# ESSAYS ON COMPETITION, INNOVATION AND GROWTH

By

**PIERCARLO ZANCHETTIN**

Thesis submitted to the School of Economics

University of Nottingham

for the degree of Doctor of Philosophy

May 2005

# Abstract

The thesis collects four essays in the fields of competition and innovation economics.

In chapter 1, we review the recent growth literature that analyses the effects of product market competition on growth. Contrary to the negative effect predicted by the early endogenous growth models, this literature emphasises that product market competition may foster innovation and growth. We argue that a common characteristic of this literature is a decrease in the intensity of technological competition relative to the early models, which seems to support the positive link between product market competition and growth.

In chapter 2, we study the effect of product market competition on growth in an endogenous growth model that maintains the intensity of R&D competition of the early models. We extend the early models by accounting for the possibility that many asymmetric firms (i.e. successive innovators) are simultaneously active in each industry. We show that an increase in competitive pressure exerts two positive effects on the incentive to innovate, which contrast the negative effect due to lower prices: the productive efficiency effect and the front loading of profits. We demonstrate circumstances in which the productive efficiency effect dominates the price effect, leading to a positive link between competition and growth.

In chapter 3, we reconsider the comparison between Bertrand and Cournot competition in a differentiated duopoly with asymmetric costs. Our main finding is that, with high degrees of cost asymmetry and/or low degrees of product differentiation, the efficient firm's and the industry profits are higher under Bertrand competition. This contrasts with Singh and Vives (1984) seminal result that, with substitute goods, equlibrium profits are always higher with Cournot competition.

In chapter 4, we study vertical integration and product innovation as interdependent strategic choices of vertically related firms. Our main finding is that, although product differentiation allows to soften product market competition and to avoid market foreclosure, the downstream market may prefer less product differentiation to prevent vertical integration. Therefore, less product innovation can be a possible social cost of a lenient antitrust policy.

# Acknowledgements

I am grateful to my supervisors, Marta Aloi and Martin Sefton, for their guidance and support over the last four years of my academic career. I am also particularly thankful to my co-authors, Vincenzo Denicolò and Arijit Mukherjee, for having broadened my interests and knowledge of economics. Special thanks go to Joanna Poyago Theotoky and Claudio Zoli for their useful comments and suggestions. I am in debt with the School of Economics of Nottingham University for having hosted me and financed my studies. Finally, my greatest thanks go to Simona, for her love, patience and support over these long four years. Any fault or imprecision in the thesis is my own responsibility.

# Author's declaration.

Chapter 2 has been published in the 2004 University of Leicester Discussion Paper Series in Economics. It has been presented at the 2002 EARIE annual conference (Madrid), and at the 2003 GEP Postgraduate Conference (Nottingham).

Chapter 3 has been presented at the 2004 EARIE annual conference (Berlin), and at the 2004 ATINER international conference on Industrial Organization and Law and Economics (Porto Carras, Greece). A previous version has been published in the 2003 University of Nottingham Discussion Paper Series in Economics.

Chapter 2 is joint work with Vincenzo Denicolò. Chapter 4 is joint work with Arijit Mukherjee. Each author has equally contributed with me in elaborating each of these chapters.

## TABLE OF CONTENTS

# Introduction

The thesis consists of a collection of essays in the fields of competition and innovation economics. Although each chapter presents an autonomous contribution to the topic, the content of the first three chapters arises from the same line of research, focusing on the effect of product market competition on innovation and growth.[1] The fourth chapter concentrates on vertical integration as an alternative source of competition that can affect product innovation in vertically related markets.

The first chapter revisits the recent growth literature which analyses the effects exerted by the intensity of competition in innovative good markets on economic growth. Contrary to the conclusions of early neo-schumpeterian models ("standard" models in the sequel), this literature emphasises that product market competition may foster growth by enhancing the pace of technical progress. These works aim at reconciling the endogenous growth theory with the available empirical evidence, suggesting a positive relationship between the intensity competition and the intensity of innovation at firm and industry levels.

The survey starts by considering how the issue is addressed in the standard models, whose main conclusion is that tougher competition is detrimental to growth since it erodes the innovator's prospective monopoly rents. In these models, the combination of intense technological competition (i.e. free-entry patent races with *leapfrogging* of the technological leader) and drastic innovations leads to a monopoly in the product market at each stage of the technical progress. As a consequence, an explicit analysis of different modes or degrees of competition in the product market is inhibited, and the argument for a negative relationship between competition and growth rests on the assumption that profit erosion is the dominant effect of a more intense competition in the product market.

Next, we group the subsequent works according to the main variation they introduce on the "standard setting": agency costs in the decision process of the innovative firms and

[1] Although chapter 3 deals with a classical problem of oligopoly theory (i.e. the comparison between price and quantity competition in a differentiated duopoly), the issue springs directly from our findings along the line of research above.

deterministic R&D technology (*agency cost models);* separation of research from development activities and flexibility issues in the implementation of new technologies (*research and development models*); tacit knowledge and internal accumulation of the new knowledge within the innovative firm (*step-by-step* and *quality-variety models*).

We concentrate on the image of the innovative process offered by each group of models relative to the highly competitive environment in the R&D activities provided by the standard models. Our main conclusion is that, in all groups of models reviewed, a reduction in the intensity of technological competition supports the positive relationship between product market competition and growth. Hence, an explicit analysis of the effect of product market competition on growth within the framework of intense technological competition which characterises the standard models is an issue left open by this literature.

The model we present in chapter 2 aims at filling this gap in the literature. We analyses the relationship between competition and growth without making any special assumption on the innovative process relative to the standard models. We employ a standard leapfrogging model, with profit maximising firms and immediate disclosure of the knowledge embodied in new technologies and protected by patents.

The basic idea of the model is to assume non-drastic innovations to create the scope for oligopolistic competition in the product market among successive innovators, and model the notion of higher competition by a switch from Cournot to Bertrand competition. We also consider a more general (reduced-form) model of product market competition which encompasses Bertrand and Cournot equilibria as special cases, and yields a continuous index of the intensity of competition.

Our model extends the early endogenous growth models by accounting for the possibility that in each period many asymmetric firms are simultaneously active in the market, what makes it possible an explicit analysis of the effects of the intensity of competition on growth. Furthermore, by assuming that innovation is sequential and cumulative, we also extend previous works in industrial organization which analyse the effects of product market competition in a single innovation framework.

Our main findings are that a rise in the intensity of competition fosters innovation and growth when innovations are large and/or competition is initially strong. Therefore, we argue that the conclusion drawn by the standard models crucially depends upon the assumption that the erosion of the innovator's rents is always the dominant effect of competition. Indeed, by reducing the equilibrium prices, a more intense competition exerts a downward pressure on industry profits, and hence on the innovator's prospective rents. However, when competition involves asymmetric firms (due to cumulative innovation and intellectual property rights) which remain active in the product market for more than one period (due to non-drastic innovations), two qualitatively new effects arise. First, more competition reduces the market share of the less productive firms, which reduces the total industry costs and counteracts the negative effect on industry profits exerted by lower prices. Second, in more competitive markets, a larger fraction of each innovator's rents accrues in the early stages of the innovative firm's life cycle. Both effects strengthen the incentive to innovate, and prevail on the opposite effect due the decrease in prices when the degree of asymmetry among firms is high (due to large innovations) and/or the intensity of competition in the market is initially high.

Although achieved in a macro-growth setting, the findings of chapter 2 highlight the following microeconomic result: in a homogeneous duopoly with linear-asymmetric cost functions, industry profits are higher under Bertrand than under Cournot competition when the efficiency gap between the two firms is sufficiently high. In contrast, a basic result in the oligopoly theory by Singh and Vives (1984) states that, in a differentiated duopoly with linear-asymmetric cost functions, both firms earn higher profits under Cournot than under Bertrand competition when products are substitutes.

In chapter 3 we re-consider the comparison between price and quantity competition in the Singh and Vives (1984) model. We find that the Singh and Vives's result is conditional on a parametric restriction along the dimension of cost asymmetry. Removing this restriction, our result of chapter 2 generalises to the standard model of a differentiated duopoly with linear demand and cost functions. More precisely, the equilibrium profit of the efficient firm and the industry profit are higher under Bertrand than under Cournot competition when the cost asymmetry is sufficiently high and/or the degree of product differentiation is sufficiently low.

The intuition of this result is based on the composition of the *price* and the *selection effects* associated with a switch from Cournot to Bertrand competition. The equilibrium prices are lower under Bertrand competition (*price effect*), and this works towards lower profits for both firms under this form of competition. However, also the market share of the inefficient firm is lower under Bertrand competition (*selection effect)*, and this works on the efficient firm's profit in the opposite direction relative to the *price effect*. Moreover, the *price effect* weakens while the *selection effect* gets stronger when either the degree of cost asymmetry increases (given any degree of product differentiation) or products are closer substitutes (for a sufficient degree of cost asymmetry). As a result, the efficient firm earns higher profits under price than under quantity competition when its efficiency advantage over the rival is sufficiently high and products are close substitutes. Finally, the *selection effect* entails more productive efficiency under price than under quantity competition, which explains the reversal of the industry profit ranking.

Whilst the previous chapters focus on product market competition, in the final chapter we concentrate on another source of competition that can affect product innovation in vertically related markets, i.e. the competitive threat of vertical integration. We consider product innovation in the downstream market as a strategic device of downstream firms facing a threat of vertical integration and market foreclosure by an upstream monopolist. We examine how product innovation affects the upstream firm's incentive to vertically integrate and foreclose the downstream market, and how the possibility of vertical integration impacts on the downstream firms' incentive to innovate.

The chapter is therefore related with two streams of literature: the literature on vertical integration, and the literature on product innovation. Our main innovation relative to the first literature is that we consider product innovation as a non-productive strategic decision of the downstream firms, showing its impact on the incentives for vertical integration and market foreclosure. Our main innovation relative to the second literature is that, besides product market competition, we account for another source of competition capable of affecting product innovation.

In our model, product innovation takes the form of horizontal product differentiation, which allows to soften competition in the final product market. Furthermore, we show that, by differentiating products, the downstream firms can eliminate market foreclosure in the eventuality of vertical integration. However, product innovation may foster the upstream firm's incentive to vertically integrate, which helps him to extract more rent from the downstream firms. In fact, we prove that both high and low degrees of product differentiation in the downstream market strengthen the upstream monopolist's incentive to vertical integrate. With strongly differentiated products, the gain from integration is higher because double marginalisation is avoided in a wider market. On the other hand, poorly differentiated products make the gain from integration higher since the integrated firm can better exploit its competitive advantage over the un-integrated downstream firm. As a consequence, the downstream firms have a strategic incentive to target intermediate degrees of product differentiation in order to prevent vertical integration.

We show circumstances in which the strategic incentive to prevent vertical integration prevails over the gain from softening product market competition, and refrains the downstream firms from investing in socially valuable innovations leading to higher degrees of product differentiation. We therefore point out that less product innovation in the final product market can be a possible social cost of a lenient antitrust policy which allows vertical integration.

# Chapter 1

# Competition, innovation, and growth: a critical survey of the literature

## 1. INTRODUCTION

Modern growth theory identifies the technical progress driven by private incentives to innovate as the main engine for growth in developed economies. The capacity of the economic and institutional systems to promote private innovative activities has come to the spotlight of both theoretical and empirical literature.

Besides the legal protection of the intellectual property rights, recent literature has focused on the relationship between the intensity of product market competition, on one hand, and the pace of the innovative process and growth, on the other hand. In particular, whilst earlier neo-schumpeterian models of endogenous growth ("standard models" in the sequel) tend to conclude that product market competition is detrimental to growth since it erodes innovators' prospective monopoly rents, recent theoretical works emphasise that more intense competition may stimulate innovation and growth.[1]

This chapter offers a critical survey of this literature. Starting from the "standard" neo-schumpeterian models (sections 2 and 3), we group the subsequent contributions according to the main variations they introduce on the setting of the standard models. We focus on the image of the innovative process offered by each class of models relative to the highly competitive environment of the R&D activities which characterises the standard models. This offers the reading key of the survey. In each class of models under study, a reduction in the intensity of technological competition supports the result that a more intense competition in the product market can be beneficial to growth.

[1]The attention of the theoretical literature towards a positive link between competition and growth has been stimulated by empirical evidence suggesting that firms tend to intensify their innovative activities in more competitive environments (see Blundell, Griffiths and van Reenen, 1995 and 1999; Nickell, 1996; Aghion, Blundell, Bloom and Griffiths, 2001).

# Chapter 1

# Competition, innovation, and growth: a critical survey of the literature

## 1. INTRODUCTION

Modern growth theory identifies the technical progress driven by private incentives to innovate as the main engine for growth in developed economies. The capacity of the economic and institutional systems to promote private innovative activities has come to the spotlight of both theoretical and empirical literature.

Besides the legal protection of the intellectual property rights, recent literature has focused on the relationship between the intensity of product market competition, on one hand, and the pace of the innovative process and growth, on the other hand. In particular, whilst earlier neo-schumpeterian models of endogenous growth ("standard models" in the sequel) tend to conclude that product market competition is detrimental to growth since it erodes innovators' prospective monopoly rents, recent theoretical works emphasise that more intense competition may stimulate innovation and growth.[1]

This chapter offers a critical survey of this literature. Starting from the "standard" neo-schumpeterian models (sections 2 and 3), we group the subsequent contributions according to the main variations they introduce on the setting of the standard models. We focus on the image of the innovative process offered by each class of models relative to the highly competitive environment of the R&D activities which characterises the standard models. This offers the reading key of the survey. In each class of models under study, a reduction in the intensity of technological competition supports the result that a more intense competition in the product market can be beneficial to growth.

[1] The attention of the theoretical literature towards a positive link between competition and growth has been stimulated by empirical evidence suggesting that firms tend to intensify their innovative activities in more competitive environments (see Blundell, Griffiths and van Reenen, 1995 and 1999; Nickell, 1996; Aghion, Blundell, Bloom and Griffiths, 2001).

Technological competition is formalized according to the tournament model of patent races with free entry. At each stage of the technical progress, a stochastic R&D technology links the instantaneous probability that the next innovation arrives (i.e. the hazard rate) to the aggregate investment in R&D, according to an increasing function. The aggregate investment results from the individual R&D efforts within a race to be first in innovating. Each patent race is a simultaneous game without information or cost advantages to any competitors (i.e. the current technological leader competes with the rivals on equal grounds). Since there is free-entry in the R&D game, the capital market is perfect, and the relevant information on the previous discoveries is disclosed by the patent system, everyone can attempt to innovate the most advanced technology.

Each successful innovator gains an efficiency advantage over previous innovators in the product market. The size of the efficiency advantage depends on the size of the innovative step. Most of the standard models assume that innovations are drastic, meaning that the size of each innovative step allows the successful innovator to monopolise the product market. To fix ideas, consider the innovative process as a progressive and constant increase in the productivity of an intermediate good used in the production of a final good. All generations of the intermediate good are perfect substitutes in the production of the final good, the only difference being the amount of identical productive services incorporated in different vintages. Then, innovations are drastic if the increase in productive services incorporated in any new vintage of intermediate good (relative to the previous vintage) is sufficiently high to allow any successful innovator to engage in monopoly pricing without the fear to be displaced by his most efficient competitor in the market (i.e. the previous innovator).

In the framework depicted above, the innovative process exhibits two main characteristics:

- at each stage of the technical progress, the current technological leader does not conduct research in equilibrium; only outsiders invest in R&D, and so the current

imitation, but no protection whatsoever from the occurrence of successive innovations. A more articulate analysis of the patent system within a growth model with quality ladders has been recentely provided by O'Donoghue and Zweimuller (2004).

leader is systematically replaced by an outsider (*leapfrogging pattern* of the technical progress)

- each successful innovator exploits his (drastic) productive advantage over the previous technological leader and monopolizes the product market (drastic innovations).

Both the disclosure of the knowledge embodied in the new discoveries (patent system) and the high intensity of technological competition (free-entry patent races) support the process of creative destruction of monopolies. This, in turn, fosters the pace of technical progress and the economic growth.

Indeed, the current technological leader (i.e. the current monopolist in the product market) has a weaker incentive to innovate than outsiders. By discovering a new technology, the leader would only gain an increase in his monopoly profit, while an outsider would gain the monopoly profit starting from the initial position of zero-profit (such a difference between the monopolist's and the outsiders' incentive to innovate is known as Arrow's *replacement effect*, see Arrow (1962)). Further, since the arrival of a new innovation is uncertain and the leader does not have first-mover advantages, the threat exerted by the outsiders' investment is not sufficiently reflected in the leader's incentive to innovate. As a result, the *replacement effect* settles the outcome of the R&D game: an outsider wins the patent race at each stage of the innovative process, gets the technological leadership and becomes the new monopolist of the product market. Moreover, the aggregate R&D investment is set in equilibrium by the outsiders' incentives to innovate, which are stronger than the incentive of the current leader. Therefore, the innovative process is fostered by the intense technological competition which characterises the standard models. On the contrary, since the monopoly profit is the outsiders' prize in each patent race, a lower level of monopoly profit would reduce the aggregate R&D investment and the rate of growth (*appropriability effect*).

Notice that the assumption of drastic innovations implies that only one firm (i.e. the technological leader) is active in the product market at each stage of the technical progress. Similarly, the standard models which allow for non-drastic innovations assume Bertrand competition in the product market. Then, since different vintages of the innovative good are perfect substitutes, a limit-pricing equilibrium arises: the technological leader (i.e. the latest innovator) drives its competitors out of the market and earns a unit-profit equal to the difference in quality between his vintage and the previous one (i.e. the size of the innovative step).[3] Again, the technological leader is the only firm active in the product market in each period.

As a consequence, in the standard models the schumpeterian trade-off between product market competition and growth is *guessed* from the negative impact a reduction of the leader's profit would exert on the incentive to innovate. More precisely, the intensity of competition in the product market is measured by the inverse of the elasticity of demand, which equals the mark-up the technological leader charges when innovations are drastic. A more elastic demand reduces the monopoly profit associated with a new innovation, and hence the outsiders' reward from innovating. Similarly, a more elastic demand reduces the technological leader's profit in the limit-pricing equilibrium arising with non-drastic innovations and price competition. In this case, the equilibrium price is unaffected (since it equals the size of the innovative step) but the equilibrium quantity decreases. In both cases, the effect of product market competition on growth, assessed through a comparative static exercise on the elasticity of demand, turns out to be negative.

This way of proceeding leaves room to several questions. First, the elasticity of demand is linked to structural (taste and/or preference) parameters. Therefore the effect on profits exerted by a change in the elasticity of demand incorporates structural changes in the economy which are difficult to associate with different degrees of competition in the product market. Second, a framework in which only one firm is active in the market can not accommodate an explicit analysis of the effects on growth exerted by different modes and/or degree of competition in the product market. Indeed, the argument of the standard models rests on

[3]In the case of process innovations, the assumptions of non-drastic innovation and price competition in the product market lead to a limit-pricing equilibrium in which the most efficient firm (i.e. the last innovator) prices at the marginal cost of the closest rival (i.e. the innovator at the previous stage of the innovative process), getting a unit-profit equal to the difference between the marginal cost of the closest rival and his marginal cost (i.e. the size of the innovative step).

the assumption that profit erosion, and the consequent *appropriability effect* which undermines the incentives to innovate, are the main effects of more competition in the product market. However, in a dynamic setting in which technical progress and intellectual property rights generate asymmetries between firms, other effects associated with the intensity of competition may become relevant, first of all the selection effect of competition against the less efficient firms. In order to capture these effects, an explicit formalisation of product market competition among successive innovators should be embedded in the framework of intense technological competition which characterises the standard model.[4]

## 3. THE MULTI-SECTOR VERSION OF THE STANDARD MODEL

In the multi-sector versions of the standard model (for instance, Caballero and Jaffe, 1993) innovations operate on a continuum of intermediate goods (sectors or industries in the sequel) employed to produce a final good. The measure of sectors is fixed (i.e. the variety of innovative goods is exogenous). Each innovation improves the quality of the specific variety of intermediate good it targets. However, the R&D activity entails inter-industry spillover: each innovation can be used directly only in the industry targeted by the innovator, but it allows successive innovators in other sectors to discover slightly better technologies. In each industry, the innovative process exhibits the leapfrogging pattern we have described above for the one-sector model. A successful innovator (outsider) replaces the previous technological leader and becomes the local monopolist in the industry targeted by his innovation.

Like in the one-sector version, also in the multi-sector version of the standard model an increase in the "intensity of product market competition" (that is, a higher elasticity of substitution among the different varieties of intermediate good in the production of the final good) diminishes the equilibrium investment in R&D and the rate of growth. In the multi-sector models, however, the negative link between competition and growth arises from the composition of three effects, one of them working in the opposite direction relative to

[4] An attempt along these lines is the model we present in the next chapter.

the overall effect:

- the (negative) *appropriability effect*, related (as in the one-sector version) to the reduction of the monopoly profit the successful innovator expects to gain in the specific industry targeted by its innovation (i.e. the direct effect of a higher elasticity of demand on the monopoly profit of the industry targeted by the innovator);
- a (positive) *productivity* (or *efficiency*) *effect*, due to the successful innovator's increased ability to exploit his efficiency advantage *inter-sectorially* (i.e. in the competition with the local monopolists of the other sectors) when the different varieties of intermediate goods are closer substitutes;[5]
- an "additional" (negative) *obsolescence effect* which operates via the same channel as the efficiency effect: if the varieties of intermediate goods are closer substitutes, the occurrence of an innovation in one industry exerts a negative impact on the profits of the local monopolist in another industry. As a consequence, the incentive to innovate in the second industry is weakened.[6]

In these models, the *productivity effect* works in the direction of a positive relation between product market competition and growth. However this effect is always dominated by the other two effects, the *appropriability* and the *obsolescence* effects, which work in the opposite direction.

[5]Using the degree of substitutability among different varieties of innovative goods as a proxy for the intensity of competition in the product market, the interpretation of the *productivity* effect is that the successful innovator in one sector can compete more freely with the local monopolists of other sectors. Since a new innovation increases the productivity of his variety relative to other varieties, the successful innovator gains from a freer competition. This, at its turn, increases the expected profit from innovating in that sector. (see Aghion and Howitt, 1998).

[6]"Additional" relative to the main *obsolescence effect* which characterises the neo-schumpeterian model of growth: the successive innovator interrupts the flow of profits to the current innovator taking his position as the monopolist in the product market.

## 4. AGENCY COSTS MODELS

This class of models (Aghion, Dewatripont and Rey, 1997 and 1999) embeds two fundamental changes in the structure of the multi-sectorial standard model.

First, in these models the R&D technology is deterministic at the innovative firms' level. Firms adopt innovations by paying a sunk cost instead of investing in a risky technology. This is upheld either by assuming that each firm employs a continuum of researchers engaged in uncorrelated risky projects, or simply by assuming that firms adopt new technologies already discovered. In any case, the highly competitive environment in the R&D activity that characterises the standard model is replaced by a process of adoption of new technologies which weakens the consequences of new discoveries on the market position of the firms active in the product market. Indeed, within the multi-sectorial framework discussed in the previous section, each firm is a local monopolist in the sector of his variety of the intermediate good. However, the arrival of an innovation in one sector of the intermediate good market does not cause the replacement of the current monopolist in this sector. Rather, the new technology is at the disposal of the current monopolist, which has only to decide the optimal adoption timing.[7]

Second, these models introduce agency costs in the decision process of the innovative firms. The separation between ownership and control shifts the decision to adopt the new technologies to the managers. Managers decide according to their own preferences, which depend positively on the private benefits associated with the control of the firm, and negatively on the private costs related to the adoption of new technologies (for instance, training costs or non-monetary costs from reorganizing the firm in order to implement the new technologies).

The presence of fixed operating costs, together with the gradual decline of profits as the adoption of new technologies is delayed (*obsolescence effect*), imply that the firm could go

[7]In the following we refer to the firms active in the product market as "the innovative firms". These firms are innovative in the sense that they adopt the new technologies, rather than in the sense of being involved in R&D competition.

bankrupt if the decision to adopt new technologies is postponed too much.[8] Then, if we assume that managers have lexicographic preferences, they will postpone the adoption of new technologies until the critical moment that precedes bankruptcy. In general, even with less extreme assumptions on managers' preferences, the managerial firm will postpone the adoption of new technologies relative to a profit-maximising firm. This has a negative effect on the aggregate growth performance of the economy since the rate of growth decreases with the average delay in adopting new technologies.

Turning to the effect of product market competition on growth, the measure of the intensity of competition is the same as in the multi-sectorial version of the standard model, that is the degree of substitutability among different varieties of the intermediate good in the production of the final good. However, in the models under study, an increase in the "parameter of competition" has a beneficial effect on growth. The reason is that a higher degree of substitutability among varieties strengthens the obsolescence effect. Therefore, for any given adoption-strategy of the new technologies, the profit flow of the managerial firm worsens. This anticipates the critical instant of bankruptcy, forcing the managers to speed-up the adoption of new technologies.

In other words, a more intense competition in the product market operates as a discipline device on the managers' slackness. The decrease in the free-cash flow available to the managers tightens the constraint conditioning the managers' behaviour (i.e the risk of bankruptcy and the consequent loss of the benefits of control), inducing them to choose a strategy closer to the profit-maximising one (that is, a faster adoption of the new technologies).

It is worth emphasising how the image of the technical progress offered by this class of models is distant from that of an innovative process driven by intense technological competition offered by the standard model. The (risky) activity of research, stimulated by the prospective of acquiring strategic advantages over the rivals, is replaced with an adoption process of new technologies forced by the threat of accumulating efficiency gaps

[8] The decline of profit as a firm delays the adoption of new technologies is due to the obsolescence effect discussed in the previous section: the firm will gradually lose competitiveness relative to the rivals that are adopting new technologies in other sectors.

leading to the failure of the firm. In other words, the positive effect on growth of a more intense competition in the product market is obtained by reducing drastically the intensity of technological competition.

## 5. MODELS WITH RESEARCH *AND* DEVELOPMENT

The main characteristic of these models (Aghion and Howitt, 1996 and 1998) consists in the parting of the research activity from the activity of development of new technologies: the first is oriented towards discovering new technological paradigms (Multi-Purpose-Technologies), while the second is concerned with the development and implementation of the new inventions.

Skilled workers can move between the research and the development sectors, as well as within the development sector across product lines generated by discoveries of different vintages. The degree of mobility of skilled workers, both within the development sector and between the two sectors, depends on the degree of specificity of the investment (in training and qualification) required to perform the development activity on a particular product line. The higher the degree of developers' mobility across product lines, the higher the speed with which the economy implements new technological paradigms. This strengthens the incentive to invest in research, which augments the pace of technological progress and the rate of growth.

The impact of product market competition on growth is assessed by using the degree of substitutability among the different product lines as the measure of the intensity of competition. A higher substitutability of products fosters both the investment in less specific activities (research in particular) and the incentive to reduce the degree of specificity in developers' training and qualification. As a consequence, skilled workers are more mobile among different product lines.

As in the agency costs models, also in the research and development models the image of the technical progress is distant from that of a process driven by intense technological competition aimed at gaining strategic advantages in the product market. Rather, these models focus on the adequate degree of flexibility of the economic systems required to implement the new technologies. Flexibility favours research and technical progress, and depends, in turn, on the institutional framework being able to promote private incentives towards a lower degree of specificity in the innovative activities and higher mobility of the skilled labour (i.e. lower switching costs).

## 6. MODELS WITH TACIT KNOWLEDGE ("INTERNAL" ACCUMULATION OF KNOWLEDGE)

In the standard neo-schumpeterian models reviewed in sections 2 and 3, the knowledge incorporated in each innovation is immediately disclosed. The new knowledge is perfectly codifiable, and the patent protection of intellectual property rights requires that the information incorporated in the new technologies is disclosed and verifiable. Therefore, the new knowledge can be utilised by any potential innovator racing for future discoveries on equal grounds with the current innovator.

By contrast, the main characteristic of the models we review in this section is that the knowledge incorporated in the new technologies remains to a great extent private information of the innovative firm. This characteristic of the innovative process, in turn, is motivated by the assumptions that innovations incorporate tacit knowledge which can be utilised only by the innovator and/or that the protection of the intellectual property rights is based on trade secrecy rather than on the patent system.

Clearly these assumptions lower the intensity of technological competition relative to the framework employed in the standard models (i.e. free-entry patent-races without any information or cost advantages for the current technological leader). However, this class of models maintains the image of the technical progress as a process driven by technological competition between innovative firms aimed at gaining competitive advantages in the product market. As a consequence, these models provide an explicit comparison among different models of competition in order to assess the effects of product market competition on growth.

In what follows we present two distinct groups of models based on the internal accumulation of knowledge: the *step-by-step* models (Aghion, Harris and Vickers, 1997; Aghion, Harris, Howitt and Vickers, 2001; Encaoua and Ulph, 2000); the *quality-variety* models (Smulders, Van de Klundert, 1995, Van de Klundert, Smulders, 1997; Peretto, 1996, 1999).

### 6.2. The step-by-step models

The step-by-step models are multi-sectors models with process innovations on the production cost of intermediate goods. There are two firms active in each sector (the current technological leader and the technological follower) competing in the market of their variety of intermediate good. The two firms are also engaged in technological competition to obtain process innovations which reduce the marginal cost of their variety. The technological leader can exploit the tacit knowledge incorporated in the more advanced technology. This enables him to target innovations which directly improve the leading technology. On the contrary, the technological follower must first engage in R&D in order to disclose the knowledge embodied in the more advanced technology before being able to innovate the leading technology. In each sector the innovative contest is restricted to the two firms active in the market, and evolves as a step-by-step run-up.

The product market of each variety is formalized as a duopoly with firms asymmetric in costs. The intensity of the product market competition is measured both via the comparison between different models of strategic interaction (i.e. Cournot competition versus Bertrand competition) and via the inverse of the elasticity of demand (which derives from the elasticity of substitution among different varieties of the intermediate good in the production of the final good)

The main results obtained by this class of models can be summarised as follows.

First, a more intense competition in the product market increases the incentive to innovate (i.e. the R&D investments) of both firms when they are "neck-and-neck" in their technological contest. This is denoted by the authors as the *escape from competition effect*, to emphasise that the incentive to gain a technological lead gets stronger when firms are symmetric (i.e. no one has a competitive advantage over the rival) and therefore they exert each other an intense competitive pressure in the product market. Then, a higher intensity of competition in the product market fosters the incentive to escape from the competitive pressure of the rival.

Second, a higher intensity of competition in the product market tends to reduce firms' incentive to innovate (especially for the technological follower) when they are distanced in the technological race. As we will see below in more details, this result is closely related to the *appropriability effect* we have discussed for the standard models.

Finally, an increase in the degree of product market competition tends to reduce the frequency of the sectors in the economy where firms are "neck-and-neck" with respect to the frequency of the sectors where firms are "distanced" in the technological race. This follows directly from the first two results, that is from the positive (resp. negative) effect that product market competition exerts on the firms' incentive to innovate starting from (and in order to exit) the "neck-and-neck" (resp. the "distanced") state. Such a *composition effect*, in turn, works in the direction of a lower rate of growth, since the frequency of the sectors where the incentives to innovate are stronger decreases.

The intuition for the first two results is based on the different structure of the incentives to innovate in the two alternative states of the technological race: the "neck-and-neck" and the "distanced" states. In the "neck-and-neck" state, the *profit effect* (i.e. the difference between a firm's expected profit from innovating and its current profit in the product market) has a weak impact on the incentives to innovate, while the *competitive threat* exerted by the rival (i.e. the difference between the profit of the winner and the profit of the loser in the technological race) exerts a stronger impact. [9] A higher intensity of competition in the product market strengthens the *competitive threat* component of the incentive to innovate. On the other hand, in the "distanced" state, the structure of the incentives to innovate is more affected by the *profit effect*, which is weakened by a higher degree of product market competition.

From the three results above it follows that the overall effect on growth of a more intense

[9] We are referring here to the classification of the incentives to innovate in a simultaneous R&D game introduced by Beath, Katsoulacos and Ulph, 1989.

competition in every sectors of the economy is ambiguous. [10] Indeed, the innovative process accelerates in the industries where firms are "neck-and-neck" but it slows down in the industries where firms are distanced. In the steady state, the aggregate effect depends on the equilibrium distribution of the sectors of the economy over the two states of technological competition. In turn, the equilibrium distribution must respect a stability condition in terms of entries into and exists from the two states. Finally, the entry- and exit- flows are affected by the intensity of the incentives to innovate in order to change the initial state, and therefore, by the initial degree of product market competition.

Therefore, the overall effect on growth of a general increase in the intensity of competition turns out to depend, critically, on the initial intensity of competition. If the initial intensity of competition is low, the distribution of the industries over the two technological states is concentrated on the "neck-and-neck" state. In this case, the *escape from competition effect* is widely diffused in the economy, while the *composition effect* is weak. Therefore a general increase in the intensity of competition exerts a positive effect on growth. On the contrary, if the initial intensity of competition is high, the economy is mostly populated by industries lying in the "distanced" state. In this case the *escape from competition effect* operates only in a few sectors, while the *composition effect* is strong. Hence a more intense competition has a negative effect on growth.

The *escape from competition effect* is the crucial result which allows the step-by-step models to obtain a positive impact on growth from a higher intensity of product market competition. The following example helps to clarify the way in which the *escape from competition effect* operates.

