurement using equation 2.17. A standard refractive index of 1.000 309 2 is used. The corrected distance is calculated as,

$$D_s = 2011.862\,6$$

which implies an atmospheric correction of + 27 mm, or + 13.4 ppm.

This result, calculated from first principles, can be compared with the corrections obtained from the nomographs and correction formula in the Geodimeter manufacturer's handbook [Anon, 1965]. The corrections obtained for the example are compared below.

Source of Correction	Atmospheric Correction
1. First Principles	+ 13.4 ppm
2. Manufacturer's Nomograph	+ 14.0 ppm
3. Manufacturer's Formula	+ 13.3 ppm

The correction calculated from first principles is the most accurate. The small disagreement between the manufacturer's formula and first principles is due to the manufacturer's assuming an average humidity. Although this difference is small it is recommended that calculated atmospheric corrections are worked from first principles. With computational aids the corrections can be rapidly calculated and humidity corrections can be considered with little extra expenditure of time.

Very great care was taken in obtaining a correction from the nomograph and a correction in error by + 0.6 ppm was obtained, the correction was + 0.7 ppm different from the value calculated from the manufacturers formula. It is suggested that nomographs should only be used to check calculated corrections for accurate E.D.M. distance measurements.

There is a trend in modern E.D.M. design to incorporate a refractive index control on the instrument panel. The atmospheric correction is determined from a nomograph and dialled into the instrument, the correction being applied automatically. For short range measurements this system is very useful, and any small error in the correction set on the instrument will not have a significant effect on the distance obtained. However, if such an instrument is used for accurate measurement of longer distances greater care should be exercised when applying the correction. It is recommended that the setting of the refractive index control during measurement is recorded as well as the atmospheric parameters. In this way the correction can be checked after the measurement, and any error rectified. Two way radios are essential for E.D.M. instruments with refractive index controls. The radio telephones enable atmospheric parameters measured at the reflector station to be transmitted to the E.D.M. operator, who by meaning the temperatures and pressures at each end of the line is able to set a suitable correction on the refractive index control. This procedure would not be necessary for the less accurate measurement of short distances over level ground, however, it is of great importance for accurate work in which longer distances are measured over undulating terrain.

2.9. The Two Wavelength Technique

The two wavelength technique of electromagnetic distance measurement makes it possible to measure distance very accurately without any measurement of atmospheric parameters. Instruments of this design have been built in Britain [Bradsell and Shipley, 1974] and the United States [Earnshaw and Hernandez, 1972] but at present have limited applications to engineering survey measurements. Such instruments adopt standard conditions in a vacuum with a unity value of standard refractive index.

The formula for atmospheric correction to E.D.M. can be rewritten for a two wavelength instrument as

$$D_s = \frac{1}{n_a} \cdot D_a \qquad \ldots\ 2.18.$$

from equation 2.17. where the standard refractive index, n_s, is equal to unity. The two wavelength method involves the simultaneous measurement of a distance with two electromagnetic waves of slightly different wavelength. As a result of dispersion these two waves will have different refractive indices for the same ambient atmospheric conditions. If D_1 and D_2 are the two distances recorded with the two waves which have ambient refractive indices of n_1 and n_2, then from equation 2.18.,

$$D_s = \frac{D_1}{n_1} = \frac{D_2}{n_2}$$

or $$D_1 = D_s\, n_1 \qquad \ldots\ 2.19.$$

and $$D_2 = D_s\, n_2 \qquad \ldots\ 2.20.$$

Which combine to give

$$D_1 - D_2 = D_s\,(n_1 - n_2) \qquad \ldots\ 2.21.$$

The Barrell and Sears formulae, equation 1.11., can be applied to give the ambient refractive index of each wave. The group refractive indices of each wave will be different, they are designated as n_{g1} and n_{g2}. Neglecting humidity we obtain,

$$n_1 - 1 = \frac{n_{g1} - 1}{\alpha\ T} \cdot \frac{P}{1013.25} \qquad \ldots\ 2.22.$$

and $$n_2 - 1 = \frac{n_{g2} - 1}{\alpha\ T} \cdot \frac{P}{1013.25} \qquad \ldots\ 2.23.$$

which combine to give the expression

$$n_1 - n_2 = \frac{(n_{g1} - n_{g2})}{\alpha\ T} \cdot \frac{P}{1013.25} \qquad \ldots\ 2.24.$$

This value of $(n_1 - n_2)$ can be substituted into equation 2.21. to give

$$D_1 - D_2 = D_s \frac{(n_{g1} - n_{g2})}{\alpha \; T} \cdot \frac{P}{1013.25} \qquad \ldots 2.25.$$

Now equation 2.22. can be re-arranged to

$$\frac{P}{1013.25 \; \alpha \; T} = \frac{n_1 - 1}{n_{g1} - 1} \qquad \ldots 2.26.$$

which can be substituted into equation 2.25. to give the expression

$$D_1 - D_2 = D_s \; (n_{g1} - n_{g2}) \; \frac{(n_1 - 1)}{(n_{g1} - 1)} \qquad \ldots 2.27.$$

Now, if we let $\beta = (n_{g1} - n_{g2})/(n_{g1} - 1)$, which is a function dependent only on the two wavelengths being used. We can write

$$D_1 - D_2 = D_s \; \beta \; (n_1 - 1) \qquad \ldots 2.28.$$

and substituting for D_s from equation 2.19. gives

$$D_1 - D_2 = \frac{D_1}{n_1} \cdot \beta \; (n_1 - 1) \qquad \ldots 2.29.$$

which re-arranged gives

$$n_1 = \frac{D_1 \; \beta}{D_1 \; \beta - (D_1 - D_2)} \qquad \ldots 2.30.$$

Substituting this value of refractive index into equation 2.19. we obtain an expression for the corrected, or true, distance D_s which is independent of the ambient refractive index of the air in which the measurement has been made

$$D_s = \frac{D_1 \beta - (D_1 - D_2)}{\beta} \qquad \ldots 2.31.$$

It should be emphasised that the two distance measurements D_1 and D_2 are the only readings that need to be taken, no meteorological observations are necessary.

The accuracy of the two wavelength technique depends on the accuracy of the measurement of the difference between the two distance readings D_1 and D_2. Considering equation 2.28.

$$D_1 - D_2 = D_s \beta (n_1 - 1)$$

The sensitivity of $(D_1 - D_2)$ to changes in refractive index, n_1, can be obtained by differentiation.

$$\delta (D_1 - D_2) = D_s \beta \delta n_1$$

If, for example, two wavelengths of 0.4047 μm and 0.5791 μm are used in the two wavelength technique, the constant β can be calculated, using values of group refractive index calculated from equation 1.15., as

$$\beta = 0.0544$$

If distance measurements are required to an accuracy of one ppm then the ambient refractive index must be known to the same degree of accuracy. To meet this requirement the resolution of the difference in measurements can be expressed as

$$\delta\ (D_1 - D_2) \quad = \quad 0.0544\ D_s \times 10^{-6}$$

A plot of this function is shown in Figure 2.7. For example, a 3000 m distance measurement accurate to one ppm would require a difference measurement better than 0.162 mm. With present phase angle measurement techniques this degree of resolution is not possible and consequently the two wavelength method is not of assistance in distance measurements of less than ten kilometres. For long range distance measurements where less accurate difference measurements are necessary the two wavelength E.D.M. is a very valuable instrument.

With improvements in phase angle measurement in short range E.D.M. it is highly probable that the two wavelength technique will improve the possible accuracy of engineering survey measurements in the future.

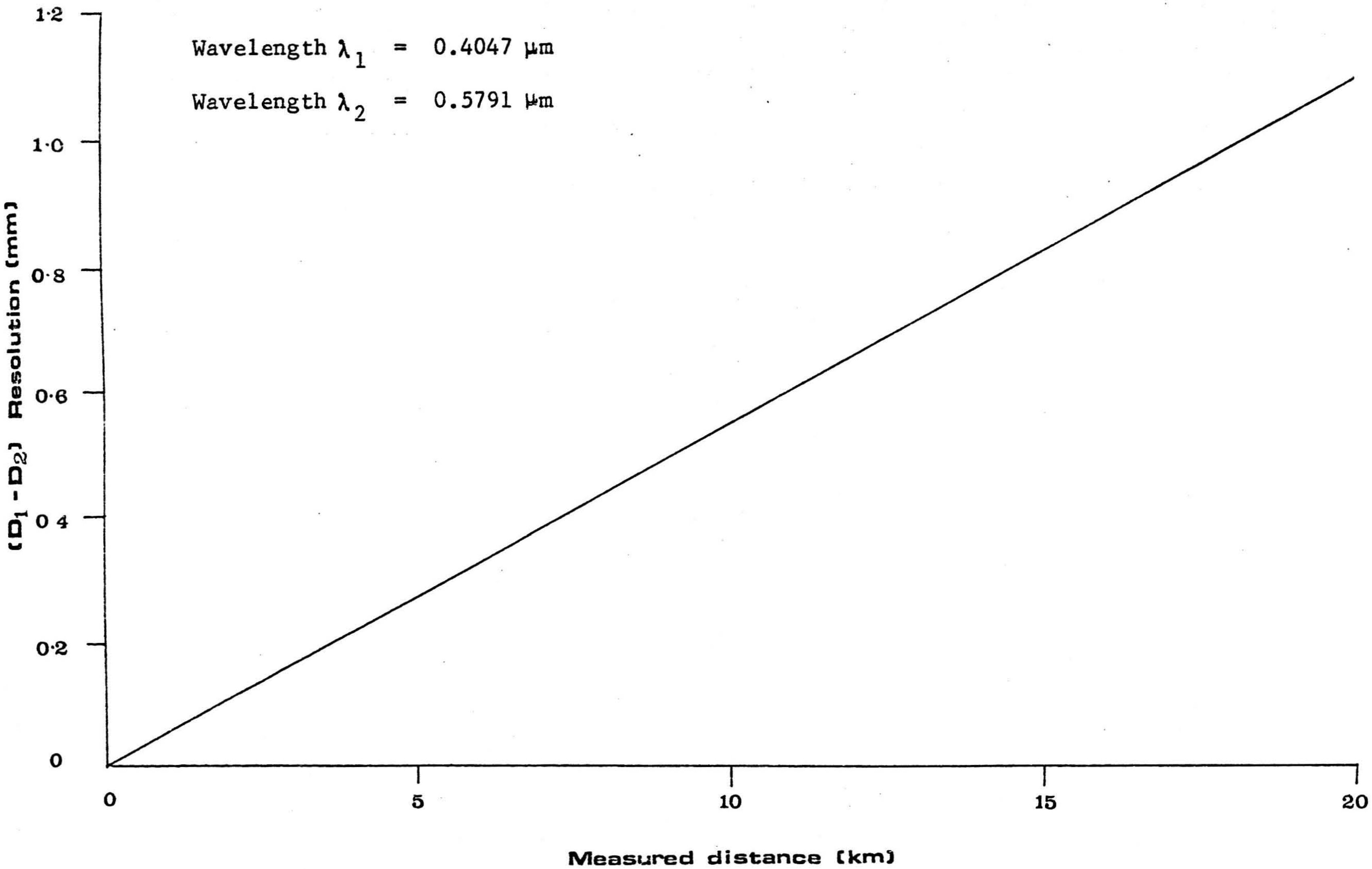


Figure 2.7. Two Wavelength Technique, Required Resolution for 1 ppm

CHAPTER THREE

EQUIPMENT FOR THE MEASUREMENT OF ATMOSPHERIC PARAMETERS

CHAPTER THREE

EQUIPMENT FOR THE MEASUREMENT OF ATMOSPHERIC PARAMETERS

3.1. Introduction

The accurate measurement of air temperature, pressure and humidity is essential if accurate refractive index calculations are to be applied to E.D.M. measurements. This Chapter describes the meteorological equipment used in experimental work, designed to obtain representative mean atmospheric parameters along the lines of sight of survey measurements.

Calibration of all meteorological instruments is of great importance in such work if accurate values of atmospheric parameters are to be obtained. Calibration is normally carried out in the laboratory, however, it is essential that the calibration of each instrument should be checked regularly whilst in use in the field. Calibration is of particular importance for temperature measuring equipment where different instruments and measuring techniques are used, in various situations, to give results which must be directly comparable.

3.2. Barometers

Three aneroid barometers were used in the experimental work, they are specified in the following table.

Barometer	Unit of Reading	Correction to Reading (mb)
A. Negretti and Zambra No. 10564	inches Hg	+4.9
B. Thommen No. 108526	mb	-11.3
C. Thommen No. 82614	mb	-1.2

Figure 3.1. The results of barometer calibration.

Barometer A was fitted with a vernier which enabled the pressure to be read to one thousandth of an inch. Barometers B and C were direct reading, with the aid of a lens, to 0.2 millibars with estimation to 0.1 millibars. The barometers are shown in Plate 3.1.

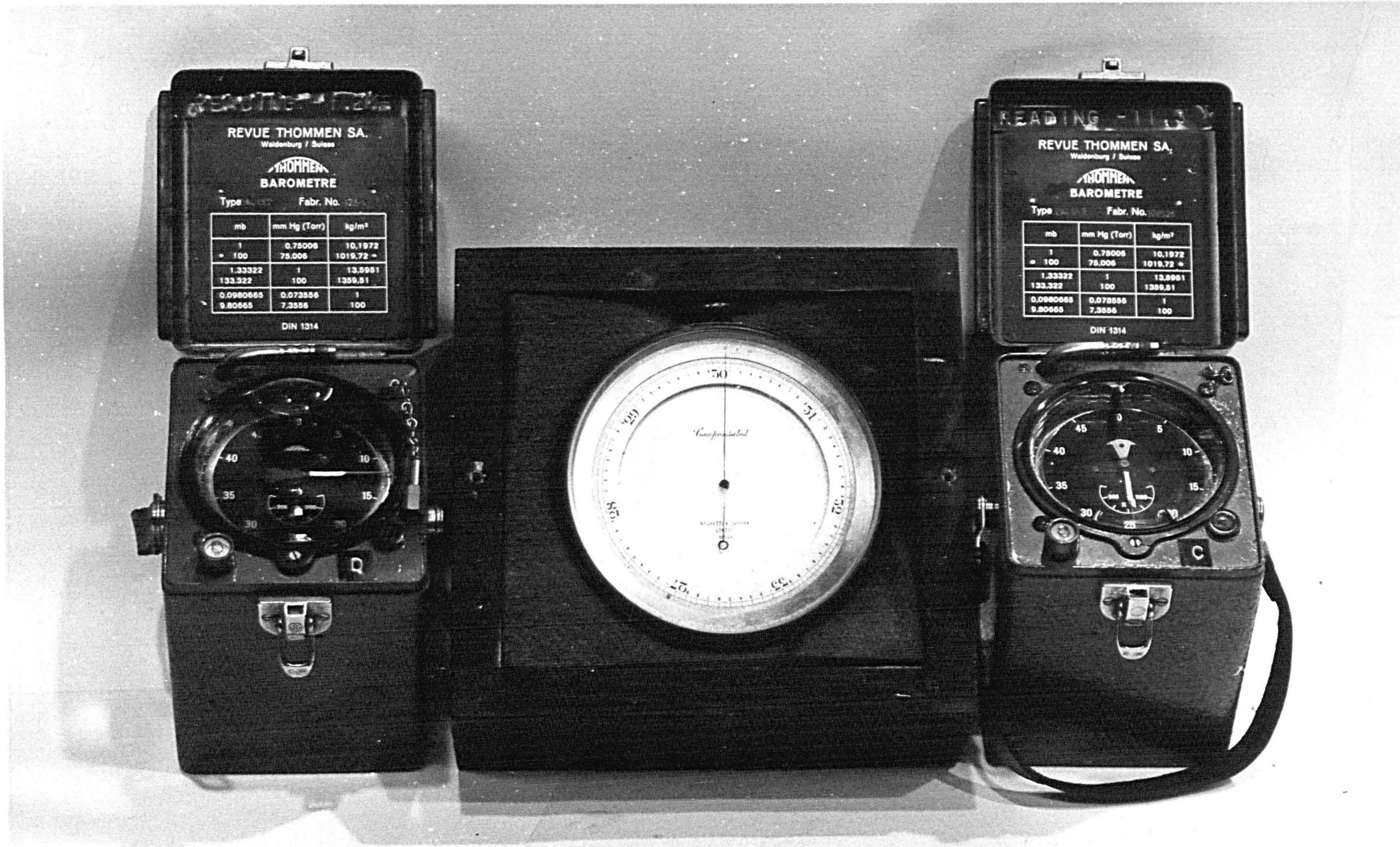
All the barometers used in the experimental work were calibrated against a laboratory standard Fortin barometer (No. M.7002), each barometer being compared in turn with the standard. During a three week period it was possible to calibrate the barometers over a range from 995 mb to 1030 mb using the variation of atmospheric pressure, due to changes in the weather.

Absolute air pressure is required for the calculation of refractive index in the correction of E.D.M. measurements, consequently it is necessary to apply the following corrections to the Fortin barometer readings during calibration.

(a) A temperature correction for the linear expansion of the barometer scale and the cubical expansion of the mercury.

PLATE 3.1.

The barometers used for experimental work.


REVUE THOMMEN SA.
Waldenburg / Suisse
THOMMEN
BAROMETRE
Type
Fabr. No.
mb
mm Hg (Torr)
kg/m²
DIN 1314
Compensated
REVUE THOMMEN SA.
Waldenburg / Suisse
THOMMEN
BAROMETRE
Type
Fabr. No.
DIN 1314
C
D

The Fortin barometer used was standard at $0^{o}C$, giving a typical value of this correction as -2.7 mb.

(b) A standard gravity correction for the change in gravity with latitude. Standard gravity is equal to 9.806 65 ms^{-2}. The correction to barometer readings in Nottingham is +0.7 mb.

(c) An altitude correction for the height of the barometer above sea level. The standard Fortin barometer used in Nottingham is approximately 30 m above sea level and in this situation the correction is negligibly small.

The values of these corrections are given in tables published by the Meteorological Office [1956]. A typical value of the total correction for the Fortin barometer, during the period of the calibration procedure, was -2 mb.

The readings of the three aneroid barometers were compared with the corrected Fortin pressure and the errors noted. The errors were found to be consistent over the pressure range, thus enabling a constant correction to be applied to all readings. The correction for each barometer is given in Figure 3.1. These corrections were applied to all subsequent pressure readings made with the aneroid barometers in the field. When the barometers were in regular use they were periodically brought together and their corrected readings compared. In this way it would have been immediately apparent if one of them had gone out of adjustment. When using the barometers care was taken to level the instruments and to allow sufficient time for the readings to stabilise before the measurements were recorded. All the aneroid barometers used were temperature compensated.

3.3. Aspirated Hygrometers

An Assman clockwork aspirated hygrometer was used as the standard for wet and dry bulb temperature measurements (Plate 3.2.). To avoid spurious readings the wet bulb muslin wick was regularly replaced; the wick was moistened with distilled water before each measurement. The hygrometer was held at arms length by the operator during temperature measurements, to avoid errors in air temperature measurement caused by heat from the body. Although the outer surfaces of the instrument are highly polished, to minimise radiant heating of the thermometers, it was also thought necessary to take air temperature measurements in the shade wherever possible.

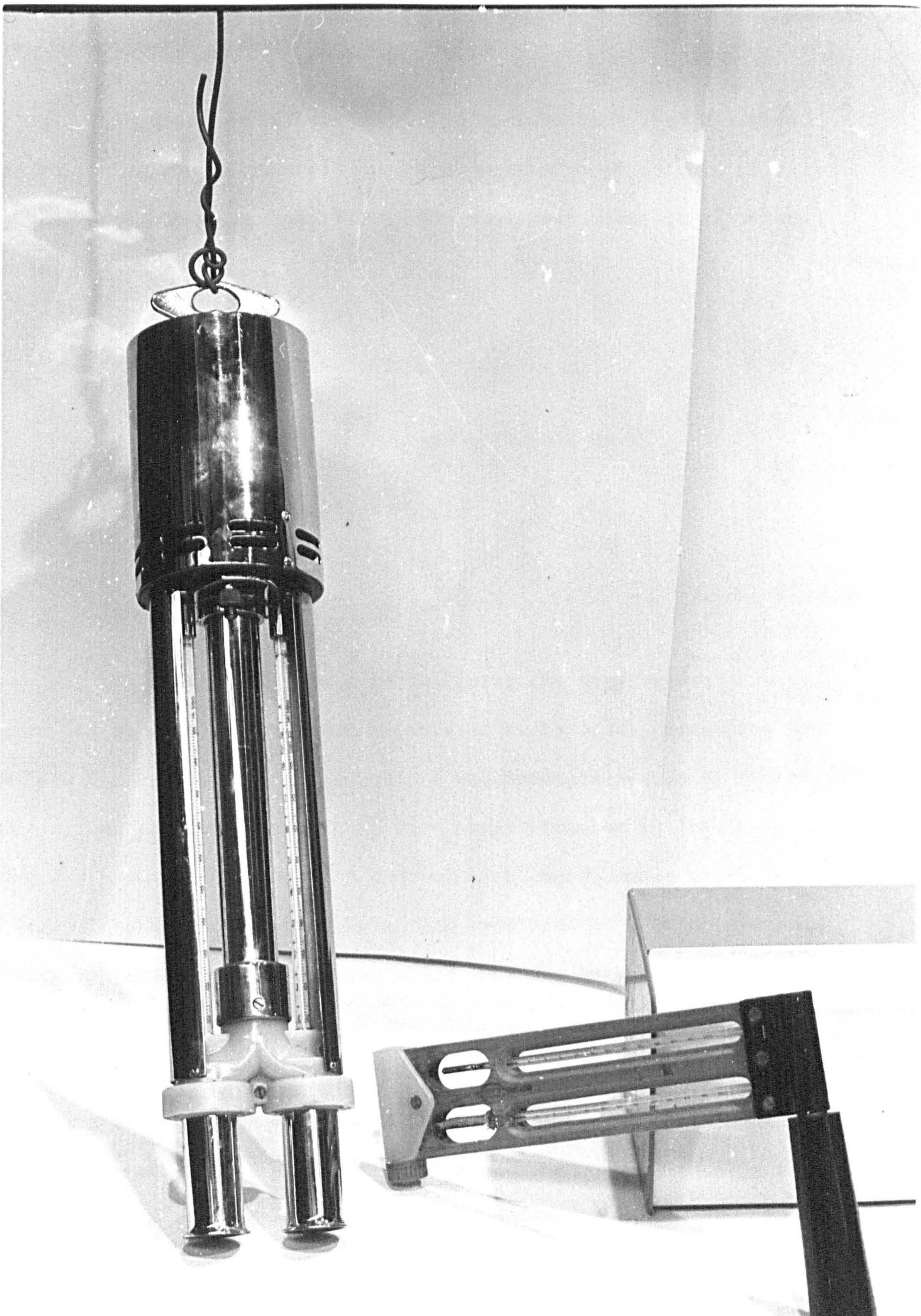
All the temperature measuring instruments were calibrated to agree with the Assman hygrometer. During experimental work the Assman was taken to the site of each separate temperature measuring instrument in turn, to ensure that all the recorded values were compatible.

Storrow whirling hygrometers were used for less accurate measurements of wet and dry bulb temperatures (Plate 3.2.). As these thermometers had no shielding against radiant heating, measurements where taken in the shade wherever possible.

PLATE 3.2.

The Assman aspirated hygrometer (left) and

a Storrow whirling hygrometer (right).



3.4. Elevated Temperature Recorders

Elevated temperature recording systems have been designed and built for the measurement of air temperatures at heights of up to ten metres above ground level. The equipment consists of three parts,

(a) a temperature measuring unit,

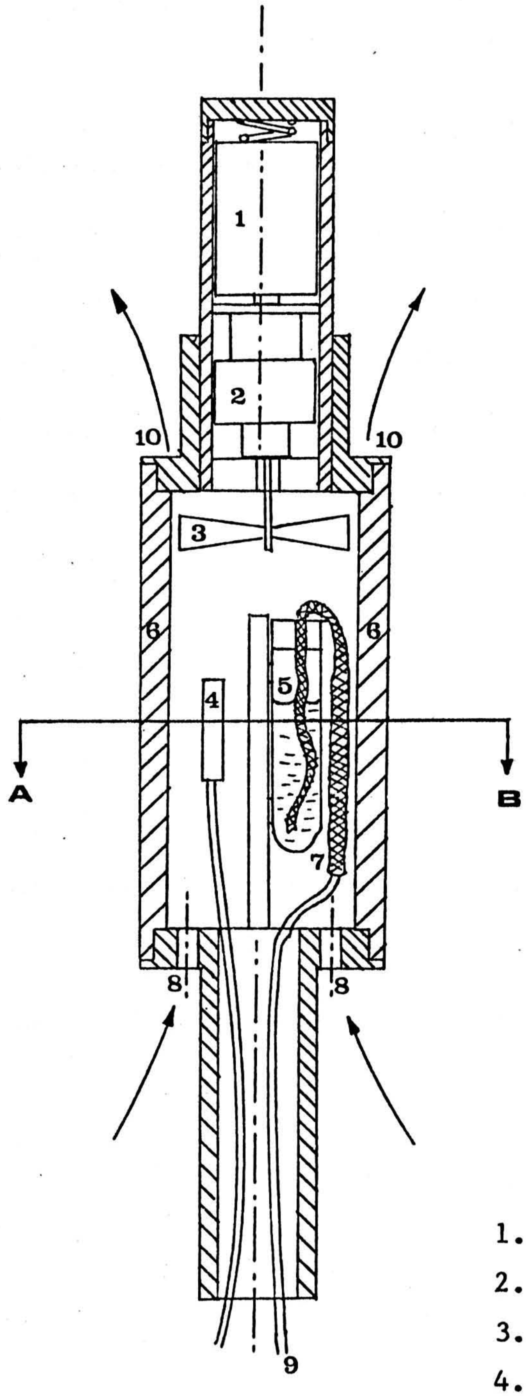
(b) a temperature reading and recording unit,

(c) a ten metre mast.

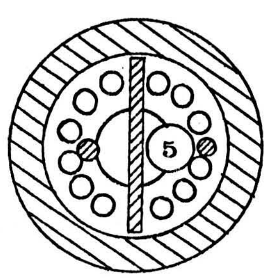
(a) Temperature measuring units

The initial objective in designing the temperature measuring units was to develop apparatus capable of continuously measuring both wet and dry bulb air temperatures. A thermocouple system of temperature sensing was adopted. Bundles of four copper-constantin junctions were used for both the hot and cold ends of each thermocouple assembly. A vacuum flask, containing melting ice, was used as a cold reference junction. The hot junctions of the wet and dry bulb thermocouples were mounted in a perspex housing (Figure 3.2.), air was drawn over the thermocouples by a small, battery operated, fan. Eleven metres of wire were used between each junction to enable ground level measurement of the thermo-electric potential generated by the thermocouples whilst mounted on the ten metre masts.

The wet bulb hot junctions were covered by a muslin wick, which was moistened from a small distilled water reservoir within the



Section A-B



Key

1. Battery
2. Motor
3. Fan
4. Dry thermocouple
5. Water reservoir
6. Perspex housing
7. Wick over wet thermocouple
8. Air inlets
9. Leads to recorder
10. Air outlet

Figure 3.2. Section of a thermocouple temperature measuring unit (half scale).

thermocouple housing. Problems were encountered in obtaining correct wet bulb temperature readings. These were due to the conduction of heat along the thermocouple wires from regions at dry bulb temperature to the thermocouple junctions at wet bulb temperature. To minimise this source of error the muslin wick was extended 70 mm down the wet bulb leads from the thermocouple junctions. However, although this remedy was successful, it had the effect of syphoning water from the reservoir and thus reducing the operating life of the wet bulb thermocouple to less than three hours. A further problem was the contamination of the dry bulb temperature readings by the water present in the thermocouple assembly. Owing to the great importance of accurate dry bulb temperature measurements in the atmospheric correction of light and infra red wave E.D.M. instruments, and the relative unimportance of humidity, the wet bulb thermocouples were abandoned.

The thermocouple assemblies were calibrated against the Assman aspirated hygrometer. The operational life of each unit was restricted by the fan battery life, which on average was eight hours. Greater operation time was obtained by connecting additional batteries in parallel. Each thermocouple housing was lined with reflective metal foil to reduce errors in the thermocouple temperature measurements caused by radiant heating. Three temperature measuring units were manufactured for use in the Llangollen experimental work (Plate 3.3.).

(b) <u>Temperature reading and recording equipment</u>

In the first experiments the potential difference between the hot and cold thermocouple junctions was measured with Cambridge portable potentiometers (Plate 3.4a.). A change in the air temperature

PLATE 3.3.

An elevated temperature recording system and a dismantled temperature measuring unit.


THERMO 1
RECORDER 1
GRANT
C
BAT 1
EVER READY
HIGH POWER
HP2

at the hot junction of approximately six degrees Celsius generated an increase in potential of one millivolt. The potentiometers could be read directly to one microvolt, which gave more than adequate temperature reading accuracy.

The thermocouples were calibrated using the portable potentiometers and a hot water bath, as described by Biswas [1959] 63. Each potentiometer was labelled and always used with the same thermocouples. The potentiometers functioned well in conjunction with the temperature measuring units, however, they required a person to operate them at all times. To overcome this disadvantage an automatic temperature recording system was required.

Two Grant minature temperature recorders were purchased for the automatic and continuous recording of the temperatures measured by the thermocouples. Each recorder was built to specifications designed to match those of the thermocouple assemblies described in the previous section. The recorders have a range of zero to 25°C, the temperature is recorded continuously on a roll of graph paper. Each recorder is powered by a 7.5 volt rechargeable battery which enables continuous running for twenty four hours. The recorders are shown in Plate 3.4b. with a specially constructed transport box, which was used to protect the instrument during outdoor use.

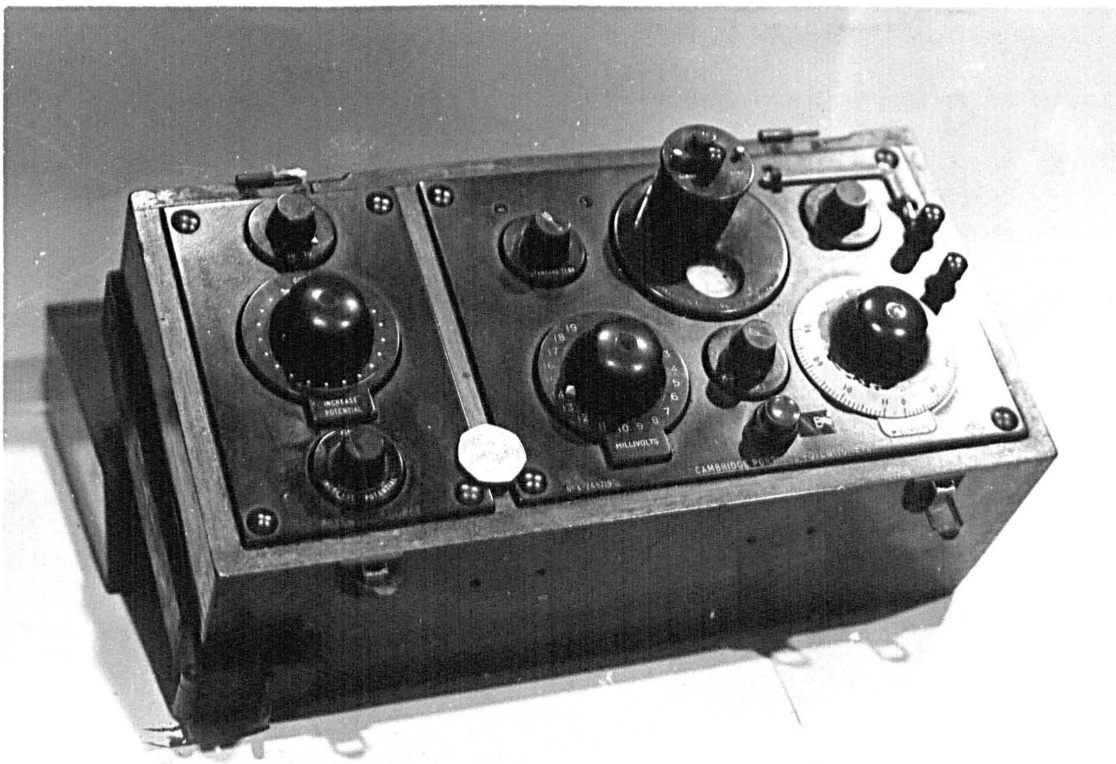
(c) <u>Ten metre masts</u>

Three masts were made to support the temperature measuring units at heights of up to ten metres above ground level at survey

PLATE 3.4.

(a) A Cambridge portable potentiometer.

(b) The Grant temperature recorders and a transport box.


INCREASE POTENTIAL
MILLIVOLTS


PLEASE DO NOT TOUCH
0 5 10 15 20 25
C
RECORDER 2
GRANT
BAT-1
0 5 10 15 20 25
C
RECORDER 1
GRANT

stations. The first masts built in 1975, were made from ten one metre sections of 25 mm diameter steel tube. Five men were required to erect each mast. Once vertical the masts were supported by three wire guy lines from the top and three from half height. A temperature measuring unit was mounted rigidly on top of each mast, and by altering the number of steel sections the thermocouple assembly could be set at the required level. Plate 3.5a. shows a steel mast erected on the golf course in Llangollen in 1975. This type of mast was difficult to transport and erect and was unstable in light winds; consequently a second type of mast was designed and two were built and used in 1976.

The second mast design utilised two five metre plastic drain pipes which were slotted together to form the upright of the mast. Three light metal rods, one metre in length, were fitted radially to the mast at half height (Plate 3.5b.). Three wires were then rigged from the top of the mast, each passing through the end of one of the radial rods, to the base of the mast. This rigging considerably reduced the flexibility of the mast and increased the resistance to wind forces. The masts stood on wooden bases which were pegged to the ground. Three nylon guy lines were used to hold the mast vertical. The temperature measuring units were hoisted up the mast on a nylon line, thus enabling variation in the heights at which temperatures could be measured. The new design of mast could be erected with ease by only two men.

On several occasions it was necessary to continuously measure and record temperature at survey instrument height, to enable this the temperature measuring units were mounted on survey tripods using a

PLATE 3.5.

(a) A 10 m steel mast on the Llangollen golf course, 1975.

(b) A 10 m plastic mast at the Sheepfold station, 1976.

specially designed adaptor and a retort clamp as shown in Plate 3.6. This arrangement could be left functioning, without an operator, for the duration of an experiment.

3.5. Radiosonde Equipment and Administration

3.5.1. Radiosonde Equipment

A tethered radiosonde was used, in experimental work in Llangollen, to measure air temperatures and pressures at heights up to 350 m above ground level. The radiosonde (Plate 3.7a.) was lifted by a hydrogen filled balloon which was tethered to the ground; the height of the balloon above the ground was controlled by a hand operated winch.

The radiosonde contained three meteorological measuring systems.

(a) An aneroid barometer for pressure measurement.

(b) A bi-metallic thermometer for dry bulb air temperature measurement.

(c) A hair hygrometer for humidity measurement.

The hygrometer was completely neglected in experimental work, there being no need for elevated humidity measurements in the correction of light wave E.D.M. The humidity correction to E.D.M. can be adequately

PLATE 3.6.

Instrument height temperature recording at the Sheepfold station, showing calibration of the temperature measuring unit with the Assman hygrometer.

derived from hygrometer readings taken at either end of the line of sight. Had humidity measurements been necessary the hygrometer would have required soaking over a water bath before each sonde flight, this would have added a further complication to the sonding procedure.

Each measuring system was mechanically linked to an electrical contact, these rested in turn on a rotating code cylinder inside the radiosonde. Changes in temperature, pressure and humidity caused the contacts to move across the outer surface of the cylinder and in so doing connect with different coded sections of the surface. Each section of the coded cylinder caused a different signal to pass to a small on-board radio transmitter, which broadcasted the coded signals to ground level. The information was transmitted in sequence, humidity first, then temperature and then pressure.

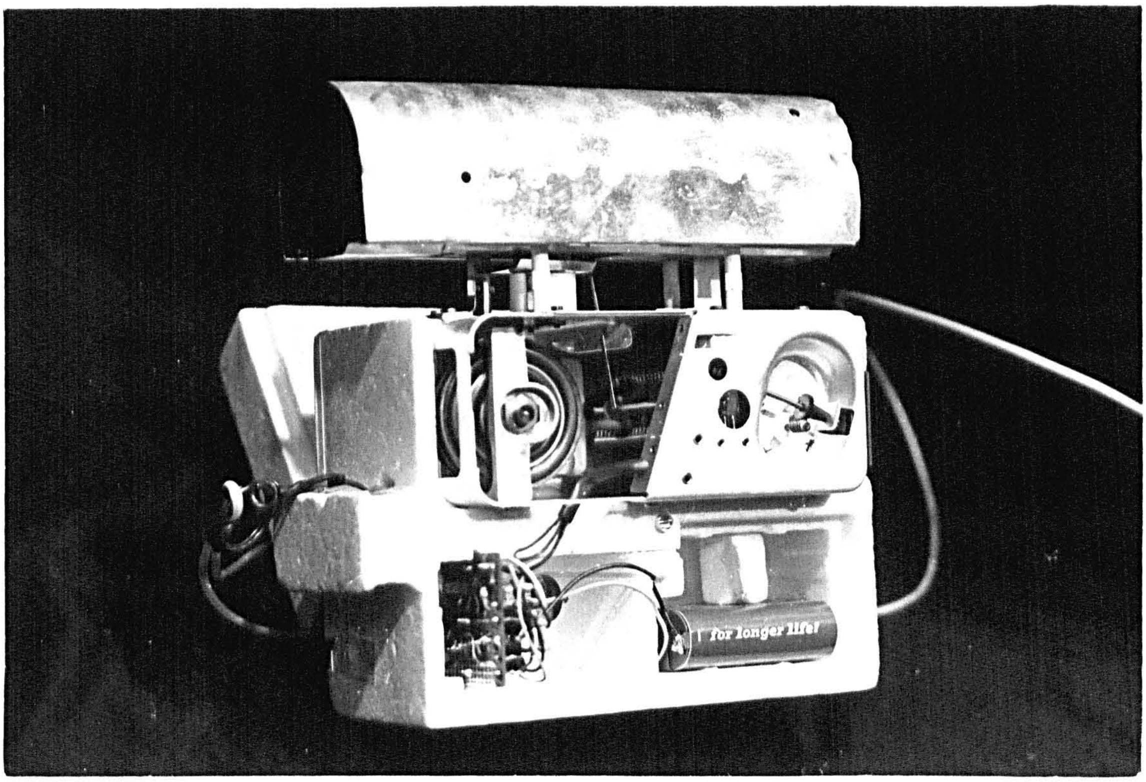
The broadcast from the radiosonde, which was in morse code, was received at ground level by an Eddystone communications receiver, model 940 (Plate 3.7b.). The received signals were recorded on magnetic tape at high speed. The recordings were later played back at low speed to facilitate the translation of the morse code into values of temperature and pressure. Both the receiver and recorder could be powered from either the mains or a small 600 W portable petrol driven generator.

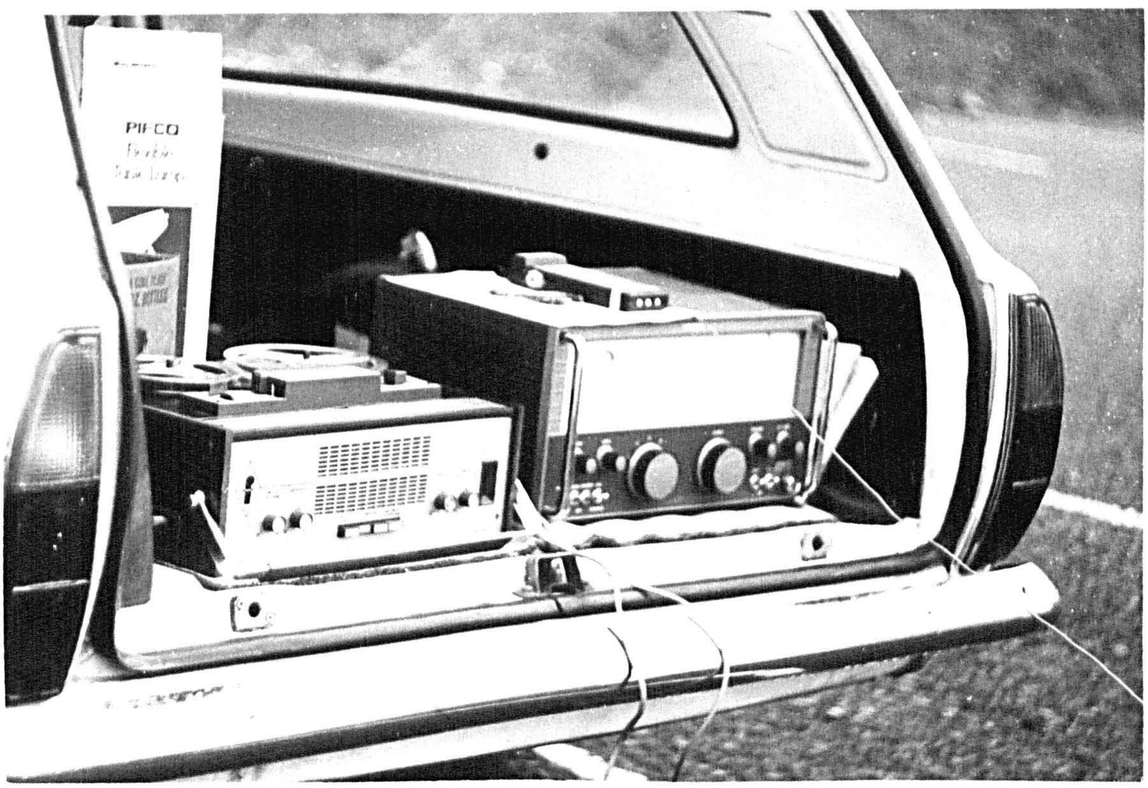
The following example is used to illustrate the translation procedure. The example considers one complete transmission of data, this cycle is repeated every ten seconds while the sonde is operating. Figure 3.3. shows the format of one batch of meteorological information from the radiosonde. There is a long blank section on the recording to

PLATE 3.7.

(a) The radiosonde.

(b) The receiving and recording equipment.


for longer life!


PIFCO

TIME

RECORDED MORSE MESSAGE	ATMOSPHERIC PARAMETER	DECODED MESSAGE	PARAMETER VALUE
BLANK			
BLANK			
BLANK			
--			
BLANK	HUMIDITY	-	-
..-			
BLANK			
BLANK			
....		7	
BLANK	TEMPERATURE	72	10.0^{o}C
...		2	
BLANK			
BLANK			
.-.		8	
BLANK	PRESSURE	85	1002.0 mb
--		5	
BLANK			
BLANK			
BLANK			
etc.			

Figure 3.3. Example translation of recorded radiosonde information.

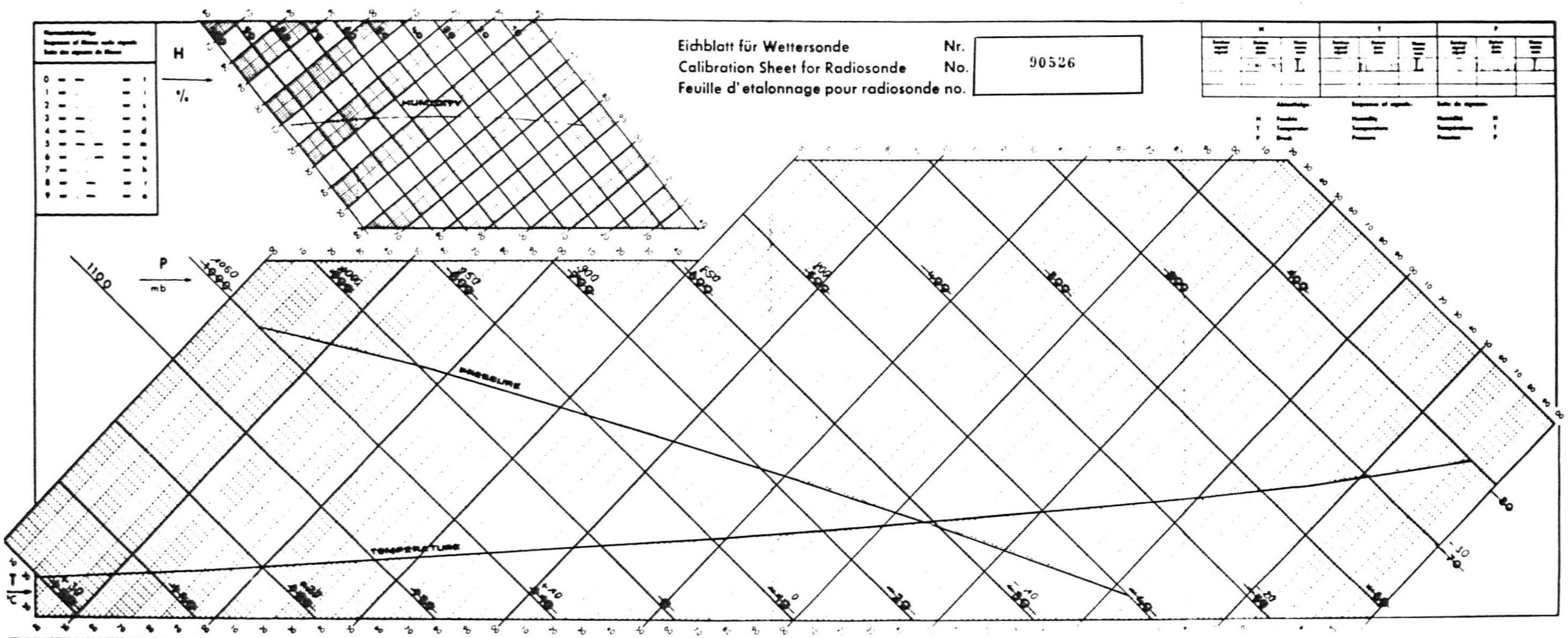
signify the start of the information cycle. The atmospheric parameters then follow in turn, each defined by two digits in morse code. The two digit figures are applied to the calibration curves, which are supplied by the manufacturer, to give the values of temperature and pressure. The calibration curves are shown in Plate 3.8. Because the radiosonde is only capable of transmitting information in discrete digital form the measuring accuracy of the meteorological information is ± 1 morse code unit. This corresponds to an accuracy of ± 0.07°C for temperature and ± 0.35 mb for pressure.

The radiosonde was lifted by a 370 gramme rubber weather balloon, which was filled with hydrogen until capable of lifting a 1.5 kg weight. The radiosonde had a mass of 0.5 kg thus giving the balloon a resultant net lift of one Newton. The balloon was tethered to the ground by two independent woven dacron lines each having a breaking load of 13.5 N and each individually capable of holding the balloon. Two lines were used as a safety precaution in the event of a line failure. The length of the line released, which determined the height of the balloon, was controlled by a hand winch, with a drum diameter of 100 mm and a gear ratio of 2.2 to 1. The winch was mounted on a survey tripod which formed a very stable base at a comfortable operating height.

The configuration of radiosonde equipment is shown in Figure 3.4. and plate 3.9. A nylon parachute was attached between the balloon and the radiosonde to protect the sonde, and third parties, in the event of the balloon bursting. There were two possible circumstances in which the radiosonde could have dropped supported by the parachute. The first

PLATE 3.8.

The radiosonde calibration curves.


Eichblatt für Wettersonde Nr.
Calibration Sheet for Radiosonde No.
Feuille d'etalonnage pour radiosonde no.
90526
H
%
P
mb
1100
PRESSURE
TEMPERATURE
HUMIDITY
T
°C

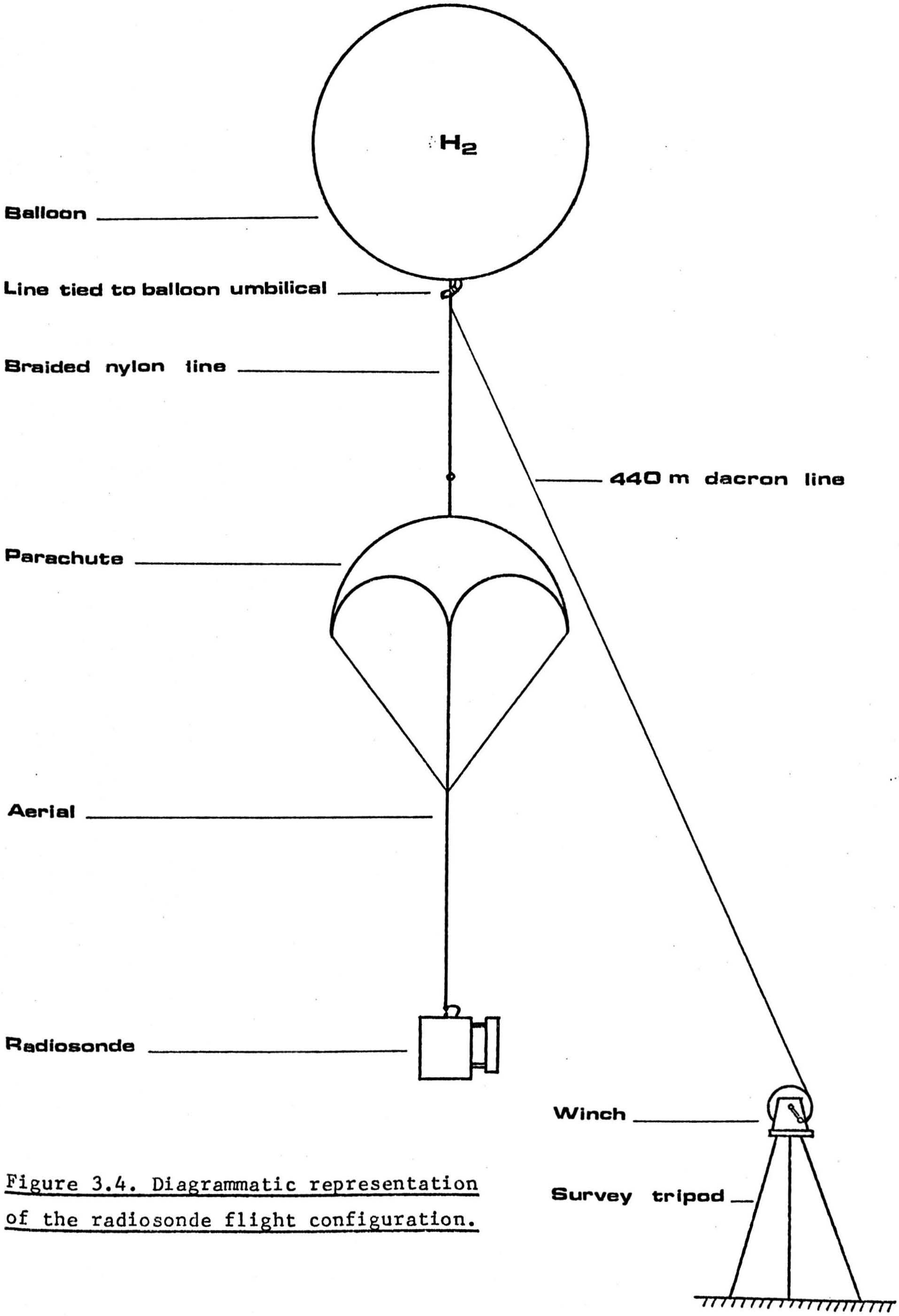
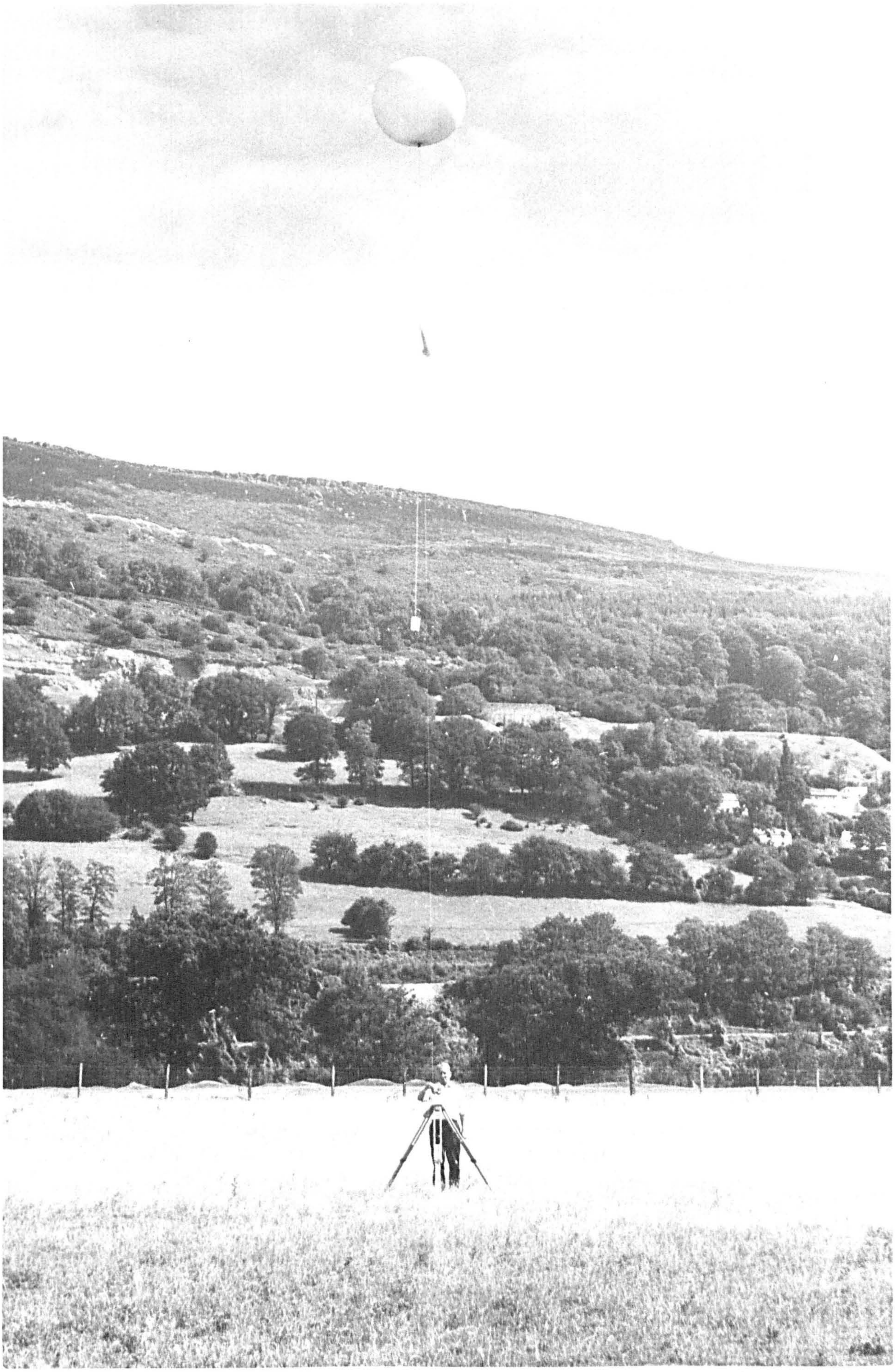


Figure 3.4. Diagrammatic representation of the radiosonde flight configuration.

