ON THE IMPACT OF INEQUALITY ON INVESTMENT AND OCCUPATIONAL CHOICE UNDER IMPERFECT CAPITAL MARKETS

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ABSTRACT

Direct finance such as stock and bond markets has provided an alternative source of funds for a large number of firms, which depends heavily on bank loans for credit previously. Bank loans are more expensive indeed than direct finance so that only those borrowers cannot access to capital markets turn to it. Such co-existence of alternative sources of finance has supplied prevalent issues for researches. The thesis focuses on investment choice and the selection of occupation with finance restriction under co-existence of finance sources. We also investigate the impact of inequality in imperfect markets.

In Chapter 2 we develop a model in which agents differ according to their endowments of working capital. They can borrow their money to the capital market to earn interest, or invest in a CRS technology, which is an abbreviation of Constant Return to Scale. The return of CRS is a linear increasing function corresponding to its input. Hence it is a completely riskless investment. Individuals also can undertake a risky project which has a fixed set-up cost. On account of finance constraint, they make their investment and occupation choice. Because of imperfection in the market, lenders cannot obtain information about the real return of the risky project from borrowers, there is credit ration and many entrepreneurs invest at a sub-optimal status. We then introduce a monitoring technology to make it possible for external lenders to observe the return of the private project. Thus lenders who have the monitoring technology can supply any amount of money to borrowers and individuals have an alternative source to raise money. Our result shows the monitoring technology improve the economy. Part of poor individuals who are pure lenders in the previous situation can afford the set-up cost of the risky project. Meanwhile part of sub-optimally investment entrepreneur reach to a optimal status.

Chapter 3 is an empirical test which is derived straightforwardly from the comparative statics results of Chapter 2. The comparative statics show that there are threshold effects of the model on income difference between the rich and the poor. In other words, if income difference is lower than a critical value, then the minimal investment level and the interest rate positively related to the mean wealth. Otherwise both of them negatively linked to the mean value. Comparing to the previous theoretical model, a significant improvement of the empirical model is that we substitute income inequality for income difference. In this chapter we discuss how the average wealth affects financial development over the different values of income inequality. We found both initial wealth and its distribution work together to determine the interest rate, the minimal investment level and consequently the size of the capital market. The empirical tests observe two important results. One is that a rise of average income does improve financial development as long as the income inequality is below a cut-off value. The other is that there is a strong threshold effect on income inequality.

Chapter 4 modify the model of Chapter 2 from a continuous investment model to a fixed one while it introduce a labour market to analyze the markets equilibrium from both physical and human capital sides. We present a static model of an economy where individuals are heterogeneous in terms of initial wealth and there are credit constraints. Individuals are endowed with time resource which they can allocate between working and leisure to maximize their utility. What’s more, individuals can choose to either sell their labour in the labour market or self-employ. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or become entrepreneurs by running a risky project. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on either to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds while the wage rate meets the labour market clearing condition. We find that an increase in the mean wealth leads to a decrease in the interest rate. In equilibrium, the wage rate rises and so does the labour time. Meanwhile, both the optimal amount of labour and the minimal requirement of labour of the project decrease. Chapter 5 is a conclusion.

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CHAPTER 1

INTRODUCTION

Researches on corporate finance had made tremendous progress over the past twenty years. There are a lot of substantial work providing a clear pattern of capital markets, financial structure, governance and their impacts on individuals’ behavior and firms’ activities. However, most of researches presume perfect market conditions such as convex preferences, perfect competition and symmetric information.. In fact, imperfect market and asymmetric information dominate the real world. Individuals come from different sides possessing specific private information. It is the main source of market imperfection in our model. For example, lenders usually do not know borrowers' exact returns of the project without paying considerable monitoring costs. Consequently, lenders may offer funds to high-quality borrowers rather than low-quality borrowers. Individuals’ wealth is one of the criteria to distinguish the low quality borrowers from those of high quality. Hence credit rationing divides individuals into different groups such that some of them obtain credits while the others not. An interesting question is how initial wealth and the distribution of wealth affect individuals' activities in the imperfect markets. The essay is revolving around this issue all the time.

In Chapter 2 we discuss imperfection in the capital market from investment point of view. Agents are distinguished by their initial wealth. They have three different investment methods. Firstly, they may lend money to others who need credit. It is equivalent to put their money to the capital market and they earn the interest. Secondly, they may have a CRS technology with constant returns. Finally, they may carry out a risky project with a fixed set-up cost. We find that the poorest individuals are pure lenders. Those low to medium wealth individuals borrow money which is equal to the liquidated value of the project to achieve a sub-optimal project. Individuals with medium to high wealth raise necessary credit to achieve an optimal project. Meanwhile, the richest individuals self-finance. Then we expand the model by introducing a monitoring technology to observe the real returns of the private project. We call the intermediary with such a monitoring technology as the bank. Since bank has exhaustive information about the private project, it is willing to lend any amount of money to entrepreneurs. Then there are two different ways to raise money in the market. Entrepreneurs may collect credit either from the capital market or the loan market. Apparently, bank loan is more expensive than its alternative since the monitoring cost. But some very poor individuals still benefit from it because they obtain a chance to raise enough money to undertake the risky project while they have to be pure lenders without bank loan. Our result shows that the poorest individuals are still pure lenders. But some relatively poor individuals now are entrepreneurs by obtaining bank loans to invest optimally. On the other hands, part of low to medium wealth agents prefer to borrow from the capital market because of the lower cost, they invest sub-optimally. Other low to medium wealth individuals choose bank loans and invest optimally. Those medium to high wealth individuals raise enough funds from the capital market to achieve an optimal investment. Those richest individuals still self-finance. While most researches focus on a fixed investment model, the innovation of our model is to analyse the issue of investment and occupation choice in the circumstance of a continue investment model. We also carry out a comparative statics analysis to reveal the relationship between the investment level, the interest rate and the average wealth.

The result of comparative statics shows there are an explicit threshold effect on the income difference between the rich and the poor. If the income difference is below the threshold then the minimal investment level and the interest rate positively related to the average wealth. Otherwise both of them are negatively linked to the average wealth. As we all known, income difference is associated with income inequality indeed. The minimal investment level and the interest rate also determine the size of financial sectors. Hence the result of comparative statics may predict the relationships between financial development, average income and income inequality. We examine such relationship by carrying out an empirical test. A significant difference between the empirical model and the theoretical one is that we present income inequality as the Gini coefficient. The empirical tests apply to a panel of 16 countries for the period 1989-2004. We find overwhelming evidence of the threshold effect. The point estimates of the two Gini thresholds are 36.6 and 37.663, respectively. It means a rise of average income would improves financial development if the Gini coefficient is lower than 37.663. As long as the coefficient is higher than this value, financial development is negatively linked to average income. Normally, empirical researches on this area have to detect the threshold value by specifying an estimated equation and the form of the equation significantly affect the empirical result. Our contribution is to use the threshold regression which automatically detects any possible threshold to avoid the problem.

In chapterChapter 4, we expand the model of Chapter 2 by introducing a labour market so that the risky project needs not only physical capital but also human capital. Unlike the continuous investment model in chapterChapter 2, here the risky project need a fixed physical investment while the human capital investment is continuous. Individual are heterogeneous in terms of initial wealth and there are credit constraints due to asymmetric information. Individuals are also endowed with time resource which they can allocate between working and leisure to maximize their utility. What’s more, individuals can choose to either sell their labour in the labour market or self-employ. Put differently, they can supply as pure wage workers or become entrepreneurs by undertaking a risky project. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on whether to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds, while the wage rate meets the labour market clearing condition. We explore the simultaneous equation at different situations and one of the comparative statics results. Our contribution is to analyse the investment choice and occupation selection when the capital market and the labour market coexist. Meanwhile most researches only focus on single market circumstance.

CHAPTER 2

IMPERFECT CAPITAL MARKETS, UNDERINVESTMENT AND THE CHOICE BETWEEN BANK AND MARKET FINANCE

ABSTRACT

We investigate investment and occupation choice in an imperfect financial market. Here agents differ according to their initial endowments. Individuals choose to invest either the capital market, the CRS technology with constant returns or a risky project which has a fixed set-up cost. The result is the poorest individuals have to be pure lenders. Those low to medium wealth individuals borrow money which is just equal to the liquidated value of the project and invest sub-optimally. Those medium to high wealth individuals can borrow sufficient funds to invest optimally. The richest individuals self-finance. Then we introduce a monitoring technology to make it possible for external lenders to observe the return of the private project. Thus individuals have an alternative credit source which is called bank loan to raise money. Our result shows that the poorest individuals are still pure lenders. But some relatively poor individuals may undertake the risky project optimally by obtaining bank loans. On the other hands, part of low to medium wealth agents prefer to the capital market because of the lower cost of direct finance, they invest sub-optimally. Meanwhile, other low to medium wealth individuals choose bank loans and invest optimally. Medium to high wealth individuals raise enough funds from the capital market to achieve an optimal investment. Those richest individuals still self-finance.

2.1 Introduction

Research on corporate finance has made tremendous progress over the past twenty years. There are substantial work on capital markets, financial structure, and their impacts on individuals' behavior and firms' activities. Most of them, for instance, the well-known Arrow-Debreu model and the Modigliani-Miller model, presume prefect market conditions so that no one suffers from imperfection problem such as asymmetric information in the market. However, the issue like asymmetric information is presented under most circumstance in the real world. For example, lenders usually do not know borrowers' exact returns of the project. Consequently, lenders may offer funds to high-quality borrowers rather than low-quality borrowers. Individuals' wealth may be one of criteria to distinguish low quality from high quality. Credit rationing thus divides individuals into different groups in that some of them obtain credits while the others not (Bester and Hellwig (1987)). A costly monitoring technology is introduced to prevent asymmetric information problem. It is an external control mechanism that gets rid of private benefit of borrowers and consequently ameliorates the problem. The situation is equivalent to the Costly State Verification framework which were original studied by Townsend (1979). Gale and Hellwig (1985) and Williamson (1986, 1987)state that the verification problem also leads to credit rationing. Generally banks have such monitoring technology. However, monitoring is obviously expensive because of its cost. So only those borrowers who cannot access direct finance turn to bank loans. There are hence two different finance sources for firms. One is direct finance such as stocks and bonds. The other is bank loans which monitor borrowers' information. The problem is why some borrowers can access the capital market while others cannot? What does the role of initial wealth play in this case? How the aggregate wealth impacts investment and interest rate? We will explore these issues in the following sections.

We investigate the imperfection on financial market for investment. Individuals differ according to their endowments of working capital. They make investment decisions among the capital market, the CRS technology with constant returns and a risky project which has a fixed set-up cost. We then introduce a monitoring technology to make it possible for external lenders to observe the returns of the private project. We will discuss the market equilibrium conditions with and without banks under different situations.

2.2 Literature review

Entrepreneurs with limited endowments seek external funds in order to undertake risky projects. However, external financing reduces their expected returns by sharing of profits. Under conditions of imperfect markets, lenders cannot observe the realized returns of projects without paying considerable monitoring costs because of asymmetric information. Therefore entrepreneurs have a strong incentive to conceal their real returns from the project. Instead, they may claim the returns that are just equal to the liquidated value of the project. Lenders thus have to differentiate low-quality agents from high quality according some features of agents, such as initial wealth. In other words, there is credit ration in the system.

Monitoring technology, which is equivalent to the Costly State Verification framework, is discussed by Townsend (1979), Gale and Hellwig (1985) and Williamson (1986, 1987). It is introduced to alleviate the information asymmetry problem. Repullo and Suarez (2000) interpret that monitoring is an external control mechanism deterring entrepreneurs from diverting project' resources toward private benefits. Williamson (1986) points out that there is a duplication of effort over direct lending when intermediation is prohibited. For instance, suppose lenders monitor only in the event of default and each borrower collects credits from several lenders, then all the lenders monitor in the case of default. Financial intermediaries such as banks thus endogenously emerge. They borrow from a large number of lenders and lend to a large number of borrowers to eliminate this duplication. The idea of monitoring is very similar to Diamond (1984) who states that financial intermediaries play a 'delegated monitoring' role. The significant difference between them is that Williamson (1986) concludes that an equilibrium may exhibit credit rationing such that presuming borrowers being identical ex ante, some receive loans and others do not. In contrast, credit rationing is not a feature of the equilibrium in Diamond (1984)'s model. Furthermore, intermediaries in Diamond (1984) are single agents, while Williamson (1986) allows individuals to choose activities given their endowments and preferences. Boyd and Prescott (1985) discuss the issue under similar conditions except that intermediaries consist of multi-agent coalitions. Keeton (1979) provides the empirical support for the existence of credit rationing. It is necessary to point out that financial intermediaries are not always endogenously presented in these models. For example, Stiglitz and Weiss (1981) takes the idea of 'bank' mainly by assumption. Our model regards banks as an existing monitoring technology and adopts a similar method of Williamson (1986).

Besides CVS(the Costly State Verification framework), our model allows capital markets coexist with banks so that agents can choose to raise money from either capital markets or banks. A number of papers develop the issue of imperfection in these circumstances. Bolton and Freixas (2000) discuss a co-existence situation that both direct finance and financial intermediaries present simultaneously. They show that in equilibrium the riskier firms, who are equivalent to the poorest individuals in our model, prefer bank loans; the safer ones propose to borrow from the capital market; and the ones in between prefer to collect money from both. The result is consistent with Stiglitz and Weiss (1981). Repullo and Suarez (2000) explore the issue from the net worth point of view. In their equilibrium, the set of firms can be divided into three groups according to the value of net worth ratios. In particular, firms with small net worth try in vain, firms with medium net worth raise money by bank loans, and firms with large net worth prefer to collect funds from the capital market. Seward (1990) and Diamond (1991) provide assumptions different from Besanko and Kanatas (1993) such that banks can observe the return of project by monitoring, while external lenders do not, though all of them allow capital markets and banks coexist. Seward (1990) permits entrepreneurs to use bank and capital markets simultaneously, but Diamond (1991) assumes borrowers rely on either banks or the capital market. Our model also allows co-existence of the capital market and banks. However, our model differs from Bolton and Freixas (2000) because we only permit individual choose one source subject to their one-off liquidation value, just like Diamond's model.

We consider a model derived from that of Bougheas (2007), except that we discuss a continuous investment model rather than a fixed investment model. Bougheas depicts a situation in which project returns drop dramatically beyond a certain level. In contrast, our model assumes the risky project has increasing returns to scale. Holmstrom and Tirole (1997) do a similar research. They study an incentive model of financial intermediation in which firms and intermediaries are capital constrained. They examine the reductions in different types of capital impact on investment, interest rates, and the forms of financing. Our model differs from theirs in several respects. Firstly, the frictions in their model are due to moral hazard rather than CVS. The moral hazard problem related to unobservability of effort. In our model, ex post informational asymmetry about project returns brings about credit ration. We then introduce a monitoring technology to make lenders observing project's returns. Secondly, they consider either a fixed size investment or a variable but without a fixed cost. In other words, the optimal investment is infinite in their set up. However, we assume that the product function is a concave curve in order to make investment finite. In particular, the sign of the first order derivative of the production function is negative while the sign of the second order is positive so that the marginal returns of the project will meet the marginal investment. It implies that there is an optimal investment level for the project. Meanwhile, the returns from CRS technology are constant. Individuals can choose investment methods among CRS technology, capital markets and the risky project with a set up cost. Another interesting difference is that distribution in our model is continuous while in their case is not. It is a significant improvement because a continuous investment model is more common.

In our model, individuals only differ from their initial endowments. The restriction of initial wealth and the existing of set-up cost of the risky project make them to choose different investment methods. For example, individuals whose wealth cannot meet the set-up cost have to invest in either the capital market or CRS technology. The Medium-wealth individuals will consider either to borrow a small amount of capital to approach the full investment level, or maintain the sub-optimal status. Since lenders cannot observe the realized returns of the risky project, entrepreneurs have a strong incentive to default and pay the liquidated value, which is far lower than the expected returns, to the lenders. Hence lenders will provide funds no more than the liquidated value of the project to any borrowers if monitoring cost is infinite. In this environment, there are some explicit thresholds which divide individuals into different groups. The credit ration in phenomenon is also discussed in an alternative model by Matsuyama (2007). The interest rate of the capital market is endogenously determined in our model.

In order to make lenders observing project's returns, financial intermediaries such as banks introduce a monitoring technology which is equivalent to the ‘costly state verification” environment and is originally studied by Townsend (1979). The monitoring technology makes it possible that the external lenders can observe the realized returns of the project. But the cost of monitoring also makes bank loans more expensive than direct finance. Individuals in the economy now have an alternative source to collect necessary money. Our result shows that the poorest individuals are still pure lenders. But some relatively poor individuals alter from lenders to entrepreneurs by obtaining bank loans to invest optimally. On the other hands, part of low to medium wealth agents prefer to the capital market because of the lower cost of direct finance, they invest sub-optimally. Meanwhile, other low to medium wealth individuals choose bank loans and invest optimally. The novelty in our analysis is that decreasing marginal returns imply that the optimal size of the project is finite, and thus we can make prediction about the level of underinvestment. We also use the model to explore how changes in the distribution affect the predictions of the model.

The set of arguments such as the mean wealth and the difference of wealth in this model have obvious effects on variables of the model, we then run comparative statics to examine how these arguments affect the model. The remainder of the paper is organized as follows. Section 2.3 introduces the general framework of the model. Section 2.4 briefly discusses the case of perfect capital markets. Section 2.5 explores the imperfect capital market conditions when financial intermediaries are prohibited. The monitoring technology is introduced in section 2.6 in which bank loans are available. Section 2.7 is a comparative statics extension of section2.5. We conclude in the last section.

2.3 The Model Framework

There is a continuum of risk-neutral agents of measure 1 indexed by i. A single good can be either invested or consumed. An individual i is endowed with W_i initially, where $W_i \in [\underline{W}, \overline{W}]$. Let $G(W)$ denotes the distribution of endowments across agents, $g(W)$ is the corresponding density function. Then the mean wealth of the economy is $\hat{W} = \int_{\underline{W}}^{\overline{W}} W_i g(W) dW_i$. The economy involves two stages, ($t=$0, 1). The individual makes an investment decision at the first stage and consumes at the next one. Three different ways of investment are available in the system. The first one is a CRS technology that yields Z (>1) for each unit input. This method is a definitely secure way to invest. Opposite to the riskless CRS, the second method is rather risky. Suppose there is a project requiring a fixed cost $K(>0)$. The positive K means that the risky project needs some basic facilities such lands and machines. The liquidated value l directly comes from liquidating these assets so that $l < K$. If the fixed cost is zero then there is no liquidated value and lenders would not provide any money when they suffer from information asymmetry. Let's consider I_i being the exclusive production investment of the project for the individual i, where $I_i \in [0,\infty]$. The total investment of the individual then must be $K + I_i$. The project succeeds with a probability p. Once succeed, it produces a return of $f(I_i)$. Otherwise it is liquidated with the value l if fails. Here we assume $f^{'}(I_i) > 0$ and $f^{''}(I_i) < 0$ so that the function curve is concave and the investment is limited. It is an important assumption because it makes sure that the marginal return of the project is decreasing. So there is an optimal investment level to maximize the profits. Lastly, the individual a can also invest in the capital market to earn R^E (>1). We'll discuss the interest rate R^E later since it is endogenously determined in the system.

There are a few restrictions on the variables of the project. Firstly, the project must be profitable. Put differently, the expected return must be higher than the investment input. Otherwise no one would undertake the risky project. Since the expected return of the risky project is

$$pf(I_i) + (1-p)l \qquad (2.1)$$

and the total investment is $K + I_i$, then

$$pf(I_i) + (1-p)l \geq K + I_i$$

Suppose each unit return of the project is $R(I_i)$, then

$$R(I_i) = \frac{pf(I_i) + (1-p)l}{K + I_i} \geq 1 \tag{2.2}$$

In addition, $R(I_i)$ must not be lower than the return from the capital market, namely R^E. Suppose $R(I_i) < R^E$, all agents would prefer to invest in the capital market and it would increase the supply of funds in the capital market. Hence the price of credit R^E would reduce until it is no higher than $R(I_i)$. In other words,

$$R(I_i) \geq R^E$$

The inequality above implies that there is a minimal level of investment in which individuals are indifferent to invest either in the risky technology or in the capital market. Let the minimal level be $\underline{I}$. It is determined by

$$R(\underline{I}) = R^E = \frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} \tag{2.3}$$

Equation (2.3) shows that the minimal investment level is a function of R^E. Let's denote it as $\underline{I}(R^E)$.

Thirdly, the conditions $f^{'}(I_i) > 0; f^{''}(I_i) < 0$ prevent the investment from infinitely increasing. So there is also a maximum investment level, I^*. It can be produced by maximizing the risky project's revenue.

The expected profit of the project is just the difference between the expected return and the total investment, that is

$$\left[pf(I_i) + (1-p)l\right] - (K + I_i)$$

The maximum investment level is obtained by taking the first order derivative of the above equation with respect to I and equating it with zero. That is

$$pf'(I_i) - 1 = 0$$

Therefore

$$f'(I^*) = 1/p \tag{2.4}$$

The optimal investment value I^* is derived from (2.4). Then the corresponding optimal return of each unit from the project is

$$R^* = \frac{pf(I^*) + (1-p)l}{K + I^*} \tag{2.5}$$

The investment falls in the scope of $\underline{I}(R_E) \le I_i \le I^*$.

Furthermore, since the connection between the minimal investment level $\underline{I}$ and the interest rate R^E relates to the investment decision, it is also necessary to reveal their relationship. The most intuitive ways is to make the first order derivative of (2.3) to $\underline{I}$. That is $\frac{dR^E}{d\underline{I}} = \frac{pf'(\underline{I}) - R^E}{(K + \underline{I})}$.[1]

The result shows that the sign of the first order derivate is determined by $pf'(\underline{I}) - R^E$. Here the first item is exactly the marginal returns of the project at the minimal investment level. We know the minimal investment $\underline{I}$ is a cut-off value where the capital market and the project yield the same returns. Hence an individual is indifferent to carry out either investment method at this point. However, according to the definition of the minimal threshold, if we add a few credits to the project, its return must be higher than the profits of the capital market, otherwise no one would choose the risky technology. Put it in another

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[1] $$\frac{dR^E}{d\underline{I}} = \frac{d\left(\dfrac{pf(\underline{I}) + (1-p)l}{K + \underline{I}}\right)}{d\underline{I}} = \frac{pf'(\underline{I})(K + \underline{I}) - [pf(\underline{I}) + (1-p)l]}{(K + \underline{I})^2}$$

$$= \frac{pf'(\underline{I}) - \dfrac{pf(\underline{I}) + (1-p)l}{K + \underline{I}}}{(K + \underline{I})} = \frac{pf'(\underline{I}) - R^E}{(K + \underline{I})}$$

way, suppose the marginal increment is $d\,\underline{I}$,

$$pf\,'(\underline{I}) - R^E = \frac{d[pf(\underline{I}) + (1-p)l]}{d\,\underline{I}} - R^E == \frac{d[pf(\underline{I}) + (1-p)l] - d\,\underline{I} \cdot R^E}{d\,\underline{I}}$$

Obviously, the marginal return of the project $d[pf(\underline{I}) + (1-p)l]$ must be higher than the marginal return from the capital market $d\,\underline{I} \cdot R^E$. Therefore $pf\,'(\underline{I}) - R^E > 0$ and $\frac{dR^E}{d\,\underline{I}} > 0$. To conclude, the minimal investment value $\underline{I}(R^E)$ must move in the same direction with R^E .

Finally, the maximal return of the capital market should not be higher than R^*, otherwise all agents would prefer to invest the capital market. Similarly, there is no way that $R^E < Z$, otherwise individuals would never lend money. In conclusion, the inequality $Z \leq R^E \leq R^*$ must be hold. It implies that the optimal return of the risky technology is higher than that of the CRS:

Assumption 1: $R^* > Z$

We presume that the maximal investment falls into the range below:

Assumption 2: $\underline{W} < K + I^* < \overline{W}$

In the following sections we will divide the model into two parts. One is discussed under the perfect market condition; the other is in the imperfect market taking account of the moral hazard problem. The latter is further distinguished by whether there is a bank in the system.

2.4 Perfect Capital Markets

Under the perfect market condition, both an individual's initial endowment and the return of the project are public information. It leaves no private benefit to entrepreneurs. Because the moral hazard problem is not present, lenders are always willing to provide full funds.

Proposition 1 *Perfect Capital Market Equilibrium:*

1) If $\hat{W} > K + I^*$ then $R^E = Z$. There is an excess supply of credits. All projects are funded at the optimal level. The total funds flows to CRS technology is $\hat{W} - (K + I^*)$
2) If $\hat{W} < K + I^*$ then $R^E = R^*$. There is an excess demand of credits. The proportion of project funded at the optimal level is equal to $\hat{W}/(K + I^*)$. No one invests CRS technology.

Proof:

Under the perfect capital market condition, all information is public. Public information eliminates dishonest behaviour. So an individual can raise funds to invest the risky technology even her initial wealth is zero.

1) The inequality $\hat{W} > K + I^*$ implies a rich economy in which the average wealth is higher than the maximal investment level. There is an excess supply of funds because the aggregate wealth exceeds the aggregate demand of credits. Competition in the capital market results in an equilibrium interest rate $R^E = Z$. Since $R^* > Z$, all agents would prefer to be entrepreneurs. All projects are fully funded at the optimal level. The aggregate funds for CRS is $\hat{W} - (K + I^*)$.[2] It is shown in Figure 2.1.

[2] The total funds flowing to the CRS technology equals to the difference between the supply and the demand of credits. In this case, the former is $\int_{K+I^*}^{\bar{W}} \left[W_i - (K + I^*)\right] g(W) dW_i$ and the latter is $\int_{\underline{W}}^{K+I^*} \left[(K + I^*) - W_i\right] g(W) dW_i$.

$$\int_{K+I^*}^{\bar{W}} \left[W_i - (K + I^*)\right] g(W) dW_i - \int_{\underline{W}}^{K+I^*} \left[(K + I^*) - W_i\right] g(W) dW_i$$

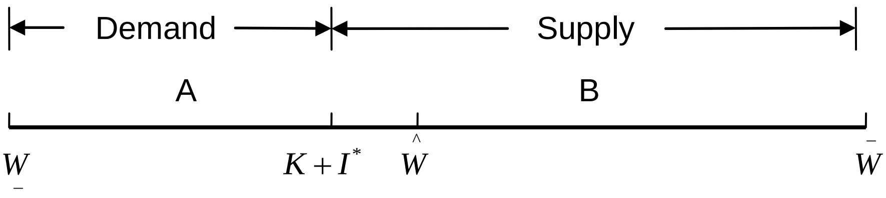

Figure 2.1

2) If inequality $\hat{W} < K + I^*$ holds, this means that is a poor economy. The capital market cannot support all projects and there is an excess demand for credits. If $R^E < R^*$ then all agents with the wealth $W_i < K + I^*$ prefer to borrow $K + I^* - W_i$ to be entrepreneurs. The demand of credits would raise the capital market returns until R^*. Alternatively, if $R^E > R^*$ then all agents prefer to invest in the capital market. It leads to an excess supply of funds so that R^E goes down to R^*. Therefore the equilibrium return of the capital market must be equal to R^*. Agents will be indifferent between investing either in the capital market or in the risky technology. The situation is exhibited in Figure 2.2.

