UNDERSTANDING THE ROLE OF VISUO-SPATIAL WORKING MEMORY IN ADULT MATHEMATICS

PAULA JANE HUBBER, BSc. MSc. F.C.C.A.

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Abstract

Mathematics is an important part of everyday adult life and requires skilled use of a variety of cognitive resources. The aim of this thesis was to explore the use of working memory in adult mathematics performance and particularly the role of visuo-spatial working memory.

In the first study, differences in working memory capacity between skilled adult mathematicians and those who have less expertise in mathematics were investigated. This involved the use of working memory span tasks that included a novel processing element that was as neutral as possible with regard to the verbal and visuo-spatial storage elements. The results of this study included the novel finding that skilled adult mathematicians have a superior ability to store visuo-spatial information within working memory whilst concurrent processing is taking place.

In the second study, measures of basic temporary visuo-spatial storage and endogenous spatial attention were used to discover whether these abilities drive the differences in visuo-spatial working memory capacity between skilled mathematicians and non-mathematicians found in Study 1. Results included the novel finding that capacity differences are not explained by basic temporary storage or endogenous spatial attention.

The relationships of visuo-spatial item memory and order memory with adult mathematics were then explored in Study 3. Results showed the ability to order visuo-spatial information, rather than memory for whether a visuo-spatial item is simply present, seems to be related to adult mathematics achievement.

Working memory capacity differences were again investigated in Study 4, in which the processing elements within the span tasks were of a more traditional verbal and visuo-spatial nature. Differences between mathematicians and non-mathematicians for general visuo-spatial ability were also examined to see whether this ability drives the relationship between visuo-spatial working memory capacity and adult mathematics performance. Contrary to the results of Study 1, mathematicians did not have superior working memory capacity to non-mathematicians in any combination of verbal and visuo-spatial storage and processing. Mathematicians therefore only seem to have superior visuo-spatial working memory capacity when the executive resources used during processing are comparatively low, as in Study 1. Adult mathematicians were also found to have superior general visuo-spatial ability to non-mathematicians, but this did not explain observed working memory capacity differences.

Finally, Study 5 explored the relative roles of the visuo-spatial sketchpad and central executive components of visuo-spatial working memory when adults solve arithmetic using different strategies. Whilst both the central executive and visuo-spatial sketchpad are used in adult arithmetic, the former was found to be used to a greater extent and particularly when counting was used to solve problems.

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Table of Contents

List of Figures

Chapter 1: Introduction

Chapter 2: Working Memory Storage Capacity

Chapter 3: Short Term Memory and Endogenous Spatial Attention

Chapter 4: Item Memory and Order Memory in the Visuo-Spatial Domain

Chapter 5: Working Memory Storage Capacity: Verbal & Visuo-Spatial Processing

Chapter 6: The Involvement of the Central Executive and Visuo-Spatial Storage in Mental Arithmetic

Chapter 7: General Discussion

List of Tables

Chapter 2: Working Memory Storage Capacity

Chapter 3: Short Term Memory and Endogenous Spatial Attention

Chapter 4: Item Memory and Order Memory in the Visuo-Spatial Domain

Chapter 5: Working Memory Storage Capacity: Verbal & Visuo-Spatial Processing

Chapter 6: The Involvement of the Central Executive and Visuo-Spatial Storage in Mental Arithmetic

Chapter 1: Introduction

1.1 General Introduction

Mathematics is an important part of everyday adult life. Its use ranges from basic arithmetic, such as calculating change in a shop, through to the more complex mathematics involved in balancing the household budget and assessing different financial products, and the advanced mathematics involved in, for example, accountancy, engineering and physics. Geary (2004) highlighted that mathematics achievement has been shown to affect employability and earning levels over and above the effects of literacy and general intelligence. However, it is estimated that around a quarter of UK adults have poor mathematics skills (Department for Business, Innovation & Skills, 2011), equating to over 8 million adults with mathematics skills below those expected of children aged between 9 and 11 years of age. Whilst many different factors may contribute to an adult's mathematics achievement, this thesis will examine the cognitive processes involved in mathematics and particularly the role of visuo-spatial working memory.

Any theoretical model attempting to explain the processes involved in mathematical cognition needs to be able to accommodate the initial sensing and encoding of information from a variety of sources, the retrieval of previously stored number facts, the selection and execution of available procedures for solving a problem, the combining of information, the temporary storage of interim calculations for later use and the output of answers. This chapter will firstly consider three existing models of mathematical cognition (section 1.2) and show that they are inadequate for explaining the complex cognitive processes involved in performing mathematics, particularly due to the absence within them of any role for working memory. Section 1.3 will then consider four prominent models of working memory. Section 1.4 will introduce the literature surrounding mathematics and working memory in both adults and children. Finally, section 1.5 outlines the aims and structure of this thesis and includes an introduction to the methods and analyses used throughout the following chapters.

1.2 Models of Mathematical Cognition

Several models of mathematical cognition exist that attempt to explain the cognitive processes involved in performing mathematics. They address issues such as the extent to which processes involved are independent or interact and whether processes are additive (for example, encoding of information is completed before calculation commences) or integrated (Campbell & Epp, 2005). Three models of mathematical cognition will now be discussed, in turn, before discussing their overall limitations.

1.2.1 Abstract Code Model

When adults attempt to solve a mathematical problem, they commonly see the initial problem in either digit form (e.g. 2 + 4 = ?) or written form (e.g. two add four = ?) or hear the information (Campbell & Epp, 2005). The abstract code model (McCloskey & Carmazza, 1985), depicted in Figure 1.1, states that this initial information is then encoded into an abstract quantity code for use by three distinct cognitive systems: comprehension; calculation; and response. The comprehension system is responsible for converting the initial input into this abstract code. The calculation system then uses an individual's memory for number facts and rules to calculate the correct answer to a problem, using the abstract code, before the response system converts the abstract code into an appropriate output format, such as digit, written word or oral form (Campbell & Epp, 2005). The model assumes that, as an abstract code is used, performance should be no different for any of the initial input forms. It also assumes independence of the three cognitive systems and that they are additive in nature. In other words, calculation only takes place once encoding into the abstract code has been completed and output only occurs after completion of calculation.

Support for this model arises from research involving brain-damaged patients and the manipulation of the format of presentation of mathematical problems. For example, patient P.S., who had difficulty in retrieving arithmetic facts from long-term memory (a permanent store of information: Craik & Lockhart, 1972; Shiffrin & Atkinson, 1969), showed no difference in the comprehension or solving of basic arithmetic problems whether sums were presented or answers given using Arabic numerals, written words or strips of paper with dots printed on them (Sokol, McCloskey, Cohen & Aliminosa, 1991). This finding was interpreted as indicating that an internal abstract code is used to represent numbers and perform calculations. According to the abstract code model, if mathematical cognition involves format-specific codes and processes, rather than an abstract code, different presentation and answering formats should illicit different amounts of errors (McCloskey, 1992). P.S. produced the same amount of errors across all formats, suggesting the use of a single abstract code.

The model does not adequately explain, however, how the calculation system actually combines rules and procedures in this abstract code to produce answers to different types of mathematical problems. For example, McNeil & Warrington (1994) investigated a patient, H.A.R., who could perform simple additions and subtractions when problems were presented orally, but when presented with written problems his performance on additions was impaired whilst his performance on subtractions was not. Therefore, McNeil & Warrington argued that the abstract code model could not adequately explain the calculation process because of the dissociation between different types of arithmetic. The model predicts that any deficits in performance should be consistent across modality of input and arithmetic type. Also, the finding that number words are named faster than digits, yet numerical magnitude judgements are faster for digits than for number words (Damian, 2004) also contradicts the model’s assumption that different input formats map onto a common abstract format. The model would predict that if number words are named faster, they are therefore converted to abstract code faster and therefore numerical judgements presented in word format should be solved faster.

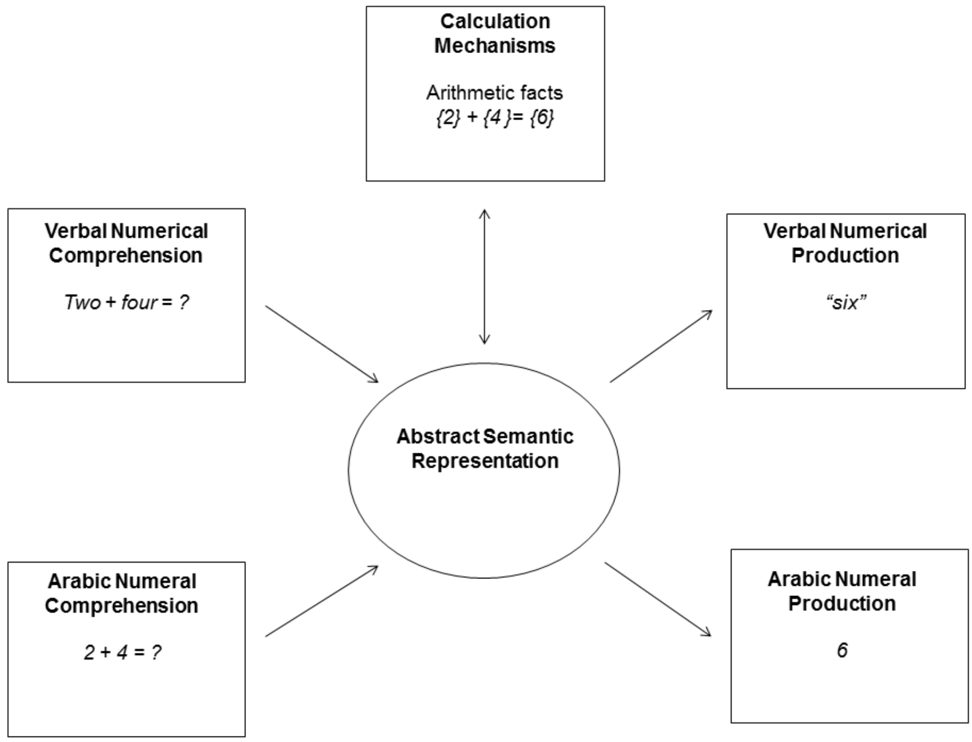

Figure 1.1: Abstract Code Model. Adapted from "Architectures for arithmetic" by J. I. Campbell & L. J. Epp, 2005, *Handbook of Mathematical Cognition*, p. 348. Copyright 2005 by Psychology Press.

1.2.2 Triple Code Model

As discussed above, McCloskey's abstract code model has been criticised for its use of an abstract code (e.g. Damian, 2004; McNeil & Warrington, 1994). Dehaene (1992) then created the triple code model, which assumes there is no need to convert information into an abstract code and that mathematical cognition uses three different types of code: visual-Arabic numbers; auditory-verbal code; and analogue magnitude representation. The model is depicted in Figure 1.2 and does not assume any interaction between the three different codes. Input is converted into the appropriate code required for the specific type of processing involved in a given mathematical problem rather than the codes all working together (Campbell & Epp, 2005).

The triple code model specifies roles for both verbal and spatial codes (Dehaene & Cohen, 1995). Arabic-number code is believed to support the input and output of digit information. The auditory-verbal code provides a representation of arithmetic facts and supports the retrieval of number facts from long-term memory. There is a direct link between Arabic numbers and verbal codes, which supports the fast retrieval of number facts from memory. This model therefore assumes that verbal, language-based representations are important for mathematics (Dehaene, 2001).

The spatial magnitude code is believed to have a role in estimation, approximate calculation and in comparing the size of numbers and is thought to give meaning to a number in relation to other numbers (Dehaene & Cohen, 1995). Numbers are situated along a number line where quantities are represented by the distribution of activations of memory. Relationships between numbers are then represented by the overlap between these activations. The magnitude code is not thought to be precise and so can be used only for approximations. If precision is required, the magnitude code must be converted into an appropriate verbal or Arabic code. Similarly, if two numbers are to be compared, the verbal or Arabic code must first be converted into a magnitude code to enable the comparison to take place. Dissociation between verbal and magnitude codes has been supported through research with patients who were able to perform approximations but were impaired on performing precise calculations (Lemer, Dehaene, Spelke & Cohen, 2003).

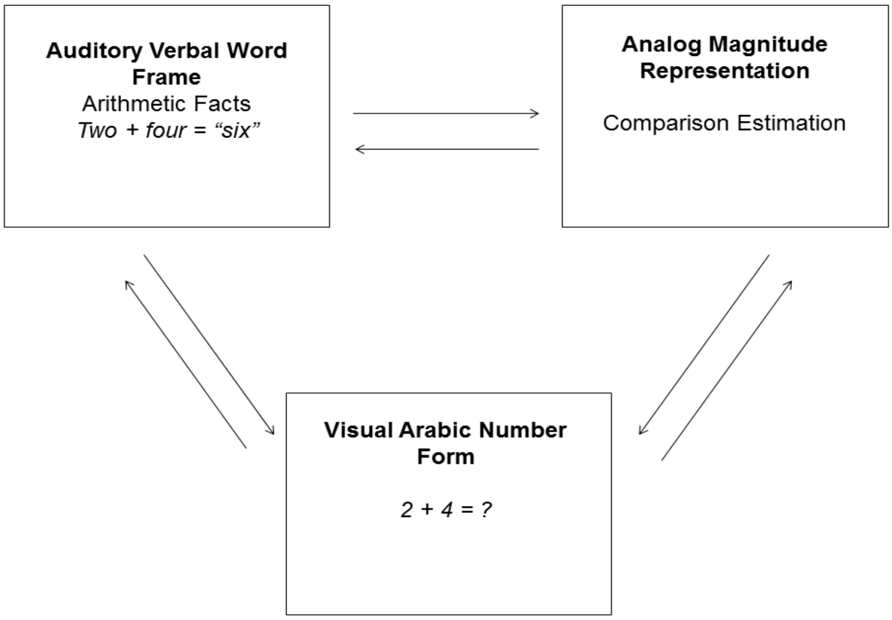

Figure 1.2: Triple Code Model. Adapted from "Architectures for arithmetic" by J. I. Campbell & L. J. Epp, 2005, *Handbook of Mathematical Cognition*, p. 349. Copyright 2005 by Psychology Press.

1.2.3 Encoding-Complex Hypothesis

The abstract code model and the triple code model have been criticised for being too simplistic as they deal mainly with retrieving arithmetic facts and do not explain the processes involved in more complex calculation (Campbell, 1994). Campbell also argued that there is evidence that the processing of number magnitude, the production of numbers verbally and the solving of simple arithmetic problems all involve formats specific to the processes required rather than an independent code proposed by the abstract code model. For example, he argued there is evidence that multiplication involves non-abstract and format specific representations (Clark & Campbell, 1991). This is supported by Noël & Seron (1993) who found that different tasks involve different representations of numbers and that these representations may also differ from individual to individual. Campbell also argued that both of the preceding models are wrong to assume that mathematical processes are additive, as there is evidence that the various processes interact rather than acting independently (Campbell & Epp, 2005).

Campbell’s encoding-complex hypothesis assumes that both verbal and visuo-spatial codes are involved in mathematics (Clark & Campbell, 1991). Systems representing both verbal and visuo-spatial information interact whilst solving mathematical problems through an associative memory network which involves the excitation and inhibition of information, within and between these systems (Campbell & Epp, 2005). Previous robust findings in the mathematical cognition literature of magnitude effects for latencies and accuracy in adults solving arithmetic problems suggest that magnitude information is accessed when retrieving number facts (e.g. Dehaene, 1989; Dehaene, Bossini & Giraux, 1993; Fias, Brysbaert, Geypens & d’Ydewalle, 1996). This suggests an interaction between verbal and visuo-spatial information (Gallistel & Gelman, 2005). Campbell (1995) suggested that activating magnitude information may assist the retrieval of number facts from long-term memory through the priming of numbers that are approximately the correct size. Following examination of number fact naming and arithmetic performance in bilinguals, Campbell & Epp (2004) concluded that the efficiency of the interaction between verbal and visuo-spatial codes depends upon the amount of previous practice using a number fact and therefore how strongly it is represented within long-term memory.

Finally, the encoding-complex hypothesis includes a role for attention, which was not included in the two preceding models. Campbell (1994) suggested that attention, defined as the “goal-directed focus on one aspect of the environment, while ignoring irrelevant aspects” (Gazzaley & Nobre 2012, p.129), is required to solve mathematics problems. The level of attention required is assumed to be dependent on the format of presentation of a problem (e.g. written word or Arabic numeral). Problems involving larger numbers and the calculation of answers may require more attention than simple arithmetic fact retrieval. However, these assumptions are yet to be systematically tested within the mathematical cognition literature.

1.2.4 Limitations of the Models of Mathematical Cognition

The abstract code model has been widely criticised for its assumption that, when solving a mathematical problem, initial information is converted into an abstract code (Campbell & Epp, 2005). Dehaene’s triple code model acknowledged this and incorporated the importance of both verbal and visuo-spatial codes in performing mathematics. However, like the abstract code model, the triple code model is additive and assumes that the domain-specific verbal and visuo-spatial codes do not interact. As discussed in section 1.2.3, evidence suggests that both domains are required for retrieving number facts from long-term memory (Dehaene, 1989; Dehaene et al., 1993; Fias et al., 1996; Gallistel & Gelman, 2005) and the encoding-complex hypothesis model assumes that the two domains interact through excitatory and inhibitory networks. This model is more detailed in how it incorporates the various types of information needed to solve problems, but, in common with the other two models, it only really attempts to explain how basic number facts are accessed in memory rather than how complex calculations are performed. Whilst a basic calculation, such as 2 + 4 = ? can be solved by quickly accessing a well-rehearsed number fact from long-term memory, solving more complex problems is not so straight forward.

Consider the sum 34 + 57 = ? This sum cannot be solved, by the majority of adults, through immediate access of a stored number fact held in memory. If adults are not able to simply answer a problem via fact retrieval, they will employ procedural strategies to break down the problem into smaller stages. The most common procedural methods used by adults are counting and decomposition (Geary, Frensch & Wiley, 1993; Hecht, 1999). In order to solve the sum 34 + 57 = ?, a form of decomposition might be used, as follows. An individual might first add the tens 30 and 50 together and retrieve the answer 80 from long-term memory. This interim result will then be held in mind temporarily. They might then add the unit 7 to the interim result (80 + 7) and then hold 87 in mind before adding the unit 4 to reach the result of 91. Whilst the calculation is performed, it is also necessary to remember the initial problem, so that constituent parts can be accessed, if it is not available in written form. Therefore, even a relatively simple problem such as an addition involving double digits requires the remembering and encoding of initial information, the ordering and performing of several steps, the accessing of number facts from long-term memory and the remembering and updating of interim calculations before outputting the final answer. Calculation requires both the storage and processing of information as well as the retrieval of learnt number facts (Adams & Hitch, 1997; Trbovich & LeFevre, 2003).

Whilst the triple code model and encoding-complex hypothesis state that verbal and spatial codes are used in mathematics, neither model adequately describes how these codes are used and combined to perform calculations or how initial information and interim calculations are stored for further use. Hitch (1978) found that forgetting interim information and details of the original problem contribute to mathematical errors and expert calculators state the importance of being able to hold interim information in mind (Butterworth, 2006). None of the models of mathematical cognition described above can account for performing a calculation that doesn't simply require the direct retrieval of an answer from memory. A more complex cognitive model is required to accommodate all of the factors involved in solving these types of problems.

McCloskey (1992) suggested that further research was required regarding the importance of verbal and spatial systems and the complex cognitive system, working memory, for performing more detailed calculations. In the years since he introduced his abstract code model, research has shown that working memory is indeed involved in performing mathematics (Raghubar, Barnes & Hecht, 2010).

1.3 Models of Working Memory

The term working memory is used to refer to the complex cognitive system that both maintains and manipulates information in order to complete a task (Bayliss, Jarrold, Baddeley & Gunn, 2005), such as the double-digit addition described above. Baddeley (2003, p. 829) also states that working memory “supports human thought processes by providing an interface between perception, long-term memory and action”. Working memory involves both the storage and processing of information, and is therefore regarded in the literature as being distinct from short-term memory which simply refers to the temporary storage of information in mind without the involvement of any processing (Baddeley, 2000). The term working memory capacity is used to refer to the amount of information that can be held within working memory whilst processing is carried out (Conway & Engle, 1996) and this capacity is generally believed to be limited (Baddeley, 2003). Short-term memory capacity is viewed as the amount of information that can be stored when no processing is involved (Bayliss, Jarrold, Gunn & Baddeley, 2003). Short-term memory performance has been found to be linked to complex cognitive tasks, but performance on working memory tasks is generally more predictive (Bayliss et al., 2003; St. Clair-Thompson & Sykes, 2010). Whether working memory capacity is limited by the actual number of items that can be stored or by attentional control is debated within the literature.

The sections that follow will discuss four alternative models of working memory. They are not intended to provide an exhaustive discussion of all of the available models of working memory but are intended to reflect the models most referred to within the mathematical cognition literature. The following sections will discuss Cowan’s embedded-process model (section 1.3.1), Barrouillet’s time-based resource-sharing model (section 1.3.2), Engle’s controlled-attention model (section 1.3.3) and the Baddeley and Hitch multi-component model (section 1.3.4). These models reflect differing views as to the relative importance of domain-general elements (such as controlled attention) and domain-specific elements (such as temporary stores) within working memory.

1.3.1 Embedded-Process Model

Cowan's embedded-process model (shown in Figure 1.3) has been used within the mathematical cognition literature to refer to how number facts may be retrieved from long-term memory (e.g. Imbo & Vandierendonck, 2007b & 2008) but has not been used to explain how more complex calculations are performed. Within this model, working memory is viewed as a subset of long-term memory. Working memory includes items activated within long-term memory and currently within the focus of attention plus items in the short-term store that are activated but not currently within the focus of attention (Cowan, 1999).

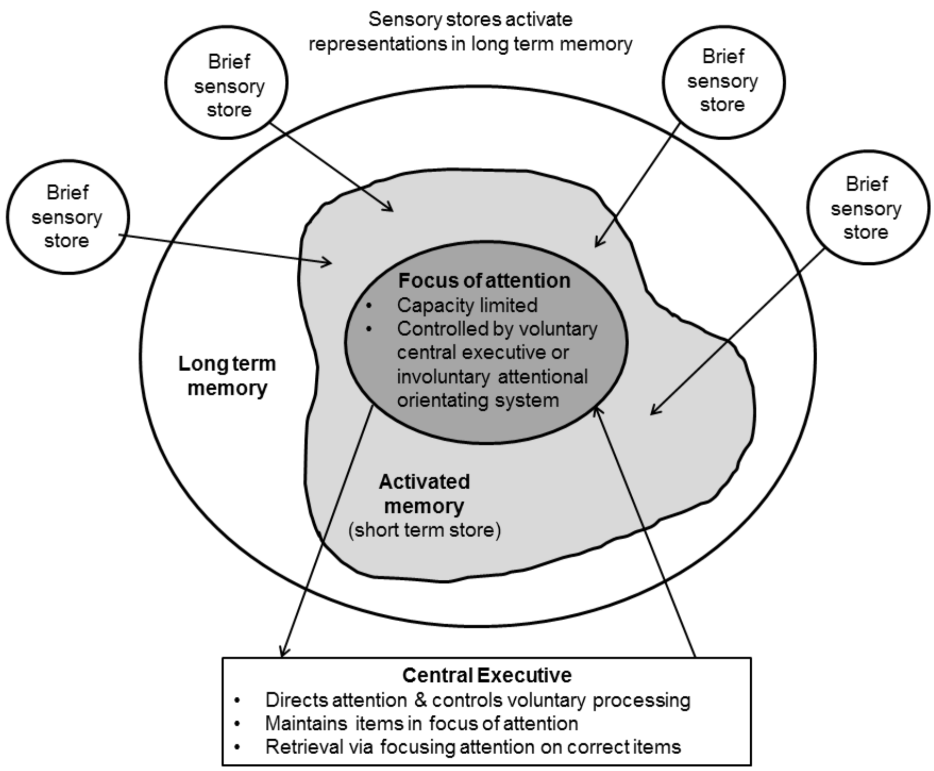

Figure 1.3: Embedded-Process Model. Adapted from "An embedded-process model of working memory" by N. Cowan, 1999, *Models of working memory: Mechanisms of active maintenance and executive control*, p. 64. Copyright by Cambridge University Press.

According to this model, information can enter working memory through sensory stores or via the central executive (the control processes within working memory: Cowan, 1988). An external stimulus enters a brief sensory store, where it can remain for several hundred milliseconds (e.g. auditory storage: Darwin, Turvey & Crowder, 1972), and involuntarily activates representations previously stored in long-term memory. These activated representations remain outside of awareness, in the temporary store, unless they are sufficiently different to previous stimuli or are regarded as important for the current task and then enter the focus of attention. The central executive also controls voluntary attention by activating items in long-term memory that are considered relevant for the task. This allows the individual to retrieve, think about and process information. Therefore, during a task such as a mathematical calculation, the information being attended to is a subset of the activated portion of long-term memory (Cowan, 1988). Only information currently within the focus of attention is available to conscious awareness (Cowan, 2000), but activated items not currently being attended to, regarded as being in the short-term store, can still be retrieved and become part of the focus of attention with a time delay. However, information in this short-term store will decay over time unless reactivated through rehearsal via the refocussing of attention. Processing, such as combining the units of a sum to give an interim total, result in the creation of new representations in long-term memory which can then be recalled when needed (Cowan, 1999).

Working memory within this model is seen as domain-general and capacity-limited. Items from various modalities, such as verbal, visuo-spatial, haptic and olfactory, are viewed as being processed in the same way and within the same system. Limits in working memory capacity are caused by limits in the capacity of the domain-general focus of attention and also the time limits on items remaining activated within the short-term store before they decay (Cowan, 1999).

The focus of attention is viewed as being limited to three to five unconnected items in adults, but this can be increased through using structure and ‘chunks’. Chunks are defined by Cowan as being “a collection of concepts that have strong associations to one another and much weaker associations to other chunks concurrently in use” (2000,

p. 89). Evidence for the capacity to attend to three to five chunks has been provided by several researchers. For example, Chen & Cowan (2009) examined adult participants' recall of word lists when items were presented singularly or in pairs, whilst the ability to rehearse items was prevented via articulatory suppression. They found that around three chunks of information could be recalled regardless of whether items were presented on their own or within the pairs. Within the visuo-spatial domain, stable recall performance across varying delays between presentation and recall of items indicated attentional capacity of around four items (Jones, Farrand, Stuart & Morris,1995). Süß, Wilhelm & Sander (2007) argued that it is the ability to combine these chunks to form new representations that is important for working memory processing and an individual's working memory capacity limits this ability to form new representations. Interference occurs between similar activated representations and the ability to inhibit distracting information is thought to also use attentional resources (Engle, Kane & Tuholski (1999).

To summarise, the embedded-process model assumes that working memory is the currently activated portion of long-term memory. To solve a mathematical problem through the direct retrieval of number facts held within long-term memory, controlled attention focuses on and retrieves the number fact most strongly activated whilst inhibiting other number facts that are activated more weakly. Where procedural methods, containing more than one step, are required, relevant long-term memory representations are activated and become part of the temporary store. Attentional resources are then used to combine these representations and create new items in long-term memory which can then be retrieved and combined with others. Working memory capacity is constrained by the capacity of the focus of attention and the amount of time representations can remain in the temporary store before they decay.

It should be noted however that, following a series of nine dual-task experiments investigating memory for domain-specific items and items that required some form of binding, Cowan has recently questioned whether the idea of a central store of information is too simplistic (Cowan, Saults & Blume, 2014). Cowan and colleagues have proposed that, as well as domain-general controlled attention being important in working memory, there is also a need to store information within domain-specific peripheral stores.

1.3.2 Time-Based Resource-Sharing Model

The time-based resource-sharing model assumes that working memory is a domain-general resource responsible for both processing and storing information, which compete for resources (Barrouillet, Bernardin & Camos, 2004).

Previously, Towse & Hitch (1995) formulated a task-switching model of working memory. This was based on their manipulation of the difficulty of and time spent on a counting task carried out by children. They found that the difficulty of the counting task did not affect the amount of information that could be stored in working memory, but the amount of time spent on the task did. The longer items had been stored in memory while processing was carried out, the worse the children's ability to recall the stored items. Towse, Hitch & Hutton (2000) also found that recall of stored items by adults was affected by the amount of time they had to be retained in memory. Barrouillet & Camos (2001) subsequently argued that by manipulating the duration of the processing tasks, Towse & Hitch had not only affected the time for which stored items had to be retained, but had also affected the cognitive difficulty of the task.

Consequently, Barrouillet & Camos (2001) proposed their time-based resource-sharing model, which incorporated both time and resource-switching. In seven experiments that manipulated both time and cognitive load in working memory span tasks involving both processing and storage, Barrouillet et al. (2004) deduced that working memory performance can be limited by the amount of attention available, the ability to switch between processing and storage of information and the time taken for items currently stored in working memory to decay. Working memory performance can also be hampered by interference caused when stored items are similar to representations created during processing (Barrouillet, Bernardin, Portrat, Vergauwe & Camos, 2007).

Within the time-based resource-sharing model, resources are viewed as domain-general because items from different modalities can disrupt each other within working memory (Barrouillet et al., 2007). This model supports the embedded-process model view that working memory is an activated portion of long-term memory, but also attempts to explain further how constraints on working memory can occur. The model includes four basic assumptions: firstly, processing and storage both require attention, which is limited; secondly, once attention has been switched away from storage to facilitate processing, stored items start to decay; thirdly, a central bottleneck affects retrieval of items as processing requires retrieval of facts from long-term memory and only one fact can be retrieved at a time; and fourthly, sharing of resources occurs through the rapid switching of attention between processing and storage. Barrouillet and colleagues argued that processing does not need to be difficult for it to have an impact on the ability to store information as even a simple processing task requires use of shared attentional resources.

A link between processing speed, working memory capacity and mathematics has been found in a study with children. Barrouillet & Lépine (2005) found that answering addition problems involved fast access to a large number of stored facts, complex procedures and demand on cognitive resources. The study involved children answering 40 single-digit addition sums and also completing tasks designed to measure verbal working memory capacity. The purpose was to investigate whether those with greater verbal working memory capacity were able to solve more problems through direct retrieval of number facts as opposed to more procedural methods. The authors found that working memory capacity affected the strategies that individuals chose to answer problems, with direct retrieval used more by those with greater capacity. Individuals with greater working memory capacity were also faster to answer problems through both retrieval and procedural methods. They also found that retrieving number facts from long-term memory was prone to interference from competing answers and suggested those with greater working memory capacity are better able to inhibit incorrect answers.

In terms of the use of procedural strategies, Barrouillet & Lépine argued that, based on the time-based resource-sharing model, a faster processing speed assists the maintenance of initial problems in memory whilst calculation occurs and also assists the holding in mind of interim calculations. With a faster processing speed, exhibited by the children with greater working memory capacity, less time was taken between encoding of the initial problem and production of an answer. For the direct retrieval of answers, they argued a greater working memory capacity might reflect stronger associations in long-term memory, which aids faster and more accurate retrieval. This argument was based on Siegler's (1996) theory that those with greater working memory capacity are able to form stronger associations between problems and their answers and therefore search long-term memory and retrieve appropriate facts more efficiently. However, they stated their results could also support views that individual differences in working memory are due to differences in the ability to activate information in long-term memory (e.g. embedded-process model: Cowan, 1995) or to control attention (e.g. controlled-attention model: Engle et al., 1999). The controlled-attention model will now be discussed in the following section.

1.3.3 Controlled-Attention Model

There is an assumption within the controlled-attention model (shown in figure 1.4), as within the embedded-process model and time-based resource-sharing model, that items in working memory represent the currently activated elements within long-term memory (Engle, Cantor & Carullo, 1992). It also assumes an important role for domain-general attention (Engle et al., 1999). According to this model, items currently within working memory have either been activated through external stimuli or produced internally through the processing of information (Engle et al., 1992).

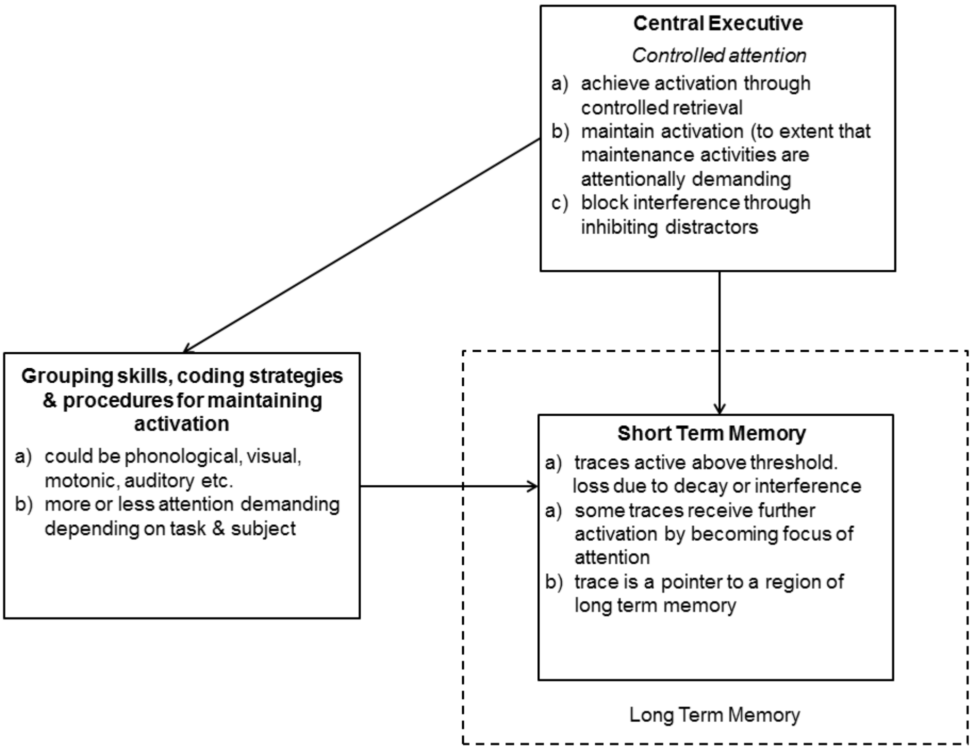

Figure 1.4: Controlled-Attention Model. Adapted from “Individual differences in working memory capacity and what they tell us about controlled attention, general fluid intelligence, and functions of the prefrontal cortex” by R. W. Engle, M. J. Kane & S. W. Tuholski, 1999, *Models of working memory: Mechanisms of active maintenance and executive control,* p. 106. Copyright 1999 by Cambridge University Press.

There is an assumption within the controlled-attention model, as with the embedded-process model, that working memory capacity is limited by the ability to use attention to maintain items in an activated state. However the controlled-attention model places greater emphasis on the importance of using attention to inhibit competing information (Engle, 2002; Kane, Conway, Hambrick & Engle, 2007). Kane & Engle (2000) investigated proactive interference in individuals with high and low working memory capacity. Proactive interference refers to how an item in memory can suffer interference from an item coded earlier (Underwood, 1957). Kane & Engle’s 192 adult participants were initially allocated to groups of high and low capacity based on upper and lower quartile performance of a larger pool of participants on an operation span task that measured the ability to store and recall words that were interleaved with mathematical problems. They found that those with greater working memory capacity were better able to reduce the effect of proactive interference when remembering lists of words. This was particularly important at the points where items were encoded to or retrieved from memory. The controlled-attention model also differs from the embedded-process model in that it highlights the role of domain-specific coding, strategies and procedures for maintaining activation of items within working memory (Engle et al., 1999).

As discussed in section 1.3, working memory is viewed as the combination of storage and processing whereas short-term memory is viewed as simply temporary storage of information (Baddeley, 2000). Within the controlled-attention model, short-term memory is seen as a subset of working memory (Engle et al., 1992) and reflects the ability to store chunks of information (Engle, 2002).

Through a latent variable analysis, Kane et al. (2004) found common variance between short-term memory and working memory performance, which represented this storage ability, and that working memory's additional, unique variance represented executive controlled attention. Verbal and visuo-spatial working memory shared between 70-85% of their variance compared to only 40% for domain-specific storage (pp. 202-203). In this study, 236 adult participants completed a battery of verbal and visuo-spatial short-term memory, working memory and reasoning tasks as well as measures of general fluid intelligence. Although the authors found controlled attention to be domain-general, their results supported a more domain-specific view of short-term storage than that found in the original embedded-process model.

Overall, the results from Kane et al. (2004) suggested the most important working memory factor was that of controlled attention: for the fast retrieval of information from long-term memory and for inhibiting distracting information. This conclusion was based on the greater shared variance between verbal and visuo-spatial working memory compared to that of short-term memory. Domain-specific short-term storage and rehearsal of items was therefore found to be of secondary importance.

In terms of performing mathematics, this model has been applied to support the view that controlled attention is important for retrieving number facts from long-term memory (Barrouillet & Lépine, 2005, section 1.3.2). The model has not, however, been used to explain the use of procedural methods to solve mathematics problems. Use of procedural methods would be supported by storage of chunks of information, such as interim answers and the initial problem, within short-term memory and accessing number facts from long-term memory. However, whereas the previous two models have assumed that storage and processing of items is wholly domain-general, the controlled attention model includes a domain-specific element. The storage and rehearsal of items in short-term memory is believed to rely on domain-specific codes and processes, which could be, for example, verbal, visual, spatial or auditory.

1.3.4 Multi-Component Model

In contrast to the previous three models which assume working memory to be an activated subset of long-term memory, the multi-component model (shown in Figure 1.5) views working memory and long-term memory as separate cognitive systems (Baddeley & Logie, 1999). Models that base working memory on an activated subset of long-term memory have been criticised for not adequately explaining how tasks requiring the use of working memory are actually carried out (Cornoldi & Vecchi, 2003). The multi-component model views working memory as a system for both storing and manipulating information, with the outcomes of manipulations then encoded into long-term memory (Baddeley & Logie, 1999).

Baddeley & Hitch originally proposed a three-component model of working memory in 1974, comprising a central executive, and two information stores: the phonological loop and visuo-spatial sketchpad. A fourth component, the episodic buffer, has been added more recently (Baddeley, 2000). Each of these components will be described in turn below.

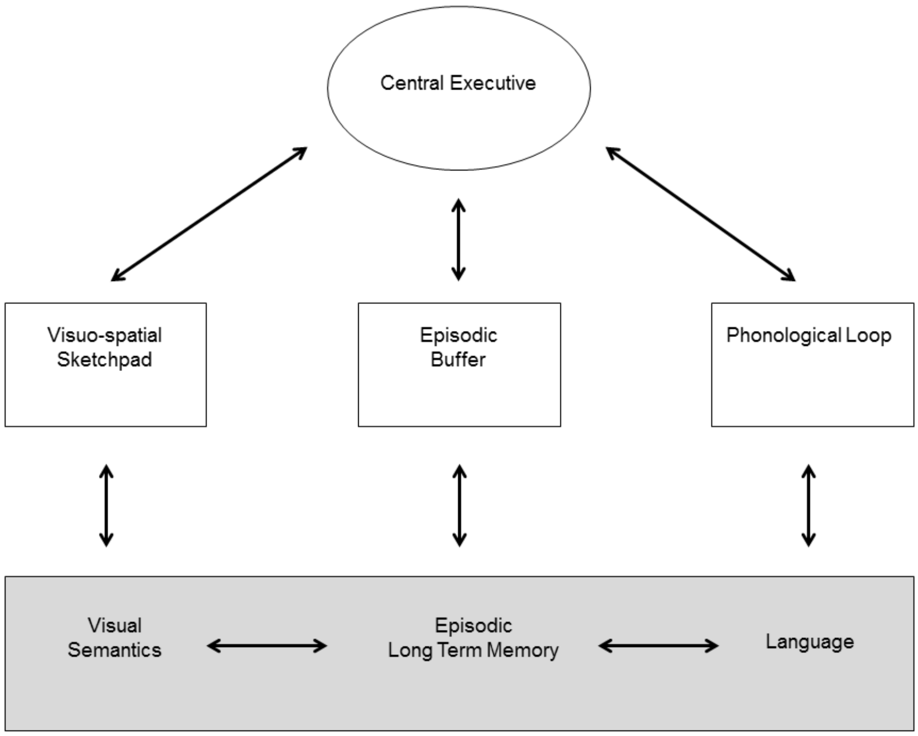

Figure 1.5: Multi-Component Model of Working Memory. Adapted from "Is working memory still working?" by A. D. Baddeley, 2002, *European Psychologist, 7*(2), p. 93. Copyright 2002 by Hogrefe Publishing.

1.3.4.1 Phonological Loop

The phonological loop has been investigated more thoroughly than the other working memory components and is believed to have evolved to enable the acquisition of language (Baddeley, 2003). It comprises a domain-specific phonological store for verbal and acoustic information and a rehearsal mechanism for refreshing and maintaining information within the store and maintaining its serial order (Baddeley, 1996). Items enter the loop either from sensory input or via the central executive (Baddeley, 2000). Phonological items have been found to decay within two to three seconds of entering the store and the rehearsal mechanism acts to keep the items refreshed through sub vocal articulation (Baddeley 1992). Capacity of the phonological loop is reached when the first item held in the store fades before the last item within the store can be rehearsed. The rehearsal mechanism has been evidenced through the word length effect, which finds that memory for words containing a greater number of syllables is poorer than for those containing fewer syllables and that this difference disappears when sub vocal rehearsal is prevented (Baddeley, Thomson & Buchanan, 1975). Support for a temporary store for phonological information, as opposed to the activation of representations in long-term memory, comes from neuropsychological evidence showing that patients with verbal short-term memory deficits can also have intact language and verbal long-term memory functions (e.g. Vallar & Baddeley, 1984).

1.3.4.2 Visuo-Spatial Sketchpad

The visuo-spatial sketchpad is domain-specific and has been described as a “mental blackboard” (Heathcote, 1994, p.27) used for temporarily storing and manipulating visual and spatial information. It is thought to be limited to three or four items (e.g. Luck & Vogel, 1997). Following criticism of the multi-component model’s inability to incorporate items other than those that are verbal, visual or spatial in nature, the theory around the visuo-spatial sketchpad has been amended to also include haptic and motor information (Baddeley, 2002). As is the case with the phonological loop, items can enter the visuo-spatial sketchpad via sensory information or via the central executive (Baddeley, 2000).

It has been proposed that the visuo-spatial sketchpad can be fractionated into two interconnected subsystems (Darling, Sala & Logie, 2009; Duff & Logie, 1999; Logie, Gilhooly & Wynn 1994): one for the storage of static visual material such as shape and colour, sometimes referred to as the inner eye; and one for dynamic spatial information such as movement and location, sometimes referred to as the inner scribe. For example, Darling et al. (2009) found a dissociation between memory for appearance and memory for location through loading both with static (viewing a display of dots on screen) and dynamic (spatial tapping) tasks during the interval between presentation and recall of items. The static secondary task interfered with memory for appearance whilst the dynamic task interfered with memory for location.

In later models of the visuo-spatial sketchpad, Logie has proposed that the inner eye can be further fractionated into a visual cache for the temporary storage of visual information and a visual buffer for the representation of visual material, whilst the inner scribe encodes spatial locations and movement (Pearson, 2001). This version of the visuo-spatial sketchpad is shown in Figure 1.6. The buffer is supported by the cache, which acts as a temporary backup store for representations no longer maintained as a conscious mental image. Maintenance of conscious images relies on attentional resources via the central executive and the inner scribe can operate independently of the visual cache and buffer or can interact with them if, for example, locations are remembered via a visual image. Whereas the phonological loop has a clear mechanism for the rehearsal and ordering of verbal items via sub vocal articulation, the mechanism for visuo-spatial rehearsal and ordering is still not clear, although Baddeley (2000) has speculated that this may occur via some form of attentional refreshing. Logie (1995), however, suggested spatial rehearsal may be performed by the inner scribe.

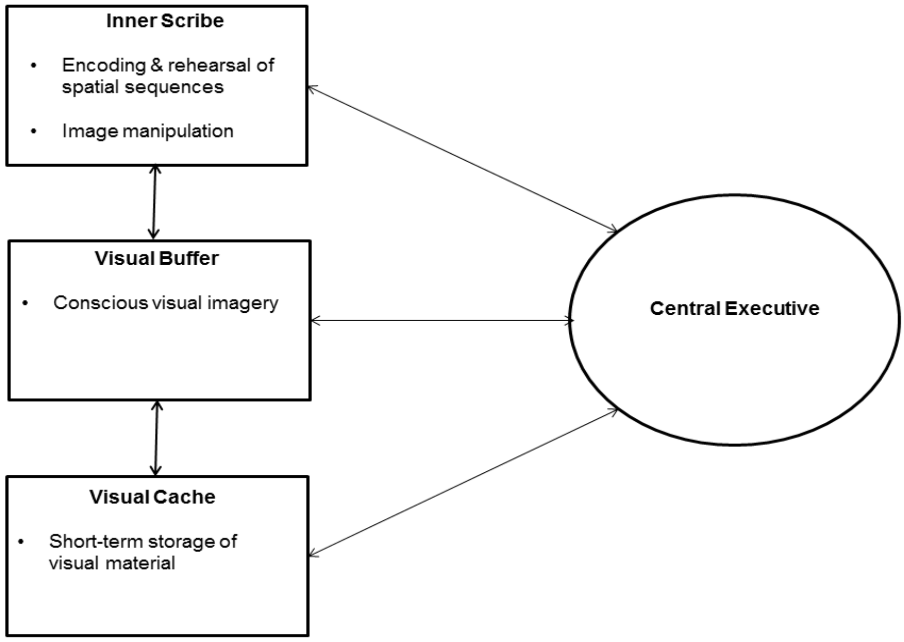

Figure 1.6: Fractionation of the visuo-spatial sketchpad and its relationship with the central executive. Adapted from “Imagery and the visuo-spatial sketchpad” by D. G. Pearson, 2001, *Working memory in perspective*, p. 52. Copyright 2001 by Psychology Press.

1.3.4.3 Central Executive

The central executive is thought to control working memory. It was originally proposed as a general pool of processing resources, with no storage capacity, and was based on the earlier supervisory activating system (SAS) of Norman and Shallice which proposed a system for applying attentional control over the processing of information (Baddeley, 2003). The SAS was assumed to be limited in terms of attentional capacity and capable of combining information from long-term memory with novel information currently held within working memory (Baddeley, 2002). However, this early view of the central executive was criticised for being ”little more than a homunculus, the little man taking all the important decisions” (Baddeley, 2003, p. 835).

Subsequent research has attempted to fractionate the central executive into more specific functions. It is still assumed to control working memory and this is achieved through switching attention from one task to another, monitoring and updating representations within working memory and inhibiting activated but irrelevant information (e.g. Miyake, Friedman, Emerson, Witzski & Howerter, 2000) as well as focusing attentional resources (Baddeley, 2002) and manipulating information held within the other components of working memory (Repovš & Baddeley, 2006). The visuo-spatial sketchpad is thought to be more strongly connected to the central executive than is the phonological loop (Miyake, Friedman, Rettinger, Shah & Hegarty (2001). This is argued because, whilst articulatory suppression can be used to disrupt only the phonological loop, secondary tasks designed to disrupt the visuo-spatial sketchpad also disrupt the central executive (e.g. Hegarty, Shah & Miyake, 2000).

1.3.4.4 Episodic Buffer

The episodic buffer was added to the multi-component model as a limited-capacity, domain-general store capable of integrating, or binding, information from the other components via the central executive (Baddeley, 2000). The central executive can access and manipulate the buffer's content through conscious awareness. The buffer is assumed to use multiple types of code to enable the binding of different types of information from across the whole system (Baddeley, 2003) and acts as an interface between the other working memory components and long-term memory (Baddeley, 2000).

Examination of the episodic buffer is beyond the scope of this thesis.

1.3.5 Comparison of the Models of Working Memory

Whilst the four theoretical models of working memory discussed in the sections above all include a role for domain-general controlled attention and the accessing of facts from long-term memory, there are fundamental differences in the way they conceptualise working memory.

The embedded-process model, time-based resource-sharing model and controlled-attention model all contain the view that working memory is an activated subset of long-term memory with capacity differences between individuals caused by attentional ability. The multi-component model, however, views working memory as a separate system to long-term memory, although it has links with long-term memory via the central executive and episodic buffer. Also, whilst the embedded-process model and time-based resource-sharing model both regard working memory as completely domain-general, the controlled-attention and multi-component models both include elements of domain-specific storage and rehearsal. The controlled-attention model assumes that items are maintained within working memory through the use of domain-specific processes and codes, dependent upon the original nature of the items. The multi-component model assumes that the short-term stores (the phonological loop and visuo-spatial sketchpad) within working memory are domain-specific and that domain-specific storage and domain-general attention are responsible for individual differences in working memory capacity (Towse & Hitch, 2007). Also, Cowan et al. (2014) now suggest that domain-specific storage, as well as domain-general attention, plays a role in working memory within the embedded-process model.

There is evidence within the working memory and mathematical cognition literature that the storage of verbal and visuo-spatial information is in fact domain-specific as assumed within the multi-component model (e.g. Bayliss et al., 2003; Jarrold & Towse, 2006; Noël, Désert, Aubrun & Seron, 2001; Shah & Miyake, 1996; Trbovich & LeFevre, 2003). For example, Shah & Miyake (1996) found that the relationship between the amount of information adults could store and recall (when storage was interleaved with processing) and performance on tests of language processing and spatial thinking varied depending upon whether verbal or visuo-spatial information was being stored. Bayliss et al. (2003) measured children's performance on working memory span tasks involving different combinations of verbal and visuo-spatial processing and storage. The patterns of correlations between performances on these different combinations led the authors to conclude that working memory consists of general resources for processing but that resources for the storage of verbal and visuo-spatial information are domain-specific.

The multi-component model also conceptualises working memory as a "cognitive workspace" (Pearson, 2001, p. 41), and is therefore perhaps better able to accommodate the storage and manipulation of numbers during the solving of mathematical problems. This model has also been extensively used in previous studies of mathematical cognition. The following section (section 1.4) will now discuss evidence for the involvement of the different components of the multi-component model of working memory in mathematics.

1.4 The Multi-Component Model of Working Memory and Mathematics

1.4.1 Overview of the Previous Literature

Previous literature examining the relationship between working memory and mathematics will be reviewed in this section. The involvement in mathematics of the central executive, phonological loop and visuo-spatial sketchpad components of working memory will be discussed. As the episodic buffer is a relatively recent introduction to the multi-component model of working memory, it has not yet been investigated in relation to mathematics. The relationship between working memory and mathematics in adults has been previously examined, but has not received as much attention as the relationship between mathematics and working memory in children. Therefore, in addition to discussing the literature involving adults, previous research considering the relationship between working memory and mathematics in children will also be considered. Examining research involving children will help inform whether any working memory elements are consistently implicated in mathematics achievement and may also shed light on resources used when adults who are less mathematically skilled perform calculations.

Research examining working memory and mathematics in children is plentiful, but largely examines performance when solving arithmetic as opposed to more complex forms of mathematics. The literature has also largely examined verbal working memory and visuo-spatial working memory, consisting of both storage and processing, rather than the individual components of the multi-component model of working memory: the central executive, phonological loop and visuospatial sketchpad. Where studies have attempted to isolate the individual components, they have largely investigated the central executive and phonological loop. The visuo-spatial sketchpad has been examined separately in children, but not in adults, although it is difficult to separate it from the central executive (Hegarty et al., 2000).

In the following sections, correlational studies (section 1.4.2), experimental studies (section 1.4.3) and studies involving those who are excellent at mathematics and those who are poor at mathematics (section 1.4.4) will be discussed.

1.4.2 Correlational Studies

Several studies have used a correlational design to explore the relationships between working memory and mathematics. Correlational studies involve participants performing tasks designed to assess working memory ability and then measure the relationship with either concurrent or future performance on standardised mathematics tests or school mathematics achievement. The majority of correlational studies have investigated verbal working memory rather than visuo-spatial working memory. They have also largely examined working memory, involving both storage and processing, rather than examining the storage and central executive elements separately.

Evidence within the literature for a relationship between verbal working memory and mathematics is mixed and, as mentioned above, largely involves research with children. Several studies have found that verbal working memory ability correlates with mathematics ability. Purpura & Ganley (2014) found that verbal working memory capacity was related to cardinality, subitizing, set comparison and number order in 6 year olds. Cardinality reflected the children’s ability to count out a smaller set of items from a larger set. Subitizing referred to the ability to recognise the number of dots presented in a series of quickly presented pictures. Number order reflected ability to state which number comes before or after another given number.

