Essays in Business Cycles: Housing Market, Adaptive Learning, and Credit Market Imperfections

by

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Abstract

In this thesis, we focus on the housing sector, which is important to the economy but is under-researched in business cycles analysis. We discuss several housing sector related issues in dynamics stochastic general equilibrium (DSGE) models.

To begin with, we conduct a sensitivity analysis using a simple DSGE model with the feature of sticky prices and a fixed housing supply, which is similar with the basic model in Iacoviello (2005) but with representative agents. Then we introduce credit market imperfections in two different ways. The first case is referred to as 'borrowing to invest', in which entrepreneurs take loans and accumulate production housing, which is a factor of production. We observe the financial accelerator (or decelerator) effect since their borrowing is related to output directly. The second case is referred to as 'borrowing to live', in which impatient households take loans to buy housing and gain utility from it. In contrast with the first case, we do not find the financial accelerator (or decelerator) effect, since the borrowing is not directly related to output anymore.

First, we add a variable housing supply, thus we can discuss the supply side effect in the housing market, including both the direct effect and the feedback effect. The direct effect is the impact of a housing technology shock, and the feedback effect is the impact of a change in new housing production, which is caused by other shocks. We find, however, that the magnitudes of these two effects are negligible under the standard setting of the housing market that is commonly used in the literature of DSGE model with housing, such as Davis and Heathcote (2005), Iacoviello and Neri (2010). The key assumption in the standard setting is that every household trades housing in a given period. An empirical examination of the U.S. housing sector suggests us to (i) re-construct the housing market and (ii) introduce the feature of time to build to new housing production. After constructing the new setting for the housing market by introducing the probability of trading housing, we find that (i) the steady state ratios from the model are consistent with their empirical targets and (ii) the magnitudes of both the direct effect and the

feedback effect are 60 times larger. Furthermore, the feature of time to build, together with the new setting of the housing market, allows us to observe overshooting behaviour on the real house price.

Second, we discuss the impact of the assumption of adaptive learning, as we are convinced that the house price bubble is partially contributed by this alternative way of forming expectations. After writing the Nottingham Learning Toolbox,[1] we find that, given the AR(1) learning model, in which variable is forecasted using its own lagged terms, the adaptive learning mechanism largely amplifies and propagates the effects of a goods sector technology shocks to the economy, and also, enlarges the impact of the time to build feature on the real house price. Furthermore, our sensitivity analysis shows that the values of initial beliefs are important to the mechanism but forecasting errors are not if the constant gain coefficient is small.

Then we consider the assumption of heterogeneous expectations. From the impulse response analysis, we find that (i) the adaptive learning mechanism also has amplification and propagation effects to the economy when only a fraction of the population are learning agents; (ii) when two types of agents have equal weights, the impulse responses from heterogeneous expectations are much closer to those from rational expectations than those from adaptive learning; (iii) when rational agents are fully rational, the adaptive learning mechanism has larger amplification and propagation effects on the economy than when rational agents are partially rational. From the sensitivity analysis, We find that fully rational agents always have larger impacts on model variables than partially rational agents.

Finally, we introduce credit market imperfections to the housing market, thus the mortgage market subjects to a costly verification problem. Our empirical analysis suggests that, while the default rate is countercyclical, the loan to value ratio is procyclical. Our impulse response analysis shows that, given a positive goods sector technology shock, the default rate is counter-

[1] The Nottingham Learning Toolbox is a series of Matlab files that can solve a general form of DSGE models under adaptive learning and heterogeneous expectations. The toolbox solves the model using the Klein's QZ decomposition method, and facilitates the impulse response analysis. The Cambridge Learning Toolbox provides helpful reference for this toolbox at the initial stage.

cyclical, but the loan to value ratio is also countercyclical. The reason we suppose is that, in our model, credit constrained households have less housing in an economic upturn, thus the volume of loans they receive also decreases, leading to a fall in the loan to value ratio. Moreover, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, we always have a positive relation between the default rate and the loan to value ratio. In order to overcome this co-movement, we show that a time-varying mean is necessary.

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Contents

1 Introduction

1.1 Motivation

In this thesis, we focus on the housing sector, which is important to the economy but is under-researched in business cycle analysis. By taking the U.S. economy as an example, here we list four reasons why we think the housing market is important. Firstly, the bursting of the house price bubble has been proposed as one of the main causes of the Great Recession, the most recent recession from December 2007 to June 2009, and the depressed housing market slows the economic recovery in the United States. Figure 1.1 shows the business cycles in U.S. real GDP over the period 1963Q1 to 2010Q4. In the period from the 1960s to the mid-1980s, the U.S. economy experienced four recessions. From the mid-1980s to the late-2000s, the volatility of the U.S. economy was largely reduced and this period was referred to as the Great Moderation, which was ended by the most recent recession begun from December 2007. Meanwhile, Figure 1.2 shows that real house price started to fall in 2007. It has been suggested that one of the causes of the Great Recession was the bust of the house price bubble, which was contributed by the declining underwriting standards and risky lending in the mortgage market. Meanwhile, the fall in the house price has led to a 7 trillion dollars loss in home equity, more than half the amount that prevailed in 2006 (Federal Reserve reports). This large decline in household wealth has weakened household consumption and thus slows the economic recovery. In addition, Figure 1.1 also shows the movement of the federal funds rate over the sample period. We can see that the monetary policy has been used to combat recession. After the beginning of each economic downturn, the central bank reduced the federal funds rate to stimulate the economy, and then raised it when the economy returned to the recovery path. In particular, during the Great Recession, the nominal interest rate has been reduced to near-zero level, referred to as the zero lower bound, indicating that there was no room for conventional monetary policy to have an impact on the real economy anymore, and thus the central bank needed to conduct non-conventional monetary policy, such as quantitative easing. Although it was officially announced that the Great Recession ended in June 2009, the federal funds rate has been kept at near-zero level, as the negative impacts of the recession on the other aspects of the economy, such as high unemployment and depressed housing market, have not yet disappeared.

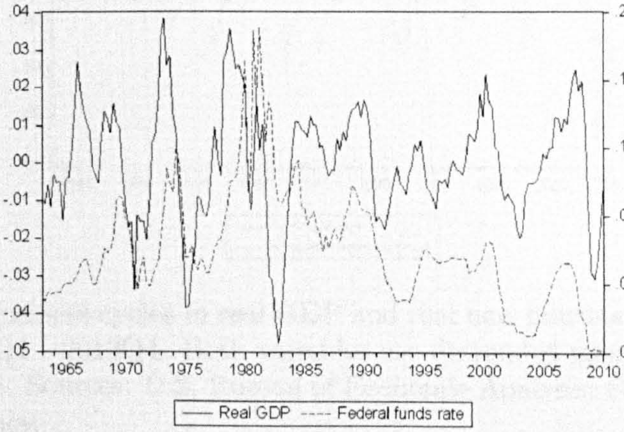

Figure 1.1: Business cycles in real GDP, and Federal funds rate. Sample period: 1963Q1 - 2010Q4. Real GDP is detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; Board of Governors of the Federal Reserve System.

Secondly, housing market related variables are volatile, i.e., residential investment may be the most volatile component of GDP and real house price is more volatile than inflation. The two main components of GDP are private consumption and private investment, which account for 70% and 13% of GDP respectively. Figure 1.3 shows business cycles in these two components over the period of $1963Q1 - 2010Q4$. We can see that the volatility of investment is higher than consumption. At a disaggregate level, business cycles in non-residential investment and residential investment are shown in Figure 1.4. We can see that residential investment, which consists of purchases of private residential structures and residential equipment, is more volatile than non-residential investment. Meanwhile, new housing production has a similar volatility with residential investment.

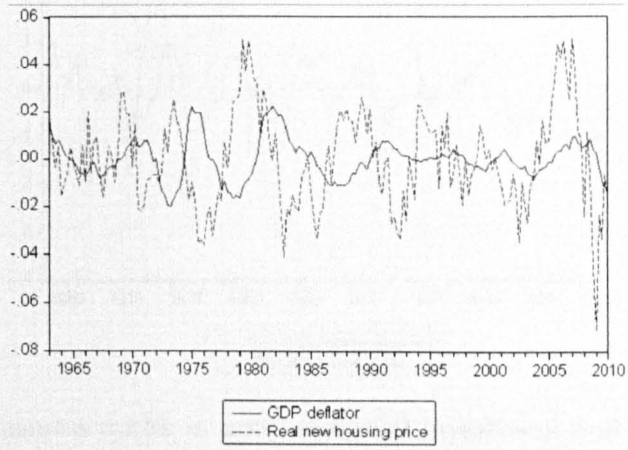

Figure 1.2: Business cycles in real GDP and real new housing price. Sample period: 1963Q1 - 2010Q4. Both variables are detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; Federal Housing Finance Agency.

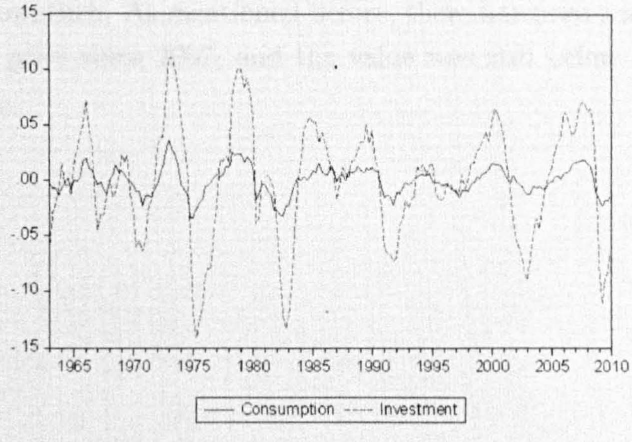

Figure 1.3: Business cycles in consumption and investment. Sample period: $1963Q1$ - $2010Q4$. Both variables are detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis.

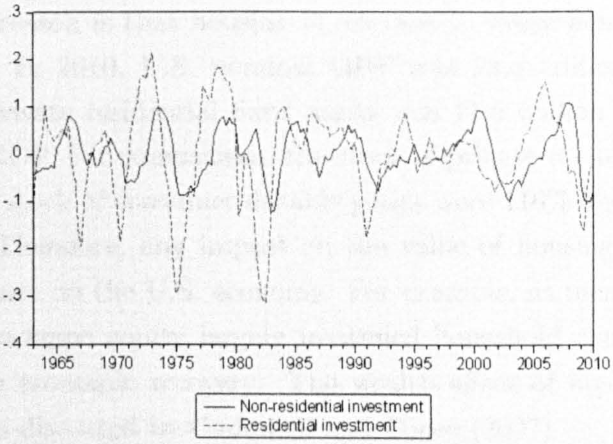

Figure 1.4: Business cycles in non-residential investment and residential investment Sample period: 1963Q1 - 2010Q4. Both variables are detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis.

Figure 1.2 shows business cycles of GDP deflator and real house price. We can see that real house price is more volatile than GDP deflator. In particular, the movement of real house price usually leads GDP deflator, indicating that the housing market may play a role during both economy upturn and downturn. As mentioned before, there has been a sharp decrease in real house price since 2007, and the value was still below the long-term trend in 2010.

The third reason is that housing is relevant to every household and its stock is large. In 2010, U.S. nominal GDP was 14.5 trillion dollars, and the stock of private residential fixed assets was 17.4 trillion dollars, which was 120% of GDP. For comparison, the stock of private nonresidential fixed assets and the stock of consumer durable goods were 116% and 32% of GDP respectively. Therefore, any impact on the value of housing assets should have an influence on the U.S. economy. For example, as mentioned earlier, the decrease in home equity largely weakened household consumption and thus slows the economic recovery. The wealth effect of housing assets on consumption is discussed by Campbell and Cocco (2007).

The final reason is that the mortgage debt outstanding is also large, as housing is usually purchased using mortgage. In 2010, the mortgage debt outstanding was around 13.8 trillion dollars, which is around 95% of GDP. We suppose that the interaction between credit market imperfections and the housing market is important to the economy. For example, better credit market conditions in the mortgage market pushes up house price and increases the value of housing assets.

Variable (2010)	**Billions of dollars**	**% of GDP**
GDP	14,526	
Stock of fixed asset		
Private nonresidential	16,803	116%
Private residential	17,397	120%
Consumer durable goods	4,581	32%
Stock of loan		
Mortgage debt outstanding	13,813	95%

This evidence suggests that the housing sector is important to the U.S. economy. This thesis attempts to discuss several housing sector related issues in dynamic stochastic general equilibrium (DSGE) models.

- To begin with, we conduct a sensitivity analysis. We illustrate a simple DSGE model with sticky prices and a fixed housing supply. Similar with Iacoviello (2005), we add credit market imperfections and discuss the dynamics of the economy, where housing also plays a role of collateral.

- Firstly, we show that the supply side effect of new housing production is largely underestimated in the standard setting of the housing market that is considered in Davids and Heathcote (2005), Iacoviello and Neri (2010). Then we develop a new setting for the housing market by introducing the probability of trading housing. Moreover, we examine the impact of the time to build feature on the real house price.

- Secondly, we examine the hypothesis that the house price bubble is, at least, partially related to the way agents form expectations. To investigate this link, we construct a leaning toolbox, and examine the impact of the small learning models on the economy. The small learning models are also discussed in Eusepi and Preston (2011), Slobodyan and Wouters (2012). In addition, we discuss the impact of heterogeneous expectations.

- Finally, we discuss the dynamics of the default rate and the loan to value ratio. We consider a costly verification problem, which is discussed in Bernanke, Gertler and Gilchrist (1999), Aoki, Proudman and Vlieghe (2004), thus default is a steady state phenomenon. Besides, we also discuss the importance of the time varying mean of the idiosyncratic shock.

1.2 Overview of the thesis

My thesis includes four chapters, and here I discuss them briefly in turn.

1.2.1 A simple DSGE model with a fixed housing supply and credit market imperfections

In the 1990s, the New Neoclassical Synthesis became the most popular way to explain short-run economic fluctuations and discuss the role of monetary and fiscal policies.[2] In this new synthesis, the economy is a dynamic general equilibrium system that deviates from an efficient allocation of resources in the short run because of sticky prices and perhaps a variety of other market imperfections. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) discuss medium-sized New Keynesian models. These models have formed the foundation for the large-sized New Keynesian models that are employed to analyse monetary policy in the central banks.

The first model we discuss is a simple DSGE model with the feature of sticky prices. In particular, we add a housing sector, as we are particularly interested in the dynamics of the real house price in response to various exogenous shocks. In this chapter, we assume that the housing supply is fixed for two reasons: (i) the housing is closed related to land, which is fixed, (ii) the supply of housing is not likely to change in the short run as it takes time to build new housing.

In DSGE models, the financial market is commonly assumed to be perfect. As this assumption is inconsistent with reality, researchers try to introduce credit market imperfections, in which borrowers need to use their assets as collateral when they take loans. A simple way to consider credit market imperfections is to assume that loans are fully secured by collateral.

There are two different approaches to introduce credit market imperfections. Firstly, borrowers accumulate a factor of production, such as goods capital or production housing, and then rent it to goods producers. Meanwhile, they take loans from patient households, and use this factor of production as collateral. In this case, the borrowing constraint is directly linked to output, i.e., a relaxed borrowing constraint leads to increases in the volume of loans and the stock of the factor of production, which in turn has a positive impact on output. This type of borrowers is discussed in Kiyotaki and Moore (1997) and Iacoviello (2005). Our second model, i.e., the simple model with entrepreneurs, also discusses this type of borrower. We refer this case to as 'borrowing to invest'. In this model, we observe the financial accelerator (decelerator) effect as the volume of loans is related to goods production

[2]The simplest form in this synthesis is a DSGE model with the feature of sticky prices, which is also referred to as a New Keynesian model.

through production housing.

Secondly, borrowers accumulate long-lasting goods, such as durable goods or domestic housing, and then gain utility from it. Meanwhile, they also take loans from patient households, and the long-lasting goods are used as collateral. In this setting, the borrowing constraint is not directly linked to output, i.e., a relaxed borrowing constraint leads to increases in the volume of loans and the stock of the long-lasting goods, which has no direct impact on output. This type of borrowers is considered in Campbell and Hercowitz (2005), Monacelli (2009), Iacoviello and Neri (2010). Our third model, i.e., the simple model with impatient households, considers the second type of borrowers. We refer this case to as 'borrowing to live'. In this model, the financial accelerator (decelerator) effect is be influential as the volume of loans is not related to goods production anymore.

1.2.2 An examination of the direct effect and the feedback effect from the variable housing supply

We suppose that the housing sector is important to the economy, but it is usually ignored in DSGE models. Davis and Heathcote (2005) begin to consider a multi-sector model featuring housing production, and their model can explain the dynamics of housing capital investment well.

When researchers raise concerns over credit market imperfections, housing is assigned another role, i.e., being collateral of loan in the credit market. An important paper that considers housing as an alternative market good is written by Iacoviello and Neri (2010). Following this work, various versions of Iacoviello and Neri (2010) model have been widely used, such as Notarpietro (2007), Paries and Notarpietro (2008), Kannan, Rabanal and Scott (2009), Christensen et al. (2009), Sellin and Walentin (2010), and the settings of the housing market are similar. We refer this setting to as 'the standard setting of the housing market' and we consider it in our benchmark model, which is a simple DSGE model with sticky prices and housing production.

After introducing a variable housing supply, we can discuss the supply side effect to the economy, including both the direct effect and the feedback effect.

While the direct effect is the impact of a housing sector technology shock, the feedback effect is the impact of a change in new housing production, caused by other shocks, such as a goods sector technology shock or a monetary policy shock. We find that, under the standard setting of the housing market, the magnitudes of these two effects from new housing production sector are negligible to the economy.

Next, we examine the U.S. housing sector using data from the U.S. Census Bureau for the period of $1968Q1 - 2009Q4$. We generate several empirical ratios, but we notice that the steady state ratios from our benchmark model cannot meet their empirical targets. We argue that this inconsistency is caused by the standard setting of the housing market. Therefore, our first contribution in this chapter is to construct a new setting for the housing market by introducing the probability of trading housing. As a result, we find that the steady state ratios are consistent with their empirical targets, and both the direct effect and the feedback effect are 60 times larger.

Meanwhile, our empirical analysis also suggests us to apply the feature of time to build to new housing production. The feature of time to build has been introduced to goods capital, such as Kydland and Prescott (1982), Gomme, Kydland and Rupert (2001), Tsoukalas (2011). For the first time, this feature is introduced to new housing production in this chapter. One important implication of the feature of time to build is that, given a goods sector technology shock or a monetary policy shock, the feedback effect of new housing production leads to overshooting behaviour for the real house price since (i) the response of new housing production has an opposite impact on the real house price against with the shock, and (ii) the feature of time to build delays this impact while the demand for housing is diminishing.

1.2.3 Adaptive learning and heterogeneous expectations

Rational expectations is a standard assumption in DSGE models, i.e., agents know the structure of the true model and the values of the model parameters, and use them to form expectations for the future. Therefore, agents are able to form beliefs that are consistent with actual outcomes. Some researchers, such as Slobodyan and Wouters (2012), argue that the assumption of rational expectations is too strong and models under rational expectations find them difficult to capture the persistence of macroeconomic variables.

The adaptive learning mechanism, an alternative way of forecasting the future, is discussed by Marcet and Sargent (1989a, 1989b), Evans and Honkapohja (1999, 2002), but they focus on the convergence of the models to the rational expectations equilibrium. In their adaptive learning mechanism, agents do not necessarily have full information about the structure of the true model and the values of the model parameters, thus they forecast the future according to their past experience, and then update their beliefs using the forecasting errors.

Since this alternative was suggested, the quantitative importance of the adaptive learning mechanism in business cycle fluctuations has been discussed in the context of DSGE models. Milani (2007) provides the first example of using Bayesian methods to estimate a DSGE model under adaptive learning, and he finds that the adaptive learning mechanism is an important source that can lead persistence to the economy. Slobodyan and Wouters (2009) find that their model under adaptive learning fit the data better than the model under rational expectation. In particular, their model can explain the data even better when only a few variables are included in the forecasting equations. The success of small learning models is shared by other researchers, such as Williams (2003), Adam (2004), Eusepi and Preston (2011), Slobodyan and Wouters (2012). Their results suggest that the model with simpler assumptions about the expectation mechanism can improve the empirical fit of the model.

In our chapter, we suppose that this way of forming expectations partially contributes to the recent house price bubble in the United States, and we discuss the impact of the assumption of adaptive learning in a two-sector DSGE model with sticky prices, housing production, the new setting of the housing market, and the feature of time to build. In particular, we explore the impacts of the AR(1) learning model and discuss the interaction between the adaptive learning mechanism and the feature of time to build. Using the

Nottingham Learning Toolbox,[3] we find that the adaptive learning mechanism largely amplifies and propagates the impact of a goods sector technology shock on the economy. Meanwhile, it enlarges the impact of the time to build feature on the real house price and allows this variable to exhibit more obvious cyclical behaviour. Besides, we suppose that a relatively higher weight on the lagged variables in the model solution is the reason for the amplification and propagation effects from the adaptive learning mechanism.

The sensitivity of the dynamics to the initial beliefs and the updating algorithms under adaptive learning are assessed by Carceles-Poveda and Giannitsarou (2007) in an univariate forward looking linear model. They find that the behaviour of macroeconomic variables depends on both the initial beliefs and the learning algorithms. We also carry out a sensitivity analysis to check the robustness of our results. We find that the values of initial beliefs are crucially important for the responses of model variables, and the forecasting errors do not have obvious impacts on the dynamics, when the value of the constant gain coefficient is relatively small, $0 - 0.05$.

While the assumption of rational expectations or adaptive learning implies that there is only one type of agents in the economy, it is more realistic to assume that we have both types of agents simultaneously, and this case is referred to as heterogenous expectations. Branch and McGough (2009), Branch and McGough (2010), Fuster, Laibson and Mendel (2010) discuss the assumption of heterogeneous expectations in which non-rational agents' beliefs are constant.

Our second contribution in this chapter is to discuss the impact of heterogeneous expectations with adaptive learning agents, who update their beliefs at the end of each period. Besides, we consider two types of rational agents: (i) partially rational agents, i.e., they do not know the existence of learning agents; (ii) fully rational agents, i.e., they know the existence of learning agents and take learners' beliefs into account. We find that, given that two

[3] The Nottingham Learning Toolbox is a series of Matlab files that can solve a general form of DSGE models under adaptive learning and heterogeneous expectations. The toolbox solves the model using the Klein's QZ decomposition method, and facilitates the impulse response analysis. The Cambridge Learning Toolbox provides helpful reference for this toolbox at the initial stage.

types of agents have equal weights, (i) the responses of variables from heterogeneous expectations are larger than those from rational expectations, (ii) the impulse responses from heterogeneous expectations are much closer to those from rational expectations than those from adaptive learning, (iii) when rational agents are fully rational, the adaptive learning mechanism has larger amplification and propagation effects on the economy than when rational agents are partially rational. Moreover, in our sensitivity analysis, fully rational agents always bring larger impacts on model variables than partially rational agents.

1.2.4 A discussion of the default rate and the endogenous loan to value ratio

The assumption of perfect credit markets is commonly seen in DSGE models. Recently, however, researchers have raised concerns over credit market imperfections. The costly verification problem or the agency problem has been introduced to the investment sector by Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999). Then Aoki, Proudman and Vlieghe (2004) discuss this problem in the housing market but they actually assume fixed default rate and loan to value ratio when they solve the model. Moreover, Iacoviello and Neri (2010) also discuss the impacts of the credit market imperfections in the housing market. In their model, the debt is fully collateralised, and there is no possibility of default.

In our chapter, we introduce the agency problem to the housing market, and focus on the default rate and the loan to value ratio. In particular, we assume that an idiosyncratic shock realise on the value of housing assets. Meanwhile, we assume that both lenders and borrowers can purchase housing. Our impulse response analysis shows that, given a positive goods sector technology shock, the response of the default rate is countercyclical, which is consistent with our empirical analysis. The loan to value ratio, however, is also countercyclical, while our empirical analysis suggests procyclical behaviour. The reason we suppose is that, in our model, credit constrained households have less housing in an economic upturn, thus the volume of loans they receive also decreases, leading to a fall in the loan to value ratio. Therefore, the inconsistency between the results from our model and empirical evidence suggests that, in the future research, we need to improve the model in a way that allows credit constrained households to obtain more housing in an economic upturn.

Furthermore, we discuss the implications of the time-varying mean of the idiosyncratic shock. Faia and Monacelli (2007) discuss this feature based on the agency problem framework of Carlstrom and Fuerst (1997). They show that, after linking the mean distribution of investment opportunities to aggregate total factor productivity, a countercyclical premium on external finance is successfully generated.

In our model, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, the structure of the model implies a positive relation between the default rate and the loan to value ratio. In consequence, if we can improve the model to have a procyclical loan to value ratio, the default rate will become procyclical as well. Therefore, we need to overcome this co-movement and to have both procyclical loan to value ratio and countercyclical default rate, as suggested by data. We show that a time-varying mean of the idiosyncratic shock is required.

1.2.5 My contributions

Chapter 2

- We consider a simple DSGE model with the feature of sticky prices, credit market imperfections, and a fixed housing supply. When the borrowing constraint is related to production housing, which is a factor of production, we observe the financial accelerator (decelerator) effect.

- When the borrowing constraint is related to domestic housing, which provides utility to owners, we do not observe the financial accelerator effect.

Chapter 3

- After introducing a variable housing supply, we can discuss the supply side effect on the economy, including both the direct effect and the feedback effect. We find that the magnitudes of these two effects are negligible in the standard setting of the housing market.

- We examine the U.S. housing sector and suggest that we should construct a new setting for the housing market and introduce the feature of time to build to new housing production.

- After constructing the new setting for the housing market, the magnitudes of the direct effect and the feedback effect are 60 times larger.

- The feature of time to build, together with the new setting of the housing market, allows us to observe cyclical behaviour on the real house price.

Chapter 4

- Our contributions to the literature of adaptive learning are that (i) the dynamic impacts of the AR(1) learning model are explored; (ii) the interaction between the adaptive learning mechanism and the feature of time to build is discussed. After writing the Nottingham Learning Toolbox, we find that the adaptive learning mechanism largely amplifies and propagates the effects of exogenous shock to the economy, and also, enlarges the impact of the time to build feature to the real house price. We also show that the amplification and propagation effects from the adaptive learning mechanism are possibly caused by a relatively higher weight on the lagged variables in the model solution.

- From the sensitivity analysis, we find that (i) the shapes of impulse responses heavily depend on the values of initial beliefs, (ii) the updating process is not crucial for the mechanism if the constant gain coefficient is small.

- We then consider the assumption of heterogeneous expectations. Our contributions to this literature are: (i) we consider learning agents under heterogeneous expectations; (ii) we compare two cases of heterogeneous expectations, i.e., one with partially rational agents and one with fully rational agents.

- From the impulse response analysis, we find that (i) when we have an equal weight on learning agents and rational agents, the impulse responses from heterogeneous expectations are much closer to those from rational expectations than those from adaptive learning; (ii) when rational agents are fully rational, the adaptive learning mechanism has a larger amplification and propagation effect on the economy than that when rational agents are partially rational.

Chapter 5

- We introduce the agency problem to the housing market. Our empirical analysis suggests that, while the default rate is countercyclical, the loan to value ratio is procyclical.

- Our impulse response analysis shows that, given a positive goods sector technology shock, the default rate is countercyclical, but the loan to value ratio is also countercyclical. The inconsistency between our results and empirical evidence suggests that we need to improve the model in a way that allows borrowers to obtain more housing in an economic upturn.

- Moreover, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, we always have a positive relation between the default rate and the loan to value ratio. In order to overcome this co-movement, we show that a time-varying mean is essential.

2 A Simple DSGE Model with A Fixed Housing Supply and Credit Market Imperfections

2.1 Introduction

In the 1990s, the New Neoclassical Synthesis, referred by Goodfriend and King (1997), emerged among macroeconomists about the best way to explain short run economic fluctuations and the role of monetary and fiscal policies. The heart of the new synthesis is the view that the economy is a dynamic general equilibrium system that deviates from an efficient allocation of resources in the short run because of sticky prices and perhaps a variety of other market imperfections. In many ways, this new synthesis forms the intellectual foundation for the analysis of monetary policy at the Federal Reserve and other central banks around the world. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) illustrate medium-sized DSGE models, which include many frictions and shocks, and these models have formed the foundation for the large-sized DSGE models that used in the central banks. Meanwhile, researchers also use small DSGE models to analyse particular questions. For example, Davis and Heathcote (2005) discuss the dynamics of U.S. house price, Millard (2011) analyses the impact of energy on the UK economy.[4]

In this chapter, we firstly illustrate a simple DSGE model with sticky prices.[5] In particular, we add a housing market, as we are particularly interested in the dynamics of the real house price in response to various exogenous shocks. In this chapter, we assume that the housing supply is fixed for two reasons: (i) the availability of land is fixed; and (ii) it takes time to build new housing. Under this assumption, we can discuss the demand side effect on the real house price, but not the supply side effect.[6] Our impulse response

[4]Blanchard (2009) and Woodford (2009) discuss the convergence in methodology in macroeconomics and explain the elements of the New Synthesis.

[5]We also refer it to as simple DSGE model or simple model in this thesis.

[6]The demand (supply) side effect is the impact of a change in the housing demand (supply) on the real house price.

analysis suggests that the real house price responds positively to a positive goods sector technology shock or a negative monetary policy shock, and is the only variable that responds to the housing preference shock. Given a positive goods sector technology shock, our sensitivity analysis suggests that, while the feature of consumption habit mainly affects consumption and output, the dynamics of the real house price are affected largely by the feature of price indexation and the alternative monetary policy rule.

While New Keynesian models have become the workhorse for the monetary policy analysis in the central banks, there are still active projects to introduce important elements into this framework. In standard New Keynesian models, the financial market is assumed to be perfect, but this assumption is inconsistent with reality. Therefore, researchers introduce credit market imperfections, in which borrowers need to use their assets as collateral to secure their loans. The main purpose of this chapter is to examine the relation between credit market imperfections and the financial accelerator effect, and also discuss how agents are affected by various exogenous shocks.

A simple way to consider the feature of credit market imperfections is to assume that loans are fully secured by collateral. In this literature, there are two types of borrowers. The first type of borrowers accumulates a factor of production, such as goods capital or production housing, and then rents it to goods producers. Meanwhile, they take loans from patient households, and use this factor of production as collateral. Therefore, the borrowing constraint is directly linked to output, i.e., a relaxed borrowing constraint leads to increases in the volume of loans and the stock of the factor of production, which in turn has a positive impact on output.

Here we discuss two papers that have studied the first type of borrowers. Kiyotaki and Moore (1997) construct a real business cycle model with two types of agents, lenders and borrowers. In the imperfect credit market, the maximum volume of loans is tightly constrained by the level of borrowers' net worth. In such an economy, goods capital is not only a factor of production, but also the collateral for loans. Therefore, borrowers' credit limits are affected by the prices of the collateralised assets, and at the same time, these prices are affected by the size of the credit limits. The dynamic interaction between credit limits and asset prices becomes a powerful transmission mechanism that temporary shocks to technology or income distribution could generate large, persistent fluctuations in output and asset prices. Iacoviello (2005) explores the interaction of credit limits and asset prices in a New Keynesian model with housing. In his model, the collateral constraint is tied to the value of real estate of firms. The reason for using housing as collateral is that a large proportion of borrowing is secured by real estate. He finds that credit market imperfections leads to (i) an increased response of output to a monetary policy shock, and (ii) a positive response of consumption to a house price shock.

Our second model, i.e., a simple DSGE model with entrepreneurs, also considers this type of borrowers. We refer this case as 'borrowing to invest'. In this model, we can discuss the financial accelerator mechanism, as the volume of loans is related to goods production through production housing. The hypothesis of the financial accelerator mechanism is that, given a positive goods sector technology shock or a negative monetary policy shock, both agents demand more housing and the real house price increases, hence entrepreneurs' borrowing constraint is relaxed and thus they accumulate more production housing, which in turn has a positive impact on output. Our impulse response analysis suggests that (i) a positive goods sector technology shock leads to decreases in entrepreneurs' housing and the volume of loans; (ii) a negative monetary policy shock increases entrepreneurs' housing and leads to a higher volume of loans; (iii) a positive housing preference shock leads to decreases in entrepreneurs' housing and the volume of loans. Then we find that the dynamics of the economy are affected if we switch off the collateral effect, indicating financial accelerator (or decelerator) effect plays a role. Furthermore, given a goods sector technology shock, our sensitivity analysis suggests that (i) we obtain the financial accelerator effect under the alternative monetary policy rule; (ii) a higher loan to value ratio strengthens the re-allocation of housing and the decrease in the volume of loans.

The second type of borrowers accumulates long-lasting goods, such as durable goods or domestic housing, and then gains utility from it. Meanwhile, they also take loans from patient households, and use these goods as collateral. In this setting, the borrowing constraint is not directly linked to output, i.e., a relaxed borrowing constraint leads to increases in the volume of loans and the stock of the long-lasting goods, which have no direct impact on output.

Here we list several papers that have discussed the second type of borrowers. Campbell and Hercowitz (2005) consider heterogeneous agents and the collateral constraints in a one-sector real business cycle model, where durable goods are used as collateral. They examine the contribution of the financial reform of relaxed collateral constraints to households borrowing and they find that the relaxation of collateral constrains can explain a large fraction of the actual volatility decline in the macroeconomy. Monacelli (2009) also considers heterogeneous agents and the collateral constraints but in a two-sector New Keynesian model. His model can explain the facts that, in response to monetary policy shocks, (i) durable and non-durable spending co-move positively, and durable spending exhibits a much larger sensitivity to the shocks. Iacoviello and Neri (2010) consider nominal rigidities, credit market frictions and housing production a two-sector model, where domestic housing is used as collateral. At business frequencies, their model matches the observation that both house prices and housing investment are strongly procyclical, volatile, and sensitive to monetary shocks. Over the longer horizons, they suggest that the house price boom in the 1970s was caused by a productivity slowdown in the housing sector and that housing demand shocks are the main reason for the recent house price boom.

Our third model, i.e., the simple DSGE model with impatient households, considers the second type of borrowers. We refer this case as 'borrowing to live'. In this model, we do not observe the financial accelerator effect as the volume of loans is not related to goods production anymore. Our impulse response analysis suggests that (i) a positive goods sector technology shock leads to a decrease in the volume of loans; (ii) a negative monetary policy shock leads to an increase in the volume of loans; (iii) a housing preference shock leads to a re-allocation of housing from patient households to impatient households and an increase in the volume of loans. Then we switch off the collateral effect to show that the financial accelerator (or decelerator) effect is not observed in this model. Moreover, given a positive goods sector technology, our sensitivity analysis suggests that (i) the alternative monetary policy rule weakens the re-allocation of housing and the decrease in the volume of loans; (ii) a higher loan to value ratio strengthens the decrease in the volume of loans and the re-allocation of housing.

Finally, we summarise our results in the following table and emphasise that (i) the financial accelerator effect is only relevant to the simple model with entrepreneurs, (ii) we observe the financial accelerator effect given a monetary policy shock, (iii) given a goods technology shock, this effect is sensitive to the monetary policy rule. In addition, the response of output to a housing preference shock depends on the model setting.

The model with E	**Housing Market**	**IRA**	**SA**
positive goods technology shock	$E \rightarrow PH$	FD	FA
negative monetary policy shock	$PH \rightarrow E$	FA	
positive housing preference shock	$E \rightarrow PH$		
The model with IH			
positive goods technology shock	$IH \rightarrow PH$	No FA/FD	
negative monetary policy shock	$PH \rightarrow IH$	No FA/FD	
positive housing preference shock	$PH \rightarrow IH$		

where E denotes entrepreneurs, IH denotes impatient households, IRA is impulse responses analysis, SA is sensitivity analysis, and FA (FD) stands for financial accelerator (decelerator) effect.

2.2 The simple model

In this section, we illustrate a simple DSGE model with sticky prices. In particular, we add a housing market, as we are particularly interested in the dynamics of the real house price in response to various exogenous shocks. In this chapter, we assume that the housing supply is fixed, similar with Iacoviello (2005). This assumption is simple and intuitive because the quantity of housing is closely related to the availability of land, which is fixed. Besides, in the short run, the supply of housing can be seen as fixed because it takes time to build new housing. Under this assumption, we can discuss the demand side effect in the housing market, i.e., the impact of a change in the housing demand, but not the supply side effect, i.e., the impact of a change in the housing supply.

2.2.1 Patient households

Patient households are infinitely lived and of measure one.[7] They consume final goods, demand domestic housing, and supply labour. They maximise their lifetime utility subject to their budget constraint. We assume that they own the profitable retail goods firms.

The patient households' lifetime utility function is

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\Gamma_c \ln \left(c_{t+k} - \varepsilon_c c_{t+k-1} \right) + j_{t+k} \ln h_{t+k} - \frac{1}{1+\gamma_n} \left(n_{t+k} \right)^{1+\gamma_n} \right)$$

where E_t is the expectation operator, β is the patient households' discount factor, c_t is patient households' consumption, ε_c measures the degree of consumption habit, Γ_c is a scaling factor, h_t is domestic housing, n_t is the supply of patient households' labour, and $\frac{1}{\gamma_n}$ is the Fisher elasticity of labour supply, i.e., the elasticity of labour supply respect to the change in the current wage rate keeping fixed marginal utility of consumption. The weight on domestic housing, j_t, follows the stationary process

$$j_t = j^{1-\rho_j} j_{t-1}^{\rho_j} e^{\varepsilon_{j,t}}, \qquad \varepsilon_{j,t} \sim N\left(0, \sigma_j^2\right) \tag{2.1}$$

Given the lifetime utility function, the patient households' marginal utility of consumption is

$$u_{c,t} = \Gamma_c \frac{1}{c_t - \varepsilon_c c_{t-1}} - \beta \varepsilon_c \Gamma_c \frac{1}{E_t c_{t+1} - \varepsilon_c c_t} \tag{2.2}$$

[7]We use 'patient households' in this representative agents model because we will add another type of agents in the later context.

which expresses the marginal utility of consumption, $u_{c,t}$, in terms of lagged, current, and future consumption.[8]

The patient households' real budget constraint shows that the real total expense (LHS) should be no more than the real total income (RHS), and is expressed as

$$c_t + q_{h,t}h_t + b_t \leq w_t n_t + q_{h,t}h_{t-1} + \frac{R_{t-1}}{\pi_{c,t}}b_{t-1} + f_t$$

where $q_{h,t}$ is the real house price, b_t is the volume of bonds purchased in period t, w_t is the real wage rate, R_{t-1} is the (gross) nominal interest rate on the bonds hold in period $t-1$, $\pi_{c,t}$ is the (gross) inflation rate, and f_t is the real profit from retail goods firms.[9] The real prices of final goods and bonds are normalised to one.

We obtain three first order conditions from the patient households' lifetime utility maximisation problem.[10] Firstly, the patient households' Euler equation is

$$u_{c,t} = \beta E_t\left(\frac{R_t}{\pi_{c,t+1}}u_{c,t+1}\right) \tag{2.3}$$

which implies that the real price of bonds in terms of the marginal utility of consumption at t is equal to the real gross return of bonds in terms of the discounted marginal utility of consumption at $t+1$. This equation is an intertemporal optimality condition that governs the optimal allocation of consumption over time.

Secondly, the equation that governs the patient households' labour supply is

$$n_t^{\gamma_n} = w_t u_{c,t} \tag{2.4}$$

which implies that the marginal disutility of labour supply at t is equal to the real wage in terms of the marginal utility consumption at t. This first

[8] If we do not have the feature of consumption habit, the marginal utility of consumption depends on current consumption only, $u_{c,t} = \frac{1}{c_t}$.

[9] We use a lagged time subscript for the variables that are predeterminedin period t. For example, we use R_{t-1} as the nominal interest rate on bonds held in the previous period, b_{t-1}, because it is already determined in period $t-1$.

[10] The patient households' lifetime utility maximisation problem is shown in Appendix.

order condition is an intratemporal optimality condition that indicates how patient households make decisions about consumption and labour supply in period t.[11]

Thirdly, the equation that governs the patient households' demand for domestic housing is

$$q_{h,t}u_{c,t} = \frac{j_t}{h_t} + \beta E_t \left(q_{h,t+1}u_{c,t+1} \right) \tag{2.5}$$

which implies that the real house price in terms of the marginal utility of consumption at t is equal to the sum of the marginal utility of domestic housing at t and the expected real house price (for resale) in terms of discounted marginal utility of consumption at $t+1$. This first order condition is an intertemporal optimality condition that describes an optimal allocation of resource between consumption and domestic housing.[12]

2.2.2 Goods production sector

In the goods production sector, we have three players: (i) final goods producers buy retail goods from individual retail goods producers, and compose them into final goods, which are ready for consumption; (ii) retail goods producers (or retailers) buy intermediate goods from intermediate goods producers, and differentiate the goods at no cost into retail goods; (iii) intermediate goods producers combine goods sector technology and labour from patient households to produce intermediate goods, which are then sold to retail goods producers.[13]

Final goods firms Final goods producers buy retail goods from individual retail goods producers, and compose them into final goods, which are

[11] The alternative way to interpretes the first order condition is that the marginal rate of substitution, $\frac{n_t^{\eta_n}}{u_{c,t}}$, is equal to the real wage.

[12] This condition indicates the choice between consumption and housing in period t, but we refer it as an intertemporal optimality condition as it involves $t+1$ term.

[13] It is equivalent to combine retail goods firms and intermediate goods firms.

ready for consumption. These firms are perfectly competitive, thus make zero profit. The main objective of this stage is to derive the individual demand curve for retailer.

Final goods producers compose retail goods into final goods according to the following production function,

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where Y_t is final goods, $Y_t(z)$ is retail goods from retail goods producer z, ε is the elasticity of substitution between differentiated varieties.

The final goods producers' real profit maximisation problem is

$$\max_{Y_t(z)} \left[Y_t - \int_0^1 \frac{P_{c,t}(z)}{P_{c,t}} Y_t(z)\, dz \right]$$

where $P_{c,t}$ is the nominal price of final goods, $\frac{P_{c,t}(z)}{P_{c,t}}$ is the real price of retail goods from retail goods producer z, $\int_0^1 \frac{P_{c,t}(z)}{P_{c,t}} Y_t(z)\, dz$ is the real total cost of buying retail goods from retail goods producers.

The aggregate nominal price level or the nominal price of final goods is expressed as

$$P_{c,t} = \left[\int_0^1 P_{c,t}(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}$$

which indicates that the nominal price of final goods is a composite of the nominal prices of retail goods.

After solving the final goods producers' real profit maximisation problem, we obtain an individual demand curve for each retailer as

$$Y_{t+k}(z) = \left(\frac{P_{c,t+k}(z)}{P_{c,t+k}} \right)^{-\varepsilon} Y_{t+k}$$

which indicates that the relative output, $\frac{Y_{t+k}(z)}{Y_{t+k}}$, depends on the relative price, $\frac{P_{c,t+k}(z)}{P_{c,t+k}}$, together with the elasticity of substitution between differentiated varieties, ε. In other words, when the relative price increases by 1%, the relative output decrease by ε%.

Retail goods firms The economy is composed of a continuum of retailers, whose total is normalised to unity. Retailers buy intermediate goods from intermediate goods producers, and differentiate the goods at no cost into retail goods $Y_t(z)$. They are monopolistic competitive, thus they are price-setters and are able to make profit by selling retail goods with a price markup. We assume that these profitable retail goods firms are owned by patient households.

Additionally, the feature of sticky prices arises from these firms. Following Calvo (1983), we assume that retailers can reset their prices optimally in a given period with probability $1-\theta$. Let $P^*_{c,t}(z)$ denote the optimal nominal price set by retailers who are able to change prices at period t. Besides, by introducing the feature of price indexation, the fraction θ of retailers, who are not able to reset their prices in period t, index their prices to past inflation with a degree of indexation ι_π, thus their prices become $P_{c,t}(z)\left(\frac{P_{c,t}}{P_{c,t-1}}\right)^{\iota_\pi}$, instead of $P_{c,t}(z)$.

The retailers' real profit maximisation problem is

$$\max_{P^*_{c,t}(z)} E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\frac{P^*_{c,t}(z)}{P_{c,t+k}} \left(\frac{P_{c,t+k-1}}{P_{c,t-1}} \right)^{\iota_\pi} Y_{t+k}(z) - \frac{P_{wc,t+k}}{P_{c,t+k}} Y_{t+k}(z) \right]$$

where $\Lambda_{t,t+k} = E_t\left(\beta^k \frac{u_{c,t+k}}{u_{c,t}}\right)$ is the stochastic discount factor, which is used to discount profit in terms of consumption,[11] $P_{wc,t}$ is the nominal price of intermediate goods, and $\frac{P_{wc,t}}{P_{c,t}}$ is the real price of intermediate goods. If a retailer is able to reset price at period t, he will set a price $P^*_{c,t}(z)$ to maximise the expected profits for all subsequent periods, taking the possibility of being unable to reset prices into account.

Substituting the individual retailer's output, $Y_{t+k}(z)$, by the individual

[11] $E_t\left(\beta^k \frac{u_{c,t+k}}{u_{c,t}}\right)$ is also known as the marginal rate of substitution between consumption at period $t+k$ and consumption at period t. It tells how the individual values consumption at period $t+k$ relative to consumption at period t.

demand curve in the profit maximisation problem, we have

$$\max_{P^*_{c,t}(z)} E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\begin{array}{c} \frac{P^*_{c,t}(z)}{P_{c,t+k}} \left(\frac{P_{c,t+k-1}}{P_{c,t-1}}\right)^{\iota_\pi} \left(\frac{P^*_{c,t}(z)}{P_{c,t+k}}\right)^{-\varepsilon} Y_{t+k} \\ -\frac{P_{wc,t+k}}{P_{c,t+k}} \left(\frac{P^*_{c,t}(z)}{P_{c,t+k}}\right)^{-\varepsilon} Y_{t+k} \end{array} \right]$$

The first order condition derived from the retailers' real profit maximisation problem is the equation of the real optimal price, and it is expressed as

$$Q_{c,t} = \frac{\varepsilon}{\varepsilon - 1} E_t \left[\frac{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}}\right)^{-\varepsilon} \frac{P_{wc,t+k}}{P_{c,t+k}}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}}\right)^{1-\varepsilon} \left(\frac{P_{c,t+k-1}}{P_{c,t-1}}\right)^{\iota_\pi}} \right] \tag{2.6}$$

in which we define the real optimal price as $Q_{c,t} = \frac{P^*_{c,t}}{P_{c,t}}$, and inflation as $\pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}}$.[15] This equation implies that the optimal price set in period t depends on the expected real price of intermediate goods, $\frac{P_{wc,t+k}}{P_{c,t+k}}$, in all subsequent periods.[16]

Given the features of sticky prices and price indexation, the nominal price level can also be written as

$$P_{c,t} = \left[\theta \left(\pi^{\iota_\pi}_{c,t-1} P_{c,t-1}\right)^{1-\varepsilon} + (1-\theta) P^{*\ 1-\varepsilon}_{c,t}\right]^{\frac{1}{1-\varepsilon}} \tag{2.7}$$

which implies that the nominal price level at period t depends on the indexed nominal price, $\pi^{\iota_\pi}_{c,t-1} P_{c,t-1}$, set by the fraction θ of retailers that are not able to reset their prices, and the optimal nominal price, $P^*_{c,t}$, set by the fraction $(1-\theta)$ of retailers that are able to reset their prices.

By combining the log-linearised form of the equations of the real optimal price and the aggregate nominal price level, we have the New Keynesian Phillips curve as

$$\widehat{\pi}_{c,t} - \iota_\pi \widehat{\pi}_{c,t-1} = \beta \left(\widehat{\pi}_{c,t+1} - \iota_\pi \widehat{\pi}_{c,t}\right) - \frac{(1-\theta)(1-\theta\beta)}{\theta} \left(\widehat{Z}_t\right)$$

[15] We have $P^*_{c,t}(z) = P^*_{c,t}$ because retail firms will set a same price if they are able to reset their prices.

[16] If the monopolistic competitive firms are intermediate goods firms, the optimal real price will depends on the real marginal costs.

which is not purely forward looking but depends on lagged inflation since a fraction θ of retailers index their prices to past inflation with an elasticity ι_π.

The real profit from retail goods firms is

$$f_t = Y_t - \frac{P_{wc,t}}{P_{c,t}} Y_t = \left(1 - \frac{1}{Z_t}\right) Y_t \tag{2.8}$$

which implies that the real profit is the difference between the real price of final goods, which is normalised to one, and the real price of intermediate goods, $\frac{P_{wc,t}}{P_{c,t}}$. Besides, $Z_t = \frac{P_{c,t}}{P_{wc,t}}$ is the price markup of retail goods firms.

Intermediate goods firms Intermediate goods producers combine exogenous goods sector technology and labour from patient households to produce intermediate goods, which are then sold to retail goods producers. We assume that intermediate goods firms are perfectly competitive, and thus they make zero profit.

The intermediate goods production function is[17]

$$Y_t = A_{c,t} \left(n_{c,t}\right)^{\mu_n} \tag{2.9}$$

where $n_{c,t}$ is patient households' labour, and μ_n is the labour share of output.[18] The goods sector technology, $A_{c,t}$, follows the stationary process

$$A_{c,t} = A_c^{1-\rho_{Ac}} A_{c,t-1}^{\rho_{Ac}} e^{\varepsilon_{Ac,t}}, \qquad \varepsilon_{Ac,t} \sim N\left(0, \sigma_{Ac}^2\right) \tag{2.10}$$

The intermediate goods producers' real profit maximisation problem is

$$\max_{n_{c,t}} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\frac{1}{Z_{t+k}} Y_{t+k} - w_{t+k} n_{c,t+k} \right)$$

[17]Precisely, we should use $Y_{c,t}$ to denote output from intermediate goods firms. As $Y_{c,t} = Y_t$ at aggregate term, we use Y_t directly for simplicity.

[18]It is equivalent to assume a fixed level of goods capital, $\overline{K}_c$, in the intermediate goods production function,

$$Y_t = A_{c,t} \left(n_{c,t}\right)^{\mu_n} \overline{K}_c^{\mu_{kc}}$$

where μ_{kc} is the goods capital share of output.

where $\Lambda_{t,t+k}$ is the stochastic discount factor, $\frac{1}{Z_t}$ is the real price of intermediate goods, $\frac{1}{Z_t}Y_t$ is the real total revenue, and $w_t n_{c,t}$ is the real total cost.

The first order condition derived from this real profit maximisation describes the intermediate goods producers' demand for patient households' labour, and it is expressed as

$$w_t = \mu_n \frac{Y_t}{Z_t n_{c,t}} \tag{2.11}$$

which implies that the real wage of labour, i.e., the marginal cost of labour, is equal to the marginal product of labour.

2.2.3 Monetary authority

The monetary authority uses the nominal interest rate as a policy instrument to affect the real economy. In our model, monetary policy is non-neutral because of the feature of sticky prices that arises from the monopolistic competition among retail goods firms. Therefore, the nominal interest rate can affect the real interest rate, thus has an impact on real variables.