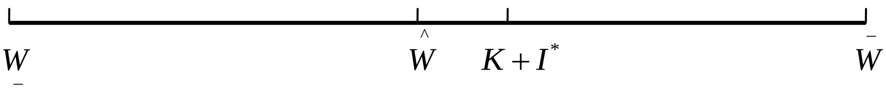

Figure 2.2

$$= \int_{\underline{W}}^{\bar{W}} \left[W_i - (K + I^*) \right] g(W) dW_i$$

$$= \int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i - (K + I^*)$$

$$= \hat{W} - (K + I^*)$$

In this case, the return of the capital market is the same as that of the project, namely $R^E = R^*$. Individuals are different between two methods and there is no distinct boundary between lenders and borrowers. The proportion of agents whose investment is the optimal level is equal to $\hat{W}/(K+I^*)$. [3]

2.5 Imperfect Capital Markets without Banking

Under the conditions of asymmetric information, the return of project is private information. Since monitoring is unavailable, entrepreneurs naturally have the incentive to report a return no more than l even though they succeed in the project. In other words, a lender maximally obtains l no matter how much the project earns. Therefore the maximum credits that a lender is willing to lend equals l/R^E. We know the requirement of being an entrepreneur is the wealth is no smaller than $K + \underline{I}(R^E)$. The equilibrium return thus defines a cut-off value that is $W^c(R^E) = K + \underline{I}(R^E) - (l/R^E)$. All agents with wealth $W_i < W^c(R^E)$ cannot access capital markets in order to raise enough money for the risky project. The conditions $l < K$ and $R^E > 1$ also implies $W^c(R^E) > 0$.

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[3] The equilibrium is achieved when the supply of credits meets the demand. Suppose the proportion of entrepreneur is H. In this case, $\int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i$ is the supply while $H \cdot (K + I^*)$ is the demand. Then $\int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i = \hat{W} = (K + I^*) \cdot H$. So the proportion of agents whose investment is the optimal level equals $\hat{W}/(K+I^*)$

In order to make the model applicable to a general situation, we assume some very poor individuals cannot borrow from capital market. That means the lowest wealth is less than the cut-off value. Formally,

Assumption 3: $\underline{W} < W^c(Z)$

The return of the risky technology is an increasing function of investment because $f'(I_i) > 0$. An individual will keep investing the project until it reaches the optimal level. Aggregate demand and supply of credits are thus determined by the optimal value I^* and the return of the capital market, R^E. The expected supply of credits in the economy is:

$$\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$$

The first item covers all individuals who are too poor to carry out the risky technology. They have to invest in either the capital market or CRS technology. The second item involves a few rich individuals whose wealth exceeds the optimal investment level. They want to lend the rest of funds or invest them in CRS. In addition, the expected demand of credit is:

$$\int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W) dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^*-W_i) g(W) dW_i$$

The first item shows those individuals who can maximally borrow l/R^E to undertake the risky project at a suboptimal investment position. The second one exhibits all agents who qualify for borrowing $K+I^*-W_i$ to achieve the optimal investment level of the project. The equilibrium of capital market is approached as the total supply of credits equals its total demand. This general situation is shown in Figure 2.3.

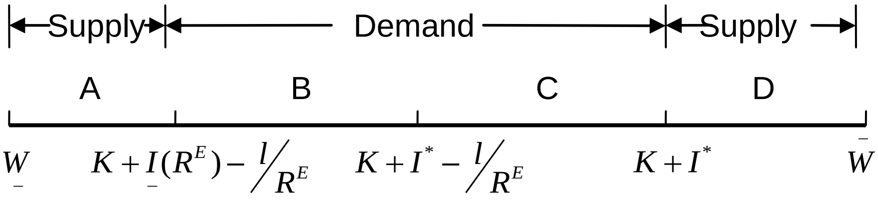

Figure 2.3

Proposition 2 *Imperfect Capital Market Equilibrium without Banking*:

1) If $$\int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} (l/Z)g(W)dW_i + \int_{K+I^*-(l/Z)}^{K+I^*} (K+I^*-W_i)g(W)dW_i$$

$$< \int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i-(K+I^*)\right]g(W)dW_i \ ,$$

then $R^E = Z$. There is an excess supply of credits in the economy. The spare funds will be invested in CRS. The proportion of suboptimal investment individuals is $\int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} g(W)dW_i$. The proportion of agents who achieve optimal level investment is $\int_{K+I^*-(l/Z)}^{\bar{W}} g(W)dW_i$. The total credits flowing into CRS is

$$\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i-(K+I^*)\right]g(W)dW_i \quad .$$

$$- \int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} (l/Z)g(W)dW_i - \int_{K+I^*-(l/Z)}^{K+I^*} (K+I^*-W_i)g(W)dW_i$$

2) If $$\int_{K+I^*-(l/R^*)}^{K+I^*} (K+I^*-W_i)g(W)dW_i > \int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i-(K+I^*)\right]g(W)dW_i$$

then $R^E = R^*$. There is an excess demand of credit. No one invest in CRS technology.

3) If $\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$

$$\leq \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W) dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^*-W_i) g(W) dW_i$$

$$\leq \int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$$

then $Z \leq R^E \leq R^*$. The proportion of the risky project with suboptimal investment is $\int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} g(W) dW_i$. The proportion of the optimal level project is $\int_{K+I^*-l/R^E}^{\bar{W}} g(W) dW_i$. There is no CRS investment.

Proof.

1) The lowest capital market return must not be less than Z, otherwise all agents would prefer to invest in CRS. When the inequality holds, there is an excess supply of funds because the projects that entrepreneurs are willing to invest. The situation reduces the interest rate R^E to Z. Entrepreneurs, regardless of the sub-optimal groups or those fully invested groups, are both funded. The proportions of them are $\int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} g(W) dW_i$ and $\int_{K+I^*-(l/Z)}^{\bar{W}} g(W) dW_i$, respectively. Spare funds are invested in CRS technology. The total amount of funds for CRS is

$$\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i \qquad .$$

$$- \int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} (l/Z) g(W) dW_i - \int_{K+I^*-(l/Z)}^{K+I^*} (K + I^* - W_i) g(W) dW_i$$

The situation is shown in Figure 2.4.

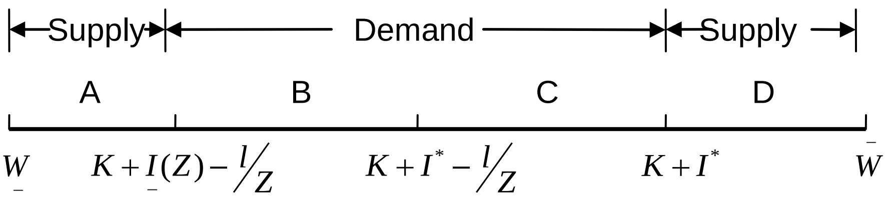

Figure 2.4

2) The highest return of capital market must be no higher than R^*, otherwise agents would never consider the risky project. When the inequality holds, there is an excess demand for credits because the amount of project that entrepreneurs are willing to invest is higher than lenders can fund. The scarcity of funds makes the interest rate increase to R^*. In addition, the condition $R^E = R^*$ means $\frac{pf(\underline{I})+(1-p)l}{K+\underline{I}} = \frac{pf(I^*)+(1-p)l}{K+I^*}$. Comparing both sides of the equation we conclude that $\underline{I} = I^*$ under this situation. In other words, the minimal investment is equal to its optimal value as long as $R^E = R^*$. The continuous invest model shrinks to a fixed model in this case. No one invests in CRS technology. This situation is shown in Figure 2.5.

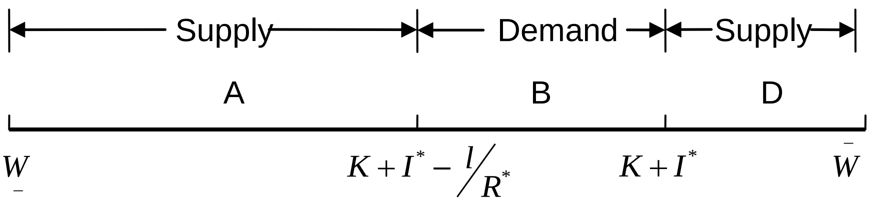

Figure 2.5

3) The minimal supply of funds in the economy is

$\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$ when $R^E = Z$, and the maximal supply of funds is $\int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$ when $R^E = R^*$. The inequality means the demand of credits in the economy varies between the maximum and the minimum value of supply. When the inequality holds, there is an excess demand of credits if $R^E = Z$ and an excess supply of credits if $R^E = R^*$. In this case R^E moves between Z and R^*. It reaches its equilibrium point as long as the total demand equals the total supply, namely the market clear point. Therefore the proportion of project funded at the optimal level is $\int_{K+I^*-l/R^E}^{\bar{W}} g(W) dW_i$. The proportion of under-investment project is $\int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} g(W) dW_i$. Because at the market clear point the supply of funds is equal to the demand, there is no spare fund for the CRS. The situation is shown in Figure 2.6.

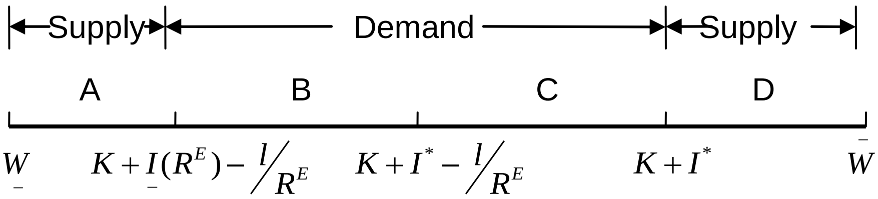

Figure 2.6

2.6 Imperfect Capital Markets with Banking

In this section we introduce a new monitoring technology which allows lenders observe borrowers' returns with a finite cost. We assume the cost is positive. The situation is equivalent to the costly state verification framework which were studied by Townsend (1979). Gale and Hellwig (1985) and Williamson (1986, 1987). They state that the verification problem leads to credit rationing. We introduce an intermediary call the bank who owns the monitoring technology. Individuals can put money into the bank or just provide fund through the capital market directly while the later do not have any monitoring technology.

Assumption 4: The monitoring cost per unit is **m**>0.

This innovation makes bank loans available for borrowers. The entrepreneurs may borrow more than l/R^E under the monitoring technology while the returns are observed by lenders. The project will be liquated if the returns are lower than expectation.[4] Suppose the loan repayment in the contract is ***P*** and the size is ***B***. The financial markets consist of two sections in this economy. One is the direct funds part, namely the capital market. The other is the loan market. Thus the lenders have an alternative way to invest their wealth. If the return of loan market are higher than that of the capital market, all lenders would prefer to the former. Credits supply of the loan market thus increases, which reduces its return. A similar process occurs when the return of the capital market is higher the loan market. Consequently there must be an equilibrium return of loan market, let's call it R_b, which produces a reasonable repayment to eliminates the gap between the returns of the two markets. In other word, a lender will be indifferent between investing the capital market and the loan market if both returns equal. That is:

$$BR^E = pP + (1-p)(l - mB) \tag{2.6}$$

The right hand side of (2.6) states if the project succeeds with the probability *p*, the returns of total loan is P. If the project is liquidated with the value *l*, the total

[4] From the point of liquidation view, individuals can only access one credit source at a time. Once they choose to borrow from the capital market, they leave no liquidation value to the loan market. So they access either the capital market or the bank loan.

monitoring cost *mB* must be deducted from the returns. So

$$P = \frac{1}{p}\left\{B\left[R^E + (1-p)m\right] - (1-p)l\right\} \quad (2.7)$$

Furthermore, the interest rate of the loan market is

$$R_b = \frac{pP + (1-p)l}{B} \quad (2.8)$$

Comparing (2.6) and (2.8) we have:

$$R_b = R^E + (1-p)m \quad (2.9)$$

Lenders are indifferent to investing in either markets under such conditions. The equation also implies that the price of loan market is more expensive than capital market because of the monitoring cost. Naturally, the capital market has priority if both markets are available.

In the previous section, individuals whose wealth is less than $K + \underline{I}(R^E) - {l}/{R^E}$ are too poor to undertake the risky technology. The introduction of the monitoring technology make it possible for them to get more than ${l}/{R^E}$. Therefore a few poor individuals may raise required money instead. They will be indifferent to lend their money or to borrow $K + I^* - W_i$ to carry out the risky project， if the following condition holds:

$$W_i R^E = p[f(I^*) - P] \quad (2.10)$$

subjects to the restriction: $W_i < W^c(R^E) = K + \underline{I}(R^E) - {l}/{R^E}$.

Equation (2.10) describes that an individual whose initial wealth is W_i will be indifferent to lend all the money or to invest in the risky project if the return of the latter, after repaying the bank loan, is equal to the former. It also implies that there is a cut-off endowment level, $W^b(R^E)$, such that all agents with endowment between $W^b(R^E)$ and $W^c(R^E)$ will be better off if they borrow

$K+I^*-W_i$ and invest in the project. However, those individuals whose wealth is lower than the cut-off value are too poor to afford the loan. They will lend out money or invest in CRS technology.

According to the definition, W_i in the left hand side of (2.10) is actually the threshold $W^b(R^E)$. Substituting (2.7) for *P* in (2.10), we have

$$W^b(R^E)\cdot R^E = p\left\{ f(I^*)-\frac{1}{p}\left\{(K+I^*-W^b(R^E))\left[R^E+(1-p)m\right]-(1-p)l\right\}\right\}$$

Rearranging the equation gives

$$W^b(R^E)=(K+I^*)\left[1-\frac{(R^*-R^E)}{(1-p)m}\right] \tag{2.11}$$

$W^b(R^E)$ from (2.11) must be smaller than $W^c(R^E)$. The reason is simple. If $W^b(R^E)\geq W^c(R^E)$, it means someone prefer bank loans rather than direct funds. But we know bank loans is much costly than direct funds. So it would never happen if $W^b(R^E)\geq W^c(R^E)$. (2.11) tells that $W^b(R^E)$ also changes simultaneously with R^E.

In fact, the loan market benefits not only the poor individuals, but also a few entrepreneurs whose endowments are between $W^c(R^E)$ and $K+I^*-{}^{l}\!/\!_{R^E}$. In the capital market, they can maximally borrow ${}^{l}\!/\!_{R^E}$ and stay in the suboptimal investment situation because of imperfect information. Now they can borrow from the loan market to meet the optimal investment level. However, the cost of the loan market is higher than that of the capital market. Agents would switch from the capital market to the loan market only if they were better off. In other words, agents will be indifferent between staying in the suboptimal investment status or making a loan to reach the optimal investment level if

$$p[f(I^*)-P]=p[f(W_i+{}^{l}\!/_{R^E}-K)-l] \qquad (2.12)$$

Under the constraint: $W^c(R^E)\le W_i<K+I^*-{}^{l}\!/_{R^E}$.

The left hand side of (2.12) shows the expected returns of a fully funded project after paying off the loan. The right hand side indicates the expected returns of a sub-optimal funded project in which the entrepreneur accesses the capital market only. Obviously, (2.12) defines another cut-off value, let's call it $W^d(R^E)$, which divides individuals with the wealth in the range of $[W^c(R^E),K+I^*-{}^{l}\!/_{R^E})$ into two groups. The first group accesses the capital market as usual and maximally borrowès ${}^{l}\!/_{R^E}$ to sustain the suboptimal funded project. Another group is better off by borrowing from the loan market and meets the optimal investment level. This situation is plotted in Figure 2.7.

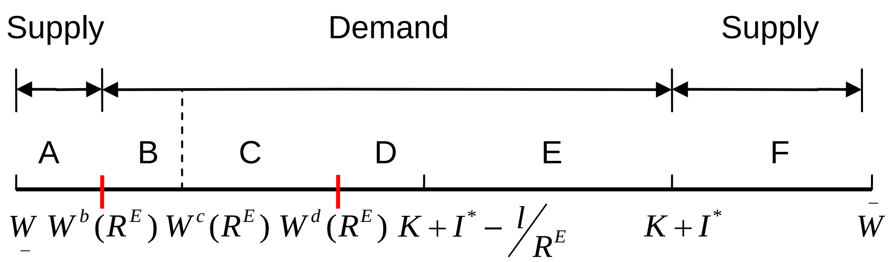

Figure 2.7

We have seen there are two extra thresholds in Figure 2.7 comparing the previous graph. Area B presents a few poor individuals who do not qualify for accessing the capital market can carry out the optimal level project by borrowing from bank. Furthermore, the individuals in either area C or area D who have performed the suboptimal funded project by raising money from the capital market can have fully funded project for the same reason. The point is that it is ambiguous which group of individuals will propose to access the loan market. One fact is that the price of the loan market is higher than that of the capital market because of the monitoring cost. The more funds the individuals borrow from the loan market, the more repayment they have. If only they benefit from the choice, they would not access the loan market. Suppose there is an individual whose wealth is in the area D, namely the scope of $[W^d(R^E), K+I^* - l/R^E)$. His wealth can be written as $W_i = K + I^* - l/R^E - \varepsilon$, where $0 < \varepsilon \le K + I^* - l/R^E - W^d(R^E)$. If he borrows l/R^E only from the capital market, his welfare, WF^C, is $WF^C = pf(I^* - \varepsilon) - pl$ [5].Alternatively, if he borrows from the loan market, his welfare, WF^L is

$$WF^L = pf(I^*) - pl - \frac{(1-p)ml}{R^E} - \varepsilon\left[R^E + (1-p)m\right]$$ [6]

There is no point switching from the capital market to the loan market if $WF^L < WF^C$. In addition, as one would expect, both WF^L and WF^C rise as the endowments increase[7]. Suppose there is a position close to $K + I^* - l/R^E$.

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[5] $WF^C = p\left[f(W_i + l/R^E - K) - l\right]$

$= p\left[f(K + I^* - l/R^E - \varepsilon + l/R^E - K) - l\right]$

$= pf(I^* - \varepsilon) - pl$

[6] $WF^L = p\left[f(I^*) - P\right]$

$= pf(I^*) - p\dfrac{1}{p}\left\{(K + I^* - W_i)\cdot[R^E + (1-p)m] - (1-p)l\right\}$

$= pf(I^*) - [K + I^* - (K + I^* - l/R^E - \varepsilon)]\cdot[R^E + (1-p)m] + (1-p)l$

$= pf(I^*) - (l/R^E + \varepsilon)\cdot[R^E + (1-p)m] + (1-p)l$

$= pf(I^*) - pl - \dfrac{(1-p)ml}{R^E} - \varepsilon\left[R^E + (1-p)m\right]$

[7] $\dfrac{\partial WF^C}{\partial \varepsilon} = \dfrac{\partial}{\partial \varepsilon}[pf(I^* - \varepsilon) - pl] = -pf'(I^* - \varepsilon) < 0$

$\dfrac{\partial WF^L}{\partial \varepsilon} = \dfrac{\partial}{\partial \varepsilon}\left\{p[f(I^*) - P]\right\} = -[R^E + (1-p)m] < 0$

In other words, $\varepsilon \to 0$. Then WF^C is approaching $pf(I^*) - pl$ while WF^L is approximating to $pf(I^*) - pl - \frac{(1-p)ml}{R^E}$. Apparently the former is larger than the latter because the sign of $\frac{(1-p)ml}{R^E}$ is positive. The result unveils a fact that at the point close to $K + I^* - {}^{l}\!/_{R^E}$, an individual prefers the suboptimal investment status to the fully funded situation because he would not benefit from switching to the loan market. The difference between two kinds of welfare reaches its maximum level, $\frac{(1-p)ml}{R^E}$, when $\varepsilon = 0$. So if W_i moves backwards the point $W^C(R^E)$, such gap will fade away. In fact, a left movement of W_i means that ε is indeed increasing. We know that both WF^L and WF^C are decreasing functions of ε. An expanding ε makes both of them shrink at different rate. WF^L reduces at the speed $\left|\frac{\partial WF^L}{\partial \varepsilon}\right| = R^E + (1-p)m$ which is a constant while the latter changes at an accelerating rate $\left|\frac{\partial WF^C}{\partial \varepsilon}\right| = pf'(I^* - \varepsilon)$. The situation illustrates that WF^C reduces faster than WF^L and eventually they will meet at the threshold $W^d(R^E)$. Individuals on the left side of the point, namely the area C in Figure 2.7, prefer to borrow from the loan market and achieve the optimal investment level. Meanwhile, individuals in the area D have no motivation to switch to the loan market because they would not benefit from it. Those individuals whose

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Both WF^L and WF^C are decreasing functions of ε. Because ε also negatively relates to W_i, they are increasing functions of W_i. The result is consistent with the expectation that the richer the individuals are, the more the profits are.

wealth is precisely $W^d(R^E)$ will be indifferent to access either the capital market or the loan market.

Finally, the supply side of the economy consists of two parts:

$$\int_{\underline{W}}^{W^b(R^E)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} (W_i - I^* - K) g(W) dW_i$$

The individuals who are too poor to invest the risky project are presented in the first item. The second item shows the rich individuals who have spare capital to supply to the markets. The demand side of the economy is divided into four parts:

$$\int_{W^b(R^E)}^{W^d(R^E)} (K + I^* - W_i) g(W) dW_i + (1-p)m \int_{W^b(R^E)}^{W^d(R^E)} (K + I^* - W_i) g(W) dW_i$$

$$+ \int_{W^d(R^E)}^{K+I^*-\frac{l}{R^E}} \frac{l}{R^E} g(W) dW_i + \int_{K+I^*-\frac{l}{R^E}}^{K+I^*} (K + I^* - W_i) g(W) dW_i$$

The first item shows those agents who access the loan market. The second item presents their monitoring cost. The third item describes those agents maximally borrow l/R^E from the capital market and stay in the suboptimal investment status. The fourth item indicates those agents who obtain full funds from the capital market.

Proposition 3

1) If $\int_{K+I^*-\frac{l}{R^*}}^{K+I^*} (K + I^* - W_i) g(W) dW_i \geq \int_{\underline{W}}^{K+I^*-\frac{l}{R^*}} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} (W_i - I^* - K) g(W) dW_i$

then $R^E = R^*$. There is an excess demand for credit. The proportion of the fully funded project is $\int_{K+I^*-\frac{l}{R^*}}^{\bar{W}} g(W)dW_i$. No one wants to make a loan or to invest in CRS technology.

2) If $\int_{K+I^*-\frac{l}{R^*}}^{K+I^*} (K+I^*-W_i)g(W)dW_i < \int_{\underline{W}}^{K+I^*-\frac{l}{R^*}} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} (W_i - I^* - K)g(W)dW_i$

Then either

a) $R^E = Z$, $R_b = Z + (1-p)m$. There is an excess supply of credit. The proportion of agents with full investment is $\int_{W^b(Z)}^{W^d(Z)} g(W)dW_i + \int_{K+I^*-l/Z}^{\bar{W}} g(W)dW_i$. The proportion of agents with under-investment is $\int_{W^d(Z)}^{K+I^*-l/Z} g(W)dW_i$. After meeting the demand of credits, the rest funds flow into CRS technology. Or

b) $Z < R^E < R_1$, where R_1 corresponds to the interest rate from the capital market when $W^b(R^E) = W^c(R^E) = W^d(R^E)$. $R_b = R^E + (1-p)m$. The proportion of the risky technology funded at the optimal level is $\int_{W^b(R^E)}^{W^d(R^E)} g(W)dW_i + \int_{K+I^*-l/R^E}^{\bar{W}} g(W)dW_i$. The proportion of the risky technology funded at the under-investment level is $\int_{W^d(R^E)}^{K+I^*-l/R^E} g(W)dW_i$. No one invests INCRS technology. Or

c) $R_1 \le R^E < R^*$. There is no banking loan. The proportion of the risky technology funded at the optimal level is $\int_{K+I^*-l/R^E}^{\bar{W}} g(W)dW_i$. The proportion of the risky technology funded at the suboptimal investment level is $\int_{W^c(R^E)}^{K+I^*-l/R^E} g(W)dW_i$. No one invests in CRS technology either.

Proof:

1) This is actually the same situation as the second case of proposition 2. The maximal interest rate never exceed R^* , otherwise no one would considers the risky technology. When the inequality holds, there is an excess demand for credits because the proportion of project that entrepreneurs are willing to carry out is higher than the proportion of project that lenders can fund. A shortage of credits cause the interest rate jump to R^* . In the previous section we have proofed that $\underline{I}(R^*) = I^*$ and consequently $W^c(R^*) = K + I^* - l/R^*$ in this situation. Regarding to the condition of $W^c(R^E) \le W^d(R^E) < K + I^* - l/R^E$, $W^d(R^E)$ disappears as soon as $R^E = R^*$. Furthermore, substituting $R^E = R^*$ for (2.11) gives $W^b(R^E) = (K + I^*)\left[1 - \frac{(R^* - R^E)}{(1-p)m}\right] = K + I^*$. If the interest rate reaches its maximal value R^*, then $W^b(R^*) = K + I^*$. It conflicts with the definition that $W^b(R^E) < W^c(R^E)$. The result implies that $W^b(R^E)$ also disappears in this case. In other words, because the price of loan is too high to afford, no one wants to borrow from banks. The group of suboptimal investment vanishes too. Finally, the proportion of the fully funded project is $\int_{K+I^*-\frac{l}{R^*}}^{\bar{W}} g(W)dW_i$.

   There is no role for CRS technology because the capital market is clear. The situation is shown in Figure 2.8.

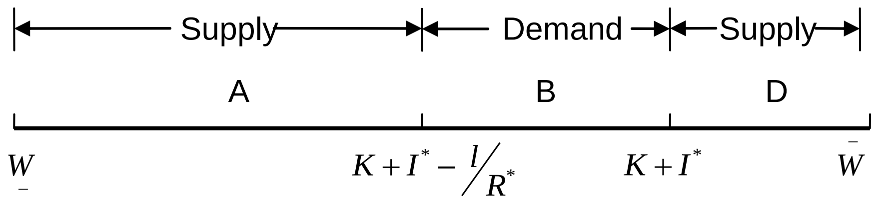

Figure 2.8

2) Looking back Figure 2.7 we find that all thresholds $W^b(R^E)$, $W^c(R^E)$ and $W^d(R^E)$ are functions of the interest rate. They must vary as the interest rate changes and consequently affect the demand and supply of funds. Therefore it is essential to determine how they react to the change of the interest rate. We have shown that $W^c(R^E)$ acts upon the change of R^E. (2.11) indicates that $W^b(R^E)$ positively relates to the interest rate. It is necessary to explore the connection between $W^d(R^E)$ and R^E.