Suppose that the tacit knowledge embedded in the leading technology (that is, the trade

[10] A general increase of the degree of competition in all sectors may result, for instance, from a lenient antitrust policy or a less effective protection of the intellectual property rights. In formal terms, the models under study assess the effect of the intensity of competition on growth either by the (usual) comparative exercise on the elasticity of demand or by comparing the equilibrium rate of growth under alternative models of competition in the product market (i.e. Bertrand vs Cournot competition). Finally, a parameter capturing the intensity of imitation of the innovations is employed in order to assess the effect exerted on growth by the legal protection of the intellectual property rights.

secrecy on the more advanced technology) remains private information of the leader only for one innovative step. In other words, the gap between the technological leader and the technological follower equals one innovative step in the industries where firms are distanced. Moreover, suppose that innovations are drastic, and therefore the technological leader monopolises the product market. Let us compare now the structure of the incentives to innovate (from both the "neck-and-neck" and the "distanced" states) under Cournot and under Bertrand competition.

To begin with, the following table describes the product market equilibrium, both in the "neck-and-neck" and in the "distanced" states, under the two alternative forms of competition.

| | Bertrand | Cournot |
|---|---|---|
| "neck-and-neck" | $\pi_1 = \pi_2 = 0$ | $\pi_1 = \pi_2 = \pi_C > 0$ |
| "distanced" | $\pi_{TL} = \pi_M$ ; $\pi_{TF} = 0$ | $\pi_{TL} = \pi_M$; $\pi_{TF} = 0$ |

If firms are "neck-and-neck" (and, therefore, they are symmetric in costs) they earn zero profits under Bertrand competition (i.e. $\pi_1 = \pi_2 = 0$), while they earn the same positive profit under Cournot competition (i.e. $\pi_1 = \pi_2 = \pi_C > 0$). If firms are "distanced", the technological leader obtains the monopoly profits (i.e. $\pi_{TL} = \pi_M$) while the follower earns zero profits ($\pi_{TF} = 0$) irrespective of the form of competition.

Now, in the "neck-and-neck" state, the comparison between the incentives to innovate under the two forms of competition can be formalised as follows:

| | | | |
|---|---|---|---|
| Bertrand | $\Rightarrow$ | $\pi_M - 0$ | for both firms |
| Cournot | $\Rightarrow$ | $\pi_M - \pi_C$ | for both firms |

In other words, starting form the "neck-and-neck" state, the value of gaining the technological leadership equals the entire monopoly profit under Bertrand competition, while it equals the difference between the monopoly profit and the current (positive) profit under Cournot. Hence the incentive to innovate is stronger under Bertrand competition.

On the contrary, the follower's incentive to innovate from the "distanced" state is stronger under Cournot than under Bertrand competition. Indeed, the follower's prize from catching-up with the leader is nil under Bertrand, since he would earn zero profit in the symmetric post-innovation equilibrium (i.e. in the "neck-and-neck" state following his eventual innovation). On the other hand, by catching-up with the leader under Cournot competition, the follower can increases his profit from zero (in the initial "distanced" state) to the positive level $\pi_C$ (in the "neck-and-neck" state following his eventual innovation). This is summarised below:

$$\text{Bertrand} \quad \Rightarrow \quad TL = ? \quad TF = 0$$

$$\text{Cournot} \quad \Rightarrow \quad TL = ? \quad TF = \pi_C$$

In what regards the technological leader's incentive to innovate from the "distanced" state, we cannot reach defined conclusions unless we fully specify the R&D game between the two firms. Looking only at the "stand-alone" incentive, the technological leader has no reason to invest in R&D since he is already the monopolist in the market.[11] However, the leader's incentive to innovate increases with the degree of "internalisation" of the competitive threat exerted by the follower.

### 6.2. The quality-variety models

The other group of models which assume internal accumulation of the knowledge embodied in the new technologies endogenises both the quality and the variety (sectors) of the innovative goods. More precisely, in these models innovations intervene on the quality of goods (vertical innovations), whilst the variety is determined in equilibrium by the degree of competition in the product market. The innovative firms incur fixed R&D costs [12], which

[11] For simplicity, we are abstracting here from the leader's incentive to innovate in order to increase his monopoly profit by using a more efficient technology (i.e. the efficiency effect associated with a further innovation by the incumbent monopolist).

[12] More precisely, sunk-costs are incurred both to innovate vertically on the quality of a given variety and to introduce a new variety in the market. Hence firms face sunk-costs of entry at the innovative stage of the game, when they choose both whether to introduce a own variety in the market and the quality level targeted for that variety. Notice that the entry-cost is endogenous, depending on the innovative strategies

have to be covered with operating profit, and sell the innovative goods in an imperfect competitive market.

The number of firms (i.e. the varieties of the innovative good), the degree of concentration in the product market and the equilibrium level of investment in R&D are determined in a similar way as in the "two stage" model of Sutton (1991). In the first stage of the game firms decide their innovation strategy (strategic investment in R&D) and whether or not to enter the market. In the second stage firms compete in the product market according to alternative models of oligopolistic competition.

In this framework, a higher competition in the product market reduces profit margins (for any given degree of market concentration), which, in turn, diminishes the equilibrium number of firms (i.e. varieties of the innovative good) according to the free-entry condition. As a result, the market share of each firm active in the market augments, which induces a positive effect on the incentives to innovate vertically on the quality of the existing varieties. The reason is that the competitive advantage offered by each innovation can now be exploited on a larger market. In other words, the strategic investment in R&D promises a relative efficiency advantage with respect to competitors which serve a larger market share, and therefore, a higher profit perspective.

## 7. CONCLUSIONS

Two main conclusions arise from our review of the literature. First, the negative effect of product market competition on growth due to the *appropriability* effect is inferred in the standard models without an explicit analysis of different modes and/or degrees of competition in the product market. The combination of intense technological competition and drastic innovations (or price competition with non-drastic innovations) leads to only one firm being active in each industry at each stage of the technical progress. Then the argument for a negative effect of product market competition on growth is based on the assumption

followed by all firms (and therefore, on the product market equilibrium arising in the subsequent stage of the game).

that profit erosion is the dominant effect associated with a more intense competition, which undermines the incentives to innovate.

Second, in all subsequent works analysed, the reversal of the effect of product market competition on growth predicted by the "standard" models is obtained in a framework characterised by less intense technological competition. This allows to soften the effect that the arrival of new innovations exerts on the position occupied in the product market by the firms involved in the innovative process.

An open issue in this literature is that of a more complete analysis of the relationship between product market competition and growth within the framework of intense technological competition which characterises the standard models. An attempt along this line is provided in the next chapter.

# Chapter 2

# Competition and growth in a Neo-Schumpeterian Model

## 1. INTRODUCTION

It has often been claimed that competition is good for innovation and growth. Indeed, what empirical evidence is available suggests an increasing, or inverted U-shaped, relationship between competition and growth.[1] However, there is no straightforward theoretical explanation for such a positive link. Quite to the contrary, early models of endogenous growth tend to conclude that tougher competition erodes the innovator's prospective monopoly rents and is therefore detrimental to growth.

This chapter aims to reconcile the Schumpeterian view that the search for monopoly rents is the primary engine of growth and empirical evidence that competition is good for growth. We argue that the conclusion drawn by early endogenous-growth models crucially depends upon the simplifying assumption that at every point in time the technological leader is the only active firm in each industry. In more highly structured models, which allow for two or more firms to be simultaneously active in the same industry, two qualitatively new effects arise – the front loading of profits and the productive efficiency effect – that can generate a positive relationship between product market competition, innovation and growth.

Any definition of competition involves the idea that more intense competition reduces the equilibrium price, thus exerting downward pressure on the innovator's prospective rents (we call this effect the *price effect*). However, in more competitive markets, a larger fraction of these rents accrues in the early stages of the innovative firm's life cycle (this we call the *front loading of profits* following Segal and Whinston (2003)) and low-cost firms have a larger portion of the market, which reduces total industry costs (*productive efficiency effect*). We find circumstances in which the productive efficiency effect dominates the price effect, namely, when the size of innovations is large and/or competition is strong. In these

[1] See Nickell (1996), Blundell, Griffiths and van Reenen (1996) and Aghion et al. (2002).

circumstances, the front loading of profits and the fact that the productive efficiency effect dominates the price effect compound to make the equilibrium rate of growth increase with the intensity of competition.

As a modeling strategy, we depart from standard quality ladder models of endogenous growth only to the extent that is necessary to allow for several firms to be simultaneously active in each industry. We therefore stick to the standard assumption that innovative technological knowledge is proprietary; this implies that firms are asymmetric in that they have access to different technologies. In early quality ladder models, the fact that only the technological leader is active in the product market rests on the assumption that either innovations are drastic (Aghion and Howitt (1992)),[2] or else firms compete *a là* Bertrand (Grossman and Helpman (1991)). To allow for a richer market structure, we focus on the case of non drastic innovations, contrasting Bertrand with Cournot competition. With asymmetric firms, the number of active firms and their respective market shares will depend on the intensity of competition, Bertrand or Cournot, and the size of innovations. (In fact we use a more general reduced-form model which encompasses the Bertrand and Cournot equilibria as special cases and yields a continuous index of the intensity of competition.)

Our model possesses a steady state in which $m+1$ firms are simultaneously active, i.e., the latest innovator and $m$ past innovators, where $m$ is endogenously determined (with Bertrand competition, $m=0$). An innovator, that does not conduct any further research, will nonetheless remain active, and reap positive profits, for $m+1$ periods (a period is the random time interval between two innovations, as in Aghion and Howitt (1992)). As new innovations arrive, the original innovator's market share shrinks but he will exit the market only after $m+1$ successive innovations have occurred. Consequently, the value of an innovation, and hence the incentive to innovate, is a weighted average of all active firms' profits, where the weights reflect the expected length of time periods, discounting, and growth. In a stationary environment with no discounting each innovator would get total industry profits over time periods, irrespective of the mode of competition.

[2] An innovation is drastic if the innovator is unconstrained by outside competition and can therefore engage in monopoly pricing.

We show that a rise in competitive pressure makes profits accrue sooner to the innovator: for example, with Cournot competition each innovator collects its rents over various time periods, whereas with Bertrand competition all of the rents are obtained in the one period starting when the innovation is achieved. In a stationary environment with no discounting, such a front loading of profits would have no effect on the incentive to innovate. In our model, however, delayed profits increase over time periods as the economy grows and firms discount future rents. The transversality condition implies that discounting prevails over growth, and so the front loading of profits raises the incentive to innovate implying that competition tends to be positively associated with growth.

The intensity of competition affects the incentive to innovate also *via* its effect on total industry profits. We decompose the effect of product market competition on industry profits into a price effect and a productive efficiency effect. The price effect is the change in industry profits that would obtain if all active firms shared the same technology. This effect is negative, i.e. more intense competition would lead to lower industry profits if firms were symmetric. With asymmetric firms, however, a rise in the intensity of competition raises the market shares of low-cost firms, and lowers those of high-cost firms. For example, under Bertrand competition all of the output is produced by the most efficient firm – the latest innovator; whereas under Cournot competition high-cost firms produce a positive share of total output. Therefore, a rise in competitive pressure improves the productive efficiency of the industry which is good for industry profits.

We identify two circumstances in which the productive efficiency effect outweighs the price effect. First, when innovations are almost drastic, the equilibrium price is close to the monopoly price irrespective of the mode of competition. In this case, the price effect is second order. However, with Cournot competition the high-cost firm holds a positive market share (when innovations are almost drastic, only two firms are active in each period); the productive efficiency effect is therefore first order. Thus, with large innovations industry profits are greater under Bertrand competition than under Cournot competition. (In fact, with large innovations industry profits are monotonically increasing in the intensity of competition). Second, we show that in the vicinity of the Bertrand equilibrium the productive efficiency effect is remarkably large: indeed, a unit decrease in the equilibrium price lowers the industry average cost by as much as one! Therefore, independently of the size of innovations, when competition is strong a further increase in the intensity of competition must increase industry profits.

The rest of the chapter is organized as follows. In Section 2, we discuss the related literature. In Section 3, we analyze the value of an innovation when innovation is sequential but innovators are not immediately displaced by the occurrence of the next innovation. We show that the incentive to innovate depends both on industry profits, and the distribution of profits across active firms. Section 4 studies how the intensity of product market competition impacts on the incentive to innovate. In Section 5, the insights obtained in Sections 3 and 4 are embedded in a simple general equilibrium endogenous growth model. Finally, Section 6 offers some concluding remarks.

## 2. RELATED LITERATURE

Our chapter is related to two different literatures: the industrial organization literature that examines the effect of product market competition on the incentive to innovate, and the recent endogenous growth literature that tries to reconcile theory and evidence on the relationship between competition and growth.

### 2.1. The industrial organization literature.—

The debate on the effect of competition on the incentive to innovate goes back to Schumpeter (1943) and Arrow (1962). Schumpeter (1943) claims that there exists a positive correlation between innovation and market power. He argues that for a variety of reasons a monopoly may likely develop and employ a more advanced technology than a competitive industry. This claim has been countered by Arrow (1962), who argues that the incentive to innovate is higher in competitive industries, because a monopolist's post-innovation profits replace his pre-innovation profits, whereas this replacement effect vanishes under competition. Moving to the case of oligopoly, Delbono and Denicolò (1990) find that Bertrand duopolists have greater incentives to innovate than Cournot duopolists when the product is homogenous. However, Bester and Petrakis (1993) and Bonanno and Haworth (1998) show that this result can be reversed with horizontal and vertical product differentiation, respectively, and Symeonidis (2003) shows that the same is true when the products are both horizontally and vertically differentiated. Qiu (1997) develops a model in which the incentive to innovate is greater with Cournot competition even if the product is homogeneous. Boone (2000, 2001) generalizes these findings and shows that the relation between competition and incentives to innovate is generally non monotone. In short, the industrial organization literature on the effect of product market competition on the value of an innovation is largely inconclusive.[3] In part, these conflicting results are due to different assumptions on the nature of technical progress (tournament or non-tournament) and on who conducts the research (incumbents or outside firms). The remaining ambiguity rests on the fact that in more highly competitive industries the technological leader has a larger market share, and this market share effect may or may not outweigh the negative effect of more intense competition on the equilibrium price.

All of these papers focus on a single innovation framework and therefore identify the incentive to innovate with the (increase in the) profits of the technological leader. We depart from this literature by modeling an infinite sequence of innovations. In our framework the incentive to innovate cannot be equated to the leader's profits, but is a weighted average of all active firms' profits. As such, the positive effect of more intense competition on the leader's market share does not translate mechanically into higher incentives to innovate, but operates only *via* the productive efficiency effect and the front loading of profits. Our contribution is to show that these indirect effects may nevertheless be substantial.

Segal and Whinston (2003) independently study a model of successive innovations in which each innovator stays active for two periods. Our analysis has many elements in common with theirs, including the front loading of profits. However, their model differs from ours in many respects; for example, they use a rectangular demand function, a fixed timing of innovations, and an exogenously given number of firms ($m = 1$). Moreover, they

[3]Equally inconclusive is the related literature on the effects of product market competition on managerial incentives: see Raith (2003).

do not compare Bertrand and Cournot competition, but focus on various practices that may or may not be anti-competitive. Notwithstanding these differences, our conclusions and theirs complement and reinforce each other.

### 2.2. The growth literature.—

A small endogenous growth literature tries to reconcile theory and empirical evidence on the relationship between competition and growth. One strand of this literature introduces agency issues into the picture (Aghion, Dewatripont and Rey (1999)). In these models, non-profit maximizing managers delay the adoption of new technologies until profits fall below a threshold level. The effect of tougher competition is to reduce profits thereby speeding up the adoption process.

In the non-tournament models of van de Klundert and Smulders (1997) and Peretto (1999), tougher competition reduces the equilibrium number of varieties and increases the size of active firms, which raises their incentive to innovate. These papers posit a positive, deterministic relationship between the level of R&D investment and the size of the innovation. In a related contribution, d'Aspremont, Dos Santos Ferreira and Gerard-Varet (2002) consider the case in which R&D investment affects the probability of success rather than the size of innovations, but still many firms can innovate simultaneously. Thus, in each period there are some firms which have successfully innovated, and others that have access only to the prior art (which is in the public domain). They compare the Cournot and Bertrand equilibria, and also analyze an intermediate case in which all successful innovators co-operatively engage in limit pricing. They show that growth is fastest in this intermediate case, and conclude that the relationship between competition and growth is inverted U-shaped.

Aghion et al. (2001) develop a general equilibrium model of step-by-step technical progress in which two firms produce horizontally differentiated products, and show that more competition (as measured by an increase in the degree of product substitutability) may be beneficial to growth.[4] In step-by-step models, firms' incentive to innovate is great-

[4]In a simplified version of the model, Aghion, Harris and Vickers (1997) parametrize the intensity of

est when they are neck-and-neck (which can never occur in leapfrogging models). In such a state, the incentive to gain a technological lead is greater when competition is intense; however, with fierce competition the fraction of industries in which firms are neck-and-neck tends to be lower. The interaction of these effects can generate an increasing, or inverted U-shaped, relationship between competition and growth. Encaoua and Ulph (2000) argue that introducing into this model the possibility of leapfrogging strengthens the positive effect of competition on growth.

The main difference between these papers and ours is that we do not make any special assumption: we use the standard leapfrogging model with profit-maximizing firms and tournament technical progress. The novelty of our analysis lies in that we allow for several firms to be simultaneously active - which requires that innovations are non-drastic and competition is Cournot rather than Bertrand.

## 3. THE INCENTIVE TO INNOVATE WITH SEQUENTIAL INNOVATIONS

In this section we analyze the key determinants of the incentive to innovate in a model of repeated innovations. We extend previous work in industrial organization by assuming that innovation is sequential and cumulative, and earlier endogenous growth models by accounting for the possibility that in each period many firms are simultaneously active.

Throughout, the following assumptions will be maintained. Innovative activity happens at a rate determined by R&D efforts. In each period $k$, where $k-1$ is the number of past innovations, there is a patent race for innovation $k$. (Time is continuous but can be divided into periods, where a period is the random time interval between two innovations.) Patent races come in a sequence in the sense that only after one innovation is achieved can the race for the next one begin. The size of innovations is exogenous but the timing of innovations is a probabilistic function of the amount invested in R&D by research firms; specifically, the R&D effort determines the expected time of successful completion of the R&D project according to a Poisson discovery process with a hazard rate $z_k$. We assume that incumbents

competition also as a switch from Cournot to Bertrand competition.

do no research; research is conducted by outsiders, and so in each period the current leader is systematically replaced.[5] As discussed at greater length in Denicolò (2001), this pattern of leapfrogging is indeed an equilibrium of a simultaneous moves R&D game with free entry if the size of innovations is not too small.[6]

To fix ideas, suppose that there is perfect, infinitely-lived patent protection, so that nobody can imitate the innovation without infringing the patent.[7] Because innovative technological knowledge is proprietary, in period $k$ only the $(k-i)$th innovator, who holds a patent on the $(k-i)$th innovation, can practice it.[8] Under the assumption that all innovations are obtained by outsiders, nobody holds multiple patents. In period $k$, innovator $k-1$ is the technological leader, but we allow for $m$ past innovators to be active. Let $\pi_{i,k}$ be the flow of profit earned by innovator $k-1-i$ in period $k$. Thus, $\pi_{0,k}$ is the technological leader's profit; $\pi_{1,k}$ is the profit of the second most efficient firm (i.e., innovator $k-2$); and so on. Later $m$ and $z_k$ will be determined endogenously (for example, $m=0$ with Bertrand

[5]The possibility that incumbents invest in R&D so that technological leadership may persist over time periods will be discussed in the concluding section.

[6]This result is well known in the literature for the case of drastic innovations, where the Arrow replacement effect leads a monopolist incumbent not to invest in R&D at equilibrium (the aggregate R&D investment is set by the outsiders' zero-profits condition). With non-drastic innovations, incumbents have the additional incentive to cumulate multiple patents in order to improve their relative position in the market (eventually gaining an unconstrained monopoly). Although such an efficiency effect strengthens incumbents' incentive to invest, the Arrow replacement effect continues to prevail if the size of innovations is not too small (see Denicolò (2001) for the case of non-drastic innovation and Bertrand competition, Zanchettin (2001) for the case of non-drastic innovation and Cournot competition). Therefore, the assumption that incumbents do not invest is not restrictive for most of our results below, which apply when innovations are non-drastic but sufficiently large.

[7]In practice, there are various means of protecting innovative technological knowledge, including patents, copyrights, secrecy, lead time etc. Typically the protection an innovator enjoys declines over time, but for simplicity we abstract from this additional source of dynamics.

[8]We follow the vast majority of endogenous growth models in ruling out patent licensing. The standard justification for this assumption is that licensing agreements between successive innovators would have anti-competitive effects and thus would be prohibited by antitrust authorities. When licensing improves productive efficiency, however, such a justification loses some of its strength. After developing our results, we discuss licensing agreements more fully in the concluding section.

competition), but for the moment we take them as given.

To determine the expected value of innovation $k$, $E(V_k)$, one must take into account that the $k$th innovator's rents will not be terminated by the occurrence of the $(k+1)$th innovation: although competition from the $(k+1)$th innovator will reduce all past innovators' market shares and profits, only the least efficient among active firms will be driven out of the market when a new innovation occurs. Thus, $E(V_k)$ is determined by the following asset condition:

$$rE(V_k) = \pi_{0,k+1} - z_{k+1}\left[E(V_k) - E(V_k^1)\right]$$

where $r$ is the interest rate. This equation says that securities issued by the leader pay the flow profit $\pi_{0,k+1}$ in period $k+1$, less the expected capital loss $z_{k+1}\left[E(V_k) - E(V_k^1)\right]$ that will be incurred when the next innovation is achieved. Such a capital loss is the difference between the value of being leader and that of being the second most efficient firm in the market, i.e. $E(V_k) - E(V_k^1)$, where $E(V_k^h)$ is the value of innovation $k$ after $h$ periods, i.e. in period $k+h$. The value of being the second most efficient firm in the market, $E(V_k^1)$, is in turn determined by the asset condition

$$rE(V_k^1) = \pi_{1,k+2} - z_{k+2}\left[E(V_k^1) - E(V_k^2)\right],$$

and so on. Eventually, after $m+1$ innovations, the $k$th innovator will exit the market, so that $E(V_k^{m+1}) = 0$. Consequently, we have

$$rE(V_k^m) = \pi_{m,k+m+1} - z_{k+m+1}E(V_k^m).$$

These $m+1$ equations can be solved recursively yielding

$$\begin{aligned} E(V_k) &= \frac{\pi_{0,k+1}}{r+z_{k+1}} + \frac{z_{k+1}}{(r+z_{k+1})}\frac{\pi_{1,k+2}}{(r+z_{k+2})} + \ldots \\ &\quad + \left[\prod_{i=1}^{m}\frac{z_{k+i}}{(r+z_{k+i})}\right]\frac{\pi_{m,k+m+1}}{(r+z_{k+m+1})} \\ &= \sum_{i=0}^{m}\left[\frac{\pi_{i,k+i+1}}{(r+z_{k+i+1})}\prod_{h=1}^{i}\frac{z_{k+h}}{(r+z_{k+h})}\right] \end{aligned} \quad (1)$$

When $m=0$, this expression reduces to the standard formula

$$E(V_k) = \frac{\pi_{0,k+1}}{(r+z_{k+1})}$$

that is, the value of the $k$th innovation is the discounted value of the innovator's profits, where the interest rate is augmented by the factor $z_{k+1}$ that captures the expected duration of the innovator's leadership. More generally, equation (1) says that the value of the $k$th innovation is the expected present value of all future profits that the innovator will get in the $m+1$ periods for which he will be active in the product market. In each period, the discount factor is augmented to keep into account the probability that the current flow of profits is terminated by the occurrence of the next innovation. Moreover, because innovation is cumulative future profits are weighted by the factors $\prod_{h=1}^{i} \frac{z_{k+h}}{(r+z_{k+h})}$, which can be interpreted as the "discounting-adjusted probabilities" that future innovations are achieved: with a Poisson discovery process, each future innovation eventually occurs with probability one, but since there is discounting, a delayed success counts less than instant success. Thus, $\prod_{h=1}^{i} \frac{z_{k+h}}{(r+z_{k+h})}$ is the "discounting-adjusted probability" that innovation $k+i$ occurs and period $k+i+1$ profits start accruing to the $k$th innovator.

More intuition on the determinants of the incentive to innovate can be gained by focusing on the case of a stationary environment in which $z_k$ and $\pi_{i,k}$ are constant across periods. In the limiting case in which $z$ tends to zero, the value of the innovation will then depend only on the technological leader's profit, $\pi_0$. This limiting case effectively corresponds to a single innovation framework, like that envisaged in the early industrial organization literature. In the polar case in which $r$ tends to 0, the value of the innovation would depend only on the sum total of firms' profits, $\Pi = \sum_{i=0}^{m} \pi_i$. In general, both industry profits and the profit distribution across firms matter.

## 4. INTENSITY OF COMPETITION AND INCENTIVE TO INNOVATE

In this section we analyze the effect of an increase in the intensity of competition on the incentive to innovate. Building on the result of the previous section, we focus on how competition affects industry profits and their distribution across firms. We identify the front loading of profits, the price effect, and the productive efficiency effect associated with a change in competitive pressure. We also demonstrate circumstances in which the productive efficiency effect dominates the price effect. To underscore that our results are independent of the details of the particular growth model that we develop below, the analysis is cast in a partial equilibrium framework.

### 4.1 Preliminaries.—

Consider an industry comprising $s+1$ asymmetric firms, indexed by $i = 0, 1, ..., s$, producing a homogeneous product. Let firms' marginal cost be constant at $c_i$ per unit, and label firms so that $c_0 < c_1 < ... < c_s$. Thus, firm 0 is the technological leader (e.g. the latest innovator), firm 1 is the leader's most efficient competitor (e.g. the penultimate innovator) and so on. The number of firms that are active in equilibrium, $m+1$, is determined endogenously; firm $m$ is the least efficient amongst active firms. Let demand be given by $X(p)$, where $p$ is price, $X$ is output, and $X(.)$ is a strictly decreasing and twice differentiable function on $[0, \bar{p}]$ and is zero on $[\bar{p}, \infty)$. It follows that inverse demand, $p(X)$, is decreasing and twice differentiable on $[0, X(0)]$. For simplicity, we assume that the following regularity condition holds: $2p'(X) + p''(X)X < 0$ on $[0, X(0)]$. This assumption of decreasing marginal revenue simplifies the exposition (in particular, it implies that the function $\Pi(X) = [p(X) - \psi]X$ is strictly concave for any constant $\psi < \bar{p}$) but is not needed for most of our results. The individual firm's profit function is $\pi_i = [p(X) - c_i]x_i$, where $x_i$ is the individual firm's output. To keep the analysis interesting, assume that $c_1$ is lower than the monopoly price associated with $c_0$, $p^M(c_0) = \arg\max[p - c_0]X(p)$. If this assumption failed, firm 0 could engage in monopoly pricing without fear of being displaced by its competitors.

### 4.2. Bertrand and Cournot competition.—

Initially we parametrize the degree of competition by a switch from Cournot to Bertrand competition. With Bertrand competition, the outcome is a limit-pricing equilibrium in which price equals the marginal cost of the second most efficient firm and all of the output is produced by the low-cost firm: $p^B = c_1$, $m^B = 0$, and $x_0^B = X^B = X(c_1)$. In a Cournot equilibrium, the first-order conditions are[9]

$$p'(X^C)x_i^C + p^C = c_i \qquad i = 0, ..., m \tag{2}$$

where $X^C = \sum_{i=0}^{m} x_i^C$ and $m^C$ is the greatest integer such that $p^C \geq c_m$ holds. For future reference, note that

$$\frac{x_i^C}{x_j^C} = \frac{p^C - c_i}{p^C - c_j} \qquad i, j = 0, ..., m$$

that is, in the Cournot equilibrium, *the ratio of any two active firms' market shares equals the ratio of their respective price-cost margins.* This relationship trivially holds also in the Bertrand equilibrium. However, in the Cournot equilibrium high-cost firms hold positive market shares, which means that the Cournot equilibrium exhibits productive inefficiency. This productive inefficiency is important to explain why industry profits can be larger under Bertrand competition, even if the Bertrand equilibrium price is lower than the Cournot price.

It is indeed well known that with Cournot competition both the equilibrium price and the number of active firms are higher than under Bertrand competition: $p^C > p^B$ and $m^C \geq 1$. Therefore, a switch from Cournot to Bertrand competition is associated with an increase in the intensity of competition. To facilitate the comparison we now present a general solution that encompasses the Bertrand and Cournot equilibria as special cases. This will also allow us to obtain a continuous index of the intensity of competition.

### 4.3. A reduced-form model.—

The intensity of competition can be measured in many different ways, but any definition of competition involves the idea that more intense competition reduces the equilibrium price of a homogeneous product. Accordingly, we use a reduced form specification in which the intensity of competition is simply identified with the (inverse of the) equilibrium price.[10] Thus, let $p$ range from the Cournot equilibrium price $p^C$ to the Bertrand equilibrium price

[9] With constant marginal costs, the assumption $2p'(X) + p''(X)X < 0$ ensures that the second order conditions are satisfied and the Cournot equilibrium is unique.

[10] Previous work on competition and growth has often measured the intensity of competition by the inverse of the elasticity of demand, which determines the size of the innovator's mark-up: see Aghion and Howitt

$p^B = c_1$ as product market competition increases. Industry output is $X(p)$. To pin down the industry equilibrium, assume that the ratio of any two firms' market shares equals the ratio of their respective price-cost margins without making any more specific assumption on the nature of product market competition:

$$\frac{x_i}{x_j} = \frac{p - c_i}{p - c_j} \qquad i, j = 0, ..., m. \tag{3}$$

The number of active firms, $m(p)$ is determined as a function of $p$ as the largest integer such that $p > c_m$ (it is understood that $x_i = 0$ when $p < c_i$). The active firms' equilibrium outputs and profits are then uniquely determined by the adding up condition $\sum_{i=0}^{m(p)} x_i = X(p)$.

To show existence and uniqueness of the solution, note that for any given $p$, $m(p)$ is uniquely determined. Equations (3) provide $m(p)$ independent conditions. Together with the adding up condition, they comprise a system of $m(p)+1$ linearly independent equations in the $m(p) + 1$ unknowns $x_0, ..., x_m$, the solution of which exists and is unique. It is easy to show that individual outputs are

$$x_i = \frac{p - c_i}{p - \hat{c}} \frac{X(p)}{[m(p) + 1]} \tag{4}$$

where $\hat{c} = \sum_{i=0}^{m(p)} \frac{c_i}{[m(p) + 1]}$ is the unweighted average of the marginal costs of active firms.

Clearly, because (3) holds both at the Bertrand and Cournot equilibria, these equilibria are reproduced for $p = p^B$ and $p = p^C$, respectively. For intermediate values of the price, our solution can be interpreted as a reduced form of a more highly structured model where firms can collude partially (Cabral (1995)), or can choose both capacities and prices (Maggi (1996), Boccard and Wauthy (2000)), or compete dynamically in quantity under convex adjustment costs (Dockner (1992)). In fact, our solution coincides with the conjectural variations equilibrium under the assumption that the conjectural variations parameter is

(1998). Our reduced-form model allows us to obtain a continuous measure of the intensity of competition that disentangles the effects of a change in the degree of competition from those associated with changes in structural (taste and/or technology) parameters that ultimately determine the elasticity of demand.

the same for all firms.[11] Alternatively, and perhaps more prudently, the solution can be thought of as an analytical tool that helps compare the Bertrand and Cournot equilibria.

### 4.4. The productive efficiency effect.—

Consider now an increase in the intensity of competition, i.e. a fall in the equilibrium price. If the number of active firms and their respective markets shares stayed constant, the fall in the equilibrium price will unambiguously reduce industry profits $\Pi = \sum_{i=0}^{s} \pi_i$. This is the *price effect.* The reason why the price effect is negative is that industry profits are $\Pi = [p(X) - \bar{c}]X$, where $\bar{c} = \sum_{i=0}^{m} \frac{x_i}{X} c_i$ is the *industry average cost*, i.e. weighted average of firms' marginal costs. With constant market shares, $\bar{c}$ is constant; since $\Pi(X)$ is quasi-concave, any fall in price must then reduce industry profits if the price is lower than the monopoly price. However, the number of active firms and their market shares change with the equilibrium price. As a consequence, $\bar{c}$ changes with the intensity of competition, and the associated change in industry costs and profits is the *productive efficiency effect.*

Formally, the change in industry profits associated with a change in the intensity of competition is[12]

$$\frac{d\Pi}{dp} = \underbrace{X + (p - \bar{c})\frac{dX}{dp}}_{\text{price effect}} \quad \underbrace{-\frac{d\bar{c}}{dp}X}_{\text{productive efficiency effect}} \tag{5}$$

We now show that an increase in the intensity of competition improves the productive efficiency of the industry; a fall in the equilibrium price, that is to say, lowers the industry average cost.

[11] To see this, note that in a conjectural variations equilibrium, the first-order conditions (2) become

$$(1 + \phi_i)p'(X)x_i + p = c_i \qquad i = 0, ..., m$$

where $\phi_i$ is the conjectural variations parameter of firm $i$. Provided that $\phi_i$ is the same for all firms, it is immediate that (3) holds at equilibrium.