The monetary policy rule, which reacts to inflation and output, is[19]

$$R_t = (R_{t-1})^{\phi_r} \pi_{c,t}^{(1-\phi_r)\phi_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\phi_r)\phi_y} e^{u_{R,t}} \tag{2.12}$$

where R_{t-1} is the lagged nominal interest rate, $\pi_{c,t}$ is the gross inflation rate, Y_t is actual output, and Y is the steady state value of output, ϕ_r, ϕ_π, ϕ_y are weights coefficients. The monetary policy shock, $u_{R,t}$, follows the stationary

[19] Alternatively, we can also assume that the monetary authority is concerned with GDP rather than output from goods sector, thus the policy rule becomes

$$R_t = (R_{t-1})^{\phi_r} \pi_{c,t}^{(1-\phi_r)\phi_\pi} \left(\frac{GDP_t}{GDP}\right)^{(1-\phi_r)\phi_y} e^{u_{R,t}}$$

where $GDP_t = Y_t + q_h IH_t$. Following Iacoviello and Neri (2010), we can use the steady state value of the real housing price, thus short run fluctuation in the real house price has no impact on GDP.

process

$$u_{R,t} = u_R^{1-\rho_{uR}} u_{R,t-1}^{\rho_{uR}} e^{\varepsilon_{uR,t}}, \qquad \varepsilon_{uR,t} \sim N\left(0, \sigma_{uR}^2\right) \tag{2.13}$$

The Fisher equation, which governs the relation between the real interest rate and the nominal interest rate, is

$$r_t = \frac{R_t}{E_t \pi_{c,t+1}} \tag{2.14}$$

which implies that the (gross) real interest rate, r_t, is equal to the nominal interest rate, R_t, adjusted by the expected inflation rate, $E_t \pi_{c,t+1}$.[20]

2.2.4 Market clearing conditions

The bonds market clearing condition is

$$b_t = 0 \tag{2.15}$$

which implies that (i) the aggregate saving is zero, and (ii) there is no borrowing between agents since we consider a model with representative agents.

The economy-wide resource constraint or the goods market clearing condition is

$$Y_t = c_t \tag{2.16}$$

which implies that the final goods are all consumed by patient households as consumption goods.

The labour market clearing condition is

$$n_t = n_{c,t} \tag{2.17}$$

which implies that the supply of patient households' labour is equal to the intermediate goods producers' demand for patient households' labour.

[20]Our monetary policy rule guarantee that the real interest rate moves in a same direction with the nominal interest rate.

The housing market clearing condition is

$$H = h_t \tag{2.18}$$

which implies that housing has a fixed supply, H, and is fully occupied.

2.2.5 Equilibrium

An equilibrium is an allocation of prices $(\pi_{c,t}, R_t, Q_{c,t}, q_{h,t}, w_t, Z_t, r_t)$, quantities $(c_t, u_{c,t}, h_t, Y_t, n_t, n_{c,t}, f_t, b_t)$, and exogenous stochastic process $\{A_{c,t}, j_t, u_{R,t}\}_{t=0}^{\infty}$ satisfying equations (2.1) – (2.18) given the initial conditions for $\pi_{c,t-1}$, R_{t-1}, c_{t-1}.

2.2.6 Calibration

Most of parameters are calibrated in a way that is consistent with Iacoviello and Neri (2010). For the patient households' discount factor, we set $\beta = 0.9925 = 1.03^{-0.25}$, implying a steady state annual real interest rate of 3 percent. The patient households' labour schedule is assumed to be flat, $\gamma_n = 0.01$, which is suggested by Iacoviello (2004), who argues that this flat labour supply curve has the virtue of rationalising the weak observed response of real wages to macroeconomic disturbance.[21] Besides, the coefficient of housing preference is set to $j = 0.12$. In the baseline calibration, we switch off the feature of consumption habit by setting $\varepsilon_c = 0$.

Patient households preference		
The patient households' discount factor	β	0.9925
The inverse of the elasticity of labour supply	γ_n	0.01
The weight on housing	j	0.12
The degree of consumption habit	ε_c	0

The share of labour in the goods production function is set to $\mu_n = 0.65$, implying that the steady state share of labour income is 65%. For the retail

[21] With $1 + \gamma_n$ approaching 1, the utility function becomes linear in leisure.

goods sector, we assume a steady state markup of 15% in the goods sector by setting $Z = 1.15$. For the degree of prices stickiness, we assume that 25% of retailers is able to re-optimise their prices in a given period by setting $\theta = 0.75$, implying that price setters can re-optimise their prices once every $\frac{1}{1-\theta} = 4$ periods. In the baseline calibration, we also switch off the feature of price indexation by setting $\iota_\pi = 0$.

Intermediate goods firms		
Labour share	μ_n	0.65
Retail goods firms		
The steady state gross markup	Z	1.15
The probability of fixed prices	θ	0.75
The degree of price indexation	ι_π	0

For the monetary policy rule, we set the weights coefficients to $\phi_r = 0.6$, $\phi_\pi = 1.5$, and $\phi_y = 0.5$, which are similar with Iacoviello and Neri (2010).[22]

Monetary policy		
The interest rate inertia	ϕ_r	0.6
The weight coefficient on inflation	ϕ_π	1.5
The weight coefficient on output	ϕ_y	0.5

As we focus on the impulse responses of variables to various temporary shocks, the autocorrelation coefficients of these shocks are set to 0.01.[23]

[22] The estimates in Iacoviello and Neri (2010) for these coefficients are $\phi_r = 0.61$, $\phi_\pi = 1.36$, and $\phi_y = 0.51$.

[23] We use 0.01, instead of 0, to facilitate Matlab programs in the future research.

Meanwhile, we set the standard deviation of all shocks to 0.01.

Autocorrelation of shocks		
Goods sector technology	ρ_{Ac}	0.01
Housing preference	ρ_j	0.01
Monetary policy	ρ_{uR}	0.01
Standard deviation of shocks		
Goods sector technology	σ_{Ac}	0.01
Housing preference	σ_j	0.01
Monetary policy	σ_{uR}	0.01

2.2.7 Impulse response analysis

In this section, we discuss how variables respond to various exogenous shocks in the simple model. We consider three types of shocks: (i) a positive goods sector technology shock, which brings extra resource to the economy; (ii) a negative monetary policy shock, which affect the intertemporal optimality condition by making current consumption cheaper in terms of future consumption; (iii) a positive housing preference shock, which affect the intertemporal optimality condition between consumption and housing by increasing the utility from domestic housing.

Goods sector technology shock Figure 2.1 shows the impulse responses of variables to a one percent positive shock in goods sector technology with persistence of $\rho_{Ac} = 0.01$.[24] A higher goods sector technology leads to a higher marginal product of labour, and thus a higher real wage and real income. The higher real income implies that households increase their consumption. Meanwhile, households also demand more housing, leading to a rise in the real house price.[25] In the labour market, employment decreases

[24] In all figures, impulse responses are measured as percentage deviations from the steady state, and horizontal axes display the number of quarters after the shock.

[25] A higher (lower) housing price implies an increased (decreased) demand for housing.

because the negative impact of the lower marginal utility of consumption dominates the positive impact of the higher real wage.[26] Moreover, a higher output (i.e., a higher supply of intermediate goods) leads to a fall in the price of intermediate goods, and thus a higher markup, as the price of final goods are sticky. Also, the lower price of intermediate goods has a negative impact on inflation, through a higher markup, as the New Keynesian Phillips curve shows that inflation is negatively related to the markup. For the monetary policy, the nominal interest rate decreases because it reacts more to the lower inflation dominates than the higher output.

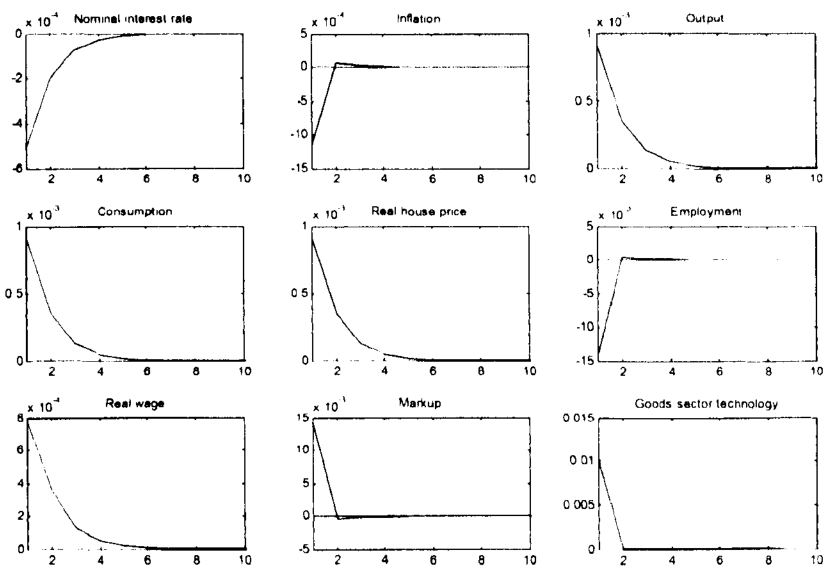

Figure 2.1: Impulse responses to a positive goods sector technology shock from the simple DSGE model.

Monetary policy shock Figure 2.2 shows the impulse responses of variables to a one percent negative monetary shock with persistence of $\rho_{uR} =$

[26]The optimality condition between consumption and labour supply indicates that a lower marginal utility of consumption leads to a lower labour supply, keeping the real wage rate constant.

0.01. A negative monetary policy shock causes a lower nominal interest rate and thus a lower real interest rate, making current consumption cheaper in terms of future consumption. This leads to a higher current consumption as households re-allocate their resource over time horizons, according to the intertemporal optimality condition. Meanwhile, because of the optimality condition between consumption and housing, a higher current consumption leads to a higher demand for housing, and thus a higher real house price. Moreover, the higher consumption causes higher levels of output, marginal product of labour, and real wage. The employment increases because the positive impact of the higher real wage dominates the negative impact of the higher marginal utility of consumption. Furthermore, given a rise in the price of intermediate goods, caused by a higher demand, the feature of sticky prices in the final goods price leads to a decrease in the markup, and thus an increase in inflation.

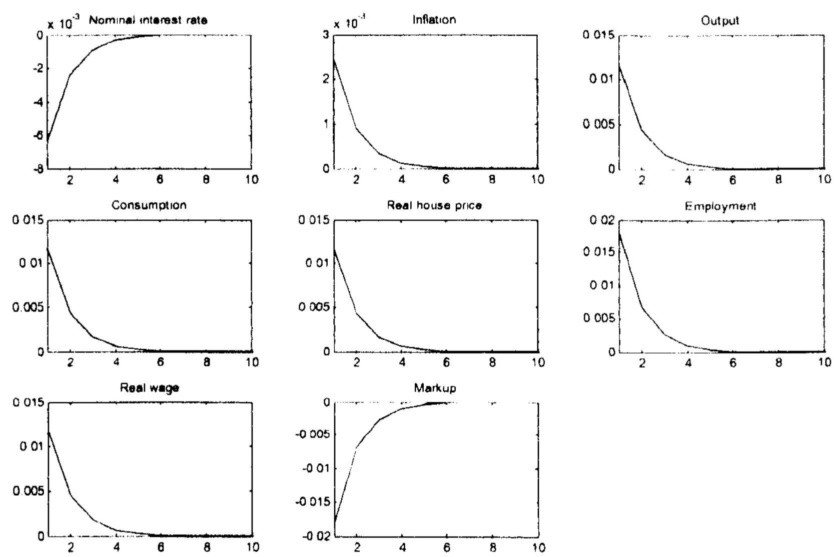

Figure 2.2: Impulse responses to a negative monetary policy shock from the simple DSGE model.

Housing preference shock Figure 2.3 shows the impulse responses of variables to a one percent positive housing preference shock with persistence of $\rho_j = 0.01$. A higher housing preference leads to higher demand for housing, and thus a higher real house price. In this simple model, however, the housing supply is fixed and households are identical, thus there is no transaction of housing and households' decisions on consumption and labour supply are not affected. Hence, the real house price is the only variable that responds to the housing preference shock in this representative agents model with a fixed housing supply.

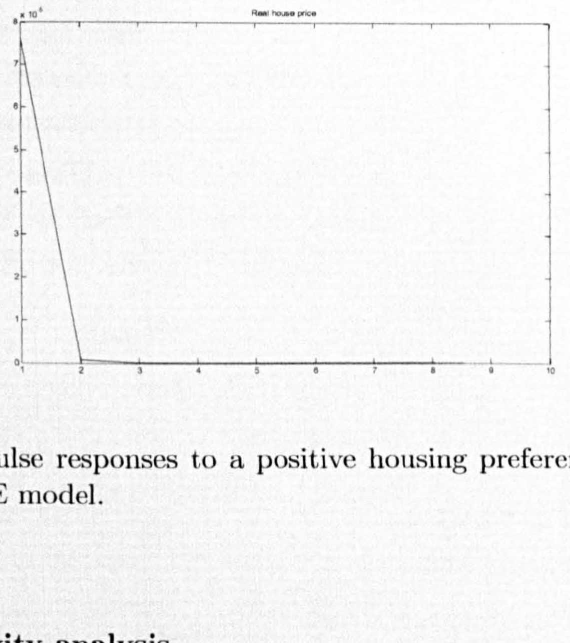

Figure 2.3: Impulse responses to a positive housing preference shock from the simple DSGE model.

2.2.8 Sensitivity analysis

In this section, we examine how the impulse responses of variables are affected by changing the values of some parameters, given a positive goods sector technology shock. We consider different degrees of consumption habit and price indexation, and an alternative monetary policy rule.[27]

[27]In this thesis, amplification means that variable reaches a higher maximum point and propagation means that variable takes longer to return to its steady state.

The feature of consumption habit The feature of consumption habit has been introduced into DSGE models to increase the internal persistence. This feature includes lagged consumption into the households' utility function, and thus the marginal utility of consumption depends on current, future, and lagged consumption. This motivates households to smooth consumption at a higher degree. Here we examine how our results are affected by introducing this feature. We set the degree of consumption habit to $\varepsilon_c = 0.32$, which is consistent with Iacoviello and Neri (2010). Figure 2.4 shows the impulse responses of variables to a one percent positive goods technology shock with persistence of $\rho_{Ac} = 0.01$ under various degrees of consumption habit. We can see that, after introducing this feature, output (consumption) increases less but its pace of returning to steady state is slower. Meanwhile, households shift a fraction of resource to housing, thus the real house price increases more. In addition, the nominal interest rate decreases further as the positive impact of the higher output is weakened. In sum, the main impact of this feature is on the dynamics of output (consumption).

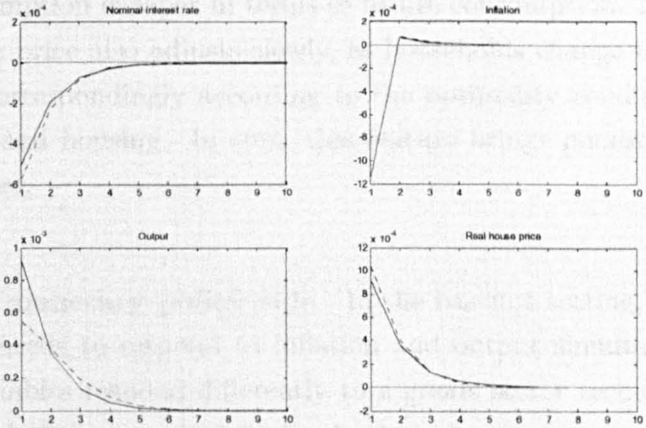

Figure 2.4: Impulse responses to a positive goods sector technology shock under various degrees of consumption habit from the simple DSGE model. The solid line is from the baseline calibration $\varepsilon_c = 0$, and the dashed line is from the alternative calibration $\varepsilon_c = 0.32$.

The feature of price indexation Next, we discuss the impact of the feature of price indexation, which is also used to increase the internal persistence of DSGE models. This feature allows price-setters to index their prices to past inflation if they are not able to reset their prices, implying that current inflation has an extended impact on the economy. We set the degree of price indexation to $\iota_\pi = 0.69$, which is also consistent with Iacoviello and Neri (2010). Figure 2.5 shows the impulse responses of variables to a one percent positive goods technology shock with persistence of $\rho_{Ac} = 0.01$ under various degrees of price indexation. We can see that inflation takes longer to return to its steady state, leading to a slower adjustment in the nominal interest rate as well. Given the slower adjustment in the nominal and thus the real interest rate, the consumption also takes longer to return to its steady state, as households need to re-allocate their resource over time according to their intertemporal optimality condition. Recall that a lower real interest rate make current consumption cheaper in terms of future consumption. Furthermore, the real house price also adjusts slowly, as households change their demand for housing correspondingly according to the optimality condition between consumption and housing. In sum, this feature brings persistent effect to model variables.

Alternative monetary policy rule In the baseline setting, the nominal interest rate needs to respond to inflation and output simultaneously, but these two variables respond differently to a goods sector technology shock. It is suggested that a stable inflation is the primary target of the central banks, thus here we examine how our results are affected if the monetary authority reacts to inflation only by setting the weight coefficient on output to $\phi_y = 0$. Figure 2.6 shows the impulse responses of variables to a one percent positive goods technology shock with persistence of $\rho_{Ac} = 0.01$ under two different monetary policy rules. When the central bank does not pay attention to output, the positive impact of the higher output on the nominal

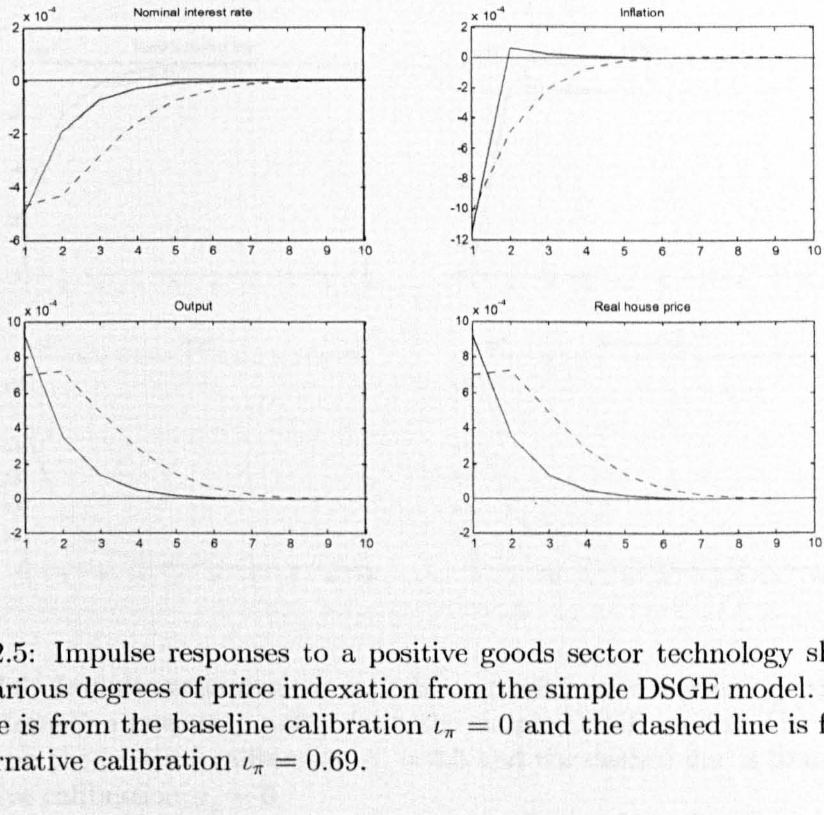

Figure 2.5: Impulse responses to a positive goods sector technology shock under various degrees of price indexation from the simple DSGE model. The solid line is from the baseline calibration $\iota_\pi = 0$ and the dashed line is from the alternative calibration $\iota_\pi = 0.69$.

interest rate disappears, and thus the nominal interest rate decreases further in response to the lower inflation. Meanwhile, the lower nominal and thus the real interest rate makes current consumption cheaper in terms of future consumption. This motivates households to increase current consumption further. Meanwhile, households also demand more housing, and thus leads to a higher real house price. In sum, if the central bank does not pay attention to output, the goods sector technology shock has larger impacts on output and the real house price.

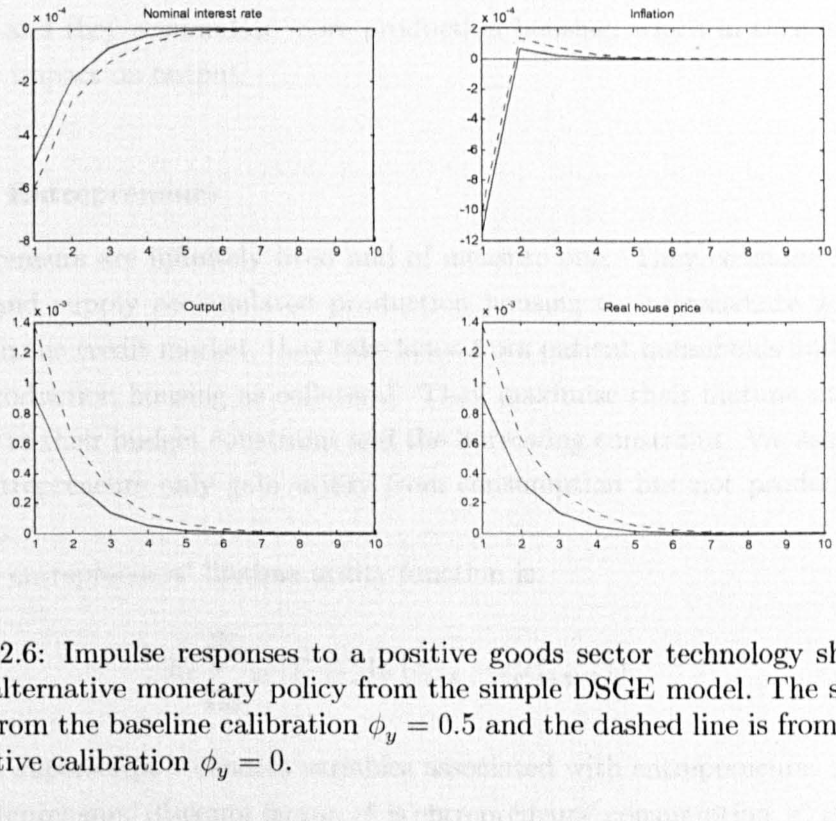

Figure 2.6: Impulse responses to a positive goods sector technology shock under alternative monetary policy from the simple DSGE model. The solid line is from the baseline calibration $\phi_y = 0.5$ and the dashed line is from the alternative calibration $\phi_y = 0$.

2.3 The simple model with entrepreneurs

In this section, we add entrepreneurs into the simple DSGE model, as we want to discuss the impact of credit market imperfections on the economy. In the credit market, entrepreneurs take loans from patient households, and the main purpose of the loans is to accumulate production housing and then rent it to intermediate goods firms. This type of agents are also considered in Kiyotaki and Moore (1997) and Iacoviello (2005), and we refer this case as 'borrowing to invest'. In this model, we can discuss the financial accelerator mechanism, as the volume of loans is related to goods production through production housing. According to the hypothesis of the financial accelerator mechanism, given a positive goods sector technology shock or a negative monetary policy shock, both agents demand more housing and the real house price increases, hence the entrepreneurs' borrowing constraint is relaxed and they accumulate more production housing, which in turn has a positive impact on output.

2.3.1 Entrepreneurs

Entrepreneurs are infinitely lived and of measure one. They consume final goods and supply accumulated production housing to intermediate goods firms. In the credit market, they take loans from patient households and use their production housing as collateral. They maximise their lifetime utility subject to their budget constraint and the borrowing constraint. We assume that entrepreneurs only gain utility from consumption but not production housing.

The entrepreneurs' lifetime utility function is

$$E_t \sum_{k=0}^{\infty} (\beta^e)^k \left(\Gamma_c^e \ln\left(c_{t+k}^e - \varepsilon_c^e c_{t+k-1}^e\right)\right)$$

where a superscript e denotes variables associated with entrepreneurs: β^e is the entrepreneurs' discount factor, c^e is entrepreneurs' consumption, ε_c^e is the degree of consumption habit for entrepreneurs, and Γ_c^e is a scaling factor.[28]

Given the utility function, the entrepreneurs' marginal utility of consumption is

[28] The condition $\beta^e < \beta \Rightarrow \lambda_b^e > 0$, where λ_b^e is the Lagrange multiplier on the borrowing constraint, implies that the borrowing constraint is binding.

$$u_{c,t}^{e} = \Gamma_{c}^{e}\frac{1}{c_{t}^{e} - \varepsilon_{c}^{e}c_{t-1}^{e}} - \beta^{e}\varepsilon_{c}^{e}\Gamma_{c}^{e}\frac{1}{E_{t}c_{t+1}^{e} - \varepsilon_{c}^{e}c_{t}^{e}} \tag{2.19}$$

which expresses the marginal utility of consumption, $u_{c,t}^{e}$, in terms of lagged, current, and future consumption.

The entrepreneurs' real budget constraint shows that the real total expense (LHS) is no more than the real total income (RHS), and is expressed as

$$c_{t}^{e} + q_{h,t}h_{t}^{e} + \frac{R_{t-1}}{\pi_{c,t}}b_{t-1}^{e} \leq q_{z,t}h_{t-1}^{e} + q_{h,t}h_{t-1}^{e} + b_{t}^{e} \tag{2.20}$$

where h^{e} is production housing, $q_{z,t}$ is the real rental price of production housing, b_{t}^{e} is the volume of loans taken from patient households, and $\frac{R_{t-1}}{\pi_{c,t}}b_{t-1}^{e}$ is the real total repayment for previous loans. We assume that entrepreneurs do not own the profitable retail goods firms and thus do not receive profit. In addition, we assume that domestic housing and production housing have a same price, and the transformation between them is costless.

Entrepreneurs take loans from patient households and use their production housing as collateral, thus their borrowing constraint is

$$E_{t}\left(\frac{R_{t}}{\pi_{c,t+1}}\right)b_{t}^{e} \leq m^{e}E_{t}\left(q_{h,t+1}h_{t}^{e}\right) \tag{2.21}$$

which shows that the (gross) real return of lending, $\frac{R_{t}}{\pi_{c,t+1}}b_{t}^{e}$, is fully secured by a fraction of the expected value of production housing, $m^{e}E_{t}\left(q_{h,t+1}h_{t}^{e}\right)$, where m^{e} is the loan to value ratio.[29] In another word, the borrowing constraint shows that the maximum volume of loans is positively related to the expected value of the collateral. This type of borrowing constraint has two features: (i) the value of the loan to value ratio is fixed; and (ii) there is no possibility of default because there is no asymmetric information and agency problem in this model.

We obtain two first order conditions from the entrepreneurs' utility max-

[29] Iacoviello (2005) assumes that, when borrowers default, lenders can repossess borrowers' asset by paying a proportional transational cost, $(1 - m^{e})E_{t}\left(q_{h,t+1}h_{t}^{e}\right)$.

imisation problem.[30] First, the entrepreneurs' Euler equation is

$$u_{c,t}^e = E_t \left(\beta^e \frac{R_t}{\pi_{c,t+1}} u_{c,t+1}^e \right) + \lambda_{b,t}^e \tag{2.22}$$

where $\lambda_{b,t}^e$ is the Lagrange multiplier on the borrowing constraint.[31] This non-standard Euler equation shows that the real price of bonds in terms of the marginal utility of consumption at t is greater than the expected gross real return of bonds in terms of the discounted marginal utility of consumption at $t+1$, i.e., $u_{c,t}^e > E_t \left(\beta^e \frac{R_t}{\pi_{c,t+1}} u_{c,t+1}^e \right)$, indicating that entrepreneurs should borrow more for current consumption.[32] There is, however, a marginal cost of tightening the borrowing constraint by holding one more unit of borrowing, $\lambda_{b,t}^e$, where the borrowing constraint is tighten by one unit and the shadow price of tightening borrowing constraint by one unit is $\lambda_{b,t}^e$. Therefore, this intertemporal optimality condition implies that, given one more unit of borrowing, the marginal gain in terms of current consumption is equal to the sum of the marginal cost in terms of future consumption and the marginal cost of tightening the borrowing constraint.[33]

Second, the equation that governs entrepreneurs' demand for production housing is

$$u_{c,t}^e q_{h,t} = E_t \left(u_{c,t+1}^e q_{z,t+1} \beta^e + u_{c,t+1}^e q_{h,t+1} \beta^e + \lambda_{b,t} m^e \left(\frac{q_{h,t+1} \pi_{c,t+1}}{R_t} \right) \right) \tag{2.23}$$

which implies that the real house price in terms of the marginal utility of consumption at t is equal to the sum of three component: (i) the expected real rental price of production housing in terms of the discounted marginal utility of consumption at $t+1$, (ii) the expected real house price (for resale) in terms of the discounted marginal utility of consumption at $t+1$, and

[30]The entrepreneurs' utility maximisation problem is shown in Appendix.

[31]The Lagrange multiplier is the shadow price of tightening or loosening the budget constraint by one unit.

[32]If the borrowing constraint is not binding, the Lagrange multiplier has a zero value, i.e., $\lambda_{b,t}^e = 0$, and we have a standard form of the Euler equation, $u_{c,t}^e = \beta^e \frac{R_t}{\pi_{c,t+1}} u_{c,t+1}^e$.

[33]In contrast, as lenders, patient households' intertemporal optimality condition is interpreted as that, given one more unit of saving, the cost in terms of current consumption is equal to the gain in terms of future consumption.

(iii) the marginal benefit of relaxing the borrowing constraint by holding one more unit of housing, $\lambda_{b,t}^{e} m^{e}\left(\frac{q_{h,t+1}\pi_{c,t+1}}{R_t}\right)$, where the borrowing constraint is loosened by $m^{e}\left(\frac{q_{h,t+1}\pi_{c,t+1}}{R_t}\right)$ unit and the shadow price of loosening borrowing constraint by one unit is $\lambda_{b,t}^{e}$.

2.3.2 Goods production sector

Intermediate goods firms In this simple DSGE model with entrepreneurs, the intermediate goods production demands lagged production housing as a factor of production, then the intermediate goods production function becomes

$$Y_t = A_{c,t}\left(n_{c,t}\right)^{\mu_n}\left(h_{t-1}^{e}\right)^{\mu_h} \tag{2.24}$$

where $n_{c,t}$ is patient households' labour, h_{t-1}^{e} is entrepreneurs' production housing from the previous period, μ_n is the labour share of output, μ_h is the housing share of output or the elasticity of output to housing.

The intermediate goods producers' real profit maximization problem becomes

$$\max_{n_{c,t},h_{t-1}^{e}} E_t\sum_{k=0}^{\infty}\Lambda_{t,t+k}\left(\frac{1}{Z_{t+k}}Y_{t+k} - w_{t+k}n_{c,t+k} - q_{z,t+k}h_{t-1+k}^{e}\right)$$

where $\frac{1}{Z_{t+k}}Y_{t+k}$ is the real total revenue, $w_t n_{c,t}$ is the real cost of patient households' labour, and $q_{z,t}h_{t-1}^{e}$ is the real cost of entrepreneurs' production housing.

We obtain two first order conditions from the intermediate goods firms' profit maximisation problem. Firstly, the equation that governs the intermediate goods firms' demand for patient households' labour is

$$w_t = \mu_n\frac{Y_t}{Z_t n_{c,t}} \tag{2.25}$$

which indicates that the real wage of patient households' labour (i.e., the real marginal cost) is equal to the marginal product of patient households' labour.

Secondly, the equation that governs the intermediate goods firms' demand for entrepreneurs' production housing is

$$q_{z,t} = \mu_h \frac{Y_t}{Z_t h^e_{t-1}} \tag{2.26}$$

which indicates that the real rental price of entrepreneurs' production housing is equal to the marginal product of entrepreneurs' production housing.[34]

2.3.3 Market clearing conditions

The loans market clearing condition is

$$b_t = b^e_t \tag{2.27}$$

which indicates that, in the loan market, the supply of loans from patient households is equal to the demand for loans from entrepreneurs, and thus total net saving is zero.

The economy-wide resource constraint or the goods market clearing condition is

$$Y_t = c_t + c^e_t \tag{2.28}$$

which indicates that final goods are consumed by both patient households and entrepreneurs.

To facilitate the summary of our model in the next section, we repeat the labour market clearing condition, which is

$$n_t = n_{c,t} \tag{2.29}$$

which implies that the supply of patient households' labour is equal to the intermediate goods producers' demand for patient households' labour.

[34]We have entrepreneurs and intermediate goods production in our model. It is equivalent to combine them as in Iacoviello (2004), except having one more equation to describe the movement of rental price.

The housing market clearing condition is

$$H = h_t + h_t^e \tag{2.30}$$

which indicates that the fixed total housing supply H are divided into domestic housing and production housing. Note that the purpose of housing is determined when it is purchased by consumer or by entrepreneurs, thus intermediate goods producers can only rent housing capital from entrepreneurs.

2.3.4 Equilibrium

An equilibrium is an allocation of prices $(\pi_{c,t},\ R_t,\ Q_{c,t},\ q_{h,t},\ w_t,\ Z_t,\ r_t,\ q_{z,t})$, quantities $\left(c_t,\ u_{c,t},\ h_t,\ Y_t,\ n_t,\ n_{c,t},\ f_t,\ b_t,\ c_t^e,\ u_{c,t}^e,\ h_t^e,\ b_t^e,\ \lambda_{b,t}^e\right)$, and exogenous stochastic process $\{A_{c,t},\ j_t,\ u_{R,t}\}_{t=0}^{\infty}$ satisfying equations (2.1) – (2.8), (2.10), (2.12) – (2.14), (2.19) – (2.30), given the initial conditions for $\pi_{c,t-1}$, R_{t-1}, c_{t-1}, c_{t-1}^e, h_{t-1}^e, b_{t-1}^e.

2.3.5 Calibration

Here we calibrate parameters that are related to entrepreneurs. We set the entrepreneurs' discount factor to $\beta^e = 0.98$, which is consistent with Iacoviello (2005). Also, we switch off the feature of consumption habit by setting $\varepsilon_c^e = 0$. In the intermediate goods production function, we set the income share of production housing to $\mu_h = 0.03$, which is also consistent with Iacoviello (2005). Following Iacoviello and Neri (2010), we set the loan to value ratio to $m^e = 0.85$, implying that the maximum volume of loans is

85% of the expected value of entrepreneurs' production housing.

Entrepreneurs' preference		
The entrepreneurs' discount factor	β^e	0.98
The degree of consumption habit	ε_c^e	0
Intermediate goods production		
The income share of real estate	μ_h	0.03
The borrowing constraint		
The loan to value ratio	m^e	0.85

2.3.6 Impulse response analysis

In this section, we discuss the impulse responses of variables to various exogenous shocks in the simple DSGE model with entrepreneurs. Similar with the previous model, we consider a positive goods sector technology shock, a negative monetary policy shock, and a positive housing preference shock respectively.

Goods sector technology shock Figure 2.7 shows the impulse responses of variables to a one percent positive shock in goods sector technology with persistence of $\rho_{Ac} = 0.01$. Similar with the result from the previous model, the nominal interest rate and inflation decrease, while output, patient households' consumption, and the real house price increase. In the housing market, the positive goods technology shock increases both agents' demand for housing. The housing supply, however, is fixed, thus it is not possible for both agents to have more housing. In Figure 2.7, we observe an increase in patient households' housing and a decrease in entrepreneurs' housing, i.e., a re-allocation of housing from entrepreneurs to patient households. This may suggest that the demand for housing from patient households is stronger than that from entrepreneurs. One possible reason is that patient households are the owner of profitable retail firms. Meanwhile, although the real house price is higher, the volume of loans decreases because the positive impact of the higher real house price is dominated by the negative impact of the lower entrepreneurs' housing.[35] Moreover, entrepreneurs decrease their consumption according to the optimality condition between consumption and housing.

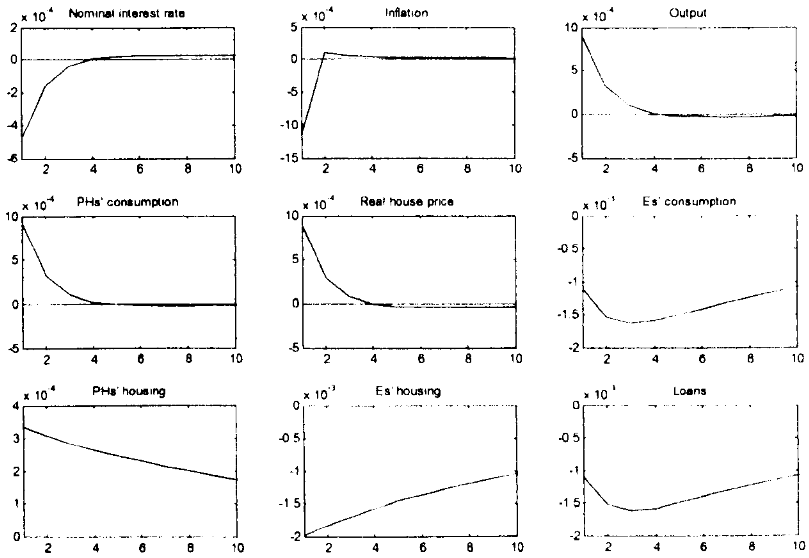

Figure 2.7: Impulse responses to a positive goods sector technology shock from the simple model with entrepreneurs.

In Figure 2.8, we switching off the collateral effect to examine how the dynamics of output is affected by setting the loan to value ratio to zero, $m^e = 0$, or by setting the volume of loan to a fixed level, $b_t^e = b^e$, given a goods sector technology shock. We observe that output decreases faster in the model with collateral effect, because the fall in entrepreneurs' production housing has a negative impact on output. Therefore, we observe the financial decelerator effect as the volume of loans is reduced by the decrease in entrepreneurs' production housing, although the real house price is rising.[36]

[35]We can also say that the lower entrepreneurs housing is caused by a lower volume of loans. Because model variables are determined simultaneously in DSGE models and they interact with each others, a clear causality is difficult to define.

[36]The financial accelerator effect suggests that, in order to amplify and propagate the impacts of the shock, the borrowing constraint should be relaxed, given the higher real house prices.

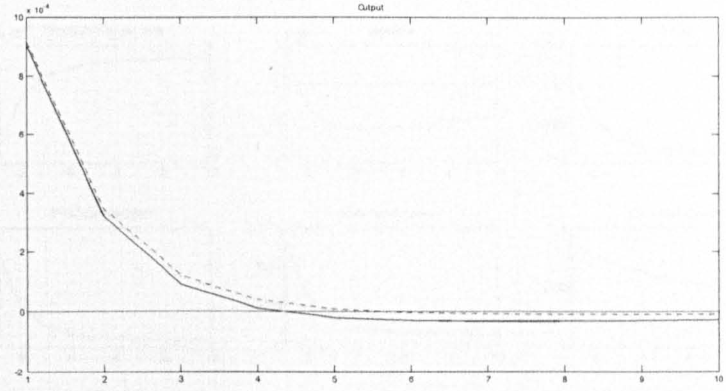

Figure 2.8: Impulse response output to a positive goods sector technology shock from the simple model with entrepreneurs. The solid line is from the baseline calibration $m^e = 0.85$, the dashed line is from the alternative calibration $m^e = 0$, and the dotted line is from $b_t^e = b^e$.

Monetary policy shock Figure 2.9 shows the impulse responses of variables to a one percent negative monetary shock with persistence of $\rho_{u\pi} = 0.01$. Similar with the results from the simple model, a negative monetary policy shock causes a lower nominal interest rate, a higher output, a higher patient households' consumption and a higher real house price. In the housing market, while both households demand more housing, reflected by a rising real house price, there is a re-allocation of housing from patient households to entrepreneurs, suggesting that entrepreneurs are more competitive in the housing market than patient households. One of the possible reasons is that patient households switch resource from housing sector to goods sector in order to have a higher consumption. Therefore, the volume of loans increases as both real house price and entrepreneurs' production housing are higher. Furthermore, entrepreneurs increase their consumption because of a lower real interest rate and a higher housing.

In Figure 2.10, we examine the impact of the collateral effect given a goods sector technology shock. We observe that output takes longer to return to its steady state in the model with collateral effect, because the rise in entrepreneurs' production housing has a further positive impact on output.

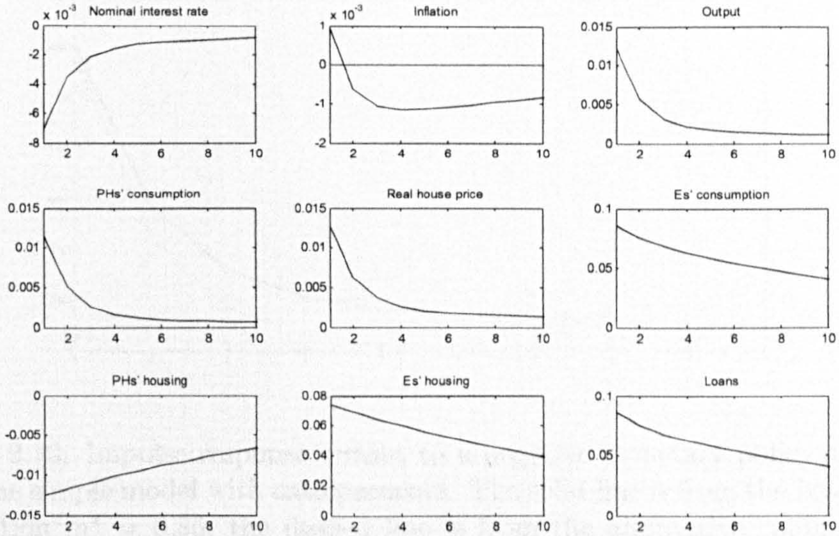

Figure 2.9: Impulse responses to a negative monetary policy shock from the simple model with entrepreneurs.

Therefore, given a monetary policy shock, we observe the financial accelerator effect, which amplifies and propagates the impacts of the shock.

Housing preference shock Figure 2.11 shows the impulse responses of variables to a one percent positive housing preference shock with persistence of $\rho_j = 0.01$. In the simple model, only real house price responds to the housing preference shock. In contrast, other variables also respond to this shock in this model. A higher patient households' housing preference indicates a higher utility from housing, thus they demand more housing, while entrepreneurs is not affected as they do not gain utility from housing. Therefore, we observe an increase in patient households' housing and a decrease in entrepreneurs' housing, i.e., a transfer of housing from entrepreneurs to patient households. As the housing preference shock does not bring extra resource to the economy, patient households need to shift their resource from consumption to housing and entrepreneurs have more resource for consumption by

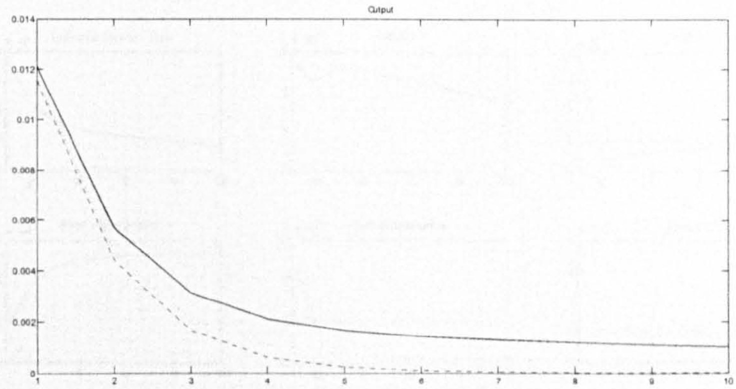

Figure 2.10: Impulse response output to a negative monetary policy shock from the simple model with entrepreneurs. The solid line is from the baseline calibration $m^e = 0.85$, the dashed line is from the alternative calibration $m^e = 0$, and the dotted line is from $b_t^e = b^e$.

selling housing. The total impact on output is positive as the positive impact of entrepreneurs consumption dominates the negative impact of patient households consumption.

2.3.7 Sensitivity analysis

In this section, we examine how the impulse responses of variables are affected by changing the values of some parameters, given a positive goods sector technology shock. We first consider an alternative monetary policy, and then examine various values of the loan to value ratio.

Alternative monetary policy rule Here we discuss how the dynamics are affected if the central bank reacts to inflation only by setting the weight coefficients on output to $\phi_y = 0$. Figure 2.12 shows the impulse responses of variables to a goods technology shock with persistence of $\rho_{Ac} = 0.01$ under two different monetary policy rules. Similar with the results from the simple model, when the nominal interest rate does not adjust for output, it decreases more in respond to the negative inflation, thus the positive responses

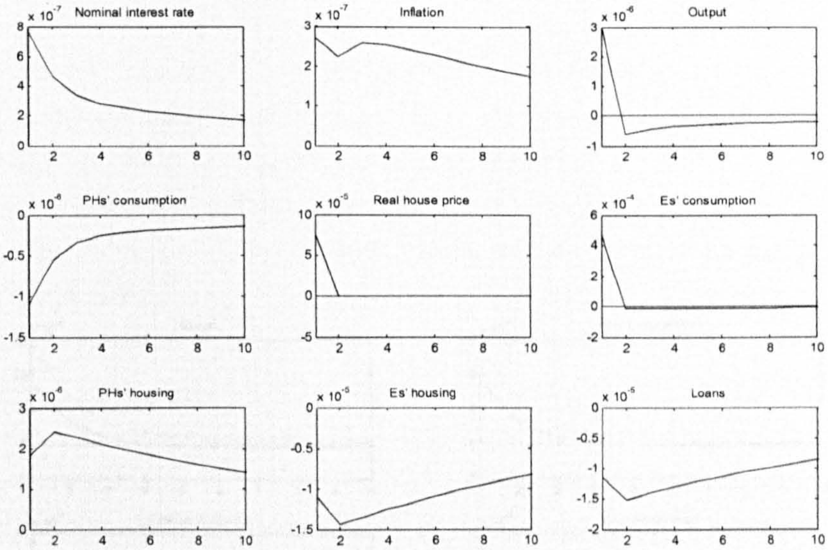

Figure 2.11: Impulse responses to a positive housing preferece shock from the simple model with entrepreneurs.

of output, patient households' consumption and the real house price are strengthened. There is, however, a big change in the housing market. When the central bank does not pay attention to output, the direction of housing movement is reversed: entrepreneurs' housing rises and patient households' housing falls. The possible reason is that, given a lower real interest rate, patient households increase their current consumption and thus their competitiveness in the housing market is weakened. As a result of the higher entrepreneurs' production housing and the higher real house price, we observe a higher volume of loans, which in turn increases entrepreneurs' consumption. Meanwhile, the higher entrepreneurs' production housing has a further positive impact on output. This is the financial accelerator effect, and we argue that, in the simple model with entrepreneurs, given a positive technology shock, the existence of this mechanism is sensitive to the parameter in the monetary policy rule.

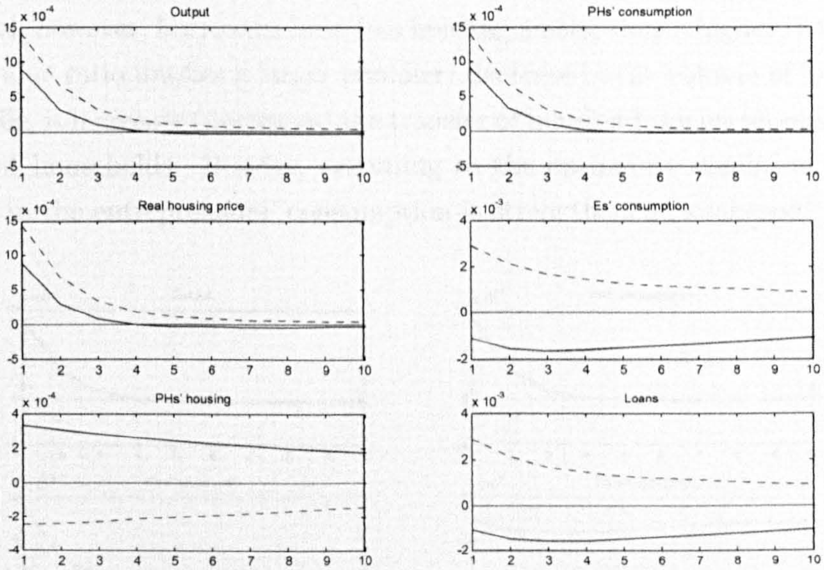

Figure 2.12: Impulse responses to a positive goods sector technology shock under the alternative monetary policy rule from the simple model with entrepreneurs. The solid line is from the baseline calibration $\phi_y = 0.5$ and the dashed line is from the alternative calibration $\phi_y = 0$.

Variation in the loan to value ratio Finally, we examine how the dynamics are affected by changing the value of the loan to value ratio. Figure 2.13 and Figure 2.14 show the impulse responses of variables to a goods technology shock with persistence of $\rho_{Ac} = 0.01$ for a higher loan to value ratio, $m^i = 0.95$, and a lower ratio, $m^i = 0.75$, respectively.

The main impact of the change in the loan to value ratio is on the housing market. Theoretically, if the housing assets that borrowers are buying are constant, a higher loan to value ratio leads to a higher volume of loans. In our model, however, borrowers have less housing assets, thus a higher (lower) loan to value ratio implies a larger (smaller) decrease in the volume of loans. Meanwhile, it increases (decreases) the transfer of housing from entrepreneurs to patient households. Besides, according to the optimality condition, the decrease in the entrepreneurs' consumption is strengthened (weakened)

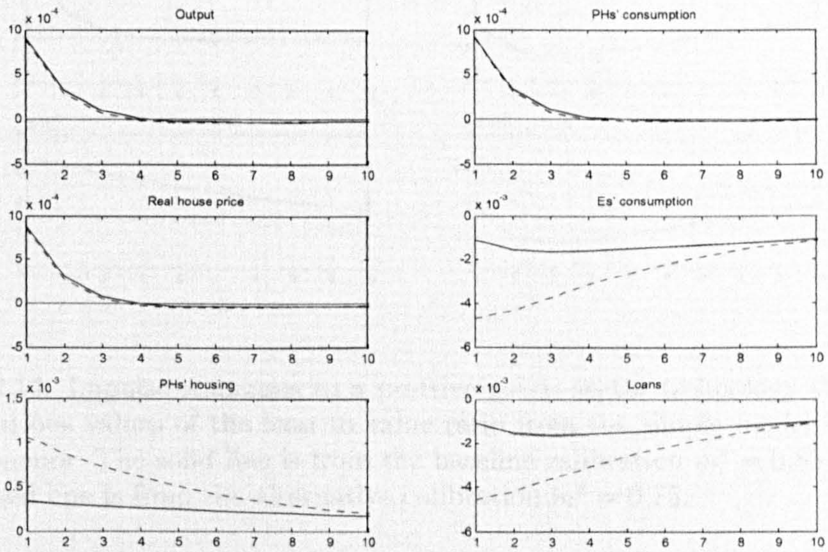

Figure 2.13: Impulse responses to a positive goods sector technology shock under various values of the loan to value ratio from the simple model with entrepreneurs. The solid line is from the baseline calibration $m^e = 0.85$ and the dashed line is from the alternative calibration $m^e = 0.95$.