Suppose an individual's wealth is located in area C plus D in Figure 2.7, namely $W^c(R^E) \le W_i < K + I^* - l/R^E$. If he borrows from the capital market, his welfare, WF^C, is $WF^C = p[f(W_i + l/R^E - K) - l]$ and $\frac{\partial WF^C}{\partial R^E} = -\frac{pl}{(R^E)^2} f'(W_i + l/R^E - K) < 0$. This first order partial derivative shows a rise in the interest rate has an opposite effect on welfare. In addition, $\frac{(\partial WF^C)^2}{\partial R^E \partial W_i} = -\frac{pl}{(R^E)^2} f''(W_i + l/R^E - K) > 0$. The second order partial derivative indicates that the loss rate of the welfare $\frac{\partial WF^C}{\partial R^E}$ is an increasing function relating to W_i. However, $\frac{\partial WF^C}{\partial R^E}$ is negative. An increase in $\frac{\partial WF^C}{\partial R^E}$ then implies a decrease in its absolute value. In other words, the welfare loss is larger near $W^d(R^E)$ rather than at the point of $K+I^*-{}^{l}\!/\!_{R^E}$. So the poor are more fragile than the richer when the interest rate changes.

Alternatively, if the individual borrows from banks, the welfare is

$WF^L = R^*(I^*+K)-(K+I^*-W_i)(1-p)m-(K+I^*-W_i)R^E$ [8]

So $\frac{\partial WF^L}{\partial R^E} = -(K+I^*-W_i)<0$ and $\frac{(\partial WF^L)^2}{\partial R^E \partial W_i} = 1>0$.

The results is completely similar to the previous one such that a rising interest rate does harm individuals' welfare and the poor suffer more than the richer.

In order to detect how $W^d(R^E)$ responds to the change of the interest rate, we need to compare individuals' welfare under this situation. Suppose the interest rate increases and $\left|\frac{\partial WF^C}{\partial R^E}\right| > \left|\frac{\partial WF^L}{\partial R^E}\right|$, it tells that the loss of raising money from the capital market is larger than that from banks. Then entrepreneurs prefer to borrow from the latter. Eventually $W^d(R^E)$ move towards right and vice versa. Unfortunately, we cannot compare the welfare loss directly at the point of $W^d(R^E)$. Since $\left|\frac{\partial WF^C}{\partial R^E}\right|$ and $\left|\frac{\partial WF^L}{\partial R^E}\right|$ are smaller as W_i increases, we can investigate the result at the point of

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[8] $WF^L = p\left[f(I^*)-P\right]$

$$= pf(I^*)-p\frac{1}{p}\left\{(K+I^*-W_i)\left[R^E+(1-p)m\right]-(1-p)l\right\}$$

$$= R^*(I^*+K)-(K+I^*-W_i)(1-p)m-(K+I^*-W_i)R^E$$

$K+I^{*}-{l}/{R^E}$ firstly. Let's consider individuals who are extremely close to this cut-off value. Substituting $W_i = K+I^{*}-{l}/{R^E}$ for $\left|\frac{\partial WF^C}{\partial R^E}\right|$ and $\left|\frac{\partial WF^L}{\partial R^E}\right|$ gives $\left|\frac{\partial WF^C}{\partial R^E}\right| = \frac{l}{R^E}\frac{1}{R^E}$ [9] and $\left|\frac{\partial WF^L}{\partial R^E}\right| = \frac{l}{R^E}$. Obviously $\left|\frac{\partial WF^C}{\partial R^E}\right| < \left|\frac{\partial WF^L}{\partial R^E}\right|$. The result reveals that at the position close to $K+I^{*}-{l}/{R^E}$ the change of welfare relative to R^E is larger if the agent borrows from the loan market than the change if she borrows from the capital market. Furthermore, since both of the sensitivities, $\left|\frac{\partial WF^C}{\partial R^E}\right|$ and $\left|\frac{\partial WF^L}{\partial R^E}\right|$, are increase as W_i decreases. Therefore the result still holds when $W_i = W^d(R^E)$ because $W^d(R^E) < K+I^{*}-{l}/{R^E}$. Suppose there is a small increment of R^E, the individuals who were indifferent to borrow from either markets now prefers to collect funds from the capital market. That is to say, $W^d(R^E)$ moves in a reverse direction of R^E. In conclusion, if R^E increases from an equilibrium situation, $W^b(R^E)$ and $W^c(R^E)$ increase as well. Meanwhile, $W^d(R^E)$ decreases instead.

a) The lowest capital markets return R^E must be no smaller than Z, otherwise all individuals prefer to invest in CRS technology. There is an excess supply of funds because the proportion of project that entrepreneurs are willing to invest is less than the proportion of project that lenders could fund. Over-supply credits reduce R^E to Z. All entrepreneurs are funded. No one would like to borrow from banks because of the high price. The proportion of individuals with

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[9] $\left|\frac{\partial WF^C}{\partial R^E}\right| = \left|-\frac{pl}{(R^E)^2} f'(I^*)\right| = \left|-\frac{pl}{(R^E)^2}\frac{1}{p}\right| = \frac{l}{R^E}\frac{1}{R^E}$

full-investment is $\int_{W^b(Z)}^{W^d(Z)} g(W)dW_i + \int_{K+I^*-l/Z}^{\bar{W}} g(W)dW_i$. The proportion of individuals with suboptimal investment is $\int_{W^d(Z)}^{K+I^*-l/Z} g(W)dW_i$. The rest funds flow into CRS .

The situation is shown in Figure 2.9.

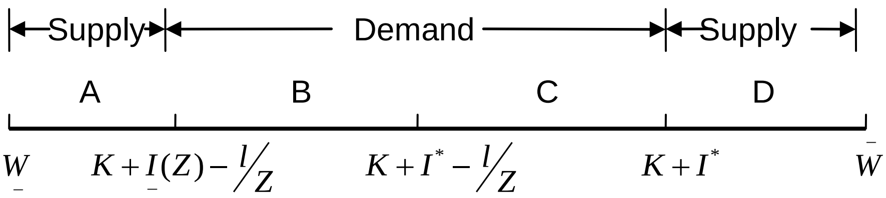

Figure 2.9

b) We have known that as R^E increases, both $W^b(R^E)$ and $W^c(R^E)$ increase while $W^d(R^E)$ decreases. $W^d(R^E)$ will inevitably meet $W^c(R^E)$ and disappear. However, it is still ambiguous whether $W^b(R^E)$ will catch $W^c(R^E)$. Eventually, three of them meet at one point. The issue can be investigated by detecting where $W^b(R^E)$ goes as long as $W^c(R^E)$ meets $W^c(R^E)$.

We know $W^d(R^E)$ is determined by (2.12),

$$p[f(I^*)-P] = p[f(W_i + l/R^E - K) - l]$$

Substituting (2.7) for the left item of (2.12) leads to

$$p[f(I^*)-P]$$

$$=(R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + W^d(R^E)[R^E + (1-p)m]$$ [10]

At the moment $W^d(R^E) = W^c(R^E)$ and $W^c(R^E) = K + \underline{I}(R^E) - l/R^E$.

Substituting (2.3) for $W^c(R^E) = K + \underline{I}(R^E) - l/R^E$ gives

$$W^c(R^E) = \frac{pf(\underline{I}(R^E)) - pl}{R^E}$$

Then $W^d(R^E) = W^c(R^E) = \dfrac{pf(\underline{I}(R^E)) - pl}{R^E}$. Substituting it for the left hand side of (2.12) gives

$$p[f(I^*) - P]$$

$$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + [pf(\underline{I}(R^E)) - pl] + W^c(R^E)(1-p)m$$ [11]

In addition, substituting $W^d(R^E) = W^c(R^E) = K + \underline{I}(R^E) - l/R^E$ for the right hand side of (2.12) obtains

$$p[f(W^d(R^E) + l/R^E - K) - l] = pf(\underline{I}(R^E)) - pl$$ [12]

Putting both left and right hand sides together gives

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[10] $p[f(I^*) - P]$

$= pf(I^*) - (K + I^* - W^d(R^E))\left[R^E + (1-p)m\right] + (1-p)l$

$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + W^d(R^E)[R^E + (1-p)m]$

[11] $p[f(I^*) - P]$

$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + W^d(R^E)[R^E + (1-p)m]$

$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + \dfrac{pf(\underline{I}(R^E)) - pl}{R^E}[R^E + (1-p)m]$

$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + [pf(\underline{I}(R^E)) - pl] + \dfrac{pf(\underline{I}(R^E)) - pl}{R^E}(1-p)m$

$= (R^* - R^E)(K+I^*) - (K+I^*)(1-p)m + pf(\underline{I}(R^E)) - pl + W^c(R^E)(1-p)m$

[12] $p[f(W^d(R^E) + l/R^E - K) - l]$

$= p[f(K + \underline{I}(R^E) - l/R^E + l/R^E - K) - l]$

$= pf(\underline{I}(R^E)) - pl$

$$(R^* - R^E)(K + I^*) - (K + I^*)(1-p)m + \lfloor pf(\underline{I}(R^E)) - pl \rfloor + W^c(R^E)(1-p)m$$

$$= pf(\underline{I}(R^E)) - pl$$

Finally, by rearranging the equation we have

$$W^d(R^E) = W^c(R^E) = (K + I^*)\left[1 - \frac{(R^* - R^E)}{(1-p)m}\right] = W^b(R^E)$$

The result reveals that as R^E increases, $W^b(R^E)$, $W^c(R^E)$ and $W^d(R^E)$ eventually meet together. It defines a cut-off interest rate of the capital market, let's call it R_1, so that any interest rates higher than the value will eliminate the demand of bank loans. In other words, the poor individuals find it is too expensive to borrow from banks. Only if $Z < R^E < R_1$, those poor individuals might benefit from the low interest rate and approach the optimal investment level. The interest rate of the loan market is $R_b = R^E + (1-p)m$. The proportion of the risky technology funded at the optimal level is $\int_{W^b(R^E)}^{W^d(R^E)} g(W)dW_i + \int_{K+I^*-l/R^E}^{\bar{W}} g(W)dW_i$.

The proportion of the risky technology funded at the under-investment level is $\int_{W^d(R^E)}^{K+I^*-l/R^E} g(W)dW_i$. No one invests in the CRS technology. The situation is shown below:

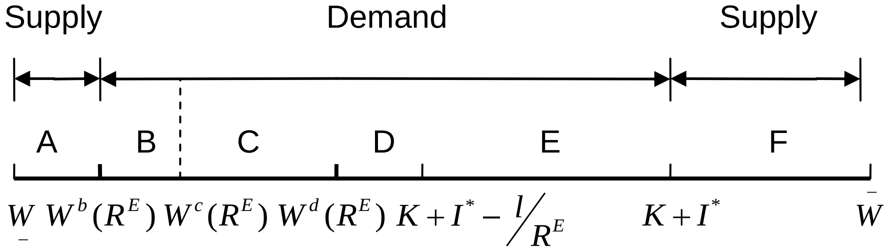

Figure 2.10

c) If $R_1 \le R^E < R^*$, the loan market disappears because the cost is too high to afford. The situation is the same as the case 3 of the proposition 2. The proportion of the risky technology funded at the optimal level is $\int_{K+I^*-l/R^E}^{\bar{W}} g(W)dW_i$. The proportion of the risky technology funded at the suboptimal level is $\int_{W^c(R^E)}^{K+I^*-l/R^E} g(W)dW_i$. There is no CRS investment as well. The situation is shown in Figure 2.11.

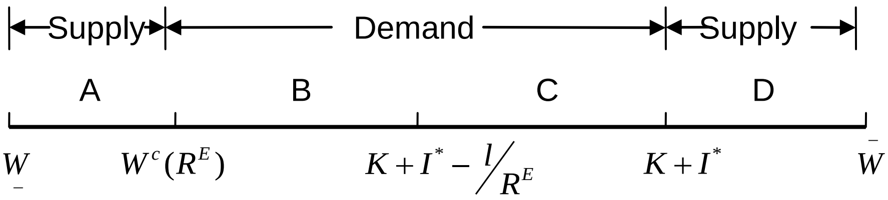

Figure 2.11

2.7 Comparative Statics, An extension of the Asymmetric Information Model

By now the model defines a few of endogenous variables such as the minimal investment level $\underline{I}$, the optimal investment level I^*, the interest rate of the capital market R^E and the maximal unit returns of the project R^*. There are also several exogenous parameters such as the mean wealth $\hat{W}$ and the income difference X which is the difference between the poorest and the richest individual.. It is interesting to explore how these parameters influence endogenous variables. In this section, we will investigate the issue under the conditions of imperfect capital markets without banking. The restriction means the information on the return of project is private and entrepreneurs can maximally collect $\frac{l}{R^E}$ from the capital market. As Figure 2.3 shows, the individuals whose wealth is less than $W^c(R^E) = K + \underline{I}(R^E) - (\frac{l}{R^E})$ cannot collect money to undertake the risky project. They will only be pure lenders. The individuals with wealth between $K + \underline{I}(R^E) - \frac{l}{R^E}$ and $K + I^* - \frac{l}{R^E}$ perform suboptimal investment and those with wealth higher than $K + I^* - \frac{l}{R^E}$ carry out full investment. A few individuals who are richer will lend spare money after they have invested in the project.

Therefore the expected credit supply is:

$$\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W) dW_i + \int_{K+I^*}^{\overline{W}} \left[W_i - (K + I^*)\right] g(W) dW_i$$

The first item covers individuals located in the area A of Figure 2.3. They have to invest in either the capital market or CRS. The second one involves the richer whose wealth exceeds the demand of a full investment project. They have to invest the rest of funds in other technologies. On the other hand, the expected demand of credits is:

$$\int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W) dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K + I^* - W_i) g(W) dW_i$$

The first item presents individuals who can maximally borrow l/R^E to invest the risky project at the suboptimal level. The second one exhibits individuals who are able to reach optimal investment level after borrowing $K + I^* - W_i$.

Lastly, we know the lowest interest rate must be no smaller than Z, otherwise all individuals would prefer to invest in CRS technology. On the other hand, the highest interest rate cannot be higher than R^*, otherwise agents would never consider the risky technology. So the interest rate must vary from Z to R^*. Which value the interest rate approaches is determined by different parameters. There are three different circumstances.

1) If $\int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} (l/Z) g(W) dW_i + \int_{K+I^*-(l/Z)}^{K+I^*} (K+I^*-W_i) g(W) dW_i$

$< \int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$ then $R^E = Z$.

If the above inequality holds, there is an excess supply of funds because the proportion of projects that entrepreneurs are willing to invest in is less than the proportion of projects that lenders can support. One of restrictions of the model is the lowest interest rate must not be smaller than Z. The interest rate then declines to its minimal value Z. The spare funds flow to CRS technology. The share of entrepreneurs with suboptimal investment is $\int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} g(W) dW_i$. The proportion of individuals who invest at the optimal level is $\int_{K+I^*-(l/Z)}^{\bar{W}} g(W) dW_i$, . The total funds for CRS technology are the difference between the supply and the demand of credits.

$$\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i$$

$$- \int_{K+\underline{I}(Z)-(l/Z)}^{K+I^*-(l/Z)} (l/Z) g(W) dW_i - \int_{K+I^*-(l/Z)}^{K+I^*} (K+I^*-W_i) g(W) dW_i$$

Equilibrium of the model is determined by the following four equations.

$$R^E = \frac{pf(\underline{I}) + (1-p)l}{K+\underline{I}} \tag{2.3}$$

$$f'(I^*) = 1/p \tag{2.4}$$

$$R^* = \frac{pf(I^*) + (1-p)l}{K+I^*} \tag{2.5}$$

$$R^E = Z \tag{2.15}$$

There are four equations respond to four variables R^E, $\underline{I}$, I^* and R^*. Thus it should have a unique solution. However, we cannot obtain a reduced form solution on account of the general form of the product function. We'll discuss its comparative statics results instead. Chiang (1984) introduce a method called Derivatives of Implicit Functions to explore the comparative statics issue for those equations without reduced forms. In the following sections we are going to find out the results by the method.

Equation (2.15) says that the interest rate is a constant. Comparing it with (2.3) gives

$$\frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} = Z \tag{2.16}$$

The minimal investment $\underline{I}$ is determined by the equation above while the optimal investment I^* is determined by equation (2.4). Notice that neither the mean value of wealth $\hat{W}$ nor the wealth difference X are presented in the simultaneous equations. It implies that they do not affect R^E and I^*. The reason is simple. No matter how $\hat{W}$ and X change, there is always an excess supply comparing to the demand as long as the inequality holds. The fundamental of the model never change in this case.

In conclusion, the results of comparative statics are shown below:

i. $\frac{d\underline{I}}{d\hat{W}} = 0$, $\frac{d\underline{I}}{dX} = 0$;

ii. $\frac{dI^*}{d\hat{W}} = 0$, $\frac{dI^*}{dX} = 0$;

iii. $\frac{dR^E}{d\hat{W}} = 0$, $\frac{dR^E}{dX} = 0$;

iv. $\frac{dR^*}{d\hat{W}} = 0$, $\frac{dR^*}{dX} = 0$

It states that both the average wealth and the difference of wealth do not change anything in this system, because they cannot change the fundamental fact of exceeding credits in this particular case.

2) If $\int_{K+I^*-(l/R^*)}^{K+I^*} (K + I^* - W_i) g(W) dW_i > \int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K + I^*)\right] g(W) dW_i$

then $R^E = R^*$. When the inequality holds, there is an excess demand of credits because the proportion of project that entrepreneurs are willing to invest is higher than the proportion of project that lenders can fund. The demand for funds pushes up the interest rate till R^*, because the maximum of the interest rate cannot be larger than R^*. In addition, comparing equation (2.3) with (2.5), it is easy to find that $\underline{I}(R^*) = I^*$ if $R^E = R^*$. In other words, the minimal investment is equal to the optimal value as long as $R^E = R^*$. Those individuals who carry out suboptimal investment completely disappear. No one will invest in CRS technology either. The solution of the model is determined by the following five equations.

$$R^E = \frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} \tag{2.3}$$

$$f'(I^*) = 1/p \tag{2.4}$$

$$R^* = \frac{pf(I^*) + (1-p)l}{K + I^*} \tag{2.5}$$

$$\underline{I}(R^*) = I^* \tag{2.17}$$

$$R^E = R^* \tag{2.18}$$

For the same reason, we only discuss its comparative statics results here. Firstly, according to (2.4) I^* is a constant in this case. It does not change any more, so does $\underline{I}$ on account of (2.17). In addition, both $\hat{W}$ and X are not shown in the simultaneous equations. It means they do not affect the equilibrium because they cannot change the fundamental fact of exceeding demand.

In conclusion, the results of comparative statics are shown below:

i. $\frac{d\underline{I}}{d\hat{W}} = 0$, $\frac{d\underline{I}}{dX} = 0$;

ii. $\frac{dI^*}{d\hat{W}} = 0$, $\frac{dI^*}{dX} = 0$;

iii. $\frac{dR^E}{d\hat{W}} = 0$, $\frac{dR^E}{dX} = 0$;

iv. $\frac{dR^*}{d\hat{W}} = 0$, $\frac{dR^*}{dX} = 0$

The results show that both I^* and $\underline{I}$ are fixed in this case. In addition, the interest rate and its maximal value are not affected by the mean wealth and indifference in wealth as well..

3) If $\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} [W_i - (K + I^*)] g(W) dW_i$

$$\leq \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E)g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^* - W_i)g(W)dW_i$$

$$\leq \int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right]g(W)dW_i$$

then $Z \leq R^E \leq R^*$.

The minimal supply of funds in the economy is $\int_{\underline{W}}^{K+\underline{I}(Z)-(l/Z)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} [W_i - (K+I^*)]g(W)dW_i$ with $R^E = Z$ while the maximal supply of funds is $\int_{\underline{W}}^{K+I^*-(l/R^*)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} [W_i - (K+I^*)]g(W)dW_i$ with $R^E = R^*$. If the inequality holds, there is an excess demand of credits if $R^E = Z$, and an excess supply of credits if $R^E = R^*$. Therefore the demand of credits varies in the range of the maximum and the minimum credits supply, and the interest rate R^E changes between Z and R^*. It approaches equilibrium at the market clear point, where the total demand is equal to the total supply. That is

$$\begin{aligned} &\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} [W_i - (K+I^*)]g(W)dW_i \\ &= \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E)g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^* - W_i)g(W)dW_i \end{aligned} \tag{2.19}$$

The total proportion of projects funded at the optimal level is $\int_{K+I^*-l/R^E}^{\bar{W}} g(W)dW_i$.

The proportion of projects in the suboptimal status is $\int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} g(W)dW_i$.

Because the demand of funds is equal to the supply, the CRS investment has no place in this case. The system is determined by the following equations

$$R^E = \frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} \tag{2.3}$$

$$f'(I^*) = 1/p \tag{2.4}$$

$$R^* = \frac{pf(I^*) + (1-p)l}{K + I^*} \tag{2.5}$$

$$\begin{aligned} &\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W) dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W) dW_i \\ &= \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W) dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^* - W_i) g(W) dW_i \end{aligned} \tag{2.19}$$

The results of comparative statics show below. The proof is in the Appendix A.1. Apparently, there are three different results depending on the value of $K+I^*$ and 2X. Here $K+I^*$ is the optimal investment level while X is the difference between the poorest and the richest individual. So $K+I^* < 2X$ point at an economy where the difference between the poorest and the richest is large and $K+I^* > 2X$ means the difference between the poorest and the richest is small.

a) If $K+I^* < 2X$, then

i. $\frac{\partial \underline{I}}{\partial \hat{W}} > 0$, $\frac{\partial \underline{I}}{\partial X} < 0$;

ii. $\frac{\partial I^*}{\partial \hat{W}} = 0$, $\frac{\partial I^*}{\partial X} = 0$;

iii. $\dfrac{\partial R^E}{\partial \hat{W}} < 0$, $\dfrac{\partial R^E}{\partial X} < 0$;

iv. $\dfrac{dR^*}{d\hat{W}} = 0$, $\dfrac{dR^*}{dX} = 0$

If the mean wealth increases, the minimal investment level goes up while the interest rate falls. The movement of the threshold $K + \underline{I} - {}^{l}\!/_{R^E}$ is ambiguous. But the second threshold $K + I^* - {}^{l}\!/_{R^E}$ shift leftwards. If the difference X increases, both the minimal investment level and the interest rate decreases. Therefore the thresholds $K + \underline{I} - {}^{l}\!/_{R^E}$ and $K + I^* - {}^{l}\!/_{R^E}$ move towards left. The fixed set-up cost has a similar effect except that it also has a negative relation with the maximal return of the risky project.

b) If $K + I^* = 2X$, then

i. $\dfrac{\partial \underline{I}}{\partial \hat{W}} = 0$, $\dfrac{\partial \underline{I}}{\partial X} < 0$;

ii. $\dfrac{\partial I^*}{\partial \hat{W}} = 0$, $\dfrac{\partial I^*}{\partial X} = 0$;

iii. $\dfrac{\partial R^E}{\partial \hat{W}} = 0$, $\dfrac{\partial R^E}{\partial X} < 0$;

iv. $\dfrac{dR^*}{d\hat{W}} = 0$, $\dfrac{dR^*}{dX} = 0$

In this case, the mean wealth does not connect to the variables in the economy. The difference X and the fix cost K impose the same effects on the system as the previous case does.

c) If $K + I^* > 2X$, then

i. $\frac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\frac{\partial \bar{I}}{\partial X} < 0$;

ii. $\frac{\partial I^*}{\partial \hat{W}} = 0$, $\frac{\partial I^*}{\partial X} = 0$;

iii. $\frac{\partial R^E}{\partial \hat{W}} > 0$, $\frac{\partial R^E}{\partial X} < 0$;

iv. $\frac{dR^*}{d\hat{W}} = 0$, $\frac{dR^*}{dX} = 0$

In this case, the mean wealth carry out an opposite effect to the economy comparing with the first case. The effects of the rest parameters are the same.

2.8 Conclusion

We investigate a model of asymmetric information on account of market imperfection. The only heterogeneity over individuals is their initial endowments. According to their restrictions of working capital, individuals make investment decision among the capital market, the CRS technology and a risky project. We find that the poorest individuals are pure lenders. Some medium-wealth individuals borrow money to reach a sub-optimal investment level while a few richer individuals collect enough money to achieve the full investment level. The richest individuals are self-financing. We then introduce a monitoring technology to make it possible for external lenders observing the returns of the private project. Thus individuals have an alternative source to raise money. Our results show that the poorest individuals are still pure lenders, some relatively poor individuals may obtain bank loan to achieve the optimal level of investment. Meanwhile, some medium wealth individuals prefer to borrow from the capital market to remain at sub-optimal investment level and some richer individuals raise necessary money from the direct finance source to achieve the full investment. The richest individuals are still self-finance.

Finally, we carry out the comparative statics to the imperfect capital market model without banking. We find that if there is an excess supply of funds then the interest rate is equal to the CRS return. Neither the average wealth nor the difference of wealth affects the system in this case. Furthermore, if there is an excess demand of credits. Then the interest rate is equal to the optimal unit return. Both the optimal and minimal investments are fixed. In addition, the interest rate and its maximal value are not affected by either the mean wealth or indifference of wealth. Only if the demand of credits varies between the two supplies above, the interest rate changes accordingly between the CRS return and the optimal return. The comparative statics results depend on the difference between the optimal investment value and the income difference.

CHAPTER 3

THE EFFECT OF AVERAGE INCOME ON FINANCIAL DEVELOPMENT: EVIDENCE FROM INCOME INEQUALITY THRESHOLD REGRESSION

ABSTRACT

The relation among income inequality, economic growth and financial development has been highly controversial for a long time. Some researchers suggest that there is an inverted-U curve between income inequality and growth (Kuznets (1955), Aghion and Bolton (1997), Galor and Tsiddon (1996)). They believe that income inequality goes up at the earlier developing stages and falls as long as the economy is fully grown. However, some empirical tests do not support such processes (Clarke et al. (2003), Barro (2000)). Furthermore, most studies only focus on relations between either economic growth and income inequality, or between financial development and inequality. Little research goes further to explore the combined effects of economic growth and income inequality on financial development. In Chapter 2, we have developed a model to detect individuals' investment decisions, given the initial endowment and the distribution of wealth. In this chapter we discuss how average wealth affects financial development over the different values of income inequality. Both initial wealth and its distribution work together to determine the interest rate, the minimal investment level and consequently the size of the capital market. This chapter examines the impact of income inequality and average income on financial development by a threshold regression. The empirical tests observe two important results. One is that the rise of average income does improve financial development as long as income inequality is below a cut-off value. The other is that there is a strong threshold effect on income inequality. In other words, the effect of average income on financial development depends on the value of income inequality.

3.1 Introduction

It's been a controversial issue about the relation among economic growth, financial development and income inequality. Some evidence shows that economic growth acts as a stimulus for financial development (Greenwood and Jovanovic (1990)). On the other hands, a lot of studies suggest that growth would push up income inequality firstly and pull it down later on (Kuznets (1955), Aghion and Bolton (1997), Galor and Tsiddon (1996)). However, little research explores the combined effect of economic growth and income inequality on financial development. In Chapter 2, we have developed a theoretical model to detect individuals' investment decisions under the different financial restrictions. In this chapter, we deliver a meaningful result that both average wealth and income inequality affect the way individuals accessing the financial market and consequently their loan sizes, namely financial development. Note it is not a dynamic model, so we cannot detect the relation between economic growth and financial development. Instead, the purpose of this chapter is to inspect how average income and income inequality influence financial development by carrying out an empirical threshold regression model.