PLATE 3.9.

The radiosonde flight configuration.



would have involved a rupture of the balloon at low level, possibly caused by collision with birds. Secondly, had the tethering lines failed the balloon and sonde would have ascended rapidly, the balloon expanding until the rubber ruptured under excessive strain. From results quoted by Herz and Tennet [1953] the type of balloon used in the experimental work had an altitude ceiling of 18,000 m. In the first case the sonde will always be recovered, however, in the second situation the balloon may drift large distances making recovery less certain.

Wind is an important factor affecting the sonding operations, it governs the strength of the balloon tethering lines and the maximum height to which the sonde can ascend. Calculations were made to assess wind effects on the balloon and they are detailed in Appendix B.

3.5.2. Radiosonde Administration

The flying of tethered radiosonde balloons is strictly controlled by law, consequently a considerable amount of administration was necessary to obtain permission to legally fly the radiosonde equipment. Civil Aviation Authority permission to fly the balloon at heights greater than sixty metres above ground level had to be obtained before the flights could commence. Permits to make flights in the Vale of Llangollen were obtained for two periods in 1976, March 24th to April 13th and July 1st to December 31st. A specimen permit is shown in Appendix D.

Under the Wireless Control Act of 1949 it was necessary to licence the radio transmitter housed in the radiosonde. The licence was obtained from the Department of Radio Regularity at the Home Office.

To indemnify against the possible damage to the person or property of a third party, in the event of the balloon bursting and dropping the sonde, albeit suspended from a parachute, insurance cover was considered necessary. An aviation insurance policy was obtained through the Bursar's Department at the University on the understanding that Civil Aviation Authority had been granted.

The Royal Air Force and the Navy, who use the Llangollen valley for helicopter training, were informed of the radiosonde activities. Despite these warnings on several occasions low flying helicopters flew down the valley during periods when the radiosonde was in flight.

3.5.3. Derivation of Radiosonde Altitude

To enable the correlation of radiosonde temperature readings with height above the launch site the altitude of the sonde was calculated from on board pressure measurements. Ground level pressure readings were taken with an aneroid barometer during the radiosonde flights, the difference between these and the radiosonde pressures being proportional to the height of the radiosonde.

The variation of pressure with height is well defined, Bomford [1971] gives,

$$\frac{dP}{dh} = -0.0342 \; \frac{P}{T} \qquad mb.m^{-1} \qquad \ldots 3.3.$$

where P = atmospheric pressure, mb,

h = height, m,

T = air temperature, K.

When equation 3.3. is integrated between two heights, h_1 and h_2, at which the pressures are P_1 and P_2, respectively, it follows that

$$h_2 - h_1 = - \ln\left(\frac{P_2}{P_1}\right) \frac{T}{0.0342} \qquad m \qquad \ldots 3.4.$$

If P_1 is the ground level pressure and P_2 is the radiosonde pressure, then the height of the sonde above ground level is given by

$$h = - \ln\left(\frac{P_2}{P_1}\right) . \frac{T}{0.0342} \qquad \ldots 3.5.$$

where T is the mean air temperature between the radiosonde and ground level.

Formulae 3.5. was used for the derivation of radiosonde height in the Llangollen experiments. As a general guide, a change in pressure of -1 mb corresponds to an increase in level of +8 m. The radiosonde pressure measurement is accurate to ± 0.35 mb which means an accuracy of the sonde height of approximately ± 3 m was possible with the barometric technique.

CHAPTER FOUR

THE DERIVATION OF MEAN ATMOSPHERIC PARAMETERS

CHAPTER FOUR

THE DERIVATION OF MEAN ATMOSPHERIC PARAMETERS

4.1. The Derivation of Mean Atmospheric Pressure

Atmospheric pressure is an essential parameter in the calculation of the ambient refractive index for the correction of E.D.M. measurements. An error of ± 3 mb in the mean pressure along an E.D.M. line of sight will cause an error of ± 1 ppm in the corrected distance measurement.

Atmospheric pressure decreases with altitude by approximately 1 mb in 8 m. Consequently barometer readings at an E.D.M. station will only be representative of the mean pressure along the line of sight if the line is level. When two survey stations have a difference in level of less than ten metres, pressure measurement at one end of the line will be sufficiently representative of the mean along the line for the correction of high accuracy distance measurements. When the difference in level is greater than this the pressure should be measured at both survey stations with calibrated barometers. The barometers should be read simultaneously whilst the E.D.M. measurement is being made.

The arithmetic mean of end station pressure measurements is normally assumed as the actual mean pressure along the line of sight and is used in the calculation of ambient refractive index. However, this is not strictly correct; the change of pressure with height, the vertical pressure gradient, is not linear but exponential. Bomford [1971] gives the following simplified relationship between pressure and height,

$$P = P_o \exp\left[\frac{-0.0342\ h}{T}\right] \qquad \ldots\ 4.1.$$

where P is the atmospheric pressure (mb) at height h (m) above sea level, when the ambient air temperature is T (K). P_o is the atmospheric pressure at mean sea level. The mean pressure between two levels h_1 and h_2 can be obtained by integrating equation 4.1. to give

$$P_{mean} = \frac{P_o}{h_2 - h_1}\left[\frac{\exp(-0.0342\ h_1/T) - \exp(-0.0342\ h_2/T)}{0.0342/T}\right] \qquad \ldots\ 4.2.$$

where h_1 is the lower level. The sea level pressure P_o can be obtained from equation 4.1. with a knowledge of the pressure and level of one of the survey stations.

The following example compares the result of a straight arithmetic mean of end station pressures and a mean by integration.

STATION A	LEVEL = 91.07 m	PRESSURE = 1003.8 mb
STATION B	LEVEL = 385.12 m	PRESSURE = 968.8 mb

MEAN AIR TEMPERATURE = 9.3°C (284.45 K)

The sea level pressure, P_o, is calculated from equation 4.1. to give

$$P_o = 1014.9\,{}^{85}\ \text{mb}$$

The mean pressure by integration is obtained from equation 4.2. which gives

$$P_{mean} = 986.1 \text{ mb} \qquad \text{(integrated mean)}$$

The arithmetic mean of the two pressures is

$$P_{mean} = 986.3 \text{ mb} \qquad \text{(arithmetic mean)}$$

from which it can be seen that there is no significant difference between the arithmetic mean and the integrated mean. This example has a greater difference in level than would normally be encountered in engineering survey networks, and consequently the error in the integrated mean pressure is exagerated.

The accuracy of the integrated mean pressure calculations is reduced by the inaccuracies in the derivation of mean air temperature between the two survey stations; an error of $\pm 1^{\circ}C$ in the temperature introduces an error of $\pm$ 0.2 mb in the mean pressure. The integrated mean cannot be calculated without a prior knowledge of the survey station levels. Considering the necessary accuracy of pressure measurement, and the errors in the integrated mean due to inadequate mean temperatures, the arithmetic mean of end station pressures is sufficiently accurate for high accuracy E.D.M. measurements.

With long lines of sight it is possible for two survey stations to be in regions of different pressure due to the weather pattern [Saastamoinen, 1968],[Crabtree, 1974]. However, for short range distance measurement this effect will be negligible.

4.2. The Derivation of Mean Humidity

Section 2.5. showed that the effect of humidity on light and infra red wave E.D.M. measurements was small. Errors in wet bulb temperature of as much as $5^{o}C$ will not have a significant effect on the calculated value of refractive index (Figure 2.6.). As refractive index is so insensitive to errors in humidity measurement, wet bulb temperatures taken at each end of a survey line will give a sufficiently accurate mean value for the atmospheric correction of distance measurements.

Wet bulb temperatures can be measured to the necessary accuracy with Storrow Whirling hygrometers. However, as aspirated hygrometers of the Assman type are necessary for the accurate measurement of dry bulb temperature it is usually convenient to measure wet bulb temperature with the same instrument.

4.3. The Derivation of Mean Air Temperature

Temperature is the atmospheric parameter which has the greatest effect on the accurate measurement of distance with light and infra red wave E.D.M. instruments. To enable a corrected distance measurement accurate to 1 ppm to be obtained the total atmospheric correction must be accurate to the same degree. This assumes that the possible instrumental errors have been accounted for and that the instrument and prims are accurately centred and levelled over the survey stations. Section 4.1. illustrated that no significant error in the atmospheric correction will arise from pressure measurement if sufficient care is taken in the calibration and use of the barometers. The effect of humidity on light and infra red wave E.D.M. is small, and it has been suggested that wet bulb temperature measurements at either end of a survey line are sufficient to give a mean humidity for the atmospheric correction (Section 4.2.). Dry bulb temperature remains as the outstanding atmospheric parameter affecting high accuracy distance measurement. An error of $\pm$ 1^{o}C in the mean temperature along a line of sight will cause an error of $\pm$ 1 ppm in the distance measurement. To obtain representative mean temperatures with this accuracy is often a difficult problem for the surveyor.

The following brief description of daytime heat transfer, to the air in the atmosphere, indicates the important role of the ground surface in determining the air temperature. The atmosphere receives all of its heat from solar energy, both directly and indirectly. The radiation from the sun is short wave, which is not absorbed to an appreciable extent by air. However it is absorbed by the ground surface, and causes an increase in the ground temperature. Some of the heat absorbed is transferred to the air layer immediately above the surface by conduction, but the heat flow is very small because of the poor conductive properties of air. A major mechanism of heat transfer to the air is the absorbtion, by the air, of long wave radiation which is emitted from the warm ground surface. Up to ninety per cent of the outgoing long wave radiation is absorbed by the atmosphere, a majority by the water vapour in the air. Finally a considerable quantity of heat is transferred to the atmosphere from the condensation of water vapour that has evaporated from the surface.

Due to the heat transfer taking place at the ground surface the near surface air temperatures are not truly representative of the temperatures in the free air of the atmosphere at the same level. Most survey measurements are taken from within the air layers immediately above the ground, using pillar or tripod mounted instruments. Consequently air temperature measurements taken at two survey stations, at instrument height, are unlikely to be representative of the air temperature along the line of sight, especially if the line passes high above the surface.

It is common practice at present, when measuring distances with E.D.M., to use the mean of the end station temperatures for the calculation of the atmospheric correction to the distance measurement. On level lines passing close above an even ground surface such temperature measurements may be sufficiently representative of the mean on-line temperature to enable an accurate determination of the mean ambient refractive index. However when a distance is measured between two mountain tops, with the line of sight passing through free air, high above the valley floor, it is less probable that end station temperatures will be representative of the mean air temperature.

The problem of obtaining representative mean temperature has been considered by many research workers, but a majority of the experimentation has been concerned with long distance measurement, usually with microwave instruments [Thompson, 1960], [Richards, 1965], [Saastamoinen, 1968]. Meade [1969] used aircraft mounted temperature sensors to record air temperature along the line of sight. The problem as it concerns short range E.D.M. has only recently become important; the increased accuracy of the instruments has made errors in refractive index significant for high accuracy measurements. At present the derivation of mean air temperature along the line of sight is the limiting factor in short range, high accuracy, distance measurement.

The investigations made by the author were designed to assess the degree to which end station temperature measurements are representative of the mean air temperature along a line of sight, and to study methods of obtaining more representative temperature measurements. The experimental work was carried out on lines of a survey network in Llangollen (Plates 4.1. and 4.2.). This network has been established, over the past five years, by members of the Mining Department at Nottingham University [Hodges, Scoble, 1974], [Hodges, Scoble, Curl, 1975]. Six permanent survey stations have been established in and surrounding the Llangollen valley. Two of the lines in the network have been used for experimental work concerning air temperature. The line Bryn Howel to Tyn y Wern was used to evaluate the accuracy of temperature measurements on a level line passing close to the ground over its entire length. This line is 2011 m long and has a difference in level of only 10 m between end stations, the line passes no more than 30 m above the ground surface. A second line between the Ridge and Sheepfold survey stations was used to evaluate the possible accuracy of mean temperature determinations on an inclined line traversing a valley. The line is 2440 m long and has a difference in level of 125 m between the survey stations. The maximum height of the line above the ground surface is 250 m, the average being 160 m (Figure 4.1.).

PLATE 4.1.

Ordnance Survey map of the Llangollen survey network.

Approximate scale 1 : 17000

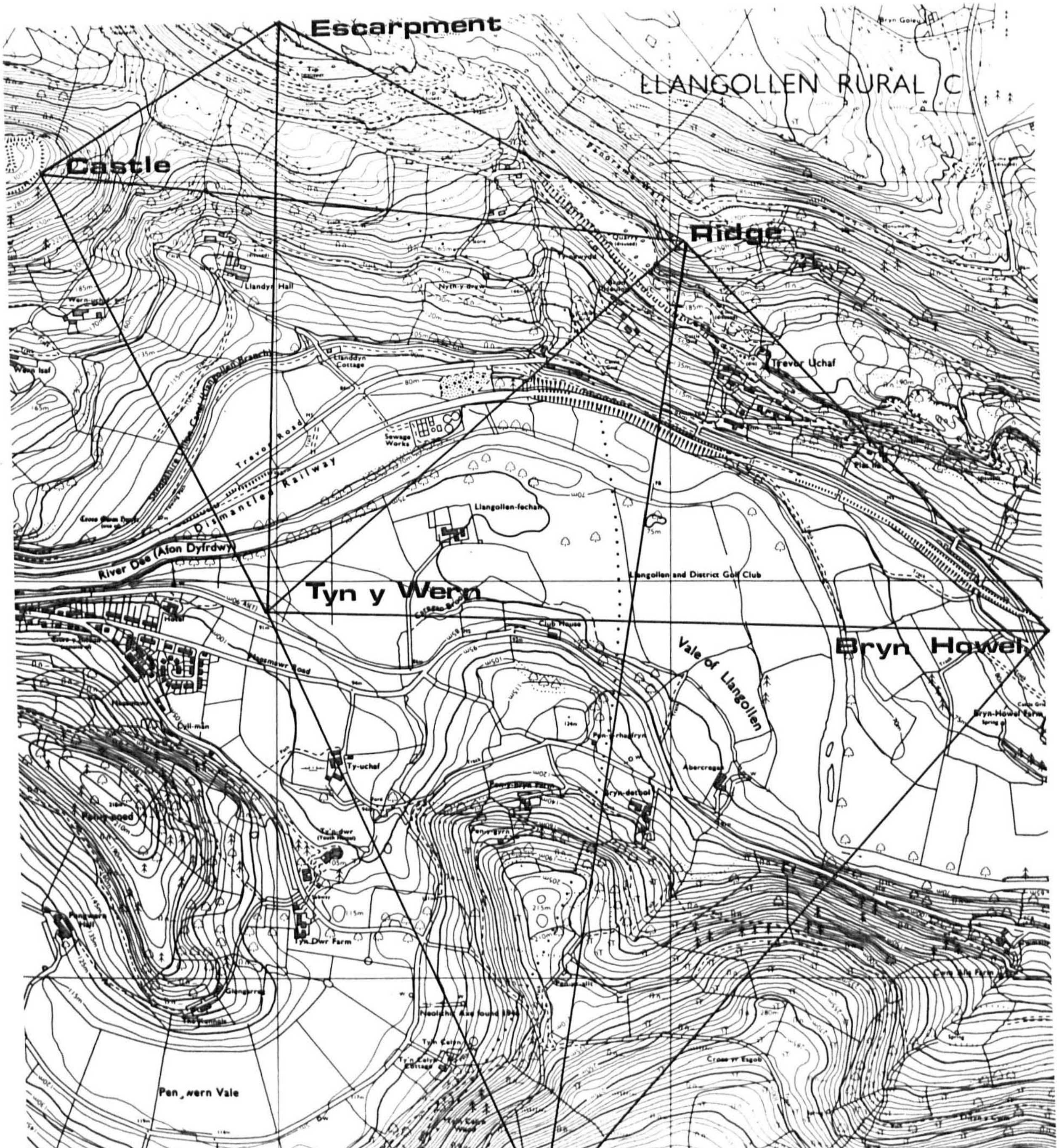
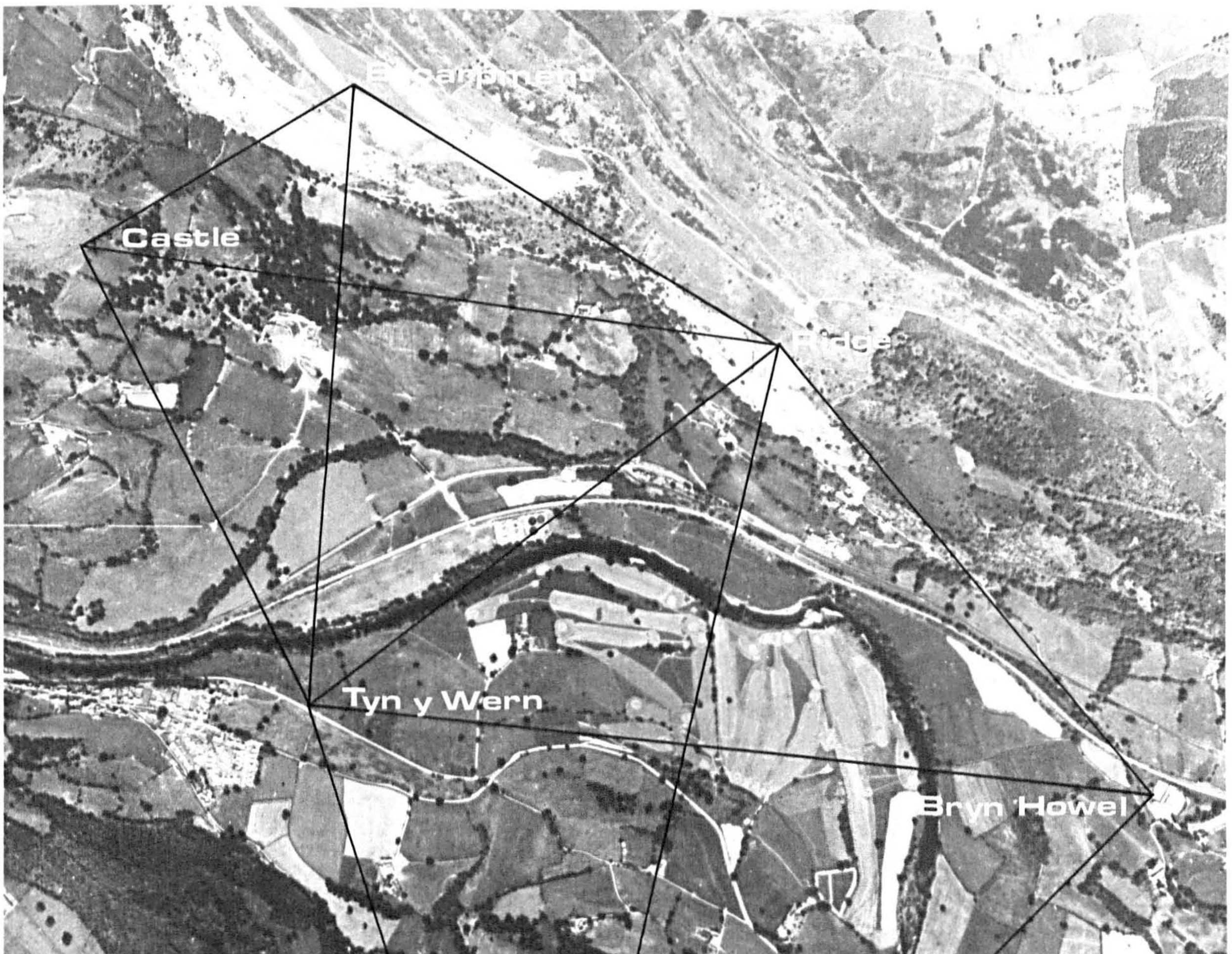

Escarpment
LLANGOLLEN RURAL C
Castle
Ridge
Llandyn Hall
Trevor Uchaf
Llanddyn Cottage
Sewage Works
Trevor Road
Dismantled Railway
Llangollen-fechan
River Dee (Afon Dyfrdwy)
Llangollen and District Golf Club
Tyn y Wern
Club House
Bryn Howel
Vale of Llangollen
Bryn-Howel Farm
Ty-uchaf
Abercrogan
Bryn-derbol
Tyn Dwr Farm
Neolithic Axe found
Tyn Celyn
Croes yr Esgob
Pen-y-wern Vale
Tan-y-graig
Sheepfold
Wireless Station

PLATE 4.2.

Aerial photograph of the Llangollen survey network.

Approximate scale 1 : 17000


Castle
Tyn y Wern
Bryn Howel
Sheepfold

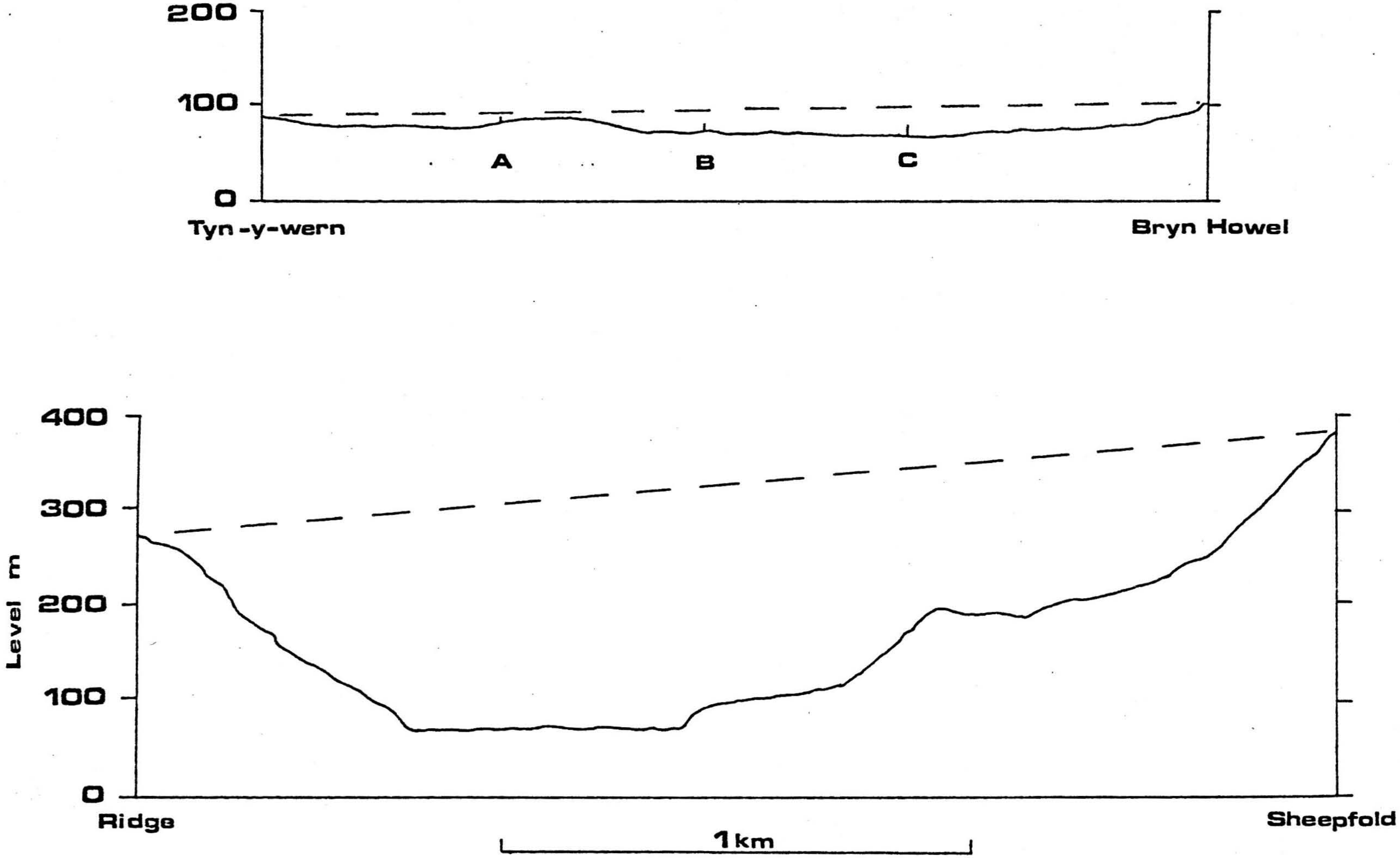


Figure 4.1. Topographic sections along the two experimental lines in Llangollen.

4.4. The Bryn Howel - Tyn y Wern Experiment

To investigate the possible errors in the evaluation of mean air temperature on a nearly level line, passing close to the ground surface, careful temperature measurements were made along the survey line Bryn Howel to Tyn y Wern. The temperature measurements were taken at the same time as the distance was measured with a Model 6 Geodimeter.

Wet and dry bulb temperatures were recorded at instrument height at both ends of the line with aspirated hygrometers. Air pressure was also measured, with calibrated barometers, at each end station. Ten metre steel masts were erected at three temporary stations, A, B and C (Figure 4.1.) which were established just below the line of sight. The dry bulb measurements were taken remotely at the top of each mast with elevated thermocouple assemblies and also at the foot of each mast with Storrow hygrometers. All the temperature measurements were synchronised and repeated every fifteen minutes.

The experimental work required eleven people to make measurements and erect the meterological masts. The instructions issued to personnel are reproduced in Appendix C.

The experimental results, for the period of the Geodimeter distance measurements, are shown in Figure 4.2. The weather during the experiment was overcast and dull. No elevated temperature values were obtained at station C because the operator made an error in the use of the potentiometer. The variations of the recorded dry bulb temperatures at each station, over the period of the distance measurement, were considered sufficiently small to enable mean temperatures to be used in

TIME (hrs) \ STATION	DRY BULB AIR TEMPERATURE °C							
	TYN Y WERN	A		B		C		BRYN HOWEL
	GROUND LEVEL	GROUND LEVEL	MAST TOP	GROUND LEVEL	MAST TOP	GROUND LEVEL	MAST TOP	GROUND LEVEL
15.30	4.3	6.0	6.1	5.8	6.4	6.0	-	4.2
15.45	4.3	6.0	5.9	6.6	6.6	6.0	-	4.2
16.00	4.3	6.0	6.1	6.6	6.3	6.3	-	4.1
16.15	4.3	6.0	5.5	6.5	5.9	6.3	-	4.2
16.30	4.3	6.0	5.9	6.5	6.1	6.0	-	4.2
MEAN °C	4.3	6.0	5.9	6.4	6.3	5.9	5.8*	4.2
HEIGHT OF LINE ABOVE STATION (m)	0 m	10 m	0 m	20 m	10 m	30 m	20 m	0 m
EXTRAPOLATED ON LINE TEMPERATURE	4.3	5.9	5.9	6.2	6.2	5.6	5.6	4.2

Mean of end station dry bulb temperatures = 4.25 °C

Mean of on line dry bulb temperatures = 5.24 °C

Mean of end station wet bulb temperature = 4.0 °C

Mean of end station pressure = 1002.6 mb

Figure 4.2. Results of Bryn Howel - Tyn y Wern Experiments, April 1975.

* extrapolated value

the calculation of the atmospheric correction. The greatest temperature variation in the one hour period was 0.7 ^{o}C at the top of the 10 m mast at B. The elevated temperature recorder at station A was at the same level as the line of sight and the elevated readings represent on line temperatures. At station B the elevated temperature recorder was 10 m below the line of sight and the recorded temperature had to be extrapolated to give an on-line value. The extrapolation was made using the vertical temperature gradients recorded between the top and bottom of the masts. At both station A and B the temperature change was -0.1 ^{o}C in the 10 m vertical interval. The ground level temperature at station C was extrapolated 30 m to the line of sight with this temperature gradient. Considering the small differences in level over which these extrapolations have been made the values obtained are considered to be the most probable temperatures on the line of sight, within the limits of accuracy that would significantly affect the experimental results.

The results show that during the experiment the end station temperatures were lower than the average measured temperature along the line. Stations A, B and C are sheltered in comparison to the end stations and it is possible that this enabled the air to gain more heat from the ground surface. The mean of the end station dry bulb temperatures was 4.25 ^{o}C, the mean of the end station and the extrapolated on-line temperatures was 5.24 ^{o}C. If it is assumed the mean, including extrapolated temperatures, is more representative of the true mean air temperature, then by taking only end station measurements the mean temperature would have been in error by approximately -1 ^{o}C.

While the temperature measurements were made the distance Tyn y Wern to Bryn Howel was measured ten times with the Model 6 Geodimeter. The mean slope distance obtained, before atmospheric correction was 2011.836 m. Figure 4.3. compares the calculated ambient refractive index, the atmospheric correction, and the corrected distance for the two experimental values of mean dry bulb temperature. The mean of end station wet bulb temperatures and pressures were used in the calculations. By including elevated dry bulb readings in the calculation of mean air temperature it was anticipated that the mean would be more representative of the air along the line of sight. When the mean temperature, including elevated values, was used in the computation of the atmospheric correction the derived value was +1.1 ppm greater than that obtained with end station temperatures alone. The use of on-line temperatures increased the corrected E.D.M. distance measurement by +2.2 mm.

Although the Bryn Howel - Tyn y Wern experiment was only conducted once, the results obtained show that the magnitude of errors in E.D.M. distance measurement, caused by unrepresentative temperature sampling, can be greater than 1 ppm. Some modern first order E.D.M. instruments have a specified proportional accuracy of ± 1 ppm, which must include both the frequency error and the possible refractive index error. However, in practice such accuracy may be difficult to realise on longer distances unless very great care is taken in the evaluation of mean ambient dry bulb temperature.

The Kern ME3000 Mekometer samples the air at the instrument station and adjusts the modulation wavelength accordingly, so that the

	END STATION TEMPERATURE MEASUREMENT ONLY	END STATION AND ELEVATED TEMPERATURE MEASUREMENT
MEAN DRY BULB TEMPERATURE, ^{o}C	4.25	5.24
MEAN WET BULB TEMPERATURE, ^{o}C	4.0	4.0
MEAN AIR PRESSURE, mb	1002.6	1002.6
MEAN MEASURED DISTANCE, m	2011.836	2011.836
AMBIENT REFRACTIVE INDEX	1.000 296 4	1.000 295 3
CORRECTED DISTANCE, m	2011.862	2011.864
ATMOSPHERIC CORRECTION, ppm	+12.8	+13.9
mm	+25.8	+28.0

Figure 4.3. Comparison of atmospheric correction to Geodimeter distance measurement, with and without mid-line temperatures.

instrument is effectively measuring in a standard atmosphere. This technique will only give accurate results if the air at the instrument station is representative of the air along the line of sight. To overcome this problem the Kern Company [Anon, 1974] suggest that the distance should be measured from both ends of the line. This method will improve the accuracy of the distance measurement but it assumes a uniform temperature distribution between the two survey stations and the absence of near surface temperature anomalies. The Bryn Howel - Tyn y Wern results have shown that this is not always the case on a nearly level line passing close to the ground surface. The same improvement in accuracy could be obtained if temperature and pressure readings were taken at both survey stations while the distance was measured in one direction only. A correction could then be applied for the difference in atmospheric parameters between the two survey stations.

4.5. The Ridge - Sheepfold Experiment

4.5.1. The Experimental Procedure

The inclined line between the Ridge and Sheepfold survey stations traverses the Llangollen valley at an average height of 160 m above the ground surface. It was anticipated that the temperature anomalies would be greatly exaggeratedon a line passing above such a distinct topographic feature. This line contrasts dramatically with the level line, passing close to the surface, between the Bryn Howel and Tyn y Wern stations (Plate 4.3.).

The programme for the experiment consisted of a series of radio-

PLATE 4.3.

View of the Llangollen valley showing the two test lines, levels in metres above Ordnance datum.


Sheepfold
385·13
to Bryn Howel
101·15
to Tyn y Wern
River Dee
Ridge
259·99

sonde flights in the centre of the valley at the same time as continuous temperature recordings were taken at each end of the line of sight. The experiments were carried out during two periods in 1976, the first was from the 24th of March to the 13th of April and the second was from the 18th to the 23rd July. The work was severely restricted in the first period by poor weather and very strong winds. The second period had better weather but wind still restricted the number of radiosonde flights that could be made.

Radiosonde flights were made from a launch site on the Vale of Llangollen golf course (OS E323900, N342000) during the first period (Plate 4.3.). On these occasions the receiving and recording equipment was mounted in an estate car in the golf club car park, approximately 200 m from the launch site. The equipment was powered by a portable petrol generator. In July the balloon was launched from a site approximately 300 m west of the golf club site in a field to the south of the Llangollen Fechan (OS E323600, N342050). At this site the receiving and recording equipment was only 50 m from the launch site and it was possible to obtain a supply of mains electricity from the Fechan. Since there was no interference from the generator the quality of the tape recordings was considerably improved. Both launch sites were in open areas where the possibility of the balloon tethering lines catching in trees was minimised.

The prevailing wind in the Vale of Llangollen is from the north west, passing along the line of the valley, the wind very rarely changes from this direction [Evans, 1976]. The launch sites were both established to the west of the line between the Ridge and Sheepfold stations, so that the sonde would be blown towards the line of sight by the wind. Since horizontal temperature changes in free air are negligibly small the horizontal positioning of the radiosonde is not critical. The correlation of the radiosonde level and the level of the line of sight, using barometric techniques, enabled sufficiently accurate determination of mid-line temperatures.

Detailed instructions issued to the personnel are included in Appendix D. Briefly, the radiosonde was attached to the hydrogen balloon which was allowed to rise, by releasing the tethering line from the winch in 40 m increments. The balloon was held steady for two minutes at each 40 m level and the radiosonde transmissions were recorded. When a maximum of 440 m of line had been released the tethering lines were wound in continuously, all the radiosonde transmissions being recorded. By adopting this procedure both discrete and continuous measurements of pressure and temperature were obtained. The maximum sonde height was determined by the length of the tethering lines (440 m) and the wind strength (Appendix B). Strong winds severely reduced the number of flights that could be made and on several occasions restricted the length of tethering line that was released. Ground level pressure and temperature were continually recorded at the launch site during flights.

Air temperature was recorded, at instrument height, at both the Ridge and Sheepfold survey stations during the radiosonde flights. In the April period these measurements were taken manually with hygrometers, however, in July a thermocouple assembly and a continuous recorder was left at each station (Plate 3.6.).

To ensure compatibility between the results of all the temperature measuring systems, they were each calibrated in situ against the same Assman hygrometer before and after each radiosonde flight.

4.5.2. Processing of Radiosonde Data

Nine radiosonde flights were made from the Llangollen valley (Figure 4.4.), with approximately two hundred readings of air pressure and temperature being recorded on magnetic tape during each flight. Temperature and pressure were transmitted and recorded sequentially. Thus each temperature measurement was paired with a pressure measurement, which was taken at the same time and with the radiosonde in the same position. The morse code recordings were decoded into digital format and then converted into values of pressure and temperature using the calibration curves (Plate 3.8.).

The temperature readings taken at the same pressure, and therefore at the same height, were grouped together and meaned. The mean temperatures were only considered if there were more than three readings taken at the corresponding pressure. Values of pressure and temperature were adjusted so that the ground level measurements agreed with the manual barometer and thermometer readings taken at the launch site. This calibration ensured compatibility of the results with other temperature measurements taken during the experiment. The radiosonde pressures were converted to give the height of the instrument above the launch site using the barometric method outlined in section 3.5.3.

FLIGHT NUMBER	DATE	TIME (hrs)	WIND	CLOUD (%)	MAXIMUM HEIGHT (m)	COMMENTS
1	1.4.76	11.16 - 11.43	STILL		65	Maximum height restricted by tethering line tangle.
2	8.4.76	15.20 - 16.04	LIGHT BREEZE	100	175	
3	12.4.76	11.05 - 11.40	STILL	100	185	Misty. Helicopter interfered with flight.
4	19.7.76	21.02 - 21.50	STILL	60	280	Dusk.
5	20.7.76	11.45 - 12.06	BREEZE	40	140	Wind restricted height.
6	22.7.76	10.35 - 11.04	LIGHT BREEZE	30	315	
7	22.7.76	11.45 - 12.07	LIGHT BREEZE	30	330	
8	22.7.76	15.10 - 15.42	BREEZE	30	-	Unreadable recording.
9	22.7.76	15.58 - 16.23	BREEZE	30	310	

Figure 4.4. Radiosonde Flight Log.

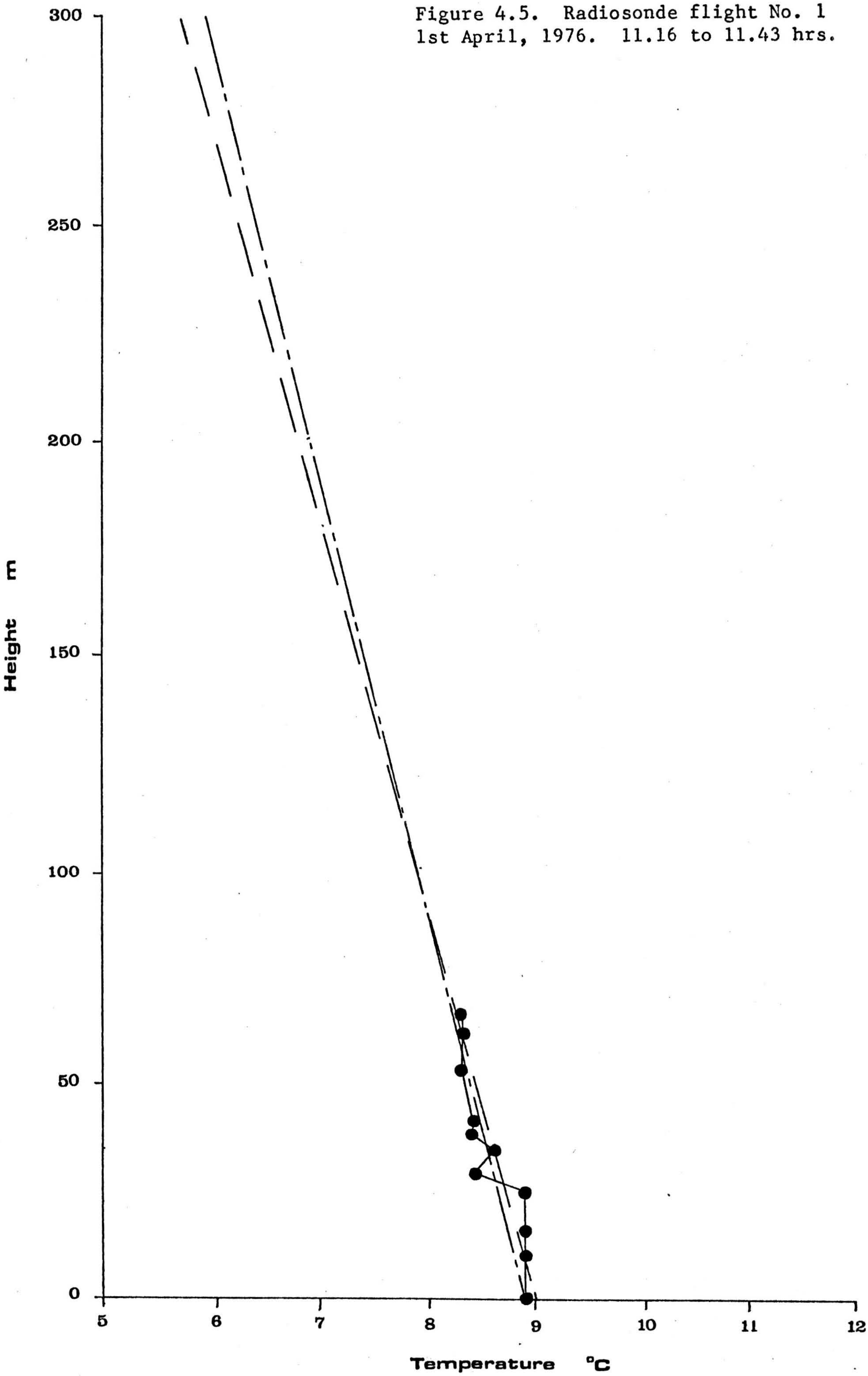
A series of mean temperatures at different heights above the ground were thus obtained from each radiosonde flight. These results are plotted as temperature versus height graphs in Figures 4.5. to 4.12. The results for the upward and downward flights have been plotted separately. No attempt has been made to plot smooth curves through the data points as this may have given a misleading representation of the results.

Figures 4.5. to 4.12. Graphs of air temperature against height.

KEY

●	Data Points for Upward Flights.
●	Data Points for Downward Flights.
—— —— ——	Linear Regression Line.
—— - —— - ——	Dry Adiabatic Lapse Rate.

Figure 4.5. Radiosonde flight No. 1
1st April, 1976. 11.16 to 11.43 hrs.



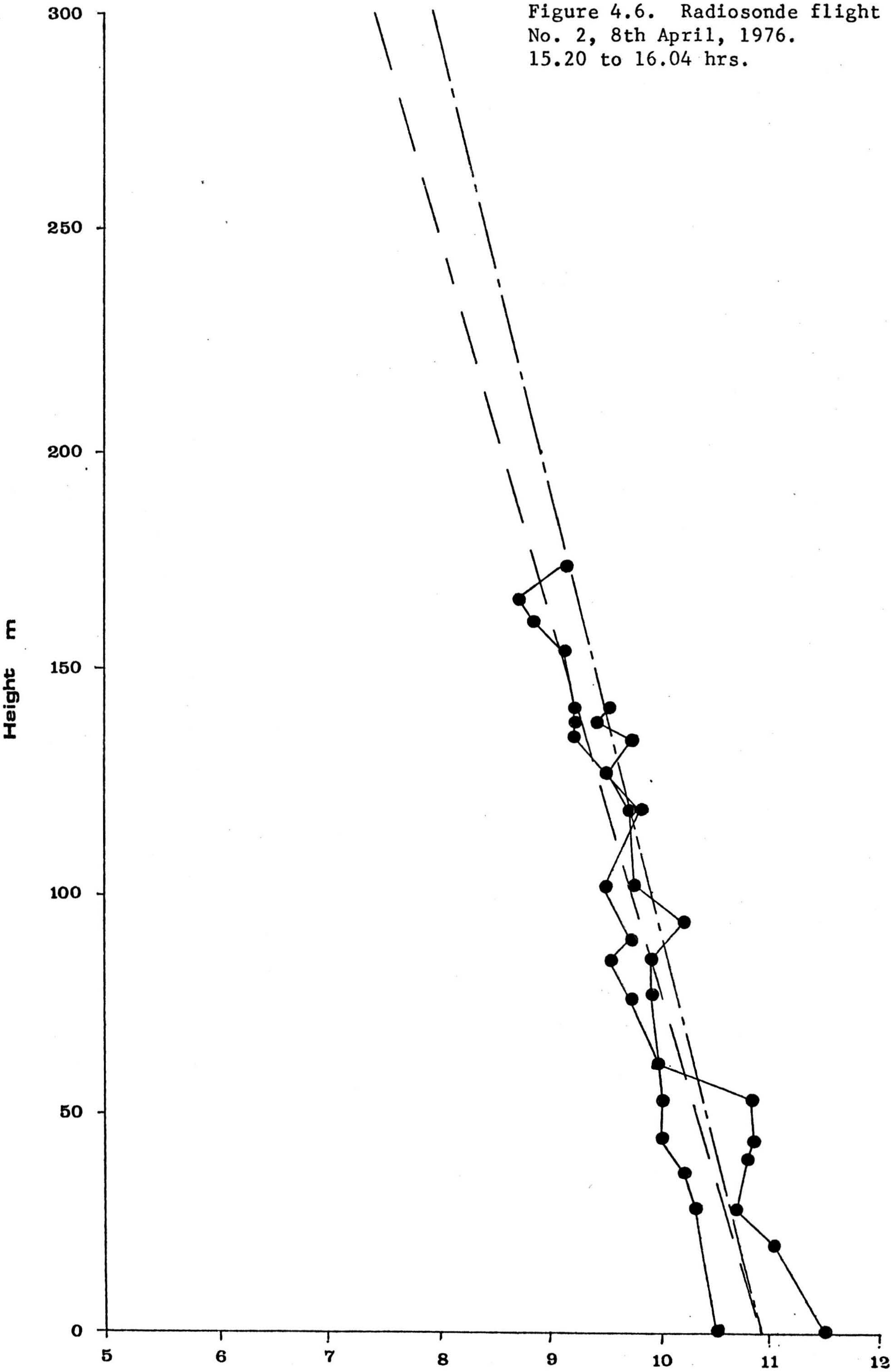


Figure 4.6. Radiosonde flight No. 2, 8th April, 1976. 15.20 to 16.04 hrs.

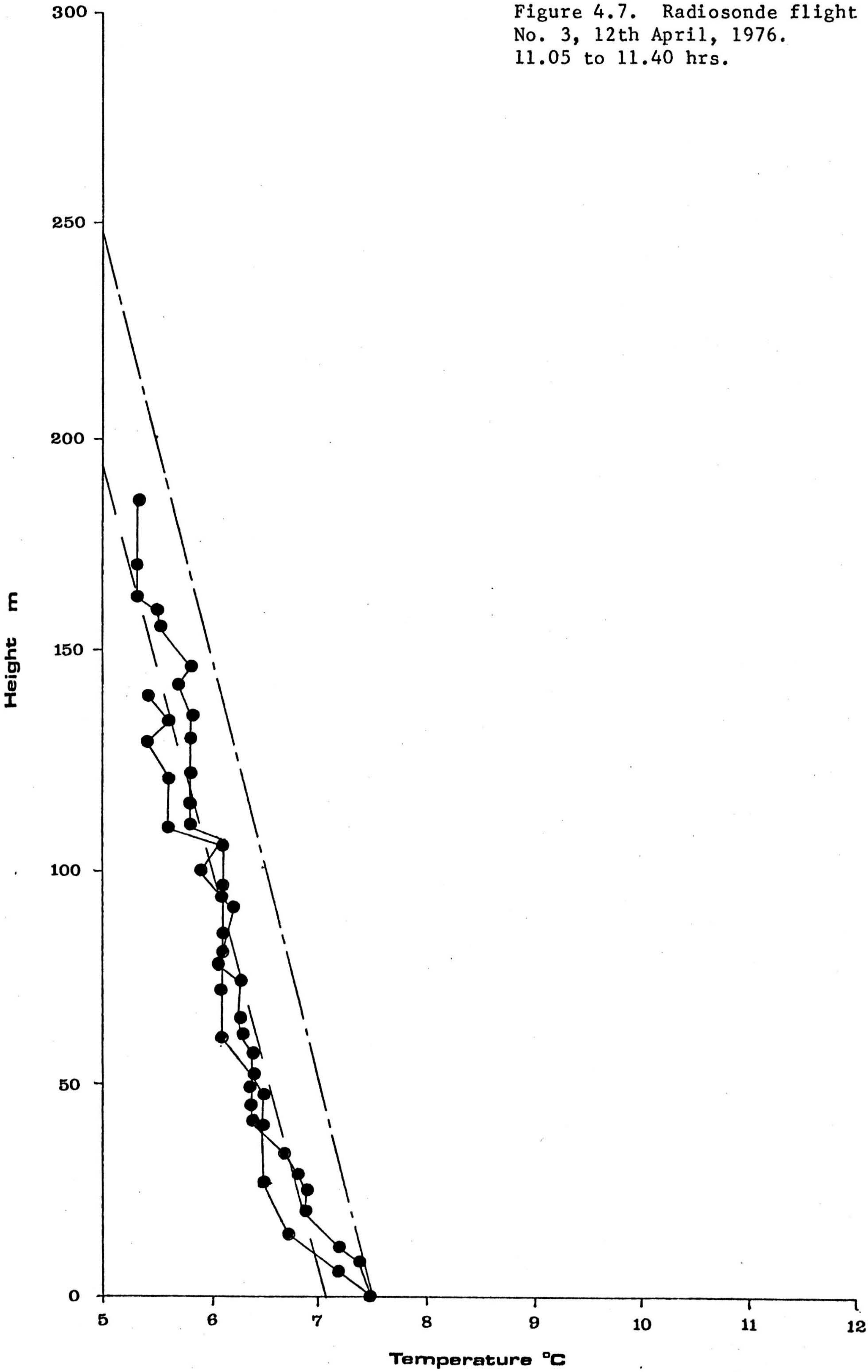


Figure 4.7. Radiosonde flight
No. 3, 12th April, 1976.
11.05 to 11.40 hrs.

