PROBLEMS IN THE LEARNING OF PHYSICS - DEVELOPMENT IN THE CONTROL OF VARIABLES

by

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C O N T E N T S

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ABSTRACT

The primary purpose of this work was to identify the process by which the 'control of variables' strategy develops.

Investigations are described which indicate that the strategies used on 'control of variables' tasks are task specific. The quasi/empirical nature of science tasks is shown to affect the use of the control strategy.

Further investigations are described that indicate the existence of a concrete operational control strategy which has as its basis negation by elimination and cancellation, and not Piaget's formal level operation of negation by neutralisation.

Two major studies, one at the Secondary School and one at the Junior School level, are described in which four parallel substages in the development of the control strategy were noted. At the concrete operational level these substages represented an increasing ability to produce a consistency between judgments and experimental results through the formation of increasingly sophisticated strategies, i.e. through attendance to first order relations. At the formal operational level the substages represented an increasing ability to compare criteria for the use of strategies, i.e. attendance to second order relations.

Neo-Piagetian procedures are applied and the calculation of the M demand for the substages tends to confirm a static model for the size of M space.

CHAPTER 1

A PRELIMINARY CONSIDERATION OF THE TOPIC - THE PROBLEM

Introduction

This study is presented in two sections. The first describes the initial work undertaken in Secondary Schools. The overall aim of this section is twofold. First it sets out to explain the reasons for use of alternative strategies to the 'control of variables' strategy. Second it attempts to explain the development of a formal operational 'control of variables' strategy. The second section describes the later work undertaken in Junior Schools. The overall aims of this section are to identify a concrete operational 'control of variables' strategy and to describe a process by which such a strategy may develop.

The clarification of the intellectual level at which, and situations in which, children may be expected to understand 'controlled' comparisons and to produce accurate conclusions from their own 'controlled' experimentation, is of prime importance. The Piagetian pronouncement that 'controlled' experimentation is a formal operational strategy (Inhelder & Piaget, 1958) may be considered to predict that not until 14 or 15 years of age is scientific proof adequately understood.* This has led 'informed' educators to avoid its use with low ability or young Secondary School pupils and with Junior School pupils. This has meant that these children have had a limited experience

* Ages quoted for Piagetian stages and operations are meant to give an approximate orientation to the reader and are not considered to be definitive.

of investigative science. Yet it has been the writer's experience that 11 year olds have a sound notion of 'fairness' in scientific investigations and can make 'fair' comparisons in the following; when testing the bounciness of tennis balls, when testing the effect of the number of turns of elastic on the distance travelled by toy tanks, or when testing the effect of the stream-lining of a boat on its speed through the water.

Several of the enquiries described in Inhelder and Piaget's 1958 volume, the Growth of Logical Thinking (G.L.T.), involve investigations into the development of the experimental control of variables. These well known investigations involve the separation of variables, i.e. the demonstration of the role of each variable, one at a time for successful solution. They are; the pendulum, the bending rods, the inclined plane, the combination of liquids and the invisible magnet tasks. For example, consider the pendulum task. The apparatus provided consists of a set of weights that may be attached to a length of string that may be lengthened or shortened. The subjects are shown how to make and oscillate the pendulum and are asked to discover what governs the frequency of oscillation. The successful subject discovers by experimenting with the material provided that, within wide limits, the frequency of oscillation of the pendulum is independent of the mass or the weight of the bob and the amplitude of swing, but is dependent upon the length of the string. In order to prove this the subject must control all the variables in the situation, except the one whose effect he/she wishes to demonstrate. This methodology or strategy is that of "holding all other things equal" and is termed the H or control strategy in the following. In addition the term 'control of variables' is used.*

The behaviours noted by Inhelder were classified under Piaget's preoperational, concrete and formal stages of development. These findings were generally corroborated in the replication studies of Lovell (1961) and Jackson (1965).

At Stage I (the preoperational stage, age 5 to 6) the subjects understood their own physical actions to be the cause of an event. The independent variables within the pendulum tended not to be considered, rather subjects showed a tendency to push the apparatus to make it go faster. Thus the only causality within the situation that was understood was an externally imposed one, i.e. the causal relations independent of their actions were not fully understood.

At Stage IIA (the early concrete operational stage, age 7 to 8), serial ordering of the independent variables began and the role of impetus was seen as a separate independent effect and could be shown. The independent variables within the pendulum task were noted as the causes of change in the

*Thus in this thesis the use of the term 'control' does not refer to the setting up of 'controls' as used frequently in investigations in the Biological Sciences, although the strategy is equivalent to the H strategy in terms of the reasoning involved. Such 'controls' tasks do not involve immediate feedback as to the effects of the manipulation of the independent variables, thus they have proved less suitable for study by Piaget's clinical method (Piaget, 1929).

dependent variables but the roles were confounded in experimentation. The subjects tended to vary all the independent variables. The subjects thus attributed causal roles to all of the independent variables.* Nevertheless, they could establish that the shorter the string the faster the frequency of oscillation.

At Stage IIB (the late concrete operational stage, age 9 to 10), the subjects accurately serially ordered the independent variables and classified and compared experiments using logical multiplication e.g., heavy weight x short length of string produces a greater frequency of oscillation than light weight x long length of string. Thus a series of experiments could be classified accurately in terms of their effects, but often the confounding effects produced inaccurate conclusions concerning the roles of the independent variables. For example, if a light weight x long length produced a smaller frequency of oscillation than a heavy weight x medium length, then a subject might conclude a direct relation for the role of weight.

At Stage IIIA (the early formal operational stage, age 11 to 14), the ambiguity due to conclusions made from experiments that confounded independent variables was realised by the subjects. They were thus able to distinguish between 'fair' and 'unfair' experiments but would often use

*It should be noted that this may not just be due to conclusions from poor experimentation, but due to intuitions about these roles. The writer has noted that the majority of 14 year olds prior to experimentation consider that all the independent variables affect the frequency of oscillation of a pendulum.

'unfair' experiments that verified their judg ments concerning the roles of the independent variables. Certain subjects controlled the weight to discover the role of length (Inhelder and Piaget, p73: ROS (12;8)). This subject controlled for the role of impetus but when controlling the length to show the effect of weight did not believe his result, that it had no effect, and returned to a judgment verification experiment by logical multiplication. The examples at this Stage in G.L.T. show two important features:

(i) The subjects lacked objectivity. Even when the subjects used controlled experimentation on several occasions and achieved the same result, they would not accept the result and this influenced their further experimentation.

(ii) The use of controlled experimentation was poorly applied in relation to the hypothesis made, although correct conclusions were made for the experiment. For example, (Inhelder and Piaget, p73: JOT (12;7)) in order to show the role of weight, performed a controlled experiment to show the role of length, yet concluded correctly that when the string was shorter the frequency of oscillation increased.

At Stage IIIB (the late formal operational stage, age 14 to 16), the subjects were capable of the spontaneous use of the H strategy, controlling all other independent variables but the one whose role was under investigation.

Piaget (1958) argues that an algebraic group, the INRC group, together with its 16 binary operations, is the underlying structure which plays a crucial role in all formal thinking, i.e. control of variables, explaining compensating systems, etc. Recent workers however, have not found evidence to support Piaget's proposals. Lunzer (1978) has presented evidence of 'formal' problems that, it is proposed, do not have this underlying structure, together with logical problems that do and yet cannot be solved by the majority of adults. Other investigators using Piaget's formal stage criteria, have found that formal thought is lacking in about 50% of adolescent and even adult populations (Friot, 1970; Higgens - Trenk & Gaite, 1971; Kohlberg & Gilligan, 1971; Lawson & Renner, 1974; McKinnon & Renner, 1971; Wollman & Karplus, 1974). In addition a study by Karplus et al (1975) of control of variables and proportional reasoning in several thousand High School subjects in Britain, Austria and the United States, found differences in their acquisition within and between countries. It appears from the above that difficult problems that do not involve the INRC group do not require formal operations as defined by Piaget. Or as appears most likely the INRC group and the 16 binary operations are not sufficient to explain all formal reasoning. Neimark (1975) concludes from the evidence that there is a more advanced intellectual functioning than concrete operational thought, but that such reasoning is not used as reliably and as universally as Piaget's writings imply. In the light of the above, it is proposed to consider the descriptions of behaviour relevant to the control of variables at each stage, where possible avoiding logical models for formal thought.

At this point it is useful to distinguish between the possible early use of controlled experimentation and the formal H strategy. The formal (Stage IIIB) H strategy is one of controlling all other independent variables but the one whose effect is to be demonstrated. This is done in turn for all the independent variables. This method differs from Stage IIIA in the fact that there, although the strategy may be used for some variables it may not be used for others, especially where the results of controlled experimentation are counter-intuitive. Thus the H strategy is not seen as a necessity at this Stage. That the subject is able to control in some instances is evidence that the inability to use the strategy is not due to the fact that it is not available, but that other strategies are seen as being applicable to the situation, e.g., verification of a judgment when it is not confirmed by an H strategy. Thus it appears that a subject using a formal H strategy must reject all other strategies and thus his/her criteria for its use may differ from early formal reasoners. Up to the formal level the H methodology used is insufficiently developed. In order to avoid any ambiguity the H or control strategy will now refer to any strategy that controls the effect of the independent variables but the one whose effect is to be demonstrated in the situation. Thus the early formal reasoner may be considered to use an H strategy in certain situations and not in others. A late formal reasoner may be considered to use the H strategy consistently for all independent variables.

There is evidence to suggest that the H strategy may develop at a concrete level in that a notion of fairness or justice develops at this Stage (Piaget, 1932), and in this light it is useful to consider a task devised by Pocklington (1976). Lunzer (1978) has suggested that a feature of the control strategy is that the subject anticipates possible ambiguities and controls for them. Pocklington's experiment provides evidence of the age at which subjects are aware of the ambiguity of variables. The apparatus consisted of a box with a single light with up to four buttons to press. A range of problems were used with increasing numbers of buttons, the more complex using all four buttons. The subject was told the following information about the buttons:

> One was a switch that causes the light to go on if it was off and vice-versa.
>
> One was an on button that would cause the light to go on if it was off but have no effect if it was already on.
>
> One was an off button that would cause the light to go off if it was on, but have not effect if it was already off.
>
> One was a neutral button that had no effect.

The task was to find out which switch was which. In order to help the subjects keep track of events, eight labels were offered. These were on, off, change, neutral, on or change, on or neutral, off or change and off or neutral. Most of the subjects (5 to 18 years of age) were successful at all but the most difficult problem, but interestingly a difference was found in the use of the labels. The last four represent labels for ambiguous situations and their use would indicate the ability of the subjects to anticipate ambiguous situations, considered by Lunzer to be a prerequisite for the use of the control strategy. At seven, no children were found to use these 'ambiguous' labels, at nine half the subjects used the labels and by eleven they were used whenever they were appropriate. It may be inferred from this that if these nine year olds had an H strategy available they might use it to control for anticipated ambiguities in the control of variables task. Certainly from this evidence it might be expected that the majority of eleven year olds should be capable of the H strategy unless, of course, something within the task misleads them from its use.

A task may be defined as misleading if alternative strategies to the correct strategy are used in its solution when the subjects are capable of the correct strategy. Subjects at Stage IIIA have the H strategy available (evidenced by Piaget's protocols in G.L.T.), thus Piaget's control of variables task may be considered to be misleading in that alternative strategies are used in its solution. This may be the case in the pendulum task in that the intuitive judgment that all the independent variables have an effect may mislead the subject to reject the results of successful H strategies. In this case the subject's objective methodology precedes his/her ability to make objective judgments. It is doubtful whether the latter ability is well developed in many adults. The pendulum task is in fact similar in nature to many tasks used in Secondary

School science courses and it is doubtful that there can be many 'control of variables' situations that are not misleading, in that all subjects bring to the situation some intuitive notion of the effects involved. The accuracy of these intuitions is not of importance. That they are likely to affect the strategy used is. As argued previously the Stage IIIB H strategy is one that is used consistently and it is proposed that it is only through the rejection of the alternative strategies that a set of formal criteria for the use of the H strategy emerges. It is therefore of importance to identify the alternative strategies that are available at the concrete and formal stages as described in the available literature.

A study of the behaviours in the control of variables tasks in G.L.T. leads to the identification of several strategies that may be used upon the independent variables within the task (see Table 1.0). In addition other strategies have been described, notably by Ervin (1960) and Wollman (1976). Where possible, reference will be made to the strategies in relation to the pendulum task described in G.L.T.

<u>The D or Difference strategy</u> is an attempt on the subjects' part to show a difference in the dependent variable due to the independent variable. As noted in the discussion of the pendulum task at Stage IIA the subjects could vary the effects of the variables to show a difference in the frequency of oscillation. The fact that the independent variables were confounded led to incorrect conclusions.

The H impetus, strategy may in effect be a D strategy but it has been included in that Piaget notes that the subjects at Stage IIA/B could show the role of impetus. Thus they would push the pendulum hard and then not push it to show the effect of this independent variable on the frequency of oscillation. That the length of string and weight remained unchanged in the situation are clear, subjects consciously attended to this is unclear.

The G.D. or Greatest Difference strategy. This strategy utilises logical multiplication . The subject varies two or more independent variables at a time in an attempt to increase the effect on the dependent variable. He concludes that one independent variable shows this large effect, e.g., large weight x short length of string > small weight x long length of string leads to the conclusion that the weight increases the frequency of oscillation. In this case the strategy is a G.D. strategy due to the fact that the subject assumes a direct relation for both the weight and length.

The T or Test of Strength strategy. This was noted by Susan Ervin (1960) in a study of seven and eight year olds using Piaget's bending rods task. She concluded that,

"children who stated they were testing hypotheses sometimes controlled known variables (rarely), sometimes varied them favourably (i.e. G.D. or D) and sometimes varied them in a direction opposed to the prediction. They described the last technique as if it were a test of strength of the variable". (Ervin, 1960; p178).

For example, in a test for the role of thickness, a subject who knew that steel rods bent less than brass rods and that heavy weights bent the rod more than light ones, initially discovered that thick rods bent less than thin ones. He then chose a thin steel rod with a light weight and predicted that it would 'go down'. He also chose a thick brass rod with heavy weights and predicted that it would 'stay up'. It should be noted that Ervin also described the occasional use of the H strategy and thus this is tentatively included at Stage IIB. It should also be noted that Case (1974b) has produced evidence that seven and eight year olds may in fact have available an H strategy.

The C or Compensation strategy. This was also evident in Ervin's study but was not identified as such. The strategy was used to show the multiplicative effects of two independent variables. In this instance the method was to make up for some inadequacy in effect of one independent variable by increasing the effect of another e.g., light weights go on thin rods and heavy weights or thick rods. Ervin felt that the response was due to a natural inference from a pair of display rods used in the study. She notes that several subjects persisted with this strategy. Wollman (1976) also noted the C strategy and noted that some subjects combined or compensated independent variables in an attempt to produce equality between compared results.

As can be seen from Table 1.0 the number of alternative strategies at Stages IIB and IIIA which may be applied to control of variables tasks, appears to be high. The study

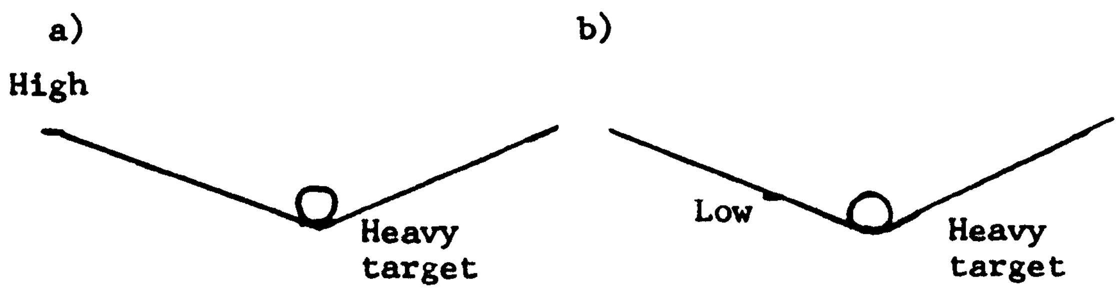
TABLE 1.0

	CONCRETE OPERATIONS		FORMAL OPERATIONS	
STAGES	IIA	IIB	IIIA	IIIB
APPROXIMATE AGES	7-8	9-10	11-13	14-16
STRATEGIES AVAILABLE	D	D	D	
		GD	GD	
		C	C	
		T the test of strength.		
		H impetus.		
	H impetus.	H	H	H

by Wollman (1976) has provided evidence of the use of several of these strategies and of the frequency of use of a consistent control of variables strategy.

Wollman's tasks (1976) involved the use of the familiar variables of weight and height in a situation where a sphere rolls down an incline and strikes a target sphere, sending it up a second incline. Wollman's target sphere task, question W.1, is presented below:-

Wollman's Target Sphere Task



W.1. You have a heavy and a light sphere. Suppose you want to find out whether it makes a difference to release a sphere from the high or low position. Look at drawings (a) and (b) and decide which spheres you would release from the high and low positions in order to find out whether the position makes a difference.

(i) Which sphere would you release from the high position? ____________

(ii) Which sphere would you release from the low position? ____________. Please explain your answer. (Space was provided).

Subjects were asked to prove (to someone who did not already know) that starting position "makes a difference in how far the target goes". The task was presented in an introduction to ensure as far as possible that subjects understood every aspect of the situation e.g., the effects of the independent variables and the different combination of spheres it was possible to use. The sample tested was aged from nine to seventeen years and consisted of 1,555 urban and suburban High School children. 90% of the suburban and 75% of the urban fifteen year old subjects showed some understanding of a controlled experiment, a large increase over the 50%, found by Kohlberg and Gilligan (1971) using Piaget's tasks and criteria.

In the study three questions were given. Question W.2, see p. 19, gave the subjects only one choice of position as one position was already chosen for the light sphere. A further question asked the subjects to judge the ambiguity of a conclusion from an uncontrolled experiment, which attended to only one variable. As would be expected from the discussion of Piaget's findings in G.L.T. this latter question was found to be easiest. The results of 44% and 5.6% of the suburban and urban nine year old samples who answered this question correctly confirm that the realisation of the ambiguity in making unfair comparisons occurs between the ages of nine and eleven (the percentages for the eleven year old samples were 54% and 30% respectively).

Wollman classified the responses in terms of 4 levels of explanation, to represent a developmental trend and these levels have been summarised below:

LEVEL 1 No display of the concept of a comparison; focus on "far" in an absolute sense.

LEVEL 2 An intuitive, inarticulate, and concrete use of the concept with focus on the mechanisms and/or the result of the comparison.

LEVEL 3 A transitional stage with recognition of the desirability of equal weights but without a "logical imperative".

LEVEL 4 Definite rejection of unequal weights as a source of logical ambiguity with focus on the logical consequences of antecedent conditions.

Wollman provides examples of the behaviours, i.e. explanations categorised at each level, instances of the H, C, G,D. and D strategies being evident amongst these. Of particular interest were the C or compensation responses noted by Wollman. He distinguished between two types. The first, a low level C response, was similar to that noted by Ervin (1960). The second, a high level C response, represented an attempt to make the target spheres travel the same distance up the other side of the ramp, i.e. the light sphere would be compensated by a high position, the heavy sphere would be compensated by a low position as in the low level C response but, at the high level, subjects argued that if the target spheres were to go the same distance, because the role of weight is known, then the role of position can be inferred. G.L.T. contains examples of the compensation strategy in the bending of rods task. In addition it has been noted by Piaget to be an important element in the development of proportionality and to be a strategy used in the solution of conservation tasks. The number of subjects using this alternative inference strategy was not clear due to Wollman's categorisation of behaviour.

It was the hypothesis that the formal H concept developed through the rejection of alternative strategies, it was thus necessary to utilise tasks similar to Wollman's in order to provide information as to the development of the H strategy in relation to the strategies that were considered inappropriate. To this end it was necessary initially to replicate Wollman's studies in the form of a pilot study and then to analyse the information in terms of the strategies utilised. These pilot and large scale replication studies are described in the following chapter.

CHAPTER 2

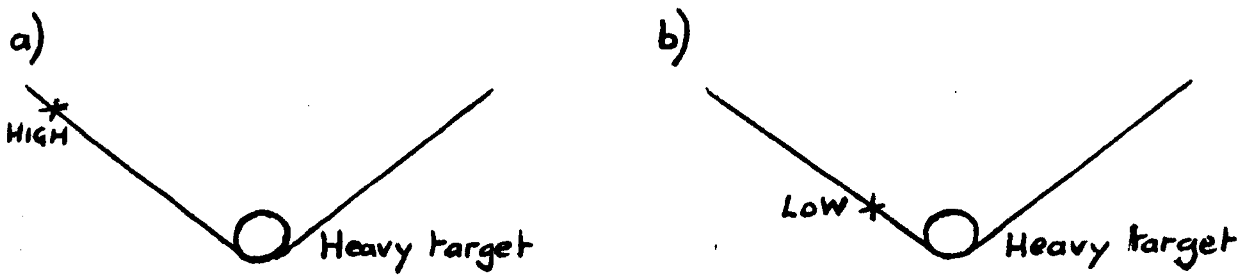
A LARGE SCALE STUDY TO DETERMINE THE STRATEGIES USED ON TASKS SIMILAR TO THE TARGET SPHERE TASK AT THE SECONDARY SCHOOL LEVEL

Introduction

In chapter 1, the target sphere task devised by Wollman (1976) was discussed. The work presented in this chapter is a large scale study of Secondary School children which, unlike Wollman's study, analyses the childrens' solutions to tasks similar to the target sphere task in terms of all the strategies used; compensation, greatest difference, difference and control strategies.

Wollman's tasks involved the proof that the height of release affects the distance travelled by the sphere. Question W1 and W2 are included below for ease of reference.

WOLLMAN'S TARGET SPHERE TASK



W1. You have a heavy and a light sphere. Suppose you want to find out whether it makes a difference to release a sphere from a high or low position. Look at drawings (a) and (b) and decide which spheres you would release from the high and low positions in order to find out whether the position makes a difference.

Which sphere would you release from the high position?

____

Which sphere would you release from the low position?

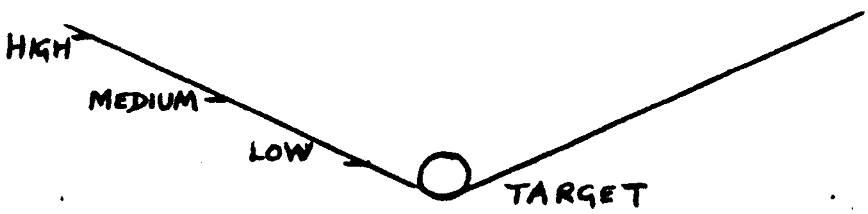
____

W2. Suppose in drawing (b) someone releases a light sphere from the low position. Then which sphere would you release from the high position in order to see whether the position makes a difference.

Light ____________ Heavy ____________

It might be argued that Wollman's questions were misleading, in that they might have been seen by a subject to imply that both spheres needed to be used, whereas the control strategy required the same sphere to be used twice. The subject was forced to release the spheres from different positions and he/she might then have considered because he/she had used one sphere, that the remaining sphere had to be used. This argument needed to be tested and to this end the format of the target sphere task was altered and is referred to as Joyes' target sphere task, questions J1 and J 2 are shown below.

JOYES' TARGET SPHERE TASK



J1. Suppose you want to find the difference the weight of the sphere makes in how far the target goes. You are going to use two spheres a heavy and a light one. Where would you start the light sphere? ____________
Where would you start the heavy sphere? ____________
Please explain your answer carefully (space was provided).

J2. Suppose you want to find the difference the weight of the sphere makes in how far the target goes. You are going to use two spheres a heavy and a light one. The light sphere is started from the low position. Where would you start the heavy sphere? ____________

Please explain your answer carefully (space was provided).

Joyes' target sphere task involves the proof that the weight of the spheres affects the distance moved by the target sphere. The subject is forced to use both spheres, but he may use any height. The choice of one height is less likely to be considered by the subject to preclude the re-use of that height. A total sample of 60 third and fourth year comprehensive school subjects was given the target sphere tasks in a pilot study. These subjects were two science classes selected by the school as being of 'average ability' for their age. The third year were streamed, the selected class being the fourth band selected from eight. The selected fourth year class were a 'good' C.S.E. physics class whose predicted examination grades were grades 1 to 4. It is possible that the sample were of above average ability for their age. Thus conclusions that assume that the samples used are representative of their age group are tentative. Half the third and fourth year subjects were given the Wollman task followed by the Joyes' target sphere task. This order of presentation was reversed for the other subjects in order to control this factor. The results are shown in Table 2.0.

TABLE 2.0 The number of subjects using control methodology on the target sphere tasks.

(Percentages in Parenthesis)			Wollman's target sphere task		Joyes' target sphere tasks	
YEAR	Mean Age in years	N	Question 1 W1	Question 2 W2	Question 1 J1	Question 2 J2
THIRD	13.6	30	10(33)	16(53)	10(33)	15(50)
FOURTH	14.7	30	15(50)	22(73)	17(57)	22(73)

The McNemar test for the significance of changes was applied to test for differences between the questions. This test was chosen because of the classificatory measurement used and because the questions were answered by the same subjects, i.e. related samples were used. The fourfold tables are presented in Appendix A and the significant differences between questions are summarised in Table 2.1.

TABLE 2.1 A summary of the significant differences found between the use of the H strategy on the target sphere questions.

Questions compared	Significant Differences	
	Year 3	Year 4
W1 x W2	*	**
W1 x J1	ns	ns
W1 x J2	*	**
W2 x J1	*	*
W2 x J2	ns	ns
J1 x J2	*	*

ns = not significant
* = significant at the 0.05 level
** = significant at the 0.01 level

Performance on the questions appeared to improve with age and a comparison of W1 and J1 for each year tested shows an almost identical number of correct solutions, the slight differences were not found to be significant. Thus it appears that W1 and J1 were found to be of equal difficulty by the subjects. Similarly W2 and J2 appear to be of equal difficulty. For the sample tested the difficulty did not therefore appear to be within the question itself, but in the subjects' inability to realise that a variable within the situation needed to be controlled.

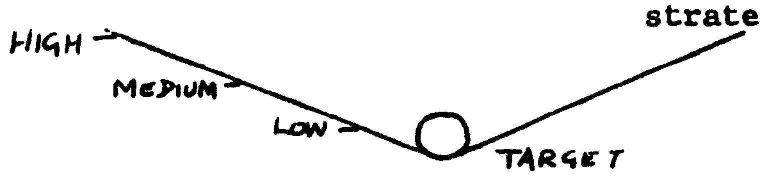
For both target sphere tasks, performance was apparently better on W2 and J2 than W1 and J1. Notably for the Wollman's target sphere task 25% more fourth year pupils were correct on W2 than W1, the difference being 20% at the third year level. This difference was significant at the 0.01 level. The greater facility of W2 was noted by Wollman who felt that this was due to the lower number of alternatives offered to the subject. W1 required the subject to make a choice from four alternatives. In W2 a choice was made for the subject, i.e. the light sphere was released from the low release position, thus only two alternatives were available, heavy sphere from high or light sphere from high. The results of the pilot study tend to refute this hypothesis and this was tested formally in the main study described in this chapter. J2 offered the subjects a choice of three heights of release not two as in W2. The responses for J2 and W2 for each year were nevertheless almost identical. Further evidence against Wollman's hypothesis is given by a comparison of J1 and W1 responses for each year, as these were almost identical. Yet W1 involved four alternatives and J1 involved six alternatives.

It may be argued that the difficulty of W1 and J1 compared to W2 and J2 was due to the fact that W1 and J1 required the subjects to make two decisions in relation to the outcome of releasing the spheres, and these anticipated outcomes had to be compared. For example, for J1 the question was as follows:-

> You are going to use two spheres, a heavy and a light one. Where would you start the light sphere? Where would you start the heavy sphere?

A two comparative decisions response would be firstly to decide to release the heavy sphere from a high position anticipating that it would knock the target far, then secondly to decide to release the light sphere from a low position anticipating that it would not knock the target very far. The anticipated outcomes of these two decisions would then be compared, i.e. that this would result in the greatest difference being shown. W2 and J2 required only one comparative decision to be made, in that one had already been made for the subject. Nevertheless, it seems more reasonable to propose that the difficulty of W1 and J1 compared to W2 and J2 was due to the strategies that each allowed. As has been mentioned in chapter 1, the compensation strategy was found to be a common strategy at the Secondary School level. Thus a common strategy was to release the light sphere from a high position and the heavy sphere from a low position. This compensation strategy could not be used for W2 and J2 in that the position of release for the light sphere was already chosen as being the low position. This strategy could be used for W1 and J1 because both decisions concerning the release of the spheres were made by the subject. Thus a child capable of both the compensation and control strategies may have chosen either strategy on W1 and J1, but not the compensation strategy on W2 and J2. Such a child might have been more likely to use the control strategy on W2 and J2 because the use of the compensation strategy was excluded by the question. This exclusion of strategies is a feature of questions that only require one comparative decision. It was the intention of the main study described in this chapter to test this exclusion of strategies hypothesis and to this end, a further question J3, shown below, was devised.

<u>JOYES' TARGET SPHERE TASK</u> (excludes the use of the greatest difference strategy).



J3. You are going to use two spheres a heavy and a light one. Someone starts the light sphere from the high position. Where would you start the heavy sphere to find out exactly how much difference the weight makes to how far the target goes? ________________

Question J3 involves the subject in making one comparative decision as in W2 and J2, but in this case the light sphere is started, not from the low position but the high position. This allows the use of the compensation strategy and it excludes the possibility of using the greatest difference strategy, because the light sphere must be released from a low position in the latter strategy.

Wollman's method of classification of the behaviours into four levels of response referred to in chapter 1 paid little attention to the major strategy used by the subject and this method was therefore not used in this study.

Examples of the compensation strategy have been included by Wollman at levels 1 and 2.

LEVEL 1.	High-Light Low-Heavy	"A light sphere would need space to move the target so you would place it at the high position. A heavy sphere would not need as much speed because of its weight so you would place it at the low position."
LEVEL 2.	High-Light Low-Heavy	"At the high position the sphere would get more speed and would be able to hit the target about as far as the heavy sphere at the low position. Where the ball wouldn't get as much speed but since it is bigger it would have more power."

Early forms of the difference strategy have been included with the compensation strategy at these levels. In addition early attempts at the control strategy have been included at level 2, as well as at levels 3 and 4.

LEVEL 2.	High-Heavy Low-Heavy	"To see if the starting point makes a difference let a heavy ball go from a high point and see how far it goes up the other side and then a heavy one from a low position and compare your results."
LEVEL 2.	High-Heavy Low-Heavy	"Heavy high because I want to see how far it will go. Heavy low because you'll find out how much higher <u>it</u> goes from the high position".

Thus distinctly different strategies have been classified at the same level in terms of the explanation given for their use. For the purposes of this study it seemed more important to classify the responses in terms of the strategy used as the difficulties the subjects encountered with the questions appeared to be one of choice of strategy, with the one comparative decision questions reducing this choice. A further classification in terms of level of explanation within each strategy to outline an increasing sophistication in the reasons for its use was undertaken, to provide evidence of the confusion between the use of the strategies.

The aims of this initial study were thus:-

1. To analyse the data in terms of the strategy utilised, i.e. the difference, the greatest difference, compensation, control strategy or other.
2. To discover whether the greater use of the control strategy on the one comparative decision question, W2, when compared to question W1, the two comparative decisions question, was due to the exclusion of the compensation strategy on W2.

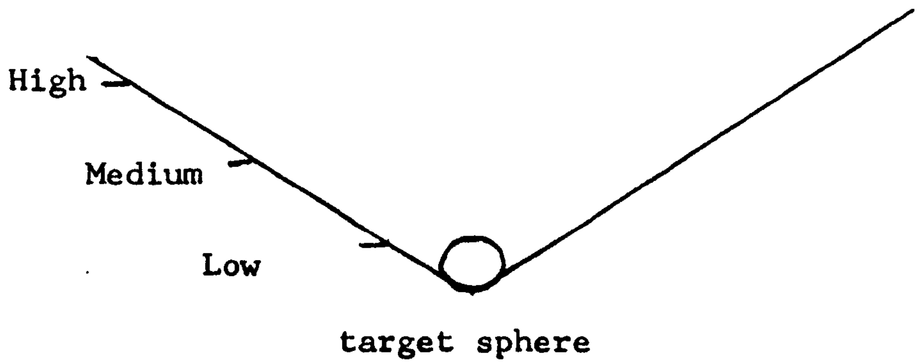
Experimental Method

The Tasks

The three tasks used are shown on the following pages. The Joyes' target sphere task is a modification of Wollman's task already referred to. The toy lorry task is similar to the target sphere task, it involves the proof that the weight of the lorries affects the depth of hole left by the pencil in the plasticine. Thus the indicator of the weight difference was static and a set of results could be shown as proof of the difference at the end of the experiment. The plasticine task presented the subject once again with the task of determining the effect of different weights, but in this instance on the depth of hole

THE TASKS

JOYES' TARGET SPHERE TASK



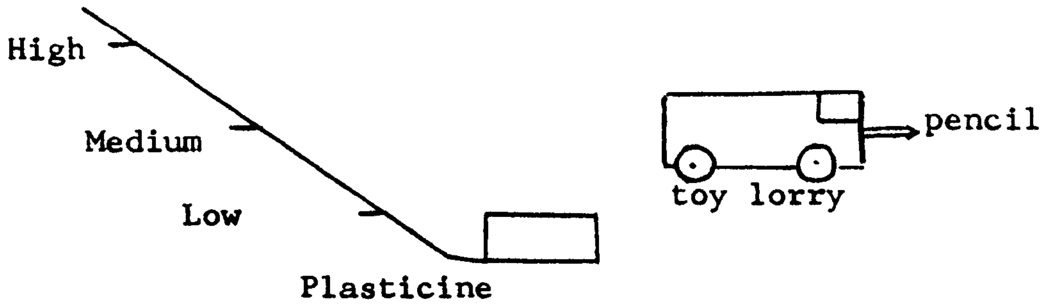
1. Suppose you want to find out how much difference the weight of the sphere makes in how far the target goes. You are going to use two spheres a heavy and a light one. Where would you start the heavy sphere? ________________. Where would you start the light sphere? ________________.

2. Someone starts the light sphere from the high position. Where would you start the heavy sphere to find out exactly how much difference the weight makes to how far the target goes? ________________ Please explain your answer carefully (space was provided).

3. Someone starts the light sphere from the low position. Where would you start the heavy sphere to find out how much difference the weight makes to how far the target goes? ________________. Please explain your answer carefully. (space was provided).

THE TASKS

THE TOY LORRY TASK



1. Suppose you want to find out how much difference the weight of the lorry makes to the depth of hole left in the plasticine. You are going to use two lorries a heavy and a light one.

   Where would you start the heavy lorry? ____________

   Where would you start the light lorry? ____________

   Please explain your answer carefully. (space was provided).

2. Someone starts the light lorry from the high position. Where would you start the heavy lorry to find out how much difference the weight makes to the depth of hole? ________________

   Please explain your answer carefully. (space was provided).

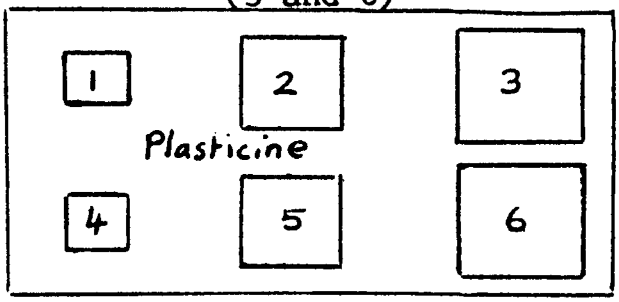
3. Someone starts the light lorry from the low position. Where would you start the heavy lorry to find out how much difference the weight makes to the depth of hole? ________________. Please explain your answer carefully. (space was provided).

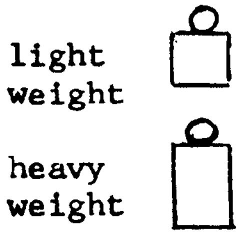
THE TASKS

THE PLASTICINE TASK

The following blocks of wood are identical:-

(1 and 4)
(2 and 5)
(3 and 6)





1. Suppose you want to find out how much difference the weight makes to the depth of hole left in the plasticine. You are going to use two weights a heavy and a light one.

   Where would you place the heavy weight? ___________

   Where would you place the light weight? ___________

   Please explain your answer carefully.

   (space was provided).

2. Someone places the light weight on Block No 1.

   Where would you place the heavy weight to find out how much difference the weight makes to the depth of hole?

   ___________. Please explain your answer carefully.

   (space was provided).

3. Someone places the light weight on Block No. 6.

   Where would you place the heavy weight to find out how much difference the weight makes to the depth of the hole? ___________. Please explain you answer carefully.

   (space was provided).

left in plasticine by blocks of wood. Small scale trials showed that due to the static nature of the task, the subject sometimes considered that, once a choice of a weight had been made on one particular block, the same block could not be chosen again because it was already occupied. The form of the task used in the study overcame this difficulty by means of three pairs of identical blocks. This still gave the subject three different alternatives as in the other tasks.

Each task contained three questions and the corresponding questions on each task were parallel in terms of the strategies they excluded. Question 1 was a two comparative decisions question where the subject had to make both decisions for the choice of start position in the target sphere and toy lorry tasks, or as to the choice of block of wood for the weights in the plasticine task. Questions 2 and 3 were one comparative decision questions. Question 2 excluded the possibility of using the greatest difference strategy and question 3 excluded the possibility of using the compensation strategy.

TABLE 2.2 The three question types.

Question No.	Type(number of decisions to be made by subject)	Strategy excluded
1	2 comparative decisions	None
2	1 comparative decision	greatest difference(G.D.)
3	1 comparative decision	compensation (C)

The question order was chosen carefully after considering the responses to a pilot study. The chosen order was intended to exclude the effect of any learning sequence on the use of the compensation or control strategies. Question 1 allowed the subject to utilise any strategy. At the age range of interest the most common error strategy had been found to be one of compensation. A subject using the compensation strategy on question 1 would have had to use another strategy if question 3 had immediately followed question 1, because the compensation strategy would now have been excluded. He might then have continued to use this non-compensation strategy on question 2 if it had followed question 3, even though he could have once again used the compensation strategy. The sequence used was the numerical one in that the compensation strategy was not excluded from use until question 3. Thus the level of the compensation responses should not have been affected by the question order. The sequence presented to the subjects might have affected the use of the greatest difference strategy in a similar manner in that its use was excluded on question 2 and this might have affected its use on question 3. This possible effect was not controlled for in that evidence from Wollman's study and from Piagetian theory suggested that the frequency of use of the greatest difference strategy would be small if used at all. In addition any subject who was likely to choose to use the greatest difference strategy on question 3 would have been most likely to be able to comprehend the compensation rather than the control strategy in that the greatest difference and compensation strategies have as their basis concrete operations. Such a subject would be unlikely to have the control strategy within his/her repertoire of strategies. It was considered that for a subject using the greatest difference strategy on question 1, the exclusion of the use of this strategy on question 2 would be most likely to lead to the use of the compensation strategy on this question, with a reversion to the use of the greatest difference strategy on question 3, because compensations were now excluded. Thus the chosen question order was considered unlikely to affect the use of the greatest difference strategy.

The Subjects

A total of 510 subjects was tested at the Secondary School level. The details of the sample are given in Table 2.3. The sample was selected at random from two Secondary Schools in Nottinghamshire.

TABLE 2.3

YEAR	MEAN AGE YEARS, MONTHS	TOTAL
1st	11.7	90
2nd	12.7	90
3rd	13.8	90
4th	14.6	90
5th	15.8	90
L6th	16.9	30
U6th	17.8	30
		Σ 510

Hypotheses

Question 1 offered the subjects 6 alternatives from which to make their choices whereas questions 2 and 3 offered the subjects only 3 alternatives. Thus it was possible to test Wollman's hypothesis concerning the effect of the number of alternatives as well as the exclusion of strategies hypothesis. Thus the following hypotheses were set up:-

1. That any significant difference found in the frequency of use of the major strategies on the three question types may be explained by the number of alternatives of start positions available from which the subject may choose (Wollman's hypothesis).
2. That any significant difference found in the frequency of use of the major strategies on the three question types may be explained in terms of the strategies excluded by the question.

Experimental Design

Each year tested was randomly selected into 3 groups of 30 subjects in years 1 to 5 and 3 groups of 10 subjects in the sixth year. Each group in a year was given a different task to complete.

Experimental Procedure

1. The effects of the variables involved in the experiment under consideration by the particular group were fully discussed with the subjects. For example, in the target sphere task the subjects were shown the apparatus, one

sphere was released from a position and the subjects saw the target sphere roll up the other side. Then the effect of the height of release of a sphere on the distance moved by the target sphere was discussed and a qualitative conclusion was reached that the higher the release position the further the target sphere would travel. A similar conclusion was reached concerning the effect of the weight of the sphere on the distance travelled by the target sphere.

2. The subjects were then given the questionnaire to complete.
3. Each question was read through with all the subjects in years 1 and 2 and they were given adequate time to complete a response.
4. The answers were briefly checked and any ambiguity in a response was checked with the subject concerned.

Results

Five main categories of response were identified from the subjects responses to the tasks. These were:-

H - using a form of control methodology or "Holding all other things equal".

C - using a form of Compensation strategy

G.D. - using a form of Greatest Difference strategy

D - using a form of Difference strategy

U - using an Unclear method or offering no response.

The three tasks, the target sphere, toy lorry and the plasticine tasks were found to be of similar difficulty by the subjects. The full set of results is presented in Appendix B.

The McNemar tests for significant changes between the tasks revealed no significant differences. This test was chosen because nominal measurements and related samples were used. The fourfold tables are presented in Appendix C.

For further analysis and the sake of clarity, it was decided to pool the results for the tasks as they apparently presented identical difficulties to the subjects. Table 2.4 shows the pooled results as a percentage response for all three question types. In order to test for any significant changes between the questions on the use of the major strategies the McNemar test for significance was chosen. The fourfold tables for each strategy are presented on the following page. The differences between performance on the three questions were found to be significant for all the strategies except between Q1 and Q2 on the use of the H strategy and Q2 and Q3 on the use of the D strategy, the least significant being at the 0.05 level.

A comparison of the frequency of use of the strategies for question 1 and question 3 shows a marked increase in the use of the D., G.D. and H strategies on question 3. (For H; $X^2 = 64.64$; $p < 0.001$, for G.D.; $X^2 = 34.22$; $p < 0.001$, for D; $X^2 = 4.92$; $p < 0.05$). This is coupled with a decrease in the use of the C strategy, significant at the 0.001 level ($X^2 = 146.01$). Thus the exclusion of the C strategy on question 3 appeared to make a significant difference to the use of the other strategies, in particular the H strategy. Question 2 offered the same number of alternatives to the subjects as

TABLE 2.4 **PERCENTAGE OF SUBJECTS RESPONDING WITH THE DIFFERENT STRATEGIES TO THE THREE QUESTION TYPES (one decimal place)**

YEAR	MEAN AGE	N	QUESTION 1 STRATEGY					QUESTION 2 STRATEGY					QUESTION 3 STRATEGY				
			H	C	GD	D	U	H	C	GD	D	U	H	C	GD	D	U
1	11.7	90	7.8	25.6	18.9	42.2	5.6	15.6	32.2	0	46.7	5.6	14.4	0	17.8	52.2	15.6
2	12.7	90	32.2	36.7	14.4	13.3	3.3	32.2	34.4	0	32.2	1.1	46.7	0	38.9	13.3	1.1
3	13.8	90	45.6	46.7	6.7	0	1.1	45.6	54.4	0	0	0	55.6	0	44.4	0	0
4	14.6	90	62.2	33.3	4.4	0	0	63.3	36.7	0	0	0	95.6	0	4.4	0	0
5	15.8	90	84.4	14.4	1.1	0	0	84.4	15.6	0	0	0	100	0	0	0	0
L6TH	16.9	30	83.3	10.0	6.7	0	0	90.0	10.0	0	0	0	100	0	0	0	0
U6TH	17.8	30	86.7	13.3	0	0	0	86.7	13.3	0	0	0	100	0	0	0	0

FOURFOLD TABLES SHOWING McNEMAR TESTS FOR SIGNIFICANCE BETWEEN QUESTION 1, 2 AND 3

H STRATEGY

Question 1	Question 3: Not H	Question 3: H
H	9	251
Not H	160	90

(X^2 = 64.64***)

Question 1	Question 2: Not H	Question 2: H
H	9	251
Not H	231	19

(X^2 = 2.89 n.s.)

Question 2	Question 3: Not H	Question 3: H
H	10	260
Not H	159	81

(X^2 = 53.85 ***)

C STRATEGY

Question 1	Question 3: Not C	Question 3: C
C	148	0
Not C	362	0

(X^2 = 146.01 ***)

Question 1	Question 2: Not C	Question 2: C
C	2	146
Not C	345	17

(X^2 = 10.32 **)

Question 2	Question 3: Not C	Question 3: C
C	163	0
Not C	347	0

(X^2 = 161.01***)

G.D. STRATEGY

Question 1	Question 3: Not GD	Question 3: GD
G.D.	12	31
Not GD	403	64

(X^2 =34.22***

Question 1	Question 2: Not GD	Question 2: GD
G.D.	43	0
Not GD	467	0

(X^2 = 35.03 ***

Question 2	Question 3: Not GD	Question 3: GD
G.D.	0	0
Not GD	415	95

(X^2 = 93.01***)

D STRATEGY

Question 1	Question 3: Not D	Question 3: D
D	2	48
Not D	449	11

(X^2= 4.92 *)

Question 1	Question 2: Not D	Question 2: D
D	3	47
Not D	436	24

(X^2 = 14.82 ***)

Question 2	Question 3: Not D	Question 3: D
D	31	40
Not D	420	19

(X^2 = 2.42 n.s.)

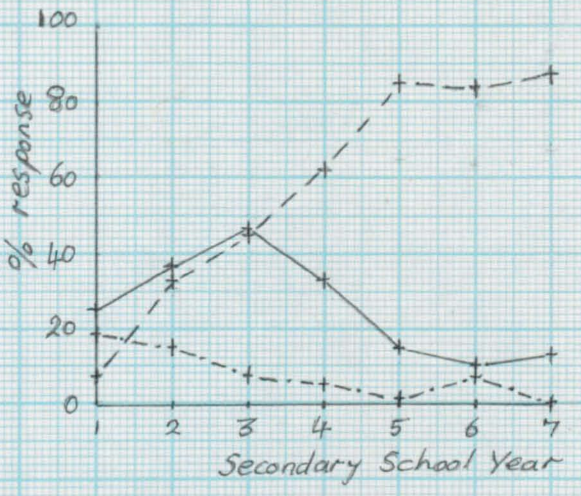
n.s. - not significant
* - significant at the .05 level
** - significant at the .01 level
*** - significant at the .001 level
(McNemar two tailed test: df = 1).

question 3, yet there was a difference significant at the 0.001 level in the use of the H strategy on these questions ($X^2 = 53.85$; $p < .001$). In addition question 2 offered less alternatives than question 1, yet there was no significant difference in the use of H strategy ($X^2 = 2.89$ n.s.). It appears then that the difference in frequency of use of the strategies on the three questions may be explained by the exclusion of strategies as proposed and not by the number of alternatives of position of start etc., available. Thus hypothesis 2 was confirmed and hypothesis 1 was refuted.