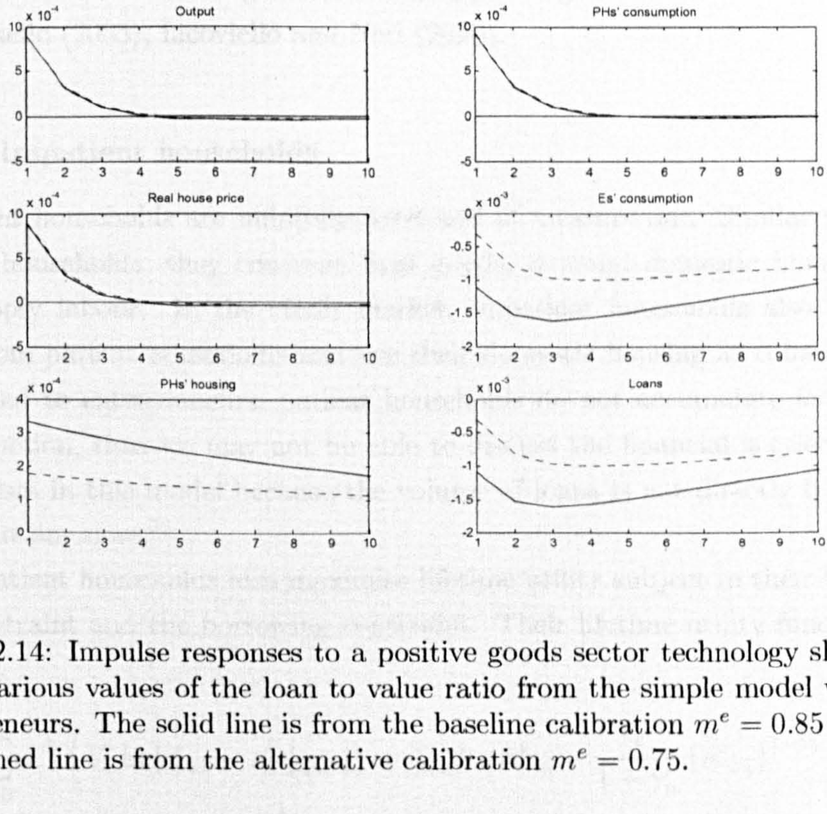

Figure 2.14: Impulse responses to a positive goods sector technology shock under various values of the loan to value ratio from the simple model with entrepreneurs. The solid line is from the baseline calibration $m^e = 0.85$ and the dashed line is from the alternative calibration $m^e = 0.75$.

2.4 The simple model with impatient households

In this section, we add another type of agents, impatient households, into the simple model. The similarity between impatient households and entrepreneurs is that both of them take loans from patient households and thus face a borrowing constraint. The difference, however, is that, while entrepreneurs demand production housing and rent it to intermediate goods firms, impatient households demand domestic housing and gain utility from it, thus we refer this case as 'borrowing to live'. This type of agents are also considered in Iacoviello (2005), Iacoviello and Neri (2010).

2.4.1 Impatient households

Impatient households are infinitely lived and of measure one. Similar with patient households, they consume final goods, demand domestic housing, and supply labour. In the credit market, impatient households also take loans from patient households and use their domestic housing as collateral. Compared to entrepreneurs, patient households do not accumulate factors of production, thus we may not be able to discuss the financial accelerator mechanism in this model because the volume of loans is not directly linked to output anymore.[37]

Impatient households also maximise lifetime utility subject to their budget constraint and the borrowing constraint. Their lifetime utility function is

$$E_t \sum_{k=0}^{\infty} \beta^{ik} \left(\Gamma_c^i \ln \left(c_{t+k}^i - \varepsilon_c^i c_{t+k-1}^i \right) + j_{t+k}^i \log h_{t+k}^i - \frac{1}{1+\gamma_n^i} \left(n_{t+k}^i \right)^{1+\gamma_n^i} \right)$$

where a superscript i denotes variables associated with impatient households: β^i is impatient households' discount factor, c_t^i is impatient households' consumption, ε_c^i is the degree of consumption habit for impatient households, Γ_c^i

[37] If we assume that borrowers have a higher preference on housing than lenders, we may observe a feedback loop between the real housing price and the volume of loans: higher real house prices loosen the borrowing constraint, and lead to a higher volume of loans, which in turn raises the demand for housing and the real house prices further.

is a scaling factor, h_t^i is impatient households' domestic housing, n_t^i is supply of impatient households' labour, and $\frac{1}{\gamma_n^i}$ is the Fisher elasticity of labour supply.[38] For simplicity, we assume that impatient households' weight on domestic housing is same as patient household's,

$$j_t^i = j_t \tag{2.31}$$

Therefore, a housing preference shock will affect both households' demand for housing.

Given the utility function, the impatient households' marginal utility of consumption is

$$u_{c,t}^i = \Gamma_c^i \frac{1}{c_t^i - \varepsilon_c^i c_{t-1}^i} - \beta^i \varepsilon_c^i \Gamma_c^i \frac{1}{E_t c_{t+1}^i - \varepsilon_c^i c_t^i} \tag{2.32}$$

which expresses the marginal utility of consumption $u_{c,t}^i$ in terms of lagged, current, and future consumption.

The impatient households' shows that the real total expense (LHS) should be no more than the real total income (RHS), and is expressed as

$$c_t^i + q_{h,t} h_t^i + \frac{R_{t-1}}{\pi_{c,t}} b_{t-1}^i \leq w_t^i n_t^i + q_{h,t} h_{t-1}^i + b_t^i \tag{2.33}$$

where b_t^i is the volume of loans taken from patient households, and $\frac{R_{t-1}}{\pi_{c,t}} b_{t-1}^i$ is the real repayment for the previous loans. We assume that impatient households do not own the profitable retail goods firms and thus they do not receive profit.

The impatient households' real borrowing constraint is

$$E_t \left(\frac{R_t}{\pi_{c,t+1}} \right) b_t^i \leq m^i E_t \left(q_{h,t+1} h_t^i \right) \tag{2.34}$$

which implies that the (gross) real return of borrowing, $E_t \left(\frac{R_t}{\pi_{c,t+1}} \right) b_t^i$, is fully secured by a fraction of the expected value of domestic housing, $m^i E_t \left(q_{h,t+1} h_t^i \right)$,

[38]The condition $\beta^i < \beta \Rightarrow \lambda_b^i > 0$, where λ_b^i is the Lagrange multiplier on the borrowing constraint, implies that the borrowing constraint is binding.

where m^i is the loan to value ratio.

We obtain three first order conditions from the impatient households utility maximisation problem.[39] Firstly, the impatient households' Euler equation is

$$u^i_{c,t} = \beta^i \frac{R_t}{\pi_{c,t+1}} u^i_{c,t+1} + \lambda^i_{b,t} \tag{2.35}$$

where $\lambda^i_{b,t}$ is the Lagrange multiplier on the borrowing constraint. The description of this equation is similar with that of entrepreneurs' Euler equation, i.e., given one more unit of borrowing, the marginal gain in terms of current consumption is equal to the sum of the marginal cost in terms of future consumption and the marginal cost of tightening the borrowing constraint .

Secondly, the equation that governs the impatient households' labour supply is

$$\left(n^i_t\right)^{\gamma^i_n} = w^i_t u^i_{c,t} \tag{2.36}$$

which implies that the marginal disutility of labour supply at t is equal to the real wage in terms of the marginal utility consumption at t.

Thirdly, the equation that governs patient households' demand for domestic housing is

$$u^i_{c,t} q_{h,t} = \frac{j_t}{h^i_t} + \beta^i u^i_{c,t+1} q_{h,t+1} + \lambda^i_{b,t} m^i \left(\frac{q_{h,t+1}\pi_{c,t+1}}{R_t}\right) \tag{2.37}$$

This is a non-standard form of housing demand equation.[40] This equation implies that the real house price in terms of the marginal utility of consumption at t is equal to the sum of three component: (i) the marginal utility of domestic housing at t, (ii) the expected real house price (for resale) in terms of the discounted marginal utility of consumption at $t+1$, and (iii) the marginal benefit of relaxing the borrowing constraint by holding one more unit of housing, $\lambda^i_{b,t} m^i \left(\frac{q_{h,t+1}\pi_{c,t+1}}{R_t}\right)$, where the borrowing constraint is loosened by $m^i \left(\frac{q_{h,t+1}\pi_{c,t+1}}{R_t}\right)$ unit and the shadow price of loosening the borrowing

[39]The impatient households' utility maximisation problem is shown in Appendix.

[40]If the borroing constraint is not binding, $\lambda^i_{b,t} = 0$, we have the stadnard form of housing demand equation, $u^i_{c,t} q_{h,t} = \frac{j_t}{h^i_t} + \beta^i u^i_{c,t+1} q_{h,t+1}$.

constraint by one more unit is $\lambda^{i}_{b,t}$.

2.4.2 Goods production sector

Intermediate goods firms Since impatient households also provide labour to intermediate goods firms, the intermediate goods production function becomes

$$Y_t = A_{c,t}\left(n_{c,t}\right)^{\alpha\mu_n}\left(n^{i}_{c,t}\right)^{(1-\alpha)\mu_n} \tag{2.38}$$

where $n_{c,t}$ is the patient households' labour, $n^{i}_{c,t}$ is the impatient households' labour, μ_n is the labour share of output, α is the patient households' share of labour income, and $1-\alpha$ is impatient households' share of labour income.

The intermediate goods firms' real profit maximization problem becomes

$$\max_{n_{c,t},n^{i}_{c,t}} E_t\sum_{k=0}^{\infty}\Lambda_{t,t+k}\left(\frac{1}{Z_{t+k}}Y_{c,t+k} - w_{t+k}n_{c,t+k} - w^{i}_{t+k}n^{i}_{c,t+k}\right)$$

where $\frac{1}{Z_t}Y_{c,t}$ is the real total revenue, $w_t n_{c,t}$ is the real cost of patient households' labour, and $w^{i}_{t}n^{i}_{c,t}$ is the real cost of impatient households' labour.

We obtain two first order conditions from the intermediate goods firms' profit maximisation problem. Firstly, the equation that governs the intermediate goods producers' demand for patient households' labour is

$$w_t = \alpha\mu_n\frac{Y_t}{Z_t n_{c,t}} \tag{2.39}$$

which indicates that the real wage of patient households' labour (i.e., the real marginal cost) is equal to the marginal product of patient households' labour.

Secondly, the equation that governs the intermediate goods producers' demand for impatient households' labour is

$$w^{i}_{t} = (1-\alpha)\,\mu_n\frac{Y_t}{Z_t n^{i}_{c,t}} \tag{2.40}$$

which indicates that the real wage of impatient households' labour is equal to the marginal product of impatient households' labour.

2.4.3 Market clearing conditions

The loans market clearing condition is

$$b_t = b_t^i \tag{2.41}$$

which indicates that the supply of loans from patient households is equal to the demand for loans from impatient households, and thus the total net saving is zero.

The economy-wide resource constraint or the goods market clearing condition is

$$Y_t = c_t + c_t^i \tag{2.42}$$

which indicates that final goods are consumed by both patient households and impatient households.

To facilitate the summary of this model in the next section, we repeat the market clearing condition for patient households' labour,

$$n_t = n_{c,t} \tag{2.43}$$

The market clearing condition for impatient households' labour is

$$n_t^i = n_{c,t}^i \tag{2.44}$$

which indicates that the supply of impatient households' labour is equal to the intermediate goods firms' demand for impatient households' labour.

The housing market clearing condition is

$$H = h_t + h_t^i \tag{2.45}$$

which indicates that the fixed housing supply H are occupied by both patient households and impatient households.

2.4.4 Equilibrium

An equilibrium is an allocation of prices $(\pi_{c,t},\ R_t,\ Q_{c,t},\ q_{h,t},\ w_t,\ Z_t,\ r_t,\ q_{z,t},\ w_t^i)$, quantities $\left(c_t,\ u_{c,t},\ h_t,\ Y_t,\ n_t,\ n_{c,t},\ f_t,\ b_t,\ c_t^i,\ u_{c,t}^i,\ h_t^i,\ n_t^i,\ n_{c,t}^i,\ b_t^i,\ \lambda_{b,t}^i,\ j_t^i\right)$, and exogenous stochastic process $\{A_{c,t},\ j_t,\ u_{R,t}\}_{t=0}^{\infty}$ satisfying equations (2.1) – (2.8), (2.10), (2.12) – (2.14), (2.31) – (2.45), given the initial conditions for $\pi_{c,t-1}$, R_{t-1}, c_{t-1}, c_{t-1}^i, h_{t-1}^i, b_{t-1}^i.

2.4.5 Calibration

In the section, we calibrate the additional parameters in the simple model with impatient households. Following Iacoviello and Neri (2010), we set the impatient households' discount factor to $\beta^i = 0.97$. Similar with patient households, the impatient households' labour schedule is also assumed to be flat, $\gamma_n^i = 0.01$. Besides, we also switch off the feature of consumption habit for impatient households by setting $\varepsilon_c^i = 0$. In addition, similar with patient households, the impatient households' wight on housing is set to $j^i = 0.12$.

Impatient households' preference		
The impatient households' discount factor	β^i	0.97
The inverse of the elasticity of labour supply	γ_n^i	0.01
The degree of consumption habit	ε_c^i	0
The weight on housing	j^i	0.12

Secondly, in the intermediate goods production function, we assume that the patient households' share of labour income is $\alpha = 0.79$, then the impatient households' share of labour income is $1 - \alpha = 0.21$. In addition, we set the loan to value ratio to $m^i = 0.85$, implying that the maximum volume of loans is 85% of the expected value of impatient households' domestic housing.

These values are consistent with Iacoviello and Neri (2010).

Intermediate goods production		
The labour income share of patient households	α	0.79
The labour income share of impatient households	$1-\alpha$	0.21
The borrowing constraint		
The loan to value ratio	m^i	0.85

2.4.6 Impulse response analysis

In this section, we discuss the impulse responses of variables to various exogenous shocks in the simple model with impatient households. Similar with the simple model, we consider a positive goods sector technology shock, a positive monetary policy shock, and a positive housing preference shock respectively.

Goods sector technology shock Figure 2.15 shows the impulse responses of variables to a one percent positive shock in goods sector technology with persistence of $\rho_{Ac} = 0.01$. Similar with the results from the simple model, the nominal interest rate and inflation decrease, while output, patient households' consumption, and the real house price increase. In the housing market, the positive goods technology shock increases both households' demand for housing, but they cannot both have a higher housing due to a fixed housing supply. In Figure 2.15, we observe a re-allocation of housing from impatient households to patient households, suggesting that the demand for housing from patient households dominates that from impatient households. As we discussed in the previous model, it may be possible that patient households own profitable retail firms. Meanwhile, although the real house price is higher, we observe a decrease in the volume of loans because the positive impact of the higher real house price is dominated by the negative impact of the lower impatient households' housing. Furthermore, since their housing has decreased, the optimality condition implies that impatient households decrease their consumption as well.

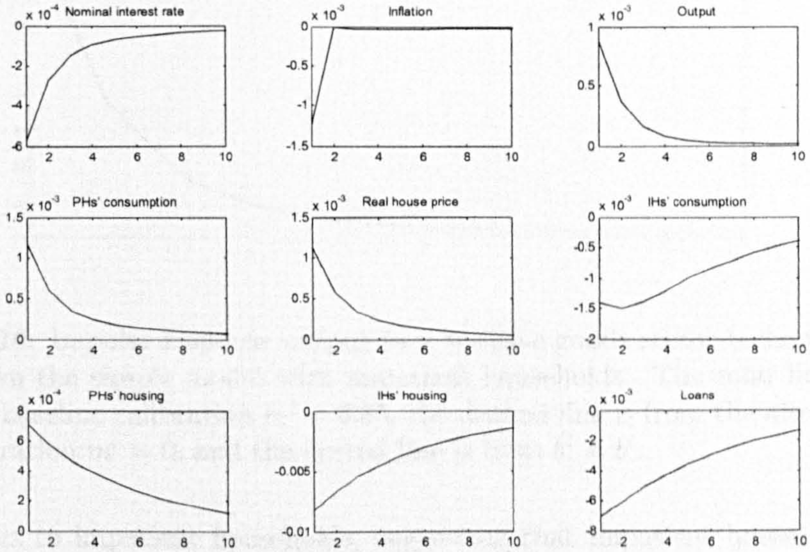

Figure 2.15: Impulse responses to a positive goods sector technology shock from the simple model with impatient households.

By switching off the collateral effect, Figure 2.16 shows that, since impatient households has a higher propensity to spend, a decrease in the volume of loans will have a negative impact on output. However, since impatient households do not accumulate factors of productions, the decrease in the volume of loans has little impact on the output from the second quarter.

Monetary policy shock Figure 2.17 shows the impulse responses of variables to a one percent negative monetary shock with persistence of $\rho_{uR} = 0.01$. Similar with the results from the simple model, a negative monetary policy shock causes a lower nominal interest rate, a higher output, a higher patient households' consumption, and a higher real house price. In the housing market, while both households demand more housing, reflected by the rising real house price, there is a re-allocation of housing from patient

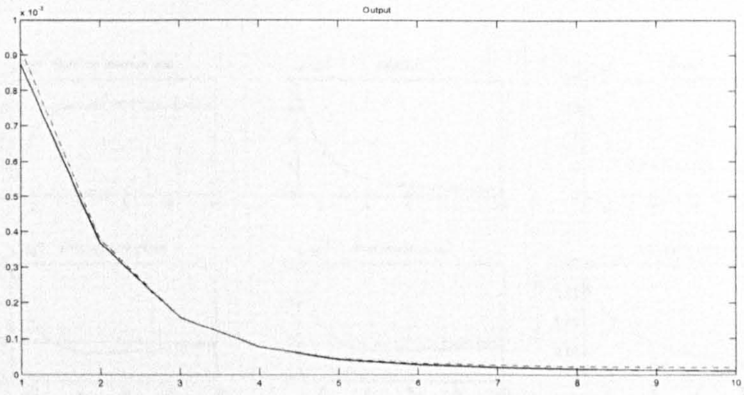

Figure 2.16: Impulse response output to a positive goods sector technology shock from the simple model with impatient households. The solid line is from the baseline calibration $m^i = 0.85$, the dashed line is from the alternative calibration $m^i = 0$, and the dotted line is from $b_t^i = b^i$.

households to impatient households, suggesting that impatient households are more competitive in the housing market than patient households. Similar with the previous model, it is possible that patient households transfer resource from housing sector to goods sector. In addition, the volume of loans increases as both real house price and impatient households' housing are higher. Furthermore, impatient households increase their consumption because of a lower real interest rate and a higher housing.

Figure 2.18 shows that the dynamics of output do not has an obvious change when we switch off the collateral effect. Therefore, a higher volume of loans pushes output slightly since impatient households has higher propensity to consume. However, since they do not accumulate factor of production, the dynamics of output are not affected obviously.

Housing preference shock Figure 2.19 shows the impulse responses of variables to a one percent positive housing preference shock with persistence of $\rho_j = 0.01$. In the housing market, when both households have a higher preference on housing, the real house price is pushed up and we observe a transfer of housing from patient households to impatient households, indicat-

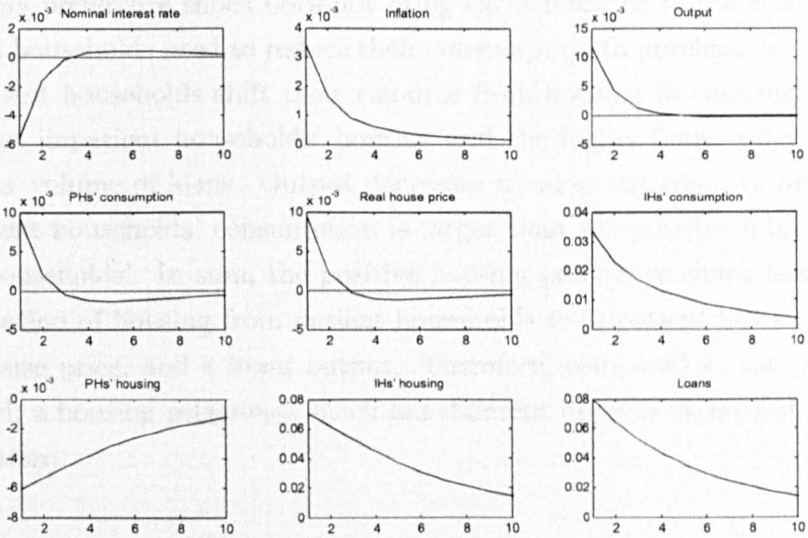

Figure 2.17: Impulse responses to a positive monetary policy shock from the simple model with impatient households.

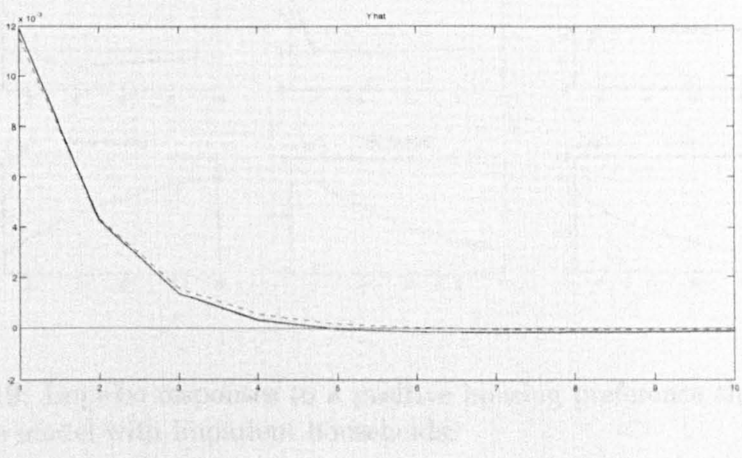

Figure 2.18: Impulse response output to a negative monetary policy shock from the simple model with impatient households. The solid line is from the baseline calibration $m^i = 0.85$, the dashed line is from the alternative calibration $m^i = 0$, and the dotted line is from $b_t^i = b^i$.

ing that impatient households' demand for housing is stronger. Moreover, as the housing preference shock does not bring extra resource to the economy, impatient households need to reduce their consumption to purchase housing, while patient households shift their resource from housing to consumption. The higher impatient households' housing and the higher house price lead to a larger volume of loans. Output decreases because the negative impact of impatient households' consumption is larger than the positive impact of patient households'. In sum, the positive housing preference shock leads to a re-allocation of housing from patient households to impatient households, higher house price, and a lower output. Therefore, compared to the previous model, a housing preference shock has different impacts on output (and consumption).

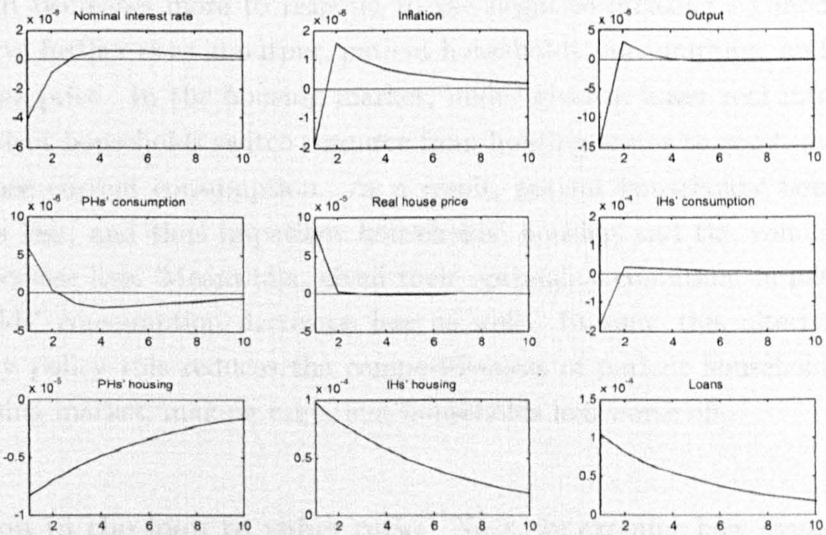

Figure 2.19: Impulse responses to a positive housing preference shock from the simple model with impatient households.

2.4.7 Sensitivity analysis

In this section, we examine how the impulse responses of variables are affected by changing the values of some parameters, given a positive goods sector technology shock. We first consider an alternative monetary policy rule, and then examine various values of the loan to value ratio.

Alternative monetary policy rule Here we discuss the impact on the dynamics if the central bank reacts to inflation only by setting the weight coefficients on output to $\phi_y = 0$. Figure 2.20 shows that the impulse responses of variables to a goods technology shock with persistence of $\rho_{Ac} = 0.01$ under two different monetary policy rules. Similar with the results from the simple model, when the nominal interest rate does not adjust for the higher output, it decreases more to respond to the negative inflation. Therefore, we observe further rises in output, patient households' consumption, and the real house price. In the housing market, under given a lower real interest rate, patient households switch resource from housing sector to goods sector to increase current consumption. As a result, patient households' housing increases less, and thus impatient households' housing and the volume of loans decrease less. Meanwhile, given their optimality condition, impatient households' consumption decreases less as well. In sum, this alternative monetary policy rule reduces the competitiveness of patient households in the housing market, making impatient households less worse off.

Variation in the loan to value ratio Next, we examine how dynamics are affected by changing the value of the loan to value ratio. Figure 2.21 and Figure 2.22 show the impulse responses of variables to a goods technology shock with persistence of $\rho_{Ac} = 0.01$ given a higher loan to value ratio, $m^i = 0.95$, and a lower one, $m^i = 0.75$, respectively. The main impact of a change in the loan to value ratio is on the housing market. When impatient households have less housing, a higher (lower) loan to value ratio strengthens (weakens) the decreases in the volume of loans. Meanwhile, it

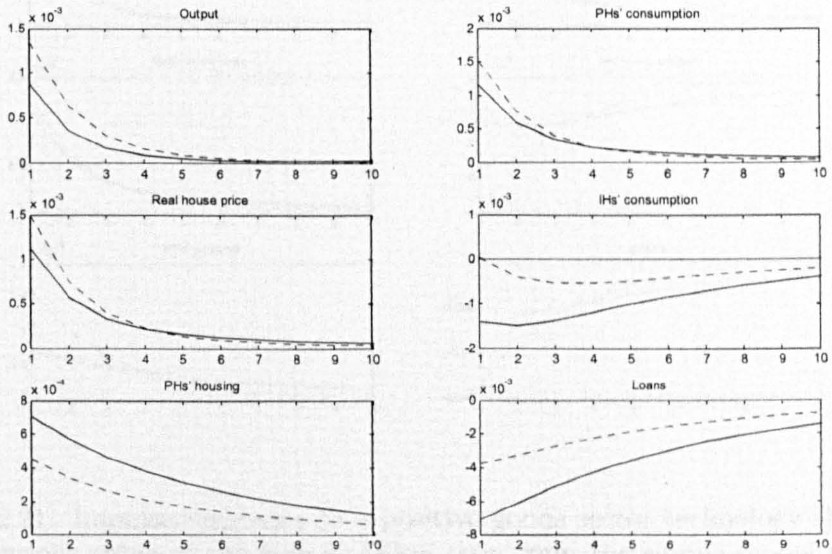

Figure 2.20: Impulse responses to a positive goods sector technology shock under alternative monetary policy from the simple model with impatient households. The solid line is from the baseline calibration $\phi_y = 0.5$ and the dashed line is from the alternative calibration $\phi_y = 0$.

increases (decreases) the transfer of housing from impatient households to patient households. In addition, similar with entrepreneurs, the decrease in the impatient households' consumption is strengthened (weakened), suggested by the optimality condition between consumption and housing.

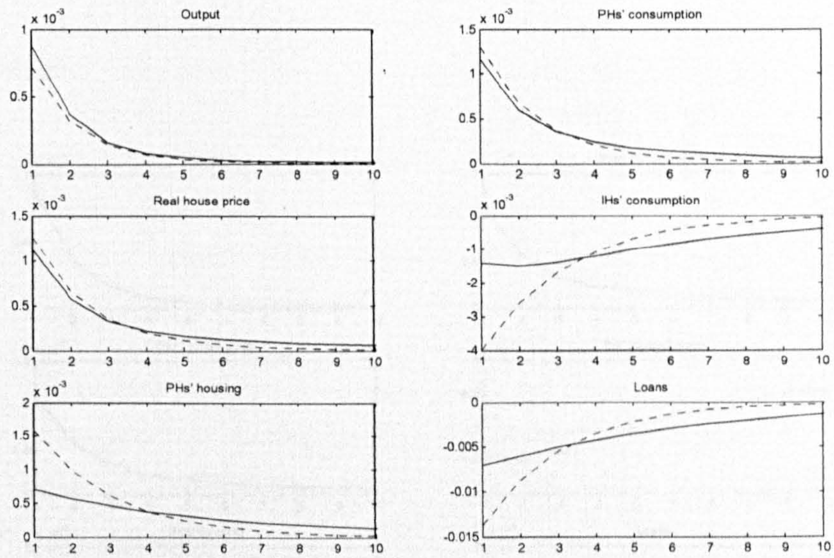

Figure 2.21: Impulse responses to a positive goods sector technology shock under various value of the loan to value ratio from the simple model with impatient households. The solid line is from the baseline calibration $m^i = 0.85$ and the dashed line is from the alternative calibration $m^i = 0.95$.

2.5 Conclusion

In this chapter, we discuss the dynamics of the real house price and the impact of credit market imperfections in a simple DSGE model with a fixed housing supply. We consider various exogenous shocks and examine how the dynamics are affected by changing the values of several parameters.

Firstly, our consider a simple DSGE model with sticky prices. Our impulse response analysis suggests that the real house price responds positively to a positive goods sector technology shock or a negative monetary policy

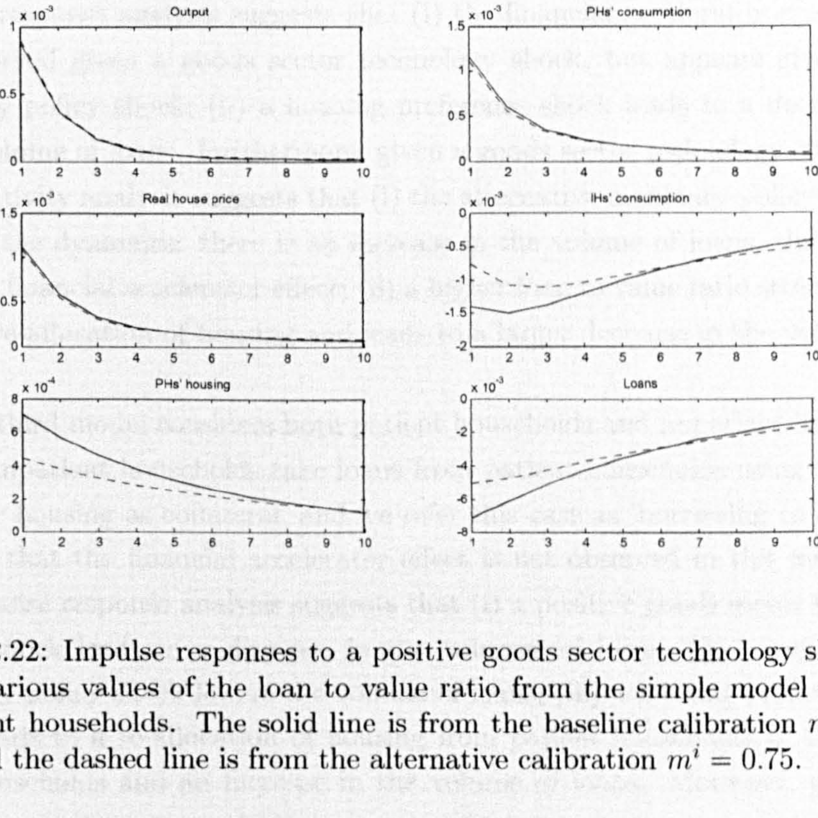

Figure 2.22: Impulse responses to a positive goods sector technology shock under various values of the loan to value ratio from the simple model with impatient households. The solid line is from the baseline calibration $m^i = 0.85$ and the dashed line is from the alternative calibration $m^i = 0.75$.

shock, and is the only variable that responds to a housing preference shock. Moreover, given a positive goods sector technology, our sensitivity analysis suggests that, while the feature of consumption habit mainly affects consumption and output, the dynamics of the real house price are affected largely by the feature of price indexation and the alternative monetary policy rule.

Our second model considers both patient households and entrepreneurs. Entrepreneurs take loans from patient households using their production housing as collateral, and we refer this case as 'borrowing to invest'. Our impulse response analysis suggests that (i) the financial accelerator effect is not observed given a goods sector technology shock, but appears given a monetary policy shock; (ii) a housing preference shock leads to a decrease in the volume of loans. Furthermore, given a goods sector technology shock, our sensitivity analysis suggests that (i) the alternative monetary policy rule reverses the dynamics: there is an increase in the volume of loans, thus we have the financial accelerator effect; (ii) a higher loan to value ratio strengthens the re-allocation of housing and leads to a larger decrease in the volume of loans.

Our third model considers both patient households and impatient households. Impatient households take loans from patient households using their domestic housing as collateral, and we refer this case as 'borrowing to live'. We find that the financial accelerator effect is not observed in this model. Our impulse response analysis suggests that (i) a positive goods sector technology shock leads to a decrease in the volumes of loans; (ii) a negative monetary policy shock lowers the volume of loans; (iii) a housing preference shock leads to a re-allocation of housing from patient households to impatient households and an increase in the volume of loans. Moreover, given a positive goods sector technology, our sensitivity analysis suggests that (i) the alternative monetary policy rule weakens the re-allocation of housing and leads to a smaller decrease in the volume of loans; (ii) a higher loan to value ratio strengthens the re-allocation of housing and leads to a larger decrease in the volume of loans.

2.A Appendix to Chapter 2

2.A.1 Lagrangian program for patient household

Patient households maximise utility subject to their budget constraint,

$$\max_{c_t,b_t,n_t,h_t} E_t \sum_{k=0}^{\infty} \left(\begin{array}{ll} \multicolumn{2}{l}{\beta^k \left(\Gamma_c \ln\left(c_{t+k} - \varepsilon_c c_{t+k-1}\right) + j_{t+k} \ln h_{t+k} - \frac{1}{1+\gamma_n}\left(n_{t+k}\right)^{1+\gamma_n}\right)} \\ +\lambda_{t+k}\beta^k & \left(\begin{array}{c} w_{t+k}n_{t+k} + q_{h,t+k}h_{t-1+k} + \frac{R_{t-1+k}}{\pi_{c,t+k}} b_{t-1+k} + f_{t+k} \\ -c_{t+k} - q_{h,t+k}h_{t+k} - b_{t+k} \end{array} \right) \end{array} \right)$$

2.A.2 Lagrangian program for entrepreneurs

Entrepreneurs maximise utility subject to their budget constraint and their borrowing constraint,

$$\max_{c_t^e,b_t^e,h_t^e} E_t \sum_{k=0}^{\infty} \left(\begin{array}{c} \left(\beta^e\right)^k \left(\Gamma_c^i \ln\left(c_{t+k}^e - \varepsilon_c^e c_{t+k-1}^e\right)\right) \\ +\lambda_{t+k}^e \left(\beta^e\right)^k \left(\begin{array}{c} q_{z,t+k}h_{t-1+k}^e + q_{h,t+k}h_{t-1+k}^e + b_{t+k}^e \\ -c_{t+k}^e - q_{h,t+k}h_{t+k}^e - \frac{R_{t-1+k}}{\pi_{c,t+k}} b_{t-1+k}^e \end{array} \right) \\ +\lambda_{b,t+k}^e \left(\beta^e\right)^k \left(m\left(\frac{q_{h,t+1+k}h_{t+k}^e \pi_{c,t+1+k}}{R_{t+k}}\right) - b_{t+k}^e\right) \end{array} \right)$$

2.A.3 Lagrangian program for impatient households

Impatient households maximise utility subject to their budget constraint and their borrowing constraint,

$$\max_{c_t^i,b_t^i,h_t^i,n_t^i} E_t \sum_{k=0}^{\infty} \left(\begin{array}{c} \left(\beta^i\right)^k \left(\Gamma_c^i \ln\left(c_{t+k}^i - \varepsilon_c^i c_{t+k-1}^i\right) + j_t^i \ln h_{t+k}^i - \frac{1}{1+\gamma_n^i}\left(n_{t+k}^i\right)^{1+\gamma_n^i}\right) \\ +\lambda_{t+k}^i \left(\beta^i\right)^k \left(\begin{array}{c} w_{t+k}^i n_{t+k}^i + q_{h,t+k}h_{t-1+k}^i + b_{t+k}^i \\ -c_{t+k}^i - q_{h,t+k}h_{t+k}^i - \frac{R_{t-1+k}}{\pi_{c,t+k}} b_{t-1+k}^i \end{array} \right) \\ +\lambda_{b,t+k}^i \left(\beta^i\right)^k \left(m\left(\frac{q_{h,t+1+k}h_{t+k}^i \pi_{c,t+1+k}}{R_{t+k}}\right) - b_{t+k}^i\right) \end{array} \right)$$

2.A.4 The stochastic discount factor

From the patient households' Euler equation, the inverse of real interest rate at time t is

$$E_t\left(\frac{\pi_{c,t+1}}{R_t}\right) = E_t\left(\beta\frac{u_{c,t+1}}{u_{c,t}}\right)$$

which is used to discount profit at time $t+1$. Furthermore, the inverse of real interest rate at time $t+1$ is

$$E_{t+1}\left(\frac{\pi_{c,t+2}}{R_{t+1}}\right) = E_{t+1}\left(\beta\frac{u_{c,t+2}}{u_{c,t+1}}\right)$$

Therefore, the product of the inverse of real interest rate from period t to $t+k$ is

$$\begin{aligned}\Lambda_{t,t+k} &= E_t\left(\frac{\pi_{c,t+1}}{R_t}\frac{\pi_{c,t+2}}{R_{t+1}}...\frac{\pi_{c,t+k}}{R_{t+k-1}}\right) = E_t\prod_{i=0}^{i=k}\frac{\pi_{c,t+i}}{R_{t+i-1}} \\ &= E_t\left(\beta\frac{u_{c,t+1}}{u_{c,t}}\beta\frac{u_{c,t+2}}{u_{c,t+1}}...\beta\frac{u_{c,t+k}}{u_{c,t+k-1}}\right) = E_t\left(\beta^k\frac{u_{c,t+k}}{u_{c,t}}\right)\end{aligned}$$

This is the stochastic discount factor. While we use β^k to discount utility, we use $\Lambda_{t,t+k}$ to discount profit in terms of utility. Note that $\Lambda_{t,t} = 1$.

2.A.5 The aggregate nominal price level

We know that a fraction θ of retailers are not able to reset price in period t. But we know that, among these retailers, we know that a fraction θ of retailers reset their prices in period $t-1$, i.e., a fraction $\theta(1-\theta)$ of retailers use the optimal price from the previous period. Following the same logic, we can rewrite the aggregate price in terms of the optimal price in period $t-k$, $k = 0, 1, 2, 3...$

$$\begin{aligned}P_{c,t} &= \left((1-\theta)\left(P^*_{c,t}\right)^{1-\varepsilon} + \theta(1-\theta)\left(P^*_{c,t-1}\right)^{1-\varepsilon} + \theta^2(1-\theta)\left(P^*_{c,t-2}\right)^{1-\varepsilon} + ...\right)^{\frac{1}{1-\varepsilon}} \\ \left(P_{c,t}\right)^{1-\varepsilon} &= (1-\theta)\left(P^*_{c,t}\right)^{1-\varepsilon} + \theta(1-\theta)\left(P^*_{c,t-1}\right)^{1-\varepsilon} + \theta^2(1-\theta)\left(P^*_{c,t-2}\right)^{1-\varepsilon} + ...\end{aligned}$$

Backward one period, gives

$$\left(P_{c,t-1}\right)^{1-\varepsilon} = (1-\theta)\left(P_{c,t}^{*}\right)^{1-\varepsilon} + \theta(1-\theta)\left(P_{c,t-1}^{*}\right)^{1-\varepsilon} + \theta^{2}(1-\theta)\left(P_{c,t-2}^{*}\right)^{1-\varepsilon} + \ldots$$

Multiply both sides by θ, gives

$$\theta\left(P_{c,t-1}\right)^{1-\varepsilon} = \theta(1-\theta)\left(P_{c,t}^{*}\right)^{1-\varepsilon} + \theta^{2}(1-\theta)\left(P_{c,t-1}^{*}\right)^{1-\varepsilon} + \theta^{3}(1-\theta)\left(P_{c,t-2}^{*}\right)^{1-\varepsilon} + \ldots$$

Then use $\left(P_{c,t}\right)^{1-\varepsilon}$ to replace all terms before $t-1$ in the previous equation, gives

$$\begin{aligned}
\left(P_{c,t}\right)^{1-\varepsilon} &= (1-\theta)\left(P_{c,t}^{*}\right)^{1-\varepsilon} + \theta\left(P_{c,t-1}\right)^{1-\varepsilon} \\
P_{c,t} &= \left[(1-\theta)\left(P_{c,t}^{*}\right)^{1-\varepsilon} + \theta\left(P_{c,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

2.A.6 The steady state values from the simple model with entrepreneurs

In order to solve the model, we need following steady state ratios: $\frac{q_z h^e}{Y}$, $\frac{q_h h^e}{Y}$, $\frac{q_z}{q_h}$, $\frac{b^e}{Y}$, $\frac{c^e}{Y}$, $\frac{c}{Y}$, $\frac{q_h h}{Y}$, $\frac{h^e}{h}$.

From the patient households' Euler equation, the steady state value of nominal interest rate is the inverse of the patient households' discount factor,

$$R = \frac{1}{\beta}$$

From the entrepreneurs' Euler equation, we have

$$\lambda_b^e = u_c^e\left(1 - \beta^e R\right)$$

This equation suggests that the borrowing constraint is binding since $\beta^e < \beta \Leftrightarrow \lambda_b^e > 0$.

From the intermediate goods production demand for housing, we have the steady state value of $\frac{q_z h^e}{Y}$,

$$\frac{q_z h^e}{Y} = \frac{\mu_h}{Z}$$

From the entrepreneurs' demand for housing, we have the steady state value of $\frac{q_h h^e}{Y}$ and $\frac{q_z}{q_h}$,

$$\frac{q_h h^e}{Y} = \beta^e \frac{q_z h^e}{Y} \frac{1}{1 - (\beta^e (1-m) + m\beta)}$$

and

$$\frac{q_z}{q_h} = \frac{1 - (\beta^e (1-m) + m\beta)}{\beta^e}$$

From the entrepreneurs' borrowing constraint, we have the steady state value of $\frac{b^e}{q_h h^e}$ and $\frac{b^e}{Y}$,

$$\frac{b^e}{q_h h^e} = \frac{m}{R}$$

and

$$\frac{b^e}{Y} = \frac{b^e}{q_h h^e} \frac{q_h h^e}{Y}$$

From the entrepreneurs' budget constraint, we have the steady state value of $\frac{c^e}{Y}$,

$$\frac{c^e}{Y} = \frac{q_z h^e}{Y} + \left(1 - \frac{1}{\beta}\right) \frac{b^e}{Y}$$

From the goods market clearing condition, we have the steady state value of $\frac{c}{Y}$,

$$\frac{c}{Y} = 1 - \frac{c^e}{Y}$$

From the patient households' demand for housing, we have the steady state value of $\frac{q_h h}{Y}$,

$$\frac{q_h h}{Y} = \frac{j}{1-\beta} \frac{c}{Y}$$

Combining the steady state value of $\frac{q_h h^e}{Y}$ and $\frac{q_h h}{Y}$, we have the steady state value of $\frac{h^e}{h}$,

$$\frac{h^e}{h} = \frac{q_h h^e}{Y} / \frac{q_h h}{Y}$$

2.A.7 The steady state values from the simple model with impatient households

In order to solve the model, we need following steady state ratios: $\frac{q_h h}{c^i}$, $\frac{q_h h^i}{c^i}$, $\frac{b^i}{c^i}$, $\frac{w^i n^i}{c^i}$, $\frac{wn}{c^i}$, $\frac{Y}{c^i}$, $\frac{c}{c^i}$, $\frac{h}{h^i}$.

From the patient households' Euler equation, the steady state value of nominal interest rate is the inverse of the patient households' discount factor,

$$R = \frac{1}{\beta}$$

From the patient households' demand for housing, we have the steady state value of $\frac{q_h h}{c}$,

$$\frac{q_h h}{c} = \frac{j}{1-\beta}$$

From the impatient households' Euler equation, we have

$$\lambda_b^i = \left(1 - \beta^i R\right) u_c^i$$

This equation suggests that the impatient households' borrowing constraint is binding since $\lambda_b^i > 0 \Leftrightarrow \beta^i < \beta$.

From the impatient households' demand for housing, we have steady state value of $\frac{q_h h^i}{c^i}$,

$$\frac{q_h h^i}{c^i} = \frac{j^i}{\left(1-\beta^i\right) - \left(1 - \beta^i R\right)\frac{m}{R}}$$

From the impatient households' borrowing constraint, we have the steady state value of $\frac{b^i}{q_h h^i}$,

$$\frac{b^i}{q_h h^i} = m\frac{1}{R}$$

Combining the steady state value of $\frac{b^i}{q_h h^i}$ and $\frac{q_h h^i}{c^i}$, we have the steady state value of $\frac{b^i}{c^i}$,

$$\frac{b^i}{c^i} = \frac{b^i}{q_h h^i}\frac{q_h h^i}{c^i}$$

From the impatient household budget constraint, we have the steady state value of $\frac{w^i n^i}{c^i}$,

$$\frac{w^i n^i}{c^i} = 1 - (1 - R)\frac{b^i}{c^i}$$

From the intermediate goods firms' demand for labour from impatient households,we have the steady state value of $\frac{w^i n^i}{Y}$,

$$\frac{w^i n^i}{Y} = \frac{(1-\alpha)\mu_n}{Z}$$

Combining the steady state value of $\frac{w^i n^i}{c^i}$ and $\frac{w^i n^i}{Y}$, we have the steady state value of $\frac{Y}{c^i}$,

$$\frac{Y}{c^i} = \frac{w^i n^i}{c^i} / \frac{w^i n^i}{Y}$$

From the goods marker clearing condition, we have the steady state value of $\frac{c}{c^i}$,

$$\frac{c}{c^i} = \frac{Y}{c^i} - 1$$

Combining the steady state value of $\frac{q_h h^i}{c^i}$, $\frac{q_h h}{c}$and $\frac{c}{c^i}$, we have the steady state value of $\frac{h^i}{h}$,

$$\frac{h^i}{h} = \frac{\frac{q_h h^i}{c^i}}{\frac{q_h h}{c}\frac{c}{c^i}}$$

2.A.8 The difference between sticky prices and flexible prices

In models without sticky prices, the retailers' real profit maximisation problem is

$$\max_{P^*_{c,t}(z)} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}\left[\frac{P^*_{c,t+k}(z)}{P_{c,t+k}}Y_{t+k}(z) - \frac{P_{wc,t+k}}{P_{c,t+k}}Y_{t+k}(z)\right]$$

where $\Lambda_{t,t+k} = E_t\left(\beta^k \frac{u_{c,t+k}}{u_{c,t}}\right)$ is the stochastic discount factor, which is used to discount profit in terms of consumption, $P_{wc,t}$ is the nominal price of intermediate goods, and $\frac{P_{wc,t}}{P_{c,t}}$ is the real price of intermediate goods. Every retailer is able to set a price $P^*_{c,t}(z)$ to maximise the expected profits in this period, by assuming the probability of fixed price to $\theta = 0$.

Substituting the individual retailer's output, $Y_{t+k}(z)$, by the individual demand curve in the profit maximisation problem, we have

$$\max_{P_{c,t}^{*}(z)} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[\begin{array}{c} \frac{P_{c,t+k}^{*}(z)}{P_{c,t+k}} \left(\frac{P_{c,t+k}^{*}(z)}{P_{c,t+k}} \right)^{-\varepsilon} Y_{t+k} \\ -\frac{P_{wc,t+k}}{P_{c,t+k}} \left(\frac{P_{c,t+k}^{*}(z)}{P_{c,t+k}} \right)^{-\varepsilon} Y_{t+k} \end{array} \right]$$

The first order condition derived from the retailers' real profit maximisation problem is the equation of the real optimal price, and it is expressed as

$$Q_{c,t} = \frac{\varepsilon}{\varepsilon - 1} E_t \left[\frac{\Lambda_{t,t} Y_t \left(\frac{P_{c,t}}{P_{c,t}} \right)^{-\varepsilon} \frac{P_{wc,t}}{P_{c,t}}}{\Lambda_{t,t} Y_t \left(\frac{P_{c,t}}{P_{c,t}} \right)^{1-\varepsilon}} \right] \quad (2.46)$$

$$Q_{c,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{P_{wc,t}}{P_{c,t}} \quad (2.47)$$

in which we define the real optimal price as $Q_{c,t} = \frac{P_{c,t}^{*}}{P_{c,t}}$. This equation implies that the optimal price set in period t depends on the real price of intermediate goods, $P_{c,t}^{*} = \frac{\varepsilon}{\varepsilon - 1} P_{wc,t}$.[41]

Then nominal price level can also be written as

$$P_{c,t} = \left[\int_0^1 P_{c,t}^{*}(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} = P_{c,t}^{*} \quad (2.48)$$

thus markup is fixed,

$$Z_t = \frac{P_{c,t}}{P_{wc,t}} = \frac{\varepsilon}{\varepsilon - 1}$$

and also the the real optimal price,

$$Q_{c,t} = \frac{P_{c,t}^{*}}{P_{c,t}} = 1$$

In this case, we are not able to derive the New Keynesian Curve that links the nominal variables to real variables.

[41] If the monopolistic competitive firms are intermediate goods firms, the optimal real price will depends on the real marginal costs.

Note that we still have a positive markup in the model since we have monopolistic competition. In contrast, under perfect competition, the elasticity of substitution between differentiated varieties is infinity, $\varepsilon = \infty$, thus the markup is one, $Z_t = 1$.

3 An Examination of the Direct Effect and the Feedback Effect from the Variable Housing Supply

3.1 Introduction

The housing sector is important to the economy, but it has been ignored in business cycle analysis for a long time. A recent paper that treats housing as alternative market goods is written by Davis and Heathcote (2005). They consider a multi-sector model featuring housing production. In their representative agents model, housing producers combine housing capital and land to produce new housing, and households gain utility from both consumption and housing. Their model can explain the dynamics of housing capital investment well but not the dynamics of the house price.

When researchers have raised concerns over credit market imperfections, an important reason for considering housing as alternative goods is that it can be used as collateral of loan in the credit market. Iacoviello and Neri (2010) estimate a two-sector model featuring nominal rigidities and credit market frictions and they find that the main reason for the persistent increase in house price is slow technological progress in new housing production. In their heterogeneous agents model, impatient households take loans from patient households but they need to use housing as collateral because of credit market imperfections, thus their borrowing constraints are related to the value of their housing assets.

Following this work, various versions of Iacoviello and Neri (2010) model have been widely used, such as Notarpietro (2007), Paries and Notarpietro (2008), Kannan, Rabanal and Scott (2009), Christensen et al. (2009), Sellin and Walentin (2010), and, in particular, the settings of the housing market are similar. We refer this setting as 'the standard setting of the housing market', which assumes that every household trades housing in a given period. We consider this standard setting in our benchmark model, which is a simple DSGE model with the feature of sticky prices and housing production. After introducing a new housing production, we are able to discuss the supply side effect on the economy, i.e., the impact of a change in housing supply, including both (i) the direct effect and (ii) the feedback effect. The direct effect is the impact of a housing sector technology shock, and the feedback effect is the impact of a change in new housing production, caused by other shocks, such as a goods sector technology shock or a monetary policy shock.