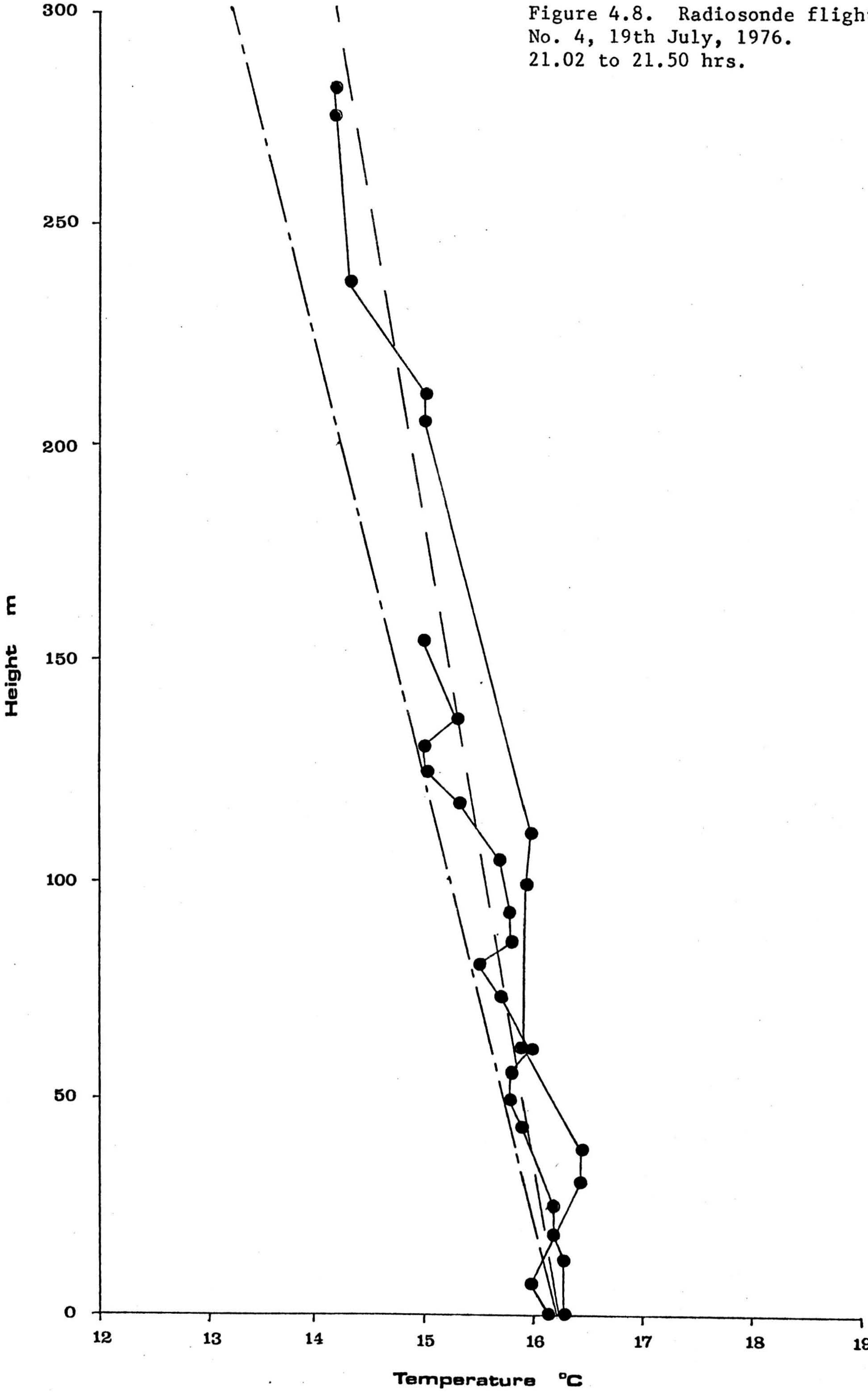
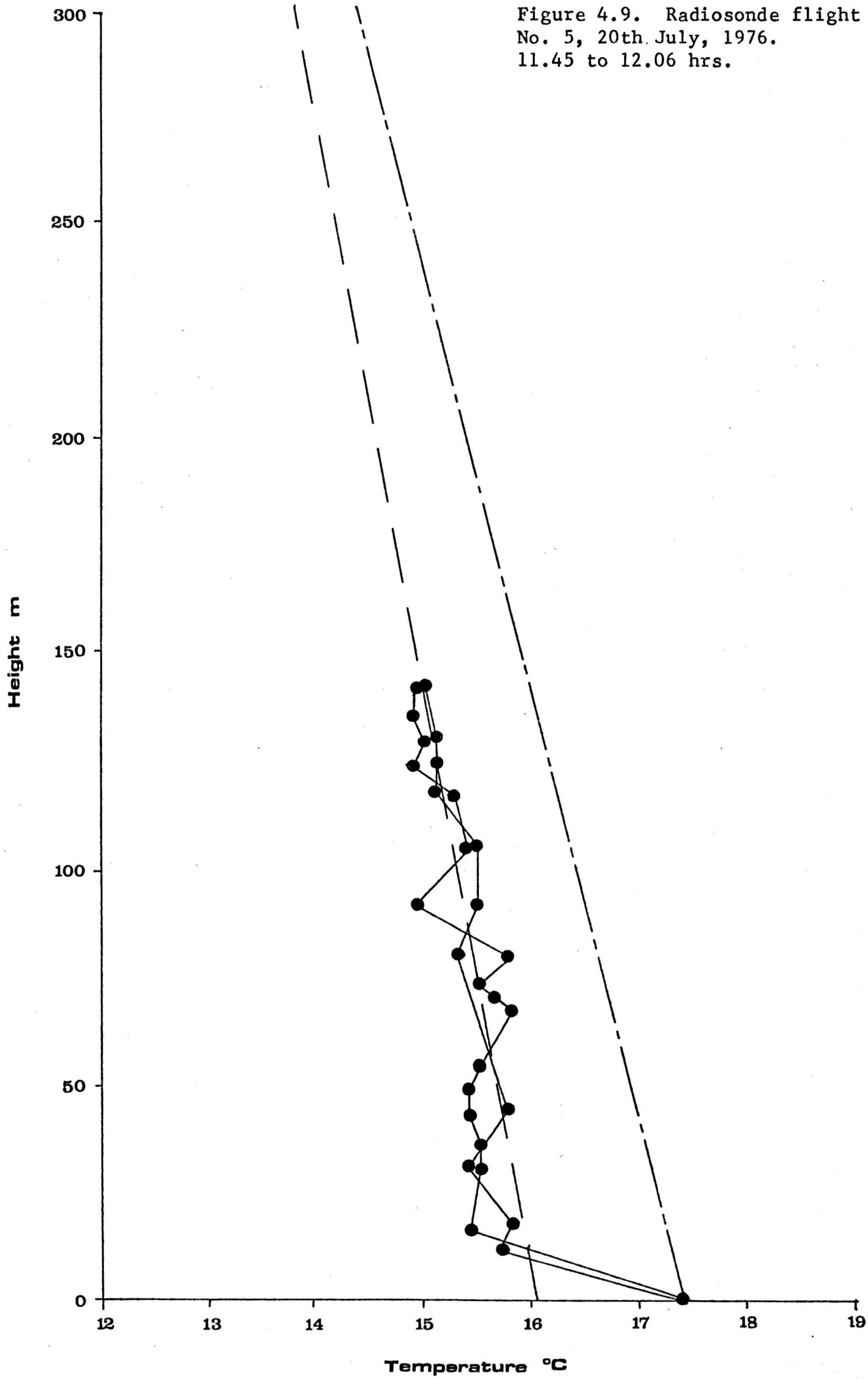


Figure 4.8. Radiosonde flight No. 4, 19th July, 1976. 21.02 to 21.50 hrs.

Figure 4.9. Radiosonde flight No. 5, 20th July, 1976. 11.45 to 12.06 hrs.



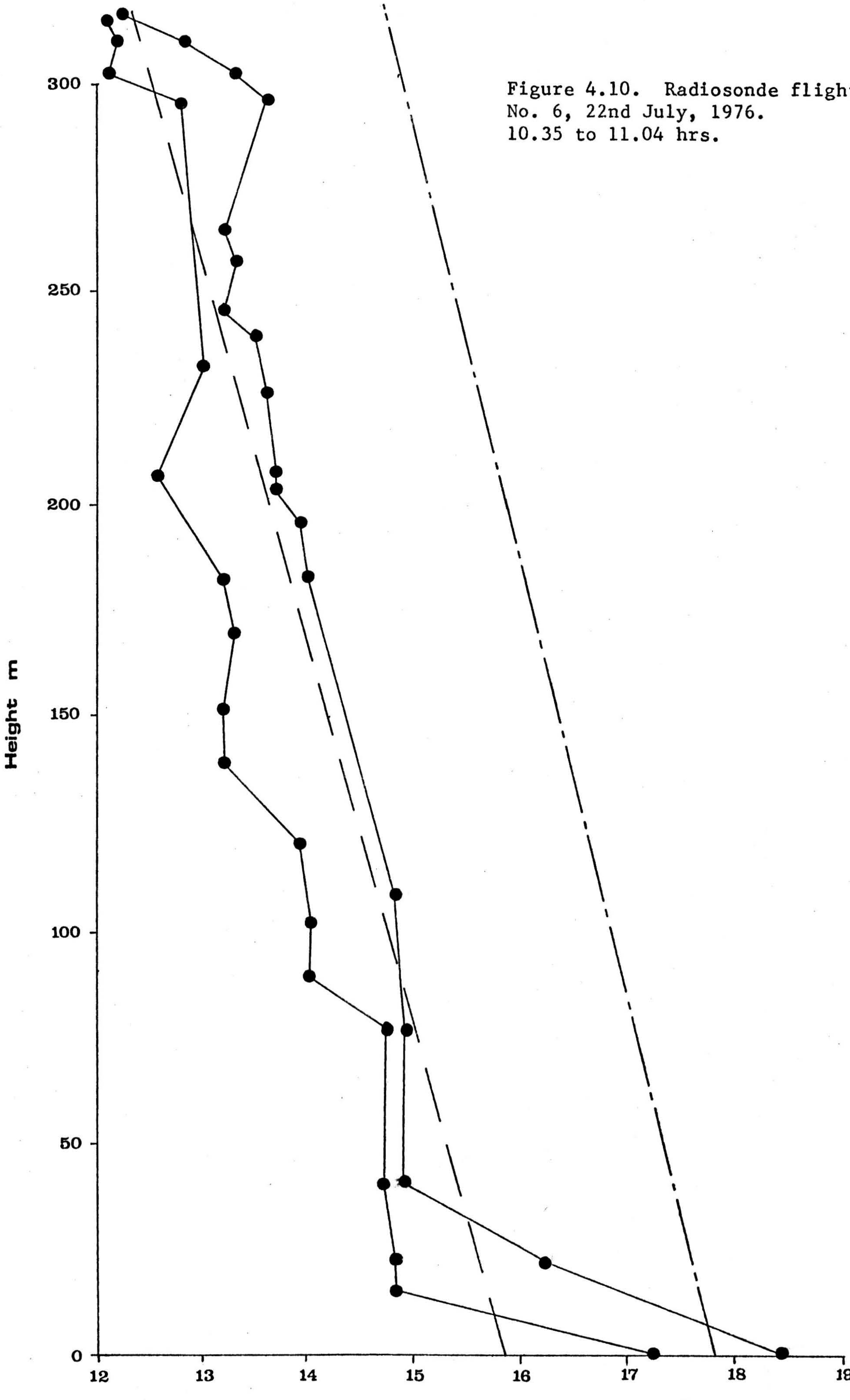


Figure 4.10. Radiosonde flight No. 6, 22nd July, 1976. 10.35 to 11.04 hrs.

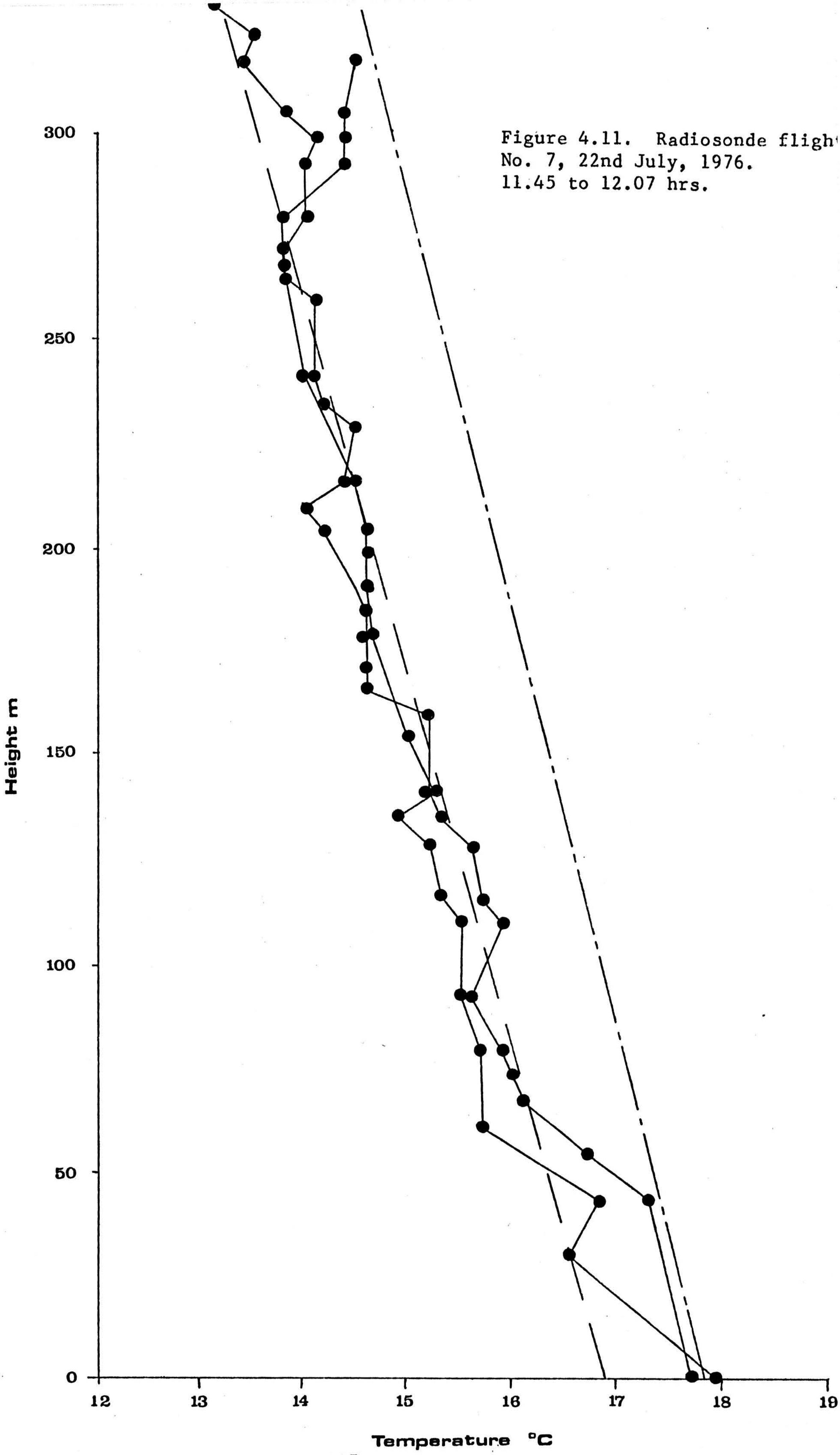


Figure 4.11. Radiosonde fligh[t]
No. 7, 22nd July, 1976.
11.45 to 12.07 hrs.

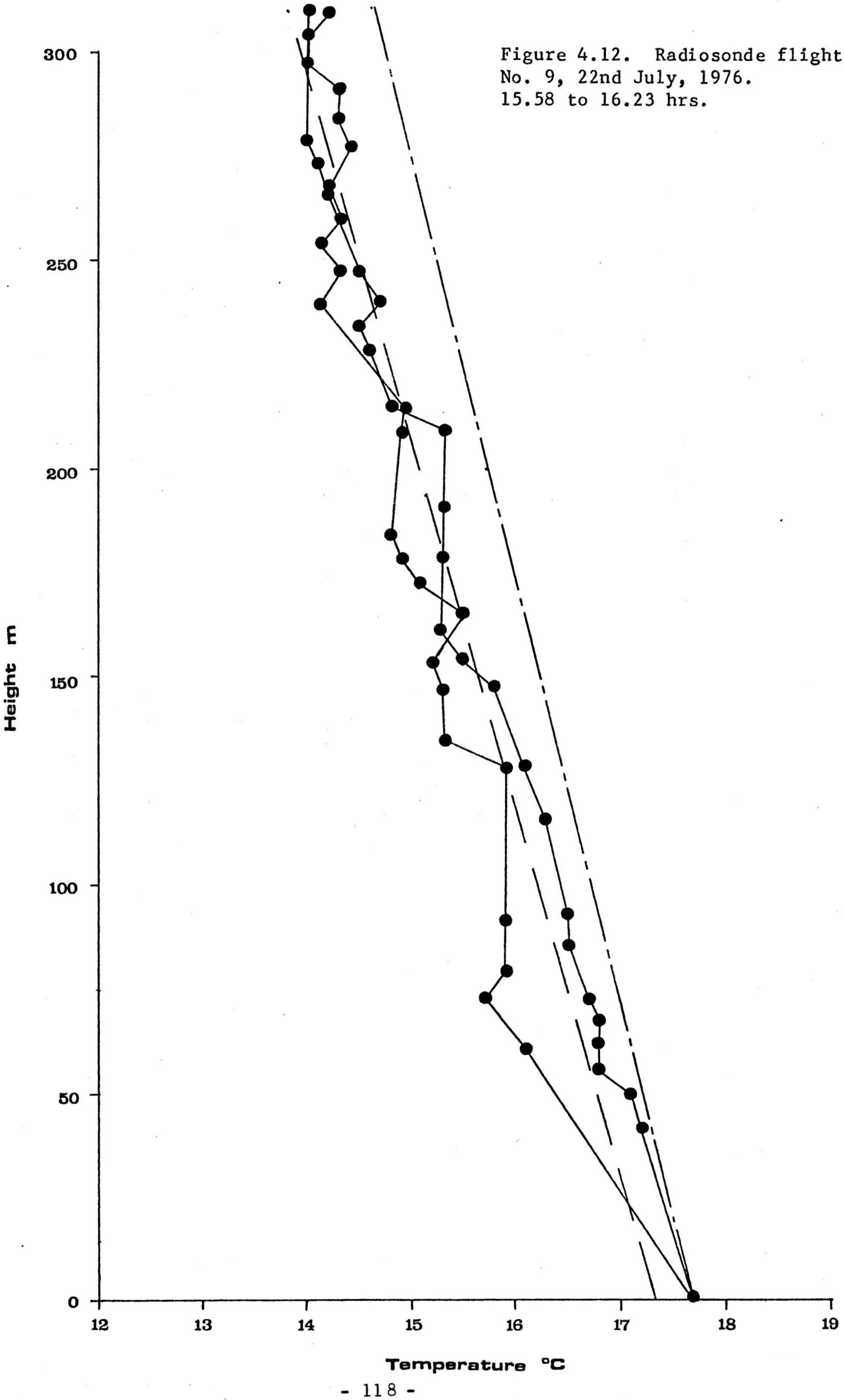


Figure 4.12. Radiosonde flight No. 9, 22nd July, 1976. 15.58 to 16.23 hrs.

4.5.3. Radiosonde Results

Wind drastically reduced the maximum height of the radiosonde flights, however, three flights reached over 300 m above the launch site level. Owing to a tangle in the tethering lines flight 1 was aborted at 65 m.

There is very good agreement between the results of the upward and downward flights. The greatest disagreement between the temperatures at the same height is approximately 1 ^{o}C, but the average was less than 0.4 ^{o}C. Perfect agreement cannot be expected because of the time lag between upward and downward flights and the dynamic nature of air temperature.

All the graphs indicate a decrease in air temperature with height above the ground surface, thus giving a negative temperature lapse rate. The lapse rate is greatest near to the ground, and this shows particularly clearly in the graph for flight 3. A line of linear regression has been plotted on each graph, both upward and downward flight data being used in the regression calculations. The gradient of the regression line represents the mean temperature lapse rate in the air at the time of the radiosonde flight. The mean lapse rates for each flight are shown in Figure 4.13., they vary between -1.15 and -0.73 ^{o}C per 100 m. However, the mean lapse rate agrees well with the dry adiabatic lapse rate of -0.976 ^{o}C per 100 m.

FLIGHT NUMBER	MEAN TIME (hrs)	LAPSE RATE ($^oC/100m$)
1	11.30	- 1.11
2	15.40	- 1.15
3	11.20	- 1.05
4	21.25	- 0.74
5	11.55	- 0.73
6	10.50	- 1.12
7	11.56	- 1.06
9	16.10	- 1.15
	MEAN =	- 1.01

Figure 4.13. Table of mean measured temperature lapse rates

A line representing an adiabatic lapse rate is plotted on each temperature versus height graph, with the ground level temperature as the origin.

The difference between the intersection of the regression line on the temperature axis and the measured ground level temperature gives the magnitude of the ground level temperature anomaly. The values of the anomalies are shown in Figure 4.14.

FLIGHT NUMBER	MEAN TIME (hrs)	GROUND LEVEL TEMPERATURE ANOMALY (^{o}C)
1	11.30	0.1
2	15.40	0.1
3	11.20	0.5
4	21.25	0.1
5	11.55	1.4
6	10.50	2.0
7	11.56	1.0
9	16.50	0.4

Figure 4.14. Table of ground level temperature anomalies at the launch site

The anomalies for flights number 6, 7 and 9 are of particular interest, these flights were made on the same day and there is a clear trend for the anomaly to decrease throughout the day time. This result agrees with those of Sheppard [1946] who showed the surface anomaly to decrease from mid morning to a minimum in mid afternoon. The large morning anomaly exists because the ground surface is warmed by incoming radiation while the air is still cool; as the day continues the air gains heat and the temperature rises. In the afternoon the air temperature reaches a maximum and the ground anomaly is small. At night the ground loses its heat and the air may be warmer than the surface, giving rise to a negative temperature anomaly. The radiosonde data shows that air temperature is only influenced by the ground to a height of approximately 50 m above the surface.

The mid point of the line between the Ridge and Sheepfold survey stations is 250 m above the radiosonde launch site. Since in free air changes of temperature are small in the horizontal plane, the radiosonde temperature measurements at a height of 250 m have been taken as temperatures on the mid point of the survey line between the two stations. When the radiosonde did not reach a height of 250 m the mid line temperature was extrapolated from the linear regression line. This is justifiable because all the radiosonde flights reached heights at which the ground effect on air temperature has been proven to be negligible, that is above 50 m, and in so doing the radiosonde gave sufficient data to indicate the temperature lapse rate above this level. The temperature versus height graphs for flights that reached greater heights have shown that away from the influence of the surface the temperature lapse rate can be considered as constant, and that it is well represented by the linear regression line plotted through the data points.

4.5.4. End Station Temperature Results

The results of dry bulb air temperature measurements, taken at 1.5 m above the survey station level, at Ridge and Sheepfold are given as graphs of temperature against time in Figures 4.15. to 4.19. The readings were taken on five different days and in total over thirty hours of continuous temperature measurements were obtained at each station.

Figures 4.15. to 4.19. Diurnal air temperature variation at 1.5 m above the surface at Ridge and Sheepfold

<u>KEY</u>

o	Temperatures at Ridge
•	Temperatures at Sheepfold
●	Mid-line radiosonde temperature

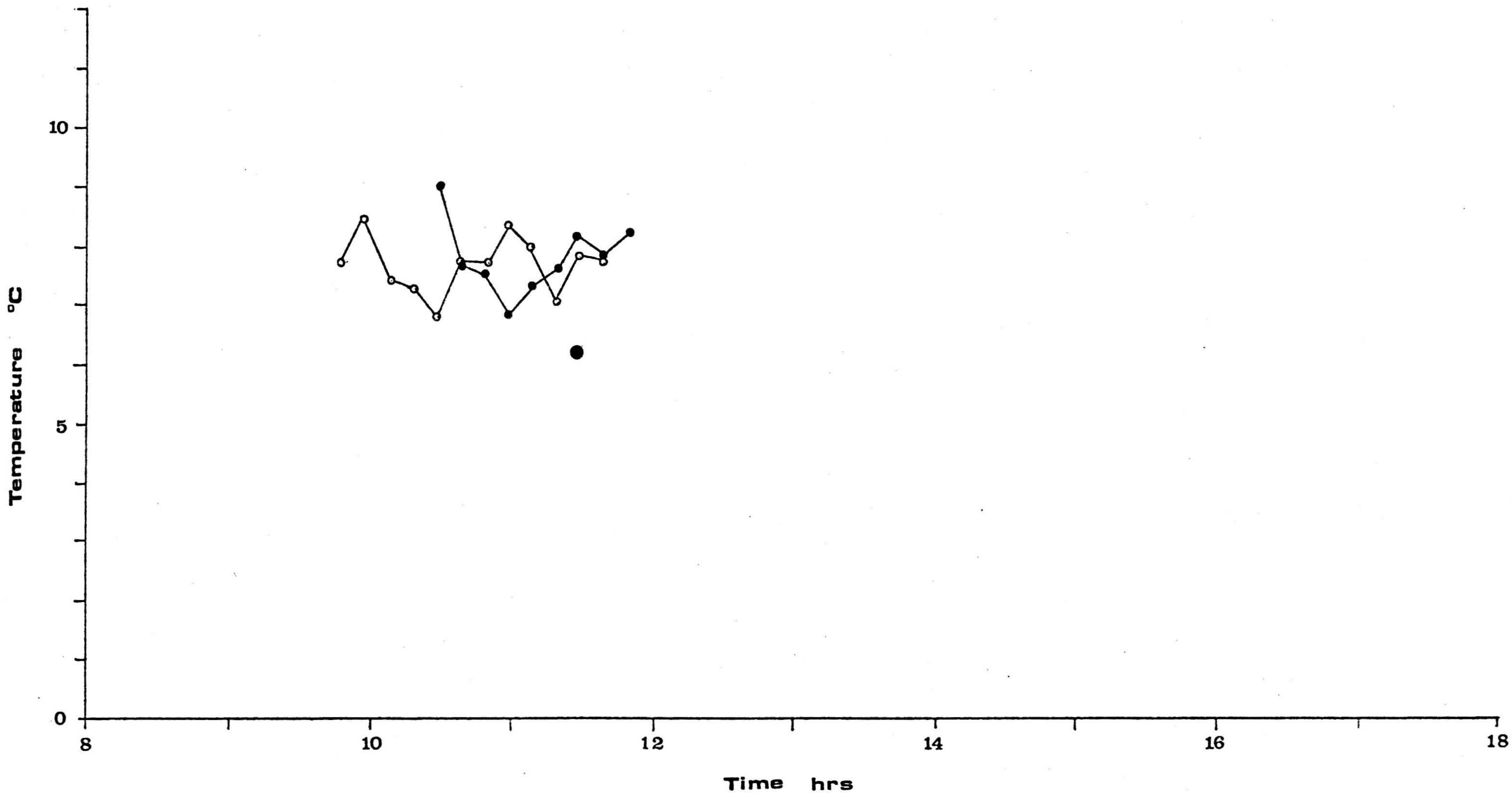


Figure 4.15. Diurnal air temperature variation at 1.5 m above the surface at Ridge (∘) and Sheepfold (•) on 1st April, 1976.

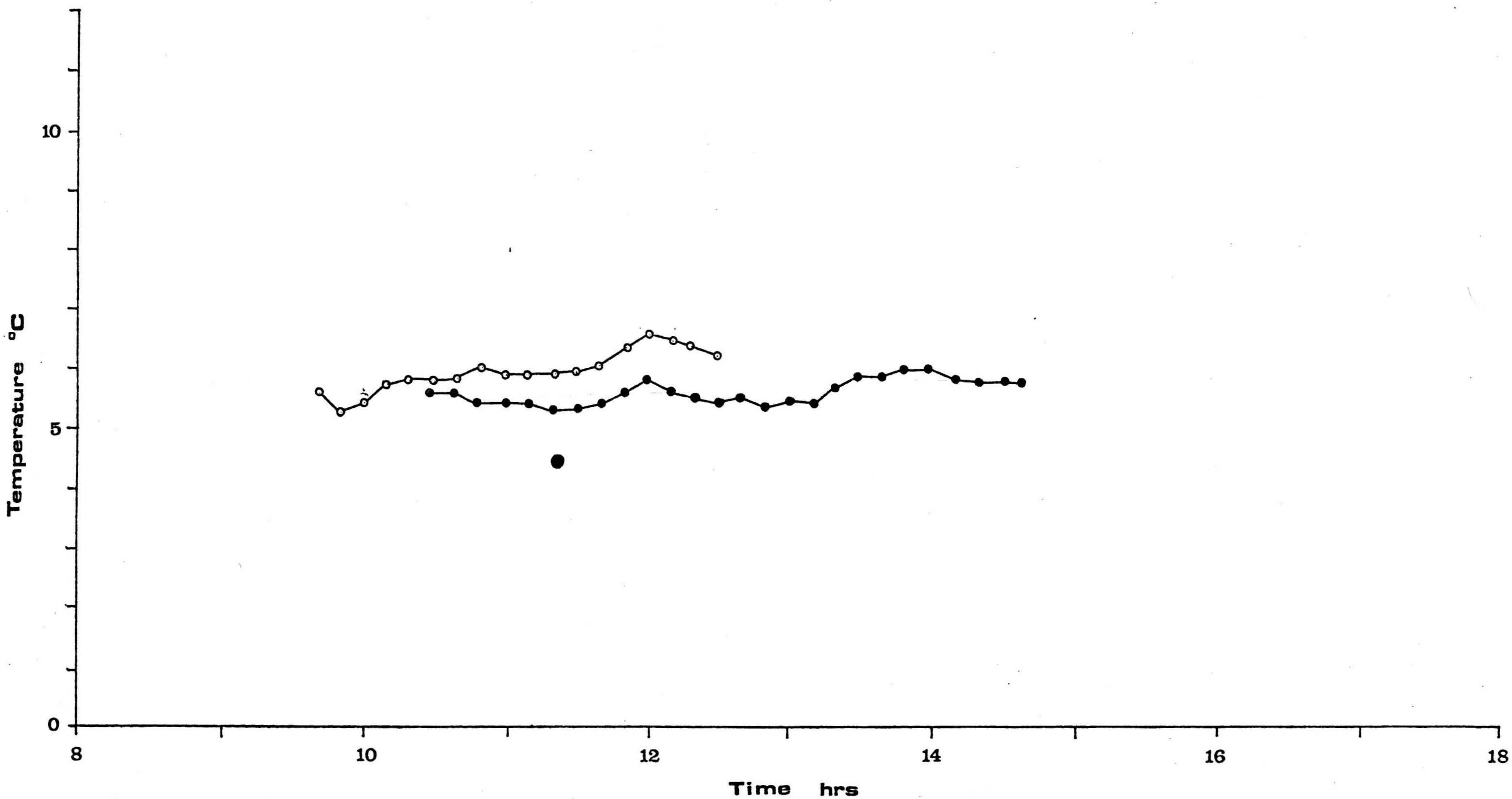


Figure 4.16. Diurnal air temperature variation at 1.5 m above the surface at Ridge (∘) and Sheepfold (•) on 12th April, 1976.

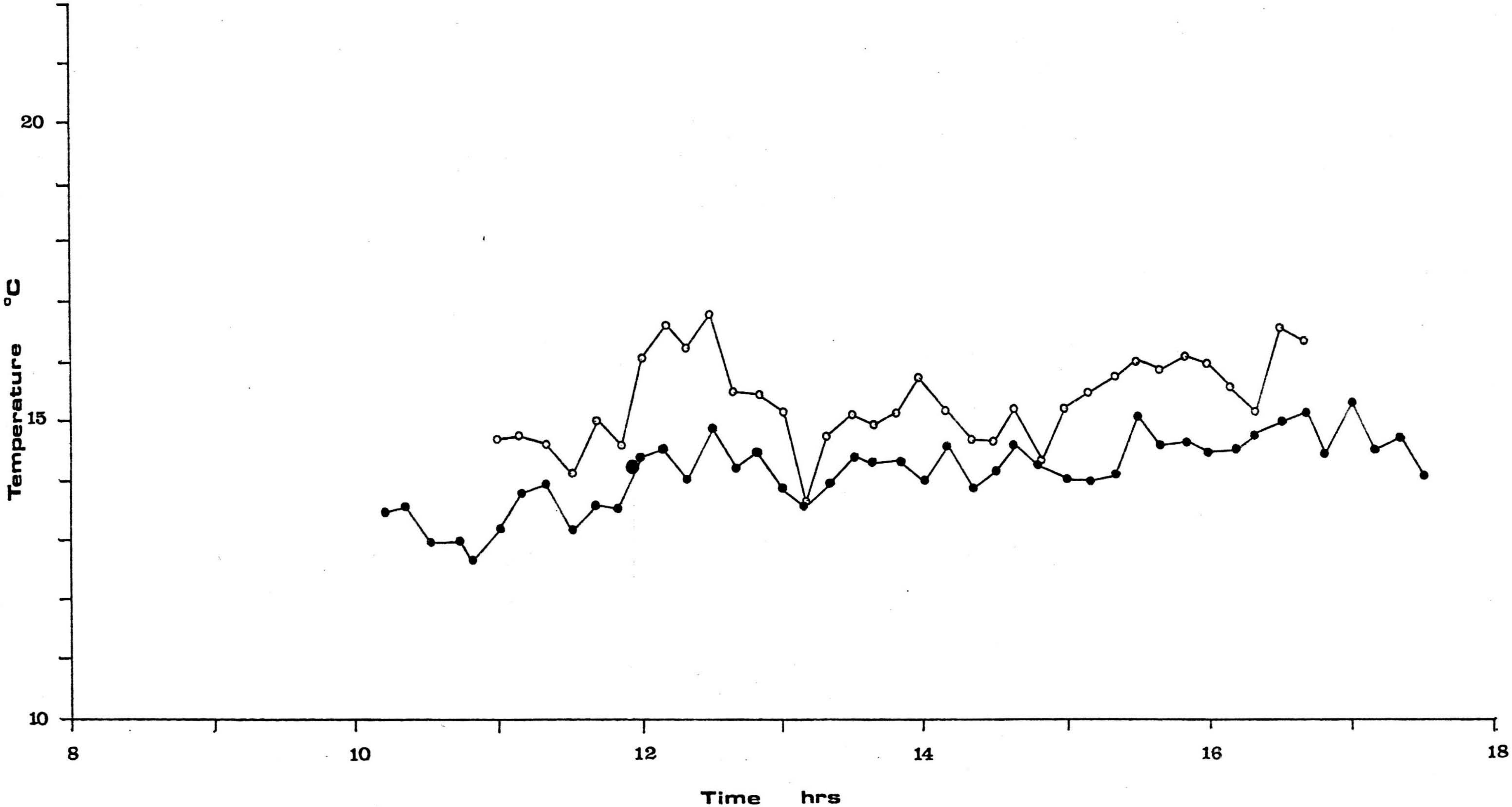


Figure 4.17. Diurnal air temperature variation at 1.5 m above the surface at Ridge (o) and Sheepfold (•) on 20th July, 1976

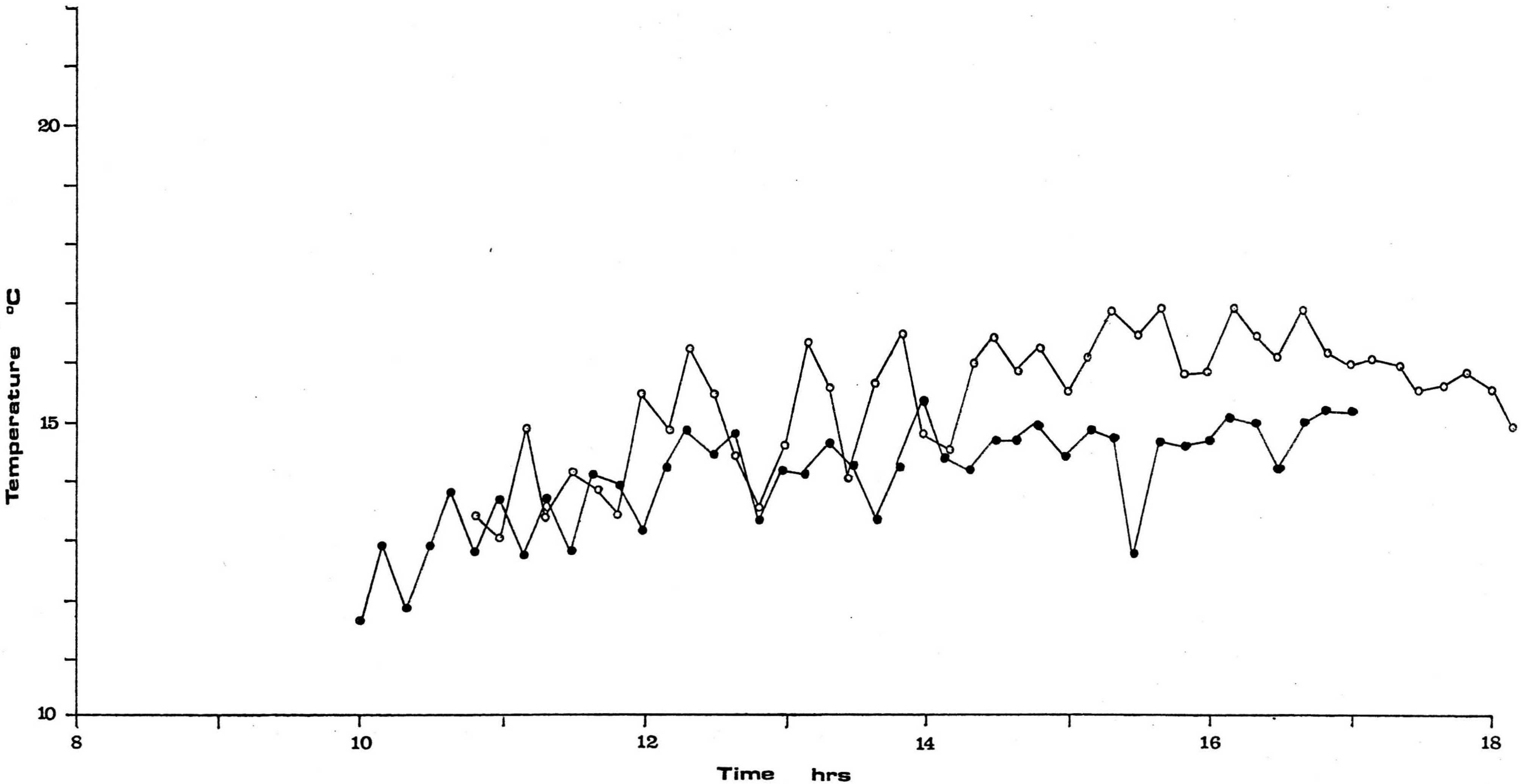


Figure 4.18. Diurnal air temperature variation at 1.5 m above the surface at Ridge (○) and Sheepfold (•) on 21st July, 1976.

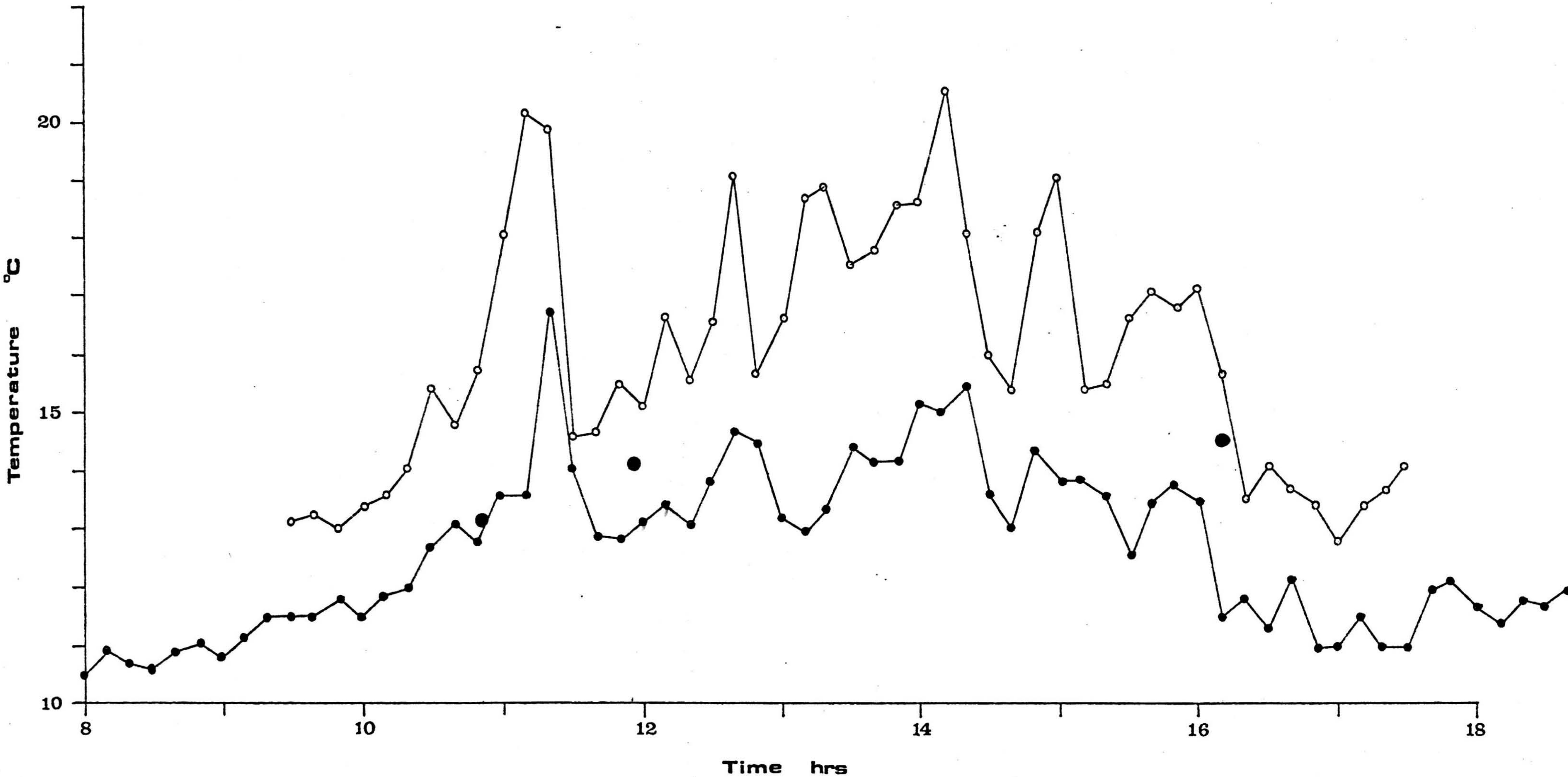


Figure 4.19. Diurnal air temperature variation at 1.5 m above the surface at Ridge (◦) and Sheepfold (•) on 22nd July, 1976.

The short term fluctuations in the air temperature are the most outstanding features of the graphs. The sudden changes show up particularly well on the graph for July 22nd (Figure 4.19.); between 11.10 and 11.20 hours, for example, the temperature at Ridge decreased by five degrees Celsius. On this day there was only 30 per cent cloud cover with frequent sunny intervals. In contrast, on the morning of April 12th (Figure 4.16.) there was 100 per cent cloud cover and a fog with very little wind; on this occasion the temperature was very stable throughout the period of the measurements. These results suggest that short term variations in air temperature at the end stations are caused by increased heating of the air during sunny periods. When there is fog the radiant heating is reduced and no short term variations of near surface air temperature occur.

There is good agreement between the temperature variations at the two survey stations. During several periods of the temperature measurements there is a noticeable time lag between temperature changes at Ridge and Sheepfold. This is clearly indicated on July 21st between 12.00 and 14.00 hours (Figure 4.18.), when the temperature pattern at Ridge leads that at Sheepfold. There was a north-west wind during the period of the experimental work and this may explain the time lapse between the temperature changes. First, any mass of warm or cold air will pass over the Ridge station and then, after a short time, will reach the Sheepfold station. Secondly, in partially overcast weather with sunny intervals, areas of clear sky will allow radiant heating of the surface and the air at Ridge and later, as the clouds move, at Sheepfold.

The theoretical difference in temperature between two survey stations can be derived from the difference in level between the stations and the ambient air temperature lapse rate. The difference in level between the Ridge and Sheepfold stations is 125 m, and by assuming an adiabatic lapse rate of -1 ^{o}C per 100 m a theoretical temperature difference of 1.25 ^{o}C can be calculated. The temperature lapse rate measurements obtained from the radiosonde results have indicated that the ambient lapse rates were near adiabatic. The actual deviations from the adiabatic lapse rates were small, and would have no significant effect on the theoretical temperature difference.

Figure 4.20. shows graphs of the Ridge minus Sheepfold temperature difference plotted against the time of day. The large variation in the difference between simultaneous temperature measurements shows clearly in the graphs for July, but the difference in April is more stable. These variations are probably a result of unequal temperature changes at each station caused by air movement and radiant heating. The theoretical temperature difference is plotted on each graph, as well as the mean temperature difference for each day. The disagreement between the theoretical and the mean temperature differences can be explained in terms of local temperature anomalies at each station, these depend on the local environment of the stations and would be exagerated in sunny, rather than overcast, weather.

A disagreement between the theoretical and practical difference between end station temperatures indicates with certainty that air temperature anomalies exist at the terminal stations. In consequence the mean of end station temperatures will be in error by at least one

Figure 4.20.

Diurnal variation of temperature difference between Ridge and Sheepfold.

KEY

____ ____ ____ ____ indicates theoretical difference

__ . __ . __ . __ . __ . __ indicates mean difference

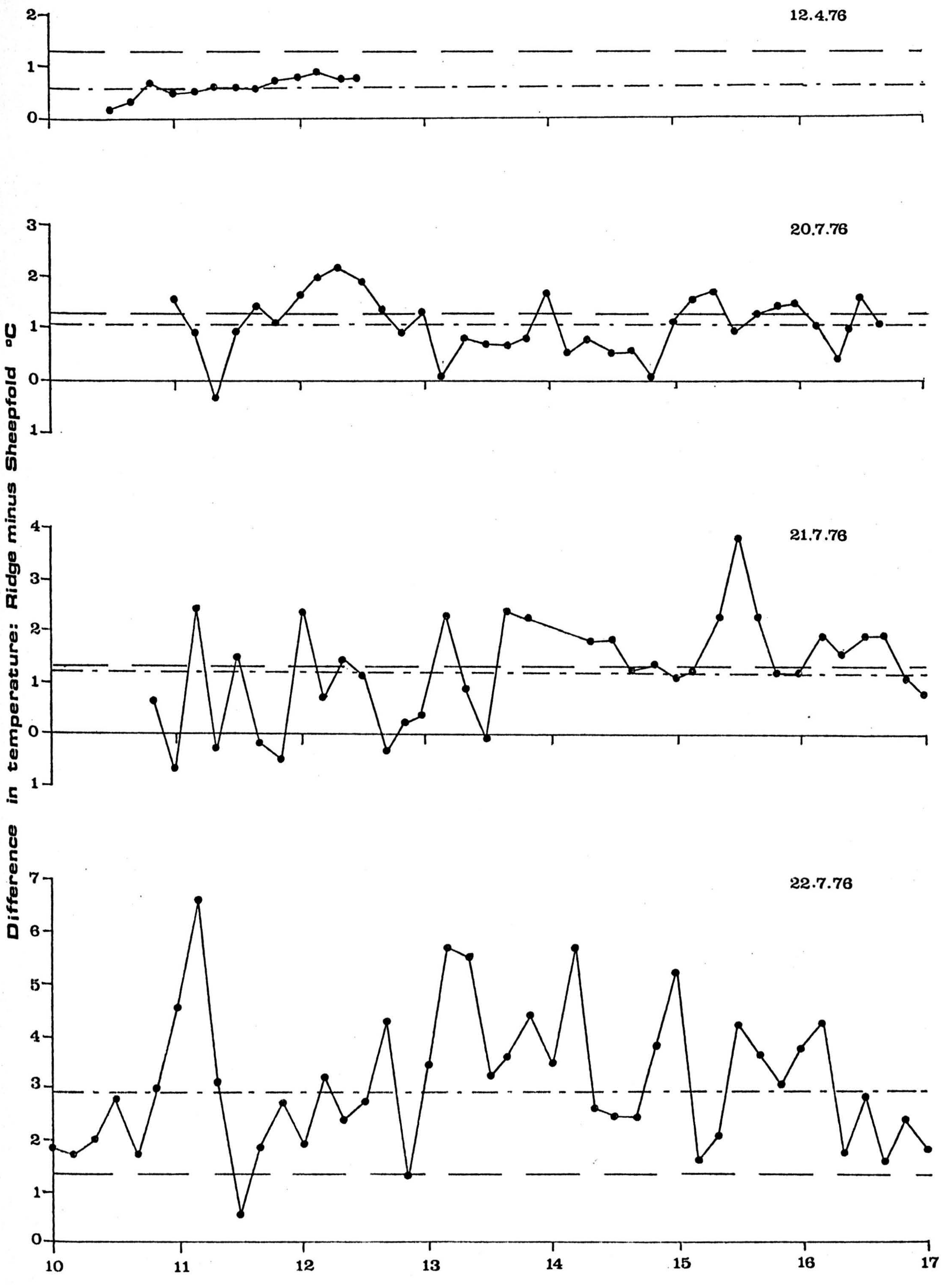


Figure 4.20. Diurnal variation of temperature difference between Ridge and Sheepfold.

half of the disagreement between the theoretical and practical temperature difference. However, if there is good agreement between theory and practice the end station anomalies will be equal and possibly zero. A study of end station temperatures can give useful information as to the suitability of these values for the determination of representative mean on-line air temperature, for the refractive index correction to E.D.M. measurements.

FLIGHT NUMBER	DATE	MEAN TIME (hrs)	END STATION TEMPERATURE RIDGE ^{o}C	SHEEPFOLD ^{o}C	MEAN ^{o}C	MID-LINE TEMPERATURE	MEAN MINUS MID-LINE ^{o}C
1	1.4.76	11.30	7.8	8.2	8.0	6.2	+1.8
2	8.4.76	15.40	8.5	6.4	7.5	8.0	-0.5
3	12.4.76	11.20	5.9	5.3	5.6	4.4	+1.2
4	19.7.76	21.25	-	-	-	14.5	-
5	20.7.76	11.55	15.2	14.0	14.6	14.2	+0.4
6	22.7.76	10.50	15.7	12.8	14.3	13.1	+1.2
7	22.7.76	11.56	15.3	13.1	14.2	14.1	+0.1
9	22.7.76	16.10	15.7	11.5	13.6	14.5	-0.9

Figure 4.21. Comparison of mid-line and end station air temperatures.

4.5.5. Comparison of End Station and Mid-Line Air Temperatures

Radiosonde temperatures at 250 m above the launch site have been taken as mid-line temperature values between Ridge and Sheepfold. These mid-line temperatures are compared with simultaneous end station temperature measurements in Figure 4.21., the mean of the end temperatures at the mean time of the radiosonde flight have been used. The mid-line radiosonde temperatures are also indicated on the graphs of diurnal temperature change at the Ridge and Sheepfold stations (Figures 4.15. to 4.19.). The average agreement between the mean of instantaneous end station temperatures and mid-line radiosonde temperature was better than 1 ^{o}C, the largest recorded difference being 1.8 ^{o}C.

During the mornings the mean of end station temperatures were greater than mid-line temperatures, in the afternoon the converse was found to be true. These results agree well with meteorological theory; in the morning the air close to the ground surface warms more rapidly than the air remote from the surface. Consequently, near surface temperatures will be greater than free air temperatures. In the afternoon the free air cools less quickly than the air near the surface and the free air temperature will be the greatest. Flight numbers five and seven were made at midday and there was good agreement between end station and mid-line air temperatures. These results suggest that the mean of end station temperatures are most representative of the mean on-line temperature at midday when atmospheric temperatures are most stable. On sunny days this result should be treated with caution due to the large short term fluctuations of end station temperatures. However, on overcast days when radiant heating of the air at the end stations is a stable minimum such an assumption appears justifiable.

Agreement between theoretical and practical inter-station temperature differences correlates well with the agreement between end station and mid-line temperatures. This is to be expected, because if end station temperature anomalies are small then temperature measurements at the end stations will have a greater probability of being representative of the mean air temperature along the line.

Figure 4.22. gives the means of end station temperatures taken over periods of 10, 30 and 60 minutes either side of the time of the radiosonde flight. By taking mean temperatures over a period of time there is no improvement in the agreement between end station and mid-line temperatures. These results indicate that it is only necessary to record end station temperatures at the time of an E.D.M. measurement, and to obtain the best results, synchronised temperature measurements should be recorded during a series of accurate E.D.M. distance measurements.

FLIGHT NUMBER	DATE	MEAN TIME (hrs)	MID-LINE TEMPERATURE °C	MEAN OF END STATION TEMPERATURE MEASUREMENTS °C			
				INSTANTANEOUS	± 10 MINS	± 30 MINS	± 60 MINS
1	1.4.76	11.30	6.2	8.0	7.5	-	-
2	8.4.76	15.40	8.0	7.5	-	-	-
3	12.4.76	11.20	4.4	5.6	5.6	5.7	5.8
4	19.7.76	21.25	14.5	-	-	-	-
5	20.7.76	11.55	14.2	14.6	14.7	14.7	14.6
6	22.7.76	10.50	13.1	14.3	14.7	15.2	14.4
7	22.7.76	11.56	14.1	14.2	14.5	14.5	15.0
9	22.7.76	16.10	14.5	13.6	13.9	14.0	13.7

Figure 4.22. The effect of the duration of temperature recording on mean air temperature.

4.6. An Assessment of the Probable Accuracy of Derived Mean Atmospheric Parameters

The accuracy with which the mean ambient refractive index can be determined, along a line between two survey stations, controls the accuracy of the atmospheric correction to E.D.M. measurements, and therefore the ultimate accuracy of the distance measurement itself. From the theory and experimental work described in this chapter it is possible to draw conclusions concerning the necessary meteorological measurements and the accuracy with which they must be taken, to achieve the most representative mean value of refractive index.

The arithmetic mean of end station pressures will be accurate to better than ± 0.4 mb, if carefully calibrated barometers are used at both ends of the line and providing that the difference in level between the stations is less than approximately 300 m. The maximum possible error in mean atmospheric pressure would contribute an inaccuracy of less than ± 0.2 ppm to a corrected E.D.M. measurement. If sufficient care is taken in the calibration and field use of precision aneroid barometers the possible errors will not significantly affect the refractive index correction of high accuracy E.D.M. measurements.

Wet bulb temperature measurements at both ends of a survey line, with aspirated hygrometers, give a sufficiently accurate determination of humidity for the refractive index correction of accurate light wave E.D.M. measurements. The effect of humidity on the calculation of ambient refractive index is so small that no significant errors in the correction will be caused by inaccuracies in the evaluation of mean on-line wet bulb temperatures.

