SELECTION OF SIMULATION VARIANCE REDUCTION TECHNIQUES THROUGH A FUZZY EXPERT SYSTEM

by

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ABSTRACT

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In this thesis, the design and development of a decision support system for the selection of a variance reduction technique for discrete event simulation studies is presented. In addition, the performance of variance reduction techniques as stand alone and combined application has been investigated.

The aim of this research is to mimic the process of human decision making through an *expert system* and also handle the ambiguity associated with representing human expert knowledge through *fuzzy logic*. The result is a fuzzy expert system which was subjected to three different validation tests, the main objective being to establish the *reasonableness* of the systems output. Although these validation tests are among the most widely accepted tests for fuzzy expert systems, the final results were not in agreement with expectations. In addition, results from the stand alone and combined application of variance reduction techniques, demonstrated that more instances of stand alone applications performed better at reducing variance than the combined

application.

The design and development of a fuzzy expert system as an advisory tool to aid simulation users, constitutes a significant contribution to the selection of variance reduction technique(s), for discrete event simulation studies. This is a novelty because it demonstrates the practicalities involved in the design and development process, which can be used on similar decision making problems by discrete event simulation researchers and practitioners using their own knowledge and experience. In addition, the application of a fuzzy expert system to this particular discrete event simulation problem, demonstrates the flexibility and usability of an alternative to the existing algorithmic approach.

Under current experimental conditions, a new specific class of systems, in particular the Crossdocking Distribution System has been identified, for which the application of variance reduction techniques, i.e. Antithetic Variates and Control Variates are beneficial for variance reduction.

AUTHORS DECLARATION

The first principle of knowledge engineering is that the problem-solving power exhibited by an intelligent agent's performance is primarily the consequence of its knowledge base, and only secondarily a consequence of the inference method employed. Expert systems must be knowledge rich even if they are methods - poor. This is an important result and one that has only recently become well understood in Artificial Intelligence. For a long time, Artificial Intelligence has focused its attentions almost exclusively on the development of clever inference methods; almost any inference method will do. The power resides in the knowledge. EDWARD FEIGENBAUM, Stanford University.

I declare that I have not been registered in any other university during my period of study at the University of Nottingham.

Journal Publication

- Adrian Adewunmi and Uwe Aickelin A Fuzzy Expert System for selecting Variance Reduction Techniques, Journal of Computing and Information Technology, 2009 (forthcoming).

- Adrian Adewunmi and Uwe Aickelin Investigating the selection of Variance Reduction Techniques: A Crossdocking distribution centre case study, Journal of Simulation Modelling Practice and Theory, 2009 (forthcoming).

Conference Attendance and Publication

- Adrian Adewunmi and Uwe Aickelin Optimisation of a Crossdocking distribution centre simulation model Proceedings of the 2008 International Simulation Multi-Conference (SCS), San Diego, USA 434-439 2008.

- Adrian Adewunmi and Uwe Aickelin Reducing variance in a Crossdocking simulation model using Common Random Numbers and Antithetic Variates Annual Operational Research Conference 50 (OR 50), York, UK 2008.

- Adrian Adewunmi, Uwe Aickelin, Mike Byrne An Investigation of Sequential Sampling Method for Crossdocking simulation output variance reduction Proceedings of the 2008 Operational Research Society 4th Simulation Workshop (SW08), Birmingham, UK 87-96 2008.

- Adrian Adewunmi and Uwe Aickelin Noise reduction technique for a Simulation Optimisation study Annual Operational Research Conference 49 (OR 49), Edinburgh, UK 2007.

- Adrian Adewunmi and Uwe Aickelin Simulation Optimisation of the Crossdock Door Assignment Problem Proceedings of the 2006 OR Society Simulation Workshop, Birmingham, UK 311-313 2006.

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Lastly, I give special thanks to my maker, who guides, provides and protects.

DEDICATION

To Anthonia Oluwatoyin Adewunmi; with Love and Pride.

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Chapter 1

Introduction

1.1 Background and Motivation

To understand various events in the world around us, scientists often resort to mathematical modelling. In many instances, however, it is easier said than done. Sometimes, the event is so complex that it defies modelling, or building a mathematical model is either an impossible or an unattractive task. Typically, a scientist can try to relax a mathematical model by using heuristics to reason about the missing information in such a model and about the event that this model represents. In other words, a complete understanding of the event does not exist and must be satisfied with an incomplete model with which to reason. It turns out that many real life models do not represent precisely the events around us, but provide enough information for us to understand such events.

Scientists, therefore, turn to building models that describe the event only partially and then reason about the rest. They use *Artificial Intelligence* which provide tools for reasoning and use these tools to understand complex or incomplete events. Artificial intelligence research in recent times has enjoyed many important successes.

Probably, the most significant success has been the development of very powerful artificial intelligence tools better known as fuzzy expert systems [129].

Fuzzy expert systems are computerised advisory programs that attempt to replicate the reasoning processes and knowledge of experts in solving specific types of problems. Fuzzy expert systems are of great interest in artificial intelligence research because of their potential to enhance productivity and to supplement work forces in many speciality areas where human experts are becoming increasingly difficult to find and retain.

The application of fuzzy expert systems is best suited to problems where specialised expertise exists for a particular problem i.e. medical diagnosis. Typically, human experts tend to specialise in rather narrow problem solving areas or tasks. These human experts tend to solve problems quickly and fairly accurately, explain what they do and judge the consistency of their own conclusions. They know when mistakes have been made, and communicate with other experts. Knowledge is a major resource in the development of fuzzy expert systems, and it often lies with only a few experts [140].

An area of expertise that is of interest to us where there exists experts and an adequate supply of literature is the field of discrete event simulation. This involves experimenting with a real world system, with the aim of understanding the behaviour of the modelled system. This behaviour of the modelled system is observed "discretely" over time [13]. Often more is known about the behaviour of the individual components of such a system than about the real system behaviour. However, the use of discrete event simulation does have its drawbacks, one of which is obtaining precise estimates of a system's performance. This can be difficult because of the variance associated with the system's output performance measures.

There are a number of methods developed specifically to deal with such situations. These methods known as variance reduction techniques, aim to reduce the variance of a selected output performance measure, using fewer or the same number of replications [67]. The selection and implementation of variance reduction techniques require a specialised knowledge of discrete event simulation output analysis and as well as simulation programming knowledge. Furthermore, the effectiveness of any variance reduction technique is unpredictable [79], and as such its anticipated efficiency can only be judged through a pilot study, which will be carried out before full experimentation.

The motivation for this research arises from an availability of expert knowledge in the application of variance reduction techniques for discrete event simulation studies. This knowledge creates the potential for the development of an advisory system using a fuzzy expert system, with the aim of assisting simulation modellers with the selection of a variance reduction technique. It is an aspiration that this thesis will demonstrate a novel application of an artificial intelligence technique such as a fuzzy expert system, within an existing domain i.e. discrete event simulation.

1.2 Aims and Scope

The aim of this thesis is two fold; first of all to investigate the performance of three stand alone variance reduction techniques, as well as two different combinations of the three techniques under consideration. Secondly, design and develop a fuzzy expert system which should be capable of suggesting a variance reduction technique based on expert knowledge. The desired outcome is to demonstrate the capability of a fuzzy expert system in assisting a simulation modeller with the selection of a variance reduction technique.

The scope of the thesis covers the use of three variance reduction techniques (common random numbers, antithetic variates and control variates) for discrete event simulation studies and three real world application domains (manufacturing systems, call centre systems and crossdocking distribution systems).

There are two research questions which are central to this thesis:

1. Is there a reduction in the variance or standard deviation value of a selected discrete event simulation output performance measure, after the application of stand - alone or combined variance reduction techniques?

2. Can a fuzzy expert system assist a simulation modeller in the selection of a variance reduction technique for discrete event simulation studies?

The next section will describe each chapter of the thesis.

1.3 Overview of Thesis

The remainder of the thesis is divided into six chapters:

- Chapter 2 begins with an examination into the background and key topics in the field of simulation. This chapter is further divided into six sections. The next section defines the main types of simulation methods, followed by a discussion on the selected application domains applied in this thesis. This is proceeded by a discussion on reducing variance in discrete event simulation and the current state of art regarding the combination of variance reduction techniques. Next, the manual selection of variance reduction techniques for discrete event simulation is presented. The chapter ends with a discussion of the problem under consideration, which has been divided into three parts:

    1. The problem of selecting of a variance reduction technique for discrete event simulation studies,

2. The need to deal with the ambiguity associated with human expert knowledge,
3. A fuzzy expert system solution approach.

- Chapter 3 is a self contained chapter that describes definitions and important literature in the field of expert systems and fuzzy logic. This chapter provides the necessary backdrop for understanding the theory of fuzzy expert systems applied in this thesis. It is intended mainly for readers not familiar with the components and processes that make up a typical fuzzy expert system. This chapter also examines the inter-relationship between expert systems and simulation as well as presents examples of its application in automating the process of simulation output analysis.

- Chapter 4 investigates the manual selection of a variance reduction technique for simulation studies. This investigation is performed through three case studies, with a view to understanding the performance of each variance reduction technique within these application domains. Specifically three application domains, these are (i) manufacturing systems (ii) call centre systems and (iii) crossdocking distribution systems will be utilised.

  In all the case studies, the three main techniques applied are (i) common random numbers (ii) antithetic variates and (iii) control variates. Furthermore, an investigation into the performance of jointly applying common random numbers and antithetic variates as well as common random numbers and control variates will be performed. This is to measure their effectiveness in variance reduction for selected output performance measures. The aim of this study is to find out which stand alone or combination of variance reduction techniques performs best under the same conditions, for the three case studies under consideration.

- Chapter 5 examines a new approach to the selection of variance reduction techniques through a fuzzy expert system. The design and development of the prototype fuzzy expert system was performed in stages. The initial stages of knowledge acquisition, the design and development, as well as verification testing of the fuzzy expert system are explained in this chapter.

- Chapter 6 chronicles in stages the design and development of a revised prototype fuzzy expert system, with the additional knowledge elicited from variance reduction experts. This revised system will implement an additional linguistic variable and additional fuzzy rules. In addition, this chapter will describe the validation testing performed on the revised fuzzy expert system.

- Chapter 7 provides concluding remarks about this research project and suggestions for future research that may arise from the work presented in this thesis.

The next chapter presents a background into the domain of simulation.

Chapter 2

Simulation Background

Within this chapter, a discussion on the subject of simulation, the different types of simulation techniques, and the selected application domains which will be applied in this thesis is presented. Following on is a discussion on the issue of dealing with variance associated with simulation output performance measures and the current state of art regarding the combination of variance reduction techniques. The penultimate section of this chapter will examine the current method of selecting variance reduction techniques which is manually performed through a pilot study. Finally, in the last section, a description of the problem statement central to this thesis is described.

2.1 Simulation

For most real world problems which could be solved analytically, the goal is to determine the best solution. However, to use analytical techniques for such problems usually requires many simplifying assumptions, resulting in solutions likely to be inferior or inadequate for implementation. Often, in such instances, the only alternative for modelling and analysis available to the decision maker is simulation.

Simulation is one of the most popular operational - research techniques, if not the most widely used. Simulation entails the use of computers to mimic, the operation of various kind of real world facility or process. This facility or process of interest is usually called a system, and in order to study it logically, a set of suppositions have to be adopted about how it works.

These suppositions, which usually take the form of mathematical or logical relationships, constitute a model that is used to try to gain some understanding of how the corresponding system behaves. If relationships which comprise the model are simple enough, it may be possible to use mathematical methods to obtain exact information on questions of interest; this is called an analytical solution. However, most real world systems are sometimes too complex to allow reasonable models to be evaluated methodically, as a result these models may be studied by means of simulation. In a simulation, a computer is used to evaluate a model numerically, and data are gathered in order to estimate the desired true characteristics of the model [80].

A simulation is the imitation of the operation of a real-world process or system over time. Whether done by hand or on a computer, simulation involves the creation of an artificial history of the system, and the examination of that artificial history to draw inferences concerning the operating characteristics of the real system. The behaviour of a system as it evolves over time is studied by developing a simulation model.

Simulation gives modellers an opportunity to test every facet of a proposed change or addition without committing resources to their achievement. By compressing or expanding the time frame, simulation allows the modeller to speed up or slow down events so that it can be thoroughly investigated. It is an indispensable problem solving methodology for the solution of many real-world problems. Simulation is used to describe and analyse the behaviour of a complex system, ask 'what-if' questions about the real system, and aid in the design of real systems. Both existing and conceptual systems can be modelled with simulation [13].

2.1.1 Classification of Simulation Models

For the purpose of this thesis, simulation modelling is categorised into one of two types; terminating simulation and steady - state simulation.

Terminating simulation is one in which the model dictates specific starting and stopping conditions as a normal indication of how the target system actually operates. As the name suggests, the simulation will terminate according to some model - specified rule or condition [64]. For instance, a commercial bank opens at 9 am with no customers present, closes its doors to customers at 9pm, and then continues operation until all customers are processed out of the system.

A steady - state simulation, on the other hand, is one in which the output performance measures to be estimated are defined in the long run; that is, over a hypothetically infinite time frame. Of course, a steady state simulation has to stop at some point, as these runs can get quite lengthy. So something needs to be done to make sure that the simulation experiment is run long enough. This can be carried out by stipulating a stopping condition for the model [64]. For example, when simulating a computer system, steady - state simulation may be more suitable, since most large computer systems do not shut down except in cases of breakdowns or maintenance.

2.1.2 Steps in a Simulation Study

Next is a description of the process for a complete simulation study and present a systematic approach, which usually consists of distinct stages. Figure 2.1 shows the steps that will comprise a typical simulation study. Similar figures and discussion of steps can be found in other sources, for example Gordon [51] and Shannon [131].

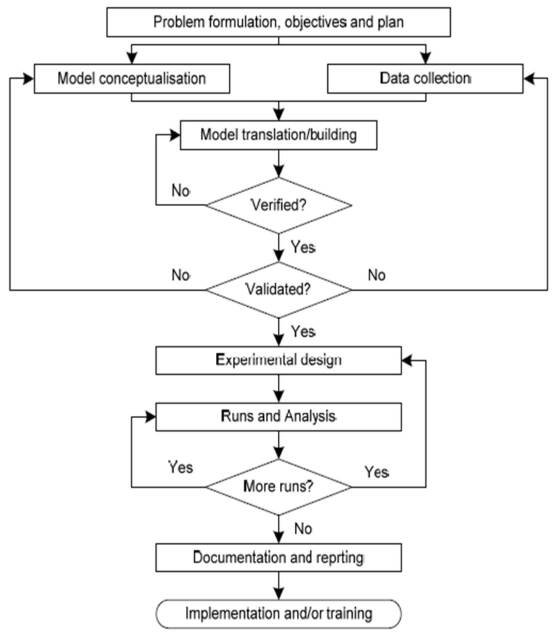

Figure 2.1

Steps in a typical simulation study (adapted from Law and Kelton (2000, Chap. 1))

Sometimes, not all simulation studies consist of all these stages or follow the order stated below, and there may even be considerable overlap between some of these stages. The initial stage of any simulation study requires an explicit statement of objectives. This should include the questions to be answered, the hypothesis to be tested, and the alternatives to be considered. Without a clear understanding and description of the problem, the chances of successfully completing the study and implementing the findings are greatly reduced. It is very likely that the original formulation of the problem under consideration will undergo many amendments as the simulation study proceeds and as more insight is gained about the complex system being studied.

The next stage is the development of the simulation model and data collection. The development of the simulation model is probably the most complicated and critical part of the simulation study. Here the essential features of the complex system under consideration is captured by logical relations. Having developed the model, the next step is put it into a form in which it can be analysed on the computer. This usually involves developing a computer program for the model.

Once the program has been developed, there is a need to determine whether the program is working properly or not. In other words, is the program doing what it should be doing? This process is called the verification step and can often be difficult, since for most simulation studies, results with which to compare the computer output will not be available at this stage. Once a level of satisfaction is gained regarding the program, the next step is validation. This validation step is another critical part of the simulation study.

During validation of the model, an assurance is sort to determine whether it realistically represents the system being analysed and whether the results from the model are dependable. The next step is experimentation, which involves working out relevant and efficient experimental conditions under which the model behaviour is examined. This stage may require parametric changes as well as structural changes to the model. This means for each system configuration of interest, a modeller will have to decide on strategic issues such as simulation run length, length of the warm up period, and the number of independent model replications.

Following the step of experimentation is the analysis of output results from the simulation experiment to decide if objectives of the study have been achieved or there is a need for additional experimentation. The final step is the documentation of the model and the associated simulation study. This should include a detailed description of the problem and the methodology used to address it, and the translation and summary of the results of the simulation output analysis into useful information. The documentation process may include recommendations for the implementation stage. The process of implementation entails making decisions that lead to changes in an existing system or to the construction of a new system on the basis of the simulation study.

2.2 Types of Simulation Techniques

Three well known simulation techniques are described within this section.

Monte Carlo simulation can be defined as a scheme employing random numbers that is *U (0,1)* random variates, which is used for solving certain stochastic or deterministic problems where the passage of time plays no substantive role. Typically, sampling from a particular distribution involves the use of random numbers, so simulation is sometimes called Monte Carlo simulation [127].

Discrete event simulation concerns the modelling of a complex system as it evolves over time by a representation in which the state variables change immediately at separate points in time. These points in time are the ones at which an event occurs, where an event is defined as an instant occurrence that may change the state of the system. Although discrete event simulation could theoretically be done by hand calculations, the amount of data that must be stored and manipulated for most real world systems dictates that discrete event simulations are done by personal computers [13].

Continuous simulation concerns the modelling over time of a complex system by a representation in which the state variable changes continuously with respect to time. Usually, continuous simulation models involve differential equations that give relationships for the rates of change of the state variables with time. If these differential equations are particularly simple, they can be solved analytically to give the values of the state variables for all values of time as a function of the values of the state variables at *0*.

For most continuous models analytical solutions are not possible, however, numerical analysis techniques such as Runge - Kutta methods are used to amalgamate the differential equations numerically, given specific values for the state variable at time *0*. Runge - Kutta methods are an important family of implicit and explicit iterative methods for the approximation of solutions for ordinary differential equation [141].

Since some systems are neither completely discrete nor completely continuous, the need may arise to construct a model with aspects of both discrete event and continuous simulation, resulting in a combined discrete - continuous simulation model. There are three fundamental types of interactions that can occur between discretely changing and continuously changing state variables [118]:

- A discrete event could cause a discrete change in the value of a continuous state variable.
- A discrete event could cause the relationship governing a continuous state variable to change at a particular time.
- A continuous state variable achieving a threshold value could cause a discrete event to occur or to be scheduled.

Discrete event simulation models have been chosen purely for the purpose of collecting data on the behaviour of the systems under consideration i.e. the manufacturing, call centre and crossdocking distribution system. This data gathering exercise will be as it is generated over a chronological sequence of events, occurring within a specific time frame. The data collected on the sequence of events will be used for variance reduction technique experimentation.

2.3 Selected Simulation Application Domains

As a prelude to discussing the three domains selected for the application of variance reduction techniques, the reason for choosing these specific domains shall be highlighted. The initial research question is to assess the performance of the application of stand alone and combined application of variance reduction techniques. This will be carried out with the assumption that all the simulation models are not the same.

The main difference between the simulation models considered within this thesis, is the assumed *randomness* inherent in each of the models. Where such randomness has been introduced by:

1. The use of probability distributions for modelling entity attributes such as inter arrival rate and machine failure.Conversely, within other models, some entity attributes have been modelled using schedules. The assumption is the use of schedules does not generate as much randomness as with the use of probability distribution.

2. The structural configuration of the simulation models under consideration i.e. the use of manual operatives, automated dispensing machines or a combination of both manual operatives and automated dispensing machines.

As a result, the manufacturing simulation is characterised by an inter arrival rate and processing time which are modelled using probability distribution, the call centre simulation an inter arrival arrival rate and processing time are based on fixed schedules. The crossdocking distribution simulation is also characterised by the use of probability distribution to model the inter arrival rate and processing time of entities.

The theoretical assumption is that by setting up these simulation models in this manner, there will be a variation in the level of model randomness. This should demonstrate the efficiency of the selected variance reduction techniques in achieving a reduction of variance for different simulation models, which are characterised by varying levels of randomness.

2.3.1 Manufacturing Simulation

Manufacturing systems involve complex and dynamic interrelationships between their many components. Materials, tools, fixtures, machines, parts, work in progress, finished goods, material handling equipment, storage systems and operators are some of the components of a manufacturing system. A decision makers task in manufacturing can be performed using one or more of the following [65]:

1. Intuitive management decisions, which rely on human experience and judgement. This method can be highly subjective on occasions. Spontaneous decisions may work in one setting but fail in others.

2. The use of expert systems that typically apply to a limited range of problems. Knowledge acquisition can sometimes be difficult.

3. Computer simulation models of manufacturing systems, which provide for the most realistic problem representation, even for very complicated scenarios.

The use of simulation has proven successful in planning and designing new plants and processes as well as for studying the behaviour of existing systems and creating consistent decision policies. The purpose of simulation in manufacturing is to gain insight, apart from generating statistics. While simulations are expected to provide numeric measures of output performance, such as throughput under a given set of conditions, the major benefit of simulation comes from the insight and understanding gained regarding the operations of the complex system under consideration [13]. When preparing to perform a manufacturing simulation study, there are a number of common system features that are included in the simulation model. These features are described below [101]:

1. Resources:

   Most manufacturing systems comprise resources, such as equipment, labour and material. The availability of these resources is usually defined by shift patterns and/or the production, arrival, or consumption of material. Resources are also often grouped together and selection rules (i.e. First In First Out or Last In First Out) used to allocate resources from these groups to perform tasks. The ability to flexibly model resources and resource groups along with their corresponding shift and breakdown patterns is a key requirement in most manufacturing simulations.

2. Processing Plans:

   In many manufacturing systems, each part type may follow its own process plan through the facility, this plan defines the routing for the processing of parts. Process plans can vary from simple, straightforward line sequences to complex networks involving parallel operations.

3. Reports:

   The process of interpreting the output results from a manufacturing simulation model is similar in most ways to any other type of stochastic model. However, there are special manufacturing focused reports that may be required. Important elements of these reports include facility metrics and job processing time as well as cost data. For example, system cycle time (how long it takes to produce one part) and total cost of resources, labour and machines.

Visualisation through animation and graphics provides major support in communication of model assumptions, system operations, and model results. Often, visualisation is a key contributor to a simulation model's credibility, which in turn leads to approval of the model's numeric output. Of course, a proper experimental design that includes the right range of experimental conditions plus a rigorous analysis of output data and, a proper statistical analysis is of utmost importance for the simulation analyst to draw correct conclusions from simulation output [13]. For many manufacturing systems, one of the reasons to represent the real system as a simulation model is due to the presence of random events. Some of the common random events encountered in manufacturing systems are:

- Processing time
- Machine set up time
- Machine down time

Many authors have discussed the proper modelling of machine down time data, examples include Williams [143] and Clark [27]. For all random events it is important to represent the distribution of randomness accurately in the simulation model. Choosing the right distribution is a very important part of the simulation process. When a known distribution cannot be found for a set of data, an experiential distribution can be used. An option to experiential distribution can be to deal with this issue of randomness during the pre - experimentation or post - experimentation stage. This can be either through simulation input data analysis or the use of variance reduction techniques [80].

2.3.2 Call Centre Simulation

In more recent times, corporations have become aware of the significant role a call centre can contribute to their operations. A call centre can be used by tele - marketers, debt collection agencies and fund raising organisations to make contact with customers by phones. These facilities can also be nominated to offer after - sales service and customer enquiry mainly through incoming calls. A typical call centre has complex interactions between several "resources and entities".

Telephone lines and human agents who call up or answer the customers calls are the key resources of a call centre, and a voice response unit can replace the human agents for some part of the incoming calls. Entities take the form of calls in a call centre. In a typical call centre the calls occupy or share the call centre's resources. Hence, the objective of call centre management team is to achieve a high service level for customers and use operative resources efficiently.

The complexity of the management and analysis of a call centre depends on several factors such as call types and waiting principles. When an incoming call entity is accepted at a call centre, and the resources are fully utilised, the arriving call may be simply lost or kept on hold in an electronic queuing system. A call may also abandon the system during the wait [66].

The main objective of most call centres is twofold; the first is to achieve a high service level i.e. to get the caller to a human agent in the shortest possible amount of time, (measured by waiting time, or in call centre terminology, service level). The second objective is to provide the customer with the appropriate information in the most efficient manner (measured by the amount of time used in handling the call). The overall aim is to minimise the time spent by the caller in the call centre, while providing the best possible service [52]. Some key performance measures include:

- Average Speed of Answer.
- Agent Utilisation.
- Abandonment Rate (Reneging).
- Average Length of Call.
- Percent Answered Without Waiting.

Balancing these objectives can be a challenging task for call - centre analysts. Furthermore, there exists a great deal of impact in the cause and effect of the performance parameters involved. For example, a small change in call routing may have a significant change on customer service. A small reduction in trunk - line capacity may cause too many lines becoming busy and potentially cause customers to abandon a call. Wrong staffing may cause long wait times, frustrated customers, and exasperated agents. These circular relationships must be defined and analysed carefully in order to achieve peak performance for the call centre [14].

The most commonly used techniques for call centre analysis are those for staffing and phone call trunking capacity calculations. A large majority of these techniques are based upon the Erlang formulation [16], which is defined as a unit of traffic intensity in a telephone system. This formulation was designed to typically find a solution to how many agents would be needed to handle the same number of calls within a single group. However, recent research had found the assumptions made using Erlang - based analysis are extremely limiting when viewed in the context of today's call centre overwhelming demand on existing capacity [14]. The Erlang formulation is based on the following assumptions:

- Every incoming call into the call centre is of the same type, it is assumed all customers typically have the same reason for making an incoming call.

- Once a incoming call enters a queue, it is never abandoned.

- Human agents handle calls on the basis of first in, first out (FIFO).

- Each human agent handles every call in exactly the same way, where for example it is assumed agents spend the same amount of time on each call.

These assumptions are rarely valid in today's call centre environment. Depending upon their individual patience for being placed on hold for a human agent, incoming callers do abandon, even if they have queued up for sometime. Agents differ in their skills and ability level and the times needed for various calls. And the reality in today's call centre is that call requests are varied in nature and may require prioritisation and complicated call handling techniques to provide better service.

In spite of this, many companies base complex staffing decisions on Erlang calculations. A well known criticism of Erlang calculations is that they consistently over - estimate staffing needs [93]. Studies have also shown that 60% to 70% of the accurable costs in call centre centres today are associated with staffing and human resources, this fact combined with the inadequacies of Erlang - based calculations, can be enormously costly to a call centre organisation [43].

Furthermore, it is clear that the application of inadequate analysis techniques could lead to poor performance evaluation when applied to a call centre that is growing in complexity. Many spreadsheet based calculations have improvised on certain aspects of the Erlang calculation to provide better practicality in the usage of such calculations for staffing. Some of them provide for an element of randomness, while others can account for abandonment [93].

Erlang - based calculations are also restrictive and sometimes incapable of analysing business questions faced by call centre analysts and managers. For example, re - engineering within call centres predominantly involves an in - depth understanding and analysis of call flow and process management. Quite simply, such problems are beyond the scope of Erlang based calculations. However, these improvements still do not provide the robustness of a solution that is provided through the use of simulation [9].

Simulation on the other hand allows call centres to represent staffing performance and analysis in the form of a simulation model. This allows all of the interrelationships between callers, human agents, call management algorithms and techniques to be explicitly defined [22]. It also ensures the modelling of human agent skills and abilities, best staffing decisions and provides an analyst with a virtual call centre that can be continually refined to answer questions about operational issues and even long term strategic business decisions. The value of Erlang-based calculations is in providing an initial input data set to feed the simulation model and there are also a variety of mathematical models for this purpose, for example Mehrotra et.al [99] as well as Mandelbaum and Shimkin [94].

2.3.3 Crossdocking Simulation

Traditionally, warehouses have had the following functions; receiving, storage, order picking and shipping. However, logistics companies have found storage and order picking to be cost intensive and this has lead to a strategy of keeping zero inventories. This strategy called crossdocking is based on a Just in Time (JIT) philosophy which eliminates the storage function in a warehouse while maintaining the receiving and shipping function [53].

Usually at a crossdocking distribution centre, trucks arrive with consignment that is sorted, consolidated, and loaded onto outbound trucks destined for customers. The customer is usually predetermined before the product arrives and as such there is no need for storage. The floor area is divided into a break up area and a build up area, where sorting and consolidation of consignment takes place, respectively. Customer order types can vary as well as the techniques for fulfilling them. The two main techniques for fulfilling orders are either through manual order picking operatives or automated order dispensers or on some occasions by both [105].

The major goals of the crossdocking distribution centre include [13]:

- How long it takes to process one day of customer orders.
- Effect of changes in order profiles.
- Truck or Trailer queuing and delays at receiving and shipping docks.
- Effectiveness of material handling systems for peak loads.
- Recovery time from short term surges.

For a crossdocking solution to succeed, the current and future operations must be understood completely. Simulation modelling can be used in building a virtual distribution facility, it can also provide additional insight beyond other design techniques. Thus, by simply building a simulation model, project team members will better understand their current and future operations. Likewise, the applicability of crossdocking can be determined with a high degree of confidence [124].

Evaluating crossdocking applications requires accurate and timely simulation modelling. Models must include sufficient detail to reflect reality. Likewise, if a model does not provide output statistics in the right format, it is difficult to make decisions based on the models results. Simulation is the next best method of giving confidence that the proposed system will meet a company's requirement [139].

In order for a simulation model to be useful, it must provide output that can be used to compare different scenarios. There are a few crossdocking performance metrics that are provided by a good model. Such as truck dock metrics i.e. the time a truck spends loading and unloading. If a truck must wait to be unloaded, then this time should be recorded to determine yard congestion. Also, crossdock door utilisation is useful when determining the number of doors required [2], [15].

For inventory moving through the system, such as pallets and cases, the time spent between receiving to shipping is an important measure of how the crossdocking system is performing. Recirculation time should also be reported for cases that miss their diverts or must be re - scanned. Throughput of the system should be reported for individual areas as well as for the entire system. Time - line graphs of throughput per hour aid in determining when peak conditions occur [84], [86].

Every crossdocking operation requires that some manual operations be performed. The randomness of these manual operations should be considered during model development. This includes collecting data on the operation itself and fitting a statistical distribution to the data. Additionally, wherever manual operators interact with automation, care should be taken to ensure modelling accuracy. In the absence of existing data, thorough sensitivity analysis should be performed [91].

Simulation has been used to determine the requirements for logistics operations, to allow continuous operations, and to provide critical decision support [74]. It has also been used to determine the way a change in the size of loading and carrying fleets would affect the performance of the system [48]. In addition, simulation has been used to validate and test the adequacy of a technique or to calculate safety stock and the proper replenishment policy [44]. Furthermore it has been used to capture the complexities in a supply chain and produce the entire warehouse or transportation link [34].

Simulation has been used in warehousing and inside distribution centres [95]. The authors Burnett et.al. [19] produced a flexible simulation model for the Ryder System, Inc., to validate automated warehouse designs, predict resource requirements, and determine operational throughput capacities for its E - channel operations. A flexible dynamic simulation model was also developed and reported by Carr and Way [24], that describes the loading, staging, travel, and unloading of rail cars at a manufacturing facility and two distribution centres. The simulation model output and analysis enabled management to optimise rail car availability and crew sizing.

Simulation models of complicated and non - automated distribution warehouses were developed by Takakuwa et.al., which consists of two phases; a program for generating parameters and another for generating a simulation program. The authors demonstrate the applicability of simulation modelling for crossdocking by illustrating an actual case study [137]. A simulation model for a universal warehouse storage using the ProModel™ simulation language was developed by Macro and Salmi [90]. The model was used to analyse the storage capacity and rack efficiency of a warehouse, the authors considered the model to be scalable and modifiable for warehouse simulation. Lately simulation is considered to be one of the most important techniques used in manufacturing and logistics systems, and is needed to meet the challenges of transportation and logistics as well as supply chain problems of today and the future [142].

2.4 Reduction of Variance in Discrete Event Simulation

The development of simulation models requires a specific knowledge that is usually acquired over time and through experience. Since most simulation output results are essentially random variables, it may be difficult to determine whether an observation is as a result of system interrelationships or the randomness inherent in simulation models.

Furthermore, simulation as a process can consume a lot of time, despite advances in computer technology. An example of a time consuming task is one that is statistically based i.e. output data analysis. However, it is known that advances in computer simulation has allowed the modelling of more complicated systems. Moreover, even when simpler systems are simulated, it can be difficult to judge the precision of simulation results.

2.4.1 Selected Simulation Output Performance Measure(s)

In general, output analysis is the examination of data generated by a simulation experimentation, and its purpose is to predict the performance of a system or to compare the performance of two or more alternative system design [79]. Output simulation analysis studies may be performed for one of the following reasons [65]:

- To determine the characteristics (mean, variance, minimum, maximum, etc) of certain variables for given input conditions, parameter values, and model configurations to analyse and understand the behaviour of a future system at the design stage.

- To compare the characteristics (mean, variance, minimum, maximum, etc) of certain variables under various input conditions, parameter values, and model configurations. Manipulation of these factors and comparison of their effects for each simulated scenario can result in finding the condition under which the system performs satisfactorily. The ultimate goal of the modeller may be either to improve the performance of the existing system or design a future system.

However, discrete event simulation models differ from one another insofar as they have different values or types of system parameters, input variables, and behavioural relationships. These varying parameters, variables, and relationships are called “factors” and the output performance measure is called “response” in statistical design terminology [7]. The decision as to which parameters are selected as fixed aspects of the simulation model and which are selected as experimental factors depends on the goals of the study rather than on the inherent form of the model. Also, in discrete event simulation studies there are usually a wide range of different responses or performance measure, which can be of interest.

As a result, output performance measure for the three different simulation models considered within this thesis have been carefully selected after considering literature which reports on the most common performance metric for judging the performance of each simulation model (i.e. Manufacturing simulation, Call Centre simulation, and Crossdocking simulation). In addition, selection of ouput performance measures have been carried out in order to achieve a research goal of reducing simulation output variance through manual experimentation. A list of selected "responses" for each discrete event simulation model can be found Chapter 4, Sections: 4.1.2.1, 4.2.2.1 and 4.3.2.1.

For simulation models, where the performance of such models is measured by its precision, i.e. mean, standard deviation, confidence interval and half width, for the selected output performance measure, it is sometimes difficult to achieve a target precision at an acceptable computational cost because of variance. This variance is usually that which is associated with the performance measure under consideration. For example, Adewunmi et.al. [3], investigated the use of the Sequential Sampling Method [80] to achieve a target variance reduction for a selected simulation output performance measure. Results from experimentation indicate that this technique for reducing variance requires a huge number of runs to achieve any success for this particular discrete event simulation model.

In a wider context, the variance associated with a discrete event simulation model or its output performance measure may be due to the inherent randomness of the complex system under study. This variance can make it difficult to get precise estimates on the actual performance of the system. Consequently, there is a need to reduce the variance associated with the simulation output value, using the same or less simulation runs, in order to achieve a desired precision [144].

2.4.2 Selected Variance Reduction Techniques

It is known that one way to generate results quicker and with more confidence for discrete event simulation models is through the use of some sophisticated statistical techniques for processing of simulation output results i.e. variance reduction techniques. "A variance reduction technique is a statistical technique for improving the precision of a simulation output performance measure without using more simulation, or, alternatively achieve a desired precision with less simulation effort" [67].

Variance reduction techniques were originally developed in the early days of computers, to be applied to Monte Carlo simulations or distribution sampling [56], [103]. They can be viewed as a means to use known information about a problem. In fact, if nothing is known about the problem, variance reduction cannot be achieved. At the other extreme, that is, complete knowledge, the variance is equal to zero and there is no need for its application.

The reduction of variance cannot be obtained from nothing, it is simply a way of not wasting information. One way to gain this information is through a **pilot simulation experiment**. Results from this pilot study can then be used to select variance reduction techniques that will refine and improve the efficiency of the full simulation. Therefore the more that is known about the behaviour of the complex system being studied, the more effective the variance reduction techniques being considered for deployment. Hence it is always important to clearly define what is known about the system and knowledge of a process to be simulated can be quantitative, qualitative, or both.