There is a famous inverted U-shape hypothesis Kuznets (1955) in the study of income inequality. In the seminal paper, Kuznets describes the transformation of an economy from agricultural society to industrial society and predicts an inverted-U shape relationship between income inequality and economic development. He suggests that income inequality will rise during the short term of the transition. As the economy grows, income inequality touches the ceiling and turns down in the long term. The process is shown in Figure 3.1.

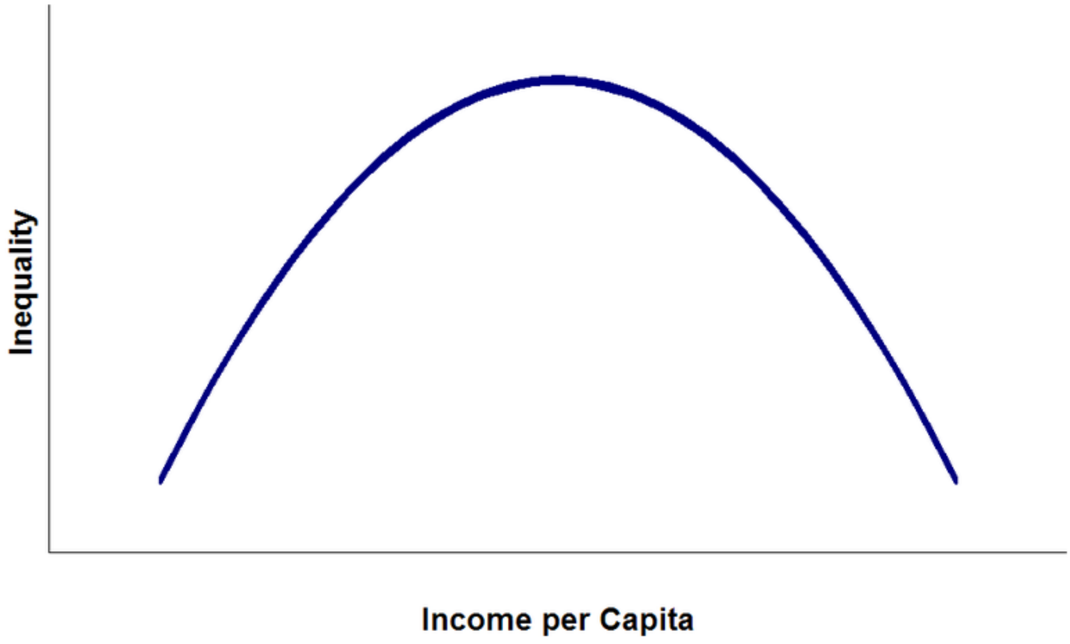

Figure 3.1 Kuznet's hypothesis

One possible interpretation is that physical capital is the major source to promote economic growth at the early stage of development. A wider income difference encourages growth by allocating credits towards those who save and invest the most. However, at the mature stage, the government redistributes income throughout different levels of society by retirement pension, health care and so on. Thus income inequality declines. Furthermore, human capital also accounts for growth at this stage because the promotion of mass education may also reduce the gap in income inequality.

The interpretation implies that the financial system plays a key role in the process. Given the initial wealth distribution, only those individuals who have sufficient physical capital are able to raise fund from the capital market to earn a higher return and consequently benefit from financial development. In this case, income inequality will be magnified across different groups. In other words, financial development, coming with the capital market imperfection, makes income inequality worse at the early stages. As the financial system is fully mature, however, market imperfection fades away and the capital market will be more equally shared over different groups. Therefore income inequality will decrease.

In Chapter 2, we develop a theoretical model to examine how individuals choose investment method to maximize their returns, given the initial wealth distribution. The results of comparative statics show that there are threshold effects on the income difference between the rich and the poor. If the income difference is lower than the critical value, then the minimal investment level and the interest rate positively related to the average wealth. Otherwise both of them negatively linked to the average wealth. We know the income difference is somehow associated with income inequality. The minimal investment level and the interest rate also determine the size of financial sectors. Therefore the model predicts a relationship between financial development, average wealth and income inequality. The purpose of this chapter is to inspect the prediction by empirical data. The innovation of our empirical test is that we adopt the method of threshold regression to avoid two major problems from other researches. Firstly, we do not arbitrarily divide countries into groups such as developing or developed countries. Secondly, we also do not deliberately select the turning-point of the development. Within the method of threshold regression, the turning-point just emerges naturally.

The rest of the paper is organized as follows. We review literature on the relationship between growth and inequality in section 2. In section 3, we recall the theoretical model developed in Chapter 2 and make an empirical extension with the Gini coefficient. An empirical model is demonstrated in section 4. The data set is shown in section 5. Then we discuss the result in section 6. Finally we conclude in section 7.

3.2 Literature Review

There are a large number of papers discussing the relationship between growth and income inequality. The famous controversial issue is the Kuznet's hypothesis (Kuznets (1955)). In his seminal paper, Kuznets describes a transformation of an economy from agricultural society to industrial society and predicted an inverted-U shape relationship between growth and income inequality. He suggests that the latter will rise at the initial stages of the transition and turns down after touching the ceiling in the long term. The argument is so prevalent that a lot of recent studies revolve around it. We list a few papers which accept or reject Kuznet's hypothesis in the table 3.1.

Accept Kuznet's hypothesis	Reject Kuznet's hypothesis
Kuznets (1955)	Saith (1983)
Ahluwalia (1976a, 1976b)	Barro (2000)
Campano and Salvatore (1988)	Clarke et al. (2003)
Galor and Tsiddon (1996)	Beck et al. (2004)
Aghion and Bolton (1997)	Andrews et al. (2009)
Jalilian (2002)	

Table 3.1

Aghion and Bolton (1997) developed a trickle-down growth and development model under the imperfect capital market circumstances. The model provided a mechanism of how the accumulation of wealth by the rich trickles down to the poor and makes the latter better off. The moral hazard with limited borrow constraint is the source of market imperfection and income inequality. This model set up a closed economy with continuum agents. Each agent lives for one period and leaves one child. At the beginning each agent is endowed with wealth randomly and one unit of labour. Then they work, invest, consume and bequeath. The only source of heterogeneity among them is the initial wealth endowments. An agent can choose either to be a worker to have a deterministic but small income, or to be an entrepreneur to obtain a higher, but uncertain revenue. The interest rate is determined endogenously. The model confirmed the trickle-down process leads to a unique stable wealth distribution under sufficiently high rates of capital accumulation. But it didn't deny government intervention because the latter brings greater opportunity equality and also accelerates the trickle-down process. The most important result is that the process of capital accumulation increases income inequality initially but reduces it later. In other words, the theoretical model supports Kuznet’s hypothesis.

Another theoretical model which supports the Kuznet’s hypothesis is provided by Galor and Tsiddon (1996). The model interprets the mechanism of the inverted U-shape relation between income inequality and per capita output from the perspective of the accumulation of human capital, instead of physical capital. It constructs a small overlapping-generations economy with perfect competition markets. An individual lives for three periods. Firstly he borrows money to invest in education for himself. After having professional skills, he then supplies inelastically efficiency units of labour and obtains wages at the competitive market in the second period. The third period is only for consumption. A single homogeneous good is produced by using capital and labour in every period. The level of the production technology is determined by the average level of human capital of the previous generation. They found the pursuit of equality at the early stage may trap the economy in a low level of investment in human capital. In contrast, inequality encourages highly educated families in society to get over the low level situation by increasing their investment in human capital. As such upper segments of society grow and income inequality becomes wider, the accumulated knowledge gradually trickles-down to the lower group in society and leads to the improvement of skills as well as the production technology. It finally improves human capital for all segments of society and thus alleviates income inequality at the mature stage of the economy. The process obviously approves Kuznet's hypothesis.

There are a lot of empirical evidences to support Kuznet's hypothesis too. Ahluwalia (1976a, 1976b) explored the nature of the relationship between economic growth and income inequality based on cross country data. He used a sample of 60 countries, including 40 developing countries, 14 developed countries and 6 socialist countries to examine the issue by multivariate regression. These 60 countries were deliberately divided into five different percentile groups, which are the top 20 percent, the middle 40 percent, the lowest 60 percent, the lowest 40 percent and the lowest 20 percent. The economic growth is indicated by the logarithm of per capita GNP. There is also a dummy variable to distinguish socialist countries from the sample because of the expectation of higher degree of equality for those countries. These five percentile segments produce five basic equations. The result of the empirical test for a quadratic relationship between the logarithm of per capita GNP and income inequality is significant. All five equations yields "correct" signs and relatively strong t-tests and coefficients of determination which consist with Kuznet's hypothesis. As a matter of fact, income shares of all groups except the top 20 percentile declined firstly and then increased as per capita GNP rose.

Campano and Salvatore (1988) re-examine the inverted U-shaped hypothesis between growth and inequality with the similar method of the previous study . Their initial motivation is to defend Ahluwalia (1976b) against Saith (1983) because the latter denies the former's empirical results. Saith indicates that the estimated parameters of the bottom group income share are unstable and statistically insignificant when the dummy variable is omitted, or the socialist countries are removed from the sample, or the estimation includes only the developing countries. Campano argues that there are five basic equations in the Ahluwalia's test. Exclusive selection for only one of the equations makes Saith’s conclusion weak. Though sometimes the bottom 20 percent of the population seems to be ignored by the development, other segments of the society are significantly affected by the process. It shows that the result is robust. Campano hence repeats the Ahluwalia's test with new data. His sample set contains 95 countries including 68 developing countries, 21 developed countries and 6 socialist countries. The results again supported Kuznet's hypothesis for the whole population. Furthermore, Jalilian (2002) reports an empirical result based on panel data of 64 countries which are composed of 43 developing and 21 developed countries. His results exhibit a strong support to Kuznets' inverted-U hypothesis.

However, there are many dissenting voices on this issue. Clarke et al. (2003) examines the relation between financial development and income inequality with panel data of 91 countries over the period 1960-1995. They predict financial development affects income inequality as long as agents migrate from the traditional sector to the modem one. If an individual can easily access the capital market, he can make the transition faster. Since the modern sector garners larger rewards for its participants, it means highly talented individuals must gain more than others. Consequently inequality will be expected higher in countries with large modern sectors and financial intermediaries. In other words, the relation between financial development and the size of the modern sector would be positive. Based on the analysis above, they construct an empirical model including linear and squared terms of the log of real per capita GDP, linear and squared terms of financial sectors and a product term of financial development and modern sector to detect their relationship. Their result shows that there is a negative relationship between the financial sector and income inequality. In other words, they didn't find the significant evidences to support the inverted U-shaped curve from Kuznet’s hypothesis.

Barro (2000) argues that there are little overall relation between inequality and growth. His empirical framework is based on conditional convergence. It considers average growth rates and average ratios of investment to GDO over three decades, 1965-75, 1975-85, and 1985-95. The estimation is applied by three-stage least squares. He also tests many control variables such as government consumption to GDP, the rule of law, an index of democracy, and the rate of inflation. He finds that the effect of log form of GDP on growth is negative for all but the poorest countries. It also negatively relates to the ratio of government consumption to GDP. On the other hands, growth is positively related to the stock of human capital. It also positively relates to the ratio of investment to GDP. This result shows that the effect of inequality on growth is negative for values of per capita GDP below $2070 (1985 U.S. dollars) and then becomes positive. Although Kuznet’s hypothesis may be an empirical regularity, he disputed that it cannot explain variations in inequality across countries or over time.

Beck et al. (2004) use the data set of 52 developing and developed countries over the period 1960 to 1999 to examine the relationship between financial development and income inequality. They find that financial development induced income inequality to fall and reduce poverty. The result didn’t support the Kuznets’ hypothesis either. The above arguments show there is no convincing evidence to support a certain relationship between growth and income inequality. It is necessary to explore the issue further.

In addition, there are some other papers which check the relationship between income inequality and economic growth from different perspectives. For instance, Bornschier (1983) analyzes two paradigms of the income inequality. The one is ‘the world economy’ paradigm and the other is 'the level of development' paradigm. The former could be regarded as globalization. It means the global economy allocates labour within industrial activities, and he analyzed inequality in a global view rather than a single country. The latter is the normal nexus between growth and income inequality across countries. One of his conclusions is that developing countries do not automatically lower their inequality in development. The reason is that the position of countries in the world economy seems to stabilize income inequality differences between these groups of countries. Under these circumstances, developing countries generally cannot be expected to reduce income inequality substantially with economic growth. Durlauf and Johnson (1995) use a method called 'regression trees' for structural-break and threshold identification. He found the marginal product of capital varies on account of the level of economic development. The results are consistent with ours in which economic growth exhibit multiple stages. Another paper evaluate the impact of financial development on changes in the distribution of income and changes in both relative and absolute poverty (Andrews et al. (2009)). They use a panel of tax data of 12 developed nations observed for between 22 and 85 years. Their results show that there is no systematic relationship between top income shares and economic growth.

No matter whether these empirical results support or reject Kuznet's hypothesis, two major problems may weaken their conclusions. Firstly, most studies divide countries into two groups (developed and less developed). An implied meaning of it is that the nature of the relationship differs according to a country's level of economic development. It is important to note, however, that the criterion to which groups a country belongs depends on a researcher's subjective category. Secondly, the form of the empirical model has considerable impact on the turning-point of the curve. The U-shape hypothesis in turns may be significant for some functional forms and not for others. For example, while Deininger and Squire (1998) reject the Kuznet's hypothesis for the fixed-effects case, they do find it survived in the pooled case for their functional form. Therefore we prefer a more natural method for the empirical model to a specified function form. The threshold model which is developed by Hansen (1999) is attractive because it allows more flexible regression functional forms by splitting data with unknown threshold values. However, Hansen's model is only effective for the exogenous threshold variables. The endogeneity problem for the threshold variables may causes coefficients estimators being inconsistent and consequently the inference is invalid. Fortunately, there is an extended framework which allows the endogenous threshold variables (Kourtellos et al. (2008), Wang and S.Lin (2010)). We will employ the endogenous threshold regression in our empirical test.

3.3 The Model Framework

In Chapter 2, we assumes that there is a continuum of risk-neutral agents of measure 1 who indexed by i. A single good can be either invested or consumed. An individual is initially endowed with a random wealth W_i, $W_i \in [\underline{W}, \bar{W}]$. Suppose the endowment is subjected to a continuous uniform distribution. Hence the cumulative distribution function is

$$G(W) = \frac{W_i - \underline{W}}{\bar{W} - \underline{W}}$$

and the corresponding density function is

$$g(W) = \frac{1}{\bar{W} - \underline{W}}.$$

In addition, the mean wealth of the economy is

$$\hat{W} = \int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i = \frac{\bar{W} + \underline{W}}{2} \qquad (3.1)$$

Let $\bar{W} - \hat{W} = \hat{W} - \underline{W} = X$. Then

$$X = \frac{\bar{W} - \underline{W}}{2} \qquad (3.2)$$

Parameter X is half of the income difference from the richest to the poorest in the economy. Consequently the range of endowment can be presented by $W_i \in [\hat{W} - X, \hat{W} + X]$. The system contains three different investment technologies. The first one is a CRS technology that yields a constant return Z(>1) for each unit input. This technology is a riskless way to invest. Another choice is to invest in the capital market and earn R^E (>1). Here the interest rate R^E is endogenously determined. The third method is the most complicated one. There is a project requiring a fixed cost K(>0) at the beginning of the period. Suppose I_i is the investment variable of the project for an individual i, $I_i \in [0, \bar{W} - K)$. The total investment of an agent is $K + I_i$. It yields $f(I_i)$ once the project succeeds with probability p, or fails with probability $1 - p$ and is thus liquidated with the value l ($l < K$). Here we assume $f'(I_i) > 0, f''(I_i) < 0$ as usual.

We have demonstrated in Chapter 2 that the model has many important features. Firstly, the maximal investment of the project, I^*, is determined by the equation $f'(I^*) = 1/p$. Secondly, due to the reason of moral hazard, a lender maximally provides l/R^E to a borrower. Hence the minimal investment value $\underline{I}$ is obtained by the equation $R^E(K + \underline{I} - l/R^E) = p[f(\underline{I}) - l]$. It means that an individual is indifferent to invest in either the capital market or the risky project at the minimal investment level. Simplifying the equation obtains

$$R^E = \frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} \tag{3.3}$$

The model is illustrated in Figure 3.2.

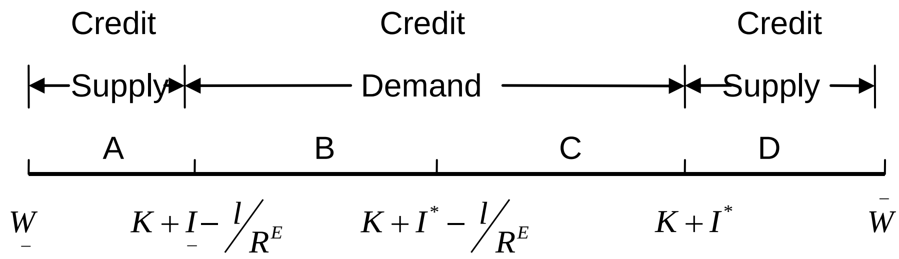

Figure 3.2

Variables $\underline{I}$ and R^E are endogenously determined in the model, while $K, l, \underline{W}, \bar{W}$ are exogenous parameters. Because I^* is given by the equation $f'(I^*) = 1/p$, it is an exogenous parameter as well.

Since the purpose of this chapter is to verify the model with the empirical experiment, the question is what the model predicts and how to test it. We will discuss these issues in detail in the following sections.

First of all, the expression $\hat{W} = \int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i$ states that $\hat{W}$ is the average wealth (also the average income) of an economy. It is easily mapped to GDP per capita in a data set. Let's examine how parameter X relates to income inequality.

We measure income inequality by the Gini coefficient which applies ratio analysis. The simplicity of the Gini coefficient makes it easy to compare across countries and to interpret. Hence it is prevalent in the study of income inequality. The Gini coefficient is based on the Lorenz curve which indicates the ratio of the incomes of the bottom proportion of all households to the total incomes, as illustrated in Figure 3.3.

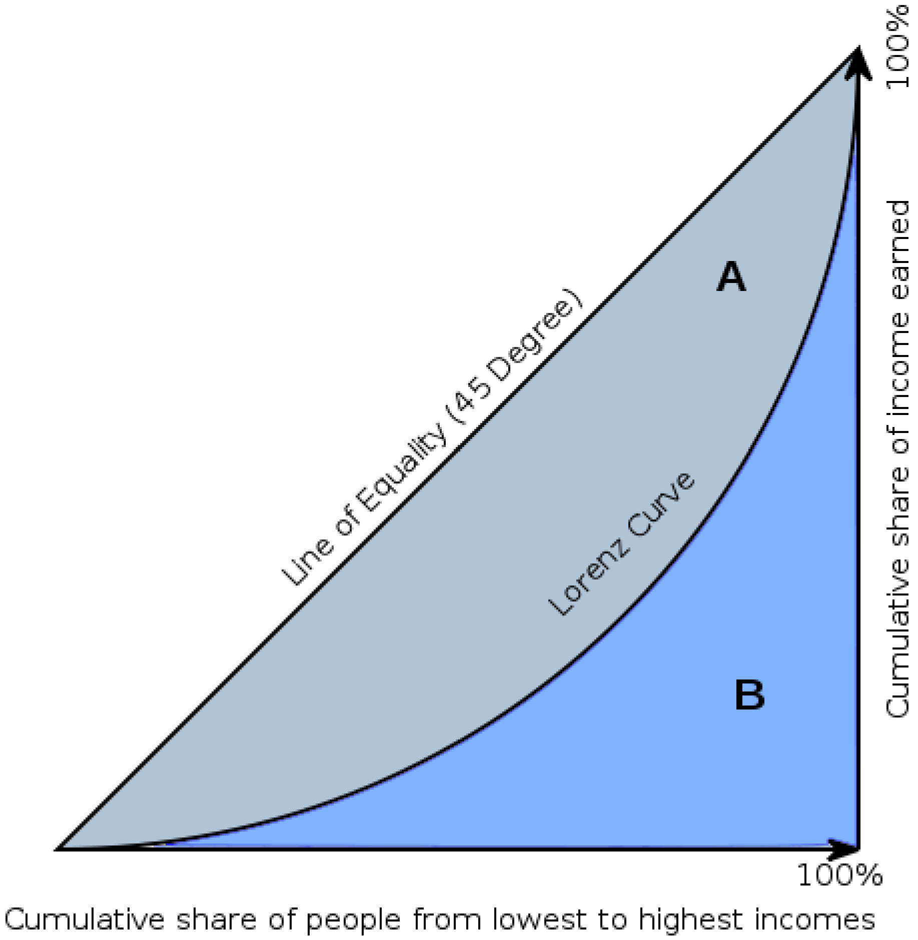

Figure 3.3

Technically, the Gini index presents the ratio of the area that lies between the line of equality and the Lorenz curve over the total area under the line of equality. In other words, $Gini = \frac{A}{A+B}$. Since A+B = 0.5, the Gini index $Gini = 2A = 1 - 2B$. Note instead of being someone wealth value, B is the integral of Lorenz curve form 0 to1. It is strictly positive. It is not so If the Lorenz curve is presented by the function $L(W)$, the value of B can be obtained by $\int_0^1 L(W) dW_i$. According to the definition of Lorenz curve, it has

$$L(W) = \frac{\int_{\underline{W}}^{W_i} W_i g(W) dW_i}{\int_{\underline{W}}^{\bar{W}} W_i g(W) dW_i} = \frac{\int_{\underline{W}}^{W_i} \frac{W_i}{\bar{W} - \underline{W}} dW_i}{\int_{\underline{W}}^{\bar{W}} \frac{W_i}{\bar{W} - \underline{W}} dW_i} = \frac{W_i^2 - \underline{W}^2}{\bar{W}^2 - \underline{W}^2} \tag{3.4}$$

So

$$B=\int_0^1 L(W)dW_i=\int_0^1\left(\frac{W_i^2-\underline{W}^2}{\bar{W}^2-\underline{W}^2}\right)dW_i=\frac{1-3\underline{W}^2}{3(\bar{W}^2-\underline{W}^2)} \tag{3.5}$$

The area of B must be strictly positive. We know that $3(\bar{W}^2-\underline{W}^2)>0$, hence $1-3\underline{W}^2>0$. The inequality puts a restriction on the lowest wealth.

Assumption 1 **:** $0<\underline{W}<\frac{1}{3}$

Therefore

$$Gini=1-2B=1-2\int_0^1 L(W)dW_i=\frac{3(\bar{W}^2+\underline{W}^2)-2}{3(\bar{W}^2-\underline{W}^2)} \tag{3.6}$$

Substituting X and $\hat{W}$ into the equation above obtains

$$Gini=\frac{3(\hat{W}^2+X^2)-1}{6\hat{W}X} \tag{3.7}$$

The equation indicates that income inequality relates to not only the gap between the rich and the poor, but also the social mean wealth.

The result of the comparative statics in Chapter 2 is

a) If $X<\frac{K+I^*}{2}$, then $\frac{\partial \underline{I}}{\partial \hat{W}}>0$, $\frac{\partial R^E}{\partial \hat{W}}>0$

b) If $X=\frac{K+I^*}{2}$, then $\frac{\partial \underline{I}}{\partial \hat{W}}=0$, $\frac{\partial R^E}{\partial \hat{W}}=0$

c) If $X>\frac{K+I^*}{2}$, then $\frac{\partial \underline{I}}{\partial \hat{W}}<0$, $\frac{\partial R^E}{\partial \hat{W}}<0$

Such result is not suitable for examined by the empirical data directly for several reasons. Firstly, X presents the income difference between the rich and the poor. It is an unobservable parameter. Secondly, it is also not a proper proxy for income inequality. The high income inequality may occur in either a poor country or a rich country. So we need a much measurable observation to replace it. Since the Gini coefficient is a function of X and $\hat{W}$, we consider to include it in our empirical model. The previous boundary condition, $X = \frac{K + I^*}{2}$, defines a threshold of income inequality such that we can obtain a relationship between income inequality and the mean wealth by substituting $X = \frac{K + I^*}{2}$ for function (3.7). Finally we have a unique threshold of Gini coefficient (see Appendix C), that is

$$Gini_C = \frac{K + I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K + I^*)\hat{W}} \tag{3.8}$$

Consequently the results of the comparative static can be expressed as:

a) If $Gini < \frac{K + I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K + I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\frac{\partial R^E}{\partial \hat{W}} < 0$ (3.9)

b) If $Gini = \frac{K + I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K + I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} = 0$, $\frac{\partial R^E}{\partial \hat{W}} = 0$ (3.10)

c) If $Gini > \frac{K + I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K + I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} > 0$, $\frac{\partial R^E}{\partial \hat{W}} > 0$ (3.11)

Let's check what financial sectors happened as the minimal investment level and the interest rate changes. The total borrowing (See Appendix B.2) indicates the size of the financial sectors. It equals the sum of the area B and C in Figure 3.2:

$$\text{Total Borrowing}=\frac{l(I^{*}-\underline{I}+\frac{l}{2R^{E}})}{2R^{E}X} \tag{3.12}$$

The income difference X is constant in equation (3.12) when we discuss the effect of changing the average wealth $\hat{W}$ on the total borrowing. Suppose both the minimal investment level $\underline{I}$ and the interest rate R^{E} increase, the denominator goes up while the numerator falls. Thus the total borrowing, i.e the size of the capital market, shrinks correspondingly. On the contrary, if both the two variables decrease, the total borrowing increases. In other words, the financial system is expanding. The first case relates to inequality (3.9), and the second one relates to inequality (3.11). The result of the comparative static hence illustrates that financial development benefits from income growth if the Gini coefficient is lower than the threshold. Once the Gini coefficient exceeds a particular cut-off value, it brings about negative effects on the financial system.

To conclude, our model predicts the following result:

$$\frac{\partial FD}{\partial \hat{W}} \geq 0, \text{ if } Gini \leq \frac{K+I^{*}}{4\hat{W}}+\frac{(\hat{W}^{2}-\frac{1}{3})}{(K+I^{*})\hat{W}} \tag{3.13}$$

$$\frac{\partial FD}{\partial \hat{W}} < 0, \text{ if } Gini > \frac{K+I^{*}}{4\hat{W}}+\frac{(\hat{W}^{2}-\frac{1}{3})}{(K+I^{*})\hat{W}} \tag{3.14}$$

The inequality (3.13) states that capital markets will benefit from the growth of average income if the income inequality is lower than a particular cut-off value. It will be worse off with the increase of average income if the income inequality exceeds the threshold. Such a conclusion coheres with the result of comparative static in Chapter 2. Suppose, for example, there is a very poor economy with low income inequality. Due to a lack of credits, the projects that the capital market can support are less than they needed. There is an excess demand on credits in the economy. It therefore has the highest borrowing cost, that is $R^E = R^*$. This is the second situation of Chapter 2. In this case, some extra wealth added to the system will alleviate the pressure of requirements for credits and drop the interest rate from R^* to R^E. Meanwhile the minimal investment level declines too. Consequently a critical boundary, $K + \underline{I} - {}^{l}\!/_{R^E}$, moves leftwards and hence capital markets is expanded. Some individuals prefer to be entrepreneurs rather than pure workers since the risky project can bring in more returns. This situation is the third case of Chapter 2. In other words, a rising average wealth leads to the expanding capital markets if the income distributes properly. By contrast, if the income inequality is higher than the cut-off value, it means the income distribution is totally distorted and it may be harmful to the economy. For instance, suppose the rich have the major wealth, an increment of wealth just output a few credits supply whereas it may create much demand—those pure workers who are recently qualified to select the risky project need at least ${}^{l}\!/_{R^E}$ capital. Therefore the demand for credits exceeds its supply. Both the interest rate and the minimal invest level rise to balance the demand and supply. Thus the size of capital markets eventually shrinks.