The Bryn Howel - Tyn y Wern experiment has shown that errors due to inadequate temperature sampling can be greater than ± 1 ppm on a line passing close to the ground over its complete length. The experimental results emphasised the importance of representative temperature measurements on a line which is within the lower 50 m of the atmosphere. In this region the heat transfers taking place at ground level can have an anomalous effect on air temperatures.

There are many conclusions, concerning the probable accuracy of the determination of mean air temperature, that can be made from the results of the Ridge - Sheepfold experiment. The radiosonde results indicate that an adiabatic, or near adiabatic, lapse rate of temperature is the norm in the lower atmosphere in which survey measurements are made. If ground level temperatures are measured beneath an E.D.M. line of sight when near surface temperature anomalies are a minimum then extrapolation of on-line air temperature is possible with an accuracy of approximately ± 0.2 ^{o}C ± 0.3 ^{o}C per 100 m difference in level. The near surface air temperature anomalies are a minimum in mid afternoon and are further reduced in overcast weather. The influence of the ground surface can cause anomalous air temperatures which extend to a height of 50 m.

Large variations of air temperature have been recorded at 1.5 m above the ground level at the end stations. These variations are greatest during weather with sunny intervals and least on overcast days. It has been suggested that the temperature variations are caused by the increased radiant heating during sunny intervals and also as a result of mass air movement.

A difference between the theoretical and the recorded temperature difference between end station air temperatures indicates the presence of end station temperature anomalies. A study of these temperature differences can indicate the suitability of end station air temperatures for the calculation of the mean on-line ambient refractive index.

The experimental results indicate that the mean of end station temperatures are most representative of on-line air temperatures during the middle of the day. The average disagreement between end station and mid-line radiosonde temperature measurements was approximately 1 ^{o}C, the mean of end station temperatures was greater than the free air temperature in the morning and less in the afternoon.

Air temperature measurements synchronised with E.D.M. measurements give the best values of mean air temperature for the calculation of the refractive index correction to distance measurements. There is no advantage in taking the mean air temperature over a period of time either side of the E.D.M. operations.

If E.D.M. measurements and the corresponding end station temperature measurements are taken in the middle of overcast days, with carefully calibrated thermometers, then the derived mean on-line air temperature, on a line such as Ridge to Sheepfold, will be accurate to ± 1 ^{o}C. This introduces an error of ± 1 ppm in the corrected EDM distance measurement. The inaccuracies in the derived mean pressure and humidity will cause errors in the corrected distance measurement of less than ± 0.2 ppm. Therefore, even with the greatest care in the derivation of the mean atmospheric parameters, on a line such as that between the Ridge and Sheepfold stations, errors of ± 1.2 ppm can occur in a corrected E.D.M. distance measurement due to the inadequacies of atmospheric sampling alone. In conclusion the following points, concerning the derivation of mean atmospheric parameters, should be considered when a high accuracy distance measurement is to be made with E.D.M. equipment. Only by taking the greatest care with meteorological measurements will it be possible to state the measured distance with confidence to an accuracy approaching ± 1 ppm.

(a) All the meteorological instruments must be calibrated before use, and at regular intervals thereafter.

(b) All meteorological observations should be made at the same time as the E.D.M. measurements.

(c) Atmospheric pressure should be measured at both ends of the line and the arithmetic mean used in the calculation of atmospheric corrections.

(d) Wet bulb temperatures should be measured at both ends of the line, with aspirated hygrometers, and the arithmetic mean used in atmospheric corrections.

(e) Thermocouple assemblies and automatic temperature recorders are recommended for end station dry bulb temperature measurements.

(f) All temperature measuring equipment should be shielded from direct sunlight.

(g) The difference between end station dry bulb temperatures should be compared with the theoretical difference to give an indication of the degree to which the mean of end station temperatures will represent the true mean on-line temperature.

(h) Equal care is necessary in obtaining representative dry bulb temperatures whatever the topography beneath the line of sight.

(j) E.D.M. measurements should be made in the middle of the day when end station temperatures are most representative.

(k) E.D.M. measurements should be made in overcast weather, when temperature anomalies are minimised.

(l) Wherever possible on-line temperatures should be measured with the aid of meteorological masts.

(m) Ground grazing lines of sight should always be avoided.

4.7. Barogenetic Temperature Determination

The rate of change of pressure with height is a well defined parameter, it depends on an accurate mean value of air temperature in the vertical column of air under consideration. By measuring air pressure at two survey stations, and knowing the difference in level between the stations, it is possible to determine the mean air temperature between the stations. An air temperature determined in this way is referred to as a 'barogenetic temperature'.

Saastamoinen [1965a]* outlines the principles of barogenetic

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* There are errors in this paper which are corrected in the correspondence of Survey Review, Number 137.

temperature calculations and explains that the method depends on the atmosphere between the two survey stations being in hydrostatic equilibrium. This is the case when adiabatic temperature lapse rates exist in the atmosphere. Bomford [1971] gives the rate of change of pressure with height as

$$\frac{dP}{dh} = - \frac{0.0342 \, P}{T(1 + 0.378e/P)} \cdot \frac{g}{g_{45}} \qquad \ldots 4.3.$$

which when integrated between the heights h_1 and h_2, at which the pressures are P_1 and P_2 respectively, gives

$$\ln . \frac{P_2}{P_1} = \frac{-0.0342 \, (h_2 - h_1)}{(g_{45}/g_m) \, T_m \, (1 + 0.378e/P)_m} \qquad \ldots 4.4.$$

where subscript m indicates the mean value of a parameter. This expression can be rewritten as,

$$T_m = \frac{0.0342 \, (h_2 - h_1)}{(g_{45}/g_m)(1 + 0.378e/P)_m \, \ln(P_1/P_2)} \qquad \ldots 4.5.$$

and if all the variables on the right hand side of the equation are known then the mean air temperature can be determined. The standard gravity, g_{45}, is equal to 9.806 65 ms^{-1} and the mean value of gravity between the two stations, g_m, can be calculated from [Brown, 1976],

$$gm = 9.780\,49 \, (1 + 0.005\,288\,38 \sin^2 \Phi - 0.000\,005\,9 \sin^2 2\Phi)$$

$$- 3.086 \times 10^{-6} \left(\frac{h_1 + h_2}{2} \right) \quad m.s^{-1} \qquad \ldots 4.6.$$

Great accuracy is required in the measurement of the air pressures, an error of ± 0.1 mb at one station introduces an error of ± 0.5 ^{o}C in the barogenetic temperature. Accuracy requirements for the other parameters are not so critical, however great care should be taken to obtain the levels of each station accurately. The following accuracies are necessary in the measured parameters to enable a barogenetic temperature to be determined to 1 ^{o}C (Figure 4.23.).

VARIABLE	ERROR	RESULTANT ERROR IN BAROGENETIC TEMP.
Pressure	± 0.1 mb	± 0.47 ^{o}C
Mean gravity	± 0.01 ms^{-2}	± 0.30 ^{o}C
Dry bulb temperatures	± 1.0 ^{o}C	± 0.04 ^{o}C
Wet bulb temperatures	± 1.0 ^{o}C	± 0.09 ^{o}C
Levels	± 0.1 m	± 0.05 ^{o}C
	TOTAL	± 0.95 ^{o}C

Figure 4.23. The necessary accuracies for barogenetic temperature calculations.

The limiting factor in barogenetic temperature calculations is the measurement of air pressures. It is most important that the pressure measurements are made at station level and thus correspond to the measured difference in level. The use of properly calibrated precision aneroid barometers is essential.

Accurate pressure measurements have been taken by the author at two Ordnance Survey triangulation stations, in Risley Park, Derbyshire. The distance between the stations was approximately 1400 m and the difference in level was 53.8 m. Wet and dry bulb temperatures were taken at each end of the line with aspirated hygrometers. The following table shows the results of the barogenetic temperature calculations for the Risley Park results (Figure 4.24.).

DATE	TIME (hrs)	MEAN D.B. TEMP. ^{o}C	BAROGENETIC TEMPERATURE ^{o}C	CLOUD COVER %	WIND
18.6.76	10.30	16.9	18.5	35	LIGHT BREEZE
21.6.76	10.00	17.3	16.4	20	LIGHT BREEZE
22.6.76	13.30	23.4	9.5	10	WINDY
28.6.76	10.00	23.3	20.3	0	CALM
30.6.76	16.00	27.0	9.8	0	STRONG WIND
14.7.76	15.00	21.9	9.0	40	WINDY

Figure 4.24. Barogenetic temperature results at Risley Park.

Three of the barogenetic temperatures agree well with the mean of end station dry bulb temperatures, and the values obtained are acceptable as possible mean on-line temperatures. However, there are large disagreements between the other barogenetic temperatures and the corresponding mean dry bulb temperatures. The good agreements were obtained on calm days, whereas the bad agreements occurred on days with strong winds. The presence of strong winds indicates that the air in the atmosphere is not in equilibrium, and therefore the theory of barogenetic temperatures is not applicable for the derivation of mean air temperature.

Even on calm days, the required accuracy of pressure measurements severely restricts the application of barogenetic temperature methods in connection with the accurate atmospheric correction of E.D.M. measurements. If the difference in level between two survey stations is unknown during an E.D.M. measurement then barogenetic temperatures cannot be calculated.

CHAPTER FIVE

THE EFFECT OF REFRACTION ON VERTICAL ANGLE MEASUREMENTS

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THE EFFECT OF REFRACTION ON VERTICAL ANGLE MEASUREMENTS

5.1. The Accurate Measurement of Vertical Angles

In a majority of survey projects the horizontal and vertical distances between stations are required for the calculation of the station co-ordinates. Using E.D.M. instruments the slope distance between survey stations can be measured with an accuracy approaching 1 mm per kilometre. To ensure accurate reduced distances either the difference in level, or the vertical angle, between the stations must be measured with an accuracy compatible with that of the slope distance measurement.

When survey lines are short, and there is an easy path between survey stations, precise spirit levelling is an acceptably accurate method for determining the difference in levels. In countries with a national survey network the levels of stations can be obtained by spirit levelling from bench marks or triangulation monuments. However, these stations may not be stable, or of sufficient accuracy, and a check levelling to correlate all the bench marks used would be necessary to ensure accurate results. When conventional levelling techniques are not practical the vertical angle between the survey stations can be measured and used in the reduction of slope distances.

To obtain accurate trigonometrical reduction of slope distances vertical angles should be measured with first order theodolites. Examples of such instruments are the Kern DKM3, which has direct reading to 0.5" with estimation to 0.1", and the Wild T3 with direct reading to 0.2". One second of arc (1") is the angle subtended by approximately 5 mm at a range of 1000 m; consequently an angular accuracy of ± 0.2" is compatible with a 1 mm positional accuracy at this range.

For the best results first order theodolites should be used mounted on survey pillars, or monuments, with constrained centring. If care is taken in selecting the sites of the survey pillars long term stability and precise repeatability of centring will be ensured. The same care should be taken to guarantee the stability and centring accuracy of theodolite targets. Centring errors are critical on short steep lines of sight. Although on a 1000 m line, inclined at 5^o to the horizontal, a combined centring error of the theodolite and target of ± 50 mm is required to introduce an error of ± 1" in the measured vertical angle.

Angle measurements from a theodolite to a target must be reduced to the vertical angle between the two survey station marks. This requires the accurate measurement of theodolites and target heights above the stations. As an example, for the line mentioned above, an error of ± 5 mm in either the height of the trunnion axis of the theodolite or the centre of the target above the respective station marks will introduce an error of ± 1" in the reduced station to station vertical angle. The reduction of vertical angles is considered in detail in section 6.2.; for the theory in this chapter the theodolite and target heights are assumed equal so that no reduction of the measured angle is necessary.

Clearly visible targets, with well defined centres, should be used to obtain the best vertical angle measurements. The design and colour of targets depends on the situation in which they are to be used and the background against which they are to be sighted.

Errors affecting vertical angle measurements can be classified as instrumental or non instrumental. The instrumental errors and the method of observing vertical angles are well documented in many publications [e.g. Bomford, 1971]. Refraction is the non instrumental error which introduces the greatest uncertainty into the measurement of vertical angles. The following section introduces the theory of the refractive bending of light, and gives a method of calculating the refraction error and the necessary correction to vertical angle measurements. Section 5.3. explains the corrections that have to be applied to vertical angle measurements because of the curvature of the earth's surface. Experimental measurements of vertical angles have been made in two test areas and the results of this work, with particular reference to refraction, are discussed in sections 5.4. and 5.5. The final section of the chapter covers in detail a method of calculating the effect of the deviation of the vertical on vertical angle measurements.

5.2. The Effect of Refraction on Vertical Angle Measurements

Fermat's Principle [Born and Wolf, 1970], known also as 'the principle of the shortest optical path' states that "the optical length

$$\int_A^B n \, dl \qquad \ldots 5.1.$$

of an actual ray between any two points A and B is shorter than any other curve that joins these points", where n is the refractive index and dl is a small part of the wave path. The principle is a statement that light waves will follow a path which requires the minimum energy expenditure.

Fermat's principle can be used to derive the equation for the curvature of an electromagnetic wave passing through a horizontally stratified atmosphere [Bomford, 1971] [Curl, 1974]

$$\frac{1}{\sigma} = - \frac{1}{n} \cdot \frac{dn}{dh} \cdot \cos \theta_R \qquad \ldots 5.2.$$

where σ is the radius of curvature of the wave path and θ_R is the inclination of the ray path to the horizontal. The refractive index gradient dn/dh can be derived by differentiating Barrell and Sears's expression for ambient refractive index (equation 1.11.) with respect to height h. Humidity is omitted from the calculations as it has an insignificant effect on the refraction of visible light waves.

$$n = \frac{(n_o - 1)}{\alpha \, T} \cdot \frac{P}{1013.25} \qquad \ldots 5.3.$$

The absolute refractive index, n_o, is used in the equation because the electromagnetic waves considered in vertical angle measurement are polychromatic (Figure 1.5.). Differentiating this equation we obtain an expression for the refractive index gradient,

$$\frac{dn}{dh} = 0.2696\,(n_o - 1)\left[\frac{1}{T} \cdot \frac{dP}{dh} - \frac{P}{T^2} \cdot \frac{dT}{dh}\right] \qquad \ldots\ 5.4.$$

This expression shows the dependence of refractive index gradient on the gradients of air pressure and temperature. The pressure gradient has been given in the following simplified form (equation 3.3.)

$$\frac{dP}{dh} = -\,0.0342\,\frac{P}{T} \qquad \ldots\ 5.5.$$

The temperature gradient in the atmosphere has been considered in detail in section 4.5., and it has been suggested that the adiabatic lapse rate is the norm, but deviations from this value are common in unstable atmospheric conditions.

If the gradients of pressure and temperature, and therefore refractive index, are constant in the air between two survey stations A and B (Figure 5.1.), the line of sight between a theodolite at A and a target at B will be an arc of a circle of radius σ. If the gradients of the atmospheric parameters change in the vertical interval between the stations, the ray path will no longer be an arc of a circle, but an irregular curve. The definition of this curve requires knowledge of the ambient temperature and pressure gradients over the entire length of the ray path. These measurements would be extremely difficult if not impossible to obtain.

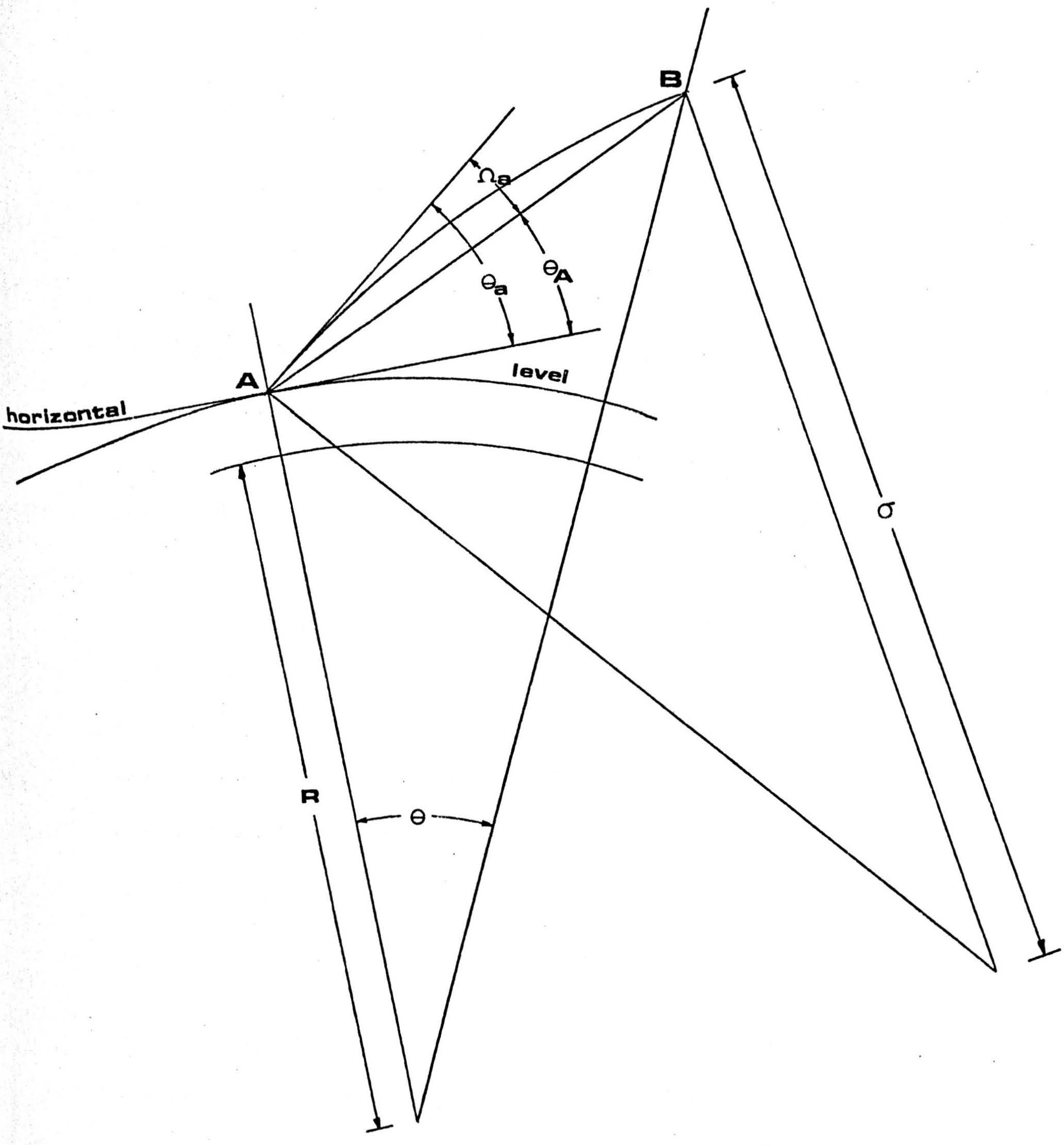


Figure 5.1. The effect of refraction on a vertical angle measurement.

The <u>linear</u> coefficient of refraction, K, is a measure of the curvature of a ray path. The coefficient is defined as the ratio of the radius of curvature of the earth, R, to the radius of curvature of the ray path, σ,

$$K = \frac{R}{\sigma} \qquad \text{... 5.6.}$$

A typical value of the linear coefficient of refraction is 0.16, which corresponds to a uniform adiabatic temperature lapse rate in typical atmospheric conditions.

The vertical angle measured with a theodolite is the angle subtended between the tangent to the path of light, from the survey target, and the horizontal at the instrument. In Figure 5.1. the upward measured angle θ_a is too great because of refractive bending of the ray path. The measured angle is in error by Ω_a, which is known as the angle of refraction. The vertical angle at A must be corrected for refraction to give the angle θ_A which is subtended between the chord AB and the horizontal at A.

$$\theta_A = \theta_a - \Omega_a \qquad \text{... 5.7.}$$

When a vertical angle is measured below the horizontal, as for example from B in Figure 5.2., the refraction error is negative and a positive correction must be applied.

$$\theta_B = \theta_b + \Omega_b \qquad \text{... 5.8.}$$

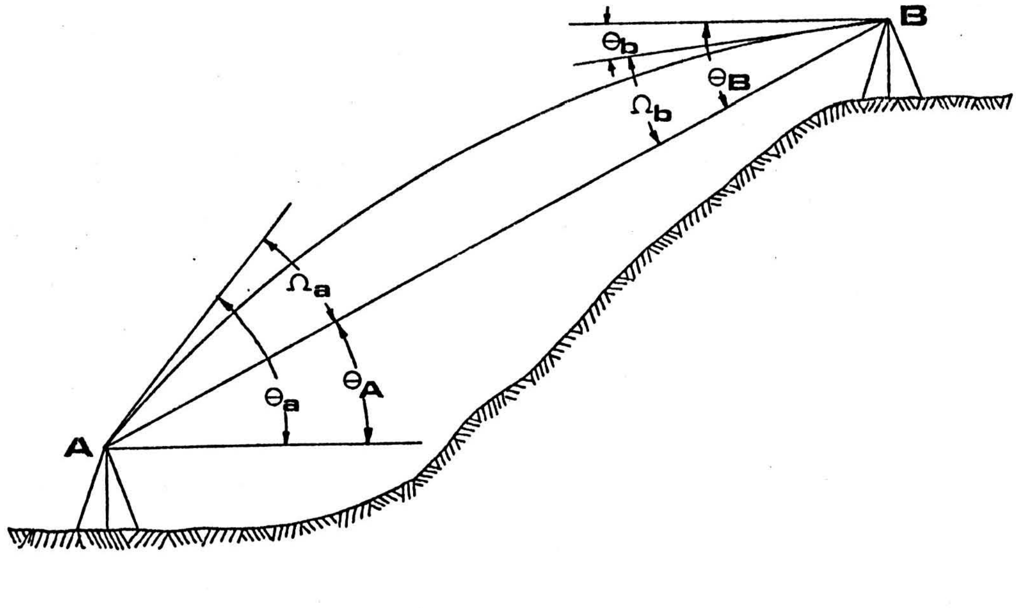


Figure 5.2. Reciprocal vertical angle measurements.

(Ignoring curvature of the earth)

The mean of simultaneous reciprocal vertical angle measurements between A and B will cancel the errors due to refraction if the angle of refraction at A is equal to that at B.

A further measure of the refractive bending of a ray is the <u>angular</u> coefficient of refraction, k, which is the ratio of the angle of refraction to the angle subtended by the two stations at the centre of the earth.

$$k = \frac{\Omega}{\theta} \qquad \text{... 5.9.}$$

The angular coefficient is equivalent to one half of the linear coefficient of refraction (Appendix E).

$$k = \frac{K}{2} \qquad \text{... 5.10.}$$

Equation 5.6. can be rearranged to give,

$$\frac{1}{\sigma} = \frac{K}{R} \qquad \text{... 5.11.}$$

and substituting from equations 5.9. and 5.10. we find,

$$\frac{1}{\sigma} = \frac{2\Omega}{R\theta} \qquad \text{... 5.12.}$$

This expression can be equated with equation 5.2. to give,

$$\Omega = -\frac{\theta R}{2n} \cdot \frac{dn}{dh} \cdot \cos \theta_R \qquad \ldots\ 5.13.$$

which will enable the calculation of the angle of refraction for a vertical angle measurement, if the atmospheric parameters can be obtained and providing a uniform refractive index gradient exists. The angle of refraction in this calculation will be in the same units as the value of the angle subtended at the centre of the earth. A typical value of the angle of refraction, assuming an adiabatic temperature lapse rate, is approximately 2.6" for a one kilometre line of sight.

Refraction of the ray path can introduce errors into horizontal, as well as vertical, angle measurements. Since horizontal changes of pressure in the atmosphere are negligibly small, it is only horizontal temperature gradients that cause refractive bending. Horizontal temperature gradients in free air are very small and only when a ray passes close to a surface, where large temperature gradients exist, will there be appreciable refractive bending of the ray path. Consequently if ground grazing lines of sight are avoided the effect of refraction will not introduce significant errors in horizontal angle measurements.

5.3. The Effect of Earth Curvature on Vertical Angle Measurements

When a theodolite is used at a station, A, in Figure 5.3., the instrument's bubble is set to the level at A and the horizontal at A is established as the reference plane for vertical angle measurements.

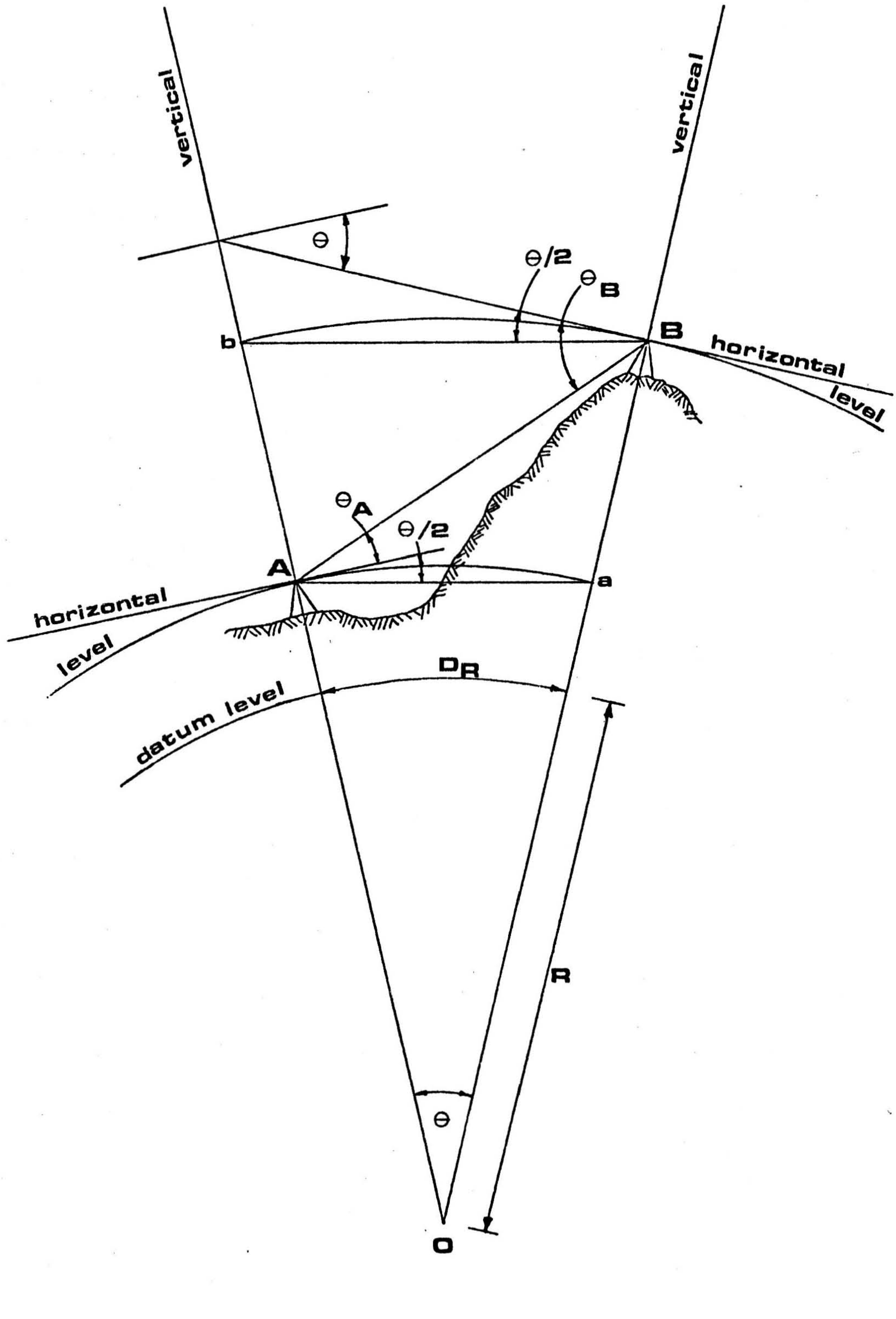


Figure 5.3. The effect of earth curvature on vertical angle measurement.

(not to scale)

Similarly the horizontal at a second station B would be the reference place for vertical angles measured from B. The horizontals at A and B will be inclined towards each other at an angle θ, where θ is the angle subtended by A and B at the centre of the earth, O. Consequently the vertical angles measured from A to B and from B to A will differ by an angle θ.

The vertical angles measured from A and B must be related to common reference planes in order that the slope distance AB can be reduced trigonometrically. These planes are parallel to the horizontal at the mid point of the line AB and pass through stations A and B, they are indicated on Figure 5.3. as Aa and Bb respectively. To calculate the corrected vertical angle, θ_{CA} at A a correction of $+\theta/2$ must be applied to the measured angle, a correction of $-\theta/2$ has to be made to the downward vertical angle at B, to give θ_{CB}. Therefore

$$\theta_{CA} = \theta_A + \theta/2 \quad \text{and} \quad \theta_{CB} = \theta_B - \theta/2 \qquad \text{... 5.14.}$$

If reciprocal angles are taken between A and B the calculation of the angle subtended at the centre of the earth is unnecessary, a mean of the measured angles at A and B will give the corrected vertical angle. Still assuming that all theodolite and target heights are the same

$$\theta_C = \frac{\theta_{CA} + \theta_{CB}}{2} = \frac{\theta_A + \theta/2 + \theta_B - \theta/2}{2} = \frac{\theta_A + \theta_B}{2} \qquad \text{... 5.15.}$$

When vertical angles are measured from only one end of a survey line it is necessary to calculate the angle θ subtended by the line at the centre of the earth in order to obtain the curvature correction, $\theta/2$. Now

$$\theta = \frac{D_R}{R} \quad \text{radians} \qquad \ldots\ 5.16.$$

where D_R is the spheroid distance between A and B and R is the local radius of the earth (Figure 5.3.). In engineering survey where distances between survey stations are less than 2000 m, and the vertical angle is less than 8^{o} above the horizontal, the slope distance can be used in equation 5.16. On steeper lines an approximate plan distance should be used in the calculation.

The curvature correction to a vertical angle on a 1000 m line is approximately 16", the correction is directly proportional to the length of the line.

The mean radius of the earth at a point is given by the following formula [Bomford, 1971]

$$R = \frac{a\ (1 - e^2)^{\frac{1}{2}}}{(1 - e^2 \sin^2 \Phi)} \qquad \ldots\ 5.17.$$

where a = the principle radius of Airy's spheroid = 6 377 563 m,

e = the eccentricity of Airy's spheroid = 0.081 66,

and Φ = the latitude of the point.

The height of survey stations above sea level will not have a significant effect on the calculation of the curvature correction to vertical angle measurements.

5.4. Vertical Angle Measurements in Risley Park, Derbyshire

A line between two Ordnance Survey triangulation stations in Risley Park, Derbyshire, has been used to investigate the effect of refraction on vertical angle measurements (Plate 5.1.). The lower station is named Risley and the upper station is Sandiacre, the plan length of the line is 1445 m and the difference in level is approximately 54 m.

The vertical angle measurements were taken with a Kern DKM3 theodolite (No. 141599). A special adaptor had to be made to mount the theodolite onto the pillars. Because the pillar tops were not sufficiently level the maximum travel of the levelling cams of the DKM3 was insufficient to permit levelling of the instrument when it was mounted directly onto the pillar. To overcome this problem a Wild tribach, which has a large movement of the levelling screws, was incorporated into the adaptor (Plate 5.2.). The adaptor ensured that the theodolite could be repeatedly centred on the pillars and that the trunnion axis of the instrument was always at the same level. The levelling screws of the Wild tribach were always set in the same position.

A specially designed perspex target (Plate 5.3.) was used at Risley Park; the target disc was 0.45 m in diameter and was sprayed with bright orange paint. Two horizontal black lines were marked on the target for vertical angle measurement, the line spacing was designed so that the theodolite cross hairs closed the gap between the lines when the target was sighted from the opposite end of the Risley/Sandiacre line with the DKM3. The target was also mounted onto the pillars by an

PLATE 5.1.

Ordnance Survey map of Risley Park showing

the Sandiacre - Risley survey line.

Scale 1 : 40000

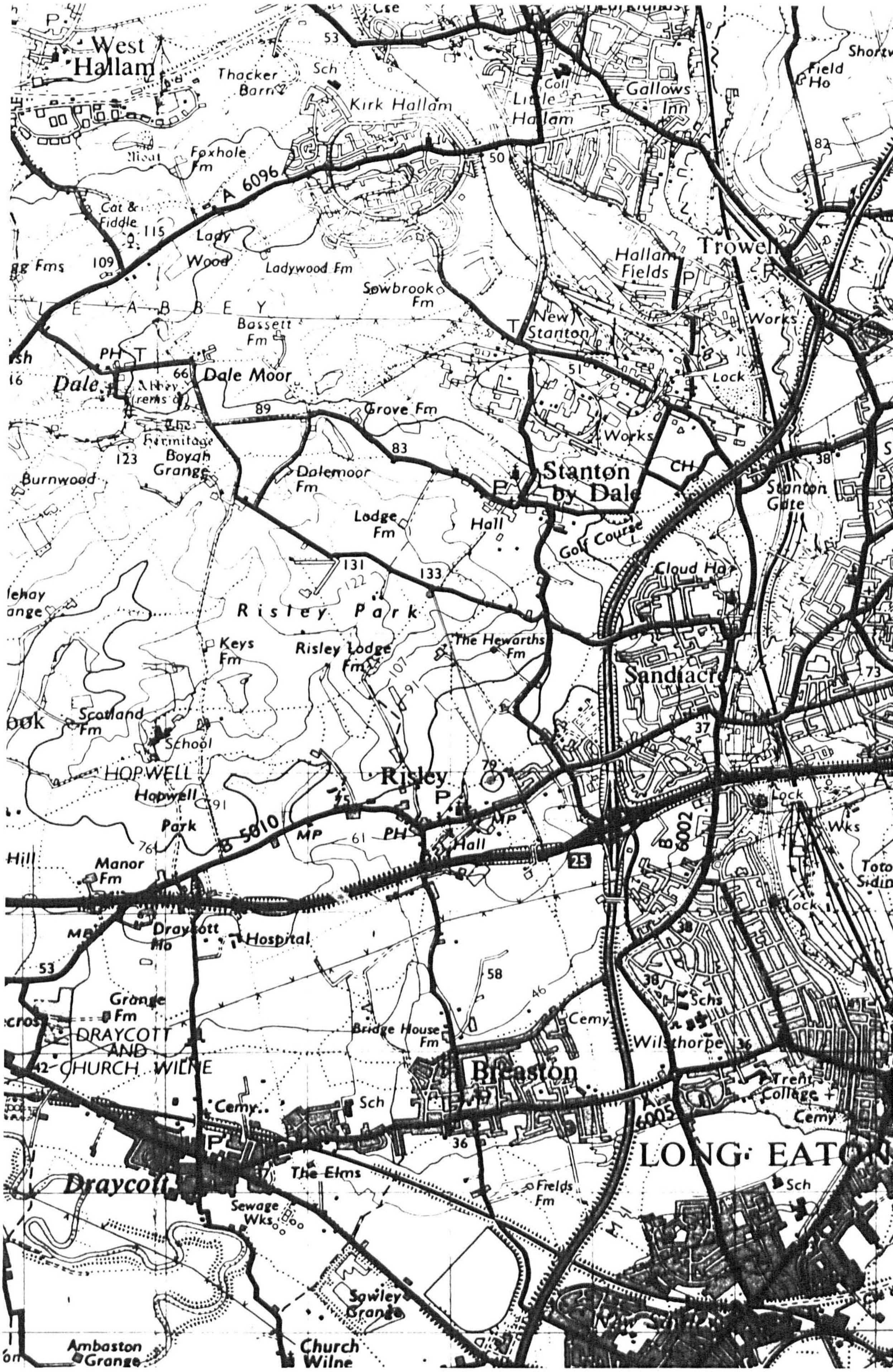

West Hallam
Thacker Barn
Sch
Kirk Hallam
Coll
Little Hallam
Gallows Inn
Field Ho
Foxhole Fm
A 6096
Cat & Fiddle
Lady Wood
Ladywood Fm
Sowbrook Fm
Trowell
Hallam Fields
Works
Bassett Fm
New Stanton
Dale
Dale Moor
Abbey (rems of)
Lock
Grove Fm
The Hermitage
Boyah Grange
Burnwood
Dalemoor Fm
Stanton by Dale
Stanton Gate
CH
Lodge Fm
Hall
Golf Course
Cloud Ho
Risley Park
Keys Fm
Risley Lodge Fm
The Hewarths Fm
Sandiacre
Scotland Fm
School
HOPWELL
Hopwell Park
Risley
B 5010
Hall
Wks
Manor Fm
Draycott Ho
Hospital
Grange Fm
DRAYCOTT AND CHURCH WILNE
Bridge House Fm
Cemy
Wilsthorpe
Breaston
Trent College
Sch
A 6005
LONG EATON
Draycott
The Elms
Fields Fm
Sewage Wks
Sawley Grange
Ambaston Grange
Church Wilne

PLATE 5.2.

The Kern DKM3 theodolite and adaptor, Risley Park.

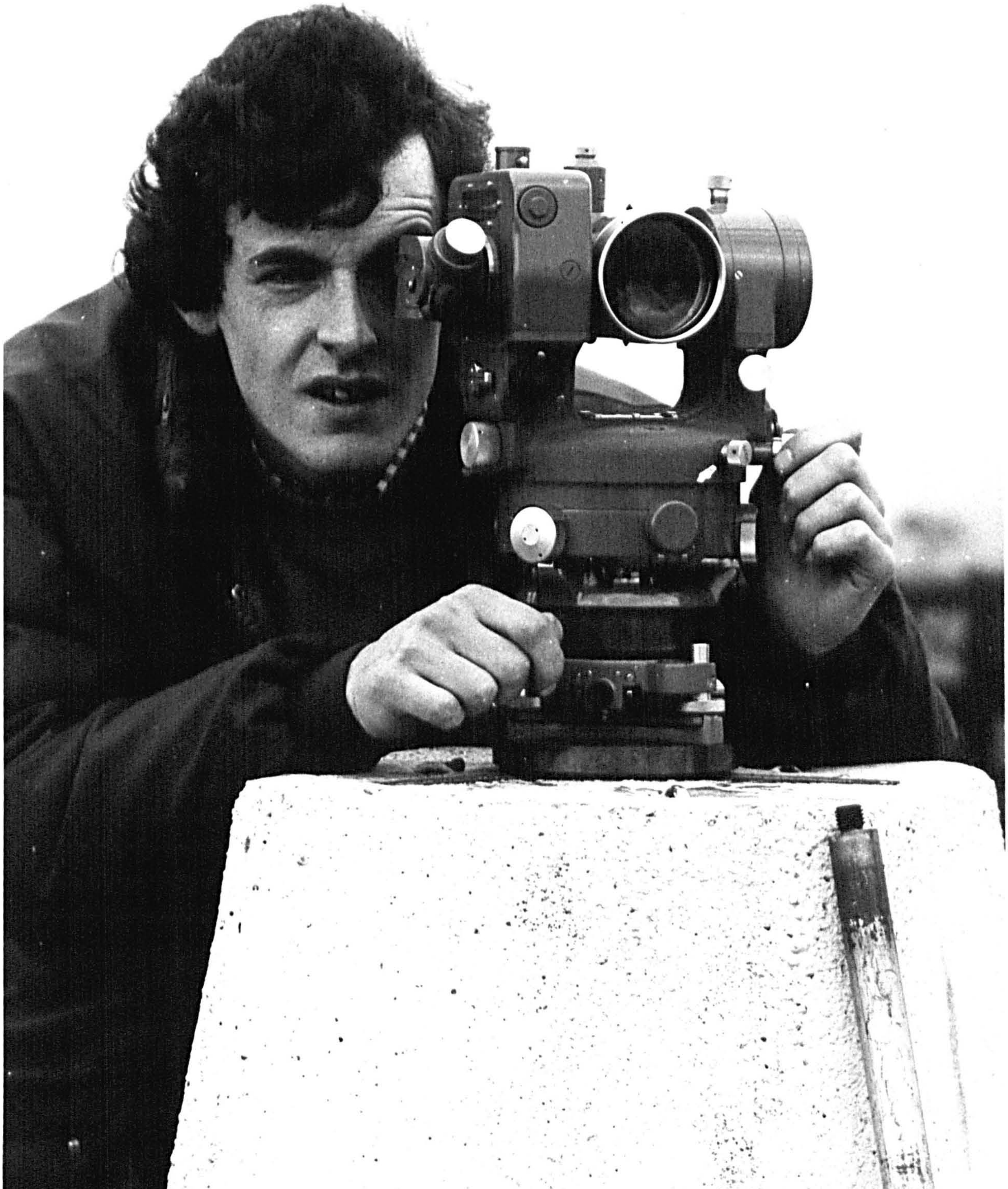


PLATE 5.3.

Survey target on the Sandiacre O.S. pillar, Risley Park.

adaptor that threaded into the pillar tops.

For the purposes of the vertical angle measurements the top surface of the theodolite adaptor set on the Risley pillar was considered as the zero datum of level. The top surface of the target adaptor at Sandiacre was the level at this station. The plan position of each station was defined as being at the centre of each adaptor. When the DKM3 was set on the Risley pillar the target was set, at the same height as the theodolite trunnion axis, on the Sandiacre pillar. However, when the theodolite was set on the Sandiacre pillar, the target at Risley was set 44 mm higher to compensate for the different heights of the two adaptors (Figure 5.4.). The target was mechanically located at each height to ensure repeatability of position.

A precise spirit levelling was made between the two stations with a Cook Troughton and Simms S500 level, fitted with a parallel plate micrometer. An invar staff was used and the double setting method of levelling was adopted. The difference in level measured between the stations was

Difference in level = 53.8007 m

The maximum probable error in the difference in level is ±10 mm, the closing error of the levellings was 2.4 mm.

The slope distance between the two stations was measured with a Tellurometer MA100 E.D.M. instrument (No. 366) and reduced to plan, using the difference in level, to give

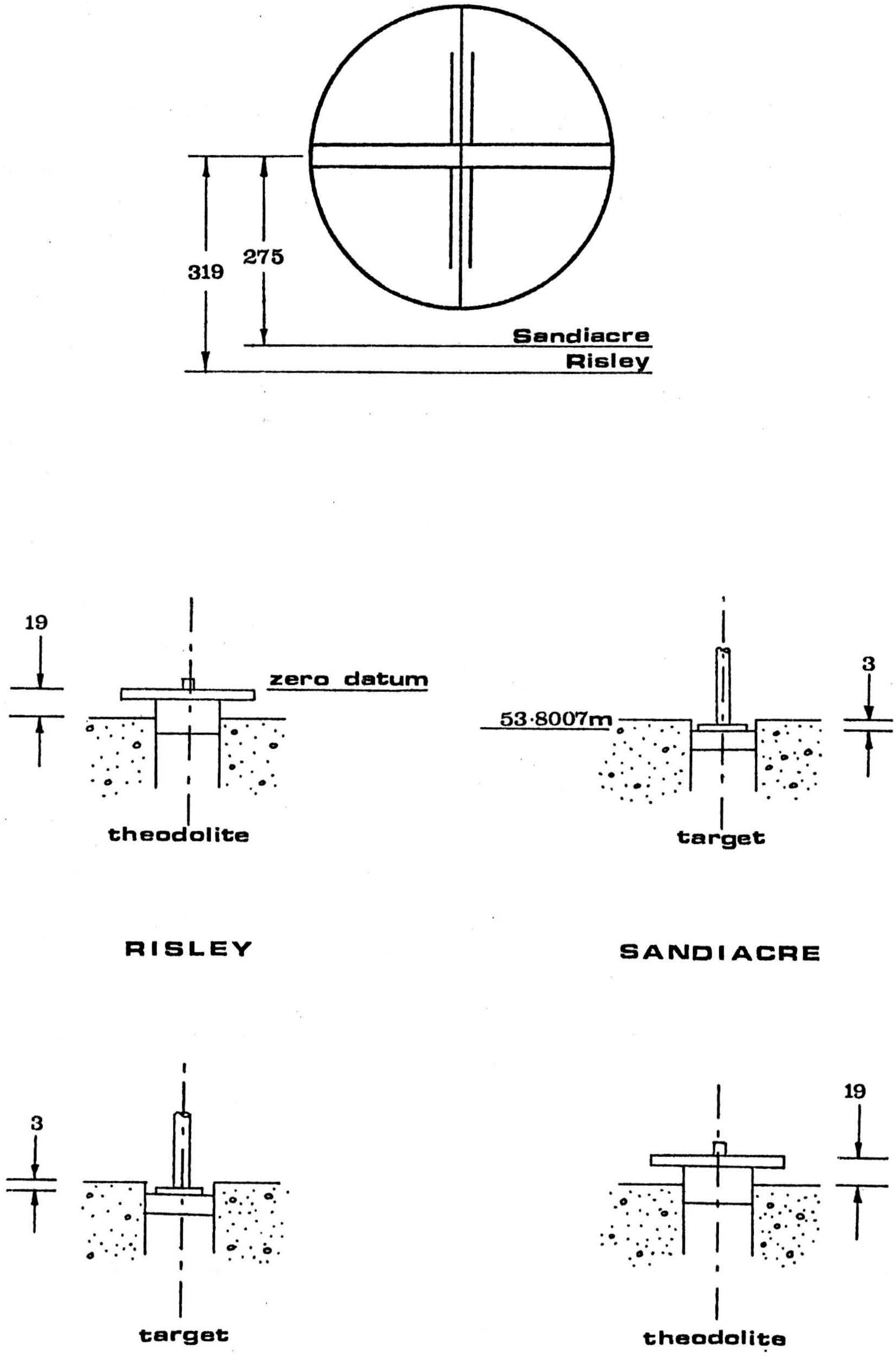


Figure 5.4. Dimensions of adaptors and target heights at Risley Park, all dimensions in millimetres.

Plan distance = 1445.0848 m

The reduced vertical angle between the two stations was determined from the plan distance and difference in level (Figure 5.5.) using

$$\theta_R = \text{arc tan}\left(\frac{\text{difference in level}}{\text{plan distance}}\right)$$

to give

$$\theta_R = 02^\circ \ 07' \ 55.7''$$

The possible errors in the slope distance measurement would not be great enough to introduce any significant errors into the derived angle, but the inaccuracies in the levelling give a possible error of ± 1" to the vertical angle. When the curvature of the earth has been considered the theoretical angles which should be measured by a theodolite at each station are,

Risley to Sandiacre $= -02^\circ \ 07' \ 32.4''$

Sandiacre to Risley $= 02^\circ \ 08' \ 19.1''$

The vertical angles between the two stations have been measured on eleven occasions, each time the angle was measured from both ends of the line with twenty face left and twenty face right readings. The mean of the measured angles are shown in Figure 5.6. It should be noted that the uphill observations are more consistent than the downhill. The difference between the measured and the theoretical vertical angle at each

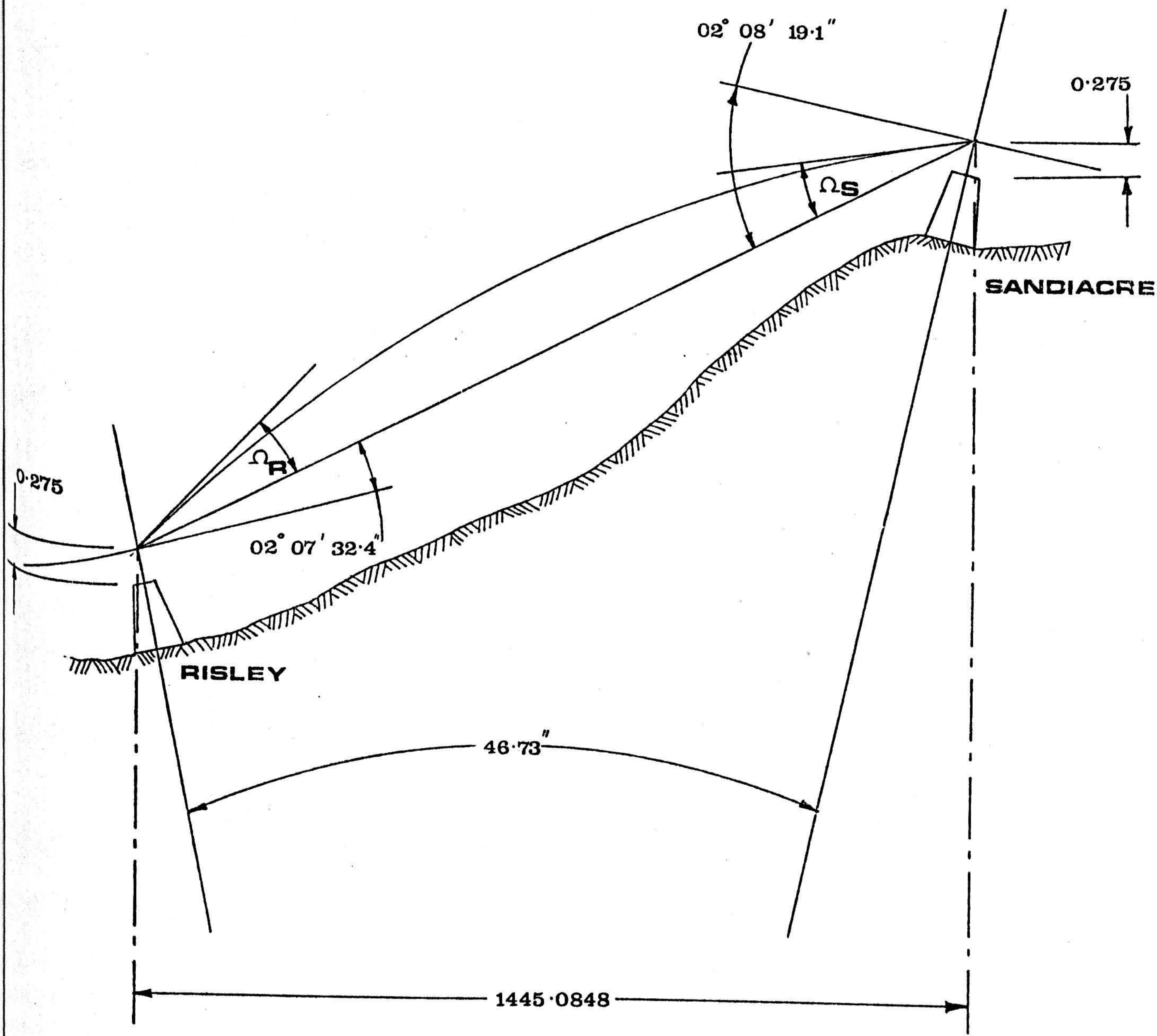


Figure 5.5. Dimensions of the Risley Park line, all distances in metres.

(not to scale)

DATE	MEAN MEASURED VERTICAL ANGLE					
	RISLEY TO SANDIACRE (+)			SANDIACRE TO RISLEY (-)		
	o	'	"	o	'	"
19.5.76	02	07	33.8	02	08	15.2
24.5.76	02	07	30.2	02	08	23.5
02.6.76	02	07	34.2	02	08	21.9
15.6.76	02	07	34.7	02	08	16.1
16.6.76	02	07	35.0	02	08	19.1
18.6.76	02	07	33.0	02	08	27.7
21.6.76	02	07	32.8	02	08	28.8
22.6.76	02	07	29.5	02	08	26.5
28.6.76	02	07	30.2	02	08	25.7
30.6.76	02	07	31.2	02	08	29.9
14.7.76	02	07	30.2	02	08	29.1

Figure 5.6. Table of vertical angle measurements at Risley Park.

end of the line is due to the refractive bending of the ray path and is equal to the angle of refraction. The angles of refraction and other experimental data are given in Figure 5.7., the standard error of the mean measured vertical angle is given as a measure of the precision of each angle measurement. The standard errors are greatest on hot days when there was a lot of shimmer in the theodolite field of view, and the precise sighting of the target was difficult.

The results from the Risley Park measurements indicate the possible variation of refraction errors in vertical angle measurement due to changes in atmospheric conditions. The theoretical angles of refraction have been calculated from the ambient atmospheric conditions at the time of each measurement (section 5.2.), an adiabatic temperature lapse rate has been assumed in these calculations. The differences between the practical and the theoretical angles of refraction are an indication that non-adiabatic lapse rates existed in the air at the time of the measurements. The agreement between theory and practice was best on overcast days when near adiabatic lapse rates could be expected. However, on occasions when there was little or no cloud cover large negative angles of refraction were recorded, these values correspond to temperature lapse rates equivalent to $-5^{o}C/100$ m. Such lapse rates are certainly possible on hot sunny days in air near to the ground surface.

