The University of Nottingham
School of Economics

Theoretical and Experimental Approaches to Institutional Design: Applications to IPO Auctions and Weighted Voting Games

by

Ping Zhang, M. Sc.

Thesis submitted to the University of Nottingham

For the degree of Doctor of Philosophy

February 2006

Table of Contents

List of Tables

List of Figures

Abstract

Multiple solutions often exist in both non-cooperative and cooperative games. In this thesis we use game theoretical arguments and experiments to examine multiplicity in two different areas, namely uniform price auctions and weighted voting games.

In the second chapter we develop a theoretical model of IPO auctions and show that when demand is discrete the tacit collusion equilibrium is obtained under a stricter condition than in the continuous format. There also exists a continuum of equilibria where investors with a higher expected valuation bid more aggressively, and as a result the market price increases with market value. The tacit collusion equilibrium is in fact an extreme case of this set. Bertrand competition, i.e, submitting a flat demand function, does not form an equilibrium in this game.

We then test our equilibrium predictions and compare the performances of uniform price auctions with fixed price offerings using laboratory experiments. In the uniform treatment, there is no evidence that the tacit collusion equilibrium has been achieved. On the contrary, subjects with higher expected value bid more aggressively. Their behaviour is close to an equilibrium derived where all players participate. The resulting market prices are significantly higher than the market value of an investor with a low value signal. As a consequence, in our experiment uniform price auctions are superior to fixed price offerings in terms of revenue raising.

In chapter four we move to weighted voting games. Power indices predict that enlargement of the voting body may affect the balance of power between the original members even if their number of votes and the decision rule remain constant. Some of the existing voters may actually gain, a phenomenon known as the *paradox of new members*. We test for this effect using laboratory experiments. We find empirical support for the paradox of new members. Our results also allow an assessment of the predictive performance of standard power indices.

Acknowledgements

First of all, I am deeply indebted to my supervisors, Dr Klaus Abbink and Spiros Bougheas. This thesis would not have been finished without their invaluable advice and suggestions. I am really grateful for their encouragement throughout my PhD study, without it I doubt if I would have been able to carry on this work. It has been three years with much freedom and without much pressure. My supervisors pass their trust and confidence to me, which helps me to gain confidence in myself. I like our instructive meetings and straightforward communications, which I will definitely miss a lot. Their guidance at different stages of both my study and my life are highly appreciated.

The same gratitude is also extended to Martin Sefton, who is a wonderful advisor. From the collaboration with Martin and Maria Montero, I learned how an idea can be produced, and how much effort should be put in, from literature digging until submitting a paper for publication. I benefit a lot from the discussions with them and their professional spirit. Research is much more than producing papers. Also I thank those who have given useful comments on our joint paper, in particular to Simon Gächter, Doug Nelson, Alex Possajennikov, Eyal Winter, participants at the Spain-Italy-Netherlands Game Theory meeting, Maastricht 2005, and workshop participants at the Centre for Decision Research and Experimental Economics (CeDEx).

I wish to express my sincere appreciation to Maria Montero and Daniel Seidmann, who have given me valuable advice and comments on this work and help me to look for future research directions. They have also given me a lot of help when I was teaching the microeconomic theory tutorial.

I began to learn game theory from Richard Cornes and Maria Montero's course. I thank them for opening a door in front of me, which leads to an interesting world. I'm also grateful to Chris Starmer, Henrik Orzen, Saileshsingh Gunessee, Robin Cubitt, Baboo Nowbutsing and Reinhard Selten, for their help and suggestions at different stages of my work. I really appreciate my examiners Hans Normann and Martin

Sefton for giving me so many valuable comments.

A heartfelt thank you to my friends and colleagues, especially Xiaojing (Karen), Lars, Tarn, Pedro, Kun, John, Paulo, Sinan, Maria, Serafeim, Cecilia, Daniel, Wang Qiang and his family who have made the life in Nottingham much easier and more enjoyable. Also thanks to all the support and academic staff. It is my pleasure to meet these excellent people. I have learnt a lot from them, not only economics, but also their ways of thinking and the attitudes to life and to people.

Financial support by the Leverhulme Centre for Research on Globalisation and Economic Policy, the University of Nottingham, and the Nuffield Foundation is gratefully acknowledged. It would be impossible to finish my PhD programme and this work without the sponsoring institutions. Also thanks CeDEx, for providing me with wonderful research atmosphere and facilities for running experiments. Also thank the participants of my pilot experiments.

I really should thank my parents, my grandmother, my sister and little Xixi for their love and constant support, the kind of love and support that come without any conditions. Finally, I own a thank you to Kaiyi, for his care and trust, for his encouraging me to pursue what I want, even if it seems unrealistic or is not good for him. I thank him for giving me courage to change my life, as he lets me feel that even if we lose the world, we still have each other.

Chapter One:

Introduction and Overview

Chapter 1:

Introduction and Overview

This thesis is a collection of essays that use game theoretical and experimental approaches on two topics related to institutional design. The first topic, which is in the area of non-cooperative game theory, is about institutions for initial public offerings [1] (IPOs). We test the previous equilibrium predictions by laboratory experiments, and the results from the experiments motivate us to derive some new equilibrium solutions of uniform price auctions for IPOs. In the experiments we also compare the performances of uniform price auctions with fixed price offerings. We then shift our interest to some cooperative games, namely weighted voting games. Laboratory experiments are conducted to test how voting power changes among existing members if the voting body is enlarged or the voting weight or voting rule is changed.

1.1 Research Methods

The use of experimental methods is now well-established in economics. Experiments allow for control over the rules of interaction, the flow of information

[1] The new shares for public sale in the primary market are called initial public offerings. It is a type of a primary offering and differs from a secondary offering, which is the public sale of previously issued securities, usually held by insiders.

and the reward system that are not permitted with field data and thus have been widely conducted for testing, refining and evaluating economic theory, generating data that might influence a specific decision, and studying new institutions for policy makers before introducing them in the field. Besides testing and refining theories about the behaviour of a single individual or small groups, economic experiments may also be used for questions about large markets and institutions (Kagel and Roth, 1995).

As Camerer (2003) points out, data are particularly important for game theory because multiple equilibria often exist, and how equilibration occurs (how subjects really behave) is not perfectly understood. Pure mathematics alone will not solve these problems. Compared to happenstance data (a by-product of ongoing uncontrolled processes), data from laboratory experiments have the advantages of replicability and control. Other researchers can verify the findings from an experiment by reproducing the experiment. On the contrary, happenstance data, which are generated from an environment where unobserved factors are constantly changing, suffer from lack of replicability and control. Thus laboratory data are especially valuable for scientific purposes because they are relatively easy to interpret due to the ability to control for other variables, and are relatively easy to be verified because of their replicability (Davis and Holt, 1993; Friedman and Sunder, 1994). Crawford (1997, page 207) summarizes the particular usefulness of experiments for testing of game theory as follows:

> Behaviour in games is notoriously sensitive to details of the environment, so that strategic models carry a heavy informational

> burden, which is often compounded in the field by an inability to observe all relevant variables. Important advances in experimental technique over the past three decades allow a control that often gives experiments a decisive advantage in identifying the relationship between behaviour and environment... For many questions, [experimental data are] the most important source of empirical information we have, and [they are] unlikely to be less reliable than casual empiricism or introspection.

When alternative solution concepts make different predictions about the effects of institutional changes, empirical evidence is particularly valuable. Of course, though laboratory data have advantages, field tests are also important in testing game theory. As Crawford (1997, page 208) argues:

> ... The experimental evidence suggests that none of the leading theoretical frameworks for analyzing games -- traditional non-cooperative game theory, cooperative game theory... -- gives a fully reliable account of behaviour by itself, but that most behaviour can be understood in terms of a synthesis of ideas from those frameworks, combined with empirical knowledge in proportions that depend in predictable ways on the environment.

It is important that we are careful in designing experiments. The laboratory environment is relative simple compared with the real world, but institutions captured in experiments should not be over simplified. One should always bear in mind the condition of parallelism when design an experiment so that the results of the experiment will carry over to the corresponding nonexperimental setting (Smith, 1982).

Given the difficulties with using field data for comparing the performance of different institutions, we propose laboratory experimentation as an alternative tool to address this issue.

1.2 Background and Motivation

1.2.1 IPO Auctions

In 1975, Ibbotson first documented the large underpricing of IPOs. He found that new shares were issued at a discount[2] that disappeared within weeks in the aftermarket. Although IPO mechanisms vary significantly across countries,[3] and their revenue and information generating abilities are different as well, underpricing is a worldwide phenomenon across markets. There is a lot of theoretical and empirical research conducted to analyse the reason for underpricing or to capture the differences of performances among various IPO mechanisms. Different reasons for the underpricing in IPOs have been offered[4] and the list is still growing. However, whatever the reason is, there is a minimum level of undercutting for achieving the purpose of underpricing. If there is a method through which this minimum level can be guaranteed, this method could be seen as a better IPO mechanism for raising revenues. Although there are some empirical papers that try to compare the outcomes of IPO mechanisms (e.g., Derrien and Womack, 2001; Kutsuna and Smith, 2001), it is very difficult to find an environment where they are compatible and comparable in a long enough period. Moreover, the results are very sensitive to both the chosen sample and the

[2] By his test, the average discount is 11.4%. Ibbotson and Jaffe (1975) also found a 16.8% average excess return relative to the market. During 1980 to 2001, the 'aggregate money' left on the table in U.S. was more than 100 billion dollars, which is nearly a quarter of the aggregate gross proceeds that the issuers raised. (Ritter and Welch, 2002).

[3] See Loughran et al. (1994) and Sherman (2002) for detailed international information.

[4] Examples include the winner's curse (Rock, 1986), signalling hypothesis (Chemmanur 1993; Grinblatt and Hwang, 1989; Welch 1989), market feedback hypothesis (Benveniste and Spindt, 1989; Sherman , 2002), boosting the aftermarket price by underwriter's analyst coverage (Michaely and Womack, 1999), etc.

econometric methods.

An environment that can be controlled efficiently would imply that the effect of noisy factors can be reduced considerably and would be ideal for this task. This was our initial motivation of using laboratory experiments for IPO analysis. Biais and Faugeron-Crouzet (2002) compare the operation of four main IPO mechanisms in a unified theoretical model; namely Bookbuilding, uniform price auction, fixed price offering and *Mise en Vente* (which will be introduced shortly). By analysing price discovery, information elicitation and strategic issues, they demonstrate the different natures of these methods and predict the equilibrium strategies for each mechanism. Their paper thus provides us a framework for the experimental analysis.

Among the four mechanisms provided in the paper, fixed price offerings and uniform price auctions are also used in other markets besides the market for IPOs. Fixed price offerings (also called public offers) probably are the simplest formats among all the IPO methods. They are the main format used in countries that rely on retail investors[5] and are widely used in conjunction with other formats. In a fixed price offering, the market price is predetermined before the public offering of shares, and investors indicate the amount they would like to buy at that price. Rationing rules (random or pro rata) are used in the case of oversubscription.

In uniform price auctions,[6] prior to the IPO, the issuer firm and the underwriter set the quantity to be offered and the reservation price. On the day of the IPO, investors

[5] Countries that use fixed price offerings include India, Mexico, Singapore, etc.
[6] Countries that use uniform price auctions include Israel, France, UK, etc. Two internet based companies OpenIPO and eCapital use uniform price auctions as well (www.openipo.com. Ord Minnett's eCapital (Australia): www.patersonord.com.au).

submit quantity-price combinations that indicate the price that they are willing to pay for obtaining the corresponding quantities. Multiple bids are allowed. The market prices are determined as the market-clearing prices according to the accumulative quantity-price schedule. All bids exceeding the market-clearing price are accepted and bidders pay the market-clearing price for all units won. The price and allocation rules will be introduced in detail in chapter two.

Fixed price offerings and uniform price auctions are both straightforward processes and the model in the paper does not change either the price or the allocation rules of the two mechanisms. The models for Bookbuilding and Mise en Vente, however, seem oversimplified compared with reality. In the real Bookbuilding process, an issuer provides prospectus that contain detailed information about the issuing company[7]. After the potential investors have indicated their interest[8] the underwriter sets the IPO price and allocates the shares based on these indications. In the Biais and Faugeron-Crouzet's model, the underwriter is assumed to act in the best interest of the issuer, and this process is modelled by a direct mechanism where the underwriter allocates shares to investors who indicate interest to buy[9]. Truth telling is a dominant strategy for all investors. However, in reality the underwriter's choice of the level of underpricing is a balancing act that takes into account of both issuers and investors' benefit (Beatty and Ritter, 1986). Many researchers argue that underwriters tend to over underprice and favour institutional investors in order to maintain their long-term

[7] Such as the company's management, as well as its financial statements.
[8] This information is usually generated during the road show, which is a tour taken by the firm preparing for an IPO in order to attract interest in the deal. In a road show, the firm managers meet prospective investors and show off their plan, and the firm's business prospects can be talked about quite openly. Only institutional investors, analysts and money managers are invited (Hoover's Online - IPO Central).
[9] By reporting a high signal to the underwriter.

relationship, so that regular investors may agree to buy shares in either a hot or a cold market[10], which avoids the possibility of undersubscription[11] thus guaranteeing the success of IPO. The direct mechanism used in the paper does not take these facts into account. So although it is still an interesting game to test if investors really truthfully report the signals that they have observed in the direct mechanism as the equilibrium predicts, the results may not have effective validity for real markets outside the laboratory.

Mise en Vente is a modified uniform price auction used in the French IPO market. Instead of using market-clearing prices, in *Mise en Vente* offering prices are determined by SBF[12] according to the aggregated demand functions computed from bids submitted by all investors. Prices are set below market-clearing prices but there is no formal explicit algorithm mapping demand into prices. Orders above the IPO price obtain pro rata execution. Biais and Faugeron-Crouzet set a price rule[13] when modelling the Mise en Vente mechanism. By doing so the process is transformed into a direct mechanism that is equivalent to their model for Bookbuilding. However, as the price rule they set is rather artificial and there is little evidence that it is similar to the implicit price rule used in the real *Mise en Vente* process, we are unable to meet the parallelism condition in evaluating the performance of *Mise en Vente*. Nevertheless this model is theoretical interesting and may shed some lights on how to improve the design of IPO institutions, which we will discuss in the following

[10] A cold market means that when offered shares start trading in the stock market, the extent of price increase is small or even negative thus investors obtain low abnormal returns. A hot market takes the opposite meaning.
[11] The situation in which investors' total demand is less than the shares that issuers supply.
[12] SBF (Société des Bourses Françaises), the bourse official, is the market authority of France who plays a pivotal role in guaranteeing the fairness of the IPO procedures.
[13] See page 22 in Biais and Faugeron-Crouzet (2002).

chapters.

We thus decide to compare the performances of the simple, well known and widely used IPO methods: namely uniform price auctions and fixed price offerings. We hope to capture some characteristics of these different methods, which may be helpful in improving the current mechanisms and designing other better institutions. And our results together with the previous empirical studies can provide useful reference for policy makers in choosing different institutions.

Another issue that we are interested in is the subjects' behaviour in the experiment especially in the uniform price auctions. Biais and Faugeron-Grouzet (2002) provide a tacit collusion equilibrium solution where the market price is as low as the reservation price. In equilibrium, the demand function of each investor is steep and identical regardless of their expected value of shares. According to this equilibrium, under the unified environment, reservation prices can be set at most at a level that equals the lowest expected value among different types of investors. They then claim that underpricing in uniform price auctions is significant due to the possibility of tacit collusion. However, the tacit collusion equilibrium does not receive much experimental or empirical support. It is possible that other equilibria exist because of strategic complementarities, though investors cannot be better off than under the tacit collusion equilibrium. In addition, the uniform price auction also has advantages such as is easy to understand and conduct, is fair in the sense that the same price is paid by winning bidders, is an even-handed allocation (it favours small bidders relative to other mechanisms such as Bookbuilding), and most importantly, can increase market competition (Friedman, 1961,1990). A more democratic allocation of shares can be achieved in uniform price auctions that allocate the stock more anonymously. Thus it is interesting to test if the tacit collusion strategy is really played by subjects in our experiments. If the answer is no, uniform price auctions may be superior to some other methods because of the advantages.

1.2.2 Voting Games

We then apply the experimental approach on weighted voting games.