[12] Because $m(p)$ jumps up at certain critical points $c_1, c_2, ..., c_m$ as $p$ increases, variables like outputs, profits etc. are piecewise differentiable. Consequently, caution should be used in differentiating those variables with respect to $p$. Any such derivative calculated at $c_j$, where $c_j$ is the critical value of $p$ at which $m$ jumps up from $j-1$ to $j$, must be interpreted as the right derivative under the conventional assumption $x_j(c_j) = 0$.

**Lemma 1** *The productive efficiency effect is positive.*

*Proof.* Simple algebra (the details are in the Appendix) shows that

$$\frac{d\bar{c}}{dp}X = \sum_{i=0}^{m(p)} (c_i - \bar{c})\frac{dx_i}{dp} = \frac{X(p)\sigma_c^2}{[m(p)+1]\,(p-\hat{c})^2} > 0 \tag{6}$$

where $\sigma_c^2$ is the variance of active firms' marginal costs. ■

The intuition behind Lemma 1 is that a rise in competitive pressure raises the market shares of low-cost firms and lowers the market shares of high-cost firms. This reduces the total cost at which any given industry output is produced. An immediate corollary of Lemma 1 is that a switch from Cournot to Bertrand competition improves the productive efficiency of the industry. This is obvious, because with Bertrand competition all of the output is produced by the low-cost firm, whereas under Cournot competition high-cost firms have positive market shares.

Before proceeding, we pause here to show that an increase in price is positively associated with an (inverse) index of the intensity of competition that is commonly used in empirical work, namely, the industry average price-cost margin $(p - \overline{c})$.[13]

**Lemma 2** *A rise in price $p$ (weakly) raises the industry average price-cost margin $(p - \overline{c})$.*

*Proof.* See the Appendix. ■

With this preliminary result in place, we are now ready to state the main results of this section.

[13] To measure the intensity of competition, Nickell (1996) and others also use indices of market concentration. In our model, however, an increase in price is negatively associated with market concentration. More precisely, let $L_X(\frac{h}{s+1}) = \sum_{i=s-h+1}^{s} \frac{x_i}{X}$ be the Lorenz curve of the output distribution, showing the proportion of industry output produced by any given percentage of the population of firms, starting from the smallest firm. Proceeding as in the proof of Proposition 3 below, it can be shown that a rise in price shifts the Lorenz curve $L_X$ up. This implies that as the price increase, market concentration falls according to any of the most commonly used measures of market concentration, like $C_n$, the sum of the market shares of the largest $n$ firms, or the Herfindhal index.

**4.5. Competition and industry profits.—**

We start focusing on the effect of product market competition on industry profits. In particular, we look for circumstances in which the productive efficiency effect dominates the price effect so that more intense competition raises industry profits.

As is clear from equation (6), if firms were symmetric ($\sigma_c^2 = 0$) the productive efficiency effect would vanish. However, with asymmetric firms, $\sigma_c^2 > 0$, the productive efficiency effect is first order. When the price effect is second order, it must therefore be dominated by the productive efficiency effect. But the price effect will, indeed, be second order when the price is close to the monopoly price. This observation leads us to the following result.

**Proposition 1** *When the marginal cost of the second most efficient firm $c_1$ is close to the monopoly price $p^M(c_0)$, industry profits are greater under Bertrand competition than under Cournot competition.*

*Proof.* The proof is in the Appendix. Here we sketch the proof of a more general claim, i.e. that when $c_1$ is close to $p^M(c_0)$, industry profits are monotonically increasing in the intensity of competition. To prove this claim, note that when $c_1$ is just below $p^M(c_0)$, $p$ must be close to $p^M(c_0)$;[14] moreover, $x_1$ must be close to zero and so $\bar{c}$ must be close to $c_0$. Using (6), equation (5) therefore reduces to

$$\begin{aligned}\frac{d\Pi}{dp} &= \underbrace{X[p^M(c_0)] + [p^M(c_0) - c_0]\frac{dX}{dp}}_{=0} - X[p^M(c_0)]\frac{\sigma_c^2}{2[p^M(c_0) - \bar{c}]^2} \\ &= -X[p^M(c_0)]\frac{\sigma_c^2}{2[p^M(c_0) - \bar{c}]^2} < 0\end{aligned}$$

Thus, industry profits monotonically decrease with price.■

The intuition is that at $c_1 = p^M(c_0)$ both Bertrand and Cournot competition yield the monopoly solution. Starting from $c_1 = p^M(c_0)$, consider now the effect of decreasing $c_1$. With Bertrand competition, the presence of firm 1 now constrains the low-cost firm (i.e., firm 0) that must price at $p = c_1$, but when $c_1$ is close to the monopoly price the effect

[14] Remember that $p$ ranges from $p^B = c_1$ to $p^C$, which cannot exceed $p^M(c_0)$.

of competition on the low-cost firm's profit is second order – the profit function is flat at $p = p^M(c_0)$. With Cournot competition a fall in $c_1$ reduces the equilibrium price less than under Bertrand competition, but now it also increases the high-cost firm's market share. Since $c_1 > c_0$, with Cournot competition the negative effect on industry profits of a fall in $c_1$ is first order, whence the result follows.

Proposition 1 might suggest that the productive efficiency effect is small and can prevail over the price effect only if the latter is negligible. Quite to the contrary, the productive efficiency effect can be surprisingly large: a unit increase in the equilibrium price can raise the average industry cost by as much as one! (Lemma 2 ensures that $\overline{c}$ cannot increase by more than one). This is indeed what happens in the vicinity of the Bertrand equilibrium – the equilibrium which most quality ladder models of endogenous growth with non-drastic innovations focus on. Consequently, starting at Bertrand competition, a small decrease in the intensity of competition will unambiguously lead to a fall in industry profits.

**Proposition 2** *Starting at the Bertrand equilibrium price, a small increase in price lowers industry profits.*

*Proof.* From (5) and (6) we get

$$\frac{d\Pi}{dp} = X + (p - \bar{c})\frac{dX}{dp} - X\frac{\sigma_c^2}{2(p - \hat{c})^2}$$

When $p$ is slightly increased starting at $p = p^B = c_1$, only two firms, 0 and 1, will be active. Consequently, $\sigma_c^2 = (c_1 - \hat{c})^2 + (c_0 - \hat{c})^2 = 2(c_1 - \hat{c})^2 = 2(p - \hat{c})^2$, whence we get

$$\frac{d\Pi}{dp} \mid_{p=p^B} = (p - \bar{c})\frac{dX}{dp} < 0. \blacksquare$$

Proposition 2 effectively shows that starting at the Bertrand equilibrium price, a small increase in price leaves the industry average price-cost margin $(p - \bar{c})$ unchanged. The intuitive reason is that when $p = c_1$, a unit increase in price raises the market share of firm 1 (the high-cost firm) from zero to $\frac{1}{c_1 - c_0}$, and this causes a unit increase in $\bar{c}$. Because the industry average price-cost margin $(p - \bar{c})$ is unchanged, a rise in price must necessarily lower industry profit.

Figures 1 and 2 illustrates Propositions 1 and 2, respectively, in the linear demand case $p = 1 - X$ with $c_0 = 0$. Figure 1 plots industry profits under Bertrand and Cournot competition as $c_1$ ranges from 0 (the symmetric case) to 0.5 (the monopoly price). Industry profits are greater with Bertrand competition for $c_1 > 0.28$. Figure 2 displays the regions in which industry profits increase or decrease with the intensity of competition as $c_1$ ranges from 0 to $\frac{1}{2}$ and $p$ ranges from $p^B = c_1$ to $p^C = \frac{1+c_1}{3}$. Although our qualitative results are local, Figures 1 and 2 show that the productive efficiency effect prevails over the price effect in a sizeable region of parameter values.

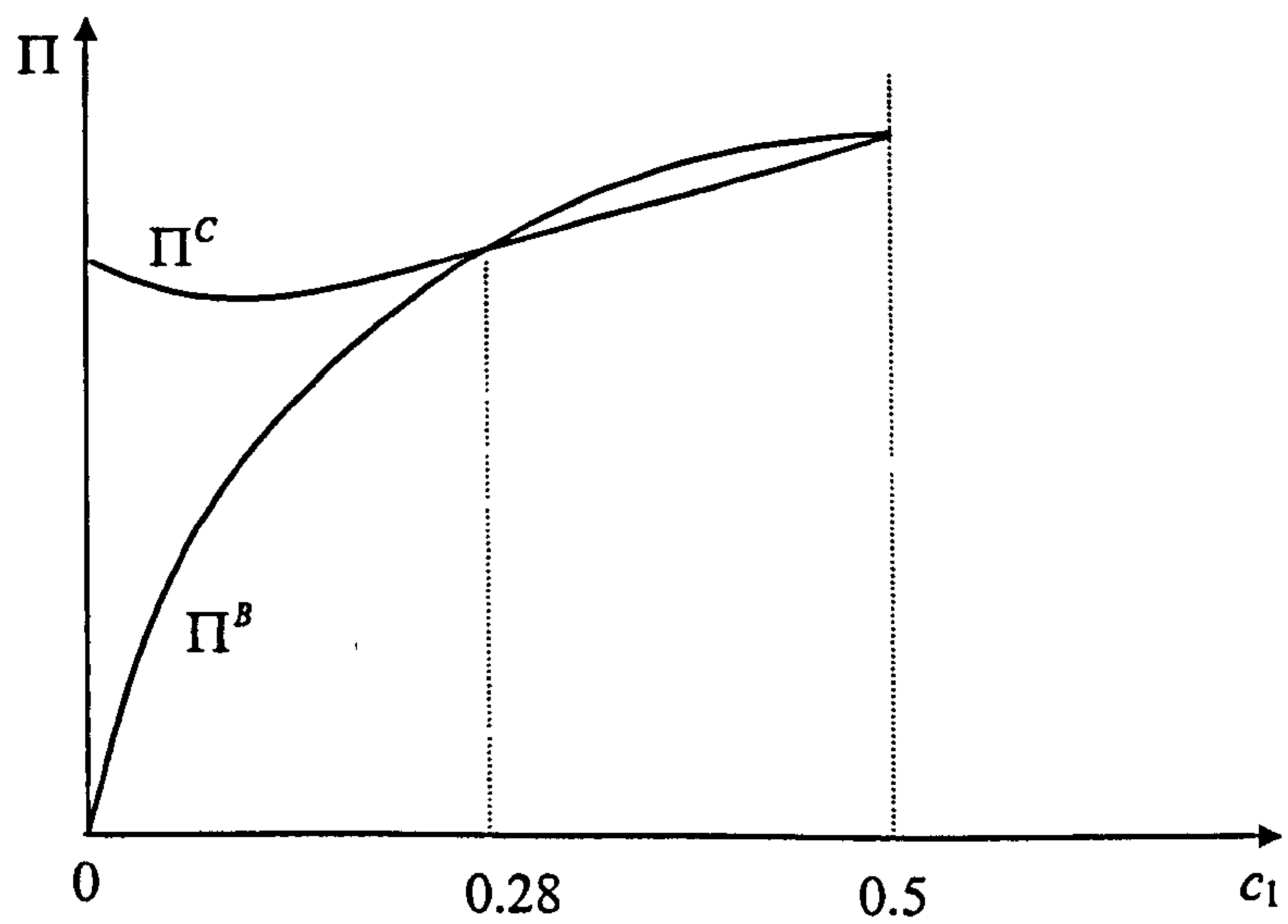


Figure 1. Industry profits under Bertrand and Cournot competititon as a function of $c_1$ when $p = 1 - X$ and $c_0 = 0$.

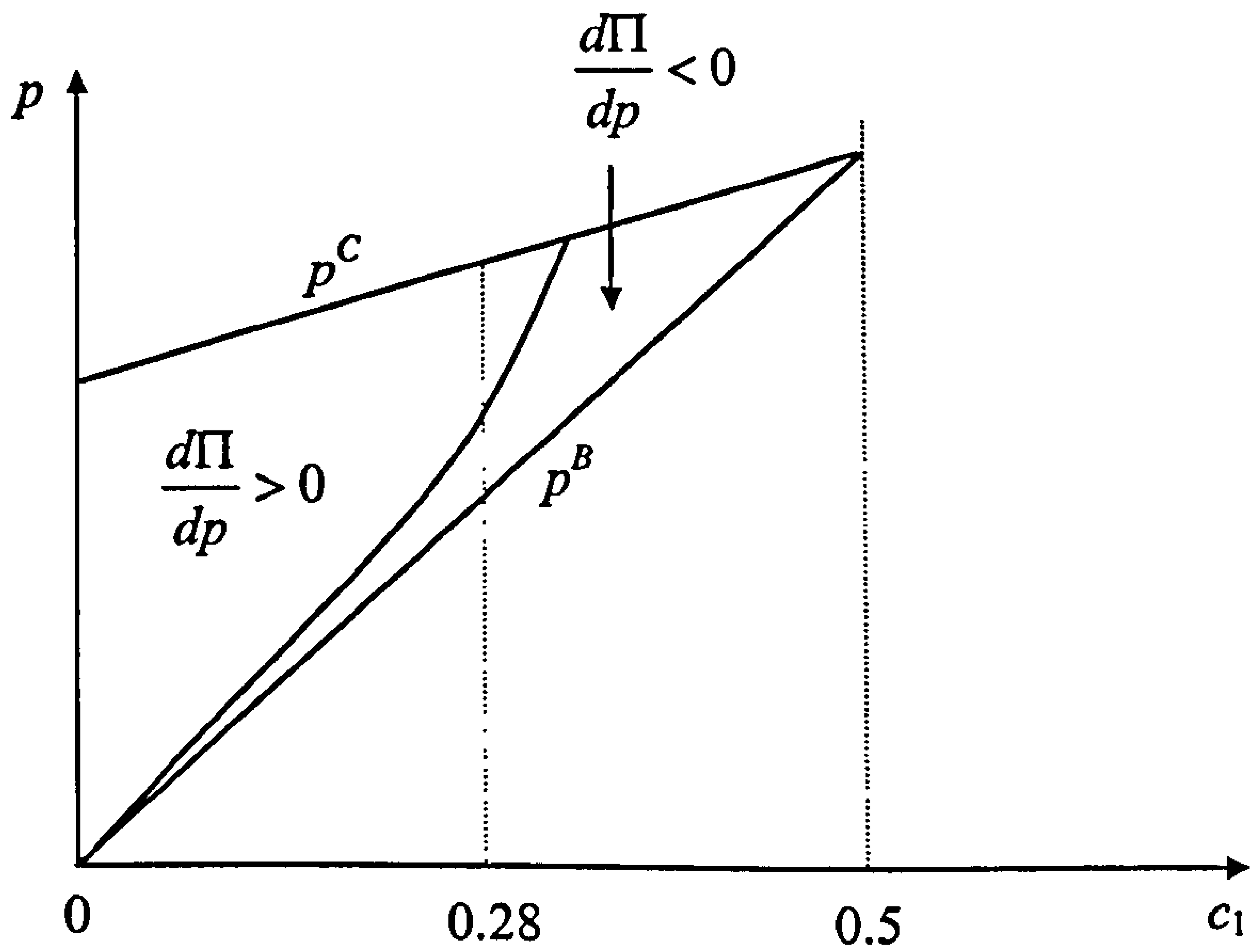


Figure 2. The effect of an increase in the equilibrium price on industry profits as a function of $c_1$and $p$ when $X = 1 - p$ and $c_0 = 0$.

#### 4.6. Competition and the distribution of profits.—

Product market competition affects not only the sum total of all firms' profits, but also the other key determinant of the incentive to innovate, namely, the distribution of profits across active firms. Accordingly, we now focus on how a rise in the intensity of competition affects the profit distribution, for any given level of industry profits.

Let the profit distribution be the $s+1$-dimensional vector $\mathbf{\Pi} = (\pi_0, \pi_1, ..., \pi_s)$. Because $c_0 < c_1 < ... < c_s$, we have $\pi_0 \geq \pi_1 \geq ... \geq \pi_s$, with strict inequalities whenever profits are strictly positive. The Lorenz curve of the profit distribution is $L_\Pi(\frac{h}{s+1}) = \sum_{i=s-h+1}^{s} \frac{\pi_i}{\Pi}$, where $\Pi$ is industry profits. It shows the proportion of industry profits earned by any given percentage of the population of firms, starting from the least profitable firm.

Our next result is that the profit distribution becomes more unequal according to the Lorenz dominance criterion as competition becomes more intense. That is to say, the Lorenz curve of the profit distribution shifts down as the intensity of competition increases. Lorenz dominance implies that profit inequality would increase as the intensity of competition increases according to a wide set of inequality measures.

**Proposition 3** *If there are at least two active firms, an increase in the intensity of competition makes the profit distribution more unequal according to the Lorenz dominance criterion.*

*Proof.* See the Appendix. ■

Proposition 3 follows from the simple fact that low-cost firms gain, and high-cost firms lose in relative terms when the market becomes more competitive. The reason is twofold: first, the market shares of low-cost firms tend to increase with the intensity of competition, and second, when the equilibrium price falls, the percentage decrease in the price-cost margin is larger for high-cost firms.

In the dynamic model of successive innovations to be developed presently, each innovator will be active, and reap positive profits, for $m + 1$ periods: in the first period after his innovation is achieved, he is the technological leader, in the second period he is the second most efficient firm, in the third period he is the third most efficient amongst active firms, and so on. Over time periods, the innovator reaps total industry profits irrespective of the intensity of competition. However, the Lorenz dominance result means that as the intensity of competition increases the innovator will get a larger proportion of his prospective rents in the first $i$ periods for which he is active, for all $i = 1, ...m$ (over $m + 1$ periods he always get 100 per cent of industry profits). This is the front loading of profits associated with more intense competition. Proposition 4 below shows that the front loading of profits tends to increase the incentive to innovate, for any given level of industry profits.

## 5. A GROWTH MODEL

We now embed the insights from the previous sections in a simple growth model. To eschew distracting assumptions, we use a one-sector version of the text-book model of Barro and Sala-i-Martin (1995, ch. 7), but the main results are more general and can be reproduced in many other models with quality improvements.[15]

**5.1. Preferences and technology.—**

The economy is populated by identical individuals whose mass is normalized to 1. Each individual has linear intertemporal preferences:

$$u(c) = \int_0^{\infty} c(t)e^{-rt}dt$$

so that the rate of time preference $r$ coincides with the equilibrium rate of interest. Each individual inelastically offers one unit of labor.

The final good $y$ is produced in a perfectly competitive market using labor (which is in fixed supply) and an intermediate good the quality of which increases over time because of technical progress. We normalize at 1 the quality of the intermediate good at time 0, and we denote by $q > 1$ the size of each innovation. In period $k$, where $k-1$ is the number of past innovations, the final good can be produced according to the following constant-returns-to-scale production function:

$$y_k = \widehat{X}_k^{\alpha} \ , 0 < \alpha < 1, \tag{7}$$

where labor input is set equal to one, $(1-\alpha)$ is the share of labor's income, and $\widehat{X}_k = \sum_{i=0}^{k} q^{i-1}x_i$ is the quality-adjusted index of a composite good which combines all past generations of intermediate goods. It is convenient to rewrite $\widehat{X}_k$ as $\widehat{X}_k = q^k X_k$, where $X_k = \sum_{i=0}^{k} q^{i-k-1}x_i$ measures the input of the composite intermediate good in efficiency units relative to the last vintage.

From the production function (7) one obtains the demand for the intermediate good (measured in efficiency units)

$$X_k = \alpha^{\frac{1}{1-\alpha}} p_k^{-\frac{1}{1-\alpha}} q^{\frac{\alpha}{1-\alpha}k} \tag{8}$$

where $p_k$ is its price.

[15] In particular, the growth model we develop exhibits scale effects, but our results would continue to hold in a model with no scale effects, provided that greater incentives to innovate lead to faster growth (see for example Howitt (1999)).

The final good may be consumed, used to produce intermediate goods, or used in research. Independently of its quality, the intermediate good is produced using the final good with a constant marginal rate of transformation that is normalized to 1.

**5.2. Technical progress.—**

In each period there is a patent race. Incumbents do no research, and there is free entry by risk-neutral outsiders. In period $k$, each firm $\ell$ participating in the race decides its R&D effort $n_{\ell k}$ to obtain the $k$th innovation. The R&D effort determines the expected time of successful completion of the R&D project according to a Poisson discovery process with a hazard rate equal to $\lambda_k n_{\ell k}$, with $\lambda_k > 0$. The projects of different firms are independent, so that the aggregate instantaneous probability of success is simply the sum of the individual probabilities. Let $n_k = \sum_\ell n_{\ell k}$ denote aggregate R&D investment in period $k$. Then, the innovation occurs according to a Poisson process with hazard rate $z_k = \lambda_k n_k$.[16]

If innovations were drastic, the technological leader would be unconstrained by outside competition and could engage in monopoly pricing, and so the model's equilibrium would be independent of the mode of competition in the product market. We therefore assume that innovations are non-drastic, which in the current setting means that

$$q \leq \frac{1}{\alpha}.$$

**5.3. Steady state.—**

In a steady-growth equilibrium the price of the (latest vintage of the) intermediate good, in terms of the consumption good, will be constant. This implies that $X_k$ will grow at rate $q^{\frac{\alpha}{1-\alpha}}$, and from (7) it then follows immediately that $y_k$ will also grow at rate $q^{\frac{\alpha}{1-\alpha}}$. This is the growth factor between periods, and we denote it by $g \equiv q^{\frac{\alpha}{1-\alpha}}$. In a steady state, output, consumption, the input of intermediate goods, profits, and R&D investment will all grow at rate $g$ between periods.

In order to guarantee the existence of a steady state with positive growth, following Barro

[16]Our results immediately extend to the case where $z_k = \lambda_k n_k^\beta$ , with $0 < \beta \leq 1$ . The case $\beta < 1$ may reflect the presence of external diseconomies in research.

and Sala-i-Martin (1995, p. 250) we assume that $\lambda_k = \lambda g^{-k}$. Because in a steady state $n_k$ grows at rate $g$ across periods, under this assumption the hazard rate $z_k = \lambda_k n_k$ can be constant across periods.

Finally, note that the following transversality condition must hold (see Barro and Sala-i-Martin, 1995, p. 248):

$$r > z(g-1).$$

If this condition is violated, consumers have an incentive to postpone consumption indefinitely.

### 5.4. Equilibrium in the product market.—

To proceed, remember that only the $k$th innovator, who holds a patent on his vintage of the good, can produce the intermediate good of vintage $k$. Independently of its quality, the intermediate good is produced using the final good on a one-to-one basis. However, in period $k$ it takes $q^{i-1}$ units of the intermediate good of vintage $k-i$ to make one unit of the intermediate good $k$ *in efficiency units*. Innovator $k-i$'s unit cost of producing the intermediate good, measured in period $k$ efficiency units, is therefore $q^{i-1}$. Thus we can proceed as if the intermediate good was homogeneous but firms had different production costs, i.e. 1 for the latest innovator, $q$ for the penultimate innovator, $q^2$ for the third latest innovator and so on.

Given the demand function (8), the Cournot equilibrium price can then be easily calculated as

$$p^C = \frac{1+q+q^2+...+q^{m^C}}{m^C+\alpha} \tag{9}$$

where $m^C$ is the largest integer such that [17]

$$\frac{1+q+q^2+...+q^m}{m+\alpha} > q^{m+1}.$$

Individual outputs can be obtained by substituting (9) into (4). Clearly, in each period low-cost firms hold larger market shares than high-cost firms. However, when innovations are non-drastic different vintages of the intermediate good will be simultaneously produced, even if older vintages are less productive.

In contrast, the Bertrand equilibrium is a limit-pricing equilibrium where the leader prices at $p^B = q$ and drives its competitors out of the market. At this limit-pricing equilibrium, there is no productive inefficiency. The corresponding profits are $\pi_{i,k}^B = 0$ for $i \geq 1$, and:

$$\pi_{0,k}^B = \Pi_k^B = (q-1)\, q^{-\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} g^k.$$

Such a Bertrand equilibrium is standard in the endogenous growth literature. The next Lemma confirms that a switch from Cournot to Bertrand captures the notion of tougher competition.

**Lemma 3** *The equilibrium price under Cournot competition is greater than under Bertrand competition.*

[17]It can be shown that $m^C$ is constant across periods. With $m_k^C + 1$ active firms in period $k$, the Cournot equilibrium price is

$$p_k^C = \frac{1+q+q^2+...+q^{m_k^C}}{m_k^C+\alpha}$$

and so $m_k^C$ is the largest integer such that

$$\frac{1+q+q^2+...+q^{m_k^C}}{m_k^C+\alpha} > q^{m_k^C+1}.$$

Because all the parameters in this inequality are constant, $m_k^C$ must be constant across periods.

*Proof.* We have

$$\begin{aligned} p^C &= \frac{1+q+q^2+...+q^m}{m+\alpha} \\ &> \frac{\alpha q+q+q^2+...+q^m}{m+\alpha} \text{ because innovations are non-drastic } (\alpha q<1) \\ &> \frac{\alpha q+mq}{m+\alpha} \text{ because } q>1 \\ &= q \\ &= p^B \quad \blacksquare \end{aligned}$$

Like in Section 4, we also use a more general reduced-form solution that encompasses the Bertrand and Cournot equilibria as special cases. In this solution, $p$ ranges from $p^B = q$ to $p^C$, and the corresponding individual outputs are given by (4). The equilibrium number of active firms other than the latest innovator, $m(p)$, is the largest integer such that $m(p) \leq \frac{\log p}{\log q}$.

**5.5. Equilibrium in the research industry.—**

To complete the derivation of the model's equilibrium, we next focus on the research sector. The expected discounted profit of an outside firm that invests $n_{\ell k}$ units of the final good in period $k$ to obtain innovation $k$, as of the start of the race and given that aggregate investment in R&D is $n_k$, is

$$\frac{\lambda_k n_{\ell k} E(V_k) - n_{\ell k}}{r + n_k \lambda_k}$$

where $E(V_k)$ is the expected value of the $k$th innovation, and is given by (1). At equilibrium, outsiders' expected net profit must be equal to zero:

$$\lambda_k E(V_k) = 1 \tag{10}$$

In a steady state, $z$ is constant and profits grow at rate $g$ between periods: $\pi_{i,k} = \pi_i g^k$, where $\pi_i = \pi_{i,0}$. Equation (1) then reduces to:

$$E(V_k) = \sum_{i=0}^{m} \frac{z^i g^{k+i+1} \pi_i}{(r+z)^{i+1}} \tag{11}$$

Equilibrium in the research industry is then determined by inserting (11) into the free-entry condition (10)

$$H(z) = \frac{1}{\lambda}. \tag{12}$$

where $H(z) \equiv \sum_{i=0}^{m} \frac{z^i q^{i+1} \pi_i}{(r+z)^{i+1}}$. Equation (12) determines the equilibrium hazard rate, $z^*$, and hence the economy's rate of growth. To see this, note that the growth factor between periods, $g$, is constant. This means that the equilibrium rate of growth is entirely determined by the expected length of each period, which in turn depends on the speed of technical progress: with an exponential distribution of the timing of success, the expected waiting time for each innovation is $\frac{1}{z}$. Consequently, an increase in $z$ is associated with faster growth.

**Lemma 4** *Assume that $\frac{q\pi_0}{r} > \frac{1}{\lambda}$. Then a unique, strictly positive equilibrium hazard rate $z^*$ exists.*

*Proof.* See the Appendix. ■

Condition $\frac{q\pi_0}{r} > \frac{1}{\lambda}$ ensures that research is sufficiently profitable that some research is conducted at equilibrium. It is easy to show that the steady state level of research, $z^*$, is an increasing function of the productivity of R&D effort $\lambda$. For any other arbitrary parameter $a$ that influences $z^*$, the sign of $\frac{\partial z^*}{\partial a}$ equals the sign of $\frac{\partial H}{\partial a}$. Using this fact, it is immediate to show that $z^*$ is a decreasing function of the rate of time preference $r$ and an increasing function the step size between innovations $q$. It is also clear that the economy's rate of growth increases with the incentives to innovate.

Our next task is to analyze the impact of a switch from Cournot to Bertrand competition, or, more generally, a rise in competitive pressure, on the economy's rate of growth.

### 5.6. Competition and growth.—

**Proposition 4** *If industry profits are weakly increasing in the intensity of competition, an increase in the intensity of competition raises the equilibrium rate of growth.*

*Proof.* Let $p$ and $p'$ denote two price levels, with $p < p'$. The move from $p'$ to $p$ corresponds to an increase in the intensity of competition. By assumption, $\Pi(p) \geq \Pi(p')$. Proposition 3 then implies that

$$\sum_{i=0}^{h} \pi_i(p) \geq \sum_{i=0}^{h} \pi_i(p') \tag{13}$$

for all $h$, with a strict inequality for at least one $h$. We also know that $m(p) \leq m(p')$. We must show that $H(p) > H(p')$, i.e.

$$\sum_{i=0}^{m(p)} \pi_i(p) \left[\frac{gz}{(r+z)}\right]^i > \sum_{i=0}^{m(p')} \pi_i(p') \left[\frac{gz}{(r+z)}\right]^i .$$

This can be rewritten as

$$\sum_{h=0}^{m(p)} \left\{ \sum_{i=0}^{h} \pi_i(p) \left[ \left(\frac{gz}{(r+z)}\right)^{h-1} - \left(\frac{gz}{(r+z)}\right)^{h} \right] \right\} >$$
$$> \sum_{i=0}^{m(p')} \left\{ \sum_{i=0}^{h} \pi_i(p') \left[ \left(\frac{gz}{(r+z)}\right)^{h-1} - \left(\frac{gz}{(r+z)}\right)^{h} \right] \right\}$$

where the terms inside square brackets are positive by the transversality condition. By inequality (13), each term inside curly brackets on the left hand side is at least as large as the corresponding term on the right hand side, with at least one strict inequality. This completes the proof of the Proposition. ∎

The intuition is as follows. We have shown in section 3 that the value of an innovation is a weighted average of all active firms' profits, $\sum_{i=0}^{m} \omega_i \pi_i$, where the weights $\omega_i$ reflect the expected length of time periods, discounting, and growth. In a steady state, the expected length of time periods is constant. The transversality condition implies that discounting prevails over growth, and so the weights are decreasing in $i$ : $\omega_0 \geq \omega_1 \geq ... \geq \omega_m$. The Lorenz dominance result (Proposition 3) shows that a rise in competitive pressure shifts profits from the least profitable firms to the most profitable ones. With declining weights, such a front loading of profits implies that the incentive to innovate $\sum_{i=0}^{m} \omega_i \pi_i$ increases with the intensity of competition, provided that industry profits do not fall. And in a neo-Schumpeterian model an increase in the incentive to innovate must cause an increase in the economy's rate of growth.

Proposition 4 leads to the following corollaries.

**Corollary 1** *If innovations are sufficiently large (i.e., if the size of the innovations, $q$, is close to $\frac{1}{\alpha}$), then the rate of growth under Bertrand competition is higher than the rate of growth under Cournot competition.*

*Proof.* Follows from Propositions 1 and 4. ■

It can be shown that as $q$ falls, eventually $\Pi^B(q) < \Pi^C(q)$. By Proposition 4, this means that the rate of growth can (but need not) be greater with Cournot competition if the size of innovations is sufficiently small. Numerical calculations show that the interval in which aggregate profits are greater under Bertrand competition, and thus more competition is surely associated with faster growth, can be quite large. Figure 3 illustrates.