The efficiency of a variance reduction technique is usually measured by a decrease in variance of the mean for a selected performance measure. It is necessary to measure the decrease of the variance due to applying a variance reduction technique. If it is required that the desired precision of the simulation output is predetermined, then the variance should be measured. Of course a variance reduction implies a standard deviation reduction and vice versa. Besides the reduction of the standard deviation or variance, it is necessary to take into account the extra computational effort that variance reduction technique may require.

In this thesis, the discussion has been restricted to a selected subset of variance reduction techniques which have proven to be the most practical in use within the discrete event simulation domain [77], [26]. Apart from being a subset of variance reduction techniques that have been reported to be successful in discrete event simulation studies, these techniques have been chosen because of the manner each of them performs variance reduction i.e. through random number manipulation or the use of prior knowledge. The potential gains which may accrue from the combination of these techniques is also worth investigating because it may increase the already existing knowledge base on such a subject.

The three selected variance reduction techniques fall into two broad categories; the first class of variance reduction techniques manipulates random numbers for each replication of the simulation experiment, thereby inducing either a positive or a negative correlation between the mean response across replications. This induced correlation can lead to a reduction in the variance of the estimate of the overall mean response. On the other hand, one side effect of this process, is that there can no longer be an assumption regarding independence between replications.

Typically, this correlation between the mean response across replications is introduced by manipulating the seeds of the random number generators. Two methods of this category of variance reduction techniques are presented. The first method, **Common Random Numbers**, only applies when comparing two or more systems. The second method, using **Antithetic Variates**, applies when estimating the response of a variable of interest [29].

The second class of variance reduction techniques incorporates a modeller's prior knowledge of the system when estimating the mean response, which can result is a possible reduction in variance. By incorporating prior knowledge about a system into the estimation of the mean, the modeller's aim is to improve the reliability of the estimate. Another variance reduction technique discussed in this section is based on this concept. For this technique, it is assumed that there is some prior statistical knowledge of the system. A method that falls into this category is **Control Variates** [112].

Again, there is no guarantee that the procedures mentioned below will actually produce a reduction in variance; in fact, there have been occasions when they have had a reverse effect. Unfortunately, inducing a correlation destroys the independence property across replications, so modellers cannot compute traditional estimates from the same set of replications for the purpose of comparison. Therefore, in practice a simulation analyst generally has no way of knowing how much variance is gained or lost by using variance reduction techniques methods. Despite these drawbacks, these techniques generally do work well and can yield substantial improvement in the precision of output estimators.

It is known that variance reduction refers to reducing the population variance of an estimator, for example where X is the estimator and a may be the variance. However, variance reduction does not necessarily affect the variability of the simulated stochastic process or complex system, considering that simulation models are characteristically variable in nature. The following literature with extensive bibliographies are recommended to readers interested in going further into the subject i.e. Nelson [107] and Kleijnen [70].

In next section is a discussion on the three variance reduction techniques that appear to have the most promise of successful application to discrete event simulation modelling.

2.4.2.1 Common Random Numbers

Usually the use of Common Random Numbers only applies when comparing two or more alternative scenarios of a single systems, it is probably the most commonly used variance reduction technique. Its popularity originates from its simplicity of implementation and general intuitive appeal. The technique of common random numbers is based on the premise that when two or more alternative systems are compared, it should be done under similar conditions [18].

The objective is to attribute any observed differences in performance measures to differences in the alternative systems, not to random fluctuations in the underlying experimental conditions. The term common random numbers comes from the use of the same random numbers within pairs of replications as a means of inducing a positive covariance between responses within a paired set of replications. *Simply using the same random number seed for each replication within the pair is usually not sufficient to induce covariance, it is important to also ensure that random numbers are synchronised i.e. the random numbers are used at the same junction and for exactly the same purpose across systems [78].*

Statistical analysis based on common random numbers is founded on a single premise. Although a correlation is being introducing between paired responses, the difference, across pairs of replications is independent. This independence is achieved by employing a different starting seed for each of the n pairs of replications. Unfortunately, there is no way to evaluate the increase or decrease in variance resulting from the use of common random numbers, other than to repeat the simulation runs without the use of the technique. It can be difficult to compute the confidence interval for both with and without common random numbers from the same set of replications as can be done with other types of variance reduction techniques. In a practical application, it could be difficult to find out whether the decision to use common random numbers is correct.

However, there are specific instances where the use of common random numbers has been guaranteed. Gal et.al. present some theoretical and practical aspects of this variance reduction technique, and discuss its efficiency as applied to production planning and inventory problems [42]. In addition, Glasserman et.al. state that "common random numbers is known to be effective for many kinds of models, but its use is considered optimal for only a limited number of model classes". They conclude that the application of common random numbers on discrete event simulation models is guaranteed to yield a variance reduction [50].

To demonstrate the concept of common random numbers, let X_a denote the response for alternative A and X_b denote the response for alternative B, while considering a single system. Let D, denote the difference between the two alternatives, i.e. $D = X_a$ - X_b.

The following equation gives the random variable *D's* variance.

$$Var(D) = Var(X_a) + Var(X_b) - 2Cov(X_a, X_b) \tag{2.1}$$

2.4.2.2 Antithetic Variates

In comparison to common random numbers, the Antithetic Variates technique reduces the variance by artificially inducing a correlation between replications of the discrete event simulation model. Unlike common random numbers, the antithetic variate technique applies when seeking to improve the performance of a single system's performance.

This approach to variance reduction makes n independent pairs of correlated replications, where the paired replications are for the same system. The idea is to create each pair of replications such that a less than expected observation in the first replication is offset by a greater than expected observation in the second, and vice versa [5], [40]. Assuming that this value is closer to the expected response than the value that would result from the same number of completed independent replications, average the two observations and use the result to derive the confidence interval.

A similar feature that antithetic variates shares with common random numbers is it can also be difficult to ascertain that it will work, and its feasibility and efficacy are perhaps even more model dependent than common random number. Another similarity it shares with common random numbers is the need for a **pilot study** to assess its usefulness in reducing variance for each specific simulation model [25].

In some situations, the use of antithetic variates has been known to yield variance reduction, and as mentioned earlier it can be model specific. In his paper, Mitchell considers the use of antithetic variates to reduce the variance of estimates obtained in the simulation of a queuing system. The results reported in this paper, show that a reduction in variance of estimates was achieved [102].

The idea of antithetic variates more formally presented. Let random variable X, denote the response from the first replication and X' denote the replication from the second replication, within a pair. The random variable Y denotes the average of these two variables, i.e., $Y = (X + X')/2$. The expected value of Y and the variance of Y are given as follows:

$$E(Y) = [E(X) + E(X')]/2 = E(X) = E(X') \tag{2.2}$$

and

$$Var(Y) = [Var(X) + Var(X') + 2Cov(X, X')]/4 \tag{2.3}$$

2.4.2.3 Control Variates

This technique is based on the use of secondary variables, called Control Variates. This technique involves incorporating prior knowledge about a specific output performance parameter within a simulation model. It does not however require advance knowledge about a parameters theoretical relationship within the model as would other techniques like indirect estimation.

As compared with common random numbers and antithetic variates, the variance reduction technique, control variates attempts to exploit the advantage of the correlation between certain input and output variables to obtain a variance reduction. Of course depending on the specific type of control variate that is being applied, the required correlation may arise naturally during the course of a simulation experiment, or might arise by using common random numbers in an auxiliary simulation experiment [78].

In order to apply the control variates technique, it has to be assumed that a theoretical relationship exists between the control variate X, and the variable of interest Y. This approach does not require that a modeller knows the exact mathematical relationship between the control variate and the variable of interest; all the knowledge needed is to only know that the values are related.

This relationship can be estimated by using the data recorded for instance from a pilot simulation study. Information from the estimated relationship is used to adjust the observed values of Y [22]. Let X be the random variable that is said to partially control the random variable Y, and hence, it is called a control variate for Y. Usually it is assumed that there is a linear relationship between the variable of interest and the control variate. The observed values of the variable of interest Y can then be corrected, by using the observed values of the control variate X, as follows:

$$\mathrm{Y}_i(\mathrm{n}) = \mathrm{Y(n)\ -\ a\ (X(n)\ -\ E(X)(n))} \tag{2.4}$$

and

$$a = \frac{Cov(Y(n), X(n))}{Var(X)} \tag{2.5}$$

Where a is the amount by which an upward or downward adjustment of the variable of interest Y is carried out, *E(X)* is the mean of X, and n is the number of replications.

In reality, there may be a very large number of possible control variates for a modeller to consider. However, it is not necessarily an ideal approach to use them all, since the variance reduction they may bring is accompanied by variance contributions associated with the need to estimate the optimal variable of interest. A method is proposed by Bauer and Wilson [62] for selecting the best subset from the available control variates under a variety of assumptions in a situation where their variances and covariances are known.

There are, however, some classes of discrete event simulation models for which the application of control variates has proven to be successful. In a recent article on the use of variance reduction techniques for manufacturing simulation by Eraslan and Dengiz [37], control variates and stratified sampling were applied for the purpose of improving selected performance measures for queuing simulation models. Results from this paper suggest that control variates yields the lowest variance for selected performance measures.

The main advantage of using control variates as a technique for variance reduction is that they are relatively easy to use. They only require that a modeller identify an appropriate control variate, which has a known, expected value and is correlated to the response of interest. Statistics about the control variate and the variable of interest must be recorded, and a simple analysis of their role in the model could suggest how they might be correlated with each other.

The same set of replications has to be used to calculate the variance with and without control variates. Thus, this will potentially eliminate the danger of producing less accurate results with a variance reduction technique. Most importantly, control variates can essentially be generated anywhere within the simulation run, so they add basically nothing to the simulation's cost; thus they will prove worthwhile even if they do not reduce the variance greatly.

2.5 Combining Variance Reduction Techniques

It is a natural idea in simulation studies to try to combine different variance reduction techniques, with the hope that their individual beneficial effort will add up to a greater magnitude of variance reduction for the estimator of interest. These combination could have a positive effect especially when several alternative system configurations are to be compared.

To obtain more variance reduction, one may want to combine variance reduction techniques simultaneously in the same simulation experiment. For example, to compare two alternative systems, one may perform n pairs of simulation runs for each system, with common random numbers across the systems and antithetic variates within each pair for each alternative system. However, even if both common random numbers and antithetic variates are individually effective, their combination could possibly be worse than using only one of them, due to the cross - correlations between the response for the first system and the corresponding antithetic response for the second system [78].

Research work on combining variance reduction techniques in simulation literature include Burt [20], who combines antithetic variates and control variates, obtaining better results than when either technique is applied individually. Another study by Kleijnen [68] involving the combination of antithetic variates and common random numbers to compare two alternative systems was performed where the results show that the combined technique can be inferior to antithetic variates and to common random numbers for this particular application.

Kleijnen [69] had also previously proposed a new combined variance reduction scheme, where a pilot study is performed first, then the combined scheme is executed in the following order, (i)antithetic variates, (ii) common random numbers and (iii) joint application of antithetic variates and common random numbers. This scheme demonstrates for this author's particular application, which technique is more superior, combined application or stand-alone application of variance reduction techniques.

In an interesting paper describing the application of variance reduction techniques on a call centre, a demonstration of variance reduction results are presented by L'Ecuyer and Buist [83]. The authors studied the combination of control variate and stratification with respect to a continuous input variable, and found that combining them requires skill and judgement on the part of the simulation modeller. Another variance reduction technique combination which was considered by L'Ecuyer and Buist, in the same paper, is stratification with common random numbers. It was concluded that proper use of common random numbers achieves variance reduction in this particular instance, where the variance of the difference of performance measures across the two systems is being considered.

A strategy for combining common random numbers and antithetic variates in an experimental design scheme based on the idea of blocking and conditions under which variance reduction is guaranteed was proposed by Schruben et.al. [130]. An analysis of the efficiency of control variates and antithetic variates in improving the performance of point and interval estimators when initial bias exists in the model is presented by Nelson [108]. The study of a pairwise combination of control variates and antithetic variates for estimating a single response in a finite - horizon model, to establish conditions for which these combinations can outperform each other was reported by Avramidis et.al. [8]. The authors achieved large gains in variance reduction for a stochastic activity network.

An investigation into incorporating common random numbers into an antithetic variates and common random number scheme and an investigation into the conditions under which a combination scheme performs better was performed by Tew et.al. [138]. See also Andradottir et.al. [4] for an analysis of combining variance reduction techniques and Kwon et.al. [75] who present three methods to combine antithetic variates and control variates.

An article by L'Ecuyer et.al. [82] presents an overview of various techniques in the variance reduction technique literature by giving a number of examples; the author also discusses common random numbers, antithetic variates, control variates and some other techniques not considered in this thesis as well as the combination of those methods. In another study, Yang and Liou [145] show that the combined use of control variates and antithetic variates yields a smaller variance than the conventional control variate estimator applied without antithetic variates.

Under consideration in this thesis are three variance reduction techniques, with which a variety of combinations have been obtained. However, the selected combinations are (1) common random numbers and antithetic variates, (2) common random numbers and control variates. This combination strategy has been chosen because of its possibility to succeed based on the characteristics shared between the variance reduction techniques. For example, in strategy (1) both techniques are based on inducing correlation between separate runs, and hopefully the positive correlation will offset the negative correlation for the use of both techniques. Unlike common random numbers and antithetic variates, control variates is concerned with a correlation between random variates. So strategy (2) seeks to take advantage of the need to synchronise random numbers and the necessity to obtain a correlation between random variates for variance reduction.

A paper by Yang and Nelson considers the evaluation of alternative model designs by constructing confidence intervals of certain differences in expected performance and then proceeds to improve such results with the use of common random numbers and control variates [146]. However, this will be the first time to our knowledge that such a strategy which combines common random numbers and control variates for variance reduction techniques has been proposed for application within the domains considered in this thesis.

2.6 Manual Selection of Variance Reduction Techniques

For a successful discrete event simulation study, there should be an analysis of the input parameters, validation and verification of the simulation model, as well as an analysis of the output data, amongst other tasks. If the modeller has enough time during the experimentation process, there can be a design of experiment stage during the pre - experimentation stage as well as consideration of the utilisation of variance reduction techniques as a post - experimentation strategy. While design of experiment serves as a tool for screening factors and gauging their effect on response, variance reduction techniques provide a means of reducing variance associated with a selected output performance measure as well as globally increase the precision of a simulation model.

There are a variety of variance reduction techniques which are targeted at either Discrete event simulation, Continuous simulation or Monte Carlo simulation. Examples include common random numbers, antithetic variates, control variates, indirect method, stratified sampling and importance sampling. Since the focus is on discrete event simulation models and applicable variance reduction techniques, and the effect of the combined application of these variance reduction techniques for discrete event simulation studies. However, without a pilot study, it was not possible to ascertain which variance reduction technique would outperform the other [80].

This highlights the problem of intelligently selecting a variance reduction technique without prior experimentation. There has been some research into the automated selection of variance reduction techniques for simulation studies from the perspective of algorithms, heuristics and mathematical formulation (pure analytical solution). However, these attempts have been purely conceptual in nature without implementation to ascertain practicality. This does not negate the possible efficiency of algorithms etc., on the contrary these approaches form the theoretical foundation for the decision support system.

One of the earliest discussions about the selection of a variance reduction technique for simulation studies but from the perspective of Monte Carlo simulation was a paper authored by McGrath and Irving [98]. In this paper the authors devoted a section to discussing about the selection of variance reduction techniques, which were classified into three groups. The opinion of McGrath and Irving is that selection of a variance reduction technique for a particular simulation model is peculiar to that model, and general procedures for selecting a variance reduction technique are difficult to follow. Furthermore, the author states that initial investigation must be performed in order to select or evaluate the usefulness of applying whichever variance reduction technique that has been selected i.e. through a pilot study. Searching for a technique forces the simulation designer into asking the basic questions of:

1. What answers are to be generated from the simulation? and

2. What is known about the behaviour of the process i.e. output analysis?.

McGrath and Irving believe that problem definition is an important consideration, before selecting a variance reduction technique for simulation. And to evaluate the practicability of the selected variance reduction technique, a modeller should do the following:

1. List all the parameters to be estimated,and

2. Determine all known knowledge on the internal workings of the process being simulated.

The main conclusion that can be drawn from this paper is, for a useful selection of a variance reduction technique for Monte Carlo simulation, or any simulation at all, a modeller has to consider prior information about the simulation model. This is prior knowledge about the behaviour of the simulation model through output analysis.

Nelson [106] proposed a new categorisation of variance reduction techniques as the basis for an algorithm to select variance reduction techniques for general simulation studies. Early in the paper, the author recognises the difficulty of selecting a variance reduction technique without performing a pilot study to ascertain the effectiveness of the selected variance reduction technique. He began examining the problem of variance reduction technique selection in [111] by proposing an efficient categorisation of variance reduction techniques.

However, in this paper, “A decomposition approach to variance reduction”, Nelson seeks to merge the research output of [109] and [110] into a proposed guideline for selecting a variance reduction technique for simulation studies. He discusses five of the most popular variance reduction techniques in simulation studies i.e. common random numbers, antithetic variates, stratified sampling, post stratified sampling, and control variates. The author sparks research interest through this paper, for an algorithm which can select a variance reduction technique for simulation by stating, “a practitioner with an in-depth knowledge of variance reduction will likely do a better job of selecting variance reduction techniques than the algorithm presented in the paper”.

However, the possibility of such an algorithm demonstrates that the use of variance reduction need not be limited to such practitioners. He goes further to state that a necessary condition for successfully applying any variance reduction technique is to have prior knowledge, which is defined to be any knowledge either known with certainty or suspected, beyond what is needed to construct the original simulation experiment. The Nelson algorithm [106] characterises variance reduction techniques as transformation from one simulation experiment to another.

Here are the three major steps in Nelson’s algorithm for the selection of a variance reduction technique:

1. Express the simulation experiment as inputs, output, sampling plan and statistics,

2. Determine available prior knowledge, and

3. Select a variance reduction technique from those that completely or partially decompose into elementary transformation.

In Nelson's opinion, a one at a time investigation of variance reduction techniques produces more dead ends and may over look some useful prior knowledge. Although a practitioner with in-depth knowledge of variance reduction can likely decide on an appropriate technique more quickly without the algorithm, the approach proposed here is the kind needed for the implementation of an intelligent selection technique. The advantage of Nelson's algorithm, as opposed to a one at a time search of variance reduction techniques is that it identifies the available prior knowledge in terms of the six classes of transformation first, and then yields all variance reduction techniques that make use of that knowledge.

In his paper, which described certain criteria for the selection and application of variance reduction techniques for discrete event simulation studies, Cheng [26], comments it is important to carefully consider the selection of a variance reduction technique before application. He also corroborates Nelson's algorithm [106] for the selection of variance reduction techniques. He comments that such an algorithm has the potential to clearly show which variance reduction technique in combination with another has the potential to succeed, and when the possibility of their combination may yield a negative reduction in variance.

2.7 The Problem

This section summarises the thesis problem statement, from two perspectives; the need to intelligently select a variance reduction technique, as well as the need to handle the ambiguity which is associated with human expert knowledge. After this will be a summary of the solution approach.

2.7.1 Selection of a Variance Reduction Technique

Generally the use of variance reduction techniques can complicate simulation output analysis, although these techniques can save computer resources by reducing the required number of replications of the simulation model, the proper use of these techniques demands skill and judgement. Furthermore, there is no way to know in advance if the application of a variance reduction technique will actually reduce variance associated with the selected performance measure.

However, in most cases these techniques are effective at reducing the variance of the estimator. Unfortunately, the antithetic variate and common random number methods suffer from the same disadvantage; there is no way in a practical application to determine whether the methods have been successful. An analysis of the improvement of model performance for both with and without antithetic variates cannot be performed from the same set of replications. Thus, short of re - executing the simulations without using antithetic variates or carrying out a pilot study using common random numbers, there is no way of determining whether their use has been beneficial [22].

Furthermore, a two-stage procedure proposed by Dudewicz and Dalal [35] and extended by Koenig and Law [72] could give a clue regarding the manual selection of variance reduction techniques. This procedure involves an initial pilot study and use of the resulting variance from the pilot study to determine the simulation effort required to achieve a desired precision. For the implementation of the manual selection of a variance reduction technique reported in this thesis, a pilot study to assess the performance of a set of variance reduction techniques on three discrete event simulation models will be performed. The result of the pilot study can then be used as a guide to making a choice of which technique is most suitable.

The potential drawback of using this approach as will be demonstrated later in this thesis, is the time and effort required to perform such an investigation. In addition, the possibility of making a wrong choice of technique if for some reason statistical results do not give enough evidence to choose a particular technique.

It should pointed out, in order to perform a pilot study, a modeller is assumed to have some knowledge about variance reduction techniques and the discrete event simulation model under consideration. It is reasonable to assume that the knowledge accumulated, at the conclusion of the pilot study, makes for an addition in building a robust simulation variance reduction knowledge base. This leads to the central question of this thesis:

- Can variance reduction technique(s) be intelligently selected for a discrete event simulation study?

Initial thoughts are there is a possibility to perform such a task. As mentioned earlier, Nelson's algorithm has set the theoretical foundation for such a task. In addition, there is an availability of knowledge on the selection and application of variance reduction techniques, as well as advancements in computerised systems for assistance with decision making. The eventual goal of such an intelligent procedure will be to assist a simulation modeller in the selection of a variance reduction technique for discrete event simulation studies.

2.7.2 Dealing with Ambiguity in Human Expert Knowledge

The ability of humans to solve problems is based on a profound understanding of the problem situation. This deep knowledge of the problem is the core structure that constitutes the interrelationship between components of any proposed solution approach, whether it be manual or computerised. This profound understanding of a problem is based on a completely integrated body of human consciousness that includes emotions, knowledge, intuition and so on.

Typically this type of knowledge for problem solving can be difficult to computerise, which is one of the issues that will be dealt with in this thesis. An example of such problems is English language, probably the most popular method of communication worldwide which can sometimes can be unclear. Comparative words like "taller" and "better" can sometimes be used loosely. This raises specific questions such as; how can the words "a little variance" and "a lot of variance" be understood by a computer, since their definition and/or interpretation is wrapped in ambiguity or uncertainty.

Dealing with such ambiguity is important during the consideration of a solution approach to the intelligent selection of a variance reduction technique. The preferred solution approach should be able to utilise the available variance reduction knowledge in assisting a modeller with the decision making process.

2.7.3 A Fuzzy Expert System Approach

The main problem a simulation modeller will face with a manual selection of variance reduction technique(s) for discrete event simulation studies, is the use of his or her own knowledge will not prove sufficient in making a selection. Furthermore, without a pilot study it is difficult to determine the expected performance of a particular technique, considering that there are quite a number of techniques to choose from. However, there are literary resources and human experts a modeller may wish to consult for directions.

Nelson's algorithm seems a possible solution approach to the variance reduction technique selection problem. However, it does not meet the criteria set for a selection method which can be flexible enough to become applicable to similar discrete event simulation decision making problems.

The availability of expert knowledge improves the possibility of exploiting a proposition whereby a decision support system is designed and developed with a view to providing assistance with the selection of a variance reduction technique. Such a system, in which the knowledge of experts is captured, represented and processed would be a suitable solution approach, and an alternative to Nelson's algorithm. Furthermore, such a computerised system should be able to handle the uncertainty associated with the elicited expert knowledge from the domain of interest. A fuzzy expert system can potentially provide a solution from two perspectives:

1. Its ability to mimic human behaviour for the purpose of decision making and,

2. Its ability to handle the imprecision, inherent in representing human knowledge in a computerised manner.

A Fuzzy expert system, as the name suggests, is a computer based decision support system that emulates the reasoning process of a human expert within a specific domain of knowledge. These systems are fundamentally built for the purpose of making the experience, understanding, and problem solving capabilities of a human expert in a particular domain available to a non expert. In addition, they can be designed for various activities, such as diagnosis, decision support, planning, or research.

Furthermore, a fuzzy expert system, also known as a fuzzy logic based expert system, provides a framework for the management of uncertainty and a systematic way for representing and inferring from imprecise rather than precise knowledge. These qualities of a fuzzy expert system have motivated this research towards mimicking the human process of selecting a variance reduction technique for discrete event simulation studies. In the next chapter, a discussion aimed at giving readers an overview of fuzzy expert systems is presented.

Chapter 3

Theory of Expert Systems

The aim of this chapter is to give the reader an understanding of the methodologies applied in this thesis. The chapter begins with a definition of expert systems and proceeds into a description of techniques for verifying and validating expert systems. This is followed by a description of the technique utilised for representing expert knowledge and dealing with uncertainty in expert knowledge within the decision support system, described in this chapter. In addition, there is a description of the theory of fuzzy expert systems and a review of selected literature, where expert systems have been applied in the domain of simulation.

3.1 Expert Systems

Expert systems have been defined in various ways, but all the definitions share a common theme suggesting that experts systems are artificial intelligence techniques used to emulate the way in which domain experts solve problems. Edward Feigenbaum, one of the best known artificial intelligence researchers, defined an expert system as follows [39]:

> "An expert system is an intelligent computer program that uses knowledge and inference procedures to solve problems that are difficult enough to require significant human expertise for their solutions. Knowledge necessary to perform at such level, plus the inference procedures used, can be thought of as a model of the expertise of the best practitioners in the field. The knowledge of an expert system consists of facts and heuristics. These facts constitute a body of information that is widely shared, publicly available, and generally agreed upon by the experts in the field. Heuristics are mostly private, little - discussed rules of good judgement (rules of plausible reasoning, rules of good guessing) that characterise expert - level decision making in the field. The performance level of the expert system is primarily a function of the size and quantity of the knowledge base it possesses."

As can be seen from the above quote, an expert system is a system that employs human knowledge stored in a computer to solve problems that usually require human expertise. Well designed systems imitate the reasoning process of experts to solve specific problems. Such systems can be used by non experts to improve their problem solving capabilities.

Expert systems are used to spread scarce knowledge resources for improved consistent results. Ultimately, such systems could probably function better than any single human expert in making judgements in a specific, usually narrow, area of expertise (referred to as a domain) [140]. Expert systems were developed as a special type of artificial intelligence technique to successfully deal with complex problems from a narrow domain such as medical disease diagnosis. The classic problem of building a general - purpose artificial intelligence program that can solve any problem has been too difficult without specific knowledge from the problem domain.

Generally, the first step in solving any problem is defining the problem area or domain to be solved. This consideration is just as true in artificial intelligence as in conventional programming (One of the earliest definitions of artificial intelligence was, and still is "making computers think like people" [30]). Expert systems make extensive use of specialised knowledge to solve problems at the level of a human expert. An expert is a person who has expertise in a certain area. That is, the expert has knowledge or special skills that are not known or available to most people and have been acquired over the passage of time.

An expert can solve problems that most people cannot solve at all or solve them much more efficiently. The knowledge in expert systems may be either expertise, or knowledge that is generally available from books, magazines, and knowledgeable people. The basic concept of an expert system involves a user supplying facts or other information to the system and receiving expert advice or expertise in response. Internally, the expert system consists of two main components, the knowledge base and the inference engine which makes the conclusions. These conclusions are the experts' systems responses to the users queries for expertise [49]. The next section is a discussion on the verification and validation of expert systems.

3.2 Expert System Verification and Validation

The most challenging stage of building an expert system is verification and validation testing. The basic motivation behind these tasks is to control performance, efficiency, and the quality of the knowledge base. The goal is to establish compliance with user expectations and system functioning. When an expert system is built via prototyping, as is the case with the fuzzy expert system (see chapter 5 & 6), each phase of the building process can be subjected to verification and validation testing rather than waiting until the end of development life cycle, as is usually done in conventional information systems.

Verification consists of putting the expert system through a procedure of tests to ensure that the system is right i.e. that the program does what it is designed to do. The internal make up of the system is checked to see that rules fire when they are supposed to fire. In this way, the technical performance of the system is checked. Validation involves testing the system to ensure that it is the right system, that it meets the experts expectations. Validation provides assurance that the suggested solutions or advice derived from the knowledge base comes close enough to those of the human expert. In other words, the validation process checks the reliability of the expert systems output [11].

Unlike verification, validation is done only when the system is operational. In other words, validation can be done only after verification. If the system is modular, each module goes through both steps of verification and validation testing, in the course of building the expert system. As mentioned earlier, validation follows from verification where its focus is on testing the behaviour of the integrated system against the users requirements. Even when each rule is individually correct, something in their interactions can cause the system to act up [11].

The need for research on the validation of expert systems is emphasised by O'Keefe et.al. [114], who proposed general descriptive methodologies for quantitative and qualitative methods of validation. The role and importance of validating expert systems is well documented in Geissman et.al. [47] and Harrison et.al. [57]. The inappropriate nature of traditional software validation techniques for expert system validation testing was observed and a research method approach was proposed by O'Leary et.al. [115].

The inadequacy of traditional software engineering validation techniques for expert systems, arises from the nature of developing expert systems. While verification and validation testing for a typical software will be performed around the end of the software development life cycle, these tasks are performed concurrently as the expert system is being developed. So as each version of the expert system is refined, i.e. prototype system to fully implemented system, the verification and validation testing exercise is performed iteratively. There are two popular approaches to verification of expert systems [10]:

1. Verify knowledge base formation, and

2. Verify knowledge base functionality.

Under knowledge base formation, the structure of the knowledge as it relates to circular or redundant rules is verified. Verification of functionality focuses on confidence and reliability of the knowledge base. No single validation technique is best for detecting all errors in an expert system. Some techniques work better for certain types of problems (e.g. diagnostic versus advisory systems), and for certain types of knowledge representation (e.g. production rule versus frames). A number of known validation testing techniques worth reviewing are:

- Face Validation: With the face validation approach, a group of developers, users, and experts evaluate the performance of the expert system at its face value [114]. The solutions provided by the system are compared with those of the human experts, and value judgements are made regarding the reliability of the results. Face validation can be used to test "chunks of knowledge" at any phase of the expert system development life cycle. It is mainly a group effort of the software engineer, the domain expert, and the end user. This ad hoc tool is quite useful for testing user-system interface, user friendliness, and explanation facilities. However, it is not a rigorous validation technique, as such, it offers no measurable assurance of performance or reliability.

- Statistical Tools: Certain expert systems may be validated using quantitative measures known as metrics. The simplest metric is the measure of reliability; Reliability = MTBF/(1 + MTBF) where MTBF is the "mean time between failures". These formulas' have been used in measuring the performance of hardware rather than software. Since the idea of "failure" in the reasoning process of an expert systems does not exist, this quantitative measures do not work well for validation.

- Test Cases: One of the most popular validation techniques is executing a set of prepared test cases on the expert system. The system results are examined for agreement with those of an expert or a panel of experts who solve the same problem. An example of a fuzzy expert system for the assessment of neonatal outcome which was validated through the use of test cases is presented in Garibaldi et.al. [46].

  In general, the test case approach is known to be more of a black-box approach, in which only system inputs and outputs are significant. This does not make it any less efficient as compared with other techniques. The point the author is putting across is the conclusions of the test case approach can be further strengthen with an additional test such as the sub - system validation test. In addition to those test cases suggested by the domain experts, test cases may be provided by the software engineer and the user. Each looks at a different aspects of the expert system in trying to make sense of it. Test cases from all three sources are considered the best combination, because they provide a more comprehensive test of the system, and also tend to increase the objectivity of the validation process [115].

Like other techniques, testing a knowledge base using test cases has its own share of limitations. When the domain expert who writes the test cases is also the tester, the credibility of the validation process can be undermined. With the domain expert involved in providing test cases, bias, perhaps based on personality conflicts or in ignoring non technical aspects such as user interface or ease of use, can filter into them, especially if the test case generation is done in an ad hoc manner.

- Turing Test: With the Turing test, an outside expert or a panel of experts evaluate solutions generated for the expert system. It is a blind evaluation of the relative merit of the solutions without a priori knowledge of whether the solutions were generated by a human expert or by an expert system. An obvious benefit of this test is elimination of bias. The Turing test also provides an objective evaluation of the human experts performance. Its simplicity and intuitive appeal makes it a popular test.

  Expert systems such as MYCIN [133] were validated using the Turing test. The effectiveness of the test is its inability to distinguish between the expert systems performance and that of the human expert who helped build the system. The main limitation of the Turing test is the extensive time it takes expert evaluators and the experts who prepare the test problem. Another limitation is the tests' inadequacy, where it considers only the accuracy and correctness of the final solution, ignoring other important aspects such as the human - machine interaction in performing tasks. Finally some critics have claimed that the test suffers from being subjective in the way test cases and expert judges are selected, which tends to adversely affect the reliability of the knowledge base [114].

- Sub - system Validation: In the application of sub - system validation, the expert system is partitioned into sub - system's or modules that are tested individually and in relationship to other modules in the system. This "divide and conquer" technique allows for easier corrections of errors and should be a plus for future maintenance and system update. However, a factor to consider when applying this technique is that not all knowledge based systems are easy to modularise. So the entire system still needs to be tested as an entity before it can be certified as being valid.

The next section describes knowledge representation and the technique for representing knowledge applied to the expert system reported in this thesis.

3.3 Knowledge Representation

Knowledge can be defined as the body of facts and principles accrued by human kind or the act, fact, or state of knowing. While this definition may be true, it is not entirely complete, as it is known that knowledge is much more than this. "Knowledge is having a familiarity with language, concepts, procedures, rules, ideas, abstractions, places, customs, facts, and associations, coupled with an ability to use these notions effectively in modelling different aspects of the world" [116].

As can be seen from the above definition, without knowledge, facts and concepts are meaningless to us and, therefore without value. The meaning of knowledge is closely related to the meaning of intelligence. Intelligence requires the possession of and access to knowledge. Based on the fact that knowledge is important and in fact fundamental for intelligent behaviour, the representation of knowledge is top priority for the development of expert systems [140]. This is so true considering that the goal is to develop an advisory system that will utilise available knowledge in order to suggest a variance reduction technique for discrete event simulation studies. A variety of knowledge representation schemes have been developed i.e. Formal Logic, Semantic Network, Frames and Production Rules.

Readers are refered to Fagin et.al. [38], Helbig [58] as well as Brachman and Levesque [17] for a detailed bibliography on knowledge representation schemes. However, a discussion on production rules will be presented, since this is the technique used to represent expert knowledge in this thesis. This form of knowledge representation has been chosen because it is the most common scheme used with the inference method of choice applied in this thesis.

Typically, before the implementation of a decision support system is considered, the responsibilities of the human expert must be translated into a coherent set of rules. This simple procedure can prove to be difficult in real world applications because of their inherent complexity. One commonly used method for expressing human knowledge is the "IF-THEN" statement, also called production rules. A major advantage of using production rules is they have the ability to represent knowledge in the same manner that humans are used to, and on the other hand, this representation is easily programmable in a number of computer languages. Thus knowledge can easily be translated into code, and the decision support system becomes easy for humans to understand and reason about its conclusions.

Knowledge can be represented at different levels of extreme. The two main extremes are shallow knowledge and deep knowledge [140]. Shallow knowledge focuses on the representation of surface level knowledge which can be used to deal with specific situations. For example, if your car has not got an MOT certificate, the car should not be driven on the road.

Shallow knowledge basically represents the input - output relationship of a system.

As such it can ideally be represented in the the form of IF-THEN types of rules. On the other hand, problem solving is based on a deep knowledge of a particular problem situation. Deep knowledge refers to the internal and underlying structure of a system and considers the interactions among the systems components. For example, if one wants to investigate a deeper level of relationship between the lack of an MOT certificate and the use of a car, in the above example. There is a need to know the constituents that make up the process of issuing an MOT certificate, i.e. MOT certificate, MOT test centre, MOT tester and a car.

This form of knowledge representation i.e production rules, characterised by shallow knowledge is appropriate in the context of linguistics because it has the ability to express human empirical and heuristic knowledge in English language. In addition, rules can be used as descriptive tools for problem solving heuristics, replacing a more formal analysis of the problem. In this sense, rules are thought of as incomplete but useful guides to make search decisions that can reduce the size of the problem space being explored. Rules are entered sequentially into the knowledge base by the expert system developer [125]. It is important to stress that while shallow knowledge is sufficient to structure rules in the form of "‘IF-THEN"’, deep knowledge provides the understanding required to make sense of the "‘IF-THEN"’ rules.

Knowledge presented as production rules is in the form of condition and action pairs: “If this condition (or premise or antecedent) occurs, THEN some action (or result, or conclusion, or consequence) will (or should) occur”. Let us consider an example:

IF you get to the end of the queue, AND the cashier asks for payment, THEN give her your credit card.