Many important decisions are taken according to "weighted voting rules". According to these rules, each member is assigned a fixed weight and the proposal is implemented if and only if the sum of weights of the members in favor of the proposal equals or exceeds some fixed real number, the quota (Ben-Yashar and Milchtaich, 2005). Examples include the Electoral College in the United States, the International Monetary Fund, and the Council of Ministers in the European Union. It is well known that the voting power of a state may be very imperfectly correlated with its voting weight (see e.g. Banzhaf, 1965). For example, a state controlling 51% of the votes has all the power under simple majority voting. The imperfect correlation between weights and power makes institutional design more complex. In order to analyze voting power a number of power indices have been proposed, the most important of which are the Shapley (1953) value or Shapley-Shubik (1964) index, and the Banzhaf (1965) index. These indices often predict unintended consequences of institutional change. A particular instance of counterintuitive effects of institutional change is the paradox of new members (Brams and Affuso, 1976). Brams and Affuso show that when a new member is added to a voting body the power indices of some original members may increase, even if the original members' weights and the decision rule remain unchanged. The reason is that the addition of the new member may present some of the existing members but not others with greater opportunities to form winning coalitions.

However, it is difficult to evaluate voting theories using empirical evidence. The reason is that in reality though one can observe how decisions have changed after changes in voting weights or rules or the size of voting body, there are always other variables that also have changed, or contribute to the final decisions. Hence unless changes in other variables can be controlled, one cannot distinguish the effect from the changes of voting weights or rules or size of voting body on their own. Hence whether the enlargements have actually increased an original member state's voting power is difficult to ascertain. A natural empirical measure of voting power is the proportion of the benefits from legislation that accrue to a voter, but in order to use this one would need information on the material consequences of enlargement of the voting body. As changes in the membership of the voting body are usually accompanied by other changes in the legislative process, it may be difficult to disentangle the effect of enlargement from changes in these other factors. Moreover, the material consequences of membership may be difficult to measure, and also be influenced by numerous other factors. This situation motivates us to apply experimental methods to examine the relation between the distribution of voting weights and voting power.

1.3 Overview

In chapter two, we use a game theoretical method to analyze the equilibria in uniform price auctions. In designing and programming the experiment, we took into account that the tacit collusion equilibrium solution of uniform price auctions derived under a continuous demand function does not apply in the discrete case. As the tacit collusion equilibrium was the only existent theoretical prediction at the time of designing the experiment, we started theoretical work to look for other possible equilibrium solutions for uniform price auctions in discrete format. We found that with discrete demand the tacit collusion equilibrium exists under a stricter condition than in the continuous format. We also found a set of a continuum of equilibria where only investors who have higher expected value of shares participate (if there is at least one such investor). According to this equilibrium solution, the market price positively relates with market value, and in the upper bound of this equilibrium set, the market price equals market value. This equilibrium and the tacit collusion equilibrium seem to be two extreme cases of a broader equilibrium set. We conjectured that there exist solutions in between, which may be more likely to be played in a real game. After conducting the experiments, we found little support for the tacit collusion equilibrium from the behaviours of the subjects. On the contrary, the shape of the aggregated demand curves is similar to the new equilibrium we found but less aggressive. We then went back to the theoretical work and proved the existence of the equilibria we conjectured, a continuum of equilibria where investors with higher expected valuation bid more aggressively, and as a result the market price increases with market value. Investors of different types can be involved in different subsets of this equilibrium. The tacit collusion equilibrium is in fact only an extreme case of this set. The resulting market price in general reflects the market value, and can be any price between the reservation price and the price that equals the market value. We have also shown that unlike in other discrete uniform price auctions, Bertrand competition, i.e. submitting a flat demand function, is not an equilibrium of this game. Previous theoretical arguments, empirical and experimental evidences are also introduced in this chapter.

In chapter three, we report the experiment that compares the performances of uniform price auctions and fixed price offerings. In our experiment the average market price in uniform price auctions is well above the expected value of an investor with a low signal. The price in fixed price offerings has to be set below that level in order to guarantee full subscription. Thus uniform price auctions are superior to fixed price offerings in terms of revenue raising. We also take a closer look at subjects' behaviour and try to capture some insights on their strategies. In the uniform price treatment investors adjust their bids according to the signals they receive during the experiment. They bid more aggressively when receiving high signals thus having higher expected market values. Hence there is no evidence that the tacit collusion equilibrium has been achieved. Among the broad equilibrium set we have derived in chapter two, the equilibrium where investors of all types participate fits the data the best. As a result the resulting market price increases with market value, compared with the fixed price that is nonadjustable with market value. Uniform price auctions are also superior to fixed price offerings in terms of information elicitation.

In chapter four, we examine weighted voting games. Power indices predict that enlargement of the voting body may affect the balance of power between the original members even if their number of votes and the decision rule remain constant. Some of the existing voters may actually gain, a phenomenon known as the *paradox of new members*. We test for this effect using laboratory experiments. In our experiment participants propose and vote on how to distribute a fixed budget among themselves. This approach allows us to use a natural empirical measure of voting power -- the average share of the budget realized by a voter -- and relate it to voting weight, decision rule, and the composition of the voting body. In particular we study three treatments, corresponding to the examples in Brams and Affuso (1976), and examine how the balance of power between the original parties is affected by the addition of a new member. We find significant discrepancies between empirical measures of voting power and standard power indices. However, in a comparative static sense the theoretical predictions of Brams and Affuso are borne out. In particular, the empirical voting power of an existing member increases with the addition of a new member, thus the paradox of new members is observed.

Finally, in chapter five we summarize our results and point out the limitations and suggest future research direction.

Chapter Two:

Equilibrium analysis in Uniform Price IPO Auctions

Chapter 2:

Equilibrium analysis in

Uniform Price IPO Auctions

2.1 Introduction

Uniform price auctions are widely used for selling multiple units of identical goods to many buyers. Examples include Treasury bills, spectrum, electricity and initial public offerings (IPOs). In uniform price auctions, each bidder submits quantity-price combinations that indicate the price that she is willing to pay for obtaining the corresponding quantities. The market-clearing price is determined according to the accumulative quantity-price schedule; all bids exceeding the market-clearing price are accepted and bidders pay the market-clearing price for all units won. A lot of research has been conducted to analyse equilibrium behaviour in uniform price auctions, and to compare the performances of the uniform price auction with other mechanisms, such as the discriminatory auction[1], the Vickrey auction, its open format multiunit English auction, and other IPO mechanisms[2]. The studies indicate both advantages and disadvantages from the use of uniform price auctions.

[1] In discriminatory auctions each bidder submits demand schedules indicating various demands at various prices. Goods are allocated from high to low bids and winning bids are filled at the bid price. It is also called pay-as-you-bid auction.

[2]For comparisons of uniform price auction with discriminatory auction, see e.g. Goswami, Noe and Rebello (1996) and Ozcan (2004a); with Vickrey auction, see e.g. Ausubel and Cramton (1998) for the theoretical predictions and Kagel and Levin(2001), Engelbrecht-Wiggans, and Reiley (Forthcoming) for the experimental evidence; with English auction, see e.g. Ausubel and Cramton (2004), Kagel and Levin (1997) and Alsemgeest, Noussair and Olson(1998); with an Bookbuilding, see e.g. Sherman(2002); for comparisons among several mechanisms, see e.g. Engelmann and Grimm(2003) and Kagel and Levin(2001). This chapter concentrates on the properties of uniform price auction and thus only the relevant results from these comparisons will be mentioned.

The criticism levelled at uniform price auctions focuses on two points. The first is that market prices may be set too high because of either the Winner's Curse (Klemperer, 2004) or information free rider problems (Sherman, 2002). The second is that market prices may be set too low because there exists a tacit collusion equilibrium in the corresponding non-cooperative game, in which bidders can strategically manage to set the market price below the competitive price resulting in low revenues for auctioneers (Back and Zender, 1993; Wilson, 1979; etc.). As revenue maximization is one of the important criteria of mechanism evaluation, this property makes the uniform price auction unfavourable to many economists[3]. However, the uniform price auction also has advantages such as: is easy to understand and conduct, is fair in the sense that the same price is paid by winning bidders, results in an even-handed allocation (it favours small bidders relative to other mechanisms such as Bookbuilding[4]), and most importantly, it can increase market competition (Friedman, 1961,1990). Compared with the high commission fee of Bookbuilding process, the cost of using uniform price auctions for IPOs is lower.[5] Rather than leaving decisions about who would get shares in an IPO to Wall Street investment houses, a more democratic allocation of shares can be achieved in uniform price auctions that allocate the stock more anonymously.

The empirical and experimental results also diverge. Experimental researches report demand reduction under both the two-unit demand with private values setting (e.g., Engelmann and Grimm, 2003; Kagel and Levin, 2000) and the large demand

[3] Another criterion is efficiency, which means the goods are allocated to the bidder who values them the most. Because auctions for IPO are common value auction, shares for sale have the same value for all the investors so efficiency is not an issue. Hence we do not discuss this criterion throughout this chapter.

[4] Bookbuilding is the most widely use IPO process, in which underwriters help companies who intend to raise money establish how many shares the market is interested in taking at what prices. The information is usually generated from the interest indicated through companies' roadshow, presentations by companies to potential investors and analysts.

[5] The Bookbuilding underwriting commission was 7 percent for more than 90 percent of US IPO (Chen and Ritter 2000; see Ausubel, 2002, p. 314). The US WR Hambrecht + Co. charges 3 - 5 percent for its online uniform price auction IPO.

common value setting (see chapter three). Collusive behaviour is hard to find in these experiments. However, when communication between bidders is introduced, subjects are able to reach the collusive equilibrium (Goswami, Noe and Rebello, 1996). In the markets for Treasury bill, compared with discriminatory auctions, uniform price auctions generate more revenue in some markets such as the US (Malvey, Archibald and Flynn, 1997) and the Mexican (Umlauf, 1993) but less revenue in the others such as the Turkish (Ozcan, 2004a). Though collusive behaviour is observed among large dealers, they are less severe than under a discriminatory setting. And in all the markets, higher participation rates are reported, which indicates that the market is widen and competition is encouraged. This offsets the influence on revenue from large investors' demand reduction. In markets for IPO, uniform price auctions have tended to yield smaller underpricing thus higher revenue than Bookbuilding (Ausubel, 2002). The main theoretical arguments together with the empirical and the experimental evidences will be discussed in detail in Section 2.2.

The divergences in performance both in real markets and experiments, and the greater volatility reported in uniform price auctions suggest that there may exist other equilibria in uniform price auctions besides the tacit collusion equilibrium. If that is the case, whether this auction is a good choice from the perspective of the auctioneer depends on the equilibria most likely to obtain under the uniform price mechanism. In this chapter, we find a set of equilibria in which the tacit collusion equilibrium is only an extreme case of the whole equilibrium set. We have shown that though demand reduction is inevitable in multiunit demand uniform price auctions, there still exist equilibria where the market price is positively related with market value; and in another extreme case, the market price can be equal to market value.

Another important issue that needs to be addressed is that previous studies on share auctions have assumed that the demand functions are continuous. While in markets for IPO increments in aggregate demand schedules of all bidders' bids are negligibly small because of a large number of participants, nevertheless it is not the same thing as a mathematical continuum. And because investors rarely submit full demand functions[6], instead, they submit a limited number of quantity-demand combinations as their bids, individual investor' demand function are in fact discrete. An equilibrium that holds in the continuous case does not necessarily work in the same way with discrete bids. It has been argued that the severe underpricing in uniform price auction predicted due to the tacit collusion equilibrium can be reduced or even eliminated with discrete settings (Kremer and Nyborg, 2004). The analysis of auctions with discrete (i.e. step) bid functions differs from that for auctions with continuous demand schedules. Hence, in this chapter, we generate the tacit collusion equilibrium and pay more attention to the equilibrium analysis for the discrete case.

In general, we show that any nonnegative price below market value (with zero reservation price) can be an equilibrium price in uniform price auctions for IPO. The set of equilibria is characterized by both Winner's Curse consideration (in the sense that the bidders who have higher expected valuations bid more aggressively) and strategic aspects (in the sense that bidders bid strategically by demand reduction).

The rest of the chapter is organized as follows. In section 2.2, we discuss previous theoretical arguments, empirical and experimental evidences. Then in section 2.3 we introduce the model on which our analysis is based on. In section 2.4, the necessary and joint sufficient conditions for the existence of an equilibrium are developed. After

[6] Either because submitting full-demand schedules is costly (in preparation or submission), or because there exists minimum increment requirement in prices. For example, the online IPO auction company WR Hambrecht+ Co. used to require at least 1/32 of a dollar in increments in bids, which has been changed to 1 cent in 2005. The traditional minimum bid quantity is 100 shares. Some other markets such as electricity auctions are discrete rather

analysing the flat demand function and tacit collusion equilibrium, we demonstrate the existence and characteristics of a new equilibrium set starting from the more specific and ending with the more general. The equilibria differ according to the type of investors that participate. We sum up our results and conclude in section 2.5.

2.2 Related literature

In markets for IPO, the real value of shares is the same for all the investors though they do not know the value at the time they place their bids. If bidders place bids according to their expected market value, it would be the bidders who are the most optimistic investors win the shares. So winning may be associated with losses when the market price exceeds the realized market value. This phenomenon is known as the Winner's Curse which frequently happens in common value auctions. In markets for IPO, however, though inexperienced investors may fall in the trap of the Winner's Curse, its likelihood can be reduced by formal warnings of the danger before the offerings; like what Google has done in its IPO. And because experienced investors know how to avoid a loss by shading their bids, the resulting market price may be even lower than the one under mechanisms where market prices are artificially set below market clearing prices and rationing is present (Bulow and Klemperer, 2002).[7]

Overpricing may also happen because of the information free rider problem. In uniform price auctions, since all the successful bids are paid at the market-clearing price, it is costless for investors to submit bids well ahead their expected values unless their bids are marginal. With a large number of bidders, the chance of a bid to be

than continuous auctions for perfectly divisible goods. For example, in the Spanish electricity market generators may submit up to twenty-five price-quantity pairs (Harbord, Fabra and von der Fehr, 2002).

[7] In this chapter and chapter three, the term "Winner's Curse" means that bidders with higher expected market value bid more aggressively, and as a result, the uninformed investors obtain the highest allocation when the

marginal is low especially for small bidders. So investors may intend to bid at high prices in order to get some allocation with certainty. These bidders are regarded as information free riders as their bids do not reveal any information about market value. Such bids are not only uninformative for valuation revelation, but might push the price of shares to unsustainable high levels. If this was to happen, speculators and professional investors might choose to short the stock, an action that would burst the balloon and quickly devalue the price of shares. This action can reduce market confidence and even result in the collapse of the market. In auctions for IPOs, over-enthusiastic bidding may happen after several new stocks have generated large profits for the investors who obtain shares in IPOs and successfully sell them when shares debut in the stock market. The breakdown of the Argentina stock market in the 1990's is one such example[8] (Sherman, 2002).

Some other properties of uniform price auctions are related to the analysis of bidders' equilibrium strategies. Vickrey (1961) first demonstrates that in single unit second-price auctions where bidders have independent private values, it is a weakly dominant strategy for each bidder to reveal her maximum willingness to pay for the good. In uniform price auctions, if there are m units of goods for sale, the market price is the m+1th highest bid price[9]. When there is only one unit for sale (m=1), the uniform-price auction reduces to a second-price sealed-bid auction. It thus seems that the multiunit uniform-price auction is a nature extension of the second-price auction. Some academics and policymakers believe that uniform price auctions inherit the attractive truth-telling property of second-price auctions. Friedman (1960; 1991), for example, suggests that the US Treasury use uniform price auctions to substitute

market value is the lowest. However, bidders do not necessarily suffer losses when the Winner's Curse occurs (Rock, 1986; Wang and Zender, 2002).

[8] When the shares started trading in the stock market, the price dropped dramatically which destroyed investors' confidence and resulted in dumping of shares (Sherman, 2002).

discriminatory auctions. He claims that in uniform price auctions bidders "need only know the maximum amount [they] are willing to pay for different quantities"(Friedman, 1991), so bidders do not have incentive to shade bids strategically. As a result, the collusive behaviour in the Treasury bill market under discriminatory auctions can be overcome and revenues for Treasury can be increased.

However, although the truth-telling property of second-price sealed-bid auction still holds for uniform price auctions in the context of *single-unit* demand (McCabe, Rassenti and Smith, 1990; Weber, 1983), it does not extend to situations with *multiunit* demand (Krishna, 2002). In the studies carried out on uniform price auctions with multi-unit supply and demand, many researchers focus on the two-unit demand model with private values. They show that sincerely bidding on the first unit and applying demand reduction on the second unit is an equilibrium strategy. Because bidders have the market power of manipulating the market price, they have incentive to shade their bids after the first-unit demand in order to enjoy a lower market price in the case their bids are marginal (Engelbrecht-Wiggans and Kahn; 1998; Krishna, 2002; Noussair 1995). In other words demand reduction is inevitable in uniform price auctions. The resulting low-revenue equilibria might be "natural outcomes even without prior experience or signalling to establish and sustain the collusion." (Engelbrecht-Wiggans and Kahn, 2005, P.508). The theoretical prediction of demand reduction behaviour is supported by experiment studies (Engelmann and Grimm, 2003; Kagel and Levin, 2000) and field experiment studies (List and Riley, 2000). In these studies, overbidding on the first unit is also frequently observed. Based on this finding, Levin (2005) demonstrates that bidding *well above* value on first-unit and zero on second-unit bids is also an equilibrium when the reservation price is zero.