Steep temperature lapse rates need only be present over small sections of the ray path to cause large errors in vertical angles due to refractive bending. Such temperature gradients are most likely to occur near to the ground surface at either end of the line of sight and for

DATE	STANDARD ERROR OF MEAN VERTICAL ANGLE (")	ANGLE OF REFRACTION (")				PRESSURE (mb)	TEMPERATURE (°C)		CLOUD (%)	WIND
		AT RISLEY	AT SANDIACRE	MEAN	THEORETICAL		DRY BULB	WET BULB		
19.5.76	0.52	+1.42	+ 3.86	+2.64	+3.53	993.7	10.1	-	100	STRONG WIND
24.5.76	0.62	-2.18	- 4.42	-3.30	+3.32	1005.1	20.4	-	0	FRESH BREEZE
2.6.76	0.81	+1.87	- 2.79	-0.46	+3.46	1010.1	15.2	-	90	LIGHT BREEZE
15.6.76	0.55	+2.28	+ 2.97	+2.62	+3.43	1004.7	15.5	13.7	100	LIGHT BREEZE
16.6.76	0.49	+2.66	+ 2.67	+2.66	+3.49	1007.4	12.8	11.2	100	LIGHT BREEZE
18.6.76	0.78	+0.62	- 8.62	-4.00	+3.39	1002.5	16.9	14.3	35	FRESH BREEZE
21.6.76	1.10	+0.45	- 9.68	-4.61	+3.36	1004.8	17.2	12.9	20	LIGHT BREEZE
22.6.76	0.83	-2.92	- 7.43	-5.17	+3.27	1008.7	23.4	17.6	10	LIGHT WIND
28.6.76	0.73	-2.20	- 6.65	-4.42	+3.32	1014.3	23.3	16.9	0	CALM
30.6.76	0.62	-1.19	-10.80	-5.99	+3.24	1014.3	27.0	18.0	0	STRONG WIND
14.7.76	1.32	-2.17	-10.06	-6.11	+3.30	1000.3	21.9	15.0	40	WIND

Figure 5.7. Results from the Risley Park Vertical Angle Measurements.

this reason survey stations are best sited on steep slopes where a minimum part of the ray path is close to the surface. Ground grazing lines of sight should always be avoided for the same reasons.

The Risley Park results show how difficult it is to obtain accurate vertical angle measurements on lines passing close to the surface, even when the greatest care has been taken to ensure precision. The effects of refraction are the greatest source of uncertainty in precisely measured vertical angles. However, if measurements are made on overcast days, when temperature lapse rates are steady and near adiabatic, the effects of refraction will be minimised. The observation of reciprocal vertical angles will further reduce the effects of refraction; but will not eliminate them unless the temperature gradient is constant and the ray path is perfectly symmetrical.

5.5. Vertical Angle Measurements in Llangollen, North Wales

All the vertical angles in the Llangollen survey network have been measured reciprocally with a Kern DKM3 first order theodolite. Specially designed targets were used for the vertical angle measurement (Plate 5.4.). The centre of the two horizontal bars of the target was set permanently at 1.500 m above the survey station level, to reduce errors introduced by inaccurate measurement of target height. Great care was taken in the measurement of the height of the theodolite trunnion axis above the instrument station level. The instrument height was read directly from the plumbing rod, and to give a check the distance from the station mark to the edge of the tripod top was measured (Plate 5.5.)

PLATE 5.4.

Llangollen survey target at the

Escarpment station.

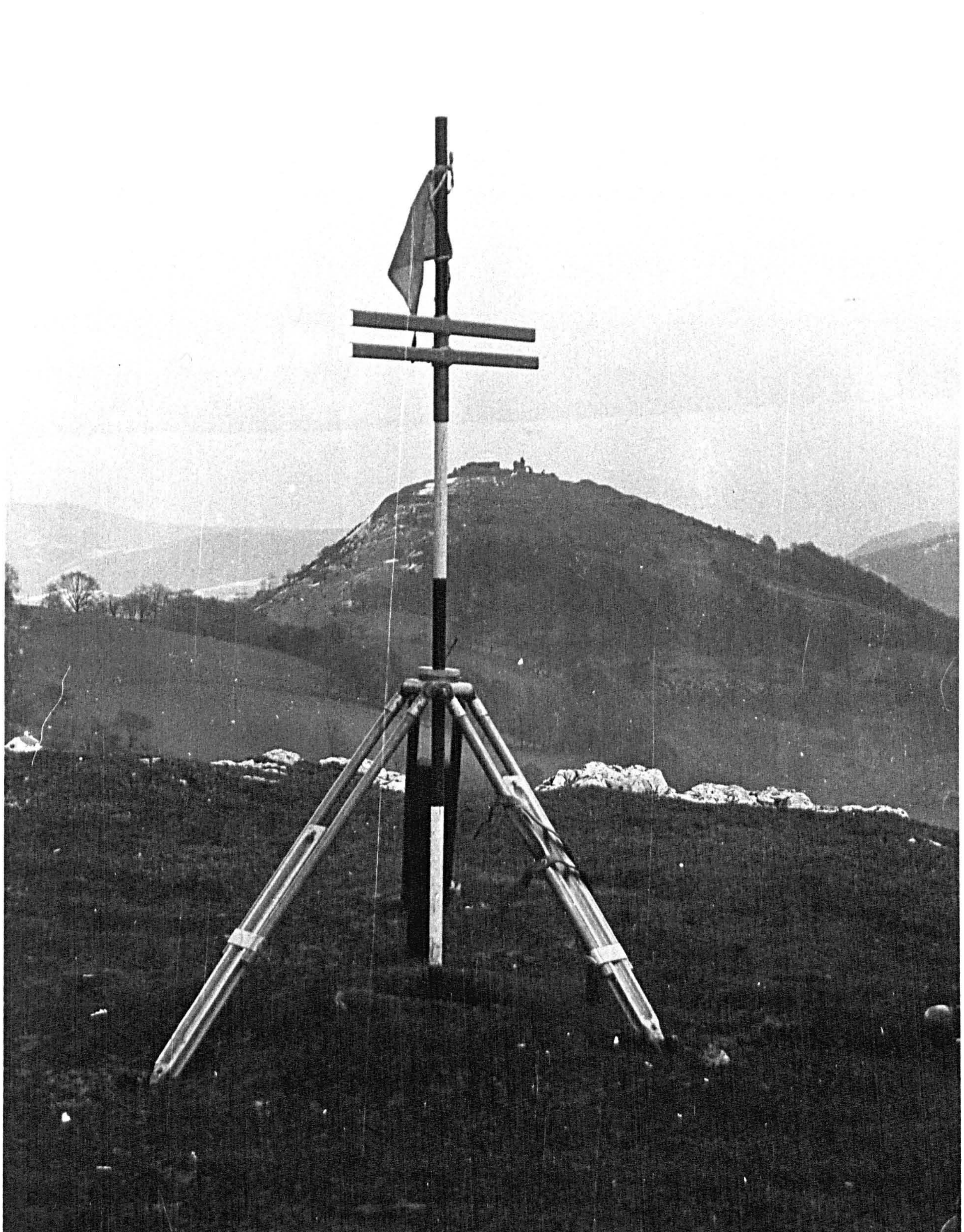


PLATE 5.5.

The Kern DKM3 theodolite at the

Ridge Station.

and the instrument height calculated. The required dimensions of the theodolite were carefully measured in the laboratory. The agreement between the calculated and the plumbing rod height was always better than 2 mm.

A minimum of five, and on average eight, vertical angle measurements were made with both face left and right readings from each end of each line in the network. It was impractical to measure simultaneous reciprocal vertical angles on each of the lines, however, measurements were taken in the middle of overcast days when the atmosphere has been found to be most stable. For the measurements between Ridge and Sheepfold and Bryn Howel and Tyn y Wern, readings were made from one end of the line and then as soon as possible from the other end. During these measurements the sky was completely overcast and it was considered that these readings would be almost as good as simultaneous reciprocal vertical angle measurements.

Figure 5.8. tabulates the results of the vertical angle measurements in Llangollen. The angle θ subtended at the centre of the spheroid has been calculated using the method given in section 5.3. The angle of refraction Ω has been obtained from,

$$\Omega = \frac{\theta - (\theta_b - \theta_a)}{2} \qquad \text{... 5.18.}$$

which is derived from equations 5.7., 5.8. and 5.14. The linear and angular coefficients of refraction have also been given. Because they are independent of the length of the survey line they should be considered when comparing the effects of refraction on the different measurements.

LINE	MEAN VERTICAL ANGLE				PLAN DISTANCE	ANGLE AT CENTRE OF SPHEROID (θ)	ANGLE OF REFRACTION			COEFFICIENT OF REFRACTION		MEAN HEIGHT OF LINE ABOVE THE SURFACE
	UPWARD SIGHT (θ_a)	DOWNWARD SIGHT (θ_b)	DIFF. ($\theta_b - \theta_a$)	MEAN	m		MEAN	THEORETICAL	DIFF.	ANGULAR k	LINEAR K	m
	° ' "	° ' "	"	° ' "		"	"	"	"			
TB	00 16 41.7	00 17 43.5	61.8	00 17 12.6	2011.838	65.0	+1.6	+5.3	3.7	0.025	0.050	10
TS	10 06 05.1	10 06 48.0	42.9	10 06 26.6	1649.581	53.3	+5.2	+4.3	0.9	0.098	0.196	90
TC	09 19 21.7	09 19 53.6	31.9	09 19 37.7	1239.037	40.0	+4.0	+3.2	0.8	0.100	0.200	50
TE	10 11 53.2	10 12 32.9	39.7	10 12 13.1	1450.833	46.9	+3.6	+3.8	0.2	0.077	0.154	70
TR	06 48 09.4	06 43 46.4	37.0	06 48 27.9	1414.907	45.7	+4.4	+3.7	0.7	0.096	0.192	60
CE	04 26 48.8	04 27 20.0	31.2	04 27 04.4	740.819	23.9	-3.6	+1.9	5.5	-0.151	-0.302	60
RC	01 12 09.4	01 12 56.2	46.8	01 12 32.8	1638.181	52.9	+3.0	+4.3	1.3	0.057	0.114	80
BR	06 30 59.7	06 31 37.5	37.8	06 31 18.6	1389.396	44.9	+3.5	+3.6	0.1	0.078	0.156	35
RS	02 55 31.1	02 56 39.5	68.4	02 56 05.3	2440.651	78.9	+5.2	+6.4	1.2	0.066	0.132	160
RE	04 46 46.3	04 47 19.3	33.0	04 47 02.8	1101.975	35.6	+1.3	+2.9	1.6	0.036	0.072	55
BS	08 29 00.7	08 29 55.0	54.3	08 29 27.9	1902.317	61.5	+3.6	+5.0	1.4	0.058	0.116	60

Figure 5.8. Vertical angle results for the Llangollen survey network.

The variation of the coefficients of refraction for different lines is clearly shown by the results. However, a change of ± 0.03 in the angular coefficient causes a change of only one second in the angle of refraction. In general the derived mean angles of refraction are in good agreement with the theoretical angles calculated from equation 5.13. by assuming a temperature lapse rate of -1 ^{o}C/100 m. The mean difference between the theoretical and measured angles of refraction was 1.6", and 1.2" if the spurious result for the line CE is neglected.

Consideration of the theoretically derived expression for the angle of refraction (equation 5.13.) indicates that the angle of refraction and therefore the coefficients of refraction depend on the cosine of the vertical angle. Consequently, the greater the vertical angle of a line, the less will be the error due to refractive bending of the ray. The vertical angles between the stations of the Llangollen network vary from zero to ten degrees above the horizontal, this gives a maximum theoretical change in the angle of refraction of two per cent which corresponds to less than 0.1" on a 1 km line. The practical results of vertical angle measurement show no clearly defined relationship between the vertical angle and the magnitude of the refractive bending of the ray path. It can be concluded that within the range of angles commonly encountered in engineering survey, say less than thirty degrees, the vertical angle of the line has no significant effect on the errors due to refraction.

Morley [1937], whose work concerned lines of 20 to 50 km in length, suggested that there was a decrease in the coefficient of refraction the closer the line of sight passed to the ground. The results of the Llangollen measurements, on lines of 750 m to 2500 m, show no clear relationship between the mean height of the line above the ground surface and the coefficient of refraction. However, a very small value of the coefficient of refraction was obtained from the measurement of the line Tyn y Wern to Bryn Howel (TB) which passes very close to the surface. High and inconsistent temperature lapse rates frequently occur on such lines, especially in sunny weather, and anomalous vertical angle readings are probable.

In conclusion it has been shown that the mean height above the ground surface and the inclination of a line have little effect on the refraction errors in vertical angle measurement. When stable adiabatic lapse rates of temperature exist along the entire ray path the angle of refraction is easily calculated, however, if variations of the lapse rate exist the calculated correction will be in error. The Radiosonde measurements conducted by the author (Chapter Four) have shown that, during the middle of the day, the adiabatic temperature lapse rate is the norm in free air, away from the influence of ground effects. Therefore the most accurate vertical angle measurements will be those where a majority of the line is in free space and no anomalous temperature gradients exist near to the surface at either the instrument or target station. These conditions are best satisfied on overcast afternoons.

When high accuracy vertical angle measurements are required in an engineering survey network the previous factors should be considered in selecting the sites for the stations. Vertical angle measurements should be taken simultaneously, or as near to simultaneously as is possible, and the mean angle of refraction derived. This value should be compared with the theoretical angle of refraction, which requires the measurement of pressure and air temperature during the vertical angle measurement. If there is good agreement between the measured and calculated angle of refraction the mean of reciprocal vertical angles can be used with greater confidence for the reduction of E.D.M. measured distances.

In conclusion, the following points should be considered when high accuracy vertical angle measurements are required:

(a) A first order theodolite, in good adjustment, should be used.

(b) Face left and face right observations are essential.

(c) The theodolite should be stable and ideally pillar mounted.

(d) The theodolite and the pillar or tripod, should be sheltered from the sun.

(e) A well defined target should be used, illuminated if necessary.

(f) Both instrument and target heights should be carefully measured.

(g) Reciprocal observations should be taken, ideally simultaneously; if not, from one end of the line and immediately afterwards from the other end.

(h) If simultaneous observations are not possible, observations should be taken from each terminal station of a line in turn, without sighting to other stations in a network.

(j) The theodolite should be repointed a minimum of five times.

(k) With micrometer instruments at least four settings should be taken and recorded for each pointing.

(l) The bubble should be reset for each pointing.

(m) Observations should be taken in overcast weather.

(n) Observations should be taken in the early afternoon to minimise refraction.

(o) Air pressure and temperature should be measured at each end of a line to enable theoretical refraction calculations.

(p) Ground grazing lines of sight should be avoided.

(q) Possible deviation of the vertical at each station should be considered (section 5.6.).

(r) Errors due to curvature of the earth should be considered and accounted for.

(s) Observations should be recorded on well designed booking sheets.

(t) The precise reduction of measured to station/station vertical angles should be used (section 6.2.).

5.6. Deviation of the Vertical Due to Local Topography

5.6.1. The effect of deviation of the vertical on vertical angle measurements

The geoid is an imaginary surface of equal gravitational potential, it corresponds to mean sea level in open oceans. The geoid is an irregular figure, changes in the surface of which are caused by density changes within the earth and changes of the topography on the surface. The geodesist can approximate the geoidal figure with a regular oblate spheroid, which is used as a reference surface (Figure 5.9.).

The vertical at a station on the surface is a line perpendicular to the geoid which passes through the station. A normal is a line passing through a station which is perpendicular to the spheroid. The angle subtended between the vertical and the normal at a station is the deviation of the vertical; the deviation is normally resolved into north/south and east/west components.

Terrestrial surveys using instruments that have been levelled with a spirit bubble are related to the vertical at a survey station, and therefore to the geoid. When the deviation of the vertical is different at two stations the reciprocal vertical angles between the stations will not be compatible, unless the magnitude and direction of the deviations can be determined.

Considering reciprocal vertical angles taken between the stations A and B in Figure 5.10. at which the deviations of the vertical, in the azimuth of AB, are ζ_A and ζ_B. The reduced vertical angle θ_R is given by

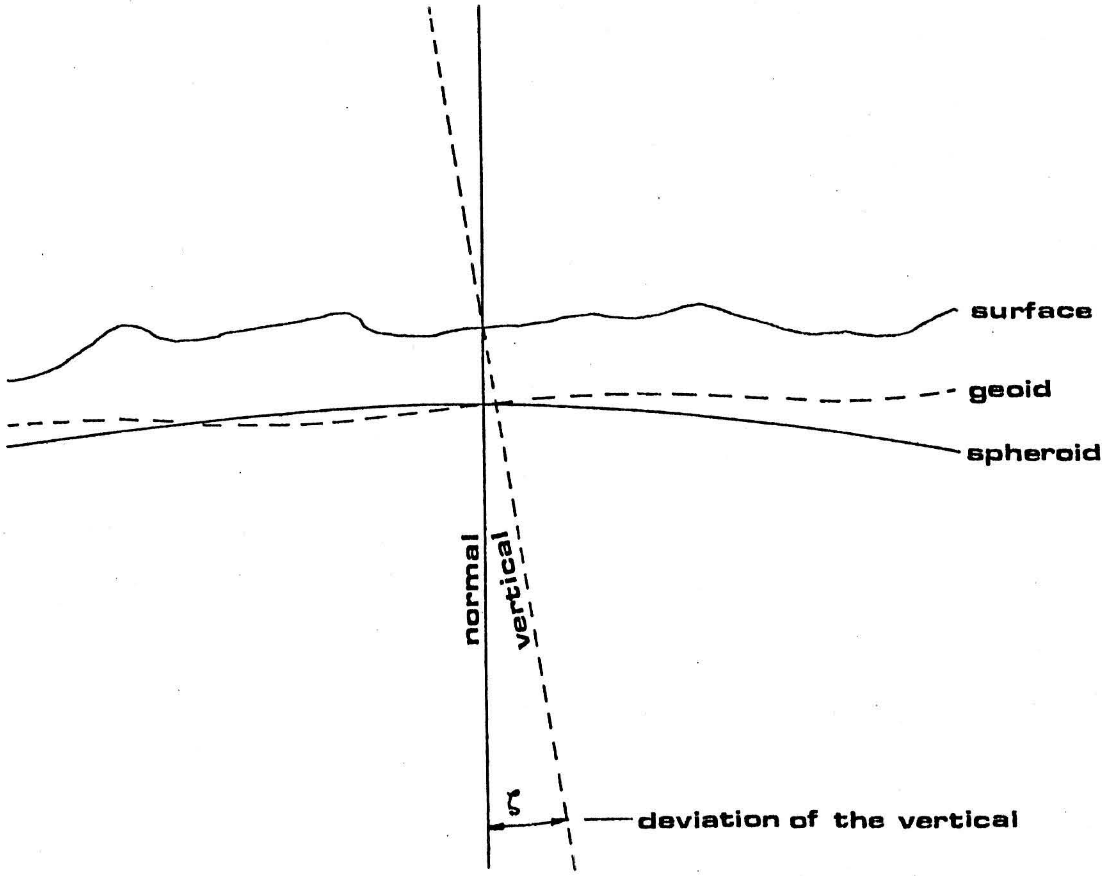


Figure 5.9. Exaggerated diagram of the geoid and deviation of the vertical.

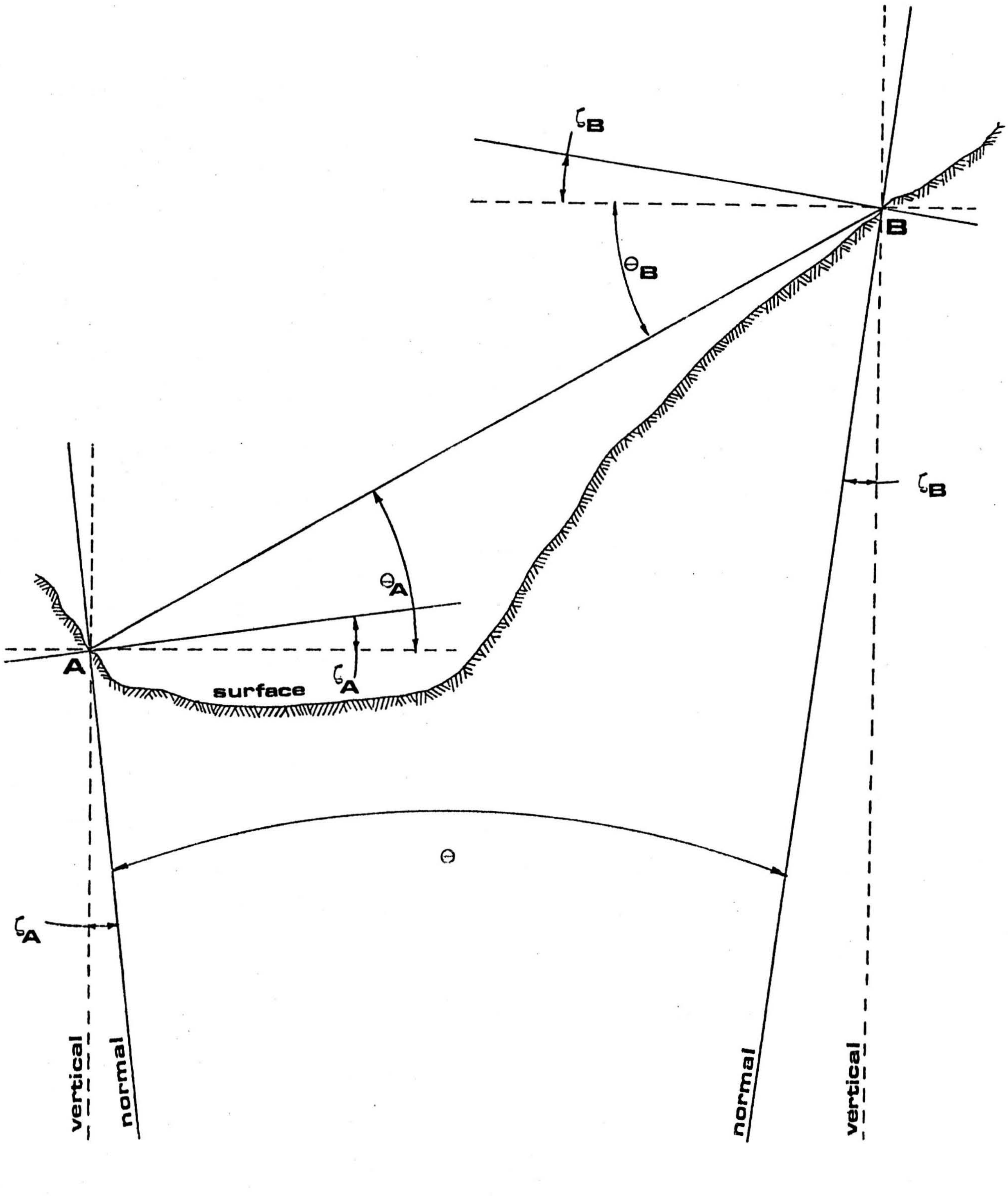


Figure 5.10. The effects of deviation of the vertical on vertical angle measurement.

$$\theta_R = \frac{\theta_A + \theta_B}{2} + \frac{\zeta_B - \zeta_A}{2} \qquad \ldots\ 5.19.$$

where the deviation of the vertical is considered positive if it displaces the vertical, below the surface, away from the other station.

The deviation of the vertical caused by massive density anomalies is of great importance to the geodesist. However, for engineering survey where the stations are relatively close together, this source of deviation causes no significant difference in the deviations at two stations, and therefore no error in reciprocal vertical angle measurements. Robbins [1963] measured the 'geodetic' deviation of the vertical at forty-three stations spaced at approximately 20 km intervals from Dover in south-east England to Tongue, on the north coast of Scotland. The results of this work (Figure 5.11.) show the magnitude of the deviation, in the meridian (N - S) and in the prime vertical (E - W), over the entire length of Great Britain. It can be seen from these graphs that the changes in the geodetic deviation over distances of less than 2 kilometres would be very small. Wherever possible Robbins established his stations in flat land to avoid the additional deviation of the vertical caused by the form of the surface topography. The engineering surveyor may often have to site stations in mountainous terrain where the deviation of the vertical due to the topography should be considered.

The following section outlines the method by which the deviations of the vertical due to the topography, at two stations in the Llangollen network, have been calculated.

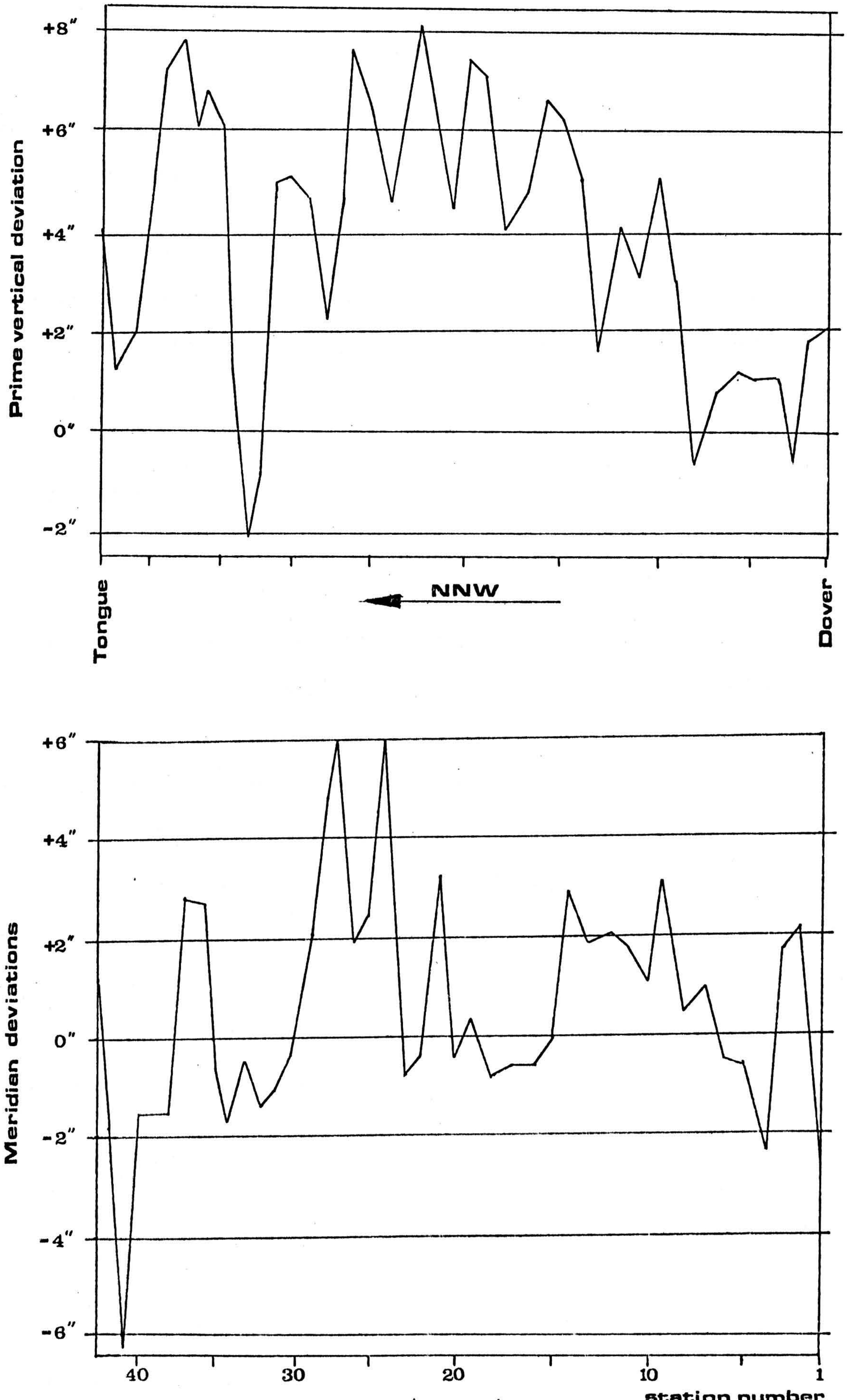


Figure 5.11. Deviations of the vertical in Great Britain (after Robbins).

5.6.2. Calculation of the horizontal acceleration due to the topography

The deviations of the vertical caused by the irregular topography surrounding the Ridge and Sheepfold stations have been calculated by the author. The same method was adopted at each station. The horizontal acceleration due to the gravitational attraction between a mass at the survey station and the mass of the surrounding topography was calculated.

The area surrounding each station was divided into 100 m squares, scaled onto a 1:10 000 Ordnance Survey map, as shown in Figure 5.12. The squares were grouped in hundreds in the four quadrants of the compass. The mean height of each 100 m square was estimated from the contours on the Ordnance Survey map to within five metres.

Geological maps of the area indicated that the volume of rock considered at the Ridge station is formed entirely of limestone extending down to sea level. The Sheepfold station is completely underlain by shale. Samples of these rocks were collected from the Llangollen area and their densities determined.

Shale density = 2707.5 $kg.m^{-3}$

Limestone density = 2612.7 $kg.m^{-3}$

Knowing the density of the rock, the mass of the column of rock below each 100 m area was calculated.

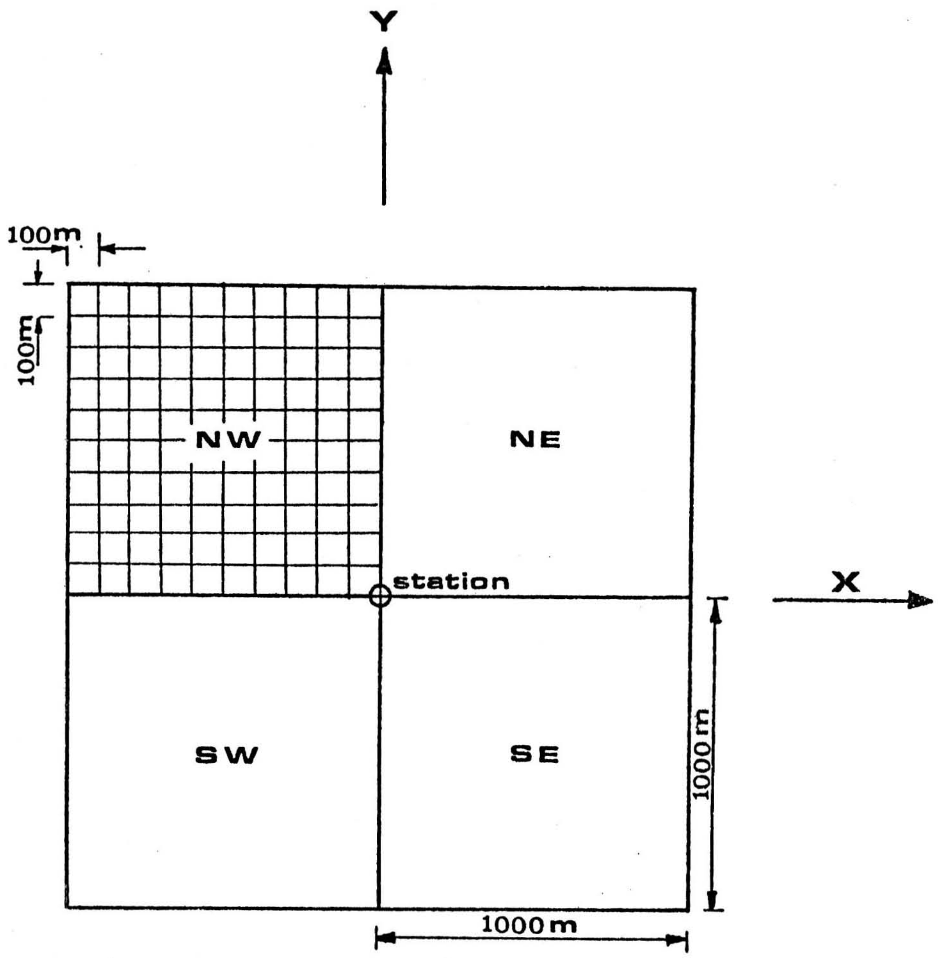


Figure 5.12. Layout of the grid system for horizontal gravity calculations.

(not to scale)

The horizontal acceleration caused at the survey stations by the presence of each rock column was determined from the laws of gravitational attraction. The acceleration 'a' caused by a mass m at a distance r from a point is given by

$$a = G \cdot \frac{m}{r^2} \qquad \ldots 5.20.$$

where G is the gravitational constant ($66.7 \times 10^{-2}\ m^3.kg.^{-1}s^{-2}$).

Figure 5.13. represents a column of rock of height H positioned relative to a survey station at S by rectangular coordinates which correspond with the Eastings and Northings of the Ordnance sheet. The survey station is a height L above mean sea level. The distance from the centre of gravity of the rock column at C to the station at S is given by

$$SC = ((x^2 + y^2) + (L - H/2)^2)^{\frac{1}{2}} \qquad \ldots 5.21.$$

The horizontal gravitational acceleration (a_h) at the station due to the presence of the rock column of mass m is given as

$$a_h = \frac{G\,m}{(SC)^2} \cdot \cos \alpha \qquad \ldots 5.22.$$

from equation 5.20.

where $\alpha = \text{arc tan } ((L - H/2)/(x^2 + y^2)^{\frac{1}{2}})$ ... 5.23.

The horizontal acceleration can be resolved into x and y components

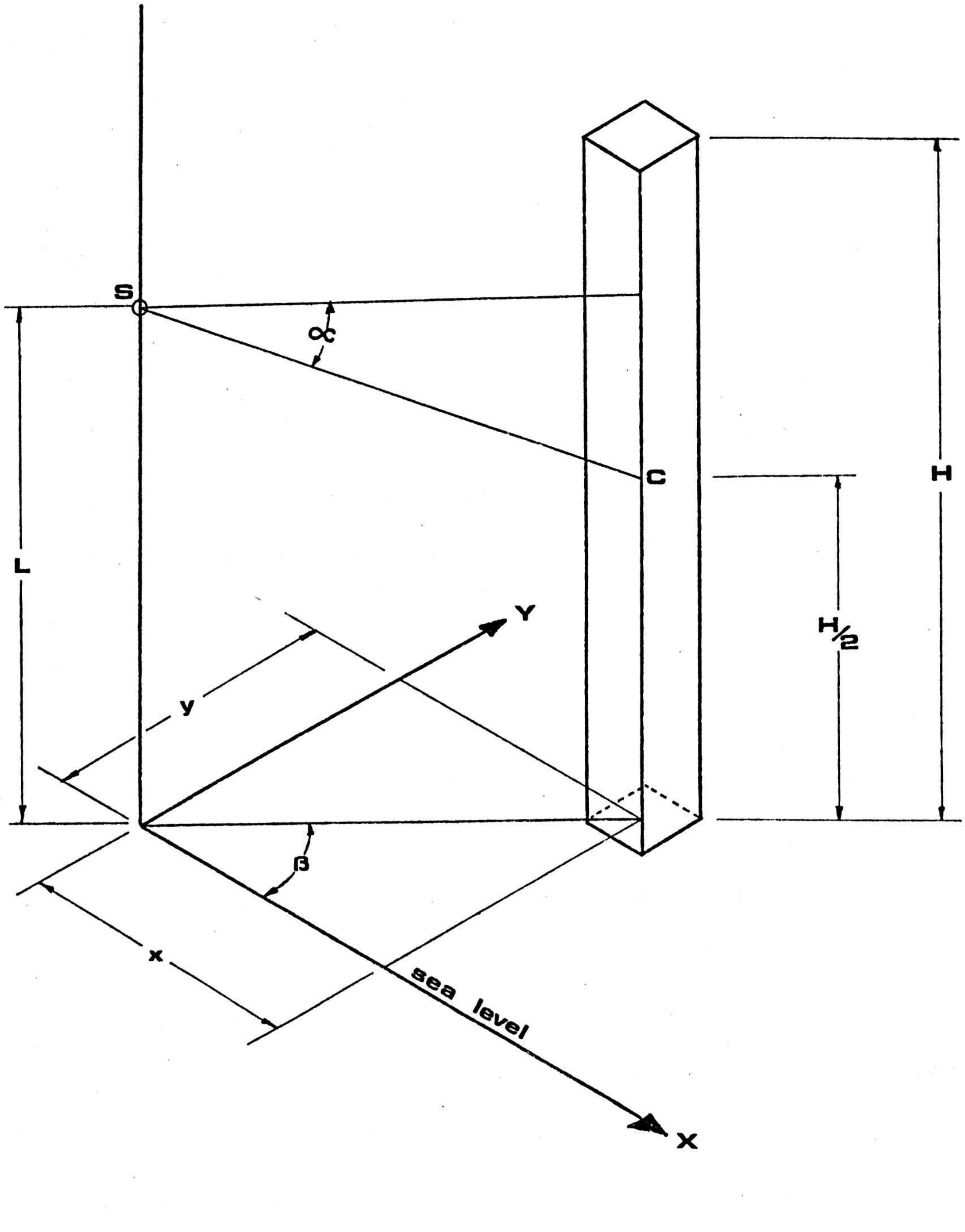


Figure 5.13. Geometry of a rock column with respect to a survey station.

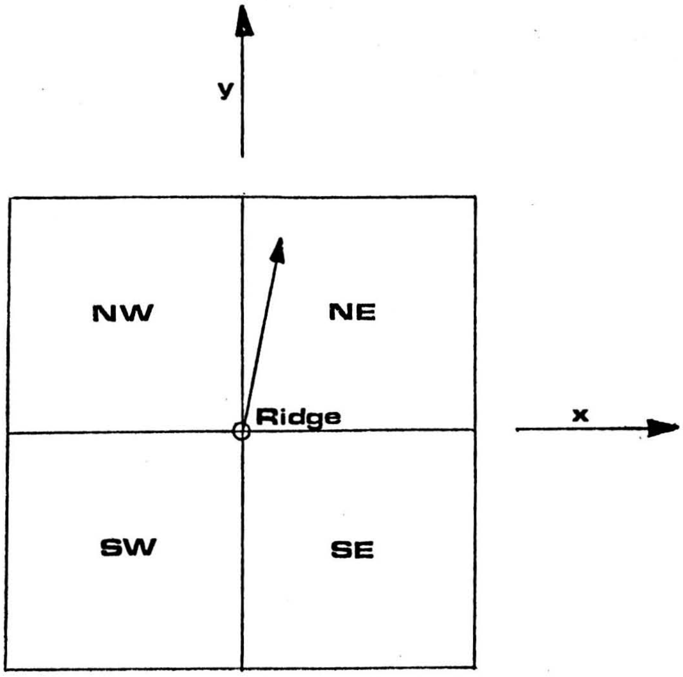
$$a_x = a_h \cos \beta \qquad \ldots\ 5.24.$$

$$a_y = a_h \sin \beta \qquad \ldots\ 5.25.$$

where $\beta = \arctan (y/x)$ ... 5.26.

A program has been written by the author, on a Wang 2200 computer, to execute the previous operations on the one hundred rock columns in each of the four quadrants adjacent to each station. The program summates the components of the horizontal acceleration and outputs the totals. The program listing and an example of the output are given in Appendix F. The components for each quadrant were manually summed to give the total direction and magnitude of the horizontal acceleration at each station due to the surrounding rock mass (Figures 5.14. and 5.15.).

The area covered at each station by the gravity calculations is sufficient to determine the horizontal acceleration due to the local topography for the calculation of the deviation of the vertical and the correction of vertical angle measurements. Figure 5.16. gives a table of the horizontal accelerations at the Ridge caused by the 100 x 100 m columns of rock to the east and west of the station. It should be noted that it is the difference between the components that affects the deviation of the vertical. A difference of ± 0.01 m Gals causes a change of ± 0.002" in the deviation of the vertical, so it is evident that beyond 1000 m from the station the effect of the topography on the deviation is negligibly small.



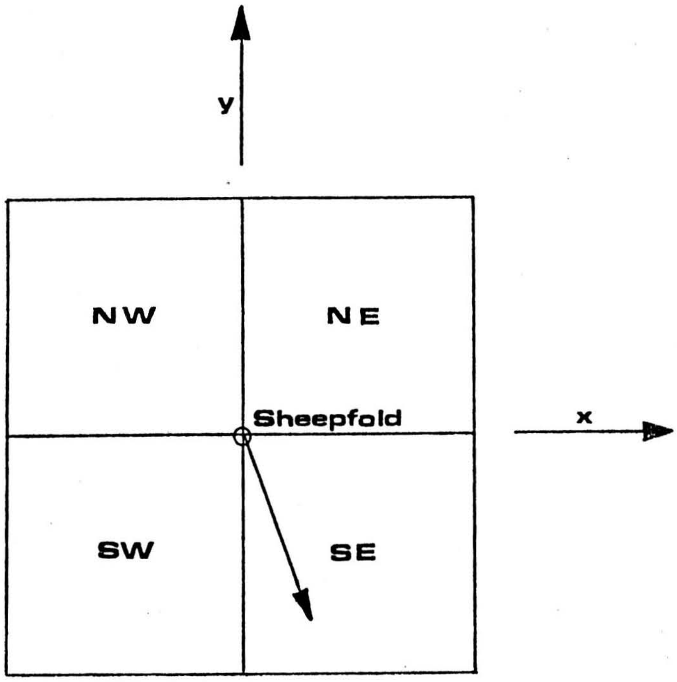
QUADRANT	HORIZONTAL ACCELERATION ($m.s^{-2}$)	
	x	y
NE	+0.000 067 104	+0.000 071 233
NW	-0.000 055 339	+0.000 067 483
SW	-0.000 024 037	-0.000 021 216
SE	+0.000 038 716	-0.000 029 005
TOTAL	+0.000 026 444	+0.000 088 495

Resultant horizontal acceleration = 0.000 092 362 $m.s^{-2}$

= 9.236 2 m.Gals

Azimuth of acceleration = N 16° 38' 15.0" E

Figure 5.14. Horizontal acceleration at Ridge caused by local topography.



QUADRANT	HORIZONTAL ACCELERATION ($m.s^{-2}$)	
	x	y
NE	+0.000 041 736	+0.000 035 410
NW	-0.000 024 060	+0.000 023 165
SW	-0.000 050 153	-0.000 055 510
SE	+0.000 053 523	-0.000 054 730
TOTAL	+0.000 021 046	-0.000 051 665

Resultant horizontal acceleration = 0.000 055 787 $m.s^{-2}$

= 5.578 7 m.Gals

Azimuth of acceleration = S 22° 09' 49.6" E

Figure 5.15. Horizontal acceleration at Sheepfold caused by local topography.

DISTANCE FROM STATION (m)	HORIZONTAL ACCELERATION m.Gals		
	EAST OF STATION	WEST OF STATION	DIFFERENCE
50	0.1180	0.1113	0.0077
150	0.2411	0.1994	0.0416
250	0.2391	0.1624	0.0767
350	0.1984	0.1190	0.0794
450	0.1625	0.0947	0.0678
550	0.1314	0.0758	0.0556
650	0.1048	0.0596	0.0452
750	0.0805	0.0480	0.0325
850	0.0619	0.0388	0.0231
950	0.0474	0.0302	0.0172

Figure 5.16. The effect of distance from the Ridge station on the calculated horizontal accelerations.

5.6.3. The measurement of the vertical acceleration due to gravity

To determine the value of the vertical component of acceleration due to gravity at the Ridge and Sheepfold stations a gravimeter traverse was made from one station to the other via two gravity stations. Because the gravimeter was levelled with a spirit bubble the vertical acceleration due to gravity was measured (Plate 5.6.). The gravity stations were situated by Ordnance Survey bench marks, at which the value of gravity had been determined by the Geological Institute. An Educator Worden gravity meter (Model 113, No. 94) was used for the traverse and very great care was taken in transporting the instrument between stations.

The gravity traverse was made in three stages; the difference in gravity between the Sheepfold station and the gravity station adjacent to the A5 main road (SJ E2479 N4115) was measured first. Secondly the difference in gravity between Ridge and the Sun Trevor gravity station (SJ E2410 N4239) was determined, and finally a check measurement was made between the two gravity stations. The results of the gravimeter measurements are shown in Figure 5.17., the calibration factor for the gravimeter was 0.4483 m.Gals/scale division.

5.6.4. The computation of the deviation of the vertical

The method of computing the deviation of the vertical, for the correction of vertical angle measurements, is illustrated by example for the Ridge survey station.

PLATE 5.6.

Gravimeter measurements at the

Sheepfold station.

3.4.76 TIME (hrs)	GRAVIMETER READINGS (scale divs.)		
	A5 GRAVITY STATION	SHEEPFOLD STATION	DIFFERENCE
14.10	581.5		
14.39	(581.45)*	349.8	-231.65
15.06	581.4	(349.8)	-231.60
15.33	(581.4)	349.8	-231.60
16.00	581.4	(349.7)	-231.70
16.22	(581.3)	349.6	-231.70
16.58	581.2		

Mean Difference = -231.65 scale divs. = -103.85 m.Gals.
Gravity value at A5 station = 981 339.49 m.Gals.
Gravity value at Sheepfold = 981 235.64 m.Gals.

3.4.76 TIME (hrs)	GRAVIMETER READINGS (scale divs.)		
	SUN TREVOR STATION	RIDGE STATION	DIFFERENCE
17.22	584.9		
17.45	(584.9)	412.1	-172.80
18.04	584.9	(412.05)	-172.85
18.22	(585.1)	412.0	-173.10
18.40	585.3	(411.95)	-173.35
18.57	(585.4)	411.9	-173.50
19.15	585.5		

Mean Difference = -173.12 scale divs. = -77.61 m.Gals.
Gravity value at Sun Trevor station = 981 341.14 m.Gals.
Gravity value at Ridge station = 981 263.53 m.Gals.

4.4.76 TIME (hrs)	GRAVIMETER READINGS (scale divs.)		
	SUN TREVOR STATION	A5 GRAVITY STATION	DIFFERENCE
10.09	588.0		
10.30	(588.15)	583.6	-4.55
10.47	588.3	(583.8)	-4.50
11.02	(588.3)	584.0	-4.30
11.18	588.3	(584.1)	-4.20
11.34	(588.35)	584.2	-4.15
11.50	588.4		

Mean Difference = -4.34 scale divs. = -1.945 m.Gals.
Difference from Geological Institute values = -1.65 m.Gals.

∴ These results are in good agreement.

Figure 5.17. Results of gravimeter traverses in Llangollen, 3rd and 4th April, 1976. (* interpolated values in brackets.)

The horizontal acceleration due to the topography surrounding the survey station has been calculated (section 5.6.2.) as,

$$a_h = 9.236\ 2 \text{ m.Gals.}$$

The vertical component of acceleration due to gravity was determined from the gravimetric survey (section 5.6.3.) as,

$$a_v = 981\ 263.53 \text{ m.Gals.}$$

Now the vertical acceleration due to gravity can be considered as the resultant of the horizontal component of gravity and the component in the plane normal to the spheroid, a_n (Figure 5.18.i.). The normal component can be calculated,

$$a_n = \sqrt{(a_v^2 + a_h^2)} \qquad \ldots 5.27.$$

to give $a_n = 981\ 263.500$ m.Gals.

The horizontal acceleration, which acts along the azimuth A (Figure 5.18.ii.) where,

$$A = N\ 16^{\circ}\ 38'\ 15.0''\ E$$

can be resolved to the horizontal acceleration a_{RS} along the azimuth B of the line from Sheepfold to Ridge, where

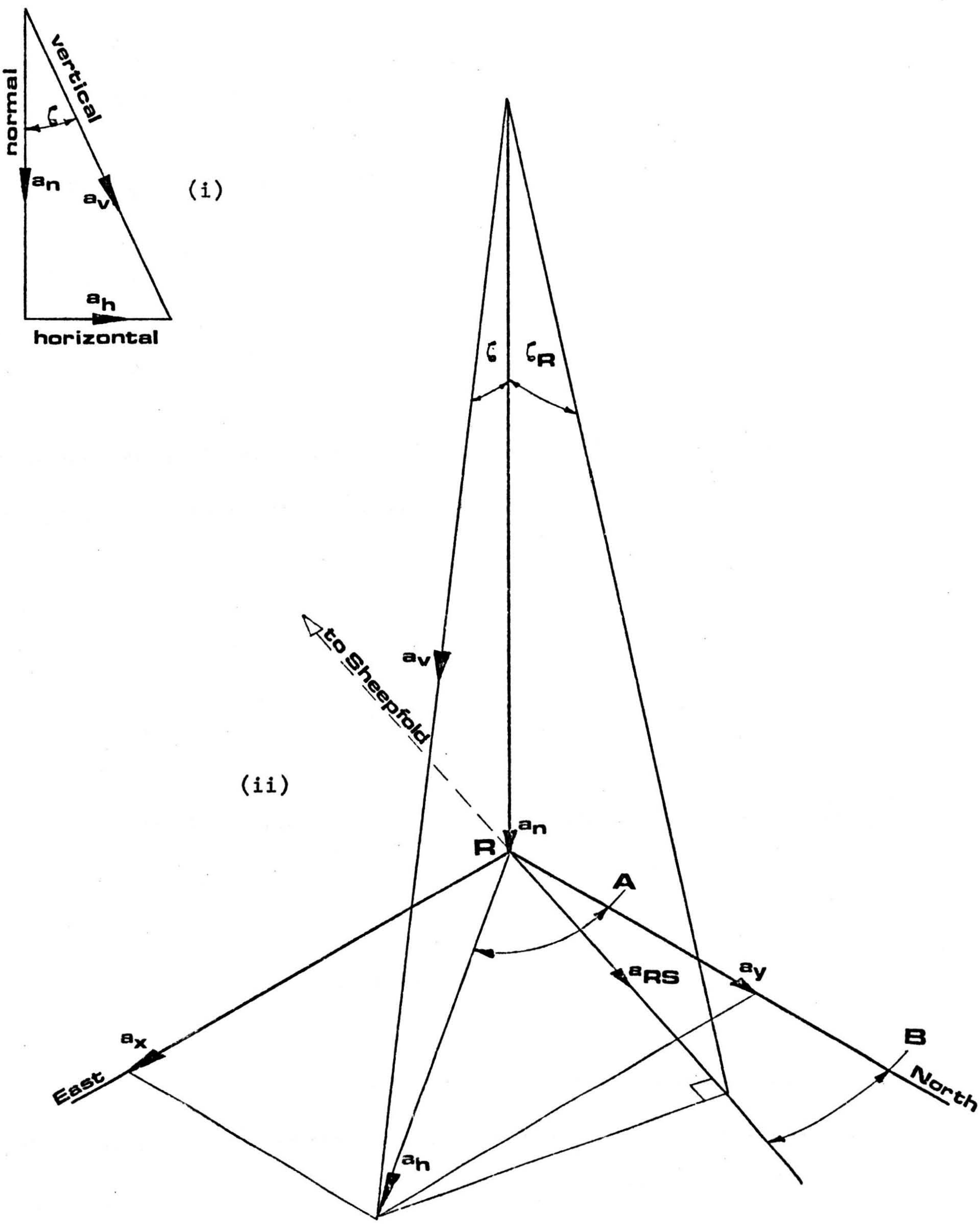


Figure 5.18. Computation of deviation of the vertical at Ridge in azimuth of Sheepfold.

$$B = N\ 07^{o}\ 10'\ 34''\ E$$

so that $a_{RS} = a_h \cos (A - B)$ ... 5.28.

which gives

$$a_{RS} = 9.110\ 5 \text{ m.Gals.}$$

The deviation of the vertical along the line Ridge to Sheepfold at Ridge ζ_R, due to the surrounding topography, can now be determined from

$$\zeta_R = \text{arc tan} \frac{a_{RS}}{a_n} \qquad \text{... 5.29.}$$

to give $\zeta_R = 1.915''$ at an azimuth N $07^{o}\ 10'\ 34''$ E.

The deviation of the vertical at Sheepfold ζ_S along the line to Ridge can be obtained from similar calculations to give

$$\zeta_S = 1.086'' \text{ at an azimuth S } 07^{o}\ 10'\ 34'' \text{ W.}$$

An error of ± 1 m.Gal. in the horizontal acceleration due to the topography introduces an error of ± 0.2" in the deviation of the vertical. 1 m.Gal. represents an error of approximately 10% in the horizontal acceleration at Ridge. Although accurate measurements of the vertical acceleration due to gravity were made in Llangollen for completeness, these measurements were not absolutely necessary because an error of ± 1000 m.Gals. causes an error of only ± 0.002" in the deviation. Consequently a mean gravity value for an area of interest would be adequate. Errors in the estimated mean rock densities of

$\pm$ 100 kg.m^{-3} would introduce an error of $\pm$ 0.3 m.Gals in the horizontal acceleration, a similar error would be introduced by inaccuracies of $\pm$ 12 m in the height of the rock columns.

From this analysis of the errors involved in the calculation of the horizontal acceleration and the deviation of the vertical, due to topographic attraction, the values of the deviation can be stated with confidence to an accuracy of $\pm$ 0.1". This is more accurate than the possible measuring accuracy of vertical angles with a Kern DKM3 first order theodolite, as used for the Llangollen survey measurements.