Each production rule in a knowledge base implements an autonomous amount of expertise that can be developed and modified independently of other rules. When combined and fed to the inference engine, a set of rules work together, yielding better results than that of the sum of results from individual rules. In reality, knowledge-based rules are not independent, but are usually highly interdependent. For example, adding a new rule may conflict with an existing rule, or it may require a revision of attributes and/or rules. The utility of production rules comes from the fact that the conditions for which each rule is appropriate are made clear and, in theory, the interactions between rules are minimised [10].

Representation of the variance reduction knowledge elicited from experts has been carried out using production rules. In addition, production rules is a popular choice in the implementation of experts systems for a number of reasons which include [49]:

- Production rules are modular: Each production rule represents a well bounded element of knowledge such that any production rule can be added, refined, and deleted independently of any other rule in a knowledge rule base. This characteristic of modularity has the advantage of clearly demonstrating how changes in production rules would impact the behaviour of the fuzzy expert system.

- Production rules can be abstract: Production rules allow for oversimplification because their conditions may be represented as models that match to a wide range of patterns. These conditions specify the relationships between items without specifying the items themselves, i.e. A is taller than B and B is taller than C, then say A is taller than C, is true for any values of A, B and C.

- Production rules cannot be directly verbalised: This feature is based on the notion that each production rule represents knowledge about the possibility of an event occurring which is not easy to articulate. A good example is a case of an individual who can drive both automatic and manual vehicles but cannot explain how this is done.

Following on in the next section will be a discussion on ambiguity in expert knowledge.

3.4 Ambiguity in Human Knowledge

Ambiguity in knowledge can be considered as the lack of adequate information to make a decision, that is the view point from the perspective of expert systems. It is a problem for such systems because it distorts the process of making the best decision and potentially can cause a bad decision to be made. It can also range from a mere lack of absolute sureness to such vagueness as to include anything less than guesswork [49]. For example, in medicine, ambiguity may ignore the best treatment for a patient or contribute to an incorrect therapy.

There are many different sources of ambiguity in problem solving, but most of them can be credited to either imperfect domain knowledge or imperfect case data. Thus, the theory of the domain may be incomplete or even invalid. Incomplete domain theories may use concepts which are not precisely defined, consider the concept of variance reduction in discrete event simulation studies. The question can be asked; how much reduction in variance is acceptable for an output performance measure?, in response, this is model specific. This is because a missile guidance system will require a lot more precision in its performance estimates as compared with an order picking function in crossdocking distribution system.

Incompleteness in knowledge necessitates the employment of rules of thumb which (unlike scientific laws) may not always give the correct result, even on simple cases. Having incomplete knowledge also means that the effects of actions are not always conventional; for example, using different quantities of simulation runs for the application of a variance reduction technique can possibly achieve different results. However, this statement cannot be validated without the performance of experimentation. Even if the domain theory is complete, an expert may find it profitable to employ heuristic methods in preference to exact methods, because of the inherent complexity of the domain [61]. In a summary, domain experts employ inexact methods for the following reasons:

- Exact methods are not known,
- Even if exact methods are known, they can be impractical, because of lack of knowledge or problems with eliciting expert knowledge, or difficulties with processing the knowledge within the time constraints set by the problem situation.

There is a broad argument among artificial intelligence researchers that inexact methods are important in many expert system applications; however, there is very little agreement concerning what form these methods should take. Opinion has tended to follow McCarty and Hayes [97] who felt that probability theory was inadequate for the task of representing ambiguity. Their argument is:

- It is not altogether clear how to deal with the interaction of probabilities with quantifiers.
- The assignment of probabilities to events requires information that is not normally available.

The above convictions that probability theory has little to say about inherently imprecise notions, such as the quantifiers "most" and "few", and that the application of probability theory requires "too many numbers" led McCarthy and Hayes [97] to propose the exploration of an alternative formalism, such as fuzzy sets for many expert system applications. The management of ambiguity in knowledge is one of the most important characteristics of an expert system. Proper handling of such vagueness in understanding brings any expert system closer to imitating experts in their decision making abilities [54], [89]. There are several types of ambiguity associated with any typical expert system [63]:

- Ambiguity in a given rule: Given a rule "if B then Q", The question arises: How true is this rule, that is, what confidence can be placed in it, and how it can contribute to the solution of the problem?. It can be said that (R_1) is usually true but not always. Hence, a numerical value (0.5) of certainty replaces the linguistic value (true).

  Another question which maybe asked that has to do with the relationship between the premise B and the conclusion Q of a rule i.e. Given that the premise has some truth, what will be the truth of the conclusion? For example, the rule "If it is hot Then wear a linen shirt" can give rise to the question: Given that it is hot, how sure can a person be that wearing a linen shirt will make his/her feel more comfortable?

- Ambiguity in the given data: When the user provides the required input data to the expert system, the truth of that data has to be taken into account. Often the user provides data in which he or she does not possess complete confidence. For the example, the user may say "I think the volume of the container is 10 meters, but I am not sure". In this case, a rank of confidence to the given data has to be assigned.

- Ambiguity in knowledge description: When reasoning with heuristic knowledge, human experts are able to give adequate and useful estimates of their confidence in conclusions. Human weigh conclusions with terms like "highly probable", "unlikely", "almost certainly", or "possible". These weights are clearly not based on careful analysis of probabilities. Instead, they are themselves heuristics derived from experience in reasoning about the problem domain [140]. So with heuristic knowledge being described in vague terms, there is a need to consider and implement some technique to handle such ambiguity inherent in knowledge within expert systems.

In the next section, a technique applied in this thesis for handling ambiguity in knowledge is discussed.

3.5 Dealing with Ambiguity in Knowledge

It is important that an expert system with the aim of mimicking the decision making abilities of human experts be able to address the problem of ambiguity in knowledge. An expert system should be conscious of an ambiguity in its conclusions to the same level of accuracy as would a human expert, and should be capable of representing this ambiguity to a user at the conclusion of each task. There are a number of different techniques for handling ambiguity in expert systems. These include; Probability Theory, Bayesian Theory, Possibility Theory, The Stanford Certainty Factor Theory, Dempster - Shafer Theory of Evidence, as well as Fuzzy Sets and Fuzzy Logic.

Readers are referred to Luger [88] and Giarratano [49] for a detailed discussion on techniques for handling ambiguity in knowledge, from the perspective of expert systems. However, discussion on Fuzzy Sets and Fuzzy Logic is presented, since this is the technique applied to the problem of ambiguity inherent in human expert knowledge which is reported in this thesis.

3.5.1 Fuzzy Sets and Fuzzy Logic

There are two assumptions that are fundamental for the use of formal set theory. The first is with respect to set membership; "for any element and a set belonging to some universe, the element is either a member of the set or else it is a member of the complement of the set". The second assumption, referred to as the law of excluded middle, states that an "element cannot belong to both a set and also a complement" [125]. These two fundamental assumptions for formal set theory are contravened in Lotfi Zadeh's fuzzy set theory [147].

In fact, the sets and reasoning laws of traditional set theory are referred to as crisp, from the viewpoint of fuzzy set theory. Zadeh's [149] main contention is that, although probability theory is appropriate for measuring ambiguity in information, it is inappropriate for measuring the meaning of information. Indeed, much of the ambiguity surrounding the use of English language words and phrases is related to lack of clarity (vagueness) rather than randomness.

The issue of vagueness is a critical point for analysing language structures and can also be important in creating a measure of confidence in production rules. Zadeh proposes possibility theory as a measure of vagueness, just as probability theory measures randomness. Zadehs' theory expresses lack of precision in a quantitative manner by introducing a set membership function that can take on real values between *[0,1]*. As compared with classical sets, elements of a fuzzy set have membership degrees to that particular set. The degree of membership to a fuzzy set indicates the certainty (or uncertainty) that the element belongs to that set. Formally defined, suppose X is the domain, or universe of discourse, and $x \in X$ is a specific element of the domain X. Then the fuzzy set A is characterised by a membership mapping function:

$$\mu\text{A:X} \longrightarrow [0,1] \tag{3.1}$$

Therefore, for all $x \in X$, $\mu A(x)$ indicates the certainty to which element x belongs to fuzzy set A. For two - valued sets, $\mu A(x)$ is either *0* or *1*. Figure 3.1 shows a set membership function for the concept of short, medium and tall male humans. Note that anyone person can belong to more than one set, for example, a male of *0.7* meters in height belongs to both the set of medium as well as the set of tall males.

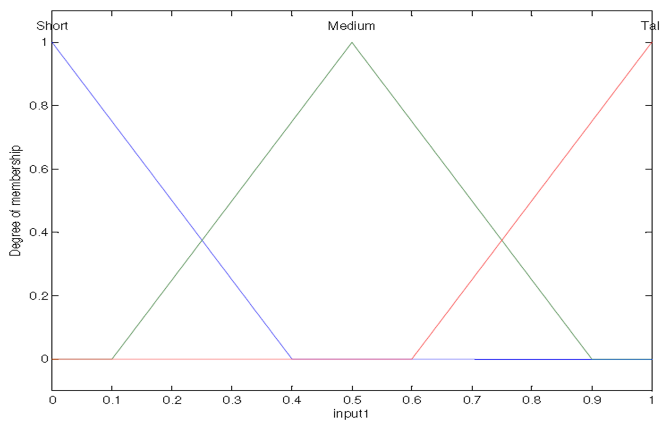

Figure 3.1

Fuzzy Set for Short, Medium, and Tall Males

The term fuzzy defines inexact knowledge or imprecise reasoning present in knowledge elicited from or used by human experts. Fuzziness measures the degree to which a condition exists, and also describes the degree of membership in fuzzy sets. As compared with probability theory, it associates uncertainty with time and specifies whether an event will occur or not. Suppose the city of Nottingham has a 60% chance of snow tomorrow, if the residents wait till tomorrow, they will see whether it snowed so the uncertainty associated with probability disappears. Unlike probability theory, fuzziness does not disperse with time. In the "60% chance it will snow tomorrow" example, the fuzzy uncertainty remains, in that, the statement remains ambiguous as to whether it will be light snow, heavy snow or moderate snow.

Fuzzy logic, then, is the approximate rather than the exact logic underlying modes of reasoning. It is based on the application of fuzzy sets rather than conventional logic. Fuzzy logic has the ability to deal with the ambiguities and undefined areas of real world knowledge. Unlike conventional logic, it can accommodate a range of values called a membership set instead of just two values (i.e. "warm" or "cold").

Fuzzy logics' history dates back to the 1920's when a Polish logician named Jan Lukasiewicz developed the principles of multivalued logic and essentially proved that statements can be functional truth values between one and zero. In 1937, philosopher Max Black applied multivariate logic to sets of objects and drew the first fuzzy set curves, Black called the sets "vague". Almost after 30 years, through the application of Lukasiewicz logic, fuzzy logic was proposed as an alternative to traditional logic by Lotfi Zadeh.

In his landmark paper "Fuzzy Sets" [147], Lotfi Zadeh summarised the idea of modelling predicates using fuzzy sets with values as follows:

> "Clearly the class of real numbers which are much greater than one, or the class of beautiful women, or the class of tall men do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined classes play an important role in human thinking, particularly in the domain of expert systems, pattern recognition, communication of information and abstraction".

Finally, the term fuzzy should not be confused with other forms of imprecision and uncertainty. There are several types of imprecision and uncertainty; fuzziness is just one aspect of it. Imprecision and uncertainty may be in aspects of measurement, probability or description. Imprecision in measurement is associated with a lack of precise knowledge. On occasions measurements can be inaccurate, inexact, or of low confidence. Imprecision as a form of probability is associated with an uncertainty about the future occurrence of events or phenomena. It concerns the likelihood of non - deterministic events. An example is a statement “It might snow later today” which exhibits a degree of randomness.

Imprecision in description is the type of imprecision addressed in fuzzy logic [147], [151]. It is the ambiguity, vagueness, or subjectivity in natural language. It is also the vagueness found in the definition of a concept or the meaning of terms such as “eclectic building” or “borderline constituent”. It is also the ambiguity in human thinking, that is, perceptions and interpretation of concepts. The nature of fuzziness and randomness are therefore quite different. They are different characteristics of ambiguity. The former conveys subjective human thinking, feelings, or language, and the latter indicates an objective statistic in the natural sciences. From a modelling stand point, fuzzy models and statistical models also possess philosophically different kinds of information; fuzzy memberships represent similarities of objects to imprecisely defined properties, while probabilities convey information about relative frequencies [129].

3.6 Theory of Fuzzy Expert Systems

As earlier defined in section 3.1 of this chapter, expert systems are primarily designed to reason through knowledge to solve problems using techniques that humans use. These systems use heuristic knowledge, rather than numbers, to control the process of solving a problem. Their knowledge is encoded and maintained separately from the computer program, and this knowledge is used to solve problems. Expert systems are also capable of explaining how a particular conclusion was reached, and why requested information is needed.

Expert systems are mainly based on rules, models or case studies. This method of classifying expert systems has its own advantages and disadvantages in the sense that these systems have different suitable application areas. One of the important issues to consider when developing expert systems is the vagueness associated with expert human knowledge. This problem can be dealt with in a variety of ways and with several techniques within the realm of expert systems, one of the most useful and popular techniques is fuzzy logic.

A fuzzy expert system is an expert system that makes use of both fuzzy set and fuzzy logic to overcome some of the inherent problems, which occur when knowledge provided by either the human expert or user is vague or incomplete. The strength of fuzzy set theory comes from its ability to describe linguistically a particular event or process, and then to represent that description with a small number of very flexible rules. In a fuzzy expert system, knowledge is contained both in its rules and in fuzzy sets, which hold general description of the properties of the event or phenomenon under consideration. A typical fuzzy expert system is made up of the following components:

- Fuzzification, which transforms the crisp inputs into degrees of match with linguistic values and defines membership functions of the fuzzy sets used in the fuzzy rules;

- Fuzzy inferencing, which performs the inference operations using fuzzy operators and an inference method on a set of fuzzy IF-THEN rules;

- Defuzzification, which transforms the fuzzy results of the inference process into a crisp output.

3.6.1 Fuzzification

The fuzzification process is concerned with finding a fuzzy representation of non-fuzzy input values. This is achieved through the application of membership functions associated with each fuzzy set in the rule space. That is, input values from the universe of discourse assign membership values to the fuzzy sets. Next is a definition of linguistic variables, hedges and membership functions.

- Linguistic Variables: Lofti Zadeh [150] introduces the concept of variable (or fuzzy variable) in 1975, which allows computation with words instead of numbers. Linguistic variables are variables with values that are words or sentences from natural language. Linguistic variables allow the translation of natural language into logical or numerical statements.

- Hedges: In natural language, nouns are frequently combined with adjectives for quantification of the noun. For example, in the phrase "good engineer", the noun "engineer" is quantified by the adjective "good", indicating an engineer who is better than average. In fuzzy systems theory, these adjectives are referred to as hedges. A hedge serves as a modifier of fuzzy values. They are implemented through subjective definitions of mathematical functions, to transform membership values in a systematic manner.

- Membership Function: A membership function, also referred to as the characteristic function of the fuzzy set, defines the fuzzy set. The function is used to associate a degree of membership of each of the elements of the domain to the corresponding fuzzy set. Membership functions for fuzzy sets can be of any shape or type as determined by experts within the domain over which the sets are defined, this in turn gives designers of fuzzy sets flexibility in selecting appropriate membership functions.

these functions must satisfy the following constraints:

- A membership function must be bounded from below by *0* and from above by *1*.
- The range of a membership function must therefore be *[0,1]*.
- For each $x \in X$, $\mu A(x)$ must be unique. That is, the same element cannot map to different degrees of membership for the same fuzzy set.

A membership function (MF), is known as a curve that defines how much each point in the input space in mapped to a membership value (or degree of membership) between *0* and *1*. The input space is sometimes referred to as the universe of discourse. The simplest membership functions are formed using straight lines. Of these, the simplest is the triangular membership function, which is a collection of three points forming a triangle. These straight line membership functions have the advantage of simplicity. Here are the four main types of membership functions considered in this thesis. Figure 3.2 illustrates the Gaussian shape membership function specified by two parameters, the Triangular membership function specified by three parameters, the Polynomial based curve open to the left named the Z - Shape and the Sigmoidal membership named S - Shape.

3.6.2 Fuzzy Inferencing

The task of the inferencing process is to map the fuzzified inputs (as received from the fuzzification process) to the rule base, and to produce a fuzzified output for each rule. That is, for the consequents in the rule output space, the degree of membership in the output sets are determined based on the degrees of membership in the input sets and the relationships between the inputs sets.

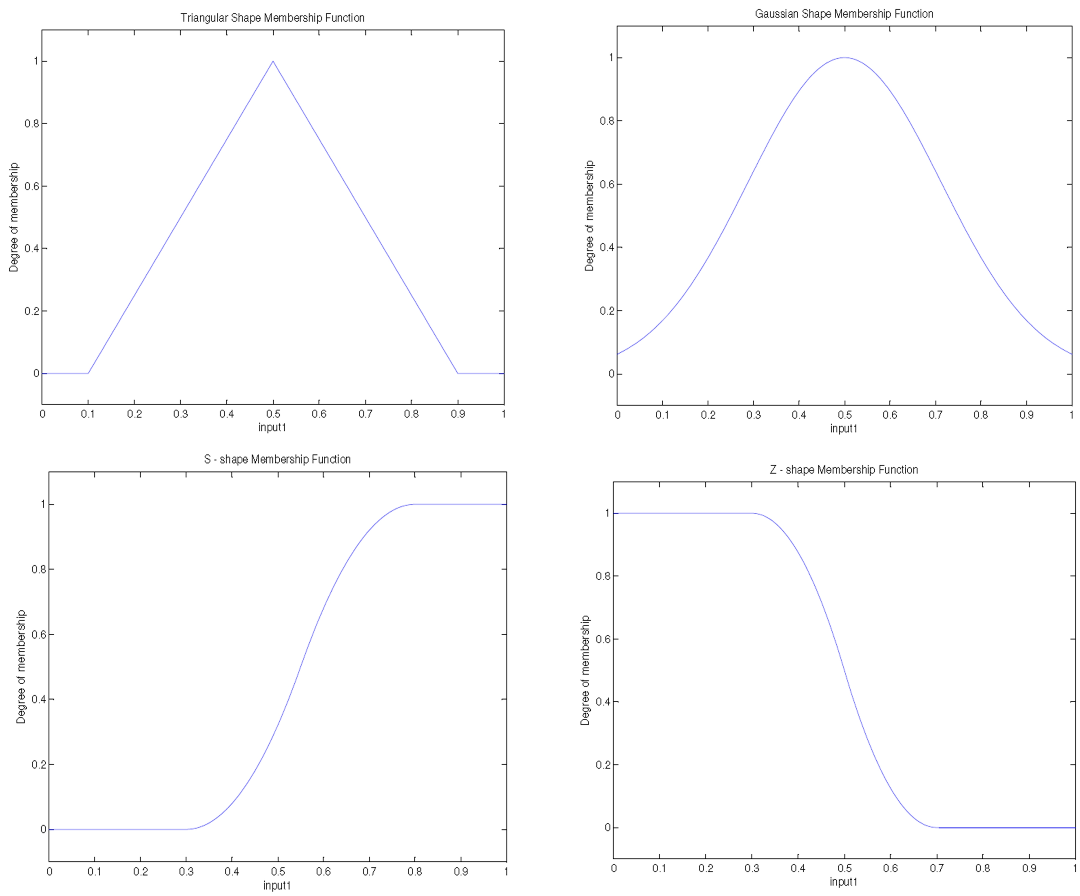

Figure 3.2

Triangular, Gaussian, S and Z-Shape Membership Function

The relationships between input sets are defined by the logical operators that combine the sets in the antecedent. The output fuzzy set in the consequent are then combined to form one overall membership function for the output of the rule. The antecedents of the fuzzy rules form the fuzzy “input space”, while the consequents form the “output space”. The input space is defined by the combination of input fuzzy sets, while the output is defined by the combination of output sets. Next is a description of fuzzy operators, fuzzy rules and fuzzy inference methods.

– Fuzzy Operators: As with crisp sets, relations and operators are equally defined for fuzzy sets. Each of these relations and operators are defined below. For this purpose let X be the domain, or the universe, and A and B are fuzzy sets defined over the domain X. Described below are some fuzzy operators:

1. Equality of fuzzy sets: For two - valued sets, these sets are equal if the two sets have exactly the same elements. For fuzzy sets, however, equality cannot be concluded if the two sets have the same elements. The degree of membership of elements to the sets must also be equal. That is, the membership of the two sets must be the same. Therefore, two fuzzy sets A and B are equal if and only if the sets have the same domain, and $\mu_A(x) = \mu_B(x)$ for all $x \in X$. That is, $A = B$.
2. Containment of fuzzy sets: For two - valued sets, $A \subset B$, if all the elements of A are also elements of B. For fuzzy sets, this definition is not complete, and the degrees of membership of elements to the sets have to be considered. Fuzzy set A is a subset of fuzzy set B if and only if $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$. That is, $A \subset B$.
3. Complement of a fuzzy set **(NOT)**: The complement of a two - valued set is simply the set containing the entire domain without the elements of that set. For fuzzy sets, the set A consists of all the elements of set A, but the membership degrees differ. Let $\bar{A}$ denote the complement of set A. Then, for all $x \subset X$, $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$.
4. Intersection of fuzzy sets **(AND)**: The intersection of two - valued sets is the set of elements occurring in both sets. Operators that implement intersection are referred to as t - norms [55]. The result of a t - norm is a set that contains all the elements of the two fuzzy sets, but with a

degree of membership that depends on the specific t - norm. A number of t - norms have been in existence, of which the min - operator is the most popular. If A and B are two fuzzy sets, then:

* Min - Operator:

$$\mu A \cap B(x) = min\left(\mu A(x), \mu B(x)\right), x \in X. \tag{3.2}$$

5. Union of fuzzy sets **(OR)**: The union of two - valued sets contains the elements of all of the sets. The same is true for fuzzy sets, but with membership that depend on the specific union operator used. These operators are referred to as s - norms [71], of which the max - operator is most used. If A and B are two fuzzy sets, then:

* Max - Operator:

$$\mu A \cup B(x) = max\left(\mu A(x), \mu B(x)\right), x \in X. \tag{3.3}$$

A graphical representation of these *AND* and *OR* operators is shown in the following figure 3.3.

– Fuzzy Rules: For fuzzy systems in general, the dynamic behaviour of that system is characterised by a set of linguistic fuzzy rules. These rules are based on the knowledge and experience of a human expert in the domain and are of the general form:

If antecedent(s) then consequent (s)

The antecedence and consequence of a fuzzy rule are propositions containing linguistic variables. In general, a fuzzy rule is expressed as:

if A is a and B is b then C is c

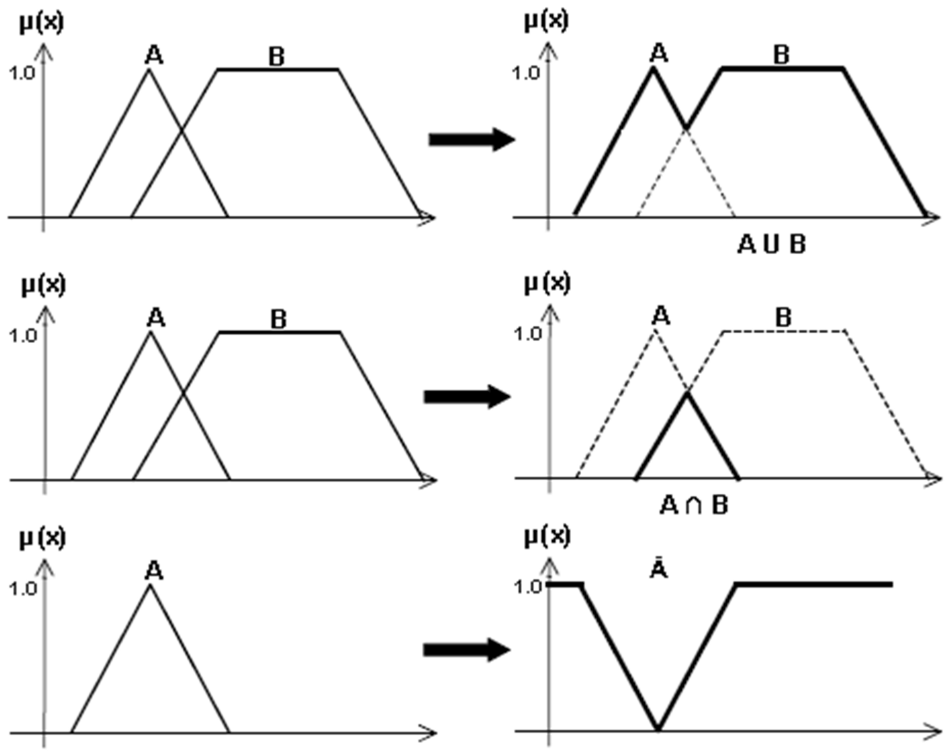

Figure 3.3

Fuzzy set operations (adapted from Asmuni (2008, Chapter 3))

For example, A and B are fuzzy sets with universe of discourse $X1$, and C is a fuzzy set with universe of discourse $X2$. Therefore, the antecedent of a rule forms a combination of fuzzy sets through application of the logic operators (i.e. complement, intersection, union). The consequent part of a rule is usually a single fuzzy set, with a corresponding membership function. Multiple fuzzy sets can also occur within the consequent, in which case they are combined using logic operators. Together, the fuzzy set and fuzzy rules form the knowledge base of a fuzzy rule - based reasoning system.

– Fuzzy inference method: Fuzzy set theory uses fuzzy inferencing to reason about linguistic variables, i.e. variables described by fuzzy sets. A number of different inference methodologies have been developed. However, two inference systems are popular; Mandani fuzzy systems [92] and Takagi-Sugeno fuzzy systems [136]. These two types of inference systems vary somewhat in the way outputs are determined.

A mamdani fuzzy system contains a knowledge base consisting of fuzzy "'IF-THEN"' rules and membership functions, together with an inference engine that applies the rules to the fuzzy input variables, generating fuzzy output variables. In mamdani fuzzy systems of inference the antecedent and consequent fuzzy sets are often chosen to be Triangular shaped or Gaussian shaped. It is also common that the input membership functions overlap in such a way that the membership values of the rule antecedents always sum to one. In this case, and if the rule is of the conjunctive form, one can interpret such a rule as defining the output value for one point in the output space [151].

The main advantages of the mamdani method are its widespread acceptance and its intuitive nature. Furthermore, it provides a natural framework to include expert knowledge in the form of linguistic rules. This knowledge can be easily combined with rules which are automatically generated from data sets that describe the relationship between input variable and output variable. Furthermore, mamdani type fuzzy expert systems possess a high degree of freedom to select the most suitable fuzzification and defuzzification components [36]. This property which allows the software engineer more flexibility with design choices is the main reason the mamdani method has been chosen as the preferred inference method.

As an alternative to mandani method, the takagi-sugeno method may be used instead. They are similar in certain respects, i.e fuzzification of input variables and applying fuzzy operators but are distinctly different in terms of their output membership functions. The output function of the takagi are singleton spikes, the implication and aggregation methods are fixed and cannot be edited. The implication is simply multiplication, and the aggregation operator just includes all of the singletons [125]. For illustration purposes, inference in mamdani's method is displayed in figure 3.4.

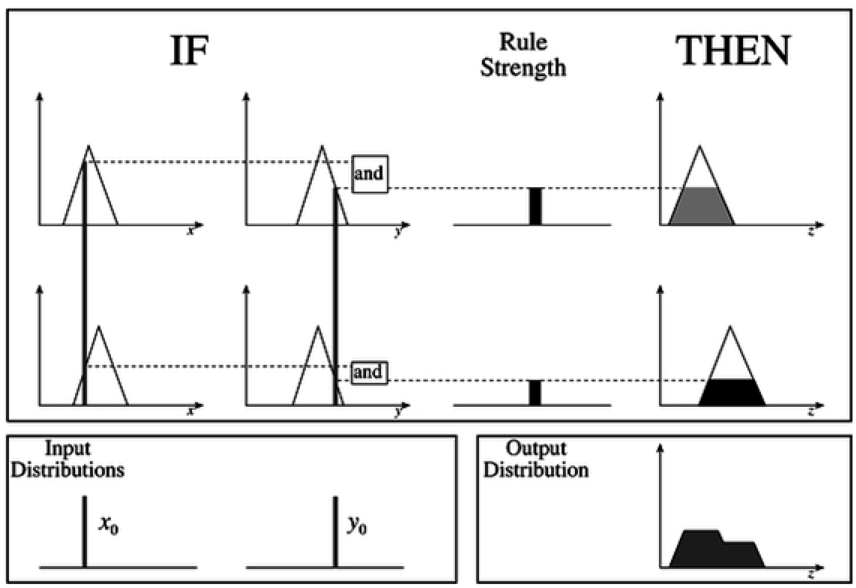

Figure 3.4

A two input, one output Mamdani inference method (adapted from Knapp (2004, Chapter 4))

3.6.3 Defuzzification

Defuzzification is the mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of non - fuzzy (crisp) control actions [30]. This process is necessary because in most practical applications crisp control actions are required. Thus, a defuzzification is necessary when fuzzy reasoning is of the type used, i.e. Mamdani type inference. A defuzzification strategy is aimed at producing a non - fuzzy control action that best represents the possibility distribution of an inferred fuzzy control action [30]. Unfortunately, there is no systematic procedure for choosing a defuzzification strategy [126]. Several defuzzification methods exist to find an approximate scalar value to represent the action to be taken, these include:

- Centroid Method: The Centroid method (also called centre of area, centre of gravity) takes the output distribution and finds its centre of area to come up with a crisp number. This method is the most prevalent and physically appealing of all defuzzification methods and is expressed mathematically as follows:

$$z^* = \frac{\int_b^a \mu(z) \cdot zdz}{\int_b^a \mu(z)dz} \tag{3.4}$$

- Mean of Maxima Method: The Mean of Maxima method is the average of maximising base variables at which the membership function reaches a maximum. This is represented mathematically by the following:

$$z^* = \frac{\int z^{'} zdz}{\int z^{'} dz} \tag{3.5}$$

  where z$^{'}$ $= (\mu_A\,(z) = \mu^*)$

- Smallest of Maxima and Largest of Maxima Methods: The Smallest of Maxima method, returns the minimum value (in terms of magnitude) at which the membership values reach the maximum. On the other hand, the Largest of Maxima method, returns the maximum value (in terms of magnitude) at which the membership values reach the maximum.

– Bisector Method: The Bisector method is the vertical line that will divide the region into sub-regions of equal area. This method satisfies the following:

$$\int_{\alpha}^{z*} \mu_A(z)dz = \int_{z*}^{\beta} \mu_A(z)dz \tag{3.6}$$

where $\alpha = min\{z|z \in Z\}, \beta = max\{z|z \in Z\}$.

Figure 3.5 shows a graphical illustration of defuzzification using the following methods; Centroid method, Mean of Maxima Method, Smallest of Maxima, Largest of Maxima Methods and Bisector Method.

3.6.4 Fuzzy Expert System Development Life Cycle

The design and development of a fuzzy expert system requires a non traditional development life cycle based on early prototyping and incremental revision of the code. The primary individuals involved in building a fuzzy expert system are the software engineer, the domain expert, and the end user. The software engineer is the artificial intelligence expert.

His or her main task is to select the software and hardware tools for the project, help the domain expert articulate the necessary knowledge, and implement that knowledge in a correct and efficient knowledge base. The domain expert provides the knowledge of the problem area. The domain expert is generally someone who has worked in the domain area and understands its problem solving techniques, and all the other skills that mark a person as an expert solver [85]. Typically, here are the stages involved in designing a fuzzy expert system:

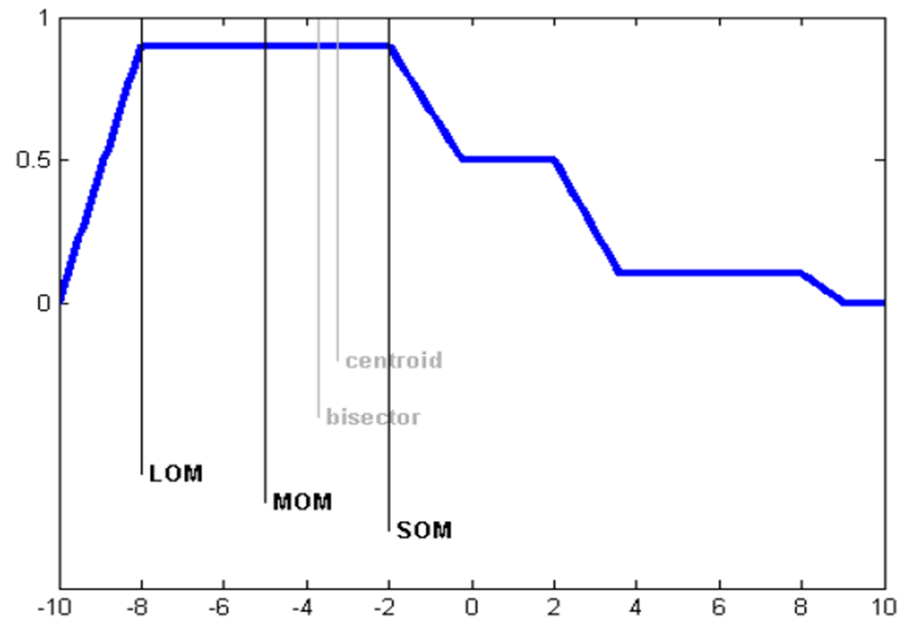

Figure 3.5

Centroid Method, Mean of Maxima Method, Smallest of Maxima Method, Largest of Maxima Method and Bisector Method of Defuzzification(adapted from Fuzzy Logic Toolbox ™ User's Guide.)

1. Generally, work begins on the expert system with the software engineer attempting to gain some familiarity with the problem domain. This is done through initial interviews with the expert and/or by observing experts during the performance of their job.

2. Next, the software engineer and expert begin the process of mining the experts' problem - solving knowledge. This is often done by giving the domain expert a series of sample problems and having him or her explain the techniques used in their solution. It is often useful for the software engineer to be a novice in the problem domain. Human experts are noto-

riously unreliable in explaining exactly what goes on in solving a complex problem. Often they forget to mention steps that have become obvious or even automatic to them after years of work in their field. Software engineers, by virtue of their relative naivetë in the domain, can spot these conceptual gaps and ask for help.

3. Once the software engineer has obtained a general overview of the problem domain and gone through several problem - solving sessions with the expert, he or she is ready to begin actual design of the system i.e. selecting several ways to represent the knowledge, and in conjuction with the expert choose the membership function shapes.

4. After making the required design selections, the software engineer builds a prototype. This prototype should be able to solve problems in a small area of the domain and provide a test bed for preliminary design assumptions.

5. Once the prototype has been implemented, the software engineer and domain expert test and refine its knowledge by giving it problems to solve and correcting its short comings. Should the assumptions made in designing the prototype prove correct, the prototype can be incrementally extended until it becomes a final system.

The overall performance of the fuzzy expert system is directly correlated to the combination of choices made at this stage of design and development. "Unfortunately, there is currently very little theoretical guidance as to which software design choices are acceptable for a particular problem or domain" [45]. Figure 3.6 shows the interconnected components that make up a fuzzy expert system. Here is a brief list of steps in the development of a typical fuzzy expert system:

1. Identify and name input linguistic variables and define their numerical ranges.
2. Identify and name output variables and define their numerical ranges.
3. Define a set of fuzzy membership functions for each of the input variables, as well as the output variables.
4. Construct the rule base that represents the control strategy.
5. Perform fuzzification of input values.
6. Perform inferencing to determine firing strength of activated rules.
7. Defuzzification, using for example the centroid method, to determine the crisp output.

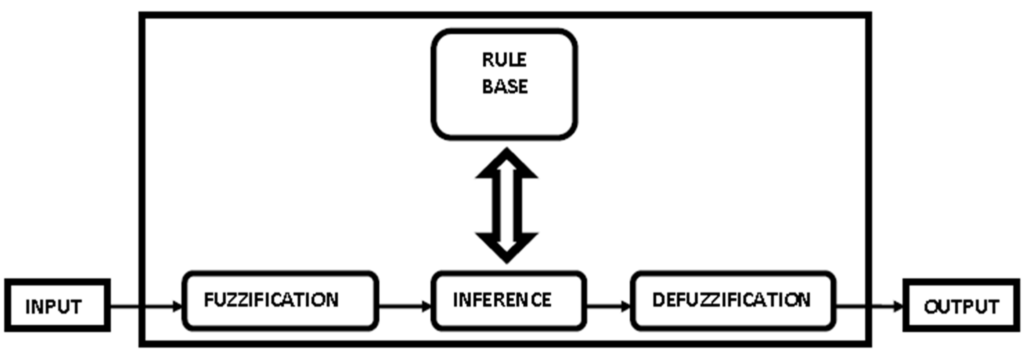

Figure 3.6

Framework of a Fuzzy Expert System

Fuzzy expert systems are built by progressive approximations with the programs mistakes leading to corrections or additions to the knowledge base. In a sense, the knowledge base is grown, rather than constructed. In the next section, is a discussion regarding the interrelationship between expert systems and simulation.