[9] The highest losing price is chosen as the market-clearing price in most of the studies on uniform price auction.

Though this equilibrium is less attractive because it relies on each bidder's belief that the other bidders also follow this strategy (Engelbrecht-Wiggans, List and Reiley, 2005), it is similar as the tacit collusion equilibrium in auctions for perfectly divisible goods (or share auctions).

In auctions for perfectly divisible goods (or share auctions) with common values it has been shown that if a bidder has a positive probability of influencing the price, in a situation where the bidder obtains allocation, then the bidder has an incentive to shade her bid (Ausubel And Cramton, 2002). Wilson (1979) and other authors (Ausubel and Cramton, 2002; Back and Zender, 1993; etc.) demonstrate the existence of equilibria which yield a significantly low sale price, because this kind of auction is subject to manipulation by the bidders and the seller obtains no advantages from increased competition. By strategically submitting steep demand schedules, bidders can manage the market price so that it is set at low levels in equilibrium. Because it takes a big price increase to increase one's allocation, collusive strategies are self-enforcing in this non-cooperative game. A continuum equilibrium resulting in low market prices exists (Maxwell, 1983) even with supply uncertainty (Back and Zender, 1993; Wang and Zender 2002). The above studies consider the existence of equilibrium under a continuous demand function and therefore develop appropriate methodologies.[10]

In auctions for IPOs, the market value of shares changes with investors' estimates about the market value of shares. The more investors observe signals indicating a high market value which, in turn, implies they would like to buy shares, the higher the market value will be. This means that the value of the shares, although is the same for every player, depends not only on each investor's signal, but also on all the other

[10] Derivative, integral and dynamic optimisation are used in the studies.

players' signals. The combination of multi-object and interdependent values in this common value auction implies that the derivation of equilibrium is a very delicate problem. For uniform price auctions for IPOs, Biais and Faugeron-Grouzet (2002) provide a tacit collusion demand function similar as the previous studies on share auctions. In equilibrium the market price is set as low as the reservation price. They claim that because the strong reaction of prices to demand leads to this tacit collusion equilibrium, the uniform price auction results in large underpricing in IPOs.

Nevertheless, though it is theoretically interesting, the tacit collusion equilibrium is not only difficult to achieve as it requires co-ordinated behaviour among bidders, but also risky for players who choose to collude. The tacit collusion strategy requires that players have the beliefs that other players also follow the same strategy. If some other players submit different demand functions, a player who follows the tacit collusion strategy may suffer a heavy loss. In addition, some researchers argue that a collusive equilibrium only exists in uniform price auctions with a continuous demand, while the discrete multi-unit auction model predicts a unique, Bertrand-like equilibrium (Harbord, Fabra and von der Fehr, 2002). Kremer and Nyborg (2004) show that the collusive equilibrium of the share auction models of Wilson and Back and Zender do not survive when bidders only make a finite number of bids. Instead, a Bertrand-like price competition is induced. In the discrete version of the Wilson/Back and Zender model, the market price in equilibrium can be as high as the market value even when investors face an uncertain supply. They suggest that this may explain why uniform price auctions are still popular despite the theoretical warnings of severe underpricing.

A common feature of the auctions where this equilibrium is observed in reality is that a relatively small number of bidders compete against each other on a relatively larger number of items; for example a spectrum auction (Engelbrecht-Wiggans and

Kahn, 2005).[11] In fact, it is relatively difficult to find statistically significant evidence of demand reduction when there are more than two bidders (Engelbrecht-Wiggans, List, and Reiley, 2005b). In an experiment with two bidders who each has two units demand and private information, though demand reduction is observed, bids for low valued units is higher than the equilibrium prediction of zero (Roumen and Vragov, 2003). The experiment conducted by Sade, Schnitzlein and Zender (2004) finds little evidence of collusive behaviour in uniform price auctions even when communication is allowed. In our experiment where the goods for sale has common value, though only a limited number of players participate, tacit collusion behaviour is hard to find as well (see chapter three). The collusion prediction also relies on the assumption that the investors know exactly the number of investors who participate in the auction. In markets for IPO where there are many investors, including a large number of retail investors (usually inexperienced), a collusion equilibrium would be more difficult to achieve. The results from the experiments may be due to subjects' failing to find the equilibrium solution of the game, or the equilibrium is difficult to achieve, but it also suggests that there might exist other equilibria that are not as extreme as the tacit collusion prediction.

Now let's review the empirical evidence on uniform price auctions. In the debate over uniform versus discriminatory pricing in Treasury bill auctions, the evidence from the Mexican Treasury bill auctions for the period 1986–1991 is consistent with Friedman's prediction (Umlauf, 1993). For the US Treasury bill market, Malvey, Archibald and Flynn (1997) report a broader allocation distribution and greater expected revenues with greater volatility in uniform price auctions during the period 1992 to 1997. Though collusive behaviour among a group of large

[11] Other collusive behaviours are reported if additionally the game is played repeatedly, such as the electricity auction in England and Wales (Wolfram, 1998), or when communication is available, such as in the experiment

investors is observed in both markets, allocation concentrations to the top dealers under the uniform price technique is significantly lower than that under the discriminatory setting. This implies that uniform price auctions attract more bidders into the market. The competition from bidders outside the cartel leads to the decline of bidders' profits. So the uniform price auction can successfully "widen the market"(Friedman, 1960) and this effect can offset at least part of the disadvantage due to the collusion behaviour.[12] Another study based on the Turkish Treasury bill market in the period from 2000-2002 also shows a higher participation rate in uniform price auctions but the superiority of discriminatory auctions in revenue raising is also reported (Ozcan, 2004a). Little evidence of collusion has been found using the individual bidder data from Finnish treasury auctions (Keloharju, Nyborg and Rydqvist, 2003. See Kremer and Nyborg, 2004). An empirical study on the Zambian foreign exchange market indicates that the uniform price auction is revenue-superior to the discriminatory auction due to higher participation. No collusive behaviour is present there because a large number of relatively small bidders are involved in the market, though demand reduction is evident (Tenorio, 1993). Though the existence of the collusive equilibria in the uniform auction is argued to be one reason for the Britain's decision of adopting a discriminatory auction format in markets for electricity (Klemperer, 2001), a report for the California power exchange concludes that a shift from a uniform to a discriminatory auction is unlikely to result in lower electricity prices (Kahn et. al, 2001; see Harbord, Fabra and von der Fehr, 2002, p5). The inconsistent results across markets in terms of revenue and the evidence of greater volatility in the US treasury market again suggest that there might exist other equilibria besides the tacit collusion equilibrium.

---

conducted by Goswami, Noe and Rebello(1996).

In the case of IPO markets, where the most commonly used method, Bookbuilding, allocates most of the shares to large institutional investors and market prices are usually set artificially far below the market value by underwriters, researchers have been looking for other practical methods for raising money and replace the current flawed system. The uniform price auction is used in some other countries such as Israel and United Kindom[13]. However, it is difficult to conclude which method is better from data produced under different circumstance across countries.[14] After Google's successful IPOs conducted by "Dutch auction"[15], both economists and companies have shown more interest and confidence relative to the Wall Street in choosing the uniform price auction rather than the traditional Bookbuilding when going public. The US investment-banking WR Hambrecht + Co. has conducted 13 IPOs successfully by uniform-price sealed-bid auctions via Internet since 1999[16]. It claims that both institutions and individual investors have equal access to bid on IPO shares, and can obtain allocations in an equal and impartial way. Because there is no preferential allocation towards big institutional investors and the market price is set at the market clearing price (or below the market clearing price if companies prefer lower offering prices to "boost" market when shares debut), uniform price auctions "demonstrated that the IPO process can be changed in ways that ultimately benefit small investors and companies that need to raise capital."(Knight, 2004, page E01). In Israel's IPO market, contrary to the steep demand function

[12] After the experiment with uniform price auctions on two-year and five-year notes that started on September 1992, the US Treasury switched entirely to the uniform-price auction in November 1998 (Ausubel, 2002).

[13] In the UK, auctions were frequently used to conduct IPO until the mid-1980s then since 1986 have fallen out of favour for some unclear reasons (Jenkinson and Ljungqvist, 2001).

[14] Underpricing is a common feature of almost every offering format in IPO. However, revenue performance of different IPO methods varies greatly across countries. In the Ritter and Welch (2002) study, average underpricing is 18.8% for a sample of 6249 IPO issued during 1980 to 2001in US market where Bookbuilding is used for almost all the IPO. Kandel et al. (1999) report an average underpricing level of 4.5 percent from December 1993 to December 1996 for uniform price auctions in Israel.

[15] Financiers call uniform price auction "Dutch auction", which has a different meaning outside financial markets.

[16] From 1999 to the second quarter of 2005, the number of IPO in the US is 1268.

provided by Biais and Faugeron-Grouzet (2002), the demand schedule is flat and elastic (Kandel, Sarig and Wohl, 1997).

In summary, the tacit collusion equilibrium does not receive much experimental or empirical support. This motivates us to look for other possible equilibrium solutions for uniform price auctions. In this chapter, we do so for IPO auctions.

2.3 The model

The model that we use in this chapter is established by Biais and Faugeron-Crouzet (2002).

In the model, a seller wants to sell homogeneous shares in the market. The amount of shares is normalized to 1[17]. There are $N \geq 2$ large institutional investors and a fringe of small retail investors as potential buyers. Each institutional investor has private information about the valuation of the shares by the market as well as a large bidding capacity. The retail investors, however, are uninformed and cannot absorb the whole issue. The private information that the institutional investors have is represented by the private signals s_i, which are identically and independently distributed and can be *high* with probability π or *low* with the complementary probability. However, each signal only reveals part of the information. The value of the shares on the secondary market increases with the number of *high* signals n. Denote the market value of one unit share as v_n when there are n high signals. Uninformed investors do not observe signals. Each informed investor can buy the

[17] The size of unit does not matter in this game.

whole issue even if the market price equals the highest possible market value[18]. The retail investors as a whole can purchase up to 1- k units shares, with $k \in [0,1]$.[19] All agents are assumed to be risk neutral. The investors have the same constant marginal value for shares.

So this is a multi-unit common value auction with incomplete information, interdependent values and asymmetric bidders. Biais and Faugeron-Crouzer (2002) follow the hypothesis that institutional investors have superior information about the value of shares, such as the information about an issuing firm's competitors, the quality of management of the issuing firm, etc. The seller is unable to know the market valuation of the shares for sale without such information. It is reasonable to assume that investors only have imperfect information about the value of shares[20]. For simplicity, in IPO models this piece of information is usually identified as *high* or *low* (or *good* or *bad*) signals indicating that the shares are favourable or unfavourable. Because a *high* signal to an investor means that the investor is in favour of (buying) the shares, more *high* signals imply a stronger demand. In securities markets, a stronger demand usually pushes the market price to higher levels. According to the *Efficient Market Hypothesis*, the stock market reveals all the information related to the market value so the market price, when shares are trading in the secondary market, can be regarded as the real market value (at that time). This is why the model assumes that the market value increases with the number of *high* signals. Another example that

[18] The authors think this assumption is reasonable "given the bidding power of the large financial institutions participating regularly to IPOs, compared to the relatively small size of most of these operations. In addition this assumption simplifies the analysis." (Biais and Faugeron-Crouzet, 2002, p15).

[19] In the real world, either because retail investors have small demand capacity or because their demand is difficult to predict, firms who go public always try to attract large institutional investors to guarantee full subscription. It is rare to rely on small retail investors to absorb all the shares of an IPO, even if the resulting market price is low. In some issues there is even a maximum subscription amount for a retail investor. Hence the assumption that the retail investors can purchase up to 1-k units shares is reasonable. Moreover, k is allowed to take a value as low as zero. In that case, the retail investors as a whole can buy the whole issue.

[20] In the fundamental work for IPO by Rock (1986), in order to emphasize the informational advantage that the market enjoys over the seller, the institutional investors as a whole are assumed to have perfect information of the

uses a similar assumption is the fundamental model for IPO established by Benveniste and Spindt (1989). Another interpretation is that the underlying market value of shares is preassigned, and the nature distributes the corresponding number of high signals to the informed investors. Then the higher the market value is, the more *high* signals there are.[21]

The price rule and the allocation rule of the auction are as follows. The seller sets a reservation price p_0 ($p_0 \geq 0$) so the market price is at least as high as p_0. If the total demand at the reservation price $D(p_0)$ exceeds the supply, the market price p_m is set at the market-clearing price, the highest bid price where demand exceeds supply.[22] Otherwise if the cumulated demand at p_0 is less than or equal to the supply, the market price is set as p_0. Formally:

$$p_m = \begin{cases} \max(p \mid D(p) > 1) & if \quad D(p_0) > 1 \\ p_0 & otherwise \end{cases} \qquad \textbf{[1]}$$

The amount bid for above the market price is fully filled, and then the shares left are prorated among the bids placed at the market price. The bids below the market price do not receive any allocation. Denote $d_i(p)$ as bidder i's cumulated demand at prices

market value of the new issue. This is not opposite to the imperfect information assumption in the current model, as the market value of shares is revealed if taking all the imperfect information that investors hold together.

21 Suppose the probability of observing either a high or a low signal is equal, signals are independent, there are three informed investors, and the market value equals one plus the number of high signals observed. The construction of private information can also be described in the following way: the market value ɵ (1, 2, 3, or 4) is drawn from a distribution where the probabilities of drawing 1, 2, 3 and 4 are 1/8, 3/8, 3/8 and 1/8 respectively, the investors are randomly ordered and the first ɵ -1 investors in the random order are given a high signal. (see Chamley (2003) for the two methods of constructions of private information).

22 Following the convention of auction theory, we use the highest losing price rather than the lowest winning price as the market-clearing price. This can also simplify our description of bidders' strategies. For example, if we use the lowest winning price, since the market price is the highest price where all the shares are sold out, instead of placing quantity 1 at some price, a player can do better if she demands marginally less than a unit at that price and bids for the rest at a lower price. Using the highest losing price, the same strategy of this player can be described as she places 1 at that price. And since there are numerous bids in real markets, the highest losing and the lowest winning prices are usually the same.

equal to or higher than p and $d_i^a(p)$ as her cumulated demand at prices *above p*, then bidder i's allocation a_i can be expressed as the following formula:[23]

$$a_i = \begin{cases} d_i^a(p_m) + [1 - \sum_{i=0}^{N} d_i^a(p_m)] \dfrac{d_i(p_m) - d_i^a(p_m)}{\sum_{i=0}^{N}[d_i(p_m) - d_i^a(p_m)]} & \text{if } D(p_0) > 1 \\ d_i(p_0) & \text{otherwise} \end{cases} \quad \textbf{[2]}$$

Where i = 0 represents the uninformed investors as a whole[24]. So if the cumulated demand at the reservation price exceeds the supply, after allocating to each bidder the amount she bids for at prices higher than the market price ($d_i^a(p_m)$, the first term of the right hand side of the equation), the rest of the shares (the multiplicand of the second term) are prorated among the bidders, each of which obtains a proportion of her bids at the market price (the numerator of the second term) over the total bids placed at the market price (the denominator of the second term). Otherwise if the total demand at p_0 is less than or equal to the supply, each bidder obtains the amount she bids for ($d_i(p_0)$).

When the realization of the market value is v_n, bidder i's payoff Π_i equals the payoff from each unit allocation, v_n - p_m, then multiplied by the number of units allocated:

$$\Pi_i = (v_n - p_m) \times a_i \quad \textbf{[3]}$$

A strategy S_i in this game is defined as a demand-price schedule $d_i(p, s_i)$ indicating how many shares (d_i) a bidder would like to bid for at price level p, under

[23] The price and allocation rules can be explained by the following example. Suppose there are two bidders (N=2) and the amount for sale is 1. Bidder 1 bids for 0.7 at price 2, and bids for another 0.3 at price 1; bidder 2 bids for 0.3, 0.5,0.2 at price 2,1 and 0 respectively. The total demand at price 2 then is 1. So bidder 1 obtains 0.7, bidder 2 obtains 0.3, and the market price is 1, the highest losing price. If instead, bidder 1 bids for 0.5 and 0.5 at prices 2 and 1 respectively, bidder 1 and 2 are allocated 0.5 and 0.3 first, then the left 0.2 shares are prorated among the two bidders according to their demand at price 1; thus they each get 1/2. So bidder 1 and 2's total allocation is 0.6 and 0.4 respectively and the market price is 1.

the observed signal s_i (s is either H, L or u representing high, low or no signal). As both the market price and the allocation are determined by bidders' demand schedules, bidder i's payoff can be written as a function of both bidder i and all the other bidders' ($-i$) demand functions:

$$\Pi_i(S_i, S_{-i}) = \Pi_i(d_i(p, s_i), d_{-i}(p, s_{-i})) \qquad \textbf{[4]}$$

An investor obtains a zero profit by demanding zero:

$$\Pi_i(0, d_{-i}(p, s_{-i})) = 0 .$$

Bidder i's problem is to maximize her expected payoff by choosing a demand schedule $d_i(p,s_i)$ conditional on the signal she observes, given the other bidders demand schedules. The function $d_i(p,s_i)$ is nonincreasing in p. We assume that the same types of investors, i.e., the investors with high or low signals and uninformed investors (called H, L and u investors respectively hereafter) are symmetric in beliefs and behaviour.