The error in the reciprocal vertical angle between Ridge and Sheepfold, due to deviation of the vertical, can be determined from equation 5.19. as

$$\text{Error} = -\frac{\zeta_S - \zeta_R}{2} \qquad \ldots\ 5.30.$$

which gives,

$$\text{Error due to deviation of the vertical} = -\frac{1.086 - 1.915}{2}$$

$$= +0.415''$$

Therefore the error in the vertical angle, due to the deviation of the vertical, between the Ridge and Sheepfold stations is less than half a second of arc.

5.6.5. Conclusions on the effect of deviation of the vertical on vertical angle measurements

An investigation has been made into the effects of deviation of the vertical on vertical angle measurements between the Ridge and Sheepfold stations in the Llangollen network. The error incurred by ignoring the deviation of this line would be +0.4", which corresponds to an error of +5 mm in the difference in level determined trigonometrically. Errors due to deviation on this line are small in comparison with refraction errors. However, if accurate corrections have been applied for refraction the deviation errors should be considered.

The example calculations have been based on the line between the Ridge and Sheepfold stations because it was on this line that the refraction experiments were conducted. This line is one for which the deviation effects are minimised, because both stations are on steep mountain slopes that face each other across a valley. If a vertical angle was measured from a station in a low flat plane, where the deviation due to topography would be zero, to a higher station such as Ridge, the vertical angle may be in error by -1".

The deviation considered in this section is that caused by the local topography. The geodetic deviation, due to undulations in the geoid, will have no effect on engineering survey measurements.

It has been shown that with good topographic maps of the area surrounding a survey station the horizontal component of acceleration due to gravity can be calculated. Although gravimeter measurements were taken in this experiment for completeness, they are not necessary if a value of local gravity is known to $\pm$ 0.01 $m.s^{-2}$. Providing that the area around a survey station is well contoured, and the geology of the underlying rocks is well understood, the computation of the deviation of the vertical may be conducted in the office. The use of a computer considerably increases the ease with which the calculations can be made.

Samples of the bed rock in the area should be collected and their density determined, as broad generalisations of density for particular rock types will not be sufficiently accurate.

CHAPTER SIX

THE REDUCTION OF ELECTROMAGNETIC DISTANCE MEASUREMENTS

CHAPTER SIX

THE REDUCTION OF ELECTROMAGNETIC DISTANCE MEASUREMENTS

6.1. Introduction

For a majority of survey tasks the difference in level and the plan distance between survey stations are required. If the difference in level is obtained by spirit levelling an E.D.M. slope measurement can be reduced to a plan distance using Pythagoras's theorem. However, if spirit levelling is impractical then the trigonometrical reduction of E.D.M. slope distance will be necessary. This requires the measurement of the vertical angle between the stations. Assume, for example, the slope length of a line is 1000 m and the difference in level is 100 m. Then the difference in level would have to be measured to an accuracy of ± 10 mm to ensure an accuracy of ± 1 mm in the reduced plan distance. The same accuracy in plan distance and difference in level could be obtained by using a measured vertical angle with a guaranteed accuracy of ± 2" of arc.

The efforts expended in accurately measuring and correcting a distance with E.D.M. will be wasted unless great care is taken in the reduction of the distance. Whichever method of reduction is used the accurate measurement of instrument and target or prism heights is essential. The following sections describe the methods of reduction and consider the possible errors in reduced distances. The results of survey measurements in the Llangollen network are given as an indication of the accuracy and compatability of results that can be obtained by using precise measurement and reduction techniques.

6.2. The Precise Reduction of Vertical Angle Measurements

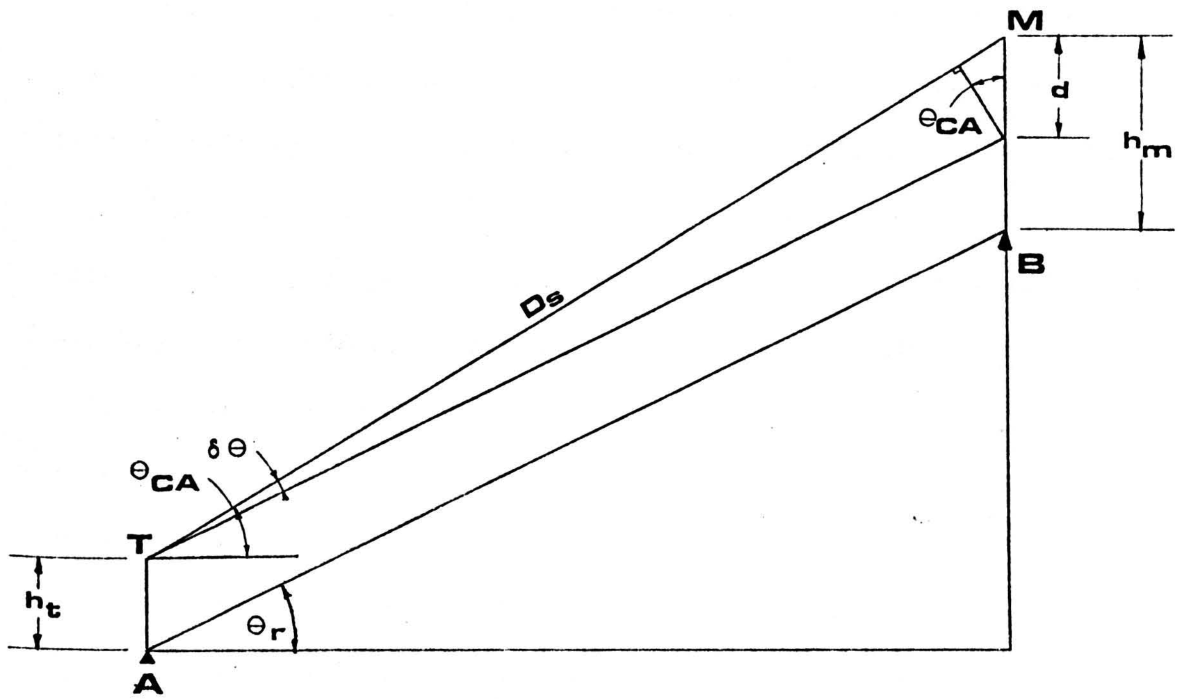
The method of reduction discussed in this section is precise, and can be used without introducing errors into the reduced distances, irrespective of the vertical angle of the line. The method described assumes that reciprocal vertical angle measurements have been observed and that the distance has been measured with an E.D.M. The instrument, target and prism heights are all assumed to be different.

Initially, each vertical angle must be corrected for the curvature of the earth and refraction, as described in Chapter Five, and then reduced to give the vertical angle between the two survey station marks. When vertical angles are measured with a theodolite at T (Figure 6.1.), height h_t above the station at A, to a target at M, height h_m above the station at B, a correction of $\delta\theta$ is necessary to give the station to station vertical angle θ_r. The correction is calculated from the following formulae which can be derived by considering Figure 6.1.

$$\delta\theta = \pm \arc\tan \left[\frac{d \cos \theta_{CA}}{D_s - d \sin \theta_{CA}} \right] \qquad \ldots 6.1.$$

where d is the magnitude of the difference between the theodolite and target heights.

$$d = \left| h_m - h_t \right| \qquad \ldots 6.2.$$



$$d = \left| h_m - h_t \right|$$

$$\delta\theta = \text{arc tan}\left[\frac{d \cos \theta_{CA}}{D_s - d \sin \theta_{CA}} \right] \qquad \ldots \; 6.1.$$

$$\theta_r = \theta_{CA} \pm \delta\theta$$

	UPWARD SIGHT	DOWNWARD SIGHT
$h_t < h_m$	$-\ \delta\theta$	$+\ \delta\theta$
$h_t > h_m$	$+\ \delta\theta$	$-\ \delta\theta$

Figure 6.1. The reduction of corrected vertical angle measurements.

D_s is the corrected E.D.M. slope distance, which is assumed to be the same as the slope distance along the theodolite line of sight. This is not strictly correct as these two distances will differ if the E.D.M. and theodolite lines of sight are not parallel. In theory the maximum difference exists when the theodolite and the E.D.M. line of sight are vertical, the difference being equal to the difference between the E.D.M. prism minus instrument height and the theodolite target minus instrument height. If this is equal to ± 1 m the error in the reduced vertical angle, on a line of 1000 m inclined at 10°, is only ± 0.04" which is insignificant. The term $d \sin \theta_{CA}$ in equation 6.1. has no significant effect on the calculated correction and can be neglected, leaving the following expression for the reduced correction.

$$\delta\theta = \pm \arc \tan \left[\frac{d \cos \theta_{CA}}{D_s} \right] \qquad \ldots 6.3.$$

Figure 6.1. indicates in which situations the corrections should be positive or negative. For a vertical angle measured below the horizontal θ_{CB} should be used in equations 6.1. and 6.3.

An approximate reduction correction can be obtained by using the following expression,

$$\delta\theta = \pm \arc \tan \left[\frac{d}{D_s} \right] \qquad \ldots 6.4.$$

There is no significant saving of computational time by using this formula and it is suggested that the precise correction should always be used in accurate survey network. As an example of the inaccuracy of the approximate method, the correction would be in error by ± 0.6" for a 1000 m line, inclined at 10° to the horizontal, where d equals 0.20 m. The error increases for lines with steeper vertical angles.

Both the upward and downward vertical angle measurements should be reduced, and then the mean station to station vertical angle should be used in the reduction of the E.D.M. slope distance.

6.3. The Precise Reduction of E.D.M. Slope Distance

Figure 6.2. represents the measurement of a slope distance between two survey stations A and B. The E.D.M. instrument is set at height h_e above the station at B and the distance is measured to the prisms at P, height h_p above the station at A. The corrected distance measurement D_s gives the distance EP; this must be reduced to give D_r, which is equal to the distance between the survey station marks. By trigonometry from Figure 6.2.

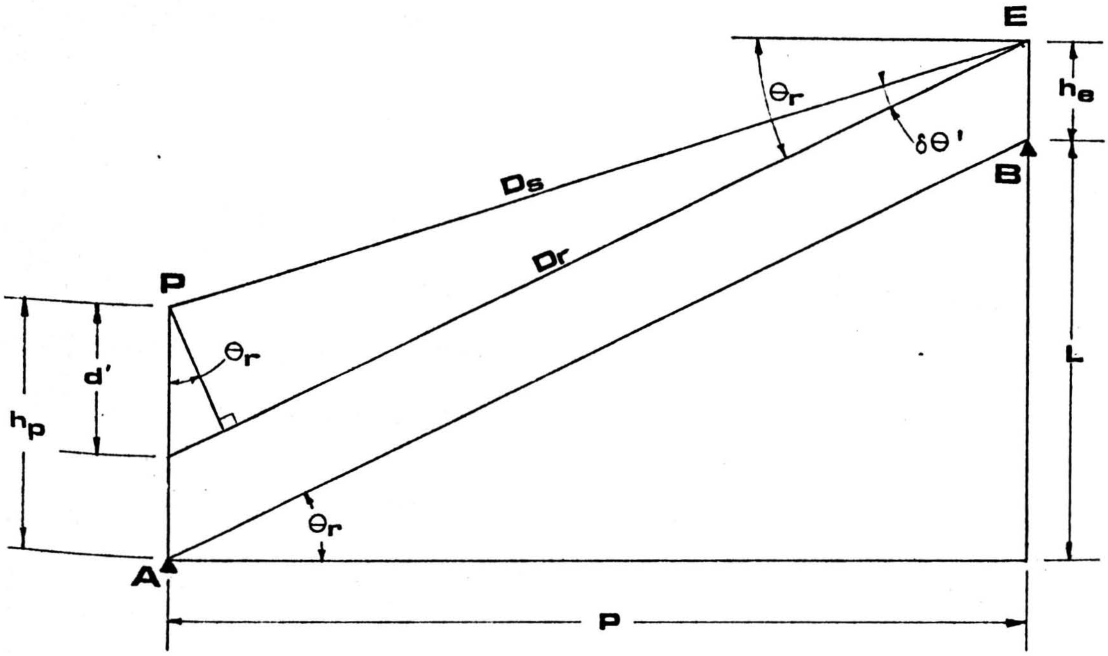
$$D_r = D_s \cos \delta\theta' + d' \sin \theta_r \qquad \ldots 6.5.$$

where $$d' = \left| h_p - h_e \right| \qquad \ldots 6.6.$$

and $$\delta\theta' = \arc \sin \left(\frac{d' \cos \theta_r}{D_s} \right) \qquad \ldots 6.7.$$

Equation 6.5. is more generally written as,

$$D_r = D_s \cos \delta\theta' \pm d' \sin \theta_r \qquad \ldots 6.8.$$



$$d' = \left| h_p - h_e \right|$$

$$\delta\theta' = \arcsin \left(\frac{d' \cos \theta_r}{D_s} \right)$$

$$D_r = D_s \cos \delta\theta' \pm d' \sin \theta_r$$

	UPWARD SIGHT	DOWNWARD SIGHT
$h_e < h_p$	$-d' \sin \theta_r$	$+d' \sin \theta_r$
$h_e > h_p$	$+d' \sin \theta_r$	$-d' \sin \theta_r$

Figure 6.2. The reduction of E.D.M. slope distance.

the sign of the last expression depending on the inclination of the line of sight, upwards or downwards, and the E.D.M. and prism heights. The signs to be used for each situation are given in the table of Figure 6.2.

The slope distance D_r and the reduced vertical angle θ_r are compatible as they both correspond to the line between the two survey stations, A and B. The slope distance is reduced to give the plan distance P and the difference in level L,

where $$P = D_r \cos \theta_r \qquad \text{... 6.9.}$$

and $$L = D_r \sin \theta_r \qquad \text{... 6.10.}$$

6.4. The Accuracy of Trigonometrically Reduced Distances

The accuracy of trigonometrically reduced plan distances and differences in level is limited by the accuracy of the E.D.M. measurement of slope distance and the accuracy of the vertical angle measurements. If the accuracy of both the distance and angle measurements are estimated the following four nomographs (Figures 6.3. to 6.6.) can be used to estimate the minimum accuracy of the reduced plan distance and difference in level. No reduced distances can be guaranteed to be more accurate than the accuracy indicated by the nomographs.

Figures 6.3. and 6.4. express diagramatically the errors in the reduced plan distance, in ppm of slope distance, due to errors in the measured vertical angle and the E.D.M. measured slope distance respectively. A family of lines are drawn for values of vertical angles from zero to the forty five degrees above or below the horizontal. The total error in the plan distance will be equal to the square root of the sum of the squares of the errors due to distance and angle measurement inaccuracies, in accordance with the theory of combination of errors.

The nomographs in Figures 6.5. and 6.6. can be used to calculate the minimum errors in reduced difference in level, as ppm of slope distance, due to errors in the measurement of vertical angles and slope distance respectively. Again the errors should be combined to give the total error in the difference in level. The use of these nomographs to assess the accuracy of reduced distances is best illustrated by example.

An E.D.M. is used to measure the distance between two survey stations, the corrected slope distance is 1000 m, with an estimated accuracy of ± 2 mm ± 2 ppm. The measured vertical angle is five degrees of arc and is accurate to ± 2". Figure 6.3. shows that for a line inclined at five degrees an error of ± 2" introduces an error of ± 0.7 ppm, of the slope distance, to the reduced plan distance. For a 1000 m line the slope distance accuracy of ± 2 mm ± 2 ppm is equivalent to ± 4 ppm which from Figure 6.4. introduces an error of ± 4 ppm, of the slope distance, in the reduced plan distance. These errors can be combined to give the accuracy of the reduced plan distance,

$$\text{accuracy of plan distance} = \left((4)^2 + (0.7)^2\right)^{\frac{1}{2}} = \pm 4.1 \quad \text{ppm}$$

which is equivalent to ± 4.1 mm for a line with a slope distance of 1000 m.

Similarly for the reduced difference in level (Figures 6.5. and 6.6.), an error of ± 9.3 ppm is introduced by the angular inaccuracy and ± 0.3 ppm by the errors in the E.D.M. measurement. These errors combine to give the accuracy of the difference in level as ± 9.3 ppm or ± 9.3 mm.

This example, which uses a line typical of many lines encountered in engineering survey, shows clearly that errors in the measurement of slope distance have the predominant effect on reduced plan distance and that the errors in vertical angles have the greatest effect on the reduced difference in level.

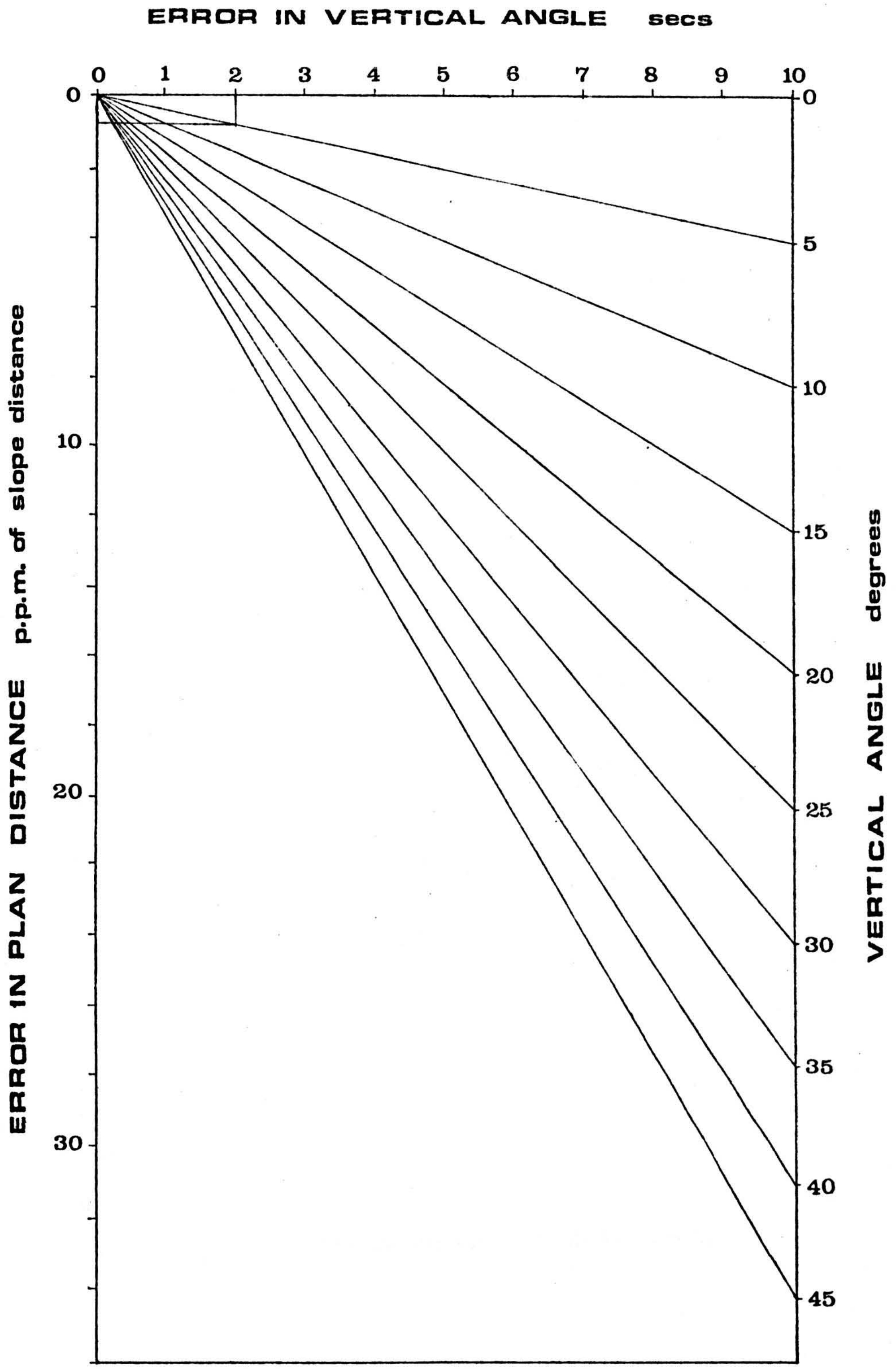


Figure 6.3. The error in reduced plan distance due to errors in the measured vertical angle.

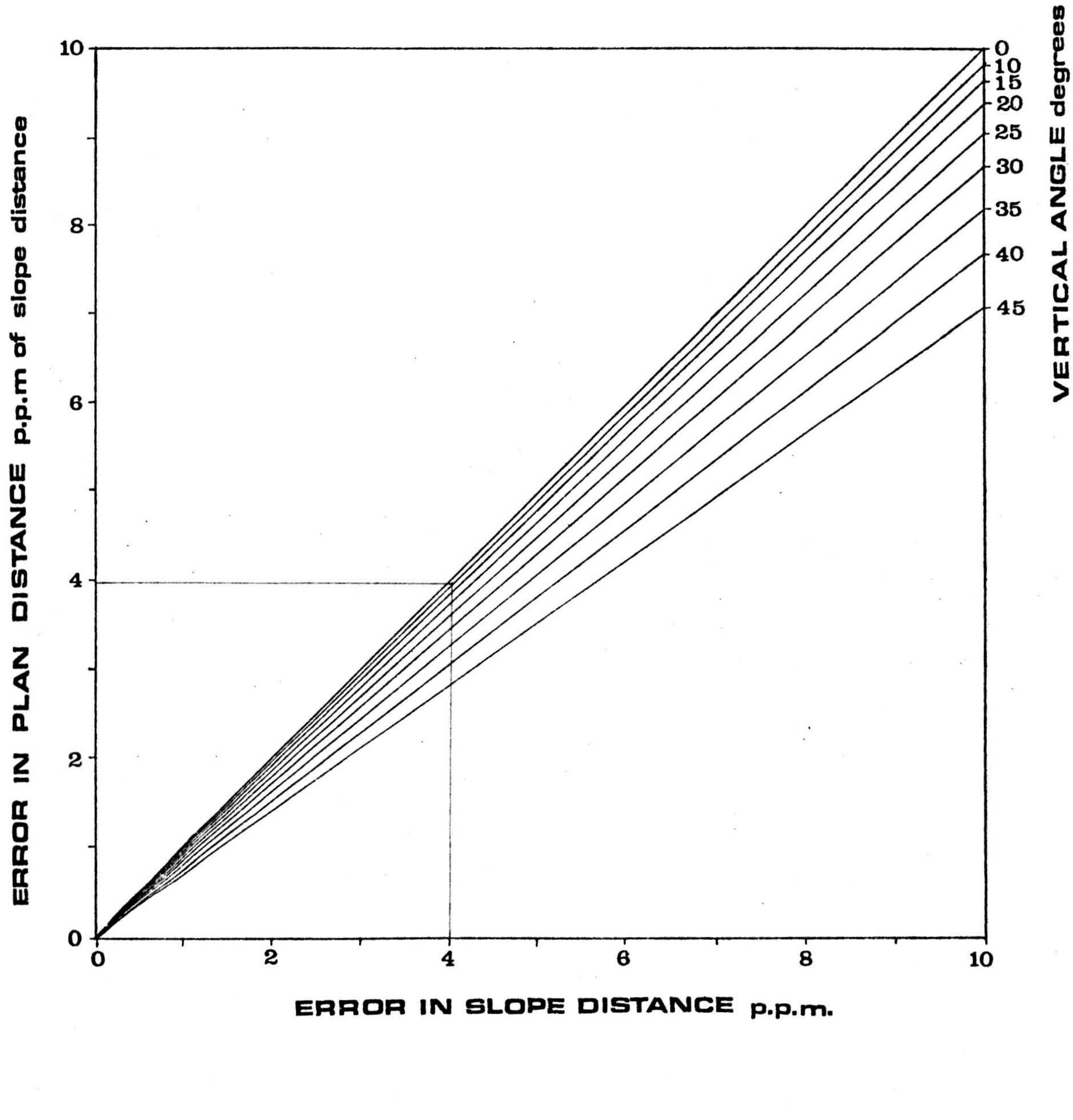


Figure 6.4. The error in reduced plan distance due to errors in the E.D.M. measurement of slope distance.

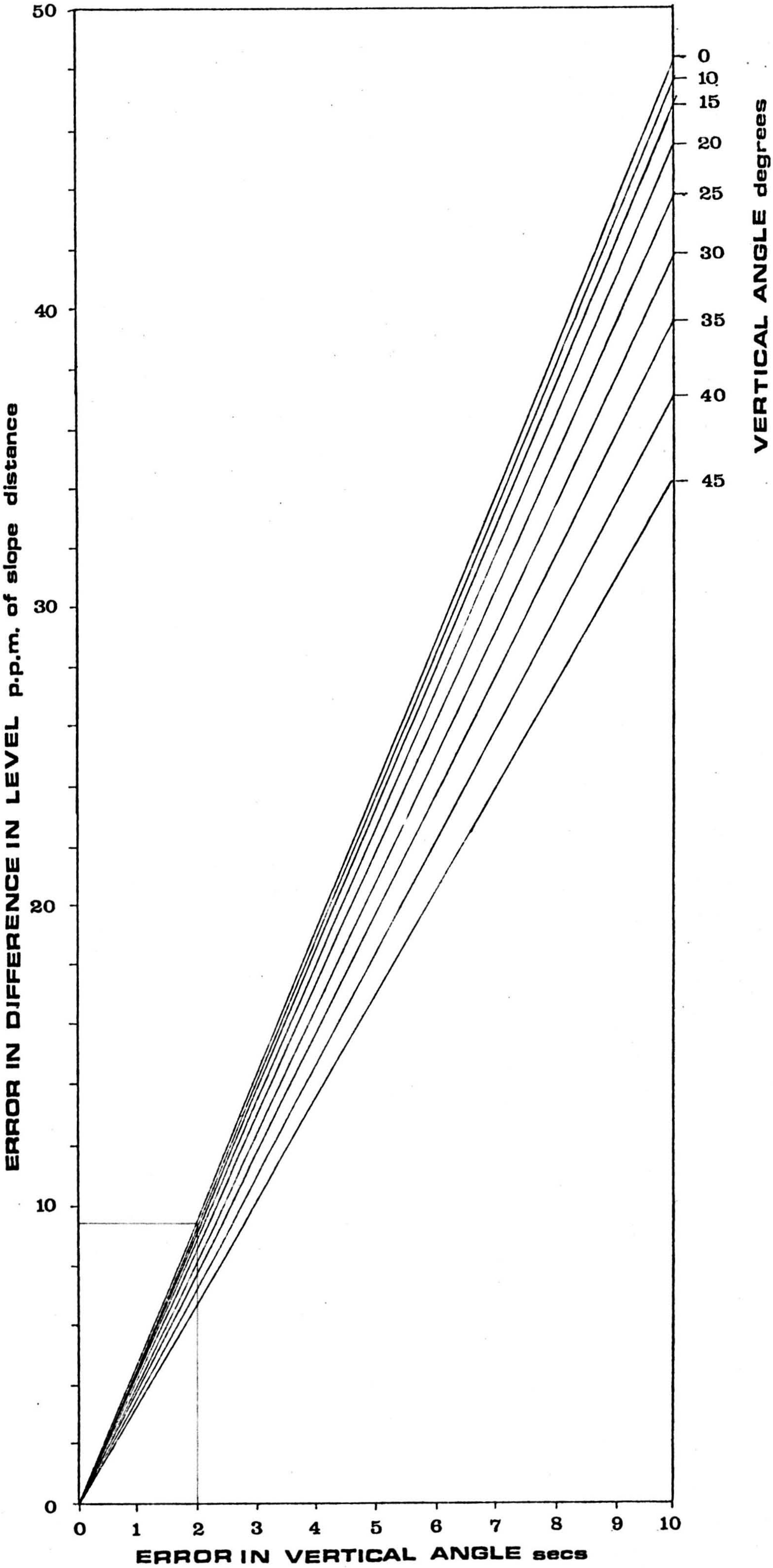


Figure 6.5. The error in reduced difference in level due to errors in the measured vertical angle.

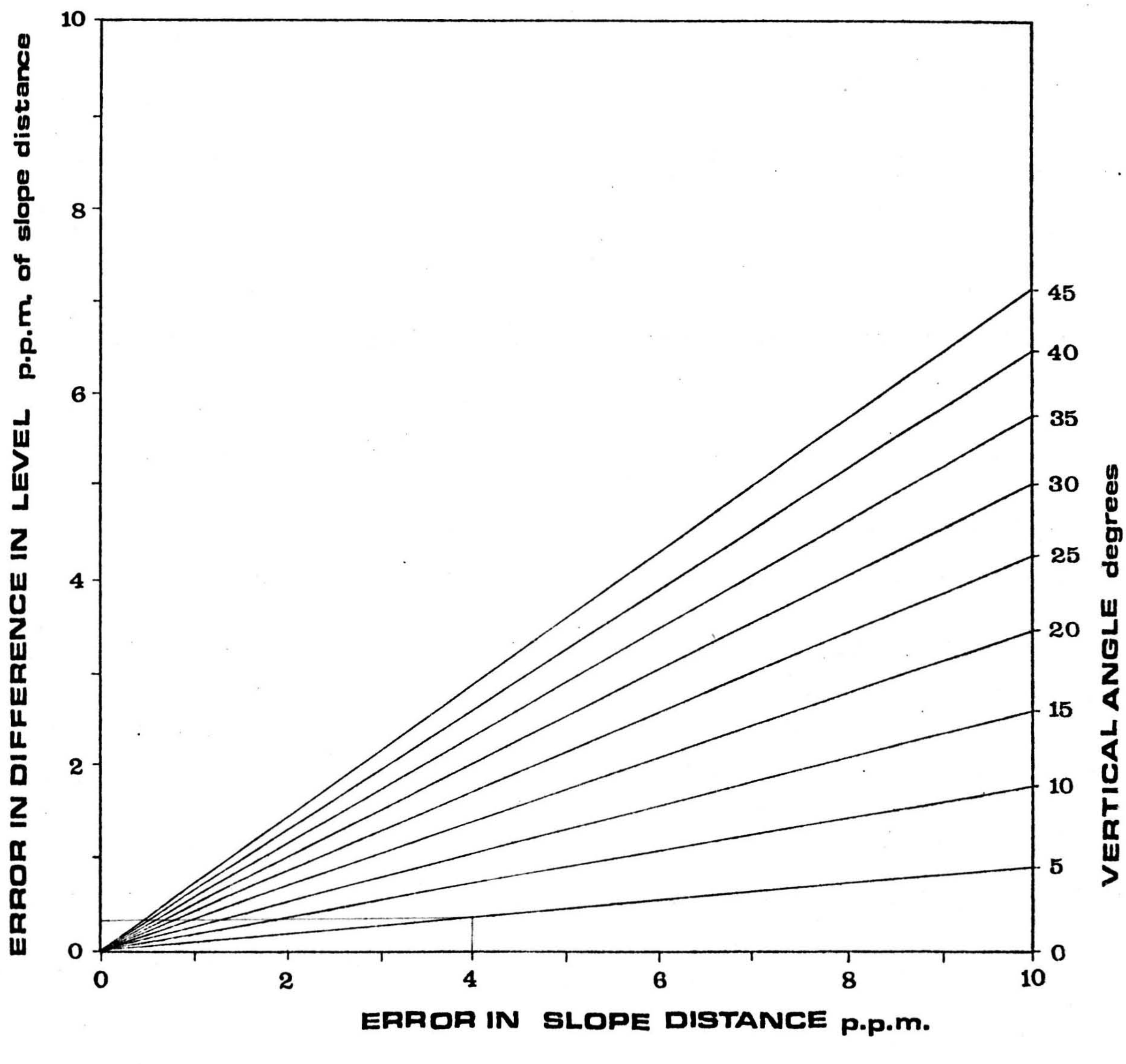


Figure 6.6. The error in reduced difference in level due to errors in the E.D.M. measurement of slope distance.

Figures 6.7. and 6.8. are examples of graphs that can be plotted to show the obtainable precision ratios of reduced plan distance and difference in level, with specified angle and distance measuring accuracy. The precision ratio for the reduced plan distance is defined as the ratio of the error in the plan distance to the plan distance, and the definition for the level difference is the ratio of the error in the difference in level to the difference in level. The vertical angle of a line and the slope distance are applied to the x and y axis of the graph and the corresponding precision ratio is interpolated from the plotted lines. These graphs are best plotted using computer methods, the derivation of the curves is given in detail in Appendix G.

The two example curves are plotted for an E.D.M. distance measuring accuracy of ± 1 mm ± 1 ppm, and an accuracy of ± 1" for the vertical angle measurements. The curves plotted are pessimistic and as such assume the worst combination of angular and distance errors. The precision ratios obtained from these graphs indicate the minimum accuracy that can be achieved with the given measuring errors, in practice the errors may compensate and the accuracy obtained will be better than that indicated from the graphs.

For example, say that both E.D.M. and vertical angle measurements have been taken on a line with a slope length of 1000 m and a vertical angle of 10^{o}. If the results of the E.D.M. measurements are estimated to be accurate to ± 1 mm ± 1 ppm and the accuracy of the vertical angle measurements is estimated to be ± 1" of arc, then the precision ratios for the trigonometrically reduced plan distance and difference in level can be obtained from the graphs of Figure 6.7. and 6.8. By applying the slope distance and the vertical angle to the axes of the graphs the following precision ratios are obtained,

Precision ratio for plan distance (Figure 6.7.),

$\delta P/P = 1:350\ 000$, approximately.

Precision ratio for difference in level (Figure 6.8.),

$\delta L/L = 1:35\ 000$, approximately.

Now the plan distance and difference in level can be calculated from the slope distance and the vertical angle to give,

Plan distance, P = 984.81 m

Difference in level, L = 173.65 m

The minimum accuracy in each of these parameters is given by the product of the precision ratio and the distance,

$$\text{Minimum accuracy of plan distance} = \frac{\delta P}{P} \times P = \pm\ 2.8\ \text{mm.}$$

$$\text{Minimum accuracy of difference in level} = \frac{\delta L}{L} \times L = \pm\ 5.0\ \text{mm.}$$

Therefore the reduced plan distance and difference in level can only be stated with confidence as,

Plan distance = 984.81 m ± 2.8 mm.

Difference in level = 173.65 m ± 5.0 mm.

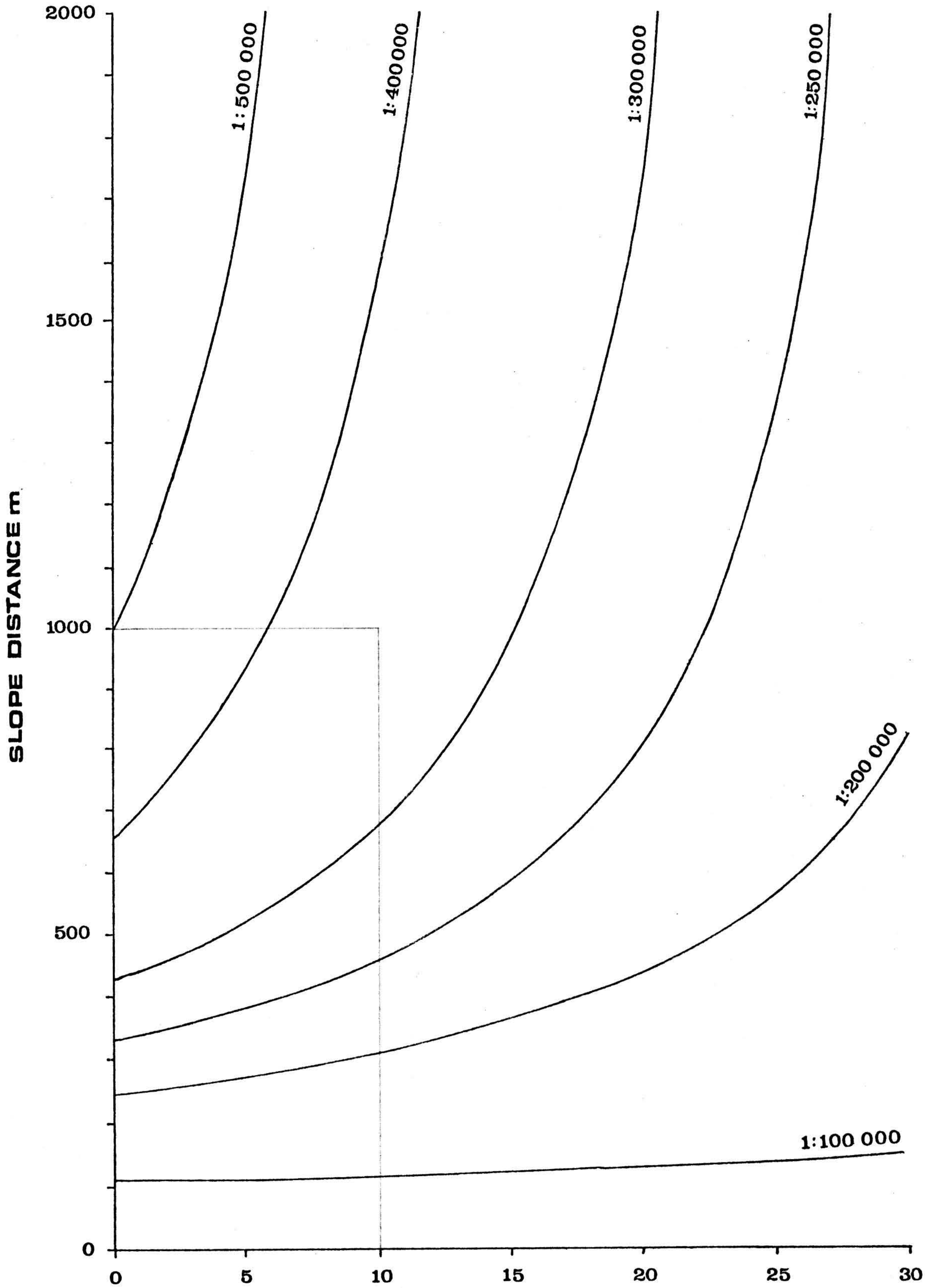


Figure 6.7. Example of precision ratios for trigonometrically reduced plan distance ($\delta P/P$).

Slope distance accuracy = ± 1 mm ± 1 ppm. Vertical angle accuracy = ± 1 sec.

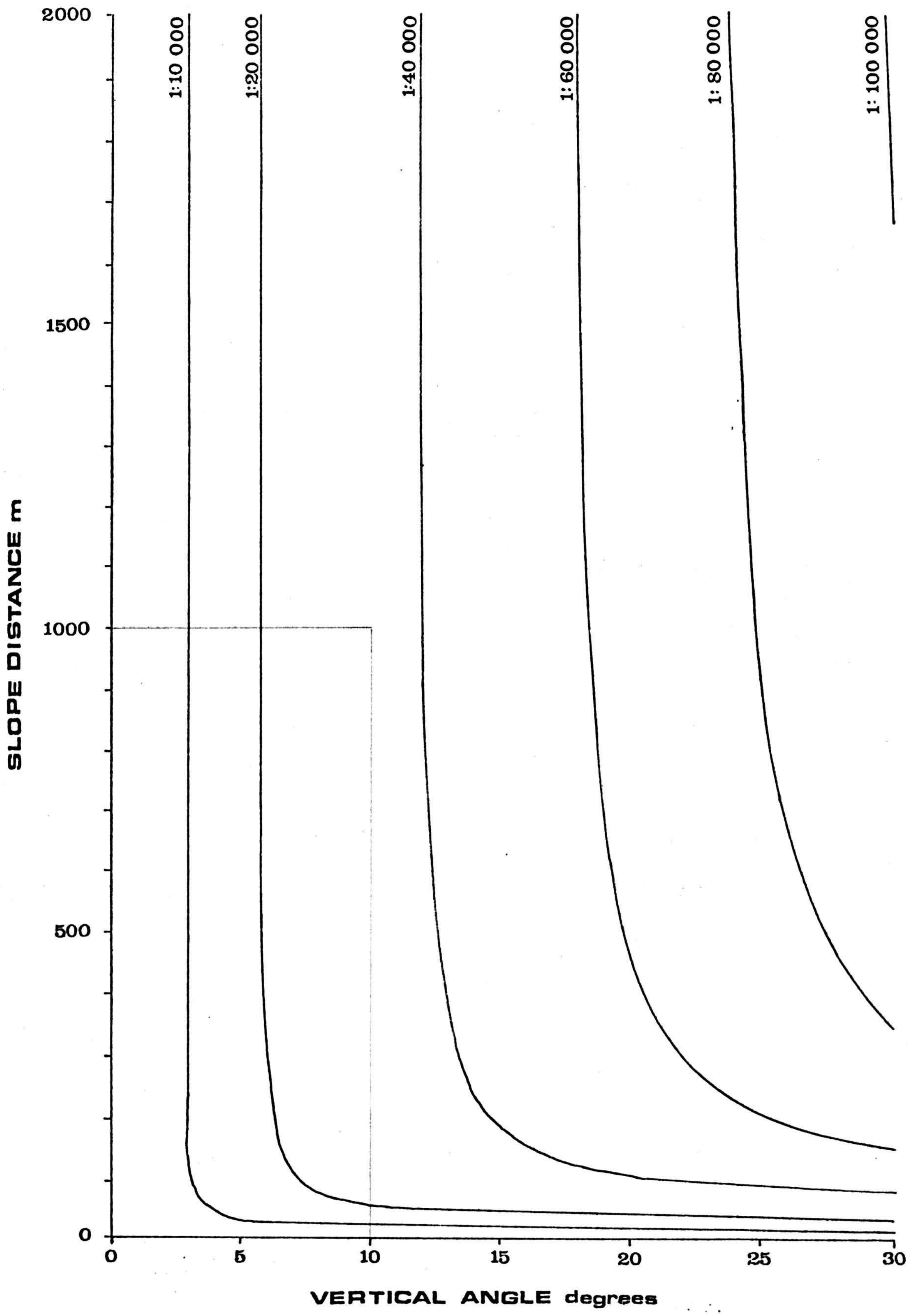


Figure 6.8. Example of precision ratios for trigonometrically reduced difference in level ($\delta L/L$).

Slope distance accuracy = ± 1 mm ± 1 ppm. Vertical angle accuracy = ± 1 sec.

6.5. Reduction to the National Grid Projection Plane

The plan distance and difference in level between two stations give the relative position of the stations in space. However, if a series of distances are measured between different survey stations they must be reduced to a projection plane before they can be combined to form the lines of a survey network. In Great Britain the Transverse Mercator Projection [Ordnance Survey, 1950] has been adopted as the national projection. Distances on the projection plane are known as grid distances, and can be derived from the spheroid distance between two points and the local scale factor.

$$\text{Grid distance} = (\text{Spheroid distance}) \times (\text{Local scale factor}) \quad \ldots\ 6.11.$$

The local scale factor can be taken from Ordnance Survey tables [1950] or calculated from the following formula.

$$\text{Local scale factor} = 0.999\ 601\ 27 + (400 - E)^2 \times 1.228 \times 10^{-8} \quad \ldots\ 6.12.$$

where E is the national grid easting of the mid point of a line. When E.D.M. distances, that have been measured to accuracies approaching ± 1 ppm, are reduced to grid distances the local scale factor should be calculated for each line.

The spheroid distance, D_R, can be calculated from the reduced slope distance, D_r, and the levels of the two stations above mean sea level, h_1 and h_2, from [Hodges, 1970],

$$D_R = D_c + \frac{D_c^{\,3}}{24R^2} \qquad \ldots\ 6.13.$$

where R is the mean local radius of the spheroid and D_c is the chord distance between the survey stations, which is given by

$$D_c = \left[\frac{(D_r - h)(D_r + h)}{(1 + h_1/R)(1 + h_2/R)} \right]^{\frac{1}{2}} \qquad \ldots\ 6.14.$$

$$\text{where} \quad h = \frac{h_1 + h_2}{2} \qquad \ldots\ 6.15.$$

The grid distances and the horizontal angle measurements taken in a survey network should be used to calculate the grid coordinates of the survey stations. No t-T correction to the horizontal angles is necessary in engineering survey when the lines of sight are usually less than 3000 m in length.

6.6. Results of Measurements in the Llangollen Network

Many measurements of horizontal angles, vertical angles and slope distances have been made on all the lines forming the Llangollen Network [Hodges and Scoble, 1974], [Hodges, Scoble and Curl, 1975]. Measurements have been taken, by members of the Mining Department, on the triangle Ridge - Bryn Howel - Sheepfold since 1971, the Tyn y Wern station was included in 1973 and the Castle and Escarpment stations in 1974. All the measurements have been made by experienced observers and the greatest care has been taken to avoid errors in each measurement.

The braced quadrilateral Ridge - Bryn Howel - Sheepfold - Tyn y Wern is considered in this section, as it is this part of the network on which refraction investigations have been based. Figure 6.9. gives the distances reduced from all the E.D.M. measured slope distances. The spheroid distances have been reduced to the National Grid using the local scale factor calculated for the mid point of each line. The horizontal angles of the quadrilateral have been derived from the grid distances and these results are compared with the mean of all the horizontal angle measurements that have been taken, with the Kern DKM3 theodolite, in Figure 6.10. The good agreement between the derived and measured angles is an indication of the precision of the E.D.M. measurement of slope distance, the reduction of the distances with measured vertical angles, and the measurement of horizontal angles.

The line between Bryn Howel and Tyn y Wern has been measured on six occasions with five different E.D.M. instruments. A weighted mean of these measurements has been used in the calculation of the grid distance, TB, the weightings have been given in accordance with the accuracy of the E.D.M. instrument, the number of times the distance was measured, and the accuracy with which the atmospheric conditions were evaluated. Particular care was taken in obtaining a representative mean temperature when the distance was measured with the Geodimeter 6 in April 1975, which explains the high weighting of this measurement. The results of these measurements are shown in Figure 6.11.

The line Bryn Howel to Tyn y Wern has been selected as a baseline for the network for two reasons, the length of the line is believed to be the most accurately known in the network, as it has been measured on six occasions with consistent results, and secondly, because the line is almost level the errors in the reduction of slope distance

LINE	SLOPE DISTANCE m	DIFFERENCE IN LEVEL m	PLAN DISTANCE m	SPHEROID DISTANCE m	MEAN N.G EASTINGS km	LOCAL SCALE FACTOR	GRID DISTANCE m
TB	2011.862	10.076	2011.837	2011.806	323.963	0.999 672 2	2011.147
BS	1923.400	284.001	1902.317	1902.248	324.324	0.999 671 5	1901.623
ST	1675.585	294.055	1649.581	1649.518	323.319	0.999 673 4	1648.980
TR	1424.954	168.912	1414.907	1414.867	323.490	0.999 673 1	1414.405
RB	1398.446	158.838	1389.396	1389.357	324.495	0.999 671 2	1388.900
RS	2443.856	125.125	2440.651	2440.527	323.850	0.999 672 4	2439.727

Figure 6.9. Reduced E.D.M. measurements in the Llangollen Network.

ANGLE	HORIZONTAL ANGLES								
	DERIVED FROM GRID DISTANCES			MEASURED WITH KERN DKM3			MEAN		
	o	'	"	o	'	"	o	'	"
RTB	43	39	14	43	39	15	43	39	14.5
TRS	40	40	55	40	40	52	40	40	53.5
SRB	50	59	52	50	59	52	50	59	52.0
RBT	44	39	59	44	40	00	44	39	59.5
TBS	49	45	13	49	45	12	49	45	12.5
RST	33	59	45	33	59	43	33	59	44.0
BSR	34	34	56	34	34	56	34	34	56.0
BTS	61	40	06	61	40	10	61	40	08.0

Figure 6.10. Comparison of derived and unadjusted measured horizontal angles (1971 - 1976).

E.D.M. INSTRUMENT	REDUCED PLAN DISTANCE (m)	WEIGHT
Geodimeter 6 (1974)	2011.834	2
Geodimeter 6 (1975)	2011.839	5
Geodimeter 6BL	2011.833	2
Geodimeter 700	2011.828	1
Kern Mekometer ME3000	2011.836	5
Tellurometer MA100	2011.842	4
Weighted Mean	2011.837	

Figure 6.11. E.D.M. slope distance measurements on line TB.

will be minimised. Using the grid length of the baseline TB and the mean of the derived and measured horizontal angles the grid distances between the stations in the quadrilateral RBST have been calculated. The results are shown in Figure 6.12. which also gives the grid distances reduced from E.D.M. measurements. The good agreement between the calculated and reduced grid distances is another indication of the high precision of all the measurements taken in the Llangollen network.

The agreement of 5 mm, or 2.0 ppm, between the reduced E.D.M. measurement of the 2440 m line between Ridge and Sheepfold, and the length of this line derived from the baseline, shows that the atmospheric corrections applied to both of these distances must be compatible. This agreement could not have been achieved if great care had not been taken in obtaining representative atmospheric conditions during all the E.D.M. measurements. Precise measurement of vertical angles and the careful reduction of E.D.M. slope distance to the National grid have also been necessary to achieve the accurate results of this survey.

LINE	GRID DISTANCE		DIFFERENCE
	REDUCED FROM E.D.M. MEASUREMENT m	CALCULATED FROM BASELINE TB m	mm
TB	2011.147	-	-
BS	1901.623	1901.632	9
ST	1648,980	1648.969	11
TR	1414.405	1414.399	6
RB	1388.900	1388.892	8
RS	2439.727	2439.732	5

Figure 6.12. Calculated and reduced distances on the National Grid.

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

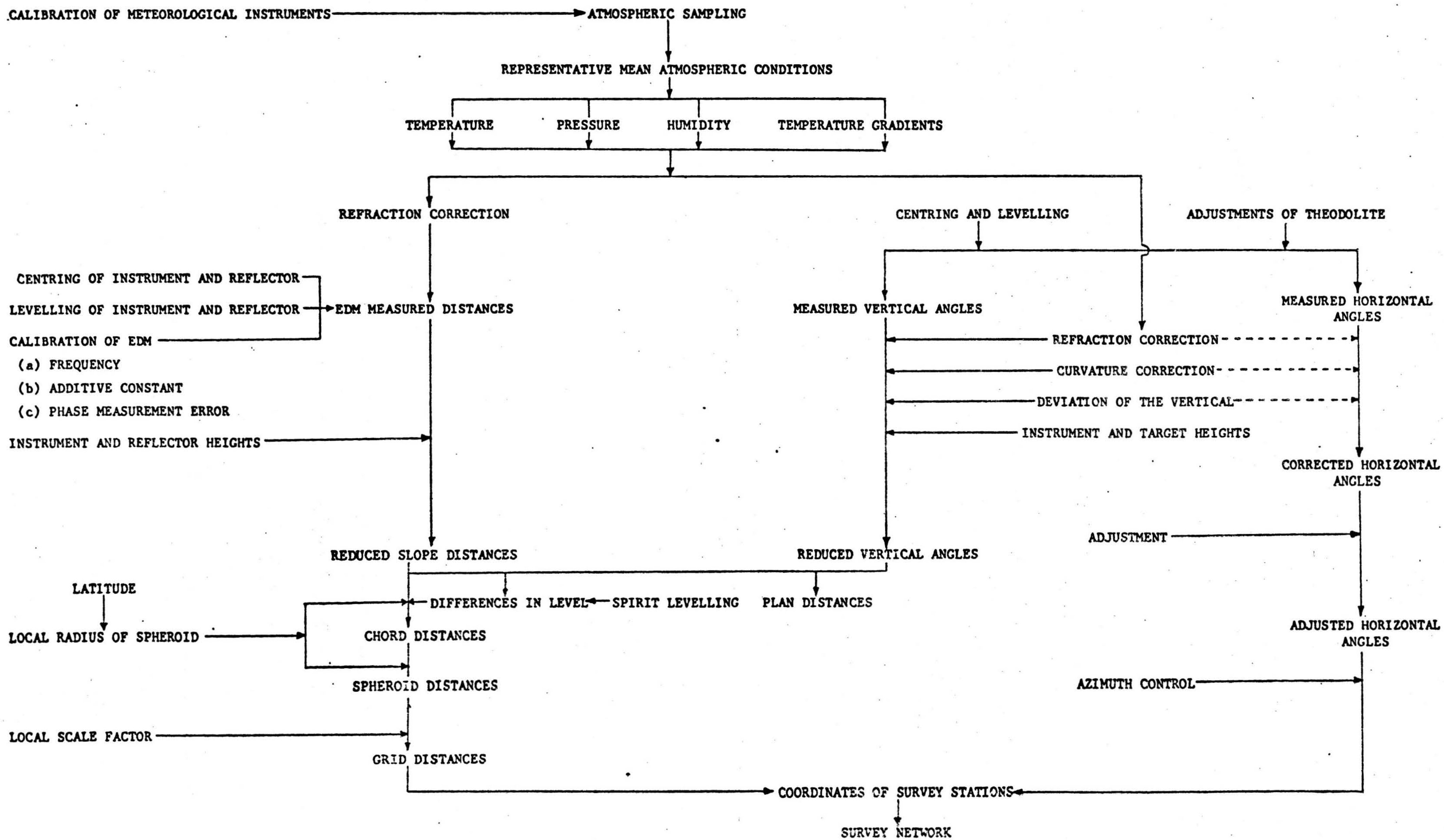
The primary objective of survey measurements is to determine the relative position of survey stations. In modern surveying it is usual to measure the slope distance between stations with electromagnetic distance measuring instruments. The slope distance is then reduced to give the plan distance by considering either the measured vertical angle or the spirit levelled difference in level between the stations. The difference in level is obtained trigonometrically when vertical angle observations are made. The plan distances are then reduced to a common referenceplane and combined with horizontal angle measurements and azimuth control to give the co-ordinates of the survey stations.