To help illustrate the differences between question 1, 2 and 3 and the use of the three major strategies, H, C and G.D. this data is presented graphically in Figures 2.1, 2.2 &2.3. A comparison of figures 2.1 and 2.2 indicates that the response pattern for question 1 is very similar to that for question 2. The exclusion of use of the G.D. strategy on question 2 resulted in an increased use of the C strategy. This increased use of the C strategy was not large, due to the low use of the G.D. strategy by the subjects on question 1, but the difference was found to be significant ($X^2 = 10.32$; $p < 0.01$). Note that there was no significant increase in the use of the H strategy when G.D. was excluded, (Q1 X Q3; $X^2 = 2.89$). It appears therefore that there is less confusion between the G.D. and H strategies than the C and H strategies. This argument appears to be confirmed by a comparison of figures 2.1 and 2.3. Question 3 excluded the use of the C strategy, and this resulted in a marked preference for the H strategy, H was found to be

GRAPHS SHOWING THE PERCENTAGE RESPONSE FOR THE MAJOR STRATEGIES ON QUESTIONS 1, 2 & 3.

Fig. 2.1 Question 1; % for strategies



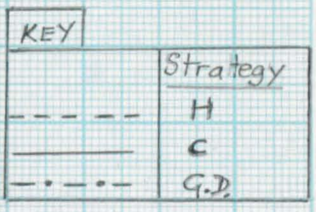


Fig. 2.2 Question 2; % for strategies

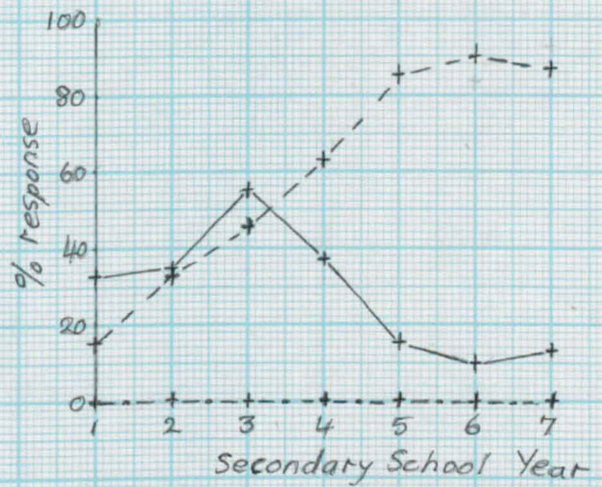
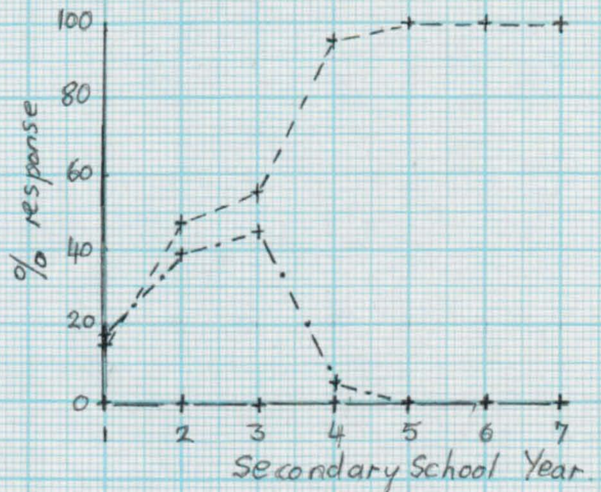


Fig. 2.3 Question 3; % for strategies



the most frequent strategy by year two. It must be pointed out that the exclusion of the G.D. strategy in the question 2 may have affected these latter results, but this has been discounted earlier in this chapter. (It should be pointed out that the predicted tendancy for those subjects who used G.D. on question 1 to use the C strategy on question 2, and to revert to the G.D. strategy on question 3, was confirmed). Thus it may be concluded that for the subjects capable of using the H strategy, the C strategy was the greatest distraction, i.e. the confusion in use was the greatest between the C and H strategies.

The analysis of the questions purely in terms of the strategy used has given a detailed view of the difference between behaviours due to the constraints of the question. This method of analysis also provided information concerning the different levels of explanation provided by the subjects for the use of the strategy.

The difference strategy, D, was the simplest strategy used by the subjects. As the examples below show the subject was purely concerned with the experimental outcome of showing a difference in the depth of hole left in the plasticine.

<u>Level of explanation</u>

<u>Examples:- D strategy</u>

<u>The Plasticine Problem</u> <u>Method</u>	<u>Explanation</u>
SCO (12.2 years). Places the heavy weight on the middle sized surface area block. The light weight on the small sized surface area block.	This will show the difference the weight makes by the depth of hole left. Subject refers to <u>EXPERIMENTAL OUTCOME</u>.

Method	Explanation
CLA (13.4 years). Places the light weight on the large block. The heavy weight on the middle block.	You would expect to see a difference in the depths. Subject refers to EXPERIMENTAL OUTCOME.

The G.D. strategy, was once again concerned primarily with the experimental outcome, as the example below shows. The subject combines the effects of both independent variables to produce the greatest difference possible.

Level of explanation

Example:- G.D. strategy

The Plasticine Problem Method	Explanation
SHEP (13.7 years). Light weight on the large block, heavy weight on the small block.	The light weight will hardly make a hole but the heavy weight will really go in deep. To make the biggest difference you can. Subject refers to EXPERIMENTAL OUTCOME.

The older subjects who used the G.D. strategy used it for a similar reason, that of concern with the experimental outcome, but as the following example shows they considered that prior knowledge as to the precise effect of the surface area might enable a calculation of the effect of weight.

Level of explanation.

Example:- G.D. strategy

The Plasticine Problem Method	Explanation
OLD (16.9 years). Light weight on the large block. Heavy weight on the small block.	If the effect of surface area were known, the effect of weight may be calculated, but it may be clearly seen. Subject refers to EXPERIMENTAL OUTCOME AND CALCULATION.

For the compensation strategy, C, five categories of explanation of use were identified as follows:-

Category - The subject refers to:-	Examples for the plasticine problem (All placed heavy-large, light-small.)
1. Initial conditions	- GUTH (14.1 years). "put the heavy weight on the large block... to make up for the largeness."
2. Method	- BRI (14.3 years). "to even out the extra weight, this makes it fair."
3. Experimental Outcome	- DOW (13.3 years). "make the blocks sink to the same level."
4. Experimental outcome and Reason	- SMI (14 years). "if they are the same depth then it means that the large weight goes in further."
5. Outcome and Reasoned Calculation	- SIM (17.4 years). "if the right ratio of weights and surface areas were used then the exact effect could be calculated."

The five categories may represent an increasing sophistication in explanation for the use of the compensation strategy. Table 2.5 shows the frequency of response for the categories of explanation for the use of the compensation strategy for questions 1 and 2.

TABLE 2.5 Number of responses for the compensation explanation categories for questions 1 and 2 (N = 90 for years 1 to 5, N=30 for years 6 & 7)

Compensation Explanation Category	Question 1 YEAR							Question 2 YEAR						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	(16)	15	12	4	-	-	-	(19)	15	14	4	-	-	-
2	7	15	(20)	7	2	-	-	9	14	(25)	9	2	-	-
3	-	3	10	(17)	5	-	-	1	2	10	(18)	6	-	-
4	-	-	-	2	(6)	-	-	-	-	-	2	(6)	-	-
5	-	-	-	-	-	3	(4)	-	-	-	-	-	3	(4)

The most frequent response for each category is encircled. There is an apparent age dependency with category of explanation for the use of compensations. It should be noted that all the subjects utilising the compensation strategy were doing so to overcome the same problem. In each case it represented an attempt to deal with the confounding variable, i.e. the surface area in the plasticine problem, and yet still enable the difference due to the weight to be shown or implied.

All the subjects classified as utilising the H strategy did so because they were concerned with the method that they were utilising. Some explicitly mentioned fair experimentation, others mentioned the need to show the true difference due to the weight as the examples below show. There were differences in the sophistication of response but further classification of this data was not possible as most subjects' explanations mentioned both their concern with the outcome and with performing a fair experiment.

Level of explanation

Examples:- H strategy The Plasticine Problem Method	Explanation
GRI (13.9 years). Heavy weight on the small block, light weight on the other small block.	If you do not use the same size block it would not go down to its true depth. Subject refers to METHOD and EXPERIMENTAL OUTCOME.
BOO (14.4 years). Heavy weight on the small block, light weight on the other small block.	You must do a fair experiment using the same sized block for the different weight. Subject refers to METHOD

Method	Explanation
MIS (15.4 years). Heavy weight on the medium block, light weight on the other medium block.	A fair experiment must let the weight reach its proper depth. Subject refers to METHOD and EXPERIMENTAL OUTCOME.

Discussion

The results confirmed hypothesis 2. It seems fairly clear that the use of a particular strategy on any of the three questions is dependent upon the strategies excluded by the question. It also appears that the frequency of use of the H strategy is dependent upon which strategy, if any, is excluded by the question. As was noted, question 2 excluded the use of the greatest difference strategy and there was no significant increase in the use of the H strategy compared to its use on question 1. Question 3 excluded the use of the compensation strategy and there was a significant increase in the use of the H strategy.

The analysis of the reasons for the use of the strategies provided information as to why the subjects used a particular strategy. The use of the H strategy is obviously dependent upon whether the subject has the strategy available and how well it is differentiated from the other strategies available. For example, if a subject is concerned with the experimental outcome he/she may use the D, G.D., C or H strategy and consider that he/she has produced the difference in the dependent variable. The D, G.D. and H strategies all show a difference even though only the H strategy shows the difference in the dependent variable due to one chosen independent variable. For the C strategy the subject may deduce that the effect of weight has been shown if there is no observed difference in the dependent variable in that the expected difference due to weight occurs. For example, in the plasticine problem, the subjects stated that the heavy weight on a large surface area may produce the same depth of hole in the plasticine as the light weight on the small surface area and that this provides proof that the heavy weight produces a larger depth of hole. This is because the subjects consider that the larger surface area stops the difference actually being observed. If the subject is concerned with the experimental method he/she may use C or H as both are 'fair'. In this way his/her choice narrows to just two strategies. It is only when the H strategy is clearly differentiated from the other strategies available in terms of the reasons for use that the subject is likely to be capable of consistent use of this strategy, i.e. he/she must have rejected all other strategies as being inadequate methods of proof.

The following factors have so far been considered to affect the use of a strategy upon the questions:-

(i) the strategies the subject has available,
(ii) the strategies excluded from use by the question,
(iii) the reasons for acceptance/rejection of the strategy as a method of proof.

Factor (iii) is concerned with how well the strategies are differentiated from each other.

The development of a fully integrated concept of H experimentation, one which when faced with the choice of alternative strategies will enable the rejection of the alternatives may be studied by means of testing factors (i) and (iii) and will be returned to in chapter 5.

What is of continuing interest are the reasons for the use of the compensation strategy. The physical situation to which the question is referring determines whether a strategy that assumes the effects of the independent variables may be used. For example if the independent variables are weight and surface area of block and the subjects know that a large weight may be compensated for by a large surface area then they are able to use the compensation strategy. However, if the effects of the independent variables are unfamiliar or unknown to the subjects, i.e. X and Y, then the subjects are not likely to compensate. This use of a physical situation that uses unfamiliar independent variables is dealt with in chapter 4.

The compensation strategy is used because it is considered a fair experiment, yet it is fair in a different way to the control strategy. The former allows an 'inadequacy' in one independent variable to be compensated for by another independent variable, confounding both effects in the effect on the dependent variable. The latter is fair because it controls one independent variable so that a 'fair' comparison can be made of the effect of the other independent variable on the dependent variable. Thus fair experimentation may be either compensation or control experimentation. The notion of fairness by compensation is to be found in general use in the handicap race or competition and one such 'fair' situation is studied in the following chapter.

CHAPTER 3

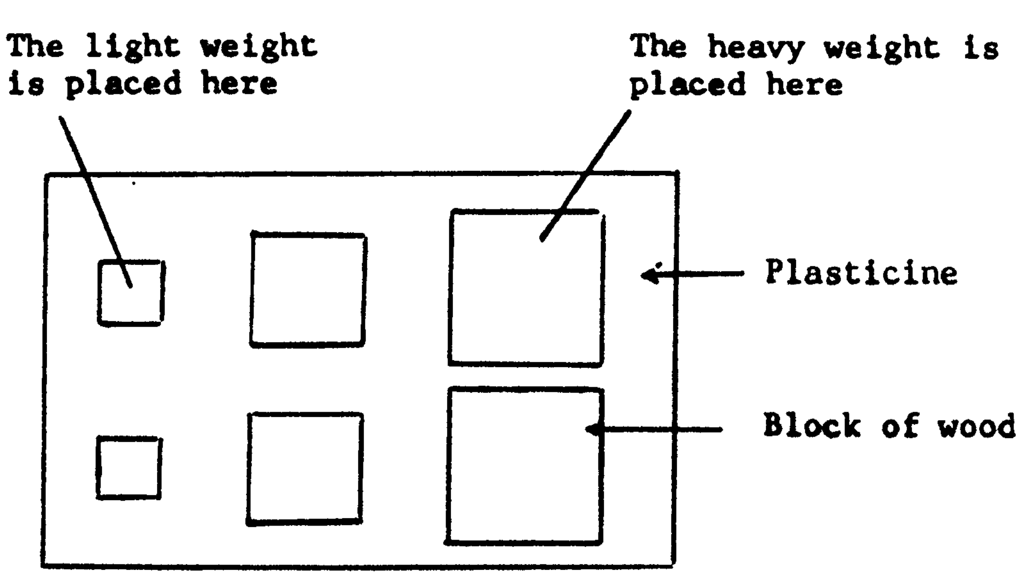
AN INVESTIGATION INTO THE NOTION OF FAIRNESS AT THE SECONDARY SCHOOL LEVEL USING THE RACING CAR AND PLASTICINE TASKS

Introduction

It is the aim of the present chapter to illustrate the different notions of fairness found at the Secondary School level. Chapter 2 confirmed that on problems that required the use of H methodology for their solution a major error strategy at the Secondary School level was the compensation strategy. It was also shown that the reasons for use of the compensation strategy C and the control strategy H were given in terms of producing fair experiments. The subjects who used the compensation and control strategies recognised the need for a fair experiment but were confused as to which type of fair experiment was required.

Consider the plasticine task for which a typical compensation strategy response is as shown in Figure 3.0.

FIGURE 3.0 A compensation strategy used on the Plasticine Task



The subject is aware that the effect of the independent variable, weight, on the depth of hole left in the plasticine has to be shown. He/she is aware of the effect of the surface area, and knows that his/her experiment must cope with this effect. The compensation approach is to make up for the inadequacy in one independent variable in its effect on the dependent variable by using the surfeit of effect due to the other independent variable. Thus a small weight has an inadequacy in its effect on the depth of hole left in the plasticine so this is compensated for by placing it on the small surface area which makes up for this. In the same way a heavy weight has a surplus of effect and this is reduced by placing it on the large surface area. The heavy weight has been handicapped and this notion of fairness, i.e. compensation fairness, is to be found in the handicap race or competition.

In horseracing, a handicap race is used to make 'fair' a race, the result of which would otherwise be a foregone conclusion. The intention is that the fastest horse, judged on past form, is given just enough saddle weight to carry so that the slowest horse can compete 'fairly' with it. The same notion is used elsewhere e.g., golf, car rallying, etc.

The control strategy, that of controlling all independent variables other than the one whose effect is to be shown, is also a fair strategy and is similar to the olympic style of race. In order to study this distinction between the two notions of fairness, two racing car tasks were devised and are shown on the following page. In both problems the subject was faced with the task of deciding upon starting positions A, B or C for the cars in order to show which was faster. The plasticine and target sphere tasks described in chapter 2 are of the racing car/saloon car type of task in that they allow the use of both types of fair strategy, compensation or control, i.e. a handicap or olympic race. Yet the racing car/saloon car task appears more obviously a situation to which a handicap race may be applied. The two racing cars task is a novel situation in that it presents the subject with a situation where he is unlikely to compensate in that he does not know which car to handicap and which car to give an advantage.

Method

The two comparative decisions plasticine task as used in chapter2, the racing car/saloon car task and the two racing cars task were used. One question was used for each task as shown below:

Key for question type

Q1 = the racing/car saloon car task

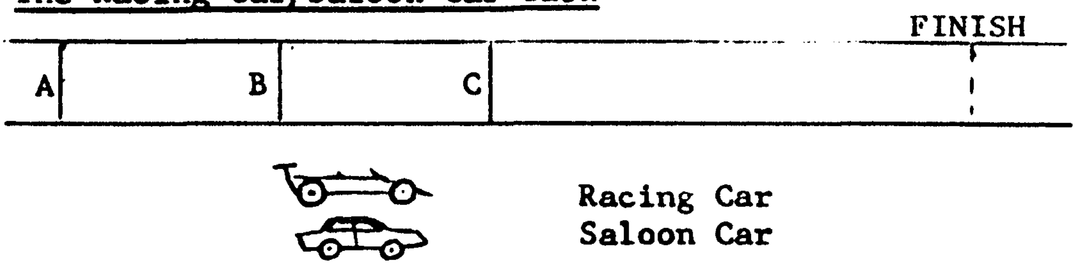
Q2 = the two racing cars task

Q3 = the plasticine task

THE RACING CAR TASKS

Question 1

The Racing Car/Saloon Car Task



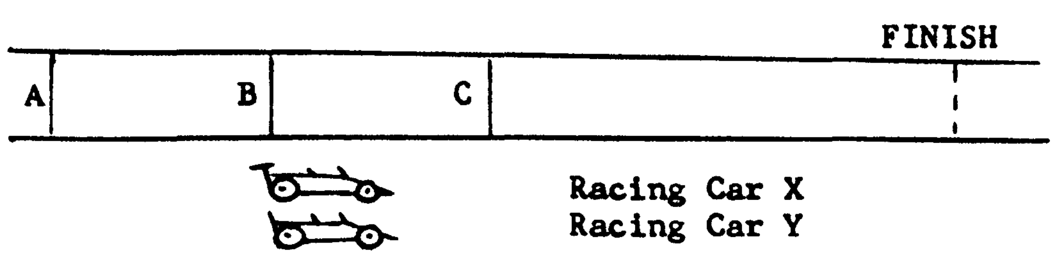
The racing car is very fast, and the saloon car less fast. You have three starting positions A, B and C. You want to race the cars to show which is the fastest.

From which position would you start the racing car? __________
From which position would you start the saloon car? __________

Please explain your answer (space was provided).

Question 2

The Two Racing Cars Task



Both racing cars are very fast. You have three starting positions A, B and C. You want to race the cars to show which is the fastest.

From which position would you start racing car X? __________
From which position would you start racing car Y? __________

Please explain your answer (space was provided).

The Subjects

A total of 90 Secondary School subjects was tested at three different age levels, the details of the sample are shown in Table 3.0.

TABLE 3.0

YEAR	MEAN AGE YEARS	TOTAL
2	12.6	30
3	13.6	30
4	14.5	30
		Σ = 90

The 2nd and 3rd year samples were mixed ability groups in a comprehensive school in Nottinghamshire. The 4th year subjects were selected at random from their age group in the same comprehensive school. The age levels were selected on the basis of the results of the study reported in chapter 2. This study provided evidence that the largest confusion between the use of the compensation and control strategies occured between 12 and 14 years of age.

Hypotheses

Three hypotheses were established for the 12 to 14 year old sample.

1) That the proportion of subjects using a control strategy on the two racing cars task would be significantly greater than the proportion doing so on the racing car/saloon car task.

2) That the proportion of subjects using a compensation strategy on the racing car/saloon car task would be significantly greater than the proportion doing so on the racing cars task.

3) That the proportion of subjects using the control or compensation strategies on the plasticine task would be the same as the proportion of subjects using these strategies on the racing car/saloon car task.

Experimental Design

In order to control for question order effects it was decided to use a latin square design, using three groups A, B and C for the three questions (see Figure 3.1). Ten subjects at each age level were randomly selected for each group.

FIGURE 3.1 The Latin Square Design

Groups	1	2	3
A	Q3	Q1	Q2
B	Q2	Q3	Q1
C	Q1	Q2	Q3

Procedure

The subjects were shown the situations and the effect of the independent variables was discussed, care was taken not to explain the solutions to the tasks. For the plasticine task the subjects were asked what effects the surface area and the weight would have on the depth hole. Reference was made to the effect of snow shoes decreasing the depth of hole left in snow. The two effects were clarified, that the "larger the surface area the shallower the hole" (for the same weight) and that the "larger the weight the larger the depth of hole" (for the same surface area). The points in parentheses were left unsaid. For the racing cars task, the effects of start position and speed of cars were discussed with reference to model cars and a long track, which the subjects were shown. The subjects were then given a questionnaire with the three questions in the correct order for their particular group.

Results

The subjects responses were classified in terms of the following categories of response.

H = using a form of control strategy
C = using a form of compensation strategy
G.D. = using a form of greatest difference strategy
D = using a form of difference strategy
U = using an unclear method or offering no response.

The full results for all strategies are shown in Table 3.1. As only the relative frequency of the use of the C and H strategies on the different questions was of prime interest it was this data that was subject to analysis. The analysis is standard for the 3 x 3 latin square, where scores of the individual children of each group are used for all cells in a row (Lewis 1968 P157-9). The method is included in the full analysis of the results in Appendix D. F ratios were calculated and where significance was at the 0.01 level, the Newman-Keuls test was used to test the significance of the difference between the questions.

TABLE 3.1 The number of subjects using the different strategies, N= 30 for each question (% in parenthesis)

SECONDARY SCHOOL YEAR	MEAN AGE (YEARS)	QUESTION NUMBER	STRATEGY USED				
			H	C	G.D.	D	U
2ND	12.6	1	4 (13.3)	21 (70)	3 (10)	2 (6.7)	0 (0)
		2	19 (63.3)	0 (0)	0 (0)	9 (30)	2 (6.7)
		3	13 (43.3)	10 (33.3)	3 (10)	3 (10)	1 (3.3)
3RD	13.6	1	9 (30)	20 (66.7)	1 (3.3)	0 (0)	0 (0)
		2	23 (76.7)	0 (0)	0 (0)	6 (20)	1 (30.3)
		3	17 (56.7)	10 (33.3)	3 (10)	0 (0)	0 (0)
4TH	14.5	1	12 (40)	17 (56.7)	1 (3.3)	0 (0)	0 (0)
		2	26 (86.7)	0 (0)	0 (0)	3 (10)	1 (3.3)
		3	22 (73.3)	7 (23.3)	1 (3.3)	0 (0)	0 (0)

For ease of reference a summary of the total scores for just the H and C strategies is shown in Table 3.3 so that this may be compared to Table 3.2 which shows the summary of the significant differences for the two strategies on the different questions.

There was a difference significant at the 0.01 level between the proportions of subjects who used the H strategy on questions 2 and 1. This higher use of the H strategy on question 2 was as predicted in hypothesis one and thus this hypothesis was confirmed. It is apparent that the subjects did perceive the olympic type race situation, i.e. the two racing cars task as one to which a control strategy was more appropriate.

TABLE 3.2 Significant Differences Between Questions (Newman-Keuls Test Values)

	H Strategy (control of variables)			C Strategy (Compensation of variables)		
SECONDARY YEAR	Q2 X Q3	Q2 X Q1	Q3 X Q1	Q1 X Q3	Q1 X Q2	Q3 X Q2
2ND	3.33*	8.33	5.00	5.76	10.98	5.22
3RD	3.40*	7.93	4.53	5.24	10.46	5.22
4TH	n.s.	7.93	5.66	5.30	9.00	3.72*

All significant at the 0.01 level but for * at the 0.05 level.

TABLE 3.3 Total Number of Subjects Using H and C Strategies on Each Question

(N = 30 per question) (% in parenthesis)

SECONDARY YEAR	H STRATEGY			C STRATEGY		
	Q1	Q2	Q3	Q1	Q2	Q3
2ND	4(13.3)	19(63.3)	13(43.3)	21(70)	0(0)	10(33.3)
3RD	9(30)	23(76.7)	17(56.3)	20(66.7)	0(0)	10(33.3)
4TH	12(40)	26(86.7)	22(73.3)	17(56.7)	0(0)	7(23.3)

The difference between the proportions of subjects who used the compensation strategy on questions 1 and 2 was found to be significant at the 0.01 level. This higher use of the compensation strategy on question 1 compared to question 2, was as predicted in hypothesis two and thus this hypothesis was confirmed. The racing car/saloon car task was perceived by the subjects as a handicap type race situation. It should be noted that no subjects used the compensation strategy on question 2 and thus the use of the strategy was considered to be inappropriate on the olympic type situation.

There was a difference, significant at the 0.01 level, between the proportion of subjects who used the compensation strategy on questions 1 and 3 which was not predicted. In addition a difference significant at the 0.01 level between the proportion of subjects who used the H strategy on questions 1 and 3 was found. The predicted similarity between questions 1 and 3 was not found, thus hypothesis three was rejected.

Discussion

The rank order for the questions showing the most frequent use of the C and H strategies is shown below:-

	1st	2nd	3rd
Rank order for the use of the C strategy	Q1	Q3	Q2
Rank order for the use of the H strategy	Q2	Q3	Q1

The data showed a significant difference in the use of the control and compensation strategies on the racing car/saloon car task question 1, and the racing cars task question 2, with the plasticine task proving of intermediate difficulty. The plasticine task was found to be similar to question 1 in that the compensation strategy was used, yet it was similar to question 2 in terms of the use of the H strategy. (If the 0.01 level of significance is taken for the comparison of differences between the plasticine task and the two racing cars task, then the differences for the use of the H strategy are not significant). Thus the tendency for the subjects to perceive the plasticine problem as a handicap race was apparent but this effect was not as large as predicted.

This study illustrates the dilemma facing a child between the ages of 12 and 14 years when attempting to answer the plasticine task. If he/she is capable of both 'fair' strategies, C and H he/she may apply either strategy. An older child who consistently uses the H strategy on the plasticine task will have resolved this confusion between the use of the 'fair' strategies. For these older children the olympic type race is differentiated from the handicap type race with the former being recognised as 'fair' experimentation in a scientific context. It has been shown that for the younger child this differentiation does not exist when the task is a science based one. This has implications for science courses which purport to teach the use of scientific methodology to discover the roles of variables in situations similar to the plasticine task. It has been shown in this study and that presented in chapter 2, that the subjects' most common error strategy at the Secondary level is the compensation strategy. Thus a child between the ages of 12 and 14 will tend to confound the effect of independent variables in order to perform a 'fair' experiment, because he/she perceives the task as one to which a handicap may be applied. If H methodology is to be taught, the difference between the two types of 'fair' methodology needs to be shown so that the child may identify the H strategy as an olympic type test and as the 'fair' methodology in a scientific context.

CHAPTER 4

THE 'BLACK-BOX' PROBLEM: A STUDY OF THE ROLE OF THE FAMILIARITY OF VARIABLES IN THE USE OF THE COMPENSATION STRATEGY

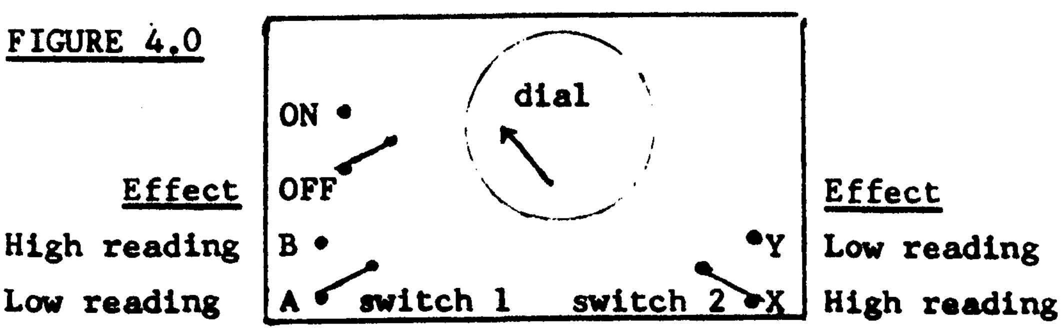
Introduction

In chapter 2 the compensation strategy was shown to be the greatest 'distractor' from the use of the control strategy at the Secondary School level. The use of the compensation strategy is obviously dependent upon whether a task contains independent variables whose functions can be seen in one sense as being compensatory. The aim of this chapter is to discover the effect a task has on the use of the compensation and control strategies if the task involves variables whose effects are initially unknown and finally, when known, are unfamiliar.

A black-box apparatus was designed for the investigation as shown in Figure 4.0.

The Black-Box Apparatus

FIGURE 4.0



Switches 1 and 2 altered the reading on the meter by altering the resistance in the circuit. An internal power supply, i.e. a 9 volt battery and an on/off switch was incorporated so that the switches could be manipulated with or without the resulting effects being shown. Switches

1 and 2 were started at AX and this gave an intermediate reading on the dial. The circuit was so designed that switch 1 gave an increase in the reading on the meter when moved from position A to B, analogous to the independent variable of weight in the plasticine task. Thus position A gave a low reading and position B a high reading. The meter had a regular scale but the original numbers were obscured. Switch 2 gave a decrease in the reading on the meter when moved from position X to Y, analogous to the effect of surface area in the plasticine problem. In addition for positions A with X and B with Y the meter gave the same reading. Thus A was compensated by X and B by Y. The greatest difference strategy could be performed with positions A and Y and B with X. This would result in the lowest and highest readings respectively. By leaving switch 2 in the same position and varying switch 1 from A to B the difference in the meter reading due to switch 1 could be found, this is the control strategy. Two tasks were devised using the black-box apparatus, these were the blind task and the meter task. In the blind task the subjects were asked to suggest an experiment to discover how much difference switch 1 makes to the meter reading and then asked what they would do to each switch. With the battery turned off the subject could manipulate the switches to familiarise himself/herself with the question but he/she was given no clue as to the relationship between switches 1 and 2. This is a blind situation where

not only are the variables abstract but the relationships between them are unknown. In the meter task the battery was turned on, the subject was then able to try the switches and discover or be shown the relationship between the switches, i.e. positions AX and BY produced no difference in reading and position AY and BX, the greatest difference in meter reading. In general they could discover that moving switch 1 from A to B increased the meter reading and moving switch 2 from X to Y reduced the meter reading by an equal amount. Given the same question as used previously, i.e. to state what positions he/she would place the switches to discover the difference switch 1 makes, it was thought that the subject would be more likely to use one of the strategies that rely upon the functions of the switches of which he/she was now aware.

It was expected that for the blind task the subjects would not be able to utilise any logical combination of the effects of the independent variables as these were unknown to the subjects. Thus they would be unable to use the C, G.D. or D strategies. It was therefore expected that the use of the H strategy would be higher on this task.

The plasticine task Q1, 2 and 3 used in chapter 3 was included in this study because it involved the more familiar independent variables of weight and surface area. Three parallel questions to the plasticine task, Q1, 2 and 3 were devised for the meter and blind tasks. It was expected that a comparison of the subject's performance on the meter and plasticine tasks would give an indication of the role of the familiarity of the variables.

The meter task allows the use of the C, G.D. and H strategies as the direction of the effects of the independent variables are known. The compensatory effects of the switches are less familiar than the compensatory effect of weight and surface area and thus it was expected that the use of the compensation strategy would be greater on the plasticine task than on the meter task.

Method

The Tasks

The meter task questions M1, M2, and M3, are shown on the following page. The same questions were used for the blind task and are referred to as B1, B2 and B3 in the following. These three questions are parallel to the plasticine task questions 1, 2 and 3 used in chapter 3 and these are referred to as P1, P2 and P3 in the following.

The Subjects

A total of 38 subjects was interviewed at the Secondary School level. The details of the sample are given in Table 4.0. The samples were randomly selected from their year groups within a Nottinghamshire Comprehensive School. The sixth form samples were advanced level students and were therefore of higher than average ability for their age. Science advanced level students accounted for seven out of the thirteen sixth form students randomly selected.

THE METER TASK

Suppose you want to know exactly how much difference switch 1 makes to the reading on the meter. You must use two positions, a B and an A position for switch 1.

Question M1) At which position would you have switch 2 when switch 1 is at the A position? __________

At which position would you have switch 2 when switch 1 is at the B position? __________

Please explain your answer carefully. (Space was provided.)

Question M2) Suppose someone has switch 1 at the A position and switch 2 at the X position. At which position would you have switch 2 when switch 1 is at the B position to find out exactly how much difference switch 1 makes to the reading on the meter? __________.

Please explain your answer carefully. (Space was provided.)

Question M3) Suppose someone has switch 1 at the A position and switch 2 at the Y position. At which position would you have switch 2 when switch 1 is at the B position to find out exactly how much difference switch 1 makes to the reading on the meter?

________________.

Please explain your answer carefully. (Space was provided.)

TABLE 4.0

YEAR	MEAN AGE YEARS	TOTAL
3RD	13.5	10
4TH	14.4	10
5TH	15.6	5
L6TH	16.7	6
U6TH	17.6	7
		$\Sigma = 38$

Hypotheses

The following hypotheses were set up in relation to the use of the compensation and control strategies.

1. There would be no use of the compensation strategy on the blind task.
2. The proportion of subjects using the control strategy on the blind task would be significantly greater than the proportion of subjects using the control strategy on the plasticine and meter tasks.
3. The proportion of subjects using the compensation strategy on the blind task B1 would be significantly less than the proportion of subjects using the compensation strategy on the plasticine task P1, due to the unknown effects of the variables.
4. The proportion of subjects using the compensation strategy on the meter task M1, would be significantly less than than the proportion of subjects using the compensation strategy on the plasticine task P1, due to the unfamiliar effects of the variables.

Hypotheses 3 and 4 were set up for question 1 as it was considered that results for questions 2 and 3 might be distorted due to the exclusion of the G.D. and C strategies.

Experimental Procedure

The subjects were interviewed and their responses tape recorded. They filled in a questionnaire containing the questions as the interview proceded. The subjects were shown the black-box and told that; switch 1 had an effect on the meter reading in that a movement from position A to B would increase or decrease the reading. A movement back from B to A would reverse this process. Switch 2 had an effect on the meter reading in that a movement from position X to Y would increase or decrease the reading. A movement back from Y to X would reverse this process. The fact that the subjects did not know the effects of switch 1 and 2 was stressed. The on/off switch was set in the off position so that the switches could be moved by the experimenter or the subject at any time to clarify the situation. The switches were always reset at AX by the experimenter after the subject had finished with the apparatus. The subjects were then given the blind task, questions B1, B2 and B3, to complete and after completion of all three questions the reasons for their choices were discussed. The subjects were then shown the effect of switch 1, that a movement from A to B increased the reading on the meter. The subjects were then shown the effect of switch 2, that a movement from X to Y would decrease the reading on the meter. The subjects were shown each effect separately, thus the experimenter used the H strategy to show these effects but did not make this specific. The experimenter did not show the G.D. or C strategies. The subjects were then given the meter task, questions M1, M2 and M3. The subjects were not allowed to use the switches, but the effects were clarified if they encountered difficulties in remembering the effects. The subjects were not corrected if they remembered the effects incorrectly. On completion the subjects' explanations were checked and discussed. Finally the subjects were shown the plasticine task. The effects were discussed as in chapter 2 and the subjects were asked to complete the plasticine task, questions P1, P2 and P3. On completion the subjects' explanations were checked and discussed.

Thus the problem order for all the subjects was:-

First - Blind Task
Second - Meter Task
Third - Plasticine Task.

This order was chosen to avoid the possible transfer of use of the compensation strategy from the plasticine task to the meter and blind tasks. The meter task necessarily had to follow the blind task.

Results

The responses were classified in terms of the three major strategies used, together with a category for any other strategy used by the subjects as shown below:-

H - using a form of control strategy
C - using a form of compensation strategy
G.D. - using a form of greatest difference strategy
O - using any other strategy than H, C or G.D.

The frequency of use of the strategies for the tasks is shown in Table 4.1. The differences in the frequency of use of the major strategies between the three tasks were tested using the McNemar test for significant changes. The fourfold tables for each strategy on each question are presented in Appendix E and a summary of the differences found to be significant is shown in Table 4.2.

TABLE 4.1 Frequency of response for the different strategies for the Blind, Meter and Plasticine Tasks. (N = 38)

STRATEGY	BLIND TASK	METER TASK	PLASTICINE TASK
	QUESTION B1	QUESTION M1	QUESTION P1
G.D.	8	12	6
C	5	5	14
H	22	21	18
O	3	0	0
STRATEGY	QUESTION B2	QUESTION M2	QUESTION P2
G.D.	3	3	0
C	8	6	8
H	26	26	28
O	1	3	2
STRATEGY	QUESTION B3	QUESTION M3	QUESTION P3
G.D.	4	16	7
C	2	0	0
H	30	21	31
O	2	1	0

TABLE 4.2 The differences found to be significant between the Blind, Meter and Plasticine Tasks. (McNemar one tailed test).

STRATEGY	QUESTION CONTRASTS	
COMPENSATION	P1 X B1*	P1 X M1*
CONTROL	P3 X M3*	B3 X M3**
GREATEST DIFFERENCE	P3 X M3*	B3 X M3**

* significant at the .025 level
** significant at the .005 level

It should be noted that for the meter and blind tasks the difference strategy was inapplicable and no subjects used this strategy on the plasticine task.

The use of the G.D. and C strategies on the blind task was unexpected. The subjects guessed at the effects of the switches on the dial and anticipated a greatest difference or compensation result. Thus hypothesis one, that there would be no use of the compensation strategy on the blind task (B1, B2 and B3) was rejected.

There was found to be no significant difference in the use of the H strategy between the three tasks for questions 1 and 2. Thus hypothesis two that the use of the H strategy would be greater on the blind task was rejected.

An unexpected difference in the use of the H strategy was noted between questions M3 and P3 and M3 and B3. The H strategy was used less frequently on question M3, the difference being significant at the 0.025 level between M3 and P3 and at the 0.005 level between M3 and B3 (for M3 X P3, $X^2 = 4.92$; for M3 X B3, $X^2 = 6.75$). This decrease in use of the H strategy was accompanied by an increase in the use of the G.D. strategy on M3 when compared to P3 and B3, the differences being significant at the 0.025 and 0.005 levels respectively, (for M3 X P3, $X^2 = 4.92$; for M3 X B3, $X^2 = 28.64$). Question M2, like P2, excluded the use of the G.D. strategy yet three subjects who incorrectly remembered the effects attempted to use this strategy on M2. It thus appears that the unfamiliarity of the independent variables within the meter task tends to result in an increased use of the greatest difference strategy.

The lower use of the C strategy on the blind task B1 compared to its use on P1 was as predicted. The difference was significant at the 0.025 level (for B1 X P1, $X^2 = 4.92$). Hypothesis three was therefore confirmed.

The decrease in the use of compensation strategy on the meter task when compared to the plasticine task was as predicted for questions M1 and P1, the difference was significant at the 0.025 level (for M1 X P1, $X^2 = 4.92$). Thus hypothesis four was confirmed.

It may be concluded from the results that the effect of the unknown functions of the variables on the blind task B1 was to reduce the use of the compensation strategy. The effect of the unfamiliarity of the functions of the variables on the meter task M1 was to reduce the use of the compensation strategy. These effects were confounded on questions B2, M2 and P2 because the use of either the G.D. or the C strategy was excluded. The strategy excluded depended upon whether the subjects anticipated the effects correctly on B2 or remembered the effects correctly on M2.

Discussion

The results of this study have shown that the effects of the unknown and of the unfamiliar functions of the variables tended to decrease the use of the compensation strategy. It can be concluded that where the effects of the independent variables are not readily intuitable as compensatory, the compensation strategy is used less fequently. It has also been shown that a subject may use the compensation strategy on a task in which the effects of the variables may only be anticipated as compensatory. The confusion between the use of the 'fair' strategies still persists in the solution of tasks where the effects of the independent variables are unknown or unfamiliar. It appears that the decreased use of the compensation strategy for the meter task results in an increased use of the greatest difference strategy. It is clear that the less frequent use of the compensation strategy due to the unfamiliarity of the effects or due to the unknown effects of the variables does not result in an increased use of the control strategy.

It may be concluded that those subjects who are confused between the use of the 'fair' strategies are equally confused by the unfamiliarity of the black-box situation. For these subjects the unfamiliarity of the variables does not appear to remove the conflict between the choice of 'fair' strategies, although the compensation strategy may be seen as less applicable The tendency is for these subjects to perceive the situation as one to which an 'unfair' strategy might apply, i.e. the greatest difference strategy.

CHAPTER 5

THE DEVELOPMENT OF THE 'CONTROL OF VARIABLES' STRATEGY AT THE SECONDARY SCHOOL LEVEL

Introduction

The work presented in this chapter is concerned with tracing the formation of the fully developed control strategy at the Secondary School level.

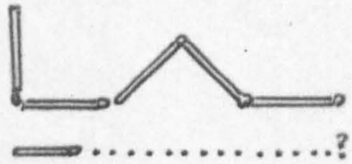
In Chapter 1 the three major strategies used on the plasticine tasks were identified together with the Piagetian stage at which they were considered to represent the typical mode of behaviour. The greatest difference strategy was noted to be a concrete operational strategy, the control strategy was noted to be primarily a formal operational strategy and the compensation strategy appeared to be a concrete/formal or transitional strategy. Thus a detailed study of adolescent behaviours on the plasticine tasks should provide evidence for the process by which transition occurs from concrete to fully developed formal reasoning, for this one aspect of formal thought. It is clear that the simultaneous application of strategies to the same problem may create conflict. Inhelder has described a mechanism for learning via conflict,

"the mechanisms bringing about improvements and progress in the various forms of equilibrium consist, first, in an application of existing schemes to an increasing variety of situations. Sooner or later, this generalization encounters resistance, mainly from the simultaneous application of another scheme; this results in two different answers to one problem and stimulates the subject seeking a certain coherence to adjust both schemes or to limit each to a particular application therefore establishing their differences and likenesses". (Inhelder et al, 1974, p256).

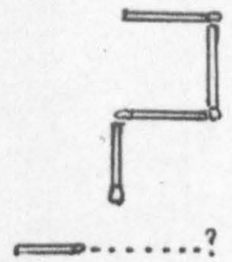
As the work reported in Chapters 2, 3 and 4 has shown, the same subject may in fact use a different strategy for different questions. In addition it has been shown in Chapter 4 that subjects appear to limit the C and H strategies to specific problems. Inhelder's dynamic mechanism for learning is described below in relation to a training task devised by Bovet (Inhelder et al, 1972) which attempted to lead children, who were capable of numerical conservation, to a grasp of the conservation of continuous quantities (normally acquired approximately 3 years later). It should be noted that the use of Inhelder's mechanism for learning, via conflict between different schemes applied to the same problem situation, does not imply that its wider application to the development of operational structures is necessarily accepted.

The following types of situation were used by Inhelder et al in which the child himself had to construct paths with matchsticks. Both the experimenter and the subject had a number of matches at their disposal, but the subject's matches were shorter than those of the experimenter and of a different colour. (Seven of the subject's red matches equalled the length of five of the experimenter's black matches). The experimenter constructed either a straight or a broken line (a "road") and asked the subject to construct a line of the same length ("just as long a road, just as far to walk," etc), e.g., see problems 1, 2 and 3.

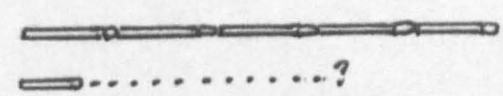
Problem 1



Problem 2



Problem 3



In each of the three problems the task was to construct a straight line of the same length underneath the road constructed by the experimenter. In problems 1 and 3 the construction took place directly underneath the experimenter's road, in problem 2 the road construction took place some distance away. The child was found to have two strategies available to cope with these problems. Firstly, an ordinal strategy involving references between the ends of the road was found to be used, e.g., in problem 3 a road might be constructed (using 7 matches) so that the start and end points were the same, a correct solution. Secondly, a strategy involving numerical references was found to be used, e.g., in problem 3 a road of 5 short matches might be constructed, an incorrect solution. Four phases in the construction of a fully developed strategy, which had as its basis the concept of conservation of length, were identified.

At phase one separation between schemes occurred, e.g., in problem 1 the child constructed a line by ordinal reference, i.e. a straight line with its extremities in coincidence with those of the zig-zag line was constructed. Although the child could state that he/she used only four short matches (one less than the experimenter) he/she would remain convinced that the two lines were of equal length. In problem 2, ordinal reference could not be made, as construction did not take place directly underneath. The child was found to use a numerical strategy in this instance.

At phase two the strategies were found to be in juxtaposition in that the conflict produced by their use was realised by the child. For example, in problem 3, the numerical strategy might be used, i.e. a parallel road of 5 short matches would be constructed. At phase two the child would realise that this was not long enough and would add more matches. Then, realising that too many matches had been used, the child would return to his/her first solution. The child was found to be confused by the contradictions involved in the use of the strategies.

At phase three compromise solutions were found to be used, i.e. inadequate partial integrations of the two strategies were made. For example, in problem 1, in order to gain the same length of road (judged on the horizontal extremities) one match might be broken in two. Thus the same number of matches could be used to construct the same 'length' of road. Another compromise solution involved ignoring the instruction to make a straight road; e.g., the subject would place one match vertically so that the same number of matches could be used without increasing the 'length'.

At phase four full integration was found to occur and this led to the understanding that a greater number of matches was needed when they were smaller and that a zig-zag reduced the horizontal length of the road even though the same number of matches was used.

Thus the separate strategies at phase one having been brought into conflict at phase two, are partially integrated at phase three, and successfully integrated at phase four.

It is apparent from the evidence in the previous chapters involving the plasticine task, that each strategy has a related criterion for use. The G.D. strategy, when it was understood, appeared to be used on the criterion of showing the 'largest difference' in the dependent variable. The H and C strategies were not clearly differentiated and this resulted in confusion when the subjects tried to choose between them. The criterion for use for either appeared to be one of performing a 'fair experiment', yet the H strategy appeared also to be used on a separate criterion of varying one independent variable at a time.