There are two differences between this model and other models for share auctions. The first difference is about the signals or expected values. In other models for share auctions (represented by Wilson/Back and Zender/Wang and Zender), bidders either observe (different) signals from a set of signals revealing all the possible states of the world, or have the same expected value (Wilson, 1979). Wilson also provides a case when the market value of shares is common knowledge to all the investors.[25] In the Biais and Faugeron-Crouzet's model, however, the type of signals is either high or low. The expected value has three levels: the expected value given a high or a low signal ($E(v \mid H)$ or $E(v \mid L)$) or the expected value given that no signal is observed ($E(v)$). Secondly, with a limited number of informed investors each submitting a

[24] There are a large number of retail investors in the market. For simplicity, they are assumed to be symmetric in behaviour, and are regarded as a single player in Biais and Faugeron-Crouzet's paper. We follow this assumption in the current paper.

limited number of bids, in this model the demand functions are discrete. As we have mentioned previously, there is work pointing out the significant difference in equilibrium predictions between models in continuous and discrete formats, and thus we cannot use the results from previous studies directly. As the discrete format is closer to the real market,[26] in this chapter we pay a lot of attention to the equilibria of the discrete case when the continuous methodology does not work.

Biais and Faugeron-Crouzet (2002) provide a tacit collusion demand function solution for this model. In equilibrium[27], all investors submit a steep demand function and as a result the market price is as low as the reservation price. In this equilibrium, all investors (the uninformed investors are taken as a whole) behave symmetrically regardless their signals and each obtains the same allocation. The function will be discussed in detail in the next section.

2.4 Equilibrium analysis

2.4.1 Conditions for equilibrium

Only equilibria in pure strategies are considered in this chapter. We assume that demand is nondecreasing in the expected value and the demand schedule of informed investors is additive. Since H and L investors in fact are the same investors with different signals, we assume that an H investor bids for no less than an L investor at any given price as they have higher expected market value:

[25] The model used by Kremer and Nyborg (2004) is the same except for the discreteness,

[26] In real markets for IPO or other share auctions, usually it is impossible to submit continuous demand functions. There exist tick size (the minimal amount of a nominal price change that is allowed) and quantity multiple in stock market. For example, in Singapore, there are five tick size categories ranging from 0.5 cents for stocks priced less than $1.00 to 10 cents for stocks priced above $10. (Comerton-Forde, Lau and McInish, 2003). The minimum order quantity for a new issue varies and is often 100 shares in US. A maximum of three bids are allowed to place in Italian treasury market (Scalia, 1996; see Kremer and Nyborg, 2004, p858).

[27] Equilibrium in this chapter refers to a Bayesian Nash Equilibrium as the game that we are discussing is a Bayesian game, i.e. a game with incomplete information.

$d(p,H) = d(p,L) + c(p)$

Where $c(p)$ is nonnegative. The equation means that if an informed investor observes a high signal, at price p she demands $c(p)$ more than when observing a low signal. For the continuous case, both $d(p,s)$ and $c(p)$ are assumed to be piecewise continuous differentiable.

Though there exist a large number of u investors, since we have assumed that u investors are symmetric in their behaviour and the number of u investors is exogenous, we can regard all the u investors as a whole and consider their joint behaviour as a single player. The representative player then is referred to as the U investor.

In general, the equilibrium strategy should satisfy the following conditions.

Condition 1 (participation condition): Each investor gains a nonnegative expected payoff in equilibrium:

$E\Pi_i \geq 0$ for each $i \in [0, N]$.

If in equilibrium the market price is p_n when there are n high signals, and an investor with signal s obtains an allocation of $a(p_n, s)$, then condition 1 requires

$$E(\Pi \mid H) = \sum_{h=0}^{N-1} \pi_h a(p_{h+1}, H)(v_{h+1} - p_{h+1}) \geq 0 \text{ for each H investor,}$$

$$E(\Pi \mid L) = \sum_{h=0}^{N-1} \pi_h a(p_h, L)(v_h - p_h) \geq 0 \text{ for each L investor,}$$

$$E(\Pi \mid U) = \sum_{h=0}^{N} \mu_h a(p_h, u)(v_h - p_h) \geq 0 \text{ for the U investor.}$$

Where π_h is the probability of h out of N-1 other investors observing high signals, μ_h is the probability of h out of N investors observing high signals.

Condition 2: A set of strategies S^* under which the payoff of investor i is Π_i is an equilibrium only if no investor can improve her expected payoff by changing her strategy S_i^*.

$E\Pi_i(S_i^*, S_{-i}^*) \geq E\Pi_i(S_i, S_{-i}^*)$ for any $i \in [0, N]$, where bidder 0 represents the U investor.

This is simply a condition for any equilibrium stating that no player could profitably deviate. In this particular model, the intuition is as follows. Recall that a strategy is a demand schedule in this game. If one investor raises the market price to a higher level in order to absorb more shares from the other players' demand reduction, Condition 2 requires that the gain from the increase in the allocation must not be sufficient to compensate for the loss from increase in the price. In the continuous case, this happens when the demand function is steep enough such that $\frac{\partial \Pi_i(d_i(p,s_i), d_{-i}(p,s_{-i}))}{\partial p^+} \leq 0$. Also, if one investor tries to lower the market price by demanding fewer shares, she has to give up a sufficient large amount of shares and the gain from a decrease in price is not sufficient to compensate for the corresponding loss. So the demand curve below the market price should be sufficiently flat. In the continuous case, this requires $\frac{\partial \Pi_i}{\partial p^-} \geq 0$. So either the demand function satisfies $\frac{\partial \Pi_i}{\partial p} = 0$, or there is a kink in the demand curve at price p. If a reservation price exists and the market price is the reservation price, then the market price is bounded in one direction and only $\frac{\partial \Pi_i}{\partial p^+} \leq 0$ is needed.

Conditions 1 and 2 are jointly sufficient for an equilibrium. They guarantee that an investor would like to participate and would not deviate. However, taking into account the market rules, in equilibrium the demand function should also satisfy the following two necessary conditions.

Condition 3 (market clearing condition): According to the price rule, the market price p_m is the highest price where the total demand exceeds supply. Hence the cumulated demand *above* p_m is no more than 1, and that *at* p_m exceeds 1 if there is excess demand at the reservation price:

$$\sum_{i=0}^{N} d_i^a(p_m) \le 1,$$

$$\sum_{i=0}^{N} d_i\ (p_m) > 1 \text{ if } D(p_0) > 1$$

Condition 4 (nonincreasing demand function): Each investor's demand function is nonincreasing in price (either downward sloping or vertical).

$d_i(p, s_i) \le d_i(p', s_i)$ if $p > p'$, for any $i \in [0,\text{N}]$ and any signal s.

This condition means that if one investor indicates that she would like to buy an amount at some price, she would also like to buy at least the same amount at a lower price. This is how the allocation rule is conducted.

Conditions 1 to 4 guarantee that each investor obtains a nonnegative profit at the equilibrium market price and cannot improve the profit further through any kind of deviation, by submitting a nonincreasing demand function. We will use the above four conditions to conduct our equilibrium analysis in the following section.

We start with analysing flat demand functions in order to see whether bidding for the entire shares at one price can form an equilibrium. Then we discuss the tacit collusion behaviour, which has been shown to be an equilibrium in this model for the continuous case and we generalize the conditions for the discrete case. After that we consider the equilibria with asymmetric strategies among different types of bidders.

We introduce the equilibria where H investors absorb all the shares before allowing L investors and then u investors entering the market.

2.4.2 Equilibria

2.4.2.1 Flat demand function

In most of the previous studies with a two-unit demand and decreasing marginal value, demand reduction is shown to be inevitable in an equilibrium. However, if bidders have a "flat demand", i.e., if bidders have the same value on both units of goods, truth revelation on both units can be an equilibrium strategy (Engelmann and Grimm, 2003). In common value auctions, a flat bid function could be an equilibrium when investors have affiliated private signals (Ausubel and Cramton, 1998) or face an uncertain supply (Back and Zender, 1993; Wang and Zender, 2002). However, the differences between these models and ours which have been described in section 2.3 make it impossible to apply the previous results to our model directly.

Suppose each bidder submits a flat demand function at price p and each obtains a positive expected payoff. Then one bidder could be better off by bidding for the same amount at a higher price to obtain all the allocation (1-k units for the U bidder) without raising the market price. Bidders compete to raise the bidding price until the expected payoff from deviation is zero. Thus bidders with lower expected valuation of shares, i.e., L and U investors, drop out first, and they can only obtain an allocation if there is no H investor in the market. Knowing this an L investor should not bid higher than v_0. In the case of all L investors placing flat demand functions, they would compete to bid higher than the others until the price reaches v_0 in order to absorb as many shares as possible when there is no high signal observed. Hence we can assume that L investors participate by bidding for 1 at v_0. Then the U investor[28] is indifferent between placing 1-k at any price below the price at which H investors place bids, as all are generating zero expected payoff for them. For simplicity we assume that the U investor also places bids at v_0. The market price and the allocation for each type of investor under these strategies are[29] (denote the number of high signals as n):

Table 2.1 Results under Flat Demand Function Strategies

Number of high signals (n)	Market price	Market Value	Allocation
>1	p	v_n	Each H investor: $1/n$
1	v_0	v_1	The H investor: 1
0	v_0	v_0	U investor: (1-k) Each L investor: k/N

Now let's examine an H investor's strategy.

For an H investor, the expected payoff is:

$$E(\Pi \mid H) = \sum_{h=1}^{N-1} \pi_h \frac{1}{h+1}\left[v_{h+1} - p\right] + \pi_0\left[v_1 - v_0\right] \quad [30] \qquad [5]$$

Where p is the price that H investors place their bids[31].

By bidding for 1 at a price higher than p, an H investor can absorb all the shares without raising the market price. The expected payoff of such an H investor is:

$$E(\Pi_d \mid H) = \sum_{h=1}^{N-1} \pi_h\left[v_{h+1} - p\right] + \pi_0\left[v_1 - v_0\right] \qquad [6]$$

[5] and [6] can be rewritten as:

$$E(\Pi \mid H) = \sum_{h=0}^{N-1} \pi_h \frac{1}{h+1}\left[v_{h+1} - p\right] + \pi_0\left[p - v_0\right] \qquad [5]'$$

[28] Recall that we regard all the uninformed investors as a whole and consider their joint behaviour. The representative player is referred to as the U investor.
[29] Here we assume the reservation price is zero for simplicity and to avoid artificial influence on investors' strategies.

$$E(\Pi_d \mid H) = \sum_{h=0}^{N-1} \pi_h [v_{h+1} - p] + \pi_0 [p - v_0] \qquad \textbf{[6]}'$$

The second terms of both equations are identical. The first term of [6]′ is in fact

$E((v - p) \mid H)$, which equals:

$$\sum_{h=0}^{N-1} \pi_h v_{h+1} - \sum_{h=0}^{N-1} \pi_h p = E(v \mid H) - p \sum_{h=0}^{N-1} \pi_h = E(v \mid H) - p$$

When p is $E(v \mid H)$, the expected market value of shares given a high signal, equals zero and equation [6]′ equals $\pi_0 [E(v \mid H) - v_0]$, which is positive.

Hence if the market price is $E(v \mid H)$, an H investor gets a zero expected payoff should she absorb all the shares. The expected payoff is positive because the market price is v_0 instead of $E(v \mid H)$ when there is only one high signal observed, which reduces the loss when the market value is low.

If p equals $E(v \mid H)$. The first term of [5]′ equals

$$\begin{aligned} E(\tau(v - E(v \mid H) \mid H) &= E((v - E(v \mid H) \mid H) \cdot E(\tau \mid H) + \mathrm{cov}(\tau, v - E(v \mid H) \mid H) \\ &= \mathrm{cov}(\tau, v - E(v \mid H) \mid H) \end{aligned} \qquad \textbf{[7]}$$

Where τ, the execution rate[32], equals $\frac{1}{n}$ (n is the number of high signals in the market). Because the more high signals there are, the higher the market value is, and the lower the proportion of shares that will be allocated to each H investor, the covariance between τ and v is negative, so the first term of [5]′ is negative. Lower market values corresponding to lower payoffs add more weight in the expected payoff than higher values.

Thus the expected payoff in equation [5] is lower than that in [6]. By bidding 1 at a price above $E(v \mid H)$, an H investor can absorb all the shares without changing the

[30] The market price when there is only one H investor should be the maximum price between L and U investors' bid prices. Without lose of generality, we assume the price is v_0 to simplify the description.

[31] Which should be no lower than $E(v)$.

market price, and improve the expected payoff to $E(\prod_d |H)$. Like in Bertrand competition, every H investor would compete until it is unprofitable to do so.

Since the second term of [5] and [6], the payoff when no other investor observes a high signal, $\pi_0[v_1 - v_0]$ is positive, H investors would compete to raise the price in order to absorb all shares, until the expected payoff from deviation when there exist other H investors (the first term of [6]) equals zero. Because every H investor would do this, it can be an equilibrium only if H investors can share the expected payoff at the price level where no one would like to overbid further, i.e. the expected payoff is zero. However, it can be shown that they each have a negative payoff (i.e., the first term of [5] is negative) at such a price level (see Part I in Appendix 2.A). If that is the case, one can be better off by bidding at a lower price and only obtaining an allocation when there is no other H investor in the market. [33] We cannot set a price that makes both bidding higher and bidding lower unprofitable.

We have shown that bidding a flat demand function cannot be an equilibrium in pure strategies for H investors. In fact, it cannot be an equilibrium in mixed strategies as well. Because if there exists a price vector of possible mixed strategies of flat demand curves where each bidder *i* bids p_i for the entire shares, such that the bidders' ex ante expected profits are positive, one investor can always increase her profit by bidding at a price above all the possible prices in the mixed strategy, so that she can absorb all the shares at the same price. From [7] we have found that if the profit from sharing, $E(\tau(v - p)|H)$, is nonnegative, it is always lower than the profit from absorbing all the shares $E[(v- p)|H]$. By bidding at a higher price, an H investor

[32] The execution rate equals a bidder's allocation divided by her total demand, i.e., the proportion that her demand is fulfilled.

[33] In the case when $\pi_0[v_1 - \max(p_u, p_l)]$ is negative, investors would stop competing when [6] equals zero. At that price [5] is already negative (See Part II in Appendix 2.A) so again we are unable to have an equilibrium where all H investors share payoffs at the highest individually affordable price.

captures the payoffs of all H investors. And we have shown that all H investors sharing at the highest individually affordable price leads to negative expected payoff for each of them and thus it cannot be an equilibrium. So now we have the following proposition:

Proposition 1: *All investors submitting flat demand functions at any price is not an equilibrium in uniform price IPO auctions.*

Nevertheless, submitting flat demand function at v_0 is an equilibrium strategy for L investors should they only obtain an allocation when there is no high signal in the market. Because both the market value (v_0) and the number of investors who share the allocation (N) are fixed, they each get zero payoffs and cannot do better given the other bidders following the same strategy. Submitting a flat demand function at a price that equals expected value may form an equilibrium if all the investors have the same information and thus have the same expected market value, or if all bidders have different market values. The existence of asymmetry and the negative relationship between market value and H investor's allocation imply that a flat demand function cannot be an equilibrium strategy in our model (see [7]). We believe this relationship is consistent with the observations in actual markets for IPOs.

Proposition 1 implies that demand reduction is inevitable. The easiest way to start looking for an equilibrium is to assume that all investors are symmetric in their behaviour. Previous researches have demonstrated that there exist a continuum of collusive equilibria in the continuous case where bidders enjoy a low price because of demand reduction (Ausubel and Cramton, 1998; Wang and Zender, 2002; Wilson, 1979; etc.). These equilibria are introduced below and then we generalize it to the discrete case.