In conclusion, financial development must be positively related to average wealth if the income inequality is small while it is negatively related to average wealth if the income inequality is higher than the critical value. However, we would not expect to observe such simple relationship in the experiment for several reasons. Firstly, in order to simplify the model, we assume that income is subjects to a uniform distribution. It is not usual in the real world. Secondly, for the same reason, we didn’t count in other factors which may affect income distribution, such as taxation, policies and laws etc, in the model. Governments often deliberately redistribute income over different social groups and decrease income inequality by taxation, subsidies and welfare benefits. These acts prevent income inequality from running too high and led to a sustainable growth. But our model still have a good interpretation power in such circumstances because we can expect to detect a strong threshold effect such that the increase of average income would distinctly stimulate financial development more in some segments of income inequality than others.

3.4 The Panel Threshold Regression with Single Exogenous Threshold

In this section we extend a model originally developed by Hansen (1999) who suggests a bootstrap procedure to estimate a threshold regression. He developed an asymptotic distribution theory for both the threshold parameter estimate and the regression slope coefficients. The TR model is given by

$$y_{i,t} = \beta_1^{'} x_{i,t} + e_{i,t}\,, \quad q_{i,t} \le \gamma \tag{3.15}$$

$$y_{i,t} = \beta_2^{'} x_{i,t} + e_{i,t}\,, \quad q_{i,t} > \gamma \tag{3.16}$$

The model considers balanced panel data which are given by $\left\{ y_{i,t}, x_{i,t}, q_{i,t} : 1 \le i \le n, 1 \le t \le T \right\}$, where $y_{i,t}$ and $q_{i,t}$ are the dependent variable and a threshold variable, respectively, and $x_{i,t}$ is a $p \times 1$ vector of independent variables. The subscript i indexes the individual and the subscript t indexes time.

The model can be written in a single equation form:

$$y_{i,t} = \beta_1^{'} x_{i,t} I(q_{i,t} \le \gamma) + \beta_2^{'} x_{i,t} I(q_{i,t} > \gamma) + e_{i,t} \tag{3.17}$$

where $I(\cdot)$ denotes the indicator function. Obviously, the observations are split into two groups which are distinguished by whether the threshold variable $q_{i,t}$ is smaller or larger than the critical value γ.

By defining $\beta = (\beta_1^{'} \quad \beta_2^{'})^{'}$ and $x_{i,t}(\gamma) = \begin{cases} x_{i,t} I(q_{i,t} \le \gamma) \\ x_{i,t} I(q_{i,t} > \gamma) \end{cases}$, a more compact form is

$$y_{i,t} = \beta^{'} x_{i,t}(\gamma) + e_{i,t} \tag{3.18}$$

Let these variables

$$y_i = \begin{bmatrix} y_{i,2} \\ \vdots \\ y_{i,T} \end{bmatrix}, \quad x_i(\gamma) = \begin{bmatrix} x_{i,2}(\gamma)^{'} \\ \vdots \\ x_{i,T}(\gamma)^{'} \end{bmatrix}, \quad e_i = \begin{bmatrix} e_{i,2} \\ \vdots \\ e_{i,T} \end{bmatrix}$$

indicate an individual's observations and errors with deleted one time period and let Y, $X(\gamma)$ and e denote the data and errors for all individuals, then equation (3.18) obtains a form as

$$Y = X(\gamma)\beta + e \tag{3.19}$$

To estimate such a TR model, Hansen provided Concentrated Least Squares (CLS) regressions based on a grid search over all threshold value $\gamma \in \{q_1, q_2, ..., q_{nT}\}$. Notice the cost of searching over values of γ is at most nT steps.

For any given γ, the equation (3.19) can be estimated by OLS. That is

$$\hat{\beta}(\gamma) = \left(X(\gamma)^{'} X(\gamma) \right)^{-1} X(\gamma)^{'} Y \tag{3.20}$$

The vector of regression residuals is

$$\hat{e}(\gamma) = Y - X(\gamma)\hat{\beta}(\gamma)$$

And the sum of squared errors is

$$S_1(\gamma) = \hat{e}(\gamma)^{'} \hat{e}(\gamma) \tag{3.21}$$

Hence the least squares estimators of γ is

$$\hat{\gamma} = \arg\min_{\gamma} S_1(\gamma) \tag{3.22}$$

Once $\hat{r}$ is obtained, the slope coefficient estimate is $\hat{\beta} = \hat{\beta}(\hat{\gamma})$. The residual vector is $\hat{e} = \hat{e}(\hat{\gamma})$ and residual variance is

$$\hat{\sigma}^2 = \frac{1}{n(T-1)}\hat{e}'\hat{e} = \frac{1}{n(T-1)}S_1(\hat{\gamma}) \tag{3.23}$$

3.5 The Panel Threshold Regression with Single Endogenous Threshold

Hansen's model virtually imposes the assumption of the threshold variables being exogenous. In our case, however, according to inequality (3.13) and (3.14) the threshold variable Gini is a function of the average income $\hat{W}$. It implies that the indicator $I(\cdot)$ in equation (3.17) is a dynamic function. Hence there is obviously an endogeneity problem for threshold variables and it causes coefficients estimators being inconsistent. Consequently the inference developed by Hansen is invalid in our model. Looking into details of the method, we find that the point is how to produce corrected sum of squared errors which get rid of the endogeneity effect. Fortunately, there is an extended framework which allows an endogenous threshold variable (Kourtellos et al. (2008), Wang and S.Lin (2010)). The method is shown below.

If we allow an endogenous threshold for equation (3.17), the model is modified as

$$y_{i,t} = \beta_1' x_{i,t} I(q_{i,t} \le \gamma) + \beta_2' x_{i,t} I(q_{i,t} > \gamma) + e_{i,t}$$

$$q_{i,t} = z_{i,t}'\pi + u_{i,t} \tag{3.24}$$

where $z_{i,t} = \begin{bmatrix} z_{1,it} & z_{2,it} \end{bmatrix}$ contains the instrument variable and $z_{2it} = x_{it}$. The equation (3.24) describes the mechanism that generates an exogenous threshold variable.

Then we carry out three steps of Concentrated Two-Stage Least Squares Method (C2SLS) to obtain the consistent estimations.

- Step 1: We estimate the parameter π in equation (3.24) by OLS.
- Step 2: We then estimate the threshold parameter γ by minimizing a C2SLS criterion using $\hat{\pi}$ obtained from the first step.

$$S_n^*\left(\beta_i(\gamma), \rho(\gamma), \gamma\right)$$
$$= \arg\min \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{i,t} - x_{i,t} I(q_{i,t} \le \gamma)\beta_1 - x_{i,t} I(q_{i,t} > \gamma)\beta_2 - \psi(q_{i,t}, z_{i,t}\gamma, \hat{\pi}) \right)^2 \quad (3.25)$$

- Step 3: we estimate the least square coefficient parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ based on $\hat{\gamma}$

The $\psi(q_{i,t}, z_{i,t}\gamma, \hat{\pi})$ in step 2 is constructed by

$$\psi(q_{i,t}, z_{i,t}\gamma, \pi) = \rho\lambda_1(\gamma - z_{i,t}\pi) I(q_{i,t} \le \gamma) + \rho\lambda_2(\gamma - z_{i,t}\pi) I(q_{i,t} > \gamma) \quad (3.26)$$

Where ρ is the covariance between $e_{i,t}$ and $u_{i,t}$.

$$\lambda_1(\gamma - z_{i,t}\pi) = -\frac{\phi(\gamma - z_{i,t}\pi)}{\varphi(\gamma - z_{i,t}\pi)}$$

$$\lambda_2(\gamma - z_{i,t}\pi) = -\frac{\phi(\gamma - z_{i,t}\pi)}{1 - \varphi(\gamma - z_{i,t}\pi)}$$

Here $\lambda_1(\gamma - z_{i,t}\pi)$ and $\lambda_2(\gamma - z_{i,t}\pi)$ are the well-known inverse Mills, which play the key role to correct the biased terms. Note if $\rho = 0$ then the bias correction items will disappear and we get Hansen's threshold regression for exogenous threshold models. It implies that the Hansen's model is a special case of a general one. If we substitute the corrected threshold parameter $\hat{\gamma}$ from the equation (3.25) for the Hansen's counterpart, his inference is still valid. So we will adopt the corrected threshold parameter $\hat{\gamma}$ in the rest part of this chapter.

3.6 The Panel Threshold Regression with Double Endogenous Thresholds Model

The Model (3.24) only has a single threshold. It is possible to have multiple thresholds. The double threshold model takes the form as

$$y_{i,t} = \beta_1^{'} x_{i,t} I(q_{i,t} \le \gamma_1) + \beta_2^{'} x_{i,t} I(\gamma_1 < q_{i,t} \le \gamma_2) + \beta_3^{'} x_{i,t} I(\gamma_2 < q_{i,t}) + e_{i,t}$$

$$q_{i,t} = z_{i,t}^{'} \pi + u_{i,t} \tag{3.27}$$

Where the thresholds are sorted so that $\gamma_1 < \gamma_2$. We will focus on the double-threshold model because the higher-order one can be extended straightforwardly from it. Hansen provides with an effective way to find out the solution.[13]

- Step 1: Let $S_1(\gamma)$ be the single threshold sum of squared errors as defined in (3.25) and let $\hat{\gamma}_1$ be the threshold estimate which minimizes $S_1(\gamma)$.
- Step 2: Fixing the first-stage estimate $\hat{\gamma}_1$, the second-stage threshold estimate is $\hat{\gamma}_2^r = \arg\min_{\gamma_2} S_2^r(\gamma_2)$.
- Step 3: Fixing the second-stage estimate $\hat{\gamma}_2^r$, the refinement estimate $\hat{\gamma}_1^r = \arg\min_{\gamma_1} S_1^r(\gamma_1)$

The first step estimate $\hat{\gamma}_1$ will be consistent for either γ_1 or γ_2 (Bai (1997), Bai and Perron (1998)). It can be used to find out the second step estimate $\hat{\gamma}_2^r$ which is asymptotically efficient (Bai (1997)). The asymptotic efficiency of $\hat{\gamma}_2^r$

[13] Note we adopt the corrected threshold parameter $\hat{\gamma}$ here so that the Hansen's method is still valid to the endogenous threshold case.

suggests that $\hat{\gamma}_1$ can be refined through the third-stage estimation. Finally the refinement estimator $\hat{\gamma}_1^r$ at the last step is also asymptotically efficient (Bai (1997)).

3.7 Determining Number of Thresholds

An important hypothesis for our test is whether the TR model is statistically significant against a simple linear specification. In the model (3.27), there are no thresholds, or one threshold, or two thresholds.

Firstly, let's examine no thresholds against one threshold. The null hypothesis of no threshold effect in (3.27) can be represented by:

$$H_0 : \beta_1 = \beta_2$$

Under the null hypothesis, it is difficult to implement such a test because the threshold parameter γ is not defined. Hansen introduces a heteroskedasticity-consistent Lagrange Multiplier (LM) boostrap procedure to test the null hypothesis of a linear specification against a TR (Hansen (1999)). His method is below.

Under the null hypothesis of no threshold, the model is

$$y_{i,t} = \beta_1' x_{i,t} + e_{i,t} \tag{3.28}$$

The coefficient parameter β_1 is estimated by OLS, yielding estimate $\tilde{\beta}_1$, residuals $\tilde{e}_{i,t}$ and sum of squared errors $S_0 = \tilde{e}_{i,t}' \tilde{e}_{i,t}$. Therefore the likelihood ratio test of H_0 is

$$F_1 = \frac{S_0 - S_1(\hat{\gamma})}{\hat{\sigma}^2} \tag{3.29}$$

where $S_1(\hat{\gamma})$ is defined in (3.25) and $\hat{\sigma}^2$ can be computed by (3.23). Under the null hypothesis , it is a linear model and the endogeneity problem of the threshold parameters does not exist. Furthermore, $S_1(\hat{\gamma})$ is defined in (3.25) which derive from the unbiased threshold parameters. Consequently, the test is still valid though it comes from Hansen's model.

Since γ is not identified under the null hypothesis, the p values are derived from a fixed bootstrap method. In this case, treat the regressors $x_{i,t}$ and the threshold variable $q_{i,t}$ as given, the bootstrap-dependent variable is generated from $N(0, \hat{e}_{i,t}^2)$. Hansen shows that this procedure yields asymptotically correct p values. The null of no threshold effect is rejected if the p values are smaller than the desired critical value.

If F_1 rejects the null of no threshold, we need a further test to discriminate between one and two thresholds. The approximate likelihood ratio test of one versus two thresholds can be based on a very similar statistic

$$F_2 = \frac{S_1(\hat{\gamma}_1) - S_2^r(\hat{\gamma}_2^r)}{\hat{\sigma}^2}$$

Where the variance estimate $\hat{\sigma}^2 = \dfrac{S_2^r(\hat{\gamma}_2^r)}{n(T-1)}$. The hypothesis of one threshold is rejected in favor of two thresholds if F_2 is large. It is worth reminding that we adopt $S_1(\hat{\gamma}_1)$, instead of $S_1^r(\hat{\gamma}_1^r)$, in the F_2 statistic because the current null hypothesis of one threshold does not need a refinement estimate. This test can be extended in a straightforward manner to a higher order model.

3.8 The Data

In order to meet the requirements of the threshold regression for balanced panel data, we find out 16 countries with continuous data over the period from

1989 to 2004 year by year. Only these countries contain full data which can be applied threshold regression. They are Argentina, Australia, Bolivia, Costa Rica, Denmark, Finland, Hungary, Italy, Jamaica, Netherlands, Norway, Poland, Sweden, United Kingdom, United States and Venezuela. Further research might cover more countries to obtain robust if the data is available.

The primary measure of financial development is 'private credit', which is defined as private credit by deposit money banks and other financial institutions over GDP. This measure excludes credits issued by the central bank and development bank. It also excludes credits to the public sector. Therefore the measure directly present credits from savers to private firms. The data comes from the World Bank Financial Infrastructure database (Beck et al. (1999)).

Secondly, the measure of income inequality is the Gini coefficient that we have shown previously. The Gini coefficient is the ratio of the area between the Lorenz curve, which is the proportion of population against the income share received, and the line of equality over the total area under the line of equality. It ranges from 0 to 1. A lower Gini coefficient presents a more equal environment while higher Gini coefficient indicates more inequality. The data come from World Income Inequality Database UNU-WIDER (2008). In addition, the model also includes the real gross domestic product per capita with constant price as the average income data. The original data of income comes from Heston et al. (2009).

Finally, the test also includes some usual control variables used in the literature on finance and growth and inequality such as trade openness, government consumption and real inflation (King and Levine (1993), Beck et al. (2000), Clarke et al. (2003)). Trade openness is the sum of exports and imports as a share of GDP. Recent research shows that it has a positive impact on financial development in economies (Baltagi et al. (2009), Demetriades and Rousseau (2010)). Government consumption was collected by government expenditure as a share of GDP. It has different effects on different stage of economy. For example, Demetriades and Rousseau (2010) demonstrate the evidence of a panel of 82 countries that government expenditure have positive effects on financial development for countries that are in the middle ranges of economic development, while it has little effect for poor countries and a strongly negative effect for the rich ones. Our data set include 16 countries which are in different development stages. Therefore the final impact of government spending is ambiguous. Both trade openness and government consumption are come from World Income Inequality Database UNU-WIDER (2008).

At last, we use inflation as the instrument variable for Gini coefficient because higher levels of inflation tend to increase inequality in an economy, while it does not correlate with real GDP per capital. The data comes from the World BankBank (2009). Details of these measures are given in Table 3.3 of Appendix C.3.

3.9 The Empirical Threshold Model

(3.13) and (3.14) tell us there are strong threshold effects between financial development and average income. Some range of the Gini coefficient might be cumbersome for the economy while other optimum values may lead to a boom in financial markets. So we will test the following equations

$$\begin{aligned} FD_{i,t} &= \alpha + \theta_1 Trade_{i,t-1} + \theta_2 Gov_{i,t-1} + \beta_1 GDP_{i,t-1} I(Gini_{i,t-1} \le \gamma_1) \\ &+ \beta_2 GDP_{i,t-1} I(\gamma_1 < Gini_{i,t-1} \le \gamma_2) + \beta_3 GDP_{i,t-1} I(Gini_{i,t-1} > \gamma_2) + e_{i,t} \end{aligned} \tag{3.30}$$

and

$$Gini_{i,t-1} = z_{11} Inflation_{i,t-1} + z_{21} Trade_{i,t-1} + z_{22} Gov_{i,t-1} + u_{i,t-1} \tag{3.31}$$

These variables are also used by Nandi (2008). Because the investment decision of the model is made at the beginning of the period, we make one year lag for all independent variables. There are also the additional regressors $Trade_{i,t-1}$, $Gov_{i,t-1}$. They can be viewed as a special case of the model by constraining the slope coefficients on these variables to be the same in the two regimes. Hence they have no effect on the distribution theory. In addition, the effect of gini coefficient on average income might be nonlinear. Before the experiment we cannot confirm how many thresholds there are. Here we assume there are two threshold values. We will test the assumption afterwards.

Test for Threshold Effects			
	Single Threshold F_1	Double Threshold F_2	Triple Threshold F_3
F_i	51.943	104.659	36.306
p-Values	0.03	0.001	0.107

Table 3.2: Tests for Threshold Effects

Our approach allows for zero, one, two and three thresholds. The test statistics F_1, F_2 and F_3, along with their bootstrap p-values, are displayed in Table 3.1. We find that the single threshold F_1 is statistically significant with the bootstrap p-values of 0.03. It means the null hypothesis of no threshold effect is rejected at 5 percent level. In other words, model (3.30) has at least one threshold. We then need to test two thresholds against a single threshold. The test for double threshold F_2 is strong significant with the bootstrap p-values of 0.001. On the other hand, the triple threshold F_3 is not statistically significant with a bootstrap p-value of 0.107. The result gives a strong support to two thresholds model in the regression. Therefore expression (3.30) is a suitable form for the test.

The point estimates of the two thresholds are 36.6 and 37.663, respectively. Table 3.2 reports the regression slope estimates and their stand errors.

Regression Estimates	
Regressor	Coefficient Estimate
GDP per Capita($Gini \leq 36.6$)	0.000024 (0.000006)
GDP per Capita($36.6 < Gini \leq 37.663$)	0.000041 (0.000010)
GDP per Capita($37.663 < Gini$)	0.000025 (0.000006)

Table 3.3 Regression Estimates: Double Threshold Model

Our major interest is how the real GDP per capita affects financial development through Gini thresholds. According to (3.13) and (3.14), we intuitively expect that average income is positively related to financial development when the Gini coefficient is small and the relationship becomes negative once the Gini coefficient is higher than the critical value. However, we do not observe such opposite signal in table 3.3. It is easy to understand because the prediction from the empirical model is restricted by a few assumptions. It usually does not happen in the real world. We normally observe a slide of financial markets on account of a business cycle or economic depression. It is unusual to find the collapse of capital markets caused by high income inequality, because most governments would try to redistribute wealth over different social groups by taxation, subsidy policies and social welfare etc. These actions effectively reduce income inequality in an economy. Therefore in the empirical test we do not expect exactly opposite signals of GDP per Capita with regards to different Gini thresholds. Instead, we focus on whether there is significant evidence to support threshold effects. If threshold effects take place then our prediction is still hold.

Since real GDP per capita is a very tiny scale comparing to credit by deposit money banks and other financial institutions to the private sector, these coefficient estimates are also small. Table 3.2 indicates two major results. One is that GDP per capita is positively related to financial development, namely private credit in our experiment. The other is that average income improves financial development with different slope values (the slope coefficient) through income inequality. For example, suppose there is 1 percent increase in real GDP per capita. If the Gini coefficient of an economy is lower than 36.6, it brings about 0.0024 percent increase in private credit. If the Gini coefficient locates between 36.6 and 37.663, the result nearly doubles than the former. If the Gini is higher than 37.663, the slope coefficient falls to the level of 0.0025. The case reveals a fact that there is an optimal Gini coefficient interval for financial development. Any values of Gini coefficient which is out of this range would be a cumbersome to the economy. This result has an intuitive interpretation. A low Gini coefficient might implies severe wealth redistribution by governments. Individuals thus lose motivation to make more money because of heavy taxes. By contrast, a high Gini coefficient means the rich take major part of the social wealth. It prevents people from starting a business because individuals will find either it is difficult to raise necessary money or the cost is very expensive. Therefore an economy would benefit from the suitable Gini coefficient.

There are also two limitations on the empirical test. Firstly, these coefficients might be too small to be of interest. For example, it states that a 1 percent increase in real GDP per capita improves private credit by 0.0024 per cent. It is because real GDP per capita is too tiny scale to private credit. However, if we change the measurement, for example, using log(GDP) instead of GDP, the meaning of the estimated equation cannot fit to the empirical model. If a further research modifies the model to substituting real GDP for real GDP per capita, it would be better. Secondly, the interval between two thresholds is too narrow, i.e from 36.6 to 37.6. It also makes the result weak in robust. If we can obtain balance data from more countries and longer time in the future, we can examine its robust again.

3.10 Conclusion

This paper examines the comparative static results of the previous chapter with an empirical model. The former states that if income difference across social groups is lower than a threshold, a rise in average income would push up the minimal investment level and the interest rate. Otherwise it would reduce both of them. Our empirical model develops further to predict that financial development must be positively related to average income if income inequality is small, while it is negatively related to average income if income inequality is higher than the critical value. We adopt a modified method of the multiple thresholds regression to the model in order to inspect this prediction.

The empirical tests are applied to a panel of 16 countries for the period 1989-2004. We find overwhelming evidence of the threshold effect. The point estimates of the two thresholds are 36.6 and 37.663 respectively. It means when income inequality is lower than 37.663, a rise of average income would improves financial development. As long as income inequality is higher than this value, financial development does not benefit from average income. Our results do not report any evidence to support Kuznet's hypothesis because we do not examine how economic growth determines income inequality. However, we cannot reject it as well because the results shows a high income inequality indeed hinders further growth.

Several extensions of our methods would be desirable. Firstly, with the restriction of the balanced panel data set we have to ignore some samples, especially those countries which do not have continuous Gini data. A larger size of samples may improve the results. Secondly, the model can be extended to a dynamic model to contain economic growth in future research.

CHAPTER 4

OCCUPATION CHOICE AND INVESTMENT DECISION WITH ASYMMETRIC INFORMATION

Abstract

We present a static model of an economy where individuals are heterogeneous in terms of initial wealth and there are credit constraints. Individuals are endowed with time resource which they can allocate between working and leisure to maximize their utility. What's more, individuals can choose to either sell their labour in the labour market or self-employ. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or become entrepreneurs by running a risky project. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on whether to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds while the wage rate meets the labour market clearing condition. We find that an increase in the mean wealth leads to a decrease in the interest rate. In equilibrium, the wage rate rises and so does the labour time. Meanwhile, both the optimal amount of labour and the minimal requirement of labour of the project decrease.

4.1 Introduction

The famous Modigliani and Miller (1958) hypothesis states that given certain conditions (for instance, perfect capital markets, no agency costs or moral hazard), the market value of a firm is not affected by its financial structure. In other word, it implies that capital structure is of no importance for production, employment and investment decisions. However, recent empirical evidence by Wadhwani (1986, 1987), Nickell and Wadhwani (1988, 1991) and Nickell and

Nicolitsas (1999) suggest that financial factors are important determinants of employment in the UK. These results conflict with the ‘independence’ and ‘irrelevance’ assumption of the M-M theorem. But they are not surprising results because the M-M theorem is restricted to perfect markets conditions. In previous chapters, we have discussed the relations between financial structure and other factors in imperfect markets. For example, in Chapter 1 we have revealed the effect of heterogeneity in terms of initial wealth on investments decisions. We have also explored the empirical evidence of the effect on income inequality in Chapter 3. Both chapters concentrate only on capital markets. In this chapter, we will introduce a few new variables to cover the issue with both the capital market and the labour market. Our contribution is to examine the investment choice and activities of lenders and borrowers when both capital market and labour market co-existence.

We present a static model of an economy where individuals are heterogeneous in terms of initial wealth and there are credit constraints due to asymmetric information. Individuals are also endowed with time resource which they can allocate between working and leisure to maximize their utility. What’s more, individuals can choose to either sell their labour in the labour market or self-employ. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or become entrepreneurs by running a risky project. Unlike the model in Chapter 2, the risky project requires both capital and labour inputs. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on whether to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds, while the wage rate meets the labour market clearing condition. We explore the simultaneous equation at different situations and one of the comparative statics results. We find that as the maximal initial wealth increases, the interest rate, the maximal and minimal labour needed by the project decreases while both wage rate and personal working time goes up.

4.2 Literature Review

The criteria guiding individuals' labour-leisure choice and investment decision making have been studied in the existing literature from different points of view, such as inflation, taxation, human capital investment, legal protection and risk.. For example, Mansoorian and Mohsin (2010) construct a model of a small open economy in which households make labour-leisure choices and investment decisions on firms They examine the effect of inflation in this framework. Ghate (2007) discusses the implications of labour-leisure choice in an equilibrium tax rate model., Kenc (2004) extends the model of Asea and Turnovsky (1998) to include the supply of labour and discusses the relationship between taxation, risk-taking and capital accumulation. He allows the supply of labour to be endogenously determined and find it significantly affects risk-taking and capital accumulation. In addition, Bodie et al. (1992) have received considerable attention because the optimal choice of labour-leisure generates a flexible investment opportunity set. Basak (1999) provides comparative static analysis of the effects of the labour-leisure choice on consumption, stock market and other factors. Galor and Zeira (1993) investigate the issue from the perspective of human capital investment. In their model, individuals are identical in terms of potential skills and preferences. However, similar to our model, their initial wealth is different. They find that individuals' wealth determines whether they invest in human capital or not. Furthermore, Balmaceda and Fischer (2010) investigate the interaction of the performance of economy, credit protection, and bankruptcy procedures with different wealth distributions. They introduce labour market frictions (?) in the model and the results show that increased labour protection leads to lower wages.

In our model, individuals are heterogeneous in terms of initial wealth. The idea that initial wealth conditions may affect an economy's prosperity in long-term is common in development literatures (e.g., Romer (1986);Loury (1981);Murphy et al. (1989a, 1989b);Matsuyama (1991); Bertola (1993)). Our paper differs from theirs because most of the papers emphasize on how a technological improvement, specifically the productivity of capital, increases returns. Instead, we consider a pure wealth effect in an imperfect capital market (similar as Galor and Zeira (1993). Furthermore, except for Murphy et al. (1989), rarely did these papers focus on income distributions . On the other hand, some papers examine the issue with regard to risk preferences. For example, Kihlstrom and Laffont (1979) construct a competitive general equilibrium model in which individuals have a choice between undertaking a risky firm or working as a pure wage worker. Individuals are distinguished by their risk attitudes. They find, that in equilibrium the more risk averse individuals choose to become workers while the less risk averse individuals become entrepreneurs. Banerjee and Newman (1991) draw a similar conclusion that the rich should take risks and the poor should not. However, we presume all individuals have the same risk preference because we concentrate on how heterogeneity of wealth impacts occupation choice and in turn endogenously determines equilibriums of the capital market and the labour market.