There are many sources of error which affect survey measurements (Figure 7.1.), but the most difficult errors to evaluate quantitively are those due to refraction. Refraction affects all optical and electro-magnetic survey measurements made in the earth's atmosphere.

Chapter One has outlined the physical principles of refraction and has explained in detail the method of calculating the ambient refractive index of air from first principles. The refractive index of air, which is the most important parameter in the atmospheric correction of E.D.M. measurements, depends upon the following factors:

FIGURE 7.1.

The effects of refraction on engineering survey networks.


CALIBRATION OF METEOROLOGICAL INSTRUMENTS
ATMOSPHERIC SAMPLING
REPRESENTATIVE MEAN ATMOSPHERIC CONDITIONS
TEMPERATURE
PRESSURE
HUMIDITY
TEMPERATURE GRADIENTS
REFRACTION CORRECTION
CENTRING AND LEVELLING
ADJUSTMENTS OF THEODOLITE
CENTRING OF INSTRUMENT AND REFLECTOR
LEVELLING OF INSTRUMENT AND REFLECTOR
EDM MEASURED DISTANCES
MEASURED VERTICAL ANGLES
MEASURED HORIZONTAL ANGLES
CALIBRATION OF EDM
(a) FREQUENCY
(b) ADDITIVE CONSTANT
(c) PHASE MEASUREMENT ERROR
REFRACTION CORRECTION
CURVATURE CORRECTION
DEVIATION OF THE VERTICAL
INSTRUMENT AND TARGET HEIGHTS
INSTRUMENT AND REFLECTOR HEIGHTS
CORRECTED HORIZONTAL ANGLES
ADJUSTMENT
REDUCED SLOPE DISTANCES
REDUCED VERTICAL ANGLES
LATITUDE
DIFFERENCES IN LEVEL
SPIRIT LEVELLING
PLAN DISTANCES
LOCAL RADIUS OF SPHEROID
CHORD DISTANCES
ADJUSTED HORIZONTAL ANGLES
SPHEROID DISTANCES
AZIMUTH CONTROL
LOCAL SCALE FACTOR
GRID DISTANCES
COORDINATES OF SURVEY STATIONS
SURVEY NETWORK

(a) The carrier wavelength of the transmitted radiation.

(b) The mean ambient air temperature.

(c) The mean ambient air pressure.

(d) The mean ambient humidity.

The refractive index of air can be calculated from either the Barrell and Sears or the Edlén formulae, which have been considered in detail in Chapter One.

The following factors affect the measuring accuracy of E.D.M. instruments:

(a) Modulation frequency.

(b) Additive constant.

(c) Phase angle measurement.

(d) Ambient refractive index.

These sources of error are discussed in detail in Chapter Two. The modulation frequency is the 'heart' of the E.D.M. technique and should be checked before all accurate measurements. An off-air frequency standard and an inexpensive frequency counter have been found suitable for frequency calibration. No E.D.M. measurement can be more accurate than the accuracy with which the additive constant is known. The constant is best determined by a series of measurements on a long multi pillar baseline. The maximum accuracy of phase angle measurement is controlled by the design of each type of E.D.M. instrument. However, the linearity of phase angle measurements should be regularly checked with a calibration bar to ensure consistency in a series of accurate distance measurements.

The mean ambient refractive index of the air along an E.D.M. ray path is necessary for the accurate atmospheric correction of distance measurements. The determination of mean refractive index, which depends on the accurate evaluation of mean air temperature, pressure and humidity, is the limiting factor on the ultimate accuracy of E.D.M. measurements. An error of $\pm$ 1°C in the temperature or $\pm$ 3 mb in the pressure would introduce an error of approximately one part per million in the refractive index and consequently an error of $\pm$ 1 mm per 1000 m in a distance measurement. Errors in the determination of humidity, by the measurement of wet bulb temperatures, have an insignificant effect on the refractive index of air for light and infra red wave E.D.M. instruments.

(Since accurate distance measurements depend on the derivation of mean atmospheric parameters it is essential that accurate meteorological equipment should be used in conjunction with E.D.M. instruments. All barometers and thermometers should be calibrated against laboratory standards before use (Chapter Three).)

When the difference in level between two survey stations is less than three hundred metres, as is normally the case in engineering survey, the mean of pressure measurements taken simultaneously at both ends of a survey line, with calibrated barometers, should not be in error by more than $\pm$ 0.4 mb. It follows that errors in a corrected distance due to the inaccuracies of pressure measurement will be less than $\pm$ 0.15 ppm.

The mean of simultaneous measurements of wet bulb temperature taken at both ends of a survey line is sufficiently accurate to give the mean humidity of the air along the line of sight for the calculation of the refractive index correction of E.D.M. measurements.

Mean air temperature is the most difficult atmospheric parameter to determine, primarily because routine temperature measurements are, for practical reasons, normally taken near to the ground surface. Section 4.3. has shown the importance of the surface in the heat transfer processes and explains that near surface measurements of temperature are unlikely to be truly representative of free air temperatures. The results of the Bryn Howel - Tyn y Wern experiment showed that errors of $\pm$ 1^{o}C were possible in the mean of ground level end station temperature measurements. This experiment emphasised the importance of representative temperature sampling on E.D.M. lines of sight passing close to the ground surface.

The Llangollen radiosonde measurements indicated that the adiabatic lapse rate, of approximately -1^{o}C per 100 m, is the norm in the lower 300 m of the earth's atmosphere. Near surface temperature anomalies were found to exist but these were least in the middle of the afternoon and were further reduced on overcast days. It is possible to extrapolate ground level temperatures, measured below a line of sight, upwards to the line of sight. The probable accuracy of such extrapolated temperatures has been estimated as $\pm$ 0.2^{o}C $\pm$ 0.3^{o}C/100 m of the extrapolated height, providing measurements are made when the surface temperature anomalies are small. Extrapolated temperatures should be considered with end station temperatures in the determination of a representative mean on-line temperature.

Investigations have been made on the line between the Ridge and Sheepfold stations, in the Llangollen survey network, to determine to what extent the mean of end station temperature measurements represents the true mean air temperature along the line. This line is 2440 m in length, has a difference in level of 125 m and has an average height of 160 m above the ground. It has been found that, on this line, on overcast days, the mean of temperatures recorded at 1.5 m above the surface at each station can be stated with confidence as being within $\pm$ 1^{o}C of the true mean air temperature. The mean of end station temperatures was less representative on sunny days when radiant heating produced near surface temperature anomalies.

The mean of end station temperatures taken at the time of E.D.M. observations should be used in the atmospheric correction of E.D.M. measurements. There is no advantage in taking the mean of temperatures recorded for a period of time before and after a distance measurement, because of the large and sudden variations of temperature which can occur at the end stations. Such variations have been recorded at stations in Llangollen (section 4.5.4.).

The combined effect of errors in the derivation of mean atmospheric conditions on an E.D.M. distance measurement, along a line such as that between the Ridge and Sheepfold stations, can be summarised as follows, assuming the measurement to have been made in the afternoon of an overcast day.

ATMOSPHERIC PARAMETER	PROBABLE ERROR IN DERIVED MEAN	ERROR INTRODUCED INTO E.D.M. MEASUREMENT (ppm)
Temperature	$\pm$ 1^{o}C	$\pm$ 1.00
Pressure	$\pm$ 0.4 mb	$\pm$ 0.15
Humidity	-	$\pm$ 0.00

It can be concluded from these figures that with the greatest care in the measurement of atmospheric parameters, on a line passing high above the ground surface, errors of approximately $\pm$ 1.2 ppm may be introduced due to the inadequacies of atmospheric sampling. The following points must be considered to achieve such high standards of accuracy in an E.D.M. distance measurement.

(a) Temperature measurements should be taken at each end of the survey line.

(b) Temperature measurements should be synchronised with E.D.M. measurements.

(c) E.D.M. and temperature measurements should be taken in the middle of the day.

(d) E.D.M. and temperature measurements should be taken in overcast weather.

(e) Ground grazing lines of sight should be avoided.

(f) Mid-line temperature measurements should be taken wherever possible.

(g) All temperature measuring instruments should be aspirated and shielded from radiant heating.

(h) Pressure reading should be taken, with calibrated barometers, at each end of the survey line.

(j) Wet bulb temperature measurements should be taken at both survey stations with aspirated hygrometers.

The use of barometer measurements to give the mean air temperature between two survey stations has been considered in section 4.7. However, experimental work has shown barogenetic temperatures to be unreliable in engineering survey, due to the requirements for very accurate barometer readings, atmospheric stability during the measurements and a prior knowledge of the difference in level between the stations.

Having obtained representative atmospheric parameters the refractive index correction to E.D.M. measurements should be obtained from first principles as described in sections 2.6. and 2.7. Manufacturer's formulae and nomographs should be used as approximate checks on the calculated correction. If E.D.M. instruments with refractive index controls are used for accurate distance measurement then the setting of the control and the measured atmospheric parameters should be recorded and the correction checked from first principles.

The two wavelength technique of distance measurement has been discussed in section 2.8. Further research should be encouraged into methods of improving the phase resolution of E.D.M. to the extent whereby the two wavelength technique can be applied to engineering survey measurements. When this method is successfully developed for short range E.D.M. the problems of obtaining representative mean refractive indexes will be considerably reduced.

Great care in the derivation of mean atmospheric parameters is only worth while if the adverse effects of all the other sources of error have been considered. Errors due to the centring of both the instrument and the reflector prisms are of paramount importance during high accuracy E.D.M. distance measurements.

When slope distance measurements are to be reduced trigonometrically, the accuracy of the vertical angle measurements must be compatible with that of the distance measurement, in order to give accurate plan distances and differences in level. Refraction can have a serious effect on the accuracy of vertical angle measurements, due to the refractive bending of the line of sight from the theodolite to the target. This refractive bending is caused by the changes of pressure and temperature with height which normally exist in the atmosphere. Assuming normal atmospheric conditions, the error in a vertical angle measurement due to refraction will be approximately ± 2.6" per 1000 m of the line of sight.

The variation of pressure with altitude is well defined, however, temperature gradients have been shown to be dependent upon the height of the air mass above the ground surface and the time of day. Only with a knowledge of the temperature gradient at every point along the line of sight, can the ray path be defined and an accurate correction for refraction applied. Such measurements of lapse rate are only possible in practice for lines of sight that pass close to the ground surface.

If vertical angle observations are made on overcast afternoons, then an approximate correction can be applied by assuming that a uniform temperature lapse rate exists (section 5.2.).

Simultaneous or neo-simultaneous reciprocal vertical angle measurements reduce refraction errors without the need for corrections. However, refraction errors will only be totally eliminated if the ray path is symmetrical, which implies the existance of a uniform temperature lapse rate.

As a result of vertical angle measurements, made by the author in Risley Park and at Llangollen (sections 5.4. and 5.5.), the following recommendations can be made to minimise the effects of refraction on accurate vertical angle measurements, with a first order theodolite.

(a) Reciprocal observations should be made, ideally simultaneously.

(b) If reciprocal observations are not possible, measurements should be taken from one station and immediately afterwards from the other station.

(c) Observations should be concentrated on one line at a time, that is only one target station should be sighted from each theodolite station.

(d) Observations should be made in overcast weather.

(e) Observations should be made in early afternoon.

(f) The angles of refraction from practical measurements should be compared with theoretical angles of refraction to indicate the suitability of the atmosphere for vertical angle measurements. This requires temperature and pressure measurements at both ends of a survey line.

(g) Ground grazing lines of sight should be avoided.

It is only by considering these factors that reduced vertical angles with an accuracy approaching ± 1" of arc will be possible.

If reciprocal observations are not possible then a correction for the curvature of the earth should be applied to the vertical angle measurement. A theoretical refraction correction should also be applied, by assuming an adiabatic lapse rate exists in the air if no other data is available.

Horizontal angle measurements will only be significantly affected by refraction if the line of sight passes close to a surface, where large horizontal temperature gradients exist. Ground, or surface, grazing lines should therefore be avoided.

The difference between the deviation of the vertical at two stations can introduce errors into vertical angle observations, (section 5.6.). It can be assumed that the geodetic deviation of the vertical is the same at all stations in a small engineering survey network. However, there will be differences in the deviations at survey stations caused by the topography surrounding each station. A study has shown that the deviation of the vertical due to the topography can be calculated providing that the area surrounding a station is accurately contoured and the geology of the underlying rocks is understood. No gravimeter measurements are required in the calculation of the deviation due to topography.

The calculated components of the deviation at the Ridge and Sheepfold stations, in the azimuth of the line between the stations, were 1.9" and 1.1" respectively. These values are unusually large because both survey stations are situated on steep mountain slopes. However, as the stations face each other across a valley the effects of the deviation are minimised and would only introduce an error of ± 0.4" in a reciprocal vertical angle measurement. The effect would be more serious if the vertical angle was observed from one end of the line only.

It is to be recommended that, when accurate vertical angle measurements are to be taken, the possible effects of deviation of the vertical due to the topography should be considered. They are only likely to be significant for the most accurate work in mountainous terrain.

The efforts expended in accurately measuring and correcting E.D.M. distances, horizontal and vertical angles will be to no avail unless great care is taken in the combination of the values to give the co-ordinates of survey stations. Chapter Six has described the precise methods of reducing E.D.M. slope distances and vertical angles to give plan distance and difference in level. The precise methods should always be used when high accuracy is required. For lines inclined at less than 45^{o} to the horizontal, errors in measured vertical angles have the greatest effect on the difference in level, while errors in the E.D.M. measurement are the dominant sources of error in the reduced plan distance.

Nomographs are given in section 6.4. which enable the calculation of the errors in plan distance and difference in level, due to errors in the measurement of slope distances and vertical angles, for any survey line (Figures 6.3. to 6.6.). Figures 6.7. and 6.8. give examples of precision ratio graphs for trigonometrically reduced plan distances and differences in level. By using this type of graph the minimum accuracy to be expected, from a series of measurements using a specific combination of E.D.M. and theodolite, can be obtained for any given survey line. Graphs of this type act as useful guides to the standards of measurement accuracy required to meet specifications of plan and level accuracy.

Great care should be taken in the measurement of E.D.M., theodolite, prism and target heights if the accuracy of distance and vertical angle measurements is not to be lost in the reduction to plan and difference in level.

All the distances in a survey network should be reduced to a common reference plane, such as the National Projection plane in Great Britain, before they are combined with horizontal angles and azimuth controls to give the co-ordinates of survey stations.

The results of the measurements in the Llangollen survey network (section 6.6.) indicate the accuracies obtainable in first order engineering surveys. These accuracies would without doubt have been improved had all the observations been made from survey pillars with constrained instrument centring. In the opinion of the author the horizontal angles of the network can be stated with confidence to better than ± 3", the E.D.M. slope distances to better than ± 5 mm, the vertical angle measurements to better than ± 4" and the reduced plan distances to better than ± 10 mm. These accuracies are pessimistic and on many of the lines far greater accuracies have been achieved.

Only by considering all possible sources of error in the complete survey system (Figure 7.1.) and by taking meticulous care in measurements and corrections, particularly those for refraction, can the co-ordinates of survey stations be stated to high standards of accuracy with absolute confidence.

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APPENDIX A

THE QUORN BASELINE

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A1. Introduction

In July 1974 facilities became available along a disused railway track at Quorn in Leicestershire, where twelve ground stations were set out in line, over a distance of 942 m. This line has proved to be particularly useful during recent times when rapidly increasing numbers of E.D.M. instruments have been introduced. Up to the present time fourteen instruments have been used at Quorn and the distances measured many times over the period July 1974 to May 1976. As new instruments have been introduced they have been checked on the Quorn baseline as part of evaluation programmes.

The purpose of this appendix is to describe the setting out of the baseline and to record the results subsequently obtained with other instruments. It should be noted that this baseline was not set out specifically for the field testing of E.D.M. instruments, but for the very practical task of enabling power cables to be cut into specified lengths.

A2. Setting Out the Nominal Distances

The object of the setting out procedure was to insert twelve survey stations at specified distances, ranging from 80 to 942 m, parallel to the average gradient of the disused railway track. These stations were intended to be used in the marking off, and cutting, of power cables into accurately known lengths. The accuracy requested for the distances between the survey stations was ± 5 mm over the full range.

The most accurate instrument available in the Mining Department at that time was a Tellurometer MA100 (No. 309) which has a claimed standard deviation of a single determination of distance due to instrumental errors of ± 1.5 mm. It is also claimed by the manufacturers that the scale error due to atmospheric refractive index determination is usually less than 2 ppm. It was considered therefore that by taking several careful measurements and using the "differential" technique explained below, the required accuracy could be met. However, before field measurements were commenced, the instrument and all ancillary equipment such as optical plummets, tripods, barometers and hygrometers were all checked in the laboratory.

The survey stations, shown in Figure A1, consisted of approximately a cubic metre of concrete sunk into the ground, with a central flat metal plate for marking out the nominal distance. A metal cylinder with a removable cap protected the measuring plate when the stations were not in use. It will be realised that it is necessary to set up tripods when using these stations, and all the results quoted are therefore for tripod mounted instruments and reflectors. Although the long term stability of these stations cannot be guaranteed, they were considered substantial enough to at least suffice for the period of the cable-cutting, a matter of three to four weeks. In fact, no perceptible movement has yet been observed.

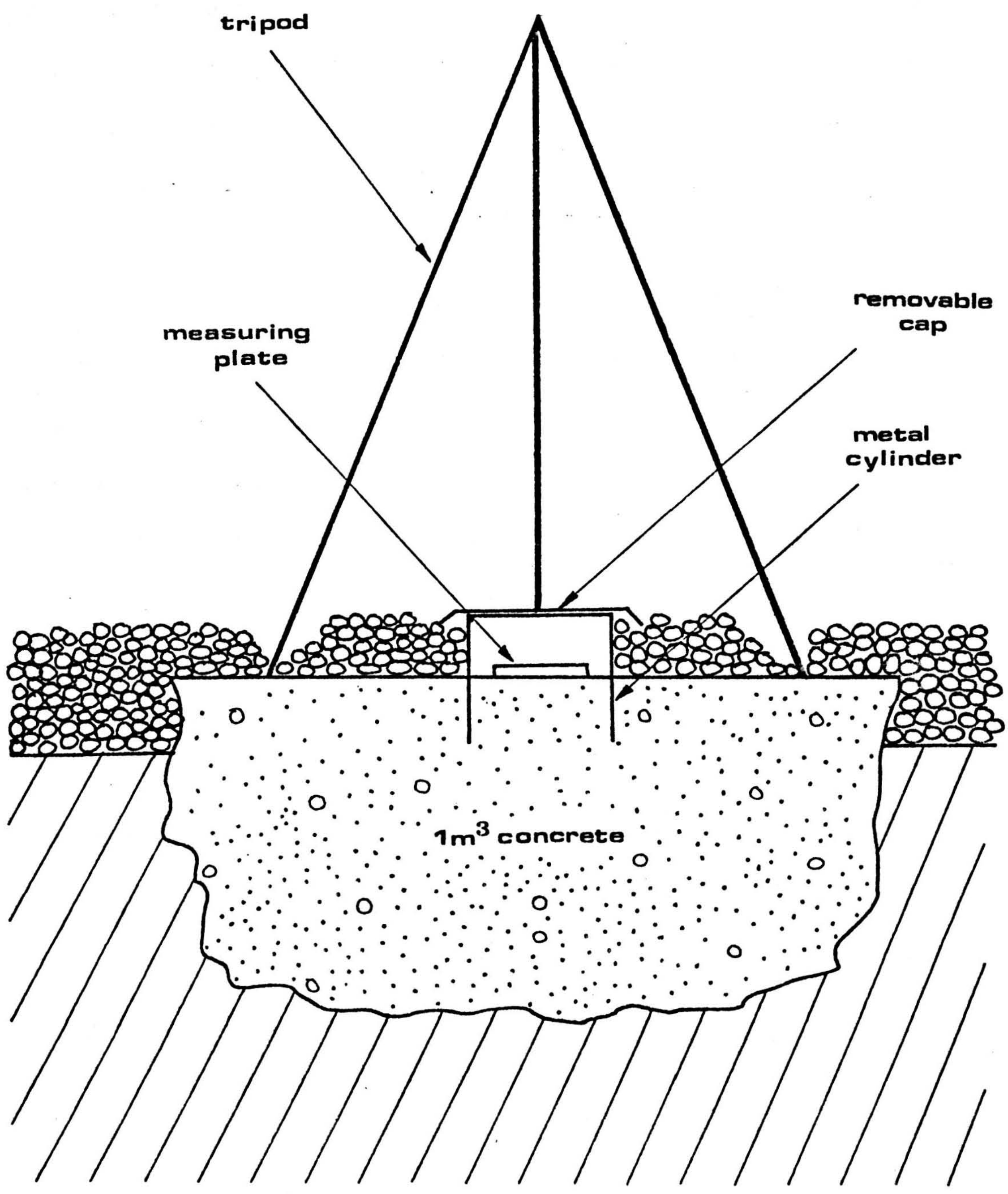
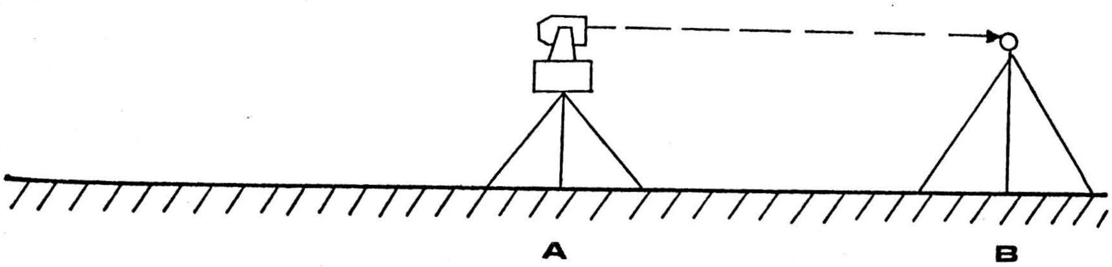


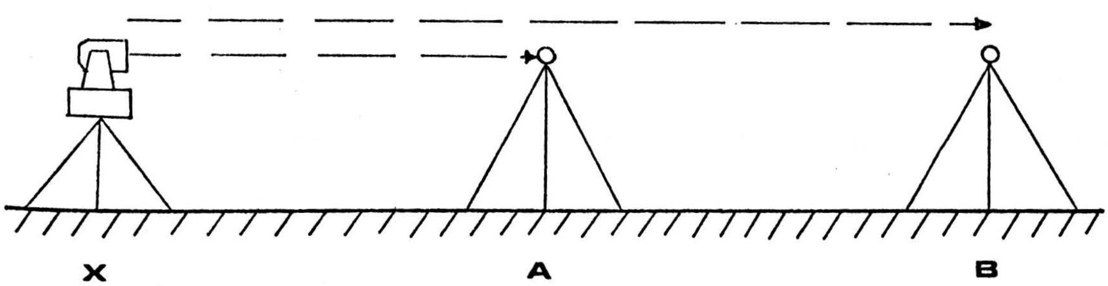
Figure A1. Construction of Stations.

Initially the survey stations were positioned approximately by steel tape, and during the setting out operations the Tellurometer MA100 measured distances to fine marks etched into the metal measuring plates. With the aid of a short steel rule the final distances were marked, at the nearest metre, by drill holes of 1 mm diameter which passed right through the metal plates so as to prevent these marks from being obliterated. All subsequent measurements have been taken from the centres of the drill holes.

The eleven specified distances were set out by direct, and in order to eliminate errors associated with the additive constant, by differential measurements, see Figure A2. During the direct measurements the Tellurometer MA100 was set up at the first station A and after allowing thirty minutes for the instrument to fully warm up, observations were taken to all the other stations in turn. A second visit to the site was made four days later. On this occasion the instrument was set up at a point X in line with the other stations but further along the track. The distance from this point to the first station (XA) was measured, followed by measurements to the other stations. The distance XA was again measured to conclude the operations. The baseline distances were then calculated by differences, for example AB = XB - XA, as explained in Figure A2, the mean value of the results obtained for the distance XA being used in the calculations. Although weather conditions during the setting out operations were stable, frequent pressure and temperature measurements were recorded. In no case were less than six sets of fine readings recorded for each distance when using the Tellurometer MA100, and in some instances, as many as fourteen sets of fine readings were recorded. In spite of these precautions the nominal



(i) Direct Measurement



(ii) Differential Measurement

AB = XB - XA

Figure A2. Direct and Differential Measurements During Setting Out Of The Quorn Baseline.

distances were set out in two working days by two people.

Following the second field visit, the instrument and all ancillary equipment were again carefully checked in the laboratory, particular attention being given to the optical plummets. The results of the setting out operations are summarised and tabulated in Figure A3. The average agreement between the direct and differential measurements was 2.3 mm, the greatest difference of 4.2 mm occurring over the distance AD (150 m) and the smallest 0.8 mm over the distance AJ (700 m). A simple arithmetic mean of the two sets of measurements was taken to be the most probable value of the distances for setting out purposes.

After the setting out work had been completed the stations were used for the measuring and marking of cables prior to cutting into the specified lengths. When this work had been completed the baseline became available for routine surveying purposes.

A3. Measurements with Other Instruments

Since the Quorn baseline became available for survey purposes in August 1974, fourteen different E.D.M. instruments have been used to remeasure the distances. These instruments are illustrated in Plates A1. In most instances the same routine has been followed, the instrument being set up at the first station A and distances measured to other stations along the base. Although the slope of the line is only 1 in 250, the same instrument and reflector heights have been maintained and no reduction to the horizontal has been attempted. This eliminates the possibility of the measured distances being influenced by reduction errors

Direct Measurements (m)		Differential Measurements (m)	Difference (mm)	Mean Distances (mm)
AB	79.9990	80.0006	+1.6	79.9998
AC	130.0201	130.0214	+1.3	130.0207
AD	150.0098	150.0056	-4.2	150.0077
AE	170.0502	170.0517	+1.5	170.0509
AF	375.0207	375.0178	-2.9	375.0192
AG	445.0207	445.0236	+2.9	445.0221
AH	600.0111	600.0082	-2.9	600.0096
AJ	700.0368	700.0360	-0.8	700.0364
AK	820.0358	820.0340	-1.8	820.0349
AL	865.0472	865.0483	+1.1	865.0477
AM	942.0450	942.0410	-4.0	942.0430
			Mean 2.3	

Figure A3. Summary of Setting Out Measurements.

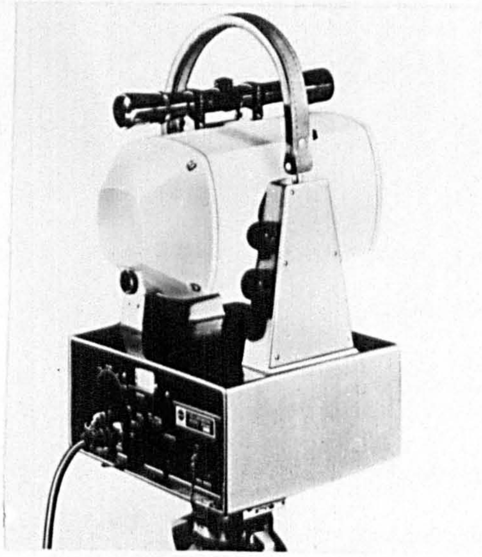
and also means that the field measurements are immediately comparable when using instruments with a refractive index control dial. The same barometers and hygrometers have been used for all measurements and these are regularly checked in the laboratory. Measurements have been taken at different times throughout the year and temperatures have ranged from below $0^{o}C$ to $+30^{o}C$.

In addition to some of the more recently introduced instruments the long established Geodimeter 6 and Wild DI10 instruments have also been used at Quorn. These particular instruments have been in regular service for over ten years in the case of the Geodimeter and six years for the DI10; a clear indication that such instruments are capable of giving excellent service over a long period of time. The use of the model 6 Geodimeter also emphasised the tremendous development which has taken place in the design of E.D.M. equipment from an operational point of view. In order to obtain a single measurement with the model 6, the operator is required to touch and adjust the instrument over forty times, to book twenty-four digital readings, and then to calculate the distance; with the Geodimeter 12 a distance is obtained almost at "the stroke of a button".

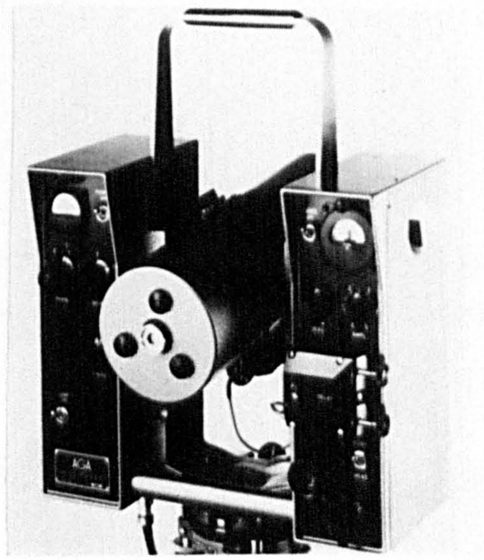
The shortest distance of 80 m was also measured by 2 m invar subtense bar methods, using a Kern DKM3 theodolite to measure the subtended angle, twenty measurements being taken from each end of the line.

PLATE A1.

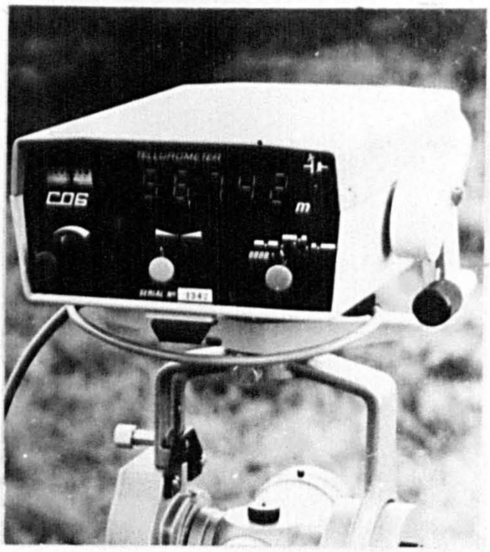
E.D.M. Instruments used on the Quorn baseline.



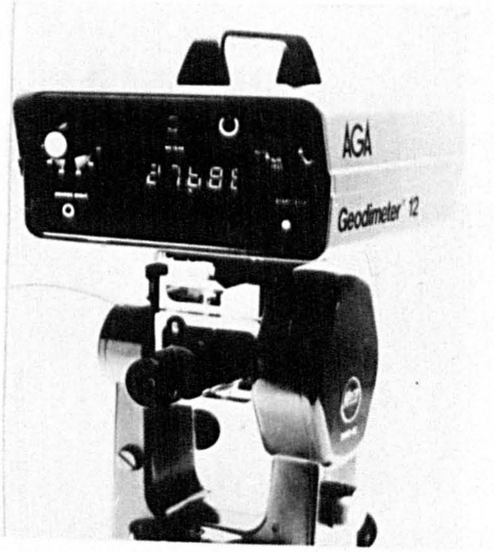
TELLUROMETER MA 100



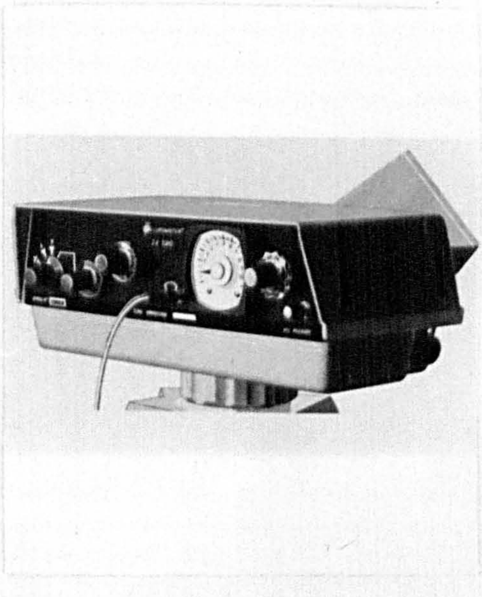
GEODIMETER 6



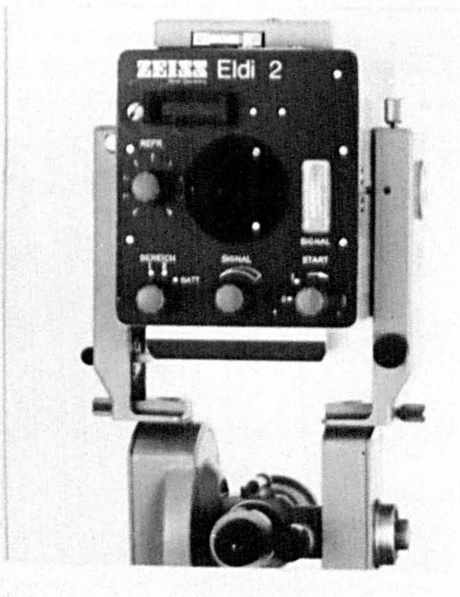
TELLUROMETER CD 6



GEODIMETER 12

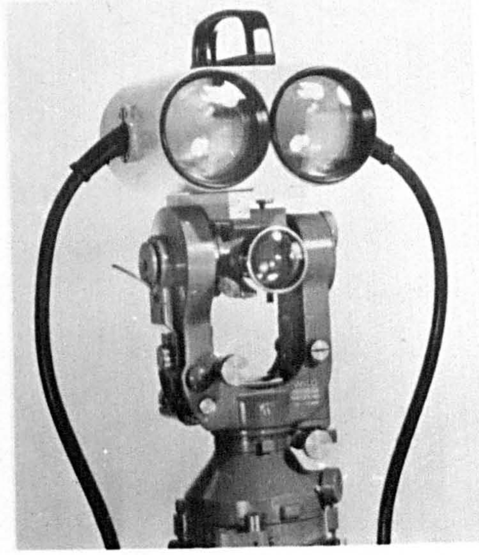


TELLUROMETER CA 1000

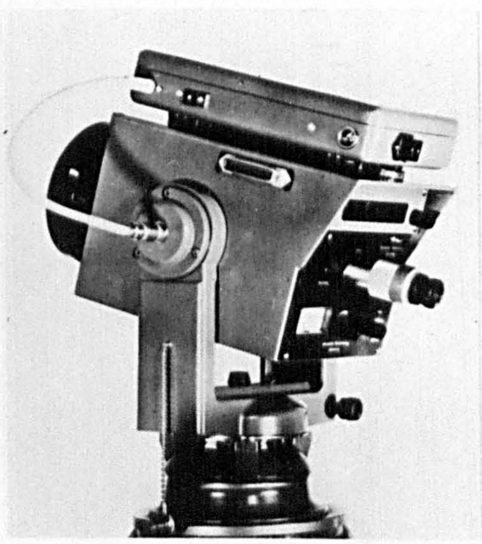


ZEISS ELDI 2

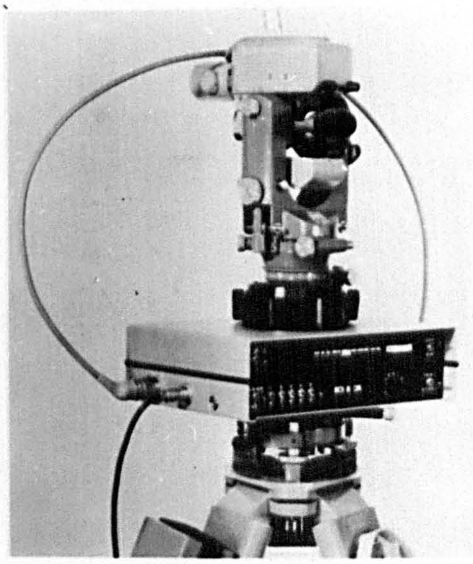
KERN ME 3000



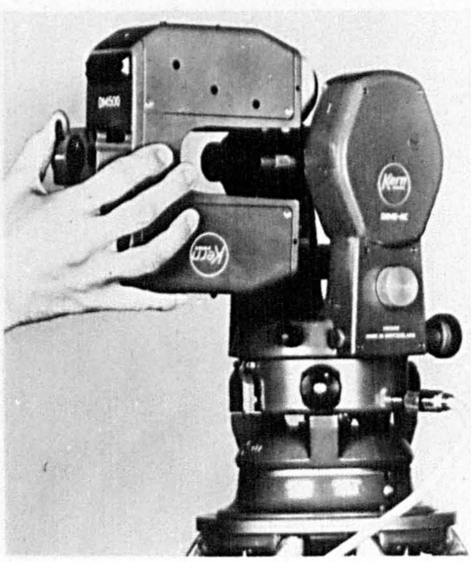
WILD DI 10



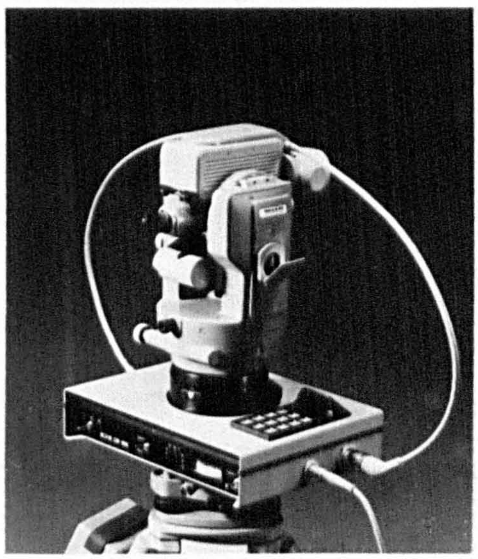
KERN DM 1000



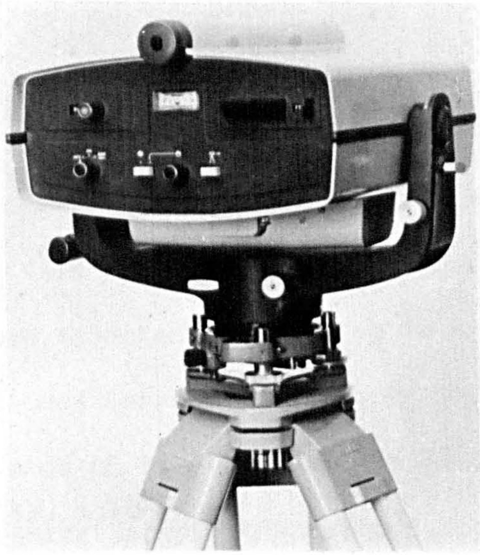
WILD DI 3



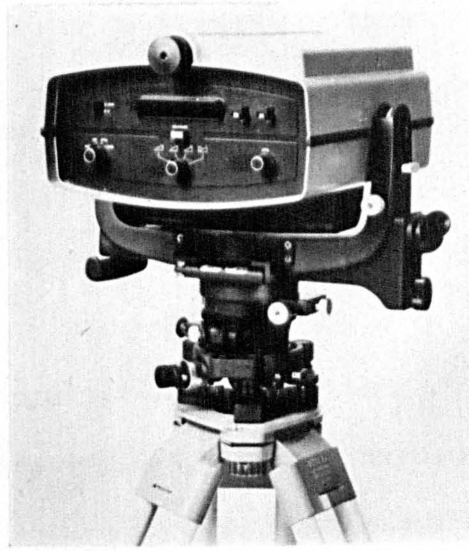
KERN DM 500



WILD DI 3S



HEWLETT-PACKARD 3805A



HEWLETT-PACKARD 3810A

A4. Results

The instruments used and the results obtained up to the present time on the Quorn baseline are listed in the table of Figure A4. No selection procedure has been adopted and therefore no observations have been rejected or adjusted in any way. One instrument taken to the base was quickly shown to be badly out of adjustment within a few minutes of the commencement of the observations; these particular measurements were terminated and are not included. The results are self-explanatory and are considered worthy of careful scrutiny.

The arithmetic means of all the E.D.M. measurements, the standard error of a single observation and of the arithmetic means have been computed; no weighting of observations has been attempted. The differences between these means and the individual measurements for each instrument have been listed in Figure A5, and the differences between the nominal and measured distances for each instrument listed in Figure A6.

The subtense bar measurement of the 80 m distance gave a result of 79.996 m, an exact agreement with the arithmetic mean of all the E.D.M. measurements. The standard error of the forty measurements of the subtended angle with the Kern DKM3 was ± 0.2", equivalent to ± 6 mm in the measured distance.

A5. Conclusions

The differences between the nominal distances set out and the

Nominal Distance July 1974	AB 80·000	AC 130·000	AD 150·000	AE 170·000	AF 375·000	AG 445·000	AH 600·000	AJ 700·000	AK 820·000	AL 865·000	AM 942·000
Tellurometer MA100 No. 366 Feb 1975	80·001	129·998	150·000	169·999	375·006	444·997	600·003	699·998	820·000	865·001	941·998
Tellurometer MA100 No. 340 June 1975	79·998	130·001	149·997	169·995	374·998	445·001	600·005	699·999	820·000	865·000	942·000
Tellurometer CA1000 M1178 R1177 June 1975						445·008					941·999
Tellurometer CD6 No. 1312 Feb 1976	79·997	130·007	149·999	170·002	374·998	444·993	600·006	700·002	820·009	865·003	942·009
Geodimeter 6 No. 6266 July 1975	79·986	129·992	149·997	169·995	374·982	444·989	600·002	699·997	819·995	864·994	941·983
Geodimeter 12 No. 3005 Jan 1975	79·990	129·995	149·993	169·997	374·996	444·991	—	699·997	—	—	941·999
Geodimeter 12 No. 3145 June 1975	79·995	129·996	149·998	169·998	374·999	444·998	600·003	699·995	819·998	864·999	941·995
Geodimeter 12 No. 3716 March 1976	79·999	130·005	150·000	170·002	375·003	445·003	600·007	700·005	820·004	865·007	942·012
Kern Mekometer ME3000 No. 197819 Nov 1975	79·995	129·999	149·997	169·998	374·999	444·998	600·002	699·999	820·000	865·001	942·002
Kern DM-1000 No. 197777 Aug 1974	79·983	129·999	—	170·002	374·993	—	599·993	—	—	—	941·982
Kern DM-500 No. 216571 May 1975	79·998	130·013	150·010	170·010	374·995	444·997					
Wild DI-10 No. 10884 Aug 1974	80·001	130·009	150·002	170 004	375·010	445·010	600·004	700·004	820·003	865·005	—
Wild DI-3 No. 31285 June 1975	79·999	129·999	149·996	169·994	—	444·995	599·994	699·992	819·993	864·994	941·986
Wild DI-3S No. 32930 May 1976	79·996	130·001	149·999	169·998	374·993	444·996	599·992	699·990	819·993	864·999	941·998
Hewlett-Packard 3805A No. 1338A00202 Dec 1975	80·001	130·007	150·007	170·008	375·007	445·006	600·008	700·006	820·011	865·008	942·012
Hewlett-Packard 3810A No. 1522A00233 May 1976	79·997	129·999	149·995	169·999	374·997	444·997	599·998	699·992	819·992	864·994	941·997
Zeiss (0) ELDI 2 No. 90357 April 1976	80·002	130·009	150·000	170·002	375·012	445·015	600·009	700·010	819·999	864·993	942·010
Subtense bar measurement May 1976	79·996 ± 6 mm.										
Mean of all E.D.M. measurements	79·996	130·002	149·999	170·000	374·999	445·000	600·002	699·999	820·000	865·000	941·999
Standard error of a single observation (± mm)	5·5	5·9	4·4	4·5	7·6	7·1	5·8	5·8	5·8	5·1	9·6
Standard error of the arithmetic mean (± mm)	1·3	1·5	1·1	1·1	2·0	1·8	1·5	1·5	1·6	1·4	2·5
Nominal minus Mean (mm)	+4	−2	+1	0	+1	0	−2	+1	0	0	+1

Figure A4. Summary of Measurements on Quorn Baseline.

All measurements in metres from tripod mounted instruments and reflectors

Mean of all EDM Measurements	AB 79·996	AC 130·002	AD 149·999	AE 170·000	AF 374·999	AG 445·000	AH 600·002	AJ 699·999	AK 820·000	AL 865·000	AM 941·999	Average Deviation (mm)
Tellurometer MA100 No. 366 Feb 1975	+5	−4	+1	−1	+7	−3	+1	−1	0	+1	−1	2·3
Tellurometer MA100 No. 340 June 1975	+2	−1	−2	−5	−1	+1	+3	0	0	0	+1	1·5
Tellurometer CA1000 M1178 R1177 June 1975						+8					0	—
Tellurometer CD6 No. 1312 Feb 1976	+1	+5	0	+2	−1	−7	+4	+3	+9	+3	+10	4·1
Geodimeter 6 No. 6266 July 1975	−10	−10	−2	−5	−17	−11	0	−2	−5	−6	−16	7·3
Geodimeter 12 No. 3005 Jan 1975	−6	−7	−6	−3	−3	−9	—	−2	—	—	0	4·5
Geodimeter 12 No. 3145 June 1975	−1	−6	−1	−2	0	−2	+1	−4	−2	−1	−4	2·2
Geodimeter 12 No. 3716 March 1976	+3	+3	+1	+2	+4	+3	+5	+6	+4	+7	+13	4·6
Kern Mekometer ME3000 No. 197819	−1	−3	−2	−2	0	−2	0	0	0	+1	+3	1·3
Kern DM-1000 No. 197777 Aug 1974	−13	−3	—	+2	−6	—	−9	—	—	—	−17	8·3
Kern DM-500 No. 216571 May 1975	+2	+11	+11	+10	−4	−3	—	—	—	—	—	6·8
Wild DI-10 No. 10884 Aug 1974	+5	+7	+3	+4	+11	+10	+2	+5	+3	+5	—	5·5
Wild DI-3 No. 31285 June 1975	+3	−3	−3	−6	—	−5	−8	−7	−7	−6	−13	6·1
Wild DI-3S No. 32930 May 1976	0	−1	0	−2	−6	−4	−10	−9	−7	−1	−1	3·7
Hewlett-Packard 3805A No. 1338A00202 Dec 1975	+5	+5	+8	+8	+8	+6	+6	+7	+11	+8	+13	7·7
Hewlett-Packard 3810A No. 1522A00233 May 1976	+1	−3	−4	−1	−2	−3	−4	−7	−8	−6	−2	3·7
Zeiss (Oberkochen) Eldi 2 No. 90357 May 1976	+6	+7	+1	+2	+13	+13	+7	+11	−1	−7	+11	7·4

Figure A5. Deviations from Arithmetic Means of E.D.M. Measurements.

All deviations in mm

Nominal Distance July 1974	AB 80·000	AC 130·000	AD 150·000	AE 170·000	AF 375·000	AG 445·000	AH 600·000	AJ 700·000	AK 820·000	AL 865·000	AM 942·000	Average Deviation mm
Tellurometer MA100 No. 366 Feb 1975	+1	−2	0	−1	+6	−3	+3	−2	0	+1	−2	1·9
Tellurometer MA100 No. 340 June 1975	−2	+1	−3	−5	−2	+1	+5	−1	0	0	0	1·8
Tellurometer CA1000 M1178 R1177 June 1975						+8					−1	4·6
Tellurometer CD6 No. 1312 Feb 1976	−3	+7	−1	+2	−2	−7	+6	+2	+9	+3	+9	4·6
Geodimeter 6 No. 6266 July 1976	−14	−8	−3	−5	−18	−11	+2	−3	−5	−6	−17	7·9
Geodimeter 12 No. 3005 Jan 1975	−10	−5	−7	−3	−4	−9	—	−3	—	—	−1	5·3
Geodimeter 12 No. 3145 June 1975	−5	−4	−2	−2	−1	−2	+3	−5	−2	−1	−5	2·9
Geodimeter 12 No. 3716 March 1976	−1	+5	0	+2	+3	+3	+7	+5	+4	+7	+12	4·5
Kern Mekometer ME3000 No. 197819 Nov 1975	−5	−1	−3	−2	−1	−2	+2	−1	0	+1	+2	1·8
Kern DM-1000 No. 197777 Aug 1974	−17	−1	—	+2	−7	—	−7	—	—	—	−18	8·7
Kern DM-500 No. 216571 May 1975	−2	+13	+10	+10	−5	−3	—	—	—	—	—	7·2
Wild DI-10 No. 10884 Aug 1974	+1	+9	+2	+4	+10	+10	+4	+4	+3	+5	—	5·2
Wild DI-3 No. 31285 June 1975	−1	−1	−4	−6	—	−5	−6	−8	−7	−6	−14	5·8
Wild DI-3S No. 32930 May 1976	−4	+1	−1	−2	−7	−4	−8	−10	−7	−1	−2	4·3
Hewlet-Packard 3805A No. 1338A00202 Dec 1975	+1	+7	+7	+8	+7	+6	+8	+6	+11	+8	+12	7·4
Hewlett-Packard 3810A No. 1522A00233 May 1976	−3	−1	−5	−1	−3	−3	−2	−8	−8	−6	−3	3·9
Zeiss (Oberkochen) Eldi 2 No. 90357 May 1976	+2	+9	0	+2	+12	+15	+9	+10	−1	−7	+10	7·0

Figure A6. Deviations from the Nominal Distances.

All deviations in mm

arithmetic means of sixteen sets of measurements involving fourteen different E.D.M. instruments average 1.1 mm. The largest difference of 4 mm occurs over the shortest distance of 80 m. These results would seem to confirm that the simple setting out procedure adopted enabled the required accuracy of ± 5 mm to be attained. The standard errors of the arithmetic means of the E.D.M. measurements average ± 1.6 mm and are fairly consistent up to the 865 m distance. Bearing in mind that all measurements were taken with tripod mounted instruments and reflectors the results are extremely satisfactory. Probably the most important source of error was that due to centring, even though optical plummets in good adjustment were employed. The siting of the line parallel to the gently sloping ground, together with the use of sensitive, frequently calibrated barometers, enabled a consistently accurate determination of the refractive index to be obtained under a wide range of weather conditions. It is considered that errors due to this source did not exceed ± 1 ppm.

The results give a good indication of the performance to be expected from the various instruments used, when measuring the distances quoted over fairly level ground and under differing weather conditions. They are all generally well within the manufacturers claims and even better results could be obtained by adopting techniques such as the differential method used in setting out the nominal distances.

If the assumption is made that the most probable values of the distances are represented by the arithmetic means of all the E.D.M. measurements, the accuracy of each instrument can be assessed. The Mekometer ME3000, as expected, gave results very close to the mean values, the average deviation being 1.3 mm; this was followed by the Tellurometer MA100 (No. 340) which gave an average deviation of 1.5 mm. It is worth noting that instruments of the same type and manufacture can give significantly different performances as shown, for example, by the results obtained with the three Geodimeter 12 instruments.

The consistency and reliability of the modern short range E.D.M. instrument has once again been proven. There would seem little difficulty in obtaining accuracies of better than ± 10 mm over distances of up to 1 km, provided that normal care is taken in correct use and maintenance. With greater distances, the correct determination of the atmospheric conditions along the line being measured may well be the limiting factor in determining the ultimate accuracy obtainable with these instruments. The use of such instruments will undoubtably reduce labour costs on many jobs and greatly improve overall efficiency. It is inconceivable that they should not be employed wherever possible in present day surveying and setting out work.

APPENDIX B

THE EFFECT OF WIND ON RADIOSONDE BALLOON FLIGHTS

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THE EFFECT OF WIND ON RADIOSONDE BALLOON FLIGHTS

The effect of wind on a tethered radiosonde balloon governs both the required strength of the tethering lines and the maximum height to which the balloon can ascend. To estimate the magnitude of these effects the wind force on the balloon, which is considered a sphere, is calculated. The wind force acting on the radiosonde and the tethering lines are not considered, the surface area of these parts being negligable in comparison to that of the balloon.

The drag force (F_D) exerted on a sphere is given by Fox and McDonald* as

$$F_D = \frac{1}{2} \cdot C_d \rho V^2 A \qquad \text{... B1.}$$

where C_d = drag coefficient (1)

ρ = air density ($kg.m^{-3}$)

V = wind velocity ($m.s^{-1}$)

and A = area exposed to wind (m^2)

For the present calculation a mean air density of 1.284 $kg.m^{-3}$ has been used. The drag coefficient for a sphere is plotted in Figure B1

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* Fox, R.W. and McDonald, A.T., "Introduction to fluid mechanics", Wiley, New York, 1973.

as a function of Reynolds Number, R_e, where

$$R_e = \frac{\rho\, Vd}{\mu} \qquad \text{... B2.}$$

in which d = diameter of a sphere (balloon) (m)

and μ = dynamic viscosity of air ($N.s.m^{-2}$)

The dynamic viscosity of air has been taken as 1.725×10^{-5} $N.s.m^{-2}$. The drag forces, at different wind velocities, have been calculated for a one metre diameter sphere from equation B1, using Figure B1 to derive the drag coefficient. The drag forces are plotted against wind velocity in Figure B2. The forces acting on a tethered balloon are shown in Figure B3. The net lift is the difference between the gross lift, from the buoyancy of the hydrogen filled balloon, and the load of the radiosonde. The resultant of the net lift and the drag force must be balanced by the tension in the tethering line.