Verbal working memory performance at 6 years of age has also been found to predict mathematics performance six months later, over and above the contribution of verbal short-term memory (Passolunghi, Vercelloni & Schadee, 2007). A relationship has also been found between verbal working memory and performing subtractions in 7 to 11 year olds (Adams & Hitch 1997) and performance on National Curriculum Assessments in 7 and 14 year olds (Gathercole, Pickering, Knight & Stegmann, 2004). Verbal and non-verbal working memory have also predicted mathematics performance at 11 and 14 years (Jarvis & Gathercole 2003). The studies mentioned so far therefore contain evidence for a relationship between verbal working memory and mathematics. However, some studies have not contained evidence of such a relationship.

Verbal working memory has not always been found to be directly related to mathematics achievement. Following a longitudinal study of children from pre-school age to age 7, Östergeren & Träff (2013) reported that measures of verbal working memory were only indirectly related to arithmetic performance through number knowledge. Pre-school measures of verbal working memory and number knowledge (including number naming and counting) were administered and structural equation modelling used to explore their relationships with arithmetic achievement at age 7. Only pre-school number knowledge directly predicted arithmetic ability. In another study, no predictive relationship was found for verbal working memory at age 8 when other measures such as attention, processing speed and non-verbal problem solving were taken into account (Fuchs et al. 2006).

Studies involving children therefore contain mixed evidence for a relationship between verbal working memory and mathematics. Only one correlational study has been found that examines this relationship in adults. Wilson & Swanson (2001) examined verbal and visuo-spatial working memory in both adults and children and found that performance on both types of working memory predicted mathematics performance, irrespective of age.

Although verbal working memory may be required for performing mathematics, these correlational studies do not tell us whether it is simply the storage of verbal information or storage while concurrently carrying out processing that is important for successfully solving mathematics problems. The studies highlighted above have examined verbal working memory rather than its constituent parts. Where the central executive and phonological loop have been explored separately, both the central executive and phonological loop components were related to arithmetic performance in 8 year olds (Holmes & Adams 2006). The authors suggested that the phonological loop may be used in the sub vocal rehearsal of stored information and for retrieving number facts. They also suggested that, as the central executive significantly predicted unique variance in performance across a wide range of curriculum subjects in their study, it may be associated with general intelligence rather than specifically with mathematics.

Correlational studies involving mathematics and visuo-spatial working memory in adults are also sparse. The Wilson & Swanson (2001) study above contained evidence that visuo-spatial working memory may be important for mathematics in adults. One further correlational study has been identified in which visuo-spatial working memory and mathematics in adults has been examined. Wei, Yuan, Chen & Zhou (2012) correlated the mathematics performance of 80 Chinese undergraduates with performance on a battery of cognitive tests. Their advanced mathematics test included algebra, statistics, functions theory, graph theory, geometry. Their cognitive battery included visuo-spatial working memory as well as other measures of visuo-spatial ability and they found performance on the working memory task correlated with ability at advanced mathematics. When results for the battery of tests were entered into a regression, visuo-spatial ability, including visuo-spatial working memory, predicted mathematics performance over and above the contribution of verbal and domain-general measures. This suggests that visuo-spatial working memory is implicated more than verbal working memory when adults perform mathematics.

There are, however, two main issues with the Wei et al. (2012) study. Firstly, whilst the authors claimed visuo-spatial working memory scores significantly correlated with mathematics performance, their working memory task did not actually include a processing element. Therefore, what they had actually measured was temporary storage in short-term memory rather than working memory capacity. Secondly, having established that when all of the visuo-spatial measures were grouped together they predicted mathematics performance, they did not go on to establish the relative importance of each of these elements separately. Therefore the unique contribution of the ability to store visuo-spatial information was not established. There is therefore some evidence within the literature involving adults that visuo-spatial working memory performance is related to mathematics performance, but, as with verbal working memory, research has largely been carried out with children.

The literature involving children consistently shows that visuo-spatial working memory and its components are implicated in mathematics. For example, Bull, Espy & Wiebe (2008) measured pre-school children's visuo-spatial short-term memory and working memory abilities and both predicted later mathematics ability at age 7. Dummontheil & Klingberg (2012) also found that non-verbal reasoning and visuo-spatial working memory measures, rather than verbal measures, predicted arithmetic ability in 6, 10, 12 and 16 year olds after two years. In terms of the visuo-spatial sketchpad and central executive components of visuo-spatial working memory, both appear to be involved in mathematics in children. Whilst Holmes & Adams (2006) found that both central executive and phonological loop ability are related to the arithmetic ability of children, the link between arithmetic and the visuo-spatial sketchpad was found to be stronger. In a 5-year longitudinal study involving children who were six years old at the start, measures of the central executive and visuo-spatial sketchpad predicted mathematics achievement, but the phonological loop did not (Geary 2011). These results therefore suggest that visuo-spatial working memory may be more important than verbal working memory for successfully performing mathematics.

It is suggested that the visuo-spatial sketchpad is used to hold and manipulate numbers (Logie et al., 1994) and for visualising numbers involved in arithmetic calculations (Seron, Pesenti, Noël, Deloche & Cornet, 1992). The visuo-spatial sketchpad is also important for representing information such as number magnitudes (Geary, 2004), which are believed to be a basis for more advanced maths skills (Holloway & Ansari, 2009).

There is also evidence from studies with children that the visuo-spatial sketchpad is split into static and dynamic components, as proposed by Darling, Sala & Logie, 2009; Duff & Logie, 1999; and Logie, Gilhooly & Wynn 1994 (section 1.3.4.2). Pickering, Gathercole, Hall & Lloyd (2001) discovered a developmental dissociation for performance on tasks that measured static and dynamic visuo-spatial storage capacity. Performance overall was superior for the storage of static information. Holmes, Adams & Hamilton (2008) examined performance, in 7-8 and 9-10 year olds, for visual and spatial storage. Dynamic, spatial storage predicted mathematics ability in the younger group, whilst static, visual storage predicted mathematics ability in the older children. The authors suggested this may reflect older children’s greater number of available strategies, including the use of verbal resources to solve problems. They may only need to maintain a visual image of the initial problem, whilst the younger children may still rely more on procedural methods and need to both store and manipulate the problem. Reuhkala (2001) had previously found that the ability to store static visuo-spatial information predicted the mathematics scores of 17 year olds.

Despite the evidence above for a link between visuo-spatial working memory and mathematics in childhood, it appears this may depend upon the type of mathematics and the age of the children solving the problems. Bull, Johnston & Roy (1999) investigated the use of the central executive and visuo-spatial sketchpad in arithmetic and concluded that the visuo-spatial sketchpad may be used more by younger children for counting, but is used less by older children who are able to access information directly from long-term memory. They proposed that the central executive is used more in middle stages of development where a choice of strategies has to be made from those available or when older children solve more complex sums. They also highlighted the use of visual imagery, stored in the visuo-spatial sketchpad, as a strategy for solving problems. Simmons, Willis & Adams (2012) concluded the visuo-spatial sketchpad was related to number writing and magnitude judgements in 5-6 and 7-8 year olds, whilst the central executive was related to addition performance in the younger group.

This variation with age and type of mathematics problem is important in the context of investigating the role of visuo-spatial working memory in mathematics performance by adults. Adults generally perform more complex mathematics than children and may use different strategies than children to solve problems and these strategies may vary depending on the type of problem. For example, adults mainly solve addition and multiplication problems via direct retrieval of number facts stored in long-term memory (Kirk & Ashcraft 2001), while subtraction relies more on procedural methods (Seyler, Kirk & Ashcraft, 2003) such as decomposition or counting down. Older children also seem to use more direct retrieval and more efficient strategies to solve problems (Imbo, Vandierendonck & Rosseel, 2007). The link between working memory and arithmetic seems to become weaker with age and the greater use of retrieval (Imbo & Vandierendonck 2008). This may be because working memory is used by younger children as they achieve representations of number facts in long-term memory, which is required until the accessing of the number facts becomes automatic.

The use of visuo-spatial working memory resources may then depend upon the type of mathematics being performed and the strategy being employed. However, it should be borne in mind that the involvement of working memory in mathematics may be different for adults and children, especially younger children, due to developmental trajectories of working memory and executive function. Also, greater use of retrieval strategies and more efficient use of procedural strategies are evident in adults solving mathematical problems (Imbo & Vandierendonck, 2008), which affects the comparability of findings from adults and children. Moreover, the mathematics carried out by adults tends to be more complex than that carried out by children and may therefore involve the use of different types of resources.

Therefore, when examining the link between mathematics performance in adults and working memory, it is important to consider the type of mathematics actually being performed.

In summary, there is mixed evidence from the correlational literature for a link between verbal working memory and mathematics. A link between visuo-spatial working memory and mathematics has been consistently found. However, the vast majority of studies discussed within this section have involved children performing arithmetic rather than the different types of mathematics performed by proficient adults. Although correlational studies can highlight whether verbal and visuo-spatial working memory capacity is related to mathematics achievement, they do not tell us whether it is the storage or processing elements of working memory, or both, that are important. Very few studies have looked separately at the central executive, phonological loop and visuo-spatial sketchpad components and this is particularly true for studies involving adults and the components of visuo-spatial working memory. The following section will now examine experimental studies within the mathematical cognition literature, which have attempted to differentiate the involvement of these components in mathematics.

1.4.3 Experimental Studies

Experimental studies used to examine working memory and mathematics largely involve dual-task studies that require participants to perform arithmetic while completing a secondary task at the same time. These secondary tasks are designed to use central executive, phonological loop or visuo-spatial sketchpad resources. If the simultaneous performance of a particular secondary task adversely affects performance on the arithmetic task, it can be deduced that the particular type of working memory resource being loaded by the secondary task was being used when answering the arithmetic problems. However, the majority of studies using dual-task methods have involved loading the central executive or phonological loop with a concurrent secondary task. Far fewer studies have involved loading the visuo-spatial sketchpad. Also, none of the studies examining the visuo-spatial sketchpad have looked at its use during the execution of different strategies to solve mathematical problems.

The central executive and the phonological loop seem to play different roles depending on the types of sums being answered. For example, when verifying whether additions statements, such as 6 + 3 = 7, are true or false, De Rammelaere, Stuyven & Vandierendonck (1999) and Lemaire (1996) found that the central executive is involved in the verification of both true and false sums, whereas the phonological loop is only involved in verifying true sums. When actually solving addition problems rather than verifying answers, the phonological loop appears to be involved in storing interim sums, and maintaining accuracy, whilst the central executive is involved in performing carry overs and retrieving number facts from long-term memory (Fürst & Hitch, 2000; Imbo, De Rammelaere & Vandierendonck, 2005; Imbo & Vandierendonck, 2007a; Imbo, Vandierendonck & De Rammelaere, 2007; Logie et al., 1994).

Imbo, Vandierendonck & Vergauwe (2007) also found that, for subtraction and multiplication problems, there was greater use of central executive and phonological resources as the number of carry overs and their value increased.

The phonological loop may also be used to temporarily store the initial addends within sums. Noël et al. (2001) manipulated the phonological and visual similarity of two numbers to be added together. Phonological similarity affected accuracy and latencies for answering the sums, whilst visual similarity of the numbers had no impact. This suggested that the phonological loop rather than the visuo-spatial sketchpad is used to store this information. The central executive may also be involved in strategy selection and execution (Imbo & Vandierendonck, 2010). As discussed in section 1.2.4, adults use a variety of strategies to solve mathematical problems if they are unable to simply retrieve the answers from long-term memory. The most common procedural methods used are decomposition and counting (Geary et al., 1993; Hecht, 1999). For example, Imbo, Duverne & Lemaire (2007) required participants to solve complex multiplications using two strategies of different difficulty whilst under no load and a central executive load. They found that central executive load led to both poorer strategy execution and the selection of the simpler strategy.

There is therefore evidence within the experimental literature that the central executive and phonological loop elements of verbal working memory are used in different types of arithmetic performed by adults and have different roles. However, in a review of the literature surrounding carrying and borrowing for complex multiplications and additions, Imbo et al. (2005) concluded that evidence for use of the visuo-spatial sketchpad was too sparse to enable conclusions to be drawn regarding its involvement and role. There is therefore a need to investigate the relative roles of the central executive and visuo-spatial sketchpad elements of visuo-spatial working memory when using different strategies to solve mathematical problems.

There is recent evidence that visuo-spatial working memory resources are used for solving additions. Cragg, Richardson, Hubber, Keeble & Gilmore (2014) examined 9-11 and 12-14 year old children as well as adults and found that all groups used working memory to solve arithmetic problems whether using direct retrieval or procedural strategies. Also, verbal and visuo-spatial secondary loads affected arithmetic performance to a similar degree. Visuo-spatial working memory also seems to be particularly important when adults use counting to solve problems (Hubber, Gilmore & Cragg: Experiment 1, 2014)[1], although this study did not separate out the relative roles of the central executive and visuo-spatial sketchpad. It involved use of an *n*-back task to load visuo-spatial working memory and it is not clear whether the detrimental effect the load had on arithmetic performance was due to the visuo-spatial nature of the secondary task or due to the need for the central executive to constantly monitor and update information. There is therefore a need to better establish the relative contributions of the central executive and visuo-spatial sketchpad when adults solve mathematics problems, although the visuo-spatial sketchpad seems to be used in solving subtractions, but not multiplications (Lee and Kang, 2002) and Logie et al. (1994) found it to be used in approximations.

In summary, researchers using dual-task studies have identified specific roles for the central executive and phonological loop in adults' mathematics performance although these experiments have only involved arithmetic rather than other forms of mathematics. The central executive seems to be involved in selecting and executing appropriate strategies for solving problems, for performing carry overs and for retrieving number facts from long-term memory. The phonological loop seems to be involved in storing initial problem information and interim totals. The role of the visuo-spatial sketchpad is, however, poorly understood. Although there is evidence for the use of visuo-spatial

[1] Hubber, Gilmore & Cragg: Experiment 1, 2014 has previously been examined because it was my MSc. thesis.

working memory by adults when solving arithmetic, the relative roles of the central executive and visuo-spatial sketchpad are not yet known and require further research

1.4.4 Studies involving those who are Excellent or Low Achieving at Mathematics

The research described so far has involved individuals with a range of mathematics ability. Another approach has been to examine groups of individuals who are either excellent or low achieving at mathematics. Examining studies involving those with mathematics difficulties will indicate whether deficits within working memory underpin mathematical problems. Examining those who excel at mathematics will show which elements of working memory are linked to greater mathematical proficiency. Several studies have included groups of individuals with mathematics difficulties (MD), although these have involved children rather than adults. Very few studies have examined those who excel at mathematics and none of these have examined working memory capacity in skilled adults.

Studies examining MD in children have largely concluded that MD has a relationship with poor visuo-spatial working memory. For example, Kyttälä (2008) measured static and dynamic visuo-spatial storage in MD children and children with both maths and reading difficulties (RD) and visuo-spatial working memory to also tap into executive processes. Performance, compared to those of typically developing children (TD), showed that the MD group were able to store less passive visuo-spatial information and that both MD and RD children had domain-general executive deficits, including difficulty with inhibiting irrelevant information. The presence of an executive component to MD problems is supported by Passolunghi & Siegel's (2004) findings that MD children have problems with inhibiting information. They also found no issue with temporary verbal storage ability. In contrast to this, however, it should be noted that at least one study has found that verbal storage is implicated in MD (Hitch & McCauley, 1991). In a meta-analysis of 18 studies comparing MD children with TD aged-matched children aged 8 to 19 years, David (2012) concluded that there is a moderate effect size for the phonological loop, but larger effect sizes for the involvement of the central executive and visuo-spatial sketchpad in mathematics. Visuo-spatial sketchpad storage seems to be more important at a younger age and a deficit in visuo-spatial working memory is linked to MD across all ages.

The few studies involving those who excel at mathematics have, however, provided mixed evidence regarding which elements of working memory are implicated in mathematics. As mentioned above, those examining working memory capacity have not involved adults.

Child mathematics prodigies, defined as children who “reach a professional level of achievement before the age of 10 or adolescence” (Ruthsatz, Ruthsatz-Stephens & Ruthsatz 2014, p.11), have been shown to have superior visuo-spatial working memory and general visuo-spatial skills to prodigies in art. Leikin, Paz-Baruch & Leikin (2013) also found a relationship between visuo-spatial working memory and mathematics in mathematically gifted adolescents. Also, a series of studies by Dark & Benbow (1990, 1991, and 1994) compared groups of adolescents who were gifted at mathematics to other groups on several short-term memory and working memory tasks. They found mixed results for the relationships between verbal and visuo-spatial working memory and mathematics.

Dark & Benbow (1990) compared groups of 12-13 year old children who were mathematically gifted, verbally gifted, average for their age and college students. Giftedness was judged based on scores for National Curriculum standardised tests (SAT-M and SAT-V) in the United States. They examined the groups’ performance on short-term memory and working memory tasks in both the verbal and visuo-spatial domains. Their verbal tasks involved digit stimuli rather than words. They concluded that mathematically gifted children have better verbal short-term memory than average and verbally gifted children, but are no different in this respect to college students. They suggested that this result may be due to the mathematically gifted being more familiar and experienced with digits. Results for visuo-spatial short-term memory found the mathematically gifted to have similar temporary storage capacity to the verbally gifted, worse capacity than the college students and only better capacity than average children of the same age. For working memory, measured by a continuous-paired association task, the mathematically gifted group performed better for recalling verbal items than all of the other groups. For visuo-spatial items, they performed better than the average and verbally gifted groups, but the same as the college students. Therefore, results suggested that both verbal working memory and visuo-spatial working memory may be implicated in mathematics achievement.

Although Dark & Benbow's 1990 study found the mathematically gifted had superior verbal working memory to other groups, their 1991 study produced different results. In the 1991 study, they compared a group of mathematically gifted children with a group who were verbally gifted and a group who were both mathematically and verbally gifted at ages 13-14 years. They expanded the number of types of stimuli used in their short-term and working memory tasks to include digits, words and letters in the verbal domain as well as spatial locations in the visuo-spatial domain. As in their 1990 study, results found that those who were gifted at mathematics had greater short-term memory digit spans than those who were only verbally gifted. However, the mathematically gifted were worse than the verbally gifted for the word span task and there was no difference between the groups for letter span. This suggested that, in the verbal domain, short-term memory differences depended upon the type of stimuli being remembered. For visuo-spatial short-term memory, unlike in the 1990 study, those who were mathematically gifted had greater capacities than those solely verbally gifted. For the working memory tasks, the mathematically gifted performed better for digit and letter stimuli, but no better than the verbally gifted for visuo-spatial or word stimuli.

In their 1994 study, Dark & Benbow investigated how memory for different types of verbal stimuli was related to mathematical and verbal giftedness in groups of 11-14 year old children. They again found that performance on a continuous-paired association working memory task involving remembering and recalling digits was related to mathematical giftedness. Performance for remembering and recalling words was related to both verbal and mathematical giftedness.

The three Dark & Benbow studies therefore consistently found that the ability to store numbers within working memory was related to mathematics achievement. However, results were inconclusive for the relationships between storing word and visuo-spatial items and mathematics. These studies did indicate though that different types of stimuli within the verbal domain may have different relationships with mathematics. However, results from the Dark & Benbow studies should be interpreted with caution. Those classified as mathematically gifted also generally performed at above average levels on measures of verbal ability and this may have affected comparisons between the mathematically gifted and other groups. Also, although their continuous-paired association task was used as a measure of working memory, it did not measure capacity in the same way as the studies discussed in section 1.4.2. Rather than simply measuring the number of items that could be stored within working memory, the continuous-paired association task measured ability to recall a fixed number of items over varying time delays. It was therefore likely to have been measuring the decay of items rather than the number of items.

A further two relevant studies have been identified involving the skills involved in advanced mathematics, although they did not measure working memory capacity. These have examined the strategies that mathematics experts employ to solve problems. Dowker (1992) investigated strategy use for solving complex multiplication estimations in 44 mathematics academics and found that they used a large variety of different strategies to solve problems. Dowker, Flood, Griffiths, Harriss & Hook (1996) compared these mathematicians to groups of accountants, Psychology students and English students. They found that, whilst all groups used diverse strategies to solve problems, the mathematicians and accountants used a larger number of different strategies, with the mathematicians solving problems the most accurately and using the most appropriate strategies. As discussed in section 1.4.3, strategy selection and execution have been found to involve the central executive (Imbo et al., 2007). Therefore, Dowker and colleagues' findings that mathematicians are better able to select and to execute appropriate strategies implicates central executive resources.

There is therefore a paucity of research involving adults who excel at mathematics. There is a need to investigate differences in working memory capacity between those adults who are excellent at mathematics and those who are less skilled to better understand which working memory resources are linked with mathematics proficiency. Section 1.4.3 contains evidence that the central executive, phonological loop and visuo-spatial sketchpad are all involved in performing mathematics, although the extent of the use of the latter by adults is not clear.

In summary, the research within this section includes evidence that there is a link between MD and deficits in visuo-spatial working memory. Relationships between verbal and visuo-spatial working memory and high mathematics achievement are unclear and also require investigation in adults. One element of working memory that may be implicated in mathematics achievement is the central executive, as it has been found to be involved in strategy selection and execution and adult mathematicians have been found to have superior strategy skills.

1.4.5 Aims of the Thesis

Taken as a whole, the previous research discussed throughout section 1.4 contains extensive evidence that working memory is involved when mathematics is performed by children and adults. There is therefore a need for a more comprehensive model of mathematical cognition that can incorporate the involvement of working memory (Raghubar et al., 2010).

Whilst there has been much research into the link between working memory and mathematics performance in children, research involving adults is sparse. Studies involving adults have also mainly involved the investigation of arithmetic as opposed to more varied, complex mathematics. Children are also in the process of learning mathematics so any involvement of working memory in mathematics in children may reflect the ability to learn mathematics rather than to actually perform it. It is therefore also necessary to examine individuals who are proficient at mathematics and are also performing different mathematics to basic arithmetic to gain a greater understanding of the link between working memory and mathematics.

Whilst a few studies have examined working memory in children who are gifted at mathematics, none have so far examined skilled adult mathematicians and working memory. Adults who are excellent at mathematics will be performing calculations in an optimal manner through the efficient execution of strategies (Dowker et al,, 1996). Comparison of adults who are excellent at mathematics to those who are less proficient will therefore give a clearer indication of which working memory resources are involved in the proficient solving of mathematical problems.

The literature involving adults, whilst providing evidence that both verbal and visuo-spatial working memory are involved in mathematics, has so far not addressed whether mathematicians have superior ability to non-mathematicians to hold verbal or visuo-spatial information, or both, within working memory whilst processing is carried out. Just because a component of working memory is used in mathematics does not necessarily mean that it significantly contributes to individual differences in mathematics ability. For example, the phonological loop has been found to be involved in holding interim sums in mind whilst performing additions (section 1.4.2) but this may be a function that adults generally perform to the same level of ability. Other elements may therefore contribute to individual differences in mathematics and they may vary depending upon the type of mathematics being performed. Examination of verbal and visuo-spatial working memory capacity differences between skilled adult mathematicians and non-mathematicians will help inform whether verbal storage, visuo-spatial storage or both within working memory contribute to mathematics achievement. Also, it is important to consider whether any superior cognitive abilities of mathematicians have developed as a result of performing more complex mathematics and more mathematics over time or whether their superior cognitive abilities enable them to be better at mathematics than the general population.

There is also a need to further understand how having good visuo-spatial working memory can assist adults when they solve mathematics problems. Within verbal working memory, the phonological loop seems to be involved in storing numerical information whilst processing is carried out (e.g. Fürst & Hitch, 2000). The central executive appears to be involved in direct retrieval and more so in procedural methods involving carry overs and in the selection and execution of appropriate strategies (e.g. Imbo & Vandierendonck, 2010). The research that exists regarding the use of visuo-spatial working memory has not adequately addressed the separation of the use of domain-general executive and domain-specific resources when performing mathematics. Visuo-spatial working memory resources have been implicated in mathematics performance by adults (e.g. Hubber et al., Experiment 1: 2014; Raghubar et al., 2010). However, it is not yet clear whether it is storage by the visuo-spatial sketchpad or domain-general elements controlled by the central executive or both, that drive this relationship. For example, in a review of the literature surrounding the relationship between working memory and mathematics in children and adults, Raghubar et al. (2010) highlighted the need to investigate the overlap between attention and working memory in relation to performance at mathematics. It may well be that the central executive and visuo-spatial sketchpad are not equally important for the proficient solving of mathematical problems.

It is also not yet clear how visuo-spatial working memory involvement may vary with the use of different types of mathematical strategies. The relative involvement of the central executive and phonological loop within verbal working memory has been examined in terms of the use of direct retrieval of answers and procedural methods such as counting and decomposition for solving problems (e.g. Fürst & Hitch, 2000; Imbo et al., 2005). However, the relative involvement of the central executive and visuo-spatial sketchpad elements of visuo-spatial working memory has not been examined in a similar way. This should be investigated to aid our understanding of how use of these components varies with different types of problem solving.

In summary, previous studies with adults have largely concentrated on arithmetic rather than the wider range of mathematical processes associated with the more varied mathematics often carried out by adults. Also, no one has previously compared the working memory capacity of adult mathematicians to that of non-mathematicians to gain insight into whether verbal or visuo-spatial working memory differences or both might contribute to individual differences in mathematical achievement. Whilst specific roles of the central executive and phonological loop within verbal working memory have been highlighted, the involvement of the visuo-spatial sketchpad within visuo-spatial working memory is rather vague. There is a need to further investigate the relative strengths of the association between visuo-spatial storage (visuo-spatial sketchpad) and executive resources (central executive) and mathematics performance in adults, to discover which elements drive any links between working memory and mathematics achievement. It is also necessary to investigate whether these relationships vary with the use of direct retrieval or different procedural methods for solving problems. Finally, there is need for a model of mathematical cognition that incorporates working memory. I will therefore investigate the following main research questions throughout this thesis:

1) **Are there any working memory capacity differences between adult mathematicians and non-mathematicians?**

2) **What drives the relationship between visuo-spatial working memory and mathematics achievement?**

3) **How does having good visuo-spatial working memory assist the proficient solving of mathematical problems?**

4) **How can working memory be incorporated within a model of mathematical cognition?**

Section 1.5 will now give an overview of the current thesis and explain how these questions will be examined.

1.5 The Current Thesis

1.5.1 Overview of the Current Thesis

This thesis contains five experimental chapters, reporting a total of six experiments. Relationships between the components of working memory and both arithmetic and more advanced calculation are examined. After initially investigating verbal and visuo-spatial working memory capacity differences between adult mathematicians and non-mathematicians, I will then concentrate on obtaining a better understanding of the role of visuo-spatial working memory in mathematics and of the relative roles of domain-general storage and domain-general executive processes in the calculation and retrieval of answers to mathematical problems.

Chapter 2 contains the first two experiments. I investigate whether mathematicians have a superior ability to non-mathematicians for storing verbal or visuo-spatial information, or both, in working memory whilst processing is taking place. Both experiments employ working memory span tasks that contain a processing element that is as neutral as possible with regard to the items to be stored. These experiments will also inform whether capacities for storing verbal and visuo-spatial information within working memory are related to mathematics achievement in adults.

In Chapter 3, I investigate, in the third experiment, whether more basic processes drive the link between visuo-spatial working memory and mathematics. I compare the performance of a group of mathematicians and a group of non-mathematicians on a visuo-spatial short-term memory and a controlled spatial attention task. I then examine whether the ability to store visuo-spatial information in working memory is still able to predict adult performance in mathematics calculation and arithmetic fluency when the ability to temporarily hold visuo-spatial information in short-term memory (with no processing) and controlled spatial attention are taken into account.

In Chapter 4, I report the fourth experiment. I again examine whether the link between visuo-spatial working memory and mathematics is driven by a more basic process. I investigate whether memory for visuo-spatial items or memory for their order correlate with calculation and arithmetic performance. This experiment employs a correlational design with adults of varying mathematics and arithmetic ability rather than employing a between-group design.

Chapter 5, I report the fifth experiment. As in Chapter 2, I investigate whether mathematicians have a superior ability to non-mathematicians for storing verbal or visuo-spatial information, or both, in working memory whilst processing is taking place. Within this experiment, however, I manipulate the type of processing task used within the working memory span tasks to see whether verbal and visuo-spatial processing impact on the ability to store verbal and visuo-spatial information within working memory. I also examine differences between the mathematicians and non-mathematicians in performing visuo-spatial mental rotation, as a measure of general visuo-spatial ability. Finally performance for visuo-spatial processing, general visuo-spatial ability and storage of visuo-spatial information within working memory is used to predict mathematics calculation ability to see whether the storage element can still uniquely predict calculation when the other two visuo-spatial elements are taken into account.

In Chapter 6, I employ a within-participant design using a dual task experiment with adults of differing mathematics ability. In this sixth experiment, I attempt to assess the relative roles of the visuo-spatial sketchpad and central executive components of visuo-spatial working memory whilst adults perform addition sums using direct retrieval and more procedural strategies.

A general discussion of the findings of the experimental chapters is found in Chapter 7. New knowledge, arising from the experimental chapters, is assessed in conjunction with the previous literature to further define the role of visuo-spatial working memory in mathematics and to consider how the various elements of the multi-component model of working memory are involved in mathematics.

To summarise, this thesis investigates working memory differences between skilled adult mathematicians and adults who are less skilled at mathematics. Evidence has been presented within section 1.4 that both verbal and visuo-spatial working memory are involved in mathematics performed by adults. Examining differences between skilled mathematicians and non-mathematicians will advance our understanding of which of these types of working memory has the strongest association with mathematics achievement. It also examines whether differences in visuo-spatial working memory capacity and the link between visuo-spatial working memory and mathematics is due to more basic domain-specific storage or more domain-general elements such as controlled attention. It builds on the previous literature to further understand the relationship between visuo-spatial working memory and mathematics and the relative contributions of visuo-spatial storage performed by the visuo-spatial sketchpad and domain-general functions performed by the central executive. Finally, it combines evidence from the experimental chapters and the previous literature to consider how the multi-component model of working memory can be used to explain mathematical cognition in adults.

1.5.2 Methods

The experimental chapters within this thesis contain a mixture of between-group, within-group, dual task and correlational designs.

In Chapters 2, 3 and 5, I compare performance of groups of skilled adult mathematicians with groups of adults who are less skilled at mathematics on working memory span tasks designed to measure working memory capacity. Although, within the literature reviewed in section 1.4, there is an indication that the storage of both verbal and visuo-spatial information within working memory play a part in mathematics, comparison of skilled and unskilled mathematicians helps inform which type of storage is most important for proficient mathematics performance. Also, whilst several previous studies have examined the relationship between working memory and mathematics in individuals with mathematical difficulties (Hitch & McCauley, 1991; Kyttälä, 2008; Passolunghi & Siegel, 2004), none has compared differences in working memory between adult mathematicians and non-mathematicians. Comparison of these different groups aids our understanding of to what extent verbal and visuo-spatial working memory are involved when adults perform complex mathematical problems.

In Chapter 4, I investigate whether memory for item and order are related to mathematics performance, using a correlational design, in participants with a range of mathematical ability. In Chapter 6, I investigate the relative roles of the central executive and visuo-spatial sketchpad in a dual-task experiment with members of the general population.

Statistical analyses throughout this thesis employ Analysis of Variance (ANOVA), Analysis of Covariance (ANCOVA), correlations and regressions. For all analyses other than ANCOVA or correlations involving non-normal distributions, *Pearson's correlations coefficient*, *r,* is used as a measure of effect size. This measure is preferred to other measures of effect sizes, such as *Cohen's d*, as it is easily interpreted because its value ranges from 0 to 1. Values for *r* are widely interpreted as follows: $r = .10$ (small effect); $r = .30$ (medium effect); $r = .50$ (large effect) (Field, 2009, p.57). For ANCOVA, effect sizes are reported using *partial* η^2 (Field, 2009). For correlations involving data with non-normal distributions, *Spearman's rho* (Field, 2009) is used to calculate effect size.

Chapter 2: Working Memory Storage Capacity

2.1 Introduction

This chapter investigates differences in the capacity to store information within working memory between adult mathematicians and adult non-mathematicians. A group of undergraduates studying mathematics are compared to a group of undergraduates who are not studying mathematics for their performance on working memory span tasks in the verbal and visuo-spatial domains.

As discussed in Chapter 1, individual differences in working memory, the ability to temporarily store and manipulate information in mind (Baddeley, 1992), have been shown to be linked to mathematics performance (e.g. Gathercole et al., 2004; Holmes & Adams, 2006; Imbo & LeFevre, 2010; Leikin et al., 2013; Wilson & Swanson, 2001). Also, Butterworth (2006) stated that expert calculators emphasise the importance of being able to hold interim information in mind whilst performing calculations and Hitch (1978) showed that both forgetting this interim information and also forgetting initial information about sums causes errors in mental arithmetic. Short-term memory performance has also been found to be linked to complex cognitive tasks, but performance on working memory tasks is generally more predictive (Bayliss, Jarrold, Gunn & Baddeley, 2003; St. Clair-Thompson & Sykes, 2010).

As discussed in Chapter 1, in terms of the specific storage components of the multi-component model of working memory (e.g. Baddeley, 2003); the phonological loop and the visuo-spatial sketchpad, research suggests that both are used in mathematics to varying degrees and for various functions. There is evidence for the use of the phonological loop in counting, fact retrieval, and the storage of intermediate results (e.g. Fürst & Hitch, 2000; Geary, 2011; Imbo & Vandierendonck, 2007a; Logie et al., 1994). Also, number facts are believed to be stored using a verbal code (Dehaene, 1992).

The majority of studies have concentrated on the use of the two storage components in basic arithmetic rather than the more advanced mathematics commonly used by older children, adults and expert mathematicians. Previous research has suggested that both verbal and visuo-spatial working memory are used in mathematics. As there is a paucity of research into their use in mathematics by adults, research with both children and adults is considered here. However, as highlighted in Chapter 1, it should be borne in mind that relationships with working memory and mathematics may be different for adults and children, especially younger children, due to developmental trajectories of working memory and executive function. Also, greater use of retrieval strategies and more efficient use of procedural strategies are evident in adults solving mathematical problems (Imbo & Vandierendonck, 2008), which will affect comparability between different age groups.

Correlational studies have shown links between working memory performance and mathematics in children (e.g. Holmes & Adams, 2006; Gathercole et al., 2004). Wei et al. (2012) reported a correlation between visuo-spatial working memory and mathematics in adult college students, but their working memory task did not include a processing element, so should really be classed as a short-term memory task. Their study also did not include a verbal working memory measure and Östergren & Träf (2013) have shown verbal working memory to be important for early childhood arithmetic ability. In the visuo-spatial domain, Leikin et al. (2013) showed adolescents gifted at mathematics have superior visuo-spatial working memory and Hubber et al. (2014: Experiment 1) found visuo-spatial working memory to be important when adults solve arithmetic problems. A meta-analysis carried out by David (2012) concluded that mathematics difficulties in children were attributable to visuo-spatial, not verbal, working memory deficits. Imbo & LeFevre (2010) found both phonological and visuo-spatial working memory resources were used when solving subtraction and multiplication problems.

The majority of the evidence for a link between working memory capacity and mathematics achievement has originated from research involving children rather than adults. This research might therefore reflect the importance of working memory for learning mathematics rather than the proficient performance of mathematics. As highlighted in Chapter 1, sections 1.4.4 and 1.4.5, it will be beneficial to examine the working memory performance of skilled adult mathematicians, because this will inform which working memory resources are associated with the proficient solving of mathematical problems.

It is yet to be shown whether adult mathematicians have superior working memory capacity to those who are less skilled at mathematics, and if so, whether this depends on the type of material to be stored. Examining differences in working memory capacity of mathematicians compared to non-mathematicians will help to inform whether capacity for remembering verbal or visuo-spatial information or both is related to mathematics performance in adults. Experiment 1 therefore examines differences in the working memory storage capacity of adult mathematicians and adult non-mathematicians in both the verbal and visuo-spatial domains.

The most relevant studies for examining the working memory storage capacity of mathematicians compared to non-mathematicians appear to be those of Dark & Benbow (1990, 1991, 1994), described in Chapter1, section 1.4.4. Dark & Benbow examined differences in the short-term memory and working memory capacity of groups of adolescents who were classed as mathematically gifted and groups who were verbally gifted, mathematically and verbally gifted, of average ability and college students. Their results were mixed in that the ability of the mathematically gifted to remember numerical stimuli was consistently superior to that of the other groups, but the group differences with regard to remembering visuo-spatial and word stimuli were mixed across their experiments. Their working memory tasks also employed a continuous paired-associates task that always involved the pairing of the storage stimuli with a letter. This meant that in their verbal storage conditions, storage of items may have been impaired by the verbal processing. Their mathematically gifted participants also had high verbal ability too. Although the Dark & Benbow studies involved adolescents rather than adults and have methodological issues, they do indicate that the mathematically talented may have superior working memory capacity dependent upon the type of stimuli being stored.

In summary, comparing groups of adult mathematics experts and non-experts should help us gain a better understanding of the nature of the relationships between the verbal and visuo-spatial domains and mathematics and help inform which components of working memory are important for mathematics rather than more simple arithmetic. The two experiments within this chapter investigate whether adult mathematicians have greater working memory storage capacity than adult non-mathematicians and whether any advantage is in general or specific only to the verbal or visuo-spatial domain.

2.2 EXPERIMENT 1

Experiment 1 investigated whether mathematicians have superior working memory storage capacity for words, numbers or visuo-spatial information by comparing the performance of mathematics undergraduates and non-mathematics undergraduates on working memory span tasks, which used to-be-remembered stimuli from each of these three categories. Words and numbers were both included in the verbal domain, as Dark & Benbow (1990, 1991, 1994) had consistently found that the mathematically gifted have superior memory for numerical items, but their results were mixed for word items. Their results may indicate that, whilst number and word items are both believed to be processed verbally, mathematicians are better able to store numerical information.

Traditionally, tasks used to measure working memory capacity have included a processing element, such as reading, performing arithmetic or judging the symmetry of pairs of objects, interweaved with to-be-remembered storage items, such as numbers, words, letters or orientation of arrows (Friedman & Miyake, 2004; Unsworth & Engle, 2007). At the end of each set, the to-be-remembered items have to be recalled, in correct serial order. Working memory tasks include a processing element as opposed to short-term memory tasks, which simply involve recalling lists of to-be-remembered items without carrying out any processing (St. Clair-Thompson & Sykes, 2010).

Shah and Miyake (1996) highlighted the relative importance of the processing and storage elements of working memory tasks. They found that the storage element is crucial to the correlation of a span task with measures of spatial or language ability, although the type of processing element is also significant. Jarrold, Tam, Baddeley & Harvey (2011) found that whether the processing element is verbal or non-verbal affects performance on the storage element. Therefore, the inclusion of a processing element, such as the traditional reading, arithmetic or object symmetry, may well impact on performance on the storage of word, numerical and visuo-spatial information. Jarrold et al. (2011) also noted that processing tasks used to investigate storage in the verbal and visuo-spatial domains usually involve different task formats for processing stimuli. One can therefore not be sure that any differences found for storage performance are definitely due to differences in storage ability rather than being caused by differences in the cognitive load of the processing elements. Therefore, choice of the type and format of the processing element for inclusion in a working memory span task is extremely important.

Because of the potential impact of the processing element on the storage element, this study used the same novel face-matching task (Burton, White & McNeill 2010) for the processing element in all conditions. This task required participants to make same or different judgements for pairs of unfamiliar faces. The nature of this task is a basic visual comparison involving no spatial transformation (Miyake et al., 2001) and containing no verbal processing and was chosen for the current study as being as neutral a processing task as possible with regard to the storage stimuli used. Face processing has been previously used in studies investigating proactive interference (Pimperton & Nation, 2010) and although face matching would seem to be a simple task, Burton, White and McNeill have shown that it is not trivially easy, with their 2010 study showing an average of 89.9% accuracy for their sample of 300 adults who completed the task. 1 in 10 trials resulted in error, despite the fact that participants were completing this task alone rather than it being embedded in a working memory task.

To summarise, Experiment 1 investigated differences between the working memory storage capacity for number, word and visuo-spatial stimuli of adult mathematicians and non-mathematicians, using working memory span tasks that utilised a consistent and as-neutral-as-possible processing element.

As Dark & Benbow (1990, 1991, 1994) consistently showed the mathematically gifted to have superior working memory capacity for numbers, mathematics undergraduates were expected to have greater working memory capacity for numerical stimuli than non-mathematics undergraduates. As mathematics has been previously shown to rely on the spatial relations of numbers and Hubber et al. (2014: Experiment 1) found visuo-spatial working memory to be important for arithmetic in adults, mathematics undergraduates were expected to have greater working memory capacity for visuo-spatial information than non-mathematics undergraduates. These hypotheses are supported theoretically by the fact that all four models of working memory discussed in Chapter 1, section 1.3, include the assumption that central executive resources are important for the efficient performance of working memory and the central executive has also been consistently found to be implicated in mathematics (section 1.4). For this reason, it was also expected that mathematicians would have superior working memory capacity in the word condition.

2.2.1 Method

2.2.1.1 Participants

G*Power 3 (Faul, Erdfelder, Lang & Buchner, 2007) suggested that a minimum sample size of 44 participants was required to detect an interaction, for two groups with three measures, with an effect size of .25. This effect size was chosen as being a medium effect size per Faul et al. (2007). This is a conservative effect size compared to the .30 medium effect size proposed by Field (2009), referred to in section 1.5.2. Throughout this thesis, statements regarding power calculations represent the power required to detect interactions. It should be noted that the power calculations give less power to detect main effects. 55 participants were recruited from undergraduates at the University of Nottingham: 27 (10 male) to a mathematics group and 28 (8 male) to a non-mathematics group. All participants received an inconvenience allowance of £6.

The mathematics group comprised 19 Mathematics and 8 Economics students. Their ages ranged from 18.33 to 30.58 years (M = 20.43; SD = 2.29). Economics students were included because degree modules for this subject contain substantial mathematics elements and all economics undergraduates had studied maths at A level.

The non-mathematics group comprised English, History and Sociology students, who were not studying mathematics modules at University. Their ages ranged from 18.67 to 28.92 years (M = 20.72; SD = 2.35). Five of the non-mathematics group were later discovered to have studied maths at A level and their data was therefore discarded. The remaining participants in the non-mathematics group had not studied mathematics for a mean of 4.29 years (SD = 2.71).

2.2.1.2 Equipment

A Viglen Pentium D computer, running Windows XP and PsychoPy 2 version 1.73.06 (Peirce, 2007), was used to present stimuli and record latencies and accuracy. Participants' responses were collected via keyboard, numeric keypad or USB mouse. Further details about response collection are given in section 2.2.1.3 below.

2.2.1.3 Working Memory Tasks

There were three working memory span tasks which had the same processing element and different storage elements, with the processing and storage elements interleaved.

For the processing element, participants were presented with two photographs of faces side by side on screen and had to make a judgement as to whether the two faces shown were different pictures of the same person or not. They pressed the 'y' key on the keyboard if the faces were the same person and the 'n' key if they were different people. Each picture was 8.5 cm wide and 9.5 cm high. The left picture was positioned -7 cm left of centre and the right positioned +7 cm right of centre. The pictures of faces were all taken from the Glasgow Unfamiliar Face Database, which shows a high internal reliability when used in a face-matching task (Burton et al., 2010). Faces presented were all white, Western, with neutral expressions and matching pairs were presented in approximately 50% of the trials. Examples of same and different pairs are included in Appendix A.

The storage element of each span task consisted of numerical, word or visuo-spatial items presented in the centre of the screen. To ensure consistency across the three storage types (Kane et al., 2004), items in each span set were taken from a group of nine possible stimuli in each condition:

Number span: Digits 1 to 9 (size 2cm, arial font, colour white on dark grey background)

Word span: Nine animal words (fly, cow, dog, bat, ape, fox, elk, hen, ram), each containing 3 letters and 1 syllable (size 2cm, arial font, colour white on dark grey background)

Visuo-spatial span: Black 3 x 3 grid in the centre of the screen (each square was 6cm wide x 6cm high) with a red dot (size 3 cm wide x 3cm high) placed in one of nine possible locations on the grid (see Figure 2.1)

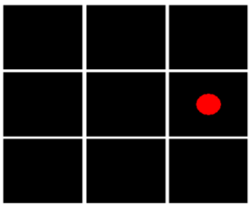

Figure 2.1: Example of storage item presentation during the visuo-spatial span task.

Each trial comprised an interleaved series of processing elements and storage items. Each pair of faces (processing element) was presented on screen for 3 seconds, although participants were still able to respond after this time, and the storage items were presented for 500 milliseconds(ms), commencing 500ms after a response had been given to the preceding pair of faces (following Kane et al., 2004). The next pair of faces was presented 500ms after the storage item disappeared from screen. At the end of each span set, once all storage items had been presented, a “ ? ” appeared in the centre of the screen that prompted the participants to recall the storage items, in their order of presentation. In the number condition, participants said the numbers aloud and the experimenter keyed the response into the USB numeric keypad. In the word condition, participants said the words out loud, the experimenter coded them and then entered them via the USB numeric keypad. In the visuo-spatial condition, a black 3 x 3 grid appeared on screen immediately after the “ ? “ and participants recalled the serial order of the red dot by clicking on the grid, using the USB mouse. Once recall was completed, the participant pressed the space bar to begin the next trial (see Figure 2.2 for an example trial sequence).

Each trial was largely experimenter-paced, rather than participant-paced, to reduce the ability of participants to utilise different strategies, such as chunking, for remembering items (Engle et al., 1992; Friedman & Miyake, 2004). The exception to this was that participants could still respond to the pairs of faces after they had disappeared from screen so that they were still able to give a response to the processing element. In practice, as will be seen in section 2.2.2.3.1, they took far less, on average, to respond to the faces than the 3 seconds allowed and there were no significant differences in response times between the two groups.

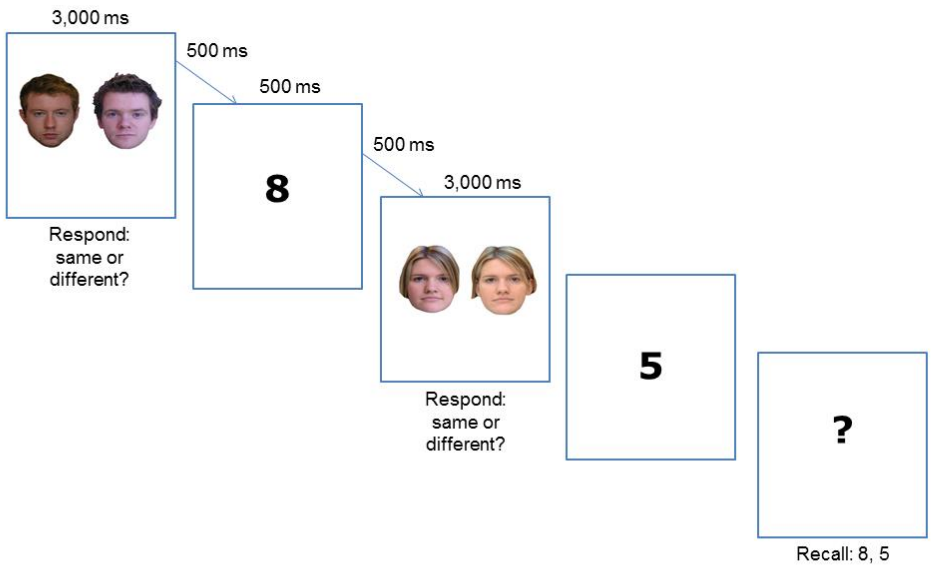

Figure 2.2: Example of a trial sequence (2 span) in the numerical condition.

Working memory span studies have traditionally presented span sets in ascending order of length (St Clair-Thompson, 2012), but Lustig, May and Hasher (2001) found that order of presentation can have an impact on span scores, suggesting that later sets are affected by interference from earlier presentations. Therefore if the longest sets, which are most important for determining a participant's span score, are presented later they are the most affected by this proactive interference. To minimise this issue (Conway et al., 2005) and also prevent participants from anticipating which span size they would have to remember next (Engle et al., 1992; Unsworth, Heitz, Schrock & Engle, 2005), span sets were presented in a random order in Experiment 1. Each of span lengths 2 to 7 was presented three times, giving 18 span sets (trials) in each of the three conditions (included in Appendix B). Each of the nine possible items within each set was presented approximately equally.

2.2.1.4 Additional Materials

Two standardised tests from the Wechsler Abbreviated Scale of Intelligence (WASI; Psychological Corporation, 1999) were administered, using the standard procedures and scores, to enable comparison of the IQ of the two groups –

WASI Matrix Reasoning (non-verbal IQ): For each item, participants were shown a matrix of coloured figures with one piece missing and had to select the missing piece from five alternatives shown below the matrix. There was no time limit for completion of the 29 items. Under the standard procedure, the test was stopped if participants scored zero on four consecutive items or scored zero on four out of five consecutive items.

WASI Vocabulary (verbal IQ): For each item, the experimenter read a word out aloud and participants had to give the meaning of the word. There was no time limit for completion of the 34 items. Under the standard procedure, the test was stopped if participants scored zero for five consecutive items.

The *Woodcock-Johnson Calculation Test* (Woodcock, McGrew & Mather, 2001) was administered, commencing with item 14, using the standard procedure, to ensure there was a difference in the maths ability of the two groups. Raw scores are reported for this test. Using pen and paper and no calculator, participants had to solve a series of mathematics calculations of increasing difficulty, which ranged from simple arithmetic, fractions and long division through to items such as matrices, integration and trigonometrical ratios. There was no time limit for completion of the 32 items. Under the standard procedure, the test was stopped if participants scored zero on six consecutive items.

2.2.1.5 Procedure

All participants were tested individually by the same experimenter and each session lasted around one hour. All participants in both groups completed the three working memory span conditions on the computer. The order in which the three conditions were presented was counterbalanced across participants and the order of presentation of span sets and the presentation of items within each set was randomized.

After initial instructions, participants practised the face-matching task, comprising same or different judgements for six pairs of faces, so they could familiarise themselves with the processing task. They then began the experiment. All three conditions that followed commenced with a practice of one 2-span set and one 3-span set comprising both processing and storage tasks, before the 18 experimental sets were administered. Participants then completed the WASI Matrix Reasoning and WASI Vocabulary tests, the order of which was counterbalanced across participants. Finally, participants completed the Woodcock-Johnson Calculation Test.

2.2.1.6 Span Scoring Method

The traditional method of scoring working memory span tasks involves assigning an absolute score (e.g. Daneman & Carpenter, 1980; Conway, Cowan, Bunting, Therriault & Minkoff, 2002). Participants are tested at ascending span lengths and their span score is taken as the span length at which they succeed a predetermined number of times, such as on 2 out of 3 attempts, before failing to meet this threshold on the next span size up. Testing usually ceases once this point has been reached. This method of scoring was considered inappropriate for the current study because it is believed to be too insensitive a measure (St. Clair-Thompson & Sykes, 2010; Unsworth & Engle, 2007).

Conway et al. (2005) evaluated four alternative methods of scoring working memory span performance: *Partial Credit Unit*, the mean proportion of elements recalled in the correct serial position; *All-or-Nothing Unit*, the proportion of sets in which all items are recalled in the correct serial position; *All-or-Nothing Load*, the sum of wholly recalled correct spans (e.g. recalling 4 span correctly 3 times gives a score of 12); *Partial Credit Load*, the sum of all items recalled correctly regardless of serial position.

Whilst load scoring is commonly used within psychology, it is rarely used in psychometrics due to its tendency for positive skews and assigning greater weight to longer lists (Conway et al., 2005). Load scoring was therefore discounted for the current study, leaving a choice between Partial Credit Unit (more commonly called Proportion Correct scoring and this term will be used going forward) and All-or-Nothing Unit.

Proportion Correct scoring showed greater reliability when Conway et al. (2005) reanalysed data from the Kane et al. (2004) study, in which 236 participants performed an operation span, reading span and counting span. St. Clair-Thompson & Sykes (2010) compared national curriculum scores for reading, writing, mathematics and science with performance on a series of short-term and working memory tasks using both absolute and proportion correct scoring methods. They found that, whilst the scoring methods were highly correlated and had similar reliability, Proportion Correct scoring provided a better predictor of ability and the authors recommended use of this method. Similarly, Friedman & Miyake (2005) recommended using Proportion Correct scoring following their comparison of four scoring methods which found this method to have greatest reliability, the best correlation with reading comprehension and verbal SAT, fewer outliers and a more normal distribution of data. Also, because it is a more continuous scoring method, Proportion Correct is more sensitive to individual differences. Finally, Unsworth & Engle (2007) reported switching to using Proportion Correct scoring, having previously used an All-or-Nothing method, due to superior psychometric properties and greater sensitivity through using information from lists that were not perfectly recalled. The current study therefore used the Proportion Correct scoring method.