Our impulse response analysis suggests that the magnitudes of both direct effect and feedback effect from new housing production is negligible to the economy. We argue that the standard setting of the housing market is the reason for our results. This standard setting assumes that every household trades housing in a given period, thus all housing is traded. As a result, the weight of new housing in the housing trading market is equal to the depreciation rate of housing, which is small. Therefore, the magnitude of the supply side effect is small.

Next, we examine the U.S. housing sector using data from the U.S. Census Bureau for the period of 1968$Q1$ - 2009$Q4$. We find that several steady state ratios from our benchmark model cannot meet their empirical targets. This inconsistency between the model and the empirical evidence motivates us to construct a new setting for the housing market, which is our first contribution in this chapter. After constructing the new setting of the housing market, those steady state ratios from the model are consistent with their empirical targets. Moreover, from impulse response analysis, we find that the response of the real house price to a housing sector technology shock is 60 times larger than that under the standard setting, implying that the standard setting largely underestimates the direct effect of new housing production on the real house price. This result may challenge one of conclusions from Iacoviello and Neri (2010), i.e., the slow growth of housing sector technology is the main cause of the persistent increase in real house price. Our chapter suggests that, when the impact of housing sector technology shock on the house price is properly estimated, the slow growth of housing sector technology becomes less important to the persistent increase in real house prices. Next, given a goods technology shock and a monetary policy shock, the feedback effect of new housing production is also 60 times larger than that under the standard setting.

Our second contribution in this chapter is to introduce the feature of time to build to new housing production, while it is commonly assumed that a housing project starts and completes within one period in the literature. Our empirical analysis suggests that a housing project usually takes several quarters to complete, supporting the introduction of the time to build feature.

The feature of time to build has been introduced to goods capital. Kydland and Prescott (1982) assume that an investment project takes four quarters to complete, and they find that this feature is crucial to obtain a persistent output movement. Gomme, Kydland and Rupert (2001) consider this feature for the production of market capital in a two-sector model and argue that this feature is essential for their model to match the cyclical properties of market and home investment. Tsoukalas (2011) considers a neoclassical investment-q model with features of time to plan and time to build for the installation of capital and show that cash flow may be important even capital markets are perfect and future investment opportunities are properly accounted for. His results suggest that investment cash flow sensitivities are not the right framework to evaluate the credit market imperfections.

For the first time, the feature of time to build is introduced to new housing production in this chapter. When we consider this feature in the benchmark model, i.e., with the standard setting of the housing market, it does not have any obvious impact on the economy, because the small weight of new housing in the housing trading market implies that a change in new housing production has a negligible impact on the economy.

In contrast, after constructing the new setting of the housing market, the feature of time to build has an obvious impact on the real house price. This is because that the weight of new housing in the housing trading market is more reasonable, thus a change in new housing production will have an obvious impact on the economy. One important implication of the feature of time to build is that, given a goods sector technology shock or a monetary policy shock, the feedback effect of new housing production leads to an overshooting behaviour for the real house price since (i) the response of new housing production has an opposite impact on the real house price against the shock, and (ii) the feature of time to build delays this effect while the demand side effect is diminishing. For example, in response to a positive goods sector technology shock, the real house price responds positively, but it falls shapely and becomes negative when new housing production begins to respond positively, before returning towards its steady state.

3.2 The benchmark model

Our benchmark model is a simple DSGE model with the feature of sticky prices and a variable housing supply. When we assume a fixed housing supply, we can only observe the demand side effect on the real house price. After introducing a new housing production sector, we can discuss the supply side effect on the economy, i.e., the impact of a change in the housing supply. The supply side effect includes (i) the direct effect, which is the impact of a housing sector technology shock on the economy, and (ii) the feedback effect, which is the impact of a change in new housing production, caused by other shocks, such as a goods sector technology shock or a monetary policy shock.[12]

3.2.1 Households

Households are infinitely lived and of measure one. They consume final goods, and demand domestic housing and supply labour. They maximise their lifetime utility subject to their budget constraint. We assume that they own the profitable retail goods productions.[13]

Similar with the Chapter 2, the households' lifetime utility function is

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\Gamma_c \ln \left(c_{t+k} - \varepsilon_c c_{t+k-1}\right) + j_{t+k} \ln h_{t+k} - \frac{1}{1+\gamma_n} \left(n_{t+k}\right)^{1+\gamma_n} \right)$$

where E_t is the expectation operator, β is the households' discount factor, c_t is households' consumption, ε_c measures the degree of consumption habit,

[12]The feedback effect shows that the response of real house price to an exogenous shock is weakened or strengthened by the response of new housing production.

[13]Households are identical to patient households in the simple DSGE model in the Chapter 2.

Γ_c is a scaling factor, h_t is domestic housing, n_t is the supply of households' labour, $\frac{1}{\gamma_n}$ is the Fisher elasticity of labour supply, and j is the weight on domestic housing.

Given the lifetime utility function, the households' marginal utility of consumption is

$$u_{c,t} = \Gamma_c \frac{1}{c_t - \varepsilon_c c_{t-1}} - \beta \varepsilon_c \Gamma_c \frac{1}{E_t c_{t+1} - \varepsilon_c c_t} \tag{3.1}$$

which expresses the marginal utility of consumption, $u_{c,t}$, in terms of lagged, current, and future consumption.

The households' real budget constraint shows that the real total expense (LHS) should be no more than the real total income (RHS), and is expressed as

$$\begin{aligned} & c_t + K_{h,t} + q_{h,t} h_t + b_t \\ \leq\ & (R_{kh,t} + 1 - \delta_{kh}) K_{h,t-1} + w_t n_t + q_{h,t} (1 - \delta_h) h_{t-1} + \frac{R_{t-1}}{\pi_{c,t}} b_{t-1} + f_t \end{aligned}$$

where $q_{h,t}$ is the real house price, b_t is the volume of bonds purchased in period t, w_t is the real wage rate, R_{t-1} is the (gross) nominal interest rate on the bond hold in period $t-1$, $\pi_{c,t}$ is the (gross) inflation rate, and f_t is the real profit from retail goods firms. For the new housing production related variables, $K_{h,t}$ is housing capital, $R_{kh,t}$ is the real rental price of housing capital, δ_{kh} and δ_h are the depreciation rates of housing capital and housing stock respectively. The real prices of final goods, bonds, and housing capital are normalised to one.

We obtain four first order conditions from the households' lifetime utility maximisation problem.[14] The first three are discussed already in the Chapter 2, and they are

$$u_{c,t} = \beta E_t \left(\frac{R_t}{\pi_{c,t+1}} u_{c,t+1} \right) \tag{3.2}$$

$$n_t^{\gamma_n} = w_t u_{c,t} \tag{3.3}$$

[14] The households' utility maximisation problem is shown in Appendix.

$$q_{h,t}u_{c,t} = \frac{j}{h_t} + \beta E_t\left(q_{h,t+1}u_{c,t+1}\right) \tag{3.4}$$

Equation 3.2 is the intertemporal optimality condition that governs the optimal allocation of consumption over time. Equation 3.3 is the intratemporal optimality condition that indicates how households make decisions about consumption and labour supply in period t. Equation 3.4 is the intratemporal optimality condition that describes an optimal allocation of resource between consumption and domestic housing in period t.

The fourth first order condition is the equation that governs households' demand for housing capital, which is expressed as

$$u_{c,t} = \beta u_{c,t+1}\left(R_{kh,t+1} + 1 - \delta_{kh}\right) \tag{3.5}$$

which implies that the real price of housing capital in terms of the marginal utility of consumption at t is equal to the sum of the expected real rental price of housing capital and the expected real price of housing capital (for reselling undepreciated housing capital) in terms of the discounted marginal utility of consumption at $t+1$. This first order condition is an intertemporal optimality condition that describes an optimal allocation of resource between consumption and housing capital.

3.2.2 Goods production sector

Similar with the Chapter 2, in the goods production sector, we have three players: (i) final goods producers buy retail goods from individual retail goods producers, and compose them into final goods, which are ready for consumption; (ii) retail goods producers (or retailers) buy intermediate goods from intermediate goods producers, and differentiate the goods at no cost into retail goods; (iii) intermediate goods producers combine goods sector technology and labour from patient households to produce intermediate goods, which are then sold to retail goods producers.

Final goods firms, retail goods firms, and intermediate goods firms All of firms were described in the Chapter 2 already, here we only list equations that are relevant to the equilibrium of this model.[15] The first order condition derived from the retailers' real profit maximisation problem is the equation of the real optimal price, and it is expressed as

$$Q_{c,t} = \frac{\varepsilon}{\varepsilon - 1} E_t \left[\frac{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}} \right)^{-\varepsilon} \frac{P_{wc,t+k}}{P_{c,t+k}}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}} \right)^{1-\varepsilon} \left(\frac{P_{c,t+k-1}}{P_{c,t-1}} \right)^{\iota_\pi}} \right] \tag{3.6}$$

in which we define the real optimal price as $Q_{c,t} = \frac{P_{c,t}^*}{P_{c,t}}$.

Given the features of sticky prices and price indexation, the nominal price level can also be written as

$$P_{c,t} = \left[\theta \left(\pi_{c,t-1}^{\iota_\pi} P_{c,t-1} \right)^{1-\varepsilon} + (1-\theta) P_{c,t}^{*\,1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{3.7}$$

The real profit from retail goods firms is

$$f_t = Y_t - \frac{P_{wc,t}}{P_{c,t}} Y_t = \left(1 - \frac{1}{Z_t} \right) Y_t \tag{3.8}$$

which implies that the real profit is the difference between the real price of final goods, which is normalised to one, and the real price of intermediate goods, $\frac{P_{wc,t}}{P_{c,t}}$.

The intermediate goods production function is

$$Y_t = A_{c,t} \left(n_{c,t} \right)^{\mu_n} \tag{3.9}$$

where $n_{c,t}$ is households' labour, μ_n is the labour share of output. The goods sector technology, $A_{c,t}$, follows the stationary process

$$A_{c,t} = A_c^{1-\rho_{Ac}} A_{c,t-1}^{\rho_{Ac}} e^{\varepsilon_{Ac,t}}, \quad \varepsilon_{Ac,t} \sim N\left(0, \sigma_{Ac}^2\right) \tag{3.10}$$

The first order condition derived from the intermediate goods firms' real

[15] We summarise all of equations that are needed to solve the model in the section of 'Equilibrium'.

profit maximisation describes the intermediate goods producers' demand for households' labour, and it is expressed as

$$w_t = \mu_n \frac{Y_t}{Z_t n_{c,t}} \tag{3.11}$$

which implies that the real wage of labour, i.e., the marginal cost of labour, is equal to the marginal product of labour.

3.2.3 Housing production sector

Housing firms Housing producers combine exogenous housing sector technology and housing capital from households to produce new housing, which are then sold to households. We assume that housing firms are perfectly competitive, thus they make zero profit. In addition, we assume flexible prices in this sector.[46]

The housing production function is

$$IH_t = A_{h,t} K_{h,t-1}^{\mu_{kh}} \tag{3.12}$$

where IH_t is new housing, $K_{h,t-1}$ is lagged housing capital, μ_h is the housing capital share of housing production.[47] The housing sector technology, $A_{h,t}$, follows the stationary process

$$A_{h,t} = A_h^{1-\rho_{Ah}} A_{h,t-1}^{\rho_{Ah}} e^{\varepsilon_{Ah,t}}, \qquad \varepsilon_{Ah,t} \sim N\left(0, \sigma_{Ah}^2\right) \tag{3.13}$$

[46]In the perfect competitive housing market, firms produce at the point where the marginal cost equals to the average total cost, $MC = ATC$. The short run supply curve (the marginal cost curve above the average total cost curve) is upward-sloping, thus a higher price (caused by a higher demand) leads to a higher supply. Due to the demand shock is temporary, the price will fall back, and the supply will also fall back to the original level.

[47]For simplicity, we only consider housing capital as the production factor, although we may need other factors such as labour, land, intermediate input.

The housing producers' real profit maximisation problem is

$$\max_{K_{h,t-1}} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(q_{h,t+k} IH_{t+k} - R_{kh,t+k} K_{h,t+k-1} \right)$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor, $q_{h,t} IH_t$ is the real total revenue, and $R_{kh,t} K_{h,t-1}$ is the real total cost.

The first order condition derived from this real profit maximisation describes the housing producers' demand for housing capital, and it is expressed as

$$R_{kh,t} = q_{h,t} \mu_{kh} \frac{IH_t}{K_{h,t-1}} \tag{3.14}$$

which implies that the real rental price of housing capital, i.e., the marginal cost of housing capital, is equal to the marginal product of housing capital.

3.2.4 Monetary authority

Similar with the Chapter 2, the monetary authority uses the nominal interest rate as a policy instrument to affect the real economy. The monetary policy rule, which reacts to inflation and output, is

$$R_t = \left(R_{t-1} \right)^{\phi_r} \pi_{c,t}^{(1-\phi_r)\phi_\pi} \left(\frac{Y_t}{Y} \right)^{(1-\phi_r)\phi_y} e^{u_{R,t}} \tag{3.15}$$

where R_{t-1} is the lagged nominal interest rate, $\pi_{c,t}$ is gross inflation rate, Y_t is actual output, and Y is the steady state value of output, ϕ_r, ϕ_π, ϕ_y are weights coefficients. The monetary policy shock, $u_{R,t}$, follows the stationary process

$$u_{R,t} = u_R^{1-\rho_{uR}} u_{R,t-1}^{\rho_{uR}} e^{\varepsilon_{uR,t}}, \qquad \varepsilon_{uR,t} \sim N\left(0, \sigma_{uR}^2\right) \tag{3.16}$$

The Fisher equation, which governs the relation between the real interest rate and the nominal interest rate, is

$$r_t = \frac{R_t}{E_t \pi_{c,t+1}} \tag{3.17}$$

which implies that the (gross) real interest rate, r_t, is equal to the nominal interest rate, R_t, adjusted by expected inflation rate, $E_t\pi_{c,t+1}$.

3.2.5 Market clearing conditions

Some clearing conditions were discussed already in the Chapter 2, but here we list them to facilitate the summary of the model. The bonds market clearing condition is

$$b_t = 0 \tag{3.18}$$

which implies that (i) the aggregate saving is zero, and (ii) there is no borrowing between agents since we consider a model with representative agents.

The economy-wide resource constraint or the goods market clearing condition is

$$Y_t = c_t + IK_{h,t} \tag{3.19}$$

which implies that the total output from goods sector is divided into consumption goods, which is consumed by households, and investment in the housing capital.

Similar with the Chapter 2, the labour market clearing condition is

$$n_t = n_{c,t} \tag{3.20}$$

which implies that the supply of households' labour is equal to the intermediate goods producers' demand for households' labour.

The housing market clearing condition now becomes

$$H_t = h_t \tag{3.21}$$

which implies that the total supply of housing, which is not fixed anymore, is equal to the total demand for housing.

Housing capital that is required in new housing production is a durable asset and depreciates at a rate of δ_{kh}. The equation that describes its accumulation process is

$$K_{h,t} = (1 - \delta_{kh}) K_{h,t-1} + IK_{h,t} \tag{3.22}$$

which implies that housing capital in the current period, $K_{h,t}$, is the sum of undepreciated housing capital from the previous period, $(1 - \delta_{kh}) K_{h,t-1}$, and new investment in the current period, $IK_{h,t}$.

Similarly, housing is also a durable assets and depreciates at a rate of δ_h. The housing stock accumulation process is

$$H_t = (1 - \delta_h) H_{t-1} + IH_t \tag{3.23}$$

which implies that total housing stock in the current period, H_t, is the sum of undepreciated housing stock from the previous period, $(1 - \delta_h) H_{t-1}$, and new housing in the current period, IH_t.

3.2.6 Equilibrium

An equilibrium is an allocation of prices $(\pi_{c,t},\ R_t,\ Q_{c,t},\ q_{h,t},\ w_t,\ Z_t,\ r_t,\ R_{kh,t})$, quantities $(c_t,\ u_{c,t},\ h_t,\ Y_t,\ n_t,\ n_{c,t},\ f_t,\ b_t,\ IH_t,\ H_t,\ K_{h,t},\ IK_{h,t})$, and exogenous stochastic process $\{A_{c,t},\ A_{h,t},\ u_{R,t}\}_{t=0}^{\infty}$ satisfying equations (3.1) – (3.23) given the initial conditions for $\pi_{c,t-1}$, R_{t-1}, c_{t-1}, H_{t-1}, $K_{h,t-1}$.

3.2.7 Calibration

Most of parameters are calibrated in a way that is consistent with Iacoviello and Neri (2010). Some of them were discussed in the Chapter 2, but here we discuss them again for convenience. For the household discount factor, we set $\beta = 0.9925 = 1.03^{-0.25}$, implying a steady state annual real interest rate of 3 percent. The households' labour schedule is assumed to be flat, $\gamma_n = 0.01$. Besides, we set the weight on housing to $j = 0.4$, as this value, together with the depreciation rates, allows our steady state ratios hit the sample average in the next section. In addition, we switch off the feature of consumption habit by setting $\varepsilon_c = 0$.

Households' preference		
The households' discount factor	β	0.9925
The inverse of the elasticity of labour supply	γ_n	0.01
The weight on housing	j	0.4
The degree of consumption habit	ε_c	0

Similar with the Chapter 2, the share of labour in the goods production function is set to $\mu_n = 0.65$, implying that the steady state share of labour income is 65%. For the retail goods sector, we assume a steady state markup of 15% in goods sector by setting $Z = 1.15$. For the degree of prices stickiness, we assume that 25% of retailers are able to re-optimise their prices in a given period by setting $\theta = 0.75$, implying that price-setters can re-optimise their prices once every $\frac{1}{1-\theta} = 4$ periods. In addition, we also switch off the feature of price indexation by setting $\iota_\pi = 0$.

Intermediate goods firms		
Labour share	μ_n	0.65
Retail goods firms		
The steady state gross markup	Z	1.15
The probability of fixed prices	θ	0.75
The degree of price indexation	ι_π	0

In the housing production function, we set the share of housing capital in housing production to $\mu_{kh} = 0.1$, implying that the steady state share of housing capital income is 10%. The depreciation rate of housing capital is set to $\delta_{kh} = 0.03$. These values are consistent with Iacoviello and Neri (2010). The depreciation rate of housing stock, however, is set to $\delta_h = 0.002$, which is supported by our empirical evidence in the next section. This parameter is usually set to $0.01 - 0.025$ in the literature, for example, it is set to 0.01 in Iacoviello and Neri (2010). One consequence of our calibration is that the impact of new housing production on the real house price is weakened.

Housing production technology		
The share of housing capital	μ_{kh}	0.1
Depreciation rates		
The depreciation rate of housing capital	δ_{kh}	0.03
The depreciation rate of housing stock	δ_h	0.002

For the monetary policy rule, we set the weight coefficients to $\phi_r = 0.6$, $\phi_\pi = 1.5$, and $\phi_y = 0.5$, which are similar with Iacoviello and Neri (2010).[18]

Monetary policy		
The interest rate inertia	ϕ_r	0.6
The weight coefficient on inflation	ϕ_π	1.5
The weight coefficient on output	ϕ_y	0.5

As we focus on the impulse responses of model variables to various temporary shocks, the autocorrelation coefficients of these shocks are set to 0.01.[19] Meanwhile, we set the standard deviations of all shocks to 0.01.

Autocorrelations of shocks		
Goods sector technology	ρ_{Ac}	0.01
Housing sector technology	ρ_{Ah}	0.01
Monetary policy	ρ_{uR}	0.01
Standard deviations of shocks		
Goods sector technology	σ_{Ac}	0.01
Housing sector technology	σ_{Ah}	0.01
Monetary policy	σ_{uR}	0.01

[18]The estimates in Iacoviello and Neri (2010) for these coefficients are $\phi_r = 0.61$, $\phi_\pi = 1.36$, and $\phi_y = 0.51$.

[19]We use 0.01, instead of 0, to facilitate Matlab programs in the future research.

3.2.8 Steady state ratios

When we calibrate the parameters, steady state ratios of the model should be consistent with the sample average. The following table summaries the sample average of the U.S. economy in the period $1947Q1 - 2011Q4$.[50]

Interpretation	Target
Annual real interest rate	3%
Consumption share	65%
Non-housing investment share	11%
Housing investment share	5%

Since we consider a closed economy and only have consumption and housing investment, we should target the modified ratios instead of these original ratios.[51] The modified ratios are summarised in the following table, and we can see that the steady state ratios from our benchmark model are close to their targets.[52]

Interpretation	Expression	Target	SS Value
Annual real interest rate	$R^4 - 1$	3%	3%
Consumption share	c/GDP'	92.6%	91.6%
Investment in housing capital	IK_h/GDP'	0.4%	0.6%
Housing investment share	qIH/GDP'	7%	7.7%

3.2.9 Impulse response analysis

In this section, we discuss how model variables respond to various exogenous shocks in the benchmark model. We consider three types of shocks: a positive housing sector technology shock, a positive goods sector technology shock, and a positive monetary policy shock. Recall that, after introducing the

[50]Sources: U.S. Bureau of Economic Analysis.

[51]If goods capital is not considered, the GDP is composed by consumption goods, investments in housing capital, and real value of new housing, i.e., $GDP' = C + IK_h + qIH$.

[52]From Iacaviello and Neri (2011), we infer that the ratio of investment in goods capital to investment in housing captial is around 43.

varied housing supply, we can discuss the supply side effect, which includes both the direct effect and the feedback effect.

Housing sector technology shock First, we discuss the direct effect of a variable housing supply. Figure 3.1 shows the impulse responses of variables to a one percent positive shock in housing sector technology with persistence of $\rho_{Ah} = 0.01$.[53] A higher housing sector technology leads to a higher new housing production, and thus a higher marginal product of housing capital. The high marginal product of housing capital implies a higher real rental price, thus a higher income and a higher consumption. Meanwhile, the higher housing supply has a negative impact on the real house price, and this is the direct effect from the housing supply on the real house price.

In particular, we notice that the impact of the housing sector technology shock on the economy is small. When new housing production increases by 1%, the real house price decreases by 0.0015%, and the responses of other variables are even smaller.

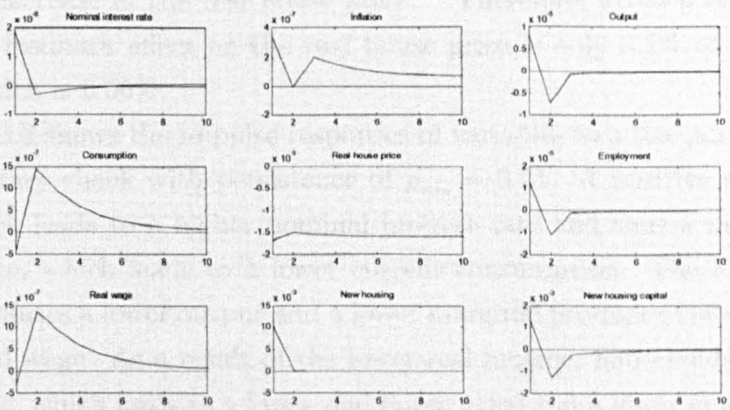

Figure 3.1: Impulse responses to a positve housing sector techology shock from the benchmark model.

[53] In all figures, impulse responses are measured as percentage deviations from the steady state, and horizontal axes display the number of quarters after the shock.

Goods sector technology shock and monetary policy shock Next, we discuss the feedback effect from the new housing production given a goods sector technology shock and a monetary policy shock respectively. Figure 3.2 shows the impulse responses of variables to a one percent positive shock in goods sector technology with persistence of $\rho_{Ac} = 0.01$. A higher goods productivity leads to a higher marginal product of labour, thus a higher real wage and a higher real income. When households have a higher real income, they demand more consumption and housing, which leads to a higher real house price. Given the higher real house price, new housing production increases and thus the marginal product of housing capital. As a result, the real rental price rise, and then leads to a higher investment in housing capital.

In particular, this figure shows that the real house price positively responds to the positive goods technology shock, but we know that the increase of the real house price is weakened by the higher new housing production. Although this (negative) feedback effect is entangled with the (positive) demand side effect of the shock on the real house price, we can infer that, when new housing production increases by 0.08%, the feedback effect leads to a 0.00012% decrease in the real house price.[54] Therefore, given a technology shock, the feedback effect on the real house price is only 0.2% of the total impact, which is 0.06%.

Figure 3.3 shows the impulse responses of variables to a one percent positive monetary shock with persistence of $\rho_{uR} = 0.01$. A positive monetary policy shock leads to a higher nominal interest rate and thus a higher real interest rate, which leads to a lower current consumption. The lower consumption causes a lower output and a lower marginal product of labour, thus a lower real wage. As a result of the lower real income, households demand less housing, which leads to a lower real house price and a lower new housing production.

Similar with the previous case, there is a feedback effect from new housing production to the real house price. This figure shows that the real house

[54]From the direct effect, we know that, when new housing production increases by 1%, the real housing price decreases by 0.0015%. Therefore, when new housing production increases by 0.08%, it should lower the real housing price by $\frac{0.0015\% * 0.08\%}{1\%} = 0.00012\%$.

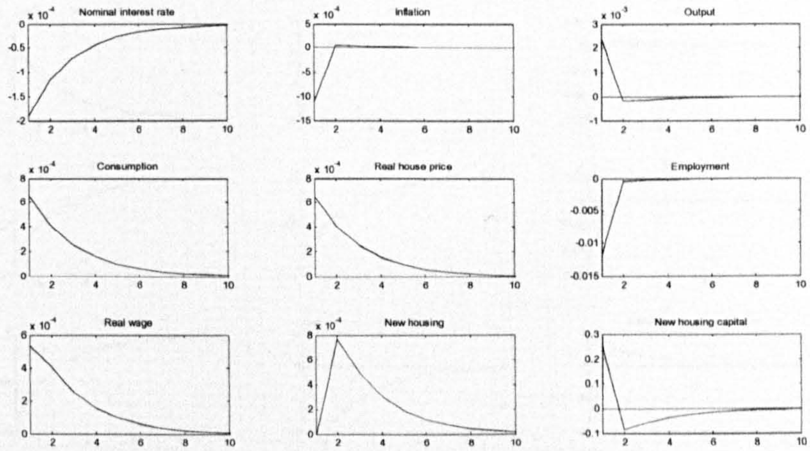

Figure 3.2: Impulse responses to a positve goods sector technology shock from the benchmark model.

price negatively responds to the a positive monetary policy shock, but we know that the decrease of the real house price is weakened by the lower new housing production. Although this (positive) feedback effect is entangled with the (negative) demand side effect of the shock on the real house price, using the same approach in the previous case, we can infer that, when new housing production decreases by 1%, the feedback effect leads to a 0.0015% increase in the real house price.[55] Therefore, given a monetary policy, the feedback effect on the real house price is also negligible, i.e., less than one percent (i.e., 0.2%) of the total effect, which is 0.8%.

The standard setting of the housing market and the supply side effect From above analysis, we know that (i) the real house price responds negatively to a positive housing technology shock but the direct effect of the housing supply on the economy is negligible; (ii) the real house price responds positively to a positive goods sector technology shock and negatively to a

[55]From the direct effect, we know that, when new housing production increases by 1%, the real housing price decreases by 0.0015%. Therefore, when new housing production increases by 1%, it should lower the real housing price by $\frac{0.0015\% * 1\%}{1\%} = 0.0015\%$.

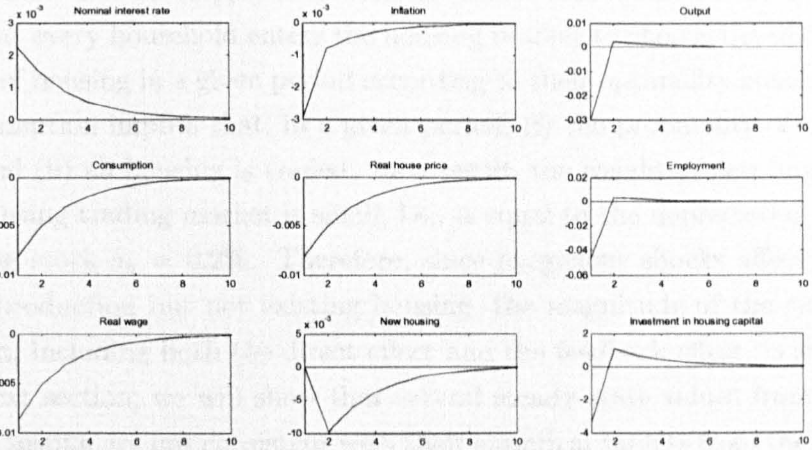

Figure 3.3: Impulse responses to a positve monetary policy shock from the benchmark model.

positive monetary policy shock, and we can infer that the feedback effect from the housing supply is negligible as well, although the feedback effect is entangled with the demand side effect. In sum, through new housing production, the supply side effect on the real house price is not important at all.

Before discussing the relation between the results and the model, we briefly describe the housing market in the benchmark model. For the demand side, every household enters the housing trading market in a given period. They sell their existing undepreciated housing and purchase the optimal quantity of housing according to their optimality condition. For the supply side, housing producers combine housing sector technology and housing capital to produce new housing, and the total housing supply includes both existing housing and new housing, according to the housing accumulation process. The housing market equilibrium condition indicates that the demand for housing is equal to the supply of housing in a given period. This setting of the housing market is widely used in the literature and can be seen as a standard setting.

We suppose that the standard setting of the housing market is the reason for our result that the supply side effect is small. This standard setting assumes that every household enters the housing market to choose the optimal quantity of housing in a given period according to their optimality condition. This assumption implies that, in a given period, (i) the probability of trade is one, and (ii) all housing is traded. As a result, the weight of new housing in the housing trading market is small, i.e., is equal to the depreciation rate of housing stock $\delta_h = 0.2\%$. Therefore, since exogenous shocks affect new housing production but not existing housing, the magnitude of the supply side effect, including both the direct effect and the feedback effect, is small. In the next section, we will show that several steady state values from this standard setting are not consistent with their empirical targets from the U.S. housing sector, and we will construct a new setting for the housing market.

3.3 An investigation of the U.S. housing sector

In this section, we examine the U.S. housing sector using data from the U.S. Census Bureau for the period of $1968Q1 - 2009Q4$. Firstly, we generate several empirical ratios and use them as targets for the steady state ratios from our theoretical model. Secondly, we calculate the average length of a housing project.

3.3.1 The empirical ratios

Firstly, we generate several empirical ratios from the U.S. housing sector. We employ three quarterly series: (a) new housing sold; (b) existing housing sold; (c) total occupied housing units. Combining these three quarterly series, we have following new quarterly series.[56]

New quarterly series	Combination method
Total housing sold (d)	(a) + (b)
$\frac{\text{New housing sold}}{\text{Total occupied housing units}}$	(a) / (c)
$\frac{\text{Total housing sold}}{\text{Total occupied housing units}}$	(d) / (c)
$\frac{\text{New housing sold}}{\text{Total housing sold}}$	(a) / (d)

Figure 3.4 plots the ratio of new housing sold to total occupied housing units. This ratio was fluctuating between 0.001 and 0.003 from 1968Q1 to 2009Q4, and the average value of this ratio is 0.002, implying that, in a given quarter, 0.2% of total occupied housing is newly built.

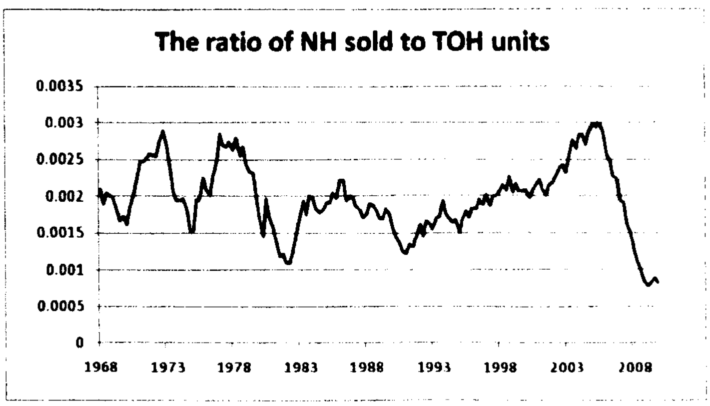

Figure 3.4: The ratio of new housing sold to total occupied housing units. Sources: U.S. Census Bureau.

Figure 3.5 plots the ratio of total housing sold to total occupied housing units. During 1968Q1 and 2009Q4, the minimum value of this ratio is 0.006 and the maximum is 0.018. The average value of this ratio is 0.0115. This suggests that, in a given quarter, 1.15% of total occupied housing is traded.

Figure 3.6 plots the ratio of new housing sold to total housing sold. We interpret this ratio as the weight of new housing in the housing trading market. This weight was between 0.2 to 0.25 before 1974. From 1974 to 2006,

[56] We use the series of total occuplied housing units instead of the series of total housing units because we assume that all housing provide utility to households in our model. Our results are not affected by the choice of these two series.

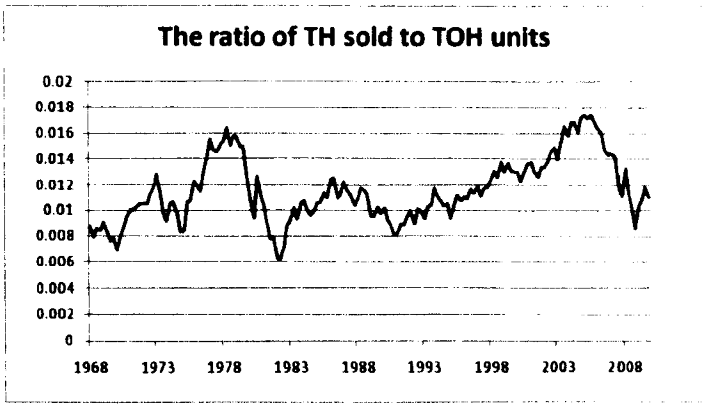

Figure 3.5: The ratio of total housing sold to total occupied housing units. Sources: U.S. Census Bureau.

this weight was fluctuating between 0.15 to 0.2. Since 2007, this weight has began to fall and was around 0.07 in 2009, reflecting a sharp decrease in new housing production since the financial crisis. The average value of this weight is 0.172 over our sample period, implying that the average weight of new housing in the housing trading market is 17.2% in a given quarter.

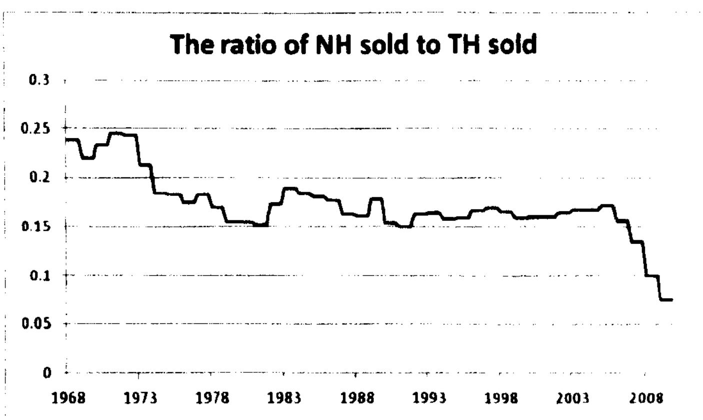

Figure 3.6: The ratio of new housing sold to total housing sold. Sources: U.S. Census Bureau.

Here we summarise above empirical ratios in the following table: in a given period, (i) 0.2% of occupied housing units are new housing, (ii) 1.15% of total occupied housing units are traded, (iii) 17.2% of traded housing is new housing. They are the targets of the steady state ratios from our benchmark model. Firstly, as we calibrate the depreciation rate of housing stock to $\delta_h = 0.002$, the steady state ratio of new housing to total housing meets its target. Secondly, since it is been assumed that all housing is traded in a given period, the ratio of total housing sold to total housing is always equal to 1. We are not able to re-calibrate any parameter to allow this ratio to hit its target. Thirdly, because of the same reason that housing sold is equal to total housing, the ratio of new housing sold to total housing sold is always equal to the first ratio, thus this ratio cannot hit its target either. Overall, while the first ratio can hit its target, the second and third ratios cannot meet their targets in the benchmark model. The inconsistency between the steady state values and the empirical ratios motivates us to construct a new setting for the housing market by introducing the possibility of trading housing.

Description	Target	SS value
$\frac{\text{New housing sold}}{\text{Total occupied housing units}}$	0.002	0.02
$\frac{\text{Total housing sold}}{\text{Total occupied housing units}}$	0.0115	1
$\frac{\text{New housing sold}}{\text{Total housing sold}}$	0.172	0.002

3.3.2 The feature of time to build

Moreover, we examine the length of time required to complete a housing project. In the U.S. housing sector, there are two stages for a housing project: (i) from authorisation to start; and (ii) from start to completion. Both stages take time. The following table shows the time required for these two steps respectively, for a building with one unit. The average time is 0.8 month for the first step and 6.2 months for the second step. These facts support us to consider the feature of time to build in new housing production, rather than using the assumption that a housing project is started and completed within one period. In general, we assume that a housing project takes 4 periods to complete, i.e., 12 months, since (i) building with one more unit takes longer at both steps, (ii) it also takes some time from completion to sold, 5.6 months on average. Therefore, our empirical analysis motivates us to introduce the feature of time to build with a 4-period lag into new housing production.

Description	Duration
From authorisation to start	0.8 month
From start to completion	6.2 months
From completion to sold	5.6 months

3.4 The benchmark model with the new setting of the housing market

In this section, we construct a new setting of the housing market that allows the steady state ratios from our model to be consistent with their empirical targets. This is our first contribution in this chapter.

3.4.1 The probability of trade and the optimal housing

In order to construct the new setting of the housing market, we introduce the Calvo (1987) assumption to the household sector. Calvo (1987) assumes that only a fraction of firms can reset their prices in a given period. This is a common assumption for the feature of sticky prices in DSGE models. In our model, we assume that households can enter the housing trading market with probability $1-\lambda_s$ in a given period.[57] Let h_t^* denote the optimal housing demanded by households who are able to trade in period t.

Then we update the housing stock accumulation process and the equilibrium condition in the housing trading market.[58] Firstly, the housing stock accumulation process becomes

$$H_t = \lambda_s (1-\delta_h) H_{t-1} + (1-\lambda_s) h_t^*$$

[57]In particular, if we set $\lambda_s = 0$, we have the standard setting, i.e., every household can trade housing in a given period. Besides, if we set $\lambda_s = 1$, every household cannot trade housing in his/her lifetime.

[58]Calvo(1987) considers the profit maximisation problem for a firm who is able to reset price, and then produce the aggregate price using the probability. Similarly, we solve the utility maximisation problem for a household who is able to trade, and then use the probability to obtain the aggregate housing using the housing accumulation process.

where h_t^* is the optimal housing, and $(1-\lambda_s)\,h_t^*$ is the total optimal housing given only a fraction $(1-\lambda_s)$ of households can trade. This equation implies that the total housing stock in period t, H_t, is composed by (i) the undepreciated housing from $t-1$ held by the fraction of households that are unable to trade, $\lambda_s\,(1-\delta_h)\,H_{t-1}$, and (ii) the optimal housing held by the fraction of households that are able to trade, $(1-\lambda_s)\,h_t^*$.

The equilibrium condition in the housing trading market is

$$(1-\lambda_s)\,h_t^* = (1-\lambda_s)\,(1-\delta_h)\,H_{t-1} + IH_t$$

In period t, households enter the housing trading market with probability $(1-\lambda_s)$. They sell their existing undepreciated housing purchased from period $t-1$, $(1-\delta_h)\,H_{t-1}$, and demand optimal housing, h_t^*. Therefore, this equation shows that, in the housing trading market, the total demand for housing is equal to the total supply of housing, which is composed by undepreciated existing housing, $(1-\lambda_s)\,(1-\delta_h)\,H_{t-1}$, and new housing, IH_t.[59]

We can rewrite this equilibrium condition as

$$(1-\lambda_s)\,(h_t^* - (1-\delta_h)\,H_{t-1}) = IH_t$$

which shows that, in the housing trading market, the demand for new housing is equal to the supply of new housing, as the difference between the optimal housing, h_t^*, and undepreciated existing housing, $(1-\delta_h)\,H_{t-1}$, is the demand for new housing, given that a fraction $(1-\lambda_s)$ of households can trade.[60]

[59] If we combine these two equations, we obtain the standard form of the housing stock accumulation process

$$H_t = (1-\delta_h)\,H_{t-1} + IH_t$$

[60] At steady state, new housing production is positive as existing housing is depreciating. In the log-linearised form, we consider the neighbourhood around the steady state. But we acknowledge the possibility of negative new housing production in reality.

3.4.2 The optimality condition between housing and consumption

The second modification is in the households' lifetime utility maximisation problem. We modify the households budget constraint and obtain an updated intertemporal optimality condition that governs households' demand for housing. When a household enters the housing trading market and decides the optimal housing h^*, he must take into account the possibility of trade and no trade in the future.[61]

The first order condition derived from the updated households' lifetime utility maximisation problem is

$$\begin{aligned}
q_{h,t}u_{c,t} &= \frac{j_t}{h_t^*} \\
&+\beta\lambda_s j_{t+1}\frac{1}{h_t^*} + (1-\lambda_s)\beta(1-\delta_h)E_t(q_{h,t+1}u_{c,t+1}) \\
&+\beta^2\lambda_s^2 j_{t+2}\frac{1}{h_t^*} + (1-\lambda_s)\lambda_s\beta^2(1-\delta_h)^2E_t(q_{h,t+2}u_{c,t+2}) \\
&+\beta^3\lambda_s^3 j_{t+3}\frac{1}{h_t^*} + (1-\lambda_s)\lambda_s^2\beta^3(1-\delta_h)^3E_t(q_{h,t+3}u_{c,t+3}) \\
&+\ldots
\end{aligned}$$

or

$$q_{h,t}u_{c,t} = \sum_{k=0}^{\infty}\lambda_s^k\beta^k\frac{j_{t+k}}{h_t^*} + \sum_{k=0}^{\infty}(1-\lambda_s)\lambda_s^k\beta^{k+1}(1-\delta_h)^{k+1}E_t(q_{h,t+k+1}u_{c,t+k+1})$$

or in recursive form

$$\begin{aligned}
q_{h,t}u_{c,t} &= \frac{1}{h_t^*}R_{1,t} + R_{2,t} \\
R_{1,t} &= j_t + \lambda_s\beta R_{1,t+1} \\
R_{2,t} &= (1-\lambda_s)\beta(1-\delta_h)E_t(q_{h,t+1}u_{c,t+1}) + \lambda_s\beta(1-\delta_h)R_{2,t+1}
\end{aligned}$$

which implies that the real house price in terms of the marginal utility of con-

[61] The updated households' utility maximisation problem is shown in the Appendix. Importantly, the bonds market clearing condition needs to be reconsidered. If we assume that households belong to a family or assume a perfect insurance market to maintain a same consumption level, the bonds market clearing condition can hold.

sumption at t equals to the sum of the marginal utility of housing at $t+k$, $k = 0, 1, 2, ...$, and the expected real house price (for resale) in terms of the discounted marginal utility of consumption at $t+k$, $k = 1, 2, ...$, taking the discount factor and the probability of trade into account. This intratemporal optimality condition describes an optimal allocation of resource between consumption and domestic housing, and links the real house price to the optimal housing, rather than the total housing stock in the standard setting of the housing market.[62]

3.4.3 Calibration

In the new setting of the housing market, the probability of no trade, λ_s, is the only new parameter we introduce. In order to make the ratio of total housing sold to total housing hit its target, we set this parameter to $\lambda_s = 0.9905$, which means that 99.05% of households do not trade housing in a given period. Therefore, the probability of trade, $1 - \lambda_s$, is 0.0095, implying only 0.95% of households enter the housing trading market in a given period. This calibration implies that each household can re-enter the housing market once over 105 (i.e., is equal to $\frac{1}{1-\lambda_s}$) quarters, i.e., 26 years interval. Meanwhile, we keep the depreciation rate of housing stock at $\delta_h = 0.002$.

Households preference		
Probability of no trade	λ_s	0.9905
Probability of trade	$1 - \lambda_s$	0.0095
Depreciation rate		
The depreciation rate of housing stock	δ_h	0.002

[62] If we assume the propability of trade to one, $\lambda_s = 0$, the first order condition becomes the standard form in the benchmark model.

$$q_{h,t} u_{c,t} = \frac{j_t}{h_t^*} + \beta (1 - \delta_h) E_t (q_{h,t+1} u_{c,t+1})$$

3.4.4 Steady state ratios

Here we discuss the steady state ratios. Firstly, by setting the depreciation rate of housing stock to $\delta_h = 0.002$, the steady state ratio of new housing sold to total housing, IH/H, is consistent with its empirical value, 0.002. Secondly, if we set probability of trade to $1 - \lambda_s = 0.0095$, together with the depreciation rate, the ratio of total housing sold to total housing, $(1 - \lambda_s)\, h^*/H$, can meet its target, 0.0115. Thirdly, given the probability of trade and the depreciation rate, the steady state ratio of new housing sold to total housing sold, $IH/(1 - \lambda_s)\, h^*$, can also hit its target, 0.172. Therefore, our new setting allows these ratios to hit their targets simultaneously.[63] In particular, the weight of new housing in the housing trading market is 17.2%, much higher than that in the standard setting, 0.2%.

Empirical ratios	Expressions	Targets	SS Values
$\frac{\text{new housing sold}}{\text{total housing}}$	$\frac{IH}{H} = \delta_h$	0.002	0.002
$\frac{\text{total housing sold}}{\text{total housing}}$	$\frac{(1-\lambda_s)h^*}{H} = \delta_h + (1 - \lambda_s)(1 - \delta_h)$	0.0115	0.0115
$\frac{\text{new housing sold}}{\text{total housing sold}}$	$\frac{IH}{(1-\lambda_s)h^*} = \frac{\delta_h}{\delta_h+(1-\lambda_s)(1-\delta_h)}$	0.172	0.172

Meanwhile, we also need to check whether other steady state ratios meet their targets. Similar with the benchmark model, when we set the depreciation rate to $\delta_h = 0.002$, we still need to set the housing preference to $j = 0.4$ to allow the following steady state ratios close to their targets, which are the sample averages between 1947Q1 and 2011Q4 from the U.S. Bureau of Economic Analysis.

Interpretation	Expressions	Targets	Values
Annual real interest rate	$R^4 - 1$	3%	3%
Consumption share	c/GDP'	93%	92.2%
Investment in housing capital	IK_h/GDP'	0.3%	0.6%
Housing investment share	qIH/GDP'	7%	7.2%

[63] If the first and second ratios hit their targets, the third ratio will do so as well.

3.4.5 Impulse response analysis

Housing sector technology shock Figure 3.7 shows the impulse responses of variables to a one percent positive housing sector technology shock with persistence of $\rho_{Ah} = 0.01$. From the lower-left panel, we can see that the response of the real house price under the new setting (dashed line) is much larger than that under the standard setting (solid line). Precisely, when new housing production increases by 1%, the real house price decreases by 0.09%, which is around 60 times larger than that under the standard setting, 0.0015%, implying that the direct effect of a variable housing supply on the real house price is largely underestimated under the standard setting. This result may challenge one of conclusions from Iacoviello and Neri (2010) that the slow growth of housing technology is the main cause of the persistent increase in the real house price. Our results suggest that, if the impact of housing sector technology on the real house price is properly measured, the rise in the real house price may not be mainly contributed by the slow growth of housing technology because an increase in the housing technology has much larger impacts on the real house price. Meanwhile, from the upper-right panel, consumption increases less as households shift resource to the housing sector because of a higher real house price.

Goods sector technology shock and monetary policy shock Figure 3.8 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence of $\rho_{Ac} = 0.01$. Since we know that the feedback effect is negligible in the benchmark model (solid line), the difference between these two lines approximates the magnitude of the feedback effect. We can see that, under the new setting (dashed line), the feedback effect has an obvious negative impact on the real house price. More precisely, given a positive goods sector technology shock, when new housing production increases by 0.08%, the feedback effect leads to a 0.0072% decrease in the real house price.[64] Therefore, the feedback effect is around 12% of the total

[64] From the direct effect, we know new housing production increases by 1%, the real house price decreases by 0.09%. Therefore, when new housing production increases by

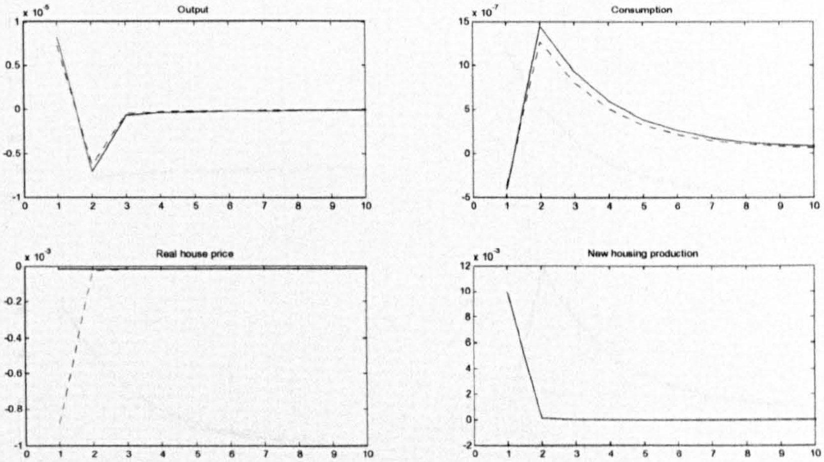

Figure 3.7: Impulse responses to a positive housing sector technology shock. The solid line is from the benchmark model and the dashed line is from the benchmark model with the new setting of the housing market.

impact, 0.06%.

Figure 3.9 shows the impulse responses of variables to a one percent positive monetary policy shock with persistence of $\rho_{uR} = 0.01$. Similar with the previous case, the difference between these two lines also approximates the magnitude of the feedback effect. Under the new setting (dashed line), the feedback effect also has an obvious impact on the real house price. More precisely, given a positive monetary policy shock, when new housing production decreases by 0.1%, the feedback effect leads to a 0.09% rise in the real house price.[65] Therefore, similar with the previous case, the feedback effect is around 11% of the total impact, 0.8%.[66] In sum, under the new setting of the housing market, the feedback effect is around 60 times more than that under the standard setting, implying that the feedback effect of a variable

0.08%, it should lower the real house price by $\frac{0.09\% * 0.08\%}{1\%} = 0.0072\%$.

[65] From the direct effect, we know new housing production increases by 1%, the real house price decreases by 0.09%. Therefore, when new housing production increases by 1%, it should lower the real house price by $\frac{0.09\% * 1\%}{1\%} = 0.09\%$.

[66] For both case, under standard setting, the feedback effect is less than one percent (i.e., 0.2%) of the total impact.