2.4.2.2 Tacit collusion equilibrium

If H, L and U investors follow the same demand function, the sum of their demand at a given price would remain unchanged regardless of the number of high signals. Hence the market price and each investor's allocation would be constant for any possible market value. To satisfy Condition 1, the highest acceptable price for H, L and U investors is $E(v|\ H)$, $E(v|\ L)$ and $E(v)$ respectively. Thus the equilibrium price should be no higher than the lowest among them, $E(v|\ L)$. Which means that to guarantee full subscription, the highest reservation price the auctioneer can set is $E(v|\ L)$. By setting up a reservation price, not only a minimum revenue is guaranteed, but can also reduce the incentive for collusion by limiting the maximum gain from collusion.

The demand function provided by Biais and Faugeron-Crouzet (1999) is one such function in the equilibrium set:

$$d(p) = \frac{1}{N+1} - \sigma(p - p_0) \text{,with } \sigma \leq \frac{1}{N(N+1)} \frac{1}{E(v \mid H) - p_0} \quad \textbf{[8]}$$ [34]

Bidders submit the same demand function regardless of their signals. The condition for σ implies that the slope of the demand function should be small enough. By submitting steep demand curves, the residual supply that a bidder faces increases only by a small amount when the price is raised by a large amount, so the gain from the increase in the allocation cannot compensate the loss from an increase in price. Because it is the marginal bid by which demand exceeds supply that determines the

[34] See Biais and Faugeron-Crouzet (2002), page 19.

market price, bidders' inframarginal bids are costless in a uniform-price auction. Thus the collusion is self-enforcing in this non-cooperative game. According to this demand function, the equilibrium market price is the reservation price, and each investor obtains an equal quota of the entire shares. If p_0 is set below $E(v|\,L)$, any price p between p_0 and $E(v|\,L)$ can be a market price in an equilibrium. In the case when p is higher than p_0, in order to satisfy Condition 2, either $\sigma = \frac{1}{N(N+1)}\frac{1}{E(v\,|\,H)-p}$ such that $\frac{\partial\Pi(p)}{\partial p} = 0$, or the demand function has a kink at price p with $\sigma < \frac{1}{N(N+1)}\frac{1}{E(v\,|\,H)-p}$ above p and $\sigma > \frac{1}{N(N+1)}\frac{1}{E(v\,|\,H)-p}$ below p. However, the equilibrium at p_0 weakly dominates all other equilibria. The condition is derived under the assumption that each u investor bids for $\frac{1}{(N+1)(1-k)} - \frac{\sigma}{1-k}(p-p_0)$ (k $\in$ [0, $\frac{N}{N+1}$]) of the shares. In general, if the fraction of shares that they bid for is r_u at p_0, the demand function of an informed investor becomes $\frac{1-(1-k)r_u}{N} - \sigma(p-p_0)$ and the equilibrium requires $\sigma \le \frac{1-(1-k)r_u}{N^2\left(E(v\,|\,H)-p_0\right)}$ (See Appendix 2.B).

The above analysis is based on the assumption that the demand function is continuous. In reality, however, investors usually only indicate the amount they would like to buy at a limited number of prices. Moreover, although there exist a large number of uninformed investors, the number of institutional investors is limited. So we should also check if the tacit collusion equilibrium still holds in the discrete case. We found that the equilibrium still exists in the discrete case, under stricter conditions.

Proposition 2: *If all types of investors are symmetric in their behaviour, in the discrete case, there exist multiple equilibria with a market price no higher than E(v|L); the price* $\bar{p}$ *from which an investor starts bidding is no lower than* $\frac{NE(v \mid H) + p_m}{N+1}$*; the demand function is vertical above the market price, and the (underlying) slope* σ *above the price* $\underline{p}$ *from which the demand curve is vertical must satisfy* $\sigma \leq \frac{\underline{p} - p_m}{N(N+1)\left(E(v \mid h) - \underline{p}\right)\left(\bar{p} - \underline{p}\right)}$.

See Appendix 2.C for the proof.

The intuition of this proposition is as follows. Since all investors start bidding at a price at least as high as $\bar{p}$, if an investor tries to absorb all the shares by bidding above the other players, the market price would be raised to at least $\bar{p}$. Then even for an H investor, when $\bar{p}$ is high enough, the expected payoff under deviation is no more than that when following the tacit collusion strategy.

The right hand side of the condition for the slope σ increases with $\underline{p}$, so it is the strictest when $\underline{p}$ equals p_m, which requires that the demand curve be vertical (σ=0). The reason is that if the total demand above the market price p_m is less than 1, at least one investor could increase her allocation by raising demand at a price (slightly) higher than p_0 and exploit the demand gap without raising (or raising by little) the market price. In the continuous case, the condition in [8] is sufficient to prevent such a deviation. But in the discrete case, the probability that one investor does not place any bid between p_m and a price higher than p_m is positive especially when the price difference is small. (See the stair curve in Figure 2.C1). Thus with a positive probability the amount left after the inframarginal bids have been filled has a positive mass, and a bidder can increase her allocation by a negligible increase in price. Only a vertical demand function can guarantee that each investor obtains $\frac{1}{N+1}$ of the total allocation in the first place and no demand is left for bids placed at p_m thus no room for a profitably deviation. However, the demand curve needn't to be vertical in the whole price range above p_m. Suppose the function is vertical between p_m and $\underline{p} > p_m$, and above $\underline{p}$ the slope σ can take a positive value. An investor can only enlarge her allocation by raising the market price to at least $\underline{p}$ (see Figure 2.C2). When the slope condition in Proposition 2 is satisfied, the gain from the increase in the allocation is not large enough to compensate the loss from the price jump. The higher $\underline{p}$ is, the higher the jump in the market price under a deviation. Since the cost of deviation needed to prevent the deviation is given, the higher the cost due to the price jump implies that a lower cost from the steep demand function is required, thus the demand curve above $\underline{p}$ could be less steep. When $\underline{p}$ reaches $\overline{p}$, σ can take any nonnegative value (the demand function is vertical between p_0 and $\overline{p}$), as no player would benefit from raising the market price to such a high level.

Note that the lowest possible price from which investors start bidding, $\frac{NE(v \mid H) + p_m}{N+1}$, is higher than $E(v| L)$ when p_m is set as $E(v| L)$. So this equilibrium is risky especially for L investors. If one investor bids more aggressively than the equilibrium strategy, the resulting market price may be raised to $\frac{NE(v \mid H) + p_m}{N+1}$, and L investors will suffer a loss. Because the demand function is generally steep (and in the extreme case it is vertical), a small amount of deviation may cause a loss for L investors. Furthermore, since this equilibrium requires that bidders know the exact number of bidders in the market, it is more reasonable for them not to play this equilibrium without this information. Following the same logic, the seller could mitigate collusion and improve revenue by adjusting supply after investors place their bids (Back and Zender, 1993).

According to the price rule of the uniform price auction, investors know that their demand schedule can influence the final market price. In the tacit collusion equilibrium, they successfully manage to keep the market price as low as possible. The demand curve in the extreme case is vertical. In this equilibrium, different type investors follow the same demand function. If symmetry only exists among the same type investors, i.e., if H and L investors have different demand functions, the market price would change with the market value. We start the equilibrium examination from the case when H investors absorb all the shares. Having only bidders with high expected market value buying shares is a realistic situation. For simplicity, from now on we assume the reservation price is zero (that is, there is no reservation price).

2.4.2.3 Equilibrium demand schedule when H investors absorb all shares

Can H investors absorb all shares by taking advantage of the higher expected value? To see this, we need to check the four conditions developed in section 2.4.1.

Let (v_1, v_2, ...v_N) be a price vector that satisfies Condition 1 as an equality for an H investor. There is no market price that lies beyond this price vector for any realization of market value in an equilibrium where all H investors absorb all shares, if such an equilibrium exists. The reason is straightforward. If for example, market price p_n lies above v_n when there are n high signals, then an H investor could lower her bid price for v_n in her demand schedule. If the other investors' demand at p_n

exceeds 1, market price is still p_n but by deviation this investor could avoid obtaining any allocation and paying more than the market value; if the other investors' demand at p_n is no more than 1, this investor's demand would be marginal and market price would be lowered, so a deviation could still benefit her. Hence the price vector ($v_1, v_2, \ldots v_N$) is the upper bound of the equilibrium price range for H investors.[35]

Thus to satisfy Condition 1, Condition 3 (the market clearing condition) requires that the total demand of H investors *above* v_n is no more than 1, when there are n high signals in the market:

$$nd^a(v_n, H) \le 1 \text{ for each } n \in [1, \text{N}] \qquad \textbf{[9]}$$

Hence an H investor's demand *above* v_n is no more than $\frac{1}{n}$. Because this relation should be satisfied for any realization of n, the demand *at* v_n (*above* v_{n-1}) is no more than 1 when there are n-1 high signals. So an H investor demands no more than $\frac{1}{n-1}$ *at* v_n (for any n>1).

If there exists an equilibrium where only H investors obtain shares, L and U investors should be unable to do better than not participating and getting a zero payoff if there is at least one high signal (we shall discuss the investors' strategy when a high signal is absent later). Under two situations this requirement can be satisfied. The first one is that the market price equals market value even without U and L investors' participation. Which means at price v_n the demand from H investors exceeds 1:

$$nd(v_n, H) > 1 \text{ for each } n \in [1, N]$$

So H investors' expected payoff is zero. If an H investor can lower market price below market value, while still keep some allocation, she would enjoy a positive

[35] The same logic also applies to an equilibrium when other types of investors participate, given that an H investor bids more aggressively than an L investor and thus the market price increases with market value.

expected payoff. So to prevent H investors from deviating, an H investor could only lower the price by giving up the whole allocation. This requires that the total demand of the other H investors is no less than 1 at v_n:

$$(n-1)d(v_n, H) \geq 1 \qquad \text{for n} \in [1, \text{N}] \qquad \textbf{[10]}$$

We have shown that equation [9] requires $d(v_n,H)$ is no more than $\frac{1}{n-1}$, so an H investor's demand *at* v_n is $\frac{1}{n-1}$ in this case. As the demand above v_{n-1} is at most $\frac{1}{n-1}$ according to equation [9], the demand function between v_{n-1} and v_n has to be vertical.

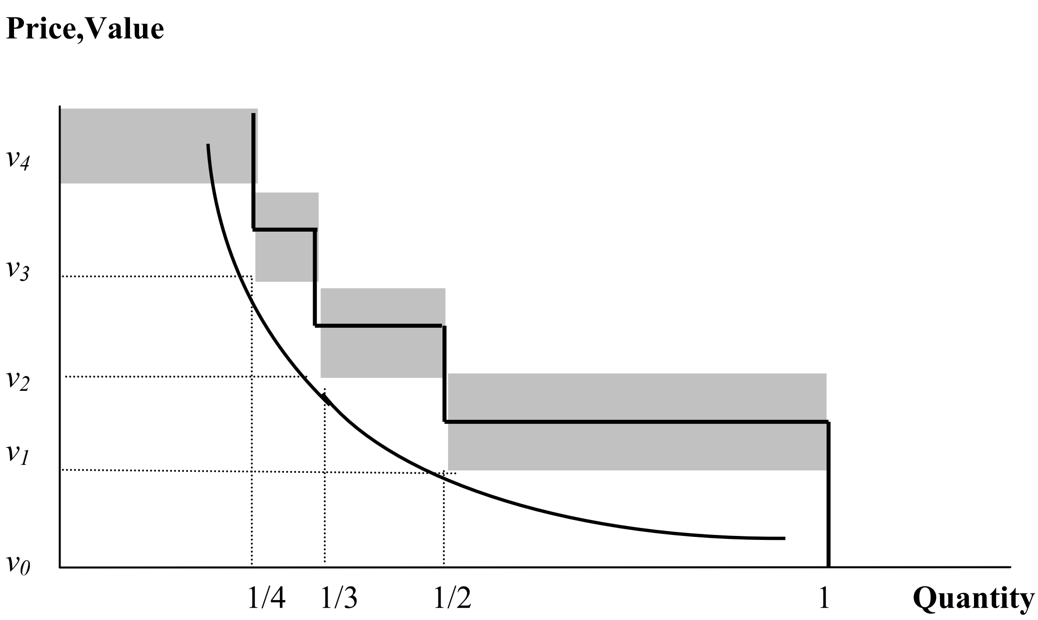
Now we check the strategies of L and U investors. H have ordered 1 at v_n if they follow the above strategy. if L or U investors place bids above v_0 they would either get no share (if they bid below v_n), or get shares but earn a nonpositive profit (if the bid is at v_n, they get a zero profit; if the bid is above v_n,, the market price would be raised to at least v_n), the expected payoff for them will be nonpositive and thus they cannot do better than by only placing bids at prices no higher than v_0. Since L and U investors can only get an allocation when the market value is v_0, they are symmetric in terms of information in the range [0, v_0]. Given what we have discussed in section 2.4.2.1, submitting a flat demand function at v_0 can be an equilibrium for them. The equilibrium in this area can also be the tacit collusion one, where all investors share the market at price zero.

So the strategy that each H investor bids for $\frac{1}{n}$ above v_n and $\frac{1}{n-1}$ at v_n, while L and U investors only place bids between zero and v_0 satisfies the four conditions and thus forms an equilibrium. As the resulting market price equals market value for any realization of $n \in [1, N]$, it is also the upper bound of the equilibrium set if multiple equilibria exist. This equilibrium can be considered as another kind of truth telling, in which an H investor anticipates that with probability π_h the market value is v_{h+1}, and the number of investors who observe high signals is h+1. If each of the H investor places $\frac{1}{h+1}$ above v_{h+1}, the expected demand above v_{h+1} is π_h = $\pi_h \frac{1}{h+1}(h+1)$. Hence the expected market demand is $1=\sum_{h=0}^{N-1} \pi_h$ above the expected market price $E(v|H)=\sum_{h=0}^{N-1}\pi_h v_{h+1}$, which is an H investor's expected market value. The demand function in this equilibrium is illustrated by the upper-right bound of the shaded area in Figure 2.1I, in which we illustrate the equilibrium when there are four informed investors.

The second situation is that the market price is lower than the market value, but the market price can be raised to at least as a high level as the market value if an L or U investor participates. In other words, though H investors' demand at v_n is no more than 1 (condition 3), there is no share left to fulfil for L or U investors at v_n:

$$nd(v_n, H) = 1 \text{ for each } n \in [1, N] \qquad \textbf{[11]}$$

Figure 2.1I Demand Curve of EH --- Discrete



Condition [11] implies that each H investor bids for $\frac{1}{n}$ at v_n for any possible realization of $n \in [1, N]$. A strategy where each H investor bids for $\frac{1}{n}$ at v_n for any $n \in [1, N]$ and places no bid between v_{n-1} and v_n (thus the demand curve between v_{n-1} and v_n is vertical) satisfies Condition 1 and 4, and both U and L investors cannot profitably deviate. According to Condition 3, the resulting market price is v_{n-1}. So it would be an equilibrium strategy if H investors cannot improve their payoff by deviating. Since the demand at v_n is already 1, and all demand above market price is fully allocated, an H investor cannot increase the allocation without raising market price to at least v_n. In that case, the payoff would be reduced to at most zero. As the demand of the other H investors at v_{n-1} is already 1 (n-1 investors each bids $\frac{1}{n-1}$ at v_{n-1}), an H investor can only lower the market price below v_{n-1} if she gives up the whole allocation. In this case, this bidder who deviates gets zero payoff. Hence an H investor cannot profitably deviate by either placing higher bids or lowering the market price. So the strategy we have described leads to an equilibrium. The lower-left bound of the shading area in Figure 2.1I describes this equilibrium.

For the same reason, any price between v_{n-1} and v_n could be a market equilibrium price depending on from which price H investors increase their demand. It can be achieved by bidding $\frac{1}{n-1}$ at any price $p_n \in [v_{n-1}, v_n]$. The bold step-bids demand curve in Figure 2.1I is such an example. When there are n high signals, the total demand at p_{n+1} is 1 and the market price is p_n. An H investor's allocation can only be increased if she raises market price to at least $p_{n+1} \geq v_n$, so the investor who does this would suffer a loss. And also, the total demand by the other n-1 H investors is 1 at p_n, so unless the investor does not place any bid at p_n at all and as a result get a zero profit, bidding less cannot reduce the market price further. Hence there exists a continuum of equilibria resulting in a market price between v_{n-1} and v_n. In this set of equilibrium, the market price increases with market value, but underpricing happens for any $n > 0$ except at the upper bound. Because if there are n high signals, H investors' demand exceeds 1 at least when the price equals v_{n-1} (n H investors each bids $\frac{1}{n-1}$ at v_{n-1}), the lower bound of the market price vector is $(v_0, v_1 ... v_{N-1})$.