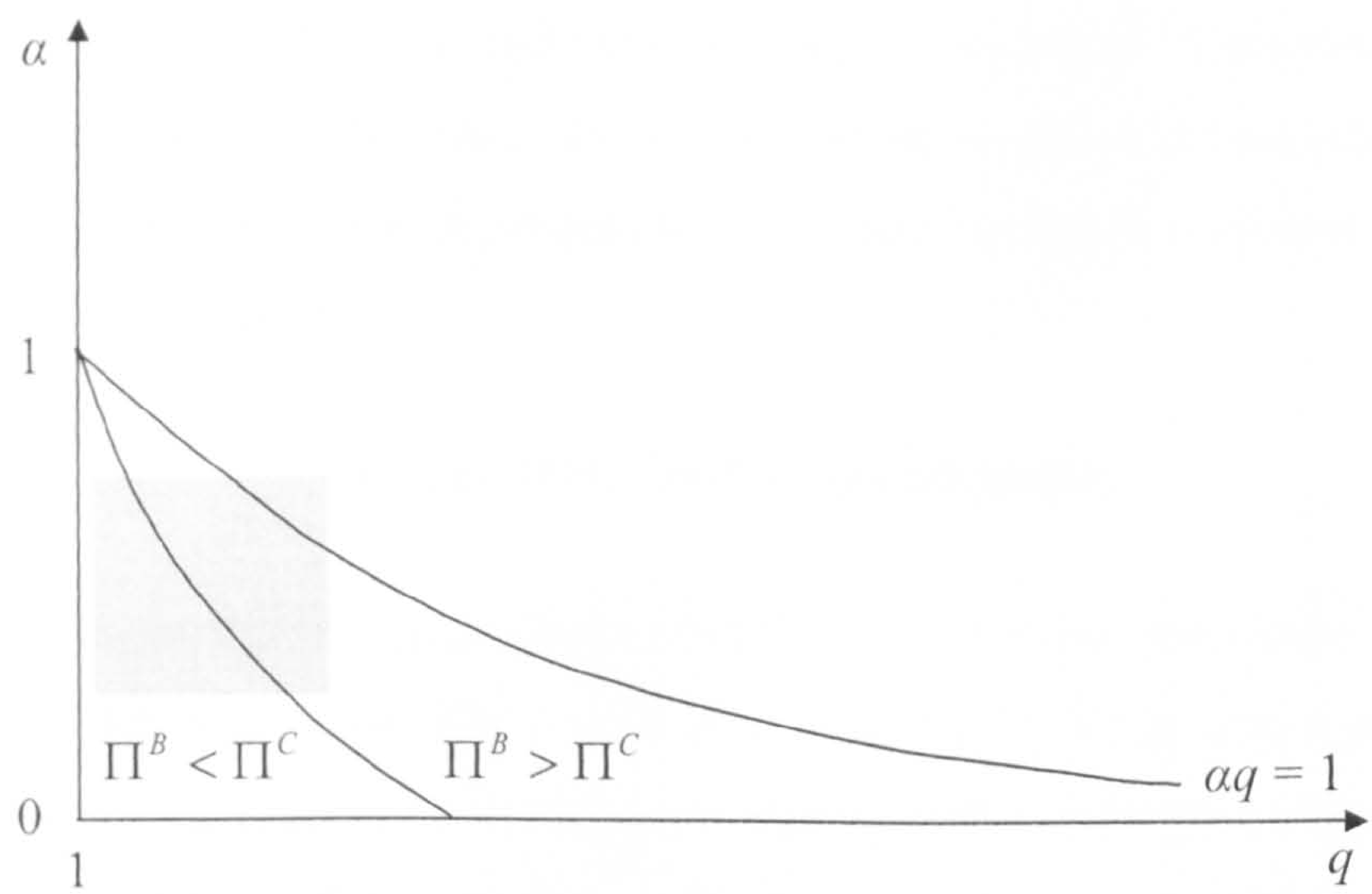


Figure 3. Industry profits under Bertrand and Cournot competition as a function of the elasticity of demand $\alpha$ and the size of innovations $q$.

Stokey (1995) notes that if innovations occur every few years, a reasonable range for $q$ is 1.02 to 1.04; if innovations occur only a couple of times per century, then a reasonable range for $q$ is 1.25 to 1.50. Barro and Sala-i-Martin (1995) note that reasonable values for $\alpha$, the share of capital income, range from 0.30 if capital is interpreted as physical capital to 0.70 if capital includes human capital. The shaded area in figure 3 corresponds to the "reasonable" range $q \in [1.02, 1.50]$ and $\alpha \in [0.30, 0.70]$. Over this range, more intense competition may well be associated with faster growth.

**Corollary 2** *If the intensity of competition is sufficiently high (i.e., $p$ is close to the Bertrand equilibrium price $q$), a further increase in the intensity of competition leads to faster growth.*

*Proof.* Follows from Propositions 2 and 4. ■

In fact, the relationship between competition and growth is monotonically increasing when the size of innovations is large. For smaller innovations, numerical calculations show that industry profits first increase and then decrease as $p$ increases. Consequently, unless the front loading of profits is sufficiently strong to outweigh the effect of tougher competition on total industry profits, the rate of growth first decreases and then increases as the intensity of competition increases.[18]

## 6. CONCLUDING REMARKS

In this chapter, we have re-considered the relationship between competition and growth in a standard neo-Schumpeterian model with improvements in the quality of products. Focusing on the case of non-drastic innovations, we have modeled the notion of lower competition by a switch from Bertrand to Cournot competition, and more generally by a decrease in the equilibrium price.

We have shown that competition is good for growth either if the size of (non-drastic) innovations is large, or if the intensity of competition is high, or both. This result follows

[18]These results hold provided that only two firms are active in the Cournot equilibrium. When innovations are still smaller, such that three firms are active under Cournot competition, the relationship between competition and growth can exhibit two local maxima.

from two qualitatively new effects – the front loading of profits and the productive efficiency effect – that arise when innovators are not immediately displaced by the occurrence of the next innovation so that two or more asymmetric firms are simultaneously active in the same industry.

We conclude with a brief discussion of repeated innovation by incumbents, patent licensing, welfare, and alternative measures of the intensity of competition.

### 6.1. Persistent leadership.—

In our model, incumbents do no research and are systematically replaced by outsiders. However, there is ample evidence that incumbents account for much of the research done, and the resulting pattern of persistent leadership is well documented in many industries. The literature that try to explain this pattern of persistent leadership recognizes that in standard quality ladder models leapfrogging is indeed an equilibrium of a simultaneous moves R&D game if the size of innovations is not too small (e.g. Denicolò (2001). In this case, the assumption that incumbents do not research is not restrictive. However, the standard model can be adapted to make room for the persistence of leadership in various ways. If, for example, it is assumed that incumbents have a first-mover advantage in the patent race starting after the latest innovation, the outcome is a pre-emption equilibrium in which all research is done by incumbents. However, the amount of research is still determined by the outsiders' zero-profit condition, and thus the incentive to innovate is driven by the same qualitative effects as in the leapfrogging equilibrium.

### 6.2. Patent licensing.—

We have followed the vast majority of endogenous growth models in ruling out patent licensing. The standard justification for this assumption is that licensing agreements between successive innovators would have anti-competitive effects and thus would be prohibited by antitrust authorities. In our model, however, patent licensing agreements could be arranged so as to improve productive efficiency with no anticompetitive effects, and so ruling them out is more restrictive an assumption than in the early models.

If patent licensing was ubiquitous, all of the output would be produced with the most efficient technology and so the productive efficiency effect would vanish. However, a variety of transaction costs impede licensing agreements. As an example, royalty licensing is possible only if the output is verifiable; when individual output is not verifiable, and only fixed-fee licensing is feasible, licensing will occur at equilibrium only if the size of innovations is sufficiently small. As another example, incomplete information over the size of the innovation can lead parties to introduce inefficient terms in the licensing agreements. In addition, innovative technological knowledge can be difficult to codify and transmit to others. These transaction costs may likely result in an equilibrium outcome in which some active firms do not use the latest generation technology. To the extent that the product market equilibrium exhibits some productive inefficiency, our qualitative results are likely to hold even if we allow for some licensing agreements.

### 6.3. Welfare.—

Although a detailed welfare analysis is outside the scope of this chapter, a few remarks are in order. Our analysis shows that an increase in the intensity of competition has two effects on social welfare, a static effect and a dynamic effect. The static effect is unambiguously positive. Indeed, for any given state of the technology, the price of the intermediate good is lower and output is greater with tougher competition. Further, if competition is Bertrand, only the most efficient firm is active in the intermediate good industry and so only the highest quality good is produced in equilibrium, ensuring that productive efficiency is achieved. The dynamic effect, that operates via the incentive to innovate and the rate of growth, is more complex. As we have seen, competition can be growth-enhancing or growth-reducing. In addition, the equilibrium rate of growth can exceed the socially optimal rate, which means that faster growth is not necessarily socially beneficial. Therefore, the overall welfare effect of tougher competition is generally ambiguous.

### 6.4. Alternative measures of the intensity of competition.—

One of the novelties of our analysis relative to most of the previous growth literature is that we have not used the elasticity of demand as a proxy of the intensity of competition. Instead, we have formalized the intensity of competition in two ways: a switch from Cournot to Bertrand competition, and a more general reduced form that uses directly (the inverse of) the equilibrium price. Both measures convey the idea of more competition as a higher degree of rivalry among firms in an oligopolistic setting, and highlight two main effects of tougher competition: the price effect and the selection effect. As noted before, the selection effect implies that our measures of the intensity of competition are positively related with market concentration, but negatively associated the industry average price-cost margin. Therefore, our formalization of the intensity of competition is appropriate to measure market power due to a less competitive behavior by oligopolistic firms, whilst it does not capture anti-competitive practices like entry barriers.[19] However, to the extent that the removal of such barriers allows more efficient entrants (i.e. current innovators) to better exploit their efficiency advantage in the product market, the main effects which drive our results (i.e. the front loading of profit, the selection, and the productive efficiency effects) will continue to work. Therefore, our qualitative results are likely to hold in such a setting.

An alternative way to formalize the intensity of competition would be to assume that products are differentiated, and cumulative process innovation generates asymmetries among firms which compete in price. The (inverse of the) degree of product differentiation would be the measure of the intensity of competition. For instance, suppose that there are two differentiated products in the market, and each successful innovator (let us stick on the assumption that innovations come from outsiders) not only disposes of a more efficient technology but can also alter consumers' perception of the degree of substitutability between his product and the rivals' (for instance, through comparative advertising). For simplicity, concentrate on the case in which the innovation size is sufficiently high such that at most two firms can be active for any degree of product differentiation. Then, as shown in Chapter 3 below, the profit of the technological leader (i.e. the current innovator) is non-monotonic

[19]Indeed, in our model, the number of active firms in the market arises endogenously from the intensity of competition and the degree of asymmetry among firms (as a result of cummulative innovations and intellectual property rights).

in the degree of product substitutability when cost asymmetry (i.e. the innovation size) is sufficiently high, whilst the profit of the technological follower (i.e. the previous innovator) always decreases with product substitutability. Provided that the current innovator can change only marginally the perceived degree of substitutability starting from an high initial level, he would choose to further decrease product differentiation. Again, the selection effect here is the driving force of more intense competition (i.e. less product differentiation in this case). Furthermore, from the intertemporal prospective, the front loading of profit would operate as in our model. Finally, more competition would entail more productive efficiency, as the technological leader would serve a large fraction of the market. To the extent that the productive efficiency effect would sufficiently contrast the price effect, a positive relation between competition, innovation and growth would arise as in our model. On the contrary, if the current innovator can marginally change the perceived degree of product substitutability starting from a low initial level, he would prefer to further increase product differentiation. Therefore, as in our model, when the intensity of competition is initially high and innovations are large, a positive relation between competition, innovation and growth is likely to arise. When the initial intensity of competition is low and innovations are small, the relationship between competition and growth tends to be negative.

## APPENDIX

**Omitted details in the proof of Lemma 1.—**

To prove the first equality in (6), i.e.

$$\frac{d\bar{c}}{dp}X = \sum_{i=0}^{m}(c_i - \bar{c})\frac{dx_i}{dp}$$

note that

$$\begin{aligned}
\frac{d\bar{c}}{dp} &= \frac{d}{dp}\sum_{i=0}^{m}\frac{x_i}{X}c_i \\
&= \frac{\sum_{i=0}^{m} c_i X \frac{dx_i}{dp} - \sum_{i=0}^{m} c_i x_i \frac{dX}{dp}}{X^2} \\
&= \frac{\sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \frac{dX}{dp}\sum_{i=0}^{m} c_i \frac{x_i}{X}}{X} \\
&= \frac{\sum_{i=0}^{m} c_i \frac{dx_i}{dp} - \sum_{i=0}^{m} \frac{dx_i}{dp}\bar{c}}{X}
\end{aligned}$$

whence the result follows immediately. Next, note that

$$\sum_{i=0}^{m}(c_i - \bar{c})\frac{dx_i}{dp} = \sum_{i=0}^{m} c_i\frac{dx_i}{dp} - \bar{c}\frac{dX}{dp}$$

From (4) we get

$$\frac{dx_i}{dp} = \frac{x_i}{X}\frac{dX}{dp} + X(p)\frac{c_i - \hat{c}}{(p - \hat{c})^2}\frac{1}{[m(p) + 1]}$$

Substituting into the above expression we get

$$\begin{aligned}
\sum_{i=0}^{m} c_i\frac{dx_i}{dp} - \bar{c}\frac{dX}{dp} &= \underbrace{\sum_{i=0}^{m} c_i\frac{x_i}{X}}_{=\bar{c}}\frac{dX}{dp} + \sum_{i=0}^{m} X(p)c_i\frac{c_i - \hat{c}}{(p - \hat{c})^2}\frac{1}{[m(p) + 1]} - \bar{c}\frac{dX}{dp} \\
&= \frac{X(p)}{[m(p) + 1]\,(p - \hat{c})^2}\sum_{i=0}^{m} c_i\,(c_i - \hat{c})
\end{aligned}$$

because the first and third term on the right hand side cancel out. But $\sum_{i=0}^{m} c_i\,(c_i - \hat{c})$ is the variance of active firms' marginal costs, $\sigma_c^2$, and so the second equality in (6) is obtained.

■

**Proof of Lemma 2.—**

From (6) we have

$$\begin{aligned}\frac{d}{dp}(p-\bar{c}) &= 1-\frac{\sum_{i=0}^{m(p)}(c_i-\hat{c})^2}{[m(p)+1]\,(p-\hat{c})^2} \\ &= \frac{\sum_{i=0}^{m(p)}[(p-\hat{c})^2-(c_i-\hat{c})^2]}{[m(p)+1]\,(p-\hat{c})^2}\geq 0\end{aligned}$$

where the inequality follows because $p \geq c_{m(p)}$. ■

**Proof of Proposition 1.—**

When $c_1 = p^M(c_0)$ we have $p^B = p^C = p^M(c_0)$ and $x_1^C = x_1^B = 0$; consequently, $\Pi^B = \Pi^C$. Starting from $c_1 = p^M(c_0)$, let us consider the effect of a small decrease in $c_1$, such that $x_1^C$ becomes positive. Because $\Pi^B(c_1) = (c_1 - c_0)X(c_1)$, we have

$$\frac{d\Pi^B}{dc_1} = X(c_1) + (c_1 - c_0)X'(c_1)$$

and so $\frac{d\Pi^B}{dc_1} = 0$ at $c_1 = p^M(c_0)$. On the other hand, when $c_1$ is just below $p^M(c_0)$, exactly two firms will be active in the Cournot equilibrium. Thus, $\Pi^C(c_1) = (p^C - c_0)x_0^C + (p^C - c_1)x_1^C$ and so differentiating we get

$$\frac{d\Pi^C}{dc_1} = \frac{dp^C}{dc_1}X^C - x_1^C + (p^C - c_0)\frac{dx_0^C}{dc_1} + (p^C - c_1)\frac{dx_1^C}{dc_1}$$

At $c_1 = p^M(c_0)$, the second and fourth term vanish, and so

$$\frac{d\Pi^C}{dc_1}\,|_{c_1=p^M(c_0)} = \frac{dp^C}{dc_1}X^C + (p^C - c_0)\frac{dx_0^C}{dc_1}$$

From the first order conditions (3) one obtains

$$\frac{dx_0^C}{dc_1} = -\frac{p'(X) + p''(X)x_0}{[p'(X)]^2}\frac{dp^C}{dc_1}.$$

At $c_1 = p^M(c_0)$ we have $x_0 = X^C$ and thus

$$\frac{d\Pi^C}{dc_1}\,|_{c_1=p^M(c_0)} = \frac{dp^C}{dc_1}X^C - (p^C - c_0)\frac{p'(X^C) + p''(X^C)X^C}{[p'(X^C)]^2}\frac{dp^C}{dc_1}.$$

But $(p^C - c_0) = -p'(X)x_0$ and so the derivative reduces to

$$\frac{d\Pi^C}{dc_1}\mid_{c_1=p^M(c_0)}= \frac{2p'(X^C)+p''(X^C)X^C}{p'(X^C)}\frac{dp^C}{dc_1}.$$

The fraction $\frac{2p'(X^C)+p''(X^C)X^C}{p'(X^C)}$ is positive given the assumption of decreasing marginal revenue. Clearly, under our assumption of constant marginal costs and decreasing marginal revenue we have $\frac{dp^C}{dc_1} > 0$.[20] It follows that

$$\frac{d\Pi^C}{dc_1}\mid_{c_1=p^M(c_0)}> 0.$$

This means that $\Pi^C(c_1)$ raises more steeply than $\Pi^B(c_1)$ in a left neighborhood of $p^M(c_0)$. By continuity, it follows that $\Pi^B(c_1) > \Pi^C(c_1)$ in a left neighborhood of $p^M(c_0)$. ■

**Proof of Proposition 3.—**

Let $p$ and $p'$ denote two price levels, with $p < p'$. The move from $p'$ to $p$ corresponds to an increase in the intensity of competition. We must show that $L_{\Pi(p)}(\frac{h}{s+1}) \leq L_{\Pi(p')}(\frac{h}{s+1})$ for all $h$, with at least one strict inequality. Note that (3) implies

$$\frac{\pi_i}{\pi_j} = \left(\frac{p-c_i}{p-c_j}\right)^2 \qquad i,j = 0,1,...,m$$

whenever firms $i$ and $j$ are active at equilibrium. Differentiating we get

$$\frac{d\frac{\pi_i}{\pi_j}}{dp} = 2\frac{(p-c_i)}{(p-c_j)^3}(c_i - c_j)$$

whence it immediately follows that

$$\frac{\pi_i(p)}{\pi_j(p)} > \frac{\pi_i(p')}{\pi_j(p')} \text{ for all } i,j = 0,1,...,m \text{ with } j > i. \tag{A1}$$

Inequalities (A1) imply

$$\frac{\pi_0(p)}{\Pi(p)} > \frac{\pi_0(p')}{\Pi(p')}$$

[20] As is well known, under Cournot competition and constant asymmetric marginal costs, the price is the same as if all firms shared the same cost $\widehat{c}$. It is also well known that in a symmetric model with decreasing marginal revenue, Cournot equilibrium price increases with the marginal cost. These two facts immediately imply that $\frac{dp^C}{dc_1} > 0$.

provided that there are at least two active firms at $p'$, whence it follows that

$$\frac{\pi_s(p)+\pi_{s-1}(p)+...+\pi_1(p)}{\Pi(p)} < \frac{\pi_s(p')+\pi_{s-1}(p')+...+\pi_1(p')}{\Pi(p')}$$

i.e., $L_{\Pi(p)}(\frac{s}{s+1}) < L_{\Pi(p')}(\frac{s}{s+1})$.

Clearly, (A1) also implies $L_{\Pi(p)}(\frac{1}{s+1}) \leq L_{\Pi(p')}(\frac{1}{s+1})$.Now suppose to the contrary that there exists $i$ such that $L_{\Pi(p)}(\frac{i}{s+1}) > L_{\Pi(p')}(\frac{i}{s+1})$, and let $h$ be the minimum value of $i$ for which inequality $L_{\Pi(p)}(\frac{i}{s+1}) > L_{\Pi(p')}(\frac{i}{s+1})$ holds, so that

$$\frac{\pi_s(p)+\pi_{s-1}(p)+...+\pi_{s-h}(p)}{\Pi(p)} > \frac{\pi_s(p')+\pi_{s-1}(p')+...+\pi_{s-h}(p')}{\Pi(p')}.$$

and

$$\frac{\pi_s(p)+\pi_{s-1}(p)+...+\pi_{s-h-1}(p)}{\Pi(p)} \leq \frac{\pi_s(p')+\pi_{s-1}(p')+...+\pi_{s-h-1}(p')}{\Pi(p')}.$$

These inequalities imply

$$\frac{\pi_{s-h-1}(p)}{\Pi(p)} < \frac{\pi_{s-h-1}(p')}{\Pi(p')} \tag{A2}$$

and that there exists at least one $j > s-h-1$ such that

$$\frac{\pi_j(p)}{\Pi(p)} > \frac{\pi_j(p')}{\Pi(p')} \tag{A3}$$

Combining (A2) and (A3) we get

$$\frac{\pi_{s-h-1}(p)}{\pi_j(p)} < \frac{\pi_{s-h-1}(p')}{\pi_j(p')}$$

but this violates (A1). This contradiction establishes the result. ■

**Proof of Lemma 4.—**

First of all, we show that $H(z)$ is monotonically decreasing in $z$. Differentiating $H(z)$ we get:

$$H'(z) = \frac{d}{dz}\sum_{i=0}^{m}\frac{z^i g^{i+1}\pi_i}{(r+z)^{i+1}} = -\sum_{i=0}^{m-1}\frac{(i+1)z^i g^i(\pi_i - g\pi_{i+1})}{(r+z)^{i+2}} - \frac{(m+1)z^m g^{m+1}\pi_m}{(r+z)^{m+2}}.$$

A sufficient condition for the derivative to be negative is that $\pi_i \geq g\pi_{i+1}$. We know from the proof of Proposition 3 that the ratio $\frac{\pi_i(p)}{\pi_{i+1}(p)}$ decreases with $p$. Moreover, because

$$\frac{\pi_i}{\pi_{i+1}} = \left(\frac{p-q^i}{p-q^{i+1}}\right)^2 \tag{A4}$$

and $q^i$ is a convex function of $i$, we have

$$\frac{\pi_i}{\pi_{i+1}} < \frac{\pi_{i+1}}{\pi_{i+2}}$$

for all $i$ and for all $p$. Consequently, it suffices to show that $\pi_0(p) \geq g\pi_1(p)$ when $p$ equals the monopoly price $\frac{1}{\alpha}$, which always exceeds the Cournot equilibrium price $p^C$. From (A4) we have

$$\frac{\pi_0\left(\frac{1}{\alpha}\right)}{\pi_1\left(\frac{1}{\alpha}\right)} = \left(\frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-q}\right)^2.$$

Therefore, we must prove that

$$\left(\frac{1-\alpha}{1-\alpha q}\right)^2 \geq g = q^{\frac{\alpha}{1-\alpha}}$$

or

$$(1-\alpha)^2 \geq q^{\frac{\alpha}{1-\alpha}}(1-\alpha q)^2$$

At $q = 1$, the weak inequality is satisfied as an equality. To conclude the proof, it suffices to show that the derivative with respect to $q$ of the right hand side of the above inequality is negative. Differentiating we get

$$\frac{d}{dq}\left[q^{\frac{\alpha}{1-\alpha}}(1-\alpha q)^2\right] = -\frac{\alpha}{1-\alpha}(1-\alpha q)\, q^{\frac{\alpha}{1-\alpha}-1}\left[(q-1)+q(1-\alpha)\right] < 0.$$

This completes the proof that $H'(z) < 0$, which implies that the equilibrium, if it exists, is unique. To show existence, note that $H(0) = \frac{g\pi_0}{r} > \frac{1}{\lambda}$ and $\lim_{z\to\infty} H(z) = 0$. Because $H(z)$ is continuous, an equilibrium exists. ■

# Chapter 3

# Differentiated duopoly with asymmetric costs: new results from a seminal model

## 1. INTRODUCTION

In a seminal paper, Singh and Vives (1984) show that, in a differentiated duopoly, quantity competition entails higher prices and profits than price competition, whereas quantities and social welfare are higher under price competition.[1] To obtain these results, however, they effectively restrict the space of parameter values by assuming *positive primary outputs* for both firms (i.e. when both prices are set at marginal costs, both firms sell positive outputs). As noted by Amir and Jin (2001), the assumption of *positive primary outputs* is crucial for the Singh and Vives's ranking of the equilibrium quantities under the two forms of competition.[2]

In this chapter, we re-consider the comparison of price and quantity competition in the Singh and Vives model allowing for a wider range of cost asymmetry between firms. The analysis reveals that price competition always leads to lower prices and larger social welfare. However, for high degrees of cost asymmetry and low degrees of product differentiation, the efficient firm's and the industry profits are higher under price than under quantity competition. Therefore, Singh and Vives's ranking of profits is reversed in a significant region of the relevant parameter space.

[1] These results hold irrespective of the nature of the goods (substitutes or complements), except that with complementary goods, the ranking of profits is reversed.

[2] This fact is apparent in the special case of a homogeneous duopoly with linear cost functions, where the assumption of *positive primary outputs* restricts attention to symmetric costs. With cost asymmetry, the inefficient firm is active in the Cournot equilibrium (provided that the efficiency gap between the two firms is not drastic) but is inactive in the limit-pricing equilibrium arising under Bertrand competition. Therefore, the Singh and Vives's ranking of the equilibrium quantities clearly does not hold. Amir and Jin (2001) nevertheless maintain the *positive primary output* assumption, and focus on the case when firms produce a mixture of complementary and substitute goods.

The intuition behind our results is as follows. When firms are asymmetric in costs, price competition not only entails lower prices (*price effect*) but also a stronger selective effect against the market share of the less efficient firm (*selection effect*) than quantity competition does. While the *price effect* works towards lower profits under Bertrand than under Cournot competition for both firms, the *selection effect* works in the opposite direction on the efficient firm's profits. Moreover, the *price effect* weakens while the *selection effect* gets stronger when either the degree of cost asymmetry increases or products are closer substitutes. As a result, the efficient firm earns higher profits under price than under quantity competition when its efficiency advantage over the rival is sufficiently high and products are close substitutes. Moreover, the *selection effect* entails greater productive efficiency under price than under quantity competition: a larger fraction of total production is allocated to the efficient firm, so that the industry average cost is lower (*productive efficiency effect*). When cost asymmetry and product substitutability are sufficiently strong, the *productive efficiency effect* prevails on the *price effect*, explaining the reversal of the industry profit ranking between the two forms of competition.

In the homogeneous goods case, the inversion of the ranking of the efficient firm's profit under the two modes of competition has been noted before by Acharyya and Marjit (1998) and Boone (2001). The present chapter generalizes these result, showing that both industry profit and the efficient firm's profit can be higher under Bertrand than under Cournot competition in a differentiated duopoly with asymmetric costs. More generally, we show how cost asymmetry and product differentiation together affect the ranking of the equilibrium profits under the two modes of competition. Our results are also related to Hackner (2000). Allowing for both vertical and horizontal product differentiation and symmetric costs, Hackner shows that the "high-quality firms" may earn higher profit with price than with quantity competition when there are more than two competitors in the market. In our chapter we show that the Singh and Vives's ranking of profits is sensitive to cost asymmetry and horizontal differentiation, irrespective of the number of competitors.[3] Moreover,

[3]Hackner's formalization of vertical differentiation can be easily reinterpreted as cost asymmetry within the Singh and Vives model. Therefore, our results show that the crucial factors for the reversal of the

we provide a clear intuition of the effects driving the ranking reversal of both industry and leader profits, which can also explain Hackner's result.[4]

Finally, our characterization of Bertrand and Cournot equilibria over the entire relevant parameter space highlights two additional results on the behaviour of the equilibrium profits when firms are asymmetric in costs. Namely, under both forms of competition, while the inefficient firm's profits always decrease as products become closer substitutes, the efficient firm's profits are non-monotonic in the degree of products differentiation. Therefore, the efficient firm may have a local incentive to reduce the degree of product differentiation. This contrasts with the standard result that arises with symmetric costs, that is, Bertrand and Cournot duopolists always gain from product differentiation (see, among others, Shy (1995), pp. 138-140).

The rest of the chapter is organized as follows. Section 2 describes the model, and characterizes Bertrand and Cournot equilibria over the relevant parameter space. Section 3 compares the two forms of competition and presents the main results of the chapter. Section 4 collects additional results on the effect of cost asymmetry on firms' incentive to differentiate products under both forms of competition. Section 5 provides some concluding remarks. All proofs are relegated to an appendix.

---

leader's profit ranking are the degree of asymmetry among firms (either in costs or in product quality) and the degree of product differentiation, and not the number of competitors in the market. Notice that Hackner assumes that all firms are active in the market under both forms of competition. On the contrary, we extend the parameter region of the model to include also the limit-pricing equilibrium that arises under Bertrand competition with strong firms asymmetry and weak product differentiation. This may explain the difference between Hackner's result and ours.

[4]Indeed, Hackner's interpretation of his result is not clear. Having shown that, when the quality differences are large, the high-quality firms may gain higher profit under Betrand, he concludes that, in such a case, the high quality firms become insulated from the competition of the low-quality segment, so that price competition may not hurt firm profits more than quantity competition. This explanation, however, cannot account for the reversal of the profit ranking. Quite to the contrary, we show that, when products are are not strongly differentiated, a high cost (or quality) advantage makes the leader "willing to compete tougher" in the market (i.e. to compete in price rather than in quantity), since the selection effect associated with price competition prevails on the price effect, leading to higher profits for the leader.

## 2. THE MODEL

We consider the following version of the Singh and Vives (1984) model.[5] On the demand side of the market, the representative consumer's utility is a symmetric-quadratic function of two products, $q_1$ and $q_2$, and a linear function of a numeraire good, $m$,

$$U = \alpha (q_1 + q_2) - \frac{1}{2} \left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + m \; .$$

The parameter $\gamma$ measures the degree of product differentiation. We consider the case of substitute goods: $0 \leq \gamma \leq 1$. $\gamma = 0$ and $\gamma = 1$ set the maximum (independent goods) and the minimum (homogeneous goods) degree of differentiation, respectively.

This utility function generates the linear system of inverse demand functions

$$p_i = \alpha - q_i - \gamma q_j \qquad [i, j = 1, 2; \; i \neq j], \tag{1}$$

whose inversion (by imposing $\gamma < 1$) leads to the direct demand system

$$q_i = \frac{1}{1 - \gamma^2} \left[(1 - \gamma)\alpha - p_i + \gamma p_j\right] \qquad [i, j = 1, 2; \; i \neq j]. \tag{2}$$

System (2) gives the direct demand functions provided that prices lead to positive demands for both goods. Discarding the trivial case with zero-demand for both goods, the region of prices where the demand for good $j$ is zero while the demand for good $i$ is positive, $\bar{R}_j$, is identified by

$$\bar{R}_j = \left\{ \begin{array}{l} p_i, p_j \geq 0 \\ (1 - \gamma)\alpha - p_j + \gamma p_i \leq 0 \\ \alpha - p_i > 0 \end{array} \right\} . \tag{3}$$

Inside region $\bar{R}_j$, the demand function of good $i$ becomes $q_i = \alpha - p_i$.

On the supply side of the market, products $q_1$ and $q_2$ are produced and supplied by firm 1 and firm 2, respectively. Both firms face linear cost functions, but firm 1 is more efficient

[5] This specification differs from Singh and Vives (1984) in that we assume symmetric demand functions. While this assumption allows us to concentrate on cost asymmetry, it is not restrictive since the relevant measure of firms' asymmetry under both modes of competition can equally reflect cost and demand asymmetries (see equation 4 below). Without loss of generality, we normalize to one the coefficients of the squared terms in the utility function (i.e. the "own quantity slopes" of the inverse demand functions).

than firm 2. We set firm 1's marginal cost at $c_1$, where $0 \leq c_1 < \alpha$ to avoid the trivial case in which neither firm has an incentive to produce. Firm 2's marginal cost, $c_2$, will lie in the range $c_2 \in [c_1, \alpha]$. We measure the degree of cost asymmetry between the two firms by the ratio:

$$x = \frac{\alpha - c_2}{\alpha - c_1}, \tag{4}$$

where $x \in [0, 1]$. As we will see below, this is the relevant measure of firms' asymmetry in order to characterize the market equilibrium under both Bertrand and Cournot competition. Clearly, $x$ decreases as the efficiency gap between the two firms increases. For $x = 1$ (i.e. $c_2 = c_1$), there is no cost asymmetry. For $x = 0$ (i.e. $c_1 < c_2 = \alpha$), firm 2 is not active in the market irrespective of the form of competition and the degree of products differentiation.

### 2.1. The relevant parameter space.—

Firm 1 and firm 2 compete either in prices or in quantities. However, the mode of competition matters only in a portion of the parameter space. First, when $\gamma = 0$, both firms are monopolists on independent segments of the market.[6] Second, for any $0 < \gamma \leq 1$, if the efficiency gap between the two firms is sufficiently high, then firm 1 can engage in monopoly pricing without bearing any competitive pressure from firm 2, which is driven out of the market irrespective of the form of competition. This is the case when

$$x \leq x^M(\gamma) = \frac{\gamma}{2}. \tag{5}$$

Equation (5) identifies an increasing *monopoly frontier*, $x^M(\gamma)$, in the space $S = \{0 \leq \gamma \leq 1; 0 \leq x \leq 1\}$ (see Figure 1). Namely, when products are closer substitutes, a lower efficiency advantage suffices for firm 1 to monopolize the market irrespective of the form of competition.[7]

[6] In this case, we get: $p_i^M = \frac{\alpha + c_i}{2}$, $q_i^M = \frac{\alpha - c_i}{2}$, and $\pi_i^M = \left(\frac{\alpha - c_i}{2}\right)^2$, where $p_i^M$, $q_i^M$ and $\pi_i^M$ are firm $i$'s monopoly price, quantity and profits, respectively ($i = 1, 2$). For $c_2 > c_1$, we clearly have: $p_1^M < p_2^M$, $q_1^M > q_2^M$, and $\pi_1^M > \pi_2^M$.

[7] On the contrary, the less efficient firm can never engage in monopoly pricing unless $\gamma = 0$. Indeed, for $\gamma \in (0, 1]$ and $x > x^M(\gamma)$, even firm 2 can price above its marginal cost and face positive demand if the rival prices at $p_1^M$. Since $p_2^M \geq p_1^M$ and $c_2 \geq c_1$, the symmetry of the demand system ensures that the same is

In our version of the model, the Singh and Vives (1984) assumption of positive primary outputs (i.e. both firms always face positive demand when both prices are set at marginal costs) is binding only for the inefficient firm, and it formally implies the parameter restriction:

$$x > \gamma. \tag{6}$$

Therefore, the parameter region considered by Singh and Vives is $S_{sv} = \{0 < \gamma \leq 1; \gamma \leq x \leq 1\}$. In this chapter, we extend the comparison of Bertrand and Cournot equilibria to any relevant combination of cost asymmetry and product differentiation. Since the mode of competition does not matter in the monopoly region (i.e. the region lying below the monopoly frontier in the space $S$), the relevant parameter space is given by $S_r = \{0 < \gamma \leq 1; \frac{\gamma}{2} < x \leq 1\}$. Figure 1 depicts the relevant parameter space $S_r$ (the area enclosed by the thick contour), the sub-region considered by Singh and Vives (the area above the dotted line), and the resulting extension of the parameter space of the model we consider in this chapter (the shaded region).