At its simplest level this was expressed as 'use the same surface area'. If the transition mechanism described by Inhelder can explain the process by which subjects reach a fully integrated concept or scheme of control then phase one should be evidenced by the use of separate criteria for the acceptance of separate strategies as methods of proof. Thus more than one strategy may be accepted on different criteria. At phase one there should be no confusion or conflict in the choice of only one strategy but at phase two the subjects should be able to realise the need to reject strategies that are in conflict with the chosen strategy. Thus the criterion used to accept one strategy should be applied to reject others.

Phase three behaviour should be abundant at the Secondary School level and should be evidenced by attempts to integrate the criterion for use of one or more strategies to other strategies. As a simple example, partial integration on the criterion of 'fair experimentation' will result in the acceptance of the C and H strategies and rejection of the G.D. strategy. The integration is only partial in that, because the criterion for the acceptance of the H strategy 'vary one thing at a time' has not also been utilised, both the C and H strategies are considered to be correct.

Phase four should occur when subjects utilise all three criteria for acceptance of the strategies in the integration and thus the compensation strategy should be rejected on the grounds of not 'varying one thing at a time'. Thus the two types of fair experimentation should be differentiated at this phase.

The recurrence of use of the C and G.D. strategies at the sixth form level evidenced in Chapters 2 and 4, suggests that this proposed final integration may be sufficient for most subjects, but for some, a higher level integration of criteria occurs.

In order to identify the criterion for use of each strategy, the criteria for the multiple use of strategies and the eventual integration of these criteria into partial or complete systems, a questionnaire was devised. This presented the subjects with examples of each strategy for the plasticine task and the subject was asked to judge the strategy's suitability as a method of showing the effect of the independent variable.

Method

The Tasks

The tasks were given in the form of a questionnaire shown on the following pages.

Q1, John's experiment involves the G.D. strategy.

Q2, Paul's experiment involves the C strategy.

This second experiment may produce the same depth of hole in the plasticine if the ratios of weights and surface areas are equal but this information is not provided and thus such an assumption is invalid.

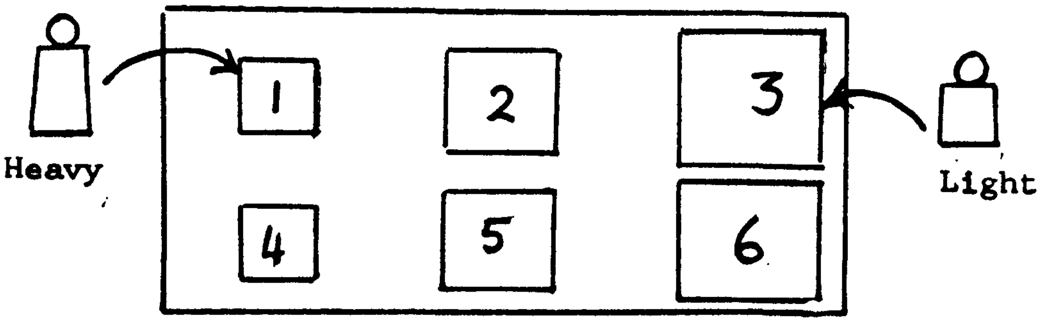
STRATEGY/TASKS

The following are experiments performed by pupils with

2 weights, a heavy one and a light one.

Q.1.

JOHNS EXPERIMENT



John places the light weight on block number 3. He also places the heavy weight on block number 1. Will this show John how much difference the weight makes to the depth of hole left in the plasticine.

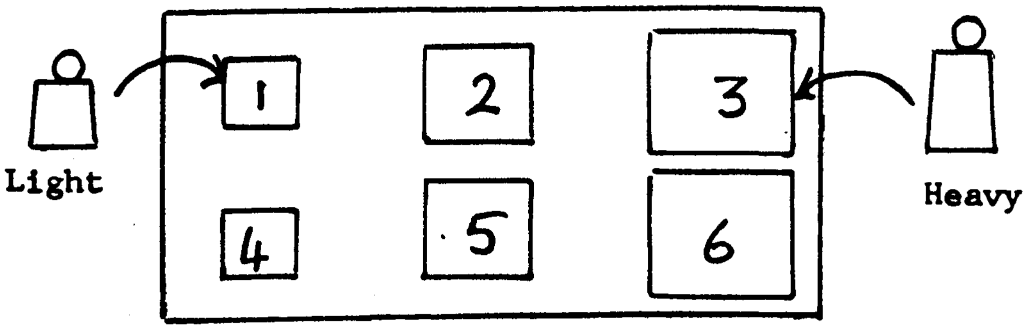
Answer YES or NO ___________

a) Please explain your answer (space was provided)

b) Explain what this experiment shows John (space was provided)

Q.2.

PAULS EXPERIMENT



Paul places the heavy weight on block number 3. He also places the light weight on block number 1. Will this show Paul how much difference the weight makes to the depth of hole left in the plasticine.

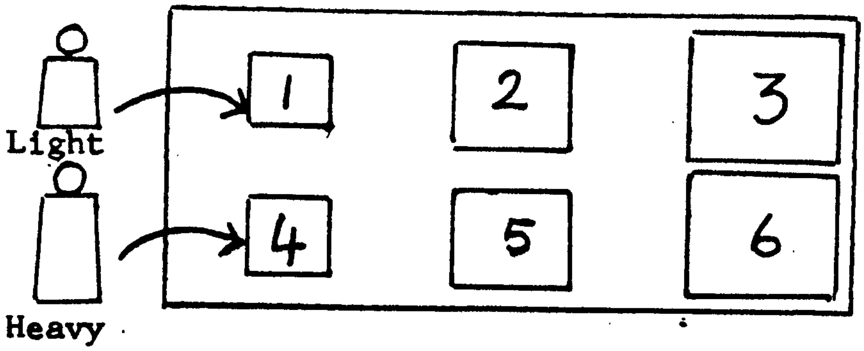
Answer YES or NO ______________

a) Please explain your answer (space was provided)

b) Explain what this experiment shows Paul (space was provided)

Q.3.,

MARYS EXPERIMENT



Mary places the light weight on block number 1. She also places the heavy weight on block number 4. (Blocks 1 and 4 are the same size). Will this show Mary how much difference the weight makes to the depth of hole left in the plasticine.

Answer YES or NO ________________

a) Please explain your answer (space was provided)

b) Explain what this experiment shows Mary (Space was provided).

Q.4.

Which experiment do you prefer in order to show how much difference the weight makes to the depth of hole left in the plasticine?______________________.

Please explain your answer.

Q3, Mary's experiment involves the H strategy.

Part (a) to each question was included to discover why the subject considered the strategy showed, or did not show, how much difference the weight made, i.e. the reasons for acceptance or rejection of the strategy. Part (b) to each question was included to discover the subject's level of understanding of the strategy e.g., subjects may not comprehend the H strategy as producing different depths of hole. This information was of importance when determining the reason for acceptance or rejection of a strategy and provided information not already given in (a). For example, a subject's answer to part (a) might simply be "it does not show how much difference the weight makes to the depth of hole left in the plasticine". That the subject rejects the strategy is clear, but his/her reason is not and part (b) can provide this information. Question 4 was intended to identify those subjects who might show confusion in choosing only one strategy, i.e. those subjects beyond phase one, in particular, those at phase two.

The Subjects

All 510 Secondary School subjects tested in the study in Chapter 2, were given the questionnaire to complete approximately 8 weeks later. Details of the sample are shown in Table 5.0.

TABLE 5.0

YEAR	MEAN AGE YEAR/MONTHS	TOTAL
1st	11.9	90
2nd	12.9	90
3rd	13.9	90
4th	14.6	90
5th	15.10	90
L6th	16.11	30
U6th	17.10	30
		Σ = 510

Hypotheses:

No formal hypotheses were set up as the study was one of identification of criteria for the acceptance of the major strategies.

In general; it was expected that the subjects had certain strategies available to them, at the Secondary level, the G.D., C and H strategies. The use of a particular strategy would depend upon those strategies the subjects accepted as showing the effect of one independent variable, i.e. the weight in the plasticine task. It was expected that these acceptance criteria would increase in sophistication throughout the Secondary level resulting in a fully developed scheme of control experimentation. It was predicted that the development of acceptance criteria could be explained in terms of conflicts resulting from the choice of one strategy out of the subject's repertoire. It was expected that the rejection of a particular strategy would result in new, more powerful, criteria for the use of the chosen strategy.

Experimental Procedure

The procedure used was identical to that used for the plasticine tasks described in Chapter 3, apart from the following two additions.

Each question was read through with all the subjects and demonstrated without showing the actual result.

The answers were briefly checked in order to identify unclear responses which were then checked with the subject concerned.

Results

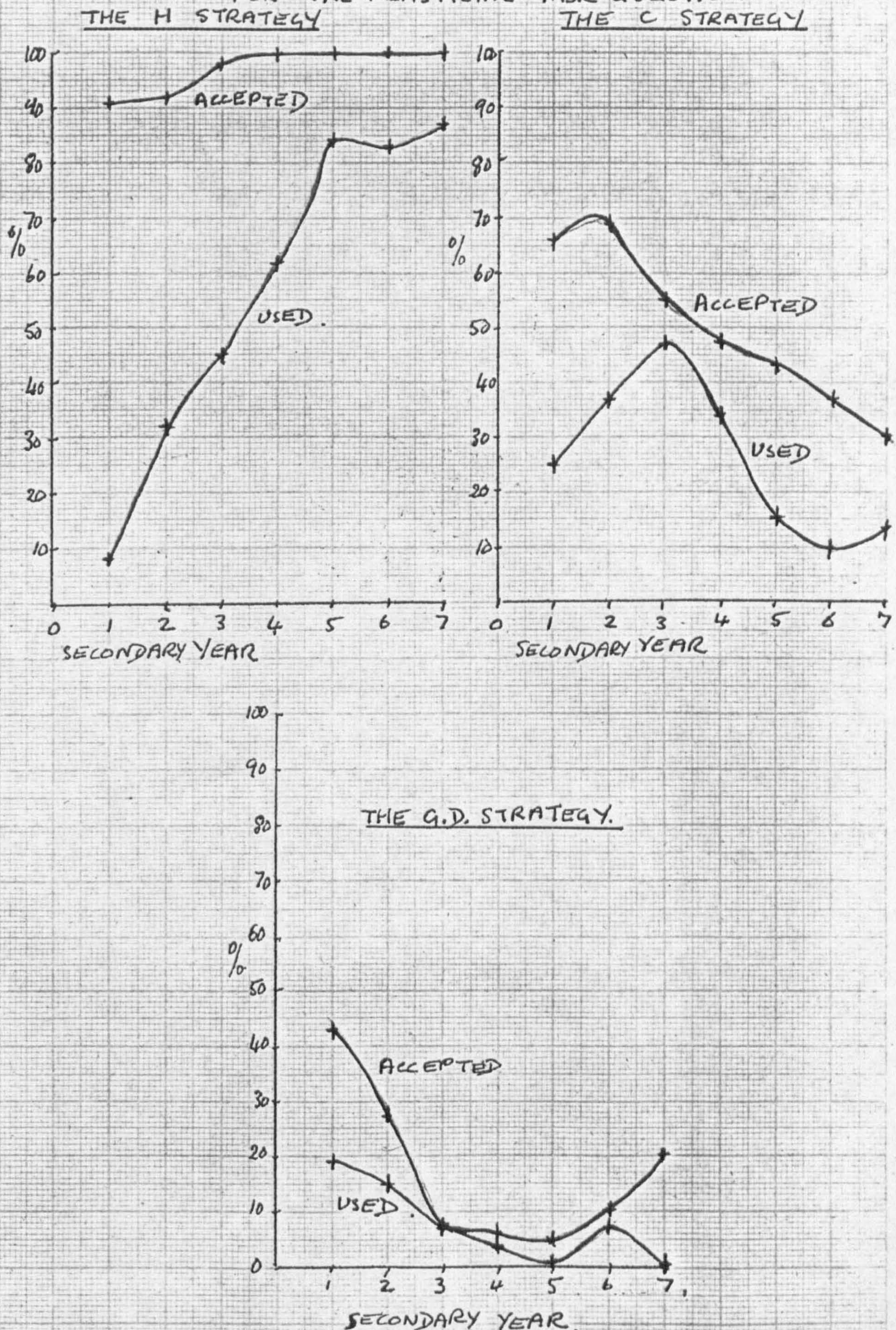
Table 5.1 shows the frequency of subjects accepting the major strategies as proof, singly or with others. It is of interest to note that no subjects in the sample tested considered C to be the only correct strategy. Eight subjects did not understand that the H strategy showed a difference in depth of hole left in the plasticine and they oonsidered only the G.D. and C strategies as correct. In fact the same eight subjects did not comprehend the compensation argument and accepted this experiment on the grounds that it used different weights. Thus all the subjects who understood the C strategy as a compensatory and

TABLE 5.1 Frequency of subjects accepting the major strategies as proof (% in parenthesis)

		YEAR						
		1	2	3	4	5	L6	U6
STRATEGIES ACCEPTED AS PROOF	(N)	(90)	(90)	(90)	(90)	(90)	(30)	(30)
G.D.		8	5	1	0	0	0	0
H		14	18	33	40	51	19	21
C&H		39	47	50	44	35	8	3
G.D. & H		8	5	6	6	0	0	0
G.D. & C&H		15	13	0	0	4	3	6
G.D. & C		6	2	0	0	0	0	0
Total No. who accept each strategy as correct								
For H		82 (91.1)	83 (92.2)	89 (98.8)	90 (100)	90 (100)	30 (100)	30 (100)
For C		60 (66.7)	62 (68.8)	50 (55.5)	44 (48.9)	39 (43.3)	11 (36.7)	9 (30)
For G.D.		37 (41.1)	25 (27.7)	7 (7.7)	6 (6.7)	4 (4.4)	3 (10)	6 (20)

fair experiment, understood the H strategy. This was to be expected for the third year sample upwards as the results in Chapter 3 showed that subjects who used the C strategy on the plasticine task could use the H strategy on the two racing cars task. Table 5.1 also shows the combined numbers of subjects who accepted each particular strategy as correct. Figure 5.0 contains graphs which compare the percentage use of each main strategy (using the data from Chapter 2 for question 1) to the percentage of subjects who accepted the strategy as

FIGURE 5.0 GRAPHS TO COMPARE THE PERCENTAGE OF SUBJECTS ACCEPTING EACH MAJOR STRATEGY AS PROOF TO THE PERCENTAGE OF SUBJECTS USING THE STRATEGY FOR THE PLASTICINE TASK QUESTION ONE.



a correct solution. (The data for question 1 was chosen for comparison because this question does not exclude the use of any of the strategies). The graphs show a disparity between the acceptance of the strategy as being a method of proof and its actual use. A detailed study of the strategy used by each subject on question 1 and the strategies accepted as correct using the present questionnaire revealed a one to one correspondence, i.e. if a subject used H on question 1 he/she accepted this strategy as correct. Thus it appeared that the use of a strategy on the plasticine task was in part determined by the strategies the subject accepted as correct. This correspondence between acceptance of a strategy and its use was an important one to determine as conclusions drawn from the criteria for acceptance of the strategies led to an explanation of the development of a more powerful scheme of control, and it is implied that this development may explain the use of a strategy.

The subjects' responses were further classified in terms of the reasons for acceptance or rejection of the strategies. Thus for each subject a profile was obtained listing all the criteria used and the strategies to which they were applied. Similar profiles were then grouped together. It was then possible to identify differing levels of the ability to judge the strategies on the criteria identified. These substages in the integration of criteria together with the strategies that were rejected and accepted are listed in Table 5.2 at the end of the chapter. The table unfolds so that it may be referred to while the chapter is read. An introduction to the features of each substage is given below. A more detailed discussion of substages 2, 3 and 4 follows this introduction.

Substage 0
(N=16)

The non-recognition of two or more strategies

At this substage the subjects failed to understand at least 2 of the strategies e.g., the C strategy was not recognised as a fair experiment. Subjects accepted a strategy if they considered that it used different weights (criterion 1) because the task required them to use different weights. Some subjects understood one strategy, usually the G.D. strategy, which was considered to show a difference in depth (criterion 2). The H strategy was not understood by any of these subjects. The subjects did not apply any criteria for the rejection of a strategy.

Substage 1 Isolated Centration (Inhelder's Phase I)
(N=20)

The isolated use of a criterion for the acceptance of its strategy (Such strategy specific criteria may be applied to 2 or more of the strategies).

At this substage the subjects understood at least 2 of the strategies and there were variations in the criteria applied for the acceptance of a strategy e.g., the H strategy was accepted on criterion 4 (use a fair experiment) by some subjects and on criterion 3 (vary one thing at a time) by others. Subjects were unable to reject strategies, because a criterion was only applied to its strategy and it was not used to compare the use of strategies e.g., a comparison of criterion 4 and criterion 2 would reveal that the G.D. strategy was unfair and this might result in its rejection. Such comparisons were not made at this substage.

Substage 2 (N= 134) Unicriterion Comparison (Inhedler's Phase II Juxtapostion)

The comparison and thus rejection of strategies on the basis of a single criterion

At this substage one criterion was applied to reject strategies, although other criteria were often applied to accept strategies. There were four types of comparison behaviours noted (A,B,C and D) and these were distinguished by the criteria used for rejection, together with the strategies to which the criterion was applied. For example, for the comparison B, application of criterion 2 (show a difference) resulted in the rejection of the C and H strategies in that these were considered to show little difference in depth of hole in the plasticine. Thus these subjects' acceptance of the G.D. strategy was modified to that of only accepting a strategy that showed a large difference.

Substages 3.1 & 3.2 (N= 108 & 129) Bicriteria Comparison (Inhelder's Phase III Partial Integration)

The application of one criterion to all three strategies with one further criterion to a maximum of two strategies for substage 3.1 and to all three strategies for substage 3.2

Partial integration was achieved because two criteria were applied to the strategies to accept or reject them.

At substage 3.1, three partial integrations were noted (A, B & C). At substage 3.2, the application of two criteria to all three strategies, two partial integrations were noted (A & B).

Substage 4.1 (N=90) Tricriteria Comparison (Inhelder's Phase IV, Integration)

The application of three criteria to each strategy

At this substage the integration achieved the rejection of all but the H strategy for one behaviour, integration A. A calculation argument was applied to also accept the C strategy for integration B and the same argument was applied to also accept the G.D. strategy for integration C. This argument assumed that the precise effect due to the difference in surface area was known and that the effect of weight could be calculated as if the same surface area was used. Thus it represents a 'calculated' H strategy applied to an ill-conceived experiment.

Substage 4.2 (N=13) Full Integration

The realisation that alternative strategies assume the effect to be demonstrated

The 'calculated' H strategy arguments applied by the subjects at substage 4 to accept the C and G.D. strategies led to a final integration, for 13 subjects of the strategies on a new 'assumptions' criterion, i.e. that a strategy should not assume the effect it is attempting to demonstrate. Thus at this substage only the H strategy was considered to be correct.

A more detailed discussion of the Substages

A distinguishing feature of substages 0 and 1 is that they represent behaviours that accept strategies, i.e. criteria are not applied in order to reject strategies. The behaviours at substages 2, 3 and 4 are of particular interest as it was at these substages that the subjects appeared to compare the strategies on the basis of the criteria for their use and thus rejection of strategies took place.

Unicriterion Comparison, Substage 2

Four behaviours were noted:

Comparison A (N=14) involved the application of criterion 2 (show a difference in depth) to all three strategies. Thus the C & H strategies were rejected as they showed no difference in depth, or less difference in depth, than the G.D. strategy.

Comparison B (N=9) again involved the application of criterion 2 to all three strategies, but in this instance only the C strategy was rejected in that it was considered to show no difference in depth.

Comparison C (N=36) involved the application of criterion 3 (vary one thing at a time) to all three strategies, thus both the G.D. and C strategies were rejected. This criterion was expressed as 'the same surface area must be used'.

Comparison D (N=75) involved the application of criterion 4 (use a fair experiment) to all three strategies, thus only the G.D. strategy was rejected. This comparison represented the most frequent behaviour and illustrates the reason for the common confusion between the use of the C and H strategies, i.e. that they were both considered fair (criterion 4).

Bicriteria Comparison Substages 3.1 and 3.2

These behaviours were more complex and involved the comparison of the three strategies on two criteria and as such, represented partial integrations in that strategies and their reasons for use were related to each other. At substage 2 comparison rather than partial integration occur ed in that the criterion for use of a strategy was applied to reject strategies and other criteria were ignored. The 'other' criteria were considered by the subjects at substages 3.1 and 3.2.

At substage 3.1, partial integrations A, B and C involved the application of one criterion to all three strategies, a second criterion was applied to one or two of the strategies.

Partial integration IA (N=15) involved the application of criterion 2 (show a difference in depth) to all three strategies. Thus strategy C was rejected in that no difference in depth was considered to be shown. Criterion 4 (use a fair experiment) was also applied to the H strategy. A typical response was:

Strategy	Response
G.D.	"is correct because it shows a large difference"(criterion2).
C	"is incorrect because it will give the same depth of hole and you can't tell the difference" (criterion 2).
H	"is correct because you can see the difference in depth and it is fair". (criteria 2 and 4).

Partial integration IB (N=39) once again involved the application of criterion 2 to all three strategies, thus the C strategy was rejected. Criterion 3 (vary one thing at a time) was applied to the G.D. and H strategies, thus the G.D. strategy was rejected and H was accepted.

> "H is correct, because the difference is shown on the same surface area".

Partial integration IC (N=54) involved the application of criterion 4 (use a fair experiment) to all three strategies. thus G.D. was rejected as unfair. Criterion 2 was noted in the responses of both the G.D.and H strategies. Thus it was realised that although G.D. showed a large difference in depth, it was unfair, whereas for H,

> "it is fair and shows the real difference in depth".

All these partial integrations have conflicts inherent in them due to the fact that all three criteria were not applied to all three strategies, e.g., in IB a consideration of criterion 4 for the C strategy would have created conflict in that it might have now been accepted as a fair strategy.

At substage 3.2 two criteria were applied to all three strategies and two partial integrations were noted.

Partial integrations IIA and B (N=51 & 78) involved the application of criterion 2 (show a difference in depth) to all three strategies, thus strategy C might be rejected. Criterion 4 (use a fair experiment) was also applied to the three strategies and thus the C and H strategies could be accepted as correct.

Partial integration IIA rejected C.

C "is fair but it is wrong because it produces no difference in depth" (criteria 2 & 4)

Partial integration IIB accepted C

C "is correct, it is fair and because there is no difference in depth you can tell the heavy weight is best at making deep holes" (criteria 2 & 4).

It was the realisation that the C strategy was fair, but that it might not show any difference in depth, that led the subjects to consider that a difference in depth might be inferred. An interesting feature of the substage 3.2 behaviour was the number of corrections to 'mistakes' made by the subjects on their C strategy responses. It thus appears that subjects were aware of the conflict produced by the use of the two criteria for the acceptance or rejection of the C strategy in that they found the acceptance or rejection of the C strategy a difficult choice to make.

Tricriteria Comparison Substage 4.1

At this substage the three criteria were applied to all three strategies; three of these integration behaviours were noted.

Integration A (N=58) involved the rejection of all strategies other than the H strategy and all three criteria are mentioned in the typically full responses made, one of which is shown below. The rejection of C is made possible by the application of criterion 3 (vary one thing at a time).

Strategy	Response
G.D.	"is incorrect it is not on the same surface area, which should be used to make a fair comparison, making a large difference is not good enough".(criteria 2, 3 and 4).
C	"is incorrect because no difference in depth is made. It is fair, but I think having the weights on the same surface area makes the difference easy to see". (criteria 2, 3 and 4).
H	"is correct, it is fair and you can see the difference when the weights are placed on the same surface area". (criteria 2, 3 and 4).

Integration B(N=19) involved the consideration that although H was the easiest method, i.e. the difference could be observed directly, the C strategy was nevertheless fair. It was argued that if the effect due to the surface area were known, then the effect due to weight could be calculated.

For integration C (N=13) this calculation argument was also applied to the G.D. strategy. A typical response at this substage is shown below.

Strategy	Response
G.D.	"is unfair and will produce a larger difference than that required. If the ratio of surface area is known then this effect may be calculated so that the true difference may be shown. I think it therefore correct".
C	"This is fair and allows for the larger weight to stick in the same distance as the smaller weight due to the surface area if the ratios are correct, if they are known, then the effect as if they are on the same surface area may be calculated. It is then correct".

H "this is fair and is the easiest method. You can see the difference because the different weights are on the same surface area, you do not have to calculate it".

Full Integration Substage 4.2

At this final substage the subjects were able to reject both the G.D. and C strategies even though they realised the calculation argument might be applied. In order to do this these subjects had to apply a new criterion, the assumptions criterion, i.e. if a strategy involves the assumption of the effects that are to be demonstrated then it is incorrect. Thirteen subjects (16 year old +) responded in this way and this represents a sophisticated notion of scientific proof. In fact several physics experiments do not meet this criterion. For example, consider a sonometer experiment to show the "laws of vibration of a fixed string", i.e. to show the following relation in a wire,

$$\text{frequency} \propto \sqrt{\text{Tension}}, \text{ for a given length of wire}$$

In order to measure the frequency of the wire under tension, a relation between frequency and length is found experimentally for a separate wire. This experimentally determined relation is then used to determine the frequency of a particular length of wire under a particular tension. A similar method is used to show that,

$$\text{frequency} \propto \frac{1}{\sqrt{\text{mass per unit length}}}$$

for a given length (L) and tension (T).

TABLE 5.3 THE NUMBER OF Ss AT EACH SUBSTAGE (% in brackets)

					SECONDARY	YEAR		
		1	2	3	4	5	L6	U6
		N=90	90	90	90	90	30	30
SUBSTAGE 0 Strategies considered correct.								
Criterion 1	G.D. & C (H found confusing) or G.D., C&H	10 (11.1)	6 (6.7)					
SUBSTAGE 1 (Strategy related to criterion in brackets)								
Criterion 2	(G.D.) C & H	4 (4.4)	2 (2.7)					
Criterion 3	G.D., C & (H)	3 (3.1)	4 (4.4)					
Criterion 4	G.D. (C & H)	4 (4.4)	3 (3.3)					
SUBSTAGE 2.1								
Comparison A	G.D.	8 (8.8)	5 (5.6)	1 (1.1)				
" B	G.D. & H	6 (6.7)	3 (3.3)					
" C	H	14 (15.6)	10 (11.1)	10 (11.1)	2 (2.2)			
" D	C & H	29 (32.2)	25 (27.7)	16 (17.7)	5 (5.6)			
SUBSTAGE 3.1								
Partial Integrations IA	G.D. & H	2 (2.2)	2 (2.2)	6 (6.7)	5 (5.6)			
IB	H		4 (4.4)	14 (15.5)	15 (16.7)	6 (6.7)		
IC	C & H	10 (11.1)	11 (12.2)	16 (17.7)	17 (18.9)			
SUBSTAGE 3.2								
Partial Integrations IIA	H		4 (4.4)	9 (10.0)	19 (21.1)	19 (21.1)		
IIB	H & C		11 (12.2)	18 (20)	22 (24.4)	27 (30)		
SUBSTAGE 4.1								
Integration A	H				5 (5.6)	24 (26.7)	14 (46.7)	15 (16.7)
B	H & C					8 (8.8)	8 (26.7)	3 (10)
C	H & C & G.D.					4 (4.4)	3 (10)	6 (20)
SUBSTAGE 4.2								
Full Integration	H					2 (2.2)	5 (16.7)	6 (20)

In the above case where no direct means of measurement of frequency of the wire is available, an already 'known' relation may be utilised.

The Substages

Table 5.3 shows the number of subjects for each year tested at each of the substages.

Substage 0 represents behaviour below that of early formal operations in that the strategies did not appear to be understood. It thus appears that the development of the 'reasons for use' of the H strategy involves four distinct substages during the formal operational level, ages 11 - 18. These are as follows:-

	Substage	Mean Age
Isolated centration	1	12.2
Unicriterion comparison	2	13.1
Bicriteria comparison	3.1 & 3.2	14.7
Tricriteria comparison	4.1 & 4.2	16.8

These substages are equivalent to Inhelder's Separation, Juxtaposition, Partial Integration and Integration Phases. Table 5.4 shows the analysis of variance applied to the chronological ages of the subjects at the substages. The differences between consecutive mean ages for the substages were found to be significant at the 0.01 level (see Table 5.5 for means, standard deviations and t-test values). Thus for this one aspect of formal thought it appears that four distinct substages may be identified.

TABLE 5.4 Analysis of variance for substages

Source of Variation	Sum of squares	d	Mean square
Between groups	145612.2	3	48537.4
Within groups	76234.8	490	155.58
Total	221847	493	

F ratio (3,490) = 311.9, $p < 0.01$

TABLE 5.5 Table of mean ages and standard deviations for the substages (t-test values shown)

SUBSTAGE	1	2	3	4
Mean Age in months	146.4	157.4	175.37	200.00
Standard Deviation	6.125	13.179	12.603	11.431
t-test Values	S1 x S2 $t_{260}=3.69$**	S2 X S3 $t_{369}=8.36$**	S3 x S4 $t_{230}=19.82$**	

** difference found to be significant at the 0.01 level.

Summary

Inhelder's mechanism of development has been identified within the behaviours. The four phases of separation, juxtaposition, partial integration and integration, were identified as the substages of isolated centration, unicriterion comparison, bicriteria comparison and tricriteria comparison. These appear to represent four substages in the development of formal reasoning for the 'control of variables' tasks.

This study was cross-sectional and yet the evidence for the substages has been interpreted as representing a developmental continuum. This kind of procedure provides at most an indication of a developmental route. The aim of the study was to explore the possibility of the application of a developmental mechanism and having done this, it may be appropriate to institute the long term, expensive process of carrying out the longitudinal studies that many writers such as Peel (1966) recommend.

TABLE 5.2 The Substages in the development of a Formal Operational 'H' Strategy

		ACCEPT	REJECT	REASON
SUBSTAGE 0 No recognition of two or more strategies.		Any strategy		Usually Criterion 1 (use a different weight)
SUBSTAGE 1 Isolated Centration Acceptance of a strategy on its criterion. 2 or more strategies recognised.		GD C H		Criterion 2 or Criterion 1 (show a difference in depth) Criterion 4 (use a fair experiment) Criterion 3 or Criterion 4 (vary one thing at a time)
SUBSTAGE 2 Unicriterion Comparison Comparison of strategies on the basis of a single criterion.	Comparison A	GD	C&H	Criterion 2
	Comparison B	GD&H	C	Criterion 2
	Comparison C	H	GD&C	Criterion 3
	Comparison D	C&H	GD	Criterion 4
SUBSTAGE 3.1 Bicriteria Comparison One criterion applied to all 3 strategies, one further criterion applied to 1 or 2 strategies.	Partial Integration IA	GD&H	C	Criterion 2 for G.D., C & H; Criterion 4 for H
	Partial Integration IB	H	C&GD	Criterion 3 for G.D. & H; Criterion 2 for G.D., C & H
	Partial Integration IC	H&C	GD	Criterion 4 for G.D., C & H; Criterion 2 for G.D. & H
SUBSTAGE 3.2 Bicriteria Comparison Two criteria applied to all 3 strategies.	Partial Integration IIA	H	C&GD	Criteria 2 & 4 for G.D.,C &H
	Partial Integration IIB	H&C	GD	Criteria 2 & 4 for G.D., C &H; but for C the difference may now be inferred.
SUBSTAGE 4.1 Tricriteria Comparison Three criteria applied to all 3 strategies.	Integration A	H	GD&C	Criteria 2,3 & 4 applied to G.D., C & H
	Integration B	H&C	GD	Criteria 2,3 & 4 applied to G.D., C & H (calculation argument applied to C)
	Integration C	H&C& GD		Criteria 2, 3 & 4 applied to G.D., C & H (calculation argument applied to G.D. & C)
SUBSTAGE 4.2 Tricriteria Comparison New criterion applied to all 3 strategies.	Full Integration	H	GD & C	'Assumptions' criterion

THE JUNIOR SCHOOL STUDIES

The following studies in Chapters 6 to 10, were completed after the Secondary School studies and arose from a need to discover the origins of H methodology.

The study in Chapter 5 showed that over 90% of the first year Secondary School subjects (mean age 11.9 years) understood the H strategy as a method of proof, even though the level of understanding was poor and thus other strategies were also accepted or preferred as methods of proof. Chapter 3 showed that of the second year Secondary School sample (mean age 12.6 years), 63% used the H strategy on the two racing cars task. It was apparent that the H strategy originated at least as early as the beginning of the formal operational level. Thus the aim of the Junior School studies was to investigate the use of the H strategy and to trace its development at the concrete operational level.

CHAPTER 6

AN ANALYSIS OF THE TENNIS BALL TASK IN RELATION TO EMPIRICAL AND QUASI-EMPIRICAL SOLUTION STRATEGIES

Introduction

The present chapter discusses the difference between a task requiring empirical proof and one that may be solved using quasi-empirical proof. An experiment is described which tests the difference between the two types of task in order to explain the child's apparent early ability to use the H strategy on the tennis ball task.

The tennis ball task which requires subjects to compare the bounce of two tennis balls under a variety of novel, but equal conditions was described by Wollman (1976). To succeed in this task the subject must realise that in order to discover which one of the two tennis balls presented is the bounciest,

> "you must drop them from the same height and let them go the same way and let them bounce on the same kind of floor". (Wollman, 1976 pl).

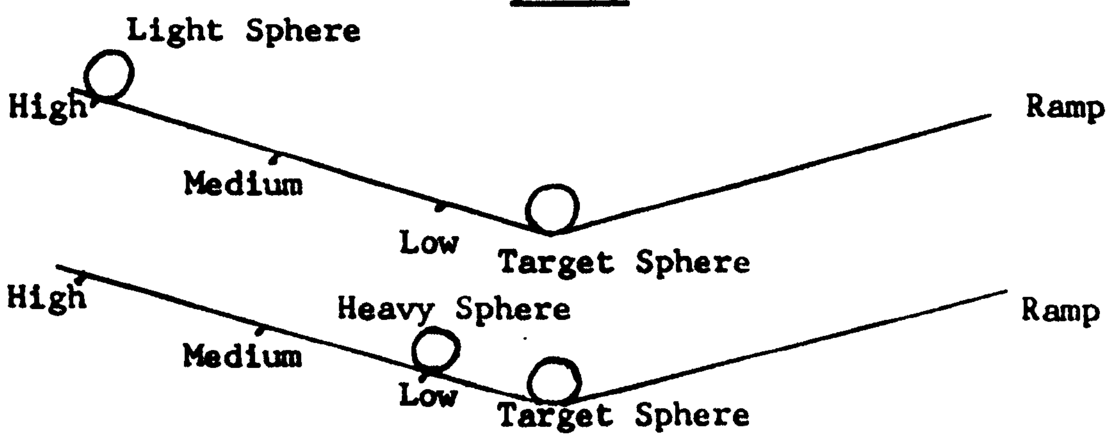
Wollman presented evidence that children (age 12-13 years) intuitively use the H strategy in this situation earlier than in the target sphere task, but he also referred to their inability to transfer this method to new situations.

> "When faced with new problems the child will begin anew, not perceiving the applicability of his prior successful solutions to other problems requiring controlled experiments". (Wollman, 1976 pl).

He suggested that the reason for this variation in the use of solutions involving the control of variables is that their use is context-dependent.

It is evident that if the H strategy is used earlier on the tennis ball task than on the target sphere task then children perceive this problem differently. The tennis ball problem presents the child with a situation where he/she is unaware of the outcome, i.e. he/she has two tennis balls, one of which is bouncier, but he/she cannot tell which by inspection only. This task lends itself to empirical proof in that the result of the proposed experiment is not already known and experimentation is a necessity in order to discover the result. The experiment is not just confirmation of prior knowledge. The target sphere task in contrast, allows confirmation of prior knowledge. It may thus be solved using quasi-empirical methodology, i.e. a methodology that involves experiment and seems to rely on observation of results for proof, but which is not purely empirical in that it assumes and uses the effect of the independent variable it sets out to establish. Of course there is a purely empirical solution to the target sphere task but, because the child has prior knowledge of the effect of the independent variable, he/she is tempted to use this knowledge. For example consider the following proof:-

FIGURE 6.0 The compensation strategy (quasi-empirical proof)



In Figure 6.0 the child releases the light sphere from a high position and the heavy sphere from a low position, so that the lack of weight of the light sphere is compensated by the increased height of release. The child will often predict that the distances travelled by the target spheres are the same. He/she will state that this indicates the effect due to the weight in that the heavy sphere is at a disadvantage yet produces the same effect as the light sphere. This compensation method is an example of a quasi-empirical proof in that the result is anticipated prior to experimentation. If the spheres were indistinguishable other than by colour, i.e. one was yellow and the other was white, then it would not be possible for the child to use quasi-empirical proof. In such a task if the child were asked to choose positions on the ramp to show which 'colour' knocked the target sphere furthest up the other side of the ramp, he/she should only be able to apply an empirical proof. Such a task is thus similar to the tennis ball task.

Any task that may be solved both quasi-empirically and empirically, named a 'quasi/empirical task' in the following, is likely to be solved either way if the child is capable of using both quasi-empirical proof and pure empirical proof, i.e. the C and H strategies. Thus if the child fails on a quasi/empirical task this is not evidence that he/she is not capable of 'control of variables' but that perhaps he/she does not 'see' the need for pure empirical proof in this situation. If the task is one that can only be solved by empirical proof, named an 'empirical task' in the following, the child can only succeed if he/she is capable of 'control of variables'. It was the aim of the study described below to show that the Wollman tennis ball task may be solved using the H strategy at an earlier age than the target sphere or plasticine tasks because the former is an empirical task and the latter are quasi/empirical task. To this end, two new tasks were devised, an empirical plasticine task and a quasi/empirical tennis ball task. The empirical plasticine task is shown with its comparable quasi/empirical task on the following page. As can be seen the fact that the two weights are labelled A and B instead of heavy and light in Q2 makes the plasticine task one of empirical proof. Wollman's tennis ball task is an empirical task and thus performance on the empirical plasticine task should be similar to the empirical tennis ball task. The quasi/empirical tennis ball task is shown after the plasticine task. The child is informed as to the result in this case, i.e. that the red tennis ball is bounciest and he/she may now answer this problem using compensations. Performance on this task should be similar to the quasi/empirical plasticine task.

Experimental Method

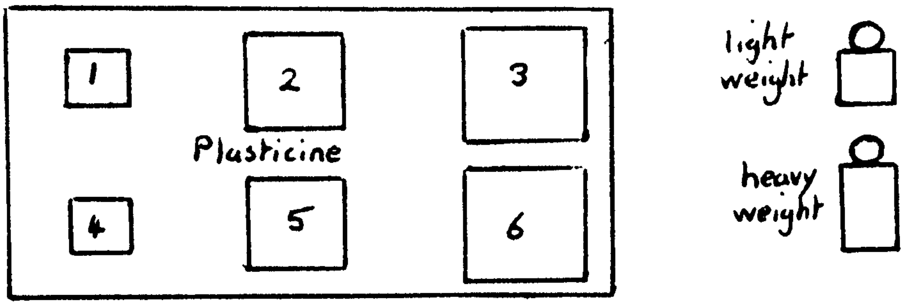
The Tasks

The two plasticine tasks Q1 and Q2 and the two tennis ball tasks Q3 and Q4, were used.

THE PLASTICINE TASKS

Q.1 QUASI/EMPIRICAL PLASTICINE TASK

Blocks of wood 3 and 6 are identical, blocks 2 and 5 are identical and blocks 1 and 4 are identical.



You have two weights a heavy and a light one.

You want to show which is the heavy one by finding out if it leaves the largest depth of hole in the plasticine.

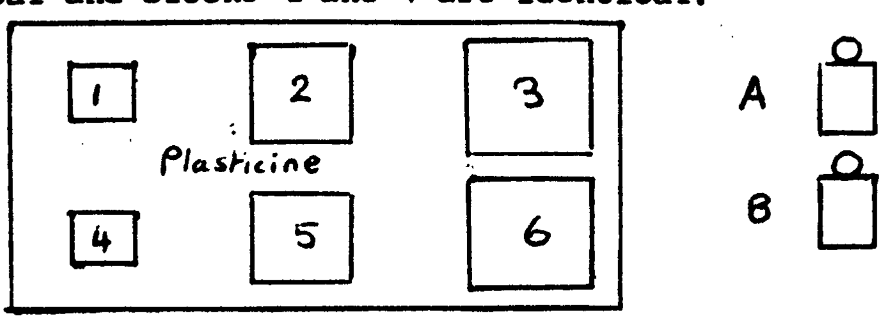
On which block of wood would you place the light weight? ________

On which block of wood would you place the heavy weight? ________

Please explain your answer carefully. (Space was provided).

Q.2 EMPIRICAL PLASTICINE TASK

Blocks of wood 3 and 6 are identical, blocks 2 and 5 are identical and blocks 1 and 4 are identical.



You have two weights A and B, one is heavy and one is light, and you do not know which. You want to find out which is the heavy one by finding out which one leaves the largest depth of hole in the plasticine.

On which block would you place weight A? ____________

On which block would you place weight B? ____________

Please explain your answer carefully. (Space was provided).

THE TENNIS BALL TASKS

Q.3 QUASI/EMPIRICAL TENNIS BALL TASK

Your have two tennis balls, a red and a blue one. The red one is bouncier than the blue one. You want to do an experiment to find out how much difference the bounciness of the balls makes to the height they bounce off the floor. You have to decide from which weight you would drop each ball, low, medium, or high.

X High

X Medium

X Low

Floor

From which height would you drop the red ball (the bouncier ball)? ____________

From which height would you drop the blue ball? ____________

Please explain your answer carefully. (Space was provided).

Q.4 EMPIRICAL TENNIS BALL TASK

You have two tennis balls, a yellow and a white one. You don't know which is the bounciest and you want to find this out by dropping them onto the floor. You have to decide from which height you would drop each ball, low, medium or high.

X High

X Medium

X Low

Floor

From which height would you drop the yellow ball? ____________

From which height would you drop the white ball? ____________

Please explain your answers carefully. (Space was provided).

Subjects

A total of 392 subjects was tested at seven different age levels the details of the sample are given in Table 6.0.

TABLE 6.0

Year	School	Mean Age Years	Total
3rd year	Junior	10.2	56
4th year	Junior	11.1	56
1st year	Secondary	12.2	56
2nd year	Secondary	13.3	56
3rd year	Secondary	14.2	56
4th year	Secondary	15.2	56
5th year	Secondary	16.1	56
			Σ = 392

Both the 3rd and 4th year Juniors were selected at random from their age group from two primary schools in Nottinghamshire. The 1st, 2nd and 3rd year Secondary sample were mixed ability groups in a Comprehensive School in the same area. The 4th and 5th year Secondary sample were selected at random from their age group in the same Comprehensive School, as mixed ability groups did not exist at this level. The age levels were selected so that subjects from the concrete operational to the formal operational level could be tested.

Hypotheses

Three hypotheses were set up:-

1) That the proportion of subjects using the control strategy on the empirical tasks would be significantly greater than on the quasi/empirical tasks.

2) That there would be no significant difference between the proportion of subjects using a control strategy on the tennis ball and plasticine empirical tasks.

3) That there would be no significant difference between the proportion of subjects using a control strategy on the tennis ball and plasticine quasi/empirical tasks.

Experimental Design

In order to control for question order effects it was decided to use a latin square design, using four groups, A, B, C and D for the four questions Q1, Q2, Q3 and Q4 (see Figure 6.1).

FIGURE 6.1

Latin Square Design

		Question Order 1	2	3	4
Groups	A	Q1	Q2	Q3	Q4
	B	Q2	Q4	Q1	Q3
	C	Q3	Q1	Q4	Q2
	D	Q4	Q3	Q2	Q1

Key for question type

Q1	=	Plasticine Task	-	Quasi/empirical
Q2	=	Plasticine Task	-	Empirical
Q3	=	Tennis Balls Task	-	Quasi/empirical
Q4	=	Tennis balls Task	-	Empirical

Experimental Procedure

The subjects were shown the two situations, the plasticine task and the tennis ball task, and the effects of the confounding variables were discussed using the procedure described in Chapter 2. For the tennis ball task the effect of dropping tennis balls from different heights was discussed so as to make clear the fact that the "higher the dropping position the higher the bounce" (for the same bounciness of tennis ball) though the point in parenthesis was not stated. The subjects were then given the questionnaire and asked to work through the four questions in order, not returning to 'correct' any previous question.

Results

The subjects' responses were classified in terms of the following categories of response:-

H - using a form of control strategy
C - using a form of compensation strategy
G.D. - using a form of greatest difference strategy
D - using a form of difference strategy
U - using an unclear method or offering no response.

All four strategies have been discussed in detail previously and examples may be found in earlier chapters. No new strategies occurred but many of the younger subjects who were classified as H gave differing reasons for using a control strategy, whether through restricted language or otherwise. These strategies were overtly control strategies and were classified as H, this will be discussed further in Chapter 7.

TABLE 6.1 The number of subjects using the different strategies (N = 56 for each question)

SCHOOL YEAR	MEAN AGE (YEARS)	QUESTION NUMBER	STRATEGY USED H	C	GD	D	U
3rd Juniors	10.2	1	0	0	6	46	4
		2	16	0	0	38	2
		3	0	0	10	44	2
		4	17	0	0	38	1
4th Juniors	11.1	1	0	2	8	44	2
		2	20	0	0	36	0
		3	0	2	10	40	4
		4	22	0	0	32	2
1st Secondary	12.2	1	3	20	7	26	0
		2	36	0	0	20	0
		3	9	22	2	23	0
		4	36	0	0	20	0
2nd Secondary	13.3	1	15	29	4	7	1
		2	44	0	0	12	0
		3	23	25	3	3	2
		4	53	0	0	3	0
3rd Secondary	14.2	1	24	29	1	2	0
		2	51	0	0	5	0
		3	25	25	3	3	0
		4	52	0	0	4	0
4th Secondary	15.2	1	34	20	2	0	0
		2	54	0	0	2	0
		3	36	20	0	0	0
		4	54	0	1	1	0
5 Secondary	16.1	1	48	8	0	0	0
		2	56	0	0	0	0
		3	50	6	0	0	0
		4	56	0	0	0	0

Table 6.1 shows the total number of subjects utilising the different strategies on the different questions. As the objective of the experiments was primarily the study of the relative frequency of the use of only the C and H strategies on the different questions it was this data that was subject to analysis.

This further analysis of variance followed the standard pattern for a 4 x 4 latin square. The Newman-Keuls test was utilised for testing the significance of the difference between two different questions, (for a discussion of the procedure see Appendix D). The full analysis is shown in Appendix F, but a summary of the significant differences for the two strategies on the different questions is shown in Table 6.3. This can be compared to the percentage use of the C and H strategies shown in Table 6.2.