3.7 Expert Systems and Simulation

So far, representing knowledge and dealing with ambiguity inherent in such knowledge has been discussed within the context of expert systems. The verification and validation of expert systems as well as the theory of fuzzy expert systems have also been presented. Reiterating from chapter 1 section 1.2, one of the objectives of this thesis is the development of a decision support system to assist simulation modellers in the selection of variance reduction techniques. These variance reduction techniques are those which have shown some success with reducing variance within discrete event simulation studies. As a prelude to describing the design and development of such a decision support system in chapters 5 & 6, through literature, the interrelationship between expert systems and simulation will be examined.

A domain which has shown success as a solution approach for assisting simulation modellers in decision making is artificial intelligence. The complexity of problems faced in real applications enable simulation to use artificial intelligence in two ways; first, the art of simulation modelling itself can possibly be enhanced by adding artificial intelligence software applications to the analysts' tools. Secondly, parts of a simulation might be modelled by an artificial intelligence application [117]. Similarly, artificial intelligence applications face large, complex problems, for which simulation could provide assistance as an add - on [33]. For example, the inference process used in artificial intelligence applications could include a simulation model. Alternatively, the input or output of an artificial intelligence application could be analysed, respectively, by a simulation model.

An early paper written in the 1980's by O'Keefe [113], discusses the linking of simulation and expert systems for the purpose of aiding the simulation modeller in decision making. The author proposes a taxonomy for combining these two techniques, and supports his idea of combination by demonstrating the similarities between simulation and expert systems. O'Keefe states that simulation and expert systems work in similar ways to provide a computer based model which aids decision making. Often, each must attempt to model the ambiguity inherent in the system under consideration, i.e. variance associated with a performance measure or ambiguity in the knowledge base of the expert system. He comments that one of the areas of potential for dual utilisation is a situation where an expert system assists the user with a finished simulation model or the development of a model. Furthermore, it is O'Keefes' view that expert systems which assist the modeller in simulation experimentation and output analysis are interesting areas of research. He attributes this to an increasing trend to hand over simulation models to users, who in some instances can be inexperienced and could need support and guidance in the use of the simulation model and the interpretation of the results of experimentation.

Shannon et.al. [132] examines the potential role of expert systems in simulation and endeavours to explore the possible impact of this interrelationship as well as predict future directions and trends. One of the future trends which is foreseen by the authors, is a desire to build into a simulation modelling system, most of the simulation study decisions that are now made by the simulation expert. As a means of demonstrating their ideas, the authors propose the development of an expert system to support decision making for a manufacturing simulation.

Examples of such decisions include, an aid for the design and analysis of factory layout and automated scheduling of machines. However, "statistically based decisions" for simulation studies was not considered.

A more recent attempt to investigate the potential gains of applying an expert system in a simulation setting is reported by Robinson et.al. [122]. The objectives of that investigation were:

1. Elicit decision making heuristics from the domain expert and represent them as "'IF-THEN"' rules within an expert system, and
2. Use a simulation model to prompt a domain expert to make decisions, which will serve as a set of examples from which an expert can learn.

The main outcome of the research by Robinson et.al. [122] is the coupling of an expert system and a simulation model for the purpose of representing human decision making in a typical simulation study. These decisions were elicited through the use of a set of simulation model examples, and the results used to train an expert system. The gains of implementing the trained expert system are: (1) The simulation model could be executed through the expert system without the intervention of human operators, and (2) It could be used to increase the knowledge of novice simulation users. However, the authors mention that the potential problem with the expert system approach is, "experts may not always be able to clearly define how they go about making complex decisions". This gives rise to the issue of handling ambiguity associated with the elicited human expert knowledge. This issue is of significant concern, when expert systems are being considered as a solution approach. As this discussion continues, an examination of other instances, where expert systems have been applied in the domain of simulation is presented.

In their paper, Anonuevo et.al. [6], describe an attempt at an automated variance reduction and output analysis procedure that interfaces with a well known simulation language. The aim is to encourage wide spread use of variance reduction techniques in simulation studies. The procedure combines the non overlapping batch means method of output analysis and the control variate variance reduction technique, automatically, in a procedure called “BMCV”. However, the paper emphasises that its aim is to consider methodological issues and experimental evaluation rather than the specifics of software implementation. The author comments that automation of variance reduction in simulation languages is probably the only hope for widespread application of variance reduction techniques in simulation studies. **Ideally an automated procedure should select and implement the most effective variance reduction technique for a particular simulation experiment under consideration.**

Decision support systems can be described as artificial intelligence tool which aid a decision maker. In particular, it has been used for interpreting simulation output and has been recognised as an area of potential for expert systems. A system to demonstrate this idea was executed by Mellichamp and Park [100]. They proposed a solution to the need for statistical experts during simulation studies by developing an expert system which could function as a statistical adviser to simulation modellers. This solution seems ideally suited for the expert system technique since the problem is well defined and an appropriate body of statistical knowledge for simulation already exists. The objective of Mellichamp and Parks’ statistical expert system is to perform appropriate computations for a variety of statistical issues including input data analysis, design of experiment, and output analysis. The authors claim the main benefit of such an expert system would do much to ensure that simulation studies incorporate appropriate statistical approaches thereby ensuring the inferences and conclusions drawn from such studies are more accurate than present.

A proposed expert system to handle both terminating and steady state systems as well as statistical analysis for a single system or comparison of two or more system is presented by Ramachandan et.al. [119]. Furthermore, a conceptual framework and capabilities of an expert system “post processor” to aid the analysis of simulation output data is described. Ramachandan et.al. conclude that statistical validation of simulation output though very vital in a simulation study, is being ignored in most simulation software. The reason for this can be attributed to the complexity of statistics for industrial users and the time consuming nature of an effective analysis. The proposed statistical expert system would aid the modeller in obtaining a validated output, while enabling an exploration of alternative system configuration for making intelligent decisions.

In conclusion, this thesis is concerned with automating the selection of a variance reduction technique for discrete event simulation studies using a fuzzy logic based expert system. By automation, it is implied that there is a deployment of a decision support system which assists users with the process of decision making. Additionally, this is the first time a fuzzy expert system is being applied to the selection of a variance reduction technique for discrete event simulation. This is also the first time to our knowledge that the issue of ambiguity in domain expert knowledge in expert systems is being considered from the perspective of decision making for discrete event simulation studies.

3.8 Chapter Summary

In summing up this chapter, there are two schools of thought on the application of expert systems for decision making in simulation studies. And in particular decisions which involve output analysis and the selection of variance reduction techniques for discrete event simulation studies. The first school of thought encourages a mathematical or heuristic approach, these include Cheng [26], Nelson [106] and MacGrath et.al. [98]. The second school of thought support an automated approach, with the use of artificial intelligence tools such as expert systems. Those in support of this alternative approach include Anonuevo and Nelson [6], Shannon et.al. [132], O'Keefe [113], Mellichamp and Park [100], Deslandres and Pierreval [31] and Robinson et.al. [123].

On the basis of literature that has been presented in chapter 2 section 2.6, a mathematical or heuristic approach has laid a firm theoretical foundation for a practical solution approach to the problem of intelligently selecting a variance reduction technique for discrete event simulation studies. However, to implement practical solutions in the area of decision making for simulation studies and in particular for output analysis, current research has turned to and succeeded in applying the expert system approach. On the other hand, there remains a gap in knowledge in regards dealing with the issue of ambiguity associated with simulation expert knowledge, when a fuzzy expert system is applied as a decision support tool for discrete event simulation studies. This thesis aims to make a contribution in closing this gap by the design and development of a fuzzy expert system for the selection of a simulation variance reduction technique, which will have the capability of handling such ambiguity in expert knowledge.

In a regular rule based system, a production rule has no concrete effect at all unless the data completely satisfies the antecedent of the rule. The operation of the inference system proceeds sequentially, with one rule firing at a time; if two rules are simultaneously satisfied, a conflict resolution policy is needed to determine which ones take precedence. In a fuzzy rule - based system, in contrast, all rules are executed during each pass through the inference system, but with linguistic variable strengths ranging from, for example, "not at all" to "completely", depending on the relative degree to which their fuzzy antecedent propositions are satisfied by the data. Thus a fuzzy - rule base system is most valuable in modelling some complex systems that can be investigated by humans because it makes use of linguistic variables as its antecedents and consequents which can be described naturally and represented by fuzzy sets and fuzzy operators like "AND", "OR" and "NOT" [140]. The point of view articulated here is that conventional approaches to the management of ambiguity in expert systems are intrinsically inadequate because they fail to come to grips with the fact that much of the ambiguity in expert systems is possibilistic rather than probabilistic in nature [148], [149], of which fuzzy logic provides a suitable solution.

Fuzzy logic provides a natural framework for the management of ambiguity in knowledge as it relates to expert systems because, in contrast to traditional techniques, it provides a systematic way of representing and inferring from imprecise rather than precise knowledge. In fact, in fuzzy sets everything is allowed to be, but need not be, as a matter of degree. The greater expressive power of fuzzy logic derives from the fact that it contains as special cases the traditional two - valued logic as well as multi - valued logic. Fuzzy expert systems are currently the most popular use of fuzzy logic with many applications now operational in a diverse range of domains. These systems have the essential characteristic of storing knowledge of human experts in a distinct form such as “IF-THEN” rules, which are easy to understand, and this provides a form of an explanation component for justifying crisp results. Furthermore, they incorporate membership functions that allow the system to be written in a more abstract level and fine tuned to improve performance. For fixed well defined knowledge, a fuzzy expert system is an efficient and effective way to represent a problem [30]. A fuzzy expert system can provide a direct means of applying human expertise and permits the knowledge as well as experience of one or more experts to be captured and stored in a computer. This knowledge can then be used by anyone who requires it. The purpose of a typical fuzzy expert system is not to replace the experts, but simply to make their knowledge and experience more widely available. Following on from this chapter will be the application of variance reduction techniques for three different application domains.

Chapter 4

Manual Selection of Variance Reduction Techniques

The aim of this thesis chapter is to answer the first research question put forward in chapter 1 section 1.2: "Is there a reduction in the variance or standard deviation value of a selected discrete event simulation output performance measure, after the application of stand - alone or combined variance reduction techniques?". The performance of a combination strategy for variance reduction techniques will also be investigated.

The manual selection of variance reduction technique(s) for discrete event simulation studies is described within this chapter. There are three techniques being applied; common random number, antithetic variates and control variates, and a combination of common random numbers and antithetic variates as well as a combination of common random numbers and control variates. The three application areas under consideration are a simple manufacturing system, a simple call centre system and a simple crossdocking distribution system.

The objective of this investigation is to determine the performance of the various variance reduction technique(s) and their combination under the same conditions i.e. similar number of replications. The assumption is that the results from experimentation will assist in guiding the selection of a variance reduction technique for each of the discrete event simulation models.

This chapter is divided into four sections:

- Simulation and the application of variance reduction techniques for a small manufacturing system,
- Simulation and the application of variance reduction techniques for a small call centre system,
- Simulation and the application of variance reduction techniques for a small crossdocking distribution centre, and
- A discussion about experimentation results.

4.1 Manufacturing System

For any selected confidence level, a narrower confidence interval is better than a wider one. The width of a confidence interval, is influenced by the variance associated with the selected output performance measure. Generally, increasing the number of replications of the simulation model has been known to be the easiest way to reduce variance (and narrow a specified confidence interval) but this may increase the simulation computational cost in complex and large systems [79].

Consequently, variance reduction techniques are used in simulation experiments to avoid using a large number of replications in order to achieve more precise results. The variance reduction techniques chosen for this study are those which have shown a possibility of succeeding when applied to discrete event simulation models [79], [13]. As this is not a full scale simulation study, but a means of collecting output data for the variance reduction experiments, this investigation will not be following the typical steps in a simulation study described in chapter 2, Section 2.1.2 of this thesis.

Typically, the simulation of manufacturing systems is performed using a commercial software, rather than through a purpose built application. In addition, the two most common criteria for selecting simulation software are modelling flexibility (ability to model any system regardless of its complexity or uniqueness) and ease of use [81]. The manufacturing simulation model has been developed using the Arena™ simulation software.

It is common that one of the activities during a simulation study is the statistical analysis of output performance measures. Since random samples from input probability distributions are used to model events in a manufacturing simulation model through time, basic simulation output data (e.g., average times in system of parts) or an estimated performance measure computed from them (e.g., average time in system from the entire simulation run) are also characterised by randomness.

Another source of manufacturing simulation model randomness which deserves a mention is unscheduled random downtime and machine failure which is also modelled using probability distributions. It is known that inherent model randomness can distort a true and fair view of the simulation model output results. Consequently, it is important to model system randomness correctly and also to design and analyse simulation experiments in a proper manner [79].

There are a number of ways of modelling random unscheduled downtimes, interested readers are directed to Chapter 13, section 3, Discrete Event System Simulation, Banks et.al. [13]. The purpose of using variance reduction techniques is to deal with the inherent randomness in the manufacturing simulation model. This is through the reduction of variance associated with any selected measure of model performance. This reduction will be gained using the same number of replications that was used to achieve the initial simulation results. Improved simulation output results obtained from the application of variance reduction techniques has been known to increase the credibility of the simulation model.

An investigation into the application of variance reduction techniques on a small manufacturing simulation model is herein presented. The purpose of the investigation is to determine whether the selected variance reduction techniques perform best as stand alone or as a combination. The simulation models under consideration have been adapted from chapter 7, Simulation with Arena, Kelton et.al. [64], purely for research purposes. Experimentation is based on the assumption that the output performance measures are of a terminating, multi scenario, single system discrete event simulation model.

The simple manufacturing system consists of parts arrival, four manufacturing cells, and parts departure. The system produces three part types, each routed through a different process plan in the system. This means that the parts do not visit individual Cells randomly, but through a predefined routing sequence.

Parts enter the manufacturing system from the left hand side, and move only in a clockwise direction, through the system. There are four manufacturing cells; Cells 1, 2, and 4 each have a single machine, however, Cell 3 has two machines.

The two machines at Cell 3 are not identical in performance capability, one of these machines is newer than the other and can perform 20% more efficiently than the other. Machine failure in Cells 1, 2, 3, and 4 in the manufacturing simulation model was represented using an *exponential distribution* with mean times in hours. Exponential distribution is a popular choice when modelling such activities in the absence of real data. A layout of the small manufacturing system under consideration is displayed in figure 4.1.

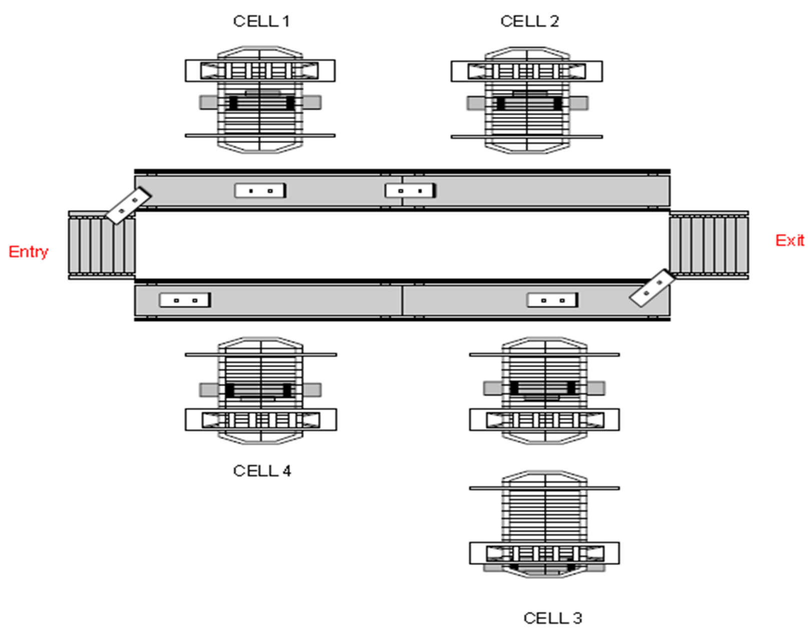

Figure 4.1

A Small Manufacturing System Layout (adapted from Kelton et.al. (2007, Chapter 7))

4.1.1 Description of Simulation Model

Within this section a description of the simulation model under consideration is presented. All process times are *triangularly distributed*, while the inter arrival times between successive part arrivals are *exponentially distributed*. These are the probability distributions which were already implemented in the simulation model, and there was no reason not to continue using them. The Arena ™ simulation model incorporates an animation feature that captures the flow of parts to and fro the cells, until they are finally disposed or exist out of the system. The inter arrival times between successive parts arrival are *exponentially distributed* with a mean of 13 minutes, while the first part arrives at time 0.

Here is a brief description of the Arena ™ control logic which underlines the animation feature. Parts arrival are generated in the create parts module. The next step is the association of a routing sequence to arriving parts. This sequence will determine the servicing route of the parts to the various machine cells. Once a part arrives at a manufacturing cell (at a station), the arriving part will queue for a machine, and is then processed by a machine.

This sequence is repeated at each of the manufacturing cells the part has to be processed. The process module for Cell 3 is slightly different from the other three Cells. This is to accommodate the two different machines, a new machine and an old machine, which process parts at different rates. Figure. 4.2 shows the animation equivalent and control logic of the small manufacturing system simulation model.

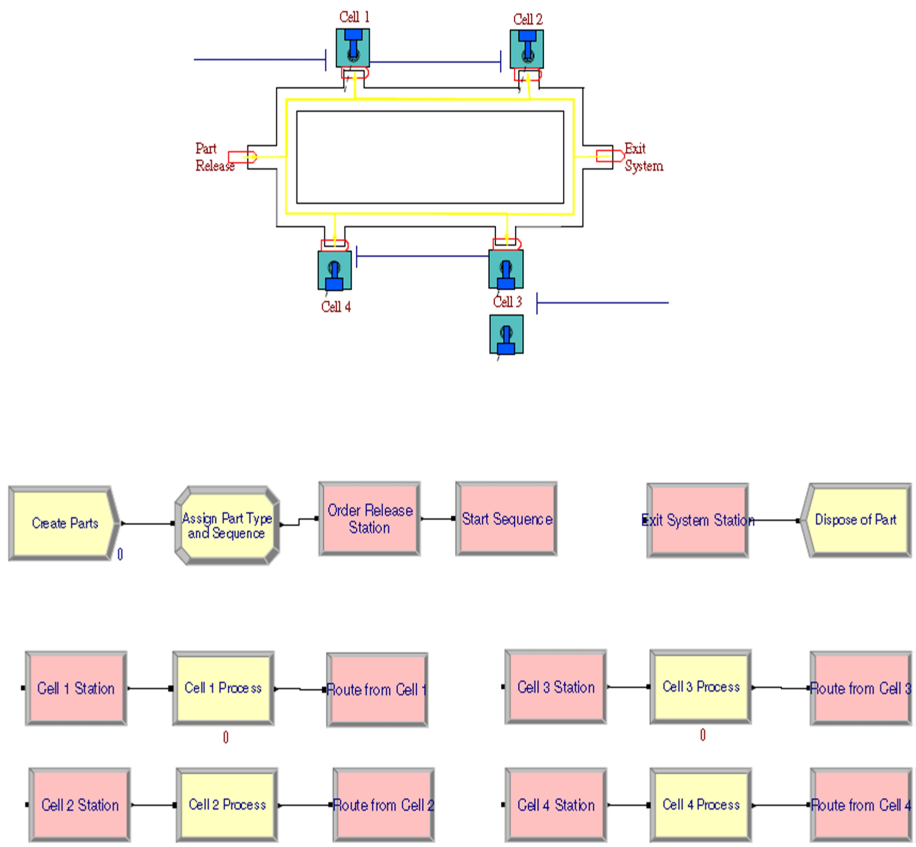

Figure 4.2

Manufacturing System Simulation Animation and Control Logic (adapted from Kelton et.al. (2007, Chapter 7))

4.1.2 Simulation Experiment

This section of the chapter which discusses the simulation experiment of the manufacturing system is divided into two parts; (a) details of the design of experiments and (b) details of the results obtained through experimentation.

4.1.2.1 Experimental Design

In designing the simulation experiment, data on time persistent performance measures was utilised for experimentation as opposed to both time and cost data. This is due mainly to the availability of time based data as opposed to cost based data during the performance of the case study. Although both types of data would have given a greater insight into the performance of the variance reduction techniques, using different classes of time based data should be sufficient for this level of experimentation. Here is a list of the three performance measures, which additionally have been labelled (Base):

- Entity Total Average Time (Base): This is the average of the total time each entity will travel over the total length of the conveyor through the manufacturing system. This consists also of its processing time, wait time etc., until it exits the system.
- Resource Utilisation (Base): This variable records the instantaneous utilisation of a resource during a specific period. It is a time persistent statistic that is weighted to take into consideration the utilisation of a resource as a function of time. This may include times that the resource was not scheduled in the system. This is an important metric for manufacturing simulation systems because it indicates the level of global resource use.
- Average Total WIP (Base): The analysis of potential bottle neck situations is one of the more important performance metric in a manufacturing system simulation. This metric records the average quantity of total work in process for each entity type.

The experimental conditions are as follows:

– Number of Replications: 10

– Warm up Period: 0

– Replication Length: 30 Days

– Terminating Condition: None

As this is a pilot study where the goal is to establish the effectiveness of the variance reduction techniques under consideration, in this instance 10 simulation replications is deemed sufficient for collecting enough data for this purpose. A period of warm up was not included because the simulation model is based on the assumption that there are no entities at the start of each day of operation. An extensive bibliography on an appropriate number of replications for simulation experimentation and such like issues can be found in Robinson et.al [120] and Hoad et.al [60].

Furthermore, issues such as the appropriate number of replications, warm up and replication length are better dealt with during full experimentation and not during a pilot study. As a terminating simulation model, there is no need to include a terminating condition because it has a scheduled time to begin and end. The performance measures have been labelled (Base), to highlight their distinction from those that have had variance reduction techniques applied and those that have not. These experiments, both simulation and variance reduction, assume that the sampled data is normally distributed.

The principle supporting the application of a variance reduction technique, is the possibility of achieving a lower variance for a simulation output performance measure whilst using the same number of replications as applied without its use.

The main simulation experiments reported within this chapter, are based the use of ten replications; it is only natural that the same number of replications be utilised for the variance reduction experiments.

4.1.2.2 Results

Reported in table 4.1, at a 95% Confidence Interval are the mean and standard deviation of the three performance measures of interest. These results will be used as a bench mark to determine whether the application of variance reduction techniques have been successful or not. The performance of each technique will be judged mainly by a reduction in variance or standard deviation at the specified replication level.

Output Performance Measure	N	Mean	StDev	95.0% C.I
Average Total WIP (Base)	10	15.637	2.760	(13.663,17.612)
Entity Total Average Time (Base)	10	9.572	1.584	(8.439,10.705)
Resource Utilisation (Base)	10	4.632	1.516	(3.548,5.717)

Table 4.1

Manufacturing System Simulation Results

4.1.3 Variance Reduction Experiments

This section of the chapter is divided into two parts; the first describes the design of the variance reduction experiments and the second details the results of the application of the variance reduction techniques, as stand alone application and as combined application. The method of combining the variance reduction techniques under consideration is an extension of that which has been proposed by Kleijnen [68].

The scheme proposed an approach where antithetic variates only and common random numbers only are applied and the combination of antithetic variates and common random numbers. However, the strategy is to apply common random numbers, antithetic variates and control variates alone, and a combination of common random numbers and antithetic variates as well as a combination of common random numbers and control variates. The main purpose is to exploit the combined application of the individual characteristics of the variance reduction techniques for the purpose of improving the precision of the selected simulation output performance measures (See Chapter 2, Section 2.5 & 2.6).

4.1.3.1 Experimental Design

The following describes the experimental settings that have been used to conduct the variance reduction technique experiments. Performance measures have been classed according to variance reduction techniques, i.e. *Average Total WIP (Base), Average Total WIP (CRN), and Average Total WIP (AV)*. This means for each performance measure, the appropriate variance reduction that has been applied to it is stated, i.e. common random number (CRN) and that which has not been treated to a variance reduction technique is labelled (Base).

Under consideration is a two scenario, single manufacturing simulation model. The scenario which has performance measures labelled (Base), is characterised by random number seeds dedicated to sources of simulation model randomness as selected by the simulation software Arena ™. The other scenario which has performance measures labelled common random number (CRN), has its identified sources of randomness, allocated dedicated random seeds by the user. So these two scenarios have unsynchronised and synchronised use of random numbers respectively [80] (See Chapter 2, Section 2.4.1).

The selected performance measures remain the same (See section 4.1.2.1 of this chapter). However, an additional performance measure *Entity Wait Time* is introduced at this stage. This performance measure will be used for the control variates experiment, with a view to applying it to adjusting upward or downwards the performance measure *Entity Total Average Time (Base)*. Initial simulation results show a linear relationship between both variables, which will be exploited for variance reduction.

Here are the hypotheses that aim to answer the research question stated at the beginning of this chapter:

1. Are the means from the two scenarios the same?
2. Are the standard deviations from the two scenarios the same?

The hypotheses that tests the true mean of the first scenario Y, $\mu 1$, against the true mean of the second scenario Z, $\mu 2$ is:

1. H_0: $\mu 1 = \mu 2$

or

2. H_1: $\mu 1 \neq \mu 2$

Test Statistic: Paired t test

The assumption is that there is a one-to-one correspondence between the values in the two samples, which is the reason for choosing a paired t test, instead of a 2 sample t test.

Significance Level: A value of $\alpha = 0.05$

The hypotheses that tests the true standard deviation of the first scenario Y, $\sigma 1$, against the true standard deviation of the second scenario Z, $\sigma 2$ are:

1. H_0: $\sigma 1 = \sigma 2$

or

2. H_1: $\sigma 1 \neq \sigma 2$

Test Statistic: F test

The F Test has been selected as the basic test for equality of variance between samples, as this is one of the most common statistical techniques for this purpose. Although an alternative test like the Levene's test could have been used, in this instance, it will not be appropriate because only two samples are being examined for equal variance.

Significance Level: A value of $\alpha = 0.05$

4.1.3.2 Common Random Numbers Results

Results from table 4.2, report the difference between means for Average Total WIP 1 (CRN) and Average Total WIP (BASE) as "not statistically significant", (T-Value = -1.22, P-Value = 0.252). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.088 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Average Total WIP 1 (CRN) has a lower standard deviation, when compared to Average Total WIP (BASE). This demonstrates a reduction in variance by the application of common random numbers on the performance measure, Average Total WIP.

Output Performance Measure	N	Mean	StDev	SE Mean
Average Total WIP 1 (CRN)	10	14.225	1.514	0.479
Average Total WIP (Base)	10	15.637	2.760	0.873

Table 4.2

Paired T-Test: Average Total WIP 1(CRN), Average Total WIP(Base)

Results from table 4.3, report the difference between means for Entity Total Average Time 1 (CRN) and Entity Total Average Time (Base) as "not statistically significant", (T-Value = -1.29, P-Value = 0.231). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.062 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Entity Total Average Time 1 (CRN) has a lower standard deviation, when compared to Entity Total Average Time (Base). This demonstrates a reduction in variance by the application of common random numbers on the performance measure, Entity Total Average Time.

Results from table 4.4, report the difference between means for Resource Utilisation 1 (CRN) and Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.07, P-Value = 0.947). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.004 is less than the α value, therefore "reject the null hypothesis", also the difference between variances is "statistically significant".

Specifically, at 10 replications, Resource Utilisation 1 (CRN) has a lower stan-

Output Performance Measure	N	Mean	StDev	SE Mean
Entity Total Average Time 1 (CRN)	10	8.717	0.818	0.259
Entity Total Average Time (Base)	10	9.572	1.584	0.501

Table 4.3

Paired T-Test: Entity Total Average Time 1(CRN), Entity Total Average Time(Base)

dard deviation, when compared to Resource Utilisation (Base). This demonstrates a reduction in variance by the application of common random numbers on the performance measure, Resource Utilisation.

Output Performance Measure	N	Mean	StDev	SE Mean
Resource Utilisation 1 (CRN)	10	4.600	0.516	0.163
Resource Utilisation (Base)	10	4.632	1.516	0.479

Table 4.4

Paired T-Test: Resource Utilisation 1(CRN), Resource Utilisation(Base)

4.1.3.3 Antithetic Variates Results

Results from table 4.5, report the difference between means for Average Total WIP 2 (AV) and Average Total WIP (Base) as “not statistically significant”, (T-Value = -1.22, P-Value = 0.252). The P value is greater than the α value, therefore “do not reject the null hypothesis”. For the F test, the P-value of 0.031 is less than the α value, therefore“reject the null hypothesis”, also the difference in variances is “statistically significant”.

Specifically, at 10 replications, Average Total WIP 2 (AV) has a lower standard deviation, when compared to Average Total WIP (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Average Total WIP.

Output Performance Measure	N	Mean	StDev	SE Mean
Average Total WIP 2 (AV)	10	14.93	1.27	0.40
Average Total WIP (Base)	10	15.64	2.76	0.87

Table 4.5

Paired T-Test: Average Total WIP 2(AV), Average Total WIP(Base)

Results from table 4.6, report the difference between means for Entity Total Average Time 2 (AV) and Entity Total Average Time (Base) as "not statistically significant", (T-Value = -1.29, P-Value = 0.231). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.022 is less than the α value, therefore "reject the null hypothesis", also the difference in variances is "statistically significant".

Specifically, at 10 replications, Entity Total Average Time 2 (AV) has a lower standard deviation, when compared to Entity Total Average Time (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Entity Total Average Time.

Results from table 4.7, report the difference between means for Resource Utilisation 2 (AV) and Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.07, P-Value = 0.947). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.101 is greater than the α value, therefore "do not reject the null hypothesis",

Output Performance Measure	N	Mean	StDev	SE Mean
Entity Total Average Time 2 (AV)	10	9.144	0.69	0.219
Entity Total Average Time (Base)	10	9.57	1.58	0.50

Table 4.6

Paired T-Test: Entity Total Average Time 2 (AV), Entity Total Average Time (Base)

also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Resource Utilisation 2 (AV) has a lower standard deviation, when compared to Resource Utilisation (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Resource Utilisation.

Output Performance Measure	N	Mean	StDev	SE Mean
Resource Utilisation 2 (AV)	10	4.616	0.852	0.270
Resource Utilisation (Base)	10	4.632	1.516	0.479

Table 4.7

Paired T-Test: Resource Utilisation 2(AV), Resource Utilisation(Base)

4.1.3.4 Control Variates Results

Results from table 4.8, report the difference between means for Average Total WIP(CV) and Average Total WIP(Base) as "not statistically significant", (T-Value = -0.01 P-Value = 0.989). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of

0.055 are greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Average Total WIP (CV) has a lower standard deviation, when compared to Average Total WIP (Base). This demonstrates a reduction in variance by the application of control variates on the performance measure, Average Total WIP.

Output Performance Measure	N	Mean	StDev	SE Mean
Average Total WIP(CV)	10	15.625	0.232	0.074
Average Total WIP(Base)	10	15.637	2.760	0.873

Table 4.8

Paired T-Test: Average WIP(CV), Average WIP (Base)

4.1.3.5 Combined Application Results

Results from table 4.9, report the difference between means for Average Total WIP (CRN+AV) and Average Total WIP (Base) as "not statistically significant", (T-Value = -1.22, P-Value = 0.252). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.009 is less than the α value, therefore "reject the null hypothesis", also and the difference between variances is "statistically significant".

Specifically, at 10 replications, Average Total WIP (CRN+AV) has a lower standard deviation, when compared to Average Total WIP (Base). This demonstrates a reduction in variance by the combined application of common random numbers and antithetic variates on the performance measure, Average Total WIP.

Output Performance Measure	N	Mean	StDev	SE Mean
Average Total WIP (CRN+AV)	10	14.578	1.060	0.335
Average Total WIP (Base)	10	15.637	2.760	0.873

Table 4.9

Paired T-Test: Average Total WIP (CRN+AV), Average Total WIP (Base)

Results for table 4.10 report the difference between means for Entity Total Average Time (CRN+AV) and Entity Total Average Time (Base) as "not statistically significant",(T-Value = -1.29, P-Value = 0.231). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.004 is less than the α value, therefore "reject the null hypothesis", also the difference between variances is "statistically significant".

Specifically, at 10 replications, Average Total Average Time (CRN+AV) has a lower standard deviation, when compared to Average Total Average Time (BASE). This demonstrates a reduction in variance by the combined application of common random numbers and antithetic variates on the performance measure, Entity Total Average Time.

Output Performance Measure	N	Mean	StDev	SE Mean
Entity Total Average Time (CRN+AV)	10	8.93	0.55	0.17
Entity Total Average Time (Base)	10	9.57	1.58	0.50

Table 4.10

Paired T-Test: Entity Total Average Time (CRN+AV), Entity Total Average Time (Base)

Results for table 4.11 report the difference between means for Resource Utilisation (CRN+AV) and Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.07, P-Value = 0.947). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.011 is less than the α value, therefore "reject the null hypothesis", also the difference in variances is "statistically significant".

Specifically, at 10 replications, Resource Utilisation (CRN+AV) has a lower standard deviation, when compared to Resource Utilisation (Base). This demonstrates a reduction in variance by the combined application of common random numbers and antithetic variates on the performance measure, Resource Utilisation.

Output Performance Measure	N	Mean	StDev	SE Mean
Resource Utilisation (CRN+AV)	10	4.608	0.598	0.189
Resource Utilisation (BASE)	10	4.632	1.516	0.479

Table 4.11

Paired T-Test: Resource Utilisation (CRN+AV), Resource Utilisation (Base)

Results from table 4.12, report the difference between means for Average Total WIP (CRN+CV) and Average Total WIP (Base) as "not statistically significant", (T-Value = -0.72, P-Value = 0.493). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, P-values of 0.001 is less than the α value, therefore "reject the null hypothesis", also the difference in variances is "statistically significant".

Specifically, at 10 replications, Average Total WIP (CRN+CV) has a lower standard deviation, when compared to Average Total WIP (Base). This demonstrates a successful reduction in variance by the combined application of common random numbers and control variates on the performance measure, Average Total WIP.

Output Performance Measure	N	Mean	StDev	SE Mean
Average WIP (CRN+CV)	10	14.925	0.801	0.253
Average WIP (Base)	10	15.637	2.760	0.873

Table 4.12

Paired T-Test: Average Total WIP (CRN+CV), Average Total WIP (Base)

4.1.4 Summary

In this section, a summary of results on the performance of each variance reduction technique on each output performance measure is presented below. By using the word “selected” in describing the results within tables 4.13, 4.14 and 4.15, it is implied that there is a variance reduction technique which is the preferred technique out of the following techniques; Common Random Numbers (CRN), Antithetic Variates (AV), Control Variates (CV), Common Random Numbers and Antithetic Variates (CRN+AV) and Common Random Numbers and Control Variates (CRN+CV), for the purpose of variance reduction. The selection of a technique is purely on the basis that it achieves the greatest reduction in variance / standard deviation for a particular output performance measure without the use of additional simulation runs, whilst applying a manual approach to the selection of a variance reduction technique.

– On the basis of the performance of the variance reduction techniques presented in Table 4.13, control variates technique (CV) was selected for the simulation output performance measure, Average Total WIP.

- On the basis of the performance of the variance reduction techniques presented in Table 4.14, the combined application of common random numbers and antithetic variates techniques (CRN+AV) was selected for the simulation output performance measure, Entity Total Average Time.
- On the basis of the performance of the variance reduction techniques presented in Table 4.15, common random numbers technique (CRN) was selected for the simulation output performance measure, Resource Utilisation.

Control Variates (CV), Common Random Numbers and Antithetic Variates (CRN+AV), and Common Random Numbers (CRN) techniques have been selected because each technique achieved the lowest standard deviation for each output performance measure, whilst using 10 simulation replications only.