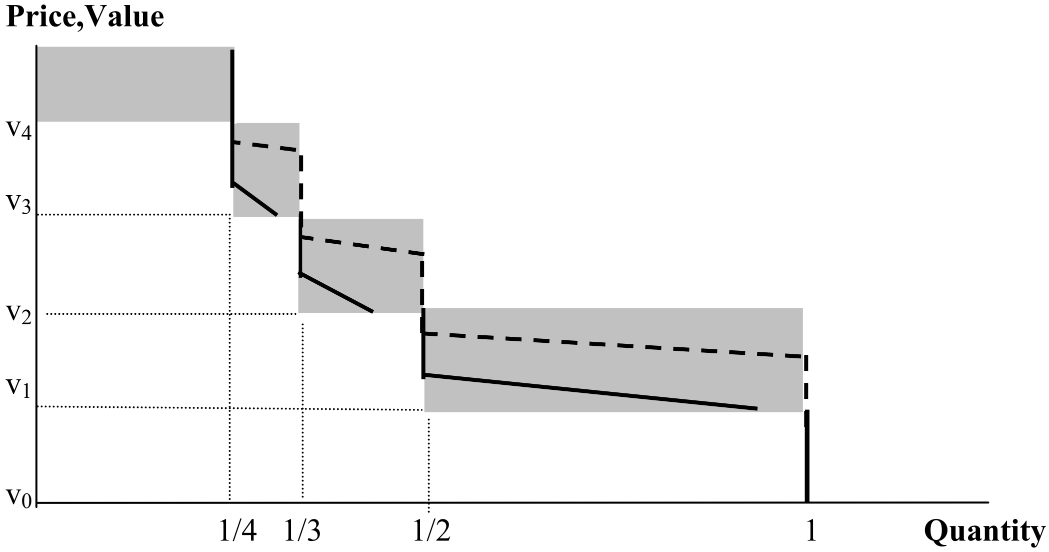
This method is not suitable for L investors because the correlation between the number of L investors and the market value is negative. If they try to share all the shares at each market value, the demand for each of them is the largest when the market value is the highest. Nevertheless, the demand curve cannot be upward sloping.

We have shown that the step-bids demand curve can be an equilibrium. In the continuous case, the demand curve below p_n (with quantity between $\frac{1}{n}$ and $\frac{1}{n-1}$) should be sufficiently flat.

Suppose the demand of each H investor at price $p \in (v_{n-1}, p_n)$ is $\min\left[\frac{1}{n} - \sigma(p - p_n), \frac{1}{n-1}\right]$ where $p_n \in (v_{n-1}, v_n]$. (See the dashed line and the continuous line in Figure 2.1II for examples). It simply implies that an H investor starts to increase her demand from market price p_n, with a nonpositive slope $-\sigma$, and the demand above price v_{n-1} is no more than $\frac{1}{n-1}$. So there is a kink in the demand function at p_n and it is differentiable only when price decreases. In order to make it unprofitable for H investors to cut the price, an H investor's profit $\prod(p, H)$ should non-decrease with the market price. This requires that $\sigma \geq \frac{1}{n(n-1)(v_n - p_n)}$ **[12]**

(Proof see Appendix 2.D). With a flat demand curve between v_{n-1} and p_n, if one H investor tries to lower the market price, her allocation would become so small that it offsets the benefit from price reduction.

Figure 2.1II Demand Curve of EH --- Partial Continuous[36]



In the discrete case, the probability that an H investor does not place any bids between p_n and a price lower than p_n is positive especially for small price gaps. So if an H investor does not increase her demand at p_n, the market price can be lowered below p_n with a positive probability without losing any allocation, except when the demand curve is horizontal at p_n (where she cannot lower market price without losing the whole allocation). This behaviour will not harm the investor who deviates, hence although in the continuous case the demand curve between demand $\frac{1}{n}$ and $\frac{1}{n-1}$ could have a flat slope, in the discrete case, the survival equilibrium demand function has to be horizontal ($\sigma = \infty$) at p_n.

The set of equilibria can be summarized by proposition 3.

Proposition 3: *If the demand of each H investor at price $p \in (v_{n-1}, p_n)$ is $\min\left[\frac{1}{n} - \sigma(p - p_n), \frac{1}{n-1}\right]$ for n >1 and equal to 1 for n =1, then H investors can absorb all the shares if $\sigma \geq \frac{1}{n(n-1)(v_n - p_n)}$ when $p_n \in (v_{n-1}, v_n]$ in the partial continuous case; and if $\sigma = \infty$ when $p_n \in [v_{n-1}, v_n]$ in the discrete case. L and U investors start bidding from price v_0 and only obtain an allocation if there is no high signal. Any price between v_{n-1} and v_n $(n>0)$ can be an equilibrium market price in this set.*

(We call the equilibrium set where H investors absorb all shares an EH equilibrium).

The lower bound of the equilibrium set weakly dominates the others. Not only because it is the most profitable equilibrium in this set, but also because it is a safe play. By choosing this strategy, an H investor obtains as large an allocation as the other H investors whatever equilibrium strategy in the EH set they choose, and with a positive probability, the other H investors do the same as her so they each enjoy the lowest possible market price in this equilibrium set. However, we don't eliminate the weakly dominated strategies because they are still Nash equilibria and in some experiments with two-unit demand, it has been observed that investors overbid on the first unit frequently (e.g. Engelbrecht-Wiggans, List and Reiley, 2005). In his textbook, David Kreps (1991) shows that ruling out weakly dominated strategies sometimes may end up picking a patently bad equilibrium. So we keep the whole equilibrium set for our future experimental examination.

[36] In this figure, we show that the demand curve below p_n (above v_{n-1}) do not need to be horizontal in the continuous case.

When N increases, the distances among the "steps" in the demand curve turn smaller so the demand curve becomes smoother. When N goes to infinity, we expect the demand curve to be convex, flatter when the price (value) is lower and the speed of turning flatter increases when the price is lower (see the curve in Figure 2.1I). The average demand schedule in IPO auctions in Israel provided by Kandel, Sarig and Wohl(1999) appears to have this shape. Also, when N goes to infinity, the difference between v_n and v_{n-1} becomes trivial so the market price is close to market value. This implies that unlike the property of the tacit collusion equilibrium, competition increases revenue. The other prediction of this equilibrium is that bidders would place more bids if the number of bidders increases because they have to include bids for any possible market value in their demand schedules. This is consistent with the evidence from the US Treasury bill market. Under the uniform price auction format, more investors participate in the market and large dealers have used new bidding strategies (compared with those under discriminatory auction) by splitting bids into more numerous smaller bids (Malvey, Archibald and Flynn, 1997).[37]

In the case when H investors absorb all the shares, the lowest possible price is v_{n-1}. If H investors try to improve their expected profits further, since they cannot get a larger allocation, they have to reduce the market price. But only by reducing their demand they can reduce the market price. Hence the total demand of H investors at v_n has to be less than 1. However, if this is the case, H investors would have to lose some shares to L and U investors, who can get a nonnegative expected payoff by compensating this demand gap. It has been argued that in multiunit uniform price auctions large bidders often make room for smaller ones by reducing demand or

[37] If bidders have the same expected value, this equilibrium reduces to the tacit collusion equilibrium. If each bidder observes a different signal, this equilibrium can be generalized in the way that n is the expected number of investors whose expected value is no lower than v_n, given the distribution of signals (which is common knowledge to all investors).

avoiding competition, especially if the smaller bidders have the ability to increase prices (Tenorio, 1997). We will check if this can be an equilibrium in the next section.

2.4.2.4 Equilibrium demand schedule when H investors absorb part of the shares

There are some facts in this game that are useful for our following equilibrium analysis.

Fact 1: If L investors share the market with H investors, the market price p_n has to be lower than the corresponding market value v_n.

Suppose that the market price equals the market value. Then to prevent an H investor from benefiting by lowering the price by bidding less, the demand of all the other investors at v_n has to be at least 1:

$$(n-1)d(v_n,H)+(N-n)d(v_n,L)+d(v_n,U)\geq 1$$

Otherwise an H investor can raise her profit from zero to a positive level. This means we also have

$(n-1)d(v_n,H)+(N-(n-1))d(v_n,L)+d(v_n,U)>1$ if $d(p,L)$ is larger than zero.

The equation implies that the total demand at price v_n is larger than 1 when there are n-1 H investors. In this case p_{n-1} equals v_n, and investors have zero profit when the realization of the market value is v_n and negative payoff when the market value is lower. So the expected payoff is negative, which violates condition 1 and thus in an equilibrium Fact 1 must hold.

Fact 2: If in an equilibrium the market price is lower than the market value, all shares should be allocated above the market price, i.e., no share is left for prorating at the market price.

According to the allocation rule, bids placed above market price are fully allocated; the shares left are prorated among the investors who bid at the market price.

So if the total demand above market price is less than 1, an investor can raise it to 1 by bidding more, while keeping her (cumulative) demand *at* the market price unchanged. Then her allocation would be increased but the market price would still remain the same. Since the market price is lower than the market value, increasing the allocation can lead to an increase in profits. Hence to prevent a profitable deviation, the cumulative demand above the market price has to be 1 and thus all the shares have to be absorbed above the market price.

Fact 3: If in equilibrium the market price is lower than the market value, and at least one type of investors is excluded from the market, the total demand at the market value has to be equal to 1.

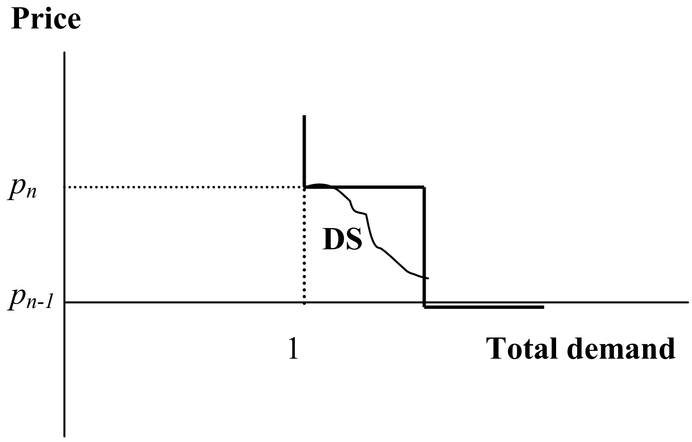
p_n lower than v_n requires that the total demand at v_n is no more than 1. If it is less than 1, then the investors who stay out of the market can get positive payoffs by raising the total demand at v_n to 1 or by a little bit less. To prevent the other investors from entering the market, the total demand of the existing investors has to be 1 at v_n.

Fact 3 also indicates that the demand function between v_n and p_n is vertical in equilibrium when not all investors participate.

Fact 4: If there exists an equilibrium where the market price is p_n when the market value is v_n, and $p_n>p_{n-1}$ for all $n>1$, there must also exist an equilibrium with the same market price levels and the demand curve between p_{n-1} and p_n vertical (see Figure 2.2).

The proof is straightforward. Suppose we have some equilibrium demand schedule (DS) under which the market price is p_n when there are n high signals. By keeping the demand curve between p_{n-1} and p_n vertical (for every $n>1$), when the realization of the number of high signals is n-1, the demand above p_{n-1} is still 1 and that at p_{n-1} is larger than 1, so both the market price and the allocation are kept at the same level as under DS. Moreover, as DS is downward sloping, it lies to the left of the vertical demand curve. So the residual supply under deviation is minimized under this piecewise vertical demand function. Thus the profit from deviation must be no more than that under DS. We shall use this piecewise vertical demand schedule to simplify our future equilibrium analysis.

Figure 2.2 A Vertical Demand Curve between p_{n-1} and p_n [38]



Fact 5: If there is no symmetric behaviour between H and L investors, the market price cannot be lowered below v_{n-1} when not all types of investors participate.

First, H investors cannot be excluded from the market in equilibrium.

To prevent H investors from participating, either the market price equals v_n, or the market price is lower than v_n but the total demand at v_n is 1 (Fact 3). Hence we should have

$$(N-n)d(v_n, L) + d(v_n, U) \geq 1$$

Because u investors in total can buy at most 1-*k*, they cannot absorb the entire shares, $d(v_n, L)$ is larger than zero.

So $(N-(n-1))d(v_n, L) + d(v_n, U) > 1$

[38] This figure illustrates that a vertical demand curve between pn-1 and pn exists in equilibrium, if a demand curve DS resulting the market price at p_n exists in equilibrium.

This means p_{n-1} is at least v_n. Since this should hold for every n>0, the expected profit for the participants is negative which violates condition 1.

So H investors must be included in equilibrium. If no symmetric behaviour exists between H and L investor (i.e. $d(p,H)=d(p,L)+c(p)$, $c(p) \neq 0$), the total demand changes with the number of high signals, so the market price changes with the market value. Similarly as before, to prevent either type of investors from participating, with n-1 H investors, we should have

$(n-1)c(v_{n-1}) \geq 1$ in the case that both L and U investors are absent; $(n-1)c(v_{n-1}) + d(v_{n-1},U) \geq 1$ in the case when L investors stay out of the market, $Nd(v_{n-1},L) + (n-1)c(v_{n-1}) \geq 1$ in the case when u investors stay out of the market.

We have assumed that $c(p)$ is nonnegative at the beginning of the equilibrium analysis[39]. If H and L investors are not symmetric in their behaviour, $c(p)$ is positive. With n H investors, the total demand at v_{n-1} is larger than 1. Thus the market price cannot be lower than v_{n-1}.

Hence if the market price could be lowered below v_{n-1}, all the investors have to participate in the market.

In the next section we allow L investors to share the market with H investors. In section II, we discuss the equilibrium where only L investors are excluded. At last, the equilibrium where all types of investors participate is introduced. With fewer restrictions, the equilibrium analysis becomes more complicated, so we focus on the equilibria in the discrete case. The equilibrium in discrete case still survives with

[39] In fact, c(p) has to be positive in this equilibrium. In the first two cases since $d(p,U) \leq 1-k$,$c(p)>0$. If $c(p) < 0$ in case 3, then we would have $Nd(v_{n-1},L)+(n-2)c(v_{n-1}) > 1$, so p_{n-2} is no lower than v_{n-1}. This violates condition 1 thus it is impossible in equilibrium.

continuous demand functions, but the reverse does not hold. As before, we still use the four conditions we have developed in section 2.4.1 to derive the following equilibria. The derivation of equilibria includes three steps. First the market clearing condition is used to satisfy the participation condition. Then another restriction such that the demand function is downward sloping or vertical is introduced. The last restriction guarantees that no player could profitable deviate. We then provide examples for each set of equilibria.

I. Equilibrium demand schedule when H and L investors share the market

Proposition 4: *There exists a continuum of equilibria with market prices between v_{n-1} and $v_n - \frac{d(v_{n-1},L)}{d(v_n,L)+c(v_n)}(v_n - p_{n-1})$ for any $n \in [1, N-1]$ when H and L investors share the market.* [40] *The U investor places bids no higher than v_0 and only obtains an allocation if there is no high signal, at a price that can be any price between 0 and v_0 in equilibrium.*

(We call this set of equilibria EHL.)

Proof: According to Facts 1 and 3, we know that the market price must be smaller than the market value when L investors participate, and the total demand of all informed investors at the corresponding market value is equal to 1:

$Nd(v_n,L)+nc(v_n)=1$ for any $n \in [1, N-1]$ **[13]**

[40] When n =0, there is no H investor in the market. L and U investors' strategy is the same as their strategies that we have described in section 2.4.2.3. An L investor would not accept a price higher than v_{N-1}, so H investors' strategy for prices over v_{N-1} is the same as the one that has been described in section 2.4.2.3 or that will be shown in the next section where H and U investors share the market.

So $c(v_n) = \dfrac{1 - Nd(v_n, L)}{n}$. Condition 1 is also satisfied when [13] is satisfied, and we have $d(v_n,L)+c(v_n) > d(v_{n-1},L)$.