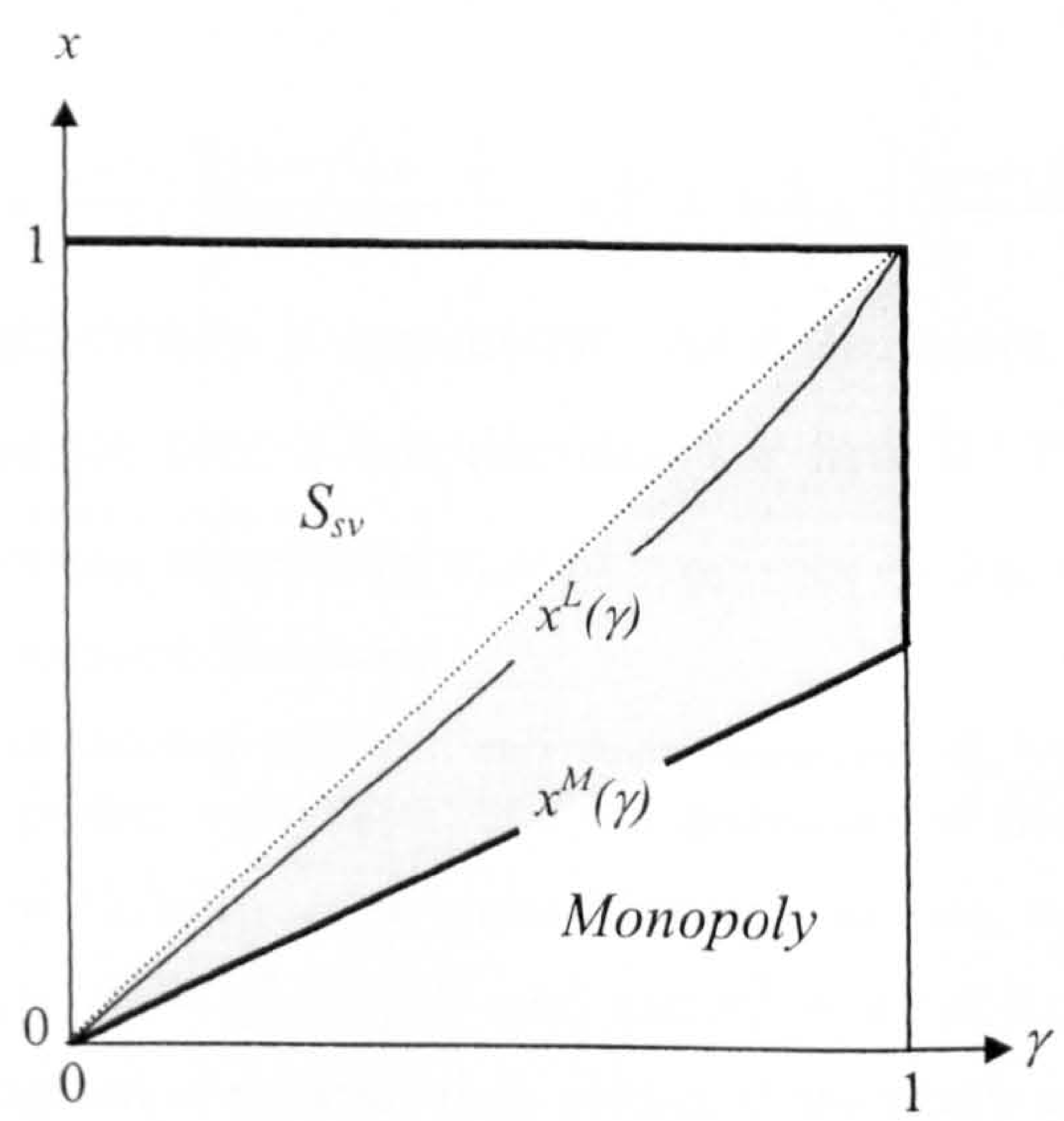


Figure 1

true for firm 1.

### 2.2. Cournot competition.—

Solving the model under Cournot competition, we get: [8]

$$q_1^C = p_1^C - c_1 = \frac{(\alpha-c_1)(2-\gamma x)}{(4-\gamma^2)}; \quad \pi_1^C = \left[\frac{(\alpha-c_1)(2-\gamma x)}{(4-\gamma^2)}\right]^2;$$
$$q_2^C = p_2^C - c_2 = \frac{(\alpha-c_1)(2x-\gamma)}{(4-\gamma^2)}; \quad \pi_2^C = \left[\frac{(\alpha-c_1)(2x-\gamma)}{(4-\gamma^2)}\right]^2. \tag{7}$$

From equation (7), as $x$ decreases below 1 (where the equilibrium is symmetric), production, unit-profit and profits increase for firm 1 but decrease for firm 2. However, the inefficient firm remains active in Cournot equilibrium over the entire space $S_r$ (i.e., given $\gamma \in (0,1]$, $q_2^C > 0$ for any $x \in (\frac{\gamma}{2}, 1]$).

### 2.3. Bertrand competition.—

Assume that $\gamma < 1$, and suppose that both firms are active in equilibrium.[9] Then, Bertrand equilibrium is characterized as follows: [10]

$$q_1^B = \frac{p_1^B-c_1}{1-\gamma^2} = \frac{(\alpha-c_1)(2-\gamma^2-\gamma x)}{(1-\gamma^2)(4-\gamma^2)}; \quad \pi_1^B = \frac{1}{1-\gamma^2}\left[\frac{(\alpha-c_1)(2-\gamma^2-\gamma x)}{4-\gamma^2}\right]^2;$$
$$q_2^B = \frac{p_2^B-c_2}{1-\gamma^2} = \frac{(\alpha-c_1)\left[(2-\gamma^2)x-\gamma\right]}{(1-\gamma^2)(4-\gamma^2)}; \quad \pi_2^B = \frac{1}{1-\gamma^2}\left[\frac{(\alpha-c_1)\left[(2-\gamma^2)x-\gamma\right]}{4-\gamma^2}\right]^2. \tag{8}$$

Again, for $x = 1$ the equilibrium is symmetric. As $x$ decreases below 1, production, unit-profit and profits increase for firm 1 but decrease for firm 2. From equation (8), we find

[8] Under Cournot, firm $i$ chooses $q_i$ to maximize $\pi_i = (\alpha - q_i - \gamma q_j - c_i)\, q_i$, by taking $q_j$ as given $(i,j = 1,2;\ i \neq j)$. This leads to the best response functions: $q_i = \max\left\{\frac{1}{2}(\alpha - c_i - \gamma q_j)\ ;0\right\}$ $(i,j = 1,2;\ i \neq j)$. Assuming an interior solution, and using equation (4), equation (7) follows.

[9] For $\gamma = 1$, products are perfect substitutes, and the derivation of Bertrand equilibrium is standard. With no cost asymmetry ($x = 1$), both prices equal the marginal cost, and each firm serves half of the market making zero-profits, i.e. $q_1^B = q_2^B = \frac{1}{2}(\alpha - c_1)$ and $\pi_1^B = \pi_2^B = 0$. With cost asymmetry ($x < 1$), the efficient firm drives the rival out of the market by pricing at the rival's marginal cost, i.e., using eq. (4), $q_1^L = (\alpha - c_1)\, x$ and $\pi_1^L = (\alpha - c_1)^2\, x\,(1-x)$.

[10] Under Bertrand, firm $i$ chooses $p_i$ to maximize $\pi_i = (p_i - c_i)\,\frac{(1-\gamma)\alpha - p_i + \gamma p_j}{1-\gamma^2}$, by taking $p_j$ as given $(i,j = 1,2;\ i \neq j)$. Focusing on an interior equilibrium, the best response functions are: $p_i = \frac{1}{2}\left[(1-\gamma)\,\alpha + c_i + \gamma p_j\right]$, $(i,j = 1,2; i \neq j)$. Equation (8) follows from the best response functions above and equation (4).

that, for any $\gamma \in (0,1)$, the inefficient firm is active in Bertrand equilibrium (i.e. $q_2^B > 0$) provided that

$$x > x^L(\gamma) = \frac{\gamma}{2-\gamma^2}. \tag{9}$$

Equation (9) defines a *limit-pricing frontier*, $x^L(\gamma)$, which is increasing in $\gamma$, meaning that any efficiency advantage of firm 1 exerts a stronger effect on the rival's market share when products are closer substitutes (see Figure 1). Moreover, it lies above the *monopoly frontier*, meaning that, for any degree of products differentiation (but $\gamma = 0$), price competition has a stronger selective effect against the market share of the inefficient firm than quantity competition. Finally, by comparing (6) and (9), it is easy to see that the assumption of *positive primary outputs* is stronger than the condition for an interior equilibrium under price competition.[11]

For $x^M(\gamma) < x \leq x^L(\gamma)$, the following limit-pricing equilibrium arises under Bertrand competition: [12]

$$\begin{aligned} &p_1^L - c_1 = \tfrac{1}{\gamma}\left[(\alpha - c_1)(\gamma - x)\right]; \quad q_1^L = \tfrac{1}{\gamma}(\alpha - c_1)x; \\ &\pi_1^L = \tfrac{1}{\gamma^2}\left[(\alpha - c_1)^2(\gamma - x)x\right]; \\ &p_2^L - c_2 = q_2^L = \pi_2^L = 0. \end{aligned} \tag{10}$$

[11] Indeed, condition (6) identifies a lower bound for $x$ which lies above the *limit-pricing frontier* for any $\gamma \in (0,1)$ (i.e. the dotted line in Figure 1).

[12] That is, the efficient firm prices good 1 at the maximum level leaving the rival zero-demand for any price of good 2 higher than the rival's marginal cost. Formally, given $\gamma$ and $x < 1$, the best response function of the efficient firm follows the expression given in footnote 9 until prices reach the boundary of the region $\overline{R}_2$, where the demand for good 2 is zero. It kinks thereafter and continues along the boundary, i.e., from eq. (3), $p_1 = \frac{1}{\gamma}\left[p_2 - (1-\gamma)\alpha\right]$. Moreover, the best response function of the inefficient firm, $p_2 = \frac{1}{2}\left[(1-\gamma)\alpha + c_2 + \gamma p_j\right]$, shifts outwards as $c_2$ increases ($x$ decreases). When the efficiency gap is sufficiently high (i.e. $x \leq x^L(\gamma)$), it crosses the best response function of the efficient firm along the boundary of region $\overline{R}_2$. Equation (10) follows from the best response functions above and equation (4).

## 3. COMPARISON OF BERTRAND AND COURNOT EQUILIBRIA

Let us denote with $S_A$ the region of the relevant parameter space where both firms are active under Bertrand competition (the area above the *limit-pricing frontier* in Figure 1), and with $S_L$ the region where Bertrand competition entails a limit-pricing equilibrium (the area between the *limit-pricing* and the *monopoly frontiers*). Singh and Vives (1984) restrict their attention to a portion of region $S_A$. In this section we extend the comparison of Bertrand and Cournot equilibria over the entire space $S_r = S_A + S_L$. We start by comparing the equilibrium prices and quantities.

**Lemma 1 (prices)** *The equilibrium prices of both firms are higher under Cournot than under Bertrand competition over the entire space $S_r$.*

**Lemma 2 (quantities)** *The efficient firm produces more under Bertrand than under Cournot competition over the entire space $S_r$. For the inefficient firm, Bertrand production exceeds Cournot production iff $x > \gamma$ (i.e. the same as condition (6)).*

Lemma 1 confirms the Singh and Vives's ranking of Bertrand and Cournot equilibrium prices over the entire space $S_r$. Namely, under price competition firms perceive a more elastic demand than under quantity competition, and this refrains them from increasing prices. Lemma 2 emphasizes the more selective effect of price competition against the market share of the inefficient firm. While the efficient firm always produces more under Bertrand competition, the inefficient firm produces less under Bertrand than under Cournot competition when the cost asymmetry is sufficiently strong and/or products are close substitutes, inside region $S_A$, as well as over the entire region $S_L$.[13]

Turning to the comparison of the equilibrium profits, we first show that the Singh and Vives's ranking holds over the entire region $S_A$.

[13] More precisely, Lemma 2 shows that the portion of region $S_A$ where both firms produce more under Bertrand than under Cournot coincides with the parameter region considered by Singh and Vives (i.e. $S_{sv}$ in Figure 1).

**Proposition 1 (Singh and Vives 1984)** *If both firms are active under Bertrand competition, they both earn higher profits under Cournot than under Bertrand competition.*

Consider now region $S_L$. Obviously, in this region the inefficient firm's profits are always higher with quantity competition (i.e. positive rather than zero). On the contrary, we prove now that the efficient firm's and the industry profits are higher under Bertrand than under Cournot competition in a significant portion of region $S_L$.

**Proposition 2 (efficient firm's profits)** *For any degree of product differentiation (but $\gamma = 0$), there exists a critical level of cost asymmetry inside region $S_L$, $\widehat{x}(\gamma) = \frac{4\gamma}{8-4\gamma^2+\gamma^4}$, such that the efficient firm's profits are higher (resp. lower) with Bertrand than with Cournot competition when $x < \widehat{x}(\gamma)$ (resp. $x > \widehat{x}(\gamma)$).*

**Proposition 3 (industry profits)** *For any degree of product differentiation (but $\gamma = 0$), there exists a critical level of cost asymmetry inside region $S_L$, $\widetilde{x}(\gamma) = \frac{\gamma(4+\gamma^2)}{8-2\gamma^2+\gamma^4}$, such that industry profits are higher (resp. lower) with Bertrand than with Cournot competition when $x < \widetilde{x}(\gamma)$ (resp. $x > \widetilde{x}(\gamma)$).*[14]

Figure 3 shows the two loci, $\widehat{x}(\gamma)$ and $\widetilde{x}(\gamma)$, and the portions of region $S_L$ where the efficient firm's and the industry profits are higher with Bertrand competition.

[14]Notice that the critical levels $\widehat{x}(\gamma)$ (Proposition 2) and $\widetilde{x}(\gamma)$ (Proposition 3) are both monotonically increasing in $\gamma$, taking values $\widetilde{x}(0) = \widehat{x}(0) = 0$ and $\widetilde{x}(1) < \widehat{x}(1) < 1$. Therefore, by Proposition 2, for any $x < \widehat{x}(1)$, we can always identify a critical degree of product differentiation inside region $S_L$, namely $\widehat{\gamma}(x) = \widehat{x}^{-1}(\gamma)$, such that the efficient firm earns higher profit under Bertrand for $\gamma > \widehat{\gamma}(x)$. Similarly, by Proposition 3, given any $x < \widetilde{x}(1)$, $\widetilde{\gamma}(x) = \widetilde{x}^{-1}(\gamma)$ sets a critical degree of product differentiation inside region $S_L$, such that industry profits are higher under Bertrand for $\gamma > \widetilde{\gamma}(x)$.

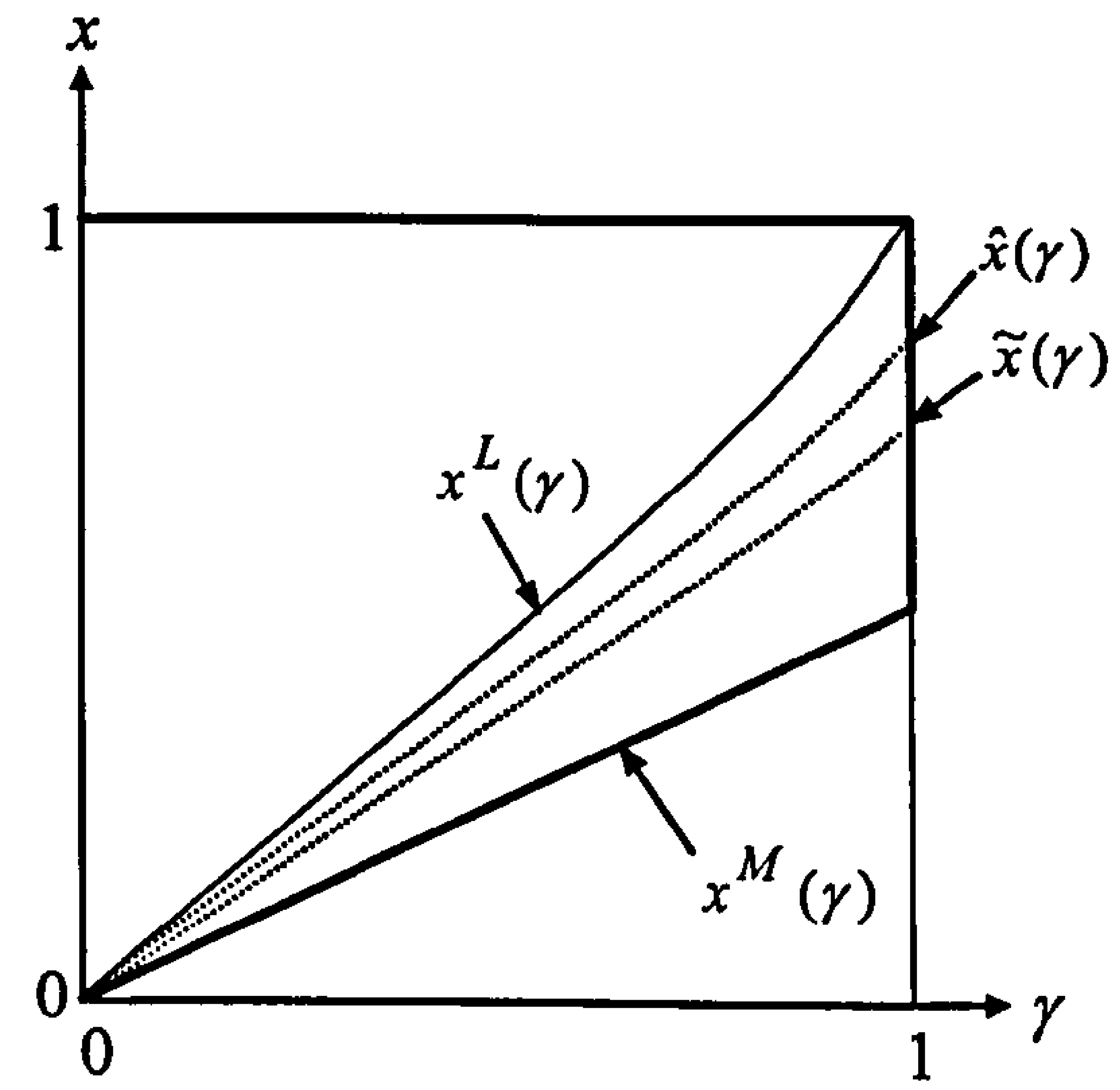


Figure 3

The interpretation of these results relies on the comparison of the effects on prices and market shares exerted by the two forms of competition over regions $S_A$ and $S_L$. On one hand, both firms face lower prices with Bertrand competition over the relevant space of the model, and this tends to make their profits lower under Bertrand than under Cournot competition (*price effect*). On the other hand, when firms are asymmetric in costs, price competition tends to reduce more the market share of the less efficient firm than quantity competition does. Obviously this tends to make the efficient firm's profits higher, and the inefficient firm's profits lower, under Bertrand than under Cournot competition (*selection effect*). Both effects work in the same direction for the inefficient firm, while they operate in opposite directions for the efficient firm. Moreover, the *price effect* weakens, whilst the *selection effect* gets stronger, when either the efficiency gap between the two firms increases or the degree of product differentiation decreases.[15] Accordingly, the *price effect* dominates the

[15]The *selection effect* reaches its maximum intensity on the limit-pricing frontier, where the market share of the inefficient firm is zero under Bertrand but still positive under Cournot. On the other hand, the *price effect* vanishes on the monopoly frontier, where the prices are identical under the two modes of competition (i.e., the monopoly price, $p_1^M$, for the efficient firm, and the marginal cost, $c_2$, for the inefficient firm).

*selection effect* (i.e. the efficient firm gets higher profits under Cournot) for mild degrees of cost asymmetry and/or strongly differentiated products (Proposition 1), whilst the *selection effect* prevails on the *price effect* (i.e. the efficient firm earns higher profits under Bertrand) when the degree of cost asymmetry is sufficiently high and/or products are close substitutes (Proposition 2). Finally, while the *price effect* works towards lower industry profits under Bertrand than under Cournot, the stronger selective mode of price competition relocates production from the inefficient to the efficient firm, working towards higher industry profits under Bertrand (*productive efficiency effect*). The *productive efficiency effect* dominates the *price effect* (i.e. industry profits are higher under Bertrand) for high degrees of cost asymmetry and/or low degrees of product differentiation (Proposition 3).

The dominance of the *selection* and *productive-efficiency effects* over the *price effect* close to the monopoly frontier can be easily perceived by the following argument. Consider a slight reduction of the efficiency gap (i.e. a small increase in $x$) or a slight increase in product differentiation (i.e. a small decrease in $\gamma$) starting from the monopoly frontier. Under Bertrand competition, the efficient firm is now forced by outside competition to price below the monopoly level. However, since the equilibrium price falls just below the monopoly price, the negative effect on its profits is only second order. In contrast, the inefficient firm holds a (slightly) positive market share under Cournot competition. Whilst the efficient firm's price decreases less than under Bertrand, the reduction in the market share of the efficient firm causes a first order negative effect on its profits. As a consequence, the efficient firm's profits are greater under Bertrand. Furthermore, being close to the monopoly frontier, the inefficient firm is forced to price at a level almost equal to its marginal cost under Cournot, so that the positive effect on its profits is second order as well. Therefore, industry profits are also greater under Bertrand.

### 3.1. Welfare.—

We conclude this section with the welfare comparison of Bertrand and Cournot equilibria over the relevant space $S_r$. From the utility function, total surplus equals

$$TS = (\alpha - c_1) q_1 + (\alpha - c_2) q_2 - \left[ \frac{1}{2} (q_1 + q_2)^2 - (1 - \gamma) q_1 q_2 \right], \quad (11)$$

while consumer surplus is given by the difference between total surplus and industry profits. Clearly, total surplus increases with quantities (as far as the marginal utility of both goods exceeds the respective marginal cost), while consumer surplus always decreases with prices. As shown by Lemma 2, Singh and Vives restrict attention to the portion of region $S_A$ where both firms produce more under Bertrand than under Cournot competition. Since prices are lower under Bertrand, they can easily conclude that both total surplus and consumer surplus are larger under Bertrand competition. Looking at the other portions of the relevant space, since prices are lower under Bertrand everywhere (see Lemma 1), the Singh and Vives's ranking of consumer surplus holds over the entire space $S_r$. This implies immediately that total surplus is also larger under Bertrand over the portion of region $S_L$ where price competition entails higher industry profits (see Proposition 3). However, the ranking of total surplus is not immediate either over the portion of region $S_A$ where the inefficient firm produces more under Cournot, or over the portion of region $S_L$ where industry profits are higher under Cournot.[16] Nevertheless, the following proposition shows that the Singh and Vives's ranking of total surplus extends over the entire space $S_r$.

**Proposition 4 (welfare)** *Total surplus is higher under Bertrand than under Cournot competition over the entire space $S_r$.*

Summarizing, while consumers always gain from a switch from Cournot to Bertrand competition, for low degrees of cost asymmetry (and/or high degrees of product differentiation) both firms lose since the *price effect* dominates the *productive efficiency effect*. The total

[16] Notice that, with imperfect substitute goods and asymmetric costs, the comparison of equilibrium prices is not conclusive for the ranking of total surplus, since lower equilibrium prices do not necessarily imply higher equilibrium quantities for both goods.

surplus rises because the increase in consumer surplus exceeds the decrease in industry profits. In contrast, when the cost asymmetry is high (and/or products are close substitutes), a switch from Cournot to Bertrand competition increases both consumer surplus and industry profits, since now the overall effect on industry profits reflects the dominance of the *productive efficiency effect* over the *price effect*.

## 4. PRODUCT DIFFERENTIATION AND THE MARKET LEADER'S PROFIT

Our characterization of Bertrand and Cournot equilibria over the entire space $S_r$ reveals immediately that, in the presence of cost asymmetry, the efficient firm's profits may be non-monotonic in the degree of product differentiation under both forms of competition. In fact, for sufficiently high degrees of cost asymmetry (i.e. low values of $x$), the efficient firm earns the monopoly profits either for $\gamma = 0$ or along the monopoly frontier. Since the profit functions are continuous under both forms of competition, in both cases the efficient firm's profits must be non-monotonic in $\gamma$ for high levels of cost asymmetry. In contrast, the standard result with symmetric costs is that, under both forms of competition, profits always decrease as products become less differentiated (see Shy (1995), pp. 138-140).

The following two lemma provide a complete characterization of the behavior of firms' equilibrium profits with respect to the degree of product differentiation, under the two forms of competition.

**Lemma 3** *Under Cournot competition, for any* $x \in (0, 1]$, *the inefficient firm's profits always decrease as* $\gamma$ *increases from* 0 *to* $min\{\gamma^M(x), 1\}$, *where* $\gamma^M(x) \equiv x^{M^{-1}}(\gamma)$. *On the contrary, there exist a threshold level* $x^C \in (x^M(1), 1)$ *and a critical locus* $\gamma^C(x) \in (0, \min\{\gamma^M(x), 1\})$, *such that the efficient firm's profits increase with* $\gamma$ *if* $x < x^C$ *and* $\gamma > \gamma^C(x)$, *and decrease with* $\gamma$ *otherwise.*

**Lemma 4** *Under Bertrand competition, for any* $x \in (0, 1]$, *the inefficient firm's profits always decrease as* $\gamma$ *increases from* 0 *to* $\gamma^L(x) \equiv x^{L^{-1}}(\gamma)$. *On the contrary, for any*

$x \in (0,1)$, *there exists a critical value* $\gamma^B(x) \in (0, \gamma^L(x))$, *such that the efficient firm's profits decrease with* $\gamma$ *from* 0 *to* $\gamma^B(x)$, *while they increase with* $\gamma$ *from* $\gamma^B(x)$ *to* $min\{\gamma^M(x), 1\}$.

As a consequence of Lemma 3 and 4, the conventional result that Bertrand and Cournot duopolists always gain from product differentiation should be amended in the presence of cost asymmetries. Indeed, when the degree of cost asymmetry is sufficiently high and/or products are initially close substitutes, the efficient firm may have a local incentive to reduce the degree of product differentiation. This is more likely to happen with Bertrand than with Cournot competition, since, under Bertrand, the efficient firm's profits are non-monotonic in $\gamma$ for any positive degree of cost asymmetry (Lemma 4), while a sufficient degree of cost asymmetry is required under Cournot (Lemma 3).

The intuition behind these results is as follows. A higher degree of product substitutability lowers the demand functions of both goods, as consumers value less any bundle of the two products in terms of the numeraire good. If firms are symmetric in costs, this leads to a new symmetric equilibrium in which firms earn lower profits under both forms of competition. However, in the presence of cost asymmetry, the demand function of the inefficient firm is shifted down more since consumers substitute the higher priced good for the lower priced one.[17] Under Cournot competition the efficient firm benefits from the larger reduction of the rival's production, which counteracts the negative effect exerted by the increase in $\gamma$ on its inverse demand function. For sufficiently high degrees of cost asymmetry, the positive effect will eventually prevail when products become sufficiently close substitutes. Under Bertrand competition, the larger decrease in the rival's demand function allows the efficient firm to capture more of the rival's demand at any given price. This counteracts the negative effect due to the adverse shift in its demand function. Now, for any degree of cost asymmetry, the positive effect will eventually prevail when products are sufficiently close substitutes.

[17] By equation (2), given an initial pair of prices, the negative shift in the demand function of each firm is inversely related to the rival's price.

## 5. CONCLUSIONS

In this chapter we have re-considered the comparison of Bertrand and Cournot competition within the standard model of a horizontally differentiated duopoly with linear demand and cost functions. Our main innovation with respect to Singh and Vives (1984) has been to enlarge the parameter space by allowing for any relevant combination of cost asymmetry and product differentiation. Our main result, that the efficient firm's and industry profits are higher under Bertrand competition for high degrees of cost asymmetry and/or low degrees of product differentiation, contrasts with the Singh and Vives's ranking of the equilibrium profits under the two forms of competition.

The intuition for this result relies on the stronger selective mode of price competition (*selection effect*), which also entails greater productive efficiency (*productive efficiency effect*). The selection and the productive efficiency effects work towards higher profits for the efficient firm and for the industry under Bertrand than under Cournot competition, contrasting the opposite effect due to the lower equilibrium prices arising under Bertrand (*price effect*). With high degrees of cost asymmetry and/or low degrees of product differentiation, the selection and the productive efficiency effects dominate the price effect, inverting the standard ranking of profits under the two forms of competition. We have also shown that, because of cost asymmetry, the efficient firm's profits may be non-monotonic in the degree of product differentiation under both modes of competition, so that a local incentive towards less differentiation may arise.

Our results have immediate applications in several strands of literature. Firstly, the IO literature that analyses the effect of product market competition on firms' incentive to innovate.[18] Both the inversion of the profits ranking between the two modes of competition, and the leader's local incentive towards less product differentiation, can significantly affect firms' incentive to innovate in multi-stage models with product or process innovation at the early stages. For instance, as shown in chapter 2, in a single innovation setting, firms'

[18] Among others, Delbono and Denicolò (1990); Bester and Petrakis (1993); Qiu (1998); Aghion and Shankerman (2000); Boone (2000).

incentive to innovate is positively related to the technological leader post-innovation profit, whereas with cumulative innovations, the incentive to innovate is linked to industry profits. Our result on the reversal of both leader's and industry profits ranking as a function of the degree of cost asymmetry (i.e. the size of innovations) and degree of product differentiation can generate interesting results in both settings. Similarly, the leader's local incentive towards less differentiation has immediate implications in a setting where process innovations generate asymmetry among firms and the degree of differentiation is endogenous, as we have briefly suggested in the conclusion of the previous chapter.

A second area of application is the literature that endogenises the strategic variable and the timing of competition in oligopolistic games (e.g. Singh and Vives (1984), Hamilton and Slutsky (1990)). For instance, using their ranking of profits between the two modes of competition, Singh and Vives (1984) show that both firms would select to quantity competition as a dominant strategy in a two stage game where they simultaneously choose the strategic variable at the first stage, and then compete in the market according to the resulting combination of strategic variables. The reversal of the efficient firm's profit ranking immediately implies that quantity is not a strictly dominant strategy for that firm when cost asymmetry and product substitutability are sufficiently high. Whether price competition can arise in equilibrium is not immediately clear, depending on the ranking of profits associated with the mixed profiles of strategic variables (i.e. price for one firm, quantity for the other). This issue is left for future research.

Finally, the reversal of the efficient firm's profit can find interesting applications in the literature on tacit collusion under the two modes of competition (e.g. Deneckere (1983), Majerus (1988), Lambertini (1997)). More specifically, modelling the demand side of the market as in Singh and Vives (1984) but assuming symmetric firms, Deneckere (1983) proves that collusion is more (resp. less) sustainable in a Bertrand supergame than in a Cournot supergame when products are weakly (resp. strongly) differentiated. Allowing for cost asymmetry in this setting, is likely to generates different results. Indeed, the reason why, in Deneckere, collusion is more sustainable with Bertrand when products are close substitutes is closely related to the standard ranking of profits between the two forms of competition.

Although the short-run deviation profit is higher under Bertrand than under Cournot, the long-run punishment is more severe under Bertrand (i.e. both firms earn lower profits in Bertrand than in Cournot one-shot game), and this second effect prevails. Now, as a consequence of our result, a sufficient degree of cost asymmetry reverses the ranking of the efficient firm's punishments from deviating under the two modes of competition. Further, with cost asymmetry, the condition for collusion sustainability is likely to reflect more the incentive to deviate of the efficient firm than that of the inefficient firm. On this basis, we guess that the Denekere's result with close substitutes may reverse along the dimension of cost asymmetry. Also the proof of this conjecture is left for future research.

# APPENDIX

## Proof of Lemma 1.—

Using equations (7) and (8), over region $S_A$ we get:

$$p_1^C - p_1^B = \frac{\gamma^2}{4-\gamma^2}(\alpha - c_1)$$

$$p_2^C - p_2^B = \frac{\gamma^2}{4-\gamma^2}(\alpha - c_1)\,x.$$

Both expressions are positive for $\gamma, x \in (0,1]$ (and hence, everywhere over region $S_A$). Turning to region $S_L$, for the inefficient firm we clearly have $p_2^C > p_2^L = c_2$. For the efficient firm, equations (7) and (10) yield:

$$p_1^C - p_1^L = \frac{(2-\gamma^2)(\alpha - c_1)}{\gamma(4-\gamma^2)}[2x - \gamma].$$

This expression is positive for $\gamma \in (0,1]$ and $x > \frac{\gamma}{2} = x^M(\gamma)$ (that is, everywhere over region $S_L$).