This paper models a simple economy with capital market imperfections which is derived from Bougheas (2007). We extend the financial equilibrium model of Bougheas by introducing a labour market to analyze the issues of occupation choice and labour-leisure decision when both physical capital and human capital are needed.

In our model, individuals are heterogeneous in terms of initial wealth. They are also endowed one unit time resource in which they can balance between working and leisure to maximize their utility. Individuals can choose either to sell their labour in the labour market or to self-employ in a risky project. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or be entrepreneurs by running a risky project. The risky project needs a set-up cost which is similar to the model of Bougheas. However, our model requires not only capital inputs but also labour inputs. Workers receive fixed wages while entrepreneurs receive risky profits. Because of capital market imperfections, lenders cannot observe the project's returns so that borrowers always have incentive to report default and pay for the liquidation value. Lenders are in turn reluctant to provide with funds more than the liquidation value. In other words, individuals can borrow only limited amounts. Consequently, the risky project that needs high level of investment is beyond the reach of the poor. Thus the poor have to sell their labour and be pure wage workers. In contrast, the richer individuals who can afford the set-up cost prefer the risky project and thus become entrepreneurs. In other words, Individuals make their decisions of whether to be entrepreneurs or wage workers by comparing the utility from the risky project with the wage work. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds, while the wage rate meets the labour market clearing conditions. In fact, classification of agents according to their activities is discussed by numerous literature. For example, Eswaran and Kotwal (1986) models an agrarian economy in which entrepreneurs subject to not only the restriction on working capital but also moral hazard. In other words, workers hired by agent need supervision. Therefore individuals in this model allocate their time across three activities. The first one is selling his labour in the market. The second one is working on his own business. The last one is supervising hired labour on his farm. Bowles (2003) makes an adaption to a modern capitalist economy of a model by Eswaran and Kotwal (1986). In Bowles's model, individuals are further sorted into six classes depends on the level of the wealth. He define as pure wage worker, mixed independent producer and wage worker, independent producer, small capitalist, pure capitalist, and rentier capitalist. Our model differs from theirs with respect to several aspects. For example, we don't present moral hazard so there is no necessary for supervision. In addition, all individuals in our model have the same utility function of leisure so that they share the same time schedule while theirs are not. Eswaran and Kotwal's model implies that richer employers even consume smaller amounts of leisure.

Banerjee and Newman (1993) model the interplay between individuals' occupational decisions and the distribution of wealth. Because of capital market imperfections, the poor individuals choose to be workers while wealthy individuals become entrepreneurs who monitor the former. In static equilibrium the occupational structure depends on distribution. Instead of simply distinguishing individuals between the wage workers and the entrepreneurs. Our model further splits entrepreneurs into three groups. A few poor entrepreneurs carry out the project only in suboptimal investment because of the capital restriction. Medium wealth entrepreneurs perform a full investment on the risky project after collecting sufficient funds from the capital market. The rich entrepreneurs not only self-finance the project but also lend the rest of money in the capital market. Such improvement reveals more details of relationship between heterogeneity of wealth and occupation choice.

A simple intuition of the relationship between wealth and labour-leisure choices is that the rich have a lower marginal utility of wealth. They prefer to enjoy more leisure and reduce their working time. Garcia-Penalosa and Turnovsky (2006) examine the issue in this way. They develop an endogenous growth model with elastic labour supply which is an extension of Romer (1986). In their framework, the growth rate and the distribution of income are jointly determined. The equilibrium labour supply determines the interest rate and in turn affect s both the capital accumulation and the distribution of income over individuals. The critical role of the wealth and the effect on the labour-leisure choice is also studied by Ortigueira (2000) and Turnovsky (2000). Our model demonstrates some evidence to support their conclusion. For example, the production function of the risky project is a function of labour. The richer an entrepreneurs is, the more workers he can hire and in turn the higher returns from the project he obtains. In other words, wealth distribution affects income distribution and capital accumulation from the side of labour supply. However, a significant difference between these models and ours is that we place the same utility function of time resource across individuals so that all individuals share the same time schedule. We then run a comparative statics analysis to explore how the mean wealth impacts on occupation and labour-leisure choices. In particular, an increase in the mean wealth leads to an increase in credit supply. In turn the interest rate goes down. Suppose that other factors of the system do not change immediately at the moment, the lower interest rate implies the cheaper borrowing cost thus a few pure wage workers become entrepreneurs and a few suboptimal investment entrepreneurs now prefer to full invest in the project. Consequently there is an increase in labour demand and the wage rate rises. Furthermore, the wage workers would like to spend more time in working because of the higher wage, so do the entrepreneurs. Put differently, the working time increases as the mean wealth goes up. On the other hand, an increasing wage rate naturally makes entrepreneurs reduce the number of employees. So the optimal amount of labour decreases as the mean wealth increases. Finally, since the minimal amount of labour positively relates to the interest rate, it is also decreases with the mean wealth..

The remainder of the paper is organized as follows. Section 4.3 introduces the general framework of the model and describes the different situations of the economy. Section 4.4 gives the simultaneous equation to determine the system in equilibrium status. Section 4.5 provides a comparative statics analysis of the last case that the demand for credits varies over the range between the maximum and the minimum value of funds supply. The last section is a conclusion.

4.3 The Model of the Imperfect Capital Market

In Chapter 1, we have illustrated the progress of how individuals make investment decisions when they are in an imperfect market environment. In that case, individuals only consider how to allocate their endowments to maximize their returns. The model does not consider the situation when individuals can choose their occupations. In this chapter, we will explore the issue of how individuals maximize their utilities by allocating both wealth and time. For simplicity, instead of the continuous investment model in Chapter 2, we adopt a fixed investment model which is essentially identical to Bougheas (2007).

There is a continuum agent indexed by i in the economy. A single capital good can be either invested or consumed. The economy has one period which lasts one unit time. At the beginning of the period the agent is endowed with a random wealth W_i, where $W_i \in [0, \bar{W}]$. The agent consumes and gets returns at the end of the period. We assume that the initial wealth follows a uniform distribution. Let *G(W)* denote the distribution of endowments across of agents, then $G(W) = \frac{W_i}{\bar{W}}$. And the density function is $g(w) = \frac{1}{\bar{W}}$. The mean wealth of the economy thus is $\hat{W} = \int_0^{\bar{W}} W_i g(W) dW_i = \frac{\bar{W}}{2}$. Individuals have to choose to whether or how much invest in one of the methods describing below, meanwhile, they have to allocate their time resource properly to maximize the utility.

The most straightforward way of arranging endowments is to lend funds in the capital market. The interest rate, which is endogenously determined in order to clear the capital market, is R^E(>1). The second way is a riskless CRS technology which produces a constant return Z(>1). The last one is a risky project. We have mentioned that it is a fixed-investment model. It suggests that the return of the project is sharply decreasing beyond a certain investment level of physical capital. Comparing to the usual fixed-investment model, we allow a continuous human capital investment while the physical capital is fixed. Put differently, the idea of 'fixed-investment' is equivalent to a fixed set-up cost, but entrepreneurs can have many employees working for them. Therefore the project needs K units working capital and n units labour, where n is a variable which indicates the total units of labour used. Then the project yields $f(n)$ with probability p , or to be liquidated with a value l otherwise. The product function is increasing and concave in n, namely, $f'(n)>0$ and $f''(n)<0$. These conditions make sure that the labour input for the project is finite.

In addition, we assume there is no moral hazard in the labour market so that no one shirks and no supervision is needed. Given all these conditions, individuals have two options to manage their time resource. They can either spend time t on working or have a rest for time s. Since each individual has one unit time, it means $t+s=1$.

Furthermore, the return of the project is private information at the imperfect capital market. Without an effective way of monitoring, entrepreneurs naturally have incentive to report a return of no more than l even if they succeed, where l is the liquidation value of the project. Thus lenders will provide funds of no more than l to any borrowers. In other words, the maximum credits that a lender lends equal $\frac{l}{R^E}$. Let's denote an individual's utility as

$$U(Y,s)=Y+u(s) \tag{4.1}$$

where Y is the present value of total earnings and $u(i)$ is the leisure function and $u'(s)>0, u''(s)<0, u'(0)=\infty$. Conditions $u'(s)>0, u''(s)<0$ indicate that the utility of leisure is finite while $u'(0)=\infty$ implies that all individuals must have some leisure. The linear utility function demonstrates that all individuals are risk-neutral. Suppose that $w(>1)$ is the price of labour and wages are paid at the outset. Individuals who do not undertake the risky project have two options. They can either rest or be a pure worker.

Suppose some individuals prefer to rest all the time. It means $s=1$ and the utility is

$$U_0 = W_i R^E + u(1) \tag{4.2}$$

$W_i R^E$ is the return from the capital market if an individual lend all the initial endowments. $u(1)$ is the utility of leisure for all the time. Alternatively, if the individual prefer to spend a few time at working, suppose it is ds, the current utility is

$$U_1 = W_i R^E + [u(1) - u'(1)ds] + wdsR^E \tag{4.3}$$

$u'(1)ds$ is the utility of leisure that the individual sacrificed for working. So $u(1)-u'(1)ds$ is the actual utility of leisure. Since we presume wages are paid at the beginning of the period, workers must lend wages to the capital market to increase their final utility. Then $wdsR^E$ is the return of this part of the investment. Naturally, hard-working individuals must be better off than those spending all the time to loaf around. Thus it comes out $U_1 - U_0 = wdsR^E - u'(1)ds > 0$. That is $u'(1) < wR^E$. It means that the marginal utility of resting all the time is always smaller than the return of lending the wage to the capital market. This encourages individuals to work. In addition, as we all know, the smallest interest rate of the model is equal to the return of CRS, namely Z. Otherwise no one would lend money to the capital market. In our case, the condition $u'(1) < wR^E$ always holds if only $u'(1) < wZ$. So we have the assumption below:

Assumption 4.1: $u'(1) < wZ$

According to the previous definition, a pure wage worker sells his labour and lends his money. The utility of a pure wage worker is

$$U_2 = R^E[wt + W_i] + u(1-t) \tag{4.4}$$

The first item of the right hand side of (4.4) describes that the individual invests both the wage and the initial endowment in the capital market. While $u(1-t)$ is the utility of leisure after deducting working time. Making a first order derivative to (4.4) gives the optimal value of working time for pure workers, which is $R^E w - u'(1-t) = 0$. Formally,

$$u'(1-t) = R^E w \tag{4.5}$$

The equation states that, a pure wage worker will stop working when the marginal utility of leisure equals the price of labour multiplying the interest rate. Thus their optimal working time t^* is determined by (4.5).

Alternatively, if the individual undertakes the risky project, then he does not sell his labour to the labour market any longer. Instead, he works for his own project. Since there is no need to monitor employees in this case, the entrepreneur devote all the working time to the project. The utility is

$$U_3 = [pf(n) + (1-p)l] - R^E[w(n-t) + (K - W_i)] + u(1-t) \tag{4.6}$$

The first term of the right hand side of (4.6) presents the returns of the risky project; the second term describes the total repayment of the capital market at the end of period. From the time resource point of view, own labour is equivalent to hired labour from the labour market. In other words, own employment is perfect substitute for hired workers. The entrepreneurs also need to pay for their own labour. So the returns of own labour must be deducted from the working capital. It is the reason that the sign of t is negative. Therefore the optimization problem is to maximize (4.6). The partial derivative of (4.6) are $U_n = pf'(n) - R^E w = 0$ and $U_t = R^E w - u'(1-t) = 0$, respectively. Rearranging the two equations gives

$$pf'(n) = R^E w \tag{4.7}$$

$$u'(1-t) = R^E w \tag{4.8}$$

Equation (4.7) states that it makes no sense to hire more labour as long as the marginal expected return of the project equals the opportunity cost of hiring one unit labour. The maximal units of labour of the project are determined by the equation. Equation (4.8) depicts that an agent will stop working as soon as the marginal utility of leisure equals the wage multiplying the interest rate. It is absolutely the same as (4.5). The result implies that all agents share the same time scheme. In other words, all individuals in the system propose to working for t^*, which is determined by (4.8).

In addition, one of the features of imperfect market is asymmetric information. In other words, the lenders cannot observe the realized returns of the risky project and the borrowers always have incentives to pretend that their returns are just l even if they succeed in the project. Therefore the lenders are willing to provide maximally $\frac{l}{R^E}$ to any borrowers. Consequently, an individual's initial endowments are tightly related to his investment and occupation choice. For example, the poor agents whose wealth is smaller than $K - wt - {l}/{R^E}$ never meet the set-up cost of the risky project. Thus they have to be pure wage workers. The decisions of the others depend on which choice yields more benefits. The difference of the utilities between being a wage worker and being an entrepreneur is

$$U_3 - U_2 = [pf(n) + (1-p)l] - R^E(K + wn) \tag{4.9}$$

The first term is the return of the risky project while the second one is the capital market return of the total investment. The latter is exactly the opportunity cost of being entrepreneurs. Given a level of labour input, if (4.9) is positive, then the returns of the risky project is higher than its opportunity cost. An individual would prefer being an entrepreneur to being a wage worker, vice versa. Only if (4.9) is equal to zero, the individual would be indifferent between being an entrepreneur and being a worker. It implies that there is a cut-off value of labour input when $U_3 = U_2$. Hence the minimal labour input $\underline{n}$ is determined by $U_3 = U_2$. Individuals whose working capital cannot meet the minimal labour level would prefer to be wage workers even though they can afford the set-up cost of the project. We will examine each term of (4.9).

Suppose

$$Y_1(n) = pf(n) + (1-p)l \tag{4.10}$$

$$Y_2(n) = R^E wn + R^E K \tag{4.11}$$

We plot (4.10) and (4.11) in Figure 4.1. Since the interest rate varies as the initial conditions change, the slope in the figures below may change as well.

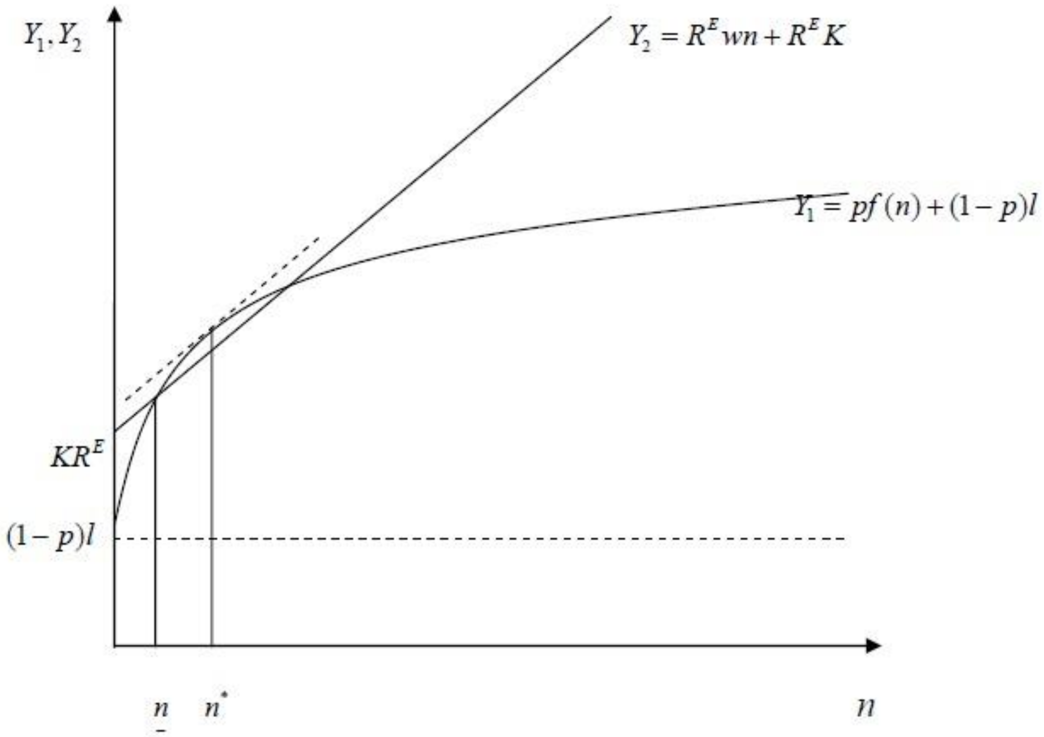

Figure 4.1

where $\underline{n}$ is the minimal labour input and n^* is the optimal level of labour. Firstly, l is the liquidation value of K. So $K > l$. We also know $R^E > 1$ while $1 - p < 1$. Hence $KR^E > (1-p)l$. Secondly, the maximal unit return of the project is

$$R^* = \frac{pf(n^*) + (1-p)l}{K + wn^*} \tag{4.12}$$

We know that the interest rate cannot exceed R^*, otherwise no one would invest in the risky project. It implies $R^E \leq \frac{pf(n^*) + (1-p)l}{K + wn^*}$. Rearranging the inequality gives $pf(n^*) + (1-p)l \geq R^E(K + wn^*)$. The left hand side of the inequality is exactly $Y_1(n^*)$ while the right hand side is $Y_2(n^*)$. Put differently, $Y_1(n^*) \geq Y_2(n^*)$. The inequality indicates that the equation $Y_1(n) = Y_2(n)$ has at least one solution. Furthermore, $Y_1'(n) = pf'(n)$ and $Y_2'(n) = R^E w$ while (4.7) tells us that $pf'(n) = R^E w$. So $Y_1'(n) = Y_2'(n)$. It means that the marginal utility of $Y_1(n)$ is exactly equal to the slope of the line (4.10). The level of the optimal labour, n^*, is determined by this equation.

Under the perfect market conditions, lenders can always observe borrowers' returns and there is no moral hazard problem. All entrepreneurs raise enough funds to carry out the project at the optimal investment level. In other words, the labour they used for the project is always n^*. The line Y_2 in Figure 4.1 lifts upwards to the position of the dash line. The minimal labour input is just equal to the optimal level. There is only one solution for the labour input, namely n^*. The imperfect market case, however, is quite different. Some entrepreneurs cannot collect necessary money to perform an optimal investment project on account of the moral hazard problem. For example, entrepreneurs whose wealth falls into the range from $K + w\underline{n} - {}^{l}\!/_{R^E}$ to $K + wn^* - {}^{l}\!/_{R^E}$ will never reach the optimal investment level $K + wn^*$, because they can borrow $\frac{l}{R^E}$ at most from the capital market. With the restriction of working capital, the amount of labour they used in the project must be smaller than n^*. The result implies that there is another cut-off level for the labour input, at which individuals are indifferent in investing in a project or being a wage worker. Therefore the difference between the utility of being an entrepreneur and the utility of being a pure worker disappears, which means $U_3 - U_2 = 0$. Put differently, there is an equation $Y_1(n) = Y_2(n)$ to meet the minimal labour level $\underline{n}$. The equation is also expressed as $pf(\underline{n}) + (1-p)l = R^E(K + w\underline{n})$. Rearranging it obtains

$$R^E = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} \tag{4.13}$$

In summary, the case is listed below

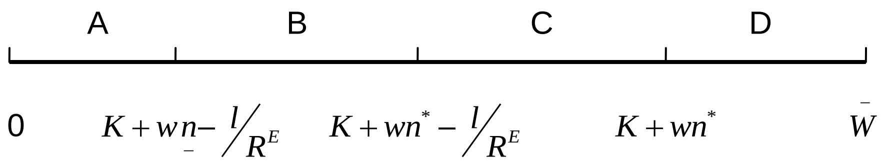

Figure 4.2

In Figure 4.2, three cut-off values divide individuals into four different groups.

i. Individuals in area A are pure wage workers and lenders. They sell their labour in the labour market and only invest in either the capital market or CRS technology.

ii. Entrepreneurs in area B borrow an amount of money $\frac{l}{R^E}$ to undertake the risky project at the suboptimal investment level.

iii. Entrepreneurs in area C can raise money to meet the requirement for full investment in the project.

iv. Individuals in area D are self-finance and invest the rest of money in either the capital market or CRS technology.

All individuals would like to work for time t^*. We also place a restriction on the maximal total investment so that

Assumption 4.2: $0 < K + wn^* < \bar{W}$

There are three different situations.

Proposition 1

a) if $\int_{K+wn^*-l/R^*}^{K+wn^*}\left[(K+wn^*)-W_i\right]g(W)dW_i >$

$$\int_0^{K+wn^*-l/R^*} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}}\left[W_i-(K+wn^*)\right]g(W)dW$$

then $R^E = R^*$.

There is an excess demand in the system. The proportion of entrepreneurs undertaking the risky project at the optimal level is $\int_{K+wn^*-l/R^E}^{\bar{W}} g(W)dW_i$. The proportion of wage workers is $\int_0^{K+wn^*-l/R^E} g(W)dW_i$.

No one invests in CRS technology

b) if $\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} l/Z\, g(W)dW_i + \int_{K+wn^*-l/Z}^{K+wn^*}\left[K+wn^*-W_i\right]g(W)dW_i <$

$$\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}}\left[W_i-(K+wn^*)\right]g(W)dW$$

then $R^E = Z$.

There is an excess supply in the economy. The proportion of entrepreneurs undertaking the risky project at suboptimal level is $\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} g(W)dW_i$. The proportion of entrepreneurs who carry out the project at optimal level is $\int_{K+wn^*-l/Z}^{\bar{W}} g(W)dW_i$. The capital which is invested in CRS is

$$\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}}\left[W_i-(K+wn^*)\right]g(W)dW$$

$$-\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} l/Z\; g(W)dW_i + \int_{K+wn^*-l/Z}^{K+wn^*} \left[K+wn^* - W_i\right] g(W)dW_i$$

c) if $\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW \le$

$$\int_{K+w\underline{n}-l/R^E}^{K+wn^*-l/R^E} l/R^E\; g(W)dW_i + \int_{K+wn^*-l/R^E}^{K+wn^*} \left[K+wn^* - W_i\right] g(W)dW_i \le$$

$$\int_0^{K+wn^*-l/R^*} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW$$

Then $Z \le R^E \le R^*$

The proportion of entrepreneurs undertaking the risky project at suboptimal level is $\int_{K+w\underline{n}-l/R^E}^{K+wn^*-l/R^E} g(W)dW_i$. The proportion of entrepreneurs carrying out the project at optimal level is $\int_{K+wn^*-l/R^E}^{\bar{W}} g(W)dW_i$.

No capital is left for CRS technology

Proof.

1) We know that the maximal interest rate cannot exceed R^*, otherwise no agents would consider the risky technology. When the inequality holds, there is an excess demand for credits because the proportion of projects that entrepreneurs are willing to invest is more than the proportion of projects that lenders can fund. Hence the return from the capital market must be equal to the highest return from the project. Here $R^E = R^*$ also means

$$R^E = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} = R^* = \frac{pf(n^*) + (1-p)l}{K + wn^*} .$$

It implies $\underline{n} = n^*$. In other words, the minimal labour input of a project is equal to its optimal value as long as $R^E = R^*$. Figure 4.3 shows the status of this case.

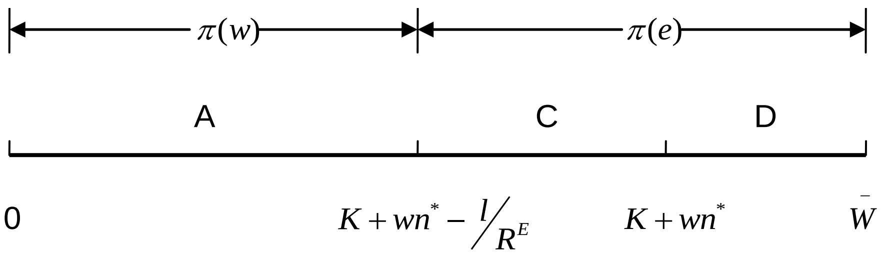

Figure 4.3

$\pi(w)$ presents the proportion of wage workers and $\pi(e)$ is the counterpart of entrepreneurs who invest in the risky technology with optimal value. Here ‘w’ is just the abbreviation of ‘worker’ and ‘e’ denote ‘entrepreneur’. In this case, area B of Figure 4.2 where entrepreneurs undertake the project at the suboptimal level disappears because individuals cannot afford the price of loans at such a high interest rate. They prefer to be wage workers instead. Consequently, there are only three groups of agents. The first one is in area A where there are only pure wage workers. The second group consists of entrepreneurs who borrow necessary money to support their full investment in projects. The rest of rich individuals are self-finance. The proportion of entrepreneurs undertaking the risky project at the optimal level is $\int_{K+wn^*-l/R^E}^{\bar{W}} g(W)dW_i$. The proportion of wage workers is $\int_{0}^{K+wn^*-l/R^E} g(W)dW_i$.

No one invests CRS technology because of the shortage of credits.

2) We also know that the lowest interest rate cannot be smaller than Z, otherwise all agents would prefer to invest in CRS technology. When the inequality holds, there is an excess supply of funds because the proportion of projects that entrepreneurs are willing to invest is less than the proportion of projects that lenders can support. Competition from the capital market puts pressure on the interest rate so that it reduces to Z. Contrast to the first case, both suboptimal investment and full investment entrepreneurs are funded. The rest of capital will be invested in CRS technology. The proportion of entrepreneurs undertaking the risky project at suboptimal level is

$\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} g(W)dW_i$. The proportion of entrepreneurs carrying out the project at optimal level is $\int_{K+wn^*-l/Z}^{\bar{W}} g(W)dW_i$. The spare capital which is invested in CRS is

$$\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW$$

$$-\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} l/Z\, g(W)dW_i + \int_{K+wn^*-l/Z}^{K+wn^*} \left[K+wn^* - W_i\right] g(W)dW_i$$

Figure 4.4 demonstrates this situation

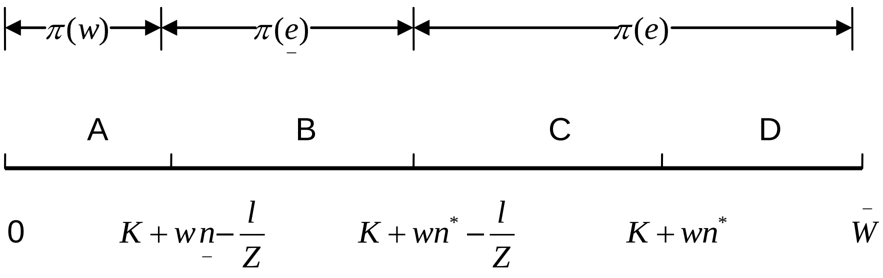

Figure 4.4

The proportion of wage workers is $\pi(w)$, the proportion of suboptimal investment entrepreneurs is $\pi(\underline{e})$ and the proportion of full-investment entrepreneurs is $\pi(e)$, respectively.

3) The minimal supply of funds in the economy is

$\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*\right] g(W)dW$ when $R^E = Z$ on account of the very low interest rate.