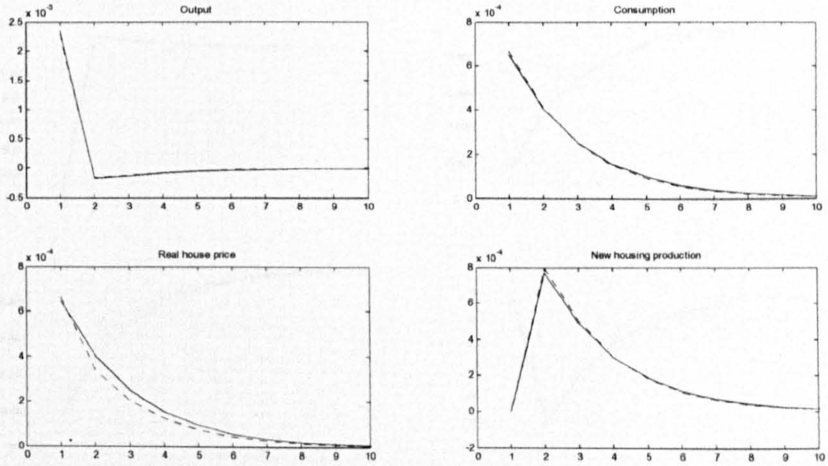

Figure 3.8: Impulse responses to a positive goods sector technology shock. The solid line is from the benchmark model and the dashed line is from the benchmark model with the new setting of the housing market.

housing supply on the real house price is also largely underestimated under the standard setting.

3.5 The feature of time to build

While it is commonly assumed that a housing project is started and completed within one period. Our previous empirical analysis suggests that a housing project takes several quarters to complete. Therefore, in this section, we relax the standard assumption and consider the feature of time to build for new housing production. The feature of time to build has been introduced to goods capital in Kydland and Prescott (1982), Wen (1998), Tsoukalas (2010), but this is the first time it is used for new housing production in our chapter. We will consider this feature to the benchmark model and the benchmark model with the new setting of the housing market respectively to discuss its impact on the dynamics.

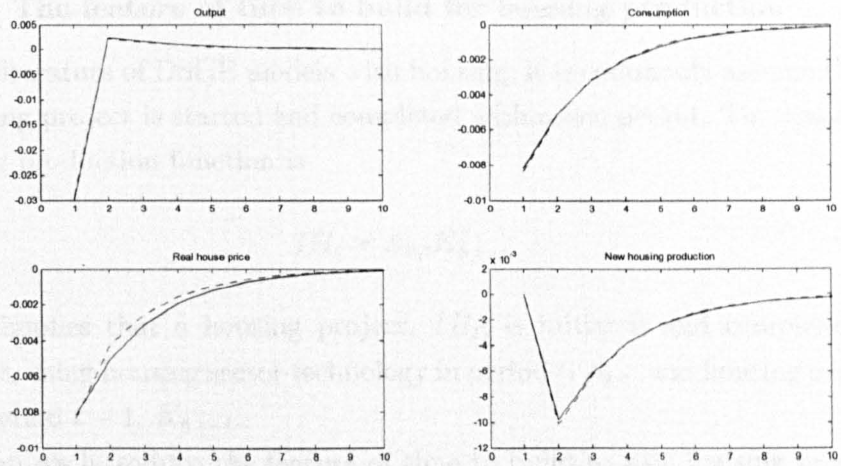

Figure 3.9: Impulse responses to a positive monetary policy shock. The solid line is from the benchmark model and the dashed line is from the benchmark model with the new setting of the housing market.

3.5.1 The feature of time to build for investment

The feature of time to build was originally introduced to goods capital. In a standard setting without this feature, a investment project takes only one period to complete. As a result, the capital stock accumulation process is

$$K_{c,t} = (1-\delta)\, K_{c,t-1} + IK_{c,t}$$

which shows that investment at period t, $IK_{c,t}$, becomes productive capital at period $t+1$, with a 1-period lag.

After introducing the feature of time to build, the investment project takes more than one period to complete. If we assume a J-period lag in the investment project, the capital stock accumulation process becomes

$$K_{c,t} = (1-\delta)\, K_{c,t-1} + IK_{c,t-(J-1)}$$

which shows that investment at period $t-(J-1)$, $IK_{c,t-(J-1)}$, becomes productive capital at period $t+1$, after J periods.

3.5.2 The feature of time to build for housing production

In the literature of DSGE models with housing, it is commonly assumed that a housing project is started and completed within one period. The standard housing production function is

$$IH_t = A_{h,t} K_{h,t-1}^{\mu_h}$$

which implies that a housing project, IH_t, is initiated and completed in period t, using housing sector technology in period t, $A_{h,t}$, and housing capital from period $t-1$, $K_{h,t-1}$.

Then we introduce the feature of time to build to new housing production.[67] If we assume a J-period lag in new housing production, the simplest case is to assume that housing has been built at $t-J$ and becomes available in period t. The sequence of this approach is that: (i) at period $t-J$, housing producers built new housing using housing capital $K_{h,t-1-J}$; (ii) at period t, housing producers make new housing available in the housing trading market. This new feature affects two equations in the model: (i) the housing production function, and (ii) the housing producers' demand for housing capital.

Firstly, the housing production function becomes

$$IH_t = A_{h,t-J} K_{h,t-1-J}^{\mu_h}$$

which implies that the new housing available in period t, IH_t, is built using the housing sector technology from period $t-J$, $A_{h,t-J}$, and the housing capital from period $t-1-J$, $K_{h,t-1-J}$.

Secondly, the housing producers' real profit maximisation problem is

$$\max_{K_{h,t-1}} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(q_{h,t+k} IH_{t+k} - R_{kh,t+k} K_{h,t+k-1} \right)$$

where $q_{h,t} IH_t$ is the real revenue and $R_{kh,t} K_{h,t-1}$ is the real cost in period

[67] Actually, we can introduce this feature twice, i.e., for housing capital and housing production respectively.

t. Without the feature of time to build, the difference between them is the profit of the housing projects initiated at period t. In contrast, given the feature of time to build, the profit of a housing project that starts at period t is the difference between its real future revenue, $q_{h,t+J} IH_{t+J}$, and its real current cost, $R_{kh,t} K_{h,t-1}$.

The first order condition derived from this real profit maximisation describes the housing producers' demand for housing capital, and it is expressed as

$$\begin{aligned} R_{kh,t} &= E_t \Lambda_{t,t+J} q_{h,t+J} A_{h,t} \mu_h K_{h,t-1}^{\mu_h - 1} \\ &= E_t \Lambda_{t,t+J} q_{h,t+J} \mu_h \frac{IH_{t+J}}{K_{h,t-1}} \end{aligned}$$

where $\Lambda_{t,t+J}$ is the stochastic discount factor, which is used to discount profit in terms of utility. This equation implies that the real rental price $R_{kh,t}$ is equal to the expected marginal product of housing capital, which depends on the future real house price $q_{h,t+J}$. Besides, this equation also implies that housing producers are concerned with the profit over each project, instead of the profit over each period.

3.5.3 Impulse response analysis for the benchmark model with the feature of time to build

First, we consider the feature of time to build in the benchmark model. We consider a 4-period lag in new housing production since the empirical data suggests that it usually takes 4 quarters to complete a housing project.

Figure 3.10 and 3.11 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence of $\rho_{A_c} = 0.01$ and a one percent positive monetary policy shock with persistence of $\rho_{uR} = 0.01$, respectively. We can see that the feature of time to build does not have any obvious impact on model variables except new housing production. This is because the weight of new housing in the housing trading market is negligible, i.e., 0.2%, in the standard setting, thus a change in new housing production has no obvious impact on the economy.

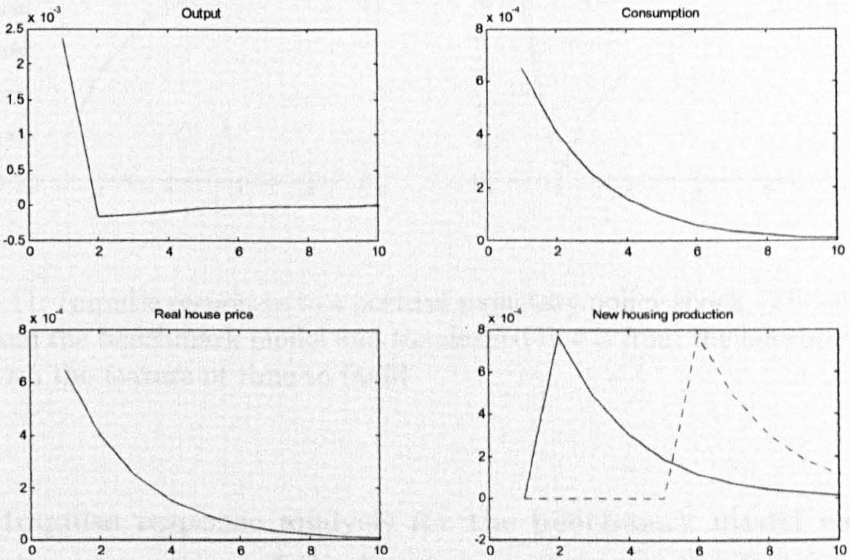

Figure 3.10: Impulse responses to a positive goods sector technology shock. The solid line is from the benchmark model and the dashed line is from the benchmark model with the feature of time to build.

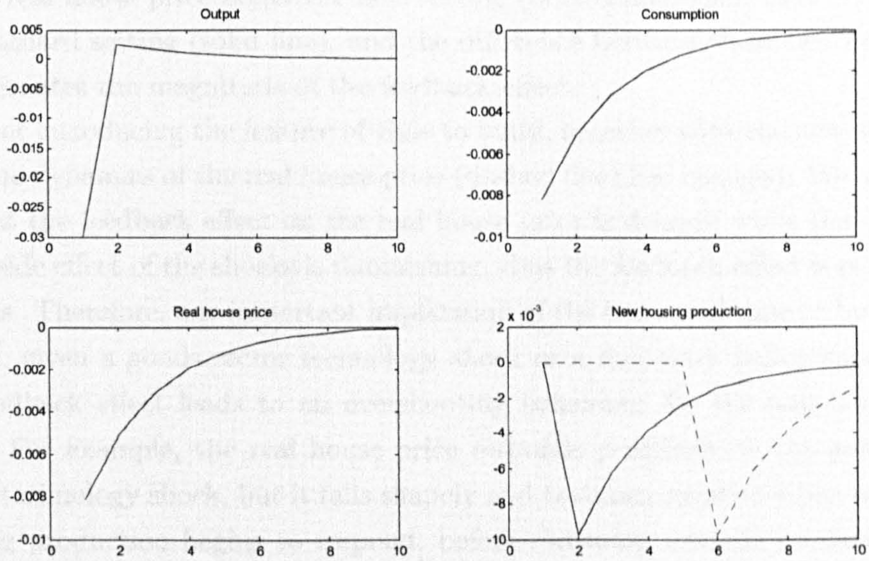

Figure 3.11: Impulse responses to a positive monetary policy shock. The solid line is from the benchmark model and the dashed line is from the benchmark model with the feature of time to build.

3.5.4 Impulse response analysis for the benchmark model with the new setting of the housing market and the feature of time to build

Then we add the feature of time to build into the benchmark model with the new setting of the housing market. We also consider a 4-period lag in new housing production.

In the new setting, the weight of new housing in the housing trading market is around 17.2%, thus a change in new housing production have a larger impact on the real house price. Figure 3.12 and 3.13 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence of $\rho_{Ac} = 0.01$ and a one percent positive monetary policy shock with persistence of $\rho_{uR} = 0.01$, respectively. Recall that, in the model without the feature of time to build, the feedback effect has a larger impact on the real house price under the new setting (dotted line) than that under the standard setting (solid line), and the difference between these two lines approximates the magnitude of the feedback effect.

After introducing the feature of time to build, together with the new setting, the dynamics of the real house price (dashed line) has changed. We can see that the feedback effect on the real house price is delayed while the demand side effect of the shocks is diminishing, thus the feedback effect is more obvious. Therefore, one important implication of the feature of time to build is that, given a goods sector technology shock or a monetary policy shock, the feedback effect leads to an overshooting behaviour for the real house price. For example, the real house price responds positively to the goods sector technology shock, but it falls shapely and becomes negative when new housing production begins to respond, before returning towards its steady state. This is because that (i) new housing production responds positively to the goods sector technology shock and this higher housing supply has a negative impact on the real house price; and (ii) the feature of time to build delays this negative feedback effect while the positive demand side effect is diminishing.

3.6 Conclusion

In this chapter, we introduce new housing production into the model, hence we can discuss the supply side effect, which includes both the direct effect and the feedback effect.

Our impulse response analysis suggests that the magnitudes of these two effects from new housing production are negligible to the economy. We suppose that the standard setting of the housing market is the reason for our results. This standard setting assumes that, in a given period, every household enters the housing market, thus all house is traded. Consequently, the weight of new housing in the housing trading market is equal to the depreciation rate of housing, which is small. Therefore, the magnitude of the supply

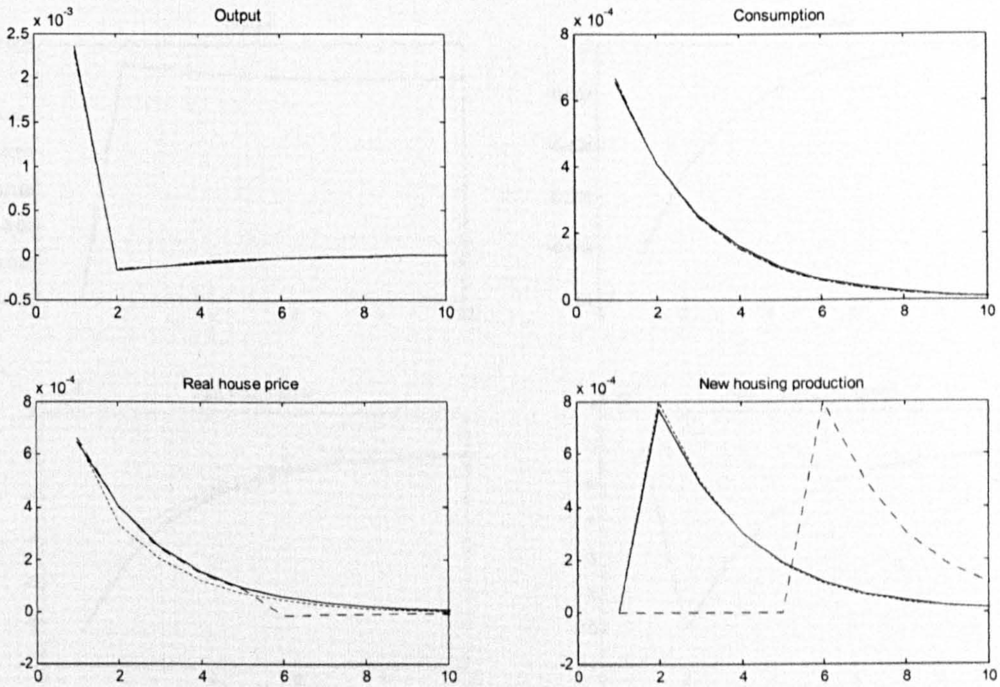

Figure 3.12: Impulse responses to a positive goods sector technology shock. The solid line is from the benchmark model, the dotted line is from the benchmark model with the new setting of the housing market, and the dashed line is from the benchmark model with the new setting of the housing market and the feature of time to build.

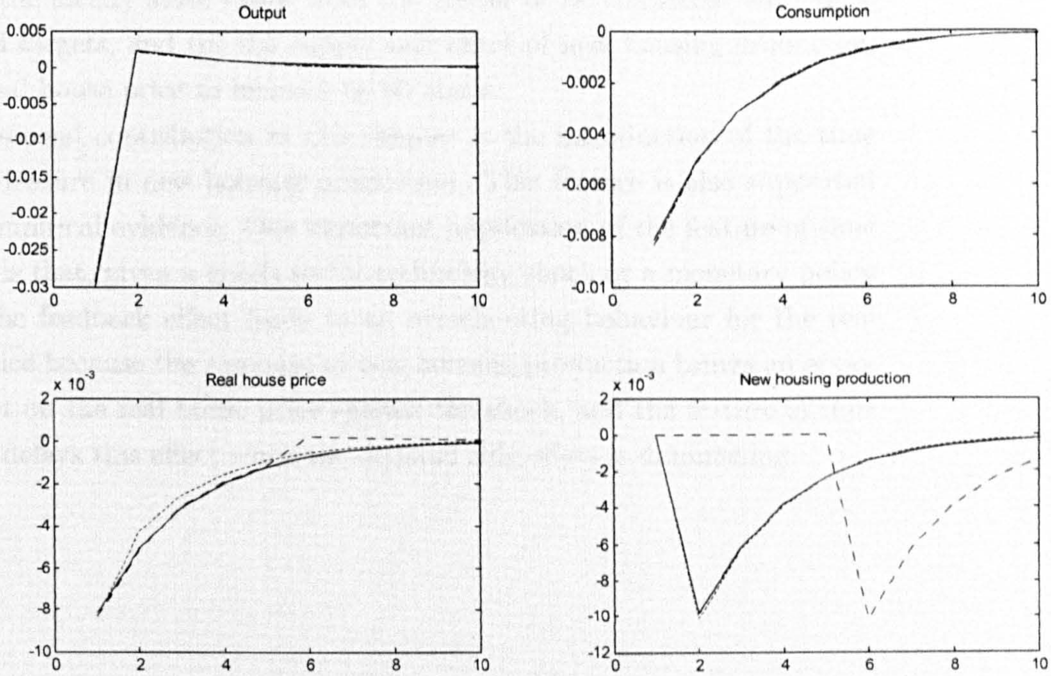

Figure 3.13: Impulse responses to a positive monetary policy shock. The solid line is from the benchmark model, the dotted line is from the benchmark model with the new setting of the housing market, and the dashed line is from the benchmark model with the new setting of the housing market and the feature of time to build.

side effect of new housing production, including the direct effect and the feedback effect, is small.

Next, our empirical analysis shows that several steady state ratios from the benchmark model cannot meet their empirical targets. This failure motivates us to construct a new setting for the housing market. As our first contribution in this chapter, the new setting of the housing market allows that (i) the steady state ratios from the model to be consistent with their empirical targets, and (ii) the supply side effect of new housing production on the real house price to increase by 60 times.

Our second contribution in this chapter is the introduction of the time to build feature in new housing production. This feature is also supported by our empirical evidence. One important implication of the feature of time to build is that, given a goods sector technology shock or a monetary policy shock, the feedback effect leads to an overshooting behaviour for the real house price because the response of new housing production brings an opposite effect on the real house price against the shock, and the feature of time to build delays this effect while the demand side effect is diminishing.

3.A Appendix to Chapter 3

3.A.1 Lagrangian program for household in the benchmark model

The benchmark model in this chapter is a simple DSGE model with the feature of sticky prices and a housing production. households maximise their lifetime utility subject to their budget constraint,

$$\max_{c_t,b_t,n_t,h_t,K_{h,t}} E_t \sum_{k=0}^{\infty} \left(\begin{array}{c} \beta^k \left(\Gamma_c \ln\left(c_{t+k} - \varepsilon_c c_{t+k-1}\right) + j_{t+k} \ln h_{t+k} - \frac{1}{1+\gamma_n}\left(n_{t+k}\right)^{1+\gamma_n} \right) \\ +\lambda_{t+k}\beta^k \left(\begin{array}{c} \left(R_{kh,t+k} + 1 - \delta_{kh}\right) K_{h,t+k-1} + w_{t+k} n_{t+k} \\ +q_{h,t+k}\left(1-\delta_h\right) h_{t+k-1} + \frac{R_{t+k-1}}{\pi_{c,t+k}} b_{t+k-1} + f_{t+k} \\ -c_{t+k} - K_{h,t+k} - q_{h,t+k} h_{t+k} - b_{t+k} \end{array} \right) \end{array} \right)$$

3.A.2 Lagrangian program for household in the benchmark model with a new setting of the housing market

The following table summarises the utility and the budget constraint in period $t+k, k = 0, 1, 2, ...,$ if the households can trade in period t. After combining these expressions, we obtain the lifetime utility function and the contemporary budget constraint.

Period t	prob of trade 1	prob of no trade 0
utility	$\ln h_t^*$	
budget constraint	$q_{h,t}\left(1-\delta_h\right)H_{t-1}-q_{h,t}h_t^*$	
Period $t+1$	prob of trade $1-\lambda_s$	prob of no trade λ_s
utility	$\ln h_{t+1}^*$	$\ln\left(1-\delta_h\right)h_t^*$
budget constraint	$q_{h,t+1}\left(1-\delta_h\right)h_t^*-q_{h,t+1}h_{t+1}^*$	$q_{h,t+1}\left(1-\delta_h\right)h_t^*-q_{h,t+1}\left(1-\delta_h\right)h_t^*$
Period $t+2$	prob of trade $\lambda_s\left(1-\lambda_s\right)$	prob of no trade λ_s^2
utility	$\ln h_{t+2}^*$	$\ln\left(1-\delta_h\right)^2 h_t^*$
budget constraint	$q_{h,t+2}\left(1-\delta_h\right)^2 h_t^*-q_{h,t+2}h_{t+2}^*$	$q_{h,t+2}\left(1-\delta_h\right)^2 h_t^*-q_{h,t+2}\left(1-\delta_h\right)^2 h_t^*$
......	......	......
Period $t+k$	prob of trade $\lambda_s^{k-1}\left(1-\lambda_s\right)$	prob of no trade λ_s^k
utility	$\ln h_{t+k}^*$	$\ln\left(1-\delta_h\right)^k h_t^*$
budget constraint	$q_{h,t+k}\left(1-\delta_h\right)^k h_t^*-q_{h,t+k}h_{t+k}^*$	$q_{h,t+k}\left(1-\delta_h\right)^k h_t^*-q_{h,t+k}\left(1-\delta_h\right)^k h_t^*$

The households maximise their lifetime utility subject to their budget constraint,

$$\max_{c_t,b_t,n_t,h_t^*,K_{h,t}} E_t\sum_{k=0}^{\infty}\left(\begin{array}{c}\beta^k\left(\begin{array}{c}\Gamma_c\ln\left(c_{t+k}-\varepsilon_c c_{t+k-1}\right)-\frac{1}{1+\gamma_n}\left(n_{t+k}\right)^{1+\gamma_n}\\+j_{t+k}\lambda_s^k\ln\left(1-\delta_h\right)^k h_t^*+j_{t+1+k}\lambda_s^k\left(1-\lambda_s\right)h_{t+1+k}^*\end{array}\right)\\+\lambda_{t+k}\beta^k\left(\begin{array}{c}\ldots+q_{h,t}\left(1-\delta_h\right)H_{t-1}-q_{h,t}h_t^*\\\lambda_s^{k-1}q_{h,t+k}\left(1-\delta_h\right)^k h_t^*-\lambda_s^k q_{h,t+k}\left(1-\delta_h\right)^k h_t^*\\-\lambda_s^{k-1}\left(1-\lambda_s\right)q_{h,t+k}h_{t+k}^*\end{array}\right)\end{array}\right)$$

3.A.3 Steady state ratios for the benchmark model

In order to solve the model, we need following steady state ratios: $\frac{IK_h}{c}$ and $\frac{Y}{c}$.

From the households' marginal utility of consumption, we have

$$u_c=\frac{1}{c}$$

since $\Gamma_c = \frac{1-\varepsilon_c}{1-\beta\varepsilon_c}$.

From the households demand for housing, we have the steady state value of $\frac{q_h h}{c}$,

$$\frac{q_h h}{c} = \frac{j}{1-\beta(1-\delta_h)}$$

From the housing market clearing condition, we have

$$h = H$$

Thus we have the steady state value of $\frac{q_h H}{c}$,

$$\frac{q_h H}{c} = \frac{q_h h}{c}$$

From the housing capital accumulation process, we have the steady state value of $\frac{IH}{H}$,

$$\frac{IH}{H} = \delta_h$$

Combining the steady state value of $\frac{q_h H}{c}$ and $\frac{IH}{H}$, we have the steady state value of $\frac{q_h IH}{c}$,

$$\frac{q_h IH}{c} = \frac{q_h H}{c}\frac{IH}{H}$$

From the households' demand for housing capital, we have the steady state value of R_{kh},

$$R_{kh} = \frac{1-\beta(1-\delta_{kh})}{\beta}$$

From the housing producers' demand for housing capital, we have the steady state value of $\frac{q_h IH}{K_{kh}}$,

$$\begin{aligned} R_{kh} &= \mu_{kh}\frac{q_h IH}{K_h} \\ \frac{q_h IH}{K_{kh}} &= \frac{R_{kh}}{\mu_{kh}} \end{aligned}$$

From the housing capital accumulation process, we have the steady state value of $\frac{IK_h}{K_h}$,

$$\frac{IK_h}{K_h} = \delta_{kh}$$

Combining the steady state value of $\frac{q_h IH}{K_h}$ and $\frac{IK_h}{K_h}$, we have the steady state value of $\frac{q_h IH}{IK_h}$,

$$\frac{q_h IH}{IK_h} = \frac{q_h IH}{K_h} / \frac{IK_h}{K_h}$$

Then combining the steady state value of $\frac{q_h IH}{c}$ and $\frac{q_h IH}{IK_h}$, we have the steady state value of $\frac{IK_h}{c}$,

$$\frac{IK_h}{c} = \frac{q_h IH}{c} / \frac{q_h IH}{IK_h}$$

From the goods market clearing condition, we have the steady state value of $\frac{Y}{c}$,

$$\frac{Y}{c} = 1 + \frac{IK_h}{c}$$

3.A.4 Targets for steady state ratios

Here are the target steady state ratios from Iacoviello and Neri (2010).

Interpretation	Expressions	Targets
Annual real interest rate	$R^4 - 1$	3%
Consumption share	c/GDP	67%
Business Investment share	$(IK_c + IK_h)/GDP$	27%
Housing investment share	qIH/GDP	6%
Housing wealth	$qH/(4 \cdot GDP)$	1.36
Business investment in non-housing sector	$K_c/(4 \cdot GDP)$	2.05
Business investment in housing sector	$K_h/(4 \cdot GDP)$	0.04
Value of land	$p_l/(4 \cdot GDP)$	0.5

where the GDP is composed as $GDP = c + IK_c + IK_h + q_h IH$. We can derive the ratio of investment in goods sector to investment in housing sector from this table: $\frac{IK_c}{IK_h} = \frac{\delta_{kc}K_c}{\delta_{kh}K_h} = \frac{\delta_{kc}\frac{K_c}{4*GDP}}{\delta_{kh}\frac{K_h}{4*GDP}} = \frac{0.025*2.05}{0.03*0.04} \approx 43$.

Our targets from empirical data are

Interpretations	Targets
Annual real interest rate	3%
Consumption share	65%
Non-housing investment share	11%
Housing investment share	5%

Since we do not have goods capital, we replace GDP by $GDP' = c + IK_h + q_h IH$. Now we produce the steady state ratios that need to be consistent with the sample average from data. Given the ratio of investment in goods sector to investment in housing sector is 43, we know that the share of investment in housing sector is $11\% * \frac{1}{44} = 0.25\%$.

The revised targets for the GDP-related steady state ratios are obtained as

$$
\begin{aligned}
\frac{c}{GDP'} &= \frac{c}{GDP}\frac{GDP}{GDP'} = 0.65 * \frac{1}{0.7025} = 92.5\% \\
\frac{IK_h}{GDP'} &= \frac{IK_h}{GDP}\frac{GDP}{GDP'} = 0.0025 * \frac{1}{0.7025} = 0.4\% \\
\frac{q_h IH}{GDP'} &= \frac{q_h IH}{GDP}\frac{GDP}{GDP'} = 0.05 * \frac{1}{0.7025} = 7\%
\end{aligned}
$$

since $\frac{GDP'}{GDP} = \frac{0.65\%+0.25\%+5\%}{0.65\%+11\%+5\%} = 70.25\%$.

Revised targets for steady state ratios are

Interpretations	Expressions	Targets
Annual real interest rate	$R^4 - 1$	3%
Consumption share	c/GDP'	92.5%
Business investment in housing sector	IK_h/GDP'	0.4%
Housing investment share	qIH/GDP'	7%

The consumption to GDP ratio is

$$
\frac{c}{GDP'} = 1/\frac{GDP'}{c} = 1/\left(1 + \frac{IK_h}{c} + \frac{q_h IH}{c}\right)
$$

The housing capital investment to GDP ratio is

$$\frac{IK_h}{GDP'} = \frac{IK_h}{c}\frac{c}{GDP'}$$

The value of new housing to GDP ratio is

$$\frac{q_h IH}{GDP'} = \frac{q_h IH}{c}\frac{c}{GDP'}$$

3.A.5 Steady state ratios for the benchmark model with the new setting of the housing market

From the housing trading market clearing condition, we have

$$\frac{IH}{(1-\lambda_s)\,h^*} = \frac{\delta_h}{\delta_h + (1-\lambda_s)\,(1-\delta_h)}$$

which is the ratio of new housing to total traded housing, i.e., the weight of new housing in the housing trading market. If we assume that every household can trade, $(1-\lambda_s) = 1$, thus the weight of new housing in the housing trading market is equal to the depreciation rate of housing, $\frac{IH}{h^*} = \delta_h$.

From the households' demand for housing, we have

$$q_h u_c h^* = \frac{j}{\frac{(1-\beta(1-\delta_h))(1-\lambda_s\beta)}{1-\lambda_s\beta(1-\delta_h)}}$$

If we assume that every household can trade, $(1-\lambda_s) = 1$, the steady state of this equation becomes $q_h u_c h^* = \frac{j}{1-\beta(1-\delta_h)}$, which is exactly same with that in the standard setting of the housing market.

4 Adaptive Learning and Heterogeneous Expectations

4.1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have been widely used in academia and the central banks for macroeconomic analysis. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) provide examples of medium-sized DSGE models. These DSGE models are built on the hypothesis of rational expectations, which assumes that agents have full information about the economy, i.e., they know the structure of the true model and the values of the model parameters, and use them to form expectations for the future. Therefore, agents are able to form beliefs that are consistent with actual outcomes. Slobodyan and Wouters (2012) argue that the assumption of rational expectations is too strong and models under rational expectations find it difficult to capture the persistence of macroeconomic variables. As a result, researchers need to use highly persistent shock or add other features, such as consumption habit and price indexation, to capture the persistence of macroeconomic variables in models under rational expectations.

The adaptive learning mechanism, an alternative way of forecasting the future, is discussed by Marcet and Sargent (1989a, 1989b), Evans and Honkapohja (1999, 2002), but they pay attention to the convergence of the models to the rational expectations equilibrium. They describe the main idea about the assumption of adaptive learning: agents do not necessarily have full information about the structure of the true model and the values of the model parameters, thus they need to use their experience to forecast the future, and then make adjustments when they realise their mistakes.

Since this alternative was suggested, the quantitative importance of the adaptive learning mechanism in business cycle fluctuations has been discussed in the context of DSGE models. Milani (2007) provides the first example of using Bayesian methods to estimate a DSGE model under adaptive learning by considering a constant learning algorithms, which gives an equal weight to newly observed data, and a correct learning model, i.e., agents are able to include correct endogenous state variables in the forecasting equations. In his article, the gain coefficient and structural and policy parameters of the economy are jointly estimated. He finds that the adaptive learning mechanism can represent a potential single source that can brings persistence to economy, and some features, such as consumption habit and price indexation, become reductant that are usually required under rational expectations to match the inertia of macroeconomic variables. Slobodyan and Wouters (2009) evaluate the empirical relevance of the adaptive learning mechanism in an estimated medium-scale DSGE model. They find that their model under adaptive learning with a correct learning model fit the data better than the model under rational expectations. In particular, if the set of variables used in the forecasting equations is limited to a list of observed macro-variables, i.e., using a small learning model, their model can explain the data better than that with a correct learning model. Overall, the adaptive learning mechanism adds additional persistence to the model but it does not systematically change the estimated structural parameters in the DSGE model.

The success of the small learning models is also shared by other researchers. Williams (2003) assumes that learning agents know the structure of the economy and also observe the exogenous processes, i.e., they use a correct learning model. In this case, he finds that the adaptive learning mechanism does not have obvious impacts on the volatility and the persistence of economic variables. When he, however, assumes that agents do not know the structure of the economy and only select several important economic indicators to generate and update their beliefs, i.e., using a small learning model, he finds that the adaptive learning mechanism has much greater propagation effects on model variables, and leads to higher volatility and persistence. The empirical relevance of small learning model is also examined. Adam (2005) finds the dynamics correlations of output and inflation match the U.S. data well when agents use a small learning model. Eusepi and Preston (2011) also consider a small learning model in their specification to explore the adaptive learning mechanism as a source of economic fluctuation. They find that, under adaptive learning, the volatility of output is comparable to that under rational expectations, but with a standard deviation of technology shock that is 20% smaller. Meanwhile, the adaptive learning mechanism provides more volatility in investment and hours. Motivated by the success of the small learning model in their previous papers, Slobodyan and Wouters (2012) explore this issue further in an estimated medium-scale DSGE model. They assume that learning agents consider AR(2) learning models and their results suggest that the model with adaptive learning can improve the empirical fit of the model. They also suggest that expectations based on small forecasting models are supported by the survey evidence on inflation expectations.

In our chapter, we discuss the impact of the assumption of adaptive learning in a two-sector DSGE model with sticky prices, housing production, the new setting of the housing market, and the feature of time to build. The main features of our adaptive learning mechanism are following: (i) we use random data that are generated under rational expectations to form initial beliefs; (ii) we consider a constant gain learning algorithm; (iii) we use a simple AR(1) learning model, which is motivated by the success of the small learning model. Our contributions to this literature are: (i) the dynamic impacts of the AR(1) learning model are explored; (ii) the interaction between the adaptive learning mechanism and the feature of time to build is discussed. After writing the Nóttingham Learning Toolbox, we find that the adaptive learning mechanism largely amplifies and propagates the impact of a goods sector technology shock on the economy. We suppose that the amplification and propagation effect added by the adaptive learning mechanism reflects from higher weights on lagged endogenous and exogenous variables in the actual law of motion compare to the rational expectations model. Meanwhile, the adaptive learning mechanism enlarges the impact of the time to build feature on the real house price and makes this variable exhibit more obvious cyclical behaviour.

The sensitivity of the dynamics to the initial beliefs and the updating algorithms under adaptive learning is assessed by Carceles-Poveda and Giannitsarou (2007) in an univariate forward looking linear model. They consider three ways of initializing initial beliefs: one that uses randomly generated data, one that is ad hoc and another that uses an appropriate distribution. They also discuss three algorithms: recursive least squares, stochastic gradient, and constant gain learning. They find that the behaviour of macroeconomic variables depends on both the initial beliefs and the learning algorithms. In particular, for the constant gain algorithms, they find that the value of the gain coefficient is crucial for determining the dynamics of the system.

For the same purpose, we carry out a sensitivity analysis to check for the robustness of our results. While our results are generally robust, we emphasise the following findings. Firstly, we find that the shapes of impulse responses heavily depend on the values of initial beliefs, implying that we should consider average initial beliefs, instead of particular initial beliefs when we make comparisons between impulse responses from rational expectations and adaptive learning models. These results also imply that, under adaptive learning, given a same exogenous shock, the response of the economy depends on the previous economic conditions, while models under rational expectations always produce exactly same responses.[68] Secondly, using the forecasting errors to update current beliefs through the updating process is a fundamental step in the adaptive learning mechanism. The value of the constant gain coefficient that are commonly used in the literature is between 0 – 0.05. Our sensitivity analysis suggests that, when the value of the constant gain coefficient is within this range, the forecasting errors do not have obvious impacts on the dynamics. In contrast, if we consider larger values, 0.1 and 0.2, i.e., agents assign more weights on recent data, the forecast errors have obvious impacts on the dynamics.

Furthermore, while there is only one type of agents in the models under rational expectations or adaptive learning, it is more realistic to assume that we have both types of agents simultaneously, and this case is referred to as heterogenous expectations. There are several works that consider the interaction between rational agents and non-rational agents in DSGE models. Branch and McGough (2009) derive aggregate IS and AS relations that

[68]For example, the growth of GDP is on its long run trend in 2002 and 2006, but an identical monetary policy shock will have different impact between these two years because households have different expectations formation.

are consistent with heterogeneous expectations from micro-founded models. In addition, they discuss the impact of heterogeneous expectations on the determinacy property of their model. In their setting, adaptive agents use constant beliefs to form expectations, and rational agents are partially rational, i.e., they do not know the existence of adaptive agents.[69] By taking one step further, Branch and McGough (2010) consider the dynamic effect of heterogeneous expectations in a DSGE model. The bounded rational agents use regression methods to obtain the coefficients, which are constant over time. Rational agents take bounded rational agents' beliefs into account. The solution of the model is called a restricted perceptions equilibria. They find that heterogeneous expectations can amplify and propagate the effect of shocks on model variables. Besides, they find that an increase in the proportion of bounded rational agents magnifies the propagation effect under the assumption of heterogeneous expectations. Fuster, Laibson and Mendel (2010) also consider the dynamic effect of heterogeneous expectations in a DSGE model. In their setting, rational agents form their expectations based upon the RE model (referred to as rational expectations), while intuitive agents form their beliefs using misspecified intuitive model (referred to as intuitive expectation). The weighted average of the two is called natural expectations. They show that, under intuitive expectations, the model variables can deviate from steady state at long horizons. In their model, the beliefs of intuitive agents are constant as they are not adjusted by the forecasting errors, and rational agents are not fully rational since they do not know the existence of intuitive agents.

In the second half of this chapter, we will discuss the impact of heterogeneous expectations with rational agents and learning agents. In our setting, the learning agents behave exactly the same way as described before. Meanwhile, we consider two types of rational agents: (i) partially rational agents, who do not know the existence of learning agents; (ii) fully rational agents, who know the existence of learning agents and take learners' beliefs into ac-

[69]The bounded rational agents use $k_{t+1} = \phi k_t$ to forecast the forward variables. The coefficients are constant over time, but expectations change when k_t changes. In contrast, the learning agents use $k_{t+1} = \phi_{t-1} k_t$, in which the beliefs ϕ_{t-1} are time-varying.

count. Our contributions to this literature are that (i) we consider learning agents under heterogeneous expectations, and (ii) we compare two cases of heterogeneous expectations, i.e., one with partially rational agents and one with fully rational agents. We obtain two findings from our impulse response analysis, given that two types of agents have equal weights. Firstly, the responses of variables from heterogeneous expectations are larger than those from rational expectations, implying that the adaptive learning mechanism also has amplification and propagation effects when only a fraction of population is learning agents. Secondly, although two types of agents have equal weights, the impulse responses from heterogeneous expectations are much closer to those from rational expectations than those from adaptive learning. This indicates that, while the adaptive learning mechanism adds amplification and propagation effects on the economy, the existence of rational agents largely weakens these effects. The economic implication of the second finding is quite interesting: when rational agents behave like learning agents in order to push up asset prices for profits, the volatility of the economy increases a great deal.[70] Thirdly, when rational agents are fully rational, the adaptive learning mechanism has a larger amplification and propagation effect on the economy than when rational agents are partially rational. The economic logic is that, if rational agents are fully rational, they should know how learners' beliefs affect the economy, and thus rational agents' beliefs will have further impacts on the economy. In our sensitivity analysis, we consider various compositions of agents, and we find that the adaptive learning mechanism has a larger amplification and propagation effect when the proportion of learning agents is higher. In addition, regardless of the compositions, fully rational agents always bring larger amplification and propagation effects on the economy than partially rational agents.

[70] A quote from the Citigroup's then-chief executive, Chuck Prince: while the music is playing, you have to dance. Large businesses are more likely to be rational than households becuase they have more information and are able to analyse it. However, they may behave like learning agents in order to obtain profit.

4.2 The adaptive learning mechanism

First of all, we discuss the assumption of rational expectations and the assumption of adaptive learning in a context of a log-linearised DSGE model.

4.2.1 DSGE model in a matrix form

In general, a log-linearised DSGE model can be summarised as

$$k_t = a_1 E_t k_{t+1} + a_2 k_{t-1} + b z_t \tag{4.1}$$

and

$$z_t = \mu + \rho z_{t-1} + \varepsilon_t \tag{4.2}$$

The vector k contains endogenous variables, including state variables (those appearing with a lag), forward variables (those appearing with a lead), and static variables.[71] Matrices a_1, a_2, b are the model parameters. The vector z contains exogenous variables, which follow an AR(1) process with an $iid(0, \sigma^2)$ disturbance ε_t. The matrix of constants, μ, can be seen as the growth rates of these exogenous variables and the matrix of autocorrelation coefficients, ρ, describes the persistence of these exogenous variables. The solution of the model is expressed as

$$k_t = T\left(k_{t-1}, z_{t-1}\right)' + V\varepsilon_t$$

which describes how the current endogenous variables, k_t, respond to lagged endogenous variables, k_{t-1}, lagged exogenous variables, z_{t-1}, and current exogenous disturbances, ε_t. The matrices of coefficients T and V are derived from the matrices of the model parameters, a_1, a_2, b.

[71] State variables and forward variables could intersect.

4.2.2 The assumption of rational expectations

In DSGE models, rational expectations (RE) usually is a standard assumption. This assumption implies that agents have full information about the economy, i.e., they know the structure of the true model and the value of the model parameters. Therefore, without exogenous disturbance, they can form beliefs (or expectations) that are consistent with actual outcomes. We refer this type of agents to as 'rational agents'.

Before solving the model, rational agents need to form beliefs about the motion of variables. Assume that rational agents form such beliefs

$$k_t = \phi^r_{t-1} \left(k_{t-1}, z_{t-1}\right)'$$

where ϕ^r_{t-1} are the coefficients in rational agents' beliefs. For simplicity, we also refer it to as beliefs directly. This is known as the perceived law of motion (PLM), which describes how variables behave in rational agents' beliefs. If we forward this PLM one period ahead, we have a matrix equation that describes how rational agents form beliefs about forward variables k_{t+1},

$$E^r_t k_{t+1} = \phi^r_{t-1} \left(k_t, z_t\right)'$$

where E^r_t denotes the expectation operator under rational expectations.[72] Then by plugging these beliefs into the matrix equation 4.1, we can solve the DSGE model and its solution is

$$k_t = T\left(\phi^r_{t-1}\right)\left(k_{t-1}, z_{t-1}\right)' + V\left(\phi^r_{t-1}\right)\varepsilon_t$$

This is known as the actual law of motion (ALM), and $T\left(\phi^r_{t-1}\right)$ and $V\left(\phi^r_{t-1}\right)$ are the matrices of coefficients in ALM. The actual law of motion describes how variables behave in reality given rational agents' beliefs. The process of transforming PLM to ALM, i.e., from ϕ^r_{t-1} to $T\left(\phi^r_{t-1}\right)$ and $V\left(\phi^r_{t-1}\right)$, is known as mapping.[73]

Under rational expectations, rational agents can form beliefs that are

[72] Under rational expecrations, the PLM is constant over time, thus we have $\phi^r_t = \phi^r_{t-1}$.
[73] Both PLM and ALM are constant over time under rational expectations.

consistent with the actual outcomes, i.e., $\phi^r_{t-1} = T\left(\phi^r_{t-1}\right)$. In other words, without exogenous disturbance, the economy behaves in the same way that rational agents perceive. Therefore, rational expectations is a convenient but strong assumption.

4.2.3 The assumption of adaptive learning

Under the assumption of adaptive learning, agents do not have full information about the economy, i.e., they do not know the structure of the true model and the value of the model parameters. Therefore, they are not able to form beliefs that are consistent with actual outcomes. Instead, we assume that they use historical data to form their beliefs, and then make updates using the forecasting errors at the end of each period, thus their beliefs are time-varying. We refer this type of agents to as 'learning agents', and we suppose that the assumption of adaptive learning is more realistic than the assumption of rational expectations.

Generating the initial beliefs Here we discuss how learning agents form their beliefs under the assumption of adaptive learning. We assume that learning agents believe that the forward variable is a linear function of the model variables. They use historical data and simple regression methods to estimate the coefficients, and then use the coefficients to form their initial beliefs.

At the beginning of period t, learning agents need to form expectations for forward variables k_{t+1}. They have information up to $t-1$, i.e., data from $T = 1, ..., t-1$, thus they estimate the initial beliefs, ϕ^l_{t-1}, using the ordinary least square method[74]

$$k_{t-1} = \phi^l_{t-1} x_{t-2} + e_{t-1} \text{ where } x_{t-2} \subseteq (k_{t-2}, z_{t-2})'$$

The values of the estimated coefficients, ϕ^l_{t-1}, which minimise the error, are given by the least squares formula, $\phi^l_{t-1} = \left(\sum_{i=1}^{t-1} x_{i-1} x'_{i-1}\right)^{-1} \sum_{i=1}^{t-1} x_{i-1} k_i$.

[74]Our results are not affected if we include a constant term.

The inclusion of variables in the independent vector, x_{t-2}, depends on the model specification, which will be discussed later.

After obtaining the coefficients, learning agents combine them with lagged endogenous and exogenous state variables to form the perceived law of motion

$$k_t = \phi_{t-1}^l \left(k_{t-1}, z_{t-1}\right)'$$

Then we forward it one period ahead to obtain the matrix equation that describes how learning agents form expectations for forward variables, k_{t+1},[75]

$$E_t^l k_{t+1} = \phi_{t-1}^l \left(k_t, z_t\right)'$$

where E_t^l denotes the expectation operator under adaptive learning. Then by plugging these beliefs into the matrix equation A, we can solve the DSGE model and its solution is

$$k_t = T\left(\phi_{t-1}^l\right)\left(k_{t-1}, z_{t-1}\right)' + V\left(\phi_{t-1}^l\right)\varepsilon_t$$

This is the actual law of motion (ALM), and $T\left(\phi_{t-1}^l\right)$ and $V\left(\phi_{t-1}^l\right)$ are the matrices of coefficients in ALM. The actual law of motion describes how variables behave in reality given learning agents' beliefs.

Since learning agents do not have full information about the economy under the assumption of adaptive learning, they are not able to form beliefs that are consistent with actual outcomes, i.e., $\phi_{t-1}^l \neq T\left(\phi_{t-1}^l\right)$. This implies that the economy behaves differently with what learning agents perceive, and forecasting errors are generated.

Updating the beliefs One important feature of the adaptive learning mechanism is the updating process. As we discussed before, learning agents

[75] At the beginning of period t, x_t is unknown, thus we do not use $E_t k_{t+1} = \phi_t^l x_t$ since ϕ_t^l is obtained after knowing x_t, which requires the expectations about k_{t+1}. Also, we do not use $E_t k_{t+1} = \left(\phi_{t-1}^l\right)^2 x_{t-1}$ as the realised value, $k_t \subset x_t$ should be different with the expected value, $\phi_{t-1}^l x_{t-1}$.

use the estimated coefficients, ϕ_{t-1}^{l}, to form expectations for forward variables, k_{t+1}. After the actual value is realised, learning agents notice that there is a difference between the actual value and the expected value, and the difference is known as the forecasting error. Learning agents partially use the forecasting error to update their beliefs, using the recursive least squares formula

$$\begin{aligned} R_t &= R_{t-1} + g\left(x_{t-1}x_{t-1}' - R_{t-1}\right) \\ \phi_t^l &= \phi_{t-1}^l + gR_t^{-1}x_{t-1}\left(k_t - x_{t-1}'\phi_{t-1}^l\right) \end{aligned}$$

where R_t is the second moment matrix, i.e., $R_t = g\sum_{i=1}^{t} x_{i-1}x_{i-1}'$, g is the gain coefficient, and $\left(k_t - x_{t-1}'\phi_{t-1}^l\right)$ is the forecasting error term.[76] The learning agents use this recursive least squares formula as the updating equations since it describes how the estimated coefficients are updated if we have one more new observation and thus one more new forecasting error.

An interesting point in the adaptive learning mechanism is that the current values of endogenous variable, k_t, is unknown at the beginning of period t, and they are affected by learning agents' beliefs about the forward terms, k_{t+1}. Once the current values, k_t, are realised, learning agents will use the forecasting errors, i.e., the differences between the actual values and the beliefs, to update their beliefs, which in turn affect the values of endogenous variables in the future. Therefore, there may be a feedback effect between the beliefs and actual outcomes, as they reinforce each other.

The learning algorithm The value of the gain coefficient is crucial for the adaptive learning mechanism. On the one hand, if we have a recursive least square gain, $g = t^{-1}$, where t is the total number of observations over time horizon, we have a recursive least squares learning algorithm. In this case, learning agents do not discount past data. They give equal weight to each observation, and thus the weight on newly observed data is decreasing as the total number of observations increases. Therefore, the effect of current

[76] The derivation of the recursive least squares formula is shown in the Appendix.

forecasting error vanishes in the limit.

On the other hand, if we have a constant gain, $g = c$, where c is a constant, we have a constant gain learning algorithm.[77] In this case, learning agents discount past data and continually pay equal attention to newly observed data. Therefore, current forecasting errors matter for learning agents' beliefs even in the limit. In particular, this algorithm represents a simple way to apply the learning mechanism if agents are concerned with potential structural changes at unknown dates.

Model specifications Here we discuss the model specifications, i.e., which variables should be included in the independent vector, x, when using regression methods to form initial beliefs or update existing beliefs.[78] On the one hand, if we assume that agents know the structure of the true model but not the values of the model parameters, they will know which variables are state variables. Therefore, they can select correct endogenous state variables to form and update their beliefs, and we refer this case to as the correct learning model.

On the other hand, if we assume that learning agents do not know the structure of the true model, they will not know which variables are state variables. Therefore, learning agents will select important variables, according to their judgements, to form and update their beliefs. At one extreme, learning agents include all variables in the regression to form and update beliefs for each variable, and we refer this case to as the full learning model. At another extreme, learning agents only include its own lagged terms to form and update beliefs for each variable, and we refer this case to as the AR(1) learning model. In the middle between these two extremes, learning agents may consider several important economic indicators, such as inflation, the nominal interest rate, output, to form and update beliefs for each variable, and we refer this case to as the small learning model.[79]

[77]The constant gain algorithms, which is widely used now, is referred as 'persistent learning dynamics' in Evans and Honkapohja (2001).

[78]In our chapter, we assume that agents are not able to observe the value of exogenous variables.

[79]Small models forecasting falls into the category of 'learning in misspecified models' in

4.2.4 Impulse response analysis

In this section, we examine how the assumption of adaptive learning affects the dynamics of the economy, by comparing the impulse responses from rational expectations and adaptive learning respectively. We build our analysis on two models from the Chapter 3. The first one is the benchmark model with the new setting of the housing market.[80] The second one is the benchmark model with the new setting of the housing market and the feature of time to build, as we also examine the interaction between the adaptive learning mechanism and the feature of time to build.

Initial beliefs, learning algorithm, and model specification The shapes of impulse responses depend on learning agents' initial beliefs, which in turn are determined by the sequence of randomly generated data. In our sensitivity analysis, we show that the impulse responses from particular initial beliefs are not representative, and thus it is misleading to compare them with those from rational expectations. In order to overcome this problem, we calculate average initial beliefs using 100 sequences of randomly generated data and then produce impulse responses.[81]

As we are convinced that the recent observations have more valuable information to forecast the future than historical ones, we use the constant gain learning algorithm. In this algorithm, learning agents discount past data and rely more on recent observations. Another reason for using this algorithm is that learning agents acknowledge the existence of structural breaks.

For the model specification, we consider an AR(1) learning model, partially motivated by the successful of small learning model in the literature. We are also convinced that ordinary households use simple techniques, rather than sophisticated models, to forecast the future. The AR(1) learning model

Evans and Honkapohja (2001).

[80]The benchmark model is a two-sector DSGE model with the feature of sticky prices and housing production.