■

## Proof of Lemma 2.—

From equations (7) and (8), over region $S_A$ we get:

$$q_1^B - q_1^C = \frac{\gamma^2(\alpha-c_1)}{(1-\gamma^2)(4-\gamma^2)}[1 - \gamma x]$$

$$q_2^B - q_2^C = \frac{\gamma^2(\alpha-c_1)}{(1-\gamma^2)(4-\gamma^2)}[x - \gamma].$$

The first expression is positive for $\gamma, x \in (0,1)$. From the second expression we find immediately that, for any $\gamma \in (0,1)$, $q_2^B - q_2^C \gtreqless 0 \iff x \gtreqless \gamma$. For $x = 1$ and $\gamma \to 1$, the model approaches a standard homogeneous duopoly with linear-symmetric cost functions (i.e. $c_2 = c_1$), and both expressions converge to the positive limit $\frac{\gamma^2(\alpha-c_1)}{(1-\gamma^2)(4-\gamma^2)}$. Turning to region $S_L$, for the inefficient firm we clearly have $q_2^C > q_2^L = 0$. For the efficient firm, equations (7) and (10) yield:

$$q_1^L - q_1^C = \frac{2(\alpha - c_1)}{\gamma(4-\gamma^2)}[2x - \gamma].$$

This expression is positive for $\gamma \in (0,1]$ and $x > \frac{\gamma}{2} = x^M(\gamma)$ (that is, everywhere over region $S_L$).

■

**Proof of Proposition 1.—**

Notice that $\gamma, x \in (0,1)$ over region $S_A$, setting aside point $(x=1, \gamma=1)$ where products are homogeneous and firms are symmetric. Clearly, in that point both firms earn higher profits under Cournot.

Consider first firm 1. Using equations (7) and (8), $\pi_1^C \geq \pi_1^B$ leads to:

$$\gamma^3[\gamma x^2 - 2x + \gamma] \leq 0.$$

Given any $\gamma \in (0,1)$, this inequality holds for $x \in [\underline{x}(\gamma), \overline{x}(\gamma)]$, where $\underline{x}(\gamma) = \frac{1}{\gamma}(1-\sqrt{1-\gamma^2})$ and $\overline{x}(\gamma) = \frac{1}{\gamma}(1+\sqrt{1-\gamma^2})$. Now, $\overline{x}(\gamma) > 1$ for $\gamma \in (0,1)$. Moreover, $\underline{x}(\gamma)$ lies below the limit-pricing frontier. Indeed, $\underline{x}(\gamma) < x^L(\gamma)$ leads to the inequality $\sqrt{1-\gamma^2}(1-\sqrt{1-\gamma^2})^2 > 0$, which is satisfied for any $\gamma \in (0,1)$. Hence, $S_A \subset [\underline{x}(\gamma), \overline{x}(\gamma)]$, so that $\pi_1^C > \pi_1^B$ everywhere over region $S_A$.

Turning to firm 2, from equations (7) and (8) we find that $\pi_2^C \geq \pi_2^B$ leads exactly to the same inequality $\gamma^3[\gamma x^2 - 2x + \gamma] \leq 0$. ■

**Proof of Proposition 2.—**

Notice first that $\gamma, x \in (0,1]$ over region $S_L$. Using equations (7) and (10), $\pi_1^L \geq \pi_1^C$ is equivalent to

$$\left(16 - 8\gamma^2 + 2\gamma^4\right)x^2 - \gamma\left(16 - 4\gamma^2 + \gamma^4\right)x + 4\gamma^2 \leq 0.$$

Given any $\gamma \in (0,1]$, this inequality is satisfied for $x \in \left[x^M(\gamma), \widehat{x}(\gamma)\right]$, where $x^M(\gamma) = \frac{\gamma}{2}$ is the monopoly frontier, and $\widehat{x}(\gamma) = \frac{\gamma}{2-\gamma^2+\frac{\gamma^4}{4}}$.

Since $x^M(\gamma) < \widehat{x}(\gamma) < x^L(\gamma)$ $(= \frac{\gamma}{2-\gamma^2})$ for any $\gamma \in (0,1]$, the critical level $\widehat{x}(\gamma)$ always lies inside region $S_L$. It is also easy to verify that $\widehat{x}(\gamma)$ monotonically increases with $\gamma$. ■

**Proof of Proposition 3.—**

From equations (7) and (10), Bertrand and Cournot industry profits over region $S_L$ are, respectively:

$$\pi^L = \frac{(\alpha-c_1)^2}{\gamma^2}[\gamma x - x^2]$$

$$\pi^C = \frac{(\alpha-c_1)^2}{(4-\gamma^2)^2}\left[x^2\left(4+\gamma^2\right) - 8\gamma x + \left(4+\gamma^2\right)\right].$$

Using these expressions, $\pi^L \geq \pi^C$ is equivalent to

$$\left[16 - 4\gamma^2 + 2\gamma^4\right] x^2 - \gamma\left(16 + \gamma^4\right) x + \gamma^2\left(4 + \gamma^2\right) \leq 0.$$

Solving in $x$, we find this inequality satisfied for $x \in \left[x^M(\gamma), \widetilde{x}(\gamma)\right]$, where $x^M(\gamma) = \frac{\gamma}{2}$ is the monopoly frontier, and $\widetilde{x}(\gamma) = \frac{\gamma(4+\gamma^2)}{8-2\gamma^2+\gamma^4}$.

It is easy to verify that $x^M(\gamma) < \widetilde{x}(\gamma) < x^L(\gamma)$ for any $\gamma \in (0,1]$, so that $\widetilde{x}(\gamma)$ always lies inside region $S_L$. Also, $\widetilde{x}(\gamma)$ is monotonically increasing in $\gamma$. ■

### Proof of Proposition 4.—

Consider first region $S_A$. Notice that $\gamma, x \in (0,1)$ over region $S_A$, setting aside point $(x = 1, \gamma = 1)$. In this point, products are homogeneous and firms are symmetric, and the welfare comparison between Bertrand and Cournot equilibria is standard.

From equations (7) and (11), the total surplus in Cournot equilibrium is:

$$TS^C = \left(\frac{\alpha - c_1}{4-\gamma^2}\right)^2 \left[(6 - \tfrac{1}{2}\gamma^2)x^2 - \left(8\gamma - \gamma^3\right)x + (6 - \tfrac{1}{2}\gamma^2)\right].$$

Similarly, from equations (8) and (11), in Bertrand equilibrium we have

$$TS^B = \left(\frac{(\alpha - c_1)}{(1-\gamma^2)(4-\gamma^2)}\right)^2 \left[\begin{array}{c} (6 - \frac{21}{2}\gamma^2 + \frac{11}{2}\gamma^4 - \gamma^6)x^2 - \left(8\gamma + 3\gamma^5 - 11\gamma^3\right)x + \\ +(6 - \frac{21}{2}\gamma^2 + \frac{11}{2}\gamma^4 - \gamma^6) \end{array}\right].$$

Imposing $TS^B > TS^C$, implies

$$(2\gamma^2 - \tfrac{3}{2}\gamma^4 - \tfrac{1}{2}\gamma^6)x^2 - \left(6\gamma^3 - 7\gamma^5 + \gamma^7\right)x + (2\gamma^2 - \tfrac{3}{2}\gamma^4 - \tfrac{1}{2}\gamma^6) > 0.$$

The discriminant of the inequality above,

$$\Delta = \gamma^4\left(60\gamma^2 + 55\gamma^6 - 85\gamma^4 + \gamma^{10} - 15\gamma^8 - 16\right),$$

is negative for any $\gamma \in (0,1)$, whilst $2\gamma^2 - \frac{1}{2}\gamma^6 - \frac{3}{2}\gamma^4 > 0$. This suffices to prove that the inequality is always satisfied for $\gamma, x \in (0,1)$.

Consider now region $S_L$. The total surplus under Cournot competition is still given by the expression above. Under Bertrand competition, equations (10) and (11) yield

$$TS^L = \frac{(\alpha - c_1)^2}{\gamma^2}\left[\gamma x - \tfrac{1}{2}x^2\right].$$

Imposing $TS^L \geq TS^C$, implies

$$-\left(8+2\gamma^2\right)x^2+16\gamma x-\tfrac{1}{2}\gamma^2(12-\gamma^2)\geq 0.$$

Given any $\gamma \in (0,1]$, this inequality is satisfied for $x \in \left[x^M(\gamma), x^{TS}(\gamma)\right]$, where $x^M(\gamma)=\frac{\gamma}{2}$ is the monopoly frontier, and $x^{TS}(\gamma)=\frac{\gamma}{2}\frac{12-\gamma^2}{4+\gamma^2}$. It is easy to verify that $x^{TS}(\gamma)>x^L(\gamma)$ $(=\frac{\gamma}{2-\gamma^2})$. This means that $S_L \subset \left[x^M(\gamma), x^{TS}(\gamma)\right]$, so that $TS^L > TS^C$ over region $S_L$. ■

### Proof of Lemma 3.—

From equation (7), we get:

$$\frac{\partial \pi_2^C}{\partial \gamma}=\frac{2(\alpha-c_1)\sqrt{\pi_2^C}}{(4-\gamma^2)^2}\left[-(4+\gamma^2)+4x\gamma\right],$$

$$\frac{\partial \pi_1^C}{\partial \gamma}=\frac{2(\alpha-c_1)\sqrt{\pi_1^C}}{(4-\gamma^2)^2}\left[-x(4+\gamma^2)+4\gamma\right].$$

From the first expression, $\frac{\partial \pi_2^C}{\partial \gamma}$ is strictly negative as far as $\pi_2^C>0$ (it equals zero on the monopoly frontier, where $\pi_2^C=0$). From the second expression, $\frac{\partial \pi_1^C}{\partial \gamma} \gtreqless 0$ is equivalent to $x \lesseqgtr \frac{4\gamma}{4+\gamma^2} \equiv x^C(\gamma)$. It is easy to verify that $x^C(\gamma)$ is monotonically increasing in $\gamma$, taking values $x^C(0)=0$ and $x^C(1)=\frac{4}{5}<1$. Moreover, $x^C(\gamma)>x^M(\gamma)=\frac{\gamma}{2}$ for any $\gamma \in (0,1)$ (i.e. $x^C(\gamma)$ always lies inside the relevant space $S_r$). This suffices to prove that, for any $x \in (0,\frac{4}{5})$, the efficient firm's profits decrease as $\gamma$ rises from 0 to $\gamma^C(x)=x^{C^{-1}}(\gamma)$, while they increase as $\gamma$ rises from $\gamma^C(x)$ to the minimum between 1 and $\gamma^M(x)=x^{M^{-1}}(\gamma)$. ■

### Proof of Lemma 4.—

We start with firm 1's profits over region $S_A$. From equation (8), we get:

$$\frac{\partial \pi_1^B}{\partial \gamma}=\frac{2(\alpha-c)(2-\gamma^2-\gamma x)}{(4-\gamma^2)^3(1-\gamma^2)^2}\left[\gamma\left(4-2\gamma^2+\gamma^4\right)-x\left(4+\gamma^2-2\gamma^4\right)\right].$$

The first term of this expression is strictly positive for $x,\gamma \in (0,1)$. Then, $\frac{\partial \pi_1^B}{\partial \gamma} \gtreqless 0$ is equivalent to $x \lesseqgtr \frac{\gamma(4-2\gamma^2+\gamma^4)}{(4+\gamma^2-2\gamma^4)} \equiv x^B(\gamma)$. It is easy to verify that $x^B(\gamma)$ is monotonically increasing in $\gamma$, taking the values $x^B(0)=0$ and $x^B(1)=1$. Moreover, $x^B(\gamma)>x^L(\gamma)=\frac{\gamma}{2-\gamma^2}$ for any $\gamma \in (0,1)$ (i.e. $x^B(\gamma)$ always lies inside region $S_A$). This suffices to prove that, for any $x \in (0,1)$, the efficient firm's profits decrease as $\gamma$ rises from 0 to $\gamma^B(x)=x^{B^{-1}}(\gamma)$, while they increase as $\gamma$ rises from $\gamma^B(x)$ to $\gamma^L(x)=x^{L^{-1}}(\gamma)$.

Turning to region $S_L$, from equation (10) we obtain:

$$\frac{\partial \pi_1^L}{\partial \gamma} = \frac{(\alpha-c)^2}{\gamma^3}[x(2x-\gamma)],$$

which is positive for any $x > x^M(\gamma) = \frac{\gamma}{2}$ (that is, everywhere over region $S_L$).

Consider now firm 2's profits over region $S_A$. From equation (8), we get

$$\frac{\partial \pi_2^B}{\partial \gamma} = \frac{2(\alpha-c)\left((2-\gamma^2)x-\gamma\right)}{(4-\gamma^2)^3(1-\gamma^2)^2}\left[x\gamma\left(4-2\gamma^2+\gamma^4\right)-\left(4+\gamma^2-2\gamma^4\right)\right].$$

The first term of this expression is positive for $1 \leq x < x^L(\gamma) = \frac{\gamma}{2-\gamma^2}$ (i.e. over region $S_A$). Then, $\frac{\partial \pi_2^B}{\partial \gamma} \leq 0$ is equivalent to $x \leq \frac{\left(4+\gamma^2-2\gamma^4\right)}{\gamma(4-2\gamma^2+\gamma^4)} = \frac{1}{x^B(\gamma)}$. Since $x^B(\gamma) \in [0,1]$, its reciprocal is always greater than 1 (it equals 1 only for $\gamma = 1$), and the inequality above is always satisfied. ■

# Chapter 4

# Vertical integration and product innovation

## 1. INTRODUCTION

One of the most controversial issues in industrial organization is market foreclosure through vertical integration.[1] Though one major advantage of vertical integration is to eliminate the problem of double marginalization, the major criticism against vertical integration is market foreclosure, which has generated a significant amount of literature to examine the competitive structure of the upstream and downstream industries and welfare.[2] However,

[1]Vertical foreclosure refers to restrictions in supply (demand) of an essential input an integrated firm would apply to its downstream (upstream) competitors, extending in this way its market power in a related market. The negative consequence in terms of welfare would arise from a reduction in consumers' welfare due to higher prices and lower quantities of the final goods.

[2]Contrary to the benign view of the so-called Chicago school (e.g. Bork (1978)), denying vertical foreclosure as an equilibrium consequence of vertical mergers, subsequent works have proved vertical foreclosure in different models of vertical integration. Salinger (1988) shows that a vertical merger in a successive oligopoly with quantity competition, causes the integrated firm to withdraw from the intermediate good market. The increased concentration raises the intermediate good price and hence the production cost of the integrated firm's downstream rivals, whose market shares fall. However, due to the avoidance of double marginalisation, the effect on the final good price is ambiguous. Riordan (1998) models a vertical merger between an upstream supplier and a dominant downstream firm. In this model, vertical integration always leads to higher prices for both the intermediate and the final products. Ordover, Saloner and Salop (1990) consider a successive duopoly model in which an integrating downstream firm is required to outbid the rival for the acquisition of an upstream supplier, and the two unintegrated firms (one upstream and one downstream) may react to the threat of foreclosure by vertically integrating as well. Vertical foreclosure arises in equilibrium provided that the increase in the input price following a vertical merger increases the joint-profit of the two unintegrated firms (whilst the individual profit of the unintegrated downstream firm always decreases), what can arise in the case of differentiated product price competition. Finally, in the recent literature following the incomplete contracts approach of Hart and Tirole (1990), market foreclosure is strictly linked to vertical integration, the latter being a means to solve the upstream firm's commitment problem of not expanding input sales in the downstream market after a contract has been signed with some costumers at monopolistic conditions.

surprisingly enough the previous literature on vertical integration has mainly concentrated on the productive activities of the firms, paying less attention to the non-productive activities of the firms and particularly, for the downstream firms. [3]

In this chapter, we consider product innovation in the downstream market as a strategic device of downstream firms facing a threat of vertical integration and market foreclosure by an upstream monopolist. We examine how horizontal product differentiation in the downstream market affects the incentive for vertical integration and market foreclosure, and how the possibility of vertical integration affects the downstream firms' incentives to differentiate products.

We use a simple model in which, without vertical integration, an upstream monopolist charges a linear price on the sole input required by two downstream firms in order to produce the final product. In the downstream market, firms compete in quantities. In this setting, vertical integration of the upstream firm with one of the two downstream firms eliminates double marginalisation in a segment of the final product market, giving the integrated firm a competitive advantage over the downstream rival. Moreover, by setting the input price, the integrated firm affects the downstream rival's costs, and it may choose to foreclose the market (i.e. monopolise the final product market).

The vertical structure of the market (i.e. vertical integration *vs* no-vertical integration) is endogenously determined by an integration game modeled as a sale-auction between the downstream firms. If the gain from vertical integrating exceeds a fixed integration cost, the upstream firm calls for offers by the downstream firms in order to integrate one of them. Then, competition between the downstream firms in the integration game allows the

See Rey and Tirole (2004) for a comprehensive survey of this literature.

[3]One exception is Baake, Ulrich and Norman (2004). They consider capital investment as a non-productive strategic decision of the upstream firm in the Hart and Tirole (1990) model. They show that banning vertical integration has the social cost of a sub-optimal level of capital investment, leading to productive inefficiency in the market. In contrast, vertical integration assures the efficient level of capital investment but output is monopolistically restricted. In this chapter, we take a different perspective by giving the downstream firms a non-productive strategic variable (i.e product innovation) in a vertical integration game.

upstream firm to appropriate more than the full surplus from integration, and reap most of the profit created in the final product market.[4] Therefore, vertical integration is a threat to the downstream firms at the initial stage of the game, when they can invest to differentiate products.

Besides the usual effect of softening competition in the downstream market, product innovation exerts two more effects in our model. Product differentiation eliminates market foreclosure under vertical integration, and it affects the possibility of vertical integration.

The elimination of market foreclosure encourages innovation in the downstream market. However, the trade-off between the benefits from eliminating market foreclosure and softening product market competition, on one hand, and the loss from vertical integration, on the other hand, makes the impact of vertical integration on innovation ambiguous. In fact, we show that whether vertical integration is more likely for higher or lower degrees of product differentiation is ambiguous and depends on the cost of integration. If the cost of integration is very small, vertical integration occurs always. If the cost of integration is moderate, vertical integration occurs for very small and very large degrees of product differentiation. If the cost of integration is sufficiently large, but not large enough to prevent

[4]Relative to the recent literature on vertical integration following Hart and Tirole (1990), our model relocates upstream firm's bargaining power from the bargaining game for the essential input to the vertical integration game. In the Hart and Tirole model (*ex-post monopolisation variant*), the upstream monopolist reaps all the industry profit by making take-or-leave-it offers of non-linear tariffs for the essential input to the downstream firms. Vertical integration (and vertical foreclosure) solves the upstream firm's commitment problem of not selling more input to other downstream firms at the disadvantage of those who bought the good at monopoly conditions at the first place (without vertical integration, the downstream firms refuse to buy at monopoly conditions in anticipation of the upstream firm's myopic incentive to sell more ex-post). Therefore, vertical integration consents the upstream firm to realise a higher industry profit (i.e. the monopoly profit), not a higher share in the industry profit at the disadvantage of the downstream firms. In our model, the upstream firm sells the input at a linear price, so that, without vertical integration, the downstream firms gain positive profits. On the other hand, the upstream firm extracts profits from the downstream firms through vertical integration, by putting them in competition to be integrated. Obviously this requires that the downstream firms cannot commit to a collusive behaviour when facing the call for integration offers from the upstream firm.

vertical integration, then vertical integration occurs for very large degrees product differentiation. Therefore, while higher product differentiation softens competition in the final goods market, it may also create the threat of vertical integration, which helps the upstream firm to extract more rent from the downstream firms. As a consequence, there are situations where the downstream market prefers relatively lower degrees of product differentiation to prevent vertical integration. So, instead of market foreclosure, we show a new possible cost of vertical integration, i.e., lower product innovation.

The rest of the chapter is organised as follows. In Section 2, we present the model, which consists of a three-stages game with the following timing: innovation stage (first stage), integration stage (second stage), market stage (final stage). In Section 3, we solve the market stage under the two alternative market structures (vertical integration *vs.* no-vertical integration), and we discuss the effect of product differentiation on market foreclosure. Section 4 analyses the vertical integration game, and shows how the vertical integration outcome depends on product differentiation and integration costs. In Section 5, we study the effects of vertical integration on product innovation. In Section 6 we point out that the possibility of vertical integration can cause the social cost of less product innovation. Finally, Section 7 provides some concluding remarks.

## 2. THE MODEL

We consider an economy with upstream and downstream markets. In the upstream market, a monopolist (firm $U$) produces the sole input needed by two downstream firms (firms $D_1$ and $D_2$) in order to produce their final products. We assume that the upstream monopolist produces the essential input at zero-costs. The downstream firms share the same production technology, which requires one unit of input in order to produce one unit of final product.

Although the downstream firms use the same production technology, their final product can be differentiated at the outset by investing in R&D. More precisely, on the demand side of the downstream market, the degree of product substitutability perceived by consumers,

$\gamma$, leads to the inverse demand system:

$$p_i = a - q_i - \gamma q_j \quad (i, j = 1, 2; \ i \neq j), \tag{1}$$

where $\gamma \in [0, 1]$.[5] With $\gamma = 0$, consumers perceive products 1 and 2 as independent goods, while $\gamma = 1$ corresponds to consumers' perception of perfect substitutes.

The perceived degree of product substitutability depends on the downstream firms' R&D effort. We set aside any strategic consideration related to R&D competition between the downstream firms. The strategic effects involved in R&D competition for product differentiation within the same duopoly model we adopt here for the downstream market, are already analyzed by Lambertini and Rossini (1998). Due to the positive externality exerted by the R&D investment of each firm on the rival's profit, a simultaneous R&D game can lead to a prisoner dilemma equilibrium. Therefore, final products can remain homogeneous even if product differentiation would increase both firms' profits.

Since R&D competition strongly complicates our model without qualitatively affecting the nature of the effects of vertical integration on product innovation (and so, our main conclusions), we adopt the simplifying assumption that only one firm can invest in R&D. By paying a fixed R&D cost $k$, the innovative firm can reduce the perceived degree of product substitutability from $\gamma = 1$ (perfect substitutes goods) to $\gamma = \widehat{\gamma} \in [0, 1)$. Without the R&D investment, products 1 and 2 are perceived as perfect substitutes by consumers.

Notice that, whilst this assumption can be justified by only one downstream firm having the capability to alter consumers' perception of product substitutability, an alternative interpretation of our model is that the downstream firms can cooperate at the R&D stage of the game. Under this interpretation, firms solve the incentive problem arising from the symmetric effect of product differentiation on the demand of both goods by reaching an agreement on the joint R&D effort. Clearly, with cooperation, the joint R&D effort will depend on the joint gain arising from product differentiation, whereas one firm's R&D effort

[5]The demand side of the downstream market is a simplified version of Singh and Vives (1984). The inverse demand sistem (1) is generated by the utility function: $U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$, where $m$ is a numeraire good.

depends only on its individual gain. However, since the downstream firms turn out to be symmetric in any respect at the innovative stage of the model, the assessment of the effect exerted by vertical integration on product differentiation is qualitatively the same.[6]

After the R&D decision is taken, the vertical integration game takes place. The upstream firm may call for (simultaneous and independent) price-offers by the downstream firms in order to integrate one of them. On the basis of the offers received, the upstream firm decides whether or not to integrate the downstream firm asking for the lowest price (in the case of tie, we assume that both downstream firms have fifty percent probability of merging with the upstream firm). We further assume that vertical integration involves a fixed cost, denoted by $E$.[7]

The outcome of the integration game sets the structure of the markets. If vertical integration does not occur, the upstream monopolist supplies the essential resource to the downstream firms charging a linear price $w_u$.[8] The input price acts as the marginal cost

[6] As we will see below, the downstream firms share identical profit expectations under any subsequent evolution of the game (i.e. vertical integration vs. no-vertical integration). Hence, the identity of the innovative firm is irrelevant. Moreover, the joint gain the two firms derive from product differentiation is always twice the individual gain of the innovative firm. Therefore, the joint incentive to differentiate products if the downstream firms can cooperate in R&D - as measured by the highest joint-R&D cost they are willing to pay in order to obtain any given degree of product differentiation - is always twice the innovative firm's incentive. Hence, our results apply immediately to the case of R&D cooperation by simply scaling-up the measure of the incentive to invest in R&D.

[7] See Hart and Tirole (1990) for the interpretation of the cost of vertical integration. Also similarly to Hart and Tirole (1990), we assume away the possibility of merger between the downstream firms. Hence, if vertical integration occurs, firm $U$ can integrate with one downstream firm only. This can be justified by the cost of integrating both downstream firms being prohibitive.

[8] Our assumption of linear pricing for the input is similar to Choi (1991), Gerstner and Hess (1995), Colangelo (1995), Economides (1998), Villas-Boas (1998), Tyagi (1999), Rao and Srinivasan (2001) and others. A similar assumption can also be found in the literature on 'access pricing' (e.g., Armstrong et al. (1996), Armstrong and Vickers (1998), De Fraja and Price (1999)) and on 'channel coordination' (e.g., Gerstner and Hess (1995)). The assumption of linear pricing may be justified by the arguments given by Rao and Srinivasan (2001) in the context of franchising. If the upstream and the downstream firms are in ongoing relationship where the demand and cost conditions vary over time, the uniform pricing of the upstream product is optimal if significant costs are involved in re-writing the contracts between the upstream

of production for both downstream firms, which finally compete á la Cournot in the downstream market. If vertical integration occurs, the downstream market is populated by a vertically integrated firm (firm $V$), and an independent firm (firm $I$). The integrated firm disposes of the essential resource at zero-cost, and sets optimally the price of the input supplied to the rival, $w_v$. Finally, the two firms compete á la Cournot in the downstream market.

Summarising, the model consists of three stages. In stage 1, the R&D decision is taken by the innovative (downstream) firm, and the degree of product differentiation is determined. In stage 2, the vertical integration game takes place, and the market structure is determined. In stage 3, the price of the essential resource is set by firm $U$ (or firm $V$, under vertical integration), and Cournot competition takes place in the downstream market. Production and profits are finally determined. Our solution concept is perfect subgame equilibrium, therefore we solve the game by backward induction starting from the market stage.

## 3. THE MARKET STAGE

At the market stage, the degree of product differentiation, $\gamma$, and the market structure (i.e. vertical integration *vs.* no-vertical integration) are already determined.

We start with the market equilibrium without vertical integration. Given the input price, $w_u$, the downstream firms ($D_1$ and $D_2$) face the same marginal cost. Hence, Cournot competition leads to a symmetric equilibrium in the downstream market, where the downstream firms produce

$$q_1^D(w_u) = q_2^D(w_u) = \frac{a - w_u}{2+\gamma},$$

and earn profits

$$\pi_1^D(w_u) = \pi_2^D(w_u) = \left[\frac{a - w_u}{2+\gamma}\right]^2.$$

The upstream monopolist faces the demand function for the essential input $\frac{2(a-w_u)}{2+\gamma}$, so that

and downstream firms.

he sets the input price, $w_u$, to maximize

$$\pi^U(w_u) = w_u \frac{2(a - w_u)}{2+\gamma}.$$

This leads to the input price:

$$w_u^* = \frac{a}{2}. \tag{2}$$

Finally, firms' equilibrium profits without vertical integration are:

$$\pi_{1,2}^D = \pi^D = \left(\frac{a}{2}\right)^2 \left[\frac{1}{2+\gamma}\right]^2, \tag{3}$$

$$\pi^U = \left(\frac{a}{2}\right)^2 \frac{2}{2+\gamma}. \tag{4}$$

We turn now to the market equilibrium under vertical integration. The integrated firm produces its final product at zero-cost, and charges the linear price $w_v$ on the input sold to the independent firm. Given $w_v$, Cournot competition leads to an asymmetric equilibrium in the downstream market, where the independent firm ($I$) and the integrated firm ($V$) produce, respectively:

$$q^I(w_v) = \frac{a(2-\gamma) - 2w_v}{4-\gamma^2}, \quad q^V(w_v) = \frac{a(2-\gamma) + \gamma w_v}{4-\gamma^2}.$$

The corresponding profits are:

$$\pi^I(w_v) = \left[\frac{a(2-\gamma) - 2w_v}{4-\gamma^2}\right]^2$$

for the independent firm, and

$$\pi^V(w_v) = \left[\frac{a(2-\gamma) + \gamma w_v}{4-\gamma^2}\right]^2 + w_v \frac{a(2-\gamma) - 2w_v}{4-\gamma^2}$$

for the integrated firm, where the second term of the integrated firm's profit comes from its sales of the essential input to the rival. The input price, $w_v$, is set by the integrated firm to maximise $\pi^V(w_v)$, leading to:

$$w_v^* = \frac{a(2-\gamma)(2\gamma + 4 - \gamma^2)}{2(8-3\gamma^2)}. \tag{5}$$

Finally, firms' equilibrium profits under vertical integration are:

$$\pi^I = \left(\frac{a}{2}\right)^2 \left[\frac{4(1-\gamma)}{(8-3\gamma^2)}\right]^2, \tag{6}$$

$$\pi^V = \left(\frac{a}{2}\right)^2 \frac{(2-\gamma)(6-\gamma)}{(8-3\gamma^2)}. \tag{7}$$

The following lemma makes some useful comparisons of the market outcomes with and without vertical integration.

**Lemma 1** *i) Unless $\gamma = 0$ or $\gamma = 1$, the input price charged to the independent firm under vertical integration is lower than the input price charged to the downstream firms without vertical integration (i.e. $w_v^* < w_u^*$ for any $\gamma \in (0,1)$). ii) Unless $\gamma = 0$, the independent firm earns lower profits under vertical integration than without vertical integration (i.e. $\pi^I < \pi^D$ for any $\gamma \in (0,1]$). iii) Both the independent firm's profit $\pi^I$ (vertical integration) and the downstream firm's profit $\pi^D$ (no-vertical integration) increase with product differentiation.*

**Proof.** *i)* Using (2) and (5), we calculate:

$$w_u^* - w_v^* = \frac{a}{2}\left[\frac{\gamma^2(1-\gamma)}{(8-3\gamma^2)}\right].$$

From the expression above it follows immediately that $w_u^* - w_v^* > 0$ for any $\gamma \in (0,1)$, while $w_u^* - w_v^* = 0$ for $\gamma = 0$ and $\gamma = 1$. [9]

*ii)* From (3) and (6), we find that $\pi^D \geq \pi^I$ is equivalent to:

$$\frac{8-4\gamma-4\gamma^2}{8-3\gamma^2} \leq 1,$$

which is strictly satisfied for $\gamma \in (0,1]$. Equality clearly holds for $\gamma = 0$.

*iii)* Differentiating (6) we get:

$$\frac{\partial \pi^I}{\partial \gamma} = -\frac{32(1-\gamma)(8+3\gamma^2-6\gamma)}{(8-3\gamma^2)^3}\left(\frac{a}{2}\right)^2,$$

[9] More precisely, while the input price with no-vertical integration is independent of $\gamma$ (see eq. (2)), it is easy to show that the input price under vertical integration is a U-shaped function of $\gamma$ in the interval $[0,1]$.

which is strictly negative for $\gamma \in [0,1)$ (it equals zero for $\gamma = 1$). Similarly, from (3) we obtain:

$$\frac{\partial \pi^D}{\partial \gamma} = -\frac{2}{(2+\gamma)^3}\left(\frac{a}{2}\right)^2$$

which is strictly negative for $\gamma \in [0,1]$. ■

### 3.1. Vertical integration and market foreclosure.—

Before proceeding to the previous stages of the game, we pause here to discuss the effect of product differentiation on the possibility of market foreclosure under vertical integration. Market foreclosure occurs if only the vertically integrated firm is active in the downstream market, i.e. $q^I(w_v^*) = 0$.

**Proposition 1** *Vertical integration leads to market foreclosure only when products are perfect substitutes (i.e. for $\gamma = 1$). In contrast, market foreclosure never occurs under vertical integration when products are differentiated (i.e. for any $\gamma \in [0,1)$).*

**Proof.** From the expression of $q^I(w_v)$, the independent firm is inactive in equilibrium iff $w_v^* \geq \frac{a(2-\gamma)}{2}$. Using equation (5), we find that $w_v^* < \frac{a(2-\gamma)}{2}$ for any $\gamma \in [0,1)$, while $w_v^* = \frac{a(2-\gamma)}{2}$ for $\gamma = 1$. That is, market foreclosure occurs only for $\gamma = 1$. In contrast, the independent firm remains active in the market for any $\gamma \in [0,1)$. ■

The interpretation of proposition 1 is as follows. The integrated firm has a strategic incentive to raise the input price charged to the independent firm, since its price and production in the downstream market increase with the rival's marginal cost. On the other hand, its sales of the essential input decrease. Intuitively, the strategic incentive is stronger the higher the degree of product substitutability (it actually vanishes if products are independent, i.e. for $\gamma = 0$). According to proposition 1, only when products are perfect substitutes the strategic incentive is strong enough to induce the integrated firm to foreclose the market and stop supplying the essential input to the rival.[10]

[10]To see this, suppose that products are perfect substitutes, and that the integrated firm engages in monopoly pricing in the product market while charging the rival the minimum input price such that rival's

Proposition 1 has an interesting implication for the previous stages of the game. Since product differentiation allows to avoid market foreclosure, it also guarantees both downstream firms positive profits under vertical integration. On one hand, the independent firm can assure a positive profit only if products are differentiated. On the other hand, by allowing the independent firm to gain a positive profit, product differentiation helps the downstream firm that vertically integrates to extract a positive profit from vertical integration even if the upstream firm has full bargaining power.