The maximal supply of funds is

$\int_0^{K+wn^*-l/R^*} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW$ when $R^E = R^*$ because of the high interest rate. The inequality means that the demand of credits in the economy varies between the maximum and the minimum value of supply. When the inequality holds, there is an excess demand of credits if $R^E = Z$ and an excess supply of credits if $R^E = R^*$. In this case R^E varies between Z and R^*. It will reach its equilibrium point as long as the total demand equals the total supply, namely the market clearing point. The proportion of entrepreneurs undertaking the risky project at the suboptimal level is $\int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} g(W)dW_i$. The proportion of entrepreneurs carrying out the project at the optimal level is $\int_{K+wn^*-\frac{l}{R^E}}^{\bar{W}} g(W)dW_i$. No one invests in CRS technology because the capital market is clear at the point that the demand for credits equals the supply. Figure 4.5 exhibits this situation

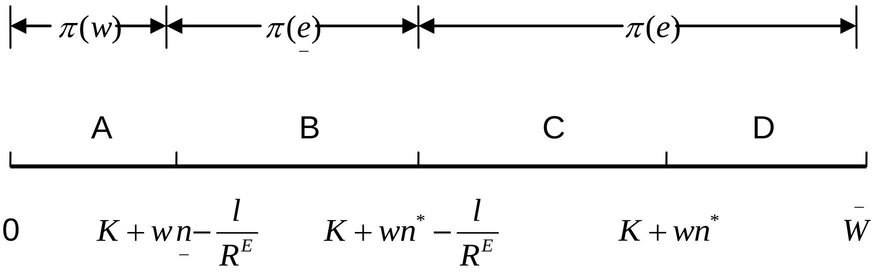

Figure 4.5

The proportion of wage workers is $\pi(w)$, the proportion of suboptimal investment entrepreneurs is $\pi(\underline{e})$ and the proportion of full-investment entrepreneurs is $\pi(e)$, respectively.

4.4 The Simultaneous Equation of Equilibrium

Proposition 1 provides a general outline of the model with three different situations. It distinguishes different cases in terms of the demand and supply of credits. However, we are also interested in what happens in the labour market. For example, how the wage rate is determined and what the optimal level of the labour used in the risky project. Similarly, we concern what factors influence the interest rate in the capital market. In this section, we will construct a series of simultaneous equations in terms of the capital market and the labour market clearing conditions.

1) If $\int_{K+wn^*-l/R^*}^{K+wn^*}\left[(K+wn^*)-W_i\right]g(W)dW_i >$

$$\int_0^{K+wn^*-l/R^*} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW$$

then $R^E = R^*$. In this case, too much borrowers pursue too little funds. Competition in the capital market pushes the interest rate up to the highest level which is equal to the returns of the risky project preformed at the optimal level. The interest rate is determined by (4.16), namely $R^E = R^* = \dfrac{pf(n^*)+(1-p)l}{K+wn^*}$. The equations of (4.13) and (4.16) also state that the minimal labour level is equal to the optimal level. It in turn implies that only the project carried out at the optimal level is profitable. Substituting $R^E = R^*$ into (4.7) and (4.8) gives

$$pf'(n^*) = R^* w \tag{4.14}$$

$$u'(1-t^*) = R^* w \tag{4.15}$$

The labour market clearing condition is that the total demand for labour, which is $\pi(e)n^*$, equals the supply of labour t^*. What's more, the capital market clearing condition is that the total demand for credits, which is $(K+wn^*)\pi(e)$, is equal to the supply $\hat{W}$.

The system is thus determined by the following equations

$$pf'(n^*) = R^* w \tag{4.14}$$

$$u'(1-t^*) = R^* w \tag{4.15}$$

$$R^* = \frac{pf(n^*)+(1-p)l}{K+wn^*} \tag{4.16}$$

$$\pi(w) + \pi(e) = 1 \tag{4.17}$$

$$\pi(w)t^* = \pi(e)(n^* - t^*) \tag{4.18}$$

$$(K+wn^*)\pi(e) = \hat{W} \tag{4.19}$$

(4.17) demonstrates that the sum of the proportion of pure wage workers and the proportion of entrepreneurs is one. (4.18) is the labour market clearing condition. The left hand side is the labour supply while the right hand side is the demand. They must equal in the market equilibrium. The last equation states that the equilibrium of the capital market. In this case, each entrepreneur needs $K + wn^*$ to carry out a full investment project. Hence the left hand side is the average demand of credits while the average supply of funds is presented by the right hand side. We finally have six endogenous variables, namely n^*, $\underline{n}$, t^*, w, $\pi(w)$ and $\pi(e)$, respectively. We also have six equations. The system should be determined by the simultaneous equation.

2) If $\int_{K+w\underline{n}-l/Z}^{K+wn^*-l/Z} l/Z \, g(W)dW_i + \int_{K+wn^*-l/Z}^{K+wn^*} \left[K + wn^* - W_i\right] g(W)dW_i <$

$$\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K + wn^*)\right] g(W)dW$$

then $R^E = Z$. Credits in this system not only meet the requirement of demand from borrowers but also flow into CRS technology. Put differently, there are so many funds that the interest rate of the capital market shrinks to the level of CRS returns. Substituting $R^E = Z$ into (4.7) and (4.8) gives

$$pf'(n^*) = Zw \tag{4.20}$$

$$u'(1-t) = Zw \tag{4.21}$$

Substituting $R^E = Z$ again into (4.13) gives

$$Z = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} \tag{4.22}$$

Apparently, the sum of proportions of all individuals in Figure 4.4 is one. So we have

$$\pi(w)+\pi(\underline{e})+\pi(e)=1 \tag{4.23}$$

From Figure 4.4 we also have $\pi(\underline{e})=\int_{K+w\underline{n}-\frac{l}{Z}}^{K+wn^*-\frac{l}{Z}} g(W)dW_i=\frac{w}{\bar{W}}(n^*-\underline{n})$

So

$$\pi(\underline{e})=\frac{w}{\bar{W}}(n^*-\underline{n}) \tag{4.24}$$

and $\pi(e)=\int_{K+wn^*-\frac{l}{Z}}^{\bar{W}} g(W)dW_i=1-\frac{K+wn^*-{}^{l}\!/_{Z}}{\bar{W}}$, that is

$$\pi(e)=1-\frac{K+wn^*-{}^{l}\!/_{Z}}{\bar{W}} \tag{4.25}$$

In addition, the labour market clearing condition is that the supply of labour equals the demand . That is

$$[\pi(w)+\pi(\underline{e})+\pi(e)]t^*=\int_{K+w\underline{n}-\frac{l}{Z}}^{K+wn^*-\frac{l}{Z}}\frac{W_i+\frac{l}{Z}-K}{w}g(W)dW_i+\pi(e)n^*$$

The left hand side of the equation is the amount of labour supply in the economy. The first item of the right hand side is the labour demanded by entrepreneurs in area B of Figure 4.4. The item $W_i+\frac{l}{Z}-K$ is the working capital and $\frac{W_i+\frac{l}{Z}-K}{w}$ is the total labour they used. The second item of the right hand side presents total labour used by the rich entrepreneurs. Substituting (4.23) into it and rearranging it gives

$$t^*=\frac{1}{w}\int_{K+w\underline{n}-\frac{l}{Z}}^{K+wn^*-\frac{l}{Z}}W_i g(W)dW_i+\frac{\frac{l}{Z}-K}{\bar{W}}(n^*-\underline{n})+\pi(e)n^*$$

Let's set $\hat{W}_{\underline{e}} = \int_{K+w\underline{n}-\frac{l}{Z}}^{K+wn^*-\frac{l}{Z}} W_i g(W) dW_i$ as the average wealth of all suboptimal investment entrepreneurs. Thus we have

$$t^* = \frac{\hat{W}_{\underline{e}}}{w} + \frac{\frac{l}{Z} - K}{\bar{W}}(n^* - \underline{n}) + \pi(e)n^* \tag{4.26}$$

It is worth mentioning that the total credits supply does not clear the demand for funds from borrowers because of the exceeding supply of funds. Finally, the system is determined by the following equations:

$$pf'(n^*) = Zw \tag{4.20}$$

$$u'(1-t) = Zw \tag{4.21}$$

$$Z = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} \tag{4.22}$$

$$\pi(w) + \pi(\underline{e}) + \pi(e) = 1 \tag{4.23}$$

$$\pi(\underline{e}) = \frac{w}{\bar{W}}(n^* - \underline{n}) \tag{4.24}$$

$$\pi(e) = 1 - \frac{K + wn^* - l/Z}{\bar{W}} \tag{4.25}$$

$$t^* = \frac{\hat{W}_{\underline{e}}}{w} + \frac{\frac{l}{Z} - K}{\bar{W}}(n^* - \underline{n}) + \pi(e)n^* \tag{4.26}$$

(4.20) demonstrates the sum of the proportions of workers and entrepreneurs is one. (4.21) is the proportion of entrepreneurs whose investment is suboptimal. (4.22) is the proportion of entrepreneurs performing full investment. (4.23), (4.24) and (4.25) are discussed above. last equation is the labour market clearing condition. We finally have seven endogenous variables, which are n^*, $\underline{n}$, t^*, w, $\pi(w)$, $\pi(\underline{e})$ and $\pi(e)$, respectively. We also have seven equations. The system should be determined by the simultaneous equation.

3) If $\int_0^{K+w\underline{n}-l/Z} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW \leq$

$$\int_{K+w\underline{n}-l/R^E}^{K+wn^*-l/R^E} l/R^E \, g(W)dW_i + \int_{K+wn^*-l/R^E}^{K+wn^*} \left[K+wn^* - W_i\right] g(W)dW_i \leq$$

$$\int_0^{K+wn^*-l/R^*} W_i g(W)dW + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K+wn^*)\right] g(W)dW$$

Then $Z \leq R^E \leq R^*$. In this case, the interest rate of the capital market varies in the range between the CRS returns and the optimal level returns from the risky project in order to clear both the capital and the labour markets. The equations (4.7) and (4.8) still hold in this case. Hence the optimal working time and the maximal labour used in the project are determined by the two equations. Equation (4.13) defines the interest rate as usual. In addition, as Figure 4.5 shows that individuals in this case are divided into three groups so that the sum of proportions of all individuals is one. We have

$$\pi(w) + \pi(\underline{e}) + \pi(e) = 1 \tag{4.27}$$

Figure 4.5 also indicates that $\pi(\underline{e}) = \int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} g(W)dW_i = \frac{w}{\bar{W}}(n^* - \underline{n})$,

so

$$\pi(\underline{e}) = \frac{w}{\bar{W}}(n^* - \underline{n}) \tag{4.28}$$

And $\pi(e) = \int_{K+wn^*-\frac{l}{R^E}}^{\bar{W}} g(W)dW_i = 1 - \dfrac{K+wn^* - \dfrac{l}{R^E}}{\bar{W}}$. That is

$$\pi(e) = 1 - \frac{K + wn^* - {l}/{R^E}}{\bar{W}} \qquad (4.29)$$

In this case, all credits are shared by the risky technology and the capital market. No one invests in CRS technology. Then the capital market clearing condition is that the supply of credit meets the demand. That is

$$\int_0^{K+w\underline{n}-\frac{l}{R^E}} W_i g(W) dW_i + \int_{K+wn^*}^{\bar{W}} \left[W_i - (K + wn^*)\right] g(W) dW_i =$$
$$\int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} \frac{l}{R^E} g(W) dW_i + \int_{K+wn^*-\frac{l}{R^E}}^{K+wn^*} \left[K + wn^* - W_i\right] g(W) dW_i$$

Rearranging the above equation gives

$$\int_0^{K+w\underline{n}-\frac{l}{R^E}} W_i g(W) dW_i + \int_{K+wn^*-\frac{l}{R^E}}^{K+wn^*} W_i g(W) dW_i + \int_{K+wn^*}^{\bar{W}} W_i g(W) dW_i =$$
$$\frac{l}{R^E} \int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} g(W) dW_i + (K + wn^*) \left[\int_{K+wn^*-\frac{l}{R^E}}^{K+wn^*} g(W) dW_i + \int_{K+wn^*}^{\bar{W}} g(W) dW_i \right]$$

The left hand side of the equation is equivalent to the average wealth of all individuals minus the average wealth of entrepreneurs who invest at the suboptimal level. Therefore the capital market equilibrium is determined by

$$\hat{W} - \hat{W}_{\underline{e}} = \frac{l}{R^E} \pi(\underline{e}) + (K + wn^*) \pi(e) \qquad (4.30)$$

where $\hat{W}_{\underline{e}} = \int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} W_i g(W) dW_i$ is the average wealth of all suboptimal investment entrepreneurs.

At last, the labour market clearing condition is

$$[\pi(w) + \pi(\underline{e}) + \pi(e)] t^* = \int_{K+w\underline{n}-\frac{l}{R^E}}^{K+wn^*-\frac{l}{R^E}} \frac{W_i + \frac{l}{R^E} - K}{w} g(W) dW_i + \pi(e) n^*$$

The left hand side of the equation is the total supply of labour in the system. The first item of the right hand side is the labour demanded by all suboptimal investment entrepreneurs. The second item is the labour demanded by full investment entrepreneurs. Rearranging the equation gives

$$wt^{*}=\hat{W}_{\underline{e}}-\frac{w}{\bar{W}}\left(K-\frac{l}{R^{E}}\right)(n^{*}-\underline{n})+\pi(e)wn^{*} \tag{4.31}$$

Thus the system is determined by the following equations:

$$pf'(n^{*})=R^{E}w \tag{4.7}$$

$$u'(1-t^{*})=R^{E}w \tag{4.8}$$

$$R^{E}=\frac{pf(\underline{n})+(1-p)l}{K+w\underline{n}} \tag{4.13}$$

$$\pi(w)+\pi(\underline{e})+\pi(e)=1 \tag{4.27}$$

$$\pi(\underline{e})=\frac{w}{\bar{W}}(n^{*}-\underline{n}) \tag{4.28}$$

$$\pi(e)=1-\frac{K+wn^{*}-\dfrac{l}{R^{E}}}{\bar{W}} \tag{4.29}$$

$$\hat{W}-\hat{W}_{\underline{e}}=\frac{l}{R^{E}}\pi(\underline{e})+(K+wn^{*})\pi(e) \tag{4.30}$$

$$wt^{*}=\hat{W}_{\underline{e}}-\frac{w}{\bar{W}}\left(K-\frac{l}{R^{E}}\right)(n^{*}-\underline{n})+\pi(e)wn^{*} \tag{4.31}$$

Here we have eight endogenous variables, n^{*}, $\underline{n}$, t^{*}, w, R^{E}, $\pi(w)$, $\pi(\underline{e})$ and $\pi(e)$, respectively. The simultaneous equations provide a solution vector to determine the situation of the system

4.5 A Case of Comparative Statics

The model defines a set of variables such as the optimal amount of labour n^{*}, the minimal level of labour $\underline{n}$, the optimal own labour t^{*}, the work wage w

and the interest rate R^E. It also comes with exogenous parameters like the maximum value of the initial endowments. In the first chapter, we discussed the comparative statics results with two important parameters, namely the mean wealth $\hat{W}$ and the half difference of wealth from the poorest to the richest X. The two parameters is integrated into one because $\hat{W} = X = \frac{\bar{W}}{2}$ in this framework. It turns out that we are interested in how the system changes as the maximal wealth changes.

In summary, the first two cases of the model are both extreme situations. For example, the interest rate equals either the maximal return of the risky project when there is an excess demand of credits, or the CRS returns when there is an excess supply of funds. Rarely do they occur in the real world. Hence we are seeking a more general situation which presents reality as much as possible. The case 3 in which the demand for credits varies over the range between the maximum and the minimum value of funds supply can meet this requirement. In this case, both the capital market and the labour market clear as long as the demand of funds equals the supply of it. In turn, the interest rate varies between Z and R^* to clear both the capital and the labour markets. We are trying to explore the effects of altering $\bar{W}$ in the system.

We have shown that the system is determined by the following equations:

$$pf'(n^*) = R^E w \tag{4.7}$$

$$u'(1-t^*) = R^E w \tag{4.8}$$

$$R^E = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} \tag{4.13}$$

$$\pi(w) + \pi(\underline{e}) + \pi(e) = 1 \tag{4.27}$$

$$\pi(\underline{e}) = \frac{w}{\bar{W}}(n^* - \underline{n}) \tag{4.28}$$

$$\pi(e) = 1 - \frac{K + wn^* - \frac{l}{R^E}}{\bar{W}} \tag{4.29}$$

$$\hat{W} - \hat{W}_{\underline{e}} = \frac{l}{R^E}\pi(\underline{e}) + (K + wn^*)\pi(e) \tag{4.30}$$

$$wt^* = \hat{W}_{\underline{e}} - \frac{w}{\bar{W}}(K - \frac{l}{R^E})(n^* - \underline{n}) + \pi(e)wn^* \tag{4.31}$$

Substituting $\pi(\underline{e})$ and $\pi(e)$ into (4.30) and (4.31) gets rid of the proportion of labour in both equations. We then obtain the capital market and the labour market clearing conditions with five endogenous variables. They are the optimal number of employees n^*, the minimal labour $\underline{n}$, the optimal working time t^*, the wage rate w and the interest rate R^E, respectively. The two markets clearing conditions with additional equations such as (4.7), (4.8) and (4.13) constitute a simultaneous equation detecting a unique solution vector for these variables. Since there are five variables coming with five equations, it should have at least one solution for the system. However, we cannot obtain a reduce form solution owning to the general function of the product. So we will explore the comparative statics results instead, just like we have done in the first chapter.

We adopted the method of Derivatives of Implicit Functions in the first chapter to explore the comparative statics issue for simultaneous equation without reduced forms. In this case, however, applying the method to a simultaneous equation with five equations requires approximately a great number of steps which may be prohibitively expensive. Hence we will adopt a simpler way to avoid such burdensome complication.

Firstly, we have the definition of the interest rate

$$R^E = \frac{pf(\underline{n}) + (1-p)l}{K + w\underline{n}} \tag{4.13}$$

Suppose there is a small increment of the maximum value of the initial wealth. It implies that the total credit supply of the system increases. The interest rate will in turn decrease, namely $\frac{\partial R^E}{\partial \overline{W}} < 0$. In addition, making a partial derivative of (4.13) to the minimal amount of labour obtains

$$\frac{\partial R^E}{\partial \underline{n}} = \frac{pf'(\underline{n})(K + w\underline{n}) - w[pf(\underline{n}) + (1-p)l]}{(K + w\underline{n})^2}$$

Substituting (4.7) into the result above gives

$$\frac{\partial R^E}{\partial \underline{n}} = \frac{pf'(\underline{n})(K + w\underline{n}) - \frac{pf'(n^*)}{R^E}[pf(\underline{n}) + (1-p)l]}{(K + w\underline{n})^2}$$

$$= \frac{pf'(\underline{n})(K + w\underline{n})R^E - pf'(n^*)[pf(\underline{n}) + (1-p)l]}{R^E(K + w\underline{n})^2}$$

Substituting (4.13) into the result again gives

$$\frac{\partial R^E}{\partial \underline{n}} = \frac{pf'(\underline{n})[pf(\underline{n}) + (1-p)l] - pf'(n^*)[pf(\underline{n}) + (1-p)l]}{R^E(K + w\underline{n})^2}$$

$$= \frac{p[pf(\underline{n}) + (1-p)l]}{R^E(K + w\underline{n})^2}[f'(\underline{n}) - f'(n^*)]$$

We know that $f'(n)$ is a decreasing function in n and $n^* > \underline{n}$, so

$f'(\underline{n}) - f'(n^*) > 0$

Finally, $\frac{\partial R^E}{\partial \underline{n}} > 0$. It states that the minimal amount of labour declines as soon as the interest rate drops. We then deduce that $\frac{\partial \underline{n}}{\partial \overline{W}} < 0$ from the fact that the interest rate negatively relates to $\overline{W}$.

Furthermore, making a partial derivative of (4.13) to wage obtains the relation between the interest rate and the wage. That is

$$\frac{\partial R^E}{\partial w} = -\frac{\underline{n}[pf(\underline{n}) + (1-p)l]}{(K + w\underline{n})^2} < 0$$

It describes that the wage rate negatively relates to the interest rate. Because of $\frac{\partial R^E}{\partial \bar{W}} < 0$, we the have $\frac{\partial w}{\partial \bar{W}} > 0$. It means that the larger the wealth is, the higher the wage is.

Furthermore, rearranging (4.7) gives

$$w = \frac{pf'(n^*)}{R^E} \tag{4.32}$$

Making a partial derivative of (4.32) gives

$\frac{\partial w}{\partial n^*} = \frac{pf''(n^*)}{R^E} < 0$, so $\frac{\partial n^*}{\partial \bar{W}} < 0$. It means that the optimal number of employees negatively relates to $\bar{W}$.

Again, rearranging (4.8) provides another reduced form definition of wage rate relating to the optimal working time. That is

$$w = \frac{u'(1-t^*)}{R^E} \tag{4.33}$$

Making a partial derivative of the above equation with respect to the optimal working time gives

$\frac{\partial w}{\partial t^*} = -\frac{u''(1-t^*)}{R^E} > 0$, so $\frac{\partial t^*}{\partial \bar{W}} > 0$. It tells that the optimal working time also positively relates to $\bar{W}$.

Finally we have $\frac{\partial R^E}{\partial \bar{W}} < 0$, $\frac{\partial \underline{n}}{\partial \bar{W}} < 0$, $\frac{\partial w}{\partial \bar{W}} > 0$, $\frac{\partial n^*}{\partial \bar{W}} < 0$ and $\frac{\partial t^*}{\partial \bar{W}} > 0$.

Unlike the model of Chapter 2, individuals' wealth is defined at the interval $\left[0, \bar{W}\right]$ in Chapter 4. Increasing $\bar{W}$ is equivalent to jointly increase in both average wealth and the distribution of wealth. In particular, an increase in the mean wealth leads to an increase in credit supply. The interest rate goes down accordingly. Suppose that other factors of the system do not change immediately at the moment, a lower interest rate implies a lower borrowing cost so that two cut-off values in Figure 4.5, both $K + w\underline{n} - \frac{l}{R^E}$ and $K + wn^* - \frac{l}{R^E}$, move towards left. In other words, a few pure wage workers become entrepreneurs and a few suboptimal investment entrepreneurs now prefer to full investment in the project. Consequently there is an increase in labour demand and thus the wage rate rises. Furthermore, the wage workers would like to spend more time on working because of the higher wage, so do the entrepreneurs. Put differently, the labour time increases as the mean wealth goes up. On the other hand, an increasing wage rate naturally makes entrepreneurs reduce the number of employees. So the optimal amount of labour decreases as the mean wealth increases. Finally, since the minimal amount of labour positively relates to the interest rate, it is also decreases with the mean wealth.

4.6. Conclusions

We present a static model of an economy where individuals are heterogeneous in terms of initial wealth and there are credit constraints due to moral hazard. Individuals are endowed with time resource which they can allocate between working and leisure to maximize their utility. What's more, individuals can choose either to sell their labour in the labour market or to self-employ. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or become entrepreneurs by running a risky project. Unlike the model in Chapter 1, the risky project requires both capital and labour inputs. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on whether to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds, while the wage rate meets the labour market clearing condition. We find that an increase in the mean wealth leads to a decrease in the interest rate. In equilibrium, the wage rate rises and so does the labour time. Meanwhile, both the optimal amount of labour and the minimal requirement of labour of the project decrease. A further development of the model can distinguish risk preferences over individuals, and extend the static model to a dynamic one to discuss the issue of economic growth.

CHAPTER 5 Conclusion

The thesis looks into the issue of imperfect market and asymmetric information and how individuals' initial wealth and the distribution of wealth affect their activities in imperfect markets. In Chapter 2 we investigate a model of asymmetric information on account of market imperfection. Individuals differ from their initial endowments. According to their restrictions of working capital, individuals make investment decision among the capital market, the CRS with constant returns and a risky project which has a fixed set-up cost. We find that the poorest individuals are pure lenders. Those low to medium wealth individuals borrow money which is equivalent to the liquidated value of the project and invest sub-optimally. Individuals with medium to high wealth can collect enough money to invest optimally. And the richest individuals self-finance. We then introduce a monitoring technology to make it possible for external lenders to observe the returns of the private project. Borrowers thus may obtain bank loans to be an alternative source of funds. Our result shows that the poorest individuals are still pure lenders. But some relatively poor individuals alter from lenders to entrepreneurs by obtaining bank loans to invest optimally. On the other hands, part of low to medium wealth agents prefer to the capital market because of the lower cost of direct finance, they invest sub-optimally. Meanwhile, other low to medium wealth individuals choose bank loans and invest optimally. Medium to high wealth individuals raise enough funds from the capital market to achieve an optimal investment. Those richest individuals still self-finance.

Chapter 3 examines one of the comparative static results of Chapter 2 by a panel of 16 countries for the period 1989-2004. The comparative static result states that if income difference across social groups is lower than a threshold, a rise in average income would push up the minimal investment level and the interest rate. Otherwise it would reduce both of them. Our empirical model develops further to predict that financial development must be positively related to average income if income inequality is small, while it is negatively related to average income if income inequality is higher than the critical value. We adopt a modified method of the multiple thresholds regression to the model in order to inspect this prediction. We find overwhelming evidence of the threshold effect. The point estimates of the two thresholds are 36.6 and 37.663 respectively. It means when income inequality is lower than 37.663, a rise of average income would improves financial development. As long as income inequality is higher than this value, financial development does not benefit from average income.

The model of Chapter 4 is derived from that of Chapter 2 except for introducing a labour market. The extension make it possible to analyze markets equilibrium conditions from both physical capital and human capital. We present a static model of an economy where individuals are heterogeneous in terms of initial wealth and there are credit constraints due to asymmetric information. Individuals are endowed with time resource which they can allocate between working and leisure to maximize their utility. What's more, individuals can choose either to sell their labour in the labour market or to self-employ. Put differently, depending on the opportunity costs of alternatives, they can supply as pure wage workers or become entrepreneurs by running a risky project. Unlike the model in Chapter 1, the risky project requires both capital and labour inputs. Workers receive fixed wages while entrepreneurs receive risky profits. Individuals make their decisions on whether to be wage workers or entrepreneurs by comparing the utility from the wage work with that from the risky project. The endogenous interest rate adjusts to the point where the supply of credits is equal to the demand for funds, while the wage rate meets the labour market clearing condition. We find that an increase in the mean wealth leads to a decrease in the interest rate. In equilibrium, the wage rate rises and so does the labour time. Meanwhile, both the optimal amount of labour and the minimal requirement of labour of the project decrease.