The minimum requirement for maximum height for the radiosonde in the Llangollen experiment was 250 m above ground level. To reach this altitude with a tethering line 440 m long the minimum acceptable inclination of the radiosonde line was 35^{o} above the horizontal. This inclination, or tether angle, is the angle (τ) subtended by the resultant force and the drag force acting on the balloon (Figure B3). Figure B4 shows the tether angle for a balloon with 1N lift as a function of the wind velocity. This graph shows that if radiosonde flights are made in winds of less than 2.5 $m.s^{-1}$ the tether angle will be greater than 35^{o}. This wind velocity will exert a drag force of less than 2N on the balloon (Figure B2), which combined with a net lift of 1N will result

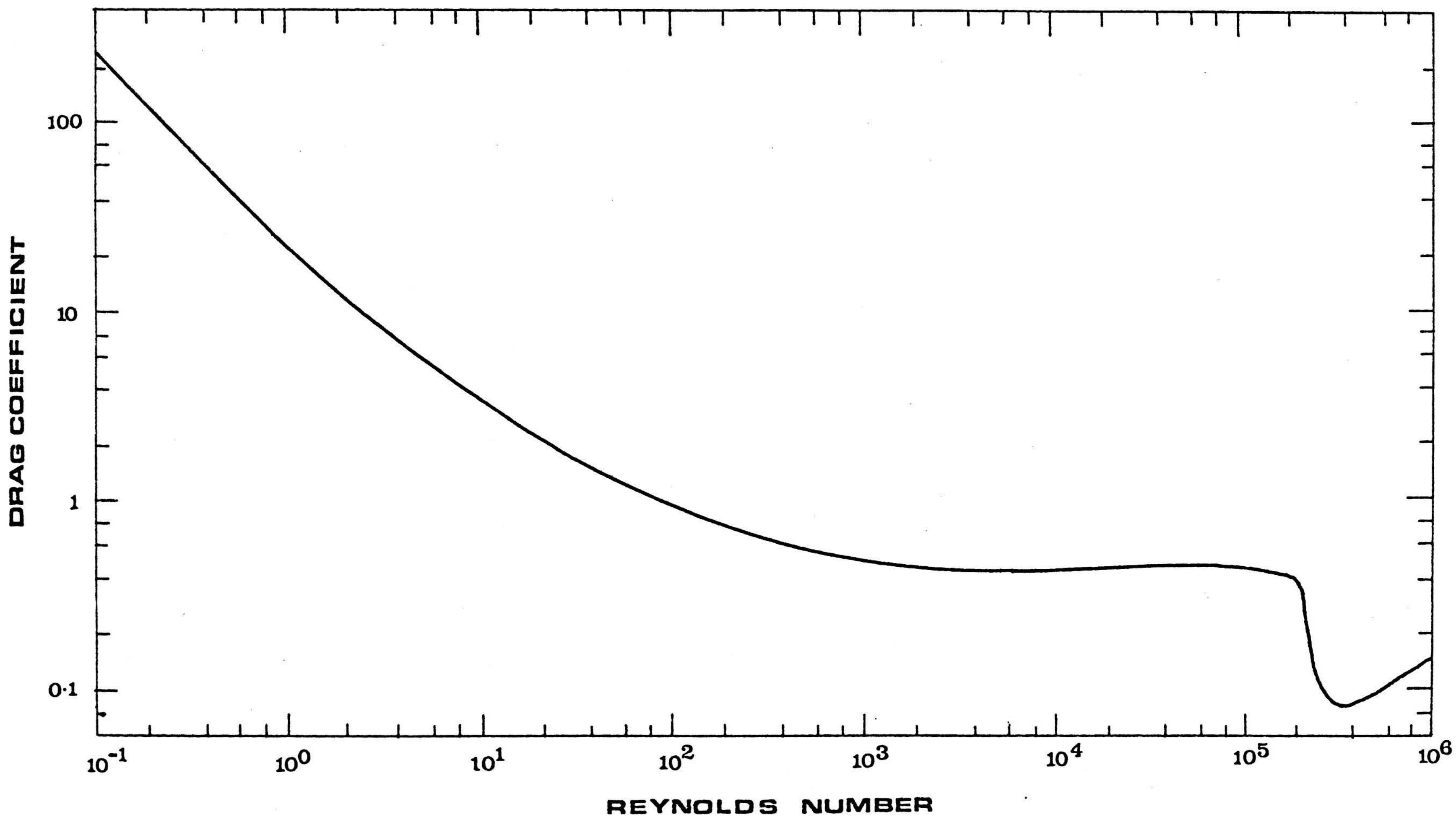

Figure B1. The Drag Coefficient of a Sphere as a Function of Reynolds Number.
(after Fox and McDonald, 1973)

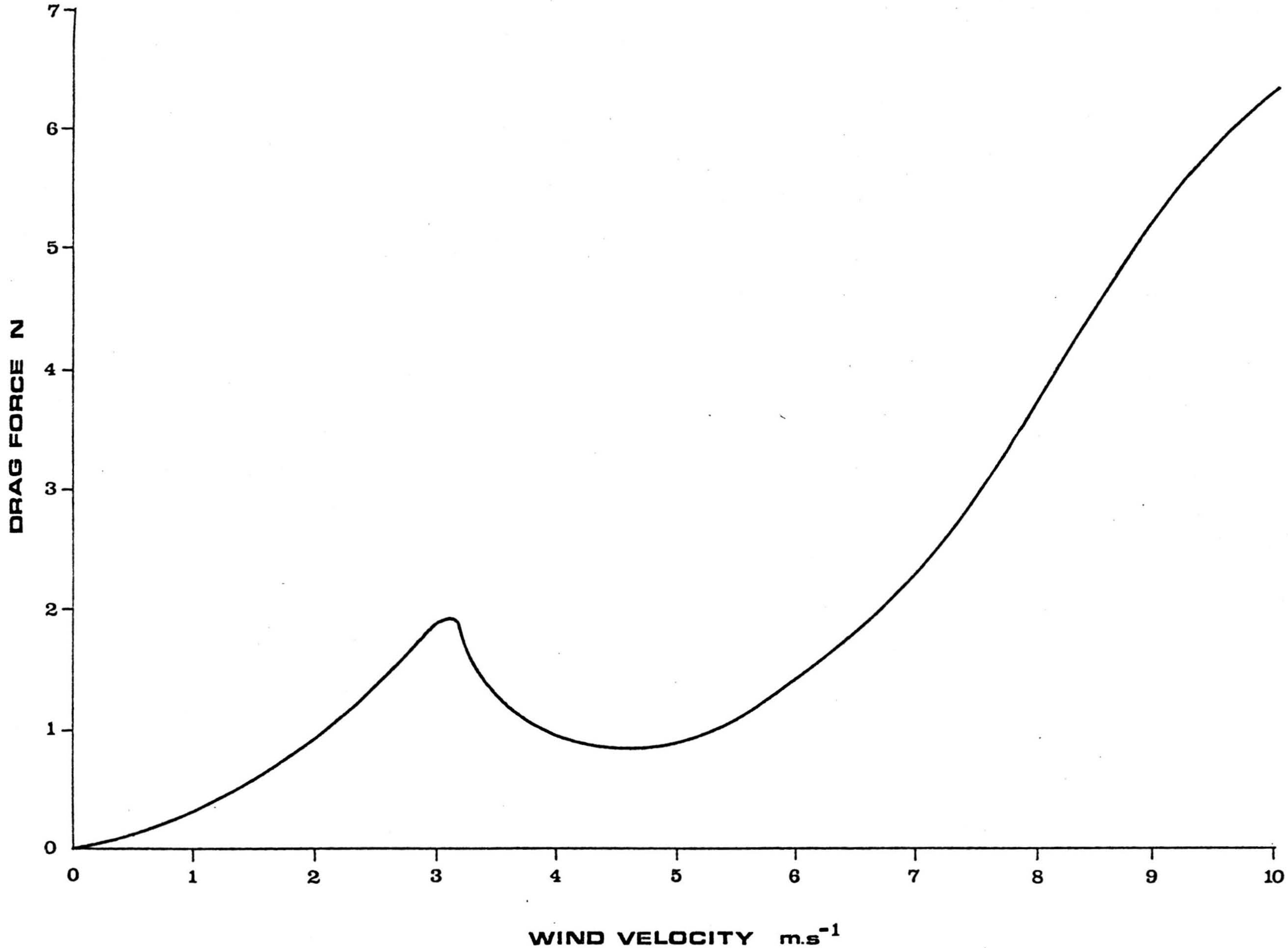

Figure B2. The Effect of Wind Velocity on Drag Force for a 1 m Diameter Balloon.

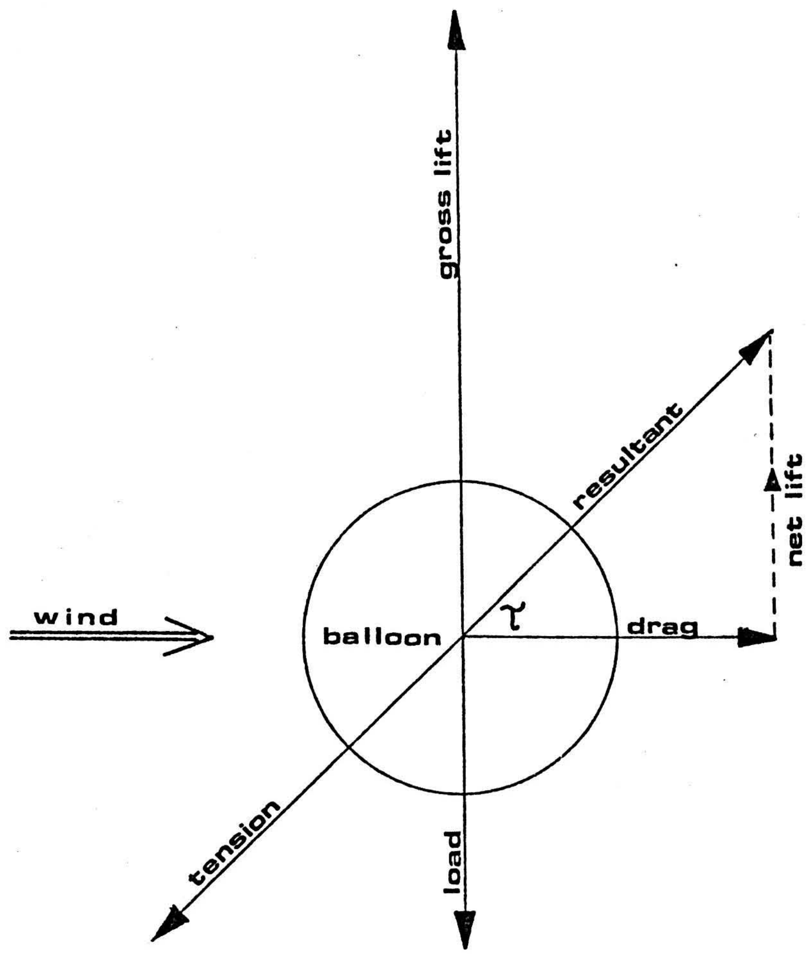

Figure B3. The Forces Acting on a Tethered Balloon.

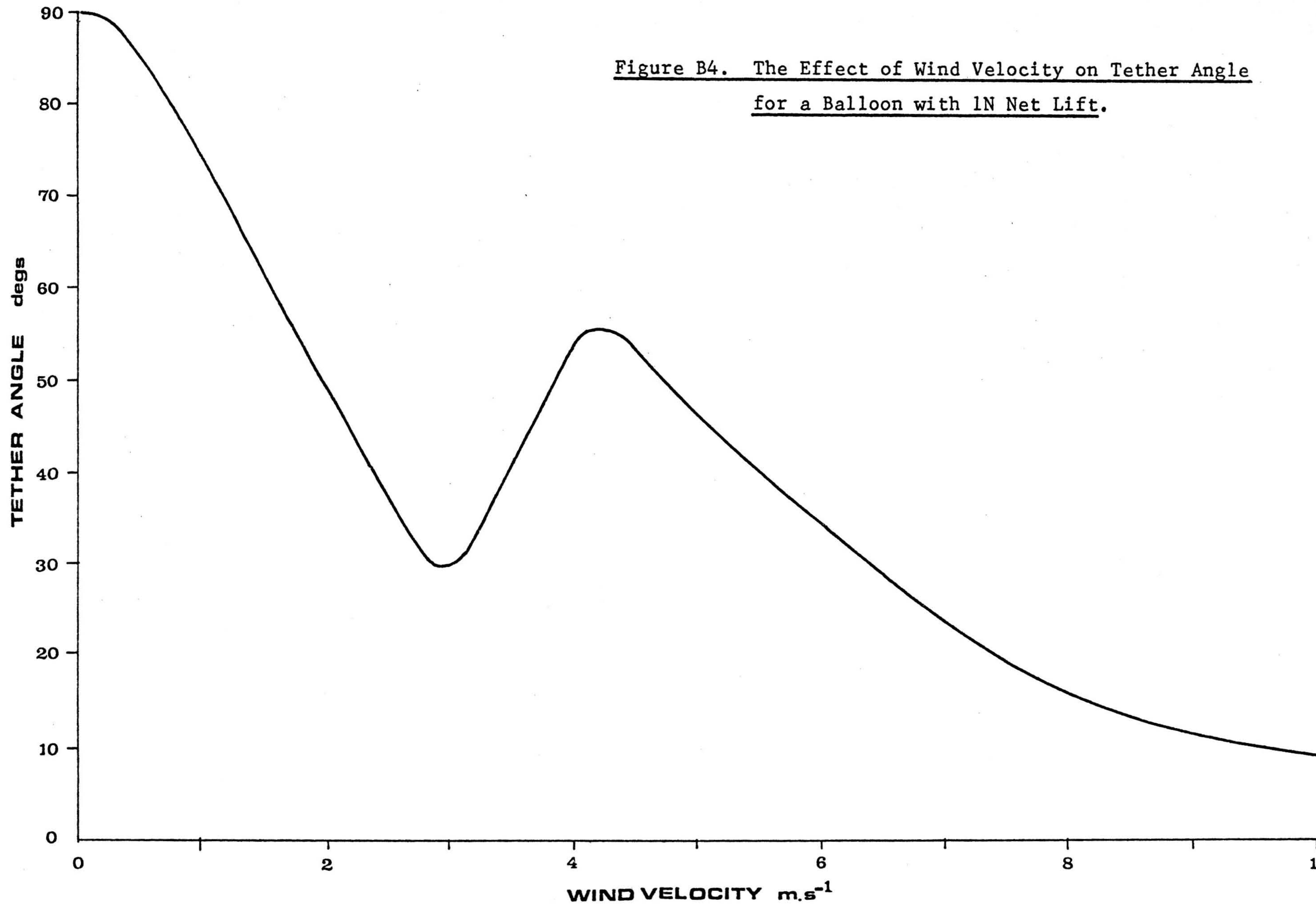

Figure B4. The Effect of Wind Velocity on Tether Angle for a Balloon with 1N Net Lift.

in a maximum tension of 2.3N in the tethering line. The breaking force of the tethering line is 13.5N, which allows a factor of safety of six. This safety margin is more than adequate to allow for the dynamic forces on the balloon from the buffeting effect caused by sudden changes of wind speed and direction.

The flight performance of the radiosonde balloon correlated well with the predictions from theory, a typical tether angle was 48^{o} allowing a maximum height of 330 m. The balloon was not flown in winds stronger than a light breeze of 2.5 $m.s^{-1}$.

APPENDIX C

INSTRUCTIONS FOR THE BRYN HOWEL - TYN Y WERN EXPERIMENT

APPENDIX C

INSTRUCTIONS FOR THE BRYN HOWEL - TYN Y WERN EXPERIMENT

The following procedure is to be followed on the afternoon of the distance measurement. The programme is split into three phases 'Setting Up', 'Measuring', and 'Dismantling'. A, B and C are the sites for the elevated temperature measurements.

PHASE		PERSONNEL	DIRECTIONS
1.a.	(i)	HODGES BELFIELD HARRIS CROFT	Proceed to Bryn Howel and set up reflector prisms.
	(ii)	HARRIS CROFT	Remain at Bryn Howel.
	(iii)	HODGES BELFIELD	Proceed to Tyn y Wern, set up tripod over station, warm up Geodimeter and Tellurometer.
1.b.	(i)	CURL WARWICK WEBB MACMILLAN NICHOLAS ADAMS ASHWORTH	Proceed to Meteorological Station A. Erect Mast.
	(ii)	WEBB	Remain at A.
	(iii)	REMAINDER	Proceed to B, Erect Mast.
	(iv)	MACMILLAN	Remain at B.
	(v)	REMAINDER	Proceed to C, Erect Mast.

	(vi)	ADAMS	Remain at C.
		ASHWORTH	
	(vii)	CURL	Return to Tyn y Wern.
		WARWICK	Return to Station A.
		NICHOLAS	Return to Station B.
2.	(i)	HODGES	At Tyn y Wern.
		CURL	Measure distance with Geodimeter
		BELFIELD	and Tellurometer, continuously. Record wet and dry bulb temperature and atmospheric pressure every 15 minutes.
	(ii)	WARWICK	At A, Measure elevated dry bulb temperatures
		WEBB	using the potentiometers and measure temperature with whirling hygrometers at 1.5 above the ground at 15 minute intervals.
	(iii)	MACMILLAN	At B, ditto.
		NICHOLAS	
	(iv)	ADAMS	At C, ditto.
		ASHWORTH	
	(v)	HARRIS	At Bryn Howel, measure wet and dry bulb
		CROFT	temperature at reflector height. Measure pressure. Both at 15 minute intervals.
3.a.	(i)	HODGES	Dismantle E.D.M. Equipment and proceed to
		BELFIELD	Bryn Howel.
	(ii)	CURL	Proceed to C.
		MACMILLAN	
		WEBB	
	(iii)	ADAMS	Remain at C.
		ASHWORTH	
	(iv)	HARRIS	Remain at Bryn Howel.
		CROFT	
	(v)	NICHOLAS	Remain at B.
	(vi)	WARWICK	Remain at A.
3.b.	(i)	HODGES	At Bryn Howel, dismantle reflectors and return
		BELFIELD	to hotel.

	HARRIS	
	CROFT	
(ii)	CURL	Dismantle mast at C. Proceed to B to
	MACMILLAN	join NICHOLAS, dismantle mast at B. Proceed
	WEBB	to A to join WARWICK, dismantle mast at A.
	ADAMS	
	ASHWORTH	Return to Hotel.

APPENDIX D

INSTRUCTIONS FOR THE RIDGE - SHEEPFOLD EXPERIMENT

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INSTRUCTIONS FOR THE RIDGE - SHEEPFOLD EXPERIMENT

Object: The aim of the experiment is to obtain representative mean atmospheric conditions along the line of sight between the Ridge and Sheepfold survey stations.

Personnel: Five people will be required to conduct the meteorological observations. They will be positioned as follows:

Station	Personnel	Name	Radio Call Sign
Sheepfold	A		UM4
Ridge	B		UM1
Radio Station	C		UM5
Launch Site	D, E		UM2, UM3

Procedure: The thermocouple assemblies will be erected at Ridge and Sheepfold by D and E before the experiment starts. When the experiment begins all personnel are to proceed to their station. At each station individual instructions are to be followed for the experimental work.

Results are to be clearly booked on the provided booking forms.

Radio Telephones: All personnel will be in verbal contact via two way radio-telephones. Operators A and B should only use their radios if any equipment malfunctions or they have a specific query on operational procedure. UM5 and UM3 will be in continual radio contact throughout the experiment.

EQUIPMENT FOR THE RIDGE - SHEEPFOLD EXPERIMENT

The following equipment will be required during the experiment. The equipment required at Sheepfold and Ridge station is the same and is only listed once for brevity.

SHEEPFOLD AND RIDGE

1 Light survey tripod.
1 Adaptor to hold thermocouple housing.
1 Aspirated thermocouple assembly.
1 New HP11 battery.
1 Grant miniature temperature recorder.
1 Vacuum flask containing ice.
1 Aneroid barometer.
1 Radio-telephone (Sheepfold UM4, Ridge UM1).
Booking forms.
Pencils.

RECEIVING AND RECORDING STATION

1 Generator (or extension lead).
1 Gallon can of fuel, (if necessary).
1 Earth peg and lead, (if necessary).
1 Radio receiver.
1 Loudspeaker.
1 Tape recorder.

2 Tape spools, 1 full.

1 Extension lead.

1 Ranging pole.

1 Radio-telephone (UM5).

1 First aid box.

1 Spare balloon.

Booking forms.

Pencils.

LAUNCH SITE

1 Civil Aviation Authority Permit.

1 Wild tripod.

1 Winch and line.

1 Radiosonde with new battery.

1 Parachute.

1 Balloon.

1 Hydrogen cylinder.

1 Spanner for cylinder.

1 Adaptor for cylinder.

1 1.5 kg weight.

1 Aneroid barometer.

1 Assman hygrometer.

1 Retort stand and clamp.

1 Pair of binoculars.

2 Radio-telephones (UM2, UM3).

Booking forms.

Pencils.

IMPORTANT:

All the meteorological instruments have been calibrated against standards. To avoid 'uncalibration' transport and handle with great care.

INSTRUCTIONS FOR METEOROLOGICAL SAMPLING AT RIDGE AND SHEEPFOLD

1. Set up the tripod over the station.

2. Attach thermocouple housing to tripod with adaptor.

3. Switch on thermocouple fan.

4. Execute operations (a) to (j) of instructions for 'operation of thermocouples and temperature recorders' which are attached.

5. Set the barometer on a level surface, remove cap on pressure pipe if necessary. Record following on the booking sheet.
   (i) Barometer number, A, C, or D.
   (ii) Barometer correction.
   (iii) Time and pressure reading.
   DO NOT APPLY A BAROMETER CORRECTION.

6. Continue all measurements until instructed to halt by radio-telephone from UM2 or UM3.

7. Follow instructions (k) to (m) of 'operation of thermocouples and temperature recorders'.

8. Dismantle equipment.

9. Return all equipment to stores.

Note. During the experiment the standard Assman Hygrometer will be brought to both stations to calibrate the thermocouple unit.

LLANGOLLEN METEOROLOGICAL SAMPLING EXPERIMENT

Date:............................ Barometer No: A ☐, C ☐, D ☐

Station:......................... Pressure Units: mb ☐, in Hg ☐

Operator:........................ Barometer correction:

Thermocouple: 1 ☐, 2 ☐

Recorder Start Time:....... hrs. Thermometer No: 1 ☐, 2 ☐, 3 ☐, 4 ☐

Recorder Stop Time:........ hrs. Temperature Units: ^{o}C ☐, ^{o}F ☐

DO NOT APPLY CORRECTION TO BAROMETER READINGS.

Time (hrs)	Barometer Reading	Temperature		Remarks
		Wet Bulb	Dry Bulb	

OPERATION OF THERMOCOUPLES AND TEMPERATURE RECORDERS

EQUIPMENT:

Grant miniature temperature recorder 0 - 25°C.
Aspirated thermocouple system.
Vacuum flask containing ice and water.

RECORDER CONTROLS:

(1) The main ON/OFF switch.
(2) A 3 position rotary switch, YELLOW/GREEN/RED.
(3) A slotted zero adjuster.

OPERATION:

(a) Place thermocouple reference junctions in the vacuum flask containing ice and water. Cork tightly.

(b) Set the temperature recorder on a level surface.

(c) Turn rotary switch to YELLOW position.

(d) Switch instrument ON.

(e) Record time, date and place of temperature readings.

(f) Adjust the yellow slotted adjuster so that the pen trace falls on the zero line OF THE GRAPH PAPER.

(g) Turn rotary switch to GREEN.

(h) Check pen trace falls within green sector of scale. If not note and then continue, but battery will need charging later.

(i) Plug thermocouple lead into SOCKET 1 at rear of recorder.

(j) Turn rotary switch to RED. Record time. Continuous temperature recording should now take place.

(k) When recording is complete turn rotary switch to YELLOW, leave for 1 minute. DO NOT READJUST.

(l) Switch instrument OFF, record time. Disconnect plug from recorder.

(m) TRANSPORT WITH CARE.

Note:

(i) Protect recorder from dust and moisture at all times.

(ii) Do not move recorder whilst it is operating.

INSTRUCTIONS FOR OPERATION OF RADIOSONDE BALLOON

NO SMOKING AT ALL TIMES

If the balloon breaks loose from its moorings take action in accordance with paragraph 2 sub-section e of the Civil Aviation Authority, Air Navigation order 1974 attached.

1. Erect tripod securely and attach winch assembly.
2. Set barometer and Assman hygrometer at ground level.
3. Check that nobody is smoking in the area. Extinguish all naked flames.
4. Remove all sharp objects, e.g. wrist watches.
5. Earth hydrogen gas cylinder.
6. Inflate the balloon with hydrogen. Fill slowly. Do not allow to touch ground.

7. HOLD THE BALLOON AT ALL TIMES.

8. When the balloon is fully inflated turn off hydrogen and remove from gas cylinder.

9. Tie balloon to winch line.

10. Attach dummy weight to balloon (1.5 kg). If the balloon lifts the weight continue with step 11, if not remove from winch and add more hydrogen until the dummy weight can be lifted.

11. Remove dummy weight from balloon and attach radiosonde and parachute by the sonde aerial to the balloon (see diagram, Figure 3.6.).

12. Radio UM5 at receiving and recording station and instruct to 'standby'.

13. Hold balloon by aerial with the radiosonde next to the ground level barometer and thermometer.

14. Switch radiosonde on by inserting the two pin plug into the socket.

15. Radio UM5, state 'radiosonde on' and check quality of morse reception.

16. Record ground level temperature and pressure.

17. Continue with sonde in this position for three minutes.

18. Release the balloon and sonde upwards, allowing the winch to run out 40 m of line (indicated by 1 mark on line per 40 m). Hold balloon in this position, locking winch with pin.

19. Allow 1 minute for the sonde to stabilise and then radio UM5 and instruct operator to start recording and to read and book digital tape position.

20. Note time, ground level temperature and pressure.

21. When notified recording is complete go to 22.

22. Repeat instructions 18, 19 and 20 until the required length of winch

cable has been released in 40 m increments (440 m maximum).

23. Radio recording station (UM5) and instruct to record continuously.

24. Lower sonde in 40 m increments to ground level, hold the sonde for 1 minute after each stage of descent.

25. Instruct recording station to note digital reading each time balloon is stationary.

26. When the sonde reaches ground level hold it near the ground level barometer and thermometer for three minutes.

27. Record ground level pressure and temperature.

28. Switch off sonde by removing plug.

29. Radio UM5, UM1, UM4 and state 'experiment complete'.

30. Disconnect sonde and parachute from the balloon.

31. CHECK AGAIN THAT THERE IS NO SMOKING IN THE AREA.

32. Release hydrogen slowly from the balloon. Totally deflate.

33. Dismantle winch assembly.

34. Return all equipment to the stores.

CIVIL AVIATION AUTHORITY

AIR NAVIGATION ORDER 1974

PERMISSION

1. The Civil Aviation Authority in exercise of its powers under Article 67(1)(a) of the Air Navigation Order, as amended, hereby grants permission to Mr S J Curl, University of Nottingham for the flying of a captive balloon in excess of 60 metres above ground level.

2. This permission is granted subject to the following conditions:

a. the said balloon shall only be flown from a site at Llangollen (1:50,000 Ordnance Survey Map 117 239 418) Clwyd;

b. the said balloon shall not be flown in excess of 1000 feet above ground level;

c. the said balloon shall not be elevated in conditions of storm or tempest nor in a visibility of less than 3 (three) kilometres nor flown within 500 feet of cloud;

d. the balloon shall not be left unattended at any time whilst in flight unless it is fitted with a device which ensures its automatic deflation if it breaks free of its moorings;

e. in the event of the balloon breaking free from its moorings the Air Traffic Control Watch Supervisor Manchester Airport shall be informed immediately (telephone number 061 489 2163) and details of the type of balloon approximate speed and direction of drift given. This information should also be passed to the nearest police station.

3. This permission shall have effect in daylight hours only from 24th March 1976 to 13th April 1976 unless previously suspended or revoked.

for the Civil Aviation Authority
this 27 day of January 1976

INSTRUCTIONS FOR USE OF RADIO RECEIVING AND RECORDING EQUIPMENT

1. Position generator as far from the motor car as possible and earth with a metal peg (or connect mains supply).

2. Set up the radiosonde receiver and recorder as described in "Operation of 27.5 MHz radiosonde receiving and recording equipment", attached. Do not connect power lead to equipment.

3. Fasten aerial to a ranging pole away from the motor car.

4. Start the generator if it is to be used. Follow instructions on the side of the generator.

5. When the generator is running smoothly attach power leads to the radio receiver and tape recorder (or switch mains ON).

6. Switch tape ON, switch receiver mains ON.

7. Switch radio-telephone ON and await instructions from the launch site to 'standby'. Wait for reception to commence and confirm quality with launch site.

8. Record signal for ten digits on the recorder at 19 $cm.s^{-1}$ when instructed to do so by the launch site. If interference from the generator is high it is advisable to switch from automatic to manual and set the level with the MIC/VOL control.

9. Record the tape position on the digital display and the time whenever instructed to do so by the launch site.

10. The maximum reading on the digital display is 395. If at any time the tape nears its end inform the launch site (UM3) and arrange to turn the tape over at a convenient time.

11. Turn off equipment and dismantle when instructed to do so from the launch site.

12. Return all equipment to the stores.

OPERATION OF 27.5 MHz RADIOSONDE RECEIVING AND RECORDING EQUIPMENT

EQUIPMENT

Eddystone Model 940 No. E236C Radio Receiver.

Fidelity Playmatic Tape Recorder.

RADIO RECEIVER

(a) Connect aerial to Al.

(b) Connect E to A2.

(c) Connect tape leads and speaker in parallel across 2.5 Ω INPUT.

(d) Connect receiver to mains (240V AC).

(e) Select frequency scale 1.

(f) Switch MAINS ON.

(g) Switch STANDBY ON.

(h) Select A.M.

(i) A.G.C. and N.L. to OFF.

(j) SELECTIVITY to MIN.

(k) Adjust volume with A.F. GAIN control.

(l) TUNE to 27.5 MHz for maximum signal strength.

TAPE RECORDER

(a) Connect to mains (240V AC).

(b) Connect input from receiver to MIC socket.

(c) Switch ON (treble switch).

(d) MIC/VOLUME to 2.

(e) GRAM to 0.

(f) Automatic button IN.

(g) Select TRACK 1-4, 2-3.

(h) SUPER IMPOSE to OFF.

(i) BASS to 0.

(j) TREBLE to 2.

(k) Select TAPE SPEED to 19 $cm.s^{-1}$.

(l) REWIND to suitable tape start position.

(m) Zero digital counter.

(n) Hold left hand lever in RECORD position, and simultaneously

(o) press record button UP.

(p) Recording should now take place.

(q) Check RECORD LEVEL.

(r) Receiving and recording should now be fully operational.

PLAYBACK

The recorded morse code from the radiosonde can be played back at $4.75\ cm.s^{-1}$ for decoding before translation into readings of humidity, temperature and pressure.

BOOKING FORM FOR RADIOSONDE RECEIVING AND RECORDING STATION

OPERATOR:

NOTES:

DATE:

STATION:

TIME HRS.	DIGITAL READING	REMARKS	TIME HRS.	DIGITAL READING	REMARKS

APPENDIX E

THE RELATIONSHIP BETWEEN THE LINEAR AND ANGULAR COEFFICIENTS OF REFRACTION

APPENDIX E

THE RELATIONSHIP BETWEEN THE LINEAR AND ANGULAR COEFFICIENTS OF REFRACTION

Referring to Figure E1, the linear coefficient of refraction, K, is given by

$$K = R/\sigma \qquad \text{... E1.}$$

and the angular coefficient of refraction, k, by

$$k = \Omega/\theta \qquad \text{... E2.}$$

Now assuming the heights of the stations, A and B, above sea level are small in comparison with the earth's radius, (the height of Mount Everest is only 0.1% of the earth's radius)

$$D = R.\theta$$

or $$\theta = D/R \qquad \text{... E3.}$$

Similarly,

$$\psi = D/\sigma \qquad \text{... E4.}$$

By simple trigonometry,

$$\Omega = \bar{\psi}/2 \qquad \ldots \text{E5.}$$

Now from equation E2,

$$k = \Omega/\theta$$

which combines with equations, E5, E4 and E3 to give

$$k = \frac{\bar{\psi}R}{2D} = \frac{1}{2}\frac{R}{\sigma}$$

Therefore from equation E1,

$$2k = K \qquad \ldots \text{E6.}$$

or

2 x angular coefficient of refraction = linear coefficient of refraction

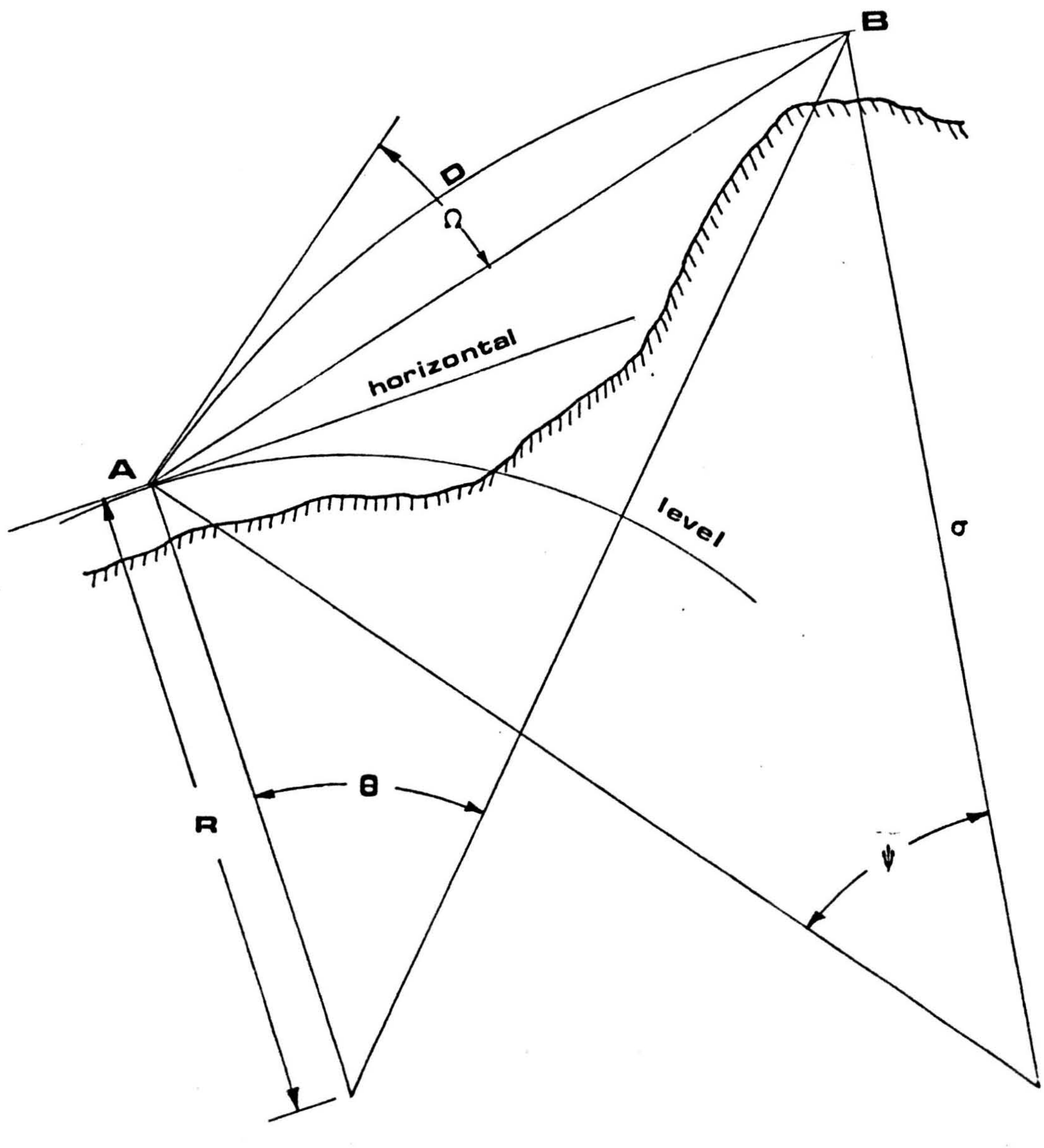


Figure E1. The Linear and Angular Coefficients of Refraction.

APPENDIX F

COMPUTER PROGRAM FOR THE CALCULATION OF HORIZONTAL ACCELERATION DUE TO GRAVITY

APPENDIX F

COMPUTER PROGRAM FOR THE CALCULATION OF HORIZONTAL ACCELERATION DUE TO GRAVITY

A computer program, written in Basic for a Wang 2200, has been used to calculate the horizontal acceleration due to gravity, caused by the mass of rock in an area 1000 m square adjacent to a survey station. This area was subdivided into one hundred 100 m square sections, the mean height of each section above sea level was stored on a data tape using the following programme:

```
10  DIM H(10,10)
20  FOR Y=1 TO 10
30  FOR X=1 TO 10
40  PRINT "ELEMENT";Y;",";X
50  INPUT H(Y,X)
60  NEXT X
70  NEXT Y
80  STOP "PREPARE DATA TAPE, AND THEN 'CONTINUE' 'EXECUTE'"
90  DATA SAVE OPEN "HEIGHTS"
100 DATA SAVE H()
110 DATA SAVE END
120 END
```

All height inputs should be in metres.

The following programme was then used to calculate the total horizontal acceleration due to gravity at a survey station.

```
10 SELECT D
20 SELECT PRINT 211(150)
30 INPUT "STATION NAME",A$
40 PRINT TAB(25);A$
50 STOP "PREPARE DATA TAPE, THEN 'CONTINUE'"
60 DIM H(10,10)
70 DATA LOAD "HEIGHTS"
80 DATA LOAD H()
90 INPUT "ROCK DENSITY",D
100 PRINTUSING 110,D
110 %      ROCK DENSITY   =  ####      KG/M!3
120 INPUT "SURVEY STATION LEVEL" ,L
130 PRINTUSING 140,L
140 %      SURVEY STATION LEVEL   =  ####      M
150 PRINT :PRINT :PRINT :PRINT
160 PRINT "  X  Y  H          M*10!6   X PART.       Y PART.
    X TOTAL       Y TOTAL"
170 G=6.670000E-11
180 F4=0
190 F5=0
200 FOR X=1 TO 10
210 FOR Y=1 TO 10
220 Y1=Y*100-50
230 X1=X*100-50
240 H1=H(X,Y)
250 V=H1*10000
260 M=V*D
270 R=(X1!2+Y1!2)!0.5
280 R1=(R!2+((H/2)-L)!2)!0.5
290 F1=(G*M)/(R1!2)
300 T=ARCTAN((((H/2_-L)!2)!0.5)/R)
310 F=F1*COS(T)
320 A=ARCTAN(Y1/X1)
```

```
330 F2=F*COS(A)
340 F3=F*SIN(A)
350 F4=F4+F2
360 F5=F5+F3
370 M=M/1000000
380 PRINTUSING 390,Y,X,H1,M,F3,F2,F5,F4
390 % ## ## ### ######### #.######### #.######### #.##
####### #.#########
400 NEXT Y
410 NEXT X
420 END
```

The following listing is an example of the output from the program. X and Y are the coordinates of a rock column of height H (m) and mass M (kg). X PART and Y PART are the accelerations due to the rock column under consideration in the X and Y direction. X TOTAL and Y TOTAL are cumulative totals of the horizontal accelerations; the total accelerations for the complete 1000 m square area are the last values of X TOTAL and Y TOTAL. These values are then used in the calculation of the resultant horizontal acceleration due to gravity. All the accelerations in the output are in units of $m.s^{-2}$.

RIDGE NE

ROCK DENSITY = 2612 KG/M!3

SURVEY STATION LEVEL = 260 M

X	Y	H	M*10!6	X PART.	Y PART.	X TOTAL	Y TOTAL
1	1	265	6923	0.000001180	0.000001180	0.000001180	0.000001180
2	1	260	6793	0.000002411	0.000000803	0.000003592	0.000001984
3	1	265	6923	0.000002391	0.000000478	0.000005983	0.000002462
4	1	275	7184	0.000001984	0.000000283	0.000007967	0.000002746
5	1	295	7707	0.000001625	0.000000180	0.000009593	0.000002926
6	1	312	8151	0.000001314	0.000000119	0.000010907	0.000003046
7	1	320	8360	0.000001048	0.000000030	0.000011956	0.000003126
8	1	310	8099	0.000000805	0.000000053	0.000012761	0.000003180
9	1	295	7707	0.000000619	0.000000036	0.000013380	0.000003216
10	1	275	7184	0.000000474	0.000000024	0.000013855	0.000003241
1	2	290	7576	0.000000896	0.000002690	0.000014752	0.000005932
2	2	295	7707	0.000002040	0.000002040	0.000016793	0.000007973
3	2	313	8177	0.000002287	0.000001372	0.000019080	0.000009345
4	2	325	8491	0.000002022	0.000000866	0.000021102	0.000010212
5	2	345	9013	0.000001709	0.000000569	0.000022812	0.000010782
6	2	340	8883	0.000001324	0.000000361	0.000024137	0.000011143
7	2	325	8491	0.000001003	0.000000231	0.000025140	0.000011374
8	2	312	8151	0.000000773	0.000000154	0.000025913	0.000011529
9	2	295	7707	0.000000596	0.000000105	0.000026510	0.000011634
10	2	290	7576	0.000000485	0.000000076	0.000026995	0.000011711
1	3	325	8491	0.000000586	0.000002932	0.000027582	0.000014643
2	3	350	9144	0.000001534	0.000002557	0.000029116	0.000017201
3	3	360	9405	0.000001855	0.000001855	0.000030972	0.000019057
4	3	364	9510	0.000001748	0.000001249	0.000032721	0.000020306
5	3	355	9275	0.000001451	0.000000806	0.000034172	0.000021112
6	3	340	8883	0.000001145	0.000000520	0.000035317	0.000021633

7	3	330	8621	0.000000909	0.000000349	0.000036227	0.000021983
8	3	310	8099	0.000000702	0.000000234	0.000036930	0.000022217
9	3	302	7890	0.000000568	0.000000167	0.000037499	0.000022384
10	3	300	7838	0.000000473	0.000000124	0.000037972	0.000022509
1	4	365	9536	0.000000376	0.000002633	0.000038348	0.000025143
2	4	385	10058	0.000001026	0.000002395	0.000039375	0.000027538
3	4	380	9928	0.000001304	0.000001825	0.000040679	0.000029364
4	4	370	9666	0.000001291	0.000001291	0.000041970	0.000030655
5	4	355	9275	0.000001131	0.000000880	0.000043102	0.000031535
6	4	335	8752	0.000000928	0.000000590	0.000044031	0.000032126
7	4	325	8491	0.000000767	0.000000413	0.000044798	0.000032540
8	4	310	8099	0.000000620	0.000000289	0.000045419	0.000032829
9	4	306	7994	0.000000519	0.000000214	0.000045939	0.000033043
10	4	304	7942	0.000000440	0.000000162	0.000046379	0.000033206
1	5	395	10320	0.000000241	0.000002176	0.000046621	0.000035382
2	5	387	10111	0.000000639	0.000001917	0.000047260	0.000037300
3	5	370	9666	0.000000840	0.000001512	0.000048101	0.000038812
4	5	360	9405	0.000000892	0.000001147	0.000048993	0.000039960
5	5	350	9144	0.000000844	0.000000844	0.000049833	0.000040805
6	5	330	8621	0.000000729	0.000000597	0.000050563	0.000041402
7	5	324	8465	0.000000636	0.000000440	0.000051205	0.000041843
8	5	310	8099	0.000000533	0.000000319	0.000051738	0.000042163
9	5	307	8020	0.000000459	0.000000243	0.000052198	0.000042406
10	5	306	7994	0.000000398	0.000000188	0.000052597	0.000042595
1	6	393	10267	0.000000150	0.000001656	0.000052748	0.000044251
2	6	378	9876	0.000000401	0.000001472	0.000053149	0.000045724
3	6	360	9405	0.000000551	0.000001212	0.000053701	0.000046937
4	6	352	9196	0.000000620	0.000000975	0.000054322	0.000047913
5	6	337	8804	0.000000609	0.000000745	0.000054931	0.000048658
6	6	327	8543	0.000000568	0.000000568	0.000055500	0.000049226
7	6	320	8360	0.000000513	0.000000434	0.000056013	0.000049661
8	6	310	8099	0.000000449	0.000000329	0.000056463	0.000049991
9	6	307	8020	0.000000398	0.000000257	0.000056861	0.000050249
10	6	306	7994	0.000000352	0.000000204	0.000057214	0.000050453
1	7	385	10058	0.000000097	0.000001261	0.000057311	0.000051714
2	7	372	9719	0.000000264	0.000001148	0.000057576	0.000052862
3	7	360	9405	0.000000381	0.000000992	0.000057958	0.000053855
4	7	345	9013	0.000000438	0.000000815	0.000058397	0.000054670
5	7	332	8674	0.000000451	0.000000652	0.000058849	0.000055323
6	7	327	8543	0.000000444	0.000000524	0.000059293	0.000055848

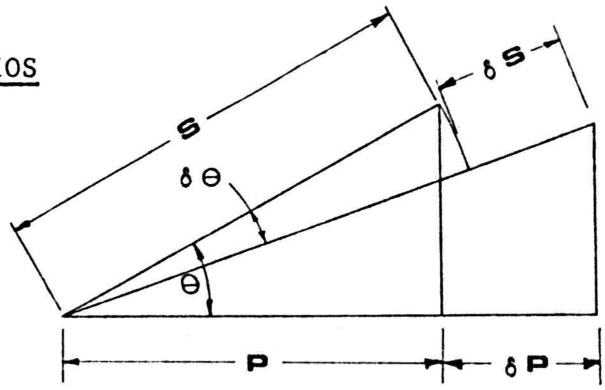
7	7	320	8360	0.000000415	0.000000415	0.000059709	0.000056263
8	7	310	8099	0.000000375	0.000000325	0.000060034	0.000056588
9	7	300	7838	0.000000332	0.000000254	0.000060417	0.000056843
10	7	302	7890	0.000000304	0.000000208	0.000060721	0.000057051
1	8	382	9980	0.000000066	0.000000992	0.000060787	0.000058043
2	8	370	9666	0.000000183	0.000000917	0.000060970	0.000058961
3	8	355	9275	0.000000268	0.000000804	0.000061239	0.000059766
4	8	340	8883	0.000000317	0.000000680	0.000061556	0.000060446
5	8	327	8543	0.000000337	0.000000562	0.000061894	0.000061009
6	8	325	8491	0.000000345	0.000000471	0.000062240	0.000061481
7	8	317	8282	0.000000332	0.000000383	0.000062572	0.000061864
8	8	305	7968	0.000000306	0.000000306	0.000062878	0.000062170
9	8	295	7707	0.000000277	0.000000245	0.000063156	0.000062415
10	8	292	7629	0.000000254	0.000000201	0.000063411	0.000062617
1	9	375	9797	0.000000046	0.000000787	0.000063457	0.000063404
2	9	360	9405	0.000000128	0.000000727	0.000063586	0.000064132
3	9	345	9013	0.000000190	0.000000649	0.000063777	0.000064781
4	9	335	8752	0.000000234	0.000000569	0.000064011	0.000065350
5	9	322	8412	0.000000255	0.000000482	0.000064266	0.000065832
6	9	315	8230	0.000000264	0.000000408	0.000064531	0.000066241
7	9	310	8099	0.000000262	0.000000343	0.000064794	0.000066585
8	9	300	7838	0.000000249	0.000000282	0.000065043	0.000066867
9	9	290	7576	0.000000230	0.000000230	0.000065274	0.000067098
10	9	282	7367	0.000000212	0.000000189	0.000065486	0.000067288
1	10	360	9405	0.000000032	0.000000621	0.000065519	0.000067909
2	10	355	9275	0.000000093	0.000000594	0.000065612	0.000068504
3	10	345	9013	0.000000143	0.000000544	0.000065756	0.000069048
4	10	330	8621	0.000000176	0.000000478	0.000065932	0.000069526
5	10	317	8282	0.000000195	0.000000413	0.000066128	0.000069940
6	10	305	7968	0.000000203	0.000000351	0.000066331	0.000070291
7	10	295	7707	0.000000203	0.000000297	0.000066535	0.000070588
8	10	290	7576	0.000000199	0.000000253	0.000066734	0.000070842
9	10	282	7367	0.000000189	0.000000212	0.000066924	0.000071054
10	10	278	7263	0.000000179	0.000000179	0.000067104	0.000071233

APPENDIX G

THE THEORY OF PRECISION RATIOS

APPENDIX G

THE THEORY OF PRECISION RATIOS



G1. Precision Ratios for Reduced Plan Distance, $\delta P/P$

The error in the reduced plan distance, δP, is maximised if the error in the slope distance is positive and the error in the vertical angle is negative. The equations for the precision ratio curves are derived using the following notation,

P = Reduced plan distance.

S = Slope distance.

θ = Vertical angle.

δP, δS, $\delta\theta$ = Errors in the variables.

Now $$P = S \cos\theta \qquad \text{... G1.}$$

and $$P + \delta P = (S + \delta S) \cos(\theta - \delta\theta) \qquad \text{... G2.}$$

therefore

$$\frac{\delta P}{P} = \frac{(S + \delta S)\cos(\theta - \delta\theta) - S.\cos\theta}{S.\cos\theta} \qquad \text{... G3.}$$

and

$$S(\delta P/P + 1) = \frac{(S + \delta S).\cos(\theta - \delta\theta)}{\cos\theta} \qquad \ldots \text{G4.}$$

Now let

$$X = \frac{\cos(\theta - \delta\theta)}{\cos\theta} \qquad \ldots \text{G5.}$$

which when substituted into G4. and rearranged gives

$$S = \frac{\delta S.X}{(\delta P/P + 1 - X)} \qquad \ldots \text{G6.}$$

Now let

$$Y = \frac{X}{(\delta P/P + 1 - X)} \qquad \ldots \text{G7.}$$

where X is derived from G5.

Therefore

$$S = \delta S.Y \qquad \ldots \text{G8.}$$

If the slope distance S has a constant error A and a proportional error B,

$$\delta S = A + BS \qquad \ldots \text{G9.}$$

which applied to G8. gives

$$S = (A + BS)Y \qquad \ldots \text{G10.}$$

or

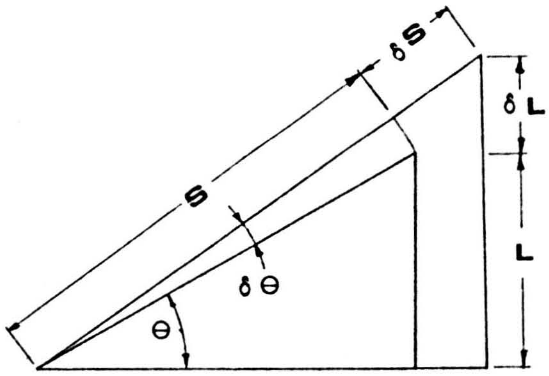
$$S = \frac{AY}{1 - B.Y} \qquad \ldots \text{G11.}$$

where Y is derived from G7.

The slope distance S can be plotted against the vertical angle θ, for constant values of $\delta\theta$ and δS (i.e. A and B), for discrete values of the precision ratio $\delta P/P$. An example of a family of curves obtained is given in Figure 6.6.

G2. <u>Precision Ratio for the Reduced Difference in Level, $\delta L/L$</u>

The error in the reduced difference in level is maximised if the error in both the measured slope distance and the measured vertical angle are positive.



The same notation is used as in the previous calculations, but in addition

L = Reduced difference in level

δL = Error in the reduced difference in level.

The derivation of the precision ratio for the difference in level is similar to that for the plan distance. The final equations, from which the precision ratio curves can be plotted are

$$X = \frac{\sin(\theta + \delta\theta)}{\sin\theta} \qquad \text{... G12.}$$

$$Y = \frac{X}{(\delta L/L + 1 - X)} \qquad \text{... G13.}$$

and $$S = \frac{AY}{1 - B.Y} \qquad \text{... G14.}$$

An example of a family of curves plotted from these equations is given in Figure 6.7.

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