Output Performance Measure	N	Mean	StDev
Average Total WIP (BASE)	10	15.637	2.760
Average Total WIP (CRN)	10	14.225	1.514
Average Total WIP (AV)	10	14.931	1.273
Average Total WIP (CV)	10	15.625	0.232
Average Total WIP (CRN+AV)	10	14.578	1.060
Average Total WIP (CRN+CV)	10	14.925	0.801

Table 4.13

One way ANOVA for Average Total WIP

Output Performance Measure	N	Mean	StDev
Entity Total Average Time (BASE)	10	9.572	1.584
Entity Total Average Time (CRN)	10	8.717	0.818
Entity Total Average Time (AV)	10	9.144	0.694
Entity Total Average Time (CRN+AV)	10	8.930	0.546

Table 4.14

One way ANOVA for Entity Total Average Time

Output Performance Measure	N	Mean	StDev
Resource Utilisation (BASE)	10	4.6323	1.5163
Resource Utilisation (CRN)	10	4.6000	0.5164
Resource Utilisation (AV)	10	4.6162	0.8523
Resource Utilisation (CRN+AV)	10	4.6081	0.5978

Table 4.15

One way ANOVA for Resource Utilisation

4.2 Call centre System

The most commonly used techniques for call centre analysis are those for staffing and phone line trunking capacity calculations. A unexpectedly large majority of these techniques are based upon Erlang calculations [16]. As previously discussed in Chapter 2 Section 2.3.2, there are many deficiencies with the Erlang calculation. With the progression towards skill based routing of inbound customer calls due to advances in technology, this technique for call centre performance analysis has become outdated since it assumes that agents have a single skill and there is no call priority [32].

On the other hand, the application of simulation ensures the modelling of human agent skills and abilities, best staffing decisions and provides an analyst with a virtual call centre that can be continually refined to answer questions about operational issues and even long term strategic decisions [83].

A close examination of a typical call centre reveals a complex interaction between several “resources” and “entities”. Entities can take the form of customers calling into the call centre and resources are the human agents that receive calls and provide some service. These incoming calls, usually classified by call types, then find their way through the call centre according to a routing plan designed to handle specific incoming call type.

While passing through the call centre, incoming calls occupy trunk lines, wait in one or several queues, abandon queues, and are redirected through interactive voice response systems until they reach their destination, the human agent. Otherwise, calls are passed from the interactive voice response system to an automatic call distributor.

An automatic call distributor is a specialised switch designed to route each call to an individual human agent; if no qualified agent is available, then the call is placed in a queue. Modern automatic call distributors' are sophisticated, allowing routing rules based on many criteria, and a queued customer may abandon without receiving service. See figure 4.3 for an illustration of the sequence of activities in typical call centre, which have just been described in this section.

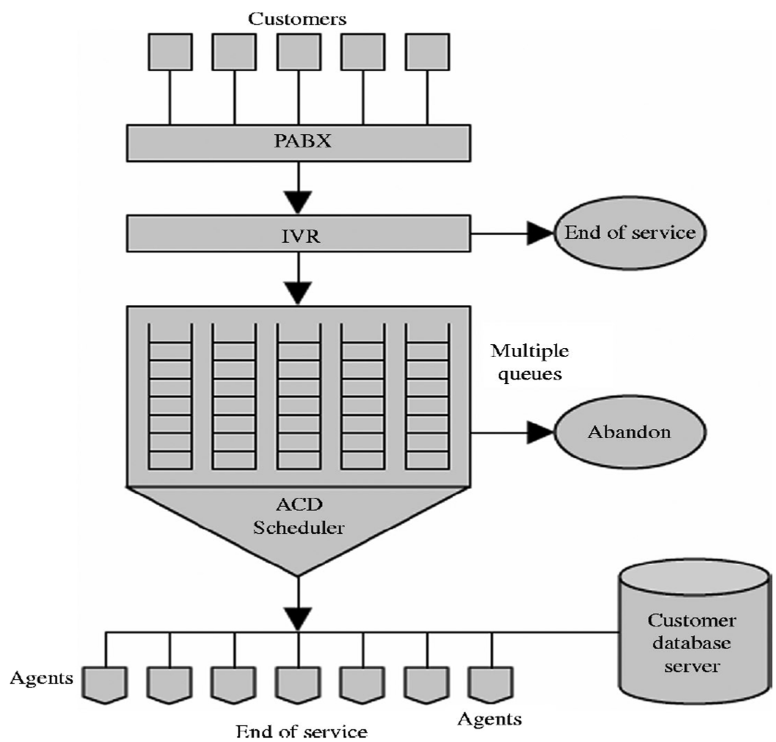

Figure 4.3

A Simple Call Centre (adapted from Doomun et.al, (2008))

Since each human agent possesses a unique skill in handling incoming calls, it is the customers request that will determine whether the agent handles the call or transfers it to another agent. Once the call is handled, it then leaves the call centre system. During all of these call handling transactions, one critical resource being consumed is time. For example time spent handling a call and the time a call spends in the system. These are important metrics to consider during the evaluation of the performance of a call centre.

4.2.1 Description of Simulation Model

The simple call centre system under consideration has been adapted from the Chapter 5, Simulation with Arena, Kelton et.al. [64]. This call centre system, although theoretical in nature, contains the essential working components of a typical real life call centre, i.e. technical support, sales and customer order status checking.

Arrival of incoming calls is generated using an arrival schedule. The purpose for using an arrival schedule instead of modelling this event using a probability distribution and a mean in minutes is to cause the system to stop creating new arrivals 11 hours into the simulation experiment. The central phone number feeds 26 truck lines, and if all 26 lines are in use, a caller gets a busy signal and may have to try again later.

An answered caller has three options: transfer to technical support, sales information, or order status inquiry. The estimated time for this activity is *uniformly distributed*; all times are in minutes. If the caller chooses technical support, the caller is requested to state which of three product types the caller requires. The percentage of requests for product types 1, 2, and 3 are 25% ,34% and 41% respectively. If a qualified technical support person is available for the selected product type, the call is automatically routed to that person. If none are currently available, the customer is placed in an electronic queue until a support person is available.

The time for all technical support calls is estimated to be *triangularly distributed* in minutes regardless of the product type. Upon the completion of the call, the customer should exit the system. Sales calls are automatically routed to the sales staff. Sales calls are modelled to be *triangularly distributed* in minutes. The triangular distribution and all other probability distributions used within this simulation model have been known to be appropriate in the absence of real data. Upon completion of the call, the satisfied customer exits the call centre system. Callers requesting order status information are automatically handled by the phone system, and there is no limit on the number the system can handle.

The estimated time for these transactions is *triangularly distributed* in minutes, with 15% of these callers opting to speak to a real person after they have received their order status. Callers opting to speak to a real person are routed to the sales staff where they wait with lower priority than sales calls. This means that if an order status call is in a queue waiting for a sales person and a new arriving sales call enters, the sales call will be given priority over the order status call and answered first.

Once these calls have been handled, callers then exit the system. The call centre hours are from 8am until 6pm, with a small proportion of the staff on duty until 7pm. Although the system closes to new calls at 6pm, all calls that enter the system by that time are answered and served. Over the course of a day there are eight technical support employees to answer technical support calls. Two are devoted to product type 1 calls; three, to product type 2 calls; and three, to product type 3 calls. There are four sales employees to answer the sales calls and those order status calls that opt to speak to a real to a real person.

In simulation terms, the "entities" for this simple call centre model are product type 1, 2 and 3. The available "resources" are the 26 trunk lines which are of a fixed capacity, and the sales and technical support staff. The skill of the sales and technical staff is modelled using schedules which show the duration during which for a fixed period, a resource is available, its capacity and skill level. The simulation model records the number of customer calls that are not able to get a trunk line and are thus rejected from entering the system similar to balking in queuing system. However, it does not consider reneging, where customers who get a trunk line initially, later hang up the phone before being served. Figure 4.4, shows an Arena TM simulation animation of the simple call centre simulation model.

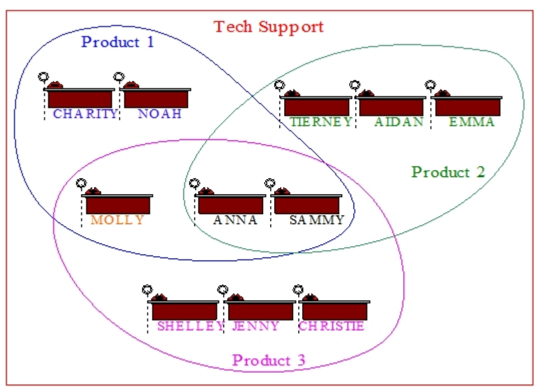

Figure 4.4

Call Centre Simulation Animation (adapted from Kelton et.al (2007, Chapter 5))

4.2.2 Simulation Experiment

This section of the chapter which discusses the simulation experiment of the call centre system is divided into two parts; (a) details of the design of experiments and (b) a discussion of the results obtained from experimentation.

4.2.2.1 Experimental Design

For the design of the call centre simulation experiments, the three output performance measures which have been chosen are both time and cost persistent in nature. Here is a list of these performance measures:

- Total Average Call Time (Base): This output performance measure records the total average time an incoming call spends in the call centre simulation system. This is sometimes known as the cycle time for each incoming call in a process. It is the difference in time between when an entity enters a process and when it exits the process.
- Total Resource Utilisation (Base): This metric records the total scheduled usage of human resources in the operation of the call centre over a specified period in time. It is a cumulative statistic that is calculated by dividing the average number busy resources by the average number scheduled resources and creating a single statistic.
- Total Resource Cost (Base): This is the total cost incurred for using a resource i.e a human agent. Usage cost is calculated based on the resource usage cost and the total units of a resource that are seized, or allocated to an entity.

The experimental conditions are as follows:

- Number of Replications: 10
- Warm up Period: 0
- Replication Length: 660 minutes (27.5 days)
- Terminating Condition: At the end of 660 minutes and no queuing incoming call, terminate the simulation.

The call centre simulation model is based on the assumption that there are no entities at the start of each day of operation and the system will have emptied itself of entities at the end of the daily cycle. For the purpose of output analysis and variance reduction experimentation, it is a terminating simulation model, although a call centre is naturally a non terminating system.

No period of warm up has been added to the experimental set up. This is because experimentation is purely on the basis of a pilot run and the main simulation experiment, when it is performed, will handle issues like initial bias and its effect on the performance of variance reduction techniques. These issues are outside the scope of this thesis. The performance measures have been labelled (Base), to highlight their distinction between those that have had variance reduction techniques applied and those that have not. These experiments, both simulation and variance reduction, assume that the sampled data is normally distributed.

4.2.2.2 Results

Reported in table 4.16, at a 95% Confidence Interval are the mean and standard deviation of the three performance measures of interest. These results will be used as a bench mark to determine whether the application of variance reduction techniques has been successful or not. This performance will be judged mainly by a reduction in variance.

4.2.3 Variance Reduction Experiments

This section of the chapter is divided into two parts; the first describes the design of the variance reduction experiments and the second details the results of the

Variable	N	Mean	StDev	95%CI
Total Resource Cost (Base)	10	21591	1652	(20409, 22773)
Total Resource Utilisation (Base)	10	26.401	0.973	(25.704, 27.097)
Total Average Call Time (Base)	10	89.15	9.09	(82.64, 95.65)

Table 4.16

Call Centre Simulation Results

application of the variance reduction techniques, as stand alone application and as combined application. The method of combining the variance reduction techniques under consideration is based on Kleijnen (1974) [68], as described in section 4.1.3 of this chapter.

4.2.3.1 Experimental Design

This section describes the experimental settings that have been used to conduct the variance reduction technique experiments on the call centre simulation model. The performance measures have been classed according to variance reduction techniques, i.e. *Total Average Call Time (Base), Total Average Call Time (CRN), and Total Average Call Time (AV).*

Under consideration as in the previous manufacturing simulation study is a two scenario, single call centre simulation model. The scenario which has performance measures labelled (Base), is characterised by random number seeds dedicated to sources of simulation model randomness as selected by the simulation software Arena ™. The other scenario which has performance measures labelled common random number (CRN), has its identified sources of randomness, allocated dedicated random seeds by the user. So these two scenarios have unsynchronised and synchronised use of random numbers [80] (See Chapter 2, Section 2.4.1).

The design of variance reduction experiments remains the same as that in the manufacturing system (section 4.1.3.1), however an additional performance measure *Total Entity Wait Time* is introduced at this stage. This performance measure will be used for the control variates experiment, with a view to adjusting the variance value of the performance measure *Total Average Call Time (Base)*.

4.2.3.2 Common Random Numbers Results

Results from table 4.17, report the difference between means for Total Resource Cost (CRN) and Total Resource Cost (Base) as “not statistically significant”, (T-Value = 0.53, P-Value = 0.607). The P value is greater than the α value, therefore “do not reject the null hypothesis”. For the F test, the P-value of 0.823 is greater than the α value, therefore “do not reject the null hypothesis”, also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Resource Cost (CRN) has a higher standard deviation, when compared to Total Resource Cost (Base). This demonstrates a negative effect in variance reduction by the application of common random numbers on the performance measure, Total Resource Cost.

Results from table 4.18, report the difference between means for Total Resource Utilisation (CRN) and Total Resource Utilisation (Base) as “not statistically significant”, (T-Value = -0.33, P-Value = 0.751). The P value is greater than the α value, therefore “do not reject the null hypothesis”. For the F test, P-value of 0.186 is greater than the α value, therefore “do not reject the null hypothesis”,

Variable	N	Mean	StDev	SE Mean
Total Resource Cost(CRN)	10	21989	1784	564
Total Resource Cost(Base)	10	21591	1652	523

Table 4.17

Paired T-Test: Total Resource Cost(CRN), Total Resource Cost(Base)

also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Resource Utilisation (CRN) has a higher standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a negative effect in variance reduction by the application of common random numbers on the performance measure, Total Resource Utilisation.

Variable	N	Mean	StDev	SE Mean
Total Resource Utilisation (CRN)	10	26.545	1.539	0.487
Total Resource Utilisation (Base)	10	26.401	0.973	0.308

Table 4.18

Paired T-Test: Total Resource Utilisation (CRN), Total Resource Utilisation (Base)

Results from table 4.19, report the difference between means for Total Average Call Time (CRN) and Total Average Call Time (Base) as “not statistically significant”, (T-Value = 0.39, P-Value = 0.704). The P value is greater than the α value, therefore“do not reject the null hypothesis”. For the F test, the P-value of 0.432 is greater than the α value, therefore “do not reject the null hypothesis”, also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Average Call Time (CRN) has a lower standard deviation, when compared to Total Average Call Time (Base). This demonstrates a reduction in variance reduction by the application of common random numbers on the performance measure, Total Average Call Time.

Variable	N	Mean	StDev	SE Mean
Total Average Call Time (CRN)	10	90.46	6.94	2.19
Total Average Call Time (Base)	10	89.15	9.09	2.88

Table 4.19

Paired T-Test: Total Average Call Time (CRN), Total Average Call Time (Base)

4.2.3.3 Antithetic Variates Results

Results from table 4.20, report the difference between means for Total Resource Cost (AV) and Total Resource Cost (Base) as "not statistically significant", (T-Value = 0.53, P-Value = 0.607). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.420 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Cost (AV) has a lower standard deviation, when compared to Total Resource Cost (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Total Resource Cost.

Results from table 4.21, report the difference between means for Total Resource Utilisation (AV) and Total Resource Utilisation (Base) as "not statistically significant", (T-Value = 0.33 , P-Value = 0.751). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value

Variable	N	Mean	StDev	SE Mean
Total Resource Cost (AV)	10	21790	1251	369
Total Resource Cost(Base)	10	21591	1652	523

Table 4.20

Paired T-Test: Total Resource Cost (AV), Total Resource Cost (Base)

of 0.763 is greater than the α value, therefore “do not reject the null hypothesis”, also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Resource Utilisation (AV) has a higher standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a negative effect in variance reduction by the application of antithetic variates on the performance measure, Total Resource Utilisation.

Variable	N	Mean	StDev	SE Mean
Total Resource Utilisation (AV)	10	26.473	1.079	0.341
Total Resource Utilisation (Base)	10	26.401	0.973	0.308

Table 4.21

Paired T-Test: Total Resource Utilisation (AV), Total Resource Utilisation (Base)

Results from table 4.22, report the difference between means for Total Average Call Time (AV) and Total Average Call Time (Base) as “not statistically significant”, (T-Value = 0.39, P-Value = 0.704). The P value is greater than the α value, therefore “do not reject the null hypothesis”. For the F test, the P-value of 0.255 is greater than the α value, therefore “do not reject the null hypothesis”, also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Average Call Time (AV) has a lower standard deviation, when compared to Total Average Call Time (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Total Average Call Time.

Variable	N	Mean	StDev	SE Mean
Total Average Call Time (AV)	10	89.80	6.12	1.94
Total Average Call Time (Base)	10	89.15	9.09	2.88

Table 4.22

Paired T-Test: Total Average Call Time (AV), Total Average Call Time (Base)

4.2.3.4 Control Variates Results

Results from table 4.23, report the difference between means for Total Average Call Time (CV) and Total Average Call Time (Base) as "not statistically significant", (T-Value = 0.00, P-Value = 1.000). The P value is greater than the α value, therefore"do not reject the null hypothesis". For the F test, P-value of 0.000 is less than the α value, therefore "reject the null hypothesis", also the difference between variances is "statistically significant".

Specifically, at 10 replications, Total Average Call Time (CV) has lower standard deviation, when compared to Total Average Call Time (Base). This demonstrates a reduction in variance by the application of control variates on the performance measure, Total Average Call Time.

Variable	N	Mean	StDev	SE Mean
Total Average Call Time (CV)	10	89.15	1.80	0.57
Total Average Call Time (Base)	10	89.15	9.09	2.88

Table 4.23

Paired T-Test: Total Average Call Time (CV), Total Average Call Time (Base)

4.2.3.5 Combined Application Results

Results from table 4.24, report the difference between means for Total Resource Cost (CRN+AV) and Total Resource Cost (Base) as "not statistically significant", (T-Value = 0.53, P-Value = 0.607). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.664 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Cost (CRN+AV) has a lower standard deviation, when compared to Total Resource Cost (Base). This demonstrates a reduction in variance by the application of common random numbers and antithetic variates on the performance measure, Total Resource Cost.

Variable	N	Mean	StDev	SE Mean
Total Resource Cost (CRN+AV)	10	21889	1423	450
Total Resource Cost(Base)	10	21591	1652	523

Table 4.24

Paired T-Test:Total Resource Cost(CRN+AV),Total Resource Cost(Base)

Results from table 4.25, report the difference between means for Total Resource Utilisation (CRN+AV) and Total Resource Utilisation (Base) as "not statistically significant", (T-Value = 0.33, P-Value = 0.751). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.424 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Utilisation (CRN+AV) has a higher standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a negative effect in variance reduction by the application of common random numbers and antithetic variates on the performance measure, Total Resource Utilisation.

Variable	N	Mean	StDev	SE Mean
Total Resource Utilisation (CRN+AV)	10	26.509	1.282	0.405
Total Resource Utilisation (Base)	10	26.480	0.960	0.290

Table 4.25

Paired T-Test: Total Resource Utilisation (CRN+AV), Total Resource Utilisation (Base)

Results from table 4.26, report the difference between means for Total Average Call Time (CRN+AV) and Total Average Call Time (Base) as "not statistically significant", (T-Value = 0.39, P-Value = 0.704). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.229 is greater than the α value, therefore "do not reject the null hypothesis", also the difference the variances is "not statistically significant".

Specifically, at 10 replications, Total Average Call Time (CRN+AV) has a lower standard deviation, when compared to Total Average Call Time (Base). This demonstrates a reduction in variance by the application of common random numbers and antithetic variates on the performance measure, Total Average Call Time.

Variable	N	Mean	StDev	SE Mean
Total Average Call Time (CRN+AV)	10	90.13	5.99	1.89
Total Average Call Time (Base)	10	89.15	9.09	2.88

Table 4.26

Paired T-Test: Total Average Call Time (CRN+AV), Total Average Call Time (Base)

Results from table 4.27, report the difference between means for Total Average Call Time (CRN+CV) and Total Average Call Time (Base) as "not statistically significant", (T-Value = 0.23, P-Value = 0.825). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.229 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Average Call Time (CRN+CV) has a lower standard deviation, when compared to Total Average Call Time (Base). This demonstrates a reduction in variance by the application of common random numbers and control variates on the performance measure, Total Average Call Time.

Variable	N	Mean	StDev	SE Mean
Total Call Average Time (CRN+CV)	10	89.80	3.68	1.16
Total Average Call Time (Base)	10	89.15	9.09	2.88

Table 4.27

Paired T-Test: Total Call Average Time (CRN+CV), Total Average Call Time (Base)

4.2.4 Summary

In this section, a summary of the results on the performance of each variance reduction technique as applied to each performance measure is presented. By using the word “selected” in describing the results within tables 4.28, 4.29 and 4.30, it is implied that there is a variance reduction technique which is the preferred technique out of the following techniques; Common Random Numbers (CRN), Antithetic Variates (AV), Control Variates (CV), Common Random Numbers and Antithetic Variates (CRN+AV) and Common Random Numbers and Control Variates (CRN+CV), for the purpose of variance reduction. The selection of a technique is purely on the basis that it achieves the greatest reduction in variance / standard deviation for a particular output performance measure without the use of additional simulation runs, whilst applying a manual approach to the selection of a variance reduction technique.

- On the basis of the performance of the variance reduction techniques presented in Table 4.28, antithetic variates technique (AV) was selected for the simulation output performance measure, Total Resource Cost.

- On the basis of the performance of the variance reduction techniques presented in Table 4.29, the stand alone and combined application of variance reduction techniques did not achieve a reduction in variance for the simulation output performance measure, Total Resource Utilisation.
- On the basis of the performance of the variance reduction techniques presented in Table 4.30, control variates technique (CV) was selected for the simulation output performance measure, Total Average Call Time.

From the results presented in this summary, Antithetic Variates (AV) and Control Variates (CV) have been selected because each technique achieved the lowest standard deviation for each output performance measure, whilst using 10 simulation replications only. However, on this occasion neither a stand alone or combined variance reduction technique achieved a reduction in variance for Total Resource Utilisation.

Output Performance Measure	N	Mean	StDev
Total Resource Cost (Base)	10	21591	1652
Total Resource Cost (CRN)	10	21989	1784
Total Resource Cost (AV)	10	21790	1251
Total Resource Cost (CRN+AV)	10	21889	1423

Table 4.28

One way ANOVA for Total Resource Cost

Factor Levels	N	Mean	StDev
Total Resource Utilisation (Base)	10	26.401	0.973
Total Resource Utilisation (CRN)	10	26.545	1.539
Total Resource Utilisation (AV)	10	26.473	1.079
Total Resource Utilisation (CRN+AV)	10	26.509	1.282

Table 4.29

One way ANOVA for Total Resource Utilisation

Factor Levels	N	Mean	StDev
Total Average Call Time (Base)	10	89.148	9.092
Total Average Call Time (CRN)	10	90.456	6.935
Total Average Call Time (CV)	10	89.149	1.803
Total Average Call Time (AV)	10	89.802	6.124
Total Average Call Time (CRN+AV)	10	90.129	5.986
Total Average Call Time (CRN+CV)	10	89.802	3.680

Table 4.30

One way ANOVA for Total Average Call Time

4.3 Crossdocking Distribution System

Many systems in areas such as manufacturing, warehousing and distribution can sometimes be too complex to model analytically, in particular, Just in Time (JIT) warehousing systems such as crossdocking can present such difficulty [21]. This is because crossdocking distribution systems operate processes which exhibit an inherent random behaviour which can potentially affect its overall expected performance. A suitable technique for modelling and analysing complex systems like crossdocking is discrete event simulation. It has the ability to evaluate and measure the performance of a variety of output performance measure values of a crossdocking distribution centre and as a result, give an insight into its random behaviour [91].

Using discrete event simulation involves utilising probabilistic distributions as part of the input parameter estimation; thus this will result in some variance associated with the output performance measure value. The greater the level of variance in the output value, the lower the precision the simulation results will contain. Within the simulation model, there are different sources of variance; these include order inter arrival time and processing time which are based on probabilistic distributions. One way to improve the performance of a crossdocking simulation model is by the application of variance reduction techniques. This will lead to an improvement in the simulation output results and a higher confidence in a typical discrete event simulation study.

Following on is a description of a typical crossdocking distribution centre. Normally, such a facility would consist of a break up area where inbound freight is received and sorted as well as a build up area which handles the task of picking customer orders for onward dispatch via out bound dock doors. The usual activities of the crossdocking distribution centre begin with the receipt of customer orders, batched by outbound destinations, at specified periods during the day. As customer orders are being received, inbound freight arranged as pallet load is being delivered through inbound doors designated according to destination.

Customer orders batched by destination can differ in volume and variety; also they are released into the order picking system at the discretion of an operator in order to even out the work load on the order picking system. Once pallet load is sorted by a floor operative i.e. during the break up process, individual items in packs of six to twelve units can be placed in totes (A plastic container which is used for holding items on the conveyor belt).

Normally, totes will begin their journey on a conveyor belt, for onward routing to the order picking area. Just before the order picking area is a set of roof high shelves where stock for replenishing the order picking area is kept. A conveyor belt runs through the order picking area and its route and speed are fixed. Figure 4.5, below provides a representation of the crossdocking distribution centre.

Within the order picking area, there are two types of order picking methods; automated dispensing machines and manual order picking operatives. These order picking resources are usually available in shifts, constrained by capacity and scheduled into order picking jobs. There is also the possibility that manual order picking operators possess different skill levels and there is a potential for automated order picking machines to breakdown.

In such a situation, it becomes important for the achievement of a smooth crossdocking operation, to pay particular attention to the order picking process

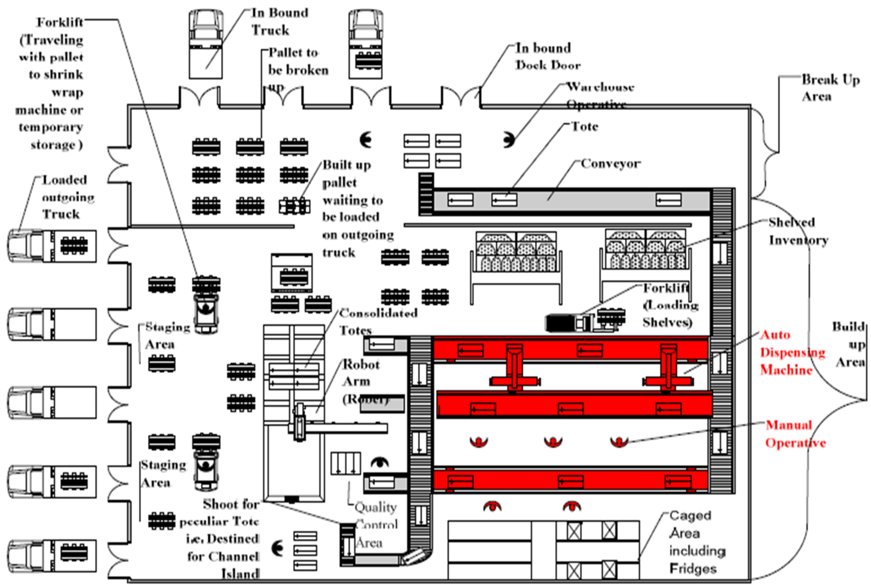

Figure 4.5

A Typical Crossdocking Distribution Centre

within the crossdocking distribution system. The order picking process essentially needs to be fulfilled with minimal interruptions and with the least amount of resource cost [86], [84]. Below figure 4.6 provides a representation of the order picking function with a crossdocking distribution centre.

4.3.1 Description of Simulation Model

A description of the order picking simulation model, which will be the scope of the crossdocking simulation study is presented. The scope of this particular study is restricted to the order picking function as a result of an initial investigation conducted at a physical crossdocking distribution centre. It was discovered that amongst the different activities performed in a distribution centre, the order picking function was judged as the most significant by management.

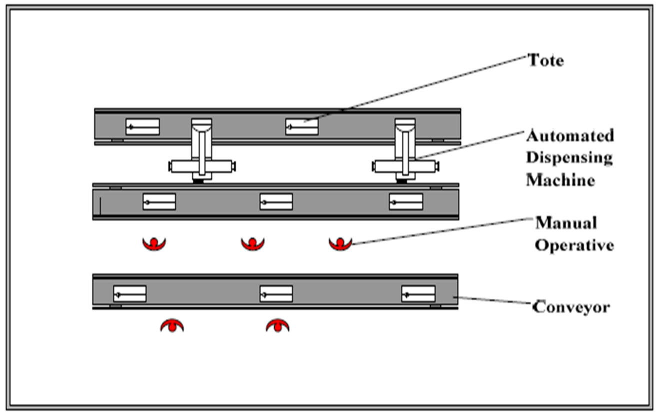

Figure 4.6

An Order Picking Process with a Crossdocking Distribution Centre

The customer order (entity) inter arrival rate is modelled using an *exponential probability* distribution, and the manual as well as the automated order picking process are modelled using *triangular probability* distribution. Customer orders are released from the left hand side of the simulation model. At the top of the model are two automated dispensing machines and at the bottom of the simulation model are two sets of manual order picking operatives, with different levels of proficiency in picking customer orders. Figure 4.7, displays a simulation animation of the order picking process crossdocking distribution centre.

4.3.2 Simulation Experiment

This section of the chapter which discusses the simulation experiment of the crossdocking distribution system is divided into two parts; (a) details of the design of experiments and (b) details of the results obtained from experimentation.

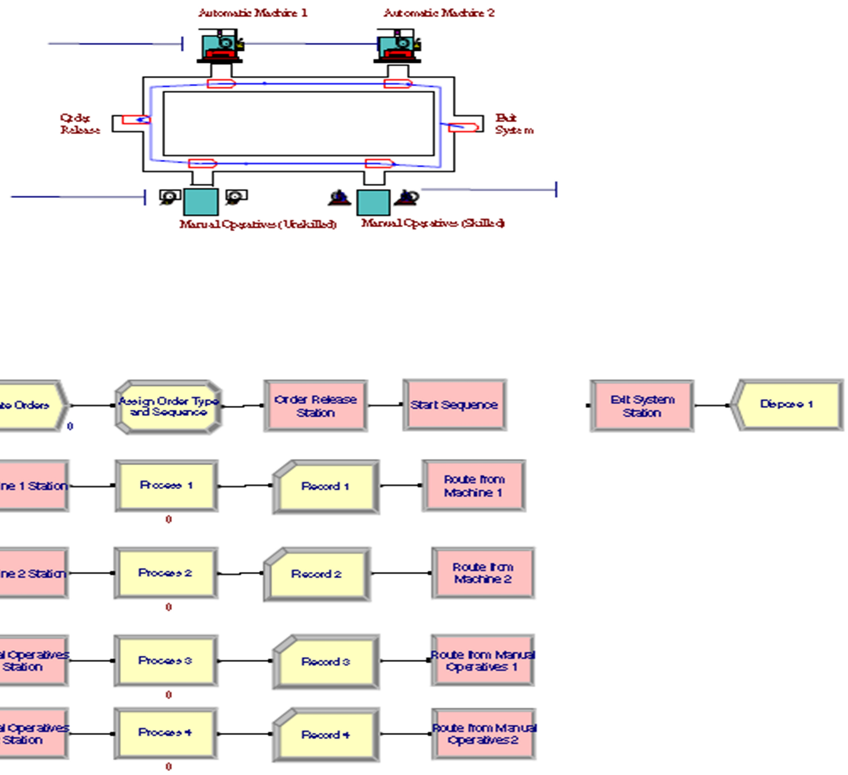

Figure 4.7

Simulation Animation of a Crossdocking Order Picking Process

4.3.2.1 Experimental Design

For the design of the crossdocking distribution simulation experiments, the following performance measures were chosen:

– Total Entity Time (Base): This variable records the total time an entity spends in the simulation system. This performance measure will accumulate data on the total time it takes for an order to be picked through the order picking system.

– Total Resource Utilisation (Base): The purpose of collecting data on resource utilisation is to have statistics on the level of usage of the resources during a specified period. In this instance, the data collected will reveal the level of automated machines and manual operative used during a picking period.

- Total Resource Cost (Base): This is a cost based statistic that records the monetary amount expended on the use of resources for a specific period. Specifically, its the monetary value of using automated machines and manual operatives for a specific period.

The experimental conditions are as follows::

- Number of Replications: 10
- Warm up Period: 0
- Replication Length: 30 Days
- Terminating Condition: None

4.3.2.2 Results

Reported in table 4.31, at a 95% Confidence Interval are the mean and standard deviation of the three performance measures of interest. These results will be used as a bench mark to determine whether the application of variance reduction technique has been successful or not. This performance will be judged mainly by a reduction in variance.

4.3.3 Variance Reduction Experiments

This section of the chapter is divided into two parts; the first describes the design of the variance reduction experiments and the second details the results of the

Variable	N	Mean	StDev	95.0% CI
Total Entity Time (Base)	10	12.445	3.072	(10.248,14.643)
Total Resource Utilisation (Base)	10	5.400	0.699	(4.900,5.900)
Total Resource Cost (Base)	10	153832	1676	(152633,155031)

Table 4.31

Crossdocking Distribution Centre Simulation Results

application of the variance reduction techniques, as stand alone application and as combined application. The method of combining the variance reduction techniques under consideration is an extension of that which is proposed by Kleijnen [68].

4.3.3.1 Experimental Design

This section describes the experimental settings that have been used to conduct the variance reduction technique experiments on the crossdocking simulation model. The performance measures have been classed according to variance reduction techniques, i.e. *Total Resource Utilisation (Base), Total Resource Utilisation (CRN), and Total Resource Utilisation (AV).*

Under consideration as in the previous call centre study is a two scenario, single call crossdocking simulation model. The scenario which has performance measures labelled (Base), is characterised by random number seeds dedicated to sources of simulation model randomness as selected by the simulation software Arena $^{\text{TM}}$. The other scenario which has performance measures labelled common random number (CRN), has its identified sources of randomness, allocated dedicated random seeds by the user. So these two scenarios have unsynchronised and synchronised use of random numbers [80] (See Chapter 2, Section 2.4.1).

The design of experiments remains the same as that in the manufacturing system (section 4.1.3.1), however an additional performance measure *Total Entity Wait Time* is introduced at this stage. This performance measure will be used for the control variates experiment, with a view to using it to adjusting the performance measure *Total Entity Time*.

4.3.3.2 Common Random Numbers Results

Results from table 4.32, report the difference between means for Total Resource Cost (CRN) and Total Resource Cost (BASE) as "not statistically significant", (T-Value = -0.02, P-Value = 0.986). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.651 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Cost (CRN) has a lower standard deviation, when compared to Total Resource Cost (Base). This demonstrates a reduction in variance by the application of common random numbers on the performance measure, Total Resource Cost.

Variable	N	Mean	StDev	SEMean
Total Resource Cost (CRN)	10	153817	1435	454
Total Resource Cost (Base)	10	153832	1676	530

Table 4.32

Paired T-Test: Total Resource Cost (CRN), Total Resource Cost(Base)

Results from table 4.33, report the difference between means for Total Resource Utilisation (CRN) and Total Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.61, P-Value = 0.555). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.770 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Utilisation (CRN) has a lower standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a reduction in variance by the application of common random numbers on the performance measure, Total Resource Utilisation.

Variable	N	Mean	StDev	SE Mean
Total Resource Utilisation (CRN)	10	5.200	0.632	0.200
Total Resource Utilisation (Base)	10	5.400	0.699	0.221

Table 4.33

Paired T-Test: Total Resource Utilisation (CRN), Total Resource Utilisation (Base)

Results from table 4.34, report the difference between means for Total Entity Time (CRN) and Total Entity Time (Base) as "not statistically significant", (T-Value = -0.10, P-Value = 0.926). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.961 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Entity Time (CRN) has a higher standard deviation, when compared to Total Entity Time (Base). This demonstrates a negative effect by the application of common random numbers on the performance measure, Total Entity Time.

Variable	N	Mean	StDev	SE Mean
Total Entity Time (CRN)	10	12.324	3.124	0.988
Total Entity Time (Base)	10	12.445	3.072	0.971

Table 4.34

Paired T-Test: Total Entity Time (CRN), Total Entity Time(Base)

4.3.3.3 Antithetic Variates Results

Results from table 4.35, report the difference between means for Total Resource Cost (AV) and Total Resource Cost (Base) as “not statistically significant”, (T-Value = -0.02, P-Value = 0.986). The P value is greater than the α value, therefore“do not reject the null hypothesis”. For the F test, the P-value of 0.651 is greater than the α value, therefore “do not reject the null hypothesis”, also the difference between variances is “not statistically significant”.