2.2.1 Results

Seven participants (3 mathematics group; 4 non-mathematics group) were excluded from the analyses for having an unacceptably high (>15%) error rate in the processing task (mathematics: 1 visuo-spatial condition, 2 word condition; non-mathematics: 2 visuo-spatial condition, 1 visuo-spatial & word conditions, 1 word condition). Omission of participants scoring < 85% on the processing element in this way is recommended (Conway et al., 2005; Unsworth et al., 2005) to ensure that unfair advantage has not been gained on the storage element through paying insufficient attention to the processing element.

A Cook's Distance score was calculated initially for each participant in each condition within the Analysis of Variance (ANOVA) described in section 2.2.2.2 to discover whether there were any influential cases that could affect the results of the ANOVA. A Cook's distance score was also calculated in a regression using storage accuracy in the three conditions to predict mathematics scores, to discover whether influential cases could affect any of the correlations reported below. One influential outlier was detected in the non-mathematics group in the visuo-spatial condition, with a Cook's Distance score >1 (Field, 2009) and this male participant's data was discarded for analysis purposes.

This left data for 24 (10 male) participants in the mathematics group and 18 (7 male) in the non-mathematics group available for analysis. This totalled 42 participants overall, which was just below the recommended minimum of 44, suggested by G-Power (Faul et al., 2007) on the basis of a small-medium effect size. However, the sections below show that 42 participants was still sufficient to detect group differences.

Controlling for gender had no significant impact on analyses and gender was therefore not controlled for in any analyses reported below. Degrees of freedom were corrected using Greenhouse-Geisser estimates of spherity where necessary.

In the sections below, results for standardised tests will be firstly reported (section 2.2.2.1), followed by results for the storage element of the working memory tasks (section 2.2.2.2), results for the processing element of the working memory tasks (section 2.2.2.3), then finally the relationship between the storage element and mathematics scores (section 2.2.2.4).

2.2.2.1 Standardised Tests

Performance of the two groups on the standardised tests was initially compared to confirm that the mathematicians were better at mathematics than the non-mathematicians and to confirm that the groups were matched for verbal and non-verbal IQ.

An independent *t*-test to compare the two groups' Woodcock-Johnson Calculation Test scores confirmed that the mathematics group (M = 25.83, SD = 2.91) were significantly better at mathematics than the non-mathematics group (M = 13.61, SD = 3.57), $t(40) = 12.22$, $p < .001$, $r = .89$. Scores for the mathematics group represented a median percentile rank compared to age norms (Woodcock, McGrew & Mather, 2001) of 93.00 (min = 81.00; max = 99.80). Scores for the non-mathematics group represented a median percentile rank compared to age norms of 45.00 (min = 7.00; max = 73.00).

Independent *t*-tests also showed that there was no significant difference between the two groups for WASI Matrix Reasoning non-verbal IQ (mathematics: M = 29.42, SD = 2.47; non-mathematics: M = 28.72, SD = 2.78), $t(40) = .86$, $p = .398$, $r = .13$ or for WASI Vocabulary, verbal IQ (mathematics: M = 61.83, SD = 6.27; non-mathematics: M = 65.17, SD = 5.73), $t(40) = -1.77$, $p = .085$, $r = .27$. Although the latter was approaching significance, controlling for verbal or non-verbal IQ made no significant difference to results or conclusions and results reported below are without controlling for IQ.

2.2.2.2 Storage Element

Proportion correct scores were first calculated for each participant for the number of storage items recalled in their correct serial position. Descriptive statistics by group are shown in Figure 3.

A 2(group: mathematics, non-mathematics) x 3(working memory storage type: number, visuo-spatial, word) mixed Analysis of Variance (ANOVA) was then performed on these scores. Results of the ANOVA showed no main effect of studying or not studying mathematics, $F(1,40) = 3.57$, $p = .066$, $r = .29$, although this was approaching significance. There was a significant main effect of storage type, $F(1.57,62.93) = 51.01$, $p < .001$, $r = .67$. Contrasts showed that scores in the number condition were significantly greater than those in the visuo-spatial condition, $F(1,40) = 51.27$, $p < .001$, $r = .75$, and the word condition, $F(1,40) = 117.45$, $p < .001$, $r = .86$. Scores in the visuo-spatial condition were also significantly greater than those in the word condition, $F(1,40) = 12.73$, $p = .001$, $r = .20$.

There was a significant group x working memory storage type interaction (see Figure 3), $F(1.57,62.93) = 6.01$, $p = .007$, $r = .30$. Tests of Bonferroni-corrected simple main effects showed that the mathematics group had significantly greater scores than the non-mathematics group in the visuo-spatial condition, $F(1,40) = 19.10$, $p < .001$, $r = .57$, but there was no significant difference in performance between the two groups in the verbal domain: word span $F(1,40) = .01$, $p = .921$, $r = .02$; number span $F(1,40) = .08$, $p = .583$, $r = .04$.[2]

[2] ANOVAs were also run using both the All-or-Nothing Unit and All-or-Nothing Load methods, which did not result in any significant changes to results or conclusions.

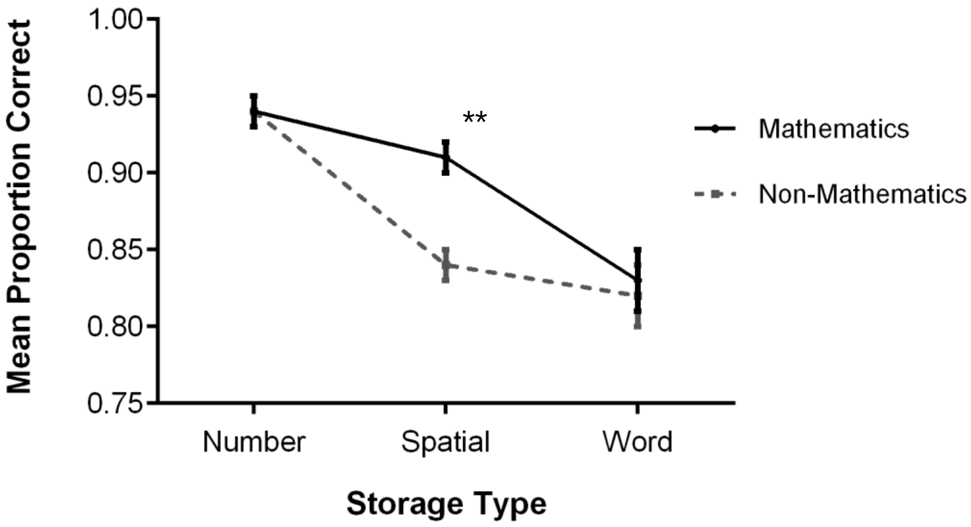

Figure 2.3: Accuracy of storage type for each participant group with S.E.M. error bars.

Word span and number span scores correlated, r = .59, p < .001, but neither storage type in the verbal domain correlated with visuo-spatial span scores: word span r_s = .16, p = .312; number span r_s = .12, p = .434.

2.2.2.3 Processing Element

Initially, mean accuracy and median RT were calculated for each participant in each of the three working memory span conditions.

Separate 2(group: mathematics, non-mathematics) x 3(working memory storage type: number, visuo-spatial, word) mixed ANOVAs were performed for each of face-matching accuracy and face-matching latencies to examine performance of the two groups on the processing element under each storage condition. Mean accuracy, mean RT and standard error by group and span type are shown in Table 2.1.

Table 2.1

Mean (M) and standard error (SE) for accuracy and reaction time in the face matching task by group in each storage type condition

Condition	Groups	Accuracy		Reaction Time(Ms)	
		M	SE	M	SE
Number	Mathematics	.94	.01	1264	63
	Non-Mathematics	.95	.01	1326	72
Visuo-spatial	Mathematics	.94	.01	1287	50
	Non-Mathematics	.96	.01	1421	86
Word	Mathematics	.93	.01	1323	63
	Non-Mathematics	.93	.01	1403	76

2.2.2.3.1 Accuracy

Results showed no main effect of studying or not studying mathematics for accuracy on the face-matching task, $F(1,40) = 2.47$, $p = .124$, $r = .24$. There was a significant main effect of storage type, $F(2,80) = 5.57$, $p = .005$, $r = .42$. Contrasts showed that there was no significant difference in accuracy on the processing element between the number and visuo-spatial storage conditions, $F(1,40) = .34$, $p = .565$, $r = .09$, but that face-matching was more accurate in both the number ($F(1,40) = 7.03$, $p = .011$, $r = .39$) and visuo-spatial ($F(1,40) = 8.65$, $p = .005$, $r = .42$) storage conditions than the word storage condition. There was no group x span type interaction, $F(2,80) = .34$, $p = .716$, $r = .09$.

2.2.2.3.1 Latencies

Results for face matching latencies showed no main effect of studying or not studying mathematics, $F(1,40) = 1.03$, $p = .316$, $r = .16$. There was a significant main effect of storage type. $F(2,80) = 3.52$, $p = .034$, $r = .21$, with simple main effects revealing latencies in the number condition were faster than those in the visuo-spatial condition $F(1,40) = 4.87$, $p = .033$, $r = .33$ and those in the word condition $F(1,40) = 7.01$, $p = .012$, $r = .39$. There was no significant difference for latencies between the visuo-spatial and word conditions $F(1,40) = .08$, $p = .774$, r

= .05. Finally, there was no group x storage type interaction, $F(2,80) = .90$, $p = .411$, $r = .15$.

2.2.2.4 Relationship of Storage Element with Mathematics Scores

There was a significant relationship between participants' Woodcock-Johnson Calculation scores and their visuo-spatial span performance, $r_s = .56$, $p < .001$, but neither storage types in the verbal domain correlated with mathematics scores: word span $r_s = .18$, $p = .264$; number span $r_s = .07$, $p = .681$.

2.2.3 Discussion

Experiment 1 investigated whether adult mathematicians have superior working memory storage capacity to adult non-mathematicians and if so whether this is in general or just specifically for number, visuo-spatial or word information, in order to discover which types of information storage within working memory have important links with mathematics.

Through the use of a consistent processing task across conditions, which was as neutral as possible with regards to the storage elements, this experiment has provided evidence that mathematicians have superior working memory capacity for the storage of items in the visuo-spatial domain, but that there is no significant difference between adult mathematicians and non-mathematicians in the verbal domain for word or numerical information. This suggests that, in terms of mathematical cognition, visuo-spatial working memory storage capacity has a significant association with mathematics. This will be considered further in the general discussion (section 2.4).

The first hypothesis was not supported because, unlike in the Dark & Benbow (1990, 1991, 1994) studies, mathematicians did not have superior working memory capacity for numerical stimuli. Dark & Benbow argued that their mathematically gifted participants may have performed better in their numerical conditions as they were more familiar with the numerical stimuli than were their other participant groups, but, as the stimuli used were basic digits, this seems unlikely. Also, their working memory task did not involve recall at varying span lengths, but measured either recall of a stimuli pairing at varying time lags between presentation and recall or simply their performance at a fixed set size of five items. Therefore, their task measured the amount of time an item could be retained or sustained performance at a fixed span length rather than measuring actual item capacity. It should also be remembered that their studies involved adolescents rather than adults, which may also account for differences between their findings for numerical storage and those of Experiment 1 in this chapter.

The most striking finding of the current study was that the group of mathematics undergraduates had significantly greater working memory capacity for visuo-spatial information than the non-mathematics group, supporting the second hypothesis. This intergroup difference was underlined by the strong correlation between participants' Woodcock-Johnson mathematics scores and visuo-spatial span proportion correct scores, which was not affected by controlling for verbal IQ, non-verbal IQ or gender. Results for the visuo-spatial condition in Experiment 1 support the findings of Wei et al. (2012) who found a correlation between visuo-spatial memory and mathematics, although they used a short-term memory task rather than a working memory task. The fact that visuo-spatial span scores correlated with mathematics scores, whereas the two verbal span scores did not, and the two verbal spans correlated with each other but neither correlated with visuo-spatial span also supports the Baddeley & Hitch multi-component model of working memory (Baddeley, 2000) which identifies the phonological loop and visuo-spatial sketchpad as separate verbal and visuo-spatial stores.

The third hypothesis was not supported in that there was no significant difference between the two groups for performance in the word condition of Experiment 1. However, this helps to make sense of the findings in the numerical condition if they are viewed within the framework of the multi-component model of working memory (Baddeley, 2000). Span tasks involving digits are generally viewed as being verbal in nature (e.g. Dark & Benbow, 1990; Baddeley, 1992), because digits are given verbal labels when stored in memory. The finding that digit span scores were greater than word span scores is consistent with previous research and likely due to word-frequency effects (Unsworth & Engle, 2007). The Experiment 1 findings that there were no differences between the two groups for storage in either the numerical or word conditions indicates no difference in phonological loop capacity for adult mathematicians and non-mathematicians.

As can be seen from Table 1, participants in both groups performed at greater accuracy levels than in Burton et al.'s (2010) study for the face-matching processing task. Burton and colleagues' participant sample comprised adults recruited from the general population, whereas the current study comprised undergraduate students, which may account for the difference in performance. Their study also included several non-white faces, whereas the current study only used white, Western faces from the database. Both accuracy and latencies were similar for mathematicians and non-mathematicians across all three storage conditions, allowing for a meaningful comparison across the storage elements. This processing task could also be readily manipulated in future research if greater difficulty was required, by, for example, increasing the number of faces presented to three or requiring matching after a delay. Overall accuracy on the face-matching task was slightly worse in the word condition than in the number and visuo-spatial conditions. This is consistent with previous findings that performance on the processing element of a working memory span task usually positively correlates with performance on the storage element (Conway et al., 2005) and performance for word storage was worse than for numbers and visuo-spatial storage, as shown in Figure 3. Similarly, processing latencies were faster in the number condition than in the word and visuo-spatial conditions, reflecting the greater performance for number storage items.

To conclude, Experiment 1 found that undergraduates studying mathematics had superior working memory storage capacity to undergraduates not studying mathematics, but that there was no significant difference between the two groups for working memory storage capacity for verbal information, as shown by results for number and word spans. These findings support the theory of separate domain-specific resources for the storage of visuo-spatial and verbal information, in line with the phonological loop and visuo-spatial sketchpad elements of the multi-component model of working memory. However, Experiment 1 used span lengths 2 to 7 across all conditions and, on examining data for the numerical condition, it was found that participants in both the mathematics and non-mathematics groups were performing at around ceiling. Whilst both groups appeared to have the same storage capacity in the numerical condition, testing at greater span lengths may have resulted in a divergence of their scores indicating a difference in capacity. Experiment 2, reported below, was therefore run using span lengths 3 to 8 in the numerical condition, to examine whether ceiling effects contributed to the results in Experiment 1.

Results therefore provide evidence for an important link between visuo-spatial working memory capacity and mathematics. Experiment 2 attempted to replicate the group difference in the visuo-spatial domain and also investigated whether potential ceiling effects had an impact in the numerical condition of Experiment 1.

2.3 EXPERIMENT 2

Experiment 2 again compared the working memory storage capacity of a group of undergraduate mathematicians and undergraduate non-mathematicians for verbal and visuo-spatial information. Only the number condition was used in the verbal domain as, in Experiment 1, the number and word conditions had shown similar patterns of association with mathematics scores and with the visuo-spatial condition. The visuo-spatial condition in Experiment 2 was identical to that used in Experiment 1 to see whether the results that mathematicians have superior visuo-spatial working memory storage capacity could be replicated. In the number condition, the span lengths used were increased to spans 3 to 8 to investigate whether ceiling effects present in the number condition had impacted the results of Experiment 1.

2.3.1 Method

2.3.1.1 Participants

54 participants were recruited from undergraduates at the university of Nottingham: 27 (9 male) to a mathematics group and 27 (9 male) to a non-mathematics group. None of the participants had taken part in Experiment 1 and all participants received an inconvenience allowance of £6.

The mathematics group comprised 15 mathematics students and 12 economics students who had studied mathematics at A level. Their ages ranged from 18.66 to 36.89 years (M = 20.88, SD = 3.53). The non-mathematics group comprised English, History, Philosophy and Sociology students who had not studied mathematics at A level. Their ages ranged from 18.78 to 22.68 years (M = 20.33, SD = .99). On average, participants in the non-mathematics group had not studied maths for 4.18 years (SD = 1.16).

2.3.1.2 Equipment

Equipment was identical to that used in Experiment 1 (section 2.2.1.2).

2.3.1.3 Working Memory Tasks

The working memory tasks in Experiment 2 were identical to those used in the number and visuo-spatial conditions of Experiment 1 (section 2.2.1.3), with the exception of span lengths 3 to 8 being used for the number condition. Span lengths 2 to 7 were again used in the visuo-spatial condition. The full list of trials used in the number condition is included in Appendix C.

2.3.1.4 Additional Materials

WASI Matrix Reasoning (WASI; Psychological Corporation, 1999) and the *Woodcock-Johnson Calculation Test* (Woodcock, McGrew &

Mather, 2001) used in Experiment 1 (section 2.2.1.4) were also administered in Experiment 2.

The *Woodcock-Johnson Math Fluency Test* (Woodcock, McGrew & Mather, 2001) was also administered as an additional mathematics assessment, using the standard procedure. Using pen and paper and no calculator, participants had to solve as many simple arithmetic problems as possible within three minutes.

WASI Vocabulary was not administered in this experiment as there was no word condition.

2.3.1.5 Procedure

All participants completed the working memory span tasks as part of an approximately hour long session with the same experimenter. The session also involved the completion of two short-term memory tasks and an attention task that will be discussed in Chapter 3.

Procedures and timings for the two working memory span conditions were identical to those in Experiment 1 (section 2.2.1.5). After completion of the working memory span tasks, participants then completed the short-term memory tasks followed by the attention task (both reported in Chapter 3) and then *WASI* Matrix Reasoning. Finally they completed the Woodcock-Johnson Calculation Test and Woodcock-Johnson Math Fluency Test, the order of which was counterbalanced.

2.3.2 Results

Three participants (2 mathematics group; 1 non-mathematics group) were later excluded from the analyses for having an unacceptably high (>15%) error rate in the processing task (mathematics: 1 number condition, 1 number & visuo-spatial conditions; non-mathematics: 1 number condition) leaving data for 25 (9 male) participants in the mathematics group and 26 (9 male) in the non-mathematics group available for analysis.

No influential outliers with a Cook's Distance score >1 (Field, 2009) were detected in either group, using the same analyses as in Experiment 1 (described in section 2.2.1). As in Experiment 1, controlling for gender had no significant impact on analyses and gender was therefore not controlled for in any analyses reported below. Degrees of freedom were corrected using Greenhouse-Geisser estimates of spherity where necessary.

In the sections below, results for standardised tests will be firstly reported (section 2.3.2.1), followed by results for the storage element of the working memory tasks (section 2.3.2.2), results for the processing element of the working memory tasks (section 2.3.2.3), then finally the association between the storage element and mathematics scores (section 2.3.2.4).

2.3.2.1 Standardised Tests

An independent *t*-test to compare the two groups' Woodcock-Johnson Calculation Test scores confirmed that the mathematics group (M = 24.80, SD = 3.50) were significantly better at mathematics than the non-mathematics group (M = 12.15, SD = 3.46), $t(49) = 12.97$, $p < .001$, $r = .88$. Scores for the mathematics group represented a median percentile rank compared to age norms (Woodcock, McGrew & Mather, 2001) of 92.00 (min = 64.00; max = 99.00). Scores for the non-mathematics group represented a median percentile rank compared to age norms of 34.00 (min = 5.00; max = 67.00).

A non-parametric Mann-Whitney test was performed to compare the two groups' Woodcock-Johnson Math Fluency scores, because the mathematics group's scores showed significant negative skew at the $p < .05$ level (Field, 2009). This showed a significantly higher performance for the mathematicians (M = 144.68, SD = 20.72) compared to the non-mathematicians (M = 113.46, SD = 16.87), $U = 90.00$, $Z = -4.43$, $p < .001$, $r = .63$. Scores for the mathematics group represented a median percentile rank compared to age norms of 89.00 (min = 21.00; max = 99.90). Scores for the non-mathematics group represented a median percentile rank compared to age norms of 43.50 (min = 3.00; max = 83.00).

An independent *t*-test showed that the mathematics group (M = 28.92, SD = 3.46) had significantly greater non-verbal IQ than the non-mathematics group (M = 26.88, SD = 3.25) when comparing their scores for WASI Matrix Reasoning. All analyses were therefore initially run controlling for WASI Matrix Reasoning scores, but this made no difference to main effects or interactions. Therefore, results reported below do not control for non-verbal IQ.

2.3.2.2 Storage Element

As in Experiment 1, proportion correct scores were first calculated for each participant for the number of storage items recalled in their correct serial position.

Before conducting the main ANOVA, scores were examined for the two groups in the number span condition for span lengths 3 to 8, to check for ceiling effects. Mean proportion correct scores (mathematics: M = .88, SD= .08; non-mathematics: M = .87, SD = .05) clearly showed that neither group was performing at ceiling and a non-parametric Mann-Whitney test showed no significant difference in proportion correct scores between the two groups, $U = 323.00$, $z = -.04$, $p = .974$. A non-parametric test was used here as the non-mathematics group's scores showed significant negative skew at the $p < .05$ level (Field, 2009).

Although a non-parametric test was used above, a parametric ANOVA is used below to examine differences in storage capacity between groups and conditions. ANOVA has frequently been found to be robust to skew in data and it has been shown that lack of normality is only problematic for the *F*-test in ANOVA in sample sizes below 40 (Field, 2009). Garcia-Marques, Garcia-Marques & Brauer (2014) also showed that transforming non-normal data for use in a 2 x 2 ANOVA renders the smallest of the main effects and interactions uninterpretable. For these reasons, a parametric ANOVA was used.

This treatment is also consistent with the previous working memory span task literature.

A 2(group: mathematics, non-mathematics) x 2(working memory storage type: number, visuo-spatial) mixed ANOVA was then performed on the proportion correct scores using span lengths 3 to 7 for both conditions, to ensure the same span lengths were being compared across conditions. Descriptive statistics for spans 3 to 7, by group, are shown in Figure 2.4.[3]

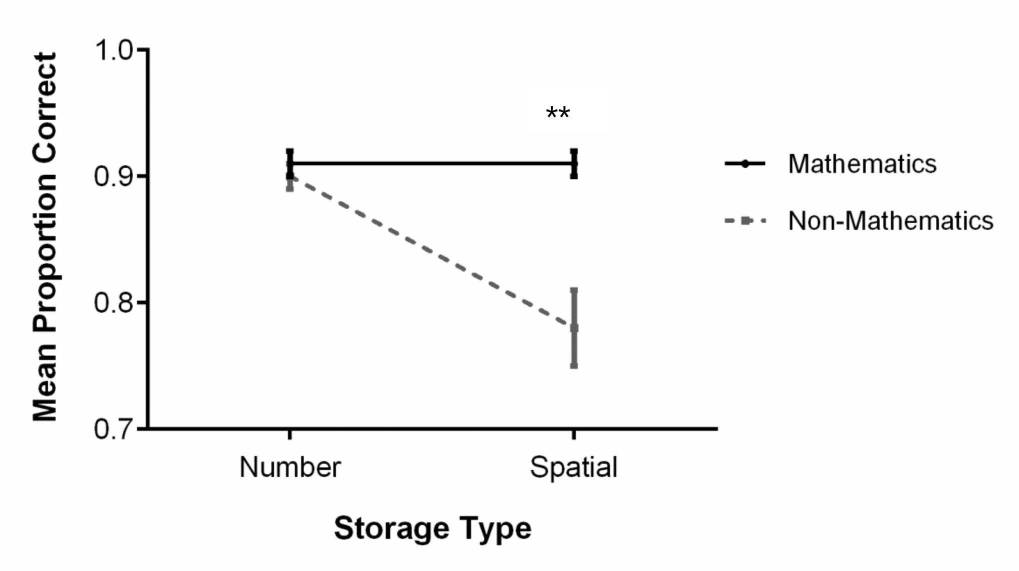

Figure 2.4: Accuracy of storage type for each participant group with S.E.M. error bars.

Results of the ANOVA showed a main effect of group, with mathematicians scoring higher overall, $F(1,49) = 8.90$, $p = .004$, $r = .39$. There was also a main effect of storage type, with performance for number span greater than that for visuo-spatial span, $F(1,49) = 29.50$, $p < .001$, $r = .61$. There was a group x storage type interaction, $F(1,49) = 24.09$, $p < .001$, $r = .57$. Tests of Bonferroni-corrected simple main effects showed that the mathematics group had significantly greater

[3] Because of the intention to exactly replicate the visuo-spatial condition and the need to increase span lengths to 3 to 8 to check for ceiling effects in the numerical condition, the fact that the span lengths across the two conditions were not comparable was overlooked at the design stage. This is why span lengths 3 to 8 were initially checked for ceiling effects in the numerical condition, before spans 3 to 7 were analysed for both conditions in the ANOVA.

visuo-spatial span scores than the non-mathematics group, $F(1,40) = 19.94$, $p < .001$, $r = .54$, but there was no significant difference in performance between the two groups for number span $F(1,49) = .07$, $p = .788$, $r = .04$.[4]

Visuo-spatial span scores also did not correlate with number span scores, $r_s = .17$, $p = .245$.

2.3.2.3 Processing Element

Initially, mean accuracy and median RT were calculated for each participant in each of the two working memory span conditions over span lengths 3 to 7, to be consistent with the storage task analysis.

A 2(group: mathematics, non-mathematics) x 2(working memory storage type: number, visuo-spatial) mixed ANOVA was performed for each of face matching accuracy and face matching latencies to examine performance of the two groups on the processing element under each storage condition. Mean accuracy, mean RT and standard error by group and span type are shown in Table 2.2.

Table 2.2

Mean (M) and standard error (SE) for accuracy and reaction time in the face matching task by group in each storage type condition

		Accuracy		Reaction Time(Ms)	
Condition	Groups	M	SE	M	SE
Number	Mathematics	.95	.01	1304	56
	Non-Mathematics	.94	.01	1252	59
Visuo-spatial	Mathematics	.95	.01	1311	50
	Non-Mathematics	.95	.01	1292	48

2.3.2.3.1 Accuracy

Results showed no significant difference in accuracy on the face matching task between groups or across the different storage conditions. There was no main effect of studying or not studying mathematics, $F(1,49) = .20$, $p = .655$, $r = .06$, no main effect of storage

[4] ANOVAs were also run using both the All-or-Nothing Unit and All-or-Nothing Load methods, which did not result in any significant changes to results or conclusions.

type, $F(1,49) = .03$, $p = .863$, $r = .22$ and no group x storage type interaction, $F(1,49) = .01$, $p = .933$, $r = .01$.

2.3.2.3.2 Latencies

Results showed no significant difference in RT on the face matching task between groups or across the different storage conditions. There was no main effect of studying or not studying mathematics, $F(1,49) = .24$, $p = .627$, $r = .07$, no main effect of storage type, $F(1,49) = 1.23$, $p = .273$, $r = .16$ and no group x storage type interaction, $F(1,49) = .58$, $p = .451$, $r = .11$.

2.3.2.4 Relationship of Storage Element with Mathematics Scores

There was a significant correlation between participants' visuo-spatial span performance and their Woodcock-Johnson Calculation scores $r_s = .57$, $p < .001$ and Woodcock-Johnson Math Fluency scores $r_s = .30$, $p = .031$. Calculation and Fluency scores correlated with each other $r_s = .69$, $p < .001$. Number span did not correlate significantly with either Calculation $r_s = .04$, $p = .763$ or with Fluency $r_s = -.17$, $p = .240$.

2.3.2.5 Serial Position Curves

The serial position curves for recall accuracy in the visuo-spatial conditions of Experiments 1 & 2 were examined, by group, to investigate whether mathematicians displayed any differences in patterns of serial recall accuracy to the non-mathematicians. Discovery of different recall patterns between the two groups may suggest that mathematicians were using different strategies for remembering the visuo-spatial locations, such as grouping items together into chunks (Engle et al., 1992; Friedman & Miyake, 2004). Only the visuo-spatial condition was examined in this way, as results for the verbal domain in Experiment 1 and Experiment 2 had shown no significant difference in capacity between the mathematicians and non-mathematicians.

Serial position curves are commonly used to examine the accuracy of recall of items in their various serial positions within a given span length. Forward recall of verbal items and visuo-spatial locations have been previously found to display a primacy effect, which is a reduction in recall accuracy after the first item in a list, and a recency effect, which is an improvement in recall for the final item, although there has been far less investigation of serial order in the visuo-spatial domain than in the verbal domain (Hurlstone, Hitch & Baddeley, 2013).

Accuracy scores from Experiments 1 and 2 were initially combined for each serial position within each span length to produce average curves across spans 2 to 7 and these are shown in Figure 2.5. Separate 2(group: mathematics, non-mathematics) x n(number of serial positions) mixed ANOVAs were then performed for accuracy at each visuo-spatial span length. Results showed that, whilst mathematicians generally displayed greater overall accuracy across the different span lengths, there were no significant differences in the patterns of the curves of the two groups, with all *p*'s for interactions at the various span lengths > .05. All comparisons reported below are Bonferroni-corrected.

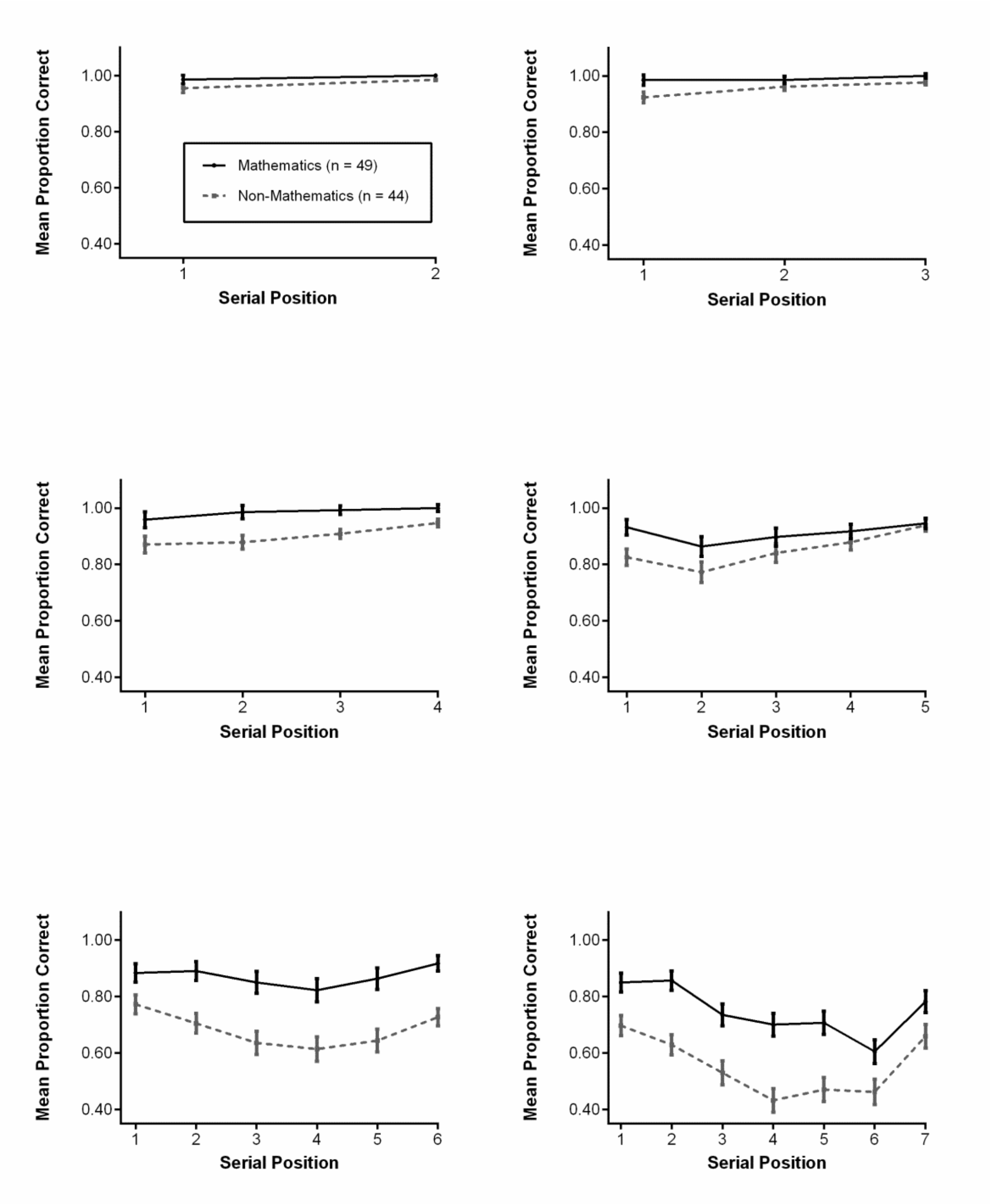

Figure 2.5: Experiments 1 & 2 combined serial position curves for each of spans 2 to 7 in the visuo-spatial span condition with S.E.M. error bars.

At span length 2, there was no significant difference between the performance of the 2 groups, $F(1,91) = 3.55$, $p = .114$, $r = .17$, but, overall, final position 2 was more accurately recalled than position 1 $F(1,91) = 6.59$, $p = .012$, $r = .26$.

Differences between the performance of the two groups began to emerge as early as span length 3, with the mathematicians showing greater accuracy overall, $F(1,91) = 5.91$, $p = .017$, $r = .25$. The recency effect also began to emerge at span length 3. There was a main effect of position, $F(1.65,150.05) = 4.23$, $p = .023$, $r = .17$, with no significant difference between positions 1 & 2 or 2 & 3 (all p's > .05), but with final position 3 more accurate than position 1 ($p = .046$).

At span length 4, mathematicians were more accurate than non-mathematicians, $F(1,91) = 12.82$, $p = .001$, $r = .35$ and there was a main effect of position, $F(1.68,153.14) = 6.59$, $p = .011$, $r = .18$. Position 1 was no different to positions 2 & 3 and 2 was no different to 3 (all p's > .05). Final position 4 was more accurate than position 1 ($p = .048$) but no more accurate than positions 2 & 3 (p's > .05).

At span length 5, mathematicians were again more accurate, $F(1,91) = 4.15$, $p = .045$, $r = .21$ and there was again a main effect of position, $F(3.31,301.11) = 7.77$, $p = < .001$, $r = .16$. Here, the primacy effect emerged, with the first position being more accurate than the second ($p = .021$). There was no significant difference between positions 2 & 3 or 3 & 4 (all p's > .05), but 2 was worse than 4 ($p = .041$). In terms of the recency effect, position 5 was no different to position 4 ($p = .303$), but 5 was better than 2 & 3 (p's < .05). The difference between positions 1 & 5 approached significance ($p = .066$).

The differences between the two groups became more apparent at span lengths 6 & 7. Span 6 showed a main effect of group, $F(1,91) = 33.49$, $p < .001$, $r = .52$ and a main effect of position, $F(3.68,334.54) = 4.33$, $p = .003$, $r = .11$. Position 1 was no different to positions 2, 5 or 6 (all p's > .05) but was more accurate than positions 3 & 4 (p's < .05). There were no significant differences between 2 & 3, 3 & 4, 4 & 5 or 5 & 6 (all p's > .05), but 4 was less accurate than 6 ($p = .038$).

Finally, span 7 again showed a main effect of group, $F(1,91) = 32.66$, $p < .001$, $r = .51$ and a main effect of position, $F(5.08,462.33) = 15.24$, $p < .001$, $r = .18$. Position 1 was no more accurate than positions 2 or 7 (both $p > .999$), but more accurate than positions 3, 4, 5 & 6 (all p's < .001). Position 2 was also more accurate than positions 3 ($p = .006$), 4, 5 & 6 (all p's < .001), but no different to 7 ($p = > .999$).

Positions 4 & 5 were no different to each other (p > .999) or to position 6 (4 & 6 p >.999; 5 & 6 p > .999), but both were less accurate than position 7 (both p's <.05). Position 7 was more accurate than position 6 (p > .001).

In summary then, mathematicians were more accurate overall than non-mathematicians from span 3 upwards, but the patterns of the serial position curves were no different between the two groups at each span length, with no significant interactions. The recency effect first emerged at span length 3, but for span lengths 3, 4, 5 & 6 it was evident through a significant difference between the final position and the position two before last, with the final and penultimate positions not significantly different. It was only at span 7 where the traditional recency effect was seen between the final & penultimate positions. The primacy effect did not appear until span 5, with the traditional difference between positions 1 & 2 apparent. However, for spans 6 & 7, the primacy effect was evident through significant differences between the first and third items, rather than the first and second.

2.3.3 Discussion

Experiment 2 attempted to replicate the Experiment 1 finding that adult mathematicians have superior visuo-spatial working memory storage capacity to adult non-mathematicians. It also investigated potential ceiling effects in the number condition of Experiment 1.

The difference in visuo-spatial working memory storage capacity replicated and visuo-spatial storage scores again correlated strongly with Woodcock-Johnson Calculations scores. They also correlated moderately with Woodcock-Johnson Math Fluency scores, which measured fluency for basic arithmetic. The results of both Experiments 1 and 2 suggest that visuo-spatial working memory storage capacity has a relationship with both basic arithmetic and more complex mathematics. This will be considered further in the general discussion (section 2.4).

In the verbal domain, the finding from Experiment 1 that there was no difference between mathematicians and non-mathematicians for memory of numerical stimuli also replicated. Importantly, the initial comparison of scores across spans 3 to 8 showed no ceiling effects and there was still no difference between the two groups at these greater span levels. The conclusion from Experiment 1 that there is no difference between adult mathematicians and non-mathematicians for working memory storage capacity in the verbal domain was supported. As in Experiment 1, correlations involving the verbal domain showed no relatedness to the visuo-spatial domain or to mathematics scores, again supporting the dissociation of verbal and visuo-spatial storage within working memory.

Serial position curves for both groups were examined to explore the nature of the group difference in the visuo-spatial condition. This analysis used combined scores of Experiments 1 and 2 and showed that differences between the two groups' visuo-spatial working memory performance was not due to different profiles of the curves. The mathematicians simply remembered more items in their correct serial positions for span lengths 3 to 7 rather than showing a different pattern of memory. Chapter 4 will explore the relative contributions of item and order memory to their superior visuo-spatial working memory.

In terms of the processing task, there was again no significant difference between the two groups in either storage domain condition. There was also no significant difference between the two conditions for processing task accuracy or latencies.

Experiment 2, then, confirmed the findings of Experiment 1 that, when using a consistent and as neutral as possible processing task across conditions, adult mathematicians have superior working memory storage capacity in the visuo-spatial domain but that there is no difference in storage capacity in the verbal domain. Analysis of visuo-spatial serial position curves indicated that mathematicians simply have a greater capacity to remember items in their correct order rather than displaying different patterns of remembering the information.

2.4 General Discussion

Chapter 2 investigated whether adult mathematicians have greater working memory storage capacity than adult non-mathematicians and whether any capacity advantage is general, or just specific to the verbal or visuo-spatial domain. Results of both Experiments 1 and 2 suggest that adult mathematicians have superior working memory storage capacity only in the visuo-spatial domain. The findings in this chapter also support the view of separable phonological and visuo-spatial resources in line with the multi-component model of working memory (Baddeley, 2000; Shah & Miyake, 1996).

The finding of a superior visuo-spatial capacity for mathematicians seems unsurprising in view of the roles of the visuo-spatial sketchpad and visuo-spatial working memory for mathematics, highlighted in the previous research discussed in Chapter 1. The correlation between visuo-spatial working memory storage capacity and Math Fluency scores (section 2.2.2.4) supports the findings of Hubber et al. (2014: Experiment 1) and Imbo & LeFevre (2010) that visuo-spatial working memory is important for basic arithmetic in adults. Lee and Kang (2002) also found that use of visuo-spatial working memory is required for performing subtractions. Despite the fact that adults solve many basic arithmetic problems using direct retrieval of answers (Imbo & Vandierendonck, 2008), Hubber et al. (2014: Experiment 1) still found that visuo-spatial working memory was used for direct retrieval.

The finding that visuo-spatial working memory storage capacity is related to more complex mathematical calculation also supports previous findings that visuo-spatial working memory seems important for procedural strategies and for the manipulation of information in mind whilst solving mathematical problems. As well as finding a role for visuo-spatial working memory in retrieval, Hubber et al. (2014: Experiment 1) also found it to be involved in the more complex procedural strategies of counting and decomposition. Previous research has also highlighted the importance of the visuo-spatial nature of initial information of calculations. For example, Landy, Brookes & Smout (2014) found that the visuo-spatial structure of algebra problems was important for their successful interpretation and Jiang, Cooper & Alibali (2014) found that spatial manipulation of the minus sign in subtraction problems affected interpretation and problem solving. Logie et al. (1994) and Seron et al. (1992) highlighted the importance of visuo-spatial working memory for visualising information about calculations and manipulating numbers in mind.

The description of the visuo-spatial sketchpad as a “mental blackboard” (Heathcote, 1994, p.27) describes its use in mathematics for the storing of visual information about a calculation and its subsequent manipulation. Indeed, Hegarty & Waller (2005) state that those with greater ability for visuo-spatial visualisation also have greater ability for problem solving, particularly for interpreting graphical representations. With the importance of visuo-spatial working memory for the holding and manipulation of information during calculation then, a greater capacity for storing visuo-spatial information would seem advantageous for performance in mathematics.

Although verbal working memory has previously been shown to be involved in counting, fact retrieval and the storing of intermediate results (e.g. Fürst & Hitch, 2000; Geary, 2011; Logie et al. 1994), the studies included in this chapter showed no advantage for mathematicians in the verbal domain. Mathematicians only appear to have an advantage in the visuo-spatial domain which is responsible for the visualisation and manipulation of material: skills which intuitively seem more required more for complex mathematics as opposed to more basic arithmetic.

Although these results indicate a visuo-spatial advantage for mathematicians, they tell us nothing about causality: that is, whether having a greater visuo-spatial working memory capacity aids the learning and performing of mathematics, or whether learning and performing mathematics leads to an advantage in developing this capacity. Previous research discussed above highlights how having superior visuo-spatial working memory storage capacity may support superior performance in mathematics, through a greater ability to represent initial calculation information and then hold and manipulate this information in mind. However, it could be possible that studying and performing mathematics for a longer period of time or at a more advanced level helps to increase visuo-spatial capacity. Future research should examine this issue. Interestingly, four out of five participants in the non-mathematics group who were discounted from the analyses in Experiment 1, because they were later found to have studied mathematics at A level, achieved visuo-spatial span scores very similar to those of the mathematics group. Although only a small number of participants, this suggests that the advantage for visuo-spatial capacity in mathematicians may be present earlier than at undergraduate study level. This and the nature of causality effects could be investigated by measuring visuo-spatial working memory span pre- and post- A level in those who study mathematics at A level and those who don’t.

Finally, Experiments 1 and 2 showed mathematicians have superior visuo-spatial storage capacity using a working memory task that employed a novel, neutral processing element not previously used in this context within the literature. Chapter 5 will investigate whether this visuo-spatial advantage still exists when the processing element has a more traditional verbal or visuo-spatial format.

2.5 Conclusion

Chapter 2 used working memory span tasks, including a novel processing element, to investigate whether adult mathematicians have superior working memory capacity to adult non-mathematicians and whether any advantage is a general one or specific to the verbal or visuo-spatial domain. Results showed an advantage in the visuo-spatial domain only, suggesting visuo-spatial storage in working memory has an important relationship with mathematics. Results suggested that this advantage is related to both basic arithmetic and more complex calculations. Results also supported the view of separable visuo-spatial and verbal storage resources. Analysis of serial position curves in the visuo-spatial domain showed there were no significant differences in the patterns of the curves of the two groups and that mathematicians were simply better overall at remembering the items in the correct order. Whether this ability is due to superior item memory, order memory or a combination of both will be investigated in Chapter 4.

Chapter 3: Short-Term Memory and Endogenous Spatial Attention

3.1 Introduction

Chapter 2 found that mathematicians have greater working memory storage capacity than non-mathematicians in the visuo-spatial domain, but not the verbal domain. However, it is possible that this could be due to group differences at a more basic level below that of working memory. This chapter investigates whether mathematicians still have an advantage for storing visuo-spatial information in working memory when ability for storing visuo-spatial information in short-term memory (with no processing) and controlled spatial attention are taken into account. Differences between adult mathematicians and non-mathematicians are examined for performance on a visuo-spatial short-term memory span task and a controlled spatial attention task before comparing group differences for visuo-spatial working memory with these two items as covariates. Analyses are also performed to discover whether visuo-spatial working memory can predict mathematics calculation and arithmetic fluency ability over and above the contributions of short-term memory and attention. This will inform whether it is the ability to hold visuo-spatial information in mind when both storage and processing are required that drives the link between visuo-spatial working memory capacity and mathematics.

There are a number of possible explanations for the superior visuo-spatial working memory performance shown by mathematicians. The working memory span tasks used throughout Chapter 2 included both processing and storage elements. It is therefore possible that the mathematicians' advantage was the result of superior processing of information, superior capacity to temporarily store information in the visuo-spatial sketchpad or the ability to combine processing and storage. It seems unlikely that the mathematicians' advantage was as a result of superior processing. The processing element used in the span tasks was as neutral as possible with regard to the verbal and visuo-spatial storage elements and was the same across all conditions. There was no significant difference between mathematicians and non-mathematicians for accuracy or RT for this processing element. This indicated that mathematicians' superior visuo-spatial working memory capacity was not due to differences in processing ability.

Mathematicians' superior visuo-spatial working memory capacity could therefore be driven by an advantage in the short-term storage of information in the visuo-spatial sketchpad during working memory use. Research has previously found that the storage of information within working memory, during processing, is required when solving mathematical problems (Adams & Hitch, 1997; Trbovich & LeFevre, 2003). There is also evidence that adults use the sketchpad to store information during calculation (Lee & Kang, 2002; Logie et al., 1994; Trbovich & LeFevre, 2003). Visuo-spatial short-term memory performance, with no processing element present, has also been found to be linked to mathematics in adults. Wei et al. (2012) found that performance on a visuo-spatial span task, containing only storage elements, correlated with mathematics ability in an adult college student sample. Although this indicates that the temporary storage of visuo-spatial information plays a role in mathematics, performance on working memory tasks, containing processing as well as storage, is generally regarded as more predictive than performance on short-term memory tasks (Bayliss et al., 2003; St. Clair-Thompson & Sykes, 2010).

No previous research has been identified that compares the relative contributions of short-term memory and working memory capacity with mathematics performance in adults. However, Bayliss et al. (2005) have found that, in children, in both the verbal and visuo-spatial domains, working memory spans were no better at predicting mathematics ability than were short-term memory spans. They also found that independently measured storage, but not processing, contributed to working memory span performance. It may therefore be that mathematicians' superior visuo-spatial working memory capacity is a result of a superior ability to store information within the sketchpad whilst processing is being carried out.

In the current chapter, performance of a group of adult mathematicians and a group of adult non-mathematicians will be compared on a visuo-spatial short-term memory span task to see whether mathematicians have an advantage for temporarily storing information within the sketchpad. Whether the mathematicians still retain their visuo-spatial working memory advantage over non-mathematicians when short-term visuo-spatial storage capacity is taken into account will also be examined to discover whether their working memory advantage is driven by an advantage in sketchpad capacity.

A further possible explanation for mathematicians' superior visuo-spatial working memory capacity could be endogenous spatial attention. Whilst exogenous attention is viewed as stimulus-directed, with attention being directed into an area by the stimulus, endogenous (controlled) attention is viewed as top-down and under an individual's control (Spence & Driver, 1994). Endogenous attention is believed to be important for refreshing items in memory and for ensuring that items remain available for further processing and/or recall (e.g. Barrouillet et al., 2007; Cowan, 2000; Engle, 2002).

Previous experimental research has suggested an overlap between endogenous attention and visuo-spatial working memory. Gazzaley & Nobre (2012) found the top-down ability to attend to and inhibit irrelevant information to be important. They highlighted evidence from fMRI studies that the prefrontal cortex (PFC) plays an important role in this. Limitations in PFC attention may affect the amount of information encoded and PFC and parietal areas may direct attention during the use of visual working memory. Attention and working memory are believed to be interdependent because working memory has a limited capacity and attention therefore regulates which items are encoded for storage (Chun & Turk-Browne, 2007). In their review paper, Awh, Vogel & Oh (2006) describe an overlap between neural systems for spatial attention & visuo-spatial working memory. Using a visuo-spatial working memory task, Awh, Jonides & Reuter-Lorenz (1998) found that when participants' ability to attend was hindered, their ability to memorise locations of stimuli was adversely affected. Their results suggest that spatial attention is the mechanism for maintaining information in visuo-spatial working memory. A review of the links between attentional control and visual memory (Astle & Scerif, 2011) also highlighted the importance of top-down attention for the development and performance of visual short-term memory as well as visual working memory.

Each of the models of working memory discussed in section 1.3 of Chapter 1 includes a role for endogenous attention. The Baddeley & Hitch multi-component model of working memory (Baddeley, 2000) proposes that maintenance of visuo-spatial items in memory occurs as a result of refreshing via endogenous attention. It is also viewed as part of the mechanism for retrieving information from long-term memory (Baddeley, 1996). The embedded-process model (Cowan, 2000) proposes that a central executive controls the focus of attention so that relevant activated items within long-term memory remain available for recall or processing. The controlled-attention model (Engle 2002) states that working memory capacity is not based on how much information can be stored, but the ability to control attention or suppress irrelevant information. Finally, the time-based-resource-sharing model (Barrouillet et al., 2004) also emphasises the importance of attention for working memory span task performance. This model states that attention has limited capacity and must be shared between processing and storage within working memory. As soon as attention is removed from an item, its representation suffers from decay over time. Refreshing a decaying item then relies upon its retrieval from memory through renewed attentional focussing. Therefore, despite the fact that these various models of working memory differ in terms of their structure, all include an important role for endogenous attention.

In summary, several prominent theories of working memory therefore include a role for endogenous attention (e.g. Baddeley, 2000; Barrouillet et al., 2004; Cowan, 2000; Engle, 2002) and there is experimental and fMRI evidence that endogenous spatial attention plays an important role in visuo-spatial working memory (Astle & Scerif, 2011; Awh et al., 1998; Awh et al., 2006; Gazzaley & Nobre 2012). With this in mind, it was decided to compare the endogenous spatial attention ability of a group of adult mathematicians and a group of adult non-mathematicians using performance on a basic Posner (1980) endogenous spatial attention task. Mathematicians were expected to have superior endogenous attention ability compared with non-mathematicians in light of the evidence outlined above demonstrating its role in visuo-spatial working memory and the fact that Chapter 2 found mathematicians to have better visuo-spatial working memory capacity. Also, according to the theoretical models of working memory discussed in Chapter 1, section 1.3, attention, controlled by the central executive, is assumed to be important for retrieving number facts stored in long-term memory. This measure of endogenous spatial attention was also used to examine whether mathematicians retain their visuo-spatial working memory capacity advantage when their spatial attention ability is controlled for. It was expected that the mathematicians' greater working memory capacity would remain when endogenous attention was controlled for. This was due to working memory performance being consistently found to have greater predictive ability than more basic measures (Bayliss et al. 2003).

In endogenous spatial attention tasks, attention is commonly measured in terms of the time taken to respond to the appearance of a target stimulus that is preceded by a central cue. This cue either indicates the position of the target (valid cue) or directs controlled attention in the opposite direction (invalid cue) (Doricchi, Macci, Silvetti & Macaluso, 2010). Participants are instructed to keep their gaze fixed on the central cue and are usually faster to respond to cues that have been validly cued than those invalidly cued. The difference in RTs between responses to targets preceded by valid cues and those preceded by invalid cues is then taken as a measure of endogenous spatial attention. A smaller difference would infer greater attentional control, as invalid cues would have caused less distraction of attention.

In summary then, this chapter investigates whether adult mathematicians still have superior visuo-spatial working memory capacity to non-mathematicians when performance on a visuo-spatial short-term memory task and an endogenous spatial attention task is taken into account. As Wei et al. (2012) had previously found a relationship between visuo-spatial short-term memory and mathematics performance in adults, it was predicted that visuo-spatial short-term memory capacity would correlate with calculation ability in the current study and that mathematicians would have greater visuo-spatial short-term memory capacity than non-mathematicians. It was, however, predicted that short-term capacity would not be related to arithmetic fluency scores because the direct retrieval of number facts from long-term memory does not require the temporary storage of information. Mathematicians were also predicted to have superior endogenous spatial attention due to previous indications in the literature that this is important for visuo-spatial working memory performance (Astle & Scerif, 2011; Awh et al., 1998; Awh et al., 2006; Gazzaley & Nobre, 2012). Endogenous spatial attention ability was expected to show a relationship with both calculation and arithmetic fluency due to several theoretical models of working memory highlighting its role in activating number facts held in long-term memory and the refreshing of information within working memory. Finally, as working memory measures are generally deemed better predictors of mathematics ability than more basic measures, it was expected that mathematicians would still have greater visuo-spatial working memory capacity when short-term memory and endogenous attention ability were controlled for.