Consider hypotheses two and three initially. Hypothesis two predicted that there would be no significant difference in the use of the control strategy on the two empirical tasks questions 2 and 4, whereas hypothesis three predicted no significant difference in the use of the control strategy on the two quasi/empirical tasks, questions 1 and 3. The results of the Newman-Keuls test for these question comparisons are shown in Table 6.4. As may be seen no significant differences in the use of control methodology were found between questions 2 and 4 or questions 1 and 3. Thus hypotheses two and three were confirmed by the data.

TABLE 6.2 Total number of subjects using control and compensation strategies on each question. (N = 56 per question).

(Percentages shown in parenthesis to one place of decimals).

	H Strategy (Control)				C Strategy (Compensation)			
YEAR	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
3 Junior	0 (0)	16 (28.6)	0 (0)	17 (30.4)	0 (0)	0 (0)	0 (0)	0 (0)
4 Junior	0 (0)	20 (35.7)	0 (0)	22 (39.3)	2 (3.6)	0 (0)	2 (3.6)	0 (0)
1 Secondary	3 (5.4)	36 (64.3)	9 (16.1)	36 (64.3)	20 (35.7)	0 (0)	22 (39.3)	0 (0)
2 Secondary	15 (26.8)	44 (78.6)	23 (41.1)	53 (94.6)	29 (51.8)	0 (0)	25 (44.6)	0 (0)
3 Secondary	24 (42.9)	51 (91.1)	25 (44.6)	52 (92.9)	29 (51.8)	0 (0)	25 (44.6)	0 (0)
4 Secondary	34 (60.7)	54 (96.4)	36 (64.3)	54 (96.4)	20 (35.7)	0 (0)	20 (35.7)	0 (0)
5 Secondary	48 (85.7)	56 (100)	50 (89.2)	56 (100)	8 (14.3)	0 (0)	6 (10.7)	0 (0)

TABLE 6.3 Significant differences between questions (Newman-Keuls test values) Comparison between empirical and quasi/empirical questions

	H Strategy (Control				C Strategy (Compensation)			
YEAR	Q4xQ1	Q4xQ3	Q2xQ1	Q2xQ3	Q1xQ2	Q1xQ4	Q3xQ2	Q3xQ4
3 Junior	7.2	7.2	6.81	6.81	ns	ns	ns	ns
4 Junior	8.75	8.75	7.95	7.95	ns	ns	ns	ns
1 Secondary	9.45	11.55	9.45	11.55	8.75	8.75	7.95	7.95
2 Secondary	10.35	13.12	7.25	10.02	11.48	11.48	9.91	9.91
3 Secondary	9.61	9.28	9.26	8.93	11.53	11.53	9.93	9.93
4 Secondary	7.37	6.63	7.37	6.63	8.02	8.02	8.02	8.02
5 Secondary	4.48	3.35*	4.48	3.35*	4.48	4.48	3.35*	3.35*

All significant at the 0.01 level but for * at the 0.05 level and except where marked ns (not significant).

TABLE 6.4

Questions compared (Control Methodology)

YEAR	Relates to Hypothesis 2: Q2 v Q4 — Difference In Means	Significance	Relates to Hypothesis 3: Q1 v Q3 — Difference in Means	Significance
3 Junior	0.018	N.S.(N_{k56}=0.43)	0.000	N.S.
4 Junior	0.036	N.S.(N_{k56}=0.81)	0.000	N.S.
1 Secondary	0.000	N.S.	0.107	N.S.(N_{k56}=2.12)
2 Secondary	0.160	N.S.(N_{k56}=3.12)	0.143	N.S.(N_{k56}=2.79)
3 Secondary	0.018	N.S.(N_{k56}=0.35)	0.017	N.S.(N_{k56}=0.33)
4 Secondary	0.000	N.S.	0.036	N.S.(N_{k56}=0.75)
5 Secondary	0.000	N.S.	0.036	N.S.(N_{k56}=1.14)

N.S. = $p > 0.01$

Hypothesis one, that the proportion of subjects using the H strategy on the empirical tasks questions 2 and 4, would be significantly greater than on the quasi/empirical tasks, questions 1 and 3, was supported by the data as shown in Table 6.5.

TABLE 6.5 Questions compared (H strategy)

YEAR	Q4 v Q1 Difference in Means	Q4 v Q1 Significance	Q4 v Q3 Difference in Means	Q4 v Q3 Significance
3 Junior	0.304	** (N_{k56}=7.3)	0.034	** (N_{k56}=7.3)
4 Junior	0.393	** (N_{k56}=8.83)	0.393	** (N_{k56}=8.83)
1 Secondary	0.482	** (N_{k56}=9.54)	0.589	** (N_{k56}=11.66)
2 Secondary	0.535	** (N_{k56}=10.44)	0.678	** (N_{k56}=13.23)
3 Secondary	0.500	** (N_{k56}=9.69)	0.483	** (N_{k56}=9.36)
4 Secondary	0.357	** (N_{k56}=7.44)	0.321	** (N_{k56}=6.69)
5 Secondary	0.143	** (N_{k56}=4.52)	0.107	* (N_{k56}=3.38)

Questions compared (H strategy)

YEAR	Q2 v Q1 Difference in Means	Q2 v Q1 Significance	Q2 v Q3 Difference in Means	Q2 v Q3 Significance
3 Junior	0.286	** (N_{k56}=6.87)	0.286	** (N_{k56}=6.71)
4 Junior	0.357	** (N_{k56}=8.02)	0.357	** (N_{k56}=8.02)
1 Secondary	0.482	** (N_{k56}=9.54)	0.589	** (N_{k56}=11.66)
2 Secondary	0.375	** (N_{k56}=7.32)	0.518	** (N_{k56}=10.11)
3 Secondary	0.482	** (N_{k56}=9.34)	0.465	** (N_{k56}=9.02)
4 Secondary	0.357	** (N_{k56}=7.44)	0.321	** (N_{k56}=6.69)
5 Secondary	0.143	** (N_{k56}=4.52)	0.107	* (N_{k56}=8.38)

** = Difference significant at the 0.01 level
* = Difference significant at the 0.05 level

There was a difference significant at the 0.01 level (test values shown in Table 6.5) for all years tested on the empirical task question 2 and question 4 compared to the quasi/empirical task question 1 and question 3, except for the fifth year sample on questions 4 vs question 3 and question 2 vs question 3, where the difference was significant at the 0.05 level.

The confirmation of the three hypotheses implies that the difficulty experienced by the subjects in the solution of tasks similar to the target sphere task, is closely related to the logical nature of the task rather than the content of the task as suggested by Wollman. When only the logical nature of the task was different, i.e. empirical compared to quasi-empirical, the subjects use of the H strategy altered significantly. When only the content was different there was found to be no significant difference in the use of the H strategy.

Discussion

It has been shown in this study that the tasks that may be solved using purely empirical proof are solved more frequently using the control strategy than are tasks that may also be solved using quasi-empirical proof. As many as 30.4% of the 10 year old (3rd year Junior) sample used the control strategy on the empirical tasks. This frequency of use was just reached by the 13 year old (2nd year Secondary) sample on the quasi/empirical task (27% on Q1, 41% on Q3). The tennis ball task as used by Wollman was a task that required purely empirical proof for solution and this appears to be the reason for this early success at the use of the control strategy. Poor transfer of the use of the control strategy from the empirical tennis ball task to other tasks which may also be solved using quasi-empirical proof, as noted by Wollman (1976), is hardly surprising. The fact that the subject has the control strategy within his repertoire is just one of the determinants for its use. If the control strategy is poorly differentiated from the other strategies allowed for by the logical nature of the task, then alternative 'fair' experimentation may be used.

The child's everyday experience provides him/her with few situations that require purely empirical proof. In science lessons in school the child is not likely to meet any of these situations. Consider as an example an experiment that might be considered to require empirical proof, the experiment to show the factors determining the time period of a pendulum. In order to perform the experiment the child has to isolate those variables that might affect the time period of the pendulum, i.e. the length of pendulum, mass of the bob, and the amplitude of swing. All children have an intuitive idea as to which factors affect the time period. Whether the child is correct concerning the effects of the independent variables is not of importance, it is the fact that he/she makes assumptions concerning these effects that makes the pendulum experiment and all other traditional science experiments that require the use of control of variables, quasi/empirical tasks. Thus as the results in Table 6.2 in this Chapter suggest, the majority of 13 year old subjects, 94.6% on question 4 in the sample used, are capable of using the control strategy when they see the need to do so. Yet 44.6% of the same 2nd year sample used the compensation strategy on one quasi/empirical task, question 3, because in order to solve the task, quasi-empirical strategies are possible and are, in the child's perception of the situation, sufficient.

CHAPTER 7

A STUDY OF THE CONTROL METHODOLOGY USED BY CONCRETE OPERATIONAL SUBJECTS

Introduction

According to Piaget the child at the beginning of the formal level makes two important discoveries:

(1) that factors can be separated out by neutralisation,
(2) that a factor can be eliminated not only for the purpose of analysing its own role, but even more important, with a view toward analysing the variations of other associated factors. (Inhelder & Piaget, 1958; p285)

In the same chapter he states that at the concrete level the child is capable of exclusion of a variable in order to see if the variable itself plays a role, but not as a means of studying the variation of another variable. The fact that Junior School subjects were found in the study reported in Chapter 6 to control the height of drop in the tennis ball task, in order to discover the variation of the intrinsic bounce variable, appears to be incompatible with the above statements. Furthermore, evidence of the use of the control strategy at the concrete level has been presented by Lawson and Wollman, 1977a; Wollman, 1976; So merville, 1974; Case, 1974b. It was thus the aim of the present chapter to undertake a detailed analysis of the reasons for use of the control strategy at the Junior School level. As was noted in Chapter 6, reasons for the use of the control strategy given in the Junior School study, although sometimes unclear, were apparently different to reasons given by older subjects.

Experimental Method

The Tasks

Three tasks all based on the empirical tennis ball task were used and these are shown on the following page. Question 1, the subject's experiment, tests what the subjects would do themselves. The other two questions ask the subjects to judge whether a method is correct or incorrect and to state their reasons for this judgment. Question 2, John's experiment, involves a deliberate manipulation to make the yellow tennis ball bounce the highest by throwing it. This is 'D' experimentation. Question 3, Peter's experiment, is an example of the test of strength experimentation first reported by S. Ervin (1960) and was referred to in Chapter 1. In this experiment Peter gives an advantage to the white tennis ball by throwing it, yet the yellow tennis ball bounces the highest, i.e. he tests the strength of the ability of the yellow tennis ball to bounce the highest by putting it at a disadvantage. Questions 2 and 3 were chosen because they represented typical concrete operational strategies that Junior School subjects could comprehend and about which they could make judgments. They were included to provide additional information as to the reasons for the use of the control strategy in terms of the acceptance or the rejection of the concrete operational strategies. Questions 2 and 3 both involve the yellow tennis ball winning. This choice was made so as to avoid the potential conflict involved when different results

THE QUESTIONS

Q.1 Subject's Experiment

Suppose you have a white and a yellow tennis ball and you want to find out which ball is the bounciest. You can find this out by either throwing or dropping the tennis balls onto the floor.

Would you throw or drop the yellow tennis ball? ___________
Would you throw or drop the white tennis ball? ___________
Please explain your answer carefully. (Space was provided).

Q.2 John's Experiment

John has a yellow and a white tennis ball and he wants to find out which ball is the bounciest. He throws the yellow tennis ball onto the floor and he drops the white tennis ball onto the floor. He finds that the yellow tennis ball bounces higher than the white tennis ball. He says that this proves that the yellow tennis ball is the bounciest.

Is he right or wrong? ___________________________
Please explain your answer carefully. (Space was provided).

Q.3 Peter's Experiment

Peter has a yellow and a white tennis ball and he wants to find out which ball is the bounciest. He throws the white tennis ball onto the floor and drops the yellow tennis ball onto the floor. He finds that the yellow tennis ball bounces higher than the white tennis ball. He says that this proves that the yellow tennis ball is the bounciest.

Is he right or wrong? ___________________________
Please explain your answer carefully. (Space was provided).

are produced by alternate experiments. Such a conflict might have produced the conclusion that both tennis balls were of equal bounce. This then may have affected the subject's answer to question 1, i.e. he/she might have dropped both the tennis balls in the same way because they were equally bouncy.

Subjects

A total of 150 subjects was tested covering 5 different age levels. The details of the sample are given in Table 7.0 below:-

TABLE 7.0

YEAR	SCHOOL	MEAN AGE (YEARS)	TOTAL
2ND YEAR	JUNIOR	9.2	30
3RD YEAR	JUNIOR	10.2	30
4TH YEAR	JUNIOR	11.1	30
1ST YEAR	SECONDARY	12.2	30
2ND YEAR	SECONDARY	13.4	30
			150

The Junior School samples were selected at random from their age group from two Primary Schools in Nottinghamshire. The 1st and 2nd year Secondary Sample were mixed ability groups in a Comprehensive School in the same area.

Experimental Design

A statistical latin square design was employed in order to control for any effect of order in the three question types e.g. see Figure 7.0.

FIGURE 7.0 Latin Square Design

ORDER

	A	B	C
GROUP 1	S	J	P
GROUP 2	J	P	S
GROUP 3	P	S	J

P = Peter's experiment
J = John's experiment
s = Subject's experiment

Experimental Procedure

The effect of throwing a tennis ball and dropping a tennis ball on the height of bounce was discussed fully with the subjects. They were asked to imagine that the experimenter was selling the yellow and white tennis balls for 5p each but that they only had 5p. They could therefore only buy one tennis ball. They were told that they wanted to buy the best tennis ball, the one that bounced the best, and that the task involved performing an experiment to find out which ball bounced the best so that they knew which one to buy. The subjects were then given the questionnaire to complete, each question was read out with the class and the subjects were given adequate time to complete a response. At the end of the test each subjects' answers were checked. Where answers were poorly written or incomplete the experimenter discussed these with the subjects to ascertain the difficulty and where possible to clarify the response.

Results

Six major categories were identified and these are included below with examples of subjects' responses.

Category 1:- No conscious strategy for experimentation for the Subject's experiment

Reasons include "I'd try them to see", "drop them because I feel like it", "to see if it bounces", "it doesn't matter just try it".

John's and Peter's experiments were often considered to be correct because the result rather than the method was considered right. For example,

TUB (8.11 years) - "Peter is right because the yellow ball bounces higher".
"John is right because the yellow ball goes higher when you throw it".

Peter's experiment was sometimes considered to be wrong because the result did not confirm the subject's expectation. For example,

COB (9.7 years) - "Peter is wrong because he said he dropped the yellow one, but he must have thrown it".

Category 2a: - Experimentation which involved a conscious manipulation of the experiment in order to obtain an anticipated result

The subjects set up a restricted hypothesis, e.g. "the yellow is the best", followed by an experimental confirmation of the hypothesis by the use of the D strategy. For example,

FITS (9 years) - "Drop the yellow ball, throw the white ball. I would throw the white ball so that it bounces high".
N.B. "Peter's right because the white tennis ball and the yellow tennis ball cannot bounce at the same height".

NEY (9.10 years) - "Throw the yellow ball, drop the white ball. I would make the yellow tennis ball bounce higher because I like it better".

Category 2b: - <u>Compensation experimentation</u>

The subjects set up a restricted hypothesis e.g. " the yellow is the best" and then judged that the other tennis ball needed some additional bounce. For example,

GER (10.6 years) - "Drop the yellow tennis ball and throw the white tennis ball, because if you drop the white ball it might not bounce at all".

OLD (12 years) - "Drop the yellow ball and throw the white ball. I would drop the yellow tennis ball because it is stronger. I would throw the white tennis ball because it doesn't go higher".

RD (12.11 years) - "Drop the yellow ball and throw the white ball. The white tennis ball might be made of harder rubber than the yellow one so it might need more of a push".

Category 3 - <u>Manipulation of the experiment due to the anticipation that the tennis balls bounce the same</u>

The restricted hypothesis led to the subjects bouncing the ball in the same way, i.e. dropping both or throwing both to make the tennis balls bounce the same. This was similar to 2a, but the restricted hypothesis was different. Subjects were not controlling to see if the balls bounce the <u>same or not,</u> but were concerned with making them bounce the same because they believed they bounced the same. For example,

FRA (9.5 years) - "Throw both tennis balls because the tennis balls bounce the same. I have tried it before".
"John's is wrong because when you drop it, it goes softly on the ground and when you throw it, it goes up higher".
Peter's was considered wrong for the same reasons, i.e. Peter's and John's were wrong because the tennis balls would not have bounced the same in their experiments.

HAS (9.8) - "Throw both balls to make it bounce higher, because they are the same and they will bounce the same height". John's was wrong for the same reason. "Peter's is wrong because the force of the white ball is heavier".

CAR (10.8 years) - "Drop both balls because all tennis balls bounce the same". John's was wrong for the same reason.

The subjectivity of CAR was obvious, he tried to make sense of Peter's experiment by considering that the yellow ball was dropped harder because the result was not in accord with his anticipated result, hence he failed to understand the test of strength argument.

Category 4: - Manipulation of the experiment in order to make the tennis balls bounce the most

The subjects considered dropping a tennis ball to be incorrect in that bounce was related to throwing for these subjects. There was no reference to doing the same thing to both tennis balls. For example,

BAS (8.10 years) - "I would throw the ball because it bounces higher".

KES (9 years) - "I would throw it and have the one which bounced the highest. Dropping it does not test it very well".

FRAN (9.5 years) - "Dropping it would not go very high to see which was best". "John is wrong dropping is not so good because throwing is harder and so it will go higher".

In the following examples John's experiment was considered correct.

WOOD (9 years) - "Throw both tennis balls because it would bounce higher". "John is right because if you throw it, it would bounce higher", i.e. he agreed with the result.

Peter was considered to be wrong because WOOD disagreed with the result, i.e. the yellow one should not have won when it was just dropped "because when you drop a ball it does not bounce higher than throwing it".
Yet WOOD throws the balls "because it will bounce higher".

TOW (9 years) - "Throw each one. Throwing makes a better bounce than dropping".
"John's is right because if you throw a tennis ball it bounces much more and because if you bounce a ball it does not go as high".

Peter's was considered to be wrong because TOW disagreed with the result.

The rejection of Peter's experiment indicates that these subjects were not attempting to show the effect of an intrinsic variable of bounce. The subjects were not attempting a control strategy.

Category 5: - A control solution in which throw was excluded

The subjects were aware of the distinction between extrinsic and intrinsic causal factors and they considered throwing to be an additional factor which had to be eliminated from the situation. The subjects did not mention making fair comparisons yet considered that throwing in order to show the intrinsic bounce of the tennis balls would introduce an extra effect. For example,

MID (9.9 years) - "I would drop it because throwing would not be bouncing it".
"John's wrong because it is more powerful throwing it".

KEV (10.8 years) - "I would drop the yellow one because if you throw it you are forcing it to bounce higher than it should. I would drop the white one for the same reason.
"John's wrong because he throws the yellow one down it will bounce too much".

CLIF (9.10 years) - "drop both, throwing doesn't let them bounce".
"John's wrong, he gave too much power to the yellow one, he made it bounce too high".

Compare these responses to the following control responses classified as Category 6.

JON (8.9 years) - "Throw both, because then they would be thrown in the same way so it is 'fair' to both".

TOD (10.1 years) - "Drop both because if you threw them you could not tell how hard you throw them. They must be given the same chance".

These responses are obviously different to category 5 responses in that they appear to be concerned with making fair comparisons and not just excluding a variable.

Category 6:- Fair comparison by doing the same thing to both tennis balls

The category 6 solutions fall into two major categories.

Category 6a: - The subjects mentioned comparisons and the construction of a fair test, but their explanations and their methodology were poor compared to category 6b, i.e. they usually threw both tennis balls not considering the strength of throw, but the subjects did not mention that throwing both was potentially an inaccurate method.

Category 6b: - The subjects were aware of the inaccuracy of throwing both tennis balls. They therefore considered throwing both at the same speed or more commonly, dropping both. A minority of subjects also considered the control of the height variable which was not referred to in the questions.

Category 6a examples - "Throw both because then they would
MAL (8.9 years) be thrown in the same way, so it is fair. You can compare them".

STOD (10 years) - "Drop both because you must do the same to both, or you will make one go higher. You could throw both as well".

Category 6b examples - "Drop both. I would drop them because
PLAT (9 years) if you throw them you might throw one harder than the other".

BELL (10.7 years) - "Drop both. I would drop the tennis balls onto the floor. If you throw the yellow tennis ball onto the floor you might have thrown the yellow tennis ball harder, so it would bounce higher. If you drop them, you can't put any extra power into either so it's even".

GRUN (10.2 years) - "Drop both, because if you throw both tennis balls you might without knowing put more power into one arm than the other, but if you just dropped them you are not using your muscles. But if you drop them you would have to drop them at the same height".

The frequency of use of categories 6a and 6b is shown in Table 7.1.

TABLE 7.1 Frequency of category 6a and 6b correct responses

	JUNIOR YEAR 2	JUNIOR YEAR 3	JUNIOR YEAR 4	SECONDARY YEAR 1	SECONDARY YEAR 2
N =	(30)	(30)	(30)	(30)	(30)
Category 6a	3 (10%)	4 (13%)	5 (17%)	7 (23%)	6 (20%)
Category 6b	7 (23%)	7 (23%)	11 (37%)	11 (37%)	15 (50%)
TOTAL	10 (33%)	11 (37%)	16 (53%)	18 (60%)	21 (70%)

Table 7.2 shows the frequency of use of the six categories.

TABLE 7.2 Frequency of use of the six categories

SCHOOL/YEAR	MEAN AGE	N	CATEGORY 1	2a	2b	3	4	5	6
JUNIOR 2	(9.2 years)	(30)	3	4	0	2	9	2	10
JUNIOR 3	(10.2 ")	(30)	0	4	1	3	2	9	11
JUNIOR 4	(11.1 ")	(30)	2	1	1	2	1	7	16
SECONDARY 1	(12.2 ")	(30)	2	1	0	1	1	7	18
SECONDARY 2	(13.4 ")	(30)	2	0	2	0	1	4	21

Table 7.3 shows the number of responses for question 1 to 3.

TABLE 7.3 Frequency of correct responses for the 3 questions (Criterion of fair experimentation)

SCHOOL/YEAR	MEAN AGE	N	QUESTION 1	QUESTION 2	QUESTION 3
JUNIOR 2	(9.2 years)	(30)	10	11	6
JUNIOR 3	(10.2 ")	(30)	11	11	5
JUNIOR 4	(11.1 ")	(30)	16	16	7
SECONDARY 1	(12.2 ")	(30)	18	17	9
SECONDARY 2	(13.4 ")	(30)	21	22	8

F ratios were calculated and the Newman-keuls test was applied to pairs of means for the questions to identify significant variances. Table 7.4 summarises the differences found to be significant.

TABLE 7.4 Significant differences in the performance on the three questions

SCHOOL/YEAR	F Ratio	P 0.01	P 0.05	Newman-Keuls test value
JUNIOR 2	1.18	ns	ns	$p > 0.05$
JUNIOR 3	1.85	ns	ns	$p > 0.05$
JUNIOR 4	3.75	ns	0.05	(John's/Peter's 3.35)
SECONDARY 1	3.2	ns	0.05	(John's/Peter's 2.902)
SECONDARY 2	9.5	0.01		(John's/Peter's 2.902) (Subject's/Peter's 5.517)

No significant difference was found between performance on the subject's question Q1 and on John's question Q2 ($p > 0.05$). Certain significant differences were found between performance on Peter's question Q3 when compared to the other two questions. These differences were due to the method used to classify the solutions as correct. The criterion used for judging successful performance for John's and Peter's questions was one of recognition that they were not control or fair solutions. Peter's question, the test of strength solution, represents a correct conclusion from an ill-conceived experiment. The majority of subjects for each age group tested considered this question correct, the minority rejecting it on the criterion of fair experimentation. Thus for each year, the total number of correct responses for question 3 was lower than for questions 1 and 2.

Discussion of Results

Categories 1 to 3 are obviously not control solutions, categories 2 and 3 are both typical concrete level manipulative solutions where the effect due to the independent variable was anticipated and formed the basis for experimentation. Categories 3 to 6 might appear to be control solutions, but distinctly different criteria were given for the use of this 'correct' solution and these are summarised below.

<u>Summary of criteria used by the subjects for the use of apparent control solutions</u>

	Subjects' Criteria
CATEGORY 3	The tennis balls have the same bounciness, so bounce them the same.
CATEGORY 4	The tennis balls should bounce the highest they can, so throw them the same.
CATEGORY 5	The tennis balls should be "left" to 'bounce' so do not throw them, drop both of them.
CATEGORY 6	The tennis ball should be compared by bouncing them in the same way, to show which is the bounciest. (For category 6b, by not throwing them both as the possibility of inconsistent strength of throw was anticipated).

Categories 3 and 4 might easily be mistaken for control solutions but are novel examples of manipulation experimentation. Category 5 and 6 reasoning is distinctly different to the other categories in that differentiation between extrinsic and intrinsic variables is implied by the responses. Category 2, 3 and 4 solutions appear to be manipulations of extrinsic variables because the influence of an intrinsic variable is not understood. For example, in category 3 the subject considered making the tennis ball bounce the same by making the extrinsic variable 'throw or drop' the same; that the two tennis balls might bounce differently due to some intrinsic factor of bounciness was not considered.

It is proposed that categories 5 and 6 represent control solutions and that categories 3 and 4 represent rather "naive" solutions which might be mistaken for control solutions. The distinction between categories 5 and 6 might be due to a restricted use of language rather than a difference in reasoning but it is proposed that category 5 indicates a form of concrete level control reasoning, i.e. the subjects consider an elimination of the extrinsic variable, exclusion of throw, in order to study the role of the intrinsic variable of bounce. That concrete level subjects can exclude factors that may be considered to be physically eliminated in order to study their role was reported by Inhelder and Piaget (1958), but as mentioned in the introduction to this chapter he considered that the exclusion of factors to study the role of other variables represented formal reasoning. It is possible that subjects may develop this skill at the concrete operational level when attempting to show the role of intrinsic factors. As Piaget states,

"the limitations of the method (of exclusion) are clear in cases where a factor cannot be physically eliminated (e.g., the weight of an object, the length of a rod, etc.). The concrete level subjects do not succeed in neutralising it, whereas stage III (early formal) subjects are able to do so (mentally) and thus calculate its effects". (Inhelder and Piaget, 1958 p284).

As mentioned previously subjects using category 6 solutions may introduce the control of the height of drop variable, a variable that may not be physically eliminated. In addition

Chapter 6 produced evidence that concrete level subjects could control this height of drop variable. Either these subjects are capable of neutralisation of the height variable, i.e. they are formal reasoners, or they are applying concrete reasoning to the height variable, i.e. they are mentally excluding some 'extra effect' due to extra height just as concrete level subjects are excluding some 'extra effect' due to 'extra throw' in category 5. From the viewpoint of concrete logic the exclusion of a difference due to a variable is possible, i.e. the subjects appear to be able to exclude 'extra height of drop'. Thus categories 5 and 6 may both be concrete level control solutions. Chapter 8 presents an attempt to differentiate between the proposed concrete and formal control solutions.

It is important to note the high proportion of 9 year old subjects who apparently used a control solution if categories 3 and 4 are mistakenly included as control solutions, i.e. if the reasons for use are not carefully analysed, see Table 7.5. Nevertheless when only categories 5 and 6 are considered as control solutions 67% of the 10 year old subjects were found to be capable of a control solution.

TABLE 7.5 Subjects apparently using control solutions

SCHOOL/YEAR	MEAN AGE	N	TOTAL FOR CATEGORIES 5 & 6		TOTAL FOR CATEGORIES 3, 4 5 & 6	
JUNIOR 2	(9.2 years)	(30)	12	40.0%	23	76.6%
JUNIOR 3	(10.2 ")	(30)	20	66.7%	25	83.3%
JUNIOR 4	(11.1 ")	(30)	24	80.0%	27	90.0%
SECONDARY 1	(12.2 ")	(30)	25	83.3%	27	90.0%
SECONDARY 2	(13.4 ")	(30)	25	83.3%	26	87.0%

CHAPTER 8

A FURTHER ANALYSIS OF THE CONTROL SOLUTIONS USED BY CONCRETE OPERATIONAL SUBJECTS

Introduction

In order to clarify the difference between control solutions possible at the concrete and formal levels the difference between the negating mechanisms used at these levels is of vital concern.

Collis (1975) points out the concrete operational level

> "child is familiar with 'undoing' (or 'inverse') operations. He knows from an early age that if, for example, one wraps up a piece of chalk one cannot see the chalk but the chalk can be made to re-appear by unwrapping. In the same way subtracting can be seen as the inverse of adding". (Collis, 1975, P39).

According to Piaget, at the formal operational level, the child is capable of both inverse and reciprocal strategies (Inhelder & Piaget, 1958). The new strategy enables the child to annul an operation by performing a reciprocal operation, thus neutralising its effect. At the concrete level the operation may only be annulled directly by removing this operation. For example, consider the beam balance experiment described by Inhelder and Piaget (1958). If a weight is added to the left hand pan the beam becomes unbalanced. The child at the concrete level may apply a negation elimination strategy by physically removing the weight. In the same way if a weight on the left hand pan is balanced by a weight on the right hand pan, an addition of weight that unbalances the system may be negated by cancellation of the operation by the concrete operational child. It should also be possible for the concrete level child to negate an addition of distance of the pan from the pivot that unbalances the beam, in order to maintain equilibrium. It seems likely, in terms of the order of development, that the elimination of all the weight in the first example to maintain equilibrium may occur before the cancellation of an additional operation to a system that has weights or distances already in equilibrium.

According to Piaget, at the formal level the child may also apply a reciprocal strategy to balance the weight added to the left hand pan by adding an equal weight to the right hand pan. The concrete level child may grasp this notion by perceptual configurations where the weighing pans are a fixed distance from the pivot but he/she is considered not to realise the equivalence of the negation and reciprocal strategies in maintaining equilibrium and that the reciprocal strategy neutralises rather than eliminates or directly cancels an operation. Thus an addition of extra weight to the right hand side of a balance in order to neutralise the addition of extra weight to the left hand side is considered to represent a higher level of reasoning than a simple cancellation of the operation by removal of the extra weight from the right hand side.

Collis (1975) found that negation by elimination was a feature of early concrete reasoning (Piaget's Stage IIA) e.g., all of what is put down can be taken up is what children at this level understand by subtraction, i.e. 3 - 3 = 0. Also, negation by cancellation was found to be a feature of later concrete reasoning (Piaget's Stage IIB) e.g., the child regards subtracting as 'destroying' the effect of an addition, i.e. y + 4 - 4 = y. This notion was also noted at Piaget's Stage IIIA, but Collis makes the distinction that the child now realises that the operation is now 'undone' by negation rather than 'destroyed' by it. However, as Collis points out,

"in practice this notion becomes difficult to distinguish behaviourally from the process at Stage IIB as it relies on distinguishing between "destroying" the effect of an operation and "undoing" the effect". (Collis,1975; P182).

By Piaget's Stage IIIB, formal operational reasoners were found to cope with the reciprocal strategy. From this evidence it appears likely that once a child distinguishes between extrinsic and intrinsic variables within the tennis ball task he/she may attempt to control the extrinsic variables in order to show the effect of the intrinsic variable of bounce. It was proposed in Chapter 7 that category 5 solutions to the tennis ball task represented evidence of a concrete control strategy. This category appears to involve negation by elimination in that 'throw' may be considered by the subject to be excluded from the situation. 'Type of floor surface', i.e. whether the tennis ball is dropped onto a soft rug or onto a hard floor, is another variable that may be used in the tennis ball situation. This too may be considered by the child to be excluded from the situation and hence its control might involve negation by elimination. 'Height of drop' of the tennis balls is an extrinsic variable that may not be eliminated, it is only 'extra height' that may be excluded from the situation in a control solution. This situation involves negation by cancellation of a difference.

It was of interest to discover whether the negation by elimination situations were solved earlier by using control solutions than the negation by cancellation situation. If evidence were found to support this then it might be implied that negation was the underlying process by which concrete subjects were able to use a control strategy. In addition if Junior School subjects were using a negation control strategy then they would not comprehend the reciprocal control strategy as used by formal reasoners. A concrete operational child should not consider as correct a strategy of increasing the height of drop for both tennis balls in order to increase the observed difference between the bounce of the tennis balls, in that this involves the reciprocal strategy, i.e. the operation of increasing the height is neutralised by increasing the height of the other tennis ball without affecting the intrinsic factor of bounce. The child who is using a negation control strategy might consider the end result of the strategy , that both tennis balls are at the same height, as correct due to symmetry. He/she should not be able to comprehend the reasoning underlying the increase of both heights but should prefer to remove the extra height, i.e. control by negation.

Experimental Method

The tennis ball questions used are shown on the following pages. Questions 1, 2 and 3 are of a parallel form except that the extrinsic independent variables differ. Table 8.0 shows the least complex negation operation allowed by each question.

TABLE 8.0 Negation operation allowed by the tennis ball questions

	TENNIS BALL QUESTION	NEGATION OPERATION ALLOWED
Q1	Type of floor surface	Negation by elimination
Q2	Throw or drop	Negation by elimination
Q3	Height	Negation by cancellation

Q4 was designed to discover whether the subject understood the reciprocal control strategy. An answer that considered such a strategy as incorrect or that suggested moving the yellow tennis ball back again to the original height of drop was considered to be evidence of the use of a negation control strategy.

THE QUESTIONS

Type of floor surface question

Q.1 Suppose you have a yellow and a white tennis ball and you want to find out which one is the bounciest. You can find this out by bouncing the tennis balls. You can use either the floor or a rug. Would you bounce the yellow tennis ball on the floor or the rug? ________
Would you bounce the white tennis ball on the floor or the rug? ______________

Please explain your answer carefully. (Space was provided).

Throw or drop question

Q.2 Suppose you have a yellow and a white tennis ball and you want to find out which one is the bounciest. You can find this out by either throwing or dropping the tennis balls onto the floor.
Would you throw or drop the yellow tennis ball? _________
Would you throw or drop the white tennis ball? __________

Please explain your answer carefully. (Space was provided).

Height question

Q.3 Suppose you have a yellow and a white tennis ball and you want to find out which one is the bounciest. You can find this out by bouncing them on the floor from either a high position or a low position.
From which position would you bounce the yellow tennis ball high or low? ________________
From which position would you bounce the white tennis ball high or low? ________________

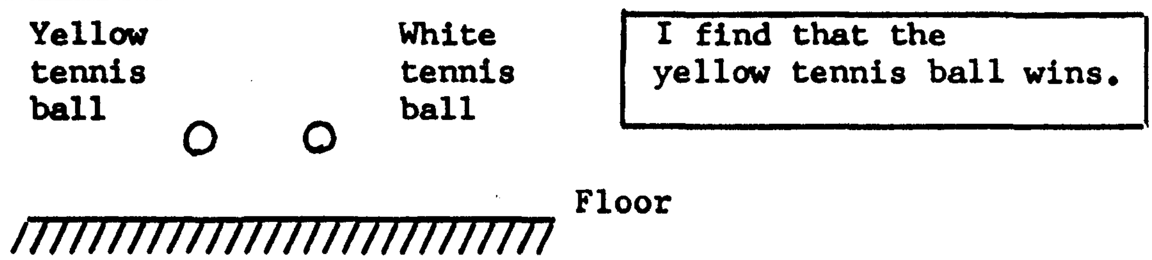
Please explain your answer carefully. (Space was provided).

THE QUESTIONS (continued)

Increasing the height problem

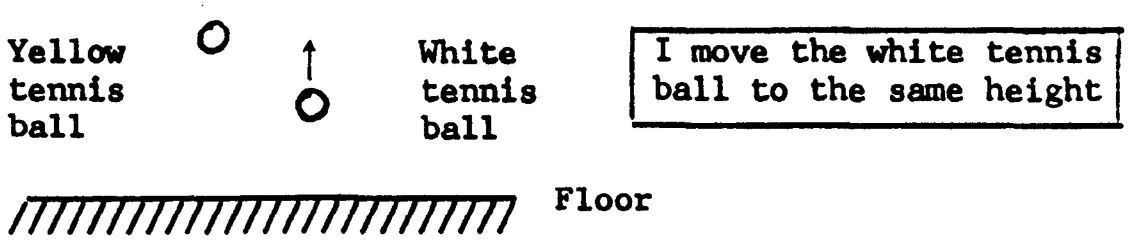
Q.4 I do the experiment shown to find out which tennis ball is the bounciest the yellow or the white one. I drop the yellow tennis ball onto the floor and drop the white tennis ball onto the floor from the same height.

Experiment One



I then do the next experiment. I increase the height of the yellow tennis ball.

Experiment Two



My friend tells me to increase the height of the white tennis ball as well. He says this will now show a bigger difference between the tennis balls than in Experiment One.

Is he right or wrong? ________________

Is this experiment a 'fair' one? ___________

Please explain your answers carefully. (Space was provided).

Which is the best experiment to show which tennis ball is the bounciest. Experiment one or two? __________.

Hypotheses

Two hypotheses were set up.

1) That the proportion of early concrete operational subjects using a control solution will be greater if the question allows the use of negation by elimination.

2) That the control solution used at the concrete operational level involves a negation strategy and that this precedes the use of the reciprocal strategy.

Subjects

A total of 450 subjects was tested at 3 different age levels, the details of the sample are given in Table 8.1.

TABLE 8.1

YEAR	SCHOOL	MEAN AGE (YEARS)	TOTAL
2nd	Primary	8.8	150
3rd	Primary	9.8	150
4th	Primary	10.7	150

The age range was selected so that concrete operational subjects could be tested. The samples were selected at random from four Primary Schools in Nottinghamshire.

Experimental Design

Three groups of 50 for each year were selected at random and each group was given a different question to complete. The tests were carried out during one school morning and all the subjects selected in a particular group in a particular school were given their questions at the same time. The remaining two groups were treated in the same manner. Only those subjects who were selected for question 3 were given question 4 to complete.

Experimental Procedure

This was identical to that described in Chapter 7.

Results

The six major categories of response described in Chapter 7 were again found. The category 6 control solutions were not divided into two categories *a* and *b*, category *a* being only a less sophisticated form of control than category *b* and applying primarily to question 2. The criteria for inclusion into category 6 were as in Chapter 7 and are shown below:-

Criteria used for inclusion of response in category 6

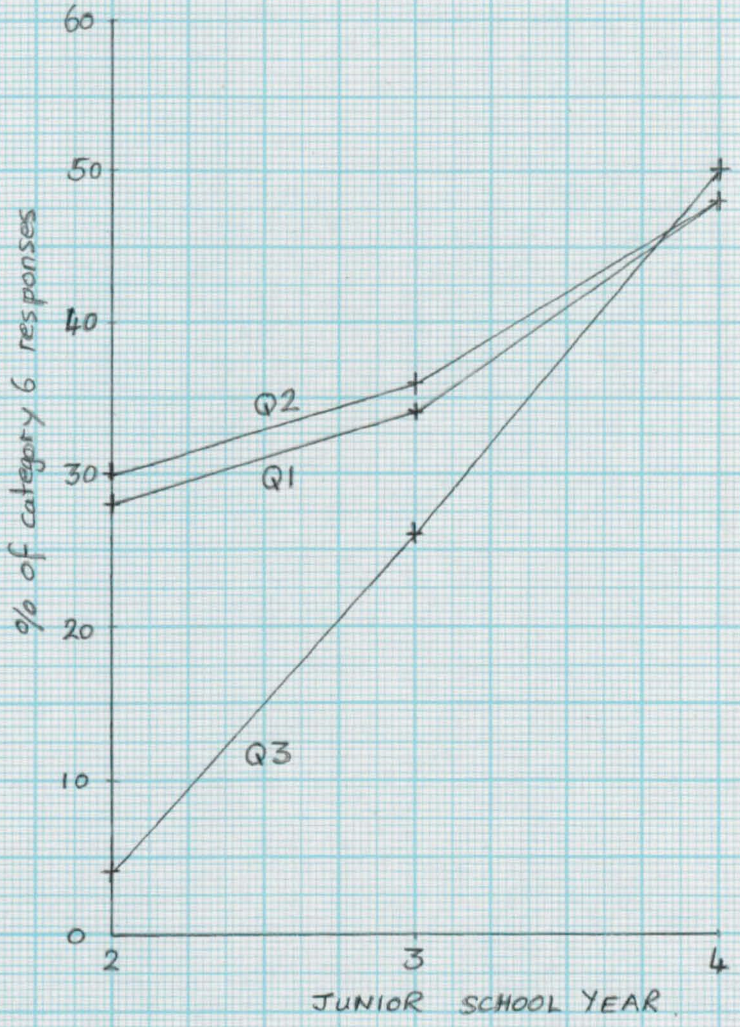
Subjects mention comparison, construction of a fair test by doing the same thing to both tennis balls.

Table 8.2 shows the frequency of use of the response categories for each of the question.

TABLE 8.2 The frequency of use of the categories for questions 1, 2 and 3

JUNIOR SCHOOL YEAR	QUESTION	N	CATEGORY 1	CATEGORY 2	CATEGORY 3	CATEGORY 4	CONTROL 5	CONTROL 6
TWO	1	(50)	6	7	3	14	6	14
	2	(50)	6	7	4	13	5	15
	3	(50)	14	13	7	12	2	2
THREE	1	(50)	5	8	7	3	10	17
	2	(50)	6	7	7	3	9	18
	3	(50)	6	6	6	10	9	13
FOUR	1	(50)	2	4	5	5	10	24
	2	(50)	1	5	5	5	10	24
	3	(50)	2	4	6	4	9	25

FIG. 8.0 A Graph to compare the % of category 6 responses on Qs 1, 2 & 3.



If categories 5 and 6 are considered control solutions then more than 50% of the subjects by the fourth year Junior School level were found to utilise a control solution, (similar to the percentage noted in Chapter 7). Category 5 solutions were obviously a form of control by negation in that the subjects' responses mentioned removing some extra bounce due to extra height or extra throw. For category 6 solutions the underlying logical reasoning was unclear.

Hypothesis one predicted that a greater proportion of early concrete operational subjects would use a control solution on question 3 when compared to questions 1 and 2. A graph showing the difference in performance on the questions for category 6 is shown in Figure 8.0. A difference between the use of control solutions on the questions was found at the second year level. F and t-tests were applied to this data and at the second year level the differences between questions 1 and 3 and questions 2 and 3 for category 6 behaviours were found to be significant at the 0.01 level. ($F_{2,147} = 4.48$; Q1 x Q3, $t_{147} = 2.95$ and Q2 x Q3, $t_{147} = 3.2$). Thus it appears that hypothesis one was confirmed. The fact that question 3, which did not allow negation by elimination, significantly affected the responses in category 6 at the early concrete level, suggests that control by negation may underlie these subjects' control solutions. This, together with the responses to question 4, tend to support hypothesis two. Table 8.3 shows those subjects using categories 5 and 6 on question 3, i.e. subjects using some form of control solution, together with their results for question 4.

TABLE 8.3 A comparison of subjects using a control solution and their understanding of the reciprocal strategy

JUNIOR YEAR	MEAN AGE YEARS	N	CATEGORY 5	CATEGORY 6	RESULTS FOR Q4	
					Reciprocal Strategy recognised as fair	Reciprocal Strategy recognised as producing a greater difference in height
2	8.8	(50)	2(4%)	2(4%)	0(0%)	0
3	9.8	(50)	9(18%)	13(26%)	4(8%)	0
4	10.7	(50)	9(18%)	25(50%)	10(20%)	0

It should be noted that no subjects using any other category of response recognised the reciprocal strategy as fair. It was obviously possible for the subjects to consider that the same height was used and therefore that the reciprocal strategy was a fair one, but the majority of the subjects found the strategy so confusing that they considered it as unfair, (90.6% of the subjects given Q3 and Q4). A typical response was the following:-

"Experiment two is incorrect, you should take away the height you gave the yellow ball then it is fair".

No subject considered that the reciprocal strategy would enable a greater difference due to the intrinsic factor of bounciness to be seen. This would suggest that the subjects were not using a reciprocal strategy in order to control the extrinsic variables but that they were using a negation strategy. Thus hypothesis two appears to be confirmed.

Discussion of Results

It is evident that a distinction can be made between concrete and formal operational control strategies. The results indicate that where the child attempts to show the effect of an intrinsic variable he is initially capable of the control of an extrinsic variable that may be more readily considered by the child to be eliminated from the situation e.g., a soft rug from a hard floor surface. Evidence was presented in the introduction to this chapter that this represents early concrete reasoning. By the late concrete operational stage (Piaget's Stage IIB), the child is capable of control of an extrinsic variable whose difference may only be cancelled e.g., height of drop. Many of these concrete control solutions, i.e. those that were classified as category 6 in Chapter 7 and the present chapter, mention making fair comparisons. It was supposed that these solutions might represent formal control solutions in that category 5 was identified as a concrete control solution and appeared to be a distinctly different response compared to category 6. But it now appears that the criterion of making fair comparisons is inadequate in the distinction between concrete and formal control solutions and that a distinction has to be made in terms of the understanding of negation and reciprocal control strategies.

CHAPTER 9

A STUDY OF THE DEVELOPMENT OF 'CONTROL OF VARIABLES' IN THE TENNIS BALL PROBLEM: - AN INTERVIEW APPROACH

Introduction

All the experimental work reported in the previous chapters, apart from the small scale Secondary School study reported in Chapter 4, relates to the development of 'control of variables' as tested by questionniare. In addition, the work described in Chapters 6, 7 and 8 has identified the tennis ball problem as being accessible to children of Junior School age, with 8% of a second year Junior School sample (mean age of 8.8 years) rising to 68% of a fourth year Junior School sample (mean age 10.7 years) being judged capable of a concrete operational level form of control methodology, (Table 8.3,Chapter 8). This chapter describes an experiment which investigates further the use of 'H' methodology on the tennis ball problem using an interview approach.