## 4. THE VERTICAL INTEGRATION GAME

Having solved the final market stage under the two alternative market structures, we are now in the position to examine the incentive for vertical integration. Recall that, at the vertical integration stage, the degree of product differentiation is already determined. We start by noting that there is a positive surplus to gain from vertical integration provided that the integrated firm's profits, net of the fixed cost of integration, exceed the joint profits of the two firms involved in the merger (i.e. the upstream monopolist and one downstream firm) without vertical integration. Let us denote with $S = \pi^V - (\pi^U + \pi^D)$ the surplus from vertical integration before the integration cost, so that the profitability condition for

production is zero. Notice that, with perfect substitutes, such a critical input price coincides with the monopoly price the integrated firm is charging in the product market (i.e. the monopoly price arising from the demand system (1) with $\gamma = 1$, and zero-marginal cost). Consider now a small decrease in the input price below the critical level. We can distinguish three effects on the integrated firm's profits. First, the equilibrium price of the final product decreases. However, the negative effect on the integrated firm's profit is only second order, since the product price falls just below the monopoly price. Second, the independent firm enters the market with a (slightly) positive production, reducing the integrated firm's production of the final good by a corresponding amount. Third, the integrated firm's sales of input increase by the same amount of the independent firm's production of the final good. Therefore, the integrated firm's sales of input equal the reduction in its sales of the final product, but the price of the input lies below the monopoly price of the final product. This means that the second (negative) effect dominates the third (positive) effect, and the integrated firm's profit decreases. The basic difference with differentiated products is that, when products are imperfect substitutes, the critical level of the input price lies above the integrated firm's monopoly price. Then, the second effect is dominated by the third effect, and the integrated firm's profit increases.

vertical integration is:

$$S > E \tag{8}$$

When condition (8) holds, each downstream firm has always an incentive to make a price-offer to be vertically integrated. If firm $D_2$ does not make any offer, it is convenient for firm $D_1$ to make an offer between $\pi^D$ and $\pi^D + (S - E)$. Since a positive surplus is left to the upstream firm, the offer will be accepted, and the bidder will gain a higher profit than under the alternative of not making any offer (without any offer, vertical integration does not occur, and both downstream firms earn $\pi^D$). Alternatively, if firm $D_2$ makes the offer above, then it is convenient for firm $D_1$ to undercut the rival's offer, since $\pi^D > \pi^I$ (see lemma 1). Furthermore, each downstream firm has always an incentive to undercut any rival's offer $O_j$ greater than $\pi^I$. By bidding above the rival, a firm ends up being the independent firm under vertical integration, earning $\pi^I$. By matching the rival's offer, it has equal chances of being independent or integrated, with expected profit $\frac{1}{2}(O_j + \pi^I)$. It is then optimal to bid just below the rival, say $O_j - \epsilon$, which assures to be integrated with a profit $O_j - \epsilon$ ($> \frac{1}{2}(O_j + \pi^I) > \pi^I$). Then, the unique Nash equilibrium pair of offers by the downstream firms is $(\pi^I, \pi^I)$. The upstream firm is left with more than the full surplus from integration, so that it will certainly call for offers at the outset, and vertical integration occurs. Notice that, due to competition in price-offers to be integrated, the downstream firm that is finally integrated reaps only its outside option under vertical integration (i.e. the equilibrium profit of the independent firm).

Assume now that condition (8) does not hold. In this case, since the net surplus from integration is negative, the upstream firm rejects any price-offer equal to (or greater than) $\pi^D$. Then, if the upstream firm calls for offers, the unique (relevant) Nash equilibrium of the game is that both downstream firms make an offer which leaves the upstream firm with negative surplus (any offer above $\pi^D - (E - S)$ will do), and the upstream firm rejects.[11] Then, vertical integration does not occur, and both downstream firms earn profit $\pi^D$. Anticipating this equilibrium outcome, the upstream firm will not call for offers at the

[11] Clearly, neither downstream firm has an incentive to deviate, and make a price-offer low enough to be acceptable by the upstream firm. The deviant would be integrated at a price below $\pi^D$.

outset.[12]

We have proved:

**Lemma 2** *If the net surplus from integration is positive (i.e. $S > E$), vertical integration always occurs, and the downstream firm involved in the merger earns the same profit as the independent firm, i.e. $\pi^I$. If the net surplus from integration is negative (i.e. $S < E$), vertical integration never occurs, so that both downstream firms earn profit $\pi^D$.*

In view of the first stage of the model, the following proposition characterises the market structure that arises after the integration stage as a function of the degree of product differentiation and the integration cost level. By lemma 2, this amounts to evaluate the sign of the net surplus from integration, $S - E$, along the range of product substitutability $\gamma \in [0, 1]$.

**Proposition 2** *a) When the integration cost is small, vertical integration occurs for any degree of product differentiation. b) When the integration cost is moderately high, vertical integration occurs only for large or for small (but not for intermediate) degrees of product differentiation. c) When the integration cost is high (but not prohibitive), vertical integration occurs only for very large degrees of product differentiation.*

**Proof.** Using equations (3), (4) and (7), the surplus from integration before the integration cost, $S$, can be written as:

$$S(\gamma) = \left(\frac{a}{2}\right)^2 \frac{8 - \gamma^2 + 2\gamma^3 + \gamma^4}{(8 - 3\gamma^2)(2 + \gamma)^2} \tag{9}$$

Inspection of (9) suffices to show that $S(\gamma) > 0$ for any $\gamma \in [0, 1]$, taking values $S(1) < S(0)$. Furthermore, we prove in Appendix 1 that $S(\gamma)$ is a U-shaped function of $\gamma$ over the interval $[0, 1]$, reaching a minimum value for $\gamma \simeq 0.61037$ (see Figure 1 below). From the shape of

[12] A qualification of this result is in order. When condition (8) does not hold, the pair of offers ($\pi^I$, $\pi^I$) still identifies a Nash equilibrium, but firms are playing weakly dominated strategies. We select it away for this reason.

$S(\gamma)$, the proof of proposition 2 is straightforward. Let us denote with $S_m$ the minimum of $S(\gamma)$. Then:

*a*) if $E < S_m$, vertical integration occurs for any $\gamma \in [0,1]$;

*b*) if $S_m < E < S(1)$, there must be two critical degrees of product differentiation, say $\gamma_{b_1}$ and $\gamma_{b_2}$ (with $\gamma_{b_1} < \gamma_{b_2}$), such that vertical integration occurs for $\gamma < \gamma_{b_1}$ and $\gamma > \gamma_{b_2}$, whilst it does not occur for $\gamma \in [\gamma_{b_1}, \gamma_{b_2}]$;

*c*) if $S(1) < E < S(0)$, there must be one critical degree of product differentiation, say $\gamma_c$ ($< \gamma_{b_1}$), such that vertical integration occurs only for $\gamma < \gamma_c$.

Finally, if $E \geq S(0)$ vertical integration never occurs (that is, $S(0)$ identifies a threshold level above which the integration cost becomes prohibitive). ■

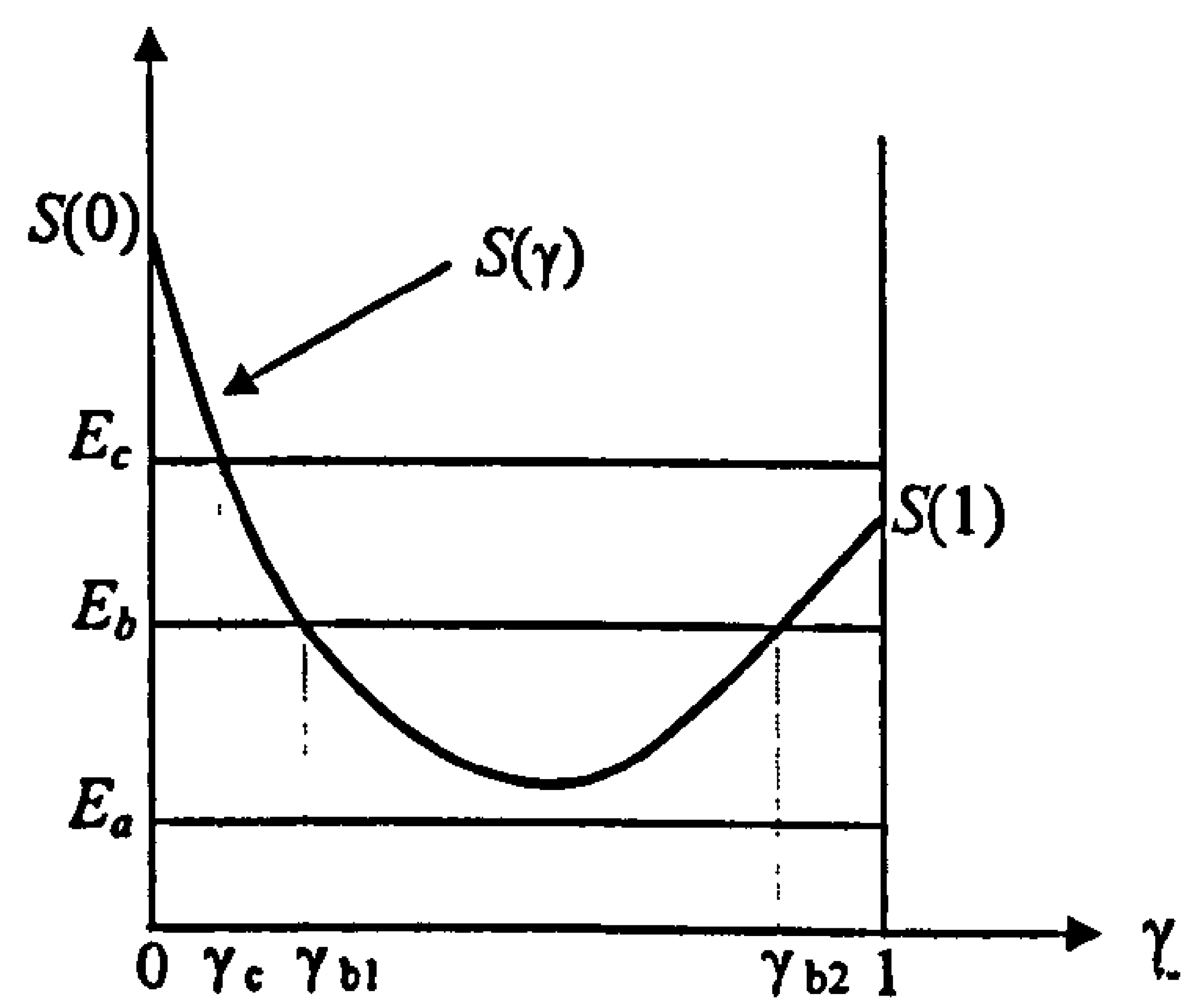


Figure 1

The interpretation of proposition 2 relies on the U-shaped behaviour of the gross surplus from integration as the degree of product differentiation decreases (see figure 1). Notice first that a positive surplus from integration may come from two sources in our model: 1) the avoidance of double marginalisation in one segment of the final product market (the one of the downstream firm that is integrated); 2) the efficiency advantage (i.e. the lower marginal cost in producing the final product) the integrated firm has relative to the independent firm in the downstream market.[13]

[13]The efficiency advantage allows the integrated firm to expand its equilibrium production of the final

Suppose now that products are independent (i.e. $\gamma = 0$). In this case, only the first source of surplus is active, since the two segments of the downstream market are isolated. As the degree of product differentiation starts decreasing (i.e. $\gamma$ starts increasing from 0), the total demand in the product market starts decreasing as well, since consumers value less any bundle of the two products relative to the numeraire good.[14] The fall in the gross surplus from integration is then explained by the lower gain from avoiding double marginalisation in a smaller market, while the second source of surplus (i.e. the efficiency advantage) is still irrelevant (since products are almost independent).[15] Only when the degree of product differentiation is sufficiently low, the second source of surplus plays a significant role. Then, the efficiency advantage of the integrated firm allows it to soften the negative effect exerted by an increase of $\gamma$ on the demand for its final product, since consumers tend to substitute the high priced product of the independent firm for the low priced product of the integrated firm. Moreover, the integrated firm benefits from the higher reduction of the rival's demand while playing the Cournot game in the product market. Hence, the integrated firm has an incentive to increase the rival's cost (by rising the input price) as the degree of product differentiation further decreases.[16] This means that the second source of surplus strengthens as the degree of product differentiation decreases. When products are sufficiently close substitutes, the second source plays a dominant role, inverting the sign of the relationship between product differentiation and surplus from integration.

good at the rival's expense. Clearly, this also has a negative effect on the component of the integrated firm's profit related to the sales of the essential input to the independent firm.

[14]See footnote 5 for the specification of consumers' preferences underlying the demand functions of the two differentiated products.

[15]This interpretation is also confirmed by the decrease in the input price under vertical integration, $w_v$, as $\gamma$ starts increasing from 0 (see the proof of lemma 1). The integrated firm tries to contrast the decrease of the demand for input by the independent firm (caused by the reduction in the global size of the product market after the increase in $\gamma$), so that its efficiency advantage over the rival decreases.

[16]This is confirmed by the U-shaped behaviour of the input price $w_v$ as the degree of product substitutability increases (see the proof of lemma 1).

## 5. VERTICAL INTEGRATION AND PRODUCT INNOVATION

In this section, we analyse the effects exerted by vertical integration on the incentive to differentiate products. As mentioned before, we assume that only one firm, say firm $D_1$, can invest in R&D aimed at product differentiation.[17] By paying a fixed R&D cost $k$, the innovative firm can reduce the perceived degree of product substitutability, $\gamma$, from 1 to $\widehat{\gamma} \in [0,1)$. On the contrary, products are perceived as perfect substitutes ($\gamma = 1$) if firm $D_1$ does not invest.

The degree of product differentiation achievable by investing in R&D, $1 - \widehat{\gamma}$, sets the effectiveness of the R&D technology (so that R&D effectiveness is higher the smaller is $\widehat{\gamma}$). Given the effectiveness of the R&D investment, we measure the incentive to differentiate products by the highest level of the R&D cost the innovative firm is willing to pay in order to obtain the associated degree of product differentiation, $\widehat{k}(\widehat{\gamma})$.

Let $\pi(\gamma)$ be the prospective profit the innovative firm expects to gain at the market stage as a function of the degree of product substitutability perceived by consumers. We clearly have:

$$\widehat{k}(\widehat{\gamma}) = \pi(\widehat{\gamma}) - \pi(1), \tag{10}$$

where the relevant profit function $\pi(\gamma)$ depends on the subsequent history of the game associated with any $\gamma$. Building upon proposition 2, we must distinguish four cases according to the level of the integration cost.

[17] Recall that, at the innovative stage of the model, the downstream firms share identical profit expectations under any subsequent evolution of the game (i.e. the independent firm's profit $\pi^I$ under the vertical integration history, the downstream firm's profit $\pi^D$ under the no-vertical integration history). Hence, the identity of the innovative firm is irrelevant. Moreover, the joint gain the two firms derive from product differentiation is always twice the individual gain of the innovative firm. Therefore, the joint incentive to differentiate products if firms $D_1$ and $D_2$ can cooperate in R&D (as measured by the highest joint-R&D cost they are willing to pay in order to obtain a given degree of product differentiation) is always twice the innovative firm's incentive when only one firm can invest with no cooperation. The results of this section immediately extend to the case of R&D cooperation by simply scaling-up the measure of the incentive to invest in R&D.

*Small integration cost (case (a) of proposition 2).* Vertical integration occurs at the second stage of the game for any degree of product substitutability. Therefore, the vertical integration outcome is independent of both the R&D effectiveness and the investment decision of the innovative firm. Since the innovative firm will end up with the independent firm's profit under vertical integration (lemma 2), the relevant profit function at the innovative stage coincides with the independent firm's profit function:

$$\pi(\gamma) = \pi^I(\gamma), \forall \gamma \in [0, 1],$$

where $\pi^I(\gamma)$ is given by equation (6). Notice that, would firm $D_1$ not invest in R&D, products are perceived as perfect substitutes by consumers, and vertical integration leads to market foreclosure (proposition 1), so that $\pi(1) = \pi^I(1) = 0$. Hence, our measure of the incentive to differentiate products becomes:

$$\widehat{k}_a(\widehat{\gamma}) = \pi^I(\widehat{\gamma}), \quad \forall \widehat{\gamma} \in [0, 1). \tag{10a}$$

Since the prospective profit of the innovative firm always coincides with the independent firm's profit under vertical integration, the incentive towards product differentiation will basically reflect the following three motives: 1) avoiding market foreclosure; 2) softening the competitive pressure of a more efficient firm (i.e. the integrated firm); 3) forcing the integrated firm to charge a lower input price.[18]

*Moderately high cost of integration (case (b) of proposition 2).* Vertical integration occurs at the second stage of the game only for large and for small, but not for intermediate, degrees of product substitutability. Consequently, the relevant profit function at the innovative stage will jump between the independent firm's and the downstream firm's equilibrium profit at both extremes of the interval of product substitutability where vertical integration does not

[18]Notice also that $\widehat{k}_a(\widehat{\gamma})$ always increases with the R&D effectiveness, as $\pi^I(\widehat{\gamma})$ monotonically decreases with $\widehat{\gamma}$ (lemma 1).

occur. Denoting such an interval with $[\gamma_{b_1}, \gamma_{b_2}]$ (as in figure 1), we have:

$$\pi(\gamma) = \begin{cases} \pi^I(\gamma) & \text{for } \gamma \in (\gamma_{b_2}, 1] \\ \pi^D(\gamma) & \text{for } \gamma \in [\gamma_{b_1}, \gamma_{b_2}] \\ \pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_{b_1}) \end{cases}$$

where $\pi^D(\gamma)$ and $\pi^I(\gamma)$ are given by equations (3) and (6), respectively. Like in the previous case, if the innovative firm does not invest in R&D, then vertical integration and market foreclosure occur at the final stage of the game, so that $\pi(1) = \pi^I(1) = 0$. Hence, our measure of the incentive to invest in R&D becomes:

$$\widehat{k}_b(\widehat{\gamma}) = \begin{cases} \pi^I(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in (\gamma_{b_2}, 1] \\ \pi^D(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}] \\ \pi^I(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in [0, \gamma_{b_1}) \end{cases} \tag{10b}$$

The outcome of the vertical integration game depends on the investment decision of the innovative firm if the R&D technology allows it to target the interval $[\gamma_{b_1}, \gamma_{b_2}]$. Hence, its incentive towards product differentiation may incorporate the additional motive of preventing vertical integration (recall that, by lemma 1, $\pi^D > \pi^I$ for any $\gamma \in (0, 1]$).

*High integration cost (case (c) of proposition 2).* Vertical integration will occur only for very low degrees of product substitutability (that is, for very high degrees of product differentiation). Therefore, the relevant profit function at the innovative stage jumps from the downstream firm's to the independent firm's equilibrium profit at the critical degree of product substitutability below which vertical integration will occur. Denoting the critical degree by $\gamma_c$ (as in figure 1), we have:

$$\pi(\gamma) = \begin{cases} \pi^D(\gamma) & \text{for } \gamma \in [\gamma_c, 1] \\ \pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_c) \ . \end{cases}$$

Contrary to the previous cases, vertical integration and market foreclosure will not occur in the subsequent stages of the game if the innovative firm decides not to invest in R&D, that is $\pi(1) = \pi^D(1) > 0$. On the other hand, vertical integration would follow the decision to invest when the resulting degree of product differentiation is very high. In other words, in the case under examination, not investing in R&D may be the only way to prevent vertical integration at the following stage of the game. Our measure of the incentive to invest in R&D is:

$$\widehat{k}_c(\widehat{\gamma}) = \begin{cases} \pi^D(\widehat{\gamma}) - \pi^D(1) & \text{for } \widehat{\gamma} \in [\gamma_c, 1] \\ \pi^I(\widehat{\gamma}) - \pi^D(1) & \text{for } \gamma \in [0, \gamma_c) \end{cases} . \tag{10c}$$

*Prohibitive integration cost (benchmark case).* If the integration cost exceeds $S(0)$ (i.e. the gross surplus from integration associated with $\gamma = 0$), vertical integration never occurs at the second stage of the game. Therefore, the innovative firm's profit function coincides with the downstream firm's profit function,

$$\pi(\gamma) = \pi^D(\gamma), \forall\, \gamma \in [0, 1],$$

and our measure of the incentive to invest in R&D becomes:

$$\widehat{k}_*(\widehat{\gamma}) = \pi^D(\widehat{\gamma}) - \pi^D(1), \quad \forall\, \widehat{\gamma} \in [0, 1). \tag{10*}$$

In this case, the vertical integration stage of the model is irrelevant for the innovative firm's incentive to differentiate products, which will only incorporate the usual motive of softening the competitive pressure of a symmetric competitor (i.e. firm $D_2$) in the product market. We use this case as a benchmark to contrast the effects on product differentiation arising from the threat of vertical integration which characterises the previous cases.[19]

We start by comparing the case of small integration costs with the benchmark case of prohibitive integration costs.

**Proposition 3** *Unless the R&D effectiveness is very low (i.e. $\widehat{\gamma}$ is very high), the innovative firm's incentive to invest in R&D is stronger when the integration cost is small (so that vertical integration always occurs) than when the integration cost is prohibitive (so that vertical integration never occurs).*

[19] As in the case of small integration costs, also with prohibitive integration costs the innovative firm's incentive to invest, $\widehat{k}_*(\widehat{\gamma})$, always increases with the R&D effectiveness, as $\pi^D(\widehat{\gamma})$ is monotonically decreasing in $\widehat{\gamma}$ (lemma 1).

**Proof.** From (10$a$) and (10$*$), $\widehat{k}_a(\widehat{\gamma}) \geq \widehat{k}_*(\widehat{\gamma})$ iff

$$\pi^I(\widehat{\gamma}) \geq \pi^D(\widehat{\gamma}) - \pi^D(1).$$

Using equations (3) and (6), the last inequality reduces to:

$$16(1-\widehat{\gamma})(6+3\widehat{\gamma})^2 - (5+\widehat{\gamma})\left(8-3\widehat{\gamma}^2\right)^2 \geq 0.$$

Calculations with Mathematica show that the polinomial on the LHS has only one real root within the admissible range $\widehat{\gamma} \in [0,1)$, that is $\widehat{\gamma}_a \simeq 0.81682$. Since $\pi^I(0) \geq \pi^D(0) - \pi^D(1)$ (recall that $\pi^I(0) = \pi^D(0)$, by lemma 1, and $\pi^D(1) > 0$), it must be:

$$\begin{aligned} \widehat{k}_a(\widehat{\gamma}) &> \widehat{k}_*(\widehat{\gamma}) \text{ for } \widehat{\gamma} \in [0, \widehat{\gamma}_a), \\ \widehat{k}_a(\widehat{\gamma}) &< \widehat{k}_*(\widehat{\gamma}) \text{ for } \widehat{\gamma} \in (\widehat{\gamma}_a, 1]. \end{aligned}$$

■

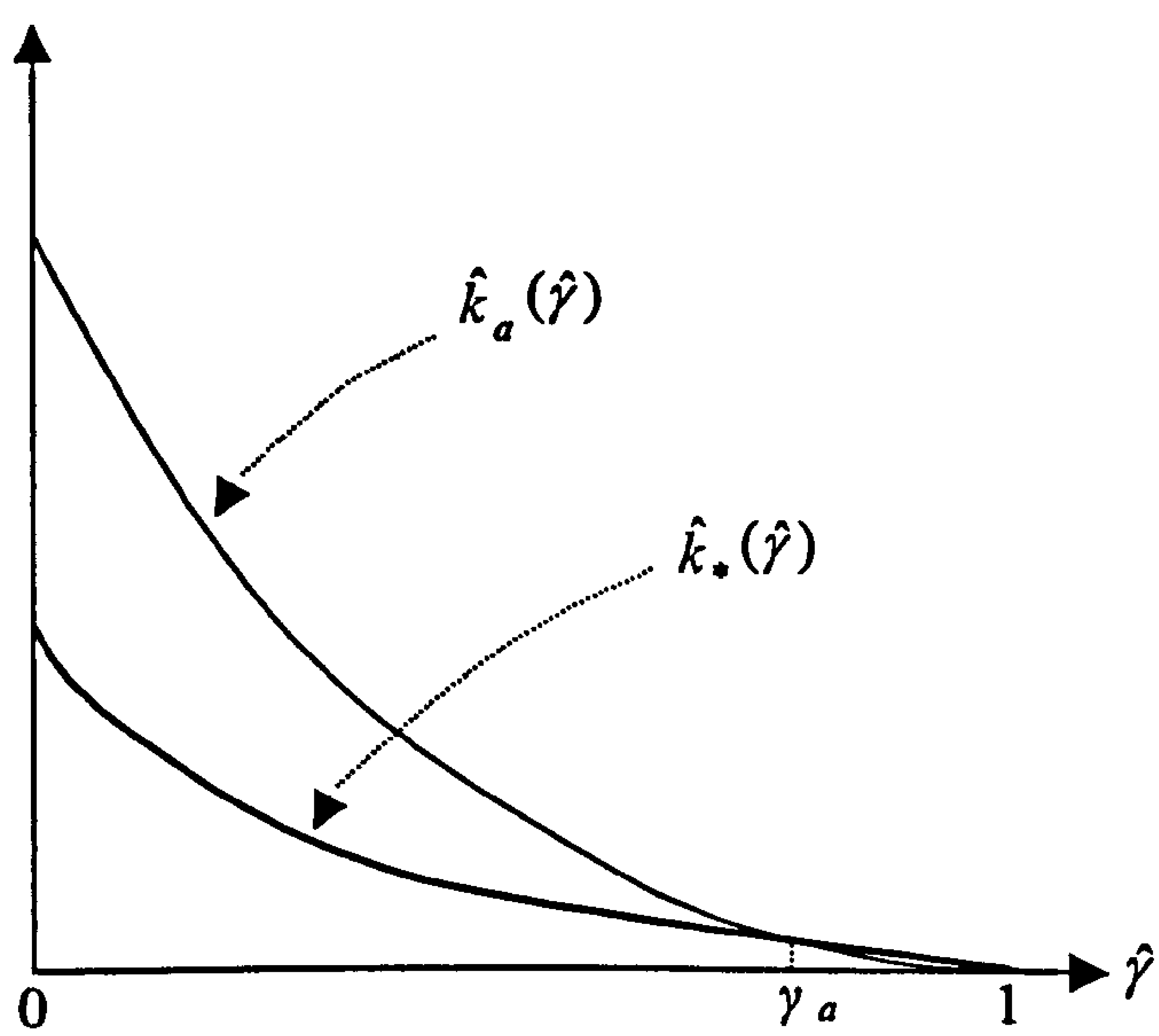


Figure 2$a$

Figure 2$a$ illustrates proposition 3. The intuition is that, when the R&D effectiveness is very low, the gain from softening the competitive pressure of a more efficient competitor (the integrated firm under vertical integration) is smaller than the gain from softening the competitive pressure of a symmetric competitor (the other downstream firm without vertical integration). Although a slight degree of product differentiation allows the independent firm to avoid market foreclosure, the resulting profit is negligible because the efficiency disadvantage relative to the integrated firm remains high when products are poorly differentiated.[20] On the contrary, when the R&D effectiveness is sufficiently high, the gain from softening the competitive pressure of the integrated firm dominates the gain from softening the competitive pressure of a symmetric competitor. The independent firm's profit is no more negligible when products are sufficiently differentiated, since both the efficiency disadvantage (up to a certain degree of differentiation) and its negative impact on the independent firm's profit sharply decrease with product differentiation. This allows the incentive to avoid market foreclosure to play a dominant role: under vertical integration, the innovative firm can guarantee a positive profit only by investing in R&D, whilst, with no-vertical integration, a positive profit arises also without investing.

Consider now the case of moderately high cost of integration. Clearly, if the innovative firm cannot target the intermediate degrees of product differentiation where vertical integration does not occur, a comparison with the benchmark case replicates exactly proposition 3. However, when the crucial interval $[\gamma_{b_1}, \gamma_{b_2}]$ can be targeted, the possibility to prevent vertical integration by product differentiation strengthens the innovative firm's incentive to invest relative to both the benchmark case and the case of small costs of integration.

**Proposition 4** *The innovative firm's incentive to invest in R&D is unambiguously strengthened by the possibility to prevent vertical integration via product differentiation which arises when the integration cost is moderately high and the R&D technology allows to target intermediate degrees of product differentiation.*

**Proof.** Assume that $\widehat{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}]$. From equations ($10b$) and ($10*$) we get:

$$\widehat{k}_b(\widehat{\gamma}) - \widehat{k}_*(\widehat{\gamma}) = \pi^D(\widehat{\gamma}) - [\pi^D(\widehat{\gamma}) - \pi^D(1)] = \pi^D(1) > 0 \, .$$

[20]In fact, the independent firm's profit function, $\pi^I(\gamma)$, is flat at $\gamma = 1$.

This proves that the incentive to invest in R&D is stronger in the case of moderately high costs of integration (where vertical integration and market foreclosure can be prevented only by investing in R&D) than in the benchmark case of prohibitive costs of integration (where vertical integration never occurs).

Similarly, using equations (10$b$) and (10$a$), we get:

$$\widehat{k}_b(\widehat{\gamma}) - \widehat{k}_a(\widehat{\gamma}) = \pi^D(\widehat{\gamma}) - \pi^I(\widehat{\gamma}) > 0 \quad \text{(by lemma 1)},$$

This proves that the incentive to invest in R&D is stronger under the case of moderately high costs of integration (where the R&D investment allows to avoid vertical integration) than under the case of small costs of integration (where vertical integration always occurs).

■

Figure 2$b$ illustrates proposition 4.

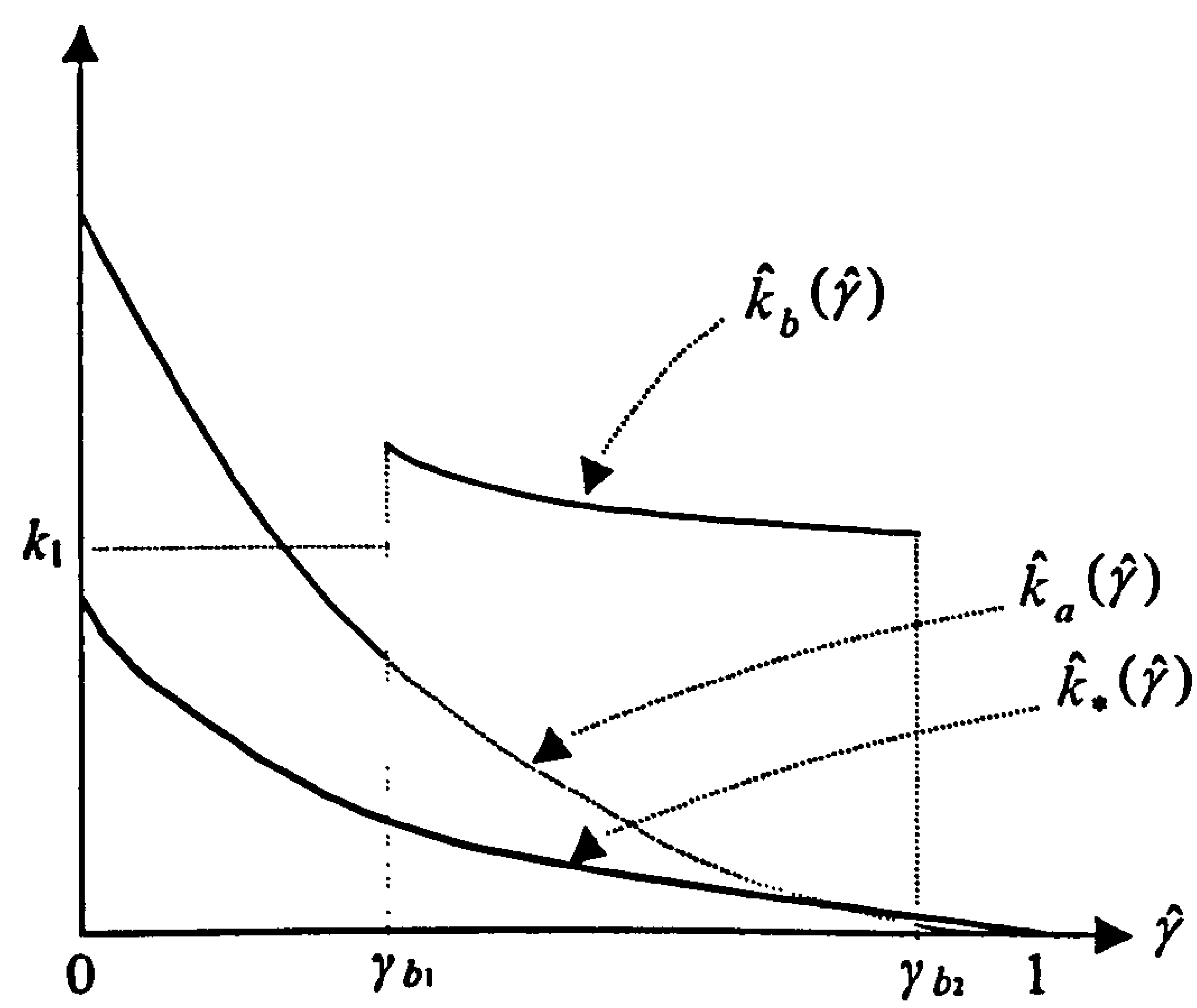


Figure 2$b$

We turn now to the case of high integration costs, where vertical integration occurs only for very high degrees of product differentiation. If the R&D effectiveness is not very high (i.e. if the achievable degree of product substitutability exceeds the critical level $\gamma_c$), the incentive to invest identically coincides with that of the benchmark case. On the contrary, if the R&D effectiveness is very high (i.e. $\hat{\gamma} < \gamma_c$), the possibility to prevent vertical integration by not-differentiating products weakens the innovative firm's incentive to invest relative to both the benchmark case and the case of small integration costs.