3.2 Method

3.2.1 Participants

As mentioned in Chapter 2, section 2.3.1.5, data for the current chapter was collected in the same experimental sessions as data for Experiment 2 of Chapter 2. Details of the participants are therefore identical to those in Experiment 2 of Chapter 2 (section 2.3.1.1), but are repeated here for ease of reference.

54 participants were recruited from undergraduates at the University of Nottingham: 27 (9 male) to a mathematics group and 27 (9 male) to a non-mathematics group. All participants received an inconvenience allowance of £6.

The mathematics group comprised 15 mathematics students and 12 economics students who had studied mathematics at A level. Their ages ranged from 18.66 to 36.89 years (M = 20.88, SD = 3.53). The non-mathematics group comprised English, History, Philosophy and Sociology students who had not studied mathematics at A level. Their ages ranged from 18.78 to 22.68 years (M = 20.33, SD = .99). On average, participants in the non-mathematics group had not studied maths for 4.18 years (SD = 1.16).

3.2.2 Equipment

A Viglen Pentium D computer, running Windows XP and PsychoPy version 1.73.06 (Peirce, 2007), was used to present stimuli and record latencies and accuracy.

3.2.3 Experimental Tasks

3.2.3.1 Short-Term Memory Task

This task consisted of a series of sequentially presented visuo-spatial storage elements. The format of the span task was identical to those of the working memory experiments in Chapter 2, except that it consisted solely of to-be-remembered storage items, with no processing element present.

A black 3 x 3 grid was presented in the centre of the screen (each square was 6cm wide x 6cm high) with a red dot (size 3 cm wide x 3cm high) placed in one of nine possible locations on the grid. Each storage item was presented on screen for 500 milliseconds, commencing 500 milliseconds after the previous item had disappeared. At the end of each span set, once all storage items had been presented, a “ ? ” appeared in the centre of the screen that prompted the participants to recall the storage items, in their order of presentation. A black 3 x 3 grid appeared on screen immediately after the “ ? “ and participants recalled the serial order of the red dot by clicking on the grid, using the USB mouse. Once recall was completed, the participant pressed the space bar to begin the next trial. Span sets, and items within each span set, were presented in a random order. Each of span lengths 3 to 8 was presented three times, giving 18 trials in each of the three conditions. Trials are included in Appendix D. Each of the nine possible items was presented approximately equally across trials.

3.2.3.2 Endogenous Spatial Attention Task

As explained in section 3.1, endogenous spatial attention was measured via a basic Posner task (Posner, 1980). It examined the time taken to respond to the appearance of a target stimulus that was preceded by a central cue. This cue either indicated the position of the target (valid cue) or directed controlled attention in the opposite direction (invalid cue) (Doricchi et al., 2010). Participants are usually faster to respond to cues that have been validly cued than those invalidly cued. The difference in RTs between responses to targets preceded by valid cues and those preceded by invalid cues is then taken as a measure of endogenous spatial attention.

For the endogenous spatial attention task, participants were sat 60cm away from the computer screen. The on-screen display (see Figure 3.1) consisted of a central cueing stimulus (a diamond shape, 1.3º wide) and peripheral squares to the left and right (1º wide), centred at 7º eccentricity, inside which a target ‘x’ appeared. The target ‘x’ was 1º in size. Initial instructions told participants to stare only at the central cue and not to move their eyes, and to respond to the appearance of target stimuli, in the peripheral squares, as quickly and accurately as possible. A response was given by pressing the space bar on the keyboard using their right index finger whenever they saw a peripheral target stimulus. In valid trials, one side of the central cue lit up, indicating that the target would appear in the square on the same side. In invalid trials, the target appeared in the square on the opposite side to the side of the cue that lit up. In neutral trials, both sides of the central cue lit up, giving no indication of whether the target would follow to the left or right. Targets appeared on the right 50% of the time for each cue type (as per Coull & Nobre, 1998; Engbert & Kliegl, 2003; Gitelman et al., 1999; Kim et al., 1999; Nobre et al., 1997). A total of 36 neutral trials, 36 invalid trials and 144 valid trials were used. This gave a total of 216 trials split into 3 identical blocks of 72 trials each. The order of trials was random within each block and across participants. All cues lit up for 100ms and targets followed cue offsets at stimulus-onset asynchronies (SOA) of 200, 400 or 800ms (Gitelman et al., 1999; Kim et al., 1999; Nobre et al., 1997). Targets were also displayed for 100ms. Each of the three SOAs was used in equal proportions within the neutral, valid and invalid trial types. All trials had a duration of two seconds, so there was a variable delay between a target appearing and the cue of the next trial. Participants could therefore not predict exactly when a cue would appear. A list of the 72 trials used in each block is included in Appendix E.

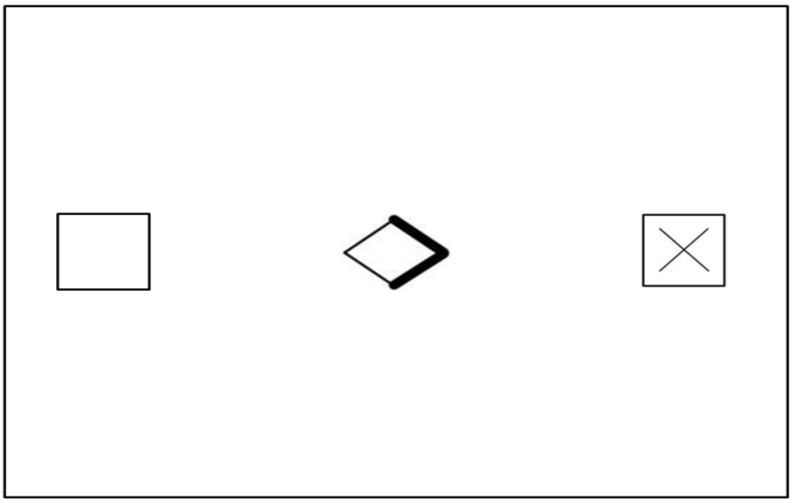

Figure 3.1: Example of screen during a valid trial in the endogenous spatial attention task.

3.2.4 Additional Materials

The *Woodcock-Johnson Calculation Test* & *Math Fluency Test* (Woodcock, McGrew & Mather, 2001) and *WASI Matrix Reasoning* (WASI; Psychological Corporation, 1999), as described in Chapter 2 (sections 2.2.1.4 & 2.3.1.4), were administered using the standard procedures to measure mathematics ability and non-verbal IQ.

3.2.5 Procedure

As mentioned in section 2.3.1.5 of Chapter 2, data for the short-term memory task, the endogenous spatial attention task and the standard mathematics tests were collected during an hour long session that also involved completion of the working memory tasks included in Experiment 2 of Chapter 2. All participants were tested individually by the same experimenter. After completion of the working memory span tasks, participants then completed the short-term memory task, followed by the attention task.

For the short-term memory task, after reading initial instructions, participants completed a practice of one 2-span set and one 3-span set, before the test sets were presented.

For the endogenous spatial attention task, after initial instructions, participants practised the task for 22 randomly presented trials. They then viewed a screen which repeated the initial instructions, before commencing the three blocks of experimental trials. A short break was allowed between blocks, if required. At the end of the task, participants were asked to self-rate for what extent of the time they had kept their gaze fixed on the central cue as instructed, using the numeric keypad, on a scale of 1 to 5, where 1 was ‘hardly any’ and 5 was ‘almost all’.

Next, participants completed WASI Matrix Reasoning. They then completed the Woodcock-Johnson Calculation Test and Woodcock-Johnson Math Fluency Test, the order of which was counterbalanced.

3.3 Results

As described in section 2.3.2 of Chapter 2, three participants (2 mathematics group; 1 non-mathematics group) were later excluded from the analyses for having an unacceptably high (>15%) error rate in the processing element of the working memory task (mathematics: 1 number condition, 1 number & spatial conditions; non-mathematics: 1 number condition) leaving data for 25 (9 male) participants in the mathematics group and 26 (9 male) in the non-mathematics group available for analysis. Although this chapter examines short-term memory and endogenous spatial attention tasks where no processing elements were present, these participants were removed for consistency with Chapter 2 analyses. This was important for the correlations with working memory and regressions involving working memory reported below.

A Cook’s Distance score was calculated in a regression using storage accuracy in the working memory and short-term memory tasks and RTs in the Posner task to predict mathematics scores, to discover whether influential cases could affect any of the analyses reported below. No influential outliers with a Cook’s Distance score >1 (Field, 2009) were detected. Controlling for gender had no significant impact on analyses and results reported below are without controlling for gender. Degrees of freedom were corrected using Greenhouse-Geisser estimates of spherity where necessary.

In the sections below, results for standardised tests will be firstly reported (section 3.3.1), followed by results for the short-term memory task (section 3.3.2.1) and for the endogenous spatial attention task (section 3.3.2.2). Section 3.3.3 will report correlations for visuo-spatial short-term memory and spatial attention with working memory. Section 3.3.4 will report results for an Analysis of Covariance (ANCOVA) where group scores for visuo-spatial working memory were compared, with visuo-spatial short-term memory and spatial attention entered as covariates. Section 3.3.5 will report relationships of visuo-spatial short-term memory and attention with mathematics scores. Finally, regression analyses to examine to what extent short-term memory, working memory and attention predicted mathematics scores are included in section 3.3.6.

3.3.1 Standardised Tests

Results for the standardised tests were reported in section 2.3.2.1 of Chapter 2, but are repeated here for ease of reference.

An independent *t*-test to compare the two groups' Woodcock-Johnson Calculation Test scores confirmed that the mathematics group (M = 24.80, SD = 3.50) were significantly better at mathematics than the non-mathematics group (M = 12.15, SD = 3.46), $t(49) = 12.97$, $p < .001$, $r = .88$. Scores for the mathematics group represented a median percentile rank compared to age norms (Woodcock, McGrew & Mather, 2001) of 92.00 (min = 64.00; max = 99.00). Scores for the non-mathematics group represented a median percentile rank compared to age norms of 34.00 (min = 5.00; max = 67.00).

A non-parametric Mann-Whitney test was performed to compare the two groups' Woodcock-Johnson Math Fluency scores, as the mathematics group's scores showed significant negative skew at the $p < .05$ level (Field, 2009). This showed significantly greater scores for the mathematicians (M = 144.68, SD = 20.72) compared to the non-mathematicians (M = 113.46, SD = 16.87), U = 90.00, Z = -4.43, p < .001, r = .63. Scores for the mathematics group represented a median percentile rank compared to age norms of 89.00 (min = 21.00; max = 99.90). Scores for the non-mathematics group represented a median percentile rank compared to age norms of 43.50 (min = 3.00; max = 83.00).

An independent *t*-test showed that the mathematics group (M = 28.92, SD = 3.46) had significantly greater non-verbal IQ than the non-mathematics group (M = 26.88, SD = 3.25) when comparing their scores for WASI Matrix Reasoning. All analyses were therefore initially run controlling for WASI Matrix Reasoning scores, but this made no difference to main effects or interactions, so results reported below do not control for non-verbal IQ.

3.3.2 Experimental Tasks

3.3.2.1 Short-Term Memory Task

Proportion correct scores were first calculated for each participant for the number of storage items recalled in their correct serial position (see section 2.2.1.6 of Chapter 2).

A non-parametric Mann-Whitney test was performed to compare the two groups' visuo-spatial short-term memory scores, because both group's scores showed significant negative skew at the p < .05 level (Field, 2009). This showed no significant difference in performance between the mathematicians (M = .88, SD = .06) and the non-mathematicians (M = .85, SD = .08), U = 234.00, Z = -1.72, p = .086, r = .25.[5]

3.3.2.2 Endogenous Spatial Attention Task

Median RTs were calculated for each participant for each category of neutral, valid and invalid trials, before calculating their Posner difference (invalid RTs minus valid RTs). A total of two

[5] Parametric independent *t*-tests were also run using both the All-or-Nothing Unit and All-or-Nothing Load methods, which did not result in any significant changes to results or conclusions.

participants in the mathematics group and three participants in the non-mathematics group failed to show a Posner difference (their valid times were longer than their invalid times), but inclusion of their reaction times made no difference to results or conclusions and they are therefore included in the analyses that follow. Mean reaction times and standard errors for all participants are shown, by group, in Table 3.1. Participants reported that they had kept their gaze fixed centrally, as required, on the majority of trials (mathematics group: M = 4.63, SD = 0 .74; non-mathematics group: M = 4.70, SD = 0.67).

Table 3.1

Descriptive statistics (mean (M) and standard error (SE)) for reaction times, in milliseconds, in the endogenous spatial attention task

	Cue Validity Type			
Group	Neutral	Valid	Invalid	Invalid minus Valid
Mathematics	348 (38)	326 (40)	351 (39)	25 (24)
Non-Mathematics	337 (41)	316 (38)	344 (43)	28 (23)

A non-parametric Mann-Whitney test showed no significant difference between the mathematicians and non-mathematicians for reaction times to respond to valid trials, U = 25.50, z = -.74, p = .463. A non-parametric test was used here as the mathematics group's reaction times showed significant positive skew at the p < .05 level (Field, 2009).

An independent t-test was then used to compare reaction times between the two groups for the Posner difference (invalid minus valid) and, again, no significant difference was found, t(49) = .91, p = .367, r = .13.

Posner difference scores (endogenous spatial attention) did not correlate with visuo-spatial short-term memory storage scores (Table 3.2).

3.3.3 Relationship of Storage & Attention with Visuo-spatial Working Memory Scores

Correlations with visuo-spatial working memory scores are reported in Table 3.2. (Working memory scores were reported in section 2.3.2.2 of Chapter 2.)

Visuo-spatial short-term memory scores correlated moderately with visuo-spatial working memory scores but Posner difference scores (endogenous spatial attention) did not.

Table 3.2

Correlations among visuo-spatial short-term memory, spatial attention, visuo-spatial working memory, mathematics calculation and mathematics fluency

	Visuo-spatial STM	Spatial attention	Visuo-spatial WM	Mathematics Calculation	Mathematics Fluency
Visuo-spatial STM					
Spatial attention	-.17				
Visuo-spatial WM	.38**	-.21			
Mathematics Calculation	.34*	-.20	.57***		
Mathematics Fluency	.09	-.36*	.30*	.69***	

Note. STM = short-term memory; WM = working memory. *p < .05; **p < .01; ***p < .001

3.3.4 Group Differences in Visuo-Spatial Working Memory when controlling for Short-Term Memory and Attention Ability

An ANCOVA was run to investigate whether mathematicians would still have greater visuo-spatial working memory storage capacity when short-term memory performance and endogenous spatial attention were taken into account. Working memory proportion correct score was entered as the dependent variable, with short-term memory and endogenous spatial attention (Posner difference) scores entered as covariates. Group (mathematicians; non-mathematicians) was entered as a fixed factor. The result showed that the covariate visuo-spatial short-term memory was significantly related to visuo-spatial working memory $F(1,47) = 13.58$, $p = .001$. The covariate endogenous spatial attention was not significantly related to visuo-spatial working memory, $F(1,47) = 1.61$, $p = .210$. When controlling for visuo-spatial short-term memory and endogenous spatial attention, the mathematicians still had significantly greater visuo-spatial working memory scores than the non-mathematicians, $F(1, 47) = 15.54$, $p < .001$, $\eta^2_p = .25$.

3.3.5 Relationship of Storage & Attention with Mathematics Scores

Correlations with mathematics scores are reported in Table 3.2. Scores for visuo-spatial working memory correlated strongly with Woodcock-Johnson Calculation scores and moderately with Woodcock-Johnson Math Fluency scores. Participants' visuo-spatial short-term memory scores correlated moderately with calculation, but did not correlate with fluency scores.

Posner difference scores (endogenous spatial attention) did not correlate with calculation scores, but did correlate moderately with fluency scores.

3.3.6 Regression Analyses to predict Mathematics Scores

Results for the visuo-spatial short-term memory and endogenous spatial attention tasks were entered into regression models with the scores for visuo-spatial working memory. This would inform whether visuo-spatial working memory storage capacity would still predict mathematics scores over and above the contributions of short-term memory and spatial attention. In both of the models, visuo-spatial short-term memory storage accuracy and Posner difference RTs (endogenous spatial attention measure) were added at step 1 and visuo-spatial working memory storage accuracy was added at step 2. Table 3.3 shows results with calculation as the dependent variable and Table 3.4 shows results with fluency as the dependent variable.

For calculation (Table 3.3), visuo-spatial short-term memory, but not endogenous spatial attention, predicted calculation score at step 1. However, once visuo-spatial working memory was added to the model at step 2, only this significantly and uniquely predicted calculation score. For fluency (Table 3.4), endogenous spatial attention, but not visuo-spatial short-term memory, predicted fluency at step 1. The addition of visuo-spatial working memory at step 2 did not significantly improve the model.

Table 3.3

Regression analysis: visuo-spatial short-term memory, endogenous spatial attention and visuo-spatial working memory predicting Woodcock-Johnson Calculation score

DV: calculation score	**B**	**SEB**	**β**
Step 1			
Constant	-7.81	12.00	
Visuo-spatial short-term memory	32.39	13.85	.32*
Endogenous spatial attention	-74.00	42.08	-.24
Step 2			
Constant	-11.55	10.65	
Visuo-spatial short-term memory	4.63	14.24	.05
Endogenous spatial attention	-43.41	38.02	-.14
Visuo-spatial working memory	32.27	8.57	.53**

Note. R^2 = .14 for Step 1 (p = .028), Δ R^2 = .20 for Step 2 (p < .001). *p < .05, **p < .001.

Table 3.4

Regression analysis: visuo-spatial short-term memory, endogenous spatial attention and visuo-spatial working memory predicting Woodcock-Johnson Math Fluency score

DV: fluency score	**B**	**SEB**	**β**
Step 1			
Constant	98.21	40.91	
Visuo-spatial short-term memory	45.91	47.22	.13
Endogenous spatial attention	-348.33	143.43	-.33*
Step 2			
Constant	91.91	40.31	
Visuo-spatial short-term memory	-.84	53.91	-.00
Endogenous spatial attention	-296.82	143.95	-.28*
Visuo-spatial working memory	54.34	32.05	.27

Note. R^2 = .12 for Step 1 (p = .050), Δ R^2 = .05 for Step 2 (p = .097), *p < .05.

3.4 Discussion

This chapter investigated whether adult mathematicians retained their superior visuo-spatial working memory capacity over non-mathematicians when the more basic abilities of visuo-spatial short-term memory storage and controlled spatial attention were taken into account. It also examined whether the link between visuo-spatial working memory capacity and mathematics still remained when short-term storage capacity and controlled attention were controlled for.

As predicted, results of the ANCOVA (section 3.3.4) showed that, when controlling for visuo-spatial short-term memory scores and endogenous spatial attention performance on the Posner task, mathematicians still had significantly greater ability to store visuo-spatial information in working memory. This therefore suggests that visuo-spatial short-term memory storage and endogenous spatial attention are not important factors in the differences between the visuo-spatial working memory capacity of mathematicians and non-mathematicians.

It also suggests that it is the ability to hold visuo-spatial information in mind whilst carrying out processing, rather than more simple storage or controlled attention, that underlies the relationship with mathematics, This pattern of results supports the general finding in the literature that working memory ability is more predictive than short-term memory ability of more complex cognitive processes (Bayliss et al., 2003; St. Clair-Thompson & Sykes, 2010). The relationship between visuo-spatial working memory capacity and calculation will be explored further in Chapter 5.

Contrary to initial predictions, comparison of the performance of the mathematicians and non-mathematicians showed no significant difference between the two groups for either endogenous spatial attention or for visuo-spatial short-term memory storage capacity. The results for the short-term memory task in this chapter appear to differ from those of the working memory task in Chapter 2.[6] For working memory, when the task involved both processing and storage elements, mathematicians showed superior storage of visuo-spatial information. However, they were not significantly better than the non-mathematicians for the storage of visuo-spatial information in the short-term memory task which contained no processing element, although this difference between the two groups did approach significance. Mathematicians seem to only have superior capacity for storing visuo-spatial information when working memory is used and therefore storage is required at the same time as processing is undertaken. Results for dual tasks requiring the retention of passive and active sets of information in memory (Oberauer 2002), found that it took around two seconds for information to be ordered sufficiently. The difference in findings between visuo-spatial short-term memory and working memory could therefore be possibly due to the inclusion of the processing element in the working memory task allowing greater time for the ordering of information before storage items were recalled. It may be

[6] It should be noted that this difference has not been tested for significance.

that mathematicians are better able to order visuo-spatial information. This will be investigated in Chapter 4.

Results for the Posner task showed no difference between mathematicians and non-mathematicians for endogenous spatial attention. Also, Posner difference scores, which reflected the slowing of RTs when targets were preceded by invalid cues as opposed to valid cues, showed no correlation with Woodcock-Johnson Calculation scores. This does not necessarily mean that endogenous spatial attention is not part of the refreshing mechanism for stored visuo-spatial items as proposed in previous literature (Astle & Scerif, 2011; Awh et al., 1998; Awh et al., 2006; Gazzaley & Nobre, 2012). It suggests rather that this type of attention does not contribute to differences in visuo-spatial working memory capacity and calculation ability between mathematicians and non-mathematicians. It may also be that the Posner task employed did not adequately measure the type of attention discussed in section 3.1. The four working memory models discussed in section 3.1 include the importance of endogenous attention for the refreshing of visuo-spatial items within working memory and also for focussing on relevant items whilst inhibiting competing information. The Posner task measured participants' ability to maintain controlled attention whilst inhibiting external, distracting visual information given by the invalid cues. This does not necessarily mimic the attentional and inhibition processes that occur for representations held internally within working memory. However, as will be discussed below, performance on the Posner task did predict ability for arithmetic fluency which, in adults, largely requires the direct accessing of facts from long-term memory.

As predicted, visuo-spatial short-term memory capacity correlated with calculations scores, but not with scores for arithmetic fluency (Table 3.2). The finding that adults' visuo-spatial short-term memory capacity correlated with calculation ability supports the previous findings of Wei et al. (2012) and also evidence that the visuo-spatial sketchpad is involved in holding information during calculation (Lee & Kang, 2002; Logie et al., 1994; Trbovich & LeFevre, 2003). Visuo-spatial short-term memory and working memory scores also correlated moderately. Although visuo-spatial short-term memory scores correlated with both calculation and visuo-spatial working memory scores, when entered into a regression model (Table 3.3) short-term memory did not significantly and uniquely predict calculations scores, but working memory did. Results therefore indicated that visuo-spatial working memory has a stronger relationship with calculation than does visuo-spatial short-term memory. As discussed in section 3.1, mathematics involves both the storage and processing of information (Adams & Hitch, 1997; Trbovich & LeFevre, 2003). Results of the regression suggest that it is the ability to store visuo-spatial information whilst processing is also taking place that drives the important relationship with calculation rather than simply the ability to temporarily store information in the visuo-spatial sketchpad.

As expected, endogenous spatial attention correlated with arithmetic fluency, but, contrary to initial predictions, there was no relationship between attention and calculation. There was also no relationship between attention and either short-term memory or working memory (Table 3.2). It therefore appears that differences in spatial endogenous attention are not important regarding the ability to manipulate or store information during calculation, but may be for retrieving answers from long-term memory.

Endogenous spatial attention was also the only element that significantly and uniquely predicted fluency scores when included in a regression model with visuo-spatial short-term memory and visuo-spatial working memory (Table 3.4). Fluency for arithmetic should largely depend on the direct retrieval of answers from memory in adults and the finding that endogenous spatial attention may be related to retrieving number facts from memory supports Campbell & Clark's Encoding-Complex Hypothesis model of numerical cognition (Campbell & Epp, 2005). This model states that number processing activates information in a variety of codes, such as verbal and visuo-spatial. Different mathematics notations may affect strategies, processes and codes utilised and fact retrieval may also involve different codes. Attention is required for both the initial encoding of numerical problems and the retrieval of answers from long-term memory (Campbell, 1994). Inhibition of alternative responses to sums within the selective attention of associative memory networks is also important for retrieving answers (Clark & Campbell, 1991). Geary & Hoard (1995) stated that individuals with mathematical deficits often have difficulty in retrieving numerical facts from long-term memory and that poor attentional control and inhibition often contribute to this problem. The relationship between inhibition and controlled attention should be investigated further to examine the role it plays in numeric fact retrieval. Results for the current study also support theories of working memory that suggest controlled attention is important for retrieval of information from long-term memory (Baddeley, 2000; Barrouillet et al., 2004; Cowan 2000, Engle, 2002).

3.5 Conclusion

Chapter 3 examined whether adult mathematicians still had superior visuo-spatial working memory capacity compared with adult non-mathematicians when more basic short-term memory storage and controlled spatial attention were taken into account. When visuo-spatial working memory scores for the two groups (from Chapter 2) were compared whilst controlling for performance on a visuo-spatial short-term memory span task and a basic Posner task, the mathematicians' working memory scores were still significantly greater than those of the non-mathematicians. Simple short-term memory storage and controlled spatial attention do not therefore seem to account for differences between the working memory capacity of adult mathematicians and non-mathematicians. Results from regression analyses showed that it is the ability to hold visuo-spatial information in mind whilst both storage and processing are required that is related to the ability to perform mathematics calculation. On the other hand, fluently retrieving arithmetic facts from long-term memory seems to rely on controlled spatial attention.

Chapter 4 will investigate whether superior memory for the ordering of visuo-spatial information contributes to mathematicians' greater visuo-spatial working memory capacity. Both Chapter 2 and the current chapter have found a link between visuo-spatial working memory and mathematics calculation, therefore this will be explored further in Chapters 5 and 6.

Chapter 4: Item Memory and Order Memory in the Visuo-Spatial Domain

4.1 Introduction

This chapter investigates whether visuo-spatial item memory or order memory or both are important for mathematics. Additional analysis of data from Chapter 2 will be discussed, together with a further experiment that was conducted. In this experiment, undergraduate students, across a range of different subjects completed two computerised tasks for visuo-spatial item memory and order memory to see whether results correlated with performance on two standard mathematics tests.

Item memory is defined by Nairne & Kelley (2004) as the ability to recognise or recall whether a specific item was present in an experimental trial. They define order memory as the ability to recognise or recall the item's position in the trial sequence. The two experiments within Chapter 2 found that mathematicians have superior working memory storage capacity in the visuo-spatial domain, but Chapter 3 discovered that this advantage for storing visuo-spatial information is not explained by short-term memory storage performance or the ability to control spatial attention. The scoring method used for the span tasks included in Chapter 2 was based on participants remembering items in their correct order, as is usual with traditional span tasks. Serial position curves, for recall accuracy in the visuo-spatial condition, showed there was no difference in the patterns of recall between the groups of adult mathematicians and non-mathematicians. However, mathematicians were better overall than non-mathematicians at the combination of remembering whether a visuo-spatial item was present in a list (item memory) and the position of that item within the list (order memory).

There is evidence that, at least in the verbal domain, memory for item and order are the result of separate cognitive processes (Majerus, Poncelet, Greffe & Van der Linden, 2006). It may be that memory for item and order are dissociated within the visuo-spatial domain and hold different importance for mathematics performance.

As with the use of working memory, there is little previous research into the use of item memory and order memory when adults perform mathematics. However, as discussed in Chapter 1, holding items in memory (item memory) is important for mathematics, with expert calculators stressing the need to hold interim calculations in mind (Butterworth, 2006) and that both forgetting these interim calculations, as well as forgetting initial information about the original calculation to be performed, contribute to errors (Hitch, 1978). Although holding interim calculations in mind is thought to involve the phonological loop (Fürst & Hitch, 2000), with the visuo-spatial sketchpad thought to be important for visualising and manipulating mathematical information (Logie et al., 1994) it seems intuitive that those more proficient at maths will have an advantage for item memory and order memory in the visuo-spatial domain. Items need to be held in memory so they can then be manipulated. Order memory seems to be important too.

In terms of order memory, Hitch (1978) found that when a problem needs to be broken down into stages, there are large individual differences in the order in which these stages are executed and that forgetting increases with the number of calculation processes involved. Pesenti (2005) reported that the knowledge of algorithms (the steps necessary to find a solution to a problem) is a major advantage for calculating prodigies over non-experts. They have greater knowledge of starting points and order of steps required for completion of a problem. They do not necessarily use different algorithms to non-experts, but find them more readily accessible. They also seem to have a superior ability for applying algorithms for one type of problem to other types of problem. Dowker et al. (1996) found that mathematicians used a larger number of appropriate strategies for solving estimation problems than did non-mathematicians and that they carried out these strategies more accurately. Montello (2005) also discussed the visuo-spatial ability of navigation as being the ability to move in a co-ordinated way, whilst keeping the initial goal in mind, not only through the environment but also with regard to non-physical problems such as '"navigating" through a math problem' (2005, p.262). Therefore order memory seems to be implicated when adults solve mathematical problems.

Both item memory and order memory in the visuo-spatial domain therefore seem potentially important for mathematics performance. With this in mind, further analyses were carried out on the results for the storage element of the visuo-spatial working memory conditions of both Experiments in Chapter 2. An additional experiment, comprising two tasks, was also conducted to further investigate the importance of visuo-spatial item memory and order memory for mathematics. Rather than comparing performance of a group of mathematicians and a group of non-mathematicians as in the previous two chapters, these tasks used a correlational design with undergraduates across a wide range of subjects. Having discovered, in Chapter 2, that mathematicians have superior visuo-spatial working memory storage capacity, use of the correlational design allowed examination of the importance of visuo-spatial item memory and order memory across a wider range of mathematical ability.

The first task that participants undertook was a process dissociation task, previously used by Nairne & Kelley (2004) and Smith & Jarrold (2013) to investigate phonological item and order within the verbal domain. It is described in detail in section 4.2.3.1. The task is based on Estes' (Lee & Estes, 1981) perturbation model. This states that when items are encoded to their positions in a list, an order error will occur when an item drifts along a list to a different position. An item error will occur when an item drifts to a different list, leading to its omission at recall or the inclusion of an item from a different list at recall. The task (Nairne & Kelley, 2004) has two blocked conditions: an inclusion condition, where participants have to recall all items presented in their correct serial position; an exclusion condition, where participants recall items present in any order (free recall) except for one item which they are told to exclude. The inclusion condition measures the number of items recalled in their correct serial position. The exclusion condition measures how many items across the block are erroneously recalled when instructed not to be recalled. This indicates an item was remembered as being present, but its order position was remembered incorrectly. Scores for the inclusion and exclusion conditions are then used to calculate item and order memory (scoring is explained in section 4.2.6.1).

The second task was a forced-choice recognition task (based on Cabeza, Anderson, Houle, Mangels & Nyberg, 2000; Kesner, Hopkins & Fineman, 1994) and is described in detail in section 4.2.3.2. Participants saw trials of six items followed by a pair of test items. For item memory they had to indicate which of the test items was present in the original set and for order memory they had to indicate which of the two items had been presented earliest in the original set. As this was a recognition task, it did not require participants to recall items in serial order.

These two tasks sought to discover whether item memory or order memory in the visuo-spatial domain, or both, are important for mathematics in adults, through investigating their relationships with scores on two standard mathematics tests. This would also inform the relative importance of item memory and order memory in the superior visuo-spatial working memory storage capacity of adult mathematicians discovered in Chapter 2. Inclusion of both tasks also allowed for a comparison of the suitably of the two methods for assessing item memory and order memory.

4.1.1 Further Analysis of Previous Data

Initially, for each of the two experiments reported in Chapter 2, an Analysis of Covariance (ANCOVA) was carried out to discover whether the mathematicians' superior visuo-spatial working memory storage capacity was due to this group simply being able to remember more visuo-spatial items, regardless of their order (item memory) or whether it was due to a greater ability to place recalled items in the correct order (order memory).

Firstly, for Experiment 1 of Chapter 2, participants' scores for the visuo-spatial storage condition were calculated for the correct recall of items in any order (item memory). Mean item memory score for the maths group was .95 (SD = .03) and mean for the non-maths group was .93 (SD = .04). A non-parametric Mann-Whitney test was performed to compare the two groups' item memory scores, because the non-mathematics group's scores showed significant negative skew and positive kurtosis at the $p < .05$ level (Field, 2009). This showed a significantly greater performance for the mathematicians, $U = 138.50$, $Z = -1.97$, $p = .049$, $r = .30$. An ANCOVA was then run with the original proportion correct score as the dependent variable, but with item memory controlled for. The result showed that the covariate item memory was significantly related to the original proportion correct score $F(1,39) = 27.28$, $p < .001$. When controlling for item memory, the mathematicians' ability to place items in the correct order (original proportion correct score) was still significantly better than that of the non-mathematicians, $F(1, 39) = 12.94$, $p = .001$, *partial* $\eta^2 = .24$. In other words, item memory was important for the mathematicians' superior performance, but recalling the items in the correct order was even more important.

The same analysis was then carried out for Experiment 2 of Chapter 2. Participants' scores for the visuo-spatial storage condition were calculated for correct recall of items but in any order (item memory). Mean item memory scores for the maths group were .96 (SD = .02) and mean for the non-maths group was .89 (SD = .08). A non-parametric Mann-Whitney test was performed to compare the two groups' item memory scores, because the non-mathematics group's scores showed significant negative skew and positive kurtosis at the $p < .05$ level (Field, 2009). This showed a significantly greater performance for the mathematicians, $U = 118.00$, $Z = -3.90$, $p < .001$, $r = .55$. An ANCOVA was then run with the original proportion correct score as the dependent variable, but with item memory controlled for. The result showed that the covariate item memory was significantly related to the original proportion correct score $F(1,48) = 84.30$, $p < .001$. When controlling for item memory, the mathematicians' greater ability to place items in the correct order (original proportion correct score) was approaching significance, $F(1, 48) = 2.84$, $p = .098$, *partial* $\eta^2 = .06$.

The two ANCOVAs reported above therefore indicated that whilst the amount of items of visuo-spatial information held in working memory is related to mathematics performance, the ability to sequence these items may have an even greater relationship with mathematics.

However, the data included in these analyses are from the results of a working memory span task and span tasks are not the best way of investigating item memory and order memory. Both ANCOVAS showed a significant relationship between item memory and proportion correct scores, suggesting ability to remember item and order were not independent of each other in the span task. Span tasks of this nature rely on participants recalling items in their correct serial order, so item memory and order memory are not really separable, which Nairne & Kelley describe as the 'process purity problem' (2004, p.114). Also, mean scores for item memory above show that participants' scores were approaching ceiling. The span tasks used involved recalling nine possible locations on a 3 x 3 grid, so it was possible for participants to perform well on item memory at the greater span lengths simply through guessing, if required. It was therefore decided to run an experiment to investigate the importance of visuo-spatial item memory and order memory using the process dissociation task and the forced-choice recognition task. These tasks attempted to separate visuo-spatial item memory and order memory and reduce the effectiveness of guessing as a strategy. Both tasks also used a correlational design, rather than the between-groups design used in Chapters 2 and 3, to allow examination of the importance of visuo-spatial item memory and order memory across a wider range of mathematical ability.

As previous research has highlighted the importance of sequencing information for adult expert mathematicians, it was predicted that item memory would correlate significantly with mathematics scores but that order memory would provide a stronger correlation.

4.2 Method

4.2.1 Participants

As the two experiments in Chapter 2 showed sample sizes of 43 and 51 participants respectively were sufficient to detect correlations with mathematics scores, 51 participants (11 male) were recruited from undergraduates at the University of Nottingham. As discussed above, participants were recruited across a range of mathematical abilities to investigate correlations between mathematics scores and visuo-spatial item memory and order memory. Psychology undergraduates received a participation credit as part of their course and undergraduates from all other disciplines received an inconvenience allowance of £6 for participation.

Participants were recruited from a variety of disciplines (30 Psychology; 8 Geography; 4 Economics; 2 Pharmacy; 1 Physics; 1 English; 1 Agriculture; 1 Law; 1 Astronomy; 1 Business Studies & French; 1 Chinese & German). Their ages ranged from 18.47 to 33.08 years ($M = 20.07$; $SD = 2.52$). On average, participants had not studied maths for 2.32 years ($SD = 2.02$).

4.2.2 Equipment

An Acer Aspire 5736Z laptop computer, running Windows 7 and PsychoPy version 1.77.01 (Peirce, 2007), was used to present stimuli and record accuracy. Participants used a USB mouse with their right hand to respond.

4.2.3 Item and Order Tasks

4.2.3.1 Process Dissociation Task

There were two conditions: an inclusion condition and an exclusion condition.

In the inclusion condition, participants had to recall the positions that a red frog (size 4cm wide by 4cm high) jumped around a blue 4 x 4 grid, positioned in the centre of the screen (each square was 6cm wide x 6cm high), and each trial consisted of 5 jumps. An example of a trial screen is shown in Figure 4.1.

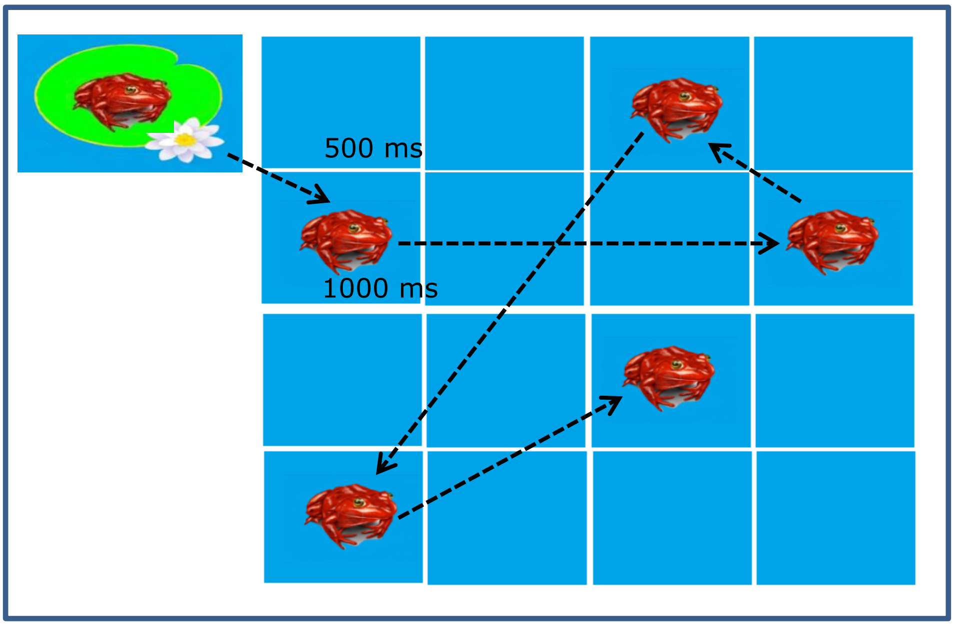

Figure 4.1: Example of a trial screen showing 5 jumps around the 4 x 4 grid, in the inclusion condition of the process dissociation task.

At the start of each trial, the frog was seen sat on a lily pad to the top left of the grid (size 6cm wide by 6cm high). The word ‘READY’ (colour red, size of 3cm, arial font) flashed on the screen twice, for 500ms with a 500ms gap in between, to indicate it was about to jump. The frog then appeared in five locations in succession on the grid. It appeared in each location for one second, with a 500ms gap between jumps. After the fifth jump, the grid disappeared and, after 500 ms, a ‘?’

appeared on screen for one second. Finally, a blank, blue 4 x 4 grid appeared and the participants were required to use the USB mouse to click on the positions that the frog had jumped to, in the same order that it had jumped. After a square had been clicked on by the participant during recall, it reduced in size to 5cm wide by 5cm high, to indicate that that square had been selected. This was done based on prior use of this task with children.

Trials in the Exclusion condition were identical to those in the Inclusion condition, except that, instead of seeing the '?' between presentation and recall, they saw a full-screen picture, for three seconds, indicating which jump to omit during recall (see example in Figure 4.2). Participants then clicked where the frog had jumped, in any order, but omitting the jump shown in the picture. Again, after a square had been clicked on by the participant during recall, it reduced in size to 5cm wide by 5cm high, to indicate that that square had been selected.

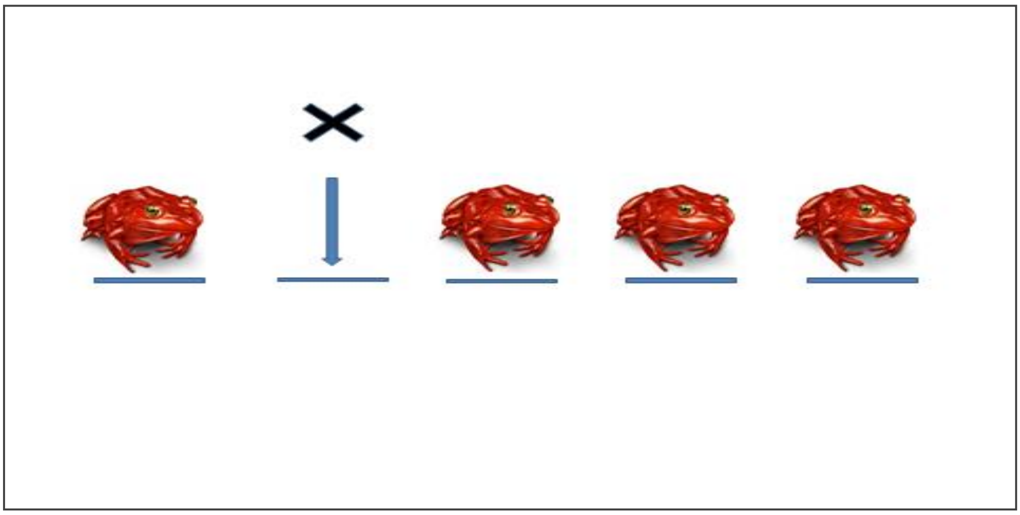

Figure 4.2: Example screen showing the second jump should be excluded from recall during a trial in the exclusion condition of the process dissociation task.

There were 15 trials in both the inclusion and exclusion conditions of the process dissociation task (included at Appendix F), with each location presented an approximately equal number of times.

4.2.3.2 Forced-Choice Recognition Task

The forced-choice recognition task included two conditions: an item memory condition and an order memory condition. In the item memory condition, participants saw a black 4 x 4 grid in the centre of the screen (each square was 6cm wide x 6cm high) and a red dot (size 3cm wide by 3cm high) was presented in different positions on the grid six times, sequentially. Each dot presentation was for 500ms with a 500ms gap between each. After the sixth presentation, there was a one second delay, before a blank 4 x 4 black grid was presented on screen with two red dots presented simultaneously on it. One red dot was present in the original 6 trial locations and one was absent from the original six. Using the USB mouse, participants had to click on the dot that was present in the trial set. The test dots remained on screen until one was selected and, after the participant had clicked on one of them, there was a one second delay before presentation of the next trial. The distance between each present and absent location was controlled to be consistent across trials, because spatial distance effects have been shown to be important in performance for recall of visuo-spatial items, with locations closer together being harder to discriminate (e.g. Awh et al., 1998). Three of the trials had a distance that was smaller than the others by one square, which was necessary to ensure each pair of present and absent items was not duplicated.

Presentation of trials in the order memory condition was identical to that in the item memory condition, except that, at test, the two red dots presented on the grid were both present in the original six trial locations and had been temporally adjacent to each other. Participants had to click on the red dot location they had seen presented earliest in the trial set.

In the forced-choice recognition task, participants completed 24 trials in the item memory condition and 20 trials in the order memory condition (included at Appendix G). For item memory, each location 1 to 16 was presented nine times within the 24 trial sets and no location was duplicated within an individual trial. At test, each serial position 1 to 6 was included as the present item four times. Locations were used as absent items approximately equal numbers of times. For order memory, each location 1 to 16 was presented an approximately equal number of times within the 20 trial sets and no location was duplicated within an individual trial. At test, each serial position pairing of 1 & 2, 2 & 3, 3 & 4, 4 & 5 and 5 & 6 was presented four times. Across each serial position pairing, test items were also spatially adjacent or not adjacent to each other an approximately equal number of times.

4.2.4 Additional Materials

The *Woodcock-Johnson Calculation Test* & *Math Fluency Test* (Woodcock, McGrew & Mather, 2001), as described in Chapter 2 (sections 2.2.1.4 & 2.3.1.4), were administered using the standard procedures to measure mathematics ability.

4.2.5 Procedure

All participants were tested individually by the same experimenter and each session lasted around forty-five minutes. All participants first completed the two computerised tasks, the order of which was counterbalanced across participants.

For the process dissociation task, participants initially read instructions for their first condition and then completed two practice trials. They then completed the experimental trials for that condition before completion of their second condition, which also included two practice trials. The order that participants completed the two conditions was counterbalanced. The order of trials and presentation of items within each trial was randomised.

For the forced-choice recognition task participants initially read instructions for their first condition and then completed two practice trials. They then completed the experimental trials before completion of their second condition, which also included two practice trials. The order that participants completed the two conditions was counterbalanced. In both conditions, the order of trials was randomised, but the items within each trial were presented in the same sequential order for each participant, to enable control over the equal presentation of adjacent serial positions, locations used and spatial proximity in test pairs of both conditions (Smyth, 1996).

Following completion of the two computerised tasks, participants then completed the Woodcock-Johnson Calculation Test and Math Fluency Test, the order of which was counterbalanced across participants.

4.2.6 Item and Order Scoring Methods

4.2.6.1 Process Dissociation Task

This paradigm used measures of item and order memory previously utilised by Nairne & Kelley (2004) and Smith & Jarrold (2013). They are considered to be purer measures of item and order memory than those involved in the more usual working memory span tasks, as they rely on dissociation between item and order. A fuller background to the scoring method used is provided in Nairne & Kelley (2004), but a summary is included below.

With traditional memory measures such as span tasks and free recall, performance is related to set sizes and materials used. It is also related to the successful combination of remembering whether each item was present (item memory) together with its position in the list (order memory) (Jarrold et al., 2011; Nairne & Kelley, 2004). Estes' (Lee & Estes, 1981) perturbation model states that items are encoded to their positions in a list. An order error will occur when an item drifts along a list to a different position and an item error will occur when an item drifts to a different list, leading to its omission at recall or the inclusion of an item from a different list at recall. The scoring method begins by calculating scores for each participant in the inclusion and exclusion conditions. The calculations apply to the retention of a particular item at position "x" within a list.

In the inclusion condition, which required recall of jumps in the order they were presented, the number of trials was calculated in which the item in position “x” was correctly recalled. Thus a greater inclusion score indicates better performance.

The exclusion condition required recall of presented items in any order (free recall), but omitting position “x”. An error occurs if the item in position “x” drifts in memory to a different position and is then recalled (Nairne & Kelley, 2004). This means the participant has remembered the item was present, but order memory has failed. For each participant, the number of trials in which the to-be-excluded item was erroneously recalled was calculated. Thus a greater exclusion score indicates poorer performance.

Scores for item and order memory were then calculated for each participant using these inclusion and exclusion scores. Nairne & Kelley (2004) explain that for the inclusion condition, remembering an item in its correct position is the probability of remembering the item was present ($Item\ memory$) multiplied by the probability of remembering its position in the list ($Order\ memory$). For the exclusion condition, participants will erroneously recall the to-be-excluded item if they remember the item was present ($Item\ memory$) but don’t remember its position in the list correctly $(1 - Order\ memory)$. This gives the following formulae:

$$Inclusion\ score = Item\ memory\ x\ Order\ memory$$

$$Exclusion\ score = Item\ memory(1 - Order\ memory)$$

These formulae can be converted, using algebra, to calculate item memory and order memory scores:

$$Item\ memory = \quad Inclusion\ score + Exclusion\ score$$

$$Order\ memory = \frac{Inclusion\ score}{Item\ memory}$$

These two formulae were therefore applied to the scores initially calculated for the inclusion and exclusion conditions, to arrive at a score for item memory and order memory for each participant. Greater scores for both item and order memory reflected better performance.

3.2.6.2 Forced-Choice Recognition Task

In both the item memory and order memory conditions, proportion correct scores were calculated for each participant for the total number of items correctly selected from the recall test pairs.

4.3 Results

A Cook's Distance score was calculated for each participant in each task, through regressions using item and order scores to predict mathematics scores, to discover whether influential cases could affect any of the analyses reported below. No influential outliers with a Cook's Distance score >1 (Field, 2009) were detected when using item memory and order memory scores to predict either mathematics scores. Controlling for gender had no significant impact on analyses and results reported below are without controlling for gender.

In the sections below, results for standardised tests will be firstly reported (section 4.3.1), followed by results for the process dissociation task (section 4.3.2.1) and results for the forced-choice recognition task (section 4.3.2.2). Finally, regression analyses will examine to what extent item memory and order memory predicted mathematics scores (section 4.3.3).

4.3.1 Standardised Tests

Mean score across participants for the Woodcock-Johnson Calculation test was 17.08 (SD = 6.04). This represented a median percentile rank compared to age norms (Woodcock, McGrew & Mather, 2001) of 62.00 (min = 3.00; max = 97.00). Mean score across participants for the Woodcock-Johnson Math Fluency test was 120.37 (SD = 23.83). This represented a median percentile rank compared to age norms of 47.00 (min = 2.00; max = 99.50). Scores for the two maths tests significantly correlated with each other, $r = .63$, $p < .001$.

4.3.2 Item and Order Tasks

4.3.2.1 Process Dissociation Task

Initially, data from the inclusion and exclusion conditions were used to calculate item memory and order memory scores for each participant, as described in section 4.2.6.1. Mean score in the inclusion condition was 13.08 (SD = 2.51) and mean number of items recalled erroneously in the exclusion condition was 1.96 (SD = 2.19). Calculated mean item memory was therefore 1.01 (SD = .17) and calculated mean order memory .87 (SD = .13).

Scores for item memory and order memory were then correlated with the two mathematics scores to discover whether performance for either type of memory was related to mathematics performance. Correlations involving order memory were carried out using Spearman's rho, as scores for order memory showed significant positive skew at the $p > .05$ level (Field, 2009). Neither item memory, $r = .22$, $p = .126$ nor order memory, $r_s = .23$, $p = .106$ significantly correlated with Woodcock-Johnson Calculation scores. Similarly, neither item memory, $r = .08$, $p = .577$ nor order memory, $r_s = .24$, $p = .096$ significantly correlated with Woodcock-Johnson Math Fluency. Item memory and order memory did not significantly correlate with each other, $r_s = -.26$, $p = .069$, suggesting dissociation of item and order memory within this task.

The exclusion condition allowed participants to recall items in any order. However, it is possible that participants still attempted to recall the items in the order in which they were presented. The data was examined to see on what proportion of trials free recall had been used and on what proportion of trials participants had still chosen to recall in serial order. It was found that the mean number of trials in which participants had used free recall was only .39 (SD = .21). The proportion of trials that participants recalled using free recall significantly negatively correlated with their order memory, $r_s = -.53$, $p < .001$. In other words, participants who used a serial recall strategy obtained greater order memory scores than those who used free recall.

The correlation between use of free recall and item memory was not significant, r = .22, p = .117. Use of free recall did not significantly correlate with calculation scores, r = .07, p = .592, or fluency scores, r = -.04, p = .785.

4.3.2.2 Forced-Choice Recognition Task

The proportion of items correctly selected from the test pairs was first calculated for each participant in each of the item memory and order memory conditions. Mean item memory was .79 (SD = .29) and mean order memory was .75 (SD = .15).

Scores for item memory and order memory were then correlated with the two mathematics scores to discover whether either was related to mathematics performance. Correlations involving item memory were carried out using Spearman's rho, as scores for item memory showed significant positive skew at the p > .05 level (Field, 2009). Item memory did not significantly correlate with either Woodcock-Johnson Calculation, r_s = .08, p = .583 or Woodcock-Johnson Math Fluency, r_s = .10, p = .490. However, order memory correlated with Woodcock-Johnson Calculation, r = .34, p = .016, although not with Woodcock-Johnson Math Fluency, r = .20, p = .198. The correlation between item memory and order memory was small but significant, r_s = .28, p = .048.

Item memory and order memory were then compared across the two tasks to assess whether they were measuring the same things. Item memory in the two tasks did not significantly correlate with each other, r_s = .04, p = .803, but order memory in the two tasks did, r_s = .32, p = .022. Item memory in the process dissociation task did not significantly correlate with order memory in the forced-choice recognition task, r = -.07, p = .638, but order memory in the process dissociation task did significantly correlate with item memory in the forced-choice recognition task, r_s = .41, p = .003.