Specifically, at 10 replications, Total Resource Cost (AV) has a lower standard deviation, when compared to Total Resource Cost (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Total Resource Cost.

Variable	N	Mean	StDev	SE Mean
Total Resource Cost(AV)	10	153825	840	226
Total Resource Cost(Base)	10	153832	1676	530

Table 4.35

Paired T-Test: Total Resource Cost(AV), Total Resource Cost(Base)

Results from table 4.36, report the difference between means for Total Resource Utilisation (AV) and Total Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.61, P-Value = 0.555). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-values of 0.148 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications,Total Resource Utilisation (AV) has a lower standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Total Resource Utilisation.

Variable	N	Mean	StDev	SE Mean
Total Resource Utilisation (AV)	10	5.300	0.422	0.133
Total Resource Utilisation (Base)	10	5.400	0.699	0.221

Table 4.36

Paired T-Test: Total Resource Utilisation (AV), Total Resource Utilisation(Base)

Results from table 4.37, report the difference between means for Total Entity Time (AV) and Total Entity Time (Base) as "not statistically significant", (T-Value = -0.10, P-Value = 0.926). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.440 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Entity Time (AV) has a lower standard deviation, when compared to Total Entity Time (Base). This demonstrates a reduction in variance by the application of antithetic variates on the performance measure, Total Entity Time.

Variable	N	Mean	StDev	SE Mean
Total Entity Time (AV)	10	12.385	2.354	0.744
Total Entity Time (Base)	10	12.445	3.072	0.971

Table 4.37

Paired T-Test: Total Entity Time (AV), Total Entity Time(Base)

4.3.3.4 Control Variates Results

Results from table 4.38, report the difference between means for Total Entity Time (CV) and Total Entity Time (Base) as "not statistically significant", (T-Value = 0.00, P-Value = 1.000). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.000 is less than the α value, therefore "reject the null hypothesis", also the difference between variances is "statistically significant".

Specifically, at 10 replications, Total Entity Time (CV) has a lower standard deviation, when compared to Total Entity Time (Base). This demonstrates a reduction in variance by the application of control variates on the performance measure, Total Entity Time.

Variable	N	Mean	StDev	SE Mean
Total Entity Time (CV)	10	12.445	0.022	0.007
Total Entity Time (Base)	10	12.445	3.072	0.971

Table 4.38

Paired T-Test: Total Entity Time (CV), Total Entity Time(Base)

4.3.3.5 Combined Application Results

Results from table 4.39, report the difference between means for Total Resource Cost (CRN+AV) and Total Resource Cost (Base) as "not statistically significant", (T-Value = -0.02, P-Value = 0.986). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.122 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Cost (CRN+AV) has a lower standard deviation, when compared to Total Resource Cost (Base). This demonstrates a reduction in variance by the application of common random numbers and control variates on the performance measure, Total Resource Cost.

Variable	N	Mean	StDev	SE Mean
Total Resource Cost(CRN+AV)	10	153821	975	308
Total Resource Cost(Base)	10	153832	1676	530

Table 4.39

Paired T-Test: Total Resource Cost (CRN+AV), Total Resource Cost (Base)

Results from table 4.40, report the difference between means for Total Resource Utilisation (CRN+AV) and Total Resource Utilisation (Base) as "not statistically significant", (T-Value = -0.61, P-Value = 0.555). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.256 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Resource Utilisation (CRN+AV) has a lower standard deviation, when compared to Total Resource Utilisation (Base). This demonstrates a variance reduction by the application of common random numbers and antithetic variates on the performance measure, Total Resource Utilisation.

Variance	N	Mean	StDev	SE Mean
Total Resource Utilisation (CRN+AV)	10	5.250	0.471	0.149
Total Resource Utilisation (Base)	10	5.400	0.699	0.221

Table 4.40

Paired T-Test: Total Resource Utilisation (CRN+AV), Total Resource Utilisation (Base)

Results from table 4.41, report the difference between means for Total Entity Time (CRN+AV) and Total Entity Time (Base) as "not statistically significant", (T-Value = -0.10, P-Value = 0.926). The P value is greater than the α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.608 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Entity Time (CRN+AV) has a lower standard deviation, when compared to Total Entity Time (Base). This demonstrates a variance reduction by the application of common random numbers and antithetic variates on the performance measure, Total Entity Time.

Results from table 4.42, report the difference between means for Total Entity Time (CRN+CV) and Total Average Call Time (Base) as "not statistically significant", (T-Value = -0.06, P-Value = 0.954). The P value is greater than the

Variable	N	Mean	StDev	SE Mean
Total Entity Time (CRN+AV)	10	12.354	2.576	0.815
Total Entity (Base)	10	12.445	3.072	0.971

Table 4.41

Paired T-Test: Total Entity Time (CRN+AV), Total Entity Time(Base)

α value, therefore "do not reject the null hypothesis". For the F test, the P-value of 0.055 is greater than the α value, therefore "do not reject the null hypothesis", also the difference between variances is "not statistically significant".

Specifically, at 10 replications, Total Entity Time (CRN+CV) has a lower standard deviation, when compared to Total Entity Time (Base). This demonstrates a reduction in variance by the application of common random numbers and control variates on the performance measure, Total Entity Time.

Variable	N	Mean	StDev	SE Mean
Total Entity Time (CRN+CV)	10	12.385	1.556	0.492
Total Entity Time (Base)	10	12.445	3.072	0.971

Table 4.42

Paired T-Test: Total Entity Time (CRN+CV), Total Entity Time(Base)

4.3.4 Summary

In this section, a summary of the results on the performance of each variance reduction technique is presented. By using the word "selected" in describing the results within tables 4.43, 4.44 and 4.45, it is implied that there is a variance reduction technique which is the preferred technique out of the following techniques; Common Random Numbers (CRN), Antithetic Variates (AV), Control Variates (CV), Common Random Numbers and Antithetic Variates (CRN+AV) and Common Random Numbers and Control Variates (CRN+CV), for the purpose of variance reduction. The selection of a technique is purely on the basis that it achieves the greatest reduction in variance / standard deviation for a particular output performance measure without the use of additional simulation runs, whilst applying a manual approach to the selection of a variance reduction technique.

- On the basis of the performance of the variance reduction techniques presented in Table 4.43, antithetic variates technique (AV) was selected for the simulation output performance measure, Total Resource Cost.
- On the basis of the performance of the variance reduction techniques presented in Table 4.44, antithetic variates technique (AV) was selected for the simulation output performance measure, Total Resource Utilisation.
- On the basis of the performance of the variance reduction techniques presented in Table 4.45, control variates technique (CV) was selected for the simulation output performance measure, Total Entity Time.

Antithetic Variates (AV), Antithetic Variates (AV) and Control Variates (CV), have been selected because each technique achieved the lowest standard deviation for each output performance measure, whilst using 10 simulation replications only.

Output Performance Measure	N	Mean	StDev
Total Resource Cost (Base)	10	153832	1676
Total Resource Cost (CRN)	10	153817	1435
Total Resource Cost (AV)	10	153825	840
Total Resource Cost (CRN+AV)	10	153821	975

Table 4.43

One way ANOVA for Total Resource Cost

Output Performance Measure	N	Mean	StDev
Total Resource Utilisation (Base)	10	5.400	0.699
Total Resource Utilisation (CRN)	10	5.200	0.633
Total Resource Utilisation (AV)	10	5.300	0.422
Total Resource Utilisation (CRN+AV)	10	5.250	0.471

Table 4.44

One way ANOVA for Total Resource Utilisation

Output Performance Measure	N	Mean	StDev
Total Entity Time (Base)	10	12.445	3.072
Total Entity Time (CRN)	10	12.324	3.124
Total Entity Time (CV)	10	12.445	0.022
Total Entity Time (AV)	10	12.385	2.354
Total Entity Time (CRN+AV)	10	12.354	2.576
Total Entity Time (CRN+CV)	10	12.385	1.556

Table 4.45

One way ANOVA for Total Entity Time

4.4 Chapter Discussion

In this chapter, the performance of three stand alone variance reduction techniques (common random numbers, antithetic variates and control variates), and two combination strategies (common random numbers and antithetic variates) and (common random numbers and control variates) was investigated. These stand alone and combined application of variance reduction techniques were used for the purpose of reducing variance in the following systems; Manufacturing System, Call Centre System and Crossdocking Distribution System. The aim of this extensive investigation is to find out in which of the domains considered within this thesis, will the variance reduction techniques considered succeed. In particular, would a stand alone application be more successful in reducing variance as compared with a combined application, where the number of replications is fixed?. A discussion based on the results from the variance reduction experiments is presented below.

As stated in the literature Law and Kelton [80], Chapter 11, Simulation Modelling and Analysis, variance reduction techniques cannot guarantee variance reduction in each simulation application, and even when it has been know to work, knowledge on the class of systems which it is provable to always work has remained rather limited. A review of results from the variance reduction experiments indicate that the amount of variance reduction by the techniques applied can vary substantially from one output performance measure to the other, as well as one simulation model to the other. In one particular instance, it was observed that the variance reduction techniques, either as a stand alone or combined application did not reduce the variance of the selected performance measure.

Among the stand alone application of the variance reduction techniques, control variates, stands out as the best technique. This is followed by antithetic variates and common random numbers. Control variates was the only technique that achieved a reduction in variance for at least one performance measure of interest, in all three application domains. This could be attributable to the fact that the strength of this technique is its ability to generate a reduction in variance by inducing a correlation between random variates. In addition, control variate has the added advantage of being able to be used on more than one variate, resulting in a greater potential for variance reduction. However, implementing the antithetic variates and common random numbers techniques required less time, and was less complex than control variates for all three domain application domains. This maybe because of the complexity in implementing control variates, where there is a need to establish some theoretical relationship between the control variate and the variable of interest.

Even though the stand alone application of the variance reduction techniques, especially control variates achieved variance reduction, further improvement is known to be possible with its combined application with another technique. From the experimental results, only on one occasion, did a combination of variance reduction techniques, i.e. common random numbers and antithetic variates produce the highest reduction in variance. This was for a performance measure attributable to the manufacturing system. In addition, on no occasion did the combination of common random numbers and control variates yield the largest variance reduction as was expected.

The results presented in this chapter are valid under the current experimental conditions, and demonstrate in particular, the effective use of antithetic variates and control variates for the purpose of variance reduction, for a class of systems which this kind of knowledge was previously unknown, i.e. a crossdocking distribution system.

With knowledge gained on the practicalities of selecting a variance reduction technique, it seems logical that the next phase of research should be the design and development of a decision support system. Such a computerised system should be able to assist users of variance reduction techniques, in the process of making a technique selection for use within a discrete event simulation study. In addition, this decision support system should fundamentally be based on existing “knowledge” on variance reduction technique selection, which can be coded into a computer. This decision support system should ultimately assist in the decision making process, but not remove the responsibility of the user in making the final decision. Following on in the next chapter with a demonstration of such a decision support system.

Chapter 5

Prototype Fuzzy Expert System

In this chapter the design and development of a fuzzy expert system which will have two input variables, 9 fuzzy rules and one output variable is presented. The fuzzy inference method that will be used for the inference process will be of the Mandani type. This chapter is important as it serves as a foundation for demonstrating the potential applicability of a fuzzy expert system approach to automating the process of selecting a variance reduction technique. In addition to being a response to the second research question posed at the beginning of this thesis; "Can a fuzzy expert system assist a simulation modeller in the selection of a variance reduction technique for discrete event simulation studies?".

This chapter has been divided into several sections which include; eliciting of expert knowledge, a description of the representation of elicited expert knowledge, specifications of the fuzzy expert system, details on the design and development of the prototype fuzzy expert system and a description of the verification testing performed on the prototype fuzzy system.

The aim of this chapter is to:

1. Elicit expert knowledge on the selection of variance reduction techniques for discrete event simulation studies,
2. Design and develop a prototype fuzzy expert system, and
3. Evaluate the prototype fuzzy expert system through verification testing.

The use of a fuzzy expert system has a variety of advantages which include, a method of handling uncertainty and vagueness in problems with unclear boundaries. Furthermore, fuzzy logic allows expert systems to combine several variables with low levels of confidence to arrive at one measure with a high level of confidence [63]. The next section is a discussion on the acquisition of expert knowledge on the selection of variance reduction techniques.

5.1 Knowledge Acquisition

The development of the fuzzy expert system was an iterative process which took place with the help of several simulation experts, who have researched the application of variance reduction techniques for discrete event simulation studies. The development process was initiated with the process of knowledge acquisition by contacting simulation experts and explaining the purpose for which their participation is required. These simulation experts have different levels of expertise in the application of variance reduction techniques. They have also applied these techniques to different real world systems modelled as discrete event simulation models. While some of these experts are knowledgeable in those techniques that utilise the manipulation of random numbers, others have done a lot of research into those techniques that are based on having prior knowledge concerning the behaviour of the simulation model. Another rich source of information on factors to consider before selecting a variance reduction technique are simulation text books, academic journal papers and conference papers, for example McGrath and Irving [98].

In order to gather expert knowledge, a manual technique was used to elicit knowledge from variance reduction technique experts who happened not reside in the UK. E mail questionnaires were dispatched with an aim to answer a central question in relation to knowledge acquisition for the prototype fuzzy expert system; "What are the important issues to consider as an expert when faced with the task of selecting a variance reduction technique?". As a result of condensing the expert's responses, it discovered that both expert's shared common views as to the issues which they would consider before selecting a variance reduction technique for discrete event simulation studies. Reported below is a summary of the elicited expert knowledge.

1. **Is there a need to use a variance reduction technique in the first instance?** There is a requirement to ascertain the need for applying such a technique, at the post experimentation stage of a typical discrete event simulation study. This is because of the time which is required to select and implement such techniques and the possibility that applying "brute force" simulation maybe sufficient. Brute force means running the simulation model without a variance reduction technique until a specified precision is achieved.

2. **Is the user of variance reduction techniques interested in comparing alternative scenarios or a single version of a simulation model?** The opinion of the experts is that a common sense answer is

available for this question. There is a popular variance reduction technique which lends itself well to the reduction of variance for alternate scenario comparison which is the common random number technique. However, the experts caution on the need to do some pre experimentation preparation such as the synchronisation of random number streams of the simulation model in order to realise its benefit. The authors Law and Kelton [80], [64] have written extensively on synchronisation of random number streams and readers are advised to read these literature for more information.

3. **Is the output performance measure being estimated an absolute value or the relative difference of two or more values?.** This relates to the above point (2) regarding the interest of the modeller. Where the focus is either the reduction of variance for an absolute value or the reduction of variance of a particular model with a view to improving its overall precision. Here the modeller is faced with the dilemma of choosing a particular variance reduction technique for either purpose.

4. **Is there a correlation between simulation input and output variables, and is there any evidence of a monotonic relationship between input and output variables?** This point is worth considering if the modeller is planning to utilise prior knowledge gained for example from a pilot simulation study. Information gained can provide useful information about the theoretical relationship between input and output variables. Thus their manipulation through an appropriate variance reduction technique can only be facilitated through the use of prior knowledge. There are variance reduction techniques such as control variates which are based on the use of prior knowledge, and exploit this knowledge to obtain an indirect estimate of the performance measure which has a smaller variance

than the direct estimate of this same value [22].

5. **What is the level of knowledge and understanding of a user of variance reduction techniques?** These techniques require a level of skill and judgement on the part of the user in relation to selection and application. It therefore makes sense that whoever is intending to use them should have even a vague idea of what they intend doing with such techniques.

6. **Is the simulation model under consideration complex enough to make the use of a variance reduction technique, insignificant or cumbersome?** The expert's view complexity in terms of the number of entities, processes, resources, and routes that constitute a simulation model. The experts are of the opinion that the application domain may or may not have a direct correlation on the variance reduction technique selected, what is important is to establish that there is a need to use a variance reduction technique and its application is feasible.

7. **Is the issue of warm up a concern to the user of a variance reduction technique?** This is possibly a factor which a modeller would want to consider in making a selection of a variance reduction technique. The removal of initial bias in simulation experiments, how long is the run length and the number of runs to be utilised for a simulation study is discussed by Robinson [121]. These factors could influence the use of a variance reduction technique depending on the level of randomness in the simulation model. Generally, the decision to apply a technique would be split between aiming to improve the performance of the simulation model at the pre - experimentation stage by considering issues like warm up or at post - experimentation stage using variance reduction techniques.

8. **Is the probability distribution used to model such events like inter arrival rate and processing rate a consideration for the selection of a variance reduction technique?** The experts ask this question with the knowledge that there are techniques that consider the probability distribution applied to modelling events in a simulation model. As such, the effectiveness of such variance reduction techniques requires an understanding of the probability distributions that underline the simulation model under consideration.

It should be pointed out that the above opinion of the two experts is based on the assumption that they do not take into consideration the type of discrete event simulation models considered in this thesis i.e. manufacturing, call centre and crossdocking distribution. Their opinion is based on making a selection for any discrete event simulation model. This approach made it easier to elicit knowledge for the selection of variance reduction techniques, especially where such knowledge did not exist in this form up until now. However, there was a consensus by the variance reduction technique experts that as a first choice, the common random number technique is chosen for experimentation. This is possibly a natural choice because of its popularity and easy of use.

After the completion of the knowledge elicitation process, the next task was the extraction of potential input variables and an output variable for the design of the prototype fuzzy expert system, a full list has been included in Appendix A.2. As a result of revising this list of possible input variables and output variable, under the guidance of the variance reduction experts the following two criteria were selected for the input variables:

a. The amount of variance associated with a particular output performance measure or simulation model,

b. The nature of the discrete event simulation model, i.e. is it made up of machines, or human operators or a combination of machines and human operators?.

While the following was selected for the output variable:

a. The type of variance reduction technique, i.e. common random numbers, antithetic variates and control variates.

In section 5.6 of this chapter, a discussion regarding the process of extracting the output variable for the prototype fuzzy expert system is presented. During the course of eliciting knowledge from both static and non static sources there were some problems. There was an initial resistance from some of the experts contacted for the research project. This could have been for a variety of reasons which may include the demand participation in such a project would entail.

However, there was some success in getting a response from the group of experts sampled for participation. Now that the knowledge acquisition stage is completed, and potential input and output variables identified, the next stage is the transfer of knowledge acquired into some form that is both consistent and understood by the fuzzy expert system. This process is called knowledge representation and is considered in the next section.

5.2 Knowledge Representation

Knowledge representation is an important step in the process of designing a fuzzy expert system. Elicited knowledge has to be represented in a knowledge base in such form that will not only be efficient to retrieve and manipulate by the fuzzy expert system but also amendable to the user. One of the most popular techniques for representing knowledge is through the use of production rules, commonly known as rules.

Rules are conditional statements that are easy to understand and write; they specify an action to be carried out, assuming a certain condition is true. They are also capable of expressing relationships between parameters or variables. Fuzzy expert systems differ from traditional "IF-THEN" programming statements because these rules are relatively independent of one another and are based on heuristics, or experimental reasoning, rather than algorithms.

Fuzzy rules represent the major building blocks of a modular representation scheme, especially when using fuzzy expert systems [63]. As a consequence, production rules has been chosen as a knowledge representation technique for the design and development of the prototype fuzzy expert system. So far, eliciting knowledge from experts in the field variance reduction for discrete event simulation studies has been discussed and highlighted the chosen knowledge representation scheme. The next section of this chapter will discuss the design and development of the prototype fuzzy expert system.

5.3 Design of Prototype Fuzzy Expert System

Within this section the different stages in designing the prototype fuzzy expert system will be discussed.

5.3.1 Fuzzification

The process of fuzzification is an important stage in the development of a fuzzy expert system. Fuzzification is where crisp quantities i.e. selected input variables are converted into fuzzy values. Below is a description of the linguistic variables and membership functions for the prototype fuzzy expert system.

The selected linguistic input variables are:

- Model Configuration, and
- Variance

While the linguistic output variable is:

- Variance Reduction Technique

5.3.1.1 Linguistic Variables

The heart of any fuzzy expert system is the idea of a linguistic variable. The first fuzzy variable is "Model Configuration", and its selected linguistic terms are "Automated", "Semi automated" and "Manual". The second fuzzy variable is Variance and its linguistic terms are "Fair amount", "Considerable amount" and "Large amount". The output fuzzy variable is variance reduction techniques, with accompanying terms like "Common random number", "Antithetic variates" and "Control variates". Table 5.1 displays the linguistic variables and associated terms for the prototype fuzzy expert system.

Linguistic Variables	
Input variables	Linguistic Terms
Model Configuration	Automated Semi - Automated Manual
Variance	Fair amount Considerable Amount Large Amount
Output variable	Linguistic Terms
Variance Reduction Techniques	Common random number Antithetic variates Control variates

Table 5.1

Linguistic variables for the prototype fuzzy expert system

5.3.1.2 Membership Function

The use of membership functions, define for each fuzzy set and each linguistic variable, the degree of membership of a crisp value in each fuzzy set. The membership function shapes differentiate the ranges suggested from the knowledge acquired, and the placement of these shapes is estimated over the universe of discourse. In addition, the number of shapes and the overlapping of these shapes is an important issue to be considered when defining membership functions.

The universe of discourse which is the range of all possible values for an input and output variable of a fuzzy expert system, will range between [0,8] for all linguistic input variables and [0,9] for linguistic output variables. Table 5.2 shows details of the selected membership functions shapes for the prototype fuzzy expert system and figure 5.1 shows the membership function for the linguistic variable “Model Configuration”.

Membership Functions(MF)		
Linguistic Terms	MF Shape	Universe of Discourse
Automated	S - Shape	[0,8]
Semi - Automated	Gaussian - Shape	[0,8]
Manual	Z - Shape	[0,8]
Fair amount	S - Shape	[0,8]
Considerable Amount	Gaussian - Shape	[0,8]
Large Amount	Z - Shape	[0,8]
Common random number	Triangular - Shape	[0,9]
Antithetic variates	Triangular - Shape	[0,9]
Control variates	Triangular - Shape	[0,9]

Table 5.2

Membership function shapes for the prototype fuzzy expert system

5.3.2 Fuzzy Inferencing

Once all crisp input and output variables have been fuzzified into their respective linguistic values, the inference engine will access the fuzzy rule base of the fuzzy expert system to derive linguistic values for the output linguistic variables. The three main components of fuzzy inferencing are:

1. Fuzzy Operators,
2. Fuzzy Rules, and the

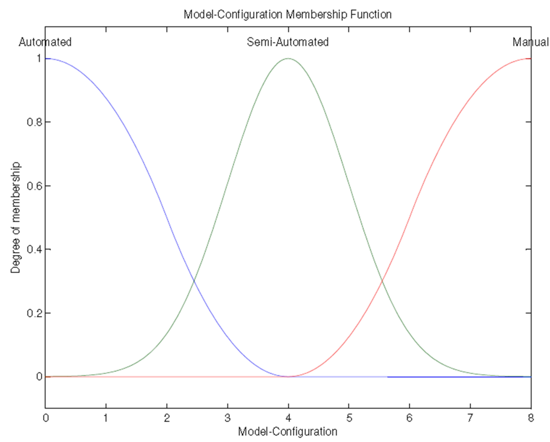

Figure 5.1

Model Configuration Membership Function for the Prototype Fuzzy Expert System

3. Inferencing Methodology

5.3.2.1 Fuzzy Operators

In this section, is a summary of the selected fuzzy operators that have been utilised in the development of the prototype fuzzy expert system. The basic connective operations in classic set theory are those of intersection and union. These operations on characteristic functions can be generalised to fuzzy sets in more than one way [30]. However, one particular generalisation, which results in operations that are usually referred to as standard fuzzy set operations, has a special significance in fuzzy set theory.

The standard operators, intersection ("AND")*min* and union ("OR") *max* are known to define the 'strongest' (most restrictive) union and the 'weakest' (least restrictive) intersection, which make it possible to calculate fuzzy sets [125]. Typically, most fuzzy expert systems make use of these fuzzy operators and their selection can be random to an extent. The choice of operators, fuzzy intersection and fuzzy union is based on its universal application to the inference method applied to the prototype system.

5.3.2.2 Fuzzy Rules

Once the process of knowledge elicitation is complete, linguistic variables have been determined and membership functions selected, the logical next step in the development of the prototype fuzzy expert system is the construction of a fuzzy rule base. This task is performed by "extracting" the fuzzy expert systems rules from the elicited human knowledge. The fuzzy associative matrix approach has been selected for the task of extracting fuzzy rules [87].

The practical application of the matrix formulation approach has been to provide a symbolic representation through a planar construction for the creation of a fuzzy rule base. This means fuzzy associative matrix is useful in representing fuzzy logic in a matrix form. These matrices can take two variables as input, mapping these inputs to a two dimensional matrix and hypothetically, it is possible to have a matrix of any number of dimensions.

To picture this lets consider table 5.3 which is in the form of a 3 x 3 matrix. The first column, contains linguistic terms "Fair Amount" ,"Considerable Amount" and "Large Amount" for the input variable "Variance", while the first row contains linguistic terms "Automated", "Semi Automated" and "Manual", for the input variable "Model Configuration". The other rows from left to right and the other columns from top to bottom contain linguistic terms "Common random numbers", "Antithetic variates" and "Control variates" for the one output variable under consideration, "Variance Reduction Technique".

The maximum number of possible rules is simply the product of the number of rows and columns, but definition of all of these rules, in some cases, may not be necessary since some (AND)/(OR) combinations may never occur in a practical situation. The primary objective of this construct is to map out the universe of possible inputs while keeping the system sufficiently under control. In this case there are nine possible logical product (AND) output conclusions, meaning nine possible fuzzy rules. The extraction of fuzzy rules should not be confused with the "formation" of fuzzy rules discussed previously. Rule formation has to do with the structure of the rules, for example "IF-THEN", while extraction deals with the contents of the rules i.e. *IF X THEN Y*. For a list of the prototype expert system's fuzzy rule base, see appendix A.1.

		Model Configuration		
	Linguistic Terms	Automated	Semi Automated	Manual
Variance	Fair Amount	CRN	CRN	AV
	Considerable Amount	CRN	AV	CV
	Large Amount	AV	CV	CV

Table 5.3

Fuzzy Rule Matrix

5.3.2.3 Inferencing Method

The reason for choosing the Mamdani method of inference is mainly because of its computational simplicity and its flexibility in experimenting with different design choices for input and output variables. This inference method produces a unique value for the consequent, using a simple computation, and computational efficiency is important for providing rapid results from prototyping which possibly can lead to a quicker user acceptance of the fuzzy expert system. In addition, the Mamdani method has a widespread acceptance and is intuitive nature.

However, there is probably no clear cut technique for selecting an inference methodology perhaps, other than considering the constituent of the problem under consideration i.e. the number of linguistic input and output variables [59]. In the next section the selected defuzzification method is presented.

5.3.3 Defuzzification

The defuzzification method which has been chosen for the design of the prototype fuzzy expert system is the centroid method. The main reason for this choice is because it is preferred in the design of fuzzy expert systems and it seems to incorporate all information in the output fuzzy set. This means the centroid method computes a value that is supported by knowledge gathered from each fired rule, and in nearly all cases as compared with other techniques, it will give the best overall expected value. In addition, the selection of centroid method is a popular design stating point for model validation [30]. Figure 5.2 shows a graphical illustration of the centroid method.

Now that the design choices have been concluded, next step is development of the prototype fuzzy expert system.

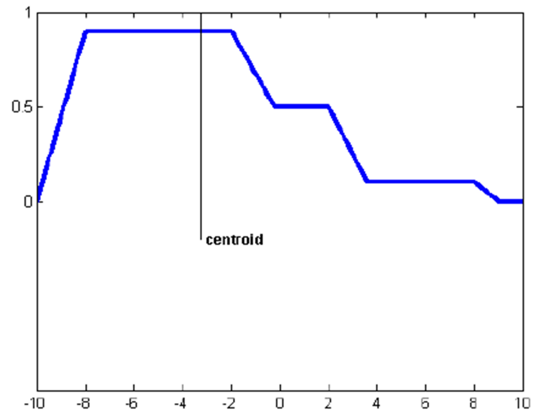

Figure 5.2

Centroid defuzzification method(adapted from Fuzzy Logic Toolbox ™ User's Guide.)

5.4 Development of Prototype Fuzzy Expert System

Within this section of the chapter the development of the prototype fuzzy expert system is described.

5.4.1 System specifications

Experimentation has been performed using a personal laptop computer with the following hardware specifications:

- Operating System: Windows Vista Home Premium™, 32-bit Operating System
- System Manufacturer: Hewlett - Packard™ & Model: Presario F500

– Processor: AMD Turion™ 64 x 2 Mobile Technology TL-50 1.60 GHz
– Memory(RAM): 1.00 GB

In addition, the fuzzy expert system has been developed using a commercial off the shelf software i.e. MATLAB ™ Fuzzy Logic Tool Box 2.2.9 by Mathworks™, Inc. USA. The Fuzzy Logic Toolbox™ 2.2.9 product extends the MATLAB™ technical computing environment with tools for designing expert systems based on fuzzy logic. The process of development is through graphical user interfaces, which guide users through the steps of fuzzy inference system design. It should be made known at this point that the words, fuzzy inference system and fuzzy expert system will be used interchangeably [135].

The toolbox lets users model complex system behaviours using simple logic rules and then implement these rules in a fuzzy inference system. There are five primary graphical user interface tools for building, editing, and observing the fuzzy expert system in the toolbox, namely [96]:

1. Fuzzy Inference System Editor
2. Membership Function Editor
3. Rule Editor
4. Rule Viewer
5. Surface Viewer

These graphical user interfaces are dynamically linked, in that changes a user makes to the fuzzy inference system editor using one of them, can affect what is seen on any of the other that are open. Users can have any or all of them open for any given system. Readers interested in understanding the functionality of each graphical user interface tool are referred to the fuzzy logic toolbox ™ users guide and Figure 5.3 illustrates the five primary GUI tools as they interact with each other.

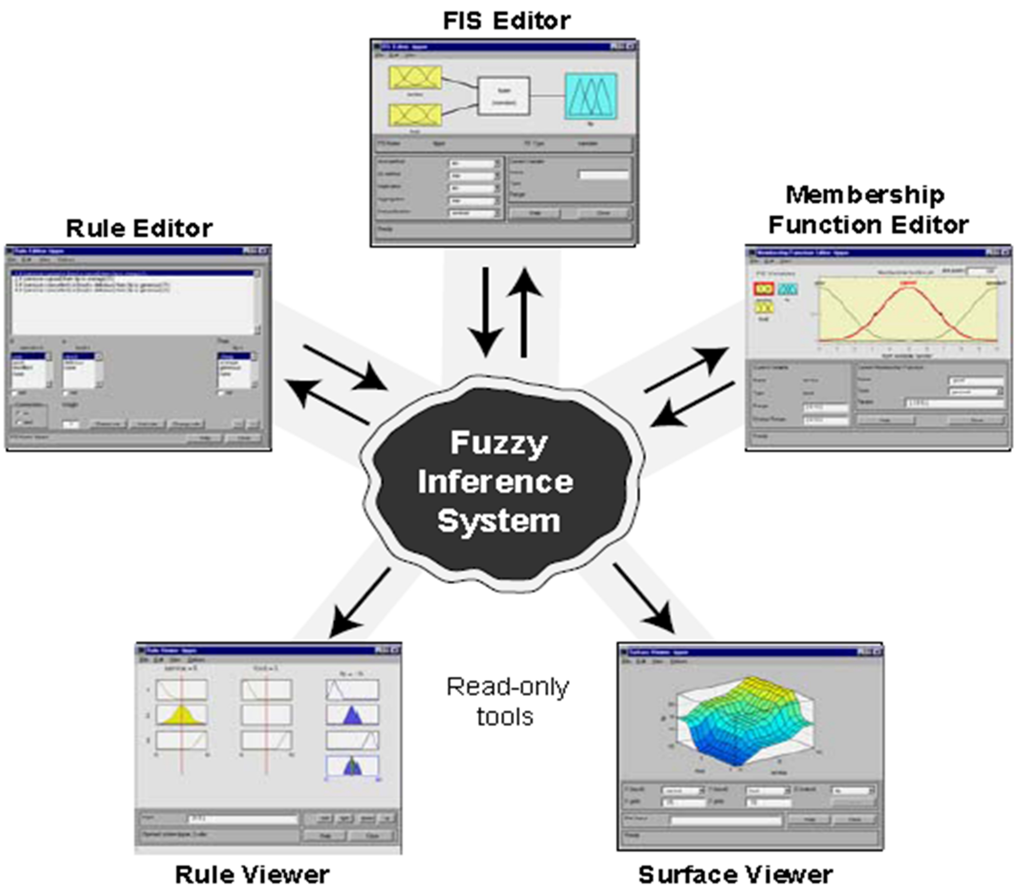

Figure 5.3

Five primary GUI tools for building Fuzzy Expert System (Adapted from Fuzzy Logic Toolbox ™ User's Guide.(Chapter 2))

5.4.2 Prototype Fuzzy Expert System [Overview]

In this section an example is considered, in order to understand how the mamdani type fuzzy inference and max-min operator works within the MATLAB™ Fuzzy Logic Toolbox™ 2.2.9 framework. The purpose of this example is two fold:

1. Demonstrate the practicalities involved in the selection of a variance reduction technique for a typical discrete simulation study.
2. Illustrate how the final crisp output is obtained from crisp input through the process of fuzzification, inferencing and defuzzification.

Next a four step process describing for the development of a prototype fuzzy expert system.

– Step 1. Determine linguistic variables and terms:

  The first step in the development of the prototype fuzzy expert system is to take the crisp inputs and determine the degree to which they belong to the appropriate fuzzy sets via membership functions. Let the two inputs be represented as linguistic variables "Model Configuration" and "Variance", and the output as linguistic variable "Variance Reduction Technique".

  The following, "Automated", "Semi Automated" and "Manual" are linguistic terms for "Model Configuration"; "Fair Amount", "Considerable Amount" and "Large Amount" are linguistic terms for "Variance"; while "Common Random Numbers", "Antithetic Variates" and "Control Variates" are linguistic terms for "Variance Reduction Techniques".

  The second step is to choose which fuzzy inference method suits the purpose, either the mamdani method or the takagi - sugeno method. As mentioned earlier, the mamdani method has been chosen.

- Step 2. Select membership functions, select fuzzy operators and defuzzification method:

  After determining the linguistic variables, the next step is to create fuzzy sets with membership functions which will spread over a universe of discourse. There are a variety of membership function shapes, however, the "Z" shape, "S" shape and Gaussian shape have been chosen for the input variables and the Triangular shape for the output variable. The universe of discourse for the input variables is [0,8] and for the output variable is [0,9]. The creation of the membership function is performed in the membership function editor.

  Next is the selection of fuzzy logical operators, the implication method as well as the aggregation method. There is a wide selection of alternatives to choose from, the *min* operator for the (AND) method as well as the *max* operator for the (OR) method have been selected. The defuzzification method which has been selected is the centroid method, which calculates the centre of area under the curve.

- Step 3. Create fuzzy rule base:

  Following on for step 2, is the creation of a fuzzy rule base, consisting of fuzzy "IF-THEN" rules. The graphical user interface capability of the fuzzy tool box allows the antecedent and consequent parts of the rules to be easily combined. As can be seen from figure 5.4, there are 9 fuzzy rules in the rule base, also displayed are their associated membership functions.