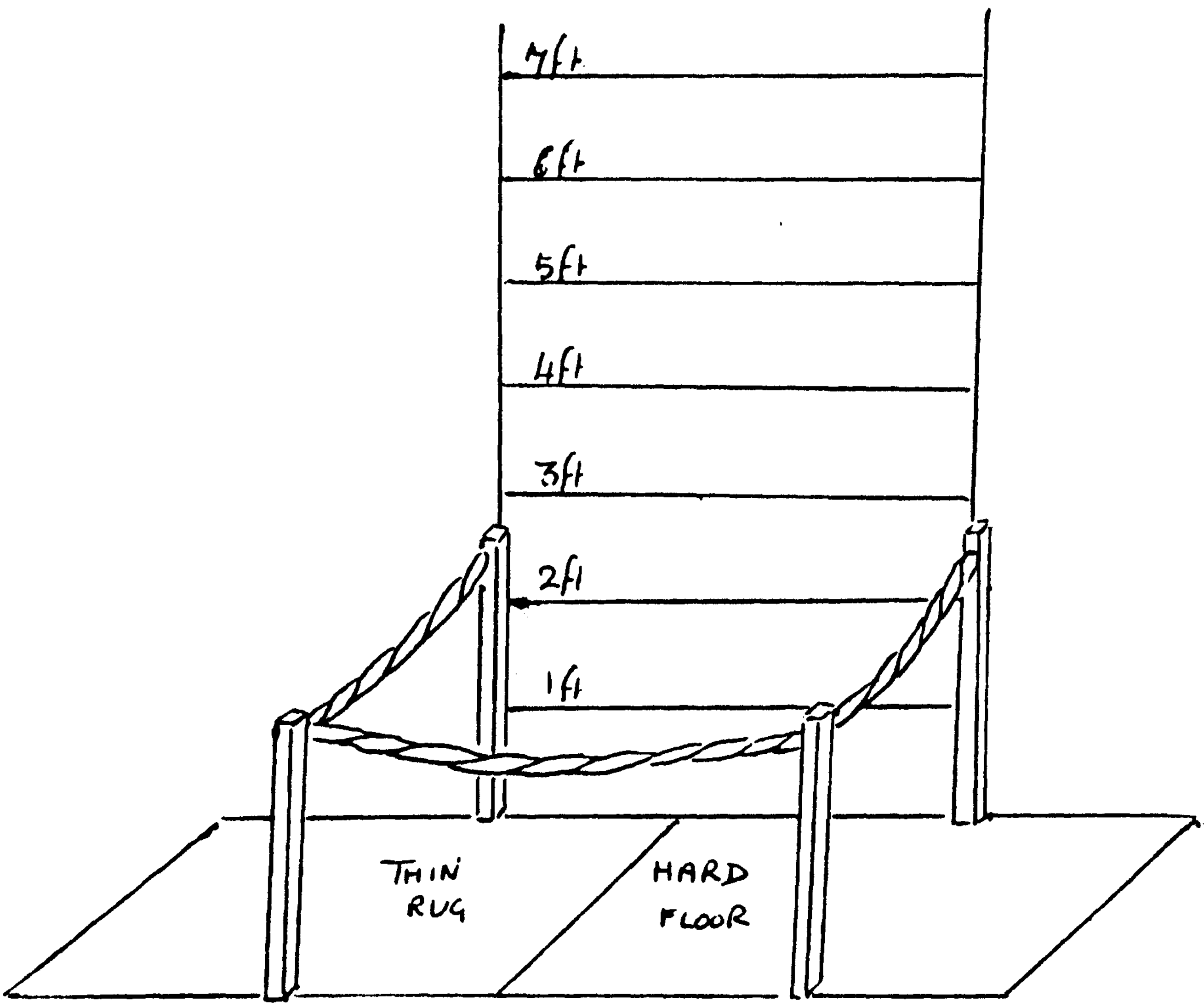
Experimental Method

Each subject was interviewed for approximately twenty minutes. The interview was recorded and the subject's exact procedure was noted by the experimenter. The interview format was as follows:-

Firstly, the effect of height at which a tennis ball was dropped and the effect of dropping a tennis ball on a hard floor surface and on a thick rug, were discussed and demonstrated. The tennis ball used in the demonstration was of a different colour to either of those used later by the subject.

Secondly, the task was described to the subject using the procedure referred to in chapter 7. The subject was then given the yellow and white tennis balls and asked to perform an experiment to find out which tennis ball was the bounciest. He was told that he could do anything he wanted with the tennis balls within the experimental area. This area, (see Figure 9.0) was divided off from the rest of the room. Half the area had a thin rug as a base and the other half a hard floor base. A scale of height in feet was attached to the wall at the rear of the experimental area. The subject was asked to stand in front of the experimental area and to perform his experiment inside the area. The experiment was designed so that the subject, not consciously considering the effect of different floor surfaces, was likely to let each ball fall onto a different surface, because he was most likely to hold each tennis ball in a separate hand. Thus the

FIGURE 9.0 THE TENNIS BALL INTERVIEW TASK; THE EXPERIMENTAL AREA



subject who intentionally controlled the floor variable would be easily identified. Each subject was questioned about his experiment in order to find out:-

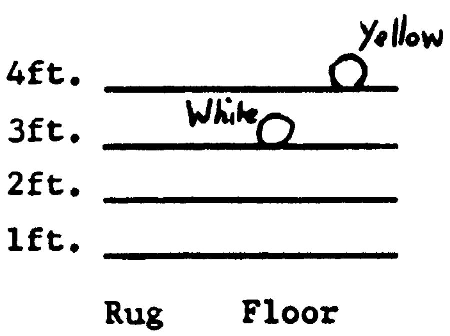
a) What he thought he actually did,
b) why he performed the experiment,
c) what the experiment actually showed.

Thirdly the subject was shown two experiments, one supposedly performed by John and one supposedly performed by Peter. John's experiment (see Figure 9.1) consisted of dropping the yellow tennis ball from four feet and the white tennis ball from three feet, both onto the hard floor surface. This is an example of the D experiment, i.e. an experiment to show, by deliberate manipulation, a difference between the tennis balls.

FIGURE 9.1

JOHN'S EXPERIMENT

(D Experimentation)

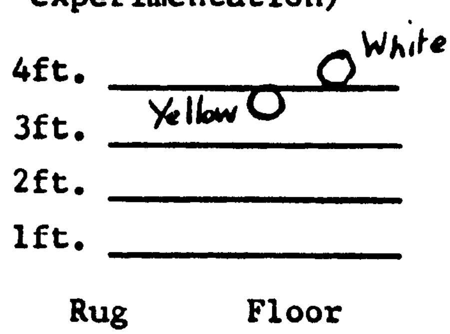


Result: Yellow bounces highest

FIGURE 9.2

PETER'S EXPERIMENT

(Test of strength experimentation)



Result: Yellow still bounces the highest.

Peter's experiment, (see Figure 9.2), consisted of dropping the white tennis ball from slightly higher than the yellow tennis ball, both dropping onto the hard floor surface. The aim of Peter's experiment was to show that the yellow tennis ball would still just win even though it was dropped from slightly lower than the white one. This is an example of the test of strength experiment, in that it tests the strength of the ability of the yellow tennis ball to bounce higher under adverse conditions. After observing each experiment the subject was asked two main questions:

Question 1: - "Do you agree with John/Peter that the experiment shows that the yellow tennis ball is the bounciest?"

Question 2: - "Do you think that the experiment was right or wrong?"

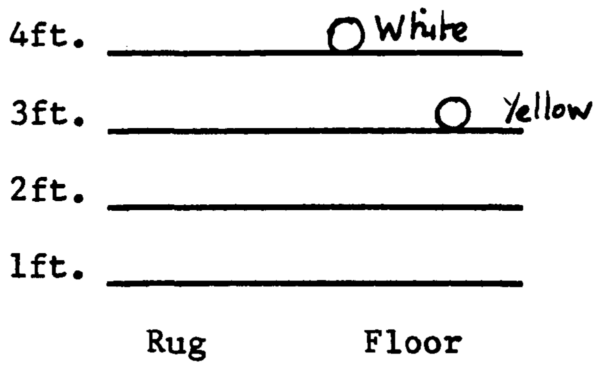
In each case the reasons behind the subject's responses were determined.

Fourthly after each subject had evaluated John's and Peter's experiments and his own, he was shown the conflict sequence. This consisted of showing the subject John's and John's reversed experiments. The latter experiment (see Figure 9.3) was the reverse of John's experiment, in that the white tennis ball was now given a height advantage.

FIGURE 9.3

JOHN'S REVERSED EXPERIMENT

(J REV) (D Experimentation)



Result: White bounces highest.

Where the subject showed ability to control the height and not the floor variable an alternative conflict sequence was used, called Paul's and Paul's reversed experiments, which consisted of alternate tennis balls winning due to dropping them onto different floor surfaces whilst controlling the height variable. The aim of these experiments was to present the subject with conflicting results due to the manipulation of one extrinsic variable, i.e. height, for those subjects not controlling this variable, and the floor surface, for those subjects controlling the height variable and not the floor variable. These procedures produced conflicting results, with the yellow and then the white tennis ball bouncing the highest. It was the intention to discover those subjects who could realise the conflict within the results of the experiments and then to discover the methods by which this conflict was resolved. It was not the aim of the study to necessarily test post hoc reasoning about the experimental results but to test the child's ability to design an experiment, in which the intrinsic effect of bounce was shown, whilst the extrinsic factors were held constant. The former aim became an essential part of the latter to provide distinctions between the early forms of reasoning, which were distinguished by the understanding of the causal relationships within the task. For example, the conflict sequences made it possible to judge if a subject was unaware of the extrinsic causal effect of height of drop within the experiments as he/she would be unable to explain why alternate tennis balls bounced higher. In addition it was possible to judge if a subject was unaware of the need for an invariant result as he/she would be unaware of the conflict involved in stating that alternate tennis balls were the bounciest.

Finally the subject was shown an 'H' experiment controlling both extrinsic variables. The aim was to determine at what level this strategy was understood.

The Subjects

126 subjects in total were interviewed. The age range tested was five to ten years, i.e., first and second year infant school pupils and first, second and third year junior school pupils. Eighteen subjects in each of the infant years and thirty subjects in each of the junior years were used.

TABLE 9.0

YEAR		MEAN AGE YEARS
INFANTS	1	5.8
"	2	6.7
JUNIOR	1	7.8
"	2	8.8
"	3	9.9

Results

Five main stages in the development of the strategy of 'holding all other things equal' were found in the age range tested.

STAGE 1 - The precausal stage. A stage in which the subjects were not aware of the distinction between the extrinsic factors and intrinsic factor of 'bounciness' or that a variation in the extrinsic factors made the balls bounce differently in the experimental situation, even when they were apparently aware of the causal relationships when discussed prior to experimentation.

STAGE 2 - The extrinsic causal stage. A stage in which the variation in the extrinsic factors of position of drop and/or floor were realised as causing the balls to bounce differently. The subjects were unaware of any variation in the instrinsic factor of 'bounciness' between the tennis balls. They did not consider the intrinsic factor at all and only considered the extrinsic factors as the cause of variation in bounce.

STAGE 3 - The extrinsic/intrinsic causal stage. A stage in which both extrinsic and intrinsic factors were realised as causing the balls to bounce differently. Thus subjects attempted some form of proof but were not able to spontaneously control any of the extrinsic factors. The test of strength experiment was often used and preferred to 'H' methodology.

STAGE 4.- The partial control of variables stage. A stage in which the subjects attempted to control the extrinsic factors. Firstly controlling height and ignoring the floor variable, secondly controlling the floor variable post hoc and thirdly controlling both extrinsic factors without realising the necessity for doing so. The majority of these subjects still considered D experimentation to be correct because it confirmed the correct result.

STAGE 5 - The control of variables stage. A stage in which subjects were spontaneously aware of both extrinsic causal factors and controlled them. They saw the necessity for the control of the extrinsic variables in order to examine the variation due to the intrinsic variable. These subjects still considered Peter's experiment (the test of strength experiment) to be correct.

These five stages were formulated by considering the subjects' total performance on all the problems, John's, Peter's and the subjects' own experiments, together with their performance on the conflict situations and 'H' methodology. The role of the conflict sequence in the analysis will be described as it relates to particular distinctions between behaviours. Table 9.1 shows the fequency of subjects at each stage.

TABLE 9.1

Evaluation of performance on the tennis ball interviews.

		INFANTS		JUNIORS		
		1ST YEAR	2ND YEAR	1ST YEAR	2ND YEAR	3RD YEAR
MEAN AGE		5.8. yrs.	6.7 yrs.	7.8 yrs.	8.8yrs.	9.9 yrs.
STAGE		N=18	N=18	N=30	N=30	N=30
PRECAUSAL	1	7(38.8)	3(16.7)		0	0
CAUSAL EXTRINSIC	2	11(61.1)	13(72.2)	16(53.3)	4(13.3)	7(23.3)
CAUSAL EXTRINSIC/ INTRINSIC	3	0	2(11.1)	5(16.7)	6(20.0)	0
PARTIAL CONTROL OF VARIABLES	4	0	0	8(26.7)	14(46.7)	12(40.0)
CONTROL OF VARIABLES	5	0	0	1(3.3)	6(20.0)	11(36.7)

(percentages shown in parenthesis are rounded up to 1 place of decimals).

The results for the analysis of variance of the chronological ages at which stages occurred are summarised in Table 9.2 below, the F test being significant at the 0.01 level.

TABLE 9.2 Analysis of variance of chronological age and stage

Source of Variation	Sum of squares	Degrees of Freedom	Mean Square
Between groups	16879.1	4	4219.78
Within groups	18036.4	121	149.06
Total	34915.5	125	

F Ratio (4,121) = 28.31, $p < 0.01$

TABLE 9.3 Means and Standard Deviations for the chronological ages at which stages occurred (t-test values for consecutive means)

	STAGE 1	2	3	4	5
Mean age in months	71.3	87.9	95.5	105.8	111.3
Standard Deviation	5.3	15.9	9.3	9.7	7.9
t-test values	(S1xS2) $t_{59}=3.24$**	(S2xS3) $t_{62}=1.66$ ns	(S3xS4) $t_{45}=3.28$**	(S4xS5) $t_{50}=2.09$**	

n.s. = not significant $p > 0.05$
* = difference significant at the 0.05 level
** = difference significant at the 0.01 level

The data in Table 9.3 indicates a trend of increasing chronological age with stage. t-tests were applied to consecutive means and the values are also shown in Table 9.3.

The only difference between consecutive means found not to be significant ($p > 0.05$) was that between stages 2 and 3 . This lack of significance can be explained by the fact that subjects at stages 2 and 3 often used similar experimental methods but applied different reasoning to the experimentation e.g., the test of strength experiment has at its basis D experimentation, which was also used by many subjects at stage 2.2. The realisation of the role of intrinsic variables within the situation at stage 3 did not result in the spontaneous use of a new methodology to show its effect. In addition, several subjects at stage 2 initially exhibited behaviour similar to stage 3, yet were unaware of the role of the intrinsic variable. The following detailed discussion of the stages provides evidence of the novel reasoning at stage 3, compared to stage 2, as it illustrates further the acquisitions of the stages.

Each stage was found to contain distinctly different categories of behaviour and Table 9.4 shows the criteria. used to identify each stage and category of behaviour and these together with their major characteristics will be discussed below. Table 9.4 is included at the end of the chapter and may be opened out so that it may be referred to more easily. The protocols for each behaviour are included in Appendix G and it is advised that they are read in conjunction with the following,where they are specifically referred to by means of an asterisk.

<u>STAGE 1</u> - <u>The Precausal Stage (N=10)</u>. This stage was subdivided into two separate categories of behaviour. The following were the major characteristics of subjects at stage 1.1 (N=3).

(a) There was no logical experimentation other than "trying the tennis balls out".

(b) The majority of subjects made an intuitive judgment concerning which tennis ball was the bounciest.

(c) There was no explanation of differing experimental results in terms of the operation of extrinsic causal factors, (criterion 2).

Stage 1.2 (N=7) was distinguished from stage 1.1 by characteristic (d).

(d) Subjects realised the necessity of an invariant result, i.e. only one tennis ball was considered to be the bounciest, (criterion 1).

The conflict sequence was crucial in determining characteristic (c) as a consistent result throughout the sequence of experiments was a common occurrence. Younger subjects when questioned after an experiment, as to why the tennis ball bounced highest, would often produce answers that were difficult to classify. The aim was to identify behaviours on the basis of criterion 2 and the conflict sequence was an alternative to leading questions such as, "did the yellow one bounce the highest because it was dropped from higher?" If a subject did not understand the causal relationship between height of drop and height bounced then he could not explain why differing results occurred in the conflict sequence. This conflict sequence also enabled the distinction to be made between stage 1.1 and 1.2 behaviour, as this presented the subjects with a variance in results.

Subjects classified as stage 1.1 appeared to accept the differing results of the two experiments, not understanding why they occurred or the need for invariance. When asked to state which tennis ball was the bounciest the subjects would refer to the immediately preceeding result, (*see the conflict sequence of stage 1.1, protocol BAR).

At stage 1.2 subjects realised the conflict involved in differing results and resolved this by choosing the tennis ball that they judged to bounce the highest throughout all the experiments, (stage 1.2 protocol NEV).

<u>STAGE 2</u> - <u>The Extrinsic Causal Stage (N=51)</u>. The major characteristics of this stage were the following:

(a) Subjects were aware of the need for an invariant result, (criterion 1).

(b) Subjects were aware of the operation of extrinsic causal factors, i.e. they knew the reason for the conflicting results in the conflict sequence (criterion 2).

(c) Subjects increasingly applied their knowledge concerning the operation of causal factors to their experimentation, noting the reason for the occurrence of a certain result as well as the result itself.

The four categories of behaviour found were distinguished by the level to which the subjects could apply their understanding of the role of the extrinsic causal factors. The subjects progressing from unsystematic experimentation, through D experimentation to finally G.D. experimentation. The additional characteristics of the categories were as follows:

Stage 2.1 (N=12) - BN(Bounce) Experimentation

(i) Subjects made no systematic experimentation other than 'trying the tennis balls out', but all subjects concentrated upon the height at which they dropped the tennis balls.

(ii) Subjects appeared to make no initial judgment other than that the result should be invariant. This lead to the conclusion that one tennis ball must be bouncier and thus equal bounciness was considered incorrect.

(iii) Subjects only stated the role of extrinsic factors after the conflict sequence or when their experimentation produced a variance in results. (*see the conflict sequence stage 2.1 protocol MIS).

Stage 2.2 (N=8) - The transition to D experimentation.

i) Through performing his/her initial unsystematic BN experiment the subject became aware of the role of the extrinsic factor of height. The subject may become aware of this role of position of drop due to the fact that the expected result was not confirmed, i.e. his/her initial judgment and the experimental result conflicted (stage 2.2 protocol MAN). Four subjects became aware of the role of position of drop by reflection upon their experimentation. (Stage 2.2 protocol BOS).

(ii) The subjects made a judgment as to which tennis ball was the bounciest related to the role of the extrinsic variable, i.e. the yellow tennis ball was the bounciest because it was dropped from a higher position. The subjects were able to perform D experimentation to confirm their result when asked to re-perform their experiment, (Criterion 3a).

MAN illustrates the confusion evidenced by all subjects at stage 2. The subjects only understood the operation of extrinsic factors and thus the same tennis ball dropped from different heights was described as two different tennis balls, i.e. one dropped from a high position was described as a better tennis ball than one dropped from a low position.

Stage 2.3 (N=25) - D experimentation.

(i) The subjects made an initial judgment as to which tennis ball was the bounciest due to intuition, i.e. it looks better, newer or harder etc.

(ii) The subjects performed a D experiment in order to confirm their initial judgment, i.e. they were able to anticipate the effect of one extrinsic variable prior to experimentation, (criterion 3b).

(iii) The experiment performed was typically:-

The yellow tennis ball was dropped from high onto either floor surface.

The white tennis ball was dropped from low onto either floor surface.

The subjects failed to anticipate the effect of the floor surface variable. Therefore the yellow tennis ball did not always bounce the highest if it was bounced onto the rug, and often bounced lower than expected.

All the subjects at stage 2.3 were capable of understanding the role of the floor variable when their experimental result conflicted with their initial judgment or when they were presented with the alternative conflict sequence. Six subjects went on to perform a G.D. experiment (see protocol WOO), 4 performed an experiment that excluded the surface variable (the rug was eliminated) and 2 used a test of strength argument. They all anticipated the effect of the floor variable post-hoc (criterion 4a) and represent transitional behaviours.

Stage 2.4 (N=6)

(i) An initial judgment was made by all subjects and then, by anticipating the role of both of the extrinsic variables, they were able to perform a G.D. experiment. (Stage 2.4 protocol MAL). (Criterion 4b).

STAGE 3 (N=13) - The Extrinsic/Intrinsic Stage. Stage 3 marked the beginning of a search for proof in terms of intrinsic factors. The two categories of behaviour at this stage were characterised by the following:

a) The subjects were aware of the need for an invariant result. (Criterion 1).

b) The subjects anticipated the operation of one extrinsic and an intrinsic causal factor. (Criteria 3b and 5).

c) The subjects were capable of anticipating the effect of the second extrinsic variable (usually the floor variable) post-hoc. (Criterion 4a).

d) The subjects were capable of understanding H experimentation but rejected this methodology when the experimental result conflicted with their judgment. (Criterion 11).

Stage 3.1 (N=8)

(i) The subjects at this stage were not capable of spontaneously controlling the extrinsic variables, but used the test of strength experiment to show the effect of the intrinsic factor of bounce. (Criterion 7). The floor variable was ignored and when this led to an unexpected result, the floor variable was controlled post-hoc in a further test of strength experiment. All subjects when guided through the conflict sequence, understood that one extrinsic variable had to be controlled, but they preferred the test of strength experiment with the floor variable controlled, to an experiment that controlled both extrinsic variables. (Criterion 9a).

Stage 3.2 (N=5)

(i) The subjects understood the test of strength experiment, (criterion 6), but performed a 'D' experiment. Subjects ignored the floor variable and where this produced an unexpected result, i.e. the subjects' initial judgment was not confirmed by the experimental result, they spontaneously performed a new experiment controlling the floor variable. Two subjects were able to spontaneously use a second D experiment that controlled floor surface and that used the same position of drop for the 'worst' tennis ball as that used for the 'best' tennis ball in the first experiment. 'Control' comparisons for the tennis balls were then made. These subjects could not perform a single H experiment. This behaviour also occurred for one subject when he was shown the conflict sequence. All subjects understood the necessity to control at least one variable after being shown the conflict sequence, but became confused when attempting to apply this to both the extrinsic variables, to show the intrinsic variable of bounce. (Criterion 9a).

STAGE 4 - The Partial Control of Variables Stage (N=34).

The distinguishing features of this stage were the following:

a) All the subjects spontaneously controlled the height variable (criterion 9b). The reason for the preference for the height variable over the floor variable was because the subjects had to overtly consider which height they had to drop the tennis ball from in order to perform any experiment.

b) The two categories at this stage were distinguished by the degree to which the subjects were capable of controlling the floor variable post-hoc (criterion 10a). At stage 4.1 no subject was capable of controlling the floor variable post-hoc. At Stage 4.2 all the subjects were capable of controlling the floor variable post-hoc.

For both categories of behaviour there were specific differences in the initial experimental methods utilised by the subjects. These were:

EXPERIMENT ONE.

The subject dropped the yellow tennis ball onto the floor and the white tennis ball onto the rug, both from the same height. The yellow tennis ball bounced the highest.

EXPERIMENT TWO.

The subjects dropped the white tennis ball onto the floor and the yellow tennis ball onto the rug, both from the same height. The white tennis ball bounced the highest.

EXPERIMENT THREE.

The subjects performed both experiments.

Table 9.5 shows the number of subjects using the different experiments for each category.

TABLE 9.5

	EXPERIMENTAL METHOD		
STAGE	1	2	3
4.1	7	2	3
4.2	14	5	3

In addition for both categories of behaviour there were four distinct levels apparent in the subjects' ability to control the floor variable.

LEVEL 1. Where the subject realised the effect of the different floor surfaces, but could not control his/her experiment for this variable.

LEVEL 2. Where the subject realised the effect of different floor surfaces in his/her experiment after the alternative conflict sequence was shown to him/her and eventually controlled the floor variable.

LEVEL 3. Where the subject realised the effect after his/her method was questioned by the experimenter and controlled for it.

LEVEL 4. Where the subject spontaneously realised his/her mistake concerning the floor variable and eventually controlled this variable.

Table 9.6 shows the relation between the level of prompting required before the subject realised the effect of the floor surface on his experiment and, in the case of levels 2, 3 and 4, controlled this variable.

TABLE 9.6 Level of guidance required before the subjects controlled the floor variable, compared to the experimental method used.

		EXPERIMENTAL METHOD		
	LEVEL	1	2	3
Stage 4.1	1	7	2	3
Stage 4.2	2	5	0	0
Stage 4.2	3	4	0	0
Stage 4.2	4	5	5	3

Stage 4.1 (N=12)

At stage 4.1 the subjects did not control the floor variable and thus all these subjects were at level 1. Subjects using experimental methods 1 and 2 ignored the effect of the floor variable. All of the subjects at stage 4.1 judged the yellow tennis ball to be the bounciest. In experiment one the yellow tennis ball bounced the highest thus the judgment and the experimental result were not in conflict. The subjects did not realise that they had manipulated the experiment so that the yellow tennis ball bounced higher until the alternative conflict sequence was shown to them. This realisation only succeeded in producing confusion in the subjects' minds as to what to do with the floor variable.

Those subjects using experimental method two were presented with an experimental result that conflicted with their judgment. They immediately realised the effect of the floor variable but they appeared unable to control this variable. One subject (*see stage 4.1 protocol SMI), after being presented with the alternative conflict sequence progressed to experimental method 3.

Experimental method three resulted from the subjects spontaneously realising that the floor variable contributed to the difference between the tennis balls shown in their first experiment. The subjects performed experiment two initially, noted that the white tennis ball bounced highest due to it falling onto the floor while the yellow tennis ball fell onto the rug and then performed experiment one. They compared the bounce of the yellow tennis ball dropped onto the floor from experiment one with the bounce of the white tennis ball dropped onto the floor from experiment two. In order to control the height variable between the two experiments, two of the subjects made careful measurements. Not one of these subjects arrived at a single experiment controlling both extrinsic variables. (stage 4.1, level 1, protocol HAV).

Stage 4.2 (N=22)

At this stage all the subjects controlled the floor variable after their initial experiment, which ignored the floor variable. The eight subjects who needed guidance before they controlled the floor variable, i.e. levels 2 and 3, apparently needed this prompting because their experimental result confirmed their initial judgment. All these subjects performed experiment one which resulted in the yellow tennis ball bouncing the highest and all these subjects judged this to be the best tennis ball. The five subjects at level 4 who performed experiment one initially apparently needed no guidance in order to perform a subsequently complete 'H' experiment. These subjects obviously had available a more fully developed 'H' scheme e.g.,

CLI 94 mths. "Actually I bounced them on the floor, if I didn't I meant to, its unfair if you don't do the same thing to both."

JOW 120 mths. "The yellow one bounced higher because it bounced on the floor. I don't know which is the bounciest, I made a mistake, I meant to do them both the same".

The five subjects who used experimental method two at level 4 all spontaneously used an 'H' methodology, subsequently all these subjects expected the yellow tennis ball to win and because their experimental result and their judgment conflicted they noted their mistake, i.e. not controlling the floor variable. The fact that these subjects were presented with the experimental result/judgment conflict within their experiment, appears to be the reason that they did not require any guidance. Once subjects had an 'H' scheme available that could control the two extrinsic variables, any mistake that was made could be highlighted by an experimental result/judgment conflict and hence easily resolved. It appears therefore that stage 4.1 subjects did not have such a scheme available. It is doubtful whether all subjects at stage 4.2 have such a scheme available, in that those subjects who used experimental method three, upon realising the experimental result/judgment conflict, did not immediately perform an 'H' experiment. These three subjects' behaviours appear to be transitional, i.e. between the use of an 'H' scheme that could control one extrinsic variable and an 'H' scheme that could control two extrinsic variables. As at stage 4.1 two of the subjects who used experimental method three performed experiment two initially, expecting the yellow tennis ball to bounce the highest. The experimental result/judgment conflict led to the subjects performing experiment one and comparing the correct controlled elements from two temporally separated experiments, i.e. yellow dropped onto the floor from experiment one, white dropped onto the floor from experiment two. One of these subjects made a careful note of the heights he dropped the tennis balls from, so that fair comparisons could be made between the two experiments. The one remaining subject operating at level 4, performed experiment one first and found that the yellow tennis ball bounced highest. Although this confirmed his initial judgment, he noted that he had not controlled the floor variable and proceeded to perform experiment two. After he had compared the correct controlled elements he proceeded to perform a complete 'H' experiment. The new behaviour at stage 4.2 is the realisation that such a temporally separated experimental comparison is not as accurate or as simple as an 'H' experiment. Two of these subjects at level 4 performed spontaneously 'H' experimentation, controlling both extrinsic variables.

These stage 4 behaviours offer an insight into the development of the 'H' scheme from its limited application to just the height variable at stage 4.1 in experiments one and two, to its application to both the height and floor variables at stage 4.2. The transition was an application of the 'H' scheme to the height variable in two temporally separated experiments followed by the application of the 'H' scheme to the floor variable post-hoc so that fair comparisons could be made. The three protocols representing stage 4 behaviours have been chosen to illustrate specifically this transition behaviour. SMI, as previously mentioned, was the one pupil at stage 4.1 behaviour who used experimental method two initially but progressed to experimental method three by the end of the interview. The other two protocols are of experimental method three behaviours at stage 4.1 and stage 4.2.

STAGE 5 - The Control of Variables Stage (N=18). The following was the major characteristic of subjects at stage 5.

a) Subjects controlled both the floor and height variables spontaneously and thus performed an 'H' experiment, (criterion 10). The reasons given stated the need to show the effect of the intrinsic variable of bounce under 'fair' conditions.

The subjects' ability to judge only an 'H' experiment as correct distinguished the two categories of behaviour at this stage.

Stage 5.1 (N=8)

(i) Subjects often accepted John's experiment as correct failing to realise that an experiment must use an 'H' methodology not just confirm the initial judgment made (criterion 8a).

(ii) Subjects rejected a controlled experiment when the result conflicted with their original judgment, i.e. when the tennis balls bounced to about the same height (criterion 11).

Stage 5.2 (N=10)

(i) Subjects rejected any unfair experiment, such as John's even when the experiment confirmed the initial judgment (criterion 8a).

(ii) Subjects accepted any experiment that controlled both extrinsic variables (criterion 11). Their decisions were made in terms of methodology used and were not made with regard to confirming a judgment.

Subjects at stage 5.1 were still concerned with confirming their initial judgment and thus controlled experimentation was still not seen as a necessity even though subjects spontaneously used this method. At stage 5.2 subjects were less concerned with whether an experiment confirmed their initial judgment than whether it utilised an 'H' methodology. These subjects were utilising a 'fair' criterion for acceptance or rejection of an experiment. Thus they were able to reject the test of strength approach on these grounds (criterion 6), the test of strength approach being a fair experiment, but not a 'scientific' approach.

Table 9.7 shows the number of subjects, for each year tested at the stages described.

Table 9.7

Frequency of subjects at each stage and category of behaviour in the development of control experimentation

STAGE AND CATEGORY OF BEHAVIOUR

		Pre-causal		Causal Extrinsic				Causal Extrinsic/ Intrinsic		Partial Control		Control	
YEAR	N	1.1	1.2	2.1	2.2	2.3	2.4	3.1	3.2	4.1	4.2	5.1	5.2
1st year Infant	18	3	4	5	3	3	0	0	0	0	0	0	0
2nd year Infant	18	0	3	4	3	6	0	1	1	0	0	0	0
1st year Junior	30	0	0	3	1	11	1	5	0	2	6	1	0
2nd year Junior	30	0	0	0	0	2	2	2	4	6	8	3	3
3rd year Junior	30	0	0	0	1	3	3	0	0	4	8	4	7

Summary

It was the intention of this study to note the behavioural stages in the development of the H strategy. It was found that eight 8 year old subjects were capable of a partial control strategy and showed evidence of the realisation of extrinsic/intrinsic factors within the experimental situation. In fact two 7 year old subjects were aware of the operation of both factors and one of these was able to spontaneously 'control' one variable. These results support the results found in Chapter 6, 7 and 8, using the questionnaire technique, for the acquisition of an H strategy at the Junior School level.

Three critical realisations which resulted in major changes in experimental behaviour were identififed and they are;

(i) the realisation of invariance between results at stage 1.2 which appeared to be crucial in the development of the realisation that experimentation was necessary;

(ii) the realisation of the role of extrinsic factors at stage 2.1, which provided an opportunity for the subject to produce consistent results by verifying their judgments;

(iii) the realisation of the role of intrinsic/extrinsic factors within the experimental situation, at stage 3, which instigated a search for a method to show the intrinsic factor's effect.

It appears that due to the interactive nature of the procedure used examples of transitional behaviours were observed, many of which have been described in this chapter. The majority of these changes in behaviour occurred spontaneously due to conflicts between the subject's judgments and their experimental results, others were transitions primed by the experimenter in that they only occurred when one of the conflict sequences was shown to the subjects. In order to describe these transitional behaviours more fully and to gain further understanding of the development of the H strategy it became necessary to precisely model these behaviours and an attempt to do this is described in Chapter 10.

TABLE 9.4 THE CRITERIA USED TO IDENTIFY THE STAGES OF BEHAVIOUR IN THE DEVELOPMENT OF CONTROL EXPERIMENTATION IN JUNIOR SCHOOL CHILDREN

NO.	CRITERIA		1.1	1.2	2.1	2.2	2.3	2.4	3.1	3.2	4.1	4.2	5.1	5.2
1	Was aware of the need for an INVARIANT result.		-	+	+	+	+	+	+	+	+	+	+	+
2	Was aware of the operation of extrinsic causal factors		-	-	+	+	+	+	+	+	+	+	+	+
3	Anticipated the effect of ONE EXTRINSIC VARIABLE.	a) Post-hoc			-	+								
		b) Ante-hoc			-	-	+	+	+	+	+	+	+	+
4	Anticipated the effect of TWO EXTRINSIC VARIABLES.	a) Post-hoc					+/-	+	+	+	+	+		
		b) Ante-hoc					-	+	+/-	+/-	+/-	+/-	+	+
5	Was aware of the operation of INTRINSIC CAUSAL Factors.							-	+	+	+	+	+	+
6	Accepted the Test of Strength argument.							-	+	+	+	+	+	+
7	Performed and preferred the Test of Strength experiment.								+					
8	Accepted a manipulative experiment if the result ...	a) Confirmed Judgment		-	+	+	+	+	+	+	+/-	+/-	+	-
		b) Conflicted with Judgment		-	+	+	+/-	+/-	-	-	-	-	-	-
9	Controlled one Variable	a) Post-hoc							+	+				
		b) Ante-hoc									+	+		
10	Controlled Two Variables	a) Post-hoc Guided:									+			
		Post-hoc Spontaneous:										+		
		b) Ante-hoc											+	+
11	Understood M experimentation but might reject it if the result was not considered to be accurate.								+	+	+	+	+	-

STAGES OF BEHAVIOUR: columns 1.1–5.2

CHAPTER 10 *

A DESCRIPTION OF THE STRATEGIES NOTED IN THE DEVELOPMENT OF THE 'CONTROL OF VARIABLES' STRATEGY, A NEO-PIAGETIAN APPROACH

Chapter 9 has described in detail the five stages that were noted in the development of the control strategy. The description, like Piaget's theory of intellectual development, was essentially a structural one rather than a dynamic or purely functional one, in that stages of competence were identified rather than the precise nature of the transitions between the stages, although points of transition were identified. This approach was essential initially to identify the acquisitions made at each stage, i.e. those underlying the functional aspects of the behaviours, the strategies, (see Table 9.4). A functional neo-Piagetian theory has been described by Pascual-Leone (1969), and has been shown to be capable of generating performance models for Piagetian tasks including control of variables (Case 1974b). It is the aim of the present chapter to identify the strategies executed by the subjects within the five stages, i.e. the static functional aspects of the development, and to describe in detail the performance of these strategies following the methods used by Noelting (1975) and Case (1974b). In addition a set of procedures are then set up to facilitate the description of the transition between these strategies, i.e. the formation of new strategies, the dynamic aspects of the development.

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* a summary of the notation used in chapters 10 & 11 is to be found in table 10.5. This table is located on the inside rear cover of the thesis and should be opened out when reading these chapters.

The information processing approach to be used in this study represents an attempt to embody the Piagetian notion of levels of operativity, in that qualitative and quantitative aspects of the use of strategies are considered. Much of the early work by McLaughlin (1963), Pascual-Leone (1969) and Case (1974b) has been primarily concerned with the quantitative aspect, i.e. counting the 'number of items' or 'chunks' of information (Miller, 1956) that must be held simultaneously in working memory in order to perform the strategy at a particular stage. Pascual-Leone postulates that the amount of processing, or total M space available, increases linearly with age, see Table 10.0.

TABLE 10.0 Pascual Leone's postulated M space developmental scale

Age	Developmental substage	M Space
3-4	Early preoperations	p + 1
5-6	Late preoperations	p + 2
7-8	Early concrete operations	p + 3
9-10	Middle concrete operations	p + 4
11-12	Late concrete-early formal Operations	p + 5
13-14	Middle formal operations	p + 6
15-16	Late formal operations	p + 7

The constant p refers to the mental effort required to activate an overlearned plan or strategy or what is referred to as an executive scheme by Pascual-Leone. There has been much evidence presented to support this linear increase in

M space, (Parkinson, 1976; Pascual-Leone, 1970; Case, 1972, 1974b; Scardamalia, 1973, 1974). Nevertheless, as Linn states

> "an operational definition of information load (M space) does not really exist and methods for assessing processing capacity are not well developed". (Linn, 1977, P366).

Although this quantitative approach has been utilised successfully to describe young children's behaviours, it becomes difficult to justify an M space value of greater than 4 or 5 units for the strategies described in the literature to date, in that 'chunking' of individual items can be applied to reduce the M demand. Case (1974b), has attempted to integrate the qualitative and quantitative aspects of development, i.e. the acquisition or application of new operations to a strategy. He has applied his procedures to a model of a concrete operational proof or control strategy. He describes the operations underlying the execution of the strategy and those underlying a conflict situation that could possibly lead to the construction of the control strategy. Nevertheless, these models of behaviour are not truly dynamic in that they do not incorporate a functional procedure that enables the description of the initial acquisition of the strategy. It appears that the procedures used by Case may only be applied to strategies already acquired as they have no functional procedure for considering arguments resulting from conflict due to the application of a strategy, a necessary feature if a model of behaviour is to be interactive and hence dynamic.

Case (1978) has proposed a new interpretation of M space in an attempt to incorporate the qualitative changes or underlying structural changes that occur at each major stage and to incorporate evidence of the necessity for an increasing M space within a stage, up to a maximum of p+ 4/5. Case proposes that at each of the Piagetian stages, i.e. sensorimotor, preoperational, concrete and formal, there can be identified an almost parallel developmental sequence of four substages.* Each successive substage requires an additional single unit of working memory reaching a maximum of four at the end of a stage. The size of M is due to the plan plus working memory and is not considered to actually change in overall size, and the reason for the apparent change is attributed to the decreasing attentional capacity required as the basic operations at each stage become automatised. The executive, which has the same function as a plan first referred to by Miller, Galanter and Pribram (1960), is considered to increase in sophistication at each substage, but to change distinctly between each stage as the underlying operations change. Case postulates that the succession of substages stems from a succession of qualitatively distinct control structures or plans and that these can be modelled by computer simulation rather than as logico mathematical systems modelled by symbolic logic. For example, consider the following for the concrete operational stage:

$$M = {}^{c1}(p1) + 1$$
$$M = {}^{c2}(p2) + 2$$
$$M = {}^{c3}(p3) + 3$$
$$M = {}^{c4}(p4) + 4$$

p1 etc = 4 executives or plans

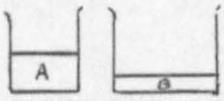
c1 etc = level of concrete operational automaticity.

*Note that Fischer's skill theory (1980) explains development as a recurring cycle of 4 levels which are applied to sensory motor, representational and abstract sets.

M apparently increases but Case considers that c4 requires less attentional capacity due to the automatisation of these operations. Hence M is not considered to increase in size being simply the portion of M available for working memory that increases.

At each stage Case provides evidence for the four substages utilising increasingly complex plans. The four substages and hence plans considered to develop during the concrete operational stage, first identified by Noelting (1975), may be identified in the stages in the development of the H strategy in the tennis ball task noted in Chapter 9. The four concrete operational substages described by Noelting are presented in Table 10.1 with the strategies used on two Piagetian tasks (Case, 1978) and on the tennis balls task. The mean ages for the tennis ball task exclude those subjects in transition between substages. There is a discrepancy between the mean ages given by Noelting (1975) and those found for the four substages for the tennis ball task. The restricted age range of the Junior School sample could explain the higher mean ages noted for substage I and substage II for the tennis ball tasks. Noelting's substages I to III appear to match closely the single substages

TABLE 10.1 Strategies observed on Piagetian Tasks and the Tennis Ball Task During the period of Concrete Operations

	TASKS				
	Pick beaker with more water 	Place stick of given size in its place in a previously constructed series. 	Prove which tennis ball is the bounciest (when position of drop and floor surface may differ)	TENNIS BALL TASK	
Noelting's Substage	STRATEGIES			JOYES' STAGE	MEAN AGE
I Isolated Centration 3 to 4½ years	Notice global appearance of water column. If it appears large, say it has more.	Decide whether stick is big or little, place it at big end or little end. (Isolated centration).	Decide on salient feature associated with bounciest, i.e. if it appears bouncy say it is. (INVARIANCE BETWEEN RESULTS NOT NOTED).	1.1	5.8
II Unidimensional Comparison 4½ to 6 years	Make careful comparison of height. Pick higher water colum as having more. (May be applied to diameter) $A_V > B_V$ if $A_H > B_H$ (A is greater volume than B if its height is greater).	Compare height of two sticks. Higher stick is one on right for each placement. Right $> X$ if $R_H > X_H$ (The right one is greater than X if its height is greater).	Bounce the tennis balls, compare the height they bounce. Choose the one that bounces the highest as the bounciest $A > B$ if $A_H > B_H$ (A is greater than B if its height is greater).	2.1	6.4
III Bidimensional Comparison 7 to 8 years	Notice both height difference and diameter difference between beakers. Attempt qualitatsive compensation. $A_H > B_H$ & $A_D < B_D$ may mean $A_V = B_V$ OR $A_H \gg B_H$ & $A_D < B_D$ may mean $A_V > B_V$	Compare height of stick to left and to right before placement. Pick spot where it is bigger than left, smaller than right. $R > X$ if $R_H > X_H$ $X > L$ if $X_H > L_H$ $\therefore R > X > L$	Anticipate the extrinsic effect of position of bounce on height of bounce. $A_H > B_H$ if $A_P > B_P$ Combine this effect with choosing the one that bounces the highest. $A > B$ if $A_H > B_H$ hence bounce A from higher than B. $A > B$ if $A_P > B_P$	2.3	7.5
IV Bidimensional Comparison with addition of third dimension 9 to 10 years	Notice both height and diameter differences. Make quantification compensation.		Anticipate a) $A_H > B_H$ if $A_{P+S} > B_{P+S}$ (extrinsic factors) b) $A_H > B_H$ if $A_B > B_B$ (intrinsic factor) c) $A > B$ if $A_H > B_H$ hence NOT $(A_{P+S} > B_{P+S})$ from (b) and (c) leaves $A > B$ if $A_B > B_B$	4&5	9

noted in the tennis ball task, with substages (1.2, 2.2, 2.4, 3.1, 3.2) marking the transitions. Noelting's substage IV may match all of the strategies that successfully anticipated the confounding effects of at least one extrinsic variable and the instrinsic variable. The correspondence of the above substages with Noelting's substages I to IV may be justified by simply considering the number of dimensions implied by the use of a strategy or by noting the number of items that the subject must hold in working memory at each step of thinking in order to perform a strategy. Initially consider the number of dimensions implied by the use of a strategy.

At stage 1.1 subjects were not aware of any conflict between invariant results (criterion 1). This strategy is one of isolated centration, Noelting's substage I. Stage 2.1 subjects were aware of the need for invariant results, i.e. comparisons between two events on one dimension were made, (substage II). In addition, the subjects used an experimental method that involved 'trying the tennis balls out' and noting the difference in heights of bounce. This B experimentation or B strategy is a comparison of the tennis balls on the single dimension of height of bounce. The subjects predicted that tennis ball A was bouncier than tennis ball B if tennis ball A bounced higher than tennis ball B, i.e. $A > B$ if $A_H > B_H$. Stage 2.3 subjects used D experimentation. They anticipated the effect of position of drop on the height of bounce (criterion 3), i.e. $A_H > B_H$ if $A_P > B_P$ and combined this with the comparison made at substage II to predict that $A > B$ if $Ap > Bp$. The use of bidimensional comparisons, <u>substage III</u> is apparent. At <u>substage IV</u> an additional dimension is included; this dimension appears to be the realisation of the role of intrinsic factors within the experiment (criterion 5). Thus, at substage IV, the single dimension of height of bounce is again attended to.

<u>The first dimension is thus</u>:

$A > B$ if $A_H > B_H$ (relation between the best tennis ball and the <u>height</u> bounced, denoted by subscript H).

This first dimension is combined with the second dimension, thus one or more of the extrinsic relations are attended to, as at substage III.

<u>Examples of the second dimension are</u>:

$A_H > B_H$ if $Ap > Bp$ (relation between the height of bounce and <u>position</u> of drop, denoted by subscript p).

$A_H > B_H$ if $A_s > B_s$ (relation between the height of bounce and the floor <u>surface</u> bounced on, denoted by subscript s).

The above two dimensions are combined with a third dimension, i.e. the role of an intrinsic factor is also attended to.

<u>The third dimension is thus</u>:

$A_H > B_H$ if $A_B > B_B$ (relation between the height of bounce and the intrinsic bounce of the tennis ball, denoted by subscript B).

Hence at substage IV the subjects realise the role of extrinsic and intrinsic factors and they may conclude $A > B$ due to $Ap > Bp$ and/ or $A_s > B_s$ and $A_B > B_B$. This clearly involves bidimensional comparison plus the third dimension of intrinsic bounce, Noelting's substage IV. Stages 4 and 5 utilised strategies that incorporated the above reasoning although it was only at stage 5 that both extrinsic variable effects were successfully anticipated and controlled. It may be argued that all the stage 4 and 5 strategies are substage IV, thus the application of this method to identify Noelting's substages leaves many strategies with the same M demand of 4 and does not distinguish between them developmentally. The alternative method in order to match the proposed stages in the tennis ball task to Noelting's substages is to calculate the strategies' M demands by using an identical method to Noelting (1978) as reported by Case (1978) which uses the procedures described by Case (1974b). For ease of reference and as an aid to comprehension of the principles involved in M demand calculation the D strategy, noted at stage 2.3 in the tennis ball task, is shown and then discussed below. The strategy is termed jvD in the following to distinguish it from strategies that are not judgment verification (jv) strategies.