APPENDIX A.1

Comparative Statics of the simultaneous equations

In this model, there are four variables R^E, $\underline{I}$, I^* and R^* and four equations.

$$R^E = \frac{pf(\underline{I}) + (1-p)l}{K + \underline{I}} \tag{2.3}$$

$$f'(I^*) = 1/p \tag{2.4}$$

$$R^* = \frac{pf(I^*) + (1-p)l}{K + I^*} \tag{2.5}$$

$$\begin{aligned} &\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} \left[W_i - (K+I^*)\right] g(W)dW_i \\ &= \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K + I^* - W_i) g(W)dW_i \end{aligned} \tag{2.19}$$

So we can have the results of the comparative statics for these simultaneous equations. Rearranging equation (2.19) gives

$$\begin{aligned} &\int_{\underline{W}}^{K+\underline{I}(R^E)-(l/R^E)} W_i g(W)dW_i + \int_{K+I^*}^{\bar{W}} W_i g(W)dW_i - \int_{K+I^*}^{\bar{W}} (K+I^*) g(W)dW_i \\ &= \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^*) g(W)dW_i - \int_{K+I^*-(l/R^E)}^{K+I^*} W_i g(W)dW_i \end{aligned}$$

So

$$\hat{W} - \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} W_i g(W)dW_i = \int_{K+\underline{I}(R^E)-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E) g(W)dW_i + \int_{K+I^*-(l/R^E)}^{\bar{W}} (K+I^*) g(W)dW_i$$

Applying the uniform distribution function on the last equation, we obtain

$$\hat{W}-\frac{(K+I^{*}-l/R^{E}+K+\underline{I}-l/R^{E})(I^{*}-\underline{I})}{4X}$$

$$=\frac{l/R^{E}}{2X}(I^{*}-\underline{I})+\frac{K+I^{*}}{2X}[(\hat{W}+X)-(K+I^{*}-l/R^{E})]$$

$$4X\hat{W}-(2K+I^{*}+\underline{I}-2l/R^{E})(I^{*}-\underline{I})$$

$$=2l/R^{E}(I^{*}-\underline{I})+2(K+I^{*})(\hat{W}+X)-2(K+I^{*})(K+I^{*}-l/R^{E})$$

$$4X\hat{W}=2(K+I^{*})(\hat{W}+X)-2(K+I^{*})(K+I^{*}-l/R^{E})+(2K+I^{*}+\underline{I})(I^{*}-\underline{I})$$

We eventually get another form of (2.19)

$$2(K+I^{*})(\hat{W}+X)-(K+I^{*})(2K+I^{*}+\underline{I}-2l/R^{E})+(K+\underline{I})(I^{*}-\underline{I})-4X\hat{W}=0 \quad (2.20)$$

Note that (2.5) is a reduced form function of R^{*}. Both $\hat{W}$ and X are not presented in this equation, so they do not make sense to R^{*}. Meanwhile, R^{*} also does not show in the rest of equations. Therefore we separate R^{*} from the method of Derivatives of Implicit Functions in order to simplify the issue.

$$2(K+I^{*})(\hat{W}+X)-(K+I^{*})(2K+I^{*}+\underline{I}-2l/R^{E})+(K+\underline{I})(I^{*}-\underline{I})-4X\hat{W}=0\,()$$

Finally, there is a 3x3 simultaneous equation.

$$R^{E}=\frac{pf(\underline{I})+(1-p)l}{K+\underline{I}} \quad (2.3)$$

$$f'(I^{*})=1/p \quad (2.4)$$

$$2(K+I^{*})(\hat{W}+X)-(K+I^{*})(2K+I^{*}+\underline{I}-2l/R^{E})+(K+\underline{I})(I^{*}-\underline{I})-4X\hat{W}=0 \quad (2.20)$$

From (2.3) we have

$$pf(\underline{I})+(1-p)l-R^{E}(K+\underline{I})=0$$

Let's set $F^{1}=pf(\underline{I})+(1-p)l-R^{E}(K+\underline{I})$

Then $\frac{\partial F^{1}}{\partial \underline{I}}=pf'(\underline{I})-R^{E}$; $\frac{\partial F^{1}}{\partial I^{*}}=0$; $\frac{\partial F^{1}}{\partial R^{E}}=-(K+\underline{I})$; $\frac{\partial F^{1}}{\partial \bar{W}}=0$; $\frac{\partial F^{1}}{\partial X}=0$;

From (2.4) we have

$$pf\,'(I^*) - 1 = 0$$

Let's set $F^2 = pf\,'(I^*) - 1$

$$\frac{\partial F^2}{\partial \underline{I}} = 0\,;\quad \frac{\partial F^2}{\partial I^*} = pf\,''(I^*)\,;\quad \frac{\partial F^2}{\partial R^E} = 0\,;\quad \frac{\partial F^2}{\partial \hat{W}} = 0\,;\quad \frac{\partial F^2}{\partial X} = 0\,;$$

From (2.20) we have

$$2(K + I^*)(\hat{W} + X) - (K + I^*)(2K + I^* + \underline{I} - 2l / R^E) + (K + \underline{I})(I^* - \underline{I}) - 4X\hat{W} = 0$$

Let's set

$$F^3 = 2(K + I^*)(\hat{W} + X) - (K + I^*)(2K + I^* + \underline{I} - 2l / R^E) + (K + \underline{I})(I^* - \underline{I}) - 4X\hat{W}$$

$$\frac{\partial F^3}{\partial \underline{I}} = -(K + I^*) - (K + 2\underline{I}) = -2(K + \underline{I})$$

$$\frac{\partial F^3}{\partial I^*} = 2(\hat{W} + X) - 2(K + I^* - l / R^E) = 2[\bar{W} - (K + I^* - l / R^E)]$$

$$\frac{\partial F^3}{\partial R^E} = -2l(K + I^*) / (R^E)^2$$

$$\frac{\partial F^3}{\partial \hat{W}} = 2(K + I^*) - 4X$$

$$\frac{\partial F^3}{\partial X} = 2(K + I^*) - 4\hat{W}$$

Here F^1 , F^2 and F^3 possess continuous derivatives.

$$|J| = \begin{vmatrix} \dfrac{\partial F^1}{\partial \underline{I}} & \dfrac{\partial F^1}{\partial I^*} & \dfrac{\partial F^1}{\partial R^E} \\ \dfrac{\partial F^2}{\partial \underline{I}} & \dfrac{\partial F^2}{\partial I^*} & \dfrac{\partial F^2}{\partial R^E} \\ \dfrac{\partial F^3}{\partial \underline{I}} & \dfrac{\partial F^3}{\partial I^*} & \dfrac{\partial F^3}{\partial R^E} \end{vmatrix} = \begin{vmatrix} pf\,'(\underline{I}) - R^E & 0 & -(K + \underline{I}) \\ 0 & pf\,''(I^*) & 0 \\ -2(K + \underline{I}) & 2[\bar{W} - (K + I^* - l / R^E)] & -2l(K + I^*) / (R^E)^2 \end{vmatrix}$$

$$= pf\,''(I^*)\begin{vmatrix} pf\,'(\underline{I}) - R^E & -(K+\underline{I}) \\ -2(K+\underline{I}) & -2l(K+I^*)/(R^E)^2 \end{vmatrix}$$

$$= -pf\,''(I^*)\left[\frac{2l(K+I^*)[pf\,'(\underline{I}) - R^E]}{(R^E)^2} + 2(K+\underline{I})^2\right]$$

Since $f\,''(I^*) < 0$ and $pf\,'(\underline{I}) - R^E > 0$, we have $|J| > 0$. Because the Jacobian $|J|$ is nonzero, we can apply the implicit function theorem to the simultaneous equation

Let's discuss comparative statics in terms of $\hat{W}$ firstly.

$$\begin{bmatrix} \dfrac{\partial F^1}{\partial \underline{I}} & \dfrac{\partial F^1}{\partial I^*} & \dfrac{\partial F^1}{\partial R^E} \\ \dfrac{\partial F^2}{\partial \underline{I}} & \dfrac{\partial F^2}{\partial I^*} & \dfrac{\partial F^2}{\partial R^E} \\ \dfrac{\partial F^3}{\partial \underline{I}} & \dfrac{\partial F^3}{\partial I^*} & \dfrac{\partial F^3}{\partial R^E} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \underline{I}}{\partial \hat{W}} \\ \dfrac{\partial I^*}{\partial \hat{W}} \\ \dfrac{\partial R^E}{\partial \hat{W}} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial F^1}{\partial \hat{W}} \\ -\dfrac{\partial F^2}{\partial \hat{W}} \\ -\dfrac{\partial F^3}{\partial \hat{W}} \end{bmatrix}$$

$$\begin{bmatrix} pf\,'(\underline{I}) - R^E & 0 & -(K+\underline{I}) \\ 0 & pf\,''(I^*) & 0 \\ -2(K+\underline{I}) & 2[\bar{W} - (K + I^* - l/R^E)] & -2l(K+I^*)/(R^E)^2 \end{bmatrix} \begin{bmatrix} \dfrac{\partial \underline{I}}{\partial \hat{W}} \\ \dfrac{\partial I^*}{\partial \hat{W}} \\ \dfrac{\partial R^E}{\partial \hat{W}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2(K+I^*) + 4X \end{bmatrix}$$

By Cramer's rule, this solution can be expressed as

$$\frac{\partial \underline{I}}{\partial \hat{W}} = \frac{|J_1|}{|J|} = \frac{\begin{vmatrix} 0 & 0 & -(K+\underline{I}) \\ 0 & pf\,''(I^*) & 0 \\ -2(K+I^*) + 4X & 2[\bar{W} - (K + I^* - l/R^E)] & -2l(K+I^*)/(R^E)^2 \end{vmatrix}}{|J|}$$

$$=-\frac{\begin{vmatrix} -(K+\underline{I}) & 0 & 0 \\ 0 & pf''(I^*) & 0 \\ -2l(K+I^*)/(R^E)^2 & 2[\bar{W}-(K+I^*-l/R^E)] & -2(K+I^*)+4X \end{vmatrix}}{|J|}$$

$$=-\frac{2pf''(I^*)(K+\underline{I})(K+I^*-2X)}{|J|}$$

Let's see what determines the sign of $K+I^*-2X$.

Here we are discussing the comparative statics in terms of $\hat{W}$. The condition implies that X is a constant. Since $\hat{W}=\underline{W}+X=\bar{W}-X$, it says both $\underline{W}$ and $\bar{W}$ must move when $\hat{W}$ changes. If both $\underline{W}$ and $\bar{W}$ have a very tiny increment, let's set ΔW. It is shown in Figure 2.12.

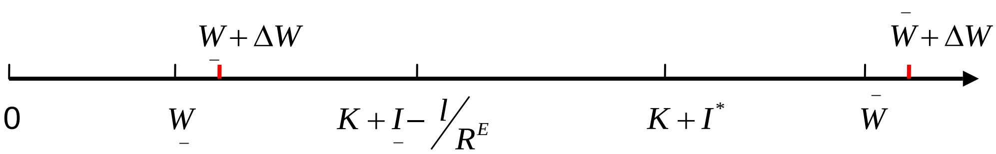

Figure 2.12

The total loss of supply funds, which is the area from $\underline{W}$ to $K+\underline{I}(R^E)-l/R^E$,is $\int_{\underline{W}}^{\underline{W}+\Delta W} W_i g(W)dW_i$. Meanwhile, the total gain of funds in the area from $K+I^*$ to $\bar{W}+\Delta W$ is $\int_{\bar{W}}^{\bar{W}+\Delta W} [W_i-(K+I^*)]g(W)dW_i$. If and only if the loss is equals to the gain, the minimal investment will not be changed. That is $\int_{\underline{W}}^{\underline{W}+\Delta W} W_i g(W)dW_i = \int_{\bar{W}}^{\bar{W}+\Delta W} [W_i-(K+I^*)]g(W)dW_i$. Solving the equation obtains

$K+I^{*}=\bar{W}-\underline{W}=2X$. Giving the conditions that $K+I^{*}=2X$ and X is a constant, this result indicates that no matter how the average wealth changes, the gain and the loss of funds supply is equal. Therefore the minimal investment value won't change in this case. That is $\frac{\partial \underline{I}}{\partial \hat{W}}=0$ when $K+I^{*}=2X$. We can also deduce that $\frac{\partial \underline{I}}{\partial \hat{W}}<0$ when $K+I^{*}<2X$ and $\frac{\partial \underline{I}}{\partial \hat{W}}>0$ when $K+I^{*}>2X$.

Similarly,

$$\frac{\partial I^{*}}{\partial \hat{W}}=\frac{|J_2|}{|J|}=\frac{\begin{vmatrix} pf'(\underline{I})-R^{E} & 0 & -(K+\underline{I}) \\ 0 & 0 & 0 \\ -2(K+\underline{I}) & -2(K+I^{*})+4X & -2l(K+I^{*})/(R^{E})^{2} \end{vmatrix}}{|J|}=0$$

and

$$\frac{\partial R^{E}}{\partial \hat{W}}=\frac{|J_3|}{|J|}=\frac{\begin{vmatrix} pf'(\underline{I})-R^{E} & 0 & 0 \\ 0 & pf''(I^{*}) & 0 \\ -2(K+\underline{I}) & 2[\bar{W}-(K+I^{*}-l/R^{E})] & -2(K+I^{*})+4X \end{vmatrix}}{|J|}$$

$$=-\frac{2pf''(I^{*})[pf'(\underline{I})-R^{E}](K+I^{*}-2X)}{|J|}$$

For the same reason $\frac{\partial R^{E}}{\partial \hat{W}}=0$ when $K+I^{*}=2X$, $\frac{\partial R^{E}}{\partial \hat{W}}>0$ when $K+I^{*}>2X$ and $\frac{\partial R^{E}}{\partial \hat{W}}<0$ when $K+I^{*}<2X$.

Secondly, Let's see how these endogenous variables changes in terms of X.

$$\begin{bmatrix} \frac{\partial F^1}{\partial \underline{I}} & \frac{\partial F^1}{\partial I^*} & \frac{\partial F^1}{\partial R^E} \\ \frac{\partial F^2}{\partial \underline{I}} & \frac{\partial F^2}{\partial I^*} & \frac{\partial F^2}{\partial R^E} \\ \frac{\partial F^3}{\partial \underline{I}} & \frac{\partial F^3}{\partial I^*} & \frac{\partial F^3}{\partial R^E} \end{bmatrix} \begin{bmatrix} \frac{\partial \underline{I}}{\partial X} \\ \frac{\partial I^*}{\partial X} \\ \frac{\partial R^E}{\partial X} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F^1}{\partial X} \\ -\frac{\partial F^2}{\partial X} \\ -\frac{\partial F^3}{\partial X} \end{bmatrix}$$

$$\begin{bmatrix} pf'(\underline{I}) - R^E & 0 & -(K+\underline{I}) \\ 0 & pf''(I^*) & 0 \\ -2(K+\underline{I}) & 2[\bar{W} - (K + I^* - l/R^E)] & -2l(K+I^*)/(R^E)^2 \end{bmatrix} \begin{bmatrix} \frac{\partial \underline{I}}{\partial X} \\ \frac{\partial I^*}{\partial X} \\ \frac{\partial R^E}{\partial X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2(K+I^*) + 4\hat{W} \end{bmatrix}$$

$$\frac{\partial \underline{I}}{\partial X} = \frac{|J_1|}{|J|} = \frac{\begin{vmatrix} 0 & 0 & -(K+\underline{I}) \\ 0 & pf''(I^*) & 0 \\ -2(K+I^*) + 4\hat{W} & 2[\bar{W} - (K + I^* - l/R^E)] & -2l(K+I^*)/(R^E)^2 \end{vmatrix}}{|J|}$$

$$= -\frac{\begin{vmatrix} -(K+\underline{I}) & 0 & 0 \\ 0 & pf''(I^*) & 0 \\ -2l(K+I^*)/(R^E)^2 & 2[\bar{W} - (K + I^* - l/R^E)] & -2(K+I^*) + 4\hat{W} \end{vmatrix}}{|J|}$$

$$= -\frac{2[(K+I^*) - 2\hat{W}]pf''(I^*)(K+\underline{I})}{|J|}$$

According to assumption 2, $\underline{W} < K + I^* < \bar{W}$. Because $2\hat{W} = \bar{W} + \underline{W}$, $(K+I^*) - 2\hat{W} = (K+I^*) - \bar{W} - \underline{W} < 0$. So $\frac{\partial \underline{I}}{\partial X} < 0$.

$$\frac{\partial I^*}{\partial X} = \frac{|J_2|}{|J|} = \frac{\begin{vmatrix} pf'(\underline{I}) - R^E & 0 & -(K+\underline{I}) \\ 0 & 0 & 0 \\ -2(K+\underline{I}) & -2(K+I^*)+4\hat{W} & -2l(K+I^*)/(R^E)^2 \end{vmatrix}}{|J|} = 0$$

$$\frac{\partial R^E}{\partial X} = \frac{|J_3|}{|J|} = \frac{\begin{vmatrix} pf'(\underline{I}) - R^E & 0 & 0 \\ 0 & pf''(I^*) & 0 \\ -2(K+\underline{I}) & 2[\bar{W}-(K+I^*-l/R^E)] & -2(K+I^*)+4\hat{W} \end{vmatrix}}{|J|}$$

$$= -\frac{2[(K+I^*)-2\hat{W}]pf''(I^*)[pf'(\underline{I})-R^E]}{|J|} < 0$$

In conclusion, there are three different cases.

a) If $K + I^* < 2X$, then

i. $\frac{\partial \underline{I}}{\partial \hat{W}} > 0$, $\frac{\partial \underline{I}}{\partial X} < 0$;

ii. $\frac{\partial I^*}{\partial \hat{W}} = 0$, $\frac{\partial I^*}{\partial X} = 0$;

iii. $\frac{\partial R^E}{\partial \hat{W}} < 0$, $\frac{\partial R^E}{\partial X} < 0$;

iv. $\frac{dR^*}{d\hat{W}} = 0$, $\frac{dR^*}{dX} = 0$

b) If $K + I^* = 2X$, then

i. $\dfrac{\partial \bar{I}}{\partial \hat{W}} = 0$, $\dfrac{\partial \bar{I}}{\partial X} < 0$;

ii. $\dfrac{\partial I^*}{\partial \hat{W}} = 0$, $\dfrac{\partial I^*}{\partial X} = 0$;

iii. $\dfrac{\partial R^E}{\partial \hat{W}} = 0$, $\dfrac{\partial R^E}{\partial X} < 0$;

iv. $\dfrac{dR^*}{d\hat{W}} = 0$, $\dfrac{dR^*}{dX} = 0$

c) If $K + I^* > 2X$, then

i. $\dfrac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\dfrac{\partial \bar{I}}{\partial X} < 0$;

ii. $\dfrac{\partial I^*}{\partial \hat{W}} = 0$, $\dfrac{\partial I^*}{\partial X} = 0$;

iii. $\dfrac{\partial R^E}{\partial \hat{W}} > 0$, $\dfrac{\partial R^E}{\partial X} < 0$;

iv. $\dfrac{dR^*}{d\hat{W}} = 0$, $\dfrac{dR^*}{dX} = 0$

APPENDIX B.1

The Threshold of Gini Coefficient

The results of the comparative statics in Chapter 2 is

a) If $X < \frac{K+I^*}{2}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} > 0$, $\frac{\partial R^E}{\partial \hat{W}} > 0$

b) If $X = \frac{K+I^*}{2}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} = 0$, $\frac{\partial R^E}{\partial \hat{W}} = 0$

c) If $X > \frac{K+I^*}{2}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\frac{\partial R^E}{\partial \hat{W}} < 0$

Since the income difference between the rich and the poor is unobservable, we have to express the result in terms of Gini coefficient and the average wealth.

Making the transition for the equation (3.7) gives

$$3X^2 - 6\hat{W}\cdot Gini \cdot X + 3\hat{W}^2 - 1 = 0$$

Then the solution of X is $X = \frac{6\hat{W}\cdot Gini \pm \sqrt{(-6\hat{W}\cdot Gini)^2 - 12(3\hat{W}^2 - 1)}}{6}$

So $X_1 = \hat{W}\cdot Gini + \sqrt{(\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})}$, $X_2 = \hat{W}\cdot Gini - \sqrt{(\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})}$

No matter what the solution of X is, the process below shows that there is a unique threshold of the income inequality.

Suppose $X_1 = \hat{W}\cdot Gini + \sqrt{(\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})}$, then

$$\frac{K+I^*}{2} = \hat{W}\cdot Gini + \sqrt{(\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})}$$

$$\frac{K+I^*}{2} - \hat{W}\cdot Gini = \sqrt{(\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})}$$

$$(\frac{K+I^*}{2} - \hat{W}\cdot Gini)^2 = (\hat{W}\cdot Gini)^2 - (\hat{W}^2 - \frac{1}{3})$$

$$(\frac{K+I^*}{2})^2-(K+I^*)\hat{W}\cdot Gini+(\hat{W}\cdot Gini)^2=(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})$$

$$(K+I^*)\hat{W}\cdot Gini=(\frac{K+I^*}{2})^2+(\hat{W}^2-\frac{1}{3})$$

$$Gini_C=\frac{(\frac{K+I^*}{2})^2+(\hat{W}^2-\frac{1}{3})}{(K+I^*)\hat{W}}=\frac{K+I^*}{4\hat{W}}+\frac{(\hat{W}^2-\frac{1}{3})}{(K+I^*)\hat{W}}$$

Suppose $X_2=\hat{W}\cdot Gini-\sqrt{(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})}$, then

$$\frac{K+I^*}{2}=\hat{W}\cdot Gini-\sqrt{(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})}$$

$$\hat{W}\cdot Gini-\frac{K+I^*}{2}=\sqrt{(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})}$$

$$(\hat{W}\cdot Gini-\frac{K+I^*}{2})^2=(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})$$

$$(\hat{W}\cdot Gini)^2-(K+I^*)\hat{W}\cdot Gini+(\frac{K+I^*}{2})^2=(\hat{W}\cdot Gini)^2-(\hat{W}^2-\frac{1}{3})$$

$$(K+I^*)\hat{W}\cdot Gini=(\frac{K+I^*}{2})^2+(\hat{W}^2-\frac{1}{3})$$

$$Gini_C=\frac{(\frac{K+I^*}{2})^2+(\hat{W}^2-\frac{1}{3})}{(K+I^*)\hat{W}}=\frac{K+I^*}{4\hat{W}}+\frac{(\hat{W}^2-\frac{1}{3})}{(K+I^*)\hat{W}}$$

Since the X has two solutions, the comparative statics may have two opposite results in terms of income inequality. For example, one of the results is

if $Gini<\frac{K+I^*}{4\hat{W}}+\frac{(\hat{W}^2-\frac{1}{3})}{(K+I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}}>0$, $\frac{\partial R^E}{\partial \hat{W}}>0$ and if $Gini > \frac{K+I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K+I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\frac{\partial R^E}{\partial \hat{W}} < 0$.

This solution indicts that the total borrowing, namely the size of capital markets, declines as the average income increases if income inequality is very low. On the contrary, if income inequality is higher than a critical value, financial development will benefit from an increase of average income. As a matter of fact, such situation would not happen in reality. Suppose there is a very poor economy with a small Gini coefficient. We assume that the Gini coefficient is lower than the threshold. It is very common at the initial stage of development in any economies. If some individuals benefit from economic development and become rich, the maximum wealth $\bar{W}$ increases. But the size of the capital market decreases on account of a small income inequality. It implies that the demand of credit decreases too. Hence there is no further development happen in the economy. Under the circumstances, no one can develop from a low income inequality situation. It is obviously ridiculous.

Consequently the unique result of the comparative static can be rewritten as:

a) If $Gini < \frac{K+I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K+I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} < 0$, $\frac{\partial R^E}{\partial \hat{W}} < 0$

b) If $Gini = \frac{K+I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K+I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} = 0$, $\frac{\partial R^E}{\partial \hat{W}} = 0$

c) If $Gini > \frac{K+I^*}{4\hat{W}} + \frac{(\hat{W}^2 - \frac{1}{3})}{(K+I^*)\hat{W}}$, then $\frac{\partial \bar{I}}{\partial \hat{W}} > 0$, $\frac{\partial R^E}{\partial \hat{W}} > 0$

APPENDIX B.2

The Size of Capital Markets--The Total Borrowing

Total Borrowing

$$= \int_{K+\underline{I}-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E)g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^*-W_i)g(W)dW_i$$

$$= \int_{K+\underline{I}-(l/R^E)}^{K+I^*-(l/R^E)} (l/R^E)g(W)dW_i + \int_{K+I^*-(l/R^E)}^{K+I^*} (K+I^*)g(W)dW_i - \int_{K+I^*-(l/R^E)}^{K+I^*} W_i g(W)dW_i$$

$$= \frac{l(I^*-\underline{I})}{R^E(\bar{W}-\underline{W})} + \frac{l(K+I^*)}{R^E(\bar{W}-\underline{W})} - \frac{1}{(\bar{W}-\underline{W})}[\frac{1}{2}(K+I^*)^2 - \frac{1}{2}(K+I^*-\frac{l}{R^E})^2]$$

$$= \frac{l(I^*-\underline{I})}{R^E(\bar{W}-\underline{W})} + \frac{l(K+I^*)}{R^E(\bar{W}-\underline{W})} - \frac{l(K+I^*)}{R^E(\bar{W}-\underline{W})} + \frac{l^2}{R^E(\bar{W}-\underline{W})2R^E}$$

$$= \frac{l(I^*-\underline{I}+\frac{l}{2R^E})}{R^E(\bar{W}-\underline{W})}$$

$$= \frac{l(I^*-\underline{I}+\frac{l}{2R^E})}{2R^E X}$$

APPENDIX B.3

Table 3.4: Variable Description and Sources

Variable	Definition	Source
Financial Development (Private Capital)	$\dfrac{(0.5)*[\frac{F(t)}{P_e(t)}+\frac{F(t-1)}{P_e(t-1)}]}{GDP(t)/P_a(t)}$, where F is credit by deposit money banks and other financial institutions to the private sector. P_e is the end of period CPI and P_a is the average CPI for the year.	Beck et al. (2009)
Gini Coefficient	The Gini Coefficient is the ratio of the area between the Lorenz curve, which is the proportion of population against the income share received, and the line of equality over the total area under the line of equality. It ranges from 0 to 1. A lower Gini coefficient presents a more equal environment while higher Gini coefficient indicates more inequalality.	UNU-WIDER (2008)
Real GDP per capita with lag	Natural logarithm of GDP per capita with lag	Alan Heston et al. (2009)
Inflation Rate	Log difference of Consumer Price Index	International Financial Statistics
Government Consumption	Government Expenditure as a share of GDP.	Bank (2009)
Trade Openness	Sum of real exports and imports as a share of real GDP	Bank (2009)

APPENDIX B.4

Table 3.5: Descriptive Statistics

	Private credit(%)	Bank asset(%)	Gini	Real GDP per capita	Gov's con	Trade openness	Inflation rate
Number of Observations	256	256	256	256	256	256	256
Mean	0.6418821	0.5949062	36.913	19854.66	16.77555	60.78473	34.40564
Std. Dev.	0.454396	0. 352172	9.504511	10597.39	4.481837	25.30989	242.6211
Minimum	0.069929	0. 1057385	23	2885.704	8.178534	15.76586	-1.166896
Maximum	1.825699	1.640301	59.862	44240.27	32.17084	129.1988	3079.81
Private credit	1						
Bank asset	0.7787	1					
Gini	-0.4051	-0.4893	1				
Real GDP per capita	0.7807	0.6415	-0.5396	1			
Gov's con	-0.2931	-0.2334	-0.3412	-0.4032	1		
Trade openness	-0.0799	0.1263	-0.0966	-0.0851	0.2401	1	
Inflation rate	-0.1416	-0.1403	0.0875	-0.1297	0.0896	-0.1559	1

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