4.3.3 Regression Analyses

Regression analyses were performed for the forced-choice recognition task, to see whether item memory and order memory could uniquely predict mathematics scores. No regressions were performed for the process dissociation task because neither type of memory correlated with either calculation or fluency.

The first regression analysis was performed for the forced-choice recognition task with scores for the Woodcock-Johnson Calculation test as the dependent variable. Table 3.1 shows the model where item memory was entered in the model first at step 1, before order memory was added at step 2. Table 3.2 shows the model where order memory was entered first at step 1, before item memory was added at step 2. Only order memory significantly predicted calculation score, regardless of the order that item memory and order memory were entered into the model.

Table 4.1

Regression analysis for item and order memory in the forced-choice recognition task predicting Woodcock-Johnson Calculation score with item memory at step 1

DV: calculation score	**B**	**SEB**	**β**
Step 1			
Constant	18.06	2.52	
Item Memory	-1.25	3.01	-.06
Step 2			
Constant	8.21	4.55	
Item Memory	-1.94	2.87	-.09
Order Memory	13.92	5.47	.35*

Note. R^2 = .00 for Step 1 (p = .681), Δ R^2 = .12 for Step 2 (p = .014). *p < .05.

Table 4.2

Regression analysis for item and order memory in the forced-choice recognition task predicting Woodcock-Johnson Calculation score with order memory at step 1

DV: calculation score	B	SEB	β
Step 1			
Constant	6.95	4.12	
Order Memory	13.56	5.41	.34*
Step 2			
Constant	8.21	4.55	
Item Memory	-1.94	2.87	-.09
Order Memory	13.92	5.47	.35*

Note. R^2 = .11 for Step 1 (p = .016), Δ R^2 = .01 for Step 2 (p = .502). *p < .05.

Similar regression models were run using Woodcock-Johnson Math Fluency scores as the dependent variable, but neither model was significant (all *p*'s > .05).

4.4 Discussion

This chapter investigated the relative importance of item memory and order memory, in the visuo-spatial domain, for mathematics. Initially, further analysis of results from both experiments in Chapter 2 suggested that, although item memory was related to mathematics, order memory was more important. This was then investigated further, through the use of a process dissociation task and a forced-choice recognition task designed to provide greater separability of these two types of memory. Neither task found a relationship between item memory and mathematics. However, the forced-choice recognition task found a relationship between order memory and mathematics calculation. This supported the initial prediction that visuo-spatial order memory would be more important for mathematics, but did not support the prediction that visuo-spatial item memory would be important.

For the process dissociation task, neither type of memory significantly correlated with mathematical performance. Correlations with both calculation and fluency scores were non-significant. However, analysis of data from the process dissociation task highlighted issues with this method of measuring item memory and order memory. Errors in the exclusion condition, used to calculate scores for both item and order memory, were low at only 13%, indicating that the task may not have been sufficiently difficult. Also, although the process dissociation task is designed to enable calculation of independent scores for item and order memory, around 2/3rds of trials in the exclusion condition resulted in participants using a serial recall strategy as opposed to free recall. The use of serial recall appeared to have impacted on their ability to exclude the correct item at test, as use of free recall negatively correlated with order memory score. In other words, the greater the use of serial recall, the greater was a participant's order memory score. The correlation between using free recall and item memory also approached significance. Item memory and order memory also moderately correlated with each other. This suggests that item and order memory may not have been truly dissociated within this task.

In summary then, the process dissociation task found that neither visuo-spatial item memory nor order memory were important for mathematical calculation or fluency in adults. However, results for both types of memory may have been affected by a lack of difficulty and the high levels of serial recall used by participants in the exclusion condition.

For the forced-choice recognition task, which was designed not to rely on the serial recall of items, item memory did not correlate with scores for either mathematics measure, but order memory correlated with calculation scores. Also, results of the regressions found that order memory significantly and uniquely predicted calculations scores, but there was no predictive relationship for item memory. Neither item memory nor order memory predicted fluency in mathematics. This corresponds with adults largely using direct retrieval of answers to solve the arithmetic included in the fluency test, rather than using procedural methods, such as decomposition or counting, that rely to a greater extent on working memory resources. Also, if some participants needed to use procedural strategies, (e.g. counting down to solve subtractions) the basic nature of the arithmetic included in the test would not require much ordering of information. Results for the forced-choice recognition task therefore indicate that it is visuo-spatial order memory, not item memory, which is important for mathematical calculation.

When looking at relationships between scores across the two tasks, it was found that the two item memory scores did not correlate, but the two order memory scores did. This implies that the two tasks were tapping into different processes for item memory. This may well reflect the difference between recall and recognition at test and suggests that, whilst the data indicated serial recall was largely utilised in the process dissociation task, serial order was not used for maintenance of items in memory in the forced-choice recognition task.

However, an interesting pattern of results emerged when comparing the item scores of one task with the order scores of the other. Item memory in the process dissociation task did not correlate with order memory in the forced-choice recognition task, but order memory in the former did correlate with item memory in the latter. Also, item and order memory correlated with each other within the forced-choice recognition task, although this was only just significant. This suggests that item memory may have been maintained serially in the forced-choice recognition task to some degree after all. However, even if this was the case, item memory still did not correlate with either calculation or fluency. It therefore appears that there were issues, to varying degrees, with both tasks regarding the dissociation of item memory and order memory and this could be due to the fact that items were presented serially in both tasks.

This pattern of results supports previous evidence that there is a preference for remembering items serially even when this is not required (e.g. Batarah, Ward, Smith, & Hayes, 2009). Whilst the verbal domain is believed by many to have separate storage and rehearsal mechanisms within the phonological loop (Baddeley, 2000) the visuo-spatial domain is less well understood in this respect. The nature of rehearsal within the visuo-spatial domain and whether it sits with the visuo-spatial sketchpad or central executive is unclear, although Baddeley (2000) suggests some form of general sequential attention may assist in maintaining serial position in memory across both domains. Rather than trying to dissociate item memory from order memory in tasks where items are presented in a serial, dynamic manner, it may be better to examine item memory by comparing performance when items are presented both serially and simultaneously. Simultaneous presentation does not readily lend itself to memory through maintaining serial order but does allow examination of whether an item is held in memory. This would also enable examination of the importance of static versus dynamic visuo-spatial working memory for mathematics (Logie et al., 1994). The role of these two types of visuo-spatial working memory for basic arithmetic are examined through use of a dual-task in Chapter 6.

Item memory then does not appear to be an important factor in mathematicians' superior visuo-spatial working memory storage capacity. Performance in the forced-choice recognition task, which did not require the recall of items in serial order, found no predictive relationship of item memory for either mathematical calculation or fluency. Results, however, suggested that order memory has a relationship with calculation and suggested that order memory contributes to mathematicians' superior visuo-spatial working memory storage capacity.

As highlighted in the introduction to this chapter, the importance of order memory for mathematics is consistent with research stating that there are individual differences in the order in which stages of solving a problem are executed (Hitch 1978). It is also consistent with Pesenti's (2005) finding that expert calculators have a greater knowledge for the order of these steps and Dowker et al.'s (1996) finding that mathematicians carry out the steps more accurately. Therefore, a greater ability for ordering information should enable better remembering of steps involved in calculation procedures and also more efficient strategy execution when solving mathematical problems. Chapter 2 discussed how visuo-spatial working memory is important for the manipulation of mathematical information and it would make sense that the ordering of information is important for this manipulation.

Although these results suggest the importance of visuo-spatial order memory for calculation, they do not tell us about the underlying mechanisms of ordering information or why mathematicians seem to have a greater ability for order memory in the visuo-spatial domain.

4.5 Conclusion

Results from working memory span tasks in Chapter 2 and a forced-choice recognition task in the current chapter have consistently indicated that those proficient at mathematics have superior ability to remember the order of information in the visuo-spatial domain. This appears to be particularly important when working memory is being used, since Chapter 3 found no difference between adult mathematicians and non-mathematicians for the storage of visuo-spatial information when short-term memory tasks, with no processing element, were employed. The ordering of visuo-spatial information also appears to be related to mathematicians' greater ability to execute ordered strategies and manipulate information when solving mathematical problems.

The working memory tasks employed so far have used processing elements that were as neutral as possible with regard to the storage elements. Chapter 5 will investigate whether mathematicians still have superior storage capacity for visuo-spatial information during working memory use when the processing elements included are verbal or visuo-spatial.

Chapter 5: Working Memory Storage Capacity: Verbal & Visuo-Spatial Processing

5.1 Introduction

This chapter examines whether the type of processing involved in working memory affects the ability of adult mathematicians and non-mathematicians to store verbal and visuo-spatial material. It also examines whether mathematicians' apparent superior capacity for storing visuo-spatial information in working memory is simply due to a greater ability to deal with visuo-spatial information.

Chapter 2 found that adult mathematicians have superior ability to store visuo-spatial, but not verbal, information when using working memory. Use of a short-term memory span task, with no processing element, in Chapter 3 found that adult mathematicians do not have superior short-term memory storage capacity in the visuo-spatial domain. Also, when short-term memory and controlled spatial attention were included as covariates in an ANCOVA, the mathematicians still retained superior visuo-spatial working memory storage capacity to the non-mathematicians. Chapter 4 found that visuo-spatial order memory, but not item memory, contributes to this superior visuo-spatial storage capacity. Moreover, both visuo-spatial working memory storage capacity and visuo-spatial order memory correlated with performance on the Woodcock-Johnson Calculation Test. It therefore seems that adult mathematicians have a greater ability to store visuo-spatial information than adult non-mathematicians when they are required to hold information in mind whilst carrying out processing. It also seems that both the ability to hold visuo-spatial information in mind whilst processing and to order visuo-spatial information are related to calculation ability.

The working memory tasks employed so far, however, have included a novel processing element: a face-matching task that was designed to be as neutral as possible with regard to the domains of the storage items. This enabled the examination of capacity for the verbal and visuo-spatial storage elements using a consistent processing element across the tasks in both domains. It also ensured that, as far as possible, the processing element did not interfere with storage in one domain more than in the other. However, previous research with adults has shown that the amount of verbal or visuo-spatial items stored within working memory varies with regard to the domain of the processing involved.

Storage of items in working memory appears to be more difficult when processing items are from the same domain. Jarrold et al. (2011) found that both verbal and visuo-spatial processing adversely affected the storage of single-syllable words, but the impact of verbal processing was greatest. It has been argued that verbal processing has a greater effect than does visuo-spatial processing on the storage of verbal items (Vergauwe, Barrouillet & Camos, 2010), because it blocks the rehearsal of verbal storage items through demands on the phonological loop. Vergauwe and colleagues did not find the same type of interference in the visuo-spatial domain, where both verbal and visuo-spatial processing produced the same level of interference for storing visuo-spatial items.

Shah & Miyake (1996) also showed that the type of processing in working memory span tasks affected storage in both the verbal and visuo-spatial domains. Visuo-spatial storage was greater when combined with verbal processing than with visuo-spatial processing and verbal storage was greater when combined with visuo-spatial processing than with verbal processing. They also found that correlations with higher level cognition tasks were greater when the working memory span tasks involved processing and storage from the same domains. Storage scores for their span task involving both visuo-spatial processing and storage were more strongly related to measures of complex spatial thinking than were scores for the span task combining verbal processing and visuo-spatial storage. Similarly, their span task involving both verbal processing and storage had a stronger relationship with language comprehension than when verbal storage was combined with visuo-spatial processing. They suggested this was due to the requirement to process and store information in the same domain within the working memory span tasks reflecting the resources required in complex cognitive tasks. For example, language comprehension predominantly requires both the processing and storage of verbal information rather than a mixture of verbal and visuo-spatial.

It may therefore be that measuring verbal and visuo-spatial storage ability when using processing tasks from the same domain will give a better indication of the relationship between mathematics and holding verbal and visuo-spatial information in mind during processing. It may also be that using working memory span tasks with verbal and visuo-spatial processing elements will produce a different pattern of results to that found in Chapter 2. The use of verbal and visuo-spatial processing elements within the span tasks employed in the current chapter will inform whether mathematicians always have superior visuo-spatial storage ability while using working memory or whether their relative ability depends upon the type of processing being carried out.

Use of a neutral processing task that does not involve verbal or visuo-spatial skills also does not perhaps truly reflect the type of processing being undertaken whilst people use working memory to solve mathematical problems. For example, Logie et al., 1994 and Hubber et al. (Experiment 1: 2014) found visuo-spatial resources are used when adults perform additions. Imbo & LeFevre (2010) found working memory resources in both the verbal and visuo-spatial domains are used when adults solve subtraction and multiplication problems. Use of verbal and visuo-spatial processing elements in the span tasks employed in the current chapter will better reflect the types of processing undertaken when adults solve mathematical problems.

5.1.1 The Current Experiment

Previous research has shown the type of processing element included in a working memory span task affects both storage performance and the link between storage and higher level cognition

(Jarrold et al., 2011; Shah & Miyake, 1996; Vergauwe et al., 2010). The current chapter therefore employs working memory span tasks with both verbal and visuo-spatial processing elements. It includes the four possible combinations of verbal and visuo-spatial processing and storage. Use of these processing elements in the current chapter allows examination of whether mathematicians still have superior capacity in the visuo-spatial domain when verbal or visuo-spatial processing is involved, as in 'real life' calculations. It will also inform whether there is still no difference between mathematicians and non-mathematicians for verbal working memory storage when the processing element is no longer neutral.

The format of the span tasks used were the same as those employed in Chapter 2, except for the type of processing elements. The verbal and visuo-spatial processing elements retained the requirement for a 'yes' or 'no' response, used in Chapter 2, to ensure the same task formats across conditions (Jarrold et al., 2011) and to ensure comparability with the Chapter 2 tasks. Each possible combination of verbal and visuo-spatial processing and storage was examined, giving four conditions: verbal processing & verbal storage; verbal processing & visuo-spatial storage; visuo-spatial processing & verbal storage; visuo-spatial processing & visuo-spatial storage. The tasks are described in detail in section 5.2.3.

The second issue examined in the current chapter is whether the apparent advantage that mathematicians have for storing visuo-spatial information while using working memory can be explained by them simply having better general ability for dealing with visuo-spatial information. Wei et al. (2012) previously found that both visuo-spatial storage capacity and general visuo-spatial ability, measured by a 3-dimensional spatial rotation task, correlated with mathematics performance in Chinese college students. However, they did not examine whether the relationship between visuo-spatial storage ability and mathematics could be explained by general visuo-spatial ability.

Previous research has implicated the use of general visuo-spatial resources in the interpretation of initial information contained in mathematical problems. Varying the spacing of operands (+, x and – signs) and the first operator (initial digit) (Jiang et al., 2014) affected the interpretation of questions and the processing of the symbols. The spatial proximity of words within algebra word problems also affects the interpretation of questions and the formulation of appropriate formulae to solve the problems (Landy et al., 2014). Pinhas, Shaki & Fischer (2014) also argued that plus and minus operands have spatial associations. Their participants had to answer simple additions and subtractions by pointing to answers along a number line presented on screen. Adults were faster to respond when additions involved responding on the right side of the number line or when subtractions involved responding to the left. When Marghetis, Núῆez & Bergen (2014) measured the movement of the computer mouse as their participants selected answers to addition and subtraction problems shown on screen, they found a similar addition-right and subtraction-left bias. Wiemers, Bekkering & Lindemann (2014) also concluded that spatial magnitude is important when solving arithmetic. They found that spatial arm movements affected addition and subtraction differently dependent on the direction of the movements. A meta-analysis (Friedman, 1995) also found moderate correlations between mathematics and general visuo-spatial ability.

For the current chapter, the performance of mathematicians and non-mathematicians was compared on the *Revised Vandenberg & Kuse Mental Rotations Test: MRT-A* (Peters et al., 1995), a test of general visuo-spatial ability involving visuo-spatial rotation. This test has been used previously across a range of subject literature as a measure of general visuo-spatial processing ability (e.g. Hausmann, Slabbekoorn, Van Goozen, Cohen-Kettenis & Güntürkün, 2000; Hedman et al., 2006; Langlois et al., 2009; Peters, Chisholm & Laeng, 1995). Delgado & Prieto (2004) also used the MRT-A to discover that visuo-spatial rotation ability predicted performance on geometry and mathematical word problems. The task is described in detail in section 5.2.4. Performance of the mathematicians and non-mathematicians will also be compared for the processing elements of the working memory span tasks.

In summary, the current chapter investigates differences between adult mathematicians and non-mathematicians in working memory storage capacity when the processing element of the span tasks used involves either visuo-spatial items or verbal items. It also examines differences between the two groups for performance on the verbal and visuo-spatial processing elements and for general visuo-spatial ability. As Chapter 2 found that mathematicians have superior visuo-spatial working memory storage capacity, it was expected that mathematicians would remember more items in their correct serial position in the two working memory span task conditions involving visuo-spatial storage: verbal processing & visuo-spatial storage and visuo-spatial processing & visuo-spatial storage. In other words, it was expected that mathematicians would have better visuo-spatial storage than the non-mathematicians whatever the domain of the processing. It was predicted that there would be no difference between the performance of the two groups in the verbal processing & verbal storage condition, as Chapter 2 found no differences in the verbal domain.

No firm prediction was made regarding differences in the visuo-spatial processing & verbal storage condition. It could be argued that there should be no difference between the two groups as it involved verbal storage. However, as it also involved visuo-spatial processing, it might be expected that the mathematicians would perform at a greater level than the non-mathematicians in this condition. This is because if the non-mathematicians find the visuo-spatial processing task harder, they may use more working memory resources to solve the processing problems leaving fewer resources available to successfully perform the verbal storage. Finally, it was expected that mathematicians would perform better than non-mathematicians for general visuo-spatial ability as measured by scores for the MRT-A. This was due to previous literature suggesting that general visuo-spatial ability may be related to mathematics (Delgado & Prieto, 2004; Friedman, 1995). The previous findings of Wei et al (2012) have also suggested that general visuo-spatial ability correlates with mathematics performance in adults. For this reason, it was also expected that mathematicians would be faster and more accurate for the visuo-spatial processing elements of the span tasks. It was expected there would be no differences between the two groups for verbal processing, as Chapter 2 found no differences in ability for verbal material.

Visuo-spatial working memory storage capacity was found to predict calculation performance in Chapter 3. As it was predicted that mathematicians would perform better than non-mathematicians for visuo-spatial working memory storage capacity, visuo-spatial processing and general visuo-spatial ability in the current chapter, these elements will be entered into a regression to see whether visuo-spatial working memory storage capacity can predict mathematics scores over and above the contribution of general visuo-spatial ability and processing.

5.2 Method

5.2.1 Participants

57 participants were recruited from the undergraduate population at the University of Nottingham: 28 (11 male) to a mathematics group and 29 (7 male) to a non-mathematics group. All participants received an inconvenience allowance of £9. None of the participants in the current chapter had taken part in the experiments included in Chapters 2 and 3.

The mathematics group comprised 20 mathematics students and 8 economics students who had studied mathematics at A level. Their ages ranged from 18.68 to 32.56 years (M = 20.83, SD = 2.68). The non-mathematics group comprised English, History, and Sociology students who had not studied mathematics at A level. Their ages ranged from 18.76 to 31.70 years (M = 20.63, SD = 2.51). On average, participants in the non-mathematics group had not studied maths for 4.22 years (SD = 1.39).

5.2.2 Equipment

An Acer Aspire 5736Z laptop computer, running Windows 7 and PsychoPy version 1.77.01 (Peirce, 2007), was used to present stimuli and record latencies and accuracy.

5.2.3 Working Memory Tasks

There were four span tasks. Each had a different combination of processing elements and storage elements, with the processing and storage elements interleaved. The four different combinations were verbal processing & verbal storage; verbal processing & visuo-spatial storage; visuo-spatial processing & verbal storage; visuo-spatial processing & visuo-spatial storage. The timings used during all four span tasks were identical to those used in the working memory span tasks of the two experiments in Chapter 2.

The visuo-spatial processing task employed spatial visualisation. It was adapted from a task used by Miyake et al. (2001). Participants saw two pictures on screen, side by side (see example in Figure 5.1). The picture on the left of each pair represented a piece of paper folded in half with a hole punched in it. Participants had to imagine opening out this piece of paper towards the dotted lines. They then had to indicate whether or not the unfolded paper would look like the picture on the right of the pair. They did this by pressing the ‘y’ key on the laptop’s keyboard for yes or the ‘n’ key for no. Figure 5.1 shows an example of a trial where the correct answer was ‘yes’. The full list of visuo-spatial processing pictures used is included in Appendix H.

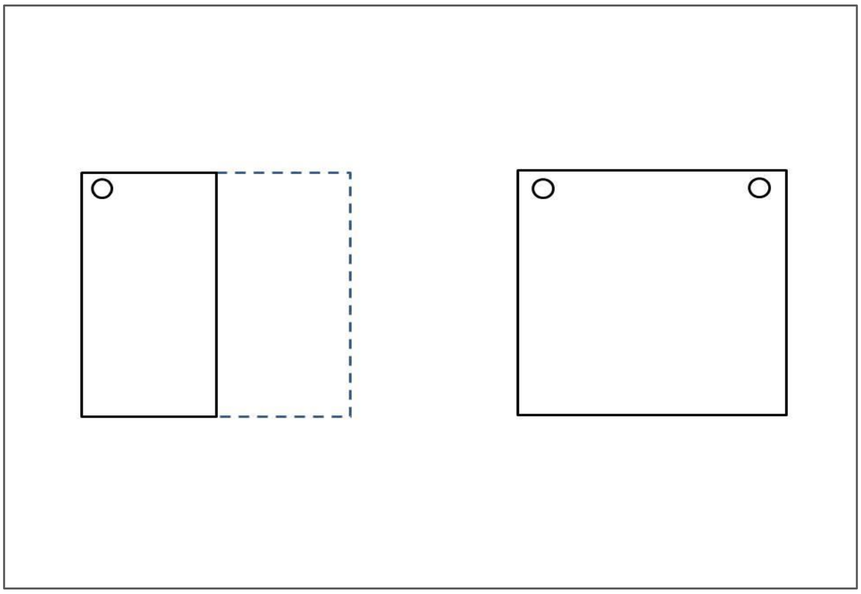

Figure 5.1: Example of a visuo-spatial processing item in the working memory span tasks.

The verbal processing task was a word rhyming judgement task, previously used in the working memory literature (e.g. Baldo & Dronkers, 2006; Gathercole, Alloway, Willis & Adams, 2006). Participants saw two English words on screen, side by side. They had to indicate whether or not the two words rhymed. They did this by pressing the ‘y’ key on the laptop’s keyboard for yes or the ‘n’ key for no. The full list of verbal processing word pairings used is included in Appendix I.

As the visuo-spatial and verbal processing items were each used in two of the conditions, two separate blocks were constructed for each processing type. No individual processing item was repeated either within or across blocks. To ensure equal difficulty of the four blocks, a pilot study was initially conducted. In this pilot study, five postgraduate Psychology students had to indicate yes or no, as described above, to each processing item within each of the four blocks. This pilot involved the processing items only, with no storage items presented. Latencies and accuracy were recorded and averaged for each block. Examination of the data from this pilot study indicated that three of the blocks were comparable for RT and accuracy, but that one of the visuo-spatial blocks was more difficult in terms of accuracy. Examination of individual items within this block indicated that four items had greater error rates than the other items and these were replaced. Also, one item within one of the two verbal blocks was replaced as it had a greater number of errors than did other items. Finally, each processing block was assigned to one of the working memory span task conditions listed in the first paragraph of the current section.

The storage items of each span task consisted either of numerical items in the verbal domain or of visuo-spatial items. The same storage items were presented as in Experiment 2 of Chapter 2. These consisted of numerical or visuo-spatial items presented in the centre of the screen. Items in each span set were taken from a group of nine possible stimuli in each condition:

Number span: Digits 1 to 9 (size 2cm, arial font, colour white on dark grey background)

Visuo-spatial span: Black 3 x 3 grid in the centre of the screen (each square was 6cm wide x 6cm high) with a red dot (size 3 cm wide x 3cm high) placed in one of nine possible locations on the grid

Each trial comprised an interleaved series of processing elements and storage items. Each processing element was presented on screen for 3 seconds, although participants were still able to respond after this time. The storage items were presented for 500 milliseconds (ms), commencing 500ms after a response had been given to the preceding processing element. The next processing element was presented 500ms after the storage item disappeared from screen. At the end of each span set, once all storage items had been presented, a “ ? ” appeared in the centre of the screen that prompted the participants to recall the storage items, in their order of presentation. In the number condition, participants said the numbers aloud and the experimenter keyed the response into the USB numeric keypad, In the visuo-spatial condition, a black 3 x 3 grid appeared on screen immediately after the “

? ” and participants recalled the serial order of the red dot by clicking on the grid, using the USB mouse. Once recall was completed, the participant pressed the space bar to begin the next trial.

Span sets, and items within each span set, were presented in a random order, as in the working memory span tasks in Chapter 2. In all four conditions, each span length from 3 to 8 was presented three times, giving 18 trials. Details of trials for all conditions are included in Appendix J. Each of the nine possible storage items within each condition was presented approximately equally.

5.2.4 Additional Materials

The *Woodcock-Johnson Calculation Test* (Woodcock, McGrew & Mather, 2001) and *WASI Matrix Reasoning* (WASI; Psychological Corporation, 1999), as described in Chapter 2 (section 2.2.1.4), were administered using the standard procedures to measure mathematics ability and non-verbal IQ. The Woodcock-Johnson Math Fluency Test was not administered because Chapter 3 found that working memory performance predicted calculation ability but not fluency.

Participants also completed the *Revised Vandenberg & Kuse Mental Rotations Test: MRT-A* (Peters et al., 1995). This was administered as a measure of general visuo-spatial ability. The MRT-A was administered using a pencil and test booklet and using the test’s standard procedure. Participants initially worked through instructions containing four practice items. They then completed 24 test items, split into two blocks, with three minutes allowed for the completion of each block. Each test item was presented horizontally, with one target item on the left and four stimulus figures on the right. All five figures consisted of 3-dimensional shapes comprising ten individual cubes. Two of the stimulus figures were rotated versions of the target figure. The other two stimulus figures were similar to the target figure, but, if rotated, could not match the target figure. Participants had to identify the two stimulus figures that could match the target figure by marking them with a cross in pencil. An answer to a test item was scored as correct, and received a score of one point, if both correct stimulus figures were identified. No marks were given for a partially correct answer if only one of the two correct items was indicated.

It is not possible to include examples of the MRT-A test items within this thesis as use of this standard test includes agreement not to reproduce it in any way. The authors do not wish test items to get into general circulation, as prior exposure to items has been found to affect test results.

5.2.5 Procedure

All participants were tested individually by the same experimenter and each session lasted around 90 minutes. All participants in both groups completed the four working memory span tasks on the computer, for each span length 3 to 8. The order in which the four conditions were presented was counterbalanced across participants. Within this counterbalancing, it was ensured that the same processing task was not presented in consecutive tasks. In other words, a task involving visuo-spatial processing was always followed by a task involving verbal processing and vice versa. The order of presentation of span sets within each task and the presentation of items within each set was randomised.

For their first and second span tasks, each participant initially practised the relevant processing task. After initial instructions, participants made yes or no judgements for six items, so they could familiarise themselves with the processing task. They then began the experiment. They commenced with a practice of one 2-span set and one 3-span set comprising both processing and storage tasks, before the 18 test sets were administered. For their third and fourth span tasks, participants followed the same procedure as described for the first and second span tasks, but omitting the initial practice of the processing element as they were already familiar with it.

After completing all four span tasks, participants then completed the WASI Matrix Reasoning and MRT-A tests, the order of which was counterbalanced across participants. Finally, participants completed the Woodcock-Johnson Calculation Test.

5.3 Results

One female participant in the non-mathematics group was excluded from the following analyses due to software failure that meant that data were not collected in the visuo-spatial processing & verbal storage condition. Six participants (1 mathematics group; 5 non-mathematics group) were also excluded for having an unacceptably high (>15%) error rate in the processing task (mathematics: 1 verbal processing & verbal storage condition; non-mathematics: 5 visuo-spatial processing & visuo-spatial storage condition) leaving data for 27 (10 male) participants in the mathematics group and 23 (7 male) in the non-mathematics group available for analysis.

A Cook's Distance score was calculated initially for each participant in each condition within the working memory ANOVAs to discover whether there were any influential cases that could affect the results of the ANOVAs. A Cook's Distance score was also calculated in a regression using storage accuracy and processing task performance in the working memory tasks and performance on the MRT-A standard test to predict mathematics scores, to discover whether influential cases could affect any of the correlations reported below. No influential outliers with a Cook's Distance score >1 (Field, 2009) were detected. Controlling for gender had no significant impact on analyses and results reported below are without controlling for gender. Degrees of freedom were corrected using Greenhouse-Geisser estimates of spherity where necessary.

In the sections below, results for standardised tests will be reported first (section 5.3.1), followed by results for the storage element of the working memory tasks (section 5.3.2) and for the processing element (section 5.3.3). A regression analysis to examine to what extent the working memory tasks and general visuo-spatial ability predicted mathematics scores are included in section 5.3.4. Finally, section 5.3.5 will compare performance on the working memory tasks in this chapter to those in Experiment 2 of Chapter 2.

5.3.1 Standardised Tests

An independent *t*-test to compare the two groups' Woodcock-Johnson Calculation Test scores confirmed that the mathematics group (M = 26.70, SD = 3.56) were significantly better at mathematics than the non-mathematics group (M = 14.57, SD = 4.86), $t(48) = 12.14$, $p < .001$, $r = .87$. Scores for the mathematics group represented a median percentile rank compared to age norms (Woodcock, McGrew & Mather, 2001) of 95.00 (min = 66.00; max = 99.90)[7]. Scores for the non-mathematics group represented a median percentile rank compared to age norms of 52.00 (min = 1.00; max = 96.00)[8].

Mathematicians (M = 12.30, SD = 4.83) performed better on the MRT-A test of general visuo-spatial ability than the non-mathematicians (M = 9.09, SD = 4.44). An independent *t*-test confirmed that this difference was significant, $t(48) = 2.43$, $p = .019$, $r = .33$. There was a significant relationship between general visuo-spatial ability and mathematics as MRT-A scores correlated with calculation scores, $r = .46$, $p = .001$.

An independent *t*-test showed that there was no significant difference between the two groups for WASI Matrix Reasoning non-verbal IQ (mathematics: M = 29.04, SD = 3.60; non-mathematics: M = 27.52, SD = 3.20), $t(48) = 1.56$, $p = .125$, $r = .22$.

5.3.2 Storage Element

Proportion correct scores were first calculated for each participant for the number of storage items recalled in their correct serial position (see section 2.2.1.6 of Chapter 2).

[7] Two outliers, with scores of 66 and 71, were detected in the mathematics group. However, neither was identified as an influential case in the following analyses and their inclusion did not affect results or conclusions.
[8] The maximum score of 96 in the non-mathematics group related to one participant. However, they were not identified as an influential case in the following analyses and their inclusion did not affect results or conclusions. The next highest score was 68.

To discover whether processing type had any effect on storage and whether there was any difference between the mathematicians and non-mathematicians for working memory storage capacity, a 2(group: mathematics, non-mathematics) x 2(working memory processing type: verbal, visuo-spatial) x 2(working memory storage type: verbal, visuo-spatial) mixed ANOVA was then performed on the proportion correct scores. Descriptive statistics, by group, are shown in Figure 5.2.

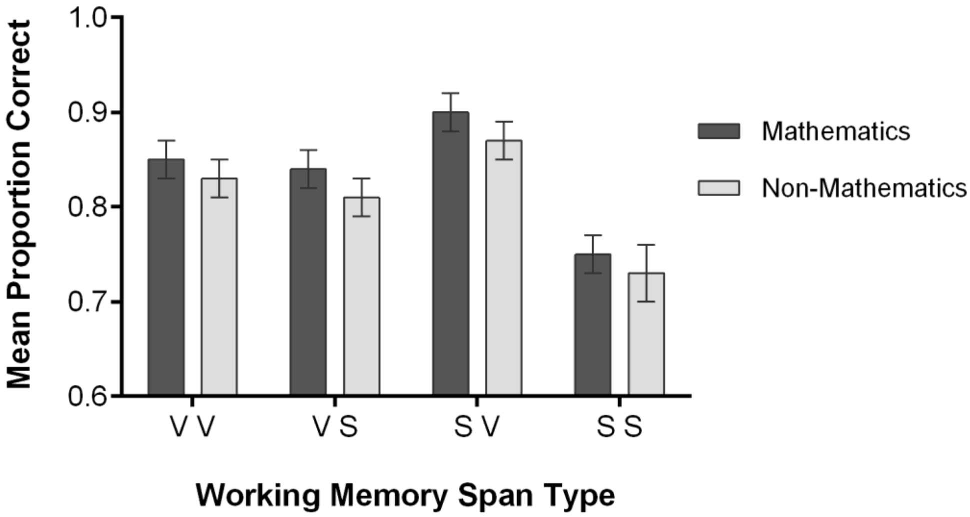

Figure 5.2: Accuracy of storage for each working memory span type for each participant group with S.E.M. error bars. On the horizontal axis, V V is verbal processing & verbal storage; V S is verbal processing & visuo-spatial storage; S V is visuo-spatial processing & verbal storage; and S S is visuo-spatial processing & visuo-spatial storage.

Results of the ANOVA showed there was no main effect of studying or not studying mathematics, $F(1,48) = 1.08$, $p = .304$, $r = .15$. There was, however, a main effect of processing type, $F(1,48) = 8.13$, $p = .006$, $r = .38$, with storage performance better overall when combined with verbal processing than with visuo-spatial processing. There was also a main effect of storage type, $F(1,48) = 31.21$, $p < .001$, $r = .63$, with storage of verbal items more accurate overall than storage of visuo-spatial items. There was no processing type x group interaction, $F(1,48) = .01$, $p = .909$, $r = .01$. In contrast to Chapter 2’s results, there was no storage type x group interaction, $F(1,48) = .02$, $p = .882$, $r = .02$. There was, however, a processing type x storage type interaction,

$F(1,48) = 47.76$, $p < .001$, $r = .71$, shown in Figure 5.3. Pairwise comparisons showed visuo-spatial storage was more accurate when paired with verbal processing than with visuo-spatial processing ($F(1,48) = 39.98$, $p < .001$, $r = .67$). However, verbal storage was more accurate when paired with visuo-spatial processing than with verbal processing ($F(1,48) = 20.35$, $p < .001$, $r = .55$). Finally, there was no processing type x storage type x group interaction, $F(1,48) = .48$, $p = .492$, $r = .10$[9].

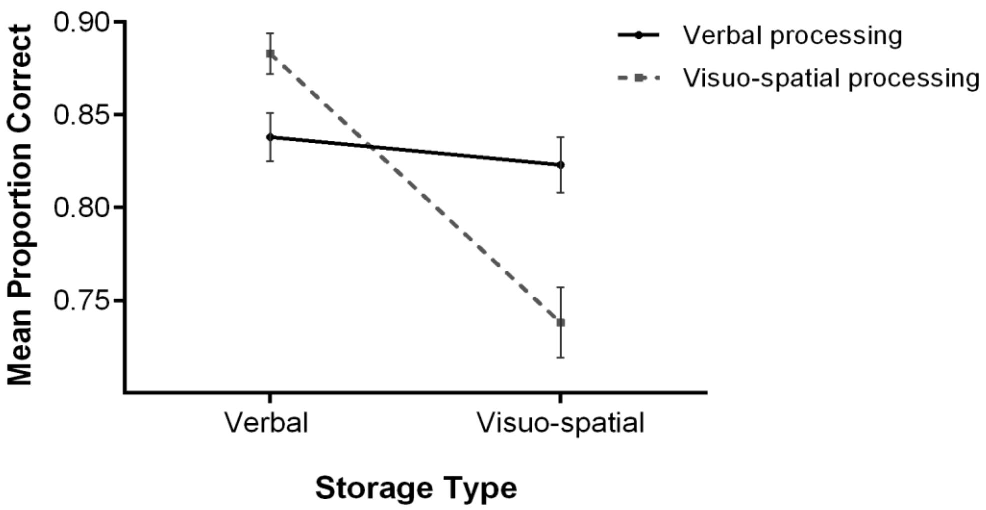

Figure 5.3: Accuracy of storage type for each processing type with S.E.M. error bars.

5.3.3 Processing Element

Initially, mean accuracy and median RT were calculated for each participant in each of the four working memory span conditions.

To discover whether there was any difference between the two groups with regard to verbal and visuo-spatial processing ability when combined with the two different types of storage, a 2(group: mathematics, non-mathematics) x 2(working memory processing type: verbal, visuo-spatial) x 2 (working memory storage type: verbal, visuo-

[9] ANOVAs were also run using both the All-or-Nothing Unit and All-or-Nothing Load methods. All-or-Nothing Unit did not result in any significant changes to results or conclusions. All-or-Nothing Load resulted in no main effect of processing type ($F(1,48)$, = .84, $p = .365$, $r = .13$, but did not significantly change any other main effects or interactions.

spatial) mixed ANOVA was performed for each of accuracy and latencies to examine performance of the two groups on the processing element of each condition. Mean accuracy, mean RT and standard error by group and span type are shown in Table 5.1.

5.3.3.1 Accuracy

Results showed no significant difference in accuracy on the processing tasks between groups or across the different working memory conditions. There was no main effect of studying or not studying mathematics, $F(1,48) = .39$, $p = .533$, $r = .09$, no main effect of storage type, $F(1,48) = 1.60$, $p = .212$, $r = .18$ and no main effect of processing type, $F(1,48) = .68$, $p = .412$, $r = .12$. There were also no significant interactions for group x processing type, $F(1,48) = .83$, $p = .367$, $r = .13$, group x storage type, $F(1,48) = .39$, $p = .537$, $r = .09$, or for processing type x storage type, $F(1,48) = 2.81$, $p = .100$, $r = .24$. Finally, there was no significant group x processing type x storage type interaction, $F(1,48) = .19$, $p = .665$, $r = .06$.

Table 5.1

Mean (M) and standard error (SE) for accuracy and reaction time in milliseconds (Ms) in the processing task by group in each working memory span task condition

Condition	Groups	Accuracy		Reaction Time(Ms)	
		M	SE	M	SE
V V	Mathematics	.97	.01	1254	46
	Non-Mathematics	.97	.01	1229	55
V S	Mathematics	.97	.01	1347	56
	Non-Mathematics	.97	.01	1306	56
S V	Mathematics	.97	.01	1225	45
	Non-Mathematics	.97	.01	1354	60
S S	Mathematics	.97	.01	1388	50
	Non-Mathematics	.95	.01	1576	103

Note. V V is verbal processing & visuo-spatial storage; V S is verbal processing & visuo-spatial storage; S V is visuo-spatial processing & verbal storage; and S S is visuo-spatial processing & visuo-spatial storage.

5.3.3.2 Latencies

Results showed no main effect of studying or not studying mathematics on latencies, $F(1,48) = .79$, $p = .379$, $r = .13$. There was, however, a main effect of processing type with verbal processing elements being answered faster than visuo-spatial processing elements, $F(1,48) = 13.63$, $p = .001$, $r = .47$. There was also a main effect of storage type, with the processing elements being answered faster overall when they were interleaved with verbal storage items compared to visuo-spatial storage items, $F(1,48) = 30.28$, $p < .001$, $r = .62$. There was a significant group x processing type interaction, $F(1,48) = 11.99$, $p = .001$, $r = .45$, shown in Figure 5.4. Pairwise comparisons showed that, for the mathematics group, there was no significant difference between latencies for the verbal and visuo-spatial processing items ($p = .866$), but that the non-mathematicians were slower to perform visuo-spatial processing than they were to perform the verbal processing ($p < .001$). Also, whilst there was no significant difference between the two groups for verbal processing task latencies ($p = .634$), the mathematics group were faster at processing visuo-spatial items and this difference approached significance ($p = .059$). There was no significant group x storage type interaction, $F(1,48) = .79$, $p = .379$, $r = .13$, no significant processing type x storage type interaction, $F(1,48) = 3.54$, $p = .066$, $r = .26$ and no significant group x processing type x storage type interaction, $F(1,48) = .02$, $p = .511$, $r = .02$.

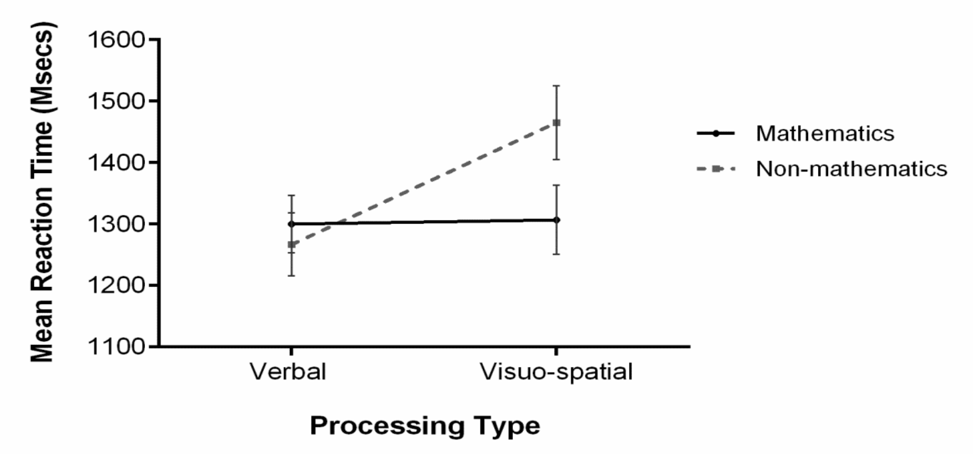

Figure 5.4: Reaction times for processing type for each participant group with S.E.M. error bars.

5.3.4 Regression Analysis to predict Mathematics Calculation Scores

In Chapter 3, only visuo-spatial working memory storage capacity uniquely and significantly predicted mathematics calculation ability when included in a regression with controlled spatial attention and visuo-spatial short-term memory storage capacity (with no processing element). A regression was performed next to discover whether visuo-spatial working memory storage capacity still uniquely and significantly predicted calculation when taking visuo-spatial processing and general visuo-spatial ability into account.

As mathematicians were faster than non-mathematicians for the visuo-spatial processing task, but there was no significant difference between the two groups for accuracy (section 5.2), only processing RT was included in the regression as a measure of visuo-spatial processing. Because of a strong correlation between accuracy in the two conditions involving visuo-spatial storage (r_s = .66, p <.001) and the two conditions measuring visuo-spatial processing RTs (r_s = .43, p = .002), storage scores and processing RTs were combined across conditions before entering them into the regression. Woodcock-Johnson Calculation Test score was the dependent variable.

Table 5.2 shows results for the regression model when MRT-A scores, for general visuo-spatial ability, were entered into the model together with visuo-spatial processing RT at Step 1, followed by visuo-spatial working memory storage at Step 2. At Step 1, only MRT-A scores significantly and uniquely predicted calculation ability. When visuo-spatial working memory storage was added at Step 2, both MRT-A and storage predicted calculation ability and there was significant improvement in the fit of the model.

Table 5.2

Regression analysis for general visuo-spatial processing (MRT-A), visuo-spatial processing latencies (RT) and visuo-spatial working memory (WM) storage predicting Woodcock-Johnson Calculation score

DV: calculation score	**B**	**SEB**	**β**
Step 1			
Constant	20.39	5.84	
MRT-A	.61	.20	.41**
Combined visuo-spatial processing RT	- 4.29	3.36	-.17
Step 2			
Constant	7.06	8.58	
MRT-A	.49	.21	.33*
Combined visuo-spatial processing RT	- 3.85	3.26	-.15
Combined visuo-spatial WM storage	17.90	8.71	.27*

Note. R^2 = .24 for Step 1 (p =.002), Δ R^2 = .06 for Step 2 (p = .045). *p < .05, **p < .01

5.3.5 Comparison of Chapter 2 and Chapter 5 Visuo-Spatial Working Memory Results

Results from the current chapter supported the findings of Chapter 2 that there is no difference between mathematicians and non-mathematicians for verbal working memory storage capacity. However, a different pattern of results to those of Chapter 2 emerged for visuo-spatial storage. In Chapter 2, mathematicians were able to store more visuo-spatial items in working memory when the span tasks included the face-matching task as an as-neutral-as-possible processing element. In the current chapter, mathematicians showed no advantage for storing visuo-spatial information in working memory when storage was combined with either verbal or visuo-spatial processing. Across the two studies it appears that, whilst participants overall found visuo-spatial storage harder when combined with visuo-spatial processing and easier when combined with verbal processing, the mathematicians found it easier than non-mathematicians to store visuo-spatial information when combined with the neutral face-matching processing task.

A statistical analysis was performed next to discover whether these assertions were correct. Visuo-spatial working memory scores from Experiment 2 in Chapter 2, with neutral processing, were compared to scores for the two working memory tasks in the current chapter that involved visuo-spatial storage. Scores for span lengths three to seven were included in the analysis to ensure consistency across conditions. Therefore, the performance of mathematicians and non-mathematicians were compared across three conditions: neutral processing & visuo-spatial storage; verbal processing & visuo-spatial storage; visuo-spatial processing & visuo-spatial storage. This investigated to what extent the three processing types affected visuo-spatial storage ability and whether mathematicians did in fact have better visuo-spatial storage ability than non-mathematicians when storage was combined with neutral processing compared to verbal or visuo-spatial processing.

The data for the tasks involving verbal processing and visuo-spatial processing were collected within participants, whereas the data for the task involving neutral processing were collected from a different group of participants. To overcome this difference, the analysis was treated as between-participants, as though data were collected from three different participant groups. Although this had less power than if all three conditions were performed by the same participants, it would still give an indication of the effect of the processing elements on visuo-spatial working memory storage capacity.

Participant numbers for the mathematics and non-mathematics groups in the three processing conditions were as follows: neutral: mathematics 25, non-mathematics 26; verbal: mathematics 27, non-mathematics 23; visuo-spatial: mathematics 27, non-mathematics 23. Initially, independent *t*-tests (shown in Table 5.3) were used to examine whether there were any differences between the profiles of the mathematics groups in Experiment 2 of Chapter 2 and the current chapter and also between the non-mathematics groups in the two chapters. For the mathematicians, there was no significant difference between the two groups for age or non-verbal IQ. Mathematicians in the current chapter had slightly higher Woodcock-Johnson Calculation Test scores than those from Chapter 2 and this difference approached significance. For the non-mathematicians, there was no significant difference between the two groups for age or non-verbal IQ. Non-mathematicians in the current chapter had slightly higher Calculation Test scores than those from Chapter 2 and this difference was just significant.

Table 5.3

Comparison of age profiles and of non-verbal IQ and Woodcock-Johnson Calculation Test scores for participants in Experiment 2 of Chapter 2 and Chapter 5

		Independent t-test result
Mathematicians	Age	$t(50) = .21$, $p = .833$
	Non-verbal IQ	$t(50) = .14$, $p = .892$
	Woodcock-Johnson Calculation	$t(50) = 1.94$, $p = .058$
Non-mathematicians	Age	$t(47) = .88$, $p = .382$
	Non-verbal IQ	$t(47) = .69$, $p = .494$
	Woodcock-Johnson Calculation	$t(47) = 2.02$, $p = .049$

A 2(group: mathematics, non-mathematics) x 3(processing type: neutral, verbal, visuo-spatial) Factorial ANOVA was performed on the visuo-spatial proportion correct scores. Descriptive statistics by group and processing type are shown in Figure 5.5.

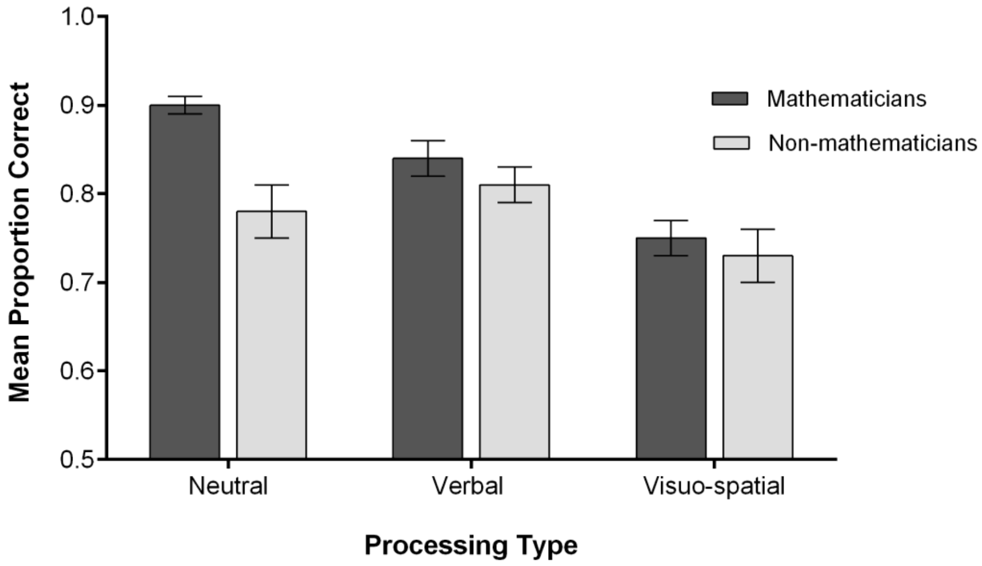

Figure 5.5: Accuracy of storage for each working memory span processing type for each participant group with S.E.M. error bars.

There was a main effect of studying or not studying mathematics, $F(1,145) = 10.08$, $p = .002$, $r = .25$, with the mathematicians having better visuo-spatial storage scores overall. The type of processing element significantly affected visuo-spatial storage ability, as there was also a main effect of processing type, $F(2,145) = 11.28$, $p < .001$, $r = .27$. Pairwise comparisons showed that visuo-spatial storage scores were greater overall when storage was combined with neutral processing than with visuo-spatial processing ($p < .001$) and greater with verbal processing than with visuo-spatial processing ($p < .001$). Storage scores were no different between the conditions using neutral and verbal processing ($p = 1.00$). There was a group x processing type interaction $F(2,145) = 3.34$, $p = .038$, $r = .15$. Tests of Bonferroni-corrected simple main effects discovered the mathematicians were better than the non-mathematicians at storing visuo-spatial information when storage was combined with the neutral processing task, $F(1, 145) = 15.65$, $p <.001$, $r = .31$. However, there was no significant difference between the two groups for visuo-spatial storage when it was combined with verbal processing, $F(1, 145) = .97$, $p = .328$, $r = .08$ or visuo-spatial processing, $F(1, 145) = .35$, $p = .557$, $r = .05$. Also, for the mathematicians, visuo-spatial storage scores were greater when storage was combined with neutral processing than with visuo-spatial processing, $F(2,145) = 12.10$, $p < .001$, $r = .28$ and when storage was combined with verbal processing than with visuo-spatial processing, $F(2,145) = 12.10$, $p = .011$, $r = .28$. Storage scores were no different between the conditions using neutral and verbal processing, $F(2,145) = 12.10$, $p = .143$, $r = .28$. For the non-mathematicians, there was no significant difference in visuo-spatial storage between any of the three conditions (all *p*s > .05).

5.4 Discussion

This chapter employed working memory span tasks, using verbal and visuo-spatial processing elements, to investigate whether the type of processing involved affected the ability of adult mathematicians and non-mathematicians to store verbal and visuo-spatial information whilst using working memory. It also investigated whether there was any difference in storage capacity or processing ability between these two groups. General visuo-spatial ability was also measured to see whether the ability to store visuo-spatial information within working memory predicted mathematics ability when general visuo-spatial ability was taken into account. Results found no significant difference between mathematicians and non-mathematicians for working memory storage capacity for any of the combinations of verbal and visuo-spatial processing and storage. However, they did find that mathematicians were faster to perform the visuo-spatial processing element of the working memory span tasks. Mathematicians also performed significantly better than the non-mathematicians on the measure of general visuo-spatial ability. Both general visuo-spatial ability and visuo-spatial storage within working memory were able to uniquely predict mathematics calculation ability. Comparison of results between Chapter 2 and the current chapter suggested that mathematicians have superior ability to store visuo-spatial information in working memory when the processing involved is neutral, but not when the processing is either verbal or visuo-spatial.

The finding, in the current chapter, that mathematicians and non-mathematicians had similar verbal storage capacity, irrespective of the type of processing, supports the results of Chapter 2. It also provides additional evidence that, whilst verbal storage has previously been found to be involved in mathematics (e.g. Fürst & Hitch, 2000; Geary, 2011; Imbo & Vandierendonck, 2007a; Logie et al., 1994), it is not an important factor in the differences between mathematicians and non-mathematicians for mathematical performance.