Working Memory Demand for the execution of a strategy

(Stage 2.3) Strategy jvD (JUDGMENT VERIFICATION BY D EXPERIMENTATION)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store)	(i) Position of drop of A(Ap)*	0
Step 2 - (Real Action) - Note position of drop of B (store)	(i) Ap (ii) Position of drop of B(Bp)*	1
Step 3 - (Real Action) - Compare positions of drop of A & B (store) make Ap > Bp	(i) Difference in positions of drop (Ap > Bp)	1
Step 4 - Note height bounced by A (store)	(i) Ap > Bp (ii) Height bounced by A(A_H)*	1
Step 5 - Note height bounced by B (store)	(i) Ap > Bp (ii) A_H (iii) Height bounced by B(B_H)*	2
Step 6 - Compare heights bounced (store) state A > B if $A_H > B_H$ if Ap > Bp	(i) Ap > Bp (ii) Difference in height of A & B	2

The M value for each step represents the figurative items in working memory. If the item is activated directly by the visual input, i.e. it is field facilitated, this is denoted by an asterisk. Such items are not involved in the working memory calculation. Strategy jvD contains 6 steps. Items generated in one step are included in subsequent steps, i.e. rehearsed if they are required at some later step. The operative schemes necessary at each step were not considered to require space in working memory for a child spontaneously using the D strategy. This reflects the automatisation of operations as suggested by Case (1978). The jvD strategy involves real and mental actions, i.e. the subject positions the tennis ball A as well as noting its position. Thus this item is field facilitated. Steps 4, 5 and 6 represent the maximum M demand of 2 for this strategy. Step 5 would have an M demand of 3 if the "height bounced by B" were not field facilitated. Thus if the child were asked to describe an experiment he/she would perform, rather than actually perform it, the M demand would increase by 1. This anticipatory M value of 3 is that predicted by the classification of this strategy as a bidimensional comparison (Noelting's substage III).

Table 10.2 summarises the strategies identified and their related stage together with the predicted M demand implied by Noelting's substages. The M demand calculated from the maximum number of items needed to be held in working memory in order to perform the strategy is also shown, and these calculations are shown in Appendix H. Strategy INV J was not noted to be a spontaneously executed strategy and thus an M demand value was not calculated.

TABLE 10.2

* Stage		Strategy	Abbreviation	Noelting's Substage	Implied M Demand	Calculated M Demand
1.1		Intuitive Judgment	IJ	I	1	0
1.2		(Invariant Judgment)	INVJ			
2.1		Bounce	BN	II	2	1
2.2	Judgment verification	D Post-hoc	JvD post-hoc			
2.3		D D(GD post-hoc) D(Cs post-hoc)	jvD jvGD post-hoc jvCs post-hoc jvT post-hoc	III	3	2
2.4		GD	jvGD	III	3	3
3.1		Test of Strength	jvT			2
		T(control of floor surface post-hoc)	jvTCs post-hoc			3
3.2		Control of floor surface	jvCs			3
			jvCs+p			3
4.1		Control of position of drop	jvCp	IV	4	2
4.2		Control of position of drop	jvCp+s1	IV	4	3
		(Control of floor surface post-hoc)	jvCp+s2			3
5.1		Control of both position of drop & floor surface	jvCp+s2	IV	4	3

* The criteria used to identify the stages are shown in table 9.4 and detailed descriptions are given in chapter 9.

It was argued that the larger M demand predicted by Noelting's substages represents that required for the formation of the strategy with the child anticipating every item in each step. It was clear from the subjects' actions and explanations that the formation of the jvD strategy involved using a judgment that tennis ball A should bounce higher than B and the argument that, because it should bounce higher, it should be placed at a higher position of drop than B. Thus the performance and formation of the strategies appeared to involve a shift in attention of working memory between two distinctly different items, figurative items and arguments. Arguments are the 'logical' reasons behind performance necessary for the formation and the post-hoc evaluation of a strategy. It appears that it is via mental attention to arguments that transition between the static strategies and hence stages occurs. It is at this level that figurative items from the execution of a strategy are evaluated in relation to the plan. For example, if a child judges that tennis ball A is bouncier than B and executes the jvD strategy to show this, he/she may realise that the figurative conclusion from the execution of the strategy, B> A, conflicts with his/her original judgment. Once the conflict is realised this must be evaluated. This appears to instigate a series of steps that lead to the modification of the strategy, i.e. the formation of a new strategy.

In the light of the above considerations procedures were set up in order to describe the subjects' total interactive behaviours on the tennis ball task interviews. Thus the procedures must allow a description of the formation, and the post evaluation of the strategies in terms of the arguments used by the child.

The Procedures

PLANS are formed, i.e. logical reasons or arguments are considered as a response to the TASK. The formation of the PLAN is directed by the task question.

The PLAN may be seen as a set of ARGUMENTS or reasons for the use of the observed strategy. Thus it incorporates the task question and anticipated relations and actions, mental or real, e.g., prove AvB by bouncing the tennis balls A and B because the one that bounces highest is best. The observed strategy and evaluation of the results are manifestations of the sequential mental and real actions necessary to carry out the PLAN. Thus the PLAN has a 'practical' and a 'reason for use' aspect, i.e. the 'strategy' and an 'argument'. In the following any mental attention directed specifically to figurative items is termed a strategy. In order to perform the PLAN the child is considered to undertake a series of mental steps, with the execution of the strategy itself being one step containing a series of substeps.

A STEP which is analogous to a subroutine in a computer program is associated with some mental or real ACT. An ACT is a mental or real reaction to the TASK.

Each STEP has a RULE which represents an instruction and relates to operations required at that step.

A PLAN may be modified into a NUPLAN through CONFLICT and thus a new strategy may be formed. This occurs when a figurative conclusion does not correspond to the expectations within the PLAN, i.e. the experimental result is unexpected (Piaget, 1971), or where it produces inconsistent results.

Figure 10.0 illustrates the 9 steps in the NUPLAN model for interactive behaviour. STEPS 1 to 3 are similar to the steps described by Newell and Simon (1972) in their general problem solving (GPS) executive program.

In step 1 figurative items are attended to and the task and the question is interpreted.

In step 2 arguments relating to figurative items and task question are attended to and a plan is formed.

Thus steps 1 and 2 are the same for all plans although the plan formed is different.

In step 3 the PLAN is executed and, as already noted, the sequence may not be automatic and appears to be structured by the PLAN, i.e. arguments are applied within the constraints of the task situation. For example, if an extrinsic variable was considered as an element in the plan then

FIGURE 10.0 THE PLAN/NUPLAN MODEL OF INTERACTIVE BEHAVIOUR

(* denotes mental attention given)

STEP	WORKING MEMORY: Relations, Plans etc. (Automatised)	WORKING MEMORY: Arguments	WORKING MEMORY: Figurative items (Strategies)	DESCRIPTION OF STEP
1. EVALUATE TASK	'Task' Question		*	Interpret the question & situation (evaluate GOAL) (Newell & Simon, 1972)
2. FORM PLAN	Plan	*		Decide upon some action (select method for GOAL) (Newell & Simon, 1972)
3. EXPERIMENT/ ACTION	Task Question		* (strategy) execution	Perform the action mental and/or real, i.e. execute the strategy (apply method to GOAL) (Newell & Simon, 1972)
4. CHECK TASK	Task Question Plan	* Figurative conclusion		Interpret the experiment in terms of the goal. (Evaluate GOAL). (Newell & Simon, 1972)
5. EVALUATE CONFLICT			* conflict resolution strategy execution	Execute the conflict resolution strategy.
6. EVALUATE PLAN		* Figurative conclusion		Interpret the conclusion to the plan and note where it is inadequate.
7. NUPLAN		* Reason for conflict + plan items		Form a new plan incorporating a method to overcome the inadequacy.
8. EXPERIMENT/ ACTION			* strategy execution	
9. CHECK TASK		* Figurative conclusion		

the first extrinsic variable to be considered appeared necessarily to be the position of drop as the child placed his/her hands in position to perform the experiment. Items included here relate to the concluding step in the execution of the strategy shown in Appendix H, page 290.

Step 3 produces a figurative conclusion which is considered at step 4 in relation to any item in the plan that directly concerns this conclusion. Where the items are equivalent, the child is able to give an answer to the task question with reasons dependent upon the PLAN utilised. Where the items are not equivalent e.g., a figurative conclusion for step 3 that $B > A$ is included with a judgment that $A > B$, then the remaining steps are instigated.

In step 5 a conflict resolution strategy is executed that considers figurative items that may be seen to be associated with the conflict, e.g. that B was dropped from a higher position than A ($Bp > A$). Items included here relate to the concluding steps shown in Appendix I which indicates the M demand of the conflict resolution strategies.

In step 6 the figurative conclusion from the previous step is attended to and relevant items of the plan are considered so that a reason for the conflict is seen in terms of the items considered in the original

PLAN e.g., if B POSITION = HIGH is the figurative conclusion from step 5 then B POSITION = HIGH made B > A is considered with the original judgment A > B and thus a logical conclusion is reached. This conclusion might be that A POSITION = HIGH might give A > B. At this step the argument represents only one element of a possible NUPLAN.

In step 7 a NUPLAN is formed incorporating the new argument (conclusion from step 6) e.g., A POSITION = HIGH makes A > B is incorporated with the judgment A > B and the plan item BOUNCE = HIGH. The NUPLAN is, if A > B make A POSITION = HIGH BOUNCE.

In step 8 the strategy relating to NUPLAN is executed note that under the direction of the plan, formation and execution of the strategy may occur simultaneously once the reasons for its use are formed. Items included here once again relate to the concluding step in the execution of the strategy shown in Appendix H.

In step 9 the figurative conclusion from step 8 is compared to a task item directly relating to it e.g., conclusion A > B from step 8 relates to the judgment A > B. If items at this step are equivalent, i.e. the conclusion from step 8 satisfies the PLAN (which itself is directed by the task question), then the behaviour ceases. An answer is given with reasons dependent upon the NUPLAN utilised. If conflict still occurs the child may repeat steps 5 to 9 to resolve this conflict. Of course the child may just give up at any of the steps past step 4.

Table 10.3 shows the PLANS and NUPLANS identified at each of the stages noted in Chapter 9. As can be seen ten NUPLANS were identified. Each utilised behaviour that may be modelled using the procedures described above. The NUPLANS, protocols and a brief explanation of each behaviour are shown on the following pages. STEP 1 is omitted for all of the models of behaviour as the result for this step was always the task question in the form: PROVE AvB or PROVE AvB, INV, when the need for invariant results was realised, stage 1.2 onwards. Each NUPLAN behaviour included the execution of a PLAN at steps 1 to 4 and through conflict the PLAN was modified or rejected and a NUPLAN was formed at step 9. Each PLAN/NUPLAN sequence represents observed behaviour and although a child's statements may be misleading and thus not reveal his/her level of thinking it may be argued that his/her actions and statements give a representation of the underlying reasoning whether typical or atypical.
It is supposed that each of the PLAN/NUPLAN models to be described represented an episode in the development of the control of variables and that a series of episodes may be used to represent a developmental sequence.

TABLE 10.3 The PLANS/NUPLANS noted at each stage and their Frequency of Use

STAGE	PLAN	FREQUENCY	NUPLAN	FREQUENCY
1.1	IJ	3		
1.2	(IJ)		INV J	7
2.1	B^N	9	B^N	3
2.2	(B^N)		jvD	8
2.3	jvD	13	jvG.D.	6
			jvC_s	4
			jvT	2
2.4	jv G.D.	6		
3.1	jvT	5	jvT⊂s (only post-hoc)	3
3.2	jvCs	2	jvC_{s+p} (only post-hoc)	3
4.1	jvCp	9	$jvCp+s_1$	3
4.2			jvCp+s1	3
	(jvCp)		(only post-hoc)	
			$jvCp+s_2$	19
5.1	$jvCp+s_2$	8		
5.2	$jvCp+s_2$	10		

The PLANS in parenthesis are those from which the NUPLAN was formed.

NUPLANS jvT⊂s, jvCs+p, $jvCp+s_1$, are behaviours that were found to be purely post-hoc modifications of an initial experiment. PLANS for these behaviour are therefore not included.

NUPLAN INV J (This involves the realisation that results should be invariant)

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. INTERPRET PLAN B	Prove AvB (task question)	BOUNCE - HIGH (experimenter's stated relation)		p+1	"You're bouncing it to see if it is higher".
3. EXPERIMENT ONE	Prove AvB BOUNCE - HIGH		A_H*	p+0 (see appendix H strategy BN)	"The white went high up there", points to scale.
and EXPERIMENT TWO			A_H Bh*	p+1	"The second time the yellow was high, it bounced up to there", points to scale.
4. CHECK TASK	Prove AvB BOUNCE - HIGH	A_H (conclusion from Bh* experiments one and two)		p+1	"they're both the best".
5. EVALUATE CONFLICT	Prove AvB BOUNCE - HIGH		A_H Bh*	p+1	"the yellow one is best, the second time it went much higher than the white one did the first time".
6. EVALUATE PLAN	Prove AvB BOUNCE - HIGH	A_H (conclusion that A is best overall) A&B (realisation that both were best)		p+2	"the yellow is best overall but both were best".
7. NUPLAN	Prove AvB (INV) BOUNCE - HIGH	INV (conclusion of invariance)		p+1	"I see, the tennis ball that is best should always be the best".

NUPLAN INV J

This NUPLAN is atypical in that subjects at stage 1 did not see the need for experimentation and thus the NUPLAN model was adapted slightly to accommodate this feature. The stage 1.2 behaviours NUPLAN INV J describes involved the subject's understanding the experimenter's PLAN, the realisation that this PLAN produced different results, and that there was a need for an invariance in the results. The behaviours arose from the use of the conflict sequence (see the experimental procedure, Chapter 9). Thus step 2 is modified from FORMPLAN to INTERPRET PLAN BN in that the experiment was not the subject's own. Step 3 is also modified to enable the observation of two experiments to be described.

At step 4 conflict was realised in that the subjects were unable to give one answer when asked, i.e. they were not centering on the last result as were stage 1.1 subjects. A re-examination of the results (the experiments were performed again), produced an OUTPUT of one result at step 6. The PLAN BOUNCE = HIGH enabled the subjects to choose the highest tennis ball overall. At step 7 subjects were able to express the realisation that results should be invariant INV. From this stage onwards the realisation of conflict between differing results was spontaneous. At stage 1.1 conflict was not realised and at stage 1.2 conflict was only realised when the experimenter persisted in asking for one result.

NUPLAN BN (This involves the realisation of the need for experiment as a method of proof through the realisation that invariance may be produced by the strategy IJ)

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. FORM PLAN IJ		BOUNCY = SALIENT FEATURE (a figurative relation)		p+1	
3. EXPERIMENT	Prove AvB INV BOUNCY=SALIENT FEATURE		A_X (stored) B_Y* .	p+1 (see appendix H similar to strategy IJ but one feature is stored)	The subject examines the tennis balls. He notes 2 features X and Y related to bounce.
4. CHECK TASK	Prove AvB INV BOUNCY=SALIENT FEATURE	conclusions A_X(A has feature X) B_Y(B has feature Y)		p+1	"I can see it the yellow one is fluffy but the white one looks hard, I can't tell".
5. EVALUATE CONFLICT	Prove AvB INV BOUNCY=SALIENT FEATURE		A_X (stored) B_Y *	p+1	The subject re-examines the tennis balls
6. EVALUATE PLAN	Prove AvB INV BOUNCY=SALIENT FEATURE	PLAN produces A & B (realisation of result variance)		p+2	"I can't tell by just looking, both cannot be best".
7. NUPLAN B	Prove AvB INV	BOUNCY = HIGH (anticipated relation) BOUNCY=SALIENT FEATURES produces conflict (conclusion from step 6)		p+2	"can I bounce them, then I can see how high they go. Just looking at them is no good".
8. EXPERIMENT	Prove AvB BOUNCY=HIGH		A_H B_H *	p+1 (see appendix H strategy BN)	Subject performs a B experiment.
9. CHECK TASK	Prove AvB INV BOUNCY=HIGH	A_H (judged to bounce highest)		p+1	"The yellow one's best it bounced highest".

NUPLAN B

The execution of PLAN IJ at step 3 produced a conflict at step 4 due to more than one feature being associated with "bounciness". By step 6 the subjects recognised that the use of PLAN (BOUNCY = SALIENT FEATURE), produced the conflict This will only occur if the subject is aware of the need for invariance of results (INV), i.e. if the task question is interpreted as PROVE AvB INV. If INV is not present in the PLAN at this point it may be realised at this step and introduced into the argument but this would increase the M demand of step 7 to p+3. Thus an alternative sequence to that presented in NUPLAN INV J may be seen in steps 1 to 6 of NUPLAN BN where invariance may be realised through the application of PLAN IJ. Such a process seems likely but was not noted and the subjects' responses indicated a familiarity with the need for invariance of results.

At step 7 a NUPLAN was formed (BOUNCY = HIGH). This arose from a realisation of the need to show an invariant answer as the previous PLAN produced inconsistent and ambiguous results. Thus the subjects saw the need to experiment. The experiment chosen was the least structured because the subjects were unaware or did not anticipate the roles of the extrinsic or intrinsic variables.

NUPLAN jvD (The formation of D experimentation through a realisation that BN experimentation may produce doubtful results. The role of the extrinsic variable of position of drop is realised.

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	N	PROTOCOL
2. FORM PLAN BN	Prove AvB INV	BOUNCY=HIGH (anticipated relation)		p+1	"Can I bounce them to see?"
3. EXPERIMENT	Prove AvB INV BOUNCY=HIGH		A_H (height bounced by A) B_H (height bounced by B)	p+1 (see appendix H, strategy BN)	Subject performs experiment
4. CHECK TASK	'Ditto'	A > B (judgment from experiment) A ≏ B (from experiment)		p+2	"The yellow was the bounciest, it just went higher. I think you can't really tell".
5. EVALUATE CONFLICT	Prove AvB INV BOUNCY=HIGH		A_P (position of drop of A) B_P (position of drop of B)	p+2 (see appendix conflict strategy I)	"I dropped the white one from higher up than the yellow one".
6. EVALUATE PLAN	'Ditto'	A > B (judgment) B POSITION=HIGH (conclusion from step 5) made B > A (extrinsic variable role realised)		p+3	"The white one was dropped from higher so it bounced higher but that's not right the yellow is best".
7. NUPLAN jvD	'Ditto'	A POSITION=HIGH (conclusion from step 6) BOUNCY=HIGH (plan item) A > B (judgment)		p+3	"The yellow from high should go high that will show it is bounciest".
8. EXPERIMENTATION	Prove AvB INV (BOUNCY=HIGH) (POSITION=HIGH) HIGH POSITION= HIGH BOUNCE		$A_P > B_P$ A_H B_H •	p+2 (see appendix H, strategy jvD)	Subject performs D experimentation
9. CHECK TASK	Prove AvB INV HIGH POSITION = HIGH BOUNCE	$A_H > B_H$ due to $A_P > B_P$ (conclusion from step 8) A > B (judgment)		p+2	"The yellow is the best now because I dropped it from higher".

NUPLAN jvD

The execution of strategy BN produced conflict due to its unsystematic nature. Those subjects who used PLAN BN at stage 2.1 did not make an initial judgment (see Chapter 9). This provides evidence for the rejection of strategy IJ at this stage and its importance in the formation of PLAN BN. For 4 subjects conflict occurred at step 4 due to the tennis balls bouncing approximately the same heights. Subjects at this stage were aware of the need for an invariant result, thus they judged one tennis ball to be the best from the results, but realised that their experiment had not shown this. The execution of the conflict resolution strategy at step 5 produced a realisation of the role of the extrinsic variable of position of drop in that this was the reason for the poor bounce of A, i.e. the white tennis ball was unintentionally dropped from slightly higher in each case.
At step 7 the subjects were able to combine their understanding

of the role of the position of drop to their judgment e.g., (A > B) and the PLAN (BOUNCY = HIGH). This resulted in NUPLAN jvD, which in all cases was followed by the execution of the jvD strategy at step 8 and the completion of the task. The use of jvD may produce conflict at step 9 in that the floor surface is ignored and its affect on the height bounced may produce a result that conflicts with the subject's judgment. This did occur in one subject's experiment, he was confused by the result but could not resolve this conflict, i.e. he was not aware of the role of the floor surface. The making of an initial judgment plays an important role in this behaviour, i.e. if no judgment is made by step 6, then the subjects are unable to make use of their understanding of the role of the extrinsic variable and the continued use of BN experimentation will produce very inconsistent results.

It appears that as the PLAN BN becomes automatised the reasons for its formation are not attended to and thus initial judgments may be made by subjects using strategy BN. This behaviour was noted for 4 subjects at stage 2.2 and for these subjects the conflict at step 4 was due to the fact that the initial judgment and the experimental result were different. A reason for the conflict due to the experiment was found in each case due to the unsystematic nature of BN experimentation. For one subject the conflict was due to the floor surface, he formed NUPLAN jvD but, because the position of drop was not attended to in the execution of the strategy, this NUPLAN was not successful in confirming the judgment. The subject showed confused behaviour at step 9 returning to just bouncing the tennis balls.

NUPLAN jvGD (This involves the realisation of the effect of the extrinsic variable of floor surface through a realisation that D experimentation may produce conflicting results

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. FORM PLAN jvD	Prove AvB INV	HIGH POSITION =HIGH BOUNCE (anticipated relation) A> B (judgment)		p+2	"The yellow is best so a high position will make the yellow one bounce higher".
3. EXPERIMENTATION	Prove AvB INV A HIGH POSITION→ A HIGH BOUNCE		$A_P > B_P$ A_H B_H*	p+2 (see Appendix H strategy jvD)	Subject performs D experimentation
4. CHECK TASK	Prove AvB A HIGH POSITION → A HIGH BOUNCE	A> B (judgment) but A=B (conclusion) or A< B (from step 3)		p+2	"It didn't work something's wrong the yellow didn't win".
5. EVALUATE CONFLICT	Prove AvB INV A HIGH POSITION → A HIGH BOUNCE		A_s B_s	p+2 (see appendix I conflict strategy 2)	"The yellow only bounced on the mat, the white was on the floor".
6. EVALUATE PLAN	Prove AvB INV A HIGH POSITION → A HIGH BOUNCE	A> B (judgment) B SURFACE = HIGH (conclusion from step 5) made B > A (extrinsic variable role realised)		p+3	"The yellow one's the best but the white one won because it was on the hard floor".
7. NUPLAN jvGD	Prove A INV A HIGH POSITION → A HIGH BOUNCE	A> B (judgment) A SURFACE = HIGH (conclusion from step 6) A HIGH POSITION→A HIGH BOUNCE (item from plan to be incorporated)		p+3	"The yellow one should drop on the hard floor as well as dropped from a high position".
8. EXPERIMENTATION	Prove AvB INV A HIGH POSITION + SURFACE→ A HIGH BOUNCE		$A_P > B_P$ $A_s > B_s$ A_H B_H *	p+3 (see appendix H, strategy jvGD)	Subject performs GD experimentation.
9. CHECK TASK	Prove AvB INV A HIGH POSITION + SURFACE→ A HIGH BOUNCE	A> B (judgment) $A_H > B_H$ due to $A_P > B_P$ (role of extrinsic variable) $A_H > B_H$ due to $A_s > B_s$ (conclusion from step 8)		p+3	"That's right the yellow's the bounciest because it bounced on the hard floor and from higher up".

NUPLAN jvGD

This NUPLAN represents typical post-hoc behaviour where a variable initially overlooked is incorporated into a modified jvD strategy. The conclusion from step 6 represents a realisation of the role of the extrinsic variable of floor surface. At step 7 the original plan which had incorporated item BOUNCE = HIGH within it at its formation, is combined with the new information concerning the effect of the floor surface. It should be noted that both NUPLAN GD and D were found to have M demands of p+3. For execution of the strategy the M demand was found to be 3 units for G.D. and 2 units for D. That the NUPLANS have the same M demands is not surprising as the PLANS are essentially the same. It is in the execution of the strategy that an extrinsic variable might be overlooked and this reduces the M demand.

NUPLAN jvC_s (The 'control' of floor surface through the realisation that D experimentation may produce conflicting results)

The model is identical to jvG.D. up to step 6

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
6. EVALUATE PLAN	Prove AvB INV A HIGH POSITION = A HIGH BOUNCE	A > B (judgment) B SURFACE = EXTRA HIGH (conclusion from 5) made B > A		p+3	"The white ball was given some more bounce by the floor so it went higher, that's what went wrong".
7. NUPLAN jvC_s	Prove AvB INV A HIGH POSITION = A HIGH BOUNCE	A > B (judgment) A HIGH POSITION = A HIGH BOUNCE (plan items) B SURFACE ≠ EXTRA BOUNCE (conclusion from 6)		p+3	"You can do what I did before, drop the yellow one from higher but you must not drop the white one like I did or the floor makes it bounce higher".
8. EXPERIMENTATION	Prove AvB INV A HIGH POSITION = A HIGH BOUNCE DIFFERENT SURFACES = DIFFERENT BOUNCE		$A_P > B_P$ $A_S = B_S$ A_H B_H =	p+3 (see appendix H strategy jvC_s)	Subject performs a D experiment but 'controls' the floor surface.
9. CHECK TASK	Prove AvB INV A HIGH POSITION = A HIGH BOUNCE DIFFERENT SURFACES = DIFFERENT BOUNCE	A > B (judgment) $A_H > B_H$ due to $A_P > B_P$ (role of extrinsic variable). B SURFACE ≠ EXTRA BOUNCE (conclusion from 8)		p+3	"The yellow bounced highest it's the best, because it was dropped from higher up but not dropped onto a different bit of floor to the white tennis ball".

NUPLAN jvCs

This 'control' or elimination of different floor surfaces represents an exclusion of an extra effect so that D experimentation may be performed without producing a conflict between the judged and experimental results. At step 6 the reason for the conflict between the judgment and the experimental results was realised, i.e. the surface on which B bounced gave it some extra height (B SURFACE = EXTRA HIGH). At step 7 this conclusion combined with the elements of PLAN jvD produced NUPLAN jvCs in which the extra effect was 'controlled' (at this level, eliminated). This was purely a simplification of the situation so that the judgment could be confirmed. Control was not used in an attempt to show the role of the intrinsic factor of bounce. It should be noted once again that the M demand for this jvNUPLAN was p+3.

NUPLAN jv T (This involves the realisation of and an attempt to show the operation of an intrinsic variable of bounce within the situation due to D experimentation producing conflicting results.)
(Floor surfaces are the same by accident)

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
The model is identical to jvD up to Step 6.					
6. EVALUATE PLAN	Prove AvB INV A HIGH POSITION = A HIGH BOUNCE	$B > A$ (judgment) but $A_P > B_P$ (experiment) B SURFACE = A SURFACE (conclusion from 5)		p+3	"The yellow one (B) bounced a bit higher than the white one, (A) the white was dropped from higher up and I think they both fell on the floor its something else that makes it bounce it harder".
7. NUPLAN jvT	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION = A HIGH BOUNCE	$B > A$ due to $B_B > A$ (conclusion of instrinsic bounce from 6). A HIGH POSITION = A HIGH BOUNCE (jvD plan item) but $B > A$ if $B_H > A_H$ (anticipated result)		p+3	"The yellow one is bouncier because its harder. You can prove this by dropping the white ball from a bit higher up, the yellow will still beat it".

NUPLAN jvT

The subject's experiment enabled the application of the test of strength argument, in that a search at step 5 for a reason for conflict produced none (by accident the floor surfaces were the same for both tennis balls). The search for a reason for conflict led to the conclusion that the judgment was incorrect and that the white tennis ball (A) was bounciest. The subjects realised that an advantage of a higher position of drop was given to the yellow tennis ball (B) and A still won. They concluded that an intrinsic bounce variable was operative within the situation. This leads to NUPLAN jvT at step 7. The subjects considered that intrinsic bounce needed to be shown thus $B > A$ due to $B_B > A_B$ was anticipated. By applying PLAN jvD, i.e. giving tennis ball A an advantage (A HIGH POSITION = A HIGH BOUNCE), a result of $B_H > A_H$ was thus considered to prove $B > A$.

In this model the subjects did not see the necessity for further experimentation as their original experiment had already shown the result. Thus NUPLAN jvT did not represent a new plan of real action, it was a re-interpretations of an old plan due to the realisation of the role of an intrinsic variable. It should be noted that unless subjects attend to the floor surface eventual experimentation using this NUPLAN may produce conflict.

NUPLAN jv T.Cs (The 'control' of the floor surface through a realisation that T experimentation produces inconsistent results

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. FORM PLAN jv.T	Prove AvB INV SHOW INTRINSIC BOUNCE	A HIGH POSITION = A HIGH BOUNCE (anticipation of relation) B > A (judgment) $B > A$ if $B_H > A_H$ (argument concerning anticipated relation)		p+3	"The yellow one (B) is bouncier than the white one (A) you can prove it by seeing if it will beat the white if the white one is dropped from higher".
3. EXPERIMENTATION	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A_H$		$A_P > B_P$ A_H B_H*	p+2 (see appendix H, strategy jvT)	Subject performed D experiment (ignored the floor surface)
4. CHECK TASK	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A_H$	B > A (judgment) but A=B (result -) or A > B (conclusion) (from 3)		p+2	"Something's wrong, the yellow should have bounced higher".
5. EVALUATE CONFLICT	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A_H$		A_s B_s	p+2 (see appendix I, conflict 2')	"I dropped the white one on the hard floor, the yellow fell on the rug".
6. EVALUATE PLAN	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A_H$	B > A due to $B_B > A_B$ (intrinsic bounce relation) even though A HIGH POSITION = A HIGH BOUNCE A SURFACE = EXTRA BOUNCE (conclusion from 5) made A > B (result)		p+4	"The yellow ball is bounciest and so should still have beat the white one, if the white one is dropped from higher. The white was made to bounce too much because it fell on the floor".
7. NUPLAN	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A$	B > A (judgment) A HIGH POSITION = A HIGH BOUNCE A SURFACE ≠ EXTRA BOUNCE $B > A$ if $B_H > A_H$ (plan item to be considered)		p+4	"I'll try it like that, the first time. You must not let the white tennis ball bounce on a different floor surface".
8. EXPERIMENTATION	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE $B > A$ if $B_H > A_H$ A SURFACE = EXTRA BOUNCE		$A_P > B$ (NOT $B_s > A_s$) (or $B_s = A_s$) A_H B_H*	p+3 (see appendix H, strategy jvTCs)	Subject performs D experimentation and 'controls' for the floor surface.
9. CHECK TASK	'Ditto'	A > B due to $A_P > B_P$ (anticipation of effect of extrinsic variable) BUT B > A due to $B_B > A_B$ (realisation of role of intrinsic variable) Because this time A SURFACE ≠ EXTRA BOUNCE ('control' of floor surface)		p+3	"It works the yellow bounces highest even though the white was dropped from higher. This time the white one did not bounce too high, it was not given more bounce by being on the floor.

NUPLAN jvT Cs

This NUPLAN is a straightforward formation and execution of the jvT, i.e. the test of strength strategy. The subjects argued at step 2, that an application of PLAN jvD to tennis ball A, i.e. A HIGH POSITION = A HIGH BOUNCE, would confirm the judgment B> A if the result was that B bounced the highest ($B_H > A_H$). These subjects must have been aware of the role of an intrinsic factor or they would not have argued in this manner and formed PLAN jvT. The execution of the strategy at step 3 produced a realisation at step 4 that the judgment was not confirmed. A reason was searched for and it was realised at step 6 that the floor surface caused tennis ball A to bounce too much (A SURFACE = EXTRA BOUNCE). The subjects arguments at this step involved considering that the intrinsic bounce ($B > A$ due to $B_B > A_B$) was not shown when A was given an advantage of position of drop (A HIGH POSITION = A HIGH BOUNCE) because A SURFACE = EXTRA BOUNCE produced the result $A > B$. The subjects decided to control this extra effect at step 7 and thus their NUPLAN was a Test of Strength experiment that controlled the floor surface. This 'control' strategy was used in order to simplify the experiment, so that the jvT strategy could be used.

NUPLAN jv C_{s+p} (The realisation that in order to show the intrinsic factor a comparison of bounce using equal positions of drop is needed (2) jvCs strategies are used and are compared)

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. FORM PLAN jvC_s	Prove AvB INV SHOW INTRINSIC BOUNCE	$A > B$ (judgment) (anticipated relations) A HIGH POSITION = A HIGH BOUNCE DIFFERENT SURFACE= DIFFERENT BOUNCE $A > B$ due to $A_B > B_B$		p+4	"I would let it bounce. I would drop them both on the floor as that makes a difference to it's bounce, with the yellow dropped from higher it will bounce highest. That will be right, the yellow is theounciest,
3. EXPERIMENTATION	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE DIFFERENT SURFACE =DIFFERENT BOUNCE		$A_p > B_p$ $A_s = B_s$ A_H B_H	p+3 (see appendix H, strategy jvC_s)	Subject performs a D experimentation but 'controls' the floor surface.
4. CHECK TASK	'Ditto'	$A \not> B$ (conclusion from 3) but $A > B$ (judgment)		p+2	"The yellow wins! I did't think it was as bouncy as that".
5. EVALUATE CONFLICT	'Ditto'		$A_s = B_s$ Ap HIGH Ap LOW	p+3 (see appendix I conflict strategy 3)	"I put the yellow one up there and the white one down there and they both fell on the floor".
6. EVALUATE PLAN	Prove AvB INV SHOW INTRINSIC BOUNCE A HIGH POSITION= A HIGH BOUNCE DIFFERENT SURFACE = DIFFERENT BOUNCE	A HIGH/FLOOR & B LOW/FLOOR gives $A_H \gg B_H$ due to $A_B > B_B$ and $A_p > B_p$ NOT $A_s > B_s$		p+4	"I did not show how they bounce. I made the yellow one bounce high by putting it up there (points to 4 ft. mark). The floor was the same".
7. NUPLAN jvC_{s+p}	'Ditto'	DIFFERENT POSITIONS = DIFFERENT BOUNCE (conclusion from 6) $A_p > B_p$ made $A_H \gg B_H$ (jvC_s plan item) $B_p > A_p$ will make $B_H > A_H$ (jvC_s plan item) A_H old $> B_H$ new will confirm $A > B$ (expected result)		p+4	"I can see if the yellow is only a bit better than the white one, by dropping the white one from as high as the yellow one was. The white one should go as high as the yellow did before."
8. EXPERIMENTATION	Prove AvB INV SHOW INTRINSIC BOUNCE (DIFFERENT POSITIONS = DIFFERENT BOUNCE) DIFFERENT SURFACE = DIFFERENT BOUNCE A HIGH POSITION = A HIGH BOUNCE		B_p new = A_p old B_s new = A_s new B_H new A_H*	p+3 (see appendix H strategy jvC_{s+p})	Subject performs D experimentation, dropping the white tennis ball from the same height as the yellow one in the first experiment.
9. CHECK TASK	'Ditto'	$A_H > B_H$ (result) due to $A_B > B_B$ (intrinsic relation) NOT ($A_s > B_s$) (extrinsic NOT ($A_p > B_p$) relations controlled)		p+4	"The yellow's best because its bounciest not because of the floor surface and the position of drop".

NUPLAN j Cs+p

The PLAN is formed at step 2. A form of centration existed within the subjects' arguments in that they excluded the floor surface in order to show the intrinsic bounce, but did not apply this idea to the position of drop. This lack of integration is typical transition behaviour (Inhelder et al, 1974)

A feature of the NUPLAN jvCs+p behaviour was that the original jvCs strategy was not rejected. Conflict was realised at step 4 in that a tennis ball was noted to bounce higher than it should. It appears that the judgment was that tennis ball A should bounce higher than B, but a large difference was not expected. At step 5 the subjects attended to the fact that only the positions of drop used were different and at step 6 it was realised that the reason for the large difference shown at step 3 was due to an intrinsic effect plus an extrinsic effect due to the higher position of A. At step 7 the use of the alternative jvCs experiment was anticipated, $B_p > A_p$ will make $B > A$. The subjects considered comparing the height bounced by A in the old experiment with the height bounced by B in the new experiment and by attending to the realisation that different positions of drop would affect the difference in the heights shown, the subjects strategy was to control the position of drop between the old and new experiments for A old and B new. The same floor surface was used in the new experiment as the PLAN jvCs was used again. One subject used this type of behaviour after he was shown the conflict sequence and thus the behaviour differed slightly, i.e. at step 4 $A > B$ and $B > A$ were considered and at step 6 item A & B (variance in result) was considered with items $A_B > B_B$, $A_p > B_p$ and NOT $A_S > B_S$ (extrinsic and intrinsic factors operative). This resulted in the formation of NUPLAN jvCs+p. None of these subjects could perform a single 'control of both variables' experiment and when they were prompted to try out one experiment, two subjects controlled the position of drop and ignored the floor surface, (jvCp) and one subject reverted to just controlling the floor surface (jvCs). It was concluded that this stage 3.2 behaviour was transitional between jvCs and jvCp.

NUPLAN jvC_{p+s1} (A realisation that floor surface must be controlled)

(2 jvCp experiments are compared)

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS	M	PROTOCOL
2. FORM PLAN jvCp	Prove AvB INV SHOW INTRINSIC BOUNCE	A > B (judgment) (anticipated relations) show $A_H > B_H$ due to $A_B > B_B$ if $A_P > B_P$ then $A_H > B_H$		p+3	"Yellow is the bounciest and it will bounce higher if it is dropped from higher I am making it bounce".
3. EXPERIMENTATION jvCp	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE		$A_P = B_P$ A_H B_H *	p+2 (see appendix H, strategy jvCp)	Subject performs experiment controllin the position of drop, ignoring the floor surface.
4. CHECK TASK	'Ditto'	A > B (judgment) B > A (conclusion) or (from 3) A = B		p+2	"Oh that's not right the yellow should have won".
5. EVALUATE CONFLICT	'Ditto'		A_S B_S	p+2 (see appendix I control strategy 2)	"It's the different floor surfaces, the white one bounced on the hard surface and the yellow one on the soft".
6. EVALUATE PLAN	'Ditto'	A > B (judgment) and so $A_H > B_H$ due to $A_B > B_B$ but $B_B > A_B$ due to $B_S > A_S$ this made $B_H > A_H$		p+4	"The yellow is the best it should have bounced highest but it didn't because the white one fell on the hard floor".
7. NUPLAN	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE	$B_S > A_S$ made B > A (result of old experiment) $A_S > B_S$ will make A > B (anticipated result of new experiment) A_S new = B_S old (conclusion from step 6 surface should be equal) A_H new > B_H old will show A>B (expected result to confirm judgment)		p+4	"I must do it like before use the same height to drop them from. But the yellow one must fall on the floor if the yellow is the bounciest then you can see the yellow's best by comparing how high the yellow one bounces on the floor this time".
8. EXPERIMENT-ATION	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE DIFFERENT FLOOR SURFACE = DIFFERENT BOUNCE		A_S new = B_S old A_P new = B_P old A_H new * B_H old	p+3 (see appendix H strategy jvC_{p+s1})	Subject performs experiment.
9. CHECK TASK	'Ditto'	A > B due to $A_B > B_B$ NOT ($A_S > B_S$) NOT ($A_P > B_P$)		p+4	

NUPLAN jvC_{p+s1}

This behaviour is similar to the behaviours involved in NUPLAN jvCs+p, but in this instance the position of drop is controlled initially. In NUPLAN jvC_{s+p} the extrinsic variable of position of drop is manipulated to verify the judgment, in NUPLAN jvCp it is controlled and the floor surface variable overlooked. The fact that the position of drop is the first variable that has to be considered suggests that this behaviour is more advanced. Conflict occurred between the initial judgment and the experimental result at step 4 because the floor variable was overlooked. By step 6 the subjects understood the reason for the conflict. At step 7 the subjects considered the following:

that $B_s > A_s$ made $B > A$ in the old experiment and
$A_s > B_s$ will make $A > B$ in a new experiment but
if A_s New = B_s Old (conclusion from 6, surfaces must be equal)
A_H new > B_H old (expected result
will show $A > B$ to confirm judgment)

Thus they were able to use their old strategy and compare 'control' elements between the two experiments.

It was noted that the 3 subjects using this behaviour at stage 4.1 could not then perform a single 'control of both variables' experiment but the 3 subjects using this behaviour at stage 4.2 could. NUPLAN jvCp+s is of course inaccurate as one subject at stage 4.2 stated "you can see it better, there is no doubt about the result when you can see them bounce in one experiment".

NUPLAN jvC_{p+s_2} (A realisation that floor surface and position of drop must be controlled (An H experiment is performed))

STEP	PLAN ITEMS	ARGUMENTS	FIGURATIVE ITEMS		U	POROTOCOL
2. FORM PLAN jvC_p	Prove AvB INV SHOW INTRINSIC BOUNCE	A > B (judgment) (anticipated relations) show $A_H > B_H$ due to $A_B > B_B$ if $A_P > B_P$ then $A_H > B_H$			p+3	"I wanted to show the yellow was best because its bounciest so I had to use the same height of drop for both or the correct bounce wouldn't be shown".
3. EXPERIMENTATION jvC_p	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE		$A_P = B_P$ A_H B_H*	(see appendix H strategy jvC_p)	p+2	Subject performs a control of position of drop experiment but ignores floor surface.
4. CHECK TASK	'Ditto'	A > B (judgment) B > A (conclusion from 3)			p+2	"That shouldn't happen the white one won".
5. EVALUATE CONFLICT	'Ditto'		A_s B_s	(see appendix I conflict strategy 2)	p+2	"I see, I dropped the white one on the hard floor, the yellow one fell on the rug".
6. EVALUATE PLAN	'Ditto'	A > B (judgment) and so $A_H > B_H$ due to $A_B > B_B$ but $B_P > A_B$ due to $B_s > A_s$ this made $B_H > A_H$			p+4	"The yellow one didn't win even though it was bounciest because the white one was dropped on the hard floor and that made it bounce higher".
7. NUPLAN	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE	$A_H > B_H$ if A > B (expected result to confirm judgment) due to $A_B > B_B$ (anticipated intrinsic variable) NOT due to $A_P > B_P$ NOT due to $A_s > B_s$ (conclusion from 6)			p+4	"The floor surface causes the problem if I want to show that the yellow is really the bounciest I have to use the same height of drop and the same floor surface".
8. EXPERIMENTATION	Prove AvB INV SHOW INTRINSIC BOUNCE DIFFERENT POSITION = DIFFERENT BOUNCE DIFFERENT FLOOR SURFACE = DIFFERENT BOUNCE		$A_P \neq B_P$ $A_s = B_s$ A_H B_H*	(see appendix H strategy $jvC_{p+s}2$)	p+3	Subject performs a control experiment.
9. CHECK TASK	'Ditto'	A > B due to $A_B > A_B$ NOT $A_P > B_P$ or $A_s > B_s$			p+4	"You see the yellow is the best its bounciest it did not bounce high because it was dropped high or because it was dropped onto the hard floor".

NUPLAN jvCp+s2

This behaviour is identical to the previous behaviour up to step 6. The subjects controlled the position of drop but overlooked the floor surface variable. The resulting conflict between the judged and experimental results produced a realisation of the role of the extrinsic variable of floor surface at step 6. The NUPLAN at step 7 involves the following items:

$A_H > B_H$ if $A > B$ (expected result to confirm judgment)

this must be due to $A_B > B_B$ (anticipated intrinsic variable)

not due to $A_P > B_P$ (anticipated extrinsic variables

not due to $A_s > B_s$ of position of drop and floor surface)

The final item represents the conclusion from step 6 and the subject is then able to perform a $jvCp+s_2$ strategy controlling both floor surface and position of drop.

Discussion

A major feature of the behaviours was the use of judgment verification strategies. The separate episodes noted in the sequence up to NUPLAN jvD highlight the 'need' for such a technique. The development appeared to follow the episodes:

(i) PLAN IJ - NUPLAN INV J
(ii) PLAN INV J - NUPLAN BN
(iii) PLAN BN - NUPLAN jvD

In (i) a need for invariance between results was realised. In (ii) this realisation of invariance led to the subjects' realisation of the need for experimentation in that PLAN INV J produced conflicting results, thus the subjects appeared not to make judgments at this stage. NUPLAN BN involved a strategy that was unstructured and this led to the conflict due to the tennis balls bouncing equal heights. The subjects' concept of invariance led to the restricted hypothesis that only one tennis ball could be bounciest and this was the reason conflict occurred. For other subjects it was argued that the use of PLAN BN was automatic and thus the reason for use in terms of the rejection of the making of judgments was forgotten. Thus PLAN BN produced conflict due to the judgment and the experimental result differing. In (iii) once the role of an extrinsic variable was realised the subjects could deliberately produce consistency and overcome conflict. The subjects attempting PLAN BN knew they had to bounce the tennis balls but they were often unsure what to do, once they had realised the role of an extrinsic variable their actions became more sure, i.e. they had a reason for performing the experiment and they anticipated consistent results.

Two types of conflict have been noted, (a) the conflict due to an expected invariance, i.e. that one tennis ball should always be best and remain invariant in experimental results, and (b) the conflict between an intuitive judgment (the anticipation of a result) and a different experimental result. It was found that all new behaviour occurred due to these two types of conflict.* The transition from NUPLAN jvD to jvCp where the position of drop was controlled successfully was of particular interest. The episodes were:

(iv) PLAN jvD - NUPLAN jvCs/jvT
(v) PLAN jvCs - NUPLAN jvCs+p_1 (PLAN jvCp)

Three post-hoc behaviours were noted for the use of PLAN jvD, these are shown in NUPLANS jvGD, jvT and jvCs. They all were noted to stem from a conflict between the judged and experimental results. NUPLAN jvCs resulted in the control of the floor surface, i.e. the rug was eliminated from the situation. NUPLAN jvT resulted in a re-interpretation of the results of the jvD experiments due to a realisation of the need to show the effect of an intrinsic variable.

It may be concluded that both the skill of the control of one variable in order to simplify the jvD experiment and the realisation of the need to show the effect of the intrinsic

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* It should be noted that F. Bryant (1982), in experiments on the development of measurement, found that agreement between strategies, rather than conflict, produced intellectual change.

variable occurred at approximately the same point in the developmental sequence. It should be noted that the test of strength episodes of behaviour were purely compromise solutions, because control of variables was not 'understood adequately' and that the control of the floor surface post-hoc occurred for the test of strength experimentation, i.e. NUPLAN TCs. Thus it appears that the strategy of control by elimination of floor surface, the simplification control strategy, was available for those subjects using the jvT strategy but that the control by cancellation of an additional position of drop was not available. The subjects who used PLAN jvCs were aware of the role of intrinsic variables and they were able to eliminate the floor surface in order to allow this effect to be shown. Thus integration between a realisation of the need to show the intrinsic variable's effect and a realisation of the confounding effects of allowing extrinsic factors to be varied was only partial at this stage, i.e. only the floor surface was controlled in order to allow the difference due to the intrinsic variable to be shown. Conflict between judged and experimental results led the subjects to consider the extrinsic variable of position of drop and hence NUPLAN jvCs+p was formed. This comparison of items from the two separate experiments represented novel post-hoc behaviour and may represent the earliest attempt to control the position of drop variable. Note that two subjects were able to perform PLAN jvCp after NUPLAN jvCs+p. These subjects ignored the floor variable immediately after using a strategy that included its control. It appears that these subjects were restricted in their application of the need to show the effect due to the intrinsic variables, i.e. the consideration of one variable at a time.