[81]To examine whether the average initial beliefs that we use is representative, we compare it with that produced using another 100 sequences, and we find that they are consistent.

is the simplest learning model, in which learning agents form beliefs for each forward variable by including its own lagged terms only.

The time sequence In the first 40 periods, we generate random data under rational expectations since learning agents need it to generate their initial beliefs. We set the values of variables to zeros in period 41 and the shock is realised in period 42. Given learning agents' initial beliefs, variables start to respond to the shock according to the actual law of motion in period 42. However, learning agents are not able to update their beliefs in period 42, since variables have zero values in the previous period. From period 43, learning agents begin to adjust their beliefs at the end of each period using the forecasting errors, and then the updated beliefs are used at the beginning of subsequent period. Under adaptive learning, both PLM and ALM are time-varying because of the updating process.

Calibration After introducing the assumption of adaptive learning, we need to calibrate the gain coefficient. Its value, however, varies across the literature.[82] Williams (2003) considers various values for the gain coefficient, $g = [0.1, 0.05, 0.03]$, and argues that the way of forming expectations has greater impacts. Carceles-Poveda and Giannitsarou (2007) show that the gain coefficient has large impacts on the evolution of variables, by considering the values of $g = [0.02, 0.2, 0.4]$. Slobodyan and Wouters (2009a) estimate a medium-sized DSGE model, and they calibrate this parameter as $g = [0.01, 0.02, 0.05]$. Milani (2007) estimates a DSGE model under the adaptive learning mechanism, and the estimated value of this parameter is 0.0183. Eusepi and Preston (2011) calibrate this parameter by minimising the sum-of-square distances between the model implied volatility of HP-detrended output and the first autocorrelation coefficients of output growth and the corresponding data moments, and this procedure gives a gain of 0.0029. In

[82] Slobodyan and Wouter (2009) suggest that, for a constant gain learning with the gain parameter g, weight of a data t periods ago is given as $g(1-g)^t$. This weight decreases by 50% in $T = -\frac{\ln 2}{\ln(1-g)} \approx \frac{0.69}{g}$ periods.

sum, according to the literature, a reasonable range of this parameter is between 0.01 and 0.05. For the baseline calibration, we set $g = 0.025$, which produces similar results of using rolling window of 40 observations, i.e., 10 years period for quarterly data. In our sensitivity analysis, we examine the impact of the gain coefficients on the dynamics by considering various values, $g = [0, 0.01, 0.05, 0.1, 0.2]$.

In addition, as we have added two features to increase the internal persistence of our model, we need to calibrate the relevant parameters. For the degree of consumption habit, the value varies in the literature, e.g., 0.65 in Christiano, Eichenbaum and Evans (2005), 0.59 in Smets and Wouters (2003), 0.7 in Smets and Wouters (2007). We set the degree of consumption habit to $\varepsilon_c = 0.32$, which is consistent with Iacoviello and Neri (2010). For the degree of price indexation, the value also varies in the literature, e.g., 1 in Christiano, Eichenbaum and Evans (2005), 0.48 in Smets and Wouters (2003), 0.24 in Smets and Wouters (2007). We set the degree of price indexation to $\iota_\pi = 0.69$, which is also consistent with Iacoviello and Neri (2010).[83]

Impulse responses from the benchmark model with the new setting of the housing market First, we consider the adaptive learning mechanism in the benchmark model with the new setting of the housing market (NHM). Figure 4.1 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$. We can observe that the responses of variables under adaptive learning (dashed line) are much larger than under rational expectations (solid line). For example, the responses of consumption, real house price, new housing production increase by around 40%, 70%, 100% respectively. This implies that the adaptive learning mechanism largely amplifies and propagates the effects of a goods sector technology shock to the economy.[84]

[83]Our sensitivity analysis, which is not included in this chapter, shows that the feature of price indexation is important for the adaptive learning mechanism to have amplification and propagation effects on model variables.

[84]The impulse responses from adaptive learning are obtained in Matlab using the Nottingham Learning Toolbox. The impulse responses from adaptive learning are not available in Dynare, since there is no updating process in this platform, i.e., the coefficients

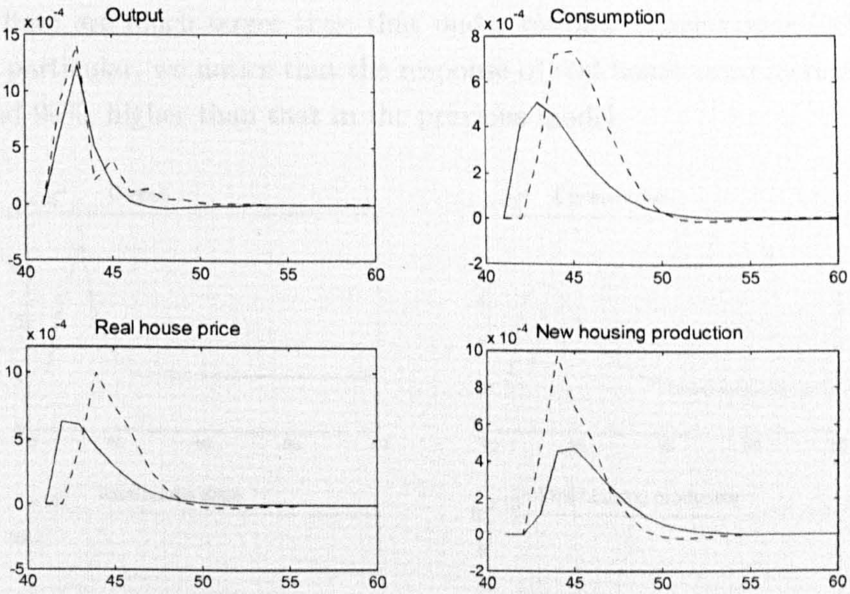

Figure 4.1: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM. The solid line is from rational expectations, and the dashed line is from adaptive learning.

Impulse responses from the benchmark model with the new setting of the housing market and the feature of time to build Next, we consider the adaptive learning mechanism in the benchmark model with the new setting of the housing market (NHM) and the feature of time to build (TTB), in which there is a 4-period lag in new housing production. Figure 4.2 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$. Similar with previous model, we can observe that the responses of variables under adaptive learning (dashed line) are much larger than that under rational expectations (solid line). In particular, we notice that the response of real house price increases by around 90%, higher than that in the previous model.

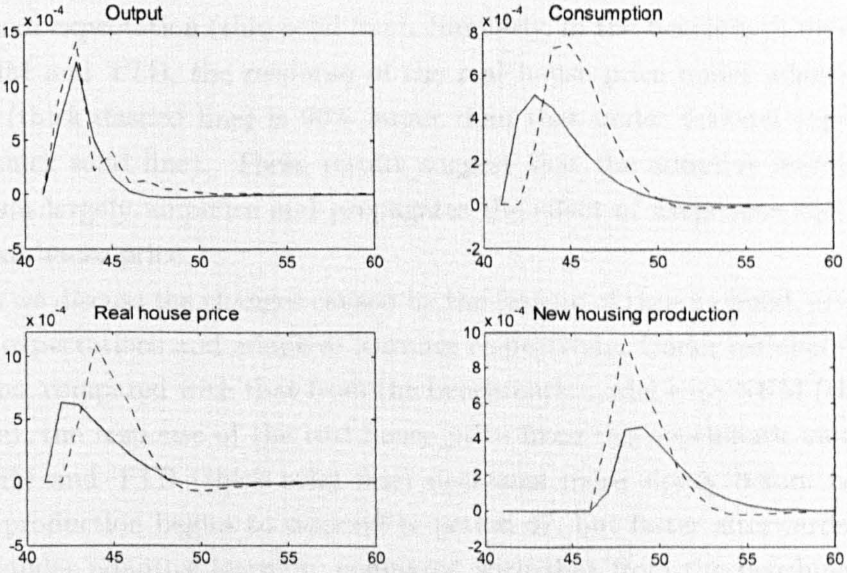

Figure 4.2: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB. The solid line is from rational expectations, and the dashed line is from adaptive learning.

are constant over time in Dynare.

The interaction between the assumption of adaptive learning and the feature of time to build We have shown that the adaptive learning mechanism has an amplification and propagation effect on the real house price in both models. However, the responses of the real house price are different between these two models, and we suppose that the difference is caused by the introduction of the time to build feature in the second model. Figure 4.3 shows the impulse responses of the real house price to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$ from both models. Firstly, we discuss the difference caused by the two alternative assumptions, rational expectations and adaptive learning, within each model. In the benchmark model with NHM, the response of the real house price under adaptive learning (thin dashed line) is 70% larger than that under rational expectation (thin solid line). Similarly, in the benchmark model with NHM and TTB, the response of the real house price under adaptive learning (thick dashed line) is 90% larger than that under rational expectation (thick solid line). These results suggest that the adaptive learning mechanism largely amplifies and propagates the effect of exogenous shocks to the real house price.

Then we discuss the changes caused by the feature of time to build, given rational expectations and adaptive learning respectively. Under rational expectations, compared with that from the benchmark model with NHM (thin solid line), the response of the real house price from the benchmark model with NHM and TTB (thick solid line) decreases more slowly before new housing production begins to respond in period 47, but faster afterwards.[85] Finally, under adaptive learning, compared with that from the benchmark model with NHM (thin dashed line), the response of the real house price from the benchmark model with NHM and TTB (thick dashed line) is around 10% higher at the peak, decreases more slowly before new housing production begins to increase. After period 47, the real house price decreases faster under adaptive learning and is around 8% lower at the though than that under rational expectations. These suggest that the feature of time to build affects the dynamics of real house price by delaying the negative impact from

[85] Thick solid line is below thin solid line after period 47.

new housing production, and its impact is enlarged by the adaptive learning mechanism.

In sum, we have several important results: (i) regardless of the feature of TTB, the adaptive learning mechanism amplifies and propagates the effect of the goods sector technology shock to the real house price; (ii) under both rational expectations and adaptive learning, the feature of TTB modifies the dynamics of the real house price; (iii) the adaptive learning mechanism amplifies the effect of the feature of TTB on the real house price, which displays obvious cyclical behaviour.

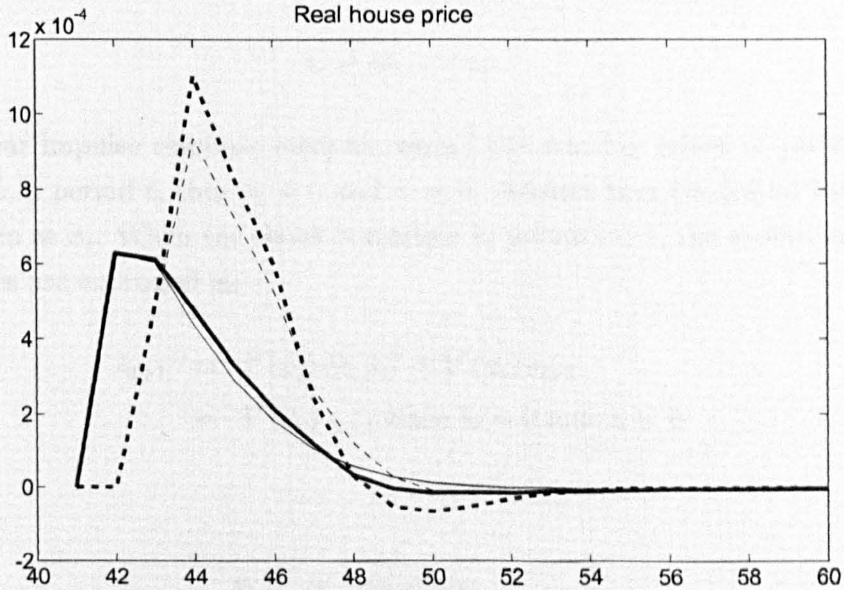

Figure 4.3: Impulse responses of real housing price to a goods sector technology shock. The thin solid line is from benchmark model with NHM under rational expectations, the thin dashed line is from benchmark model with NHM under adpative learning. The thick solid line is from benchmark model with NHM andTTB under rational expectations, and the thick dashed line is from benchmark model with NHM and TTB under adpative learning.

A discussion about the amplification and propagation effects from the adaptive learning mechanism Here we discuss the reasons why the adaptive learning mechanism can provide amplification and propagation effects on the economy. Recall that the actual law of motion is

$$k_t = T\left(\phi_{t-1}\right)\left(k_{t-1}, z_{t-1}\right)' + V\left(\phi_{t-1}\right)\varepsilon_t$$

which shows that the current values of endogenous variables, k_t, depend on the lagged values of endogenous and exogenous variables, k_{t-1} and z_{t-1}, and the current values of disturbance terms, ε_t. The matrices of coefficients, $T\left(\phi_t\right)$ and $V\left(\phi_t\right)$, indicate the weights on these two components respectively. Meanwhile, the exogenous variables have following processes

$$z_t = \rho z_{t-1} + \varepsilon_t$$

In our impulse response analysis, we set the starting values of variables to zeros in period t, thus $k_t = 0$ and $z_t = 0$. Assume that the initial beliefs are given as ϕ_t. When the shock is realised in period $t+1$, the evolutions of variables are expressed as

$$\begin{aligned} k_{t+1} &= T\left(\phi_t\right)\left(k_t, z_t\right)' + V\left(\phi_t\right)\varepsilon_{t+1} \\ &= V\left(\phi_t\right)\varepsilon_{t+1} \text{ since } k_t = 0 \text{ and } z_t = 0 \end{aligned}$$

and

$$\begin{aligned} z_{t+1} &= \rho z_t + \varepsilon_{t+1} \\ &= \varepsilon_{t+1} \text{ since } z_t = 0 \end{aligned}$$

which show that the values of endogenous variables at $t+1$, k_{t+1}, depend on the values of disturbance terms at $t+1$, ε_{t+1}, together with the weight, $V\left(\phi_t\right)$. A higher value of $V\left(\phi_t\right)$ indicates a larger initial response of variables.

If the exogenous shock is realised in period $t+1$ and does not have any persistence afterwards, the disturbance terms in subsequent periods are zero, i.e., $\varepsilon_{t+i} = 0$ for $i = 2, ..., \infty$, and thus the evolutions of variables in period

$t+2$ are expressed as

$$\begin{aligned} k_{t+2} &= T\left(\phi_{t+1}\right)\left[k_{t+1}, z_{t+1}\right]' + V\left(\phi_{t+1}\right)\varepsilon_{t+2} \\ &= T\left(\phi_{t+1}\right)\left[k_{t+1}, z_{t+1}\right]' \text{ since } \varepsilon_{t+2} = 0 \\ &= T\left(\phi_{t+1}\right)\left[V\left(\phi_t\right)\varepsilon_{t+1}, \varepsilon_{t+1}\right]' \end{aligned}$$

and

$$\begin{aligned} z_{t+2} &= \rho z_{t+1} + \varepsilon_{t+2} \\ &= \rho z_{t+1} \text{ since } \varepsilon_{t+2} = 0 \\ &= \rho\varepsilon_{t+1} \end{aligned}$$

We can see that the magnitude of the impulse responses in period $t+2$ actually depends on $T\left(\phi_{t+1}\right)$ and $V\left(\phi_t\right)$. If we take $V\left(\phi_t\right)$ as given, a higher value of $T\left(\phi_{t+1}\right)$ indicates a larger response in period $t+2$.

As we have different actual laws of motion under rational expectations and adaptive learning, we suppose that the shapes of impulse responses are determined by the matrices of coefficients, $T(\phi)$ and $V(\phi)$. Here we take the real house price as an example. Figure 4.1 shows that, compared with that under rational expectations, the real house price responds less in the first period (period 42), and then begins to catch up in the second period (period 43) and achieves a higher peak later under adaptive learning.

We starts from period $t+1$. Under rational expectations, the motion of variables is

$$k_{t+1} = V\left(\phi_t^r\right)\varepsilon_{t+1}$$

Under adaptive learning, the motion of variable is

$$k_{t+1} = V\left(\phi_t^l\right)\varepsilon_{t+1}$$

We infer that, given a same disturbance term, the real house price responds less under adaptive learning in period $t+1$ because the weight on disturbance term is lower under adaptive learning, i.e., $V\left(\phi_t^r\right) > V\left(\phi_t^l\right)$.

In period $t+2$, under rational expectations, the motion of variables is

$$k_{t+2} = T\left(\phi_{t+1}^{r}\right)\left[V\left(\phi_{t}^{r}\right)\varepsilon_{t+1}, \varepsilon_{t+1}\right]'$$

Under adaptive learning, the motion of variable is

$$k_{t+2} = T\left(\phi_{t+1}^{l}\right)\left[V\left(\phi_{t}^{l}\right)\varepsilon_{t+1}, \varepsilon_{t+1}\right]'$$

Given $V\left(\phi_{t}^{r}\right) > V\left(\phi_{t}^{l}\right)$, the real house price responds more under adaptive learning in period $t+2$ only if $T\left(\phi_{t+1}^{r}\right) < T\left(\phi_{t+1}^{l}\right)$, i.e., the actual law of motion has a higher weight on the values of lagged endogenous and exogenous variables under adaptive learning.

Here we summarise the reasons for the different responses of the real house price under rational expectations and adaptive learning. Under adaptive learning, learning agents' perceived law of motion is generated using historical data, and this may mean that the actual law of motion has a relatively higher weight on the lagged variables, $T\left(\phi_{t+i}^{l}\right)$, and a relatively lower wight on the contemporary disturbance term, $V\left(\phi_{t+i}^{l}\right)$. As we discussed above, this is why the real house price responds less in the first place, and then catches up and reaches a higher peak. Therefore, since it makes the motion of variables to rely more on lagged variables, the adaptive learning mechanism has amplification and propagation effects on the economy.

4.2.5 Sensitivity analysis

In this section, we carry out a sensitivity analysis to check the robustness of our result that the adaptive learning mechanism have amplification and propagation effects on the economy. We use the benchmark model with the new setting of the housing market and the feature of time to build. We will examine how the impulse responses are affected by various values of particular initial beliefs and the constant gain coefficient.

Particular initial beliefs Here we discuss why we consider average initial beliefs, instead of particular initial beliefs that are obtained from a particular sequence of randomly generated data. Figure 4.4 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$, given 20 particular initial beliefs. We can observe that the responses of variables varies a great deal, indicating that the initial beliefs, which are determined by the sequences of randomly generated data, are crucially important for the shapes of impulse responses. The first implication is that, given a same exogenous shock, the economy evolves diversely given different history under adaptive learning. In contrast, under rational expectations, we always obtain same impulse responses regardless the history. The second implication is that it is misleading to compare the impulse responses from adaptive learning using particular initial beliefs with those from rational expectations, as the particular initial beliefs are not representative. To overcome this problem, we calculate average initial beliefs using 100 sequences of randomly generated data.

The constant gain coefficient Then we examine how the impulse responses are affected by various values of the gain coefficient. Firstly, we consider values, $g = [0, 0.01, 0.05]$, that are close to our baseline calibration, $g = 0.025$.[86] Figure 4.5 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$. We can observe that the magnitude of the amplification and propagation effect from the adaptive learning mechanism is positively related to the value of the gain coefficient, but it is difficult to observe the difference among the impulse responses from these cases. This suggests that, when its value is within a range of $g \in [0, 0.05]$ that widely used in the literature, the constant gain coefficient does not have an obvious impact on the shapes of impulse responses, implying the forecasting errors through the updating process play a minor role in the adaptive learning mechanism.

[86]While $g = 0$ implies no updating process, $g = [0.01, 0.025, 0.05]$ approximates 25 years, 10 years, 5 years rolling windows respectively for quarterly data.

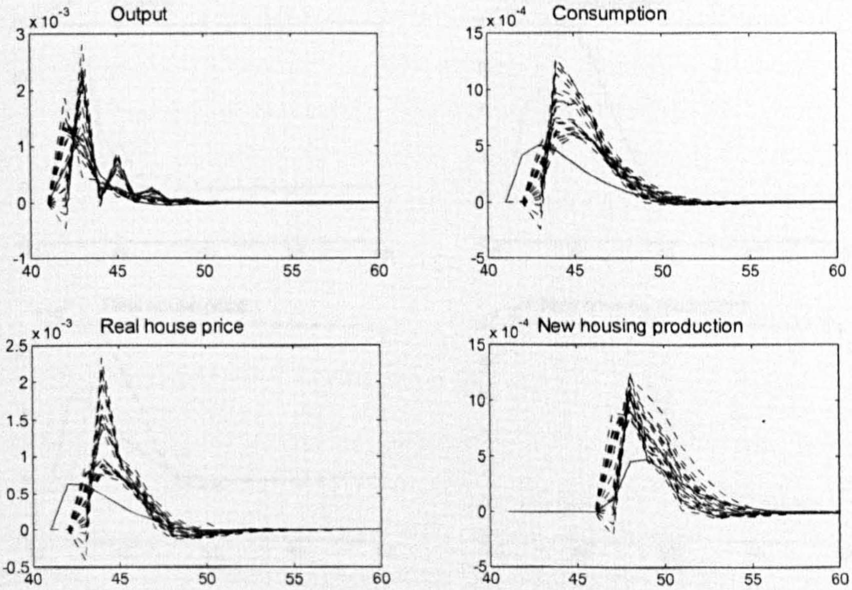

Figure 4.4: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB. The solid line is from rational expectations, and the dashed line is from adaptive learning.

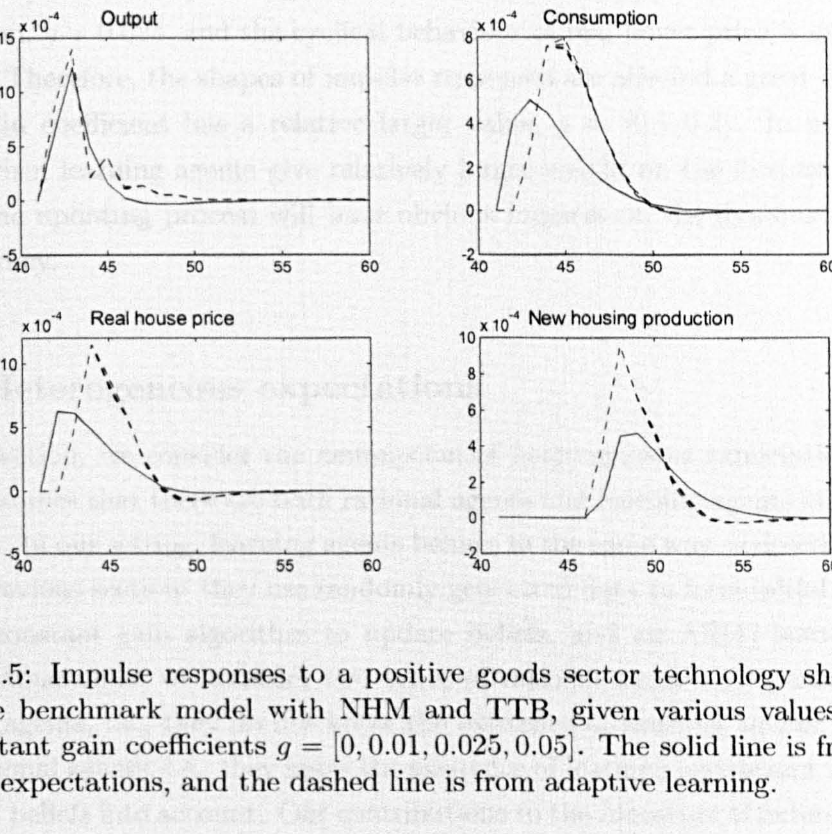

Figure 4.5: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB, given various values of the constant gain coefficients $g = [0, 0.01, 0.025, 0.05]$. The solid line is from rational expectations, and the dashed line is from adaptive learning.

Then we examine the impact of the gain coefficient with larger values, $g = [0.025, 0.1, 0.2]$.[87] Figure 4.6 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{A_c} = 0.01$. The figure shows that, when $g = 0.1$, which approximates 2.5 years of rolling window, the response of consumption is amplified by around 15% more than that in the baseline calibration, $g = 0.025$. When $g = 0.2$, which approximates 1.25 years of rolling window, the responses of consumption and real house price are amplified by around 50% more than that in the baseline calibration, $g = 0.025$, and the cyclical behaviour of real house price is more obvious. Therefore, the shapes of impulse responses are affected a great deal if the gain coefficient has a relative larger value, $g = [0.1, 0.2]$. In other words, when learning agents give relatively larger weight on the forecasting errors, the updating process will have obvious impacts on the dynamics of the economy.

4.3 Heterogeneous expectations

In this section, we consider the assumption of heterogeneous expectations, which assumes that there are both rational agents and learning agents in the economy. In our setting, learning agents behave in the same way as described in the previous section: they use randomly generated data to form initial beliefs, a constant gain algorithm to update beliefs, and an AR(1) learning model. Meanwhile, we consider two types of rational agents: (i) partially rational agents, i.e., they do not know the existence of learning agents; (ii) fully rational agents, i.e., they know the existence of learning agents and take learners' beliefs into account. Our contributions to the literature of heterogeneous expectations are: (i) we consider learning agents under heterogeneous expectations; (ii) we compare two cases of heterogeneous expectations, i.e., one with partially rational agents and one with fully rational agents.

[87] We suppose that $g = 0.4$, which approximates 2.5 quarters rolling window, is too large to consider, as learning agents are not likely to use a very short histroy, i.e., less than one year. If we consider such value, we are able to show that this coefficient has a large impact on dynamics, consistent with Carceles-Poveda and Giannitsarou (2007).

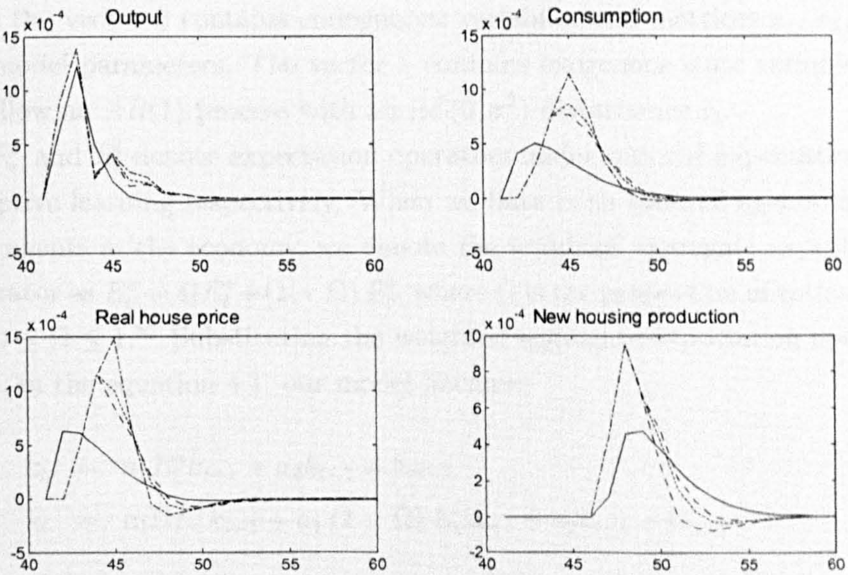

Figure 4.6: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB, given various values of the constant gain coefficients $g = [0.025, 0.1, 0.2]$. The solid line is from rational expectations, the dashed line, the dotted line, and the dash-dot line are from adaptive learning given $g = [0.025, 0.1, 0.2]$, respectively.

4.3.1 Homogeneous expectations and heterogeneous expectations

Under homogeneous expectations, a DSGE model can be summarised into a general form as

$$k_t = a_1 E_t k_{t+1} + a_2 k_{t-1} + b z_{t-1}$$

and

$$z_t = \mu + \rho z_{t-1} + \varepsilon_t$$

in which the vector k contains endogenous variables, and matrices a_1, a_2, b are the model parameters. The vector z contains exogenous state variables, which follow an $AR(1)$ process with an $iid\,(0, \sigma^2)$ disturbance ε_t.

Let E_t^r and E_t^l denote expectation operators under rational expectations and adaptive learning respectively. When we have both rational agents and learning agents in the economy, we denote the weighted aggregate expectation operator as $E_t^a = \Omega E_t^r + (1 - \Omega)\, E_t^l$, where Ω is the proportion of rational agents, $0 \leq \Omega \leq 1$.[88] Substituting the weighted aggregate expectation operator, E_t^a, in the equation 4.1, our model becomes

$$\begin{aligned} k_t &= a_1 E_t^a k_{t+1} + a_2 k_{t-1} + b z_{t-1} \\ k_t &= a_1 \Omega E_t^r k_{t+1} + a_1 (1 - \Omega)\, E_t^l k_{t+1} + a_2 k_{t-1} + b z_{t-1} \end{aligned}$$

and

$$z_t = \mu + \rho z_{t-1} + \varepsilon_t$$

Therefore, both rational agents' and learning agents' expectations are required before solving the model.

4.3.2 The first case of heterogeneous expectations: partially rational agents

Firstly, we assume that rational agents are partially rational, i.e., they know the structure of the model and the values of the model parameters but do not

[88] At two extremes, we have rational expectations if all agents are rational agents, $\Omega = 1$, and adaptive learning if all agents are learning agents, $\Omega = 0$.

know the existence of learning agents. In this case, rational agents believe that everyone is rational in the economy, and thus they form beliefs that are consistent with the solution of the model under rational expectations. They cannot realise their mistakes and make adjustments, thus their beliefs are constant over time. Meanwhile, learning agents use historical data to form their initial beliefs and then update their beliefs using the forecasting errors at the end of each period.[89]

We express rational agents' beliefs as $E_t^r k_{t+1} = \phi^r [k_t, z_t]'$, and learning agents' beliefs as $E_t^l k_{t+1} = \phi_{t-1}^l x_t$. After plugging these two expressions into the model, we have

$$k_t = a_1 \Omega \phi^r [k_t, z_t]' + a_1 (1 - \Omega) \phi_{t-1}^l [k_t, z_t]' + a_2 k_{t-1} + b z_t$$

and

$$z_t = \mu + \rho z_{t-1} + \varepsilon_t$$

While rational agents' beliefs, ϕ^r, are constant over time, learning agents' beliefs, ϕ_t^l, are time-varying. After solving the model, i.e., transfer the perceived law of motion to the actual law of motion, we know how the economy responds to exogenous shocks.

4.3.3 The second case of heterogeneous expectations: fully rational agents

Secondly, we assume that rational agents are fully rational, i.e., they know the existence of learning agents and also take learners' beliefs into account. Learning agents behave in a same way described in the previous case. As fully rational agents take learners' beliefs into account, they can form beliefs that are consistent with actual outcomes.

The time sequence is following: (i) in the initial period, learning agents use historical data to form their initial beliefs; (ii) at the end of each subsequent period, learning agents update their beliefs using the forecasting errors

[89] Branch and McGough (2009.2010) consider rational agents and the other type of agents that use fixed beliefs.

through a constant gain algorithm; (iii) at the beginning of each period, rational agents take learning agents' beliefs into account, and then form beliefs that are consistent with actual outcomes. Since learning agents' beliefs are time-varying, rational agents' beliefs are also time-varying.[90]

We express learning agents' beliefs as $E_t^l k_{t+1} = \phi_{t-1}^l [k_t, z_t]'$. After plugging this expression into the model, we have

$$k_t = a_1 \Omega E_t^r k_{t+1} + a_1 (1 - \Omega) \phi_{t-1}^l [k_t, z_t]' + a_2 k_{t-1} + b z_{t-1}$$

and

$$z_t = \mu + \rho z_{t-1} + \varepsilon_t$$

After transferring the perceived law of motion to the actual law of motion, we know how the economy will respond to exogenous shocks.

4.3.4 Calibration

Under heterogeneous expectations, we need to calibrate the proportion of rational agents, Ω. Similar with the gain coefficient, we calibrate it in a way that is consistent with the literature. Fuster, Laibson and Mendel (2010) assume that rational expectations and intuitive expectations are equally weighted, i.e., $\Omega = 0.5$. Branch and McGough (2009) consider various values, $\Omega = [1, 0.99, 0.9, 0.7]$, to examine the determinacy properties of their model. Branch and McGough (2010) also use various values, $\Omega = [0.1, 0.3, 0.5, 0.7, 0.9]$, to show that a decrease in the proportion of rational agents magnifies the model's propagation mechanism. Therefore, as there is no convincing empirical evidence to calibrate this parameter, we set it to $\Omega = 0.5$ in the baseline calibration, implying that a half of the population is rational agents and the other half is learning agents.

[90] For comparison, Branch and McGough (2010) assume that the model has a constant solution, which is referred to as restricted perceptions equilibria.

4.3.5 Impulse response analysis

Here we compare the impulse responses from heterogeneous expectations with those from rational expectations and adaptive learning. We use average initial beliefs, which are obtained from 100 sequences of randomly generated data. Figure 4.7 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{A_c} = 0.01$, given $\Omega = 0.5$, i.e., 50% of the population is rational agents. Firstly, the impulse responses from heterogeneous expectations are larger than that from rational expectations, for example, consumption responds around 8% more, and real house price responds around 6% more. These results imply that the adaptive learning mechanism also have amplification and propagation effects when only a fraction of the population is learning agents. Secondly, when two types of agents have equal weights, the impulse responses from heterogeneous expectations are much closer to those from rational expectations. This indicates that, while the adaptive learning mechanism adds amplification and propagation effects to the economy, the existence of rational agents weakens this effect a great deal. This result suggests that, if rational agents behave in the same way as learning agents, the volatility of the economy will increase a great deal. For example, agents may want to push up the assets prices in order to obtain profit even if they know there may be a bubble.

Thirdly, we also notice that there are differences between the impulse responses from two cases of heterogeneous expectations. The comparison results show that, when rational agents are fully rational (dot-dash line), the adaptive learning mechanism has a larger amplification and propagation effect than when rational agents are partially rational (dotted line). The economic logic is that, if rational agents know the existence of learning agents and take learners' beliefs into account, they will know how learners' beliefs affect the economy. Given this knowledge, fully rational agents form beliefs that have further impacts on the economy. Here we use an example to illustrate our argument. Assume there is no exogenous shock at period t. Learning agents expect that the real house price will increase by 10% in next period according to their historical experience. In the first case, partially rational agents do not know the existence of learning agents, and they expect no change in the real house price. Therefore, the actual change in the real house price should be positive, as learning agents increase their demand for housing. In contrast, in the second case, fully rational agents know learning agents's beliefs, and also know the positive impact of these beliefs on the real house price, thus they form beliefs that are consistent with actual change in the real house price, which is positive. In other words, fully rational agents also expect an increase in the real house price in next period. As a result, we can infer that the actual change in the real house price is larger in the second case than in the first case, since the demand for housing is higher in the second case, i.e., both fully-rational agents and learning agents demand more housing.

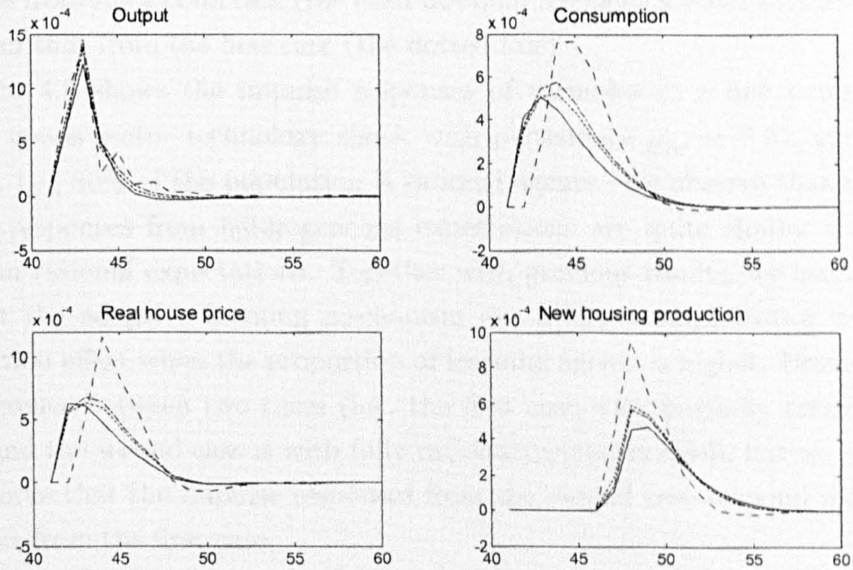

Figure 4.7: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB, given $\Omega = 0.5$. The solid line is from rational expectations, and the dashed line is from adaptive learning, the dotted line is from the first case of HE, and the dash-dot line is from the second case of HE.

4.3.6 Sensitivity analysis

In this section, we examine how the impulse responses from heterogeneous expectations are affected by a change in the proportion of rational agents. We consider two alternative values, $\Omega = [0.2,\ 0.8]$. Also, we discuss the difference between the first case of HE, i.e., partially rational agents, and the second case of HE, i.e., fully rational agents.

Figure 4.8 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$, given $\Omega = 0.2$, i.e., 20% of the population are rational agents. We observe that the impulse responses from heterogeneous expectations are larger than that from rational expectations, for example, both consumption and the real house price respond around 30% more. As expected, we observe that the impulse responses from the second case (the dash-dot line) responds around 10% more than than that from the first case (the dotted line).

Figure 4.9 shows the impulse responses of variables to a one percent positive goods sector technology shock with persistence $\rho_{Ac} = 0.01$, given $\Omega = 0.8$, i.e., 80% of the population is rational agents. We observe that the impulse responses from heterogeneous expectations are quite similar with that from rational expectations. Together with previous results, we can argue that the adaptive learning mechanism has a larger amplification and propagation effect when the proportion of learning agents is higher. Besides, the difference between two cases (i.e., the first case with partially rational agents and the second case is with fully rational agents) is small, but we can still observe that the impulse responses from the second case respond more than that from the first case.

4.4 Conclusion

In this chapter, we discuss the impact of the adaptive learning mechanism, which is an alternative way to form expectations for the future, in a two-sector DSGE model with the new setting of the housing market and the feature of time to build. The main features of our adaptive learning mech-

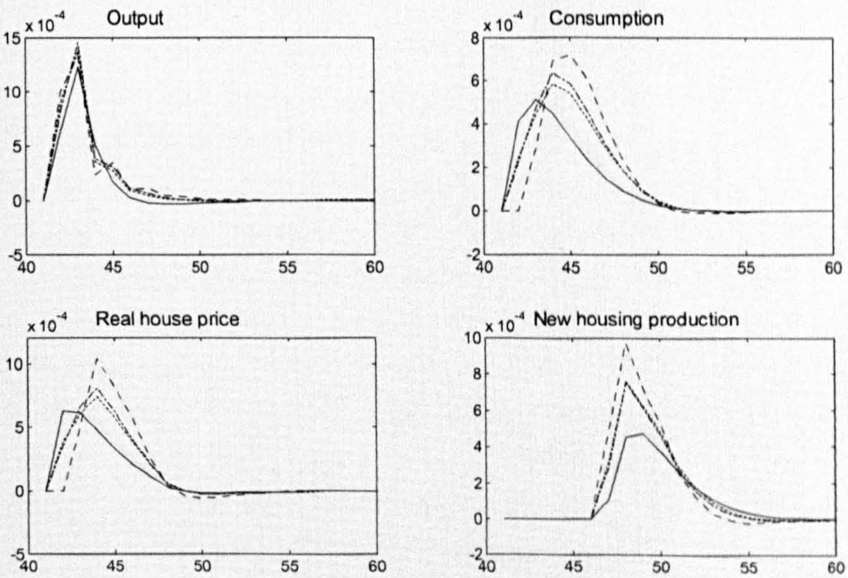

Figure 4.8: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB, given $\Omega = 0.2$. The solid line is from rational expectations, and the dashed line is from adaptive learning, the dotted line is from the first case of HE, and the dash-dot line is from the second case of HE.

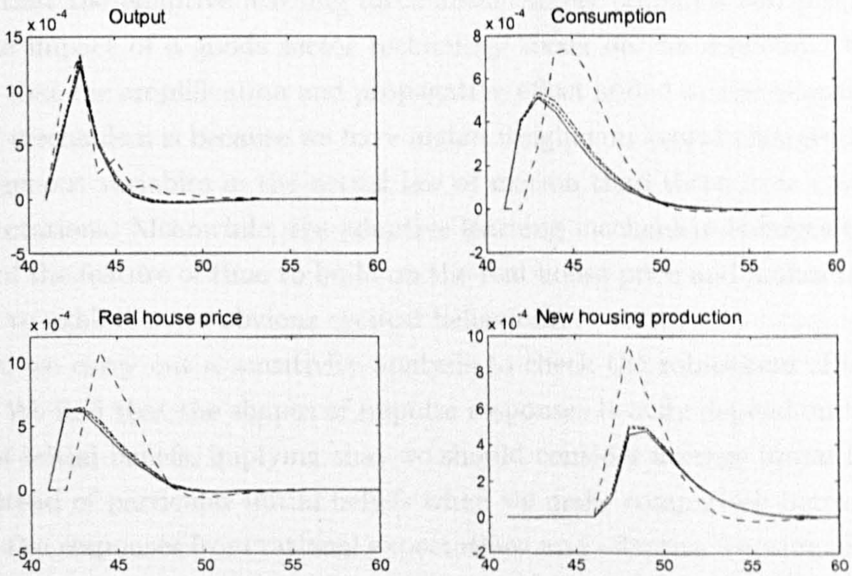

Figure 4.9: Impulse responses to a positive goods sector technology shock from the benchmark model with NHM and TTB, given $\Omega = 0.8$. The solid line is from rational expectations, and the dashed line is from adaptive learning, the dotted line is from the first case of HE, and the dash-dot line is from the second case of HE.

anism are the followings: (i) we use random data that are generated under rational expectations to form initial beliefs; (ii) we consider a constant gain learning algorithm; (iii) we use a simple $AR(1)$ learning model, which is motivated by the success of small learning model. Our contributions to this literature are that we consider the dynamic impacts of the $AR(1)$ learning model and discuss the interaction between the adaptive learning mechanism and the feature of time to build. Using the Nottingham Learning Toolbox, we find that the adaptive learning mechanism largely amplifies and propagates the impact of a goods sector technology shock on the economy. We suppose that the amplification and propagation effect added by the adaptive learning mechanism is because we have higher weights on lagged endogenous and exogenous variables in the actual law of motion than those from rational expectations. Meanwhile, the adaptive learning mechanism enlarges the impact of the feature of time to build on the real house price and makes this variable to exhibit more obvious cyclical behaviour.

Next, we carry out a sensitivity analysis to check the robustness of our results. We find that the shapes of impulse responses heavily depend on the values of initial beliefs, implying that we should consider average initial beliefs, instead of particular initial beliefs when we make comparison between the impulse responses from rational expectations and adaptive learning. Besides, our sensitivity analysis suggests that, when the value of constant gain coefficient is relatively small, $0 - 0.05$, the forecasting errors do not have obvious impacts on the impulse responses. In contrast, if we consider larger values, 0.1 and 0.2, implying that agents assign higher weights on recent data, the forecast errors have obvious impacts.

Last, we consider the assumption of heterogeneous expectations, i.e., there are both rational agents and learning agents in the economy. Learning agents behave in the same way as described before. Meanwhile, we consider two types of rational agents: (i) partially rational agents, i.e., they do not know the existence of learning agents; (ii) fully rational agents, i.e., they know the existence of learning agents and take learners' beliefs into account. Our contributions to this literature are: (i) we consider learning agents under heterogeneous expectations; (ii) we compare two cases of heterogeneous expectations, i.e., one with partially rational agents and one with fully rational agents. We obtain two findings from our impulse response analysis, given that two types of agents have equal weights. Firstly, the responses of variables from heterogeneous expectations are larger than that from rational expectations, implying that the adaptive learning mechanism also has amplification and propagation effects when only a fraction of the population is learning agents. Secondly, when two types of agents have equal weights, the impulse responses from heterogeneous expectations are much closer to that from rational expectations than that from adaptive learning. This indicates that, while the adaptive learning mechanism adds amplification and propagation effects on the economy, rational agents may play role of reducing economic volatility. Thirdly, when rational agents are fully rational, i.e., they know the existence of learning agents and take learners' beliefs into account, the adaptive learning mechanism has a larger amplification and propagation effect on the economy than that when rational agents are partially rational. The economic logic is that, if rational agents are fully rational, they will know how learners' beliefs affect the economy, and thus rational agents' beliefs will have further impacts on the economy. This result holds in the sensitivity analysis.

4.A Appendix to Chapter 4

4.A.1 The recursive least squares learning algorithm

In fitting the equation $k_t = \phi_{t-1}x_{t-1} + e_{t-1}$ using data $T = 1, ..., t$. The value of the coefficient ϕ_t which minimises the error is given by the least squares formula

$$\phi_t = \left(\sum_{i=1}^{t} x_{i-1}x'_{i-1}\right)^{-1} \sum_{i=1}^{t} x_{i-1}k_i$$

If we consider matrix form, then k_{t-1} is $m * 1$, x_{t-1} is $n * 1$, ϕ_t is $m * n$. Remember in econometrics, if the model is $y_t = \beta x_t + \varepsilon_t$, $t = 1, ..., n$, the OLS estimator for β is $\widehat{\beta} = \frac{\sum_{i=1}^{n} x_t y_t}{\sum_{i=1}^{n} x_t^2}$.

Here we derive the recursive form for the OLS estimator. Let

$$S_t = \sum_{i=1}^{t} x_{i-1}x'_{i-1}$$

and we have that

$$S_t = S_{t-1} + x_{t-1}x'_{t-1}$$

Then we can write

$$
\begin{aligned}
\phi_t &= \left. \sum_{i=1}^{t} x_{i-1}x'_{i-1} \right)^{-1} \sum_{i=1}^{t} x_{i-1}k_i \\
&= (S_t)^{-1} \sum_{i=1}^{t} x_{i-1}k_i \\
&= (S_t)^{-1} \sum_{i=1}^{t} x_{i-1}k_i + (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i - (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i \\
&= \phi_{t-1} + (S_t)^{-1} \sum_{i=1}^{t} x_{i-1}k_i - (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i \\
&= \phi_{t-1} + (S_t)^{-1} x_{t-1}k_t + (S_t)^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i - (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i \\
&= \phi_{t-1} + (S_t)^{-1} x_{t-1}k_t + (S_t)^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i - (S_t)^{-1} \frac{S_t}{S_{t-1}} \sum_{i=1}^{t-1} x_{i-1}k_i \\
&= \phi_{t-1} + (S_t)^{-1} x_{t-1}k_t + \left(\frac{S_{t-1}}{S_t} - 1 \right) (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i
\end{aligned}
$$

since $S_t = S_{t-1} + x_{t-1}x'_{t-1} \rightarrow \frac{S_{t-1}}{S_t} - 1 = -\frac{x_{t-1}x'_{t-1}}{S_t}$, thus

$$
\begin{aligned}
\phi_t &= \phi_{t-1} + (S_t)^{-1} x_{t-1}k_t - \frac{x_{t-1}x'_{t-1}}{S_t} (S_{t-1})^{-1} \sum_{i=1}^{t-1} x_{i-1}k_i \\
&= \phi_{t-1} + (S_t)^{-1} x_{t-1}k_t - \frac{x_{t-1}x'_{t-1}}{S_t} \phi_{t-1} \\
&= \phi_{t-1} + (S_t)^{-1} x_{t-1} \left(k_t - x'_{t-1}\phi_{t-1}\right)
\end{aligned}
$$

Now we have a recursive form, but some steps are required to obtain a general form.

To write the above RLS algorithm as it usually appears in the adaptive learning literature, we can define $R_t = S_t/t$. Using the definition of R_t, we get

$$
\begin{aligned}
R_t &= \frac{1}{t}S_t = \frac{1}{t}\left(S_{t-1} + x_{t-1}x'_{t-1}\right) = \frac{1}{t}S_{t-1} + \frac{1}{t}x_{t-1}x'_{t-1} \\
&= \frac{1}{t-1}S_{t-1} - \frac{1}{t-1}S_{t-1} + \frac{1}{t}S_{t-1} + \frac{1}{t}x_{t-1}x'_{t-1} \\
&= \frac{1}{t-1}S_{t-1} - \frac{1}{t\left(t-1\right)}S_{t-1} + \frac{1}{t}x_{t-1}x'_{t-1} \\
&= \frac{1}{t-1}S_{t-1} + \frac{1}{t}\left(x_{t-1}x'_{t-1} - \frac{1}{t-1}S_{t-1}\right) \\
&= R_{t-1} + \frac{1}{t}\left(x_{t-1}x'_{t-1} - R_{t-1}\right)
\end{aligned}
$$

and the recursion becomes

$$R_t = R_{t-1} + \frac{1}{t}\left(x_{t-1}x'_{t-1} - R_{t-1}\right)$$

and

$$\phi_t = \phi_{t-1} + \frac{1}{t}R_t^{-1}x_{t-1}\left(k_t - x'_{t-1}\phi_{t-1}\right)$$

which is the recursive least squares learning algorithm that appears in our chapter.

4.A.2 The Matlab files in the Nottingham Learning Toolbox

- model.m: rewrite the DSGE models into a form that can be applied to this toolbox. Unlike Dynare, we cannot write the model directly into the Matlab, we need to rewrite the model using the lag dummy method before applying it to the learning toolbox,
- newsolab.m: solve the model
- pvalue.m and vmap.m: rewrite the solutions in a more friendly way for (i) comparing with Dynare results and (ii) facilitating following programs
- LEARNING.M: defines the setting for learning, e.g., shock sequence,

number of periods for the simulations, initial periods, gain coefficient, choice of small models.

- IRF_RUN_RE.M: produce impulse responses functions under rational expectations
- IRF_RUN_AL.M: produce impulse responses functions under adaptive learning
- IRF_RUN_HETERO1.M: produce impulse responses functions under heterogeneous expectations with partially rational agents
- IRF_RUN_HETERO2.M: produce impulse responses functions under heterogeneous expectations with fully rational agents
- IRF_DO_PLOTS.M: plots the results of impulse responses functions
- IRF_DO_PLOTS_CH4.M: plots the results of impulse responses functions for chapter 4
- INI_BEL.m: generate initial beliefs and second moment matrix
- INI_BEL_DO_AVERAGE.m: generate average initial beliefs and average second moment matrix
- hpf.m: applies the Hodrick-Prescott filter to a series, with HP parameter lambda_hp.
- IRF_STATISTICS.M: summarise the statistics of impulse responses functions

4.A.3 The solution method in the Nottingham Learning Toolbox

The classification of variables We classify the variables into several group according to the Dynare User Guide.

- static variables: variables that have t term.

- purely predetermined variables: variables that have $t-1$ term or have both t and $t-1$ terms
- purely forward-looking variables: variables that have $t+1$ term or have both t and $t+1$ terms
- predetermined and forward-looking variables: variables that have both $t-1$ term and $t+1$ or have t, $t-1$, and $t+1$ terms
- state variables: variables that have $t-1$ term.

Rewriting and solving the model The Nottingham learning toolbox uses the Klein solution method, thus we need to rewrite our mode using the lag dummy method. We define x as

$$\begin{aligned} x_t &= [z_{t-1}, k_{t-1}, k_t, f_t, v_t] \\ x_{t+1} &= [z_t, k_t, k_{t+1}, f_{t+1}, v_{t+1}] \end{aligned}$$

where z is exogenous variable, k is state variable, f is forward variable, and v is static variable. Note that (i) the state variable appears twice, and we refer the first group as dummy variables, (ii) if a variable has both $t-1$ and $t+1$ terms, we treat it as state variable.