The finding that there was no difference between the two groups for visuo-spatial storage capacity, in the current chapter, did not support initial predictions. For both the verbal processing & visuo-spatial storage and the visuo-spatial processing & visuo-spatial storage conditions, there was no difference between the two groups for visuo-spatial storage accuracy. This contrasts with the findings of Chapter 2, where mathematicians showed superior visuo-spatial storage when this was combined with the as neutral as possible processing element. Section 5.3.5 above described the comparison of results across the three processing type conditions, albeit with the caveat that the analysis treated all three conditions as between-participants. This analysis indicated that, overall, participants found visuo-spatial storage more difficult when it was combined with visuo-spatial processing than with verbal or neutral processing. However, the mathematicians were better than the non-mathematicians at storing visuo-spatial information when it was combined with neutral processing. To confirm these findings, a within-participants experiment should be run to measure visuo-spatial storage with all three processing conditions.

One explanation for the difference in findings might be differences in participant characteristics. However, the participant profiles of the mathematics and non-mathematics groups used in Chapter 2 and the current chapter were very similar (section 5.3.5). Although the non-mathematicians involved in the current chapter had greater Woodcock-Johnson calculation scores than the non-mathematicians from Chapter 2, the difference in scores was very small and only just significant. The calculation scores of the mathematicians from the current chapter were also slightly greater than those from Chapter 2 and this difference was close to significance. It therefore seems unlikely that this slight difference in calculation ability is responsible for the different patterns of results found in the two studies. Therefore, the only substantial differences between the methods employed were the types of processing elements included in the working memory span tasks.

It is important to consider the level of central executive involvement in the neutral, verbal and visuo-spatial processing tasks employed. The central executive component of working memory is used both during working memory tasks and in mathematics. In terms of using working memory, the central executive is involved in controlling attention, switching between tasks, memory updating and retrieving information from long-term memory (Baddeley, 2003). In terms of mathematical cognition, it has been found to be used in the verification of sums (De Rammelaere, Stuyven & Vandierendonck 1999; 2001) and in mental addition (Logie et al., 1994). Also, Bull et al. (1999) found that performance on the Wisconsin Card Sorting Task, a general measure of executive functioning, was related to children's performance on a mathematics test. It is therefore important to consider the level of central executive involvement within the processing element of the span tasks used.

As well as differing according to content domain, it has also been argued that processing tasks differ according to their level of central executive involvement. Miyake et al., (2001) carried out a latent variable analysis and fractionated visuo-spatial processing tasks into three types: *perceptual speed*; *spatial relations*; and *spatial visualisation*. According to Miyake et al. (2001), *perceptual speed* involves the efficiency with which an individual can make basic perceptual judgements and involves visual comparisons rather than spatial manipulations. *Spatial relations* involve transformations, such as the rotation of objects. Finally, *spatial visualisation* requires complex mental manipulation of spatial objects. Their study involved 167 adults performing two executive function tasks (Tower of Hanoi & random number generation), two visuo-spatial short-term memory and two visuo-spatial working memory tasks as well as two tasks for each of the three visuo-spatial processing factors described above. Confirmatory factor analysis revealed that central executive resources were implicated in both visuo-spatial short-term and working memory storage. Structural equation modelling then indicated that, whilst visuo-spatial storage was involved in all three visuo-spatial processing factors, spatial visualisation had the highest involvement of central executive processes, spatial relations involved the central executive to a lesser degree and that perceptual speed involved the least executive resources.

This distinction is relevant to the type of visuo-spatial processing task used in Chapter 2, because the face-matching task was a form of perceptual speed task. It comprised basic visual comparison, with little spatial content, and therefore had a low level of central executive involvement. The type of spatial visualisation task used in the visuo-spatial processing & visuo-spatial storage condition (the paper folding task) involves more central executive resources than does the basic visual comparison task (the face-matching task) used in the neutral processing condition or the phonological task (rhyming words) used in the verbal condition.

It therefore seems logical that the mathematicians performed better for visuo-spatial storage when it was combined with processing in the verbal domain or with a visual task involving lower levels of central executive resources. Mathematicians performed significantly worse when the processing element involved the greatest level of central executive resources. This was not the case, however, for the non-mathematicians who performed no differently for visuo-spatial storage whatever the type of processing involved. It seems that the non-mathematicians' worse ability to store visuo-spatial information, as shown in the neutral condition of Chapter 2, prevented them from being able to take advantage of less domain interference and store a larger amount of this material when visuo-spatial storage was combined with verbal or neutral processing.

Results across this thesis have consistently found no difference between mathematicians and non-mathematicians for verbal working memory storage capacity. However, the mathematicians' superior ability to store visuo-spatial information was only apparent when more working memory resources were available in the neutral processing condition within Chapter 2, which used a comparatively low level of central executive resources (section 5.3.5). This has implications in terms of mathematical cognition. If mathematicians are more efficient at ordering visuo-spatial information (as found in Chapter 4) and more efficient at remembering and applying calculation strategies (Dowker et al., 1996; Pesenti, 2005), all of which require working memory resources, mathematicians will have greater resources available to use their visuo-spatial storage advantage to hold, visualise and manipulate numbers during calculation (Geary, 2004; Heathcote, 1994; Seron et al., 1992). They will also have more working memory resources available to use their superior general visuo-spatial ability (section 5.3.1) to solve mathematical problems. This will be discussed further in the concluding chapter, Chapter 7.

General visuo-spatial processing ability, visuo-spatial working memory storage capacity and speed of visuo-spatial processing all seem to have a relationship with calculation performance. However, the regression analyses reported in section 5.3.4 showed general visuo-spatial ability and visuo-spatial storage significantly and uniquely predicted calculation scores, but there was no predictive relationship for speed of visuo-spatial processing. The relationship between general visuo-spatial ability and calculation supports the previous research of Wei et al. (2012) who found that this ability correlated with mathematics, although the authors did not then attempt to discover whether general visuo-spatial ability could uniquely predict mathematics performance. It also supports previous findings that general visuo-spatial processing has a role in complex mathematics such as algebra (Landy et al., 2014) and interpreting graphs (Hegarty & Waller, 2005) and generally in mathematics (Friedman, 1995). The finding that visuo-spatial working memory storage capacity also significantly and uniquely predicted calculation when added to the models suggests that mathematicians' superior capacity cannot just be explained by a better general ability to deal with visuo-spatial information.

Although the results from Chapters 2, 3 and the current chapter found a relationship between general visuo-spatial ability, visuo-spatial storage in working memory and mathematics, the results for the current chapter also suggest that the level of central executive involvement may contribute to visuo-spatial working memory differences between mathematicians and non-mathematicians. It seems that, through achieving a more efficient use of domain-general executive resources via better ordering of information and application of available strategies, mathematicians are then able to use their superior visuo-spatial abilities to store and manipulate information within working memory in order to solve complex mathematics problems. The relative roles of the central executive and visuo-spatial storage in adult mathematics will be examined in Chapter 6.

5.5 Conclusion

The current chapter investigated differences between adult mathematicians and non-mathematicians in verbal and visuo-spatial working memory capacity when the processing elements of the tasks involved were either verbal or visuo-spatial. Performance between mathematicians and non-mathematicians for general visuo-spatial processing ability was also compared. Results found that, as in Chapter 2, which used an as neutral as possible processing task, there was no difference between mathematicians or non-mathematicians for verbal storage capacity. Contrary to the results of Chapter 2, mathematicians did not display superior capacity in the visuo-spatial domain. Comparison of results across Experiment 2 of Chapter 2 and the current chapter suggested that mathematicians only have superior working memory storage capacity in the visuo-spatial domain when the processing involved uses a comparatively low level of central executive resources. The relative roles of the central executive and visuo-spatial storage in mathematics will be investigated further in Chapter 6. Mathematicians also displayed superior general visuo-spatial ability. Finally, a regression analysis found that both visuo-spatial working memory storage capacity and general visuo-spatial processing ability significantly and uniquely predicted mathematics calculation scores.

Chapter 6: The Involvement of the Central Executive and Visuo-Spatial Storage in Mental Arithmetic

6.1 Introduction

This chapter investigates the relative roles of the central executive and visuo-spatial storage in adults performing single digit and double digit additions, using a dual task methodology. Although the previous chapters have consistently found a relationship between visuo-spatial working memory and mathematics, the extent to which the central executive and visuo-spatial sketchpad are involved is not clear. The current chapter therefore attempts to discover the relative roles of the central executive and visuo-spatial storage.

Chapter 2 found that adult mathematicians have superior visuo-spatial working memory storage capacity to adult non-mathematicians using a working memory span task that contained a processing element with a comparatively low level of central executive involvement (Miyake et al., 2001). Chapter 3 indicated that it is this ability to store visuo-spatial information whilst also carrying out processing rather than simply short-term storage ability that predicts mathematics calculation performance. Chapter 3 also found that endogenous spatial attention, believed to be a function of the central executive (e.g. Baddeley, 2002; Cowan, 1995), predicted performance for arithmetic fluency, but not calculation. Therefore, Chapter 2 indicated a relationship between visuo-spatial working memory storage capacity, requiring both central executive and visuo-spatial sketchpad resources, and calculation and Chapter 3 indicated a relationship between the central executive and arithmetic fluency. Finally, Chapter 5 found no difference between adult mathematicians and non-mathematicians for visuo-spatial working memory storage capacity when the visuo-spatial span task used contained a processing element with a comparatively high level of central executive involvement (Miyake et al., 2001). Visuo-spatial working memory storage capacity did still, however, uniquely predict calculation ability, as did general visuo-spatial ability. The rotation task used to measure general visuo-spatial ability was one with a comparatively moderate level of central executive involvement (Miyake et al., 2001). Inclusion of processing elements with differing levels of central executive involvement within the working memory span tasks of Chapters 2 and 5 resulted in different patterns of results for visuo-spatial storage capacity differences between adult mathematicians and non-mathematicians. The relative involvement of the visuo-spatial sketchpad and the central executive within visuo-spatial working memory therefore appears to differ depending upon the type of mathematics being performed, the type of processing required and also depending upon whether direct retrieval of answers or procedural methods are being employed.

As discussed in Chapter 1, section 1.4, there is evidence within the previous literature that adults use visuo-spatial working memory when solving arithmetic problems. It seems likely that the involvement of visuo-spatial working memory varies according to the arithmetic strategy employed. Visuo-spatial representation and processing are likely to be particularly important for counting, which emphasizes the ordinal sequence of numbers. Similarly, decomposition strategies, which involve partitioning, storing, and recombining numbers, are likely to require visuo-spatial involvement. In contrast, it has been proposed that known addition facts are stored in a verbal, not visuo-spatial, code (Dehaene, 1992), and therefore retrieval of facts from memory should not require visuo-spatial working memory.

As well as strategy use, other factors such as problem size are also likely to influence the extent and nature of working memory involvement. A common feature of mental arithmetic is the problem-size effect, whereby error rates and reaction times increase with problem sizes (e.g., De Rammelaere et al., 1999; Seyler et al., 2003). Previous research has largely concentrated on single-digit arithmetic (LeFevre, DeStefano, Coleman, & Shanahan, 2005), although problems involving double digits are likely to be more dependent on working memory because they often require holding interim sums and carry-overs in working memory (Imbo, Duverne, & Lemaire, 2007). The effect of problem size on strategy has been investigated in single-digit arithmetic (Imbo, Vandierendonck, & Rosseel, 2007) and Hubber et al. (Experiment 1: 2014) investigated problem size in both double-digit and single-digit additions.

Whilst the previous chapters have investigated between-group differences for visuo-spatial working memory storage capacity and correlational evidence for its relationship with mathematics, the current chapter investigates the use of visuo-spatial working memory when actually performing mathematics. Hubber et al. (Experiment 1: 2014) investigated the role of visuo-spatial working memory in solving single-digit and double-digit arithmetic problems using three different strategies. Adult participants, from the general population, answered addition problems, using counting, decomposition and direct retrieval of number facts from memory, whilst under visuo-spatial load in an *n*-back dual task. Participants were slower to answer the problems and less accurate in the visuo-spatial load condition for all three strategies, but the slowing was greatest for counting. This implied that visuo-spatial working memory is used for all three strategies, but is particularly important when answering sums by counting. Participants were also slower to answer double-digit than single-digit sums and this difference was more pronounced when counting was used.

The findings from Hubber et al. (Experiment 1: 2014) suggest a role for visuo-spatial working memory in arithmetic and that it was recruited to different extents by different strategies and different problem sizes. However, the nature of the visuo-spatial *n*-back task used meant that it was unclear whether it was the demands of simply holding visuo-spatial information online, or controlling and manipulating this information that was interfering with solving the addition problems. According to the Baddeley and Hitch multi-component model of working memory (Baddeley, 2000; 2003) these two processes rely on different components of working memory: holding visuo-spatial information online is the function of the visuo-spatial sketchpad, which acts as a temporary store for visual and spatial information, whereas controlling and manipulating information in memory is the function of the central executive. This is responsible for attentional control and for the co-ordination of the visuo-spatial sketchpad and the phonological loop. The *n*-back secondary task used in Hubber et al. (Experiment 1: 2014) placed a load on both the visuo-spatial sketchpad and the central executive due to the requirement to continuously monitor and update a sequence in working memory. Use of the dual tasks within the current chapter, designed to separately load the central executive and visuo-spatial sketchpad, will help inform which of these working memory components are involved in actually performing arithmetic when different methods are used to solve the problems. Undergraduates from a range of different subjects used different strategies to answer addition sums whilst performing standard separate secondary tasks designed to independently load the central executive and visuo-spatial sketchpad. This should help inform the relative contribution of these two working memory elements for performing additions with each strategy type.

With regard to the visuo-spatial sketchpad, several researchers have proposed its fractionation, with two sub-systems: one, a visual system which holds information such as shape and colour and another which holds information about movement and spatial relations (Baddeley, 2003; Bull et al., 1999; Logie et al., 1994). Moreover, Pickering et al. (2001) suggested that the visuo-spatial sketchpad is fractionated between static and dynamic functions, rather than by visual and spatial, as a result of the discovery of a developmental dissociation in performance in the static and dynamic conditions of their experiments. Previous studies investigating the role of the visuo-spatial sketchpad in arithmetic have concentrated on loading its static, visual element during dual-task experiments (Imbo & LeFevre, 2010; Lee & Kang, 2002; Trbovich & LeFevre, 2003), such as remembering a pattern of asterisks. However, as suggested in Hubber et al. (Experiment 1: 2014) and Reuhkala (2001), the dynamic, spatial element of the visuo-spatial sketchpad also appears to be involved in mental arithmetic. Hegarty and Kozhevnikov (1999) found that the use of schematic spatial representations, as opposed to pictorial representations, was positively correlated with achievement in mathematical problem solving in 11 to 13 year olds. Also, the span tasks used in Chapters 2, 3 and 5 and the item and order memory tasks used in Chapter 4 all measured dynamic resources as presentation of visuo-spatial items were sequential rather than simultaneous. To systematically address the influence of maintaining static and dynamic visuo-spatial information on mental arithmetic, half of the participants in the present experiment completed a visuo-spatial sketchpad secondary task that involved maintaining static visuo-spatial information while the other half completed a dynamic visuo-spatial sketchpad secondary task. Both groups were also given the same central executive secondary task.

6.1.1 The Current Experiment

The current experiment investigated the extent to which the central executive and visuo-spatial sketchpad components of visuo-spatial working memory are used when adults solve mental arithmetic using direct retrieval, counting and decomposition strategies. It also investigated their comparative roles for answering single-digit and double-digit sums. Participants answered addition sums whilst completing no secondary task (described in section 6.2.3.1), a secondary task designed to load the central executive (described in section 6.2.3.2) and a secondary task designed to load the visuo-spatial sketchpad (described in section 6.2.3.3). The latter task involved participants completing either a dynamic or static visuo-spatial version. Recruitment of undergraduate participants across a range of subject areas allowed examination of the importance of the central executive and visuo-spatial sketchpad across a wide range of mathematical ability.

A distinction has been previously made in the literature between strategy selection and strategy execution, through the use of choice and no-choice conditions in experiments (e.g. Imbo, Duverne & Lemaire, 2007; Imbo & Vandierendonck, 2007b). In choice conditions, participants are able to choose which strategy they use to solve problems, allowing the investigation of strategy selection under different experimental conditions. In no-choice conditions, participants are given instructions as to which strategy they should use for solving problems, thus facilitating the investigation of strategy execution, with the complication of initial strategy selection removed, although this does remove an element of ecological validity (Imbo, Duverne & Lemaire, 2007). The present experiment used a no-choice method to enable the investigation of strategy execution under different working memory load conditions. As no-choice conditions rely on participants adhering to strategies they have been instructed to use, participants in the current experiment were asked to self-rate, after answering each set of problems, to what extent they had used the strategy required. Seyler et al. (2003) highlighted issues with self-report, including a possible feeling of obligation to report the use of a strategy that hadn't in fact been used, and underlined the importance of a "silent control" (p. 1,343) to provide a reassurance regarding strategy use and self-report. To this effect, in addition to the removal of participants who self-rated strategy adherence as '1', indicating they had hardly used the required strategy, overall results for the present experiment were checked to confirm that reaction times were fastest when using the direct retrieval strategy (Imbo & Vandierendonck, 2008; Imbo & Vandierendonck, 2010; Seyler et al., 2003) and that answers were most accurate when using the procedural strategies of counting or decomposition (Imbo & Vandierendonck, 2010).

A random letter generation task was selected to load the central executive. This required making a letter series random, rather than simply producing a serial string of letters. Random letter generation involves constant attention and switching between retrieval plans to avoid automaticity of previously learnt sequences, such as alphabet order, which is controlled by the central executive (Baddeley, 1996). It involves keeping track of items, and the inhibition of familiar sequences, requiring planning and control by the central executive (Logie et al., 1994). De Rammelaere et al. (1999) also found that random generation at a fixed rate involved the central executive.

Although random letter generation contains a verbal element and therefore involves the phonological loop, this task has previously been shown to load the central executive over and above any verbal involvement. Logie et al. (1994) found that random letter generation caused far more disruption to performance on an arithmetic task than did loading the phonological loop via articulatory suppression. Imbo & Vandierendonck (2007a) also found that, during a dual task, loading the phonological loop both passively and actively had little effect on performing additions.

Previous studies have used manual responding to high and low audible tones (e.g. Imbo & Vandierendonck, 2007b) and tapping a random pattern on a keyboard (e.g. De Rammelaere et al., 2001) as a central executive secondary task. However, both of these tasks require hand movements and these have been found to disrupt adult counting (Imbo, Vandierendonck & Fias, 2011). Therefore, based on the evidence above, a random letter generation task was deemed the most appropriate to load the central executive in the current experiment.

The following predictions were made for the current experiment. Firstly, it was predicted that central executive load would hinder direct retrieval of answers, but that visuo-spatial sketchpad load would not. This was predicted as Chapter 3 showed endogenous spatial attention, controlled by the central executive, predicted arithmetic fluency, which should largely reflect the direct retrieval of number facts rather than calculation in adults. Also, the four models of working memory discussed in Chapter 1 (section 1.3) all include a role for the central executive in retrieving number facts from long-term memory. Visuo-spatial short-term memory capacity of the visuo-spatial sketchpad did not predict arithmetic fluency in Chapter 3.

Secondly, in terms of the procedural strategies; decomposition and counting, it was expected that loading both the central executive and the visuo-spatial sketchpad would hinder arithmetic performance. Both of these strategies involve calculation rather than simply retrieving number facts and Chapter 5 found that visuo-spatial working memory capacity predicted calculation performance. Visuo-spatial working memory includes both the central executive involved in processing and the visuo-spatial sketchpad for the storage of information whilst the processing is carried out.

Thirdly, no firm predictions were made as to the relative importance of the central executive and visuo-spatial sketchpad. Previous chapters have consistently found visuo-spatial working memory to be implicated in calculation performance and Miyake et al. (2001) found the visuo-spatial sketchpad to be strongly linked to the central executive. However, previous studies have largely loaded visuo-spatial working memory as a whole rather than attempting to differentiate the involvement of the central executive and visuo-spatial sketchpad.

Fourthly, as previous studies have found a relationship between both static and dynamic visuo-spatial load and arithmetic, it was predicted that both would hinder answering the sums by the two procedural methods: decomposition and counting.

Fifthly, it was expected that the dynamic visuo-spatial secondary task would have more impact on decomposition and counting than would the static task. Chapter 4 found that the ordering of dynamic visuo-spatial information predicted calculation performance, but not arithmetic fluency. It was therefore expected that the two procedural methods involving calculation, requiring several steps, rather than the direct retrieval of number facts would show greater decrement in performance under a dynamic load.

Finally, it was expected that where central executive and visuo-spatial sketchpad load affected arithmetic performance, the impact would be greater for double-digit compared to single digit sums, following the previous finding that visuo-spatial working memory load had greater impact on answering double-digit sums (Hubber et al.: Experiment 1, 2014).

6.2 Method

6.2.1 Participants

G*Power 3 (Faul et al., 2007) suggested that a minimum sample size of 22 participants was required to detect an interaction, for two groups with nine measures (3 strategies x 3 working memory load types), with an effect size of .25. As in Chapter 2, this effect size was chosen as being a medium effect size per Faul et al. (2007). 45 participants were recruited from undergraduates at the University of Nottingham: 22 (6 male) were allocated to the static visuo-spatial task group and 23 (5 male) to the dynamic visuo-spatial task group on an alternate basis. Participants received either a course credit or a £6 inconvenience allowance for taking part in the study.

The static task group comprised participants from a variety of disciplines (11 Psychology; 5 Mathematics, 4 English, 1 Chemistry; 1 History). Their ages ranged from 18.27 years to 33.42 years (M = 19.75; SD = 3.18). On average they had not studied maths for 2.15 years (SD = 3.18).

The dynamic task group also comprised participants from a variety of disciplines (9 Psychology; 6 Mathematics, 4 English, 1 Physics; 1 Engineering; 1 French; 1 German). Their ages ranged from 18.10 years to 21.84 years (M = 19.26; SD = .98). On average they had not studied maths for 1.52 years (SD = 1.39).

6.2.2 Equipment

A Viglen Pentium D computer, running Windows XP and PsychoPy Version 1.73.06 (Peirce, 2007), was used to present stimuli and record latencies and accuracy.

Responses to the sums presented were made using a USB numeric keypad, whilst responses to the secondary visuo-spatial working memory task were made using a mouse. Responses to the central executive secondary task were recorded using a digital sound recorder. Participants used their right hand to use the keypad and their left hand to use the mouse.

6.2.3 Experimental Tasks

Participants answered 20 addition problems in each combination of answering strategy and working memory load type, giving a total of nine blocks (retrieval with sum-only, visuo-spatial, central executive; counting with sum-only, visuo-spatial, central executive; decomposition with sum-only, visuo-spatial, central executive). The way conditions were presented is depicted in Figure 6.1. The addition task (section 6.2.3.1), central executive secondary task (section 6.2.3.2) and visuo-spatial secondary task (section 6.2.3.3) are now described in the following sections.

6.2.3.1 Addition Task

Participants were required to answer arithmetic problems using three different strategies: retrieval, counting and decomposition. For example, for 7 + 6 = : Retrieval – give answer directly from memory; counting – from 7, count upwards 6 times; decomposition – first, add 3 onto 7 to get to 10, then add remaining units to get to the answer. Each problem contained two numbers and was presented horizontally, with the larger number on the left (e.g. 12 + 6 =). Nine sets of 20 experimental problems were used, resulting in 180 experimental problems. Participants were also given eight practice trials for each strategy. Within each problem set, half comprised solely single digit (1 to 9 omitting 0), and half comprised a double-digit number (max 29) on the left and a single-digit number on the right. The averages for sum totals were the same across each problem set. The current experiment used the same strategies— retrieval, decomposition, and counting—

and the same sets of addition problems (see Appendix K) as those in Hubber et al.: Experiment 1 (2014).[10]

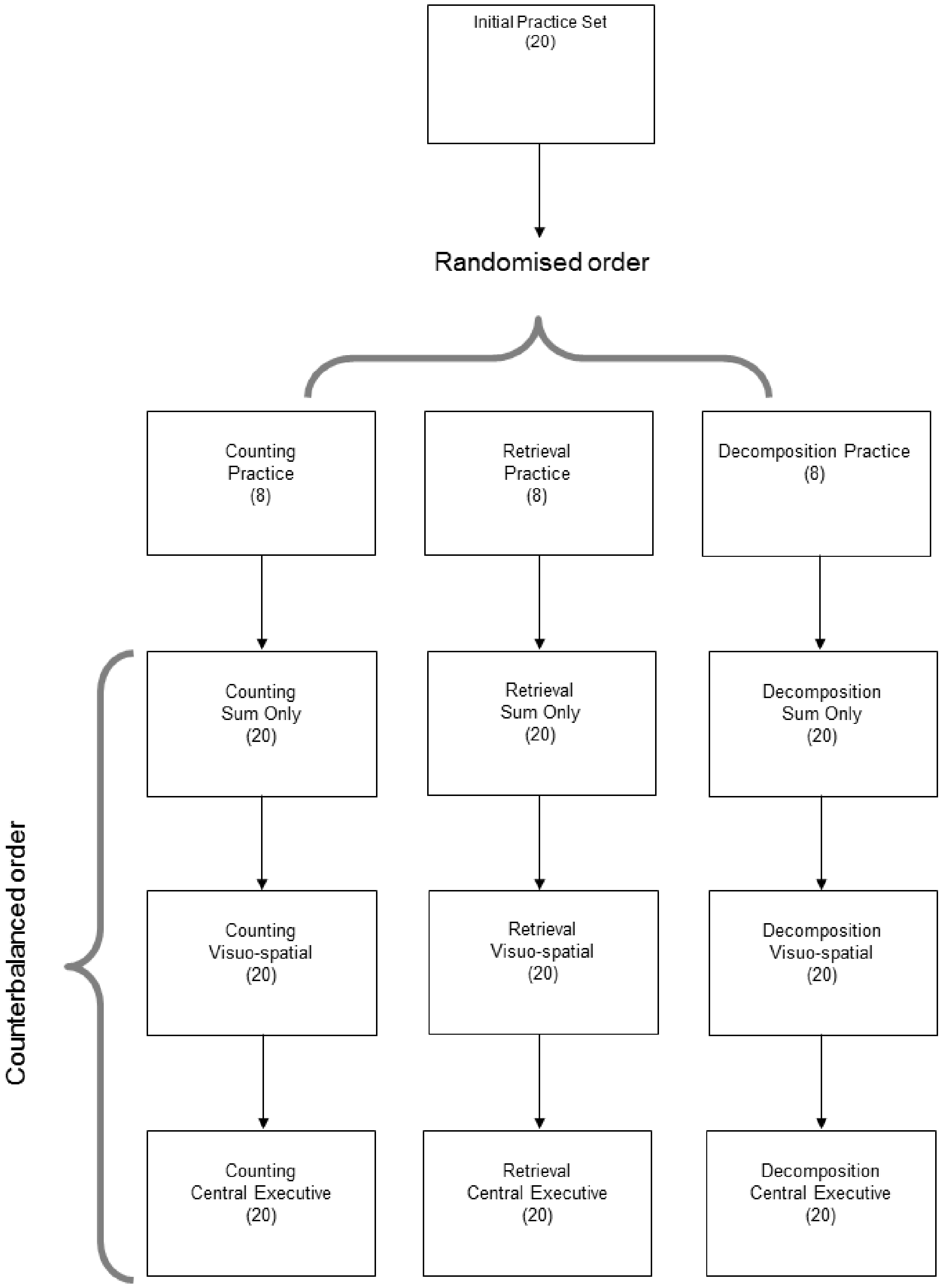

Figure 6.1: Task structure. Participants completed all three working memory conditions for a single strategy before moving onto the next strategy.

[10] In order to ensure that all nine problem sets were matched for mean size of the second addend as well as mean sum total, 8 problems were removed from the analysis leaving a total of 172 experimental trials. This was not required in Hubber et al. (Experiment 1: 2014) because the combination of problem sets with strategy/working memory condition was counterbalanced, something that was not possible in in the current study due to the experimental software used.

6.2.3.2 Central Executive Secondary Task

Participants were required to say letters from the alphabet out loud, at random, in time to a metronome set to one beat per second. Letter generation was continuous from the presentation of the first sum to the answering of the final sum in each block. Participants were instructed to avoid strings of letters, such as “a, b, c, d” and were not given a starting letter. Performance on the central executive task was measured by producing a score for randomness of the spoken letters, using RGCalc (Towse & Neil, 1998). The adjacency score measured the percentage of occasions that a spoken letter was directly followed by one of its immediate neighbours in the alphabet.

6.2.3.3 Visuo-Spatial Sketchpad Secondary Task

For both the static and dynamic groups, participants were required to memorize the positions of four red dots on a 4 × 4 black grid, presented in the centre of the screen. The two groups performed the same dots trials, but these were presented differently for the two groups, as described below in sections 6.2.3.3.1 and 6.2.3.3.2. Section 6.2.3.3.3 describes a pilot study carried out to ensure equal difficulty of visuo-spatial load on the blocks containing different types of arithmetic.

6.2.3.3.1 Static Version

In the static version of the task, the grid and all four dots on the grid were presented at the same time, for a total of two seconds. Immediately after, an addition problem was presented which participants had to answer using the required strategy for that block. As soon as the problem had been answered, a blank black grid was presented in the centre of the screen, and participants had to use the mouse to indicate the position of the four red dots, by clicking on the computer screen. The position of the mouse clicks was recorded by PsychoPy (Peirce, 2007). Once the mouse had been clicked four times, the next set of dots to remember was immediately presented. Performance was measured by calculating each participant’s proportion correct score for the number of dot positions remembered for each of the three strategies.

6.2.3.3.2 Dynamic Version

Participants in the dynamic group saw the same grid and sets of dots, but the dots were presented one at a time, for 0.5 seconds each. Once they had answered the problem, participants were required to use the mouse to indicate the position of the dots in the order that they were presented on a blank black grid. Performance was measured by calculating each participant's proportion correct score for the number of dot positions remembered, in the correct order, for each of the three strategies.

6.2.3.3.3 Pilot Study

Three blocks of visuo-spatial sketchpad task trials were required: one for each of the different arithmetic strategies. To ensure equal difficulty of the three blocks, a pilot study was initially conducted. In this pilot study, six postgraduate Psychology students each performed the dynamic version of the visuo-spatial sketchpad task trials for all three blocks, without answering any addition sums. The dynamic version was used for piloting as dynamic visuo-spatial tasks are generally found to be more difficult than static ones (e.g. Pickering et al., 2001; Reuhkala, 2001) and so the dynamic version was likely to result in greater variability between the blocks. As described for the dynamic group above (section 6.2.3.3.2), the dots in each trial were presented one at a time, for 0.5 seconds each. After presentation of the fourth dot, a blank black grid was presented in the centre of the screen, and participants had to use the mouse to indicate the position of the four red dots, by clicking on the computer screen. Performance was measured by calculating each participant's proportion correct score for the number of dot positions remembered, in the correct order, for each of the three blocks. Box plots were then created to highlight any trials where accuracy was greater than two standard deviations from the mean. As a result, two trials within the decomposition block were found to be more difficult than the other trials and they were then amended to be simpler.

6.2.4 Additional Materials

The *Woodcock-Johnson Math Fluency Test* (Woodcock, McGrew & Mather, 2001) and *WASI Matrix Reasoning* (WASI; Psychological Corporation, 1999) were administered using the standard procedures to measure mathematics fluency and non-verbal IQ, as described in Chapter 2 (sections 2.3.1.4 & 2.3.1.5).

6.2.5 Procedure

All participants were tested individually by the same experimenter and each session lasted around 50 minutes. Participants began by answering a set of 20 practice problems, using a free choice of strategy, before practising the visuo-spatial sketchpad and central executive tasks. They then began the experiment. The order in which the three strategies were used was assigned randomly, and participants completed all three working memory conditions for a single strategy (order counterbalanced) before moving onto the next strategy. Participants were told to give equal attention to the addition problems and the working memory tasks. The addition problems remained on screen whilst participants worked out the answer using the required strategy. Reaction time was measured from the time the problem appeared until the first digit of the answer was pressed. After keying the answer to the problem, the participant pressed enter, which immediately triggered the appearance of the next problem, in the sum only and central executive conditions, or the grid in the visuo-spatial condition. The order of trials within each block was randomised across participants.

At the end of each set of 20 problems, participants were instructed to self-rate on how many of the problems they had used the required strategy to answer, using the numeric keypad, on a scale of 1 to 5, where 1 was “hardly any”, and 5 was “almost all”.

Following completion of the computerised task, participants completed WASI Matrix Reasoning followed by Woodcock-Johnson Math Fluency.

6.3 Results

Of the 45 participants, two were removed from the static group (one male, one female) and three from the dynamic group (all female) as they had a self-rating of “1” at some point on the strategy check. This left data for 20 participants in the static group (5 male) and 20 participants in the dynamic group (5 male). The remaining 40 participants reported that they had used the required strategies on the majority of trials (retrieval, M = 4.68, SD = 0.67; decomposition, M = 4.35, SD = 0.72; counting, M = 4.44, SD = 0.62).

A Cook’s Distance score was calculated initially for each participant in each condition within all of the ANOVAs reported in the sections below to discover whether there were any influential cases that could affect the results of the ANOVAs. No influential outliers with a Cook’s Distance score >1 (Field, 2009) were detected. Controlling for gender had no significant impact on analyses and results reported below are without controlling for gender. Degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity where necessary.

In the sections below, results for standardised tests will be firstly reported (section 6.3.1), followed by results for comparison of the static and dynamic visuo-spatial groups in the visuo-spatial condition only (section 6.3.2) to examine whether the two visuo-spatial groups performed differently. Section 6.3.3 reports results for the arithmetic task. Performance on the central executive and visuo-spatial secondary tasks are reported in section 6.3.4.

6.3.1 Standardised Tests

An independent t-test to compare the two groups’ Woodcock-Johnson Math Fluency Test scores confirmed there was no significant difference between the static group (M = 121.80, SD= 27.78) and dynamic group (M = 126.55, SD = 20.55), for mathematical fluency $t(38) = -.62$, $p = .542$, $r = .10$. Scores for the static group represented a median percentile rank compared to age norms (Woodcock, McGrew &

Mather, 2001) of 53.00 (min = 8.00; max = 99.00). Scores for the dynamic group represented a median percentile rank compared to age norms of 62.50 (min = 12.00; max = 96.00).

An independent *t*-test showed that there was no significant difference between the two groups for WASI Matrix Reasoning non-verbal IQ (static: M = 26.10, SD = 3.11; dynamic: M =27.25, SD = 4.87), $t(38) = -.89$, $p = .379$, $r = .14$.

6.3.2 Comparison of Static and Dynamic Visuo-Spatial Groups

Initially, reaction times and accuracy for the arithmetic problems in the visuo-spatial condition only were analysed in two separate 3 (strategy: retrieval, decomposition, counting) × 2 (problem size: single digit, double digit) × 2 (visuo-spatial group: static, dynamic) mixed-design ANOVAs. This was performed to examine whether the two visuo-spatial task groups performed differently.

There was no main effect of visuo-spatial group on either RT, $F(1, 38) = .00$, $p = .968$, $r = .01$ or accuracy, $F(1, 38) = 1.07$, $p = .307$, $r = .17$, nor any significant interactions involving visuo-spatial group (all *p*s > .05).

As there were no significant main effects of group or interactions involving group, the data were collapsed across group for the analysis of arithmetic task performance.

6.3.3 Arithmetic Task

Reaction times and accuracy for the full arithmetic task were then analysed in two separate 3 (strategy: retrieval, decomposition, counting) × 3 (working memory type: sum-only, visuo-spatial, central executive) × 2 (problem size: single digit, double digit) repeated measures ANOVAs. Mean latencies, mean accuracy, and standard errors are shown in Table 6.1.

Table 6.1

Descriptive statistics for the arithmetic task. Reaction time (RT) is shown in milliseconds (ms)

		RT (ms): Double digit	RT (ms): Single digit	Accuracy: Double digit	Accuracy: Single digit
Strategy	Working Memory Load	M (SE)	M (SE)	M (SE)	M (SE)
Retrieval	Sum-only	1556 (84)	1176 (47)	.90 (.02)	.94 (.01)
	Visuo-spatial	1954 (126)	1683 (113)	.87 (.02)	.94 (.01)
	Central Executive	3452 (187)	2662 (150)	.82 (.03)	.89 (.02)
Decomposition	Sum-only	2916 (192)	2656 (183)	.92 (.02)	.94 (.01)
	Visuo-spatial	3691 (349)	2786 (240)	.94 (.01)	.97 (.01)
	Central Executive	6054 (486)	4277 (377)	.90 (.02)	.95 (.01)
Counting	Sum-only	4351 (183)	2358 (125)	.89 (.02)	.97 (.01)
	Visuo-spatial	4557 (276)	3006 (202)	.93 (.02)	.98 (.01)
	Central Executive	9655 (1511)	5833 (883)	.86 (.03)	.95 (.01)

6.3.3.1 Latencies

There was a significant main effect of working memory load type on RT, $F(1.11, 43.34) = 33.30$, $p < .001$, $r = .66$. Post hoc tests revealed that problems were solved more quickly in the sum-only condition than in the visuo-spatial load condition ($p = .007$), which in turn was faster than the central executive load condition ($p < .001$). There was a significant main effect of strategy on RT, $F(1.33, 51.72) = 34.28$, $p < .001$, $r = .63$. Problems were solved more quickly using retrieval than using decomposition ($p < .001$), which was faster than counting ($p = .002$). There was also a significant main effect of problem size, $F(1, 39) = 134.82$, $p < .001$, $r = 88$, with slower responses for double-digit than for single-digit problems.

There was a significant interaction between working memory load type and strategy, $F(1.08, 41.91) = 5.71$, $p = 0.019$, $r = .35$, suggesting that the secondary tasks had different effects on RT depending upon which arithmetic strategy was used. Tests of simple main effects demonstrated that there was a significant effect of working memory load type for each arithmetic strategy [retrieval, $F(2, 38) = 83.54$, $p <.001$, $r = .83$; decomposition, $F(2, 38) = 29.43$, $p <.001$, $r = .66$; counting, $F(2, 38) = 7.43$, $p = .002$, $r = .40$]. For all strategies, problems were solved faster in the sum-only condition than in the visuo-spatial condition, (ps $\leq .05$) and faster in the visuo-spatial condition than in the central executive condition (ps $<.001$). However, contrasts revealed a greater difference between the central executive and visuo-spatial conditions for counting than for retrieval, $F(1, 39) = 6.59$, $p = .014$, $r = 38$, or decomposition, $F(1, 39) = 4.92$, $p = .032$, $r = .33$, and for decomposition compared with retrieval $F(1, 39) = 7.39$, $p = .010$, $r = .40$. As shown in Figure 6.2, these contrasts reflect the fact that the central executive condition increased RTs more for the counting strategy than it did for the decomposition and retrieval strategies. Contrasts also showed there was a greater difference between the sum-only and visuo-spatial conditions for decomposition than for retrieval, $F(1, 39) = 6.34$, $p = .016$, $r = .37$, but not counting, $F(1, 39) = .05$, $p = .825$, $r = .04$. There was no significant difference in the slowing of latencies between the sum-only and visuo-spatial conditions for decomposition and counting, $F(1, 39) = .04$, $p = .835$, $r = .03$. There was no three-way interaction between strategy, working memory, and problem size, $F(1.35, 52.45) = 3.31$, $p = .063$, $r = .24$.

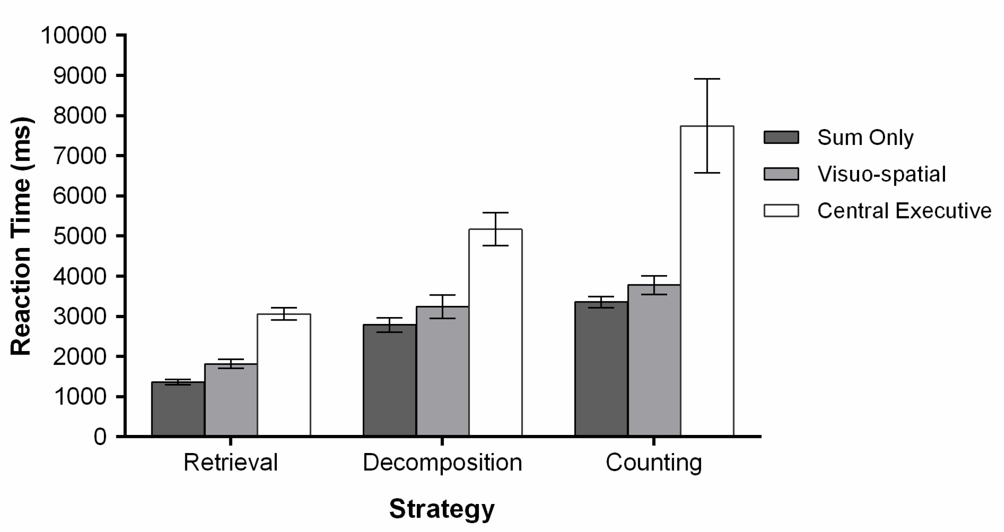

Figure 6.2: Arithmetic strategy and working memory condition interaction with S.E.M. error bars.

6.3.3.2 Accuracy

There was a significant main effect of working memory load type, $F(2, 78) = 11.95$, $p <.001$, $r = .36$. Post hoc tests revealed that arithmetic problems were solved more accurately in the sum-only ($p = .001$) and visuo-spatial ($p = .001$) conditions than in the central executive condition but that there was no significant difference in accuracy between sum-only and visuo-spatial ($p = .82$) conditions.

There was also a main effect of strategy, $F(1.54, 59.98) = 12.52$, $p <.001$, $r = .42$. Post hoc tests revealed that both counting ($p = .009$) and decomposition ($p <.001$) were more accurate than retrieval and that there was no significant difference in accuracy between counting and decomposition ($p = .92$). A significant main effect of problem size, $F(1, 39) = 42.54$, $p <.001$, $r = .72$, demonstrated that single-digit sums were solved more accurately than double-digit sums. There were no significant interactions: strategy x working memory load type, $F(3.19, 124.30) = 1.88$, $p = .133$, $r = .12$; strategy x problem size, $F(2, 78) = 3.08$, $p = .051$, $r = .19$; working memory type x problem size, $F(2, 78) = .63$, $p = .535$, $r = .09$; strategy x working memory type x problem size, $F(4, 156) = .81$, $p = .520$, $r = .07$.

6.3.4 Analysis of Secondary Tasks Performance

6.3.4.1 Central Executive Secondary Task

For the central executive task, a one-way ANOVA was carried out to compare performance for each of the three strategies (retrieval, decomposition, counting). Due to the design of the central executive task, performance could not be compared for single- and double-digit trials separately.

Mean adjacency scores (standard errors) for the random letter generation task, when using each arithmetic strategy, were as follows: retrieval, .22 (.02); decomposition, .20 (.01); counting, .21 (.02). There was no main effect of strategy, $F(1.50, 58.66) = 0.39$, $p = .622$, $r = .08$, showing that participants performed similarly on the central executive task irrespective of which addition strategy they were using.

6.3.4.1 Visuo-Spatial Secondary Task

For the visuo-spatial secondary task, a 3 (strategy: retrieval, counting, decomposition) × 2 (problem size: single digit, double digit) mixed ANOVA, with visuo-spatial task (static, dynamic) as a between-subjects factor, was performed.

There was a main effect of visuo-spatial task group, with participants in the static group performing significantly more accurately than those in the dynamic group, $F(1, 38) = 42.71$, $p < .001$, $r = .73$. There was also a significant main effect of strategy, $F(2, 76) = 24.28$, $p < .001$, $r = .49$. Post hoc tests revealed that performance in the visuo-spatial task was better whilst using retrieval than whilst using decomposition ($p < .001$) and counting ($p < .001$), but that there was no significant difference between performance whilst using decomposition and counting ($p = 1.00$). There was also a main effect of problem size, $F(1, 38) = 40.79$, $p < .001$ $r = .72$, with performance less accurate when answering problems containing double digits.

Although there was a main effect of visuo-spatial task group, this did not interact with strategy, $F(2, 76) = 1.51$, $p = .227$ $r = .14$, showing that participants in the dynamic group found the visuo-spatial task harder than those in the static group, no matter which arithmetic strategy was used. There was no visuo-spatial task × problem size interaction, $F(1, 38) = .19$, $p = .664$, $r = .07$. There was, however, a visuo-spatial task group × strategy × problem size interaction, $F(2, 76) = 4.60$, $p = .013$, $r = .24$. As shown in Figure 3, this was driven by a smaller difference in accuracy between the visuo-spatial task groups when retrieving single digit sums.

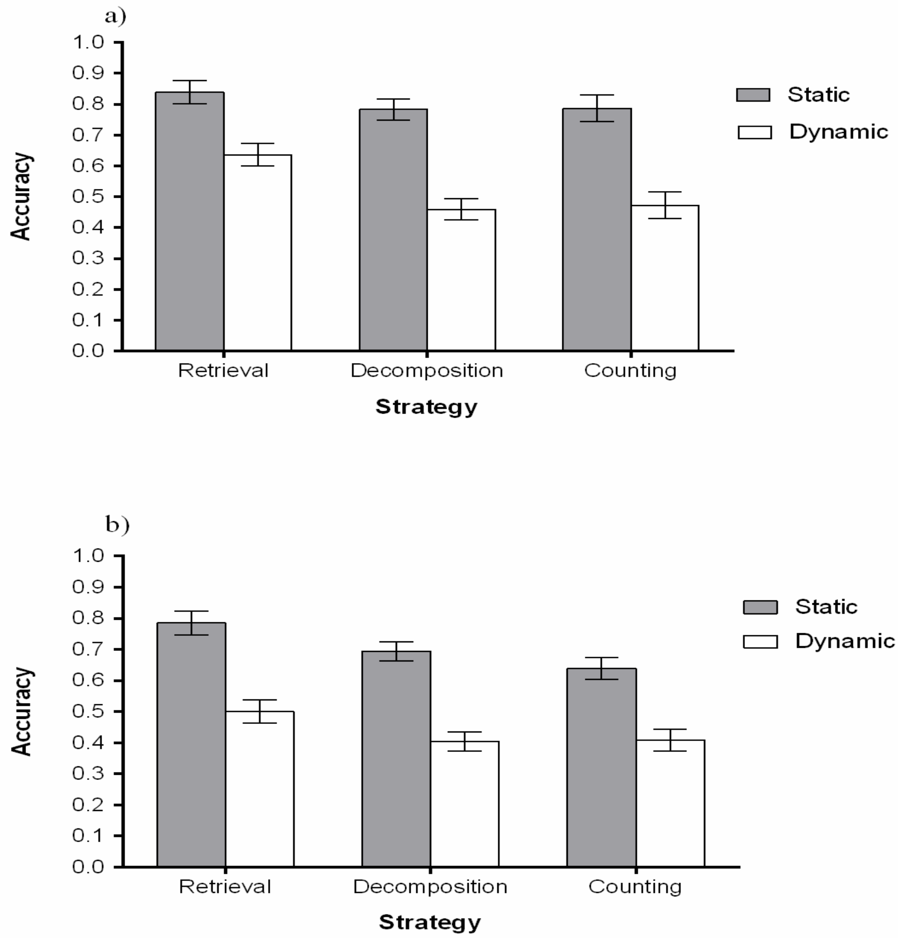

Figure 6.3: Accuracy in the secondary visuo-spatial task, for both dynamic and static groups, whilst answering (a) single-digit and (b) double-digit sums, with S.E.M. error bars.

6.4 Discussion

This chapter investigated the relative involvement of the central executive and visuo-spatial storage in adult arithmetic. Using a dual task methodology, participants answered addition sums, using three different strategies, whilst performing no secondary task, a central executive secondary task and a visuo-spatial sketchpad secondary task.

Results showed that the central executive load produced a greater impairment on arithmetic performance than the visuo-spatial sketchpad load in terms of both slower and less accurate responses. Moreover, the effect of central executive load slowed performance to a greater extent for counting than for decomposition and retrieval, and this was not due to a differential speed/accuracy or task trade-off across strategies. This clarifies the findings of Hubber et al. (Experiment 1: 2014) and indicates that the slowed counting in their experiment was likely to be due to increased load on the central executive, rather than the visuo-spatial nature of their *n*-back task.

The visuo-spatial task in the current study did not influence accuracy on the arithmetic task compared to the sum-only condition, but it did slow performance, albeit to a lesser extent than the central executive condition. It is not possible to completely rule out that this slowing was due to the general demands of performing a secondary task. However, it appears that maintaining visuo-spatial information in the visuo-spatial sketchpad plays a small role in solving addition problems whatever the strategy. This contradicted initial predictions that visuo-spatial storage would be required for procedural methods but not direct retrieval, based on the previous finding that short-term visuo-spatial storage predicted calculation performance but not fluency (Chapter 3). It appears then, that visuo-spatial storage is required for direct retrieval but that it is not an important factor contributing to individual differences in arithmetic fluency.

Similar patterns of performance on the arithmetic task were observed for both the static and dynamic visuo-spatial task groups. This contradicted initial expectations that the dynamic task would interfere more with the two procedural methods, as Chapter 4 had shown that memory for the ordering of visuo-spatial items, which is a dynamic process, predicted calculation performance. However, the types of sums included in the current study were far simpler than the majority of items included in the Woodcock-Johnson Calculation Test which was used to measure calculation ability in Chapter 4. Therefore, the different pattern of findings may well be due to the relative complexity of the mathematics measures used.

The dynamic group performed worse on the visuo-spatial task itself. This reflects the fact that the dynamic task is more difficult, requiring maintenance of the order as well as location of the stimuli. Better secondary visuo-spatial task performance for the retrieval strategy may reflect the fact that the visuo-spatial information did not have to be maintained for as long in this condition, as sums were answered faster using retrieval than when using decomposition or counting.

As expected, and in line with the findings of Hubber et al. (Experiment 1: 2014), results also confirmed that working memory load decreases performance to a greater extent when solving sums involving double digits than in those only involving single digits, suggesting it plays a greater role in more complex sums. This was true for both visuo-spatial and central executive load.

The present results show the central executive to be involved in counting, decomposition, and retrieval strategies, but to be particularly important for counting. This is consistent with a number of studies demonstrating that procedural strategies rely on the central executive to a greater extent than retrieval strategies (Hecht, 2002; Imbo & Vandierendonck, 2007b). The role of the central executive in counting is probably due to the need to store, switch between, and update several different pieces of information. For example, to solve the problem 9 + 4, it is necessary to store the size of the first addend, to increment this total as each counting step is performed (10, 11, 12, 13), and to maintain and update a record of the number of count steps made (1, 2, 3, 4). The coordination of information in memory such as this is known to be a key function of the central executive (Logie et al., 1994). The greater impact of the central executive than the dynamic visuo-spatial task on counting may also suggest that it is the central executive, rather than the visuo-spatial sketchpad store, that is important for the ordering of information in visuo-spatial working memory.

The central executive secondary task impaired performance on decomposition strategies, but to a lesser extent than counting. On the one hand this might be surprising because, like counting strategies, decomposition also involves the temporary storage and manipulation of several pieces of numerical information. However, it is possible that some elements of a given decomposition strategy relied on the recall of known facts and thus may have been less reliant on executive processes. Moreover, participants reported using different decomposition methods in the study, including estimating to the nearest 10 then subtracting, adding to the nearest 10, then adding units to get to the answer and also, where the initial addend was double digit, adding the units of the two addends first, before adding the product to the initial decade number. Thus, the use of these somewhat different strategies may have served to mask the overall effects of working memory that were observed. Although the study was designed to investigate strategy execution, there appears to have been an element of strategy selection within the decomposition condition, and this use of different methods should be investigated further, as decomposition strategies may differ in their reliance on working memory resources.

In contrast to previous research (Hecht, 2002; Imbo & Vandierendonck, 2007b), this study suggested that even direct retrieval of numerical facts relied on central executive processes to some extent. It is plausible that the use of more difficult two-digit addition problems may have caused participants to use strategies other than retrieval for these problems. However, it was found that there was a significant impact of central executive load for both the single- and double-digit problems. Single-digit addition problems are well learned, and educated adult participants, such as those involved in this study, should be able to directly retrieve these solutions. Retrieval of known facts involves more than just looking up an answer in long-term memory. Although there are some differences among models, it is generally believed that number facts are stored in a network of associations, such that a number pair (e.g., 6 + 7) will be associated with several possible solutions, with differing strengths (Ashcraft, 1992; Campbell, 1995; Siegler & Shrager, 1984). For individuals who are able to retrieve an answer correctly, the correct answer will have the strongest association; however, other surrounding answers may have weaker associations. Therefore in order to retrieve an answer to a known fact, it is necessary to select the appropriate fact and suppress others. This is in line with the controlled-attention model of working memory that considers inhibition to be an important element in fact retrieval (Engle, 2002; Kane et al., 2007) and also the time-based resource-sharing model that states similar items stored within long-term memory can cause interference (Barrouillet et al., 2007). In particular it is known that the answers to multiplication facts (i.e., 6 × 7 = 42) will interfere with retrieving the correct answer to known addition facts (i.e., 6 + 7 = 13) and vice versa (Campbell, 1987; LeFevre, Bisanz & Mrkonjic, 1988). It is likely that suppressing incorrect responses will be one process that requires central executive involvement in solving problems by retrieval.