TABLE 10.4 M demand values for the execution of the strategies together with the FORMPLANS & NUPLANS

PLAN	STRATEGY EXECUTION	FORMPLAN (anticipated formation)	NUPLAN (post-hoc formation)
IJ	0		
INV J			
B	1	1	2
jvD	2	2	3
jv GD	3	3	3
jvT	2	3	3
(jvTCs)	3		4
jvCs	3	3 or 4	3 or 4
(jvCs+p)	3		4
jvCp	2	3	4
($jvCp+s_1$)	3		4
$jvCp+s_2$	3	4	4

(The PLANS in parenthesis were only noted as post-hoc behaviours and did not have associated FORMPLANS).

(N.B.: FORMPLAN jvCs has an M demand of 4 units if the effect of an intrinsic variable is anticipated, if only extrinsic variables are considered then M = 3. Similarly for NUPLAN jvCs, M = 3 or 4).

The M demands for the NUPLANS and FORMPLANS for the behaviours are shown in Table 10.4 . It can be seen that the transition behaviour jvCs was found to require an equivalent M space to the successful FORMPLAN $jvCp+s_2$. Thus although integration was incomplete at this stage, the extrinsic, intrinsic and judgment items needed to be attended to.

It should be noted that PLAN jvCp was found in two instances to develop from jvCs+p and thus the NUPLAN jvCs+p M demand of 4 units is given for this behaviour. It therefore appears that an M space of 4 units is required for the proposed developmental sequence from PLAN jvD to PLAN jvCp. This evidence tends to justify the placement of stages 4 and 5 (PLAN jvCs+p onwards) at Noelting's substage IV in that this is considered to require an M space of 4 units. It appears also that the transition behaviours PLAN jvTCs and PLAN jvCs require an M space of 4.

It is clear that the calculation of the M space required for the formation of a strategy, in particular the post-hoc formation, i.e. NUPLAN behaviour, has a greater predictive power for the developmental level of these behaviours than the M demand value calculated from the execution of the strategy. Note that the M space required for execution of the strategy that used the control of the position of drop (jvCp) was calculated to be only 2 units. Yet it was expected that the strategy was an example of Noelting's substage IV and this should have the M demand noted for the NUPLAN of

4 units. Thus the M space required for the initial formation of a PLAN may be the true indicator of its difficulty.

It appears in the following instances that the NUPLAN required a larger M space than the FORMPLAN.

FORMPLAN	NUPLAN
BN;M = 1	BN;M = 2
jvD; M = 2	jvD; M = 3
jvCp; M = 3	jvCs+p; M = 4

This indicates further that the initial formation of a PLAN appears to be the most demanding point in that behaviour. The later formation of the PLAN, i.e. its re-use appears to be less demanding.

To summarise the points concerning M space, it appears likely that:

(i) M space must be equal to 4 units for the control strategy transition behaviours to occur (NUPLAN jvTCs, jvCs onwards);

(ii) M space may be greater for the NUPLAN than the FORMPLAN, i.e. transition to a behaviour often appears to be more demanding than the formation or the execution of the behaviour;

(iii) M space required for strategy execution appears to be a less adequate predictor of the order of attainment and the demands of a strategy than the value given by NUPLAN, i.e. the number of arguments considered that are the 'initial reasons for use' of the strategy.

With specific reference to Noelting's substages the predicted M demands for each of the substages are equivalent to the M demands noted for the NUPLAN arguments.

Noelting's Substage		M DEMAND	STRATEGY	NUPLAN M DEMAND
I	Isolated Centration	1	IJ	1*
II	Unidimensional Comparison	2	B	2
III	Bidimensional Comparison	3	jvD	3
IV	Bidimensional Comparison & third dimension	4	jvCs to jvCp+s_2	4

(*this value was not calculated but was assumed to be 1 unit).

Noelting's substage IV may be defined for the tennis ball task as a substage in which the intrinsic and extrinsic factors are considered. The work presented in this chapter tends to support Case's M demand theory (Case, 1978), in that Noelting's 4 substages have been identified. The predicted maximum M demand of 4 units for stage IV behaviours is also confirmed. This evidence does not contradict Pascual-Leone's theory in that this also predicts that subjects at substage IV have available an M space of 4 units, although M spaces larger than 4 units are predicted for later stages of development. It is interesting to note that the M demand of the NUPLAN is greater than that for the FORMPLAN by 1 unit until the NUPLAN M demand equals 4 units, it then becomes no larger as the FORMPLAN increases to 4 units. It appears that a maximum has been reached, at least for this particular task. This point will be returned to in the concluding chapter.

The noted points of transition from Noelting's substage III to IV, i.e. the NUPLAN episodes that were truly transitional and not just compromise or more sophisticated forms of simpler strategies, provide an insight into possible techniques for the acquisition of the control of both variables (substage IV). The obvious educational implications of this noted development of a 'control of variables' strategy at the concrete operational level will be discussed in the concluding chapter and related to the development, noted in Chapter 5, of a formal operational notion of control.

CONCLUSION:

PSYCHOLOGICAL AND EDUCATIONAL IMPLICATIONS

Introduction

The experimental work reported in the previous chapters represents the gradual development of the writer's thoughts in the area. It is pertinent at this stage to more fully describe the relation between several of the diverse studies presented in the preceding chapters and the major studies of the development of control of variables described in Chapters 5, 9 and 10. The relation between these three chapters is of major importance to this study, and will be discussed in the main body of this chapter.

The study had as its starting point a task, Wollman's target sphere task (Wollman, 1976). In the replication study described in Chapter 2, similar tasks were found to be solved by 49% of subjects aged from 11 to 18 years, (see Appendix B, Q1), using strategies other than the correct H strategy. Further study of this and similar tasks revealed that the majority of Secondary School children were capable of using three major strategies on these tasks, i.e. the H, C and G.D. strategies. More importantly it appeared that Secondary School children had available all three strategies. This was confirmed by the fact that the C and H strategies appeared to be task specific for the 12 to 15 year old sample tested. The C strategy was more readily applied to a task that could be perceived of as a handicap competition. The H strategy was more readily perceived of as applying to an olympic type competition. Thus it appeared that the problem was not that the H strategy was unavailable to the Secondary School child, but that alternative strategies were also available, which affected the use of the correct strategy. It was apparent that the alternate use of these strategies would lead to conflict and the eventual rejection of all other strategies but the H strategy. (At this level the control strategy may be considered to be a formal operational one). This led to a study of the development of the use of the H strategy for an 11 to 18 year old sample for which four stages in the integration of the criteria for use of the strategies were identified.

The fact that the H strategy was accepted by 90% of 11 year olds (see Table 5.1), led to a consideration of the age at which, and the process by which, this strategy developed. The tennis ball task was utilised in that this was noted by Wollman (1976) to be successfully solved by children of Junior School age. It was noted that such a task is an empirical task, whereas the plasticine and target sphere tasks used in the Secondary School studies are quasi/empirical tasks. An empirical task needs to be solved using empirical proof in that the relation to be determined is unknown. The quasi/empirical task appeared to be solved using strategies other than the H strategy because the relation to be determined was already known, thus the necessity for empirical proof was not realised. This study reported in Chapter 6, revealed that 30% of the 9 and 10 year old sample (mean age 10.2 years) could use a form of H strategy.

Studies of the H strategy utilised by 8 to 11 year olds, reported in Chapter 8, revealed that they were using a concrete form of control strategy, i.e. negation by elimination for the floor surface variable in the tennis ball task, and negation by cancellation of a difference for the position of drop variable in the tennis ball task. These subjects did not appear to be able to consider the neutralisation of an effect (Piaget's formal operational control strategy). In addition these subjects did not realise that a larger difference in bounce between the tennis balls could be shown by increasing, by an equal amount, the position of drop for both tennis balls. Such a notion would involve the consideration of the relations between the functions of the variables and this understanding of the H strategy is thus a formal operational one. That 30% of 8 and 9 year old subjects (mean age 8.8 years) could use a 'concrete' control strategy was apparent. In addition the fact that the child's ideas of moral 'fairness' or 'justice' develop at about 9 years of age (Piaget, 1932), suggested that it was at this age that the strategy developed.

The Junior School interviews using the tennis ball task, described in Chapter 9, were undertaken in order to discover the process by which the concrete operational control strategy developed.

The choice of an interview technique was partly dictated by the typically poor written responses from children of Junior School age, but more importantly it was felt that only by observing the subject's interactions with the physical situation could a true picture of a child's reasoning be determined, particularly at the concrete operational level.

The interactive nature of the behaviours, described in Chapter 9, necessarily led to the setting up of procedures in Chapter 10 that could describe in detail the dynamic behaviours. This information processing approach utilised in Chapter 10 revealed four distinct substages in the development of a 'concrete operational' control strategy.

The developmental substages

The four substages in the development of the concrete operational control strategy and the four substages in the development of the formal operational control strategy are presented in Table 11.0.

Three initial realisations marked the transitions between the concrete 'control' substages:

(i) The transition from substage 1 to 2 was marked by the realisation of the need for invariance in results and led to B experimentation.

TABLE 11.0 The Eight Substages

	Concrete 'Control' Substages	Mean Age Years.Months
1	Isolated centration	5.8
2	Unidimensional comparison	6.4
3	Bidimensional comparison	7.5
4	Bidimensional comparison, + 1 extra dimension.	9.0
	Formal 'Control' Substages	
1	Isolated centration	12.2
2	Unicriterion comparison	13.1
3	Bicriteria comparison	14.7
4	Tricriteria comparison	16.8

(ii) The transition from substage 2 to 3 was marked by a realisation of the role of extrinsic variables within the experiment and led to all jv experimentation.

(iii) The transition from substage 3 to 4 was marked by a realisation of the role of intrinsic as well as extrinsic variables within the experiment, which led to control experimentation.

It appears that the strategies applied to the 'control of variables' tasks at the formal level are developed at the concrete level. The compensation strategy was not found to be used by the subjects in the tennis ball interviews, although such a strategy is available to children at the concrete operational level. The study reported in Chapter 7 suggests that this strategy is not used widely until approximately 12 years of age (39% of a sample with mean age 12.1 years). This suggests that these concrete strategies are applied to a variety of tasks and tend to become task specific at the early formal level.

For the H strategy one specific task is the 'olympic' type situation or empirical task. For the C strategy one specific task is the 'handicap' race situation, one version of a quasi/empirical task. For the G.D. strategy one specific task appears to be the meter task, another version of a quasi/empirical task, where the roles of the variables are unfamiliar and thus are not obviously compensatory. Examples of the specific role of the G.D. strategy are to be found in G.L.T. where the child, after achieving an unexpected result, i.e. it contradicts his/her intuitions, manipulates the experiment to show what he/she wants.
It appears most likely that the other strategies identified at the concrete level are task specific e.g., the test of strength strategy appears only to be used in experimentation.

At the end of the concrete operational stage therefore, the subjects appear to have available PLANS and are aware of their 'practical' and 'reasons for use' aspects, i.e. aware of the strategies and arguments. The criterion for use of a particular strategy may be considered to be a simplified statement of its argument. The substages in the development of the 'control of variables' at the formal operational level involve comparison of the criteria for use of the strategies, i.e. the simplified arguments. Lunzer (1965) argues, in a consideration of the 'control of variables' tasks and the 'proportionality' tasks described in G.L.T., that a feature of formal reasoning is the elaboration of second order relations, that is, relations between relations. First order relations are those achieved at the concrete level as they are merely relations between objects. In this light the concrete substages may be viewed as phases in the ability to handle first order relations or strategies, i.e. NUPLAN behaviours. The formal substages may be viewed as phases in the ability to handle second order relations or 'arguments' for the use of the strategies.

It is not proposed to discuss in detail the developmental processes as these have been dealt with at length in Chapters 5, 9 and 10. Specific points concerning transition will be discussed when referring to the educational implications of this work. It must be stressed that the substage at which a child may be, at a particular instant, is dependent upon the level of operativity he/she has reached.

M Demand Considerations

The two major proposals for M space, the linear increase theory of Pascual-Leone (1969) and the static theory of Case (1978) were discussed in the introduction to Chapter 10. These theories are easily distinguished by the size of working memory they predict at the concrete and formal operational levels. Table 11.1 compares the size of working memory predicted by Pascual-Leone's theory and Case's theory, with that found for the tennis ball task.

TABLE 11.1

Age	Developmental Substage	Size of Working Memory Predicted by Pascual-Leone.	Size of Working Memory Predicted by Case	Size of Working Memory in tennis ball tasks.
6-8	Late pre-operations/ early concrete operations	2 3	1 2 3	1 2 3
9-10	Middle concrete operations	4	. 4	4
11-12	Late concrete/ early formal operations	5	1	0
13-14	Middle formal operations	6	2 3	1 2
15-16	Late formal operations	7	4	3 4

The value for working memory shown in each case is the space left available once the plan has been attended to. The values for the tennis ball task at the concrete level were established in Chapter 10 and are the NUPLAN values. The values for formal operational substages have not been established and are considered below. It is proposed that the only items included in formal operational level M space calculations should be those that relate to the comparison of arguments, or criteria, for the use of the strategies. The recognition or the 'understanding' of a strategy is considered to be concrete level behaviour and it is considered that at the formal level such recognition is automatic. Thus Isolated Centration, substage 1, has a formal working memory of 0, in that the behaviour was a simple recognition of strategies.

Unicriterion comparison, substage 2, involved the recognition of strategies, but one criterion was then applied to the two remaining strategies, one at a time. It is proposed that the working memory required for each application of the criterion is 1 unit.

Bicriteria comparison, substage 3, involved the application of two criteria to the strategies, one at a time and hence it is proposed that the working memory required for each application of the two criteria is 2 units.

Tricriteria comparison, substage 4 involved the application of the criteria to each strategy, one at a time, and thus it appears that the size of working memory required for this may be 3 units.

The formation of a new criterion at substage 4.2 may require a further 1 unit of working memory. The 'assumptions' criterion appears to be applied as an addition at this level in order to resolve the conflict due to applying all three criteria, in that this integration does not reject any strategies. Thus it appears that this substage may require 4 units of working memory.

The proposed working memory values for the formal operational substage are tentative. The analysis of the M demands for the concrete operational development of the

H strategy revealed that the NUPLAN, i.e. the initial formation of a PLAN, appeared to be the most reliable indicator of the M value for a behaviour. This value gives the size of working memory required before a child may successfully progress to a new behaviour. It was noted that this NUPLAN value appeared to be, in general, 1 unit larger than the value calculated for the execution of a strategy. It might be the case that the M values postulated above for the formal operational substages are 1 unit higher, in that these values were calculated in relation to the application of the criteria and not in relation to the arguments arising from conflicts due to the comparisons. It is apparent that a study of the transitions between the formal substages is required before M values can be satisfactorily calculated.

A structured interview technique that involves the presentation of the inherent conflicts, due to application of the criteria, at the different substages would be likely to reveal the dynamic aspects of the formal level development and hence enable a NUPLAN, M demand calculation. In spite of the tentative nature of the M space values shown in Table 11.1, it appears that M space increases with age in accordance with Case's proposals rather than Pascual-Leone's.

The parallel developmental sequences at the concrete and formal levels with their increasingly sophisticated plans adds further support to Case's proposals. The size of working memory available to be applied to arguments in the development of strategies, appears to increase at the Junior School level from 1 to 4 units. At the formal level the size of working memory available to contrast the arguments for use of the strategies also appears to increase to a maximum of 4 units.

<u>Methodological Problems</u>

Before concluding by pointing out some of the educational implications of this study, it is necessary to indicate and clarify some of the more significant methodological problems inherent in some of the studies in this thesis.

First and perhaps the most important limitation applies to the proposed concrete and formal developmental sequences for the H strategy, in that the studies upon which the sequences were based involved cross-sectional testing of age groups. The need for longitudinal studies in relation to the Secondary School development was mentioned in the concluding remarks to Chapter 5. Although longitudinal studies are to be recommended the many practical difficulties make this procedure a difficult one to put into practice. The structured interview technique used in Chapter 9, which allowed subjects to make post-hoc corrections, was found to provide evidence of transitional behaviour. This approach, although not as reliable as longitudinal studies, provided evidence of the existence and order of stages due to these noted transitions between stages.

A second limitation was due to the use of 6th form subjects for the samples of 16 to 19 year olds. The statutory

School leaving age for the subjects tested was 16 years of age and thus the samples of children over 16 years of age were of higher than average ability. Thus the results for these subjects may represent a potential, not achieved by the majority of the general population. Thus substage 4 for the formal operational stage may represent a behaviour only achieved by the minority of the adult population.

Thirdly it should be noted that, wherever possible, subject's responses were checked if they were unclear, in order to guard against mis-classifying the level of reasoning. In addition, whenever younger subjects (below 12 years of age) were tested, each question was read through with the group being tested, but no help was given with written responses. Thus the questionnaire responses for the youngest subjects may represent levels of reasoning below their potential. Nevertheless, the interview technique used in Chapter 9 tends to support the data found by questionnaire for the early use of the H strategy.

Educational Implications

Strong educational implications may be derived from these findings in relation to the age at which, and the manner in which, children should be introduced to 'tasks' involving the 'control of variables'. It appears that although the concrete level control strategy does not involve the 'formal logic' of the more mature strategy of control by neutralisation, it may be used to study the role of an independent variable by excluding other confounding independent variables. The acquisition of this skill was shown to be due to the child making and testing his/her own judgments. These 'incorrect' strategies e.g., D experimentation, G.D. experimentation, etc. play an important role in the development of the 'correct' control strategy. The acquisition of the control strategy at the concrete level was noted to be an active exploratory process in which the child discovers relationships and develops strategies through post-hoc corrections. Such a learning process is typically concrete and has been noted by Inhelder et al (1974) in the study of number, length and class inclusion. It seems likely that an approach which encouraged the making of judgments which was followed by individual experimentation, would in itself aid the development of the control strategy, in that conflicts between the judged and experimental results would occur. The emphasis is on individual experimentation in this suggested approach, in that it appears that the child must take an active part in the testing of his judgments in order for post-hoc corrections to be made to his/her strategies. Specific conflicts similar to the conflict and alternative conflict sequences used in Chapter 9 may be introduced where the child's experimentation does not produce a conflict. This approach has two aims .

Firstly, it enables the child to become familiar with a variety of independent variables in a variety of tasks and hence develops an awareness of the operation of intrinsic variables within certain tasks e.g., the bending rods task used in G.L.T. and in the tennis balls tasks used in this thesis.

Secondly, it enables the development of a concrete control strategy that may be applied to tasks, initially to simplify experimentation and then, finally, to control extrinsic variables in order to show the effect of an intrinsic variable.

The choice of task appears to be of prime importance at the concrete level and it appears that it must satisfy three criteria.

Firstly, the task should involve the determination of the effect of an intrinsic variable.

Secondly, the task should involve the use of a limited number of familiar variables, preferably including one extrinsic variable that may be removed by the subjects, e.g., the rug in the tennis ball task, as it was this variable that was controlled initially by the subjects.

Thirdly, the task should involve immediate feedback so that experiments may be immediately evaluated and post-hoc corrections made and then evaluated.

Many tasks exist of the above type e.g., comparison of the 'speeds' of two similar toy racing cars on identical tracks whose slopes and lengths may be altered.

At the formal level the criteria for use of the strategies need to be identified. To this end task specific problems may be introduced so that the child's or class's D, G.D., C and H solutions to these problems may be contrasted. Such a method should reduce the use of the D and G.D. strategies on tasks requiring 'fair' solutions. Further comparisons between the use of the C and H strategies, in terms of 'handicap' and 'olympic' type competition should reduce the use of the C strategy in 'fair' tasks in science. More importantly, these comparisons should aid the development of more sophisticated criteria for the use of the H strategy in relation to the rejected strategies. It should be noted that the emphasis in these formal level proposals for the advancement of the development of the formal 'H' strategy is not upon experimentation, but upon reflection upon the use of already acquired strategies.

A common feature of both the concrete and formal 'training' proposals is the use of alternative strategies, i.e. 'mistakes'. Early mistakes, such as making judgments and verifying them, and later ones, such as using the compensation strategy, were found to play a crucial role in the development of the H strategy and it appears that they should be 'encouraged' and utilised rather than penalised by educators.

M demand was noted to play a crucial role in the development of the concrete and formal H strategies and careful task analysis must be undertaken so that the M demand is reduced to a minimum for any task utilised.

In summary, it may be stated that educators armed with prior knowledge concerning the developmental sequences noted at the concrete and formal levels for the H strategy should be able to introduce tasks that aid these sequences. It is to be hoped therefore that individualised investigative science tasks, especially physics tasks, would be introduced at the third and fourth year Junior School level by such 'informed' educators.

It should be noted that Elkind's (1972) view that instruction in controlled experimentation should generally not be introduced until adolescence, is not supported by the studies presented in this thesis. It appears that the instruction of controlled experimentation using techniques similar to those proposed above could be introduced at approximately 9 years of age.

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APPENDIX A

FOURFOLD TABLES SHOWING McNEMAR TESTS FOR SIGNIFICANT CHANGES BETWEEN QUESTIONS.

(The Binomial test was applied as the expected cell frequencies were less than 5, i.e., $\frac{1}{2}(A+D) < 5$.

THIRD YEAR

W1 \ W2	Not H	H
H	0	10
Not H	14	6

$p = 0.016$*

W1 \ J1	Not H	H
H	0	10
Not H	20	0

$p > 0.05$ n.s.

W1 \ J2	Not H	H
H	0	10
Not H	15	5

$p = 0.031$*

W2 \ J1	Not H	H
H	6	10
Not H	14	0

$p = 0.016$ *

FOURTH YEAR

W1 \ W2	Not H	H
H	0	15
Not H	8	7

$p = 0.008$ **

W1 \ J1	Not H	H
H	0	15
Not H	13	2

$p > 0.05$ n.s.

W1 \ J2	Not H	H
H	0	15
Not H	8	7

$p = 0.008$**

W2 \ J1	Not H	H
H	5	17
Not H	8	0

$p = 0.031$*

(continued......./over)

Appendix A continued....

THIRD YEAR

W2

J2	Not H	H
H	1	15
Not H	14	0

$p > 0.05$ n.s

J1

J2	Not H	H
H	0	10
Not H	15	5

$p = 0.031$*

FOURTH YEAR

W2

J2	Not H	H
H	0	22
Not H	8	0

$p > 0.05$ n.s.

J1

J2	Not H	H
H	0	17
Not H	8	5

$p = 0.031$*

Key

n.s. - the differences were not significant
* - the differences were significant at the 0.05 level
** - the differences were significant at the 0.01 level.

APPENDIX B

FREQUENCY OF USE OF THE STRATEGIES ON THE TASKS

YEAR	TASK	N	QUESTION ONE					QUESTION TWO					QUESTION THREE				
			H	C	GD	D	U	H	C	GD	D	U	H	C	GD	D	U
1	Target Sphere	30	2	8	6	13	1	4	9	0	16	1	4	0	6	15	5
	Toy Lorry	30	2	7	5	14	2	5	10	0	13	2	5	0	4	16	5
	Plasticine	30	3	8	6	11	2	5	10	0	13	2	4	0	6	16	4
2	Target Sphere	30	9	12	4	4	1	9	10	0	10	1	14	0	11	4	1
	Toy Lorry	30	10	10	5	4	1	10	11	0	9	0	14	0	12	4	0
	Plasticine	30	10	11	4	4	1	10	10	0	10	0	14	0	12	4	0
3	Target Sphere	30	14	15	1	0	0	14	16	0	0	0	16	0	14	0	0
	Toy Lorry	30	14	13	2	0	1	14	16	0	0	0	17	0	13	0	0
	Plasticine	30	13	14	3	0	0	13	17	0	0	0	17	0	13	0	0
4	Target Sphere	30	18	10	2	0	0	18	12	0	0	0	28	0	2	0	0
	Toy Lorry	30	19	10	1	0	0	19	11	0	0	0	29	0	1	0	0
	Plasticine	30	19	10	1	0	0	20	10	0	0	0	29	0	1	0	0
5	Target Sphere	30	25	5	0	0	0	25	5	0	0	0	30	0	0	0	0
	Toy Lorry	30	25	4	1	0	0	25	5	0	0	0	30	0	0	0	0
	Plasticine	30	26	4	0	0	0	26	4	0	0	0	30	0	0	0	0
6 (Lower 6th)	Target Sphere	10	8	2	0	0	0	9	1	0	0	0	10	0	0	0	0
	Toy Lorry	10	8	1	1	0	0	9	1	0	0	0	10	0	0	0	0
	Plasticine	10	9	0	1	0	0	9	1	0	0	0	10	0	0	0	0
6 (Upper 6th)	Target Sphere	10	9	1	0	0	0	9	1	0	0	0	10	0	0	0	0
	Toy Lorry	10	8	2	0	0	0	8	2	0	0	0	10	0	0	0	0
	Plasticine	10	9	1	0	0	0	9	1	0	0	0	10	0	0	0	0

APPENDIX C

FOURFOLD TABLES SHOWING McNEMAR TESTS FOR SIGNIFICANT CHANGES BETWEEN THE TARGET SPHERE, TOY LORRY AND PLASTICINE TASKS.

QUESTION 1

Target Sphere Task (rows) × Toy Lorry Task (columns)

	Not H	H
H	5	80
Not H	79	6

$X^2 = 0$ n.s.

	Not C	C
C	9	44
Not C	114	3

$X^2 = 2.08$ n.s.

	Not GD	GD
GD	4	9
Not GD	151	6

$X^2 = 0.1$ n.s.

	Not D	D
D	3	14
Not D	149	4

p = 0.5 n.s.

Target Sphere Task (rows) × Plasticine Task (columns)

	Not H	H
H	5	80
Not H	76	9

$X^2 = 0.64$ n.s.

	Not C	C
C	9	44
Not C	113	4

$X^2 = 1.23$ n.s.

	Not GD	GD
GD	4	9
Not GD	151	6

$X^2 = 0.1$ n.s.

	Not D	D
D	5	12
Not D	150	3

p = 0.36 n.s.

Toy Lorry Task (rows) × Plasticine Task (columns)

	Not H	H
H	6	80
Not H	75	9

$X^2 = 0.27$ n.s.

	Not C	C
C	3	44
Not C	119	4

p = 0.5 n.s.

	Not GD	GD
GD	4	11
Not GD	151	4

$X^2 = 0$ n.s.

	Not D	D
D	6	12
Not D	149	3

p = 0.25 n.s.

The Binomial test was applied when the expected cell frequencies were less than 5, i.e., $\frac{1}{2}(A + D) < 5$.

(continued........../over)

Appendix C continued....

QUESTION 2

Toy Lorry Task (rows: Target Sphere Task)

	Not H	H
H	8	80
Not H	72	10

$X^2 = 0.05$ n.s.

	Not C	C
C	4	50
Not C	110	6

$X^2 = 0.1$ n.s.

	Not D	D
D	6	20
Not D	142	2

p = 0.15 n.s.

Plasticine Task (rows: Target Sphere Task)

	Not H	H
H	8	80
Not H	70	12

$X^2 = 0.45$ n.s.

	Not C	C
C	5	49
Not C	112	4

p = 0.5 n.s.

	Not D	D
D	6	20
Not D	141	3

p = 0.25 n.s.

Plasticine Task (rows: Toy Lorry Task)

	Not H	H
H	10	80
Not H	68	12

$X^2 = 0.05$ n.s.

	Not C	C
C	6	50
Not C	111	3

p = 0.25 n.s.

	Not D	D
D	3	19
Not D	144	4

p = 0.5 n.s.

QUESTION 3

Toy Lorry Task (rows: Target Sphere Task)

	Not H	H
H	4	108
Not H	51	7

$X^2 = 0.36$ n.s.

	Not GD	GD
GD	4	29
Not GD	136	1

p = 0.19 n.s.

	Not D	D
D	2	17
Not D	148	3

p = 0.5 n.s.

Plasticine Task (rows: Target Sphere Task)

	Not H	H
H	4	108
Not H	52	6

$X^2 = 0.1$ n.s.

	Not GD	GD
GD	4	29
Not GD	134	3

p = 0.5 n.s.

	Not D	D
D	2	17
Not D	148	3

p = 0.5 n.s.

Plasticine Task (rows: Toy Lorry Task)

	Not H	H
H	7	108
Not H	49	6

$X^2 = 0$ n.s.

	Not GD	GD
GD	2	28
Not GD	136	4

p=0.34 n.s.

	Not D	D
D	3	17
Not D	147	3

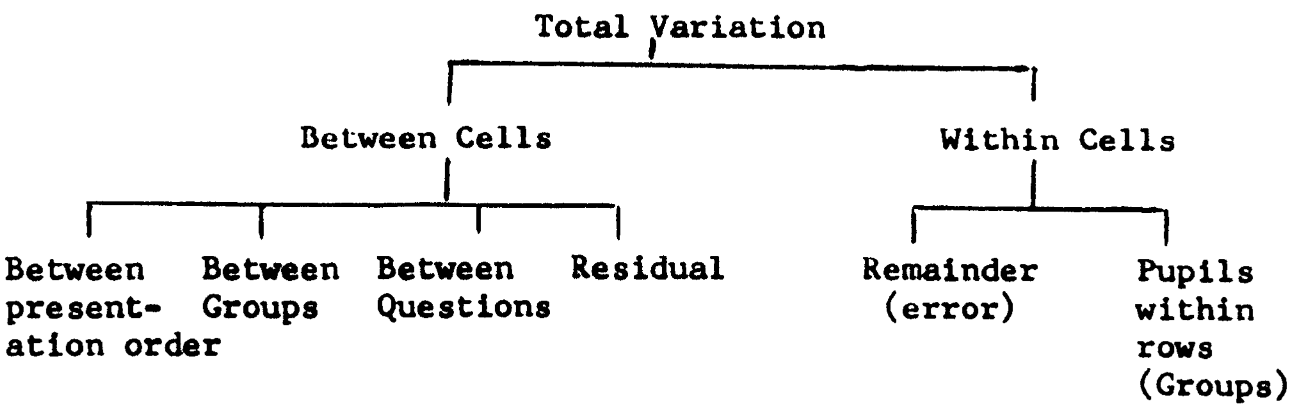
p = 0.67 n.s.

n.s. = the differences were not found to be significant, $p > 0.05$

APPENDIX D

ANALYSIS OF VARIANCE FOR THE LATIN SQUARE

The total sum of the squares was first resolved into sums for between cells and within cells, the sums for lists, groups, treatments appearing as components of the former, as shown below.



The within cells sum was resolved into the effect of the same persons nested within row, and a remainder or error sum of squares. The F ratio calculation uses the mean square error term as denominator rather than the mean square residual term (Mitchell-Dayton, 1970, p144; Lindquist, 1953, p260 & p279) The Newman-Keuls test was utilised for testing the significance of the difference between two questions and calculated from the ranked question means, as this is the most conservative approach available. Multiple t tests were not used since pairs of means which differ by several rank positions have true probability levels which may be enormous compared with the nominal risk of a type 1 error, i.e. rejection of the null hypothesis when it is true. The use of the Newman-Keuls procedure results in all pairwise contrasts being tested at the same level, in this case $p < .01$. This is directly under the control of the researcher since the studentized range statistic takes into account the rank separation of the pair of means entering into a contrast. Thus the risk of a type 1 error is constant and is reduced. The mean scores were used to rank order the questions and then pairwise contrasts were made when a general significant difference was found e.g.,

$$\frac{\text{Q3 mean} - \text{Q2 mean}}{\sqrt{m_s \text{ error}/30}}$$

The denominator used was the standard error for the mean based on 30 observations. A high level of significance for the F tests was set in order to avoid a type 1 error, thus the 0.01 level was selected.

APPENDIX D (continued)

ANALYSIS OF VARIANCE OF 3 X 3 LATIN SQUARE DATA

SECOND YEAR (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.27	2	0.135	1.25	n.s.
GROUPS	0.07	2			
QUESTIONS	3.8	2	1.9	17.59	$p < .01$
RESIDUAL	0.06	2			
PERSONS P WITHIN ROWS	11.53	27			
REMAINDER (ERROR)	5.87	54	0.108		
TOTAL	21.6				

Rank Order of Question Means

Q2 = 0.633

Q3 = 0.433

Q1 = 0.133

Newman-Keuls Test
Significant pairwise contrasts

Q2 V_S Q3 = 3.33 $p < 0.05$

Q2 V_S Q1 = 8.33 $p < 0.01$

Q3 V_S Q1 = 5.00 $p < 0.01$

SECOND YEAR (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.022	2	0.11	0.09	n.s.
GROUPS	0.022	2			
QUESTIONS	7.35	2	3.675	30.12	$p < .01$
RESIDUALS	0.026	2			
PERSONS P WITHIN ROWS	6.3	27			
REMAINDER (ERROR)	6.6	54	0.122		
TOTAL	20.32				

Rank Order of Question Means

Q1 = 0.700

Q3 = 0.333

Q2 = 0.000

Newman-Keuls Test
Siginificant pairwise contrasts

Q1 V_S Q3 = 5.76 $p < 0.01$

Q1 V_S Q2 = 10.98 $p < 0.01$

Q3 V_S Q2 = 5.22 $p < 0.01$

APPENDIX D (continued)

ANALYSIS OF VARIANCE OF 3 X 3 LATIN SQUARE DATA

THIRD YEAR (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.022	2	0.011	0.106	n.s.
GROUPS	0.022	2			
QUESTIONS	3.289	2	1.645	15.817	$p < .01$
RESIDUAL	0.087	2			
PERSONS WITHIN ROWS	13.3	27			
REMAINDER (ERROR)	5.6	54	0.104		
TOTAL	27.32				

Rank Order of Question Means

Q2 = 0.767

Q3 = 0.567

Q1 = 0.300

Newman-Keuls Test
Significant pairwise contrasts

Q2 V_S Q3 = 3.40 $p < 0.05$

Q2 V_S Q1 = 7.93 $p < 0.01$

Q3 V_S Q1 = 4.53 $p < 0.01$

THIRD YEAR (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	p
PRESENTATION	0.066	2	0.033	0.27	n.s.
GROUPS	0.066	2			
QUESTIONS	6.667	2	3.334	27.278	$p < .01$
RESIDUAL	0.001	2			
PERSONS WITHIN ROWS	6.6	27			
REMAINDER (ERROR)	6.6	54	0.122		
TOTAL	20.00				

Rank Order of Question Means

Q1 = 0.667

Q3 = 0.333

Q2 = 0.000

Newman-Keuls Test
Significant pairwise contrasts

Q1 V_S Q3 = 5.24 $p < 0.01$

Q1 V_S Q2 = 10.46 $p < 0.01$

Q3 V_S Q2 = 5.22 $p < 0.01$

APPENDIX D (continued)

ANALYSIS OF VARIANCE OF 3 X 3 LATIN SQUARE DATA

FOURTH YEAR (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.2	2	0.1	0.962	n.s.
GROUPS	0.067	2			
QUESTIONS	3.467	2	1.734	16.673	$p < .01$
RESIDUAL	0.066	2			
PERSONS WITHIN ROWS	10.6	27			
REMAINDER (ERROR)	5.6	54	0.104		
TOTAL	20.00				

Rank Order of Question Means

Q2 = 0.867

Q3 = 0.733

Q1 = 0.400

Newman-Keuls Test
Sigificant pairwise contrasts

Q2 V_S Q3 = 2.28 n.s.

Q2 V_S Q1 = 7.93 $p < .01$

Q3 V_S Q1 = 5.66 $p < .01$

FOURTH YEAR (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.067	2	0.034	0.287	n.s.
GROUPS	0.067	2			
QUESTIONS	4.867	2	2.434	20.537	$p < .01$
RESIDUAL	0.001	2			
PERSONS WITHIN ROWS	6.199	27			
REMAINDER (ERROR)	6.4	54	0.119		
TOTAL	17.6				

Rank Order of Question Means

Q1 = 0.567

Q3 = 0.233

Q2 = 0.000

Newman-Keuls Test
Significant pairwise contrasts

Q1 V_S Q3 = 5.3 $p < .01$

Q1 V_S Q2 = 9.0 $p < .01$

Q3 V_S Q2 = 3.72 $p < .05$

APPENDIX E

FOURFOLD TABLES SHOWING McNEMAR TESTS FOR SIGNIFICANCE BETWEEN THE BLIND, METER AND PLASTICINE TASKS

QUESTION 1

H STRATEGY

plasticine \ blind	Not H	H
H	4	14
Not H	12	8

$X^2 = 0.75$ n.s.

plasticine \ meter	Not H	H
H	4	14
Not H	13	7

$X^2 = 0.36$ n.s.

blind \ meter	Not H	H
H	3	19
Not H	14	2

p = 0.5 n.s.

C STRATEGY

plasticine \ blind	Not C	C
C	11	3
Not C	22	2

$X^2 = 4.92$*

plasticine \ meter	Not C	C
C	11	3
Not C	22	2

$X^2 = 4.92$*

blind \ meter	Not C	C
C	3	2
Not C	30	3

$X^2 = 0$ n.s

G.D. STRATEGY

plasticine \ blind	Not GD	GD
GD	5	1
NotGD	21	7

$X^2 = 0.08$ n.s.

plasticine \ meter	Not GD	GD
GD	5	1
Not GD	21	11

$X^2 = 1.56$ n.s.

blind \ meter	Not GD	GD
GD	1	7
Not GD	25	5

p = 0.188 n.s.

McNemar one tailed test df = 1 was used if the expected frequency (cells A + D) was A + D $\geq$ 5, if not the Bionomial test was used.

Key

n.s. - not significant
* - significant at the 0.025 level
** - significant at the 0.005 level

(continued ....over/)

Appendix E continued....

QUESTION 2

H STRATEGY

plasticine \ blind	Not H	H
H	6	22
Not H	6	4

$X^2 = 0.1$ n.s.

plasticine \ meter	Not H	H
H	6	22
Not H	6	4

$X^2 = 0.1$ n.s.

blind \ meter	Not H	H
H	4	22
Not H	8	4

$p = 0.637$ n.s.

C STRATEGY

plasticine \ blind	Not C	C
C	3	5
Not C	27	3

$p = 0.656$ n.s.

plasticine \ meter	Not C	C
C	5	3
Not C	27	3

$p = 0.363$ n.s.

blind \ meter	Not C	C
C	5	3
Not C	27	3

$p = 0.812$ n.s.

G.D. STRATEGY

plasticine \ blind	Not GD	GD
GD	0	0
Not GD	35	3

$p > 0.5$ n.s.

plasticine \ meter	Not GD	GD
GD	0	0
Not GD	35	3

$p > 0.5$ n.s.

blind \ meter	Not GD	GD
GD	1	2
Not GD	34	1

$p > 0.5$ n.s.

QUESTION 3

H STRATEGY

plasticine \ blind	Not H	H
H	2	28
Not H	5	3

$p = 0.5$ n.s.

plasticine \ meter	Not H	H
H	11	19
Not H	6	2

$X^2 = 4.92$*

blind \ meter	Not H	H
H	11	20
Not H	6	1

$X^2 = 6.75$**

C STRATEGY

plasticine \ blind	Not C	C
C	0	0
Not C	36	2

$p > 0.5$ n.s.

plasticine \ meter	Not C	C
C	0	0
Not C	38	0

n.s.

blind \ meter	Not C	C
C	2	0
Not C	36	0

$p > 0.5$ n.s.

G.D. STRATEGY

plasticine \ blind	Not GD	GD
GD	3	4
Not GD	31	0

$p > 0.5$ n.s.

plasticine \ meter	Not GD	GD
GD	2	5
Not GD	20	11

$X^2 = 4.92$*

blind \ meter	Not GD	GD
GD	1	3
Not GD	21	13

$X^2 = 8.64$**

APPENDIX F

ANALYSIS OF VARIANCE OF 4 X 4 LATIN SQUARE DATA

THIRD YEAR JUNIORS (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARE	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.16	3	0.053	2.3	ns
GROUPS	0.08	3			
QUESTIONS	4.87	3	1.623	70.56	$p < 0.01$
RESIDUAL	0.38	6			
PERSONS P WITHIN ROWS	7.51	52			
REMAINDER (ERROR)	15.13	156	0.097		
TOTAL	28.13				

Rank Order of Question Means

Q4 = 0.304
Q2 = 0.286
Q1 = 0.000
Q3 = 0.000

Newman-Keuls Test
Significant pairwise contrasts

Q4 V_S Q1)
Q4 V_S Q3) = 7.3 $p < 0.01$

Q2 V_S Q1)
Q2 V_S Q3) = 6.87 $p < 0.01$

THIRD YEAR JUNIORS (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0	3	-		
GROUPS	0	3	-		
QUESTIONS	0	3	-		
RESIDUAL	0	6	-		
PERSON P WITHIN ROWS	0	52	-		
REMAINDER (ERROR)	0	156	-		
TOTAL	0				

ANALYSIS OF VARIANCE OF 4 x 4 LATIN SQUARE DATA

FOURTH YEAR JUNIORS (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.018	3	0.007	0.27	ns
GROUPS	0.16	3			
QUESTIONS	7.91	3	2.64	101.54	$p < 0.01$
RESIDUAL	0.18	6			
PERSON P WITHIN ROWS	8.49	52			
REMAINDER (ERROR)	17.37	156	0.111		
TOTAL	34.128				

Rank Order of question means

Q4	=	0.393
Q2	=	0.357
Q1	=	0.000
Q3	=	0.000

Newman-Keuls test

Significant pairwise contrasts

Q4 Vs Q1) Q4 Vs Q3)	= 8.83 $p<0.01$
Q2 Vs Q1) Q2 Vs Q3)	= 8.02 $p<0.01$

FOURTH YEAR JUNIOR (COMPENSATION METHODOLOGY)

SOUR OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.036	3	0.012	3.41	ns
GROUPS	0.071	3			
QUESTIONS	0.071	3	0.024	6.83	ns
RESIDUAL	0.46	6			
PERSON P WITHIN ROWS	0.96	52			
REMAINDER (ERROR)	2.32	156	0.015		
TOTAL	3.912				

ANALYSIS OF VARIANCE OF 4 x 4 LATIN SQUARE DATA

1ST YEAR SECONDARY (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	P
PRESENTATION ORDER	0.18	3	0.06	1.77	ns
GROUPS	0.07	3			
QUESTIONS	16.21	3	5.40	159.32	$p<.01$
RESIDUAL	0.61	6			
PERSONS P WITHIN ROWS	13.06	52			
REMAINDER (ERROR)	22.37	156	0.143		
TOTAL	52.50				

Rank Order of Question Means			Newman-Keuls Test Significant pairwise contrasts		
Q4	=	0.643	Q4 Vs Q1	= 9.54	$p<.01$
Q2	=	0.643	Q4 Vs Q3	= 11.66	$p<.01$
Q1	=	0.161	Q2 Vs Q1	= 9.54	$p<.01$
Q3	=	0.054	Q2 Vs Q3	= 11.66	$p<.01$

FIRST YEAR SECONDARY (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df.	MEAN SQUARE	F Ratio	P
PRESENTATION ORDER	0.09	3	0.03	1.14	ns
GROUPS	0.09	3			
QUESTIONS	7.91	3	2.64	100.43	$p<.001$
RESIDUAL	0.18	6			
PERSONS P WITHIN ROWS	8.51	52			
REMAINDER (ERROR)	17.35	156	0.111		
TOTAL	34.13				

Rank order of question means			Newman-Keuls test Significant pairwise contrasts		
Q1	=	0.393	Q1 Vs Q2	= 8.83	$p < 0.01$
Q3	=	0.357	Q1 Vs Q4	= 8.83	$p < 0.01$
Q2	=	0.000	Q3 Vs Q2	= 8.02	$p < 0.01$
Q4	=	0.000	Q3 Vs Q4	= 8.02	$p < 0.01$

ANALYSIS OF VARIANCE OF 4 X 4 LATIN SQUARE DATA

SECOND YEAR SECONDARY (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.16	3	0.053	1.52	ns
GROUPS	0.12	3			
QUESTIONS	16.83	3	5.61	161.12	$p < 0.01$
RESIDUAL	0.17	6			
PERSONS P WITHIN ROWS	13.38	52			
REMAINDER (ERROR)	22.98	156	0.147		
TOTAL	53.64				

Rank order of question means

Q4	=	0.946
Q2	=	0.786
Q1	=	0.411
Q3	=	0.268

Newman-Keuls test
Significant pairwise contrasts

Q4 v_S Q1	=	10.44	$p < 0.01$
Q4 v_S Q3	=	13.23	$p < 0.01$
Q2 v_S Q1	=	7.32	$p < 0.01$
Q2 v_S Q3	=	10.11	$p < 0.01$

SECOND YEAR SECONDARY (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.16	3	0.053	2.01	ns
GROUPS	0.05	3			
QUESTIONS	13.16	3	4.387	165.93	$p < 0.01$
RESIDUAL	0.18	6			
PERSONS P WITHIN ROWS	9.98	52			
REMAINDER (ERROR)	17.45	156	0.112		
TOTAL	40.98				

Rank Order of question means

Q1	=	0.518
Q3	=	0.447
Q2	=	0.000
Q4	=	0.000

Newman-Keuls test
Significant pairwise contrasts

Q1 v_S Q2	=	11.58	$p < 0.01$
Q1 v_S Q4	=	11.58	$p < 0.01$
Q3 v_S Q2	=	10.00	$p < 0.01$
Q3 v_S Q4	=	10.00	$p < 0.01$

ANALYSIS OF VARIANCE OF 4 X 4 LATIN SQUARE DATA

THIRD YEAR SECONDARY (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.04	3	0.013	0.37	ns
GROUPS	0.04	3			
QUESTIONS	13.04	3	4.347	122.81	$p<0.01$
RESIDUAL	0.17	6			
PERSONS P WITHIN ROWS	12.21	52			
REMAINDER (ERROR)	23.36	156	0.149		
TOTAL	48.86				

Rank order of question means

Q4 = 0.929
Q2 = 0.911
Q3 = 0.446
Q1 = 0.429

Newman-Keuls test
Siginificant pairwise contrasts

Q4 V_S Q3 = 9.36 $p<0.01$
Q4 V_S Q1 = 9.69 $p<0.01$
Q2 V_S Q3 = 9.02 $p<0.01$
Q2 V_S Q1 = 9.34 $p<0.01$

THIRD YEAR SECONDARY (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.05	3	0.0167	0.64	ns
GROUPS	0.02	3			
QUESTIONS	13.16	3	4.387	167.07	$p<0.01$
RESIDUAL	0.18	6			
PERSONS P WITHIN ROWS	10.24	52			
REMAINDER (ERROR)	17.33	156	0.111		
TOTAL	40.98				

Rank order of questions means

Q1 = 0.518
Q3 = 0.446
Q2 = 0.000
Q4 = 0.000

Newman-Keuls test
Siginificant pairwise contrasts

Q1 V_S Q2 = 11.63 $p<0.01$
Q1 V_S Q4 = 11.63 $p<0.01$
Q3 V_S Q2 = 10.02 $p<0.01$
Q3 V_S Q4 = 10.02 $p<0.01$

ANALYSIS OF VARIANCE OF 4 X 4 LATIN SQUARE DATA

FOURTH YEAR SECONDARY (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.20	3	0.067	2.21	ns
GROUPS	0.20	3			
QUESTIONS	6.48	3	2.16	71.1	$p < 0.01$
RESIDUAL	0.53	6			
PERSONS P WITHIN ROWS	9.09	52			
REMAINDER (ERROR)	20.05	156	0.139		
TOTAL	36.55				

Rank order of question means

Q4 = 0.964
Q2 = 0.964
Q3 = 0.643
Q1 = 0.607

Newman-Keuls test
Significant pairwise contrasts

Q4 V_S Q3 = 6.69 $p < 0.01$
Q4 V_S Q1 = 7.44 $p < 0.01$
Q2 V_S Q3 = 6.69 $p < 0.01$
Q2 V_S Q1 = 7.44 $p < 0.01$

FOURTH YEAR SECONDARY (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0	3	0	0	ns
GROUPS	0.14	3			
QUESTIONS	7.14	3	2.38	91.54	$p < 0.01$
RESIDUAL	0.43	6			
PERSONS P WITHIN ROWS	8.18	52			
REMAINDER (ERROR)	16.97	156	0.109		
TOTAL	32.86				

Rank order of question means

Q1 = 0.357
Q3 = 0.357
Q2 = 0.000
Q4 = 0.000

Newman-Keuls test
Siginificant pairwise contrasts

Q1 V_S Q2 = 8.09 $p < 0.01$
Q1 V_S Q3 = 8.09 $p < 0.01$
Q3 V_S Q2 = 8.09 $p < 0.01$
Q3 V_S Q4 = 8.09 $p < 0.01$

ANALYSIS OF VARIANCE OF 4 X 4 LATIN SQUARE DATA

FIFTH YEAR SECONDARY (CONTROL METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.05	3	0.17	1.29	ns
GROUPS	0.05	3			
QUESTIONS	0.91	3	0.303	22.9	$p < 0.01$
RESIDUAL	0.11	6			
PERSONS P WITHIN ROWS	3.27	52			
REMAINDER (ERROR)	8.73	156	0.056		
TOTAL	13.12				

Rank order of question means

Q4 = 1.00
Q2 = 1.00
Q3 = 0.893
Q1 = 0.857

Newman-Keuls test
Significant pairwise contrasts

Q4 V_S Q3 = 3.38 $p < 0.05$
Q4 V_S Q1 = 4.52 $p < 0.01$
Q2 V_S Q3 = 3.38 $p < 0.05$
Q2 V_S Q1 = 4.52 $p < 0.01$

FIFTH YEAR SECONDARY (COMPENSATION METHODOLOGY)

SOURCE OF VARIATION	SUM OF SQUARES	df	MEAN SQUARE	F Ratio	p
PRESENTATION ORDER	0.05	3	0.017	1.29	ns
GROUPS	0.05	3			
QUESTIONS	0.91	3	0.303	22.9	$p < 0.01$
RESIDUAL	0.11	6			
PERSONS P WITHIN ROWS	3.27	52			
REMAINDER (ERROR)	8.73	156	0.056		
TOTAL	13.12				

Rank order of question means

Q1 = 0.143
Q3 = 0.107
Q2 = 0.000
Q4 = 0.000

Newman-Keuls test
Significant pairwise contrasts

Q1 V_S Q2 = 4.52 $p < 0.01$
Q1 V_S Q4 = 4.52 $p < 0.01$
Q3 V_S Q2 = 3.38 $p < 0.05$
Q3 V_S Q4 = 3.38 $p < 0.05$

A P P E N D I X

G

PROTOCOLS
FOR THE STUDY DESCRIBED IN
CHAPTER 9

PROTOCOL

Stage 1.1   BAR (75 months)

The Subject's Experiment

S I think the yellow one is the bounciest
E How might you show this?
S I don't know, feel it?
E What about showing how well it bounces if it's bouncier than the white one you should be able to show this?