- Step 4. Evaluate fuzzy rules using Rule viewer and Surface viewer:

  The Rule Viewer in figure 5.4, displays a road map of the whole fuzzy

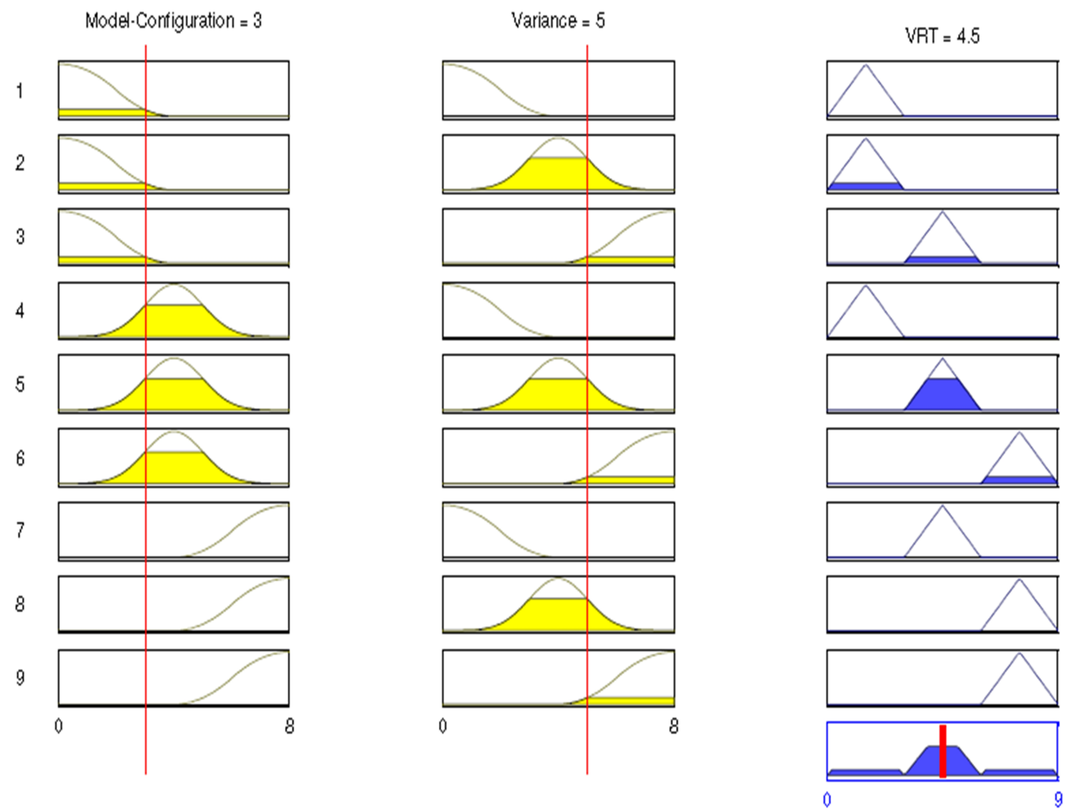

Figure 5.4

Evaluation of fuzzy rules for the prototype fuzzy expert system

inference process, and is displayed is a single figure window with 28 plots nested in it. The three plots across the top of the figure represent the antecedent and consequent of the first rule. Each rule is a row of plots, and each column is a variable. The rule numbers are displayed on the left of each row.

* The first two columns of plots (the eighteen yellow plots) show the membership functions referenced by the antecedent, or the if - part of each rule.
* The third column of plots (the nine blue plots) shows the membership

functions referenced by the consequent, or the then-part of each rule.

∗ The tenth plot in the third column of plots represents the aggregate weighted decision for the given inference system. This decision will depend on the input values for the system. The defuzzified output is displayed as a bold vertical line on this plot and the variables as well as their current values are displayed on top of the columns.

For the two - input fuzzy expert system, the following input vector is entered, [3,5], for example. It will return a value of [4.5] in the rule viewer editor. From figure 5.4, the process with which the fuzzy expert system arrives at its conclusion can be seen, where four fuzzy rules have fired. Rule 5, "If (Model Configuration is Semi Automated) and (Variance is Considerable Amount) then (Variance Reduction Technique is Antithetic Variates)" is shown as having the strongest influence on the single output fuzzy set. The linguistic interpretation of the value [4.5] is a suggestion to the user to select the Antithetic Variates technique. The rule viewer allows users to interpret the entire fuzzy inference process at once, and also how the shape of certain membership functions influence the overall result. In a sense, it presents a sort of micro view of the fuzzy inference system.

A user can see the entire output surface of a fuzzy expert system, the entire span of the output fuzzy set based on the entire span of the input fuzzy set, through the surface viewer. Displayed in Figure 5.5, is the surface viewer, where a three-dimensional curve represents the mapping from model configuration and variance to variance reduction technique. This surface plot which was generated by the 9 rules of the fuzzy expert system which represents a two - input one - output case, and shows the relationship between system's input and output variables.

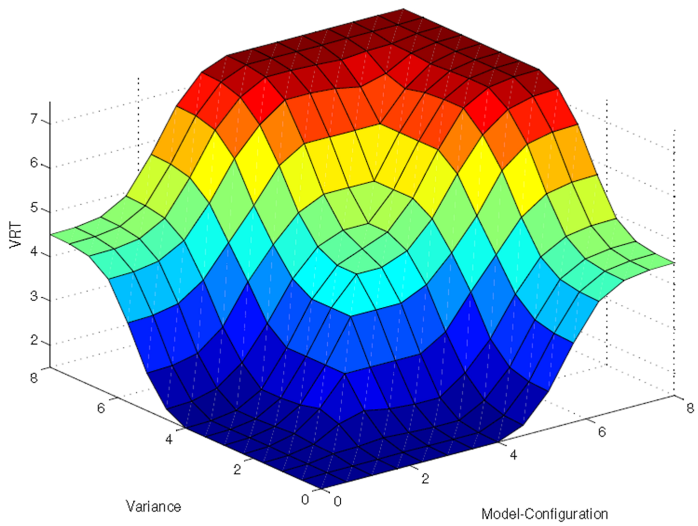

Figure 5.5

Surface Plot of Variables for the prototype fuzzy expert system

This concludes a description of the design and development as well as an illustration of the functionality of the prototype fuzzy expert system. For the problem of selecting a variance reduction technique, it can be argued that a quick lookup table could have solved the problem quicker. However, in solving an entire class of similar decision - making problems like the one under consideration, a fuzzy expert system may provide an appropriate tool as a solution approach, given the ease with which a system can be quickly developed and modified. The next section of this chapter will discuss the verification testing exercise performed on the prototype fuzzy expert system.

5.5 Prototype Fuzzy Expert System Verification

One of the more challenging tasks in the prototype fuzzy expert system development life cycle is verification. The basic motivation behind verifying the expert system is to determine the quality of the knowledge base, and achieve a level of agreement with users' expectation. In addition, if a fuzzy expert system is built through the process of prototyping, each stage of the development process, i.e. determination of linguistic input variables and membership functions, formation of fuzzy rules, choosing an inferencing methodology, and the selection of a defuzzification technique can be verified during the development process rather than at the end [46].

It is a known fact, there is no single verification technique that is best at detecting all the errors in a fuzzy expert system [114]. Some techniques have a better chance of detecting error with certain types of systems, others may be more proficient in detecting anomalies in certain types of knowledge representation schemes.

Described in chapter 3 section 3.2 are the main types of verification techniques for expert systems. The verification testing of the functionality of the knowledge base provides a means to determine the level of real world credibility the system can achieve when scrutinised by simulation variance reduction experts. The key criteria for verifying the prototype system are software graphical user interface ease of use (face appeal), accuracy of linguistic input and output variables, and the reasonableness of the extracted fuzzy rules. The term reasonableness means checking the structure of knowledge within the fuzzy rules as well as circular or redundant rules.

Four experts in the field of simulation, who are well qualified in the domain of variance reduction cooperated in the development of the prototype fuzzy expert system. Two of the four experts were consulted to provide knowledge on steps an expert would take in the selection of a variance reduction technique. This knowledge combined with relevant academic literature were used to "extract" input and output variables, linguistic terms as well as a set of nine fuzzy rules.

The selection of an inference methodology, fuzzy operators and a defuzzification technique are highlighted in chapter 5 section 5.3, but this was not an immediate concern to the two other experts carrying out the verification task. This is because the purpose of the verification exercise is to ascertain that the requirements of the system have been met and not to determine the accuracy of the prototype fuzzy expert systems output, at this stage of system development.

The task of verification testing was performed by two experts in the field of discrete event simulation, with substantial experience in the application of variance reduction techniques. The reason for using simulation experts for the verification exercise is because of their knowledge about the subject domain, and as such they are able to evaluate on the "surface" the fuzzy expert system. The process of verification was carried out in a two stage process.

First, a set of closed questions were used to gauge the opinion of the experts on the key verification criteria mentioned previously. The next stage, was a walk through of the prototype fuzzy expert system, to give the experts an opportunity to examine the system and interact with its components. The main activity during this stage of the exercise was to get the experts to scrutinise the rule base as well as the linguistic variables and terms as well as comment on which components they agreed with and which they did not agree with.

In response to the closed questions, here is a summary of expert A's response:

- Expert A agreed with the opinion that the selection and application of variance reduction technique requires a certain amount of skill and judgement in simulation and statistics techniques. He stated that, the major reason why the study and application of variance reduction techniques remains clearly in the realm of academia is because of the practicability of using such techniques. So today, modellers focus on getting simulation output results as opposed to improving such results after experimentation.
- Expert A also stressed that, while there are numerous problems that a fuzzy expert system could provide a solution in the domain of simulation, narrowing it down to the selection of a variance reduction technique will provide a chance to evaluate its performance on this particular problem and perhaps pave the way for a more generalised application of expert system methodology in simulation.
- Expert A did stress the idea of variance reduction, as being a subset of the subject; design of experiments (DOE). In his opinion, it is not a good idea to approach this topic as a whole. In response, the comment was the research strategy was to narrow interest within the domain of DOE to investigating the problem of selecting a variance reduction technique for discrete event simulation studies.
- Expert A concluded that the input variables were not sufficient to capture the type of factors a modeller would consider when deciding to choose a variance reduction technique for a simulation study. He suggested including variables such as mean response or model response, single or multiple

scenario and existence of correlation between input variable and output variable. These would better reflect a more exhaustive list of input variables to consider for the design of the fuzzy expert system. Expert A also reviewed the fuzzy rule base and found them reasonable but declined to give a final judgement until the fuzzy expert system had undergone a validation test.

Here is a summary of the expert B's verification tests:

- Expert B thought the input variables were adequate for the purpose but she thought there was a need to consider some other factors, such as; is there a consideration for a single scenario or multiple scenario's of the same model. She wanted to know the criteria used to order the output variables' linguistic terms and the response was this is based on the perceived order of implementation complexity of the variance reduction techniques under consideration i.e. common random numbers then antithetic variates then control variates (CRN $\Rightarrow$ AV $\Rightarrow$ CV).
- While Expert B has a good understanding of the problem of selecting a variance reduction technique, the terminology of fuzzification, defuzzification, and fuzzy inferencing had to be explained. Expert B wanted to know what knowledge based was used for the "extraction" of the fuzzy rule set, and the response was it is based on elicited expert knowledge, surveyed literature and variance reduction experimental results. Her thoughts concerning the nine fuzzy rules was that these rules are adequate for this particular size of problem, however, she would be interested in knowing if there would be a difference in results if a larger set of rules was used for the same problem.

At the end of the verification process, the conclusion was:

- Revise the number of input variables,
- Revise the fuzzy rule set, and
- Consider the implementation of a facility within the fuzzy expert system, which could gather users requirements and explain the basis upon which the system is making a suggested solution.

In conclusion, although face verification may not be a rigorous statistical verification technique, it does offer an assurance of the reliability that can be placed on a fuzzy expert system as it evolves from a prototype system to a fully fledged working system. This ad hoc tool is quite useful for testing user - system interface, and user friendliness from both the view point of experts and potential users.

5.6 Chapter Discussion

In this chapter, the design, development and verification of a prototype fuzzy expert system for the selection of a variance reduction technique has been presented. This Mamdani type expert system, consists of nine fuzzy IF-THEN rules, two input variables and one output variable. The membership function used is mainly of the Gaussian and Triangular shape and the method of defuzzification is the centroid technique. In this section, a summary of the prototype fuzzy expert systems design choices will be presented. In addition, to a discussion on the outcome of verification tests performed on the prototype fuzzy system.

1. Design of the prototype fuzzy expert system:

- Already, the elicitation of knowledge and selection of input variables for the fuzzy system has been extensively discussed in section 5.1 of this chapter. However, the process of choosing the linguistic terms for the output variable, variance reduction technique has not been fully explained. The aim of developing the fuzzy expert system is to assist in the selection of a variance reduction technique, known through literature (Law [78], Pegden et.al. [22]) and experimentation (Sabuncuoglu et.al [128]) to succeed on most occasions in variance reduction, in particular for discrete event simulation models.

  The decision to choose common random numbers, antithetic variates, control variates, was based mainly on the fact that these are the techniques used for the manual selection study reported in chapter 4 of this thesis and they are known to reduce variance on most occasions. In addition, it seemed a reasonable choice to consider those techniques which are not advanced or specialised at this stage of research, thus providing an excellent starting point for the validation of the fuzzy expert system.

- The choice of which membership function and its associated shape will specify the linguistic variable involved in the design process of any fuzzy expert system has been an ongoing research problem. For most fuzzy expert systems, the fuzzy sets that will have to be defined are on occasion identifiable. Other times, they will have to be determined by knowledge acquisition from an expert. Once the names of the fuzzy sets have been established, one must consider their membership functions and shapes.

  For most problems, where a fuzzy expert system is a selected solution approach, the assumption is that the membership functions are either Gaussian or Triangular in shape. Figure A.1 (Appendix A.3) shows the concept “APPROXIMATELY 4” as a Gaussian and Triangular shaped membership function, the difference is apparent. In reality, these membership function shapes have been known to be insensitive to inaccuracies in the representation of linguistic terms [30]. As a result, it is such elasticity that makes fuzzy expert systems robust which is an important quality when fuzzy systems are initially prototyped.

A choice regarding the shape of the membership function and range of the universe of discourse has been based on available expert knowledge about the problem of selecting a variance reduction technique. Subsequent fine tuning of design choices will be by trial and error as and when it is required.

2. Verification Testing:

After the completion of verification tests on the fuzzy expert system, several aspects of the results deserve more discussion. The first is the recommendation from the experts that a revision of the number of linguistic input variables become more inclusive of other seemingly important factors that should be considered when selecting a variance reduction technique. At present there are two - input variables and one - output variable fuzzy expert system, with a fixed number of three linguistic terms for each variable. Naturally an increase in input variables will lead to an increase in the number of fuzzy rules.

It is unknown if an increase in the number of rules for this particular fuzzy system will improve performance. It is known from the literature i.e. Law and Kelton [80] that the performance of a variance reduction technique is dependent on the simulation model under consideration. Using this as a guide, it was decided first of all to add one additional input variable and to select this input variable based on its perceived importance to variance reduction as a whole. Furthermore, the aim of variance reduction for discrete event simulation is two fold; the obvious reduction of variance and the use of the same number of runs or less to achieve this purpose. As a consequence, the logical choice was to include "Quantity of Replications" as the next linguistic input variable.

The issue of an explanatory module came up during the verification exercise. Both experts queried the non existence of an explanatory module to give users of the fuzzy expert system an understanding of how the fuzzy expert system works as well as an explanation of how the system arrives at a conclusion. This module was not intentionally omitted, the research plan was to initially gain sufficient assurance regarding the performance of the fuzzy expert system through verification and validation testing. And then proceed to tackle human centric problems like the development of an explanatory module.

As mentioned earlier, a commercial software was used for the purpose of demonstrating the concept of automating the selection of a variance reduction technique. The aim being to demonstrate the capabilities of using a fuzzy expert system, developed through the process of rapid prototyping. As well as see what it may look like, as viewed by experts and potential users. A major advantage of prototyping is it allows project developers to have a good idea of the feasibility of an application as a solution approach before full scale implementation is performed. Another advantage of developing fuzzy expert systems through prototyping is it provides an initial system with enough functionality that, although it is not the final system, it may be put in the field on an extended trial basis.

This early exploration of a prototype fuzzy expert system yielded more knowledge about the domain problem, highlighted potential problems with the system, and provided a measure of credibility that the eventual final system may perform its desired function [140]. Finally, although this system is acceptable as a functioning prototype, the need to revise the number of linguistic input variables and investigate alternative design choices has necessitated further improvement and a future modification of the prototype fuzzy expert system.

The design and development of the revised fuzzy expert system is described in the next chapter.

Chapter 6

Revised Fuzzy Expert System

This chapter of the thesis will describe the design and development of a revised fuzzy expert system. This system has been developed as a follow up to the prototype fuzzy expert system described in the previous chapter (Chapter 5). Although the prototype system demonstrated the potential to use a fuzzy expert system for the automated selection of a variance reduction technique, results from verification tests suggest including new knowledge acquired from further knowledge elicitation.

The objective of this chapter is two fold; first, is the design and development of a revised fuzzy expert system. Second is carrying out three validation tests on the revised fuzzy expert system as follows:

1. User agreement: The hypothesis being tested is whether there is an agreement by users on the usability of the revised fuzzy expert system.

2. Validation of the revised fuzzy expert system output against the results of the manual selection of variance reduction techniques.

3. Sub-system validity: The third stage of validation is to assess the sensi-

tivity of the revised fuzzy expert system to changes in its parameters, and also compare the performance of the prototype fuzzy expert system with the revised fuzzy expert system. Primarily, the objective is determining whether using different quantities of fuzzy rules in either system will have an effect on suggested solutions.

In the next section, the additional knowledge acquired after the development of the prototype fuzzy expert system will be presented.

6.1 Knowledge Acquisition and Representation

It is known that knowledge is central to the the development of any fuzzy expert system. In this section, additional knowledge elicited from simulation experts on the selection of variance reduction techniques will be presented. Earlier on during this research, two US based simulation experts were interviewed on the selection of variance reduction techniques for discrete event simulation studies. Of particular interest was in any heuristic which is usually applied by these experts in making a technique selection.

Later on, 2 UK experts were interviewed to find out if the application domain in which a simulation model is based, has a bearing on the selection of a variance reduction technique. The reason for enquiring was to establish from these experts if a simulation model in perhaps the manufacturing sector, would have a different variance reduction technique applied to it as would say a simulation model in the call centre sector. The interview approach was unstructured, whereby a set of open questions where used as a guide to elicit expert knowledge. The aim was to gather as much knowledge on each question from the interviewee, and then revise the response with the interviewee. This technique of gathering and sifting knowledge proved effective in pin pointing the most important facts from the knowledge gathered.

Here are three main criteria for the selection of a variance reduction technique suggested by the two UK simulation experts, which is inclusive of the two already suggested by the US simulation experts:

a. The amount of variance associated with a particular output performance measure or simulation model,

b. The nature of the simulation model, i.e. is it made up of machines, human operators or a combination of machines and human operators?,

c. The number of simulation replications which are available for experimentation.

The UK experts are of the opinion that the number of replications used in a simulation is worth including, since there is a possibility that additional replications may produce a reduction in variance, but perhaps not to the same magnitude as would a variance reduction technique. The consensus among the experts is that no universally accepted approach exists for the selection of a variance reduction technique.

The most popular approach is that which has been proposed by Law and Kelton [80], which involves using a pilot study to determine the efficiency of any variance reduction technique under consideration. They also agreed a simulation modeller should have some knowledge about the simulation model and the variance reduction techniques to be applied. Furthermore, simulation modellers interested in the application of variance reduction techniques tend to naturally try common random numbers, before considering the use of other techniques.

The next task in the revised fuzzy expert system development life cycle is selecting a technique for representing the additional expert knowledge that been elicited. In the previous chapter, production rules were used to represent the expert knowledge elicited to develop the prototype fuzzy expert system, the same technique will be used for the revised fuzzy expert system.

6.2 Design of Revised Fuzzy Expert System

Most of the fundamental fuzzy system assumptions used in the design and development of the prototype fuzzy expert system will be adopted in the design and development of the revised fuzzy expert system. This is due to the prototype fuzzy expert system demonstrating the ability to model the automated selection of a variance reduction technique, subject to some fine tuning in relation to the results from the verification tests.

The revised system will consist of a three - input, one - output, 27 rule, mamdani inference fuzzy expert system. The design choices for the fuzzy operators, fuzzy inference method and defuzzification method will remain the same as those used in the development of the prototype fuzzy expert system. Below are the changes introduced to the specification of the revised fuzzy expert system.

An additional linguistic variable has been added to the existing set of input variables which have been used for the design of the prototype fuzzy expert system. This additional variable has been added based on the advice of experts who participated in the development of the fuzzy expert system and knowledge gained from experiments reported in chapter 4 of this thesis. This new input variable is “The Quantity of Replications”. Herein are the revised input variables for the fuzzy expert system:

- Model Configuration
- Amount of Variance
- Quantity of Replication

And the output variable remains the same:

- Variance Reduction Technique

In chapter 5, section 5.3.1.1, the linguistic terms which are applicable to the variables for the prototype fuzzy expert system are presented. Since the only change is the additional variable, here are its linguistic terms; "A little", "Average" and "A lot". In addition, all the membership function shapes will be based on the Gaussian shape. This will be for all input and output membership functions, and the design choice is based on advice from the expert on fuzzy systems. By using only the Gaussian shape, there is a possibility to fine tune the revised system later on with the application of only the Triangular shape. The universe of discourse for the variable "Quantity of Replications" and all other variables specified for the revised fuzzy expert system will range between [0,8]. Figure 6.1 shows the membership function for "Model Configuration" and "Quantity of Replications", and the membership function for "Amount of Variance" and "Variance Reduction Technique".

It is known that usually, an increase in the number of linguistic variables can lead to a corresponding increase in the number of fuzzy rules. This is purely because the number of possible combinations of antecedence which can be linked to a consequence has increased. Lets consider table 6.1, where the first and second column, contain the linguistic terms "Manual" , "Semi-Automated" and "Automated" for the input variable "Model Configuration", and "A Little",

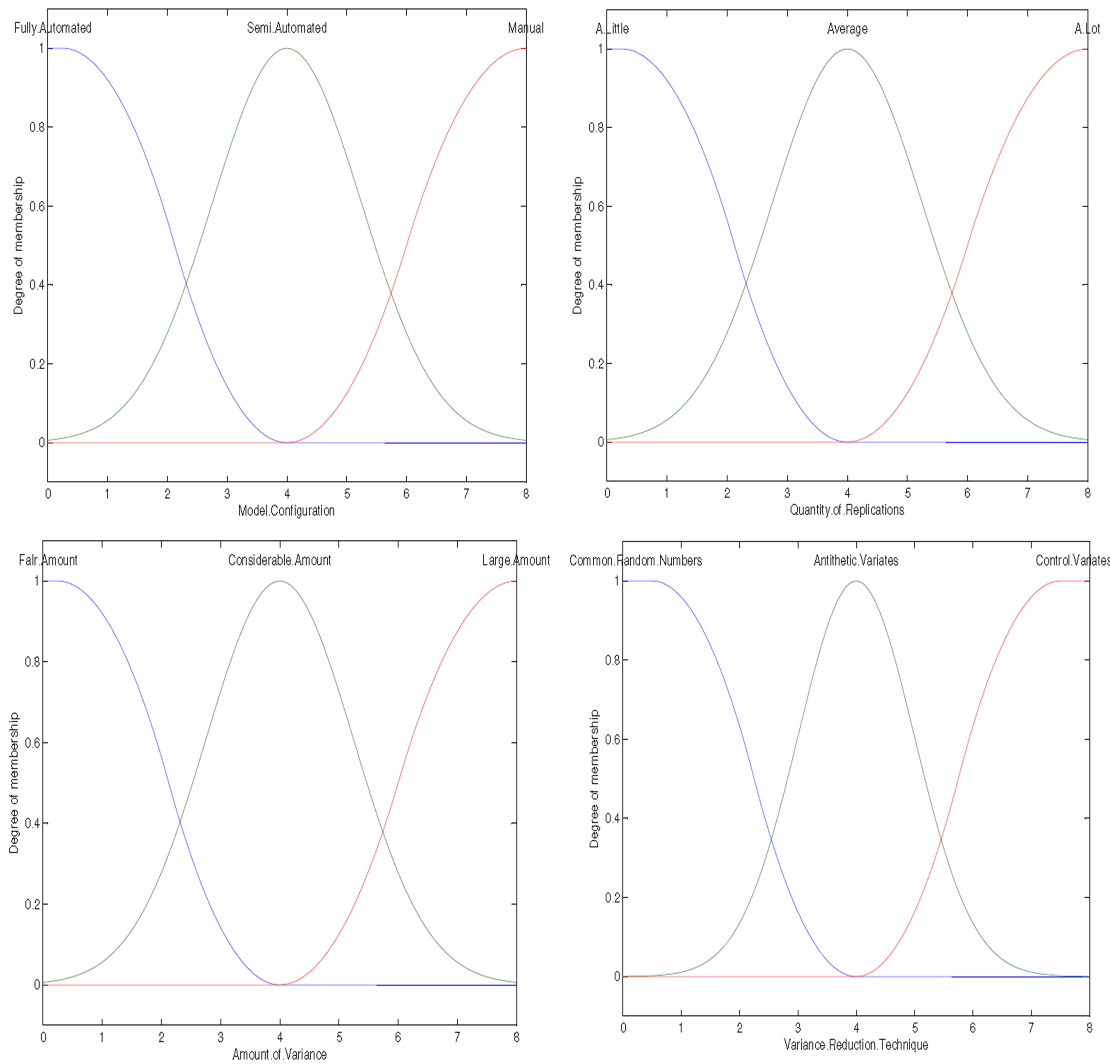

Figure 6.1

Input and Output Variable Membership Functions for the revised Fuzzy expert system

"Average", "A Lot" for the input variable "Quantity of Replications", respectively. The second row contains the linguistic terms "Fair Amount", "Considerable Amount" and "Large Amount", for the input variable "Variance".

From the third row and third column, spanning from left to right and top to bottom, contain linguistic terms “Common random numbers”, “Antithetic variates” and “Control variates” for the one output variable under consideration, “Variance Reduction Technique”. The primary objective of this construct is to map out the universe of possible outputs while keeping the system sufficiently under control. In this case there are twenty seven possible logical product (AND) output conclusions, meaning twenty seven possible fuzzy rules. A full list of twenty seven rules that make up the revised fuzzy expert system rule base can be found in the Appendix B.1.

Model Configuration	Quantity of Replications	Variance		
		Fair Amount	Considerable Amount	Large Amount
Manual	A Little	AV	CV	CV
	Average	CV	CV	CV
	A Lot	CV	CV	CV
Semi-Automated	A Little	CRN	AV	AV
	Average	AV	AV	AV
	A Lot	AV	AV	CV
Automated	A Little	CRN	CRN	CRN
	Average	CRN	CRN	CRN
	A Lot	CRN	CRN	AV

Table 6.1

Fuzzy Rule Matrix

The weight of each rule has been set at a default of 1. This is because sufficient knowledge is not available at this present time to make a decision on the appropriate ranking for each fuzzy rule, in order of importance. In addition, while some applications of fuzzy expert systems would benefit from a rule weighting system, it is not certain this would be useful for the revised fuzzy expert system. The main design changes which will be implemented in the next section have been highlighted. Next is a description of the development of the revised fuzzy expert system.

6.3 Development of Revised Fuzzy Expert System

Within this section, the development of the revised fuzzy expert system is described. This will be performed using the same system specification (see chapter 5 section 5.4.1) as was used for the prototype expert system. Next is a four step process for the development of the revised fuzzy expert system.

- Step 1: Determine linguistic variables and terms:

  The initial step in the development of the revised fuzzy expert system is to declare the linguistic variables and their associated terms. Let the three inputs be represented as linguistic variables “Model Configuration”, “Amount of Variance”, and “Quantity of Replications”. And let the output linguistic variable be “Variance Reduction Technique”. The linguistic terms for “Model Configuration” are “Automated”, “Semi Automated” and “Manual”. The linguistic terms for “Quantity of Variance” are “Fair Amount”, “Considerable Amount” and “Large Amount”. The linguistic terms for “Quantity of Replications” are “A little”, “Average” and “A lot”.

The linguistic terms for "Variance Reduction Techniques" are "Common Random Numbers", "Antithetic Variates" and "Control Variates".

– Step 2: Select membership functions, select fuzzy operators and defuzzification method:

After determining the linguistic variables and terms, the next step is to create the fuzzy set with membership function that will be spread over a universe of discourse. There are a variety of membership function shapes, however, the "Z" shape, "S" shape and Gaussian shape for the input variables and the Gaussian and "Z" shape for the output variable have been chosen. The universe of discourse for all the input variables is [0,8] and for the output variable is [0,8]. The creation of the membership function is performed in the membership function editor.

These design choices are similar to those used for the prototype expert expert system except, there is an additional membership function and the universe of discourse for the output variable has changed as well. Next is the selection of fuzzy logical operators; the *min* operator for the (AND) method as well as the *max* operator for the (OR) method will be used. The input for the defuzzification process is a fuzzy set and the output is a single number. The selected method for defuzzification is the centroid method which calculates the centre of the area under the curve. The next task is the development of the fuzzy rule base which is based on expert knowledge and the knowledge gained from the manual selection experiments performed in chapter 4.

– Step 3: Create fuzzy rule base:

Now that the linguistic variables and terms as well as the membership functions have been created, the next step is to create the fuzzy rule base, consisting of fuzzy IF-THEN rules. The rule editor is used to create the rules of a fuzzy inference system structure stored in a file, and also used to inspect the rules being used by a fuzzy inference system.

- Step 4: Evaluate fuzzy rules using Rule viewer and Surface viewer:

  Perhaps the most important aspect in the revised fuzzy expert system development life cycle is the evaluation of fuzzy rules and the generation of an output surface which lets us examine the relationship between any input and output variable combination. The rule viewer displays, in one screen, all parts of the fuzzy inference process from inputs to outputs. Each row of plots corresponds to one rule, and each column of plots corresponds to either an input variable (yellow, on the left) or an output variable (blue, on the right).

  The rule viewer in figure 6.2, displays a global view of the whole fuzzy inference process, through a single figure window with 109 plots nested in it. The three plots across the top of the figure represent the antecedent and consequent of the first rule. Each rule is a row of plots, and each column is a variable. The rule numbers are displayed on the left of each row. The last plot in the column of blue plots shows the results of applying the defuzzification method.

  * The first three columns of plots (the eighty one yellow plots) show the membership functions referenced by the antecedent, or the if-part of each rule.
  * The fourth column of plots (the twenty seven blue plots) shows the membership functions referenced by the consequent, or the then-part

of each rule.

∗ The twenty eighth plot in the fourth column of plots represents the aggregate weighted decision for the given inference system, and will depend on the input values for the system. The defuzzified output is displayed as a bold vertical line on this plot, and the variables as well as their current values are displayed on top of the columns.

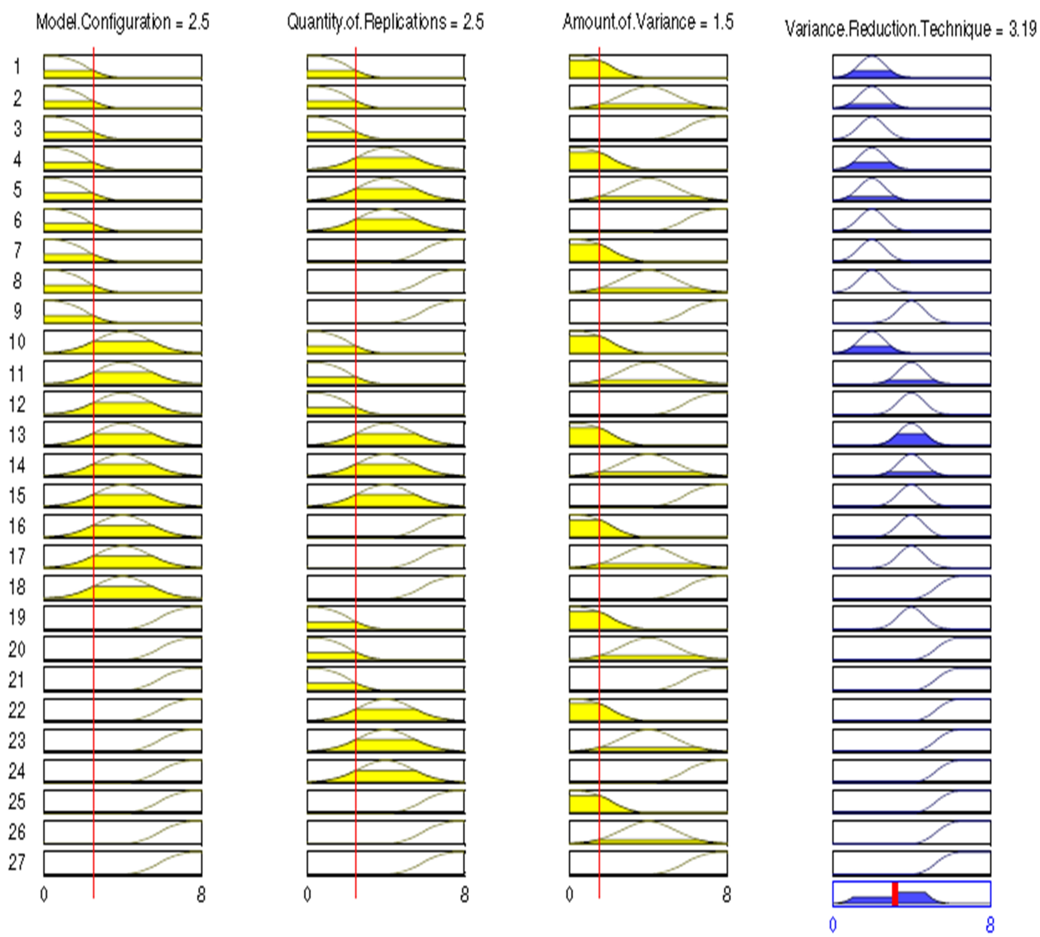

Figure 6.2

Evaluation of fuzzy rules for the revised fuzzy expert system

For the three - input fuzzy expert system, an input vector, [2.5, 2.5, 1.5], is entered for example. It will return a value of [3.19] in the rule viewer editor. From figure 6.2, the process with which the fuzzy expert system arrives at its conclusion is shown, where eight fuzzy rules have fired. Rule 13, "If (Model Configuration is Semi Automated) and (Quantity of Replications is Average) and (Amount of Variance is Fair Amount) then (Variance Reduction Technique is Antithetic Variates)" is shown as having the strongest influence on the single output fuzzy set. The linguistic interpretation of the value [3.19] is a suggestion to the user to select the Antithetic Variates technique.

The surface viewer which displays a three - dimensional curve that represents the mapping from input variables to output variable, displays a view of the output surface for the fuzzy expert system. Due to the fact that this curve represents a two - input variable one - output variable case, displaying results might be hindered by this constraint.

The solution to this problem is to "fix" one of the input variables so that the two - input variable one-output case, is relaxed to a three - input variable one - output case for analysis purposes. Figure 6.3 shows the output surface for the mapping of "Amount of Variance" and "Quantity of Replications" to "Variance Reduction" Technique, where "Amount of Variance" is the fixed input variable. Figure 6.4 shows the output surface for the mapping of "Amount of Variance" and "Amount of Replications" to "Variance Reduction" Technique, where "Amount of Variance" is the fixed input variable.

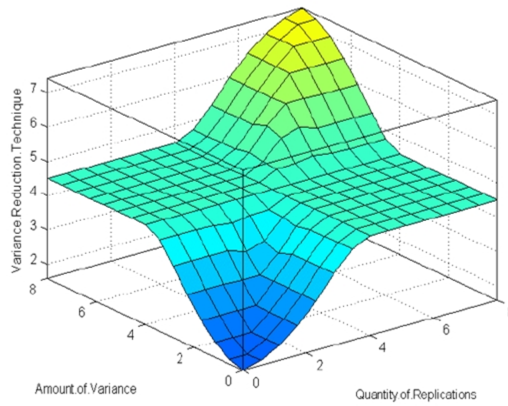

Figure 6.3

Surface plot 1 for the revised fuzzy expert system

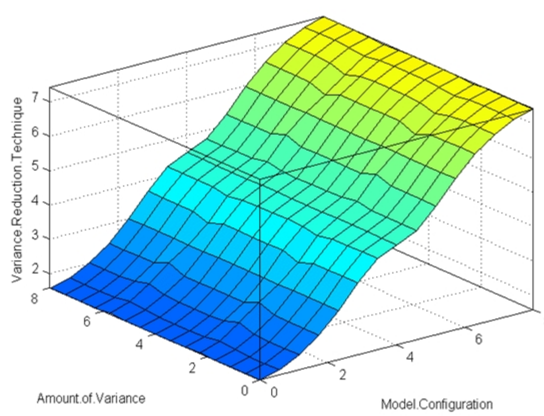

Figure 6.4

Surface plot 2 for the revised fuzzy expert system

This concludes the design and development of the revised fuzzy expert system. In the next section, the reasonableness of the results produced by the revised fuzzy expert system will be assessed through validation testing.

6.4 Revised Fuzzy Expert System Validation

In this section, the revised fuzzy expert system will be subjected to validation testing, mainly with the intention of establishing the reasonableness of the results that it produces. The purpose of the fuzzy expert system is to act as a decision support system for the selection of a variance reduction technique for discrete event simulation studies.