In contrast to the large impact of central executive load on mental arithmetic, the visuo-spatial sketch pad only appeared to play a small role. This contribution was similar across all three strategies, which suggests that the visuo-spatial sketch pad may have been involved in holding the sum in mind, rather than in performing the different strategies themselves. Participants in the current study were well-educated adults rather than children and were asked to solve addition problems involving adding a single digit. It is possible that these problems were simple enough for participants to be able to solve them without recourse to visuo-spatial working memory. Indeed, Chapters 2 and 3, which also involved adult participants, found that short-term visuo-spatial storage within working memory predicted ability for calculation but not arithmetic fluency. Perhaps more complex problems (e.g. algebra: Landy et al., 2014; interpreting graphs: Hegarty & Waller, 2005; negative numbers: Robert & LeFevre, 2013) or those involving different operations (e.g. subtraction: Lee & Kang, 2002) may have required more visuo-spatial working memory involvement.

A second possible explanation for the apparent lack of involvement of visuo-spatial storage is that adult participants have alternative methods available for solving arithmetic problems. So while participants may use visuo-spatial storage for holding numerical information in some situations, verbal storage may be available as an alternative. Thus when participants are prevented from using visuo-spatial storage, due to the dual task, they fall back onto using verbal storage. Similarly, Seron et al. (1992) found that there are wide individual differences in the extent to which participants report visualizing numbers. It is possible that there are individual differences between which of the storage systems is the preferred and which the backup one is, although Chapters 2 and 5 found no difference between adult mathematicians and non-mathematicians for verbal working memory storage capacity. Contrasting participants' performance on arithmetic problems with different types of load would be a valuable avenue to explore these possible individual differences.

6.5 Conclusion

The current chapter investigated the relative roles of the central executive and visuo-spatial storage when adults solve arithmetic problems using three different strategies.

The results have shown clearly that the working memory system in general is heavily involved in the performance of even simple arithmetic in adults. The central executive load had a greater impact on the performance of all addition strategies than visuo-spatial storage load. Counting placed more demands on this aspect of working memory than other strategies. While visuo-spatial storage does not appear to be as important for mental addition, it may play a role in other types of arithmetic such as subtraction. It may also play a role in more advanced mathematics, such as algebra, interpreting graphs and the use of negative numbers.

Chapter 7: General Discussion

7.1 Introduction

In Chapter 1, I set out four main research questions that I intended to investigate throughout this thesis. Chapters 2 to 6 then reported six experiments designed to answer those questions. Sections 7.2 to 7.5 of the current chapter will now discuss each of these main questions in light of the evidence from the experimental chapters. Section 7.6 will then include a discussion around limitations of this thesis together with suggestions for future research. Finally, section 7.7 will summarise my conclusions.

7.2 Are there Working Memory Capacity Differences between Adult Mathematicians and Non-Mathematicians?

Within Chapter 1, I discussed previous evidence that adults use both verbal and visuo-spatial working memory when solving mathematical problems (e.g. Raghubar et al., 2010; Wilson & Swanson, 2001). I also explained how the controlled-attention and multi-component models of working memory include an assumption that the storage of verbal and visuo-spatial information takes place in domain-specific stores rather than relying on entirely domain-general resources. However, no one had compared the working memory capacity of skilled adult mathematicians to the capacity of those less skilled at mathematics to discover whether the verbal or visuo-spatial domain or both are related to mathematics achievement. Differences between adult mathematicians and non-mathematicians for verbal and visuo-spatial working memory capacity were therefore investigated within two chapters of this thesis.

Chapter 2 employed working memory span tasks involving a face-matching processing task (Burton et al., 2010) that was as neutral as possible with regard to the storage items, to ensure consistency across the storage-type conditions of the experiments. Capacity for storing visuo-spatial information and both numbers and words in the verbal domain was examined. Results for Experiment 1 showed mathematicians had superior capacity for storing visuo-spatial information within working memory, but were no better than the non-mathematicians for storing verbal information. Also, visuo-spatial working memory scores correlated with mathematics ability (measured by the Woodcock-Johnson Calculation Test), but verbal working memory scores did not. Experiment 2 again examined working memory capacity differences for numbers and visuo-spatial information, with different participants to those in Experiment 1, and found a very similar pattern of results. Comparison of the serial position curves of the mathematicians and non-mathematicians for the visuo-spatial conditions in the two experiments showed no difference in the profiles of their curves at any span length. Mathematicians were simply able to store more information in total. There was also no difference between the mathematicians and non-mathematicians for performance on the neutral face-matching processing element.

The pattern of results from both experiments in Chapter 2 supported the view within the controlled-attention and multi-component models of working memory that the storage of information within working memory is indeed domain-specific. Taken overall, these results suggested that adult mathematicians have superior capacity for storing visuo-spatial information within working memory, but not for verbal information. Results also suggested that mathematicians do not have any advantage for remembering numerical information compared to other material within the verbal domain. Therefore, despite the fact that both verbal and visuo-spatial working memory have been previously found to be implicated in adults solving mathematical problems, the link between mathematics and visuo-spatial working memory appears to be strongest in adults. A different pattern of results to those in Chapter 2 was, however, found in Chapter 5.

Chapter 5 also compared performance of a group of mathematicians to a group of non-mathematicians on working memory span tasks, but this time, instead of using a neutral processing element, the tasks employed verbal and visuo-spatial processing. There was again no difference between the two groups for the storage of verbal material within working memory, but the mathematicians were no better at storing visuo-spatial information either. This supported previous findings in the literature that the type of processing undertaken affects storage ability (e.g. Jarrold et al., 2011; Shah & Miyake, 1996).

Comparison of the results from the conditions involving visuo-spatial storage in Chapter 5 and Experiment 2 of Chapter 2 confirmed that the mathematicians only had superior ability to the non-mathematicians for storing visuo-spatial information within working memory when storage was combined with neutral processing. Moreover, the storage ability of the non-mathematicians for visuo-spatial information did not vary with the type of processing, whereas the mathematicians exhibited better storage when combined with verbal or neutral processing when compared to visuo-spatial processing. As the visuo-spatial processing task contained a greater level of central executive resources than the neutral and verbal tasks (Miyake et al., 2001) it appears that the level of central executive resources required in a processing task within working memory is an important factor in mathematicians being able to take advantage of their superior visuo-spatial storage ability. Also, whereas the mathematicians were no faster at performing the neutral and verbal processing tasks, they were significantly faster at the visuo-spatial processing task.

As the comparison of visuo-spatial working memory scores across Chapters 2 and 5 found mathematicians to have superior capacity only when the central executive resources involved in processing were comparatively low, it might be expected that mathematicians would also have superior visuo-spatial short-term memory scores when no processing was present. This was not the case, however, when visuo-spatial short term memory performance was measured in Chapter 3, resulting in no significant difference between the groups of mathematicians and non-mathematicians (section 3.3.2.1). I see two possible explanations for this. Firstly, the mathematicians displayed higher mean visuo-spatial short-term memory scores than the non-mathematicians and this difference was approaching significance using a non-parametric test. As discussed in section 2.2.1.1 of Chapter 2, the power calculations within this thesis state the number of participants required to detect interactions and that there is less power for detecting main effects. This lack of significance may therefore be the result of a lack of power. Use of a larger sample size within the Chapter 3 experiment may have enabled the detection of a significant main effect for visuo-spatial short-term memory capacity. Secondly, whilst the short-term memory task in Chapter 3 involved no processing, the working memory task in Chapter 2 required constant switching between the processing and storage elements of the task. It may be that the mathematicians used central executive resources more efficiently than the non-mathematicians, in the neutral processing condition, whilst combining processing and storage and this resulted in a greater availability of working memory resources to store visuo-spatial information. A large central executive load in the visuo-spatial processing condition of Chapter 5 may have caused this advantage in central executive efficiency to disappear.

In summary, the results for the working memory tasks in Chapters 2 and 5 suggested no difference between skilled adult mathematicians and non-mathematicians for either verbal processing or storage ability within working memory. Although verbal working memory has been previously found to be used in mathematics (Wilson & Swanson, 2001) and the phonological loop has been found to be used to store interim results during calculation (Fürst & Hitch, 2000), individual differences within the verbal domain do not seem to contribute to individual differences in adult mathematics achievement. Within the visuo-spatial domain, however, mathematicians seem to be faster at processing visuo-spatial information within working memory and seem to have a greater ability to store visuo-spatial information when the central executive resources required for processing are comparatively fewer. Results from the experiments within Chapter 2 suggest that storage within working memory is domain-specific, but that the domain-general central executive resources required for processing need to be relatively low in order that mathematicians are able to utilise their visuo-spatial storage advantage. The implications of this for solving mathematical problems will be discussed in section 7.4.

7.3 What drives the Relationship between Visuo-Spatial Working Memory and Mathematics Achievement?

Having found that mathematicians had superior visuo-spatial working memory capacity when storage was combined with neutral processing in Chapter 2, I then examined, in Chapter 3, the contributions to this advantage of more basic elements.

Visuo-spatial working memory involves both storage and processing (Adams & Hitch, 1997). According to the multi-component model of working memory, temporary storage of visuo-spatial information occurs within the visuo-spatial sketchpad and the central executive controls working memory (e.g. Baddeley, 2002). The latter includes processing, shifting between tasks and cognitive flexibility, monitoring and updating, retrieving information and controlling attention.

Chapter 3 therefore examined differences between adult mathematicians and non-mathematicians for performance on a visuo-spatial short-term memory task which measured capacity of the visuo-spatial sketchpad when no concurrent processing was taking place. Differences in endogenous (controlled) attention were also examined through a Posner (1980) task. Endogenous attention has been previously found to be important for visuo-spatial working memory performance (Astle & Scerif, 2011; Awh et al., 1998; Awh et al., 2006; Gazzaley & Nobre, 2012). However, I found no significant difference between the mathematicians and non-mathematicians for performance on either task. Moreover, the mathematicians retained their superior visuo-spatial working memory advantage when working memory scores were included in an ANCOVA which controlled for short-term memory and endogenous spatial attention performance. Also, when short-term memory, endogenous attention and working memory were included in a regression analysis only the latter uniquely and significantly predicted mathematics calculation ability. These results suggested that basic temporary storage and endogenous attention do not drive the difference in visuo-spatial working memory capacity between mathematicians and non-mathematicians.

The finding that short-term memory did not drive working memory differences supported the view within the literature that working memory performance is more predictive of ability in complex cognitive tasks such as mathematics (Bayliss et al., 2003; St. Clair-Thompson & Sykes, 2010). According to the multi-component model of working memory, central executive resources are important in the processing that takes place within working memory (e.g. Repovš & Baddeley, 2006). However, results from Chapter 3 indicate that differences in attention controlled by the central executive are not responsible for differences in mathematical ability. The contribution of a further element was then examined in Chapter 5.

Chapter 5 included a measurement of general visuo-spatial ability: the MRT-A test (Peters et al., 2005). This was to discover whether mathematicians' superior visuo-spatial working memory capacity was simply due to a greater ability to deal with and manipulate visuo-spatial material. The test involved the mental rotation of 3-dimensional objects and had a greater level of central executive involvement than the neutral processing element employed in Chapter 2 and the verbal processing element in Chapter 5, but a lower level than the visuo-spatial processing element of the working memory span tasks used in Chapter 5 (Miyake et al., 2001). Mathematicians performed significantly better than the non-mathematicians on this test, suggesting they have better general visuo-spatial ability. However, when included in a regression together with visuo-spatial processing speed and visuo-spatial working memory capacity, both general visuo-spatial ability and visuo-spatial working memory capacity uniquely and significantly predicted mathematics calculation ability. This provided evidence that mathematicians' superior visuo-spatial working memory capacity is not simply driven by general visuo-spatial processing.

A forced-choice recognition task employed in Chapter 4 found the ability to order visuo-spatial information was related to mathematics performance. The ability to correctly choose which of a pair of visuo-spatial locations that had previously been presented in a larger set had appeared earliest predicted calculation ability. Simply remembering which of a pair of locations was present in a set of previously presented items (item memory) did not predict calculation ability. Further research, measuring performance of mathematicians and non-mathematicians for order memory and visuo-spatial working memory involving neutral processing within the same study, would also enable examination of whether visuo-spatial working memory capacity is still able to predict mathematics performance when the ability to order visuo-spatial information is taken into account.

There is a debate within the literature as to whether the order of visuo-spatial items is maintained within the visuo-spatial sketchpad or by the central executive. Logie (1995) suggests that the inner scribe of the visuo-spatial sketchpad is responsible for order maintenance, whilst Baddeley (2000) suggests it may be a function of the central executive via controlled attention. Although this thesis did not set out to resolve this issue, the experimental evidence it contains perhaps points more towards the central executive. If the ordering of visuo-spatial information takes place within the visuo-spatial sketchpad and ordering ability predicts mathematics ability (Chapter 4), it could be expected that mathematicians would have superior visuo-spatial short-term memory. This was not the case, however. Also, mathematicians were only better than non-mathematicians at recalling, in their correct order, visuo-spatial items stored within working memory when the processing element was neutral and had the lowest level of central executive involvement. When the processing element was the visualisation task with the highest level of central executive load, this advantage disappeared. This suggests that loading the central executive may hamper the mathematicians' superior ordering ability.

The view that the central executive is responsible for the ordering of visuo-spatial information has recently been supported by Allen, Baddeley & Hitch (2014). Their series of three experiments found that, whilst a secondary task loaded central executive resources, the final to-be-remembered- item in a visuo-spatial sequence was stored in a privileged state. However, there was a disruption in the memory for earlier items. This could be explored further through comparing performance on the forced-choice recognition task (Chapter 4) in a no load and a central executive load condition.

The involvement of the central executive in ordering visuo-spatial information may therefore contribute to the relationship between the central executive and the visuo-spatial sketchpad being greater than that between the central executive and the phonological loop (Miyake et al., 2001). The phonological loop, rather than the central executive, performs the ordering of verbal items via articulatory rehearsal (Baddeley et al., 1975). This may explain why mathematicians did not have superior verbal working memory capacity to the non-mathematicians. With increasing evidence for the central executive's role in the link between visuo-spatial working memory and mathematics in adults, Chapter 6 investigated the relative roles of the central executive and visuo-spatial sketchpad when adults actually performed mathematics.

Chapter 6 measured the performance of adults when solving single-digit and double-digit additions whilst simply solving the sums, solving them under visuo-spatial sketchpad load and solving them under central executive load. Participants also used three different strategies for solving the problems. Although loading the visuo-spatial sketchpad decreased performance across all of the conditions, the impact of loading the central executive was greater and particularly so when counting was used. This again supported the view that the central executive is an important component in the link between visuo-spatial working memory and mathematics.

In summary, the more basic ability of temporary storage within the visuo-sketchpad was not found to contribute to visuo-spatial working memory capacity differences between mathematicians and non-mathematicians, suggesting that differences lie at a higher cognitive level involving the central executive. However, ability for controlled spatial attention, believed to be performed by the central executive, did not drive the working memory differences either. Whilst general visuo-spatial ability, previously found to involve the central executive (Miyake et al., 2001), did not explain differences in the storage of visuo-spatial information within working memory, it did predict performance at mathematical calculation, as did visuo-spatial working memory storage capacity. Loading the central executive and the visuo-spatial sketchpad whilst adults actually solved mathematical problems found the central executive played a greater role than did the visuo-spatial sketchpad. Also, the ability to order visuo-spatial items was found to predict mathematics achievement and experimental evidence within this thesis perhaps points to the central executive being responsible for this ordering.

The evidence discussed within this section and section 7.2 above consistently suggests that visuo-spatial working memory capacity predicts mathematics performance in adults and that the central executive plays a crucial role. How mathematicians' superior visuo-spatial working memory enables their proficient solving of mathematical problems will now be discussed in the next section.

7.4 How does having Good Visuo-Spatial Working Memory assist the Proficient Solving of Mathematical Problems?

The experimental chapters within this thesis have consistently provided evidence for a role for visuo-spatial working memory when adults perform mathematics. Several elements of visuo-spatial working memory have also been discovered to be linked to the proficient solving of mathematical problems.

Mathematicians have been found to have greater general visuo-spatial ability to non-mathematicians and also to be able to store more visuo-spatial items within working memory when processing involves a low involvement of the central executive. Several chapters have also found that the ability to store visuo-spatial items within working memory, general visuo-spatial ability and the ability to order visuo-spatial items predicts mathematics performance. Both experiments within Chapter 2 found a strong correlation between visuo-spatial working memory capacity and mathematics ability and, in Chapter 5, both this capacity and general visuo-spatial ability significantly and uniquely predicted mathematics performance. In Chapter 4, visuo-spatial order memory, but not item memory, predicted ability in mathematics. Chapter 6 showed that loading both the central executive and visuo-spatial sketchpad hampered the solving of addition problems. However, experimental results throughout this thesis have indicated that the importance of visuo-spatial working memory, and its components, when adults perform mathematics varies depending upon the type of mathematics being performed.

Two different measures of mathematics ability have been employed throughout the experimental chapters. *Woodcock-Johnson Math Fluency Test* measured how many simple arithmetic problems participants could answer correctly within three minutes. It consisted of basic addition, subtraction and multiplication problems that the majority of adults would be able to solve by directly retrieving number facts stored within long-term memory, without the need for any form of calculation to be performed. *Woodcock-Johnson Calculation Test* measured participants' ability to attempt more complex mathematics that required procedural steps to be followed to solve the problems. The items increased in difficulty and ranged from solving double-digit additions and long division through to decimals and fractions then onto items such as matrices, simultaneous equations and trigonometry. Different relationships were found between visuo-spatial working memory and the two mathematical tests, suggesting the former plays a different role when direct fact retrieval or calculation is required.

In Chapter 3, when basic short-term storage in the visuo-spatial sketchpad and controlled spatial attention via the central executive were taken into account, visuo-spatial working memory capacity predicted calculation ability, but not retrieval fluency for answering arithmetic problems. Fluency scores were only predicted by controlled spatial attention. In Chapter 5, the ability to store visuo-spatial information within working memory again predicted calculation ability. These results indicate that visuo-spatial working memory, involving both storage and processing, predicts the ability to perform mathematical calculations whilst the ability to control spatial attention predicts the ability to retrieve number facts from long-term memory. Therefore, the efficient use of different elements of visuo-spatial working memory assists the successful answering of different types of mathematical problems.

The direct retrieval of answers from long-term memory seems to be supported by the use of controlled attention via the central executive. Previous research has found that those who are more proficient at mathematics tend to rely more on direct retrieval of number facts as a strategy for solving mathematical problems than do those who are less proficient (Imbo et al., 2007). Also, those with greater working memory capacity tend to use direct retrieval more (Barrouillet & Lépine, 2005). Group differences for the Math Fluency Test in Experiment 2 of Chapter 2 (section 2.3.2.1) showed that mathematicians were significantly better than non-mathematicians for retrieving arithmetic answers directly from memory. With controlled attention performance predicting arithmetic fluency, it therefore seems that good controlled attention is related to fluency in retrieving number facts from long-term memory.

Efficient use of central executive resources appears to assist mathematicians in answering more complex problems that require a greater amount of visuo-spatial working memory resources. Retrieving number facts directly from long-term memory requires less visuo-spatial working memory resources than using procedural methods to answer problems (Hubber et al., Experiment 1: 2014) and therefore leaves more resources available for the processing and storage of information. This is also true of efficient strategy selection and execution (Imbo et al., 2007; Imbo & Vandierendonck, 2010). Dowker (1992) and Dowker et al. (1996) found expert mathematicians used a wider variety of strategies to solve problems and were also better at selecting and executing the most appropriate strategies. Otsuka & Osaka (2014) have recently published a similar finding within the verbal domain. In their dual task study, they found that skilled mathematics performers reduced their reliance on working memory resources through choosing strategies involving less use of the phonological loop. As the direct retrieval of number facts, carry overs and strategy selection and execution require central executive resources (e.g. Fürst & Hitch 2000; Imbo & Vandierendonck, 2007a & b; Imbo et al., 2007) and mathematicians are more efficient at performing these elements, they therefore have more working memory resources available to take advantage of their superior visuo-spatial working memory storage capacity which only became apparent when the requirement for central executive resources was lower (Chapter 5, section 5.3.5).

Therefore, when presented with a mathematical problem that requires calculation using a procedure with more than one step, those proficient at mathematics are better able to retrieve required number facts from memory (Chapter 2, section 2.3.2.1), and better able to select the appropriate strategy and to execute it (Dowker, 1992; Dowker et al., 1996). This leaves more working memory resources available for mathematicians to use their superior ability to store visuo-spatial information whilst carrying out processing (Chapter 2) and their superior general visuo-spatial ability (Chapter 5) in any calculation that is required. The extent to which these elements are then required to solve a problem seems to depend upon the type of calculation taking place.

Visuo-spatial working memory has been previously found to be used in solving additions (Hubber et al., Experiment 1: 2014), subtractions (Lee & Kang, 2002), approximations (Logie et al., 1994), interpreting initial questions and operands (Jiang et al., 2014; Landy et al., 2014; Pinhas et al., 2014), interpreting graphs (Hegarty & Waller, 2005) and using visual images to solve problems (Bull et al., 1999; Holmes at al., 2008). The spatial magnitude of numbers has also been implicated in solving arithmetic (Marghetis et al., 2014; Wiemers et al., 2014). The greater ability of mathematicians to store visuo-spatial information within working memory when executive resources are used efficiently and their better general visuo-spatial ability should therefore aid mathematicians in their superior performance at these types of mathematics. Following the finding that visuo-spatial working memory is used by adults for solving addition problems (Hubber et al., Experiment 1: 2014), Chapter 6 of this thesis then attempted to understand the relative roles of the visuo-spatial sketchpad and central executive.

Chapter 6 examined performance on solving single digit and double digit additions, using retrieval, decomposition and counting strategies, whilst under no load, loading the visuo-spatial sketchpad and loading the central executive. Results showed that both the visuo-spatial sketchpad and central executive were used when solving the problems using all of the strategies and more so for double-digit (involving carry overs) compared to single-digit sums. However, loading the central executive produced a greater decrement in performance compared to the visuo-spatial sketchpad in terms of both speed and accuracy and particularly in terms of speed of counting.

As discussed in section 6.4, the finding of a greater involvement of the central executive in counting supports the need to store, order, switch between and continually update information within working memory. It also ties with the finding in Chapter 4 that the ability to order visuo-spatial information predicts mathematics ability. Moreover, it supports the view that strategies requiring several steps to solve a problem rely on more working memory resources than those involving fewer steps (e.g. Hecht, 2002; Imbo & Vandierendonck, 2007b).

Given the links between mathematics ability and general visuo-spatial skills (e.g. Chapter 5; Friedman, 1995; Wei et al., 2012), it is perhaps surprising that there was such a small effect of maintaining visuo-spatial information within the visuo-spatial sketchpad on arithmetic performance. This finding contrasts with previous evidence showing relationships between arithmetic performance and visuo-spatial working memory tasks (Dumontheil & Klingberg, 2012; Heathcote, 1994; Reuhkala, 2001; Simmons et al., 2012; Trbovich & LeFevre, 2003). There are perhaps two possible explanations for the limited involvement of visuo-spatial storage. First, participants in Chapter 6 were well-educated adults rather than children, and were asked to solve addition problems involving adding a single digit. It is possible that these problems were simple enough for participants to be able to solve them without recourse to visuo-spatial working memory. Indeed, Chapter 3, which also involved adult participants, found that visuo-spatial storage during the use of working memory only predicted performance on complex mathematics (*Woodcock-Johnson Calculation*) rather than basic arithmetic fluency (*Woodcock-Johnson Math Fluency*). Perhaps more complex problems or those involving different operations may have required more visuo-spatial working memory involvement (e.g. algebra: Landy et al., 2014; graphs: Hegarty & Waller, 2005; subtraction: Lee & Kang, 2002). Studies which involved multiple arithmetical operations and allowed participants to use a wider range of strategies would be needed to better understand the involvement of all components of working memory in arithmetic. Future studies should also investigate the relative roles of working memory components for different forms of more complex mathematics.

In summary, visuo-spatial working memory has been previously found to be involved in a variety of types of mathematics (e.g. Landy et al., 2014, Hegarty & Waller, 2005, Lee & Kang, 2002). Good visuo-spatial working memory seems to be linked to mathematics performance in different ways depending on the type of problems being solved. Controlled attention ability via the central executive predicts mathematics ability for retrieval fluency. Skilled mathematicians are better at retrieving number facts from long-term memory (Chapter 2) and at selecting and executing appropriate strategies (Dowker, 1992; Dowker et al., 1996), both of which lessen the requirement of working memory resources for solving mathematical problems. As Chapter 5 (section 5.3.5) found that mathematicians have a superior ability to store visuo-spatial information in working memory when fewer central executive resources are required for processing, more efficient use of retrieval and appropriate strategies should enable mathematicians to use their superior visuo-spatial working memory storage capacity and superior general visuo-spatial ability for solving problems. Chapter 6 showed that central executive resources are required more than visuo-spatial sketchpad resources in solving addition problems, particularly when solving them using counting which requires several steps and the continuous monitoring and updating of information. Future research should further examine the roles of the components of working memory when performing different types of mathematics, as the relative requirements of central executive, phonological and visuo-spatial resources may vary with the type of problem being solved.

7.5 How can Working Memory be incorporated within a Model of Mathematical Cognition?

As discussed in Chapter 1, section 1.2.4, the abstract code (McCloskey, 1992), triple code (Dehaene, 1992) and encoding-complex hypothesis (Clark & Campbell, 1991) models of mathematical cognition do not include a role for working memory. This thesis, however, has consistently found a relationship between working memory performance and mathematics achievement, and particularly between visuo-spatial working memory and mathematics. There is therefore a need to incorporate working memory within a cognitive model that explains the processes involved when an individual solves a mathematical problem. This was also proposed in a review of mathematics and working memory (Raghubar et al., 2010).

Chapters 3 and 5 of this thesis both found that the ability to store visuo-spatial items within working memory, during concurrent processing, predicted performance for mathematical calculation. It was consistently found that the ability to store verbal items during concurrent processing did not predict mathematics scores. Although these results have supported a domain-dissociation for storage during the use of working memory, in line with the multi-component model, they have also highlighted the involvement of the central executive.

Results within Chapter 3 showed it was the ability to store visuo-spatial items within working memory (when processing was taking place) rather than simple short-term memory (without processing) that predicted mathematics scores. Whilst there were clearly individual differences across participants regarding the amount of information that could be stored temporarily within the visuo-spatial sketchpad, supporting the view that it is limited by capacity, capacity only predicted mathematics achievement when the central executive was involved to a greater extent through the use of working memory.

The link between the central executive and mathematics was also highlighted in Chapters 5 and 6. Chapter 5 results showed that the level of central executive involvement in the processing element within working memory affected the ability of mathematicians to store visuo-spatial information (section 5.3.5). Mathematicians could store significantly more visuo-spatial information when the involvement of the central executive was comparatively low, whereas the level of central executive involvement in processing made no difference to the visuo-spatial storage ability of the non-mathematicians. Chapter 6 results showed that the central executive was used more than the visuo-spatial sketchpad when adults solved addition problems and particularly when counting, involving several steps and the continual monitoring and updating of information within working memory, was employed as a strategy. Evidence within this thesis therefore also supports the assumption within the multi-component model that working memory ability is affected by individual differences in performance of the central executive.

The embedded-process (Cowan, 1999) and controlled-attention (Engle et a., 1992) models also state the importance of limitations in central executive performance in terms of individual differences in working memory capacity, but both place an emphasis on the role of controlled attention. Chapter 3 of this thesis, however, found no differences between mathematicians and non-mathematicians for performance on a controlled attention task and controlled attention only predicted ability to retrieve number facts from long-term memory (*Woodcock-Johnson Math Fluency Test*) and not ability to perform calculations (*Woodcock-Johnson Calculation Test*). These results suggested that controlled attention ability is important for retrieving mathematical facts, in support of these two models, but that the ability to retrieve facts does not explain the superior calculation ability of mathematicians.

The time-based resource-sharing model suggests that controlled attention and also the time taken to process items within working memory affects the amount of information that can be stored because of temporal decay (Barrouillet et al., 2004). However, although mathematicians were faster to process visuo-spatial items within working memory (Chapter 5) this faster processing speed did not explain differences in calculation ability (section 5.3.4). Also, in Chapter 2, mathematicians were able to store more visuo-spatial information in working memory than the non-mathematicians even though they were no faster to carry out the processing element of the task. Functions of the central executive other than controlled attention and speed of processing therefore appear to contribute to differences in mathematical calculation achievements.

Whilst the multi-component model of working memory agrees that the central executive controls the retrieval of facts from long-term memory (Baddeley, 2002) it also includes other roles for the central executive, such as switching, updating and inhibition (Miyake et al., 2000), manipulating information (Repovš & Baddeley, 2006) and possibly the ordering of visuo-spatial information (Allen et al., 2014; Baddeley, 2000). Support for the latter function was discussed in section 7.3. This thesis has therefore provided evidence that the central executive performance plays a role in the differences between mathematicians and non-mathematicians, but that individual differences in controlled attention or processing speed do not explain differences in calculation ability. Differences between mathematicians and non-mathematicians in the other executive functions should therefore be explored. This will be discussed further in section 7.6.

As this thesis contains evidence of roles for domain-specific storage and central executive processes within working memory in mathematics that differ between fact-based retrieval and more complex calculation, any model of mathematical cognition should include roles for both that depend upon the type of mathematical problem being solved. Figure 7.1 depicts how the multi-component model of working memory can be used to explain the various processes involved in solving mathematical problems, based on the previous literature and findings within this thesis.

Within Figure 7.1, elements underlined are those for which I have found evidence within this thesis that they are related to mathematics performance in adults.

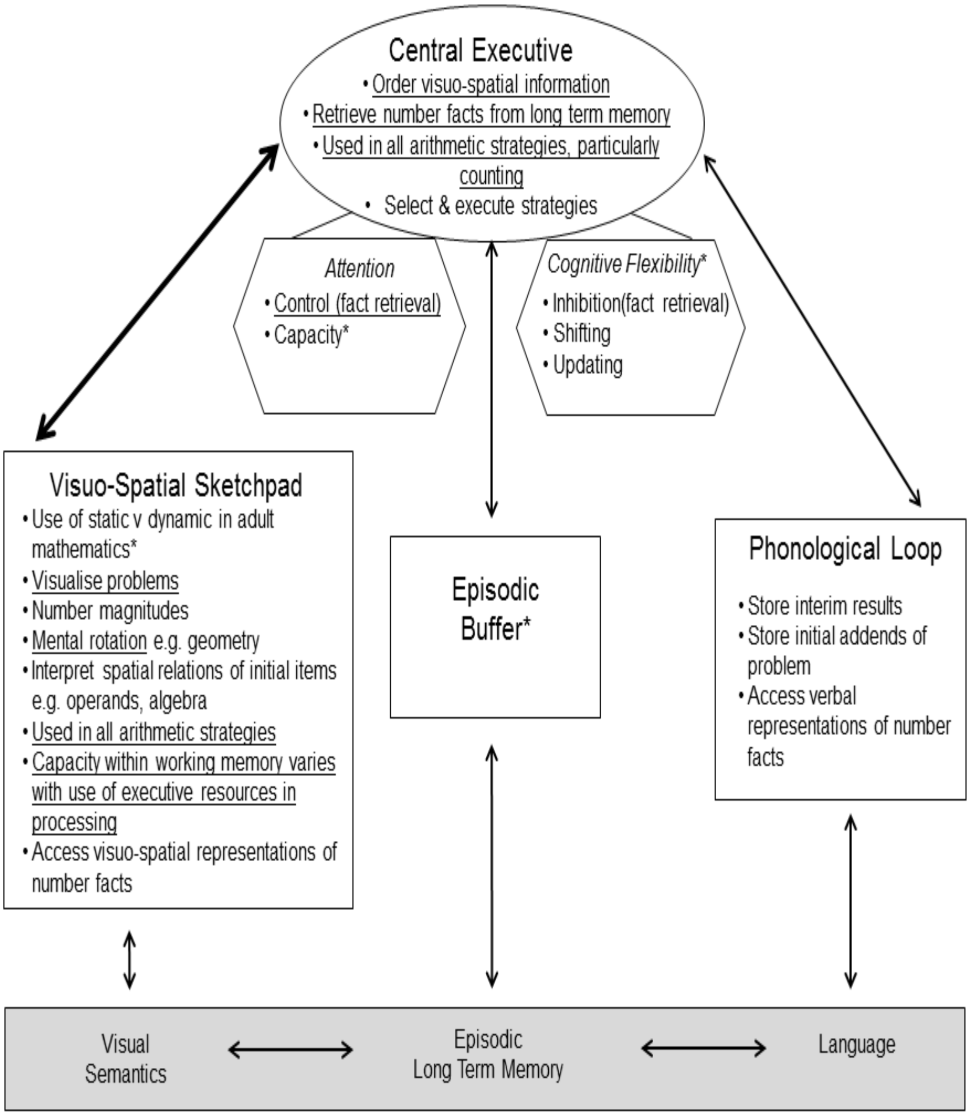

Figure 7.1: Use of the multi-component model of working memory to depict the processes involved in solving mathematical problems. *Note:* Items marked with a * indicate items that require further investigation. Underlining indicates there is evidence within this thesis that the item is related to mathematics performance in adults.

The bold, black line between the central executive and the visuo-spatial sketchpad highlights the stronger link association these two components than that between the central executive and the phonological loop (Miyake at al., 2001). It also highlights the fact that, within this thesis, I have found the link between the central executive and the visuo-spatial sketchpad to be crucial with regard to the amount of visuo-spatial information that can be stored within working memory and its impact on visuo-spatial working memory capacity differences between mathematicians and non-mathematicians (Chapter 5, section 5.3.5). The link between the central executive and visuo-spatial sketchpad also seems to be relevant with regard to the ordering of visuo-spatial information (Chapter 4) and mathematicians' superior ability at mental rotation, as evidenced by performance on the MRT-A (Peters et al., 1995) in Chapter 5.

Although results from this thesis support the view of domain-specific storage for verbal and visuo-spatial information, in line with the multi-component model, it seems too simplistic to view central executive resources as purely domain-general. For example, whilst the phonological loop is thought to be responsible for the ordering of verbal information, the central executive appears to be responsible for ordering visuo-spatial information, as mentioned above.

Items marked with a * within Figure 7.1 indicate elements that require further investigation to fully understand their importance when adults perform mathematics. These elements, cognitive flexibility (inhibition, updating and shifting), the relative use of static and dynamic visuo-spatial sketchpad resources and the involvement of the episodic buffer will be discussed in section 7.6.

7.6 Limitations and Future Studies

This section will discuss limitations within the experimental chapters of this thesis, with a view to informing future research. I will discuss issues surrounding causality with regard to the relationship between visuo-spatial working memory and mathematics (section 7.6.1), the use of static versus dynamic visuo-spatial resources (section 7.6.2) and whether visuo-spatial working memory storage capacity differences are due to differing ability for encoding or retrieval of information (section 7.6.3). In section 7.6.4 I will consider the need for a better understanding of the role of visuo-spatial working memory in different types of mathematics. I will then explain the need to further examine the contribution of the various elements of the central executive (section 7.6.5) and the role of the episodic buffer (section

7.6.6) in the relationship between visuo-spatial working memory and mathematics in adults. Finally, in section 7.6.7, I discuss an issue surrounding the use of standardised mathematics tests throughout this thesis.

7.6.1 Causality

Throughout this thesis the experimental evidence suggests that the ability to store visuo-spatial information within working memory, to order visuo-spatial information, controlled spatial attention ability and general visuo-spatial ability are related to mathematics achievement. However, the fact that a relationship exists cannot be used to imply causality. Whilst it is tempting to assume that a greater ability in these elements leads to superior mathematical achievement, it could be that adults skilled at mathematics have developed these superior abilities as a result of studying more advanced mathematics or as a result of studying mathematics for a longer period of time than those who are less proficient at mathematics.

Although longitudinal evidence exists within the literature involving children that those with better visuo-spatial working memory capacity go on to be more proficient at mathematics (e.g. Bull et al., 2008; Geary, 2011; Dumontheil &Klingberg, 2012), the direction of the relationship has not been investigated in adults who are studying or doing more complex mathematics. The direction of causality in adults has also not been investigated for the ordering of visuo-spatial information, general visuo-spatial ability or controlled attention. This should be the subject of future studies.

7.6.2 Visuo-spatial Sketchpad: Static v Dynamic

As discussed in Chapter 1 (section 1.3.4.2) it has been proposed that the visuo-spatial sketchpad is fractionated into subsystems (Darling et al., 2009; Duff & Logie, 1999; Logie et al., 1994): one for the storage of static visual material; one for the storage of dynamic information such as movement. Other than in Chapter 6, this thesis has employed tasks requiring the maintenance of dynamic visuo-spatial information through tracking the movement of objects on screen. Therefore evidence for a link between visuo-spatial working memory and adult mathematics performance has largely reflected dynamic visuo-spatial working memory.

In the visuo-spatial sketchpad load condition within Chapter 6 (section 6.3.3.2) participants were required to remember either a static visuo-spatial display of objects or a dynamic display where the objects moved. Although the two types of visuo-spatial memory had the same impact on arithmetic performance, those in the static condition performed significantly better on this secondary task, suggesting that memory for static visuo-spatial information may be easier. This corresponds to previous findings with children that static information is the easiest to store (e.g. Pickering et al., 2001). Also, in the experiments within Chapters 2 and 5, which measured visuo-spatial working memory capacity, anecdotally, the non-mathematicians reported rehearsing in mind the movement of the objects in the order they were presented on screen whereas many of the mathematicians reported forming a static shape, made up of the different locations, within memory (although they were unable to explain how they had maintained the objects' order).

Differences in visuo-spatial working memory capacity and the ability of this to predict mathematics test scores may therefore reflect a difference in strategy for remembering visuo-spatial information between static and dynamic formats and hence the visual cache and inner scribe (Logie, 1995). Holmes et al. (2008) have previously found that the ability to maintain static visual images in memory predicted mathematics ability in older children whilst the maintenance of dynamic images predicted mathematics ability in younger children. The use of static and dynamic visuo-spatial working memory and their relative relationships with mathematics has not been systematically investigated in adults and should be an area for further exploration.

7.6.3 Differences in Visuo-Spatial Working Memory Capacity: Encoding or Retrieval?

One of the main questions examined within this thesis was whether there are any differences between the working memory capacity of mathematicians and non-mathematicians. In Chapter 2, having found that mathematicians had superior ability to store visuo-spatial information within working memory when the processing required was a neutral as possible, I then examined the serial position curves of the two groups for visuo-spatial storage (section 2.3.2.5). The serial position curves showed that there were no significant differences between the two groups in the patterns of the curves, but that mathematicians simply displayed greater overall accuracy for remembering and recalling visuo-spatial items. Whilst this showed that the mathematicians had greater overall capacity, it did not shed light on whether the mathematicians' advantage was due to superior encoding of visuo-spatial items, superior ability to recall items or a combination of both.

Unsworth, Spillers & Brewer (2012) have suggested that adults with greater working memory capacity are better at retrieving previously formed representations from memory when recall is required. Although this research involved memory for verbal categorical information and cannot necessarily be generalised to the visuo-spatial domain, their experiments involving cued and free recall of word lists resulted in those with lower working memory capacity failing to use appropriate strategies to access stored information. Unsworth and colleagues concluded that those with greater working memory capacity are more efficient at retrieving items from memory, but also acknowledged the need to examine, in the future, efficiency at the encoding stage. The working memory span task employed within Chapter 2 could be repeated with loading of visuo-spatial working memory at encoding and retrieval to investigate the relative effects on performance of the mathematicians and non-mathematicians.

7.6.4 Different Types of Mathematics

Having found within Chapter 5 that the level of central executive involvement in processing within working memory affects the ability to store visuo-spatial information, results from Chapter 6 showed that the central executive has the greatest involvement in procedural strategies for solving mathematical problems and particularly for counting. This suggested that the level of involvement of visuo-spatial working memory and its components varies with the type of strategy employed. Previous research has also suggested that the use of visuo-spatial working memory varies with the type of mathematics problems being solved. For example, visuo-spatial working memory has been implicated in interpreting graphical information (Hegarty & Waller, 2005), solving problems involving approximations (Logie et al., 1994) and additions (Hubber et al., 2014: Experiment 1) and interpreting initial operands (Jiang et al., 2014). There is also evidence that it is used for solving subtractions, but not multiplications (Lee & Kang, 2002).

The *Woodcock-Johnson Calculation Test*, used to measure calculation ability throughout the experimental chapters of this thesis, included a variety of different types of mathematical problems (e.g. additions, fractions, simultaneous equations, long division, integrations, matrices and trigonometry). Results from the experimental chapters have implicated visuo-spatial working memory capacity and performance on this mathematical test, but do not give any indication of relationships with the different individual types of problems contained within the test. For example, solving long divisions involves several steps and the storage of interim results (implicating the central executive and phonological loop: e.g. Chapter 6; Fürst & Hitch, 2000) whereas trigonometry involves angles and might be expected to be more visuo-spatial in nature.

It would be useful to discover how visuo-spatial working memory capacity and general visuo-spatial ability are related to different forms of mathematics problems because this could help highlight which individuals may struggle to learn a specific type of mathematics.

7.6.5 The Central Executive

In Chapter 5, I found that the level of central executive involvement in processing affected storage of visuo-spatial information in working memory and, in Chapter 6, load on the central executive had more of a detrimental effect on performing additions than did load on the visuo-spatial sketchpad. The ability to order visuo-spatial information (Chapter 4) and ability for controlled spatial attention (Chapter 3) seem to be functions of the central executive that are related to mathematics achievement. However, there are other functions of the central executive that have not been explored within this thesis that may be important for solving mathematical problems.

7.6.5.1 Attention

In terms of attention, the models of working memory discussed in Chapter 1 (section 1.3) all suggest that attentional capacity limits working memory capacity. The use of procedural strategies to solve mathematical problems requires performing several steps and therefore the ability to attend to more than one item at a time. Mathematics ability may therefore be constrained by the limit of an individual’s attentional capacity. The Posner (1980) task employed in Chapter 3 measured the ability to control attention for one item at a time rather than the amount of items that can simultaneously be maintained within the focus of attention. Future research should explore differences between mathematicians and non-mathematicians for the number of items that can be maintained within working memory in the face of central executive load (e.g. Allen et al., 2014).

7.6.5.2 Cognitive Flexibility

Other functions of the central executive were highlighted in Chapter 1 (section 1.3.4.3). These included switching, monitoring and updating, inhibition and manipulation of information (e.g. Miyake et al., 2000; Repovš & Baddeley, 2006). Individuals may be able to compensate for poor knowledge of mathematical strategies with good working memory capacity, and executive function skills may mediate the relationship between basic numerical representations and mathematics outcomes (Gilmore et al., 2013). Solving mathematical problems using procedural strategies involves switching between different tasks (such as storage and processing) and the monitoring and updating of interim totals. Solving mathematical problems by directly retrieving answers from long-term memory involves inhibiting competing but incorrect answers (e.g. 2 x 4 = 6). Examining these executive functions in the future will aid our understanding of which of them drive the importance of the central executive when adults solve mathematical problems.

7.6.6 The Episodic Buffer

In Chapter 2 (section 1.3.4.4), I described how the episodic buffer is a domain-general store capable of integrating, or binding, information from the other components of working memory and that the central executive can access and manipulate the buffer's content through conscious awareness (Baddeley, 2000).

An emerging line of research is examining how memory for verbal items can be improved by presenting visuo-spatial information alongside the verbal information (e.g. Darling, Allen, Havelka, Campbell & Rattray, 2012; Darling & Havelka, 2010; Darling, Parker, Goodhall, Havelka & Allen, 2014). The authors have provided evidence that memory for digits can be enhanced through presenting them in familiar spatial locations. It may therefore be that mathematicians' superior visuo-spatial skills and visuo-spatial storage capacity within working memory can be used to support and enhance the use of verbal resources during calculation via the episodic buffer.

7.6.7 Use of Standardised Mathematical Tests

Throughout the experimental chapters, two standardised tests of mathematical ability have been employed: *Woodcock-Johnson Calculation* and *Woodcock-Johnson Math Fluency*. These measures were described in detail in Chapter 2, sections 2.2.1.4 and 2.3.1.4 and were used to measure an individual's ability for advanced mathematics calculation and more basic arithmetic fluency respectively.

Where extreme group designs are employed, differences between groups for performance on standardised tests can be exaggerated (A. D. Baddeley, personal communication, June 24, 2014). Items within the *Woodcock-Johnson Calculation Test* increased in difficulty from start to finish and later items consisted of mathematical problems that will only have been previously encountered by individuals who have studied mathematics at A level and beyond. Therefore, the groups of unskilled mathematicians within Chapters 2, 3 and 5 were required to answer some problems of which they had no prior experience.

This is not deemed to be too great an issue, however, because the unskilled mathematicians tended to fail at items before they reached those requiring A level knowledge. On average, the non-mathematicians began to make errors around the 13^{th} item. This and the surrounding questions involved long division, multiplication of decimals, addition and division using fractions and arithmetic involving negative numbers. They would have previously encountered all of these types of problems for GCSE mathematics. The link between the ability to store visuo-spatial information within working memory and mathematics could however be explored again using the tasks within Chapters 2, 3 and 5 with a more continuous sample of mathematics ability, to confirm the findings. This was not an issue for the *Woodcock-Johnson Math Fluency Test*, as this involved basic arithmetic and all participants would have had prior experience of all of the types of problems it contained.

Use of extreme groups (such as expert mathematicians and those poorer at mathematics) may also make generalisation to the wider population difficult (Conway, Kane & Bunting, 2005). Therefore, findings may not apply across a full range of mathematics ability. To provide balance and greater generalizability, Chapters 4 and 6 contained experiments involving adult participants with a more continuous range of mathematical ability.

7.7 General Conclusions of the Thesis

Throughout this thesis I have aimed to discover whether there are any domain-specific or domain-general differences between the working memory capacity of adult mathematicians and non-mathematicians, what drives visuo-spatial working memory capacity predicting mathematics achievement in adults, how can having good visuo-spatial working memory support the proficient solving of mathematical problems and how can working memory be accommodated within a model of mathematical cognition.

I have presented a novel finding within this thesis that mathematicians have superior capacity for storing visuo-spatial information within working memory, but only when the processing demands within working memory have a low level of central executive involvement. I have also consistently found that there is no difference between mathematicians and non-mathematicians for the amount of verbal information that can be stored within working memory.

Another novel finding is that, in adults, the fact that visuo-spatial working memory capacity predicts mathematics achievement is not driven by simple basic storage ability within the visuo-spatial sketchpad or general ability to deal with visuo-spatial material. Also, whilst controlled spatial attention ability was found to predict the ability directly retrieve number facts from long-term memory, it did not drive visuo-spatial working memory capacity predicting complex calculation ability in adults. The ability to order visuo-spatial information, however, did predict calculation ability. It may also be that attentional capacity limits, cognitive flexibility and the episodic buffer are important for proficiency in mathematics and these areas should be explored in the future.

In terms of how having good visuo-spatial working memory is related to adults performing well at mathematics, I have presented evidence within this thesis that it depends upon the type of mathematics being performed and the strategies used. The central executive was found to be used in the retrieval of number facts from long-term memory, but more so in procedural methods, such as counting which requires several steps. Adult mathematicians seem to be more efficient at using central executive resources to access stored number facts, select and execute appropriate strategies and deal with the carry-over of interim results. This means they then have greater working memory resources available to use their greater ability to store visuo-spatial information within working memory and generally deal with visuo-spatial material to solve mathematical problems.

Finally, I have provided strong evidence that visuo-spatial working memory capacity and mathematics ability are related and there is therefore a need to include working memory within any model of mathematical cognition. I have suggested how the multi-component model of working memory can be used as a basis for such a model.

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Appendix A

Examples of same and different pairs of faces used as the processing element of the working memory span tasks in Chapter 2.

Same




Different




Appendix B

List of span sets used for the working memory span tasks in the number, visuo-spatial and word conditions in Experiment 1 of Chapter 2 and in the visuo-spatial condition in Experiment 2 of Chapter 2.

Number Condition

Span Length	Storage Task: Numbers Presented	Processing Task: Faces same or different
2	2 5	1 same, 1 different
2	3 8	1 same, 1 different
2	7 9	1 same, 1 different
3	1 4 7	2 same, 1 different
3	2 6 9	1 same, 2 different
3	1 3 8	2 same, 1 different
4	2 3 5 8	2 same, 2 different
4	2 4 6 7	3 same, 1 different
4	1 3 6 8	1 same, 3 different
5	1 4 6 7 9	2 same, 3 different
5	1 2 5 6 8	3 same, 2 different
5	2 4 5 8 9	2 same, 3 different
6	1 3 4 6 7 9	2 same, 4 different
6	2 4 5 7 8 9	4 same, 2 different
6	1 3 4 5 7 9	3 same, 3 different
7	1 2 3 5 6 8 9	4 same, 3 different
7	2 3 4 5 6 7 8	3 same, 4 different
7	1 3 4 5 6 7 9	4 same, 3 different

Visuo-Spatial Condition

7	8	9
4	5	6
1	2	3

Span Length	Storage Task: Locations of red dot on 3 x 3 grid	Processing Task: Faces same or different
2	2 9	1 same, 1 different
2	1 4	1 same, 1 different
2	6 8	1 same, 1 different
3	2 5 7	2 same, 1 different
3	1 3 6	1 same, 2 different
3	2 4 9	2 same, 1 different
4	2 4 7 8	2 same, 2 different
4	1 3 5 8	3 same, 1 different
4	3 4 7 9	1 same, 3 different
5	2 4 6 7 9	2 same, 3 different
5	1 3 5 6 8	3 same, 2 different
5	2 4 5 7 9	2 same, 3 different
6	1 3 4 5 7 8	2 same, 4 different
6	1 2 4 6 8 9	4 same, 2 different
6	2 3 5 6 7 9	3 same, 3 different
7	1 2 3 5 6 7 8	4 same, 3 different
7	1 3 4 5 6 8 9	3 same, 4 different
7	1 3 5 6 7 8 9	4 same, 3 different

Word Condition

Span Length	Storage Task: Words Presented	Processing Task: Faces same or different
2	fly cow	1 same, 1 different
2	bat dog	1 same, 1 different
2	hen ape	1 same, 1 different
3	elk fox ram	2 same, 1 different
3	bat cow hen	1 same, 2 different
3	dog ape fly	2 same, 1 different
4	fox cow ram bat	2 same, 2 different
4	ape fly elk dog	3 same, 1 different
4	hen elk cow dog	1 same, 3 different
5	ram fox fly ape bat	2 same, 3 different
5	fox elk hen ram dog	3 same, 2 different
5	fly bat fox hen ape	2 same, 3 different
6	cow elk ram fly ape fox	2 same, 4 different
6	cow dog bat elk hen ram	4 same, 2 different
6	fly dog ape hen ram cow	3 same, 3 different
7	fly dog bat fox elk hen ram	4 same, 3 different
7	cow dog bat ape fox elk hen	3 same, 4 different
7	fly cow bat ape fox elk ram	4 same, 3 different

Appendix C

Span sets used in the number condition of Experiment 2 in Chapter 2.

Number Span

Span Length	**Storage Task: Numbers Presented**	**Processing Task: Faces same or different**
3	1 4 7	2 same, 1 different
3	2 6 9	1 same, 2 different
3	1 3 8	2 same, 1 different
4	2 3 5 8	2 same, 2 different
4	2 4 6 7	3 same, 1 different
4	1 3 6 8	1 same, 3 different
5	1 3 6 7 9	2 same, 3 different
5	1 2 5 6 8	3 same, 2 different
5	2 4 5 8 9	2 same, 3 different
6	1 3 4 6 7 9	2 same, 4 different
6	2 4 5 7 8 9	4 same, 2 different
6	1 3 4 5 7 9	3 same, 3 different
7	1 2 3 5 6 8 9	4 same, 3 different
7	2 3 4 5 6 7 8	3 same, 4 different
7	1 3 4 5 6 7 9	4 same, 3 different
8	1 3 4 5 6 7 8 9	4 same, 4 different
8	1 2 3 4 5 7 8 9	4 same, 4 different
8	2 3 4 5 6 7 8 9	4 same, 4 different

Appendix D

Span sets used in Chapter 3, visuo-spatial short-term memory task.