Secondly, Condition 4 requires both L and the H investors' demand functions to be downward sloping or vertical:

$d(p_n,L)$ and

$$d(p_n,H) = d(p_n,L) + c(p_n) = \frac{1+(n-N)d(p_n,L)}{n}$$

must be nonincreasing with n. **[14]**

At last, Condition 2 requires that both types of investors are unable to profitably deviate. For simplicity, we focus on the case described in Fact 4 where investors submit piecewise vertical demand functions. Suppose investors' demand functions satisfy condition [13] and $Nd(p_n,L)+(n-1)c(p_n)=1$ for any $n \in [1,N-1]$, where p_n is no less than v_{n-1}. Then the demand curve between p_{n-1} and p_n is vertical (See Figure 2.3). When the realization of the number of high signals is n, the demand of all the investors above p_n is 1 and is $1+c(v_{n-1})$ at p_n, as is illustrated in Figure 2.3. Under condition [13], an allocation can only be increased if the price is raised above v_n, which leads to a negative payoff. So we only need one more condition such that it is unprofitable to cut the price:

$$(v_n-p_n)\,[d(v_n,L)+c(v_n)] \geq (v_n-p_{n-1})\,d(v_{n-1},L) \qquad \textbf{[15]}$$

Condition [15] is a simple transformation of the statement that by lowering the market price to p_{n-1} and taking all the residual supply above price p_{n-1} the payoff is no more than the one when following the previous stated strategy for an H investor.[41] Or, it is unprofitable for an H investor to pretend to be an L investor. If [15] is satisfied, investors also do not have an incentive to lower the market price further, say, to p_{n-2}

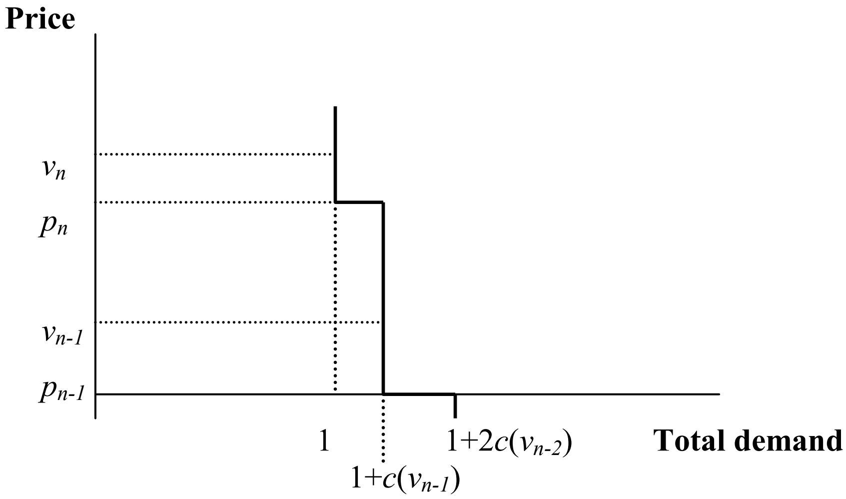
[41] If this condition is satisfied, L investors would not be able to profitable deviate as well.

under some conditions (See Appendix 2.E). [15] also indicates that

$p_n \le v_n - \dfrac{d(v_{n-1},L)}{d(v_n,L)+c(v_n)}(v_n - p_{n-1})$ for any $n \in [1, N-1]$. Together with Fact 5, the

market price p_n lies in $[v_{n-1},\ v_n - \dfrac{d(v_{n-1},L)}{d(v_n,L)+c(v_n)}(v_n - p_{n-1})]$ for any $n \in [1, N-1]$.

Figure 2.3 Demand Curve (part) in EHL



□

Notice that p_n can only be equal to v_n if $d(,L)$ is zero (EH then), which is consistent with what we have shown in Fact 1. The more allocation that H investors lose to L investors, the lower the market price could be. Because $v_n - \dfrac{d(v_{n-1},L)}{d(v_n,L)+c(v_n)}(v_n - p_{n-1})$ increases with p_{n-1}, a lower p_{n-1} requires a lower p_n.

Hence if conditions [13] to [15] are satisfied, we can have a continuum set of equilibria described in Proposition 4. The simplest example is that each L investor places bids according to a vertical demand function, i.e. $d(p,L)$ is a constant[42]; while H investors demand an additional amount of $\dfrac{1-Nd(,L)}{n}$ in the price range $(p_n, p_{n+1}]$. For

[42] When $d(p,L)=0$, the equilibrium is EH; $d(p,L)$ is a positive constant smaller than $\min[\dfrac{1}{N + n(v_{n-1} - v_{n-2})/(v_n - v_{n-1})}](n \in [2, N-1])$ such that the upper bound of p_n is no less than v_{n-1}. If each high

any $n \in [1, N-1]$, p_n is located within the range $[v_{n-1}, v_n - \frac{nd(,L)}{1-(N-n)d(,L)}(v_n - p_{n-1})]$.

Another example is that each L investor's demand function is $d(p_n,L) = \frac{K}{N(n+K)}$ and an H investor demands an extra amount $\frac{1}{n+K}$ at any $p \in (p_n, p_{n+1}]$, where $p_n \in [v_{n-1}, v_n - \frac{K(n+K)}{(n-1+K)(N+K)}(v_n - p_{n-1})]$ for any $n \in [1, N-1]$ and K is a positive constant.[43] In both examples the total demand above p_n (thus at v_n) is 1, demand functions are downward sloping, and condition [15] is also satisfied so no one could profitably deviate. The upper bound of the price range in both examples decreases with an L investors' allocation, which means that the larger the allocation that H investors lose to L investors, the lower that market price should be. Also, the market price could equal the market value only if an L investor does not get any allocation ($d(,L)=0$, or $K=0$). Because L investors have lower expected market value, they only participate when there exists (sufficient) underpricing.

Next, in order to make u investors' behaviour clearer, we consider what the equilibrium is if L investors are excluded from the market, i.e. $d(p,L) = 0$ for $p > v_0$.

II. Equilibrium demand schedule when H and u investors share the market

Proposition 5: *There exists a continuum of equilibria with market prices between v_{n-1} and v_n for any $n \in [1, N]$ when H and u investors share the market*[44]*. L investors*

signal increases by the same amount the market value, i.e., if $v_{n-1}-v_{n-2} = v_n-v_{n-1}$, the maximum value of $d(,L)$ reduces to $1/(2N-1)$.

[43] K is no higher than $[(N-1)(v_n - v_{n-1})/(v_{n-1} - p_{n-1})] - 1, n \in [1, N-1]$ such that the upper bound of the price range is no lower than v_{n-1}. If each additional high signal increases by the same amount the market value, the condition is satisfied if K is no higher than N-2.

[44] When $n=0$, there is no H investor in the market. L and U investors' strategies are the same as their strategies that we have described in section 2.4.2.3.

place bids no higher than v_0 and only obtain an allocation if there is no high signal , at a price that can be any price between 0 and v_0 in equilibrium.

(We call this set of equilibria EHU.)

Proof: To prevent L investors from entering the market, the demand at v_n should be 1 if $p_n<v_n$ (Fact 3);

$nc(v_n) + d(v_n, U) = 1$ for any $n \in [1, N]$ if $p_n<v_n$ **[16a]**

If the market price equals the market value, to prevent an H investor or the U investor from deviating, the other players' total demand has to be at least 1 at price v_n, but the total demand at v_n when there are n-1 H investors should be no more than 1, so

$(n-1)c(v_n) + d(v_n, U) = 1$ for any $n \in [1, N]$ if $p_n = v_n$ **[16b]**

thus $c(v_n) \leq \dfrac{1}{n-1}$ for any $n \in [0, N]$.

Also, the demand functions should be non-increasing in the price. So the U investor's demand and each H investor's demand at p_n must be no higher than that at p_{n-1}:

$d(p_n, U) \leq d(p_{n-1}, U)$ and

$c(p_n) \leq c(p_{n-1})$

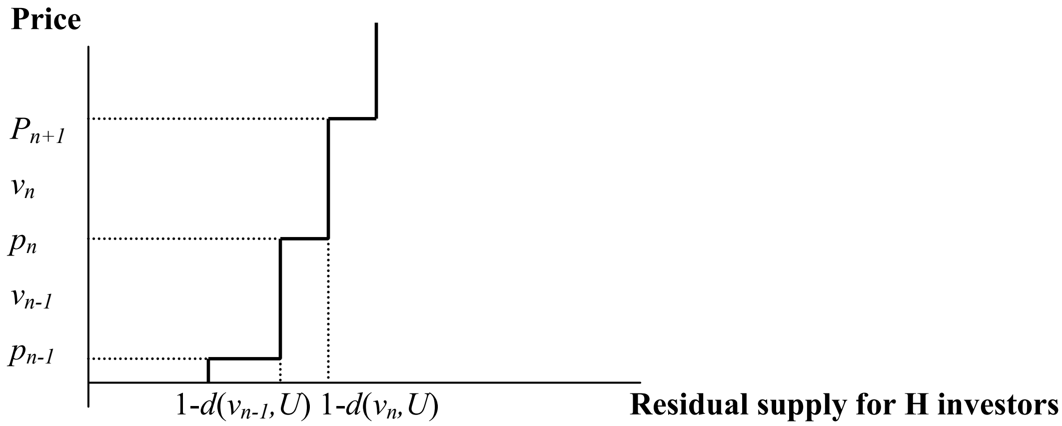
(i.e. $\dfrac{1-d(p_n,U)}{n} \leq \dfrac{1-d(p_{n-1},U)}{n-1}$ which requires $\dfrac{d(p_{n-1},U)-d(p_n,U)}{1-d(p_n,U)} \leq \dfrac{1}{n}$)

So

$0 \leq d(p_{n-1},U) - d(p_n,U) \leq c(p_n)$. **[17]**

Similarly, as before, to be a NE, no player could be better off by any deviation. For simplicity, we only consider the situation when the demand curve is vertical between (p_{n-1}, p_n] (Fact 4). The residual supply faced by all the H investors, which equals 1 minus the demand of the U investor, is illustrated in Figure 2.4. Here each H investor faces a similar scenario as that under EH, when considering to absorb all the residual supply above the market price p_n.

Figure 2.4 Residual Supply of H investors in EHU



Consider the following strategy: the U investor bids for $d(v_n,U) \in [0,1-k]$ and each H investor bids for $\frac{1-d(v_n,U)}{n}$ in $(p_n,p_{n+1}]$ for any $n \in [1,N]$. Under this strategy, the total demand at price v_n is already 1 when the market value is v_n, so no investor could be better off by raising the price in order to increase the allocation. To make it unprofitable to lower the market price, for an H investor, the payoff under this strategy should be no less than that when lowering price to p_{n-1} and taking all the residual supply at that price level:

$$(v_n - p_n)\frac{1-d(v_n,U)}{n} \geq (v_n - p_{n-1})(1-d(v_{n-1},U)-(n-1)c(v_{n-1}))$$

This is always satisfied because $d(v_{n-1},U)+(n-1)c(v_{n-1})=1$ (from [16]) so the right hand side is zero. An H investor can only reduce the market price by giving up the whole allocation.

To make a deviation by the U investor unprofitable, when she gives up part of the allocation and lowers market price to p_{n-1}, the profit from the deviation should be no more than that under the above strategy:

$$(v_n - p_n)d(v_n, U) \geq (v_n - p_{n-1})(1 - nc(v_{n-1})) \qquad \textbf{[18]}$$

The condition is satisfied if $d(v_{n-1}, U)$ is less than $c(v_{n-1})$ (thus no residual share is left at p_{n-1}), otherwise it requires $\frac{d(v_n, U)}{d(v_{n-1}, U) - c(v_{n-1})} \geq \frac{v_n - p_{n-1}}{v_n - p_n}$. It can be shown that if [18] is satisfied, an even lower market price is also unprofitable. (see Appendix 2.E).

When the market price p_n equals v_n, conditions [16] to [18] are satisfied when $c(v_n)$ equals $d(v_n, U)$ equals $\frac{1}{n}$. Under this strategy, the demand of the other players is 1 at v_n, and the total demand above v_n is also 1, so no investor can be better off by either reducing price or increasing the allocation. Similar as in EH, if every investor bids for $\frac{1}{n-1}$ at any price $p_n \in [v_{n-1}, v_n]$ while keeping the demand above p_n as $\frac{1}{n}$, the strategy is a NE resulting in a market price at p_n. In fact, if the market price is lower than v_n, the total demand at v_n is 1 (Fact 3). Investors choose a price p_n from which to increase demand and the market price will be p_n. The equilibrium is similar as the EH with one more H investor, i.e. where the u investors as a whole bid the same as an H investor.

So when H and U investors share the market, the price can be any between v_{n-1} and v_n in a NE. □

Here is another example of the possible equilibrium strategies. Suppose in equilibrium $d(p, U)$ is a constant between zero and $1-k$. Then H investors' strategy is similar to that in EH, except that now the supply they face is 1-$d(,U)$ instead of 1. So instead of $\frac{1}{n}$ in proposition 3, each H investor bids for $\frac{1-d(,U)}{n}$ in (p_n,p_{n+1}], and for the same reason as before, they are unable to profitably deviate.

We still assume that the demand curve between p_{n-1} and p_n is vertical. The residual supply to the U investors is $\frac{nd(,U)-1}{n-1}$ above p_{n-1} when there are n high signals. Condition [18] requires

$$d(,U) \leq \frac{1}{1+(n-1)\frac{p_n - p_{n-1}}{v_n - p_{n-1}}}$$

The right hand side decreases with p_n, so the condition is satisfied as least when $p_n = v_n$, i.e. $d(,U) \leq \frac{1}{n}$. Since the U investor's demand curve is constant, it is satisfied at least when $d(,U) \leq \frac{1}{N}$.

III Equilibrium demand schedule where all investors participate

Now we are ready to check the equilibrium when all the investors participate. According to Fact 2, we should have the following relationship in equilibrium:

$$Nd^a(p_n, L) + nc^a(p_n) + d^a(p_n, U) = 1 \qquad \textbf{[19]}$$

The equilibria that we have proved so far can all be nested in this equation. In the tacit collusion equilibrium, where the market price is constant regardless of the market value, this equation can be satisfied only if $c^a(p_n)$ is zero, so that the left hand side of the equation does not relate to *n*. Because H and L investors are symmetric in their bidding behaviour. In the set of EH equilibria, both $d^a(p_n, L)$ and $d^a(p_n, U)$ are zero, so $c^a(p_n) = d^a(p_n, H)$ has to be $\frac{1}{n}$. To satisfy the participation condition, p_n should be no higher than v_n. To prevent the other type of investors from entering the market, subject to Fact 2, the lowest possible price in equilibrium is v_{n-1}. EHL and EHU are the equilibria when $d^a(p_n, U)$ and $d^a(p_n, L)$ equal zero respectively.

When all the investors participate, in condition [19] $p_n < v_n$ (Fact 1) and $d^a(p_n, L)$, $c^a(p_n)$ and $d^a(p_n, U)$ are all positive.

From the previous proofs we see that the restriction for the lowest possible price, v_{n-1}, is subject to Fact 3. The demand at the price that equals the market value has to be at least 1 in order to keep one type of investor out of the market. However, when all the investors participate, we do not need this restriction, so it is possible to have a lower market price than v_{n-1}.

As before, the second restriction is that the demand functions of all types investors do not increase with price. So for any $n \in [1, N]$

$d(p_n, L) \le d(p_{n-1}, L)$,

$d(p_n, U) \le d(p_{n-1}, U)$ and

$d(p_n, L) + c(p_n) \le d(p_{n-1}, L) + c(p_{n-1})$ i.e.

$$\frac{1 - d(p_n, U) - (N - n)d(p_n, L)}{n} \le \frac{1 - d(p_{n-1}, U) - (N - (n-1))d(p_{n-1}, L)}{n-1} \qquad \textbf{[20]}$$

At last, no bidder should be able to improve the payoff by any kind of deviation. We still assume that investors' demand functions are vertical between p_{n-1} and p_n. So the best way to deviate is to raise or to lower price to p_i ($i \neq n$) and absorb all the residual supply at that price level. Hence to ensure an equilibrium, the profit of an investor under that strategy must be no higher than that under any other price quantity combination:

$(v_n - p_n)d^a(p_n, s) \geq (v_n - p_i)r_n(p_i, s)$ for any $i \neq$ n, $i \in [0, N]$ **[21]**

Let s represents signals, can be either H, L or U ($d(p,H)=d(p,L) + c(p)$); $r_n(p_i,s)$ is the residual supply for a bidder who has signal s at price p_i when there are n high signals.

In Fact 1, we have shown that the market price should be lower than the corresponding market value should all the investors participate. What is the highest affordable market price then? From [21] we have

$$p_n \leq v_n - \max_{S,i}\left(\frac{(v_n - p_i)r_n(p_i, s)}{d^a(p_n, s)}\right) \text{ where } i \in [0, n-1]$$

The right hand side iecreases with p_i^*, the price (together with the choice of s) that maximizes the second term of the right hand side. Since this relationship exists for any realization of n, if the price discount is large for some market value, it should be also large in the other possible market values[45]. It also decreases with the ratio of $\frac{r_n(p_i, s)}{d^a(p_n, s)}$, so that the larger the allocation an investor has to give up should she try to lower the price, the higher the market price could be; and the more allocation an investor could gain should she raise the market price, the lower the market price should be.