Before solving the model using the Klein method, we need to write it into following form

$$AA * x_{t+1} = BB * x_t$$

From the Klein method, the solution form is

$$\begin{pmatrix} z_{t-1} \\ k_{t-1} \end{pmatrix} = PP * \begin{pmatrix} z_{t-2} \\ k_{t-2} \end{pmatrix} = \begin{pmatrix} pp11 & pp12 \\ pp21 & pp22 \end{pmatrix} * \begin{pmatrix} z_{t-2} \\ k_{t-2} \end{pmatrix}$$

and

$$\begin{pmatrix} k_t \\ f_t \\ v_t \end{pmatrix} = FF * \begin{pmatrix} z_{t-1} \\ k_{t-1} \end{pmatrix} = \begin{pmatrix} ff11 & ff12 \\ ff21 & ff22 \\ ff31 & ff32 \end{pmatrix} * \begin{pmatrix} z_{t-1} \\ k_{t-1} \end{pmatrix}$$

then we arrange the solution as

$$\begin{pmatrix} k_t \\ k_t \\ f_t \\ v_t \end{pmatrix} = Pbar * \begin{pmatrix} z_{t-1} \\ k_{t-1} \\ k_{t-1} \\ f_{t-1} \\ v_{t-1} \end{pmatrix} = \begin{pmatrix} pp21 & pp22 & 0 & 0 & 0 \\ ff11 & ff12 & 0 & 0 & 0 \\ ff21 & ff22 & 0 & 0 & 0 \\ ff31 & ff32 & 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} z_{t-1} \\ k_{t-1} \\ k_{t-1} \\ f_{t-1} \\ v_{t-1} \end{pmatrix}$$

Note that $\begin{pmatrix} pp21 & pp22 \end{pmatrix}$ is same as $\begin{pmatrix} ff11 & ff12 \end{pmatrix}$. We can compare the values in $Pbar$ with the solution from Dynare to check the accuracy of the model file in Matlab.

Generating initial beliefs For learning agents, we generate initial beliefs Phi using randomly generated data,

$$\begin{pmatrix} k_t \\ f_t \\ v_t \end{pmatrix} = Phi * [k_{t-1}, f_{t-1}, v_{t-1}]'$$

We need to fit Phi in KK, which is a matrix consistent with x, then we have

$$\begin{pmatrix} z_t \\ k_t \\ k_{t+1} \\ f_{t+1} \\ v_{t+1} \end{pmatrix} = KK * \begin{pmatrix} z_{t-1} \\ k_{t-1} \\ k_t \\ f_t \\ v_t \end{pmatrix}, \text{ where } KK = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & phi & phi & phi \\ 0 & 0 & phi & phi & phi \\ 0 & 0 & phi & phi & phi \end{pmatrix}$$

then we can replace $t+1$ terms.

For the partially rational agents in the first case of heterogeneous expectations, we assume that they use solution under rational expectations as their initial beliefs. Here we transfer the solution under rational expectation $Pbar$ to partially rational agents' belief LL, which is also a matrix consistent with

x_t,

$$\begin{pmatrix} z_t \\ k_t \\ k_{t+1} \\ f_{t+1} \\ v_{t+1} \end{pmatrix} = LL * \begin{pmatrix} z_{t-1} \\ k_{t-1} \\ k_t \\ f_t \\ v_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ ff11 * \rho & 0 & ff12 & 0 & 0 \\ ff21 * \rho & 0 & ff22 & 0 & 0 \\ ff31 * \rho & 0 & ff32 & 0 & 0 \end{pmatrix} * \begin{pmatrix} z_{t-1} \\ k_{t-1} \\ k_t \\ f_t \\ v_t \end{pmatrix}$$

where we need to add ρ since $z_t = \rho z_{t-1}$.

Rational expectations and adaptive learning After writing the model using the lagged dummy method, we can solve this model under rational expectations and adaptive learning respectively. We need to write the model into

$$\begin{aligned} & MM * [z_t, k_t, 0, 0, 0]' + MMM * [0, 0, k_{t+1}, f_{t+1}, v_{t+1}]' \\ = \; & NN * [z_{t-1}, k_{t-1}, k_t, f_t, v_t] \end{aligned}$$

where MMM represents the expectational terms

Under rational expectations, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM + MMM \\ BB &= NN \end{aligned}$$

Under adaptive learning, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM \\ BB &= NN - MMM * KK \end{aligned}$$

Heterogeneous expectations Under heterogeneous expectations, we rewrite the model into a more general form as

$$\begin{aligned} & MM * [z_t, k_t, 0, 0, 0]' + MMR * [0, 0, k_{t+1}, f_{t+1}, v_{t+1}]' \\ & +MML * [0, 0, k_{t+1}, f_{t+1}, v_{t+1}]' \\ = & NN * [z_{t-1}, k_{t-1}, k_t, f_t, v_t] \end{aligned}$$

Actually, given this general form, we can solve the model under rational expectations, adaptive learning, the first case and the second case of heterogeneous expectations.

If we want to solve the model under rational expectations, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM + MMR + MML \\ BB &= NN \end{aligned}$$

If we want to solve the model under adaptive learning, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM \\ BB &= NN - MMR * KK - MML * KK \end{aligned}$$

If we want to solve the model under the first case of heterogeneous expectation, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM \\ BB &= NN - MMR * LL - MML * KK \end{aligned}$$

If we want to solve the model under the second case of heterogeneous expectation, the matrices of the coefficients for the model to be solved are

$$\begin{aligned} AA &= MM + MMR \\ BB &= NN - MML * KK \end{aligned}$$

5 A Discussion of the Default Rate and the Loan to Value Ratio

5.1 Introduction

The assumption of perfect credit market is commonly used in DSGE models. Recently, however, researchers have raised concerns over the feature of credit market imperfections, as they suggested that (i) it is consistent with reality and (ii) the interaction between credit limits and asset prices has important impacts on dynamics.

The seminal work by Bernanke, Gertler and Gilchrist (1999) introduces the costly verification problem (or the agency problem) to producers of final goods in a DSGE model. They successfully show that credit market imperfections can amplify and propagate the impacts of shocks to the economy, and they refer this feature as the financial accelerator mechanism. Another closely related work by Carlstrom and Fuerst (1997) also considers a debt contract problem, but in a model with flexible prices. They assume that the agency problem applies to producers of investment goods, who produce capital directly from output goods. Their credit mechanism can explain the output movement in reality.

The agency problem has been introduced to the housing market by Aoki, Proudman and Vlieghe (2004), and they show that this mechanism amplifies and propagates the effect of monetary policy shocks on housing investment, house price and consumption. They, however, keep the default rate constant when they solve their model. Besides, they assume that credit unconstrained households (lenders) rent housing from homeowners (borrowers), who take loans to purchase housing. Because homeowners are the only agents that are buying housing, a higher demand for housing in an economic upturn implies that a higher volume of loans is guaranteed. Moreover, Iacoviello and Neri (2010) also discuss the impacts of the credit market imperfections on the housing market. In their model, impatient households take loans to purchase housing, and they need to provide collateral because of credit market imperfections. Hence, housing also plays a role of collateral, and the borrowing constraint is related to housing values. In their model, however, the debt is fully collateralised, and there is no possibility of default. They calibrate the loan to value ratio using the empirical value, but we suppose that, in the model without the possibility of default, the appropriate loan to value ratio should be fixed at one.

In this chapter, we introduce the agency problem into the housing market, and we are interested in the dynamics of the default rate and the loan to value ratio. In particular, we assume that an idiosyncratic shock affects the housing assets directly. Specifically, the value of housing assets can be affected by many possible random events, such as building a new school and discovering natural resources.[91] Meanwhile, we assume that both lenders and borrowers can purchase housing. Our impulse response analysis shows that, given a positive goods sector technology shock, the default rate exhibits countercyclical behaviour, which is consistent with our empirical analysis. The loan to value ratio, however, is also countercyclical, while our empirical analysis suggests procyclical behaviour. The reason for this is that, in an economic upturn, credit constrained households have less housing in the housing market, thus the volume of loans they receive also decreases, leading to a fall in the loan to value ratio. Therefore, the inconsistency between the result from our model and empirical evidence suggests that, in the future research, we need to improve the model in a way that allows credit constrained households to obtain more housing in an economic upturn.

Next, we discuss the implications of introducing a time-varying mean of the idiosyncratic shock, by assuming that idiosyncratic shocks that are above steady state mean to the housing assets are more likely to realise in an economic upturn. For example, government invests more in infrastructure and education when the economy is booming, causing positive impact on house price in more regions. Faia and Monacelli (2007) also discuss this feature, based on the agency problem framework of Carlstrom and Fuerst (1997). In order to generate a countercyclical premium on external finance, they link the mean of the idiosyncratic shock to aggregate total factor productivity.

[91] For example, Besley and Muller (2012) discuss the impact of violence on house prices in Northern Ireland.

In our chapter, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, the structure of the model implies a positive relation between the default rate and the loan to value ratio, thus if we can improve the model to have a procyclical loan to value ratio, the default rate will become procyclical as well. Therefore, in order to overcome this co-movement and to have both a procyclical loan to value ratio and a countercyclical default rate, as suggested by data, we should consider a time-varying mean of the idiosyncratic shock.

5.2 Empirical analysis

In this section, we examine the cyclicality of several housing market variables compared to U.S. real GDP using a sample period 1973Q1 to 2010Q4. Figure 5.1 and Figure 5.2 show that both mortgage loan amount and housing value display procyclical behaviour, and their volatilities are higher than that of real GDP. Figure 5.3 shows that the loan to value ratio is also procyclical, implying that the mortgage loan amount is more procyclical than the housing value.[92] Meanwhile, the volatility of the loan to value ratio is less than that of real GDP and the average loan to value ratio is 75% over the sample period. Finally, Figure 5.4 shows that the mortgage delinquency rate was not closely related to real GDP before the 1980s. From 1985, however, this rate has exhibited countercyclical behaviour. In particular, the delinquency rate has increased sharply during the financial crisis, starting from 2007. Meanwhile, the foreclosure rate has a similar pattern, but with a lower rate. Overall, a procyclical loan to value ratio and a countercyclical delinquency rate (or a countercyclical foreclosure rate) are two facts that we attempt to reproduce from our model.

[92] The loan to value ratio is a ratio of mortgage loan amount to housing value.

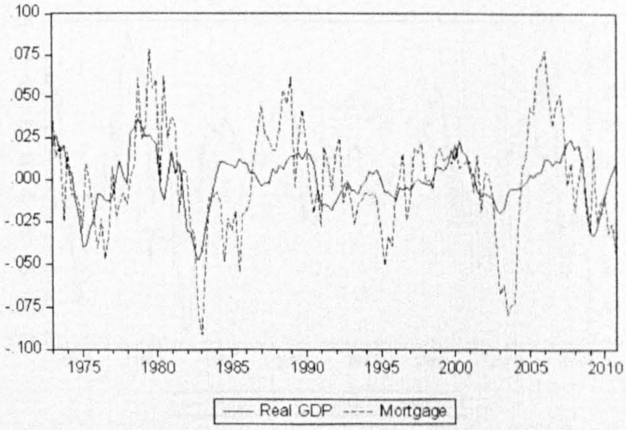

Figure 5.1: Business cycles in U.S. real GDP and mortgage loan amount. Sample period: 1973Q1 - 2010Q4. Both variables are detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; Federal Housing Finance Agency.

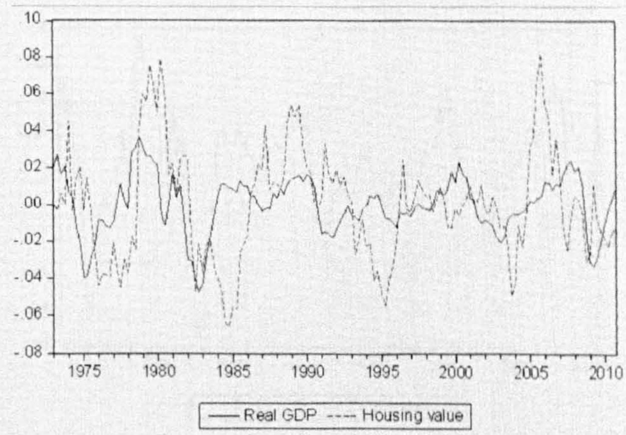

Figure 5.2: Business cycles in U.S. real GDP and purchase price of housing. Sample period: 1973Q1 - 2010Q4. Both variables are detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; Federal Housing Finance Agency.

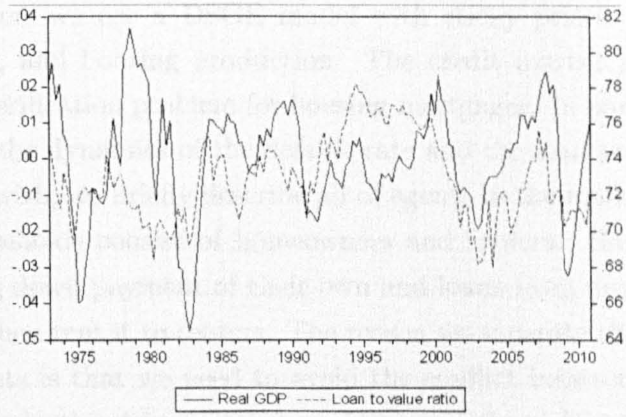

Figure 5.3: Business cycles in U.S. real GDP and the loan to value ratio. Sample period: 1973Q1 - 2010Q4. U.S. real GDP is detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; Federal Housing Finance Agency.

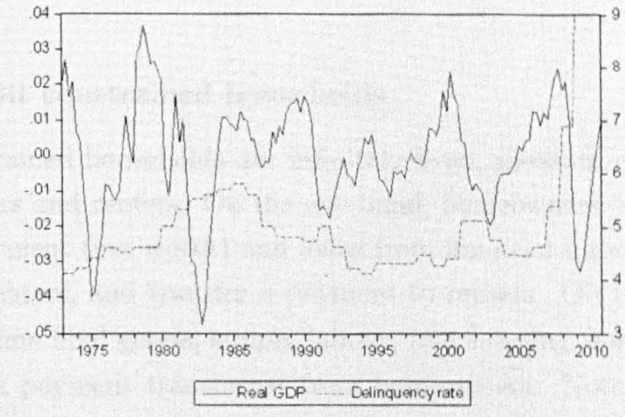

Figure 5.4: Business cycles in U.S. real GDP and the mortgage delinquency rate. Sample period: 1973Q1 - 2010Q4. U.S. real GDP is detrended using the Hodrick-Prescott filter. Sources: U.S. Bureau of Economic Analysis; U.S. Census Bereau; Mortgage Bankers Association of America.

5.3 The Model

In this chapter, we use a DSGE model with sticky prices, credit market imperfections, and housing production. The credit market is subject to a costly state verification problem for housing mortgages. In particular, we are interested in the dynamics of the default rate and the loan to value ratio.

To begin with, we briefly describe all of agents in the model. Credit constrained households consist of homeowners and renters. Homeowners buy housing using down payment of their own and loans from financial intermediaries, and then rent it to renters. The reason we separate these households into two agents is that we need to avoid the conflict between the risk neutral borrowers in the debt contract problem and the risk averse consumers in the utility maximisation problem. Financial intermediaries take deposits from credit unconstrained households and provide loans to credit constrained households. In the supply side, we have a goods production sector, which includes final goods firms, retail goods firms, and intermediate goods firms. Besides, we also have a housing production sector. Finally, the monetary authority uses the nominal interest rate as a policy instrument to affect the real economy.

5.3.1 Credit constrained households

Credit constrained households are infinitely lived, measure one and consist of homeowners and renters. On the one hand, homeowners buy housing using down payment (net worth) and loans from financial intermediaries, rent housing to renters, and transfer a payment to renters. On the other hand, renters consume final goods, supply labour, rent housing from homeowners, and receive a payment transferred from homeowners. Note that rent and transfer payment can be seen as internal flows of funds within credit constrained households.

Homeowners' decisions on default and non-default At the beginning of period t, homeowners take loans from financial intermediaries, together with down payment (or net worth) of their own, to buy housing, which is rented to renters within the period. The borrowing equation shows that the volume of loans is the difference between the real value of housing assets and the down payment,

$$b_t^b = q_{h,t} h_t^b - nw_t^b \tag{5.1}$$

where $q_{h,t}$ is the real house price, $q_{h,t} h_t^b$ is the real value of housing assets that homeowners are buying, nw_t^b is the homeowners' down payment, and b_t^b is the volume of loans taken from financial intermediaries.

In the beginning of period $t + 1$, homeowners need to repay the loans taken in the previous period. Since housing is depreciating at a rate δ_h, the undepreciated housing assets own by the homeowner are $(1 - \delta_h) h_t^b$. We assume that an idiosyncratic shock, ω_{t+1}, with a time-invariant mean, ω_m, affects the undepreciated housing assets, thus the real idiosyncratic value of undepreciated housing assets is $\omega_{t+1} q_{h,t+1} (1 - \delta_h) h_t^b$.

In this model, default is a steady state phenomenon, as homeowners can default if the value of collateralised assets is less than the loan repayment. Therefore, lenders provide the loans with a contractual (gross) interest rate, $R_{l,t}$, which is higher than the riskless (gross) interest rate, R_t, to cover their loss from default. Hence, the real repayment for the loans taken in period t is $\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$, where $\pi_{c,t+1}$ is the (gross) inflation rate.

When we combine the real idiosyncratic value of undepreciated housing assets and the real repayment, there is a threshold level of the shock, $\overline{\omega}_t$, such that,[93]

$$\overline{\omega}_{t+1} q_{h,t+1} (1 - \delta_h) h_t^b = \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b \tag{5.2}$$

At this threshold level, the real value of undepreciated housing assets, $\overline{\omega}_{t+1} q_{h,t+1} (1 - \delta_h) h_t^b$, is equal to the real repayment for the loans, $\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$. Therefore, homeowners are indifferent between default and non-default, as we assume that borrowers' housing assets are taken by lenders if they default.

Given this threshold level, we can infer that homeowners' decisions de-

[93]For simplicity, we ignore the expectation operator E_t in this chapter.

pend on the value of the idiosyncratic shock,

$$\omega_{t+1} < \overline{\omega}_{t+1} \Rightarrow \overline{\omega}_{t+1} q_{h,t+1} (1-\delta_h) h_t^b < \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b \Rightarrow \text{Default}$$
$$\omega_{t+1} > \overline{\omega}_{t+1} \Rightarrow \overline{\omega}_{t+1} q_{h,t+1} (1-\delta_h) h_t^b > \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b \Rightarrow \text{No Default}$$

In other words, when the actual level is less than the threshold level, $\omega_{t+1} < \overline{\omega}_{t+1}$, the real value of undepreciated housing assets is less than the real repayment, thus homeowner will default. Therefore, the probability of default is

$$F(\overline{\omega}_{t+1}) = \int_{\min}^{\overline{\omega}_{t+1}} f(\omega)\, d\omega = \Pr(\omega_{t+1} < \overline{\omega}_{t+1})$$

where $F(\overline{\omega}_{t+1})$ is the cumulative distribution function of the idiosyncratic shock. In contrast, if the actual level is greater than the threshold level, $\omega_{t+1} > \overline{\omega}_{t+1}$, the real value of undepreciated housing assets is greater than the real repayment, thus homeowners will not default, and the probability of no default is $1 - F(\overline{\omega}_{t+1})$.

Meanwhile, the loan to value ratio is defined as

$$m_t = \frac{\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b}{\omega_m q_{h,t+1} (1-\delta_h) h_t^b} \tag{5.3}$$

which is the ratio of the expected real total debt obligations to the expected real average value of undepreciated housing assets. Note that, in our model, the loan to value ratio is not a choice variable.[94]

If we combine equations 5.2 and 5.3, we have a relation among the loan

[94] This definition of the loan to value ratio is forward looking. Alternatively, a contemporary loan to value ratio can be defined as

$$m_t' = \frac{b_t^b}{\omega_m q_{h,t} h_t^b}$$

which is the ratio of the volumen of loan to average housing value. The way of defining the loan to value ratio. however, has little impacts on the dynamics of the model.

to value ratio, the threshold level, and the mean of the idiosyncratic shock,

$$\overline{\omega}_{t+1} = m_t \omega_m$$

which shows that, if the threshold level and the mean are fixed (i.e., the probability of default is fixed), the loan to value ratio should be fixed as well.

The allocation of the undepreciated housing assets Here we discuss the allocation of the undepreciated housing assets between homeowners (borrowers) and financial intermediaries (lenders) when homeowners decide to default or not default.

If homeowners default and do not repay the loans, financial intermediaries will take over the housing assets and pay a monitoring cost, which is a fraction μ_m of the value of undepreciated housing assets. Meanwhile, homeowners can keep the rent payment from renters. The following table summarises what borrowers and lenders get if borrowers default.

Borrowers' gain	$F(\overline{\omega}_{t+1}) \cdot q_{r,t} h_t^b$
Lenders' gross gain	$G(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$
Lenders' net gain	$(1-\mu_m) G(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$

where $q_{r,t}$ is the real rental price of housing, $G(\overline{\omega}_{t+1}) = \int_{\min}^{\overline{\omega}_{t+1}} \omega f(\omega)\, d\omega$ is the conditional mean of the idiosyncratic shock given borrowers default.

If homeowners do not default, they will repay the loans to lenders with the contractual interest rate. The following table summarises what borrowers and lenders get if borrowers do not default.

Borrowers' gain	$(\omega_m - G(\overline{\omega}_{t+1})) \cdot q_{h,t+1}(1-\delta_h) h_t^b$ $- (1 - F(\overline{\omega}_{t+1})) \cdot \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b + (1 - F(\overline{\omega}_{t+1})) \cdot q_{r,t} h_t^b$
Lenders' gain	$(1 - F(\overline{\omega}_{t+1})) \cdot \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$

where $\omega_m = \int_{\min}^{\max} \omega f(\omega)\, d\omega$ is the unconditional mean of the idiosyncratic shock.

Then we summarise what borrowers and lenders get from this debt contract problem. Firstly, borrowers' gain is summarised as

Default	$F(\overline{\omega}_{t+1}) \cdot q_{r,t} h_t^b$
No default	$\begin{bmatrix} (\omega_m - G(\overline{\omega}_{t+1})) \\ -\overline{\omega}_{t+1}(1 - F(\overline{\omega}_{t+1})) \end{bmatrix} \cdot q_{h,t+1}(1-\delta_h) h_t^b$ $+ (1 - F(\overline{\omega}_{t+1})) \cdot q_{r,t} h_{t-1}^b$
Total	$B(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b + q_{r,t} h_t^b$

where $B(\overline{\omega}_{t+1}) = (\omega_m - G(\overline{\omega}_{t+1})) - \overline{\omega}_{t+1}(1 - F(\overline{\omega}_{t+1}))$ is the borrowers' share of the real value of undepreciated housing asset.[95]

Secondly, lenders' gross gain is summarised as

Default	$G(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$
No default	$\overline{\omega}_{t+1}(1 - F(\overline{\omega}_{t+1})) \cdot q_{h,t+1}(1-\delta_h) h_t^b$
Total	$\Gamma(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$

where $\Gamma(\overline{\omega}_{t+1}) = G(\overline{\omega}_{t+1}) + (1 - F(\overline{\omega}_{t+1}))\overline{\omega}_{t+1}$ is the lenders' gross share of the real value of undepreciated housing assets, before subtracting the monitoring cost.

Thirdly, lenders' net gain is summarised as

Default	$(1-\mu_m) G(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$
No default	$\overline{\omega}_{t+1}(1 - F(\overline{\omega}_{t+1})) \cdot q_{h,t+1}(1-\delta_h) h_t^b$
Total	$L(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$

[95] Recall that the equation of the threshold level is

$$\overline{\omega}_{t+1} q_{h,t+1}(1-\delta_h) h_t^b = \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$$

thus we can replace $(1 - F(\overline{\omega}_{t+1})) \cdot \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$ by $\overline{\omega}_{t+1}(1 - F(\overline{\omega}_{t+1})) \cdot q_{h,t+1}(1-\delta_h) h_t^b$ in borrowers' gain given no default.

where $L(\overline{\omega}_{t+1}) = (1-\mu_m) G(\overline{\omega}_{t+1}) + (1 - F(\overline{\omega}_{t+1}))\overline{\omega}_{t+1}$ is the lenders' net share of the real value of undepreciated housing assets after subtracting the monitoring cost.

The financial intermediaries The lenders in this model are financial intermediaries, who take deposits from credit unconstrained households and provide loans to credit constrained households. For simplicity, we assume a perfectly competitive market for financial intermediaries, thus they make zero profit. We also assume that they facilitate the loans market at zero cost. On the one hand, financial intermediaries take saving, b_t, from credit unconstrained households in period t, and then return the saving plus the riskless interest rate, $\frac{R_t}{\pi_{c,t+1}} b_t$, in period $t+1$. On the other hand, financial intermediaries provide loans, b_t^b, to credit constrained households in period t, and receive their net share of the value of undepreciated housing assets, $L(\overline{\omega}_{t+1}) \cdot q_{h,t+1}(1-\delta_h) h_t^b$, in period $t+1$. Therefore, the lenders' participation constraint is

$$L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) h_t^b \geq \frac{R_t}{\pi_{c,t+1}} b_t \tag{5.4}$$

where $L(\overline{\omega}_{t+1})$ is the lenders' net share of the real value of undepreciated housing assets. This equation indicates that the lenders' net gain from the debt contract problem should be no less than the real repayment to savers.

The homeowners' debt contract problem Then we are ready to describe the debt contract problem. As summrised before, borrowers' gain from this problem is

$$B(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) h_t^b + q_{r,t} h_t^b$$

Then borrowers (homeowners) maximise their expected return subject to the lenders' participation constraint 5.4.

We obtained two first order conditions from the homeowners' debt contract problem.[96] Firstly, the equation that determines the threshold level of

[96] The homeowners' debt contract problem is shown in the Appendix.

the idiosyncratic shock is

$$-B'(\overline{\omega}_{t+1}) = \lambda^b_{b,t} L'(\overline{\omega}_{t+1}) \tag{5.5}$$

where $\lambda^b_{b,t}$ is the Lagrange multiplier on the lender's participation constraint. The LHS, $-B'(\overline{\omega}_{t+1})$, is the marginal cost to borrowers' return given that the threshold level $\overline{\omega}_{t+1}$ increase by one more unit, as $B(\overline{\omega}_{t+1})$ is decreasing in $\overline{\omega}_{t+1}$. On the RHS, $L'(\overline{\omega}_{t+1})$ the marginal benefit to lenders' return given that the threshold level $\overline{\omega}_{t+1}$ increase by one more unit, since $L(\overline{\omega}_{t+1})$ is an increasing function of $\overline{\omega}_{t+1}$. Therefore, the participation constraint is loosened by $L'(\overline{\omega}_{t+1})$ unit, and thus $\lambda^b_{b,t} L'(\overline{\omega}_{t+1})$ is the marginal benefit to borrowers, as the shadow price of loosening the constraint by one unit is $\lambda^b_{b,t}$. In sum, this equation implies that, given that the threshold level increases by one more unit, the marginal cost is equal to the marginal benefit of a loosened participation constraint.

Secondly, the equation that determines the homeowners' demand for housing is

$$B(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) + q_{r,t} = -\lambda^b_{b,t}\left[L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) - \frac{R_t}{\pi_{c,t+1}} q_{h,t}\right] \tag{5.6}$$

The LHS is the marginal benefit to borrowers given that their housing increases by one more unit. Meanwhile, given that borrowers' housing increases by one more unit, the increase in the lenders' return, $L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h)$, is less than the increase in debt obligation, $\frac{R_t}{\pi_{c,t+1}} q_{h,t}$, given that the net worth does not change, thus the participation constraint is tightened by $L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) - \frac{R_t}{\pi_{c,t+1}} q_{h,t}$ unit.[97] As the shadow price of tightening

[97] The paticipation constraint is

$$L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) h^b_t = \frac{R_t}{\pi_{c,t+1}}\left(q_{h,t} h^b_t - nw_t\right)$$

thus we have such an inequality, given nw_t is positive.

$$L(\overline{\omega}_{t+1}) q_{h,t+1}(1-\delta_h) < \frac{R_t}{\pi_{c,t+1}} q_{h,t}$$

the participation constraint by one unit is $\lambda_{b,t}^{b}$, the RHS is the marginal cost of a tightened participation constraint to borrowers. In sum, this equation indicates that, given that borrowers' housing increases by one more unit, the marginal benefit is equal to the marginal cost of a tightened participation constraint.

The allocation of homeowners' return Here we discuss how homeowners deal with their real return received in period t,

$$B(\overline{\omega}_t)\, q_{h,t}(1-\delta_h)\, h_{t-1}^{b} + q_{r,t-1} h_{t-1}^{b}$$

which is composed by the homeowners' share of the value of undepreciated housing assets and the rent from renters. We assume that homeowners divide the real return into two parts.[98] Firstly, homeowners keep a fraction γ_{nw} of real return as down payment (or net worth), nw_t^b, which is used to purchase housing in period t, together with the loans taken from financial intermediaries. The equation of the down payment is

$$nw_t^b = \gamma_{nw} \cdot \left[B(\overline{\omega}_t)\, q_{h,t}(1-\delta_h)\, h_{t-1}^{b} + q_{r,t-1} h_{t-1}^{b}\right] \tag{5.7}$$

Secondly, homeowners give a fraction $(1-\gamma_{nw})$ of their real return to

[98] Aoki, Proudman and Vligeghe (2004) assume that the transfer D_t is a function of the leverage ratio.

$$D_t = \chi\left(\frac{nw_t}{q_{h,t} h_t^b}\right)$$

where $\chi'(\phi) > 0$ and $\chi(\phi) = D$. Here $\phi = \frac{nw}{q_h h^b} = 0.7$ is the leverage ratio in the steady state, and D is the level of dividend consistent with ϕ.

The log-linearised form is

$$\widehat{D}_t = s\left(\widehat{nw}_t - \widehat{q}_{h,t} - \widehat{h}_t^b\right)$$

where $s = \frac{\chi'(\phi)}{\chi(\phi)}\phi$. The adjustment factor s on the dividend rule is set at 3, which is consistent the estimated average elasticity of mortgage equity withdrawal with respect to the net worth ratio. In other words, the amount of equity withdrawn will increase by 3% if the net worth of the aggregate UK household sector rises by 1%.

renters as a transfer payment, tr_t^b. The equation of the transfer payment is

$$tr_t^b = (1-\gamma_{nw}) \cdot \left[B(\overline{\omega}_t) q_{h,t} (1-\delta_h) h_{t-1}^b + q_{r,t-1} h_{t-1}^b\right] \tag{5.8}$$

Renters Renters also live in credit constrained households. They consume goods, rent housing from homeowners, receive a transfer payment from homeowners, supply labour to intermediate goods firms, and receive wage. They maximise their lifetime utility subject to their budget constraint.

The renters' lifetime utility function is

$$E_t \sum_{k=0}^{\infty} \left(\beta^i\right)^k \left(\Gamma_c^i \ln\left(c_{t+k}^i - \varepsilon_c^i c_{t+k-1}^i\right) + j^i \ln h_{t+k}^i - \frac{1}{1+\gamma_n^i}\left(n_{t+k}^i\right)^{1+\gamma_n^i}\right)$$

where we use superscript i to denote variables associated to renters. In the utility function, E_t is the expectation operator, β^i is the renters' discount factor, c_t^i is renters' consumption, ε_c measures the degree of consumption habit, Γ_c^i is a scaling factor, h_t^i is domestic housing, j^i is the weight on domestic housing, n_t^i is the supply of renters' labour, and $\frac{1}{\gamma_n^i}$ is the Fisher elasticity of labour supply.

Given the lifetime utility function, the renters' marginal utility of consumption is

$$u_{c,t}^i = \Gamma_c^i \frac{1}{c_t^i - \varepsilon_c^i c_{t-1}^i} - \beta^i \varepsilon_c^i \Gamma_c^i \frac{1}{E_t c_{t+1}^i - \varepsilon_c^i c_t^i} \tag{5.9}$$

which expresses the marginal utility of consumption $u_{c,t}$ in terms of lagged, current, and future consumption.

The renters' real budget constraint shows that the total expense (LHS) should be no more than the total income (RHS), and is expressed as

$$c_t^i + q_{r,t} h_t^i \leq w_t^i n_t^i + tr_t^b \tag{5.10}$$

where $q_{r,t}$ is the real rental price of housing, w_t^i is the real wage rate, tr_t^b is the real transfer payment from homeowners. We assume that renters do not save into or borrow from financial intermediaries, thus they spend all available resource in a given period.

We obtain two first order conditions from the renters' utility maximisation problem.[99] Firstly, the equation that governs the renters' labour supply is

$$\left(n_t^i\right)^{\gamma_n^i} = u_{c,t}^i w_t^i \tag{5.11}$$

which implies that the marginal disutility of labour supply at t is equal to the real wage in terms of the marginal utility consumption at t. This first order condition is an intratemporal optimality condition that indicates how renters make decisions about consumption and labour supply in period t.

Secondly, the equation that governs the renters' demand for domestic housing is

$$u_{c,t}^i q_{r,t} = \frac{j^i}{h_t^i} \tag{5.12}$$

which implies that the real rental price of housing in terms of the marginal utility of consumption at t is equal to the marginal utility of domestic housing at t. This first order condition is also an intratemporal optimality condition that describes an optimal allocation of resource between consumption and domestic housing.

5.3.2 The idiosyncratic shock

We have assumed that an idiosyncratic shock, ω_t, affects the undepreciated housing assets, $(1-\delta_h)\,h_{t-1}$, and thus the real idiosyncratic value of undepreciated housing assets is $q_{h,t}\omega_t\,(1-\delta_h)\,h_{t-1}$. Following Faia and Monacelli (2005), and Morozumi (2010), we assume that the idiosyncratic shock ω_t is independently distributed (across homeowners and time) with a uniform distribution and has positive support.[100] We also assume that the idiosyncratic shock has a time-invariant mean with value of one, $\omega_m = 1$, and the range of distribution is fixed at $2\rho_\omega$. Therefore, the uniform distribution of the

[99] The renters' utility maximisation is shown in the Appendix.

[100] Bernanke, Gertler and Gilchrist (1999) consider a log normal distribution.

idiosyncratic shock is

$$\omega_t \sim U\left(\omega_m - \rho_\omega, \omega_m + \rho_\omega\right)$$

The probability density function is

$$f\left(\overline{\omega}_t\right) = F'\left(\overline{\omega}_t\right) = \frac{1}{\max - \min} = \frac{1}{\left(\omega_m + \rho_\omega\right) - \left(\omega_m - \rho_\omega\right)} = \frac{1}{2\rho_\omega}$$

where $F'\left(\overline{\omega}_t\right) > 0$. Note that we also use 'min' and 'max' to denote the lower boundary and upper boundary of the distribution. For a uniform distribution, the probability density function is a constant and depends on the range of the distribution, $2\rho_\omega$, only.

The cumulative density function is

$$F\left(\overline{\omega}_t\right) = \int_{\min}^{\overline{\omega}_t} f\left(\omega\right) d\omega = \int_{\omega_m - \rho_\omega}^{\overline{\omega}_t} \frac{1}{2\rho_\omega} d\omega = \frac{\overline{\omega}_t}{2\rho_\omega} - \frac{\omega_m - \rho_\omega}{2\rho_\omega} \tag{5.13}$$

where $F\left(\overline{\omega}_t\right) \geq 0$, $\lim_{\overline{\omega}_t \to \min} F\left(\overline{\omega}_t\right) \to 0$, and $\lim_{\overline{\omega}_t \to \max} F\left(\overline{\omega}_t\right) \to 1$. This function can be interpreted as the probability of default or the default rate, which depends on the threshold level, and the probability density function.

The probability of no default is

$$1 - F\left(\overline{\omega}_t\right) = \int_{\overline{\omega}_t}^{\max} f\left(\omega\right) d\omega = \frac{\omega_m + \rho_\omega}{2\rho_\omega} - \frac{\overline{\omega}_t}{2\rho_\omega}$$

since the sum of the probability of default and the probability of no default is always equal to one.

The conditional mean given default is

$$G\left(\overline{\omega}_t\right) = \int_{\min}^{\overline{\omega}_t} \omega f\left(\omega\right) d\omega = \int_{\omega_m - \rho_\omega}^{\overline{\omega}_t} \omega \frac{1}{2\rho_\omega} d\omega = \frac{\overline{\omega}_t^2}{4\rho_\omega} - \frac{\left(\omega_m - \rho_\omega\right)^2}{4\rho_\omega} \tag{5.14}$$

where $G\left(\overline{\omega}_t\right) \geq 0$, $\lim_{\overline{\omega}_t \to \min} G\left(\overline{\omega}_t\right) \to 0$, and $\lim_{\overline{\omega}_t \to \max} G\left(\overline{\omega}_t\right) \to \omega_m$. This function depends on the threshold level and the probability density function.

The derivative of the conditional mean given default is

$$G'(\overline{\omega}_t) = \overline{\omega}_t f(\overline{\omega}_t) = \frac{\overline{\omega}_t}{2\rho_\omega} \quad (5.15)$$

since both ρ_ω and $\overline{\omega}_t$ are positive, we have $G'(\overline{\omega}_t) > 0$, indicating that $G(\overline{\omega}_t)$ is an increasing function of $\overline{\omega}_t$. The second derivative is $G''(\overline{\omega}_t) = \frac{1}{2\rho_\omega} > 0$, indicating $G(\overline{\omega}_t)$ is convex at the threshold level.

The conditional mean given no default is

$$\omega_m - G(\overline{\omega}_t) = \int_{\overline{\omega}_t}^{\min} \omega f(\omega)\, d\omega$$

since we assume that the time-invariant unconditional mean of the idiosyncratic shock is ω_m.

The lender's gross share is

$$\begin{aligned} \Gamma(\overline{\omega}_t) &= \int_{\min}^{\overline{\omega}_t} \omega f(\omega)\, d\omega + \overline{\omega}_t \int_{\overline{\omega}_t}^{\max} f(\omega)\, d\omega \\ &= G(\overline{\omega}_t) + \overline{\omega}_t (1 - F(\overline{\omega}_t)) \end{aligned} \quad (5.16)$$

where $\Gamma(\overline{\omega}_t) \geq 0$, $\lim_{\overline{\omega}_t \to \min} \Gamma(\overline{\omega}_t) \to \min$, and $\lim_{\overline{\omega}_t \to \max} \Gamma(\overline{\omega}_t) \to \omega_m$. Precisely, this is the lenders' gross share of the real value of the undepreciated housing assets at period t, before taking the monitoring costs into account.

The derivative of lender's gross share is

$$\Gamma'(\overline{\omega}_t) = 1 - F(\overline{\omega}_t) \quad (5.17)$$

since $(1 - F(\overline{\omega}_t)) > 0$, we know that $\Gamma'(\overline{\omega}_t)$ is an increasing function of $\overline{\omega}_t$. In addition, the second derivative is $\Gamma''(\overline{\omega}_t) = -f(\overline{\omega}_t) < 0$, hence $\Gamma(\overline{\omega}_t)$ is concave at the threshold level.

The lenders' net share is

$$\begin{aligned} L(\overline{\omega}_t) &= \int\limits_{\min}^{\overline{\omega}_t} \omega f(\omega)\, d\omega + \overline{\omega}_t \int\limits_{\overline{\omega}_t}^{\max} f(\omega)\, d\omega - \mu_m \int\limits_{\min}^{\overline{\omega}_t} \omega f(\omega)\, d\omega \\ &= \Gamma(\overline{\omega}_t) - \mu_m G(\overline{\omega}_t) \end{aligned} \tag{5.18}$$

where $L(\overline{\omega}_t) \geq 0$, $\lim_{\overline{\omega}_t \to \min} L(\overline{\omega}_t) \to \min$, and $\lim_{\overline{\omega}_t \to \max} L(\overline{\omega}_t) \to (1-\mu_m)\omega_m$. Precisely, this is the lenders' net share of the real value of the undepreciated housing assets at period t, after taking the monitoring costs into account.

The differential of lenders' net share is

$$L'(\overline{\omega}_t) = \Gamma'(\overline{\omega}_t) - \mu_m G'(\overline{\omega}_t) \tag{5.19}$$

There is an optimal value, $\overline{\omega}_t^* = \frac{\omega_m + \rho_\omega}{1+\mu_m}$, that maximises the lenders' net share. We infer that it is a local maximum as $L''(\overline{\omega}_t^*) < 0$. Since $L(\overline{\omega}_t)$ in increasing on $(\min, \overline{\omega}_t^*)$ and decreasing on $(\overline{\omega}_t^*, \max)$, lenders never choose $\overline{\omega}_t > \overline{\omega}_t^*$. Besides, our steady state value of $\overline{\omega}_t$ is far less than the optimal value, thus $L(\overline{\omega}_t)$ in increasing in $\overline{\omega}_t$ and concave around the neighborhood of the steady state.

The borrowers' share is

$$\begin{aligned} B(\overline{\omega}_t) &= \omega_m - \int\limits_{\min}^{\overline{\omega}_t} \omega f(\omega)\, d\omega - \overline{\omega}_t \int\limits_{\overline{\omega}_t}^{\max} f(\omega)\, d\omega \\ &= (\omega_m - G(\overline{\omega}_t)) - (1 - F(\overline{\omega}_t))\,\overline{\omega}_t \\ &= \omega_m - \Gamma(\overline{\omega}_t) \end{aligned} \tag{5.20}$$

where $B(\overline{\omega}_t) \geq 0$, $\lim_{\overline{\omega}_t \to \min} B(\overline{\omega}_t) \to \omega_m - \min$, and $\lim_{\overline{\omega}_t \to \max} B(\overline{\omega}_t) \to 0$. Precisely, this is the borrowers' share of the real value of undepreciated housing assets at period t.

The differential of borrower's share

$$\begin{aligned} B'(\overline{\omega}_t) &= -G'(\overline{\omega}_t) - (1 - F(\overline{\omega}_t)) + F'(\overline{\omega}_t)\,\overline{\omega}_t \\ &= -(1 - F(\overline{\omega}_t)) \end{aligned} \tag{5.21}$$

since $-\left(1-F\left(\overline{\omega}_t\right)\right)<0$, we know that $B'\left(\overline{\omega}_t\right)$ is a decreasing function of $\overline{\omega}_t$. Otherwise, if $B\left(\overline{\omega}_t\right)$ is increasing in $\overline{\omega}_t$, homeowners' will always choose $\overline{\omega}_t=\max$, thus the probability of default is always equal to one. In addition, the second derivative is $B''\left(\overline{\omega}_t\right)=f\left(\overline{\omega}_t\right)>0$, indicating that $B\left(\overline{\omega}_t\right)$ is convex at the threshold level.

5.3.3 Credit unconstrained households

Credit unconstrained households are infinitely lived and of measure one. They supply labour, consume final goods, demand domestic housing, accumulate goods capital and housing capital. They maximise lifetime utility subject to their budget constraint. We assume that they own the profitable retail goods firms.[101]

Similar with renters, the credit unconstrained households' lifetime utility function is

$$E_t\sum_{k=0}^{\infty}\beta^k\left(\Gamma_c\ln\left(c_{t+k}-\varepsilon_c c_{t+k-1}\right)+j\ln h_{t+k}-\frac{1}{1+\gamma_n}\left(n_{t+k}\right)^{1+\gamma_n}\right)$$

where E_t is the expectation operator, β is the credit unconstrained households' discount factor, c_t is credit unconstrained households' consumption, ε_c measures the degree of consumption habit, Γ_c is a scaling factor, h_t is domestic housing, j is the weight on domestic housing, n_t is the supply of credit unconstrained households' labour, and $\frac{1}{\gamma_n}$ is the Fisher elasticity of labour supply.

Given the lifetime utility function, the credit unconstrained households' marginal utility of consumption is

$$u_{c,t}=\Gamma_c\frac{1}{c_t-\varepsilon_c c_{t-1}}-\beta\varepsilon_c\Gamma_c\frac{1}{E_t c_{t+1}-\varepsilon_c c_t} \tag{5.22}$$

which expresses the marginal utility of consumption $u_{c,t}$ in terms of lagged, current, and future consumption.

[101] Credit unconstrained households behave in the same way as patient households in our Chapter 2.

The credit unconstrained households' real budget constraint shows that the total expense (LHS) should be no more than the total income (RHS), and is expressed as

$$\begin{aligned} & c_t + K_{c,t} + K_{h,t} + q_{h,t}h_t + b_t \\ & + \tfrac{\eta_{kh}}{2}\left(\tfrac{K_{h,t}}{K_{h,t-1}} - 1\right)^2 K_{h,t-1} + \tfrac{\eta_{kc}}{2}\left(\tfrac{K_{c,t}}{K_{c,t-1}} - 1\right)^2 K_{c,t-1} \\ & \leq \left(R_{kc,t} + 1 - \delta_{kc}\right) K_{c,t-1} + \left(R_{kh,t} + 1 - \delta_{kh}\right) K_{h,t-1} \\ & + w_t n_t + \left(\omega_m\right) q_{h,t}\left(1 - \delta_h\right) h_{t-1} + f_t + \tfrac{R_{t-1}}{\pi_{c,t}} b_{t-1} \end{aligned}$$

where $q_{h,t}$ is the real house price, b_t is real bonds (or saving), w_t is the real wage rate, R_{t-1} is the (gross) nominal return on the bonds held in the previous period or the (gross) nominal interest rate, $\pi_{c,t}$ is the (gross) inflation rate, and f_t is the real profit from retail goods firms. For the capital related variables, $K_{h,t}$ is housing capital, $K_{c,t}$ is goods capital, $R_{kh,t}$ and $R_{kc,t}$ are the real rental price of housing capital and goods capital, δ_{kh}, δ_{kc}, δ_h are the depreciation rates for housing capital, goods capital, and housing stock. The real price of goods capital and housing capital is normalised to one, same as the real price of final goods.[102] The last two terms on the LHS are the adjustment costs for goods capital and housing capital respectively, and η_{kh} and η_{kc} are the coefficients of adjustment costs.

We obtain five first order conditions from the credit unconstrained households' lifetime utility maximisation problem.[103] The first three were discussed in the Chapter 2,

$$u_{c,t} = \beta E_t\left(\frac{R_t}{\pi_{c,t+1}} u_{c,t+1}\right) \tag{5.23}$$

$$n_t^{\gamma_n} = w_t u_{c,t} \tag{5.24}$$

$$u_{c,t} q_{h,t} = \frac{j}{h_t} + \beta\left(1 - \delta_h\right)\omega_m E_t\left(u_{c,t+1} q_{h,t+1}\right) \tag{5.25}$$

where the first equation is the Euler equation that governs the optimal allocation of consumption over time, the second equation is an intratemporal

[102] The real prices of housing capital and goods capital are normalised to one, but the real rental prices of them are not.

[103] The credit unconstrained households' utility maximisation problem is shown in the Appendix.

optimality condition that indicates the trade-off between consumption and labour supply, and the third equation is an optimality condition that describes the optimal allocation of resource between consumption and domestic housing.

The fourth first order condition governs the credit unconstrained households' demand for housing capital, and it is expressed as

$$\begin{aligned} u_{c,t} &= \beta E_t \left(u_{c,t+1}\left(R_{kh,t+1}+1-\delta_{kh}\right)\right) - u_{c,t}\eta_{kh}\left(\frac{K_{h,t}}{K_{h,t-1}}-1\right) \\ &\quad -\beta E_t\left[u_{c,t+1}\frac{\eta_{kh}}{2}\left(\frac{K_{h,t+1}}{K_{h,t}}-1\right)^2 - u_{c,t+1}\eta_{kh}\left(\frac{K_{h,t+1}}{K_{h,t}}-1\right)\frac{K_{h,t+1}}{K_{h,t}}\right] \end{aligned} \quad (5.26)$$

which implies that the real price of housing capital in terms of the marginal utility of consumption at t is equal to the sum of the expected real rental price of housing capital and the expected real price of housing capital (for reselling undepreciated housing capital) in terms of the discounted marginal utility of consumption at $t+1$, subtracting the adjustment costs on housing capital. This first order condition is an optimality condition that describes the optimal allocation of resource between consumption and housing capital.

Finally, the equation that governs the credit unconstrained households' demand for housing capital is

$$\begin{aligned} u_{c,t} &= \beta E_t \left(u_{c,t+1}\left(R_{kc,t+1}+1-\delta_{kc}\right)\right) - u_{c,t}\eta_{kc}\left(\frac{K_{c,t}}{K_{c,t-1}}-1\right) \\ &\quad -\beta E_t\left[u_{c,t+1}\frac{\eta_{kc}}{2}\left(\frac{K_{c,t+1}}{K_{c,t}}-1\right)^2 - u_{c,t+1}\eta_{kc}\left(\frac{K_{c,t+1}}{K_{c,t}}-1\right)\frac{K_{c,t+1}}{K_{c,t}}\right] \end{aligned} \quad (5.27)$$

which implies that the real price of goods capital in terms of the marginal utility of consumption at t is equal to the sum of the expected real rental price of goods capital and the expected real price of goods capital (for reselling undepreciated goods capital) in terms of the discounted marginal utility of consumption at $t+1$, subtracting the adjustment costs on goods capital. This first order condition is an optimality condition that describes the optimal allocation of resource between consumption and goods capital.

5.3.4 Goods production sector

In the goods production sector, we have three players: (i) final goods producers buy retail goods from individual retail goods producers, and compose them into final goods, which are ready for consumption; (ii) retail goods producers (or retailers) buy intermediate goods from intermediate goods producers, and differentiate the goods at no cost into retail goods; (iii) intermediate goods producers combine goods sector technology and labour from patient households to produce intermediate goods, which are then sold to retail goods producers.

Final goods firms and retail goods firms As these two type of firms were described in the Chapter 2 already, here we only list equations that are relevant to the equilibrium of the model. The first order condition derived from the retailers' real profit maximisation problem is the equation of the real optimal price, and it is expressed as

$$Q_{c,t} = \frac{\varepsilon}{\varepsilon - 1} E_t \left[\frac{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}}\right)^{-\varepsilon} \frac{P_{wc,t+k}}{P_{c,t+k}}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} \left(\frac{P_{c,t}}{P_{c,t+k}}\right)^{1-\varepsilon} \left(\frac{P_{c,t+k-1}}{P_{c,t-1}}\right)^{\iota_\pi}} \right] \tag{5.28}$$

in which we define the real optimal price as $Q_{c,t} = \frac{P^*_{c,t}}{P_{c,t}}$.

Given the features of sticky prices and price indexation, the nominal price level can also be written as

$$P_{c,t} = \left[\theta \left(\pi^{\iota_\pi}_{c,t-1} P_{c,t-1}\right)^{1-\varepsilon} + (1-\theta) P^{*\,1-\varepsilon}_{c,t}\right]^{\frac{1}{1-\varepsilon}} \tag{5.29}$$

The real profit from retail goods firms is

$$f_t = Y_t - \frac{P_{wc,t}}{P_{c,t}} Y_t = \left(1 - \frac{1}{Z_t}\right) Y_t \tag{5.30}$$

which implies that the real profit is the difference between the real price of final goods, which is normalised to one, and the real price of intermediate goods, $\frac{P_{wc,t}}{P_{c,t}}$.