7	8	9
4	5	6
1	2	3

Span Length	Storage Task: Locations of red dot on 3 x 3 grid
3	2 4 6
3	3 5 9
3	1 5 7
4	2 4 5 8
4	3 6 7 9
4	1 5 7 8
5	3 4 6 8 9
5	1 2 4 6 9
5	1 3 5 7 8
6	2 3 4 6 8 9
6	1 3 4 6 7 9
6	1 2 5 7 8 9
7	1 2 4 5 6 8 9
7	2 3 4 6 7 8 9
7	1 2 3 4 5 7 8
8	1 2 3 4 5 6 7 9
8	1 2 3 5 6 7 8 9
8	1 2 3 4 5 6 7 8

Appendix E

Trials used in Chapter 3, endogenous spatial attention task.

Trial	Cue Validity	Target Position	SOA (milliseconds)
1	Neutral	Right	200
2	Neutral	Right	200
3	Neutral	Right	400
4	Neutral	Right	400
5	Neutral	Right	800
6	Neutral	Right	800
7	Neutral	Left	200
8	Neutral	Left	200
9	Neutral	Left	400
10	Neutral	Left	400
11	Neutral	Left	800
12	Neutral	Left	800
13	Invalid	Right	200
14	Invalid	Right	200
15	Invalid	Right	400
16	Invalid	Right	400
17	Invalid	Right	800
18	Invalid	Right	800
19	Invalid	Left	200
20	Invalid	Left	200
21	Invalid	Left	400
22	Invalid	Left	400
23	Invalid	Left	800
24	Invalid	Left	800
25	Valid	Left	200
26	Valid	Left	200
27	Valid	Left	200
28	Valid	Left	200
29	Valid	Left	200
30	Valid	Left	200
31	Valid	Left	200
32	Valid	Left	200
33	Valid	Left	400
34	Valid	Left	400
35	Valid	Left	400
36	Valid	Left	400
37	Valid	Left	400
38	Valid	Left	400
39	Valid	Left	400
40	Valid	Left	400
41	Valid	Left	800

Trial	Cue Validity	Target Position	SOA (milliseconds)
44	Valid	Left	800
45	Valid	Left	800
46	Valid	Left	800
47	Valid	Left	800
48	Valid	Left	800
49	Valid	Right	200
50	Valid	Right	200
51	Valid	Right	200
52	Valid	Right	200
53	Valid	Right	200
54	Valid	Right	200
55	Valid	Right	200
56	Valid	Right	200
57	Valid	Right	400
58	Valid	Right	400
59	Valid	Right	400
60	Valid	Right	400
61	Valid	Right	400
62	Valid	Right	400
63	Valid	Right	400
64	Valid	Right	400
65	Valid	Right	800
66	Valid	Right	800
67	Valid	Right	800
68	Valid	Right	800
69	Valid	Right	800
70	Valid	Right	800
71	Valid	Right	800
72	Valid	Right	800

Appendix F

List of span sets used in Chapter 4, process dissociation task.

13	**14**	**15**	**16**
9	**10**	**11**	**12**
5	**6**	**7**	**8**
1	**2**	**3**	**4**

Inclusion condition

Trial	Locations of Frog on 4 x 4 grid
1	1 3 7 10 16
2	2 5 8 11 14
3	4 6 9 11 13
4	1 7 12 13 15
5	2 3 6 10 14
6	4 5 7 9 15
7	1 6 9 11 16
8	2 8 10 14 15
9	3 4 5 12 13
10	1 6 8 12 16
11	3 7 10 12 14
12	2 5 9 13 16
13	1 4 7 11 13
14	2 6 8 11 15
15	3 5 9 10 16

Exclusion condition

Trial	Locations of Frog on 4 x 4 grid
1	4 8 10 14 15
2	1 2 6 11 16
3	3 5 7 11 14
4	2 7 9 12 15
5	3 6 10 15 16
6	2 4 5 11 14
7	1 5 6 12 13
8	3 8 9 11 15
9	4 7 10 13 16
10	1 3 8 10 16
11	2 6 7 11 13
12	1 8 9 10 16
13	2 5 11 13 14
14	1 4 6 12 15
15	3 5 8 9 14

Appendix G

List of span sets used in Chapter 4, forced-choice recognition task.

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

Item Memory Condition

Trial	Locations of red dot on 4 x 4 grid	Test Items Present	Test Items Absent
1	7 12 2 11 1 14	14	16
2	16 1 6 5 4 10	5	3
3	4 7 9 11 15 1	4	6
4	11 5 1 16 8 13	1	10
5	12 7 3 15 8 1	7	14
6	1 6 3 15 10 13	10	8
7	14 12 5 6 1 9	6	15
8	15 8 5 13 11 2	11	4
9	6 1 14 12 4 16	16	7
10	2 7 14 8 15 9	2	4
11	2 3 6 12 13 8	12	14
12	15 7 10 2 12 6	15	8
13	5 4 11 2 9 14	9	7
14	10 16 13 2 5 11	13	6
15	4 3 14 16 10 7	3	12
16	3 10 12 14 6 15	15	9
17	9 3 5 16 8 11	8	2
18	7 3 13 15 9 10	10	16
19	9 13 11 4 16 7	13	5
20	8 12 2 4 5 14	2	9

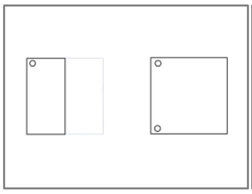
Trial	Locations of red dot on 4 x 4 grid	Test Items Present	Absent
21	13 2 1 6 12 9	1	3
22	13 3 10 4 14 8	4	11
23	11 8 9 6 16 4	11	13
24	10 7 16 5 3 15	7	1

Order Memory Condition

Trial	Locations of red dot on 4 x 4 grid	Test Items	
1	7 12 2 11 1 14	7	12
2	16 1 6 5 4 10	1	6
3	4 7 9 11 15 1	9	11
4	11 5 1 16 8 13	16	8
5	12 7 3 15 8 1	8	1
6	1 6 3 15 10 13	6	3
7	14 12 5 6 1 9	14	12
8	15 8 5 13 11 2	5	13
9	6 1 14 12 4 16	12	4
10	2 7 14 8 15 9	15	9
11	2 3 6 12 13 8	2	3
12	15 7 10 2 12 6	7	10
13	5 4 11 2 9 14	11	2
14	10 16 13 2 5 11	2	5
15	4 3 14 16 10 7	10	7
16	3 10 12 14 6 15	3	10
17	9 3 5 16 8 11	3	5
18	7 3 13 15 9 10	13	15
19	9 13 11 4 16 7	4	16
20	8 12 2 4 5 14	5	14

Appendix H

Images used in the visuo-spatial processing element in Chapter 5.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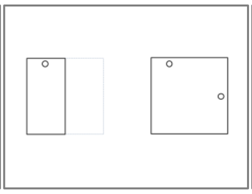
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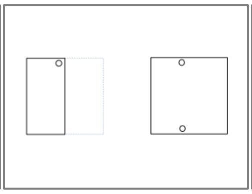
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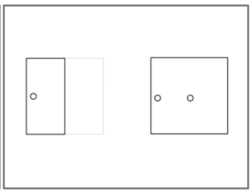
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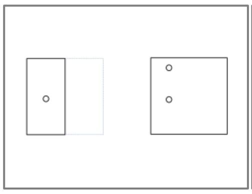
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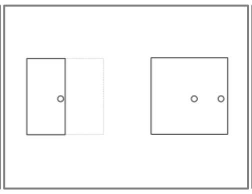
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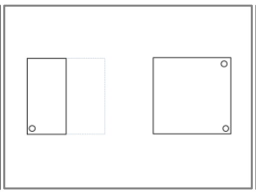
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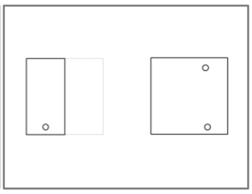
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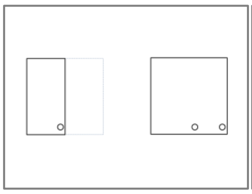
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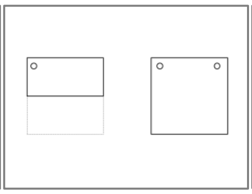
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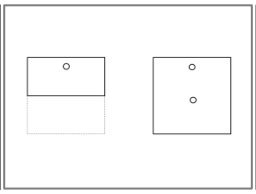
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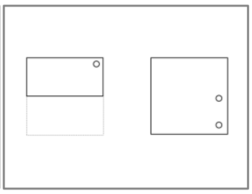
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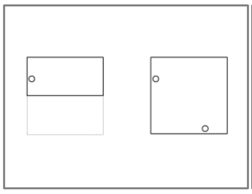
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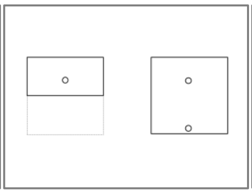
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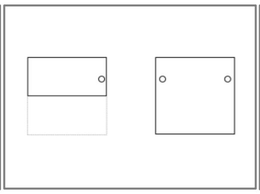
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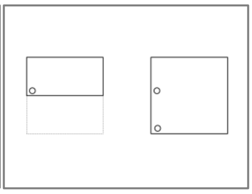
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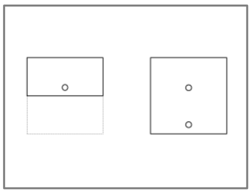
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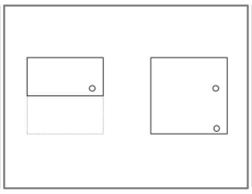
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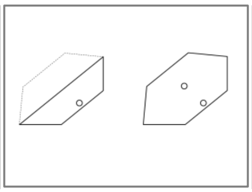
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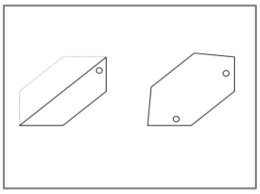
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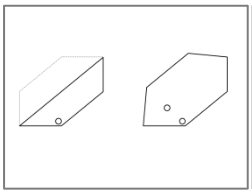
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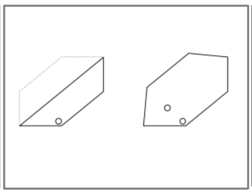
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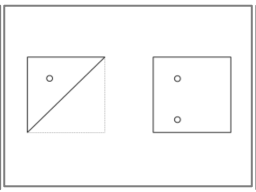
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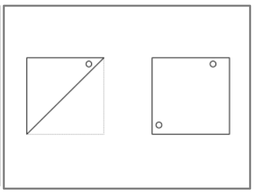
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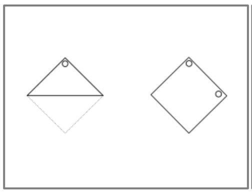
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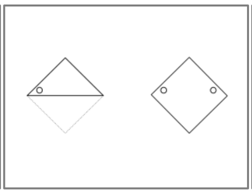
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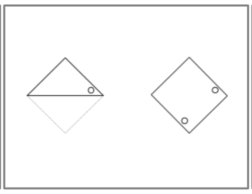
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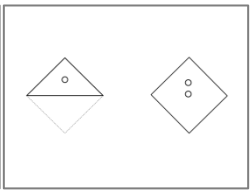
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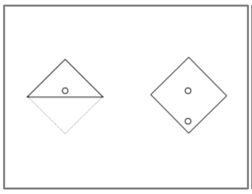
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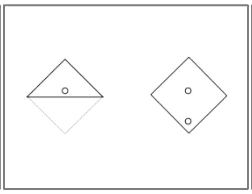
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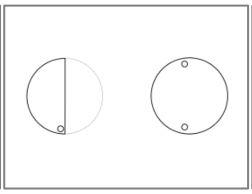
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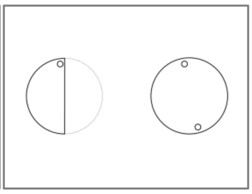
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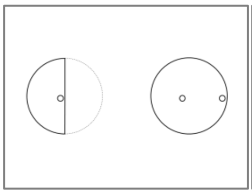
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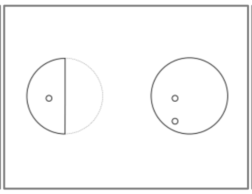
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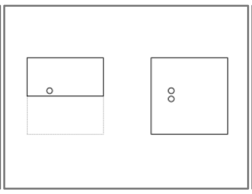
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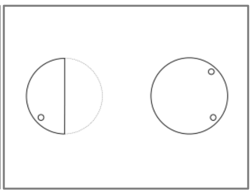
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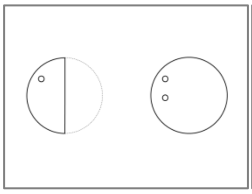
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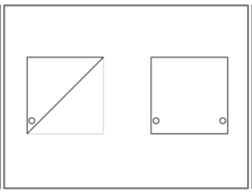
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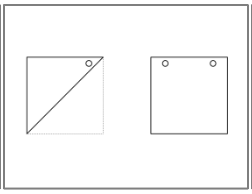
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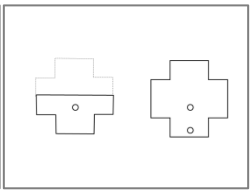
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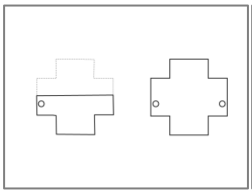
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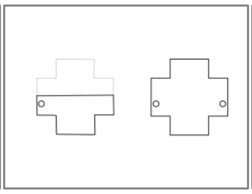
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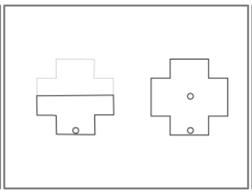
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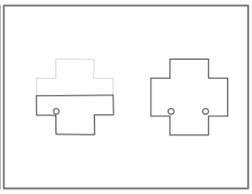
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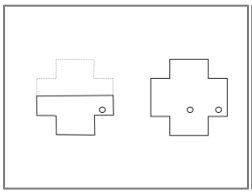
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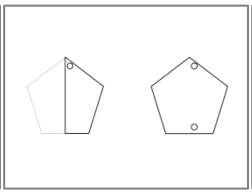
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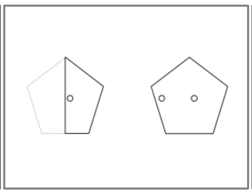
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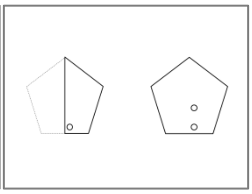
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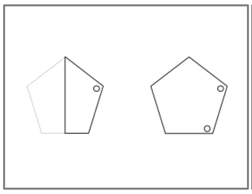
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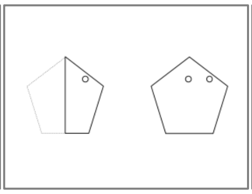
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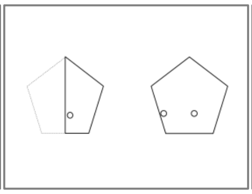
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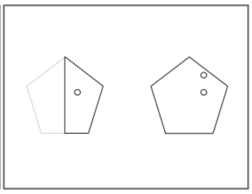
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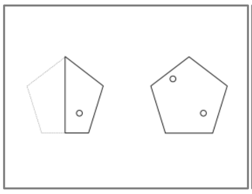
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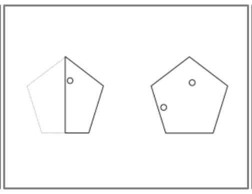
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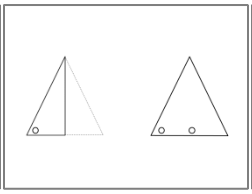
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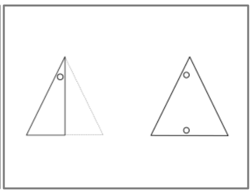
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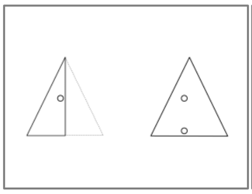
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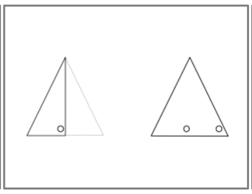
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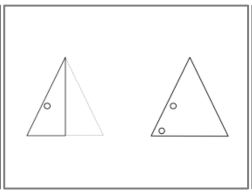
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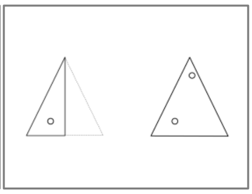
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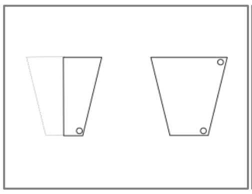
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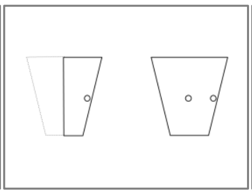
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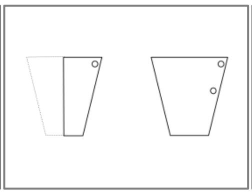
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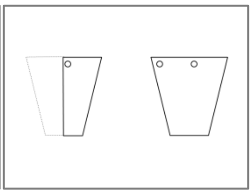
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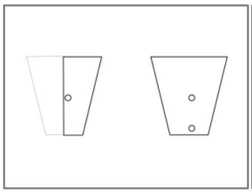
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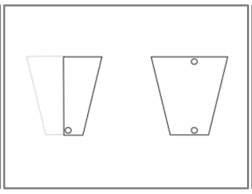
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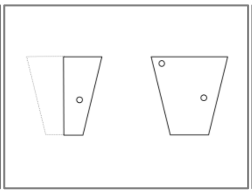
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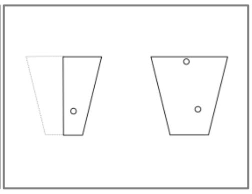
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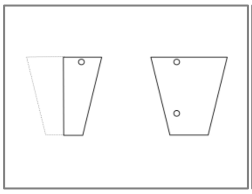
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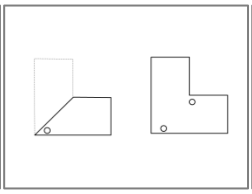
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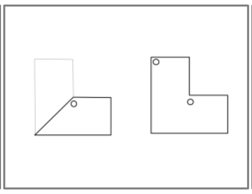
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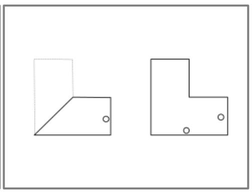
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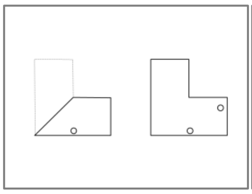
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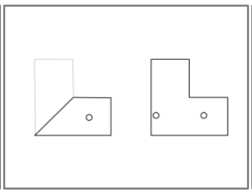
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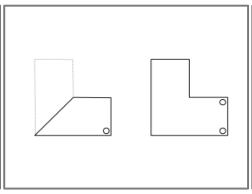
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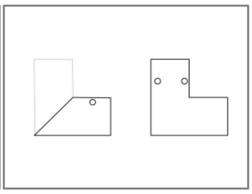
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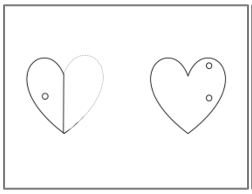
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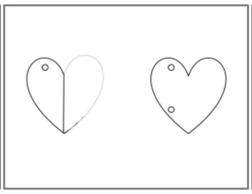
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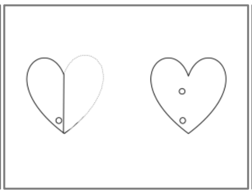
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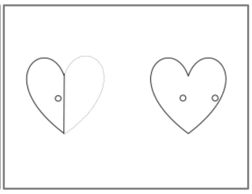
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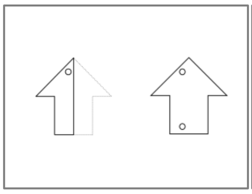
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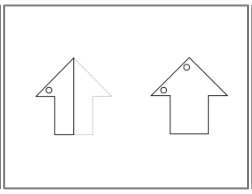
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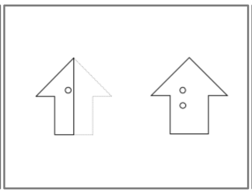
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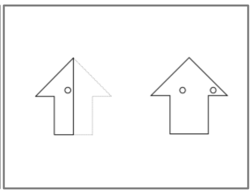
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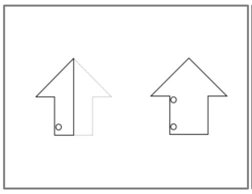
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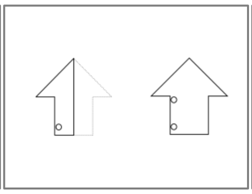
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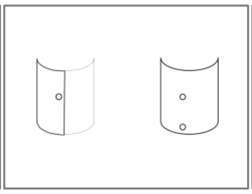
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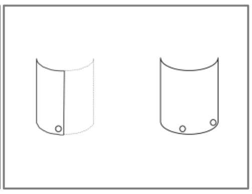
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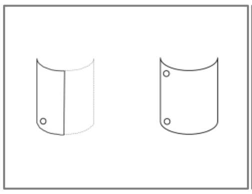
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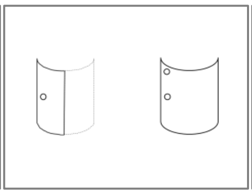
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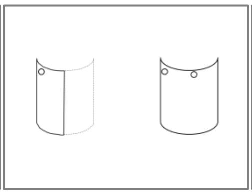
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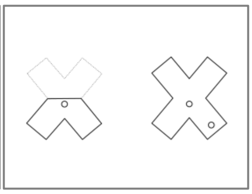
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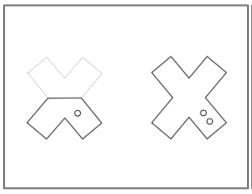
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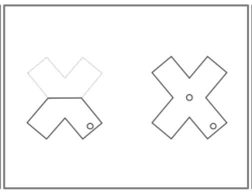
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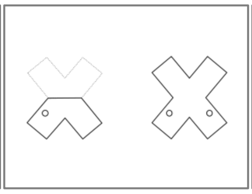
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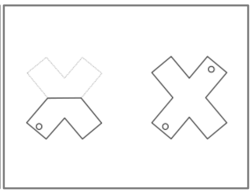
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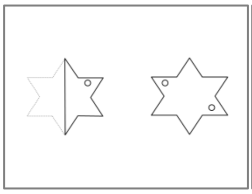
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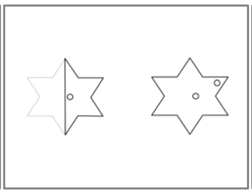
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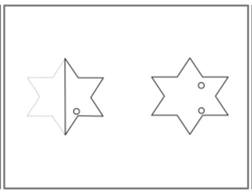
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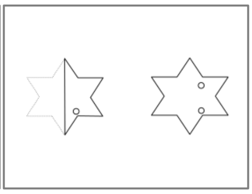
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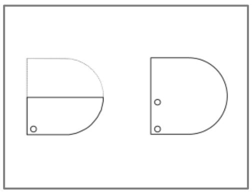
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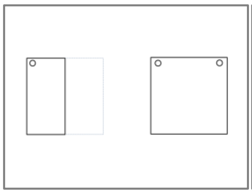
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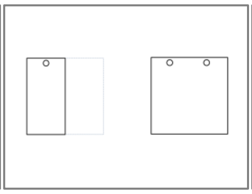
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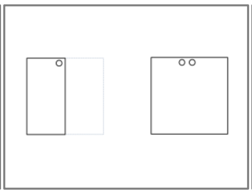
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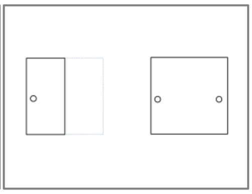
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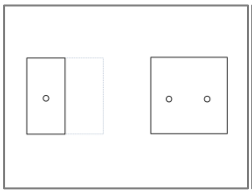
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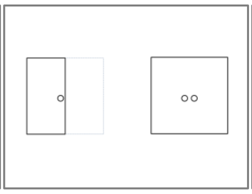
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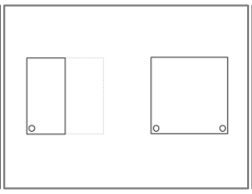
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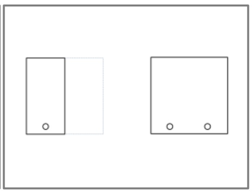
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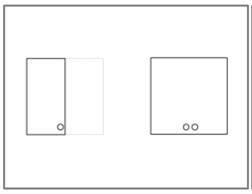
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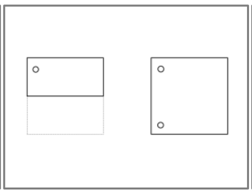
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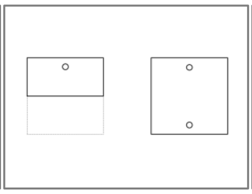
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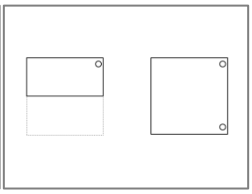
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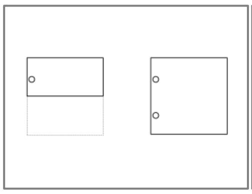
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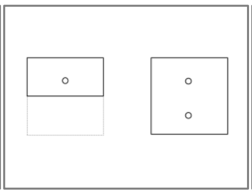
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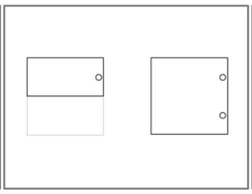
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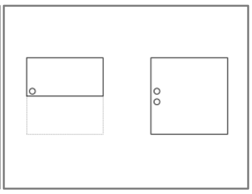
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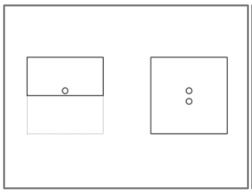
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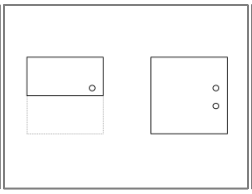
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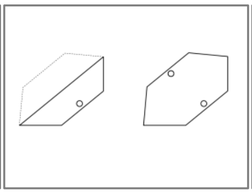
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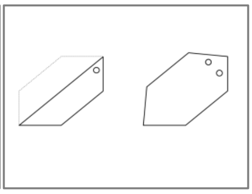
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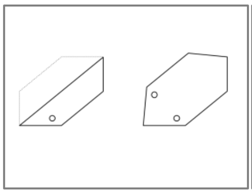
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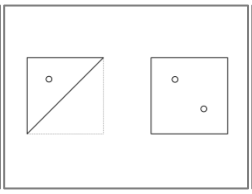
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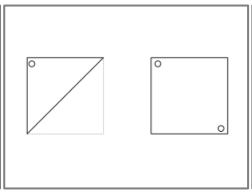
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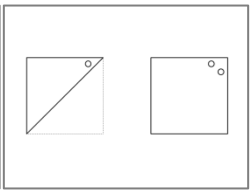
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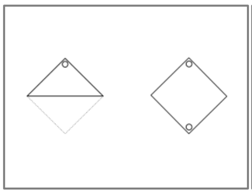
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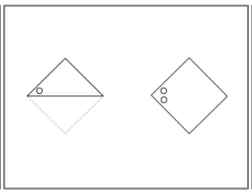
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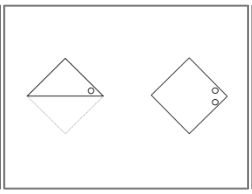
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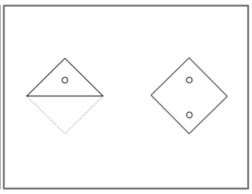
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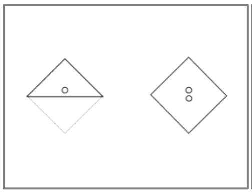
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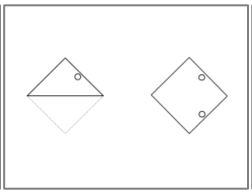
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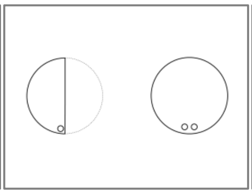
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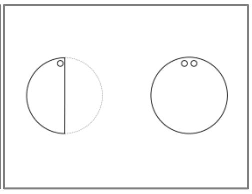
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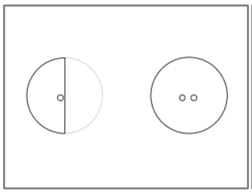
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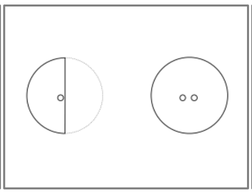
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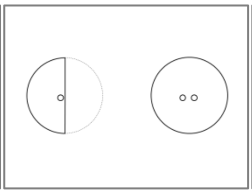
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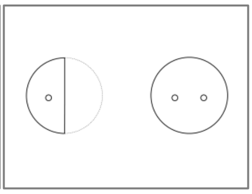
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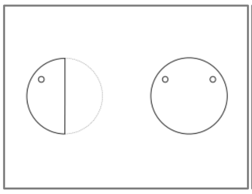
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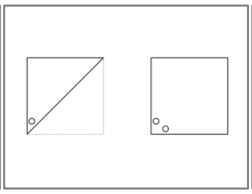
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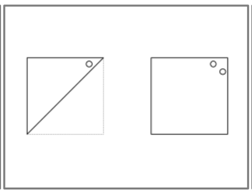
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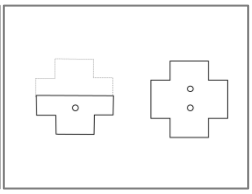
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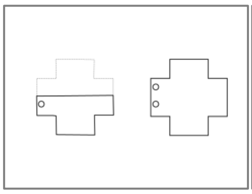
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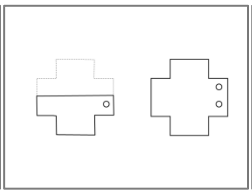
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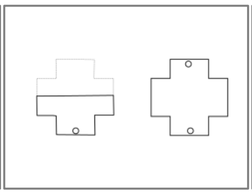
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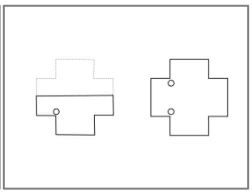
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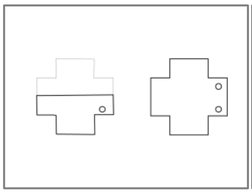
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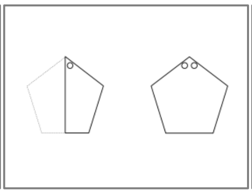
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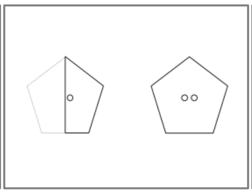
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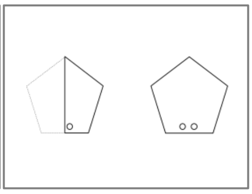
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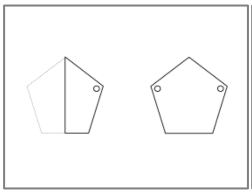
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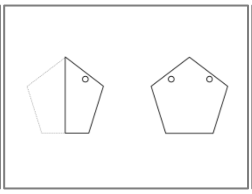
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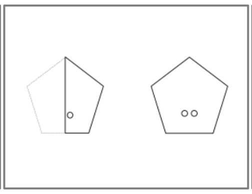
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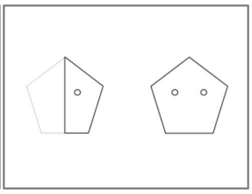
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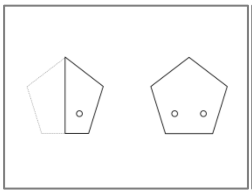
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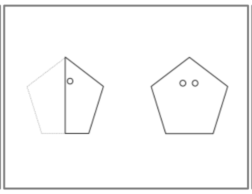
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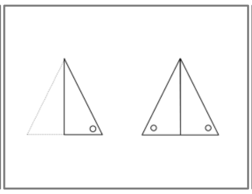
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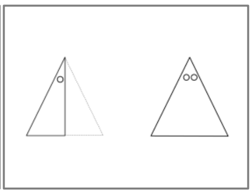
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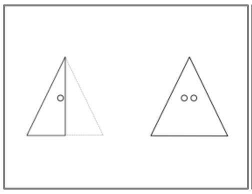
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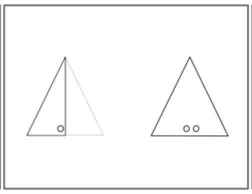
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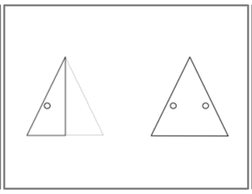
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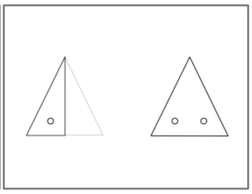
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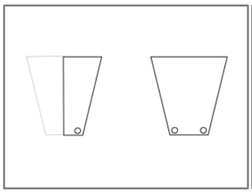
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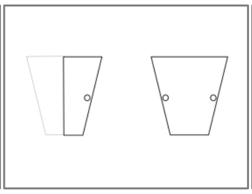
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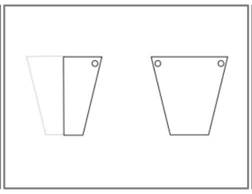
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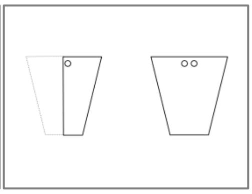
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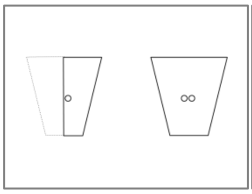
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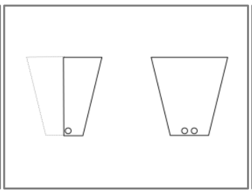
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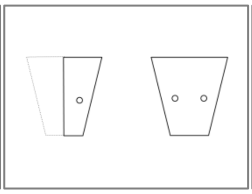
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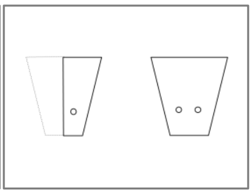
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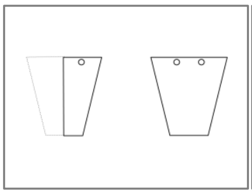
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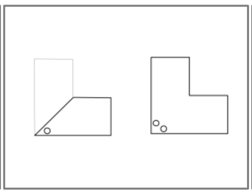
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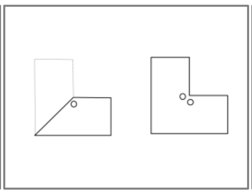
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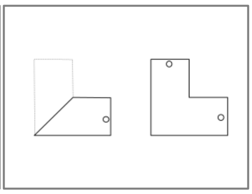
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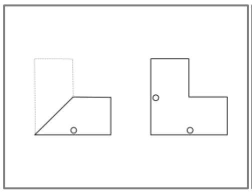
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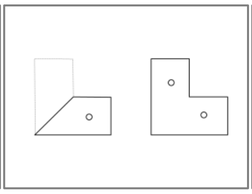
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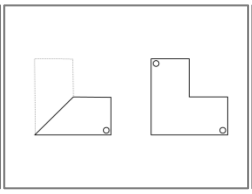
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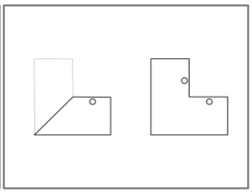
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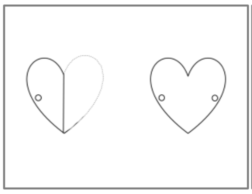
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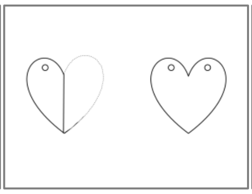
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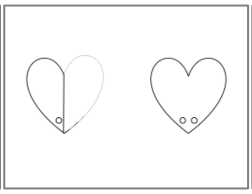
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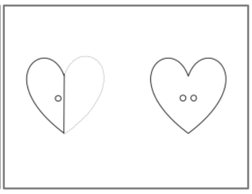
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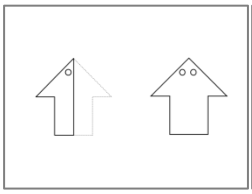
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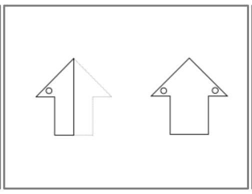
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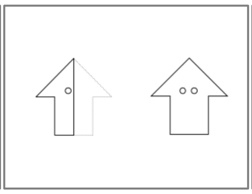
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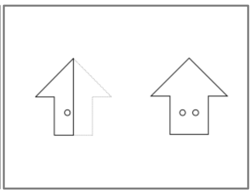
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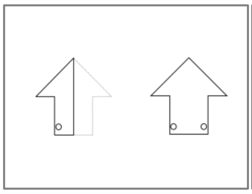
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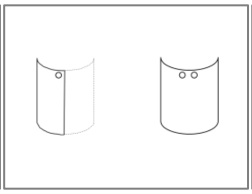
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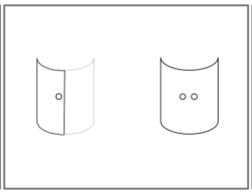
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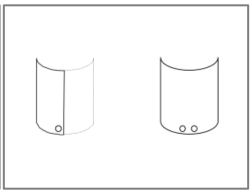
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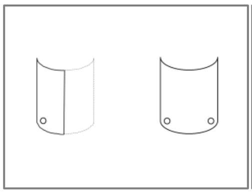
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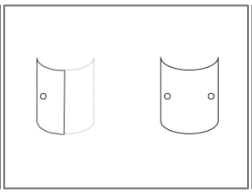
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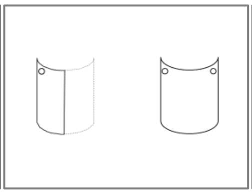
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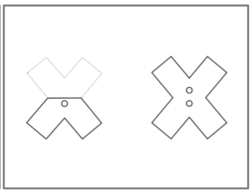
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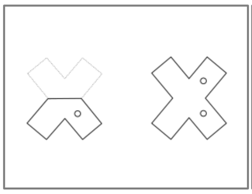
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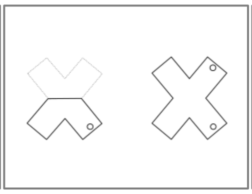
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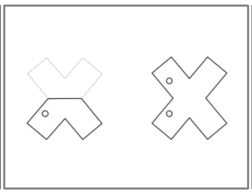
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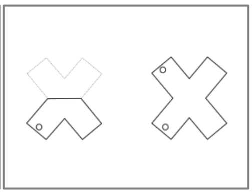
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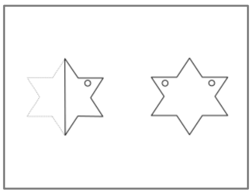
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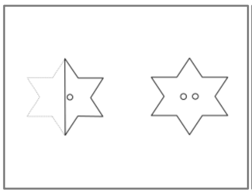
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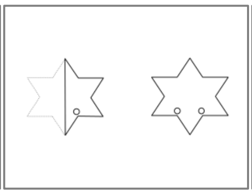
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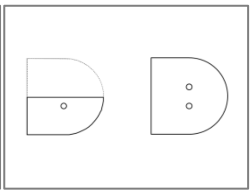
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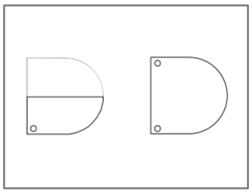
yes101

Appendix I

Word pairs used in the verbal processing element in Chapter 5.

Rhyming Pair	Word Pairs Used	
1	taught	bought
2	lace	case
3	wrote	boat
4	law	sore
5	crane	brain
6	goat	throat
7	rode	load
8	drain	mane
9	cold	mould
10	try	sigh
11	male	hail
12	weak	seek
13	corn	fawn
14	mate	bait
15	keen	bean
16	whale	tail
17	tight	bite
18	role	bowl
19	cool	rule
20	flew	too
21	soul	hole
22	light	kite
23	vile	style
24	heard	bird
25	home	loam
26	hurt	dirt
27	sort	caught
28	blame	aim
29	hoard	ford

30	weigh	tray
31	crate	great
32	heal	reel
33	grime	thyme
34	crawl	ball
35	piece	grease
36	height	white
37	haze	phase
38	climb	time
39	nerd	purred
40	wrench	bench
41	lost	frost
42	yule	spool
43	band	planned
44	cake	break
45	rack	quack
46	tossed	cost
47	cause	draws
48	fund	shunned
49	gown	noun
50	scene	queen
51	nursed	thirst
52	dial	file
53	late	eight
54	ghost	roast
55	numb	gum
56	stir	blur
57	stunt	front
58	loose	juice
59	chrome	comb
60	stoat	vote
61	west	chest
62	rest	guest

63	worst	first
64	place	grace
65	blue	shoe
66	school	mule
67	flue	ewe
68	stew	zoo
69	drawn	born
70	while	nile
71	aisle	guile
72	cling	string
73	ache	bake
74	work	jerk
75	stalk	cork
76	fault	salt
77	skirt	squirt
78	mace	trace
79	rake	steak
80	cart	heart
81	stilt	quilt
82	please	tease
83	freeze	sees
84	queue	glue
85	fame	shame
86	flight	site
87	fall	trawl
88	meant	bent
89	lean	gene
90	show	toe
91	grow	though
92	ends	lens
93	game	maim
94	fold	scold
95	tonne	fun

96	curt	shirt
97	waist	haste
98	rile	mile
99	pile	smile

Non-Rhyming Pair	Word Pairs Used	
1	pair	brake
2	flat	chart
3	train	blank
4	mast	hold
5	fine	think
6	deer	right
7	lamp	sold
8	veer	plant
9	rail	palm
10	take	hand
11	like	vest
12	chair	past
13	pound	fight
14	green	wrought
15	grown	use
16	count	turf
17	hind	vole
18	pale	near
19	lamb	pain
20	tench	brown
21	barge	face
22	flow	rock
23	tough	roof
24	wine	pert
25	race	bark
26	hunt	door
27	brace	nought
28	gel	phone
29	reach	tent

30	form	touch
31	frog	delve
32	part	soon
33	gnome	risk
34	teach	bone
35	foot	sauce
36	high	beach
37	twice	flame
38	clown	hence
39	zoom	brush
40	croft	preach
41	ouch	moist
42	prime	truck
43	loaves	wished
44	trance	gust
45	large	chased
46	mouth	dealt
47	odds	grange
48	press	grout
49	quaint	least
50	shape	purse
51	coin	lock
52	dress	grape
53	square	jinx
54	mope	north
55	wave	took
56	dream	kick
57	pelt	cream
58	bell	foam
59	mint	veil
60	dawn	ship
61	spoon	bless
62	glossed	rate
63	cope	warm
64	hall	wish

65	taste	isle
66	wand	guard
67	mist	quote
68	draught	hook
69	clause	oust
70	dank	each
71	voice	chains
72	arch	jaunt
73	crest	thorn
74	heap	floss
75	bloke	pawn
76	pug	drum
77	pint	tone
78	call	wheel
79	does	geek
80	mouse	frame
81	joke	rust
82	vents	help
83	trough	verb
84	lime	search
85	jest	bleed
86	grouse	clutch
87	frets	chore
88	shunt	scarce
89	twelve	sheer
90	wealth	best
91	faint	aide
92	drunk	hearth
93	worse	fox
94	hot	bend
95	peel	made
96	witch	blast
97	balm	rant
98	teal	hem
99	feel	ditch

Appendix J

List of span sets used in Chapter 5

Verbal processing - verbal storage condition

Span Length	Storage Task: Numbers Presented	Processing task: Words Pairs Rhyme Yes or No
3	1 4 7	2 rhyme, 1 no rhyme
3	2 6 9	1 rhyme, 2 no rhyme
3	1 3 8	2 rhyme, 1 no rhyme
4	2 3 5 8	2 rhyme, 2 no rhyme
4	2 4 6 7	3 rhyme, 1 no rhyme
4	1 3 6 8	1 rhyme, 3 no rhyme
5	1 3 6 7 9	2 rhyme, 3 no rhyme
5	1 2 5 6 8	3 rhyme, 2 no rhyme
5	2 4 5 8 9	2 rhyme, 3 no rhyme
6	1 3 4 6 7 9	2 rhyme, 4 no rhyme
6	2 4 5 7 8 9	4 rhyme, 2 no rhyme
6	1 3 4 5 7 9	3 rhyme, 3 no rhyme
7	1 2 3 5 6 8 9	4 rhyme, 3 no rhyme
7	2 3 4 5 6 7 8	3 rhyme, 4 no rhyme
7	1 3 4 5 6 7 9	4 rhyme, 3 no rhyme
8	1 2 4 5 6 7 8 9	4 rhyme, 4 no rhyme
8	1 2 3 4 5 7 8 9	4 rhyme, 4 no rhyme
8	2 3 4 5 6 7 8 9	4 rhyme, 4 no rhyme

Verbal processing – visuo-spatial storage condition

7	8	9
4	5	6
1	2	3

Span Length	Storage Task: Locations of red dot on 3 x 3 grid	Processing task: Visualisation Yes or No
3	2 4 6	2 rhyme, 1 no rhyme
3	3 5 9	1 rhyme, 2 no rhyme
3	1 5 7	1 rhyme, 2 no rhyme
4	2 4 5 8	2 rhyme, 2 no rhyme
4	3 6 7 9	3 rhyme, 1 no rhyme
4	1 5 7 8	1 rhyme, 3 no rhyme
5	3 4 6 8 9	2 rhyme, 3 no rhyme
5	1 2 4 6 9	3 rhyme, 2 no rhyme
5	1 3 5 7 8	3 rhyme, 2 no rhyme
6	2 3 4 6 8 9	2 rhyme, 4 no rhyme
6	1 3 4 6 7 9	4 rhyme, 2 no rhyme
6	1 2 5 7 8 9	3 rhyme, 3 no rhyme
7	1 2 4 5 6 8 9	4 rhyme, 3 no rhyme
7	2 3 4 6 7 8 9	3 rhyme, 4 no rhyme
7	1 2 3 4 5 7 8	3 rhyme, 4 no rhyme
8	1 2 3 4 5 6 7 9	4 rhyme, 4 no rhyme
8	1 2 3 5 6 7 8 9	4 rhyme, 4 no rhyme
8	1 2 3 4 5 6 7 8	4 rhyme, 4 no rhyme

Visuo-spatial processing - verbal storage condition

Span Length	Storage Task: Numbers Presented	Processing task: Words Pairs Rhyme Yes or No
3	1 5 8	2 rhyme, 1 no rhyme
3	6 7 9	1 rhyme, 2 no rhyme
3	3 4 8	2 rhyme, 1 no rhyme
4	1 2 4 8	2 rhyme, 2 no rhyme
4	2 5 7 9	3 rhyme, 1 no rhyme
4	1 3 4 9	1 rhyme, 3 no rhyme
5	1 3 4 6 8	2 rhyme, 3 no rhyme
5	2 5 6 7 9	3 rhyme, 2 no rhyme
5	1 3 4 5 8	3 rhyme, 2 no rhyme
6	1 2 4 5 7 9	2 rhyme, 4 no rhyme
6	2 3 4 6 7 8	4 rhyme, 2 no rhyme
6	1 3 5 6 8 9	3 rhyme, 3 no rhyme
7	1 2 5 6 7 8 9	4 rhyme, 3 no rhyme
7	2 3 4 5 6 7 9	3 rhyme, 4 no rhyme
7	1 2 3 5 6 7 9	3 rhyme, 4 no rhyme
8	2 3 4 5 6 7 8 9	4 rhyme, 4 no rhyme
8	1 2 3 4 6 7 8 9	4 rhyme, 4 no rhyme
8	1 2 3 4 5 6 7 8	4 rhyme, 4 no rhyme

Visuo-spatial processing – visuo-spatial storage condition

7	8	9
4	5	6
1	2	3

Span Length	Storage Task: Locations of red dot on 3 x 3 grid	Processing task: Visualisation Yes or No
3	2 4 6	2 rhyme, 1 no rhyme
3	3 5 9	1 rhyme, 2 no rhyme
3	1 5 7	1 rhyme, 2 no rhyme
4	2 4 5 8	2 rhyme, 2 no rhyme
4	3 6 7 9	3 rhyme, 1 no rhyme
4	1 5 7 8	1 rhyme, 3 no rhyme
5	3 4 6 8 9	2 rhyme, 3 no rhyme
5	1 2 4 6 9	3 rhyme, 2 no rhyme
5	1 3 5 7 8	3 rhyme, 2 no rhyme
6	2 3 4 6 8 9	2 rhyme, 4 no rhyme
6	1 3 4 6 7 9	4 rhyme, 2 no rhyme
6	1 2 5 7 8 9	3 rhyme, 3 no rhyme
7	1 2 4 5 6 8 9	4 rhyme, 3 no rhyme
7	2 3 4 6 7 8 9	3 rhyme, 4 no rhyme
7	1 2 3 4 5 7 8	3 rhyme, 4 no rhyme
8	1 2 3 4 5 6 7 9	4 rhyme, 4 no rhyme
8	1 2 3 5 6 7 8 9	4 rhyme, 4 no rhyme
8	1 2 3 4 5 6 7 8	4 rhyme, 4 no rhyme

Appendix K

Additions used in Chapter 6

	Retrieval	**Counting**	**Decomposition**
	Sum Only	Sum Only	Sum Only
1.	6 + 2	7 + 2	6 + 3
2.	5 + 2	5 + 2	5 + 3
3.	5 + 3	4 + 2	6 + 4
4.	5 + 4	8 + 4	8 + 5
5.	6 + 3	5 + 3	6 + 3
6.	7 + 5	6 + 2	7 + 5
7.	9 + 7	8 + 7	7 + 6
8.	8 + 7	8 + 5	8 + 6
9.	8 + 2	8 + 4	8 + 7
10.	9 + 3	9 + 5	9 + 4
11.	13 + 8	11 + 8	16 + 7
12.	14 + 3	12 + 7	17 + 9
13.	19 + 4	17 + 6	18 + 4
14.	21 + 5	19 + 9	19 + 3
15.	23 + 6	19 + 7	21 + 6
16.	24 + 3	21 + 8	22 + 5
17.	25 + 6	22 + 7	22 + 8
18.	25 + 8	23 + 6	23 + 6
19.	27 + 4	28 + 8	24 + 4
20.	28 + 7	28 + 9	26 + 3
	Visuo-spatial	Visuo-spatial	Visuo-spatial
1.	6 + 2	6 + 2	6 + 3
2.	5 + 4	7 + 2	5 + 4
3.	7 + 4	8 + 3	4 + 3
4.	4 + 3	4 + 3	8 + 5
5.	5 + 2	9 + 5	6 + 3
6.	8 + 5	5 + 3	7 + 6
7.	6 + 3	6 + 3	9 + 7
8.	7 + 5	7 + 4	7 + 2
9.	9 + 8	8 + 2	8 + 3
10.	8 + 3	9 + 7	9 + 4
11.	12 + 7	12 + 6	13 + 7
12.	17 + 3	14 + 5	14 + 5
13.	18 + 6	16 + 5	16 + 4
14.	19 + 5	18 + 7	17 + 9
15.	21 + 6	24 + 4	19 + 7
16.	22 + 7	25 + 4	23 + 6
17.	26 + 4	25 + 9	24 + 6
18.	27 + 8	27 + 6	24 + 7
19.	27 + 5	28 + 7	26 + 8
20.	29 + 6	28 + 6	27 + 8
	Central executive	Central executive	Central executive
1.	6 + 2	4 + 3	6 + 3
2.	4 + 2	6 + 3	4 + 3
3.	9 + 5	7 + 2	5 + 4
4.	5 + 4	5 + 4	8 + 4
5.	6 + 4	6 + 5	9 + 5
6.	6 + 2	6 + 2	5 + 3
7.	8 + 7	6 + 4	7 + 2
8.	7 + 5	8 + 5	6 + 2
9.	9 + 8	9 + 7	7 + 5
10.	9 + 4	9 + 3	9 + 7
11.	13 + 7	11 + 4	13 + 6
12.	15 + 3	12 + 5	14 + 9
13.	16 + 3	16 + 8	18 + 8
14.	18 + 5	19 + 7	19 + 6
15.	19 + 7	22 + 7	19 + 9
16.	21 + 7	23 + 5	21 + 4
17.	23 + 7	24 + 7	22 + 7
18.	28 + 6	26 + 6	24 + 5
19.	28 + 7	28 + 9	27 + 8
20.	29 + 5	28 + 8	28 + 8