This test of validity will be from three perspectives. First of all, a test of user agreement on the reasonableness of results which the revised fuzzy expert system produces upon interrogation. This will be followed by an investigation into establishing the possibility that the fuzzy expert system makes a selection similar to those made through the use of manual selection experimentation. Finally, there will be a study into any differences in results when design choices such as the shape of the membership function changes and as well as the number of fuzzy rules are changed.

1. Test of user agreement.

   In chapter 3 section 3.2, one of the techniques mentioned for the validation of fuzzy expert systems is the use of statistical tests. The first validity test for the revised fuzzy expert system will be a test of agreement by potential users. In the field of non parametric statistics, Cohen's (1960) [28] kappa statistic (κ) has been used to quantify the level of agreement between two raters in placing persons, items, or other elements into two or more categories. Kappa is a chance - corrected measure of agreement between two raters, each of whom independently classifies each sample of subjects into one of a set of mutually exclusive and exhaustive categories.

Fleiss (1971) [41] extended the measure to include multiple raters, denoting it the "generalized kappa statistic". Fleiss's extension can be interpreted as a chance - corrected measure of agreement among three or more raters, each of whom independently classifies a sample of subjects into one of a set of mutually exclusive and exhaustive categories. The following have been proposed by Landis and Koch [76] as standards for interpreting the strength of agreement for the kappa κ coefficient:

- ≤ 0 poor,
- .01-.20 slight,
- .21-.40 fair,
- .41-.60 moderate,
- .61-.80 substantial, and
- .81-1 almost perfect.

Next, the hypothesis below is tested, using the Fleiss kappa statistic function [23] which has been implemented in Matlab^{TM}.

H_0: degree of agreement between raters which would be expected not by chance is $\kappa > 0 < 1$

H_1: degree of agreement between raters which would be expected by chance is $\kappa \leq 0$

The α value is 0.05.

Results from statistical testing show; Fleiss's (overall) kappa = -0.0495, kappa error = 0.038, kappa C.I. (95%) = [-0.0693, -0.0297]. The value for the z statistic = -1.2732 and P-value = 0.2030. From the above results, the P-value is greater than the α value and the kappa error (κ) value is less than zero, therefore"do not reject the null hypothesis", also the observed agreement by raters is "poor" and this is not by chance. In conclusion, there is a non agreement amongst raters on the "reasonableness" of the results produced by the revised fuzzy expert system, and the level of non agreement observed is not due to a coincidence of random sampling.

2. Validation of the revised fuzzy expert system's output.

   The aim of this validation test is to determine whether the fuzzy expert system can make the same selection of a variance reduction technique, as could be achieved with the use of manual experimentation (Chapter 4). All nine performance measures from chapter 4 of this thesis have been selected, and the variance reduction technique selected by the fuzzy expert system and those through manual experimentation are compared under the same conditions. In three out of nine cases, the fuzzy expert system agreed with results from manual experimentation as shown in table 6.2.

3. Sub-system validation.

   The purpose of the sub-system validation test is investigate the effect of changing design choices already made for the revised fuzzy expert system. The sub-system validation experiments has been performed from two perspectives. First, will a change in the number of rules have any effect on the revised fuzzy expert systems?. Second, will the results of the system change if the membership function shape change?.

   – Rules: For this experiment, under consideration will be a prototype

Simulation Model	Performance Measure	Fuzzy Expert System	Manual Selection
Manufacturing System	Average Total WIP	Antithetic Variates	Control Variates
	Entity Total Average Time	Antithetic Variates	Common Random Numbers, and Antithetic Variates
	Resource Utilisation	Antithetic Variates	Common Random Numbers
Crossdocking Distribution System	Total Entity Time	Antithetic Variates	Control Variates
	Total Resource Utilisation	Antithetic Variates	Antithetic Variates
	Total Resource Cost	Antithetic Variates	Antithetic Variates
Call Centre System	Total Resource Cost	Control Variates	Antithetic Variates
	Total Resource Utilisation	Control Variates	NO VRT
	Total Average Call Time	Control Variates	Control Variates

Table 6.2

Comparison of results between the fuzzy expert system and manual selection experiments fuzzy expert system with 9 rules and a revised fuzzy expert system with 27 rules. Twenty randomly generated test cases will be used to determine whether the results from both systems are the same or different (see appendix B.3 for the full list of test cases). The hypothesis being tested is:

H_0: Fuzzy expert system (27 Rule) mean of defuzzification results = Fuzzy expert system (9 Rule) mean of defuzzification results

H_1: Fuzzy expert system (27 Rule) mean of defuzzification results $\neq$ Fuzzy expert system (9 Rule) mean of defuzzification results

Statistical test: The statistical which will be used is the 2 Sample t test. This is because inferences are being made about the difference between two population means, based on data from two independent random samples.

The α value = 0.05

Results from table 6.3, report the difference between the means of defuzzification results for the Fuzzy expert system (27 Rules) and the Fuzzy expert system (9 Rules) as (T-Value = -0.86, P-Value = 0.395). The P value is greater than the α value, therefore "do not reject the null hypothesis", also the difference is "not statistically significant". For this particular fuzzy expert system described within this thesis, no remarkable difference exists in the use of either 27 rules or 9 rules.

- Membership function shape: This validation test will involve evaluating the effect of changing the shape of the membership functions. The following settings were used for designing the experiments:
  - Fuzzy Expert System (1): Gaussian,"Z" and "S" shape membership function and the centroid defuzzification method,

	Number of Cases	Mean	Standard Deviation	SE Mean
Fuzzy expert system (27 Rules)	20	4.44	1.45	0.32
Fuzzy expert system (9 Rules)	20	4.87	1.72	0.38

Table 6.3

2 Sample T-Test of difference: Fuzzy expert system (27 Rules) and Fuzzy expert system (9 Rules)

* Fuzzy Expert System (2): Triangular, "R" and "L" shape membership function and the centroid defuzzification method. The "R" and "L" shapes are special cases of the trapezoid membership function (See figure B.1, in Appendix B.2).

Table 6.4 shows details of the selected membership functions shapes for fuzzy expert system (1) and Table 6.5 shows the same for the fuzzy expert system (2). These are the two alternative fuzzy expert systems utilised for validation testing.

The hypothesis being tested is:

H_0: Fuzzy expert system (1) mean of defuzzification results = Fuzzy expert system (2) mean of defuzzification results

H_1: Fuzzy expert system (1) mean of defuzzification results $\neq$ Fuzzy expert system (2) mean of defuzzification results

Statistical test: The statistical which will be used is the 2 Sample t test. This is because inferences are being made about the difference between two population means, based on data from two independent random samples.

The α value = 0.05

Membership Functions(MF)		
Linguistic Terms	MF Shape	Universe of Discourse
Automated	S - Shape	[0,9]
Semi - Automated	Gaussian - Shape	[0,9]
Manual	Z - Shape	[0,9]
A Little	S - Shape	[0,9]
Average	Gaussian - Shape	[0,9]
A Lot	Z - Shape	[0,9]
Fair amount	S - Shape	[0,9]
Considerable Amount	Gaussian - Shape	[0,9]
Large Amount	Z - Shape	[0,9]
Common random number	S - Shape	[0,9]
Antithetic variates	Gaussian - Shape	[0,9]
Control variates	Z - Shape	[0,9]

Table 6.4

Membership function shapes for the fuzzy expert system (1)

Results from table 6.6, report the difference between means for Fuzzy expert system 1 and Fuzzy expert system 2 (T-Value = 0.04, P-Value = 0.966). The P value is greater than the α value, therefore “do not reject the null hypothesis”, also the difference is “not statistically significant”. A change in the membership function shape for the revised fuzzy expert system, did not result in a significant change in the crisp output results generated by the system.

Membership Functions(MF)		
Linguistic Terms	MF Shape	Universe of Discourse
Automated	L - Shape	[0,9]
Semi - Automated	Triangular - Shape	[0,9]
Manual	R - shape	[0,9]
A Little	L - Shape	[0,9]
Average	Triangular Shape	[0,9]
A Lot	R - Shape	[0,9]
Fair amount	L - Shape	[0,9]
Considerable Amount	Triangular - Shape	[0,9]
Large Amount	R - Shape	[0,9]
Common random number	L - Shape	[0,9]
Antithetic variates	Triangular - Shape	[0,9]
Control variates	R - Shape	[0,9]

Table 6.5

Membership function shapes for the fuzzy expert system (2)

	Number of Cases	Mean	Standard Deviation	SE Mean
Fuzzy expert system 1	20	4.36	1.32	0.30
Fuzzy expert system 2	20	4.34	1.81	0.40

Table 6.6

2 Sample T-Test of difference: Fuzzy expert system 1 and Fuzzy expert system 2

6.5 Chapter Discussion

This chapter has described the design and development of a revised fuzzy expert system, and most importantly the validation tests carried out on such a system. Below is a discussion regarding the outcome of validation tests performed on the revised fuzzy expert system.

– Inter - raters agreement: From the results presented in section 6.4 of this chapter, there was a non agreement between raters on the reasonableness of the crisp results produced by the revised fuzzy expert system. It is possible that the reason for this result could be the unavailability of an explanation function within the revised expert system. This explanatory function would have provided an interpretation of the rationale behind the decision making process of the fuzzy expert system.

  Another possible cause could be the perception of raters towards a decision support system for discrete event simulation studies. Participants involved in validation experimentation were all potential academic users of this system. Their immediate reaction to a proposed fuzzy expert system for the selection of a variance reduction technique was one of astonishment. In addition, O'Keefe [114] highlights this same issue in his paper, stating that a potential hindrance to expert system validation testing is a situation where users refuse to use systems that fail to consider human centric factors.

  In particular, potential users were of the idea that developing a system to assist with making this particular decision, assumes the "rigour" of the process of selecting a variance reduction technique has been dispensed with. However, it was pointed out that a lot of this rigour is captured as knowledge within the brains of experts. Thus elicitation and representation of this knowledge in a computer system will enable inexperienced users of variance reduction techniques, an opportunity to have such knowledge at hand. Furthermore, the application of a fuzzy expert system provides an alternative solution approach instead of known heuristics or algorithms.

- Comparison of variance reduction technique selection results: The results from the output comparison between the fuzzy expert system approach and the manual experimentation approach contradicted expectations. Here are some reasons which may have caused this:

  * Choice of Test Cases: In an ideal situation, test cases representing results from a number of experts on the problem of selecting a variance reduction technique would be available for use during validation testing of a fuzzy expert system. Unfortunately, test cases for this particular decision making problem do not exist, and the unavailability of experts and/or practitioners did not allow for the generation of a wide range of test cases.

    Furthermore, test cases were generated randomly by the designer of the fuzzy expert system, these set of random test cases may have biased the success of validation tests. This bias could have been introduced into the test cases through the limited skill and knowledge which the fuzzy expert system designer possesses in generating test cases. For a more inclusive fuzzy system validation, O'Keefe [114] suggests test cases should be randomly selected using stratified sampling, i.e. randomly selecting test cases within each identifiable result type.

  * Quality of Test Cases: The coverage of test cases used in validation testing affects the amount of confidence placed on a fuzzy expert sys-

tem [114]. This is because a representative mix of obscure and complex case studies which even experts would find difficult would ideally be adequate for validation testing. Unfortunately, the test cases used for validation cannot be guaranteed to be representative of an ideal mix that should have been used.

– Sub - system validation: Unlike the previous two validation tests, both sub - system validation tests to determine whether there is a difference in the mean of defuzzification results did not contradict known literature i.e. Cox [30], therefore it can be assumed that the revised fuzzy expert system is tolerant of estimates not only in the variance reduction technique selection problem space, but also in the representation of its corresponding fuzzy sets.

In a summary, it is well known that a research prototype with medium performance provides sufficient indication that a standard approach has been utilised in the design and development of an expert system and as such is regarded as acceptable [114]. The capability of the revised fuzzy expert system in aiding a simulation user in the selection of a variance reduction technique has been demonstrated. This system may not have performed convincingly during validation testing, but it demonstrates a logical framework for use by discrete event simulation users. In particular, those interested in designing and developing fuzzy expert systems as a decision support tool. Next is the concluding chapter of this thesis.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

Usually during a discrete event simulation study, there are a variety of decisions to be made at the pre and post experimentation stages. Such decisions include input analysis, design of experiments and output analysis. Our interest is in output analysis with particular focus on variance reduction techniques and how the selection of such techniques can be enhanced through the use of a decision support system. In addition, the performance of stand alone as well as a combined application of variance reduction techniques have been of interested.

Within chapter 4 of this thesis, the selection of a variance reduction technique using the pilot study approach recommended by Law and Kelton [80] was performed. While it was quite a cumbersome task, for nine performance measures, to perform such a large number of experiments; the pilot study generated a variance reduction selection with a good measure of statistically validity. The aim of this exhaustive investigation was to find out in which of the domains considered within this thesis, will the variance reduction techniques considered succeed. In particular, would a stand alone technique be more successful in reducing variance as compared with a combined application of techniques.

Among the stand alone variance reduction techniques, control variates, outperformed all other techniques. This is followed by antithetic variates and common random numbers. In addition, control variates was the only technique which achieved a reduction in variance for at least one performance measure of interest, in all three application domains. On the other hand, only on one occasion, did a combination of variance reduction techniques, i.e. common random numbers and antithetic variates produce the highest reduction in variance. Additionally, antithetic variates and control variates as stand alone application, demonstrated guaranteed variance reduction under existing experimental conditions for the crossdocking distribution centre simulation model, where this knowledge was previously unknown.

Apart from investigating the performance of variance reduction techniques, this thesis has endeavoured to investigate the possibility of designing, developing and performing validation tests on a fuzzy expert system for the selection of variance reduction techniques. From the results presented in chapter 6, section 6.4 of this chapter, there was a non agreement between raters on the reasonableness of the crisp results produced by the revised fuzzy expert system.

Results from evaluating the effect of using the revised fuzzy expert system to make a variance reduction selection, under the same conditions as those used for the manual selection experimentation (pilot study) were not as expected. The fuzzy expert system suggested the same variance reduction technique as the manual selection experimentation on just three occasions. As a result of comparing a 27 rule fuzzy expert system with a 9 rule fuzzy system, there is sufficient evidence to conclude that there does not exist a major difference between the mean of both systems defuzzification results, and a change in the membership function shape did not result in a corresponding change in the defuzzification results from the revised fuzzy expert system.

The fuzzy expert system reported in this thesis began as a research prototype and validation tests have not been intrinsically performed to quantitatively measure the systems performance, but simply as part of an overall evaluation to measure the systems value as a decision support system for the selection of variance reduction techniques within the discrete event simulation domain. The validation test results contradict expected results and expert opinion, but the design and development process provides a systematic framework for the application of fuzzy expert systems within the discrete event simulation domain.

It should be pointed out that the design and development of fuzzy expert systems is an iterative process. As stated, "Expert systems are built by progressive approximations with the programs mistakes leading to corrections or additions to the knowledge base." [88]. The step by step fuzzy expert system design and development paradigm used within this thesis can be adapted into many forms. Further inputs and outputs and alternative criterion can easily be added should users of discrete event simulation use such a system for decision support purposes.

Furthermore, much of the research carried out in this thesis focused on investigating the possibility of using a fuzzy expert system for the selection of variance reduction technique. The system did not answer this question to a satisfactory level, but it was successful in demonstrating its ability to be flexible enough to cope with changes in criteria agreed upon i.e. two - input, one - output and three - input, one - output modelling.

Within this thesis, the design and development of a fuzzy expert system which can aid a user in the selection of a variance reduction technique for discrete event simulation studies has been described. As far as it is known, there have been several attempts to apply expert systems to decision making for discrete event simulation i.e. Mellichamp and Park [100], Deslandres and Pierreval [31] as well as Moser [104]. However, this is the first time, a fuzzy logic based expert system is being applied as a decision support tool for the selection of a variance reduction technique for discrete event simulation studies.

The aim of this research has been to provide users of discrete discrete event simulation with a tool that can assist them in making decisions regarding the selection of a variance reduction technique. This has been in the form of a fuzzy expert system, which has the added advantage over a typical expert system by the incorporation of fuzzy logic into the expert system. The use of fuzzy logic has assisted in improving the precision of the knowledge elicited from experts which forms the foundation of the fuzzy expert systems' rule base.

The intention of this thesis has been to demonstrate the potential of a fuzzy expert system for this purpose, which in future can be applied to other statistical decisions involved in discrete event simulation studies i.e. design of experiment, input analysis etc. The ultimate goal will be to achieve an integration of a fuzzy expert system for making statistical decision into a discrete event simulation software. It is believed that only by incorporating a variance reduction advisory system into a general purpose discrete event simulation package will all of the potential benefits of variance reduction be fully realised.

7.2 Critical Review

Within this section, a discussion on the limitations of this thesis is presented.

- Determining the mathematical relationship between discrete event simulation variables: An observation which was made at the conclusion of variance reduction experimentation is, theoretically variance reduction can only be guaranteed for certain situations, with knowledge of the monotonic and/or non monotonic relationship between simulation input and output. Unfortunately, this was not considered during the manual selection of variance reduction technique investigation.

  As later discovered during the design and development of the fuzzy expert system, the relationship between variables has an influence on the selection of the fuzzy expert system variables and linguistic terms as well as the shape of the slope for the output surface. In addition, an understanding of the mathematical relationship between fuzzy input and output variables may have indicated which variables may have been redundant and as such debilitated the firing of its corresponding fuzzy rules.

- Generating Membership Function: One of the major disadvantages of a fuzzy expert system solution approach lies in defining the membership functions for each input and output variable which requires some form of expert knowledge. The efficiency or accuracy of the fuzzy expert system is then proportional to the designers expertise in the application domain. In cases where expert knowledge is unavailable, defining the membership functions is a major hurdle. Several methods have been proposed for automatic generation of membership functions. These methods for selecting membership functions include (i) Neural Networks, and (ii) Genetic Algorithms [12].

In spite of the fact that there are sophisticated methods of selecting membership functions as mentioned above, using the available knowledge about the problem of selecting a variance reduction techniques and then fine tuning the membership function as and when it is required was the preferred choice. Under current conditions, this starting point seemed ideal because it will not cause a hindrance through the complexity associated with the use of automated techniques and it will make it easier to perform validation testing. In the future, there is the possibility to study the design and implementation of membership functions for a fuzzy expert system using one such automated method earlier mentioned.

– Selecting Membership Function Shape: Usually, in the design of membership functions for fuzzy expert systems, its shape is defined in advance and then only their position on the universe of discourse is tuned. In her paper, Koprinkova [73] comments that the shape of the membership function influences the shape of the fuzzy expert systems output surface, and hence affects the performance of the selected inference method. Given a fuzzy expert system, how can the membership function shapes which will result in its best performance be determined, if membership functions of a certain shape e.g., Triangle, Trapezoid or Gaussian are a constrain?.

  In the literature, there is no standard technique for selecting membership function shapes, although available expert knowledge can be used for this purpose. Consequently its a case of trial and error till the most ideal shape that boosts performance is found. It is known that changes in the membership functions shape can improve dynamics of the fuzzy expert system under consideration, thus the membership functions shape parameters can serve as tuning parameters for the system [1]. As a result, a review of the sub - system validation tests will need to be performed under the guidance of experts to establish which are the most suitable shapes for this particular fuzzy expert system.

- Some potential features which would been incorporated into a fully functional fuzzy expert system:

  * Combined variance reduction techniques: The design and development of a fuzzy expert system to suggest not just a stand alone technique but a combined technique was considered during this research study, but not it was pursued. The main reason for excluding a combined technique as a selection choice for the revised fuzzy expert system was because the required expert participation and knowledge was unavailable. As a result, it would be unfair to expect the fuzzy expert system to make such a selection. However, the inclusion of combined variance reduction techniques within the output variable of the revised fuzzy expert system will be a consideration for the future.

  * Fuzzy expert system graphical user interface: A significant feature of the fuzzy expert system which would have increased its capability as a decision support tool, and improved its acceptability with potential users is an input and output graphical user interface. This user interface would define the manner in which the fuzzy expert system elicits user input requirements and capture the problem requirements of users as well as explain its conclusions accompanied with reasons for its suggested solutions which can be displayed in a textual form.

    The design and development of the fuzzy expert system presented in this thesis has evolved through a number of developmental iterations to reach its current state. Although the graphical user interface has not been implemented in this instance, there is enough evidence to be certain it is a design feature that is mandatory for the next edition of this system.

7.3 Summary of Contributions

Within this section, is a summary of key contributions to knowledge.

– Originality involves encountering an established idea or view point or method in one part of a discipline, and then taking the idea or view point or method for a walk and putting it down somewhere else, i.e. applying it in a different context or a different purpose. The **originality** of this research within the field of computer science and artificial intelligence, is taking the method "fuzzy expert system" and applying it in a different context and purpose, "as an advisory system for the selection of a variance reduction technique for discrete event simulation studies".

  The main original **contribution** of this thesis is in formulating and setting out the steps necessary to implement a fuzzy expert system for the selection of a variance reduction technique. These steps include; knowledge acquisition, design choices for the fuzzy expert system, as well as verification and validation testing. A logical approach to the design and development of the fuzzy expert system, while considering methodological issues and performing experimental evaluation has been presented in this thesis.

– The **contribution** to discrete event simulation knowledge is investigating the performance of variance reduction techniques as stand alone and combined application on three discrete event discrete event simulation models which are each based on different domains and the discovery under current experimental conditions, the effectiveness of antithetic variates and control variates for the purpose of variance reduction, for a class of systems i.e. a crossdocking distribution system, which was previously unknown.

7.4 Future Work

Traditionally, there are more problems to solve than there are experts to solve them. In the future, fuzzy expert systems are expected to significantly reduce simulation project overheads and generate more timely information, by reducing the extensive use of human analyst [140]. A fuzzy expert system provides the chance to deploy expert knowledge in a computerised setting for the selection of a variance reduction technique. In addition to the added benefit of representing ambiguity in human knowledge by modelling complex systems using linguistic variables and linguistic terms. Here is a summary of the future direction of this thesis.

- Combined application of common random numbers and control variates: In the literature, Kleijnen [68] described the combination of common random numbers and antithetic variates for a queuing system, and its effectiveness in that particular setting. These additional variance reductions by the combination of techniques may be important for certain practical applications. Variance reductions results also indicate that a combination of common random numbers and antithetic variates did achieve a reduction in variance. One of the objectives of manual selection experimentation is to establish the effectiveness of combining common random numbers and control variates, with a view to gauging its performance as applied to the domains considered within this thesis. In addition, on no occasion did the combination of common random numbers and control variates yield the largest variance reduction as expected.

It is interesting that theoretically, this combination ought to perform well because of the applied strategy of combination. This strategy is based on the principle that combining two techniques with different characteristics, should achieve a greater reduction in variance. However, it is not certain at this time the reason for this deviation from expectation but future research will investigate this deviation with a view to including the combined application of variance reduction techniques as a fuzzy expert system linguistic output variable.

- Fuzzy expert system graphical user interface: Within this thesis, the design and development framework for a fuzzy expert system has been presented. In addition, some understanding into the nature of decisions made during a typical discrete event simulation study has been gained. Future work will be to develop a fuzzy expert system from scratch (See Chapter 3, section 3.6.4) using a popular programming language such as Java. This will incorporate an input and output graphical user interface function. This system can also be used as a flexible template as well as a demonstration of a more practical implementation of a fuzzy expert system for this type of decision making problem.

- Extensively validate the performance of the fuzzy expert system (Test Cases): In an ideal world, one could design and develop a fuzzy expert system and test its performance against actual test cases with known predictable outcomes. In reality, this is not often possible because full implementation of such systems is usually not immediate. Although the test case approach to fuzzy expert system validation is theoretically desirable, a set of test cases based on expert opinion for the selection of variance reduction techniques does not exist to our knowledge. One might consider the generation of test cases for this purpose, with two assumptions in mind [134].

1. First, the set of test cases has to be both sufficiently accurate and of a good quality in precision and domain coverage.
2. Second, the fuzzy expert system may not provide feasible real - time recommendations or predictions at the early stages of development, since its improvement is iterative.

This does not reduce the effectiveness of using test cases as a means of validation, but its potency will be much appreciated when accompanied with a careful consideration for the generation of test cases, which will be one of the areas of focus in the future.

– Extensively validate the performance of the fuzzy expert system (Panel of Experts): On occasions, it is sometimes possible to perform fuzzy expert system validation testing against an autonomous panel of experts [114]. This panel is considered to provide well thought out recommendations, predictions, or diagnoses against which the outputs from the fuzzy expert system can be statistically compared. This is a fairly common technique in the field of artificial intelligence; however the panel of experts needed for such an assessment must not be connected to system development [134].

In future during the design stage, there should be consideration for two panels of experts. A panel dedicated to providing expert knowledge and another dedicated to the verification and validation testing of the fuzzy expert system.

- An Intelligence based Simulator Tool: One frequently noted limitation of discrete event simulation is the required knowledge and time devoted by modeller's in creating models, specifying alternative parametric and structural instances, designing the sampling experiment, and making sense out of the discrete event simulation output data. Any effort in the direction of assisting in an advisory capacity in regards to these tasks will enhance the utility of discrete event simulation. Ideally, all that should be expected of a discrete event simulation modeller should be knowledge about the system under study (its components, relationships, and boundaries), the goals of the study, and the degree of precision required in the output results.

  One possible solution can be the development of an "intelligence based simulator tool". It should be capable of providing the user with an automatically constructed model of the system, an easy to use validation option, an automatically performed analysis of the output, user - desired output statistics in readable formats, and the capability to advise on suitable techniques to generate improved results. A truly intelligence based simulator tool should minimise the task of the human modeller and should support him or her with capabilities similar to those of a team of expert human discrete event simulation specialists.

The future of discrete event simulation and fuzzy expert systems lies mainly in the integration of these methodologies for decision making. While there may not be a need to focus on improving existing means of inferencing, there is a need to harvest the vast knowledge of discrete event simulation experts into fuzzy expert systems for use by novice modellers and for the posterity of discrete event simulation research. This research has primarily investigated the application of a fuzzy expert system for the computerised selection of a variance reduction technique for discrete event discrete event simulation studies. In the future, there is scope to exploit the use of a fuzzy expert system in making decisions regarding input analysis, design of experiments and the optimisation of discrete event simulation models.

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Appendix A

Prototype Fuzzy Expert System

A.1 Rule Base

1. If (Model-Configuration is Automated) and (Variance is Fair-Amount) then (VRT is CRN) (1)
2. If (Model-Configuration is Automated) and (Variance is Considerable-Amount) then (VRT is CRN) (1)
3. If (Model-Configuration is Automated) and (Variance is Large-Amount) then (VRT is AV) (1)
4. If (Model-Configuration is Semi-Automated) and (Variance is Fair-Amount) then (VRT is CRN) (1)
5. If (Model-Configuration is Semi-Automated) and (Variance is Considerable-Amount) then (VRT is AV) (1)
6. If (Model-Configuration is Semi-Automated) and (Variance is Large-Amount) then (VRT is CV) (1)

7. If (Model-Configuration is Manual) and (Variance is Fair-Amount) then (VRT is AV) (1)
8. If (Model-Configuration is Manual) and (Variance is Considerable-Amount) then (VRT is CV) (1)
9. If (Model-Configuration is Manual) and (Variance is Large-Amount) then (VRT is CV) (1)

A.2 List of Extracted Variables

Assumptions

– Simulation modeller has enough time to implement variance reduction techniques.

– Simulation modeller has a reasonable knowledge of the variance reduction techniques under consideration.

– Simulation modeller is "ONLY" considering variance reduction techniques which involve:

  * the manipulation of random numbers,
  * the consideration of prior knowledge,
  * consideration of probabilistic estimations of events within the simulation model.

Input Variables

1. Categories of Discrete Event Simulation models [Stochastic Process, Query System]

2. Complexity of Discrete Event Simulation models [Simple, Moderate, Complex]
3. Nature of Discrete Event Simulation models [Terminating, Steady State]
4. Type of Discrete Event Simulation models runs [Multiple Replication, Long Run]
5. Starting Condition of Discrete Event Simulation models [Warm up, No Warm Up]
6. Terminating Condition of Discrete Event Simulation models [No. of Replication, Amount of Error]
7. Discrete Event Simulation model size [Simple, Moderate, Complex]
8. Nature of Input data [Normal, Non Normal]
9. Availability of Data [No Data, Some Data, All Data]
10. Modelling Inter Arrival Rate [Poisson, Exponential, Uniform, Others]
11. Modelling Processing Rate [Normal, Uniform, Triangular, Others]
12. Correlation between input and output variable[Unknown, Assumed, Known]
13. Can random number manipulation [None, Partial, Full]
14. Number of Alternative Scenario's [Single scenario, Multiple scenario]
15. Computational time [A little, Moderate, A lot]
16. Amount of Variance[Small, Reasonable, Large]
17. Configuration of Discrete Event Simulation model [Automated, Manual, Automated and Manual]

Output Variables

1. Variance reduction techniques [Unknown, Sequential Sampling, CRN, AV, CV, CRN+AV, CRN+CV]

A.3 Representing "APPROXIMATELY 4" using Gaussian and Triangular shapes

Figure A.1 shows the difference in representing "APPROXIMATELY 4" with Gaussian and Triangular shapes.

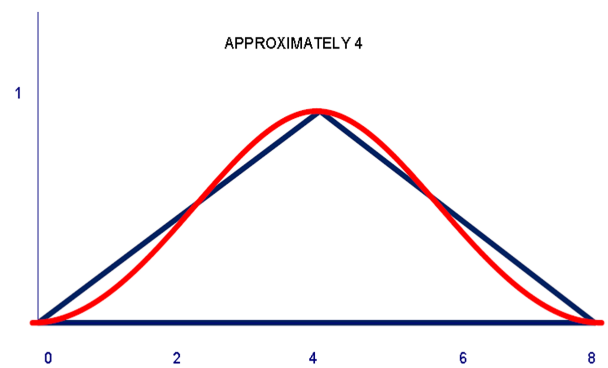

Figure A.1

The difference in representing "APPROXIMATELY 4" with Gaussian and Triangular shapes

Appendix B

Revised Fuzzy Expert System

B.1 Rule Base

1. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Little) and

   (Amount.of.Variance is Fair.Amount) then

   (Variance.Reduction.Technique is Common.Random.Numbers)(1)

2. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Little) and

   (Amount.of.Variance is Considerable.Amount) then

   (Variance.Reduction.Technique is Common.Random.Numbers)(1)

3. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Little) and

   (Amount.of.Variance is Large.Amount) then

   (Variance.Reduction.Technique is Common.Random.Numbers(1)

4. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is Average) and
   (Amount.of.Variance is Fair.Amount) then
   (Variance.Reduction.Technique is Common.Random.Numbers)(1)
5. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is Average) and
   (Amount.of.Variance is Considerable.Amount) then
   (Variance.Reduction.Technique is Common.Random.Numbers)(1)
6. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is Average) and
   (Amount.of.Variance is Large.Amount) then
   (Variance.Reduction.Technique is Common.Random.Numbers)(1)
7. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Lot) and
   (Amount.of.Variance is Fair.Amount) then
   (Variance.Reduction.Technique is Common.Random.Numbers)(1)
8. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Lot) and
   (Amount.of.Variance is Considerable.Amount) then
   (Variance.Reduction.Technique is Common.Random.Numbers)(1)
9. If (Model.Configuration is Fully.Automated) and (Quantity.of.Replications is A.Lot) and
   (Amount.of.Variance is Large.Amount) then
   (Variance.Reduction.Technique is Antithetic.Variates)(1)

10. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Little) and
    (Amount.of.Variance is Fair.Amount) then
    (Variance.Reduction.Technique is Common.Random.Numbers)(1)
11. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Little) and
    (Amount.of.Variance is Considerable.Amount) then
    (Variance.Reduction.Technique is Antithetic.Variates)(1)
12. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Little) and
    (Amount.of.Variance is Large.Amount) then
    (Variance.Reduction.Technique is Antithetic.Variates)(1)
13. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Fair.Amount) then
    (Variance.Reduction.Technique is Antithetic.Variates)(1)
14. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Considerable.Amount) then
    (Variance.Reduction.Technique is Antithetic.Variates)(1)
15. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Large.Amount) then
    (Variance.Reduction.Technique is Antithetic.Variates)(1)

16. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Lot) and

    (Amount.of.Variance is Fair.Amount) then

    (Variance.Reduction.Technique is Antithetic.Variates)(1)

17. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Lot) and

    (Amount.of.Variance is Considerable.Amount) then

    (Variance.Reduction.Technique is Antithetic.Variates)(1)

18. If (Model.Configuration is Semi.Automated) and (Quantity.of.Replications is A.Lot) and

    (Amount.of.Variance is Large.Amount) then

    (Variance.Reduction.Technique is Control.Variates)(1)

19. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Little) and

    (Amount.of.Variance is Fair.Amount) then

    (Variance.Reduction.Technique is Antithetic.Variates)(1)

20. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Little) and

    (Amount.of.Variance is Considerable.Amount) then

    (Variance.Reduction.Technique is Control.Variates)(1)

21. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Little) and

    (Amount.of.Variance is Large.Amount) then

    (Variance.Reduction.Technique is Control.Variates)(1)

22. If (Model.Configuration is Manual) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Fair.Amount) then
    (Variance.Reduction.Technique is Control.Variates)(1)

23. If (Model.Configuration is Manual) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Considerable.Amount) then
    (Variance.Reduction.Technique is Control.Variates)(1)

24. If (Model.Configuration is Manual) and (Quantity.of.Replications is Average) and
    (Amount.of.Variance is Large.Amount) then
    (Variance.Reduction.Technique is Control.Variates)(1)

25. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Lot) and
    (Amount.of.Variance is Fair.Amount) then
    (Variance.Reduction.Technique is Control.Variates)(1)

26. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Lot) and
    (Amount.of.Variance is Considerable.Amount) then
    (Variance.Reduction.Technique is Control.Variates)(1)

27. If (Model.Configuration is Manual) and (Quantity.of.Replications is A.Lot) and
    (Amount.of.Variance is Large.Amount) then
    (Variance.Reduction.Technique is Control.Variates(1)

B.2 "R" AND "L" shape membership function

Figure B.1 are two special cases of the trapezoid shape membership function, which are called the R-shape and the L-shape.

- R-shape with parameters: c = d = $+\infty$
- L-shape with parameters: a = b $-\infty$

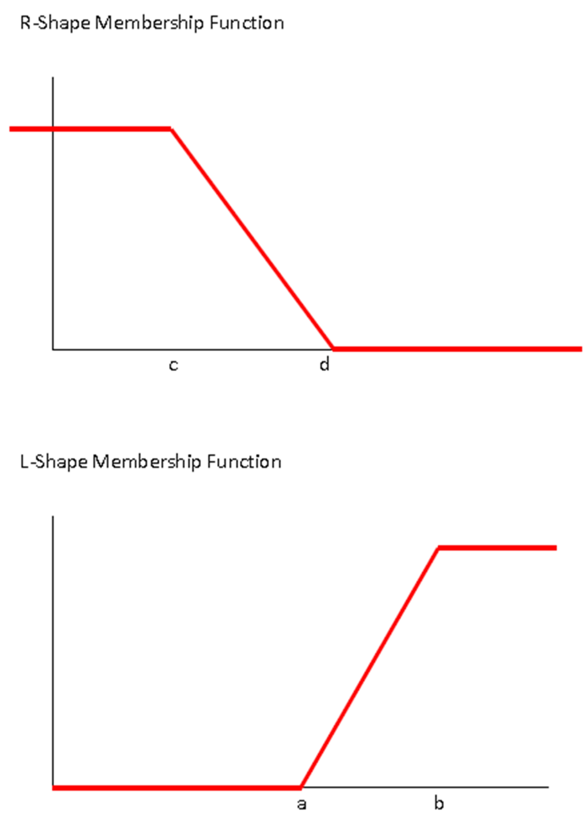

Figure B.1

"R" AND "L" shape membership function

B.3 List of Validation Test Cases

Test Case	Model Configuration	Quantity of Replications	Variance
1	1	4	5
2	2	6	3
3	8	5	7
4	3	2	4
5	6	5	2
6	7	4	1
7	6	3	7
8	4	8	3
9	2	1	6
10	7	5	8
11	3	4	6
12	3	6	5
13	1	5	6
14	4	2	2
15	6	5	3
16	5	4	5
17	3	3	8
18	6	8	4
19	5	1	2
20	8	5	2

Table B.1

List of Test Cases