Intermediate goods firms Intermediate goods producers combine exogenous goods sector technology and labour from credit unconstrained households and credit constrained households to produce intermediate goods, which are then sold to retail goods producers. We assume that intermediate goods firms are perfectly competitive.

The intermediate goods production function is

$$Y_t = A_{c,t}\left((n_t)^{\alpha}\left(n_t^i\right)^{1-\alpha}\right)^{\mu_n} K_{c,t-1}^{\mu_{kc}} \tag{5.31}$$

where Y_t is intermediate goods, n_t and n_t^i are labour from credit unconstrained households and credit constrained households respectively, $K_{c,t-1}$ is lagged goods capital, μ_n is the labour share of output, μ_{kc} is the capital share of output, α is the credit unconstrained households' share of labour income. The goods sector technology $A_{c,t}$ follows the stationary process

$$A_{c,t} = A_c^{1-\rho_{Ac}} A_{c,t-1}^{\rho_{Ac}} e^{\varepsilon_{Ac,t}}, \qquad \varepsilon_{Ac,t} \sim N\left(0, \sigma_{Ac}^2\right) \tag{5.32}$$

The intermediate goods producers' real profit maximisation problem is

$$\max_{n_t, n_t^i, K_{c,t-1}} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}\left(\frac{1}{Z_{t+k}} Y_{t+k} - w_{t+k} n_{t+k} - w_{t+k}^i n_{t+k}^i - R_{kc,t+k} K_{c,t+k-1}\right)$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor, $w_t n_t$ is the real cost of labour from credit unconstrained households, $w_t^i n_t^i$ is the real cost of labour from credit constrained households, $R_{kc,t} K_{c,t-1}$ is the real cost of goods capital.

We obtain three first order conditions from the intermediate goods firms' profit maximisation problem. Firstly, the equation that describes the intermediate goods producers' demand for labour from credit unconstrained households is

$$w_t = \mu_n \alpha \frac{Y_{c,t}}{Z_t n_{c,t}} \tag{5.33}$$

which implies that the real wage of labour from credit unconstrained households, w_t, is equal to the marginal product of labour from credit unconstrained households, $\mu_n \alpha \frac{Y_{c,t}}{Z_t n_{c,t}}$.

Secondly, the equation that describes the intermediate goods producers' demand for labour from credit constrained households is

$$w_t^i = \mu_n (1-\alpha) \frac{Y_t}{Z_t n_t^i} \tag{5.34}$$

which implies that the real wage of labour from credit constrained households, w_t^i, is equal to the marginal product of labour from credit constrained households, $\mu_n (1-\alpha) \frac{Y_t}{Z_t n_t^i}$.

Thirdly, the equation that describes the intermediate goods producers' demand for goods capital is

$$R_{kc,t} = \mu_{kc} \frac{Y_t}{Z_t K_{c,t-1}} \tag{5.35}$$

which implies that the real rental price of goods capital, $R_{kc,t}$, is equal to the marginal product of goods capital, $\mu_{kc} \frac{Y_t}{Z_t K_{c,t-1}}$.

5.3.5 Housing production sector

Housing firms Similar with the Chapter 3, here we consider a housing production sector. Housing producers combine exogenous housing sector technology and housing capital from households to produce new housing, which are then sold to households. We assume that housing firms are perfectly competitive, thus they make zero profit. In addition, we assume flexible prices in this sector.

The housing production function is

$$IH_t = A_h K_{h,t-1}^{\mu_{kh}} \tag{5.36}$$

where IH_t is new housing, $K_{h,t-1}$ is lagged housing capital, μ_h is the housing capital share of housing production, and A_h is the housing sector technology.

The housing producers' real profit maximisation problem is

$$\max_{K_{h,t-1}} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(q_{h,t+k} IH_{t+k} - R_{kh,t+k} K_{h,t+k-1} \right)$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor, $q_{h,t}IH_t$ is the real total revenue, and $R_{kh,t}K_{h,t-1}$ is the real total cost.

The first order condition derived from this real profit maximisation describes the housing producers' demand for housing capital, and it is expressed as

$$R_{kh,t} = q_{h,t}\mu_{kh}\frac{IH_t}{K_{h,t-1}} \tag{5.37}$$

which implies that the marginal cost of housing capital, i.e., the real rental price of housing capital, is equal to the marginal product of housing capital.

5.3.6 Monetary authority

Similar with previous chapters, the monetary authority uses the nominal interest rate as a policy instrument to affect the real economy and monetary policy is non-neutral because of the feature of sticky prices that arises from the monopolistic competition in retail goods firms. As a result, the nominal interest rate can affect the real interest rate, thus has a impact on real variables.

The monetary policy rule, which reacts to inflation and output, is

$$R_t = (R_{t-1})^{\phi_r}\,\pi_{c,t}^{(1-\phi_r)\phi_\pi}\left(\frac{Y_t}{Y}\right)^{(1-\phi_r)\phi_y} \tag{5.38}$$

where R_{t-1} is the lagged nominal interest rate, $\pi_{c,t}$ is gross inflation rate, Y_t is actual output, and Y is the steady state value of output, ϕ_r, ϕ_π, ϕ_y are weights coefficients.

The Fisher equation, which governs the relation between the real interest rate and the nominal interest rate, is

$$r_t = \frac{R_t}{E_t\pi_{c,t+1}} \tag{5.39}$$

which implies that the (gross) real interest rate, r_t, is equal to the nominal interest rate, R_t, adjusted by the expected inflation rate, $E_t\pi_{c,t+1}$.

The premium on external finance, $R_{p,t}$, is the difference between the contractual lending rate, $R_{l,t}$, and the riskless saving rate, R_t,

$$R_{p,t} = R_{l,t} - R_t \tag{5.40}$$

5.3.7 Market clearing conditions

The bonds market clearing condition is

$$b_t^b = b_t \tag{5.41}$$

which implies that, in the loans market, the demand for loans from homeowners in credit constraint households is equal to the supply of loans from credit unconstraint households.

The economy-wide constraint or the goods market clearing condition is

$$\begin{aligned} & c_t + c_t^i + IK_{c,t} + IK_{h,t} \\ = \; & Y_t - \left[\mu_m G\left(\overline{\omega}_t\right) q_{h,t}\left(1-\delta_h\right) h_{t-1}\right] \\ & - \frac{\eta_{kh}}{2}\left(\frac{K_{h,t}}{K_{h,t-1}} - 1\right)^2 K_{h,t-1} - \frac{\eta_{kc}}{2}\left(\frac{K_{c,t}}{K_{c,t-1}} - 1\right)^2 K_{c,t-1} \end{aligned} \tag{5.42}$$

which implies that the total output from goods sector is divided into consumption goods, which is consumed by households, and investment in the goods capital and housing capital, subtracting monitoring costs and adjustment costs.

The housing market clearing condition is

$$H_t = h_t + h_t^b \tag{5.43}$$

which implies that the total housing stock is owned by credit unconstraint households and homeowners in credit constrained households.

The rental housing market clearing condition is

$$h_t^i = h_t^b \tag{5.44}$$

which implies that the demand for rental housing from renters is equal to the supply of rental housing from homeowners.

The goods capital accumulation is

$$K_{c,t} = (1 - \delta_{kc}) K_{c,t-1} + IK_{c,t} \tag{5.45}$$

which implies that goods capital in the current period, $K_{c,t}$, is the sum of undepreciated goods capital from the previous period, $(1 - \delta_{kc}) K_{c,t-1}$, and new investment in the current period, $IK_{c,t}$.

The housing capital accumulation is

$$K_{h,t} = (1 - \delta_{kh}) K_{h,t-1} + IK_{h,t} \tag{5.46}$$

which implies that housing capital in the current period, $K_{h,t}$, is the sum of undepreciated housing capital, $(1 - \delta_{kh}) K_{h,t-1}$, from the previous period and new investment in the current period, $IK_{h,t}$.

The housing stock accumulation is

$$H_t = (1 - \delta_h) H_{t-1} + IH_t \tag{5.47}$$

which implies that the total housing stock in the current period, H_t, is the sum of undepreciated housing stock from the previous period, $(1 - \delta_h) H_{t-1}$, and new housing in the current period, IH_t.

5.3.8 Competitive equilibrium

An equilibrium is an allocation of prices ($\pi_{c,t}$, R_t, $Q_{c,t}$, $q_{h,t}$, w_t, Z_t, r_t, $R_{kh,t}$, $R_{kc,t}$, $R_{p,t}$, $R_{l,t}$, $q_{r,t}$, w_t^i), quantities (c_t, $u_{c,t}$, h_t, Y_t, n_t, f_t, b_t, IH_t, H_t, $K_{h,t}$, $IK_{h,t}$, $K_{c,t}$, $IK_{c,t}$, b_t^b, h_t^b, nw_t^b, tr_t, $\lambda_{b,t}^b$, $\overline{\omega}_t$, m_t, c_t^i, $u_{c,t}^i$, h_t^i, n_t^i, $F(\overline{\omega}_t)$, $G(\overline{\omega}_t)$, $G'(\overline{\omega}_t)$, $\Gamma(\overline{\omega}_t)$, $\Gamma'(\overline{\omega}_t)$, $L(\overline{\omega}_t)$, $L'(\overline{\omega}_t)$, $B(\overline{\omega}_t)$, $B'(\overline{\omega}_t)$), and exogenous stochastic process $\{A_{c,t}\}_{t=0}^{\infty}$ satisfying equations (5.1) – (5.47) given the initial conditions for c_{t-1}, h_{t-1}, $\pi_{c,t}$, R_{t-1}, H_{t-1}, $K_{h,t-1}$, $K_{c,t-1}$, h_{t-1}^b, $R_{l,t-1}$, $q_{r,t-1}$, c_{t-1}^i, h_{t-1}^i.

5.3.9 Calibration

Firstly, we calibrate the coefficients that are related to the debt contract problem and the idiosyncratic shock. Following Bernanke, Gertler and Gilchrist (1999), we set the coefficient of the monitoring costs to $\mu_m = 0.12$, implying a 12% of realised value of collateral lost in default. Next, we set the quarterly steady state default rate to $F(\bar{\omega}) = 0.008$ as the Federal Reserve Bank of New York shows that 3.2% of prime mortgages were in foreclosure at the end of 2011. Besides, we set the loan to value ratio to $m = 0.75$, consistent with the sample average from Federal Housing Finance Agency. For the idiosyncratic shock, we set the unconditional mean to $\omega_m = 1$, consistent with Bernanke, Gertler and Gilchrist (1999).

The debt contract problem		
The coefficient of monitoring cost	μ_m	0.12
The mortgage default rate	$F(\bar{\omega})$	0.008
The loan to value ratio	m	0.75
The idiosyncratic shock		
The unconditional mean of shock	ω_m	1

For credit unconstrained households, we set their discount factor to $\beta = 0.9925 = 1.03^{-0.25}$, implying a steady state annual real interest rate of 3 percent. Their labour supply coefficient is set to $\gamma_n = 0.52$, implying that the Fisher elasticity of labour supply is $\frac{1}{\gamma_n} = 1.92$. The degree of consumption habit is set to $\varepsilon_c = 0.32$. These values are consistent with Iacoviello and Neri (2010). Moreover, we set their weight on housing to $j = 0.08$.

Credit unconstrained households preference		
Households discount factor	β	0.9925
The inverse of labour supply elasticity	γ_n	0.52
The degree of consumption habit	ε_c	0.32
The coefficient of housing preference	j	0.08

Following Iacoviello and Neri (2010), for credit constrained households, we set their discount factor to $\beta = 0.97$. Their labour supply coefficient is set to $\gamma_n = 0.51$, implying that the Fisher elasticity of labour supply is $\frac{1}{\gamma_n} = 1.96$. The degree of consumption habit is set to $\varepsilon_c = 0.58$. In order to hit the sample average, we set the housing preference coefficient to $j^i = 1$, which is higher than that of credit unconstrained households.

Credit constrained households preference		
The households discount factor	β^i	0.97
The inverse of labour supply elasticity	γ_n^i	0.51
The degree of consumption habit	ε_c^i	0.58
The coefficient of housing preference	j^i	1

In the intermediate goods production, the share of labour in goods production function is set to $\mu_n = 0.65$, implying that the steady state share of labour income is 65%. Meanwhile, the share of capital is set to $\mu_c = 0.35$. In addition, intermediate goods firms demand labour from both credit unconstrained households and credit constrained households, and we set the shares to 0.35 and 0.65 respectively, as the Residential Finance Survey from the U.S. Census Bureau suggests that 35% of housing owners do not have mortgages.

Intermediate goods production		
The share of capital	μ_c	0.35
The share of labour	μ_n	0.65
The sharehare of credit unconstrained households	α	0.35

For the retail goods sector, we assume a steady state markup of 15% in goods sector by setting $X = 1.15$. For the degree of prices stickiness, we assume that 17% of retailers are able to re-optimise their prices in each period by setting $\theta = 0.83$, implying that price-setters can re-optimise their prices once every $\frac{1}{1-\theta} = 6$ periods. The degree of price indexation is set to

$\iota_\pi = 0.69$. These values are also consistent with Iacoviello and Neri (2010).

Goods sector sticky price		
The steady state gross markup	Z	1.15
Probability of fixed price	θ	0.83
The degree of price indexation	ι_π	0.69

Following Iacoviello and Neri (2010), the share of housing capital in the housing production function is set to $\mu_h = 0.1$, implying that the steady state share of housing capital income is 10%.

Housing production technology		
The share of housing capital	μ_h	0.1

For the monetary policy rule, we set the weights coefficients to $\phi_r = 0.6$, $\phi_\pi = 1.5$, and $\phi_y = 0.5$, which are similar with Iacoviello and Neri (2010).[104]

Monetary policy		
The interest rate inertia	ϕ_r	0.6
The weight coefficient on inflation	ϕ_π	1.5
The weight coefficient on output	ϕ_y	0.5

The depreciation rates of housing, housing capital and goods capital are set to $\delta_h = 0.01$, $\delta_{kh} = 0.03$, and $\delta_{kc} = 0.025$, respectively. These values are consistent with Iacoviello and Neri (2010).

Depreciation rates		
Housing	δ_h	0.01
Housing capital	δ_{kh}	0.03
Goods capital	δ_{kc}	0.025

We also consider adjustment costs in our model. Similar with Iacoviello and Neri (2010), the adjustment coefficients for housing capital and goods

[104]The estimates in Iacoviello and Neri (2010) for these coefficients are $\phi_r = 0.61$, $\phi_\pi = 1.36$, and $\phi_y = 0.51$.

capital are set to $\eta_{kh} = 11$ and $\eta_{kc} = 15$ respectively.

Adjustment costs		
Housing capital	η_{kh}	11
Goods capital	η_{kc}	15

At this stage, we focus on the impulse responses of model variables to a temporary goods sector shock, thus the autocorrelation coefficient of the shock is set to 0.01. Meanwhile, we set the standard deviation of the shock to 0.01.

Autocorrelation of shock		
Goods sector technology	ρ_{Ac}	0.01
Standard deviation of shock		
Goods sector technology	σ_{Ac}	0.01

5.3.10 Steady state ratios

Firstly, the steady state annual real interest rate is targeted to 3%, which is consistent with literature. The data from the Federal Housing Finance Agency suggests that the average annual mortgage rate is 3% higher than the average annual yield on 6-month treasury bill, thus the target for the external finance premium is 3%.[105] This steady state ratio, however, from our model is 0.4%, which is lower than its target.[106] Meanwhile, the Residential Finance Survey from the U.S. Census Bureau suggests that 35% of housing is without mortgage, and 65% of housing is with mortgage, thus the ratio of housing with mortgage to housing without mortgage is 1.86. By setting the coefficients of housing preference of both households to $j = 0.08$ and $j' = 1$,

[105] Data from Federal Housing Finance Agency suggests that, over the period 1973 – 2011, the average annual mortgage rate is 8.7%, which is 3% higher than the average annual yield on 6-month treasury bill, 5.7%.

[106] We can only calibrate the loan to value ratio or the lending rate. We cannot calibrate both of them.

this target is also achieved.

Interpretation	Expression	Target	SS Value
The annual real interest rate	$R^4 - 1$	3%	3%
The annual premium	$((R_l)^4 - 1) - (R^4 - 1)$	3%	0.4%
Housing composition	h^b/h	1.86	1.89

The targets for macroeconomic variables are sample averages between $1947Q1$ and $2011Q4$ from the U.S. Bureau of Economic Analysis. The following table shows that the values of steady state ratios are close to their targets, given the calibration in the previous section.[107]

Interpretation	Expression	Target	SS Value
The share of consumption	$(c + c^i)/GDP$	80%	72%
The share of investment	$(IK_c + IK_h)/GDP$	14%	22%
The share of housing investment	$q_h IH/GDP$	6%	6%

5.3.11 Impulse response analysis

In this section, we discuss the dynamics of the model. Figure 5.5 and 5.6 show impulse responses of variables to an one percent positive shock in goods sector technology with persistence of $\rho_{Ac} = 0.01$.[108]

Firstly, the responses of macroeconomic variables are standard: a higher goods sector productivity leads to a higher output, a lower inflation, a lower nominal interest rate, and a higher real house price. In the housing market, although there is an increase in new housing production, we do not observe an increase in housing for both households. Instead, as credit unconstrained households are competitive in the housing market, they not only obtain newly produced housing, but take a fraction of housing from credit constrained households. Therefore, we observe a transfer of housing from

[107]GDP is composed by consumption goods, investments in goods capital and housing capital, and real value of new housing, i.e., $GDP = c + c' + IK_c + IK_h + qIH$.

[108]In all figures, impulse responses are measured as percentage deviations from the steady state, and horizontal axes display the number of quarters after the shock.

credit constrained households to credit unconstrained households. For credit constraint households, the value of housing assets they can buy is decreasing, since the negative impact of having less housing dominates the positive impact of the higher real house price.

Secondly, in the credit market, the decrease in the volume of loans from financial intermediaries to credit constrained households is larger than the fall in the expected value of undepreciated housing assets, leading to a lower loan to value ratio.[109] In addition, the decrease in the loan to value ratio implies that the contractual lending rate decreases more than the riskless saving rate, leading to a decrease in the premium of external finance.[110] Moreover, the decrease in the loan to value ratio leads to a lower threshold level, which leads to a decrease in the default rate.

Finally, because of the optimality condition between housing and consumption, credit unconstrained households increase their consumption as they obtain more housing. For the same reason, credit constrained households need to decrease their consumption to get a higher marginal utility, as they have a higher marginal utility from housing due to the transfer of housing from them to credit unconstrained households.

In sum, given a positive goods sector technology shock, the default rate displays countercyclical behaviour, which is consistent with our empirical evidence discussed at the beginning. The loan to value ratio, however, also displays countercyclical behaviour, which is not consistent with our empirical evidence.[111] The reason we suppose is that, in reality, similar with credit unconstrained households, credit constrained households also have more housing in an economic upturn, and then the value of their housing assets is also higher, leading to an increase in the volume of loans, together with a higher real house price. Therefore, the inconsistency between the result from our

[109] Initially, the loan to value ratio responds positively , but we focus on the scenario that the responses of variables have stablised.

[110] The equation that shows the direct relation between the lending rate and the loan to value ratio is

$$\frac{R_l}{R} = \frac{\overline{\omega}}{L(\overline{\omega})} = \frac{m\omega_m}{L(\overline{\omega})}$$

[111] Given a monetary policy shock, these two variables also move together.

model and empirical evidence suggests that, in the future research, we need to modify the model in a way that allows credit constrained households to have more housing in an economic upturn.

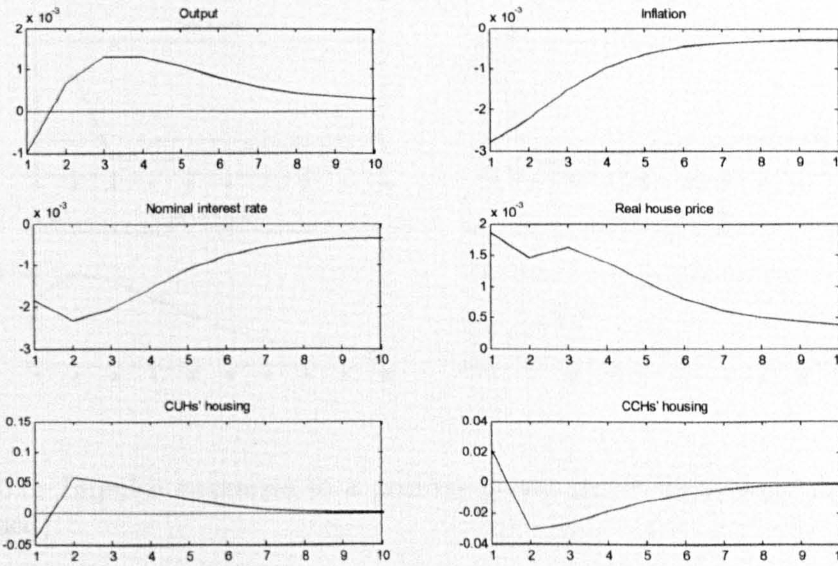

Figure 5.5: Impulse responses to a positive goods sector technology shock.

5.3.12 A discussion of the mean of the idiosyncratic shock

In this section, we discuss the relation between the default rate and the loan to value ratio. While empirical analysis shows a procyclical loan to value ratio and a countercyclical default rate, our impulse response analysis suggests that both of them display countercyclical behaviour given a positive goods sector technology shock. If we want to have a procyclical loan to value ratio, we should improve the model to allow credit constrained households to have more housing in an economic upturn (i.e., make them more competitive in the housing market). The setting of this model, however, implies that these two variables move in a same direction in response to any exogenous shock. The reason for this co-movement is the assumption of the time-invariant

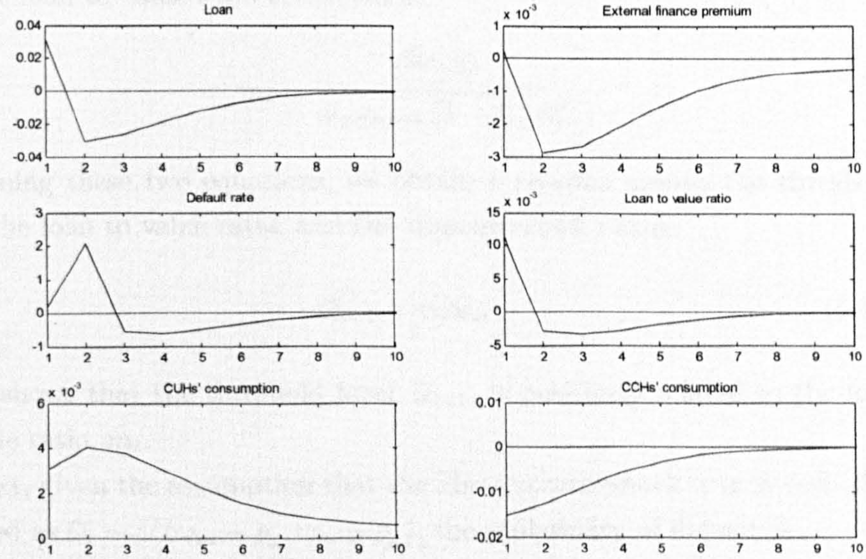

Figure 5.6: Impulse responses to a positive goods sector technology shock. (continued)

unconditional mean of the idiosyncratic shock ω_m. Then we show that a time-varying mean is able to break this co-movement. In this case, we actually assume that large (small) idiosyncratic shocks are more likely to occur in an economic upturn (downturn). For example, in an economic upturn, local governments have more resource to improve environment and reduce crime, leading positive impacts on house prices in the area.

The time-invariant mean ω_m Firstly, we discuss the implication of the time-invariant mean on the relation between the default rate and the loan to value ratio. Recall that the threshold level of the idiosyncratic shock is

$$\overline{\omega}_{t+1} = \frac{\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b}{q_{h,t+1}\left(1-\delta_h\right) h_t^b}$$

and the loan to value ratio is defined as

$$m_t = \frac{\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b}{\omega_m q_{h,t+1} (1 - \delta_h) h_t^b}$$

Combining these two equations, we obtain a relation among the threshold level, the loan to value ratio, and the time-invariant mean,

$$\overline{\omega}_{t+1} = m_t \omega_m \tag{5.48}$$

which shows that the threshold level, $\overline{\omega}_{t+1}$, is positively related to the loan to value ratio, m_t.

Next, given the assumption that the idiosyncratic shock is uniformly distributed as $\overline{\omega}_t \sim U(\omega_m - \rho_\omega, \omega_m + \rho_\omega)$, the probability of default is

$$F(\overline{\omega}_t) = \frac{\overline{\omega}_t}{2\rho_\omega} - \frac{\omega_m - \rho_\omega}{2\rho_\omega}$$

which expresses the default rate in terms of the threshold level, $\overline{\omega}_t$, and the lower boundary of the idiosyncratic shock, $\omega_m - \rho_\omega$. Since the coefficient $\frac{1}{2\rho_\omega}$ is positive, this equation implies a positive relation between the threshold level and the default rate,

$$\Delta\overline{\omega}_t \leftrightarrow \Delta F(\overline{\omega}_t) \text{ since } \frac{1}{2\rho_\omega} > 0$$

If we substitute the threshold level $\overline{\omega}_t$ using equation 5.48, we obtain

$$F(\overline{\omega}_t) = \frac{m_{t-1}\omega_m}{2\rho_\omega} - \frac{\omega_m - \rho_\omega}{2\rho_\omega}$$

which links the default rate to the loan to value ratio and the time-invariant mean. Since the coefficient $\frac{\omega_m}{2\rho_\omega}$ is positive, this equation implies a positive relation between the loan to value ratio and the default rate,

$$\Delta m_{t-1} \leftrightarrow \Delta F(\overline{\omega}_t) \text{ since } \frac{\omega_m}{2\rho_\omega} > 0$$

In other words, the default rate and the loan to value ratios move in the same direction in response to an exogenous shock when the unconditional mean of the idiosyncratic shock is time-invariant. Therefore, in order to make the loan to value ratio exhibit procyclical, the default rate should also become procyclical.

The time-varying mean $\omega_{m,t}$ In order to break up the co-movement between the default rate and the loan to value ratio, one approach is to introduce a time-varying mean of the idiosyncratic shock, $\omega_{m,t}$.

Firstly, the threshold level is

$$\overline{\omega}_{t+1} q_{h,t+1} \left(1 - \delta_h\right) h_t^b = \frac{R_{l,t}}{\pi_{c,t+1}} b_t^b$$

and the loan to value ratio becomes

$$m_t = \frac{\frac{R_{l,t}}{\pi_{c,t+1}} b_t^b}{\omega_{m,t+1} q_{h,t+1} \left(1 - \delta_h\right) h_t^b}$$

Combining these two equations, we obtained the relation among the threshold level, the loan to value ratio, and the time-varying mean,

$$\overline{\omega}_{t+1} = m_t \omega_{m,t+1} \tag{5.49}$$

which shows that the threshold level, $\overline{\omega}_{t+1}$, is positively related to the loan to value ratio, m_t, and the time-varying mean, $\omega_{m,t+1}$.

Next, when the idiosyncratic shock is assumed to be uniformly distributed with a time-varying mean, $\overline{\omega}_t \sim U\left(\omega_{m,t} - \rho_\omega, \omega_{m,t} + \rho_\omega\right)$, the default rate becomes

$$F\left(\overline{\omega}_t\right) = \frac{\overline{\omega}_t}{2\rho_\omega} - \frac{\omega_{m,t} - \rho_\omega}{2\rho_\omega}$$

which expresses the default rate in terms of the threshold level, $\overline{\omega}_t$, and the time-varying minimum of the idiosyncratic shock, $\omega_{m,t} - \rho_\omega$. This equation indicates that the time-varying mean plays an important role in the relation between the threshold level and the default rate,

$$\begin{aligned}\Delta\overline{\omega}_t &> \Delta\omega_{m,t} \Rightarrow \Delta\overline{\omega}_t \leftrightarrow \Delta F\left(\overline{\omega}_t\right)\\ \Delta\overline{\omega}_t &< \Delta\omega_{m,t} \Rightarrow \Delta\overline{\omega}_t \leftrightarrow -\Delta F\left(\overline{\omega}_t\right)\end{aligned}$$

In other words, if the change in the threshold is more (less) than the change in the time-varying mean, we have a *positive (negative) relation between the threshold level and the default rate*. Therefore, if the change in the mean is greater than the change in the threshold level, we will have a decreasing default rate while the threshold level is rising.

If we substitute the threshold level, $\overline{\omega}_t$, using equation 5.49, we obtain

$$F\left(\overline{\omega}_t\right) = \frac{m_{t-1}\omega_{m,t}}{2\rho_\omega} - \frac{\omega_{m,t} - \rho_\omega}{2\rho_\omega}$$

which links the default rate to the loan to value ratio and the time-varying mean. Their relation is more obvious if we log-linearise it,

$$\widehat{F\left(\overline{\omega}_t\right)} = \frac{m\omega_m}{2\rho_\omega F\left(\overline{\omega}\right)}\widehat{m}_{t-1} - \frac{\omega_m\left(1-m\right)}{2\rho_\omega F\left(\overline{\omega}\right)}\widehat{\omega}_{m,t}$$

which shows that the time-varying mean also play an important role in the relation between the default rate and the loan to value ratio,

$$\begin{aligned}m\widehat{m}_{t-1} &> \left(1-m\right)\widehat{\omega}_{m,t} \Rightarrow \Delta\widehat{m}_{t-1} \leftrightarrow \Delta\widehat{F\left(\overline{\omega}_t\right)}\\ m\widehat{m}_{t-1} &< \left(1-m\right)\widehat{\omega}_{m,t} \Rightarrow \Delta\widehat{m}_{t-1} \leftrightarrow -\Delta\widehat{F\left(\overline{\omega}_t\right)}\end{aligned}$$

In other words, if the change in the loan to value ratio is more (less) than the change in the time-varying mean, taking the coefficients into account, we have a positive (negative) relation between the loan to value ratio and the default rate. Thus, only when the time-varying mean is more procyclical than the loan to value ratio, we can have a countercyclical default rate simultaneously.

Therefore, the introduction of the time-varying mean can break the positive relation between the loan to value ratio and the default rate. In the future research, if we improve the model to produce a procyclical loan to value ratio, the time-varying mean is necessary to obtain a countercyclical default rate.

5.4 Conclusion

In this chapter, we introduce the agency problem into the housing market in a DSGE model, and then default becomes a steady state phenomenon as borrowers can walk away without repaying their debt. Since the loan to value ratio is closely related to the default rate, we can discuss the dynamics of the endogenous loan to value ratio in this model. Our empirical analysis suggests that, while the default rate is countercyclical, the loan to value ratio is procyclical.

Our impulse response analysis shows that, given a positive goods sector technology shock, the default rate exhibits countercyclical behaviour. The loan to value ratio, however, is also countercyclical. The reason is that, in an economic upturn, credit constrained households are not competitive in the housing market, and thus their housing is reduced. Meanwhile, the volume of loans they receive also decreases, leading to a fall in the loan to value ratio. The inconsistency between our results and empirical evidence suggests that, in the future research, in order to have a procyclical loan to value ratio, we need to improve the model in a way that allows credit constrained households to have more housing in an economic upturn. Moreover, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, the structure of our model implies a positive relation between the default rate and the loan to value ratio. As a result, if we can improve the model to have a procyclical loan to value ratio, the default rate will become procyclical as well. In order to overcome this problem and to have both procyclical loan to value ratio and countercyclical default rate, as suggested by the data, we should consider a time-varying mean of the idiosyncratic shock.

5.A Appendix to Chapter 5

5.A.1 The debt contract problem for credit unconstrained homeowners

Homeowners maximise the return of holding housing assets subject to lenders' participation constraint,

$$\max_{\overline{\omega}_{t+1}, h_t^b} E_t \left(\begin{array}{cc} & B\left(\overline{\omega}_{t+1}\right) q_{h,t+1}\left(1-\delta_h\right) h_t^b + q_{r,t} h_t^b \\ +\lambda_{b,t}^b & \left(\begin{array}{c} L\left(\overline{\omega}_{t+1}\right) q_{h,t+1}\left(1-\delta_h\right) h_t^b \\ -\frac{R_t}{\pi_{c,t-1}}\left(q_{h,t} h_t^b - nw_t^b\right) \end{array} \right) \end{array} \right)$$

5.A.2 Lagrangian program for renters

Renters maximise utility subject to their budget constraint,

$$\max_{c_t^i, h_t^i, n_t^i} E_t \sum_{k=0}^{\infty} \left(\begin{array}{cc} & \left(\beta^i\right)^k \left(\Gamma_c^i \ln\left(c_{t+k}^i - \varepsilon_c^i c_{t+k-1}^i\right) + j_{t+k}^i \ln h_{t+k}^i - \frac{1}{1+\gamma_n^i}\left(n_{t+k}^i\right)^{1+\gamma_n^i}\right) \\ +\lambda_{t+k}^i \left(\beta^i\right)^k & \left(\begin{array}{c} w_{t+k}^i n_{t+k}^i + tr_{t+k}^b - c_{t+k}^i - q_{r,t+k} h_{t+k}^i \\ -\frac{\eta_h^i}{2}\left(\frac{h_{t+k}^i}{h_{t+k-1}^i} - 1\right)^2 q_{h,t+k} h_{t+k-1}^i \end{array} \right) \end{array} \right)$$

5.A.3 Lagrangian program for credit unconstrained households

Credit unconstrained households maximise utility subject to their budget constraint,

$$\max_{\substack{c_t,b_t,n_t,h_t,\\ K_{h,t},K_{h,t}}} E_t \sum_{k=0}^{\infty} \left(\begin{array}{c} \beta^k \left(\Gamma_c \ln\left(c_{t+k} - \varepsilon_c c_{t+k-1}\right) + j_{t+k} \ln h_{t+k} - \frac{1}{1+\gamma_n} \left(n_{t+k}\right)^{1+\gamma_n}\right) \\ +\lambda_{t+k}\beta^k \left(\begin{array}{c} \left(R_{kc,t+k} + 1 - \delta_{kc}\right) K_{c,t+k-1} + \left(R_{kh,t+k} + 1 - \delta_{kh}\right) K_{h,t+k-1} \\ +w_{t+k} n_{t+k} + \omega_m q_{h,t+k} \left(1 - \delta_h\right) h_{t+k-1} \\ +\frac{R_{t+k-1}}{\pi_{c,t+k}} b_{t+k-1} + f_{t+k} \\ -c_{t+k} - K_{c,t+k} - K_{h,t+k} - q_{h,t+k} h_{t+k} - b_{t+k} \\ -\frac{\eta_h}{2} \left(\frac{h_{t+k}}{h_{t+k-1}} - 1\right)^2 q_{h,t+k} h_{t+k-1} - \frac{\eta_{kh}}{2} \left(\frac{K_{h,t+k}}{K_{h,t+k-1}} - 1\right)^2 K_{h,t+k-1} \\ -\frac{\eta_{kc}}{2} \left(\frac{K_{c,t+k}}{K_{c,t+k-1}} - 1\right)^2 K_{c,t+k-1} \end{array} \right) \end{array} \right)$$

5.A.4 Steady state ratios

In order to solve the model, we need following steady state values: $\frac{q_h h^b}{Y}$, $\frac{q_r h^i}{Y}$, $\frac{q_r}{q_h}$, $\frac{b^b}{Y}$, $\frac{nw^b}{Y}$, $\frac{tr^b}{Y}$, $\frac{w^i n^i}{Y}$, $\frac{c^i}{Y}$, $\frac{c}{Y}$, $\frac{IK_h}{Y}$.

From the credit unconstrained households' Euler equation, we have

$$R = \frac{1}{\beta}$$

From the threshold level of the idiosyncratic shock, we have

$$\overline{\omega} q_h \left(1 - \delta_h\right) h^b = R_l b^b$$

From the definition of the loan to value ratio, we have

$$m = \frac{R_l b^b}{\omega_m q_h \left(1 - \delta_h\right) h^b}$$

Combining these equations, we have the relation among the loan to value ratio, the unconditional mean of the idiosyncratic shock, and the threshold level of the shock, thus we have the steady state value of $\overline{\omega}$, after calibrating $\omega_m = 1$ and $m = 0.75$,

$$m\omega_m = \overline{\omega}$$

From the probability of default, after calibrating $F\left(\overline{\omega}\right) = 0.008$, we have the steady state value of ρ_{ω},

$$\rho_{\omega} = \frac{\overline{\omega} - \omega_m}{2F(\overline{\omega}) - 1}$$

Then we can following steady state ratios: $G(\overline{\omega})$, $G'(\overline{\omega})$, $L(\overline{\omega})$, $L'(\overline{\omega})$, $B(\overline{\omega})$, $B'(\overline{\omega})$.

From the lenders' participation constraint, we have

$$L(\overline{\omega}) q_h (1 - \delta_h) h^b = Rb^b$$

Combining the lenders' participation constraint and the definition of the loan to value ratio, we can solve the steady state value of external finance premium, $\frac{R_l}{R}$,

$$\frac{\overline{\omega}}{L(\overline{\omega})} = \frac{R_l}{R}$$

From the first order condition with respect to $\overline{\omega}_{t+1}$, we have the steady state value of λ_b^b,

$$\lambda_b^b = -\frac{B'(\overline{\omega})}{L'(\overline{\omega})}$$

From the first order condition with respect to h_t^b, we have the steady state value of $\frac{q_r}{q_h}$,

$$\frac{q_r}{q_h} = -\left[\lambda_b^b L(\overline{\omega})(1 - \delta_h) - R\right] - B(\overline{\omega})(1 - \delta_h)$$

Combing the lenders' participation constraint, the borrowing equation, and the equation of net worth, we can solve the steady state value of γ_{nw},

$$\gamma_{nw} = \frac{1 - \frac{L(\overline{\omega})(1-\delta_h)}{R}}{B(\overline{\omega})(1 - \delta_h) + \frac{q_r}{q_h}}$$

From the homeowners' demand for housing (the FOC w.r.t h_t^i), we have the steady state value of $\frac{q_r h^i}{Y}$,

$$\frac{q_r h^i}{Y} = j^i \frac{c^i}{Y}$$

which is a function of $\frac{c^i}{Y}$, while j^i can be calibrated.

Combining the steady state value of $\frac{q_r h^b}{Y}$ and $\frac{q_r}{q_h}$, we have the steady state value of $\frac{q_h h^b}{Y}$,

$$\frac{q_h h^b}{Y} = \frac{q_r h^b}{Y} / \frac{q_r}{q_h}$$

which is a function of $\frac{q_r h^b}{Y}$, and thus in term of $\frac{c^i}{Y}$.

From the lenders' participation constraint, we have the steady state value of $\frac{b^b}{Y}$,

$$\frac{b^b}{Y} = L(\overline{\omega}) \frac{(1-\delta_h)}{R} \frac{q_h h^b}{Y}$$

which is a function of $\frac{q_h h^b}{Y}$, and thus in term of $\frac{c^i}{Y}$.

From the equation of net worth, we have the steady state value of $\frac{nw^b}{Y}$,

$$\frac{nw^b}{Y} = \gamma_{nw} \cdot \left[B(\overline{\omega})(1-\delta_h) \frac{q_h h^b}{Y} + \frac{q_r h^b}{Y} \right]$$

which is a function of $\frac{q_h h^b}{Y}$, and thus in term of $\frac{c^i}{Y}$.

From the equation of transfer to renters, we have the steady state value of $\frac{tr^b}{Y}$,

$$\frac{tr^b}{Y} = (1-\gamma_{nw}) \cdot \left[B(\overline{\omega})(1-\delta_h) \frac{q_h h^b}{Y} + \frac{q_r h^b}{Y} \right]$$

which is a function of $\frac{q_h h^b}{Y}$, and thus in term of $\frac{c^i}{Y}$. Although it is not easy to prove using equations, but values from Matlab show that

$$\frac{tr^b}{Y} = \frac{q_r h^b}{Y}$$

which means that, at steady state, the transfer is equal to the rent payment.

From the intermediate goods firms' demand for labour, we have the steady state value of $\frac{w^i n^i}{Y}$,

$$\frac{w^i n^i}{Y} = \frac{(\mu_n)(1-\alpha)}{Z}$$

From the renter's budget constraint, we have

$$\frac{w^i n^i}{Y} + \frac{tr^b}{Y} = \frac{c^i}{Y} + \frac{q_r h^i}{Y}$$

Then combine the equation of transfer to renters, the renter's budget constraint, the intermediate goods firms' demand for labour, and the FOC w.r.t h_t^i, we can solve the steady state value of $\frac{c^i}{Y}$,

$$\frac{c^i}{Y} = \frac{\frac{w^i n^i}{Y}}{\left[1 + j^i - (1-\gamma_{nw}) \cdot \left(\frac{B(\bar{\omega})(1-\delta_h)}{\frac{q_r}{q_h}} + 1\right) j^i\right]}$$

In this equation, the relation between $\frac{c^i}{Y}$ and j^i is not explicit. However, the relation between $\frac{q_r h^i}{Y}$ and j^i is much clear

$$\frac{q_r h^i}{Y} = j^i \frac{c^i}{Y} = \frac{\frac{w^i n^i}{Y}}{\left[\frac{1}{j^i} + 1 - (1-\gamma_{nw}) \cdot \left(\frac{B(\bar{\omega})(1-\delta_h)}{\frac{q_r}{q_h}} + 1\right)\right]}$$

We can see that the lower j^i, the larger the denominator, the smaller the faction. Therefore, if we want a higher $\frac{q_r h^i}{Y}$, we should have a larger j^i. Thus we can solve the steady state ratios of $\frac{q_r h^i}{Y}$, $\frac{q_h h^b}{Y}$, $\frac{b^b}{Y}$, $\frac{nw^b}{Y}$, $\frac{tr^b}{Y}$.

From the credit unconstrained households' demand for goods capital, we have the steady state value of R_{kc},

$$R_{kc} = \frac{1 - \beta(1-\delta_{kc})}{\beta}$$

From the intermediate goods producers' demand for goods capital, we have the steady state value of $\frac{K_c}{Y}$,

$$\frac{K_c}{Y} = \frac{\mu_{kc}}{Z R_{kc}}$$

From the goods capital accumulation, we have the steady state value of $\frac{IK_c}{K_c}$,

$$\frac{IK_c}{K_c} = \delta_{kc}$$

Combining the steady state value of $\frac{K_c}{Y}$ and $\frac{IK_c}{K_c}$, we have the steady state value of $\frac{IK_c}{Y}$,

$$\frac{IK_c}{Y} = \frac{K_c}{Y}\frac{IK_c}{K_c}$$

From the credit unconstrained households' demand for housing capital, we have the steady state value of R_{kh},

$$R_{kh} = \frac{1 - \beta(1 - \delta_{kh})}{\beta}$$

From the housing producers' demand for housing capital, we have the steady state value of $\frac{q_h IH}{K_h}$,

$$\frac{q_h IH}{K_h} = \frac{R_{kh}}{\mu_{kh}}$$

From the housing capital accumulation, we have the steady state value of $\frac{IK_h}{K_h}$,

$$\frac{IK_h}{K_h} = \delta_{kh}$$

Combining the steady state value of $\frac{q_h IH}{K_h}$ and $\frac{IK_h}{K_h}$, we have the steady state value of $\frac{q_h IH}{IK_h}$,

$$\frac{q_h IH}{IK_h} = \frac{q_h IH}{K_h} / \frac{IK_h}{K_h}$$

From the credit unconstrained households' marginal utility of consumption, we have

$$u_c = \frac{1}{c}$$

From the credit unconstrained households' demand for housing, we have the steady state value of $\frac{q_h h}{c}$,

$$\frac{q_h h}{c} = \frac{j}{(1 - \beta(1 - \delta_h)\omega_m)}$$

Combining the steady state value of $\frac{q_h h}{c}$ and $\frac{c}{Y}$, we have the steady state value of $\frac{q_h h}{Y}$,

$$\frac{q_h h}{Y} = \frac{q_h h}{c}\frac{c}{Y}$$

which is a function of $\frac{c}{Y}$.

From the housing market clearing condition, we have

$$h + h^b = H$$

Thus gives

$$\frac{q_h H}{Y} = \frac{q_h h}{Y} + \frac{q_h h^b}{Y}$$

which is a function of $\frac{q_h h}{Y}$, and thus in term of $\frac{c}{Y}$.

From the housing capital accumulation process, we have the steady state value of $\frac{IH}{H}$,

$$\frac{IH}{H} = \delta_h$$

Thus combining the steady state value of $\frac{q_h H}{Y}$ and $\frac{IH}{H}$, we have the steady state value of $\frac{q_h IH}{Y}$,

$$\frac{q_h IH}{Y} = \frac{q_h H}{Y}\frac{IH}{H}$$

which is a function of $\frac{q_h H}{Y}$, and thus in term of $\frac{c}{Y}$.

Combining the steady state value of $\frac{q_h IH}{Y}$ and $\frac{q_h IH}{IK_h}$, we have the steady state value of $\frac{IK_h}{Y}$,

$$\frac{IK_h}{Y} = \frac{q_h IH}{Y} / \frac{q_h IH}{IK_h}$$

which is a function of $\frac{q_h IH}{Y}$, and thus in term of $\frac{c}{Y}$.

Combining the steady state value of $\frac{IK_h}{Y}$ and $\frac{IK_h}{K_h}$, we have the steady state value of $\frac{K_h}{Y}$,

$$\frac{K_h}{Y} = \frac{IK_h}{Y} / \frac{IK_h}{K_h}$$

From the goods market clearing condition, we have

$$\frac{IK_h}{Y} + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} = 1$$

Together with other equations, gives

$$\begin{aligned}
\frac{q_h IH}{Y} / \frac{q_h IH}{IK_h} + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} &= 1 \\
\frac{q_h H}{Y} \frac{IH}{H} / \frac{q_h IH}{IK_h} + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} &= 1 \\
\left(\frac{q_h h}{Y} + \frac{q_h h^b}{Y} \right) \frac{IH}{H} / \frac{q_h IH}{IK_h} + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} &= 1 \\
\left(\frac{q_h h}{c} \frac{c}{Y} + \frac{q_h h^b}{Y} \right) \frac{IH}{H} / \frac{q_h IH}{IK_h} + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} &= 1 \\
\frac{\frac{q_h h}{c} \frac{IH}{H}}{\frac{q_h IH}{IK_h}} \frac{c}{Y} + \frac{\frac{q_h h^b}{Y} \frac{IH}{H}}{\frac{q_h IH}{IK_h}} \Bigg) + \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_c}{Y} &= 1
\end{aligned}$$

Now we can solve the steady state value of $\frac{c}{Y}$,

$$\frac{c}{Y} = \frac{1 - \frac{c^i}{Y} - \frac{IK_c}{Y} - \frac{\frac{q_h h^b}{Y} \frac{IH}{H}}{\frac{q_h IH}{IK_h}}}{\frac{\frac{q_h h}{c} \frac{IH}{H}}{\frac{q_h IH}{IK_h}} + 1}$$

thus we can solve $\frac{q_h h}{Y}$, $\frac{q_h H}{Y}$, $\frac{q_h IH}{Y}$, $\frac{IK_h}{Y}$, $\frac{K_h}{Y}$.

Combining the steady state value of $\frac{q_h h^b}{Y}$ and $\frac{q_h h}{Y}$, we have the steady state value of $\frac{h^b}{h}$,

$$\frac{h^b}{h} = \frac{q_h h^b}{Y} / \frac{q_h h}{Y}$$

This is the ratio of housing owned by mortgage holders to housing owned by non-mortgage holder. The empirical target for this ratio is $\frac{65\%}{35\%} = 1.86$.

From the definition of GDP, we have the steady state value of $\frac{GDP}{Y}$,

$$\frac{GDP}{Y} = \frac{c}{Y} + \frac{c^i}{Y} + \frac{IK_h}{Y} + \frac{IK_c}{Y} + \frac{q_h IH}{Y}$$

Then the GDP-related steady state ratio are

$$\frac{c}{GDP} = \frac{c}{Y} / \frac{GDP}{Y}$$

and

$$\frac{IK_h + IK_c}{GDP} = \left(\frac{IK_h}{Y} + \frac{IK_c}{Y} \right) / \frac{GDP}{Y}$$

and

$$\frac{q_h IH}{GDP} = \frac{q_h IH}{Y} / \frac{GDP}{Y}$$

6 Concluding Remarks

Our thesis focuses on the several housing sector related issues in the context of DSGE models, as the housing sector is important to the economy but usually ignored in business cycles analysis.

To begin with, we conduct a sensitivity analysis using a simple DSGE model with the feature of sticky prices, credit market imperfections, and a fixed housing supply. When the borrowing constraint is related to production housing, which is a factor of production, we find the financial accelerator effect when the central banks are concerned with inflation only, given a goods sector technology shock. When the borrowing constraint, however, is related to domestic housing, which provides utility to owners, we do not observe the financial accelerator effect, given a goods sector technology shock. Our results suggest that the financial accelerator effect does not necessarily appear given the presence of credit market imperfections.

In the chapter 2, we introduce a variable housing supply, thus we can discuss the supply side effect in the housing market, including the direct effect and the feedback effect. We find that the magnitudes of these two effects are negligible under the standard setting of the housing market. Then we examine the U.S. housing sector and suggest that we should (i) construct a new setting for the housing market and (ii) introduce the feature of time to build to new housing production. After constructing the new setting of the housing market, the magnitudes of the direct effect and the feedback effect are 60 times larger, suggesting that the supply side effects are largely underestimated in the literature. Moreover, the feature of time to build, together with the new setting of the housing market, allows us to obtain cyclical behaviour for the real house price. This result may explain the high volatility of real house price. In addition, it is suggested that the setting of the feature of time to build is too simple in our chapter. In our future research, we can consider a more realistic setting by assuming that only a fraction of housing is built in each period before its completion, but our results should not be affected.

Next, while rational expectations is a standard assumption in DSGE models, we consider the assumption of adaptive learning, as we are convinced that this alternative way of forming expectations partially contributed to the house price bubble. In particular, we explore the impact of the AR(1) learning model and discuss the interaction between the adaptive learning mechanism and the feature of time to build. Moreover, we also discuss the assumption of heterogeneous expectations. Specifically, we consider learning agents under heterogeneous expectations and compare two cases of heterogeneous expectations, i.e., one with partially rational agents and one with fully rational agents. In future research, we plan to (i) explore the impact of other learning models; (ii) examine the determinants of the amplification and propagation effects from the adaptive learning mechanism; (iii) allow an endogenous transformation from rational agents to learning agents.

Finally, we introduce the agency problem into the housing market. Our analysis shows that, given a positive goods sector technology shock, the default rate exhibits countercyclical behaviour, which is consistent with our empirical analysis. The loan to value ratio, however, is also countercyclical, while our empirical analysis suggests procyclical behaviour. The reason we suppose is that, in an economic upturn, credit constrained households have less housing in the housing market, thus the volume of loans they receive also decreases, leading to a decrease in the loan to value ratio. Therefore, the inconsistency between the results from our model and empirical evidence suggests that, in the future research, we need to improve the model in a way that allows credit constrained households to have more housing during economic upturn. Moreover, we illustrate that, when the mean of the idiosyncratic shock is time-invariant, the structure of the model implies a positive relation between the default rate and the loan to value ratio. Then we show that a time-varying mean of the idiosyncratic shock is necessary to overcome this co-movement.

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