BAR quickly holds up both tennis balls and then drops them, the yellow dropping on the floor, the white onto the rug. The yellow tennis ball bounced slightly higher.

E What did you do?
S I dropped them onto the ground
E Did they bounce on the rug or the floor?
S I don't know
E What height did you drop them from?
S I don't know, the yellow is the best anyway.
E Is that what your experiment showed?
S I think so.

Peter's and John's experiments

BAR did not understand Peter's or John's experiments but he thought they were correct because he agreed with their results.

S He thinks the same as me and he is right.

The Conflict Sequence

*E (showed John's reversed)
Do you agree with John that the experiment shows that the white tennis ball is the bounciest?
S Yes the white one is bounciest.
E Why does it bounce the highest?
S Because it is the best

*E (Showed John's experiment)
Do you agree with John that the experiment shows that the yellow tennis ball is the bounciest?
S Yes the yellow one is the best
E Why does the yellow one now bounce the highest?
S Because it is the best
E So which is the best ball?
S The yellow one. (The subject is centering on the last result)
E Oh! The yellow one is? (Showed John's reversed experiment)
S Now, the white one is
E So which is the best ball overall?
S The white one,(The subject was centering on the last result) and the yellow one last time
E You keep changing your mind which is the best tennis ball?
S The White one that time, then before, the yellow one.

(continued...../

Protocol BAR continued...

E They can't both be the best ball
S But they are
E Why does the yellow one win sometimes and then the white one win sometimes?
S I don't know
E Which tennis ball are you going to buy then?
S I don't know ... I think the yellow one I thought it was the best
E Watch what I do, now (Showed John's reversed experiment) Did I drop one from higher?
S Yes, the white one
E Which one bounced the highest?
S The white one
E So which one is the bounciest?
S The white one
E Watch what I do now (showed John's experiment) Did I drop one from higher?
S Yes, the yellow one
E Which one bounced the highest?
S The yellow one
E So which one is the bounciest?
S The yellow one, that's what I thought to begin with, it looks bounciest, bigger

The 'H' Experiment

BAR did not understand the experiment.

S The yellow one just bounced higher
E Where did I drop the yellow one from, higher or lower than the white one?
S Lower? I don't know.

PROTOCOL

Stage 1.2 NEV (70 months)

The Subject's Experiment

NEV did not know what to do, but on prompting he threw both tennis balls onto the soft rug.

E Why not bounce the tennis balls to see if one goes higher?
S I see if it bounces high its best (Does an experiment)
The yellow one bounced highest
E What did you do?
S I threw them
E Did you throw one from a higher height than the other?
S I don't think so
E Could you have done?
S Yes, it won't matter
E Where did you throw them onto?
S The rug
E Why did you throw them onto the rug rather than the floor?
S No real reason
E Would it be alright if you had thrown one onto the floor and one onto the rug?
S Yes
E What does your experiment prove?
E The yellow one's the best
E Why does it prove that?
S Because it went higher, its newest anyway.

Peter's and John's experiments

NEV thought John and Peter were right in saying that the yellow tennis ball was the bounciest.

S The yellow one won, so he's right
(Note that the subject just considered the fact that both his observed result and what John and Peter observed as their result are the same)

The Conflict Sequence

* E After each experiment say which ball you think is the bounciest (Showed John's and John's reversed experiments alternately)

S The white one, the yellow one, the white one, the yellow one
E Which is the bounciest tennis ball?
S The yellow one
E (showed John's reversed experiment)
Which is the bounciest tennis ball?
S The white one
E So which is the bounciest tennis ball?
S ....
E Do you know?
S No, they're both best
E Why does the yellow tennis ball win sometimes and the white tennis ball win other times?

(continued...../

Protocol NEV continued...

S I don't know
E Watch what I do

(Showed John's experiment)

E Did I drop one from higher?
S Yes, the yellow one
E Which one bounced the highest
S The yellow one
E Why did it bounce the highest?
S Because it's bounciest
E Watch what I do now (Showed John's reversed experiment) Which one bounced the highest?
S The white one
E So which one is the bounciest?
S The white one
E Why does the white tennis ball win sometimes and the yellow tennis ball win other times?
S I don't know
E Which tennis ball are you going to buy then?
S The yellow one, it can't be both, I think one should win all the time, I don't get it
E Why the yellow one?
S It bounced better overall, it went the highest. The white one only made it up to there (points to 3 ft mark), the yellow went right up to there, once (points to the 6ft mark)

The 'H' Experiment

S What does it show? I think the yellow one bounced the best but I'm not sure.

PROTOCOL

Stage 2.1   MIS (83 months)

The Subject's Experiment

MIS dropped the yellow and the white tennis balls from about the same height, the yellow bounced onto the floor the white onto the rug. The yellow one bounced the highest.

E Where did you drop the yellow one from, higher or lower than the white one?
S I can't remember
E Where did you drop the tennis balls onto?
S Did they both drop here ... (points to the rug) ... I don't know
E Which tennis ball is the best?
S The yellow one
E Why does it bounce the highest?
S Because it is the bounciest.

Peter's and John's Experiments

MIS accepted both experiments because they confirmed his result. The experimental methods are not appreciated.

S John said he thought the yellow was the best and he showed it. He is right

The Conflict Sequence

*E (Showed John's reversed experiment)
Which is the best ball?
S He's right the white one is the best that time
E Why does it bounce the highest?
S I'm not sure
E (Showed John's experiment)
Which is the best ball?
S The yellow one, now. I see why?
E Why does it bounce the highest?
S Because it is the best and that time it was dropped from highest.
E (Showed John's reversed experiment)
S The white one is the best because it was dropped from higher
E So which is the best tennis ball
S The yellow one, I think it was really the best. It bounced once up here (MIS pointed to the 4 ft mark) the white only went here (MIS pointed to the 3 ft mark).

The 'H' Experiment

S That's no good, they both went about the same
E They might be of equal bounce.
S You have to choose one, one is better than another

PROTOCOL

Stage 2.2 MAN (88 months)

The Subject's Experiment

MAN dropped the white tennis ball from slightly higher than the yellow tennis ball. The yellow tennis ball was dropped onto the rug, the white tennis ball was dropped onto the floor. The white tennis ball bounced slightly higher.

S The yellow tennis ball is the best, but it didn't bounce the highest
E What do you mean?
S I guessed the yellow one was the best, but I didn't show it. I will now

MAN dropped the yellow from higher than the white. The yellow onto the floor the white onto the rug.

S There, the yellow tennis ball is the best
E Why did the white tennis ball bounce the highest the first time?
S I made a mistake I bounced it from higher
E Why did the yellow tennis ball bounce higher the second time?
S Because I dropped it from higher
E Is this yellow tennis ball (higher) bouncier than the yellow tennis ball (lower)
S Yes, because it's higher
E So is this one up here different to this one down here?
S Yes
E Why is it different?
S Because the one that's highest bounces higher
E So its a different ball is it?
S Yes
E But its the same yellow ball
S Yes ... No, because one's higher
E Your telling me that this ball (yellow high) is a different ball to that ball (yellow low)
S Yes

Peter's and John's Experiments

MAN judges both Peter's and John's experiments to be correct because they assumed the same correct result (that the yellow was the best tennis ball).

The Conflict Sequence

E (Showed John's experiment)
Do you agree with John that the experiment showed that the yellow tennis ball is the bounciest?
S Yes, it went highest, he made sure
E Do you think that his experiment was right or wrong ?
S It was right
E (Showed John's Reversed Experiment)
S He showed that the white one was the bounciest, he's right. It was better the first time because the yellow ball is really the bounciest

(continued ..../

The 'H' Experiment

S That didn't show much. The yellow ball was not put high enough that's why it didn't bounce well

PROTOCOL

Stage 2.2 BOS (95 months)

The Subject's Experiment

This subject made no initial judgment.
BOS dropped the yellow tennis ball from slightly higher than the white tennis ball, the yellow dropped onto the rug and the white onto the floor. The yellow tennis ball just bounced highest.

E Where did you drop the yellow one from, higher or lower than the yellow one?
S From higher, I think
E Why did you do that?
S I didn't really mean to, I just bounced them, but it made the yellow bounce well
E Where did you drop the tennis balls onto?
S The yellow one onto the rug and the white one onto the floor, I think
E Why did you do that?
S Because it bounced a bit higher, but it was difficult to tell
E Are you sure?
S I think so
E Show me again. (The subject has now made a judgment)
S I will do this to make sure I can see the yellow is the best, I was wrong to just let them drop

This time the yellow tennis ball is dropped from 4 ft onto the floor and the white tennis ball is dropped from 3 ft onto the rug.

S It did that time
E Why did it do it that time?
S I did it like that (BOS showed that the yellow was dropped from higher onto the floor).
E Why did you do it like that this time?
S I wanted to show the yellow one is the bounciest so I did it from higher so the yellow one was dropped from here

Peter's and John's Experiments

Peter's experiment was not understood but it was accepted because the yellow tennis ball was judged to be the bounciest. John's experiment was understood.

S John made sure the yellow one went the highest. He is right, it is the best.

The Conflict Sequence

E (Showed John's reversed experiment)
Do you agree with John that the experiment showed that the white tennis ball is the bounciest?
S He's right he showed the white one is the bounciest
E Why does it bounce the highest?
S Because it was dropped from higher, it is the best

(continued ....../

BOS protocol continued ...

E (Showed John's experiment)
Do you agree with John that the experiment showed that the yellow tennis ball is the bounciest?
S He's right he showed it, the yellow one is the bounciest
E Why does it bounce the highest?
S Because it was dropped from highest
E So which is really the best ball?
S Well I think it must be the yellow one. I thought so after my go and it looks like it bounced the highest

<u>The 'H' Experiment</u>

S He didn't show it very well. He didn't put the yellow one here (BOS pointed to the 4 ft. mark)
E Why should he do that?
S To show it better

PROTOCOL

Stage 2.3 WOO (105 months)
D experimentation transitional to G.D.

The Subject's Experiment

WOO dropped the yellow tennis ball from higher than the white one and they both dropped onto the floor. The yellow tennis ball bounced highest.

S The yellow one is the bounciest. I knew it was and I have shown it
E Where did you drop the yellow one from, higher or lower than the white one?
S Higher than the white one, so I could show it would bounce the highest
E Where did you drop them onto
S I can't remember, the floor I think, I don't think it matters.

Peter's and John's Experiments

WOO accepted Peter's experiment because he assumed that the yellow tennis ball was the bounciest. He understood John's experiment.

S He did what I did to make sure the yellow was the bounciest by putting it higher up

The Alternative Conflict Sequence

E (Showed Paul's experiment)
Which is the best ball?
S The yellow one, because it dropped onto the floor, the white one dropped onto the rug
E (Showed Paul's reversed experiment)
Which is the best ball?
S The white one, because it dropped onto the floor, the other one dropped onto the rug. This experiment is wrong the first was right, the yellow is the best. What he should have done is this,

WOO performed a G.D. experiment he dropped the yellow from high onto the floor and white from low onto the rug.

S You see the yellow is the best and you can see it is

The 'H' Experiment

S That's not too good
E What's wrong with the experiment?
S Well, you saw what I did, that's better, it shows the difference better.

PROTOCOL

Stage 2.4 MAL (110 months)

The Subject's Experiment

MAL dropped the yellow tennis ball from higher than the white tennis ball. The yellow tennis ball was dropped onto the floor the white was dropped onto the rug.

S The yellow is the best, it won
E Where did you drop the yellow one from, higher or lower than the white one?
S Higher
E Why did you do that?
S The yellow is the best so I put it higher up so I could show it was
E Where did you drop the yellow one onto, the floor or the rug?
S On the floor, it bounces best on the floor. The white one does not bounce as well on the rug so I dropped it there

Peter's and John's Experiments

MAL accepted Peter's experiment because he assumed that the yellow tennis ball was the bounciest. He understood John's experiment.

S He could have dropped the yellow one onto the floor and the white one onto the rug like I did. He's right though, the yellow one is the best

The Conflict Sequence

E (Showed John's reversed experiment)
S That's alright, if he thought the white one was the best he should really have dropped it on the floor as well as dropping it from high up
E (Showed John's experiment)
S That's alright, like I said it is better to drop the one that is best on the floor
E But I thought you said the yellow one was the bounciest and you have just said that the first of John's experiments was right
S Yes
E But he found the white one was the bounciest
S Yes, well he said it was the best and he showed it was.

The 'H' Experiment

S Yes, the yellow was the best, but it didn't go as high as mine, it would be better to do what I did

PROTOCOL

Stage 3.1 PAV (96 months)

The Subject's Experiment

PAV dropped the white tennis ball from slightly higher than the yellow tennis ball. The white tennis ball dropped onto the floor, the yellow one onto the rug.

S The yellow one is the best, it didn't start as high but it went just as high
E Did it go just as high? I thought the white one bounced highest
S Mmm ... yes, it was the floor that did it, it made it better than it should! I'll do this now

PAV dropped the white tennis ball from slightly higher than the yellow tennis ball, both dropped onto the floor.

E What did you do differently that time?
S I dropped them both onto the floor, the white from slightly higher
E Why did you do that?
S To see if the yellow tennis ball could still beat the white one. The floor didn't make the white one go higher that time. The yellow is the best

Peter's and John's Experiments

PAV understood both Peter's and John's experiments and accepted John's experiment because it showed the correct result.

The Conflict Sequence

E (Showed John's reversed experiment)
Which is the best ball?
S The white one
E Why does it bounce the highest?
S The white one was made to bounce highest because it was dropped from higher. I think he is wrong
E (Showed John's experiment)
Which is the best ball?
S The yellow one. I see he made the yellow one bounce the highest that time
E Is that experiment a good one then?
S Yes it is alright, he was right they fell to the floor and showed the right result

The 'H' Experiment

E Showed an 'H' experiment dropping both tennis balls onto the floor from the same height.
S That's not really right they both fell onto the floor I think, the yellow bounces highest, you couldn't see it very clearly
E Which experiment do you prefer to show which tennis ball is the bounciest?
S What I did, like the last one (H experiment) but the white one dropped from a bit higher to see if the yellow one still beats it.

PROTOCOL

Stage 3.2 LEY (118 months)

The Subject's Experiment

LEY dropped the yellow tennis ball from higher than the white tennis ball. The yellow dropped onto the rug, the white onto the floor. The white tennis ball bounced the highest.

S The yellow is the best
E Which tennis ball actually bounced the highest?
S The white one, because it bounced on the floor
E Where did you drop the yellow tennis ball from, higher or lower than the white one?
S From higher, I thought it should bounce the highest. I should have done this:-

LEY dropped the yellow tennis ball from higher than the white tennis ball, both being dropped onto the floor.

S The yellow bounced really well
E Why did you do that?
S They both bounced onto the floor, the rug didn't stop the yellow one bouncing
E Why did the yellow tennis ball bounce highest?
S It's the best, it bounces best

Peter's and John's Experiments

S Peter's is good, he doesn't let the white one win, and shows the yellow one is still best.
John's is what I did, it's O.K. not as good as Peter's though
E Why?
S I'm not sure, John makes the yellow one win, Peter's doesn't
E Is it wrong to make one win?
S Not if it is the right one

The Conflict Sequence

E (Showed John's reversed experiment)
John says this experiment proves that the white one is bounciest
S He's wrong
E But it bounces highest
S Yes, but it was dropped from higher
E (Showed John's experiment)
S That's right, the yellow is the best
E Where was it dropped from?
S From higher than the white one
E So why did it bounce highest?
S Because it was dropped from higher. I see ... he's wrong then, it should bounce higher because it's the best.

(continued ..../

LEY protocol continued ...

The 'H' Experiment

E Showed a complete 'H' experiment dropping both tennis balls onto the floor from the same height
S Yes the yellow is the bounciest, you could see it that time
E So which experiment is the best then
S Well you must drop them from the same height or you are not showing which is really the best
E Do it then

LEY drops them from the same height but onto different floor surface.

E Where did you drop them onto?
S I don't know ... the yellow one was the best.

PROTOCOL

Stage 4.1 Level 1 Experimental Method Two
SMI (112 months)

The Subject's Experiment

SMI dropped the yellow and white tennis balls from the same height, the white tennis ball dropped onto the floor, the yellow tennis ball dropped onto the rug. The white tennis ball bounced the highest.

S The white ball is the bounciest (he looks puzzled)
E What's wrong?
S Well I thought the yellow one would be the bounciest. I see the white one bounced on the floor that made it bounce higher
E So which tennis ball is the bounciest?
S I'm not sure, I dropped them from the same height to be fair to both, but the white bounced on the floor
E What experiment might you do then?
S I'm not sure

Peter's and John's Experiments

Peter's was considered correct and SMI liked the test of strength experiment.

S That's right he really showed the yellow was the best. The white should have bounced much higher because it was dropped from higher.
John's was considered incorrect because he did not control the height variable.

The Alternative Conflict Sequence

E (Showed Paul's experiment)
S That's alright I think, he dropped them from the same height and the yellow one bounced highest. No the floor is not the same
E (Showed Paul's reversed experiment)
S The white one bounced the highest because it bounced on the floor, that's not right
E Why did it bounce the highest?
S Because it fell on the floor
E Does that show that the yellow tennis ball is bounciest?
S No, not really
E So which tennis ball is the bounciest?
S The yellow one
E How do you know that?
S You would have to do this:-

SMI performs experiment two, followed by experiment one.

S You see in the first one the white was on the floor and it went to that mark (about 2 ft). In this one the yellow one was on the floor and it went here (about 2.5 ft)
E How does your experiments show which tennis ball is the bounciest?

(continued......./

S It's fair you see them bounce on the floor
E Can you think of just one experiment to show this?
S I've done it, I've shown which is the bounciest

The 'H' Experiment

E (Showed the 'H' experiment)
S I see, the yellow was the bounciest
E Why did it bounce the highest?
S Because it was the best
E Is this experiment correct?
S Yes, it is a fair experiment.

PROTOCOL

Stage 4.1 Level 1 Experimental Method Three

HAV (108 months)

The Subject's Experiment

HAV dropped the yellow and white tennis balls from the same height. The yellow one fell onto the rug, the white one fell onto the floor

S Oh, that's not right, the white one fell onto the floor that's why it went higher. I'll do this:-

HAV dropped the yellow and white tennis balls from the same height, the white one fell onto the rug, the yellow one fell onto the floor.

S Now you can see, the yellow one is the best
E How does that show it?
S What do you mean, it bounces highest
E Well, why did you do the second experiment?
S Well I thought you should drop them from the same height but the first time the yellow didn't bounce its best because it was on the rug. The second time the yellow did bounce its best because it was on the floor.
E I think I see, so the yellow one is the bounciest because it bounced highest in the second experiment
S Not just that, you have to look at how high the white one went on the floor in the first one and how high the yellow one went on the floor in the second one
E Can you think of just one experiment to show this?
S ... No I can't, I've shown which is best!

Peter's and John's Experiments

Peter's was considered correct and HAV rejected John's experiment as unfair.

The Alternative Conflict Sequence

E (showed Paul's experiment)
S Well that's not right, the yellow one is the bounciest but he made it bounce highest
E (Showed Paul's reversed experiment)
S That's wrong, it's not fair, the white one was made to win

The 'H' Experiment

E (Showed an 'H' Experiment)
S The yellow one didn't bounce as high as I thought it should
E Is the experiment correct?
S Well, it's fair
E What do you mean?
S They were both dropped from the same height onto the floor
E So it is correct?
S I think so, its fair, but the yellow one is better than that.

Stage 4.2 Level 4 Experimental Method Three

MAW (118 months)

The Subject's Experiment

MAW dropped the yellow and white tennis balls from the same height, the yellow one fell onto the rug, the white one fell onto the floor.

S I'm wrong, can I do another one?
E Yes

MAW dropped the yellow and white tennis balls from the same height. The white one fell onto the rug, the yellow one fell onto the floor.

S That's better, the yellow one's the bounciest
E Why do you think it's the bounciest?
S You look at how high the white one bounced on the floor at first and then how high the yellow one bounced on the floor. Can I do another experiment?
E Yes

MAW dropped the yellow and white tennis balls from the same height onto the floor.

E Why did you do that?
S It's a lot better you can see it better
E What does your experiment show?
S The tennis balls both bounced on the floor and they fell from the same height. You just have to look at how they bounce

Peter's and John's Experiment

Peter's was considered correct and John's experiment was rejected.

S John's made the yellow one the best, it is the best but he made it because it bounced onto the floor and the white bounced onto the rug.

The Alternative Conflict Sequence

E (Showed Paul's experiment)
S He's right the yellow is the best. What he did was wrong, the white one only bounced on the rug
E (Showed Paul's reversed experiment)
S He's wrong the white one bounced on the floor and the other one fell on the rug. If he did both experiments like I did that would be right.

The 'H' Experiment

E (Showed an 'H' experiment)
S The yellow is the best, he's right, it's fair. You can see which is the best because they both fell onto the floor from the same height.

PROTOCOL

Stage 5.1 JAM (111 months)

The Subject's Experiment

JAM controlled both the height and floor variables but could give no reason for controlling the height variable. The white tennis ball bounced the highest due to the fact that they just fell onto the rug.

S The white one's the bounciest
E Did you drop the white one from higher than the yellow one or ... ?
S No, both the same height
E Why did you do that?
S I don't know I felt like doing it
E Where did you drop the tennis balls onto?
S Both fell onto the rug
E Why did you do that?
S If I did one on the floor and one on the rug, the one on the floor would bounce the highest and I don't want that

Peter's and John's experiments

Peter's was considered correct but John's was rejected because it used different heights.

E Why is John's wrong?
S The yellow ball was made to go higher, the yellow was dropped from highest

The Conflict Sequence

E (Showed John's reversed experiment)
Which is the bounciest?
S The white one
E Is his experiment correct?
S Yes, the white one is the best and he found this
E (Showed John's experiment)
Which is the bounciest?
S The yellow one
E Is his experiment correct?
S Not really he made the yellow one the bounciest. The white one is the bounciest
E So it's wrong to make one bounce highest?
S Yes
E (Showed John's reversed experiment)
Which is the bounciest?
S The white one
E Does he make the white one the bounciest?
S Yes he does it fell onto the floor and the yellow one only fell onto the rug. So that's not right

(Continued......./

JAM protocol continued ...

The 'H' Experiment

S That's good it is a good experiment
E What do you mean?
S They both bounced on the floor from the same height
They were not made to bounce
E So that experiment is right
S I don't think the result is, I found the white one was the bounciest and you found the yellow one was the bounciest. Perhaps it's wrong.

Stage 5.2 ARM (110 months)

The Subject's Experiment

ARM controlled both height and floor variables and the yellow tennis ball bounced the highest.

S The yellow one's the best
E Was the yellow one dropped from higher than the white one?
S No, they were both the same height
E Why did you do that?
S It's obvious, it's fair, I need to find out which tennis ball is bounciest
E On what floor surface did you drop them?
S I dropped them both onto the floor
E Why did you do that?
S Because they bounce well on the floor and not on the carpet
E Would it be alright if I dropped one on the floor and one on the carpet?
S No, it would be wrong, you'd be cheating. The one on the floor would go the highest

Peter's and John's Experiments

E (Showed John's experiment)
S It's rubbish, he did them from different heights
E Why is that wrong?
S Because the yellow one would bounce more than the white one. The height does it.
E (Showed Peter's experiment)
S He's right, if the yellow one still won when it was dropped from lower it must be the bounciest
E But I thought you said that you must drop them from the same height, Peter didn't
S Mmm ... in Peter's experiment it's alright, in John's it's not because he's cheating and Peter is not

The 'H' Experiment

S That's right. There's no cheating both the yellow and the white were left to bounce
E Which bounced the highest?
S I'm not sure, there was not much difference. It doesn't matter what you did was right.

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APPENDIX H

Working Memory Demands for strategy execution

(Stage 1.1) Strategy IJ (INTUITIVE JUDGMENT)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Look for salient feature of tennis ball A. If there is one and this is related to bounciness (Bouncy means X) say it is bouncy.	(i)"fluffiness" of A A_X*	0
Step 2 - Look for salient feature of tennis ball B, etc. (Bouncy means Y)	(i) "fluffiness" of B. $B_{NOT\ X}$* or "newness" B. B_Y	0

(Stage 2.1) Strategy BN(BOUNCE EXPERIMENTATION)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Note height bounced by A (store).	(i) Height bounced by A (A_H)*	0
Step 2 - Note height bounced by B (store).	(i) A_H (ii) Height bounced by B (B_H)*	1
Step 3 - Select larger height and state that the ball that bounces the highest is the bounciest. (Bouncy means Highest)	(i) A_H (ii) B_H*	1

(Stage 2.3) Strategy ivD (D EXPERIMENTATION)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store)	(i) Position of drop of A(Ap)*	0
Step 2 - (Real Action) - Note position of drop of B (store).	(i) Ap (ii) Position of drop of B (Bp)*	1
Step 3 - (Real Action) - Compare positions of drop of A & B (store) (Make Ap > Bp)	(i) Difference in positions of drop (Ap>Bp)	1
Step 4 - Note height bounced by A (store).	(i) Ap>Bp (ii) Height bounced by A(A_H)*	1

(Appendix H continued over ..../)

Appendix H continued...

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 5 - Note height bounced by B (store).	(i) $A_p > B_p$ (ii) A_H (iii) Height bounced by B(B_H)*	2
Step 6 - Compare heights (store), state A >B if A_H>B and $A_H > B_H$ if $A_p > B_p$	(i) $A_p > B_p$ (ii) Difference in height of A & B	2

Stage 2.4) Strategy ivG.D.(GREATEST DIFFERENCE EXPERIMENTATION)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store).	(i) Position of drop of A(Ap)*	0
Step 2 - (Real Action) - Note position of drop of B (store).	(i) Ap (ii) Position of drop of B(Bp)*	1
Step 3 - (Real Action) - Compare positions of drop of A & B (store) make $A_p > B_p$	(i) Difference in position of drop ($A_p > B_p$)	1
Step 4 - (Real Action) - Note floor surface to be dropped on by A (store)	(i) $A_p > B_p$ (ii) Floor surface dropped on by A. (A_s) *	1
Step 5 - (Real Action) - Note floor surface to be dropped on by B (store).	(i) $A_p > B_p$ (ii) As (iii) Floor surface dropped on by B(Bs)*	2
Step 6 - (Real Action) - Compare floor surface dropped on by A & B (store) make $A_s > B_s$	(i) $A_p > B_p$ (ii) Difference in position of drop ($A_s > B_s$) *	2
Step 7 - Note height of bounce of A (store)	(i) $A_p > B_p$ (ii) $A_s > B_s$ (iii) Height of bounce of A (A_H) *	2
Step 8 - Note height of bounce of B (store).	(i) $A_p > B_p$ (ii) $A_s > B_s$ (iii) A_H (iv) Height of bounce B(B_H) *	3

(Appendix H continued over....../)

Appendix H continued ...

Steps involved	Items in Working Memory (i.e being attended to)	MEMORY DEMAND
Step 9 - Compare height of bounce (store) of A & B State A > B if $A_H > B_H$ and $A_H > B_H$ if $A_p > B_p$ and $A_H > B_H$ if $A_s > B_s$	(i) $A_p > B_p$ (ii) $A_s > B_s$ (iii) Difference in height of bounce of A&B.	3

(Stage 3,1) Strategy ivT (TEST OF STRENGTH)
(floor surface ignored)

Steps involved	Items in Working Memory (i.e being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store)	(i) Position of drop of A (A_p)*	0
Step 2 - (Real Action) - Note position of drop of B (store).	(i) A_p (ii) Position of drop of B(B_p)*	1
Step 3 - (Real Action) - Compare positions of drop of A & B if B > A make $A_p > B_p$ (store)	(i) Difference in positions of drop ($A_p > B_p$)	1
Step 4 - Note height of bounce of A (store)	(i) $A_p > B_p$ (ii) Height of bounce of A (A_H) *	1
Step 5 - Note height of bounce of B (store)	(i) $A_p > B_p$ (ii) A_H (iii) Height of bounce of B(B_H)*	2
Step 6 - Compare height of bounce of A & B (store) state B > A if $B_H > A_H$ due to $B_B > A_B$ and NOT ($B_p > A_p$)	(i) $A_p > B_p$ (ii) Difference in height of bounce of A & B.	2

(State 3,1) Strategy ivTCs (TEST OF STRENGTH
(floor surface attended to)

Steps involved	Items in Working Memory (i.e being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A	(i) Position of drop of A(A_p)*	0
Step 2 - (Real Action) - Note position of drop of B.	(i) A_p (ii) Position of drop of B(B_p)*	1
Step 3 - (Real Action) - Compare position of drop of A & B if B > A make $A_p > B_p$	(i) Difference in position of drop ($A_p > B_p$)	1
Step 4 - (Real Action) - Note floor surface to be dropped on by A (store)	(i) $A_p > B_p$ (ii) Floor surface dropped on by A (A_s) *	1

(Appendix H continued over ...../)

Appendix H continued ...

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 5 - (Real Action) - Note floor surface to be dropped on by B (store)	(i) Ap > Bp (ii) As (iii) Floor surface dropped on by B(Bs)*	2
Step 6 - (Real Action) - Compare floor surfaces to be dropped on make As > Bs or As = Bs i.e. NOT Bs > As (store)	(i) Ap > Bp (ii) Difference in floor surface dropped on NOT(B_s > As)	2
Step 7 - (Real Action) - Note height of bounce A (store)	(i) Ap > Bp (ii) NOT (Bs > As) (iii) Height bounced by A(A_H)*	2
Step 8 - (Real Action) - Note height of bounce B (store)	(i) Ap > Bp (ii) NOT (Bs > As) (iii) A_H (iv) Height bounced by B (B_H)*	2
Step 9 - Compare heights bounced by A & B; B > A if $B_H > A_H$ due to $B_B > A_B$ NOT (Bp > Ap or Bs > As)	(i) Ap > Bp (ii) NOT(Bs > As) (iii) Difference in height of bounce of A & B.	3

Stage 3.2) Strategy ivCs(D EXPERIMENTATION & CONTROL OF FLOOR SURFACE

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store)	(i) Position of drop of A(Ap)*	0
Step 2 - (Real Action) Note position of drop of B (store)	(i) Ap (ii) Position of drop of B(Bp)*	1
Step 3 - (Real Action) - Compare positions of drop of A & B (store) make Ap > Bp	(i) Difference in positions of drop (Ap > Bp)	1
Step 4 - (Real Action) - Note floor surface to be dropped on by A (store)	(i) Ap > Bp (ii) Floor surface dropped on by A (As)*	1
Step 5 - (Real Action) - Note floor surface to be dropped on by B (store)	(i) Ap > Bp (ii) As (iii) Floor surface dropped on by B(Bs) *	2

(Appendix H continued over ....../)

Appendix H continued ...

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 6 - (Real Action) - Compare floor surface to be dropped on by A & B. make $A_S = B_S$ (store)	(i) $A_P > B_P$ (ii) Difference in floor surfaces dropped on ($A_S = B_S$)	2
Step 7 - Note height bounced by A (store)	(i) $A_P > B_P$ (ii) $A_S = B_S$ (iii) Height bounced by A(A_H)*	2
Step 8 - Note height bounced by B (store)	(i) $A_P > B_P$ (ii) $A_S = B_S$ (iii) A_H (iv) Height bounced by B(B_H)*	3
Step 9 - Compare heights of bounce of A & B state A> B if $A_H > B_H$ due to $A_P > B_P$	(1) $A_P > B_P$ (ii) $A_P > B_P$ (iii) Difference in height of bounce of A & B	3

Stage 3.2) Strategy ivCs+p(CONTROL POSITION OF DROP BY USING TWO CONTROL OF FLOOR SURFACE EXPERIMENTS)

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Note height bounced by A in first experiment (store).	(i) Height bounced by A (A_H old)	1
Step 2 - Note position of drop of A in the first experiment (store)	(i) A_H(old) (ii) Position of drop of A in old experiment (Ap old)	2
Step 3 - (Real Action) - Note position of drop of B in new experiment (store)	(i) $A_H$old (ii) Ap old (iii) Position of drop of B in new experiment (Bp new)*	2
Step 4 - (Real Action) - Compare position of drop A old and B new, make positions the same (store).	(i) $A_H$old (ii) Difference between positions of drop (Ap old = Bp new)	2
Step 5 - Note position of drop of A in new experiment (store)	(i) $A_H$old (ii) Ap old = Bp new (iii) Position of drop of A in new experiment (Ap new)*	2

(Appendix H continued over...../)

Appendix H continued...

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 6 - (Real Action) - Compare position of drop of A & B in new experiment, make Bp new > Ap new (store).	(i) A_H old (ii) Difference in position of drop (Ap old = Bp new) > Ap new	2
Step 7 - (Real Action) - Note floor surface to be dropped on by A (store)	(i) A_H old (ii) (Ap old = Bp new) > Ap new (iii) Floor surface dropped on by A(As new) *	2
Step 8 - (Real Action) - Note floor surface to be dropped on by B in new experiment (store)	(i) A_H old (ii) (Ap old = Bp new) > Ap new (iii) As new (iv) Floor surface dropped on by B (Bs new) *	3
Step 9 - (Real Action) - Compare floor surface dropped on by A & B (make A_s = Bs)(store)	(i) A_H old (ii) (Ap old = Bp new) > Ap new (iii) Difference in floor surface dropped on (A_s new = Bs new)	3
Step 10 - Note height of bounce of B new (store)	(i) A_H old (ii) (Ap old = Bp new) > Ap new (iii) A_s new = Bs new (iv) Height of bounce of B (B_H new) *	3
Step 11 - Compare height of B new to A old state A > B if $A_H > B_H$ due to $A_B > B_B$ NOT $A_p > B_p$	(i) (Ap old = Bp new) > Ap new (ii) A_s new = Bs new (iii) Difference in height of B_H new and A_H old	3

(Stage 4.1) Strategy iv Cp (CONTROL OF POSITION OF DROP BUT THE FLOOR SURFACE IS OVERLOOKED)

Step involved	Item in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - (Real Action) - Note position of drop of A (store)	(i) Position of drop of A(Ap)*	0
Step 2 - (Real Action) - Note position of drop of B (store)	(i) Ap (ii) Position of drop of B(Bp)*	1

(Appendix H continued over...../)

Steps involved	Item in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 3 - (Real Action) - Compare positions of A & B (store) make Ap = Bp	(i) Difference in positions of drop (Ap = Bp)	1
Step 4 - Note height of bounce of A (store)	(i) Ap = Bp (ii) Height of bounce of A (A_H)*	1
Step 5 - Note height of bounce of B (store)	(i) Ap = Bp (ii) A_H (iii) Height of bounce of B(B_H)*	2
Step 6 - Compare height of bounce of A & B (store) state A > B if $A_H > B_H$ due to $A_B > B_B$ $A_H = B_H$ if Ap = Bp	(i) Ap = Bp (ii) Difference in heights of bounce A & B	2

(Stage 4.2) Strategy]vCp+s] (CONTROL OF POSITION OF DROP AND CONTROL OF FLOOR SURFACE BY MEANS OF COMPARISON OF TWO]vCp EXPERIMENTS)

Steps involved	Item in Working Memory (i.e being attended to)	MEMORY DEMAND
Step 1 - Note height bounced by B in old experiment (store)	(i) Height bounced by B (B_H old)	1
Step 2 - Note position of drop of B in old experiment (store)	(i) B_H old (ii) Position of drop of B (Bp old)	2
Step 3 - Note floor surface dropped on by B in old experiment (store)	(i) B_H old (ii) Bp old (iii) Floor surface dropped on by B(B_s old)	3
Step 4 - Note floor surface dropped on by A in new experiment (store)	(i) B_H old (ii) Bp old (iii) B_s old (iv) Floor surface dropped on by A (A_s new)*	3
Step 5 - Compare floor surfaces dropped on make A_s new = B_s old (store)	(i) B_H old (ii) Bp old (iii) Difference in floor surfaces (A_s new = B_s old)	3

(Appendix H continued over ..../)

Appendix H continued ...

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 6 - Note floor surface dropped on by B in new experiment (store)	(i) B_H old (ii) B_p old (iii) A_s new = B_s old (iv) Floor surface dropped on by B (B_s new) *	3
Step 7 - Compare floor surface dropped on by A& B in new experiment make $A_s > B_s$ (store)	(i) B_H old (ii) B_p old (iii)(A_s new = B_s old)$>$ B_s new	3
Step 8 - Note position of A in new experiment (store)	(i) B_H old (ii) B_p old (iii)(A_s new = B_s old)$>$ B_s new (iv) Position of drop of A (A_p new)*	3
Step 9 - Compare position of drop of A in new exerpiment with B in old experiment make A_p new = B_p old (store)	(i) B_H old (ii)(A_s new = B_s old)$>$ B_s new (iii) A_p new = B_p old	3
Step 10 - Note position of drop of B in new experiment(store)	(i) B_H old (ii) (A_s new = B_s old)$>$ B_s new (iii) A_p new = B_p old (iv) Position of drop of B (B_p) *	3
Step 11 - Compare position of drop of B in new experiment with A in new experiment make A_p = B_p(store)	(i) B_H old (ii) (A_s new = B_s old)$>$ B_s new (iii) (A_p new = B_p old) = B_p new	3
Step 12 - Note Height bounced by A in new experiment (store)	(i) B_H old (ii) (A_s new = B_s old)$>$ B_s new (iii) (A_p new = B_p old) = B_p new (iv) Height bounced by A(A_H)*	3
Step 13 - Compare height bounced by B in old experiment with A in new experiment	(i) (A_s new = B_s old) $>$ B_s new (ii) (A_p new = B_p old) = B_p new (iii) Difference in heights bounced B_H old and A_H new	3

(Appendix H continued over ....../)

Appendix H continued ...

(Stage 5) Strategy ivCp+s₂ (CONTROL OF POSITION OF DROP AND FLOOR SURFACE

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 -(Real Action) - Note position of drop of A (store)	(i) Position of drop of A(Ap) *	0
Step 2 - (Real Action) - Note position of drop of B (store)	(i) Ap (ii) Position of drop of B(Bp)*	1
Step 3 - (Real Action) - Compare positions of drop A & B (store) make Ap = Bp	(i) Difference in positions of drop (Ap = Bp)	1
Step 4 - (Real Action) - Note floor surface to be dropped on by A (store)	(i) Ap = Bp (ii) Floor surface dropped on by A(A_s)*	1
Step 5 - (Real Action) - Note floor surface to be dropped on by B (store)	(i) Ap = Bp (ii) A_s (iii) Floor surface dropped on by B(B_s)*	2
Step 6 - (Real Action) - Compare floor surface dropped on by A & B (store) $A_s = B_s$	(i) Ap = Bp (ii) Difference in floor surface dropped on ($A_s = B_s$)	1
Step 7 - Note height of bounce of A (store)	(i) Ap = Bp (ii) $A_s = B_s$ (iii) Height of bounce of A(A_H)*	2
Step 8 - Note height of bounce of B (store)	(i) Ap = Bp (ii) $A_s = B_s$ (iii) A_H (iv) Height of bounce of B(B_H)*	3
Step 9 - Compare height of bounce of A & B (store) state A > B if $A_H > B_H$ due to $A_B > B_B$ and NOT due to $A_p = B_p$ and NOT due to $A_s = B_s$	(i) Ap = Bp (ii) $A_s = B_s$ (iii) Difference in height of bounce A_H & B_H	3

APPENDIX I

Working Memory Demands for executing the resolution of conflict strategies

Conflict Strategy 1

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Note position of drop of A (store)	(i) Position of drop of A (Ap)	1
Step 2 - Note position of drop of B (store)	(i) Ap (ii) Position of drop of B(Bp)	2
Step 3 - Compare positions of drop (store)	(i) Difference in positions of drop Bp > Ap	1

Conflict Strategy 2

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Note floor surface dropped on by A (store)	(i) floor surface dropped on by A (As)	1
Step 2 - Note floor surface dropped on by B (store)	(i) As (ii) Floor surface dropped on by B (Bs)	2
Step 3 - Compare floor surfaces dropped on by A & B (store)	(i) Difference in floor surface dropped on Bs > As	1

Conflict Strategy 3

Steps involved	Items in Working Memory (i.e. being attended to)	MEMORY DEMAND
Step 1 - Note floor surface dropped on by A (store)	(i) Floor surface dropped on by A(As)	1
Step 2 - Note floor surface dropped on by B (store)	(i) As (ii) Floor surface dropped on by B (Bs)	2
Step 3 - Compare floor surface dropped on by A & B (store)	(i) Difference in floor surface dropped on (As = Bs)	1
Step 4 - Note position of drop of A (store)	(i) As = Bs (ii) Position of drop of A(Ap)	2
Step 5 - Note position of drop of B (store)	(i) As = Bs (ii) Ap (iii) Position of drop of B(Bp)	3

TABLE 10.5 NOTATION USED IN CHAPTERS 10 & 11

Symbols	Meaning and examples
For tennis balls	
A	A tennis ball e.g., A may represent the yellow tennis ball.
B	The other tennis ball e.g., B represents the white tennis ball if A represents the yellow tennis ball.
For experimental variables	
Subscript H	The height bounced by the tennis ball e.g., A_H represents the height bounced by tennis ball A.
Subscript p	The position of drop of a tennis ball e.g., A_p represents the position of drop of tennis ball A.
Subscript s	The type of floor surface dropped onto by a tennis ball e.g., A_s represents the floor surface that tennis ball A drops onto.
Subscript B	The intrinsic bounce of a tennis ball e.g., A_B represents the intrinsic bounce of tennis ball A.
For PLANS, NUPLANS and Strategies (listed in order of increasing sophistication)	
IJ	Signifies that only an INTUITIVE JUDGMENT was made, no experimentation took place.
INV J	Signifies that an INVARIANT JUDGMENT was made.
BN	Signifies that a BOUNCE strategy was used.
lower case jv	Signifies that the PLAN/NUPLAN is one of judgment verification. ALL PLANS/NUPLANS past BN involve jv.
jvD.	Signifies that a DIFFERENCE strategy was used.
jvG.D.	Signifies that a GREATEST DIFFERENCE strategy was used.
jvC.	Signifies that a CONTROL strategy was used. The subscript denotes the extrinsic variable or variables controlled e.g., jvC_s - control of floor surface only. jvC_p - control of position of drop only. jvC_{p+s} - control of both extrinsic variables.
jvT.	Signifies that a TEST OF STRENGTH was used. This may be combined with a control strategy e.g., jvTCs - test of strength and the control of the floor surface only.