**Proposition 5** *The innovative firm's incentive to invest in R&D is unambiguously weakened by the possibility to prevent vertical integration by not-differentiating products which arises when the integration cost is high and the R&D effectiveness leads to very high degree of product differentiation.*

**Proof.** Assume that $\hat{\gamma} < \gamma_c$. From equations ($10c$) and ($10_*$), we get:

$$\begin{aligned} \widehat{k}_c(\widehat{\gamma}) - \widehat{k}_*(\widehat{\gamma}) &= \left[\pi^I(\widehat{\gamma}) - \pi^D(1)\right] - \left[\pi^D(\widehat{\gamma}) - \pi^D(1)\right] \\ &= \pi^I(\widehat{\gamma}) - \pi^D(\widehat{\gamma}) < 0 \quad \text{(by lemma 1).} \end{aligned}$$

This proves that the incentive to invest is lower under in the case of high integration costs (where the R&D investment leads to vertical integration) than under the benchmark case of prohibitive integration cost (where vertical integration never occurs).

Similarly, from equations ($10c$) and ($10a$), we have:

$$\widehat{k}_c(\widehat{\gamma}) - \widehat{k}_a(\widehat{\gamma}) = \left[\pi^I(\widehat{\gamma}) - \pi^D(1)\right] - \pi^I(\widehat{\gamma}) = -\pi^D(1) < 0.$$

This prove that the incentive to invest is lower in the case of high integration costs (where vertical integration can be avoided only by not-investing in R&D) than in the case of small integration costs (where vertical integration always occurs). ■

Figure $2c$ illustrates proposition 5.

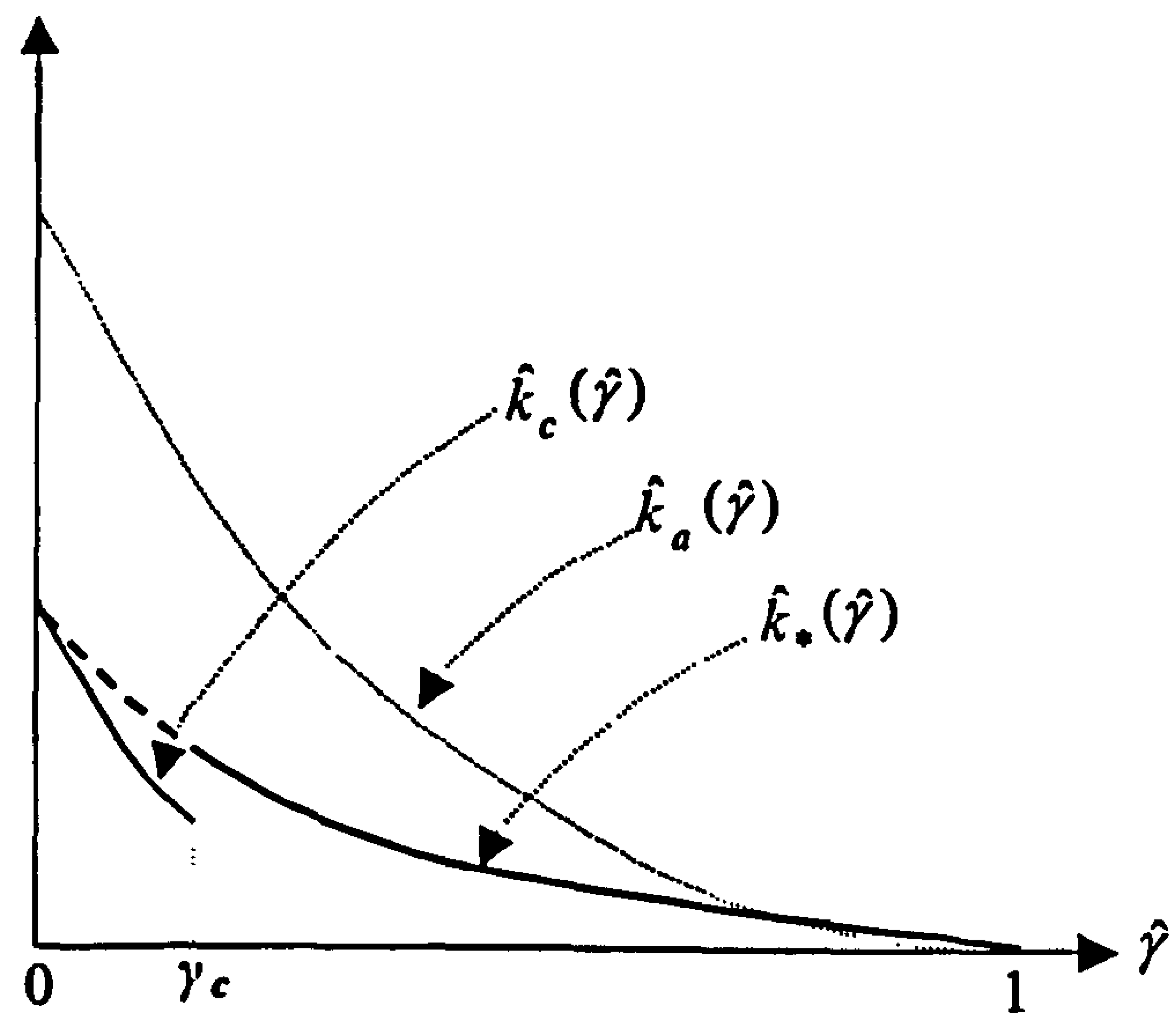


Figure 2$c$

To sum up, a prospective threat of vertical integration may have either positive or negative effects on the downstream firms' incentive to differentiate products. The nature of the effects crucially depends on the effect of product differentiation on the upstream firm's incentive to vertically integrate, as well as on the effectiveness of the R&D technology. When vertical integration is an unavoidable outcome because of small integration costs, product differentiation allows to soften the competitive pressure of the integrated firm in the product market. This has a greater value than softening the competitive pressure of a symmetric competitor (what motivates product differentiation in the benchmark case) only if products can be sufficiently differentiated. With higher integration costs, the incentive to differentiate products incorporates the strategic motive of preventing vertical integration. Both strong and weak degrees of product differentiation foster the upstream firm's incentive to vertically integrate. With strongly differentiated products, the surplus from integration is high since double marginalisation is avoided in a wider market, whilst, with poorly differentiated products, the integrated firm can better exploit its efficiency advantage over the independent firm. Then, the downstream firms have a strategic interest in targeting intermediate degrees of product differentiation, to deter the upstream firm from vertically integrating. Finally, very high costs of integration impede vertical integration unless products are strongly differentiated.[21] This gives the downstream firms a strategic motive to avoid high degrees of differentiation.

The following two examples further clarify our results.

Example 1.—

Suppose that the R&D investment allows the innovative firm to obtain (exactly) the degree of product substitutability $\widehat{\gamma} = \gamma_{b_1}$ (see figure 2b). Given the "point-to-point" nature of the R&D technology, in equilibrium we will observe either no-differentiation (i.e. $\gamma = 1$) if the innovative firm does not invest in R&D, or the degree of product differentiation $1 - \gamma_{b_1}$ (i.e. $\gamma = \gamma_{b_1}$) if the innovative firm invests. Assume that the R&D cost, $k_1$, is sufficiently high such that $\widehat{k}_a(\gamma_{b_1}) < k_1 < \widehat{k}_b(\gamma_{b_1})$. Then, inspection of figure 2b, immediately reveals that we will observe product differentiation in the downstream market only when moderately high costs of integration give the innovative firm a strategic incentive to invest in R&D in order to deter vertical integration.

[21] In this case, the surplus from integration exceeds the integration cost only when the market size is wide because products are strongly differentiated.

**Example 2.—**

Suppose that the R&D investment allows the innovative firm to reduce the degree of product substitutability up to a minimum level $\widehat{\gamma}_2$, with $\widehat{\gamma}_2$ slightly lower than $\gamma_c$ (see firgure 3). Hence, if the innovative firm decides to invest in R&D, it can select the optimal degree of product differentiation in the range $(0, 1 - \widehat{\gamma}_2]$. Let the fixed R&D cost, $k_2$, be sufficiently low such that $k_2 < \widehat{k}_c(\gamma_c)$. Clearly, the innovative firm will choose the degree of differentiation to maximize $\widehat{k}(\widehat{\gamma}) - k_2 = \pi(\widehat{\gamma}) - \pi(1) - k_2$. Then, inspection of figure 3 reveals that, whilst products will be differentiated in all cases, the innovative firm will select the maximum degree of differentiation, $1 - \widehat{\gamma}_2$, only in the cases of small and prohibitive integration costs. On the contrary, the incentive to avoid vertical integration will lead it to select lower degrees of differentiation in both cases of high and moderately high integration costs, i.e. $1 - \gamma_c$ and $1 - \gamma_{b_1}$, respectively.

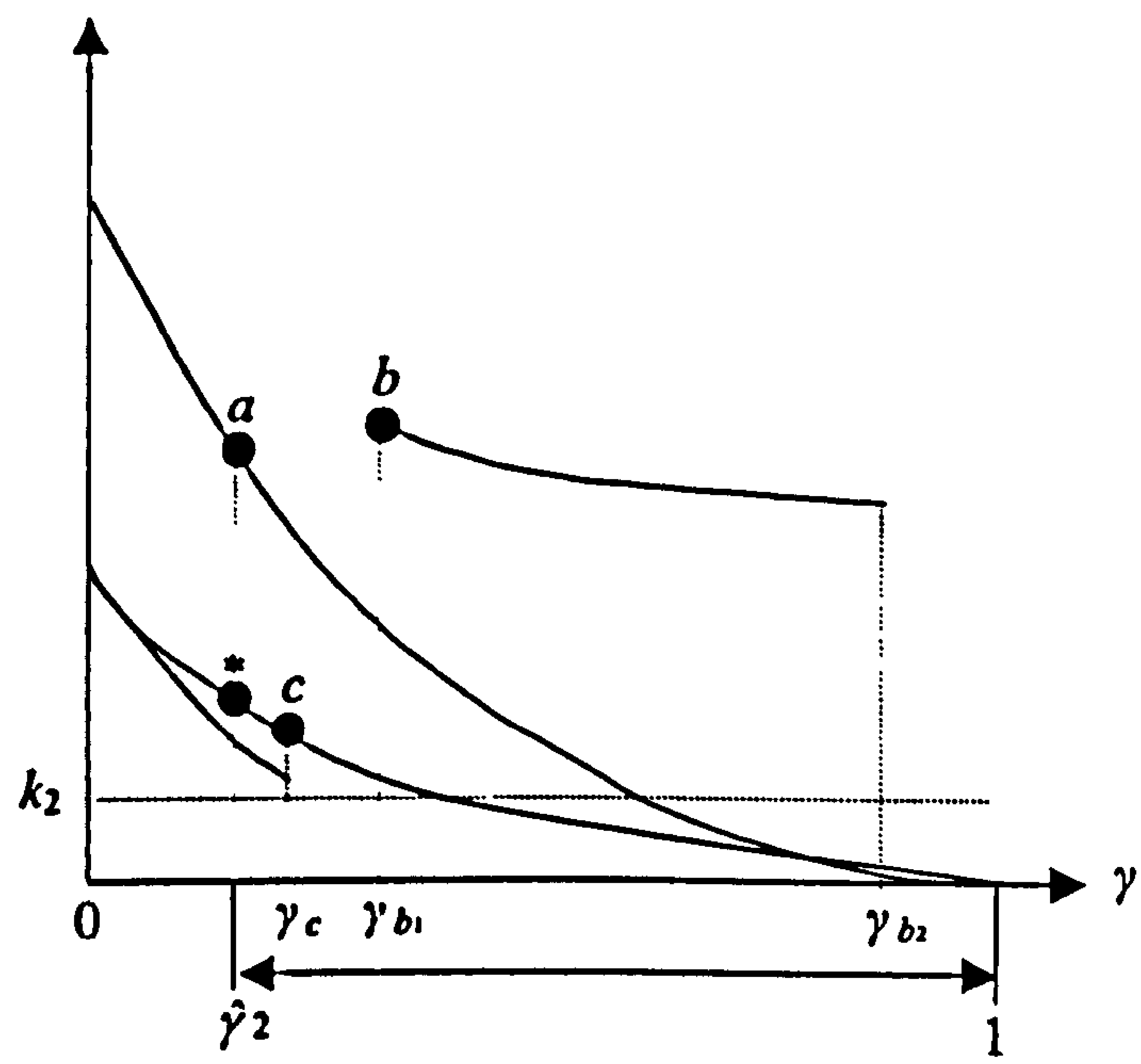


Figure 3. Optimal degrees of product differentiation (example 2) when integration costs are: small (a); moderately high (b); high (c); prohibitive (*).

## 6. A new welfare loss from vertical integration

The previous results suggest that a threat of vertical integration faced by an innovative firm vertically related to a monopolistic supplier may decrease welfare by discouraging socially valuable innovations. The simplest way to show this is to reconsider the example 2 above, where the innovative firm can select the optimal degree of product differentiation up to a maximum level $1 - \hat{\gamma}_2$. Suppose that the integration cost is high enough to make the innovative firm's incentive to prevent vertical integration active (i.e. consider either case b) or case c) in figure 3). As we have seen before, the innovative firm will deter vertical integration by choosing a lower degree of product differentiation relative to the benchmark case where the threat of vertical integration is absent. If we re-interpret the benchmark as the case of a severe antitrust policy which bans vertical mergers, we can say that a lenient antitrust policy will cause a lower degree of product differentiation in the case under consideration, whilst the vertical structure of the market is identical in the two policy regimes (i.e. no-vertical integration). On the other hand, it is easy to prove that social welfare, measured by the total surplus generated in the market, is higher when products are more differentiated. Consider first industry profits. Profits in the downstream market increase with product differentiation (see lemma 1 (point *iii)*). Similarly, inspection of equation (4) suffices to see that also the upstream firm's profit increases. Hence, industry profits are higher with more differentiation. Consider now the consumer surplus. As shown in Appendix 2, the consumer surplus can be expressed in terms of the equilibrium quantities as: [22]

$$CS = \frac{1}{2}[(q_1^D)^2 + (q_2^D)^2 + 2\gamma q_1^D q_2^D].$$

Since $q_1^D = q_2^D = \frac{a}{2}\frac{1}{2+\gamma}$ in the symmetric equilibrium without vertical integration, we get:

$$CS = (1+\gamma)\left(\frac{a}{2}\frac{1}{2+\gamma}\right)^2.$$

[22] More precisely, the expression above gives the consumer surplus as a function of the consumer's optimal demands of goods $q_1$ and $q_2$ at given prices.

Then, we evaluate:

$$\frac{\partial CS}{\partial \gamma} = -\left(\frac{\alpha}{2}\right)^2 \frac{\gamma}{(2+\gamma)^2} < 0,$$

that is, the consumer surplus increases with product differentiation. Intuitively, consumers' preference for variety and the increase in equilibrium quantities compound to increase consumers' welfare even if equilibrium prices increase. Therefore, in our example, a lenient antitrust policy would allow the threat of vertical integration to reduce product differentiation, causing a reduction in total surplus and welfare. It is worth noting that, although more product differentiation and vertical integration would generate a positive surplus for the integrated firm (and for society), as far as the R&D cost is sunk before the integration game takes place the upstream firm does not have any credible strategy to induce more innovation in the downstream market.[23] On the contrary, the upstream firm could solve the inefficiency if vertical integration could be credibly contracted before the innovation takes place. Whilst we have highlighted here the possibility that vertical integration leads to this new form of inefficiency, we leave for future research the study of the upstream firm's incentives and strategies to prevent it.

## 7. CONCLUSIONS

In this chapter we have studied vertical integration and product innovation as interdependent strategic choices of vertically related firms. Our main innovation with respect to the previous literature on vertical integration is that we have considered product differentiation as a non-productive strategic decision of the downstream firms, showing its impact on the incentives for vertical integration and market foreclosure.[24] Our main innovation relative to the literature on product innovation, is that, besides product market competition, we have accounted for another source of competition capable of affecting product innovation

[23] Indeed, any commitment to vertically integrate the innovative firm at more favorable conditions in exchange of more innovation would be non-credible.

[24] Previous works have analysed the incentives to vertically integrate and foreclose the downstream market when the final products are differentiated (e.g. Ordover, Saloner and Salop (1990), Economides (1994), Colangelo (1995), Hackner (2001)). However, product differentiation is exogenous in all these studies.

by innovative firms vertically related to a monopolistic supplier, i.e. the threat of vertical integration.[25] Due to the downstream firms' inability to commit to a cooperative behaviour if asked for integration offers, the monopolistic supplier can use vertical integration as a means to reap profits in the downstream market. As a consequence, the incentive to differentiate products in the downstream market incorporates the strategic motive of preventing vertical integration. Our main finding has been that, although product differentiation allows to soften product market competition and to avoid market foreclosure under vertical integration, the strategic motive of preventing vertical integration may lead to less innovation in the downstream market. Indeed, the monopolist's incentive to vertically integrate strengthens with both high and low degrees of product differentiation, so that the downstream firms may find it convenient to refrain from adopting (profit and welfare enhancing) innovations leading to strongly differentiated products. Therefore, instead of market foreclosure, we have shown a new possible welfare loss from vertical integration, i.e. less product innovation.

[25]The literature on product innovation has focused on the effects of product market competition and R&D competition on the incentive to innovate (see Chapters 1 and 2 for a detailed reference to this literature). For instance, Lambertini and Rossini (1998) and Lin and Saggi (2002)) focus on R&D and product market competition in a setting similar to ours, where product innovation entails horizontal differentiation in a linear differentiated duopoly. A related literature analyses product differentiation in both the upstream and the downstream market under alternative vertical structures of the industry (e.g. Pepall and Norman (2001), Belleflamme and Toulemonde (2003), Matsushima (2004)). Also this literature essentially concentrates on the relationship between product differentiation and the intensity of product market competition. In this chapter we have pointed out a different source of competition that can affect product innovation when the innovative firms depend on an upstream monopolist for the supply of an essential resource.

## APPENDIX

**Appendix 1.—**

We prove that the surplus form integration before the integration cost, $S$, is a U-shaped function of the degree of product differentiation over the range $\gamma \in [0,1]$. From equation (9), we calculate:

$$\frac{\partial S(\gamma)}{\partial \gamma} = \left(\frac{a}{2}\right)^2 \frac{2\left(-64+32\gamma+96\gamma^2+40\gamma^3-\gamma^4-3\gamma^5\right)}{\left(8-3\gamma^2\right)^2\left(2+\gamma\right)^3}.$$

Since $\left(8-3\gamma^2\right)^2(2+\gamma)^3 > 0$ for $\gamma \in [0,1]$,

$$sign\left\{\frac{\partial s(\gamma)}{\partial \gamma}\right\} = sign\left\{-64+32\gamma+96\gamma^2+40\gamma^3-\gamma^4-3\gamma^5\right\}.$$

Using Mathematica, we find that the polynomial on the RHS has an unique real root within the admissible range $[0,1]$, that is $\gamma_m \simeq 0.61037$. Since $\frac{\partial S(\gamma)}{\partial \gamma}$ is continuous over $[0,1]$, and takes values $\frac{\partial S(\gamma)}{\partial \gamma}|_{\gamma=0} = -0.25\left(\frac{a}{2}\right)^2 < 0$ and $\frac{\partial s(\gamma)}{\partial \gamma}|_{\gamma=1} = 0.2963\left(\frac{a}{2}\right)^2 > 0$, then it must be negative for $\gamma < \gamma_m$ and positive for $\gamma > \gamma_m$. Finally, with simple calculations we get: $S(0) = \frac{1}{4}\left(\frac{a}{2}\right)^2$ and $S(1) = \frac{10}{45}\left(\frac{a}{2}\right)^2$, so that $S(0) > S(1)$.

**Appendix 2.—**

The representative consumer's optimisation problem is:

$$\begin{aligned} &\text{Max } U = a(q_1+q_2) - \tfrac{1}{2}(q_1^2+q_2^2+2\gamma q_1 q_2) + m \\ &\text{s.t.} \quad p_1 q_1 + p_2 q_2 + m = I \end{aligned}$$

where $I$ is the consumer's income in units of the numeraire good $(m)$.

From the first order conditions $p_i = a - q_i - \gamma q_j$ $(i,j = 1,2;\ i \neq j)$ and the budget constraint, we get:

$$m = I - a(\widehat{q}_1 + \widehat{q}_2) + (\widehat{q}_1^2 + \widehat{q}_2^2 + 2\gamma \widehat{q}_1 \widehat{q}_2),$$

where $\widehat{q}_i$ denotes the consumer's optimal demand of good $i$ at given prices.

Substituting for $m$ into the utility function, we get:

$$\widehat{U} = I + \frac{1}{2}(\widehat{q}_1^2 + \widehat{q}_2^2 + 2\gamma \widehat{q}_1 \widehat{q}_2).$$

Finally, the consumer surplus is:

$$CS = \widehat{U} - I = \frac{1}{2}(\widehat{q}_1^2 + \widehat{q}_2^2 + 2\gamma\widehat{q}_1\widehat{q}_2).$$

## REFERENCES


[1] Acharyya, R. and S. Marjit, 1998, To liberalize or not to liberalize an LDC-market with an inefficient incumbent, International Review of Economics and Finance, 7, 277-296

[2] Aghion, P., R.W. Blundell, N. Bloom and R. Griffiths, 2001, Empirical estimates of the relationship between product market competition and innovation, mimeo, CEPR/ECARES Workshop on Product Markets, Financial Markets and the Pace of Innovation in Europe (Bruxelles, may 2001)

[3] Aghion, P., M. Dewatripont and P.Rey, 1997, Corporate governance, competition policy and industrial policy, European Economic Review, 41, 771-782

[4] Aghion, P., M. Dewatripont and P.Rey, 1999, Competition, financial discipline and growth, Review of Economic Studies, 66, 825-852

[5] Aghion, P., C. Harris and J. Vickers, 1997, Competition and growth with step-by-step innovation: An example, European Economic Review, 41, 771-782.

[6] Aghion, P., C. Harris, P. Howitt and J. Vickers, 2001, Competition and growth with step-by-step innovation, Review of Economic Studies, 68, 467-492

[7] Aghion, P. and P. Howitt, 1992, A model of growth through creative destruction, Econometrica 60, 323-351

[8] Aghion, P. and P. Howitt, 1996, Research and development in the growth process, Journal of Economic Growth, 1, 49-73

[9] Aghion, P. and P. Howitt, 1998, Endogenous Growth Theory, Cambridge (Mass.), MIT Press

[10] Aghion, P. and M. Shankerman, 2000, A model of market enhancing infrastructure, CEPR Discussion Paper No. 2461

[11] Amir, R. and J.Y. Jin, 2001, Cournot and Bertrand equilibria compared: substitutability, complementarity and concavity, International Journal of Industrial Organization 19, 303-317.

[12] Armstrong, M., C. Doyle and J. Vickers, 1996, The access pricing problem: a synthesis, The Journal of Industrial Economics, 44, 131-150

[13] Armstrong, M. and J. Vickers, 1998, The access pricing problem with deregulation: a note, The Journal of Industrial Economics, 46, 115-121

[14] Arrow, K., 1962, Economic welfare and the allocation of resources for invention, in: R. Nelson, ed., The rate and direction of innovative activity, Princeton, Princeton University Press

[15] Baake, P., U. Kamecke and H. T. Normann, 2004, Vertical integration with capital precommitment: monopolization vs. inefficiency, International Journal of Industrial Organization, 22, 185-192

[16] Barro, R. J. and X. Sala-i-Martin, 1995, Economic growth, New York, McGraw-Hill

[17] Beath, J., Y. Katsoulacos and D. Ulph, 1989, Strategic R&D policy, The Economic Journal, 99, 74-83

[18] Belleflamme, P. and E. Toulemonde, 2003, Product differentiation in successive vertical oligopolies, Canadian Journal of Economics, 36, 523-545

[19] Bester H. and E. Petrakis, 1993, The incentives for cost reduction in a differentiated industry, International Journal of Industrial Organization, 11, 519-534

[20] Blundell, R., R.Griffith and J. van Reenen, 1995, Dynamic count models of technological innovations, The Economic Journal, 105, 333-344

[21] Blundell, R., R.Griffith and J. van Reenen, 1999, Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms, The Review of Economic Studies, 66, 529-554.

[22] Boccard, N. and X. Wauthy, 2000, Bertrand and Cournot outcomes: further results, Economics Letters, 68, 279-285

[23] Bonanno G. and B. Haworth, 1998, Intensity of competition and the choice between product and process innovation, International Journal of Industrial Organization 16, 495-510

[24] Boone J. , 2000, Competitive Pressure: The Effects on Investments in Product and Process Innovation, Rand Journal of Economics 31, 549-569

[25] Boone J., 2001, Intensity of competition and the incentive to innovate, International Journal of Industrial Organization 19, 705-726

[26] Bork, R., 1978, Antitrust Paradox, New York, Basic Books

[27] Bousquet, A., H. Cremer, M. Ivaldi and M. Wolkowicz, 1998, Risk sharing in licensing, International Journal of Industrial Organization, 16, 535-154

[28] Buehler, S. and A. Shmutzler, 2003, Downstream investment in oligopoly, Socioeconomic Institute, University of Zurich, Working Paper N. 0310

[29] Caballero, R.J. and A.B. Jaffee, 1993, How high are the giants' shoulders: An empirical assessment of knowledge spillovers and creative destruction in a model of economic growth, in NBER Macroeconomics Annual, MIT Press, Cambridge (Mass.)

[30] Cabral, L., 1995, Conjectural variations as a reduced form, Economics Letters 49, 397-402

[31] Choi, S. C., 1991, Price competition in a channel structure with a common retailer, Marketing Science, 10, 271-296

[32] Colangelo, G., 1995, Vertical vs. Horizontal integration: pre-emptive merging, The Journal of Industrial Economics, 48, 323-337

[33] d'Aspremont, C., Dos Santos Ferreira, R. and L. A. Gerard Varet, 2002, Strategic R&D investment, competition toughness  and growth, mimeo, CORE.

[34] De Fraja, G. and C. W. Price, 1999, Regulation and access pricing: comparison of regulated regimes, Scottish Journal of Political Economy, 46, 1-16

[35] Delbono F. and V. Denicolò, 1990, R&D investment in a symmetric and homogeneous oligopoly: Bertrand vs Cournot, International Journal of Industrial Organization, 8, 297-313

[36] Deneckere, R., 1983, Duopoly supergames with product differentiation, Economics Letters, 11, 37-42

[37] Denicolò V., 2001, Growth with non-drastic innovations and the persistence of leadership, European Economic Review 45, 1399-1413

[38] Dixit, A.K.,1979, A model of duopoly suggesting a theory of entry barriers, Bell Journal of Economics 10, 20-32

[39] Dockner, 1992, A dynamic theory of conjectural variations, The Journal of Industrial Economics, 40, 377-395

[40] Economides, N., 1994, The incentive for vertical integration, Stern School of Business, New York University, Working Paper EC-94-05

[41] Economides, N., 1998, The incentive for non-price discrimination by an input monopolist, International Journal of Industrial Organization, 16, 271-284

[42] Encaoua, D., and D.Ulph, 2000, Catching-up or leapfrogging? The effects of competition on innovation and growth, CNRS, Collection Les Cahiers De l'Innovation, Cahier no. 00017

[43] Gerstner, E. and H. Hess, 1995, Pull promotions and channel coordination, Marketing Science, 14, 43 - 60

[44] Grossman, G. and E. Helpman, 1991, Innovation and growth in the global economy, Cambridge (Mass.), MIT Press

[45] Hackner, J., 2000, A note on price and quantity competition in differentiated oligopolies, Journal of Economic Theory 93, 233-239

[46] Hackner, J., 2001, Vertical integration and competition policy, Department of Economics, University of Stockholm, Working Paper N. 0101

[47] Hamilton, J. and S. Slutsky, 1990, Endogenous timing in duopoly games: Stackelberg or Cournot equilibria, Games and Economic Behaviour, 2, 29-47

[48] Hart, O. and J. Tirole, 1990, Vertical integration and market foreclosure, Brookings Papers on Economic Activity, 205 - 276, and 285-286

[49] Howitt, P., 1999, Steady endogenous growth with population and R&D inputs growing, Journal of Political Economy, 107, 715-730

[50] Lambertini, L., 1997, Prisoners' dilemma in duopoly (super) games, Journal of Economic Theory, 77, 181-191

[51] Lambertini, L. and G. Rossini, 1998, Product homogeneity as a prisoner's dilemma in a duopoly with R&D, Economics Letters, 58, 297-301.

[52] Lin, P. and K. Saggi, 2002, Product differentiation, process R&D, and the nature of market competition, European Economic Review, 46, 201-211

[53] Maggi, G., 1996, Strategic trade policies with endogenous mode of competition, American Economic Review 86, 237-258

[54] Majerus, D., 1988, Price vs quantity competition in oligopoly supergames, Economics Letters, 27, 293-297

[55] Matsushima, N., 2004, Technology of upstream firms and equilibrium product differentiation, International Journal of Industrial Organization, 22, 1091-1114

[56] Nickell, S., 1996, Competition and corporate performance, Journal of Political Economy, 104, 724-746

[57] O' Donoghue, T., and J. Zweimuller, 2004, Patents in a Model of Endogenous Growth, Journal of Economic Growth, 9, 81-123

[58] Ordover, J.A., G. Saloner and S. C. Salop, 1990, Equilibrium vertical foreclosure, American Economic Review, 80, 127-142

[59] Oz Shy, 2000, Industrial Organization: Theory and Applications, The MIT Press, Cambridge

[60] Pepall, L.and G. Norman, 2001, Product differentiation and upstream-downstream relations, Journal of Economics and Management Strategy, 10, 201-233

[61] Peretto P., 1996, Sunk Costs, Market Structure, and Growth, International Economic Review 37, no. 4, 895-923

[62] Peretto P., 1999, Cost reduction, entry, and the interdependence of market structure and economic growth, Journal of Monetary Economics 43, 173-195

[63] Qiu L., 1997, On the dynamic efficiency of Bertrand and Cournot equilibria, Journal of Economic Theory 75, 213-229

[64] Raith, M., 2003, Competition, risk and managerial incentives, American Economic Review, 93, 1425-1436

[65] Rey, P. and J. Tirole (2004), A premier on foreclosure, forthcoming in M. Amstrong and R. H. Porter (Eds.), Handbook of Industrial Organization, Vol. 3, North-Holland, Amsterdam

[66] Rao, R. and S. Srinivasan, 2001, An analysis of advertising royalty in franchise contracts, Journal of Marketing Channels, 8, 85-118

[67] Riordan, M. H., 1998, Anticompetitive vertical integration by a dominant firm, American Economic Review, 88, 1232-1248

[68] Salinger, M., 1988, Vertical mergers and *market foreclosure,* The Quarterly Journal of Economics, 103, 345-356

[69] Schumpeter, J., 1942, Capitalism, socialism and democracy, London, Allen and Unwin

[70] Segal and Whinston, 2003, Antitrust in innovative industries, mimeo

[71] Segerstrom, P., Anant, C. and E. Dinopoulos, 1990, A Schumpeterian model of the product cycle, American Economic Review 80, 1077-1091

[72] Singh, N. and X.Vives, 1984, Price and quantity competition in a differentiated duopoly, Rand Journal of Economics 15, 546-554

[73] Smulders, S., and T. Van de Klundert, 1995, Imperfect competition, concentration and growth with firm-specific R&D, European Economic Review, 39, 139-160

[74] Stokey, N., 1995, R&D and economic growth, Review of Economic Studies, 62, 469-489

[75] Sutton, J., 1991, Sunk costs and market structure: price competition, advertising, and the evolution of concentration, Cambridge (Mass.), MIT Press

[76] Symeonidis, G., 2003, Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R&D, International Journal of Industrial Organization 21, 39-55

[77] Tyagi, R. K., 1999, On the effects of downstream entry, Management Science, 45, 59-73

[78] Van de Klundert, T. , and S. Smulders, 1997, Growth, competition and welfare, Scandinavian Journal of Economics, 99, 99-118

[79] Vickers, J.,1995, Concepts of competition, Oxford Economic Papers 47, 1-23

[80] Villas-Boas, J. M., 1998, Product line design for a distribution channel, Marketing Science, 17, 156-169

[81] Zanchettin, P., 2001, Competizione, Innovazione tecnologica e crescita, University of Siena Discussion Papers (Quaderni del Dipartimento di Economia Politica), Discussion Paper N. 327