If we introduce a restriction such that an investor cannot improve profit by lowering (or raising) price further to, say, p_{n-2}(or p_{n+2}), when she cannot be better off by lowering (or raising) the price to p_{n-1}(or p_{n+1})(see Appendix 2.E), then we only need to check if [21] is satisfied at the "neighbour prices", i.e., at the price p_{n-1} (for n>0) and p_{n+1} (for n<N). Then [21] can be rewritten as:

[45] Suppose a set of choice (s,i) maximizes $\frac{r_n(p_i, s)(v_n - p_i)}{d^a(p_n, s)}$. Then the larger that v_n-p_i is (indicating the larger price discount for v_i), the smaller that p_n should be.

$$\frac{v_n - p_n}{v_n - p_{n-1}} \geq \max\left(\frac{r_n(p_{n-1}, s)}{d^a(p_n, s)}\right)^{46}, \text{ and } \frac{v_n - p_n}{v_n - p_{n+1}} \geq \max\left(\frac{r_n(p_{n+1}, s)}{d^a(p_n, s)}\right)^{47} \text{if } p_{n+1} < v_n.$$

If $p_{n+1} \geq v_n$, investors will not try to increase price and we do not need the second inequality. The right hand sides in both inequalities represent the maximum ratio that each player's residual supply given a deviation against the allocation without a deviation, for the signals H, L or U.

No other restrictions are needed for an equilibrium. We introduce an example below to explore this complicated situation. Consider the following strategy:

Each L investor submits a vertical demand function, i.e., the demand is constant, say, $K \in [0, \frac{1}{N}]$,at any prices. The extra demand of an H investor, $c^a(p)$, is the same as that of the U investor.

According to[19]:

$NK + (n+1)\, c^a(p_n) = 1$

So $c^a(p_n) = \frac{1 - NK}{n+1}$, which satisfies condition [20].

Then [21] requires

$$(v_n - p_n)K \geq (v_n - p_{n+1})\left(K + \frac{1 - NK}{n+2}\right),\ n \in [0, N-1]$$

The above inequality simply implies the condition that an L investor gets no more profit by deviating than by following the stated strategy. It can be shown that an L investor benefits the most from a deviation by increasing price to p_{n+1} compared with

[46] The right hand side takes the maximum value among all the ratios for different signals, which are $\frac{d^a(p_{n-1}, L)}{d^a(p_n, L) + c^a(p_n)}, \frac{d^a(p_{n-1}, L) - c^a(p_{n-1})}{d^a(p_n, L)}, \frac{d^a(p_{n-1}, U) - c^a(p_{n-1})}{d^a(p_n, U)}$ for an H, L or U investor respectively.

[47] The right hand side takes the maximum value among all the ratios for different signals, which are $\frac{d^a(p_{n+1}, L) + 2c^a(p_{n+1})}{d^a(p_n, L) + c^a(p_n)}, \frac{d^a(p_{n+1}, L) + c^a(p_{n+1})}{d^a(p_n, L)}, \frac{d^a(p_{n+1}, U) + c^a(p_{n+1})}{d^a(p_n, U)}$ for an H, L or U investor respectively.

either an H or U investor. [48]So if it were still unprofitable for her to deviate, no other player would like to change strategy. It is also unprofitable to further increase the allocation by raising the price even higher if this restriction is satisfied (see Appendix 2.E). [21] also requires that

$$(v_n - p_n)(K + \frac{1-NK}{n+1}) \geq (v_n - p_{n-1})K \text{ , } n \in [1, N]$$

The above inequality makes it unprofitable to deviate in this way even for an H investor. When deviating by lowering price to p_{n-1}, an H investor benefits the most than either an L or the U investor. It is also unprofitable to decrease price further if this restriction is satisfied (see Appendix 2.E).

Rearranging them we have

$$\frac{v_n - p_{n+1}}{(n+2)(p_{n+1} - p_n)} \leq \frac{K}{1-NK} \leq \frac{v_n - p_n}{(n+1)(p_n - p_{n-1})} \qquad \textbf{[22]}$$

When [22] is satisfied, the strategy we have stated forms an equilibrium.

Considering condition [22] further. $\frac{K}{1-NK}$ increases with K, an L investor's demand. The higher K is, the larger $\frac{v_n - p_{n+1}}{p_{n+1} - p_n}$ could be and $\frac{v_n - p_n}{p_n - p_{n-1}}$ should be for any realization of n. This implies that the price could and should be discounted more (larger v_n-p_{n+1} and v_n-p_n) and the closer the neighbour prices should be located (smaller p_{n+1} - p_n and p_n-p_{n-1}), because the lowest market price p_0 is bounded. The higher the price discount H investors try to achieve, the more allocation they have to lose to L investors. In the extreme case, when K equals $\frac{1}{N}$, p_n should equal p_{n-1} so the price should be constant. This is consistent with the tacit collusion equilibrium

[48] In other words, $\frac{r_n(p_{n+1}, s)}{d^a(p_n, s)}$, takes the maximum value for signal L.

resulting in the market price at the lowest possible level p_0. In the other extreme case, when K equals zero, p_n should be no higher than v_n but no lower than v_{n-1} (as v_n-p_{n+1} is nonpositive for every n when K is zero), this is consistent with the equilibria EH and EHU. For values of K in between, we expect that the market price can be any one between p_0 and $v_n - \max_{S,i}(\frac{(v_n - p_i)r_n(p_i,s)}{d^a(p_n,s)})$, i.e. the highest affordable price when the market value is v_n which we have gotten before from [21]. Because with higher n the larger $\frac{v_n - p_{n+1}}{p_{n+1} - p_n}$ could be and $\frac{v_n - p_n}{p_n - p_{n-1}}$ should be, higher market value corresponds to severe price discounts at least when each high signal contributes the same amount to the market value.

Proposition 6: T*here exists a continuum of equilibria where all investors participate resulting in a market price in the range from zero to* v_n *for any* $n \in [0, N]$.

Note that the equilibrium strategy takes into account of the Winner's Curse, so the resulting market price in equilibrium is no more than the corresponding market value for any realization of the number of high signals. It also includes a demand reduction aspect. Hence with only exception the upper bound of the equilibrium set where price always equals value, underpricing always takes place.

2.5. Conclusions

The previous sections have provided five sets of equilibria, where the first four can all be nested in the last one. Here we summarize them for convenience.

1. There is no equilibrium under a flat demand function in uniform price IPO auctions where bidders are asymmetric.

2. A continuum of tacit collusion equilibria exist in the model in both the continuous and the discrete cases with a market price between the reservation price and the expected value given a low signal. The equilibrium that results in the market price at the reservation price level weakly dominates the other tacit collusion equilibria.

3. In the equilibrium where H investors absorb all shares, the market price can be any price between v_{n-1} and v_n. However, the strategy that leads to market price at v_{n-1} weakly dominates the other strategies in this set.

4. In the equilibrium where H and L investors share the market, the market price can be any price between v_{n-1} and $v_n - \frac{d(p_{n-1}, L)}{d(p_n, L) + c(p_n)}(v_n - p_{n-1})$. The larger the allocation that H investors lose to L investors, the lower the upper bound of the price range can be.

5. In the equilibrium where H and u investors absorb all the shares, the market price can be any price between v_{n-1} and v_n.

6. In the equilibrium where all types of investors participate, the market price can be any price between the zero and v_n (with zero reservation price). Equilibria 2 to 5 are all special cases in this equilibrium set.

For the price range between zero and v_0 (in the case where there is no reservation price), either a flat demand function or tacit collusion can be an equilibrium for L and U investors. Because investors have the same information in this range, i.e., the market value is at least v_0, they can behave symmetrically.

Hence not only there exist multiple equlibria resulting in different market prices, but a certain market price can result from different equilibrium strategies. The volatility observed in actual uniform price auctions may be explained by the existence of a continuum equilibrium set. Among these equilibria, the tacit collusion equilibrium where H and L investors share the market equally regardless their signals is the most profitable strategy for the investors as a whole. However, it is also the most "risky" one in the sense that if some investors bid more aggressively than the equilibrium strategy, or intend to follow the strategies under the other equilibrium sets, the market price may be raised too high. As the demand function is generally steep in this equilibrium, a small amount increase may be large enough to result in a jump of the market price. In other words, playing this equilibrium requires that each bidder believes that her rivals will not raise demand at a price higher than her expected market value. This may happen if a small number of bidders play repeatedly (e.g. the electricity market in England and Wale, see Wolfram, 1998). However, as many institutional investors and individual investors are involved in buying initial public offerings, the collusion of a small number of parties is not a likely scenario. But an equilibrium is an equilibrium. Potentially it may be realized. As Ausubel (2002) argues, any auction procedures need to be used in conjunction with good anticollusion mechanisms. In general, in auctions where either a small number of bidders participate, or some bidders are significant in size relative to the auction volume, the competition is limited and the auctioneer needs to address the potential exercise of market power in the auction design (Ausubel and Cramton, 2004). A policy that might be effective for the seller is to choose the quantity after investors place their bids. This would allow the seller to "pick off" any high "inframarginal" bids that are submitted, so it might eliminate the collusive equilibria. The Mexican Treasury has this option, which may explain Umlauf's results that collusive behaviour is mitigated in uniform price auction compared with the discriminatory auction. The French sealed-bid auction *Offre à Prix Minimal*, a modification of a uniform price auction is recommended by some researchers according to both theoretical analysis (McDonald and Jacquillat, 1974) and empirical evidence (Biais and Faugeron-Crouzet 1999; Derrien and Womack,2001). In the settings of this auction, the market price is set according to the accumulated demand schedule, usually lower than the market-clearing price; such that the investors' ability of managing the market price is weaken. Widening the market is also an effective way to prevent collusive behaviour. Attracting more investors by including small investors not only makes it impossible for all investors to follow the tacit collusion equilibrium, but also makes it difficult for some large investors to collude because of the large uncertainty in demand. So a good auction design should encourage small investors to enter the market by policies such as allowing smaller investors the opportunity to participate in the auction by purchasing a smaller block of shares, and lower the threshold for opening accounts to bid. For example, Google has lowered the minimum number of shares individuals can bid on to 5 from the traditional 100, but it was still hard for unsophisticated investors to open accounts in order to compete.

On the other hand, demand reduction, as we have shown in the equilibrium analysis, does exist in every equilibrium set, where the vertical demand functions between two neighbour prices in equilibrium is similar as the tacit collusion strategy, but in "local areas" only. So all the equilibrium sets have the underpricing character, though the extent of the price discount is different. This suggests that underpricing in auctions for IPOs is taken by investors who determine the market-clearing price. This claim is contrary to the signalling hypothesis, which predicts that underpricing is taken by firms who signal their high quality to investors in order to get more capital equity in the subsequent offerings.[49] From investors' demand functions and resulting market clearing prices, one can get some insights of the real market value of shares. So the information revealed upon the completion of the auction may explain the usual price increase in IPO shares when they start trading in stock markets.

Another issue is that the market return prior to an offering may impact investors' choices of strategy. The empirical work carried out by Derrien and Womack (2001) shows that market return and volatility are significant ex ante predictors of the level of underpricing in French IPOs. We then expect investors to choose a more conservative strategy in cold markets and a more aggressive strategy in hot markets[50]. Moreover, in a hot market a large number of inexperienced bidders may place bids at very high levels to ensure some allocation, in order to obtain the high abnormal return on the first trading day. When the number of such bidders is sufficiently large, the market price may be raised too high. Hence, it may be necessary to introduce some rules to manage the possible crazy behaviour, and, ideally, to induce investors to choose the strategy that results in higher profits for the seller and a wider distribution of shares. In *Offre à Prix Minimal* auctions, bids that are higher than a certain level are ruled out, so those "free-ride" bids without providing any information on market value can be eliminated. Google has issued a warning to investors stating that over-enthusiastic bidding might push the price of shares to an unsustainable level that has no relationship with the actual value of the company. This education seems to have a positive effect in the sense that Google has chosen a reasonable offering price[51] where

[49] The empirical findings in French IPO markets show that the signalling hypothesis can explain underpricing in fixed-price offerings, but it is not the reason for underpricing in auctions (Faugeron-Crouzet, Ginglinger and Vijayraghavan, 2000).

[50] A cold market means that when offered shares start trading in the stock market, the extent of price increase is small or even negative thus investors obtain low abnormal returns. A hot market takes the opposite meaning.

[51] In the IPO auction, Google has chosen to allocate shares to meet only 75 percent of the demand at $85. The close price of the shares' first trading day was $100.33 (Vise, 2004). A soar of $15 in price when the shares debut "looks" good for the company as it shows big market confidence and interest; it is also a reward to the investors who showed their interest and successfully bought shares in the first place; and, though Google may not meant to

a large number of inexperienced investors show enthusiasm for participating in its IPO, though it may discourage some potential (inexperienced) investors as well.

We have demonstrated a continuum set of equilibria in pure strategies in this chapter. There might exist other equilibrium in mixed strategies that we have not explored. However, it seems implausible that people use mixed strategy in IPO auctions. In the next chapter, we will present some results from our experiment. Hopefully it may shed some light on the possible strategies that investors may choose in real markets for IPOs.

do that, it gave as good as it got from the Wall Street, who wishes a failure of the uniform price auction and at least a low reward to investors, even after Google's success (Sloan,2004).

Appendix 2.A:

Part I:

$\sum_{h=1}^{N-1}\pi_h\frac{1}{h+1}[v_{h+1}-p]$ ***is negative when*** $\sum_{h=1}^{N-1}\pi_h[v_{h+1}-p]$ ***is zero.***

Proof: Because both items strictly decrease with p, the statement is true if $\sum_{h=1}^{N-1}\pi_h[v_{h+1}-p]$ is positive when $\sum_{h=1}^{N-1}\pi_h\frac{1}{h+1}[v_{h+1}-p]$ is zero. So it is equivalent to prove that $\sum_{h=1}^{N-1}\pi_h[v_{h+1}-p^0]$ is positive, i.e., $\frac{\sum_{h=1}^{N-1}\pi_h v_{h+1}}{\sum_{h=1}^{N-1}\pi_h} > p^0$, where p^0 denotes a price such that $\sum_{h=1}^{N-1}\pi_h\frac{1}{h+1}[v_{h+1}-p]$ equals zero. Solving for p^0 we get:

$$p^0 = \frac{\sum_{h=1}^{N-1}\frac{\pi_h v_{h+1}}{h+1}}{\sum_{h=1}^{N-1}\frac{\pi_h}{h+1}}$$

Hence we need to prove that $\frac{\sum_{h=1}^{N-1}\pi_h v_{h+1}}{\sum_{h=1}^{N-1}\pi_h} > \frac{\sum_{h=1}^{N-1}\frac{\pi_h v_{h+1}}{h+1}}{\sum_{h=1}^{N-1}\frac{\pi_h}{h+1}}$. By rearranging the inequality what we need to prove is that $\sum_{h=1}^{N-1}\pi_h v_{h+1}\sum_{h=1}^{N-1}\frac{\pi_h}{h+1} - \sum_{h=1}^{N-1}\pi_h\sum_{h=1}^{N-1}\frac{\pi_h v_{h+1}}{h+1} > 0$. The left hand side of this inequality equals

$$\sum_{h=1}^{N-1}\sum_{i=1}^{N-1}\pi_h v_{h+1}\frac{\pi_i}{i+1} - \sum_{h=1}^{N-1}\sum_{i=1}^{N-1}\frac{\pi_h v_{h+1}}{h+1}\pi_i = \sum_{h=1}^{N-1}\sum_{i=1}^{N-1}\pi_i\pi_h v_{h+1}\left(\frac{1}{i+1}-\frac{1}{h+1}\right) \quad \textbf{[23]}$$

Both i and h take the value of each integer between 1 and N-1, and thus in the expanded expression there will be N-1 times h (for $h = i$), $\frac{(N-1)^2-(N-1)}{2}$ times h (for $h > i$) and $\frac{(N-1)^2-(N-1)}{2}$ times i (for $i>h$) terms, respectively. The terms corresponding to $h = i$ in the expanded expression are equal to zero. For each combination of $i = x< h = y$, we have an inverse combination of $i = y> h =x$. The sum of each pair of these combinations is $\pi_x \pi_y c(v_{y+1} - v_{x+1})$ where c equals $\frac{1}{x+1} - \frac{1}{y+1}$.

Because x< y, c is positive, and so is $v_{ly+1} - v_{x+1}$. Hence equation [23] is positive. □

Part II:

When $\pi_0 [v_1 - \max(p_U, p_l)]$ is negative, [5] is negative when [6] equals zero.

$$E(\Pi \mid H) = \sum_{h=1}^{N-1} \pi_h \frac{1}{h+1} [v_{h+1} - p] + \pi_0 [v_1 - \max(p_U, p_L)] \quad \textbf{[5]}$$

$$E(\Pi_d \mid H) = \sum_{h=1}^{N-1} \pi_h [v_{h+1} - p] + \pi_0 [v_1 - \max(p_U, p_L)] \quad \textbf{[6]}$$

which can be written as:

$$E(\Pi \mid H) = \sum_{h=0}^{N-1} \pi_h \frac{1}{h+1} [v_{h+1} - p] + \pi_0 [p - \max(p_U, p_L)] \quad \textbf{[5]}'$$

$$E(\Pi_d \mid H) = \sum_{h=0}^{N-1} \pi_h [v_{h+1} - p] + \pi_0 [p - \max(p_U, p_L)] \quad \textbf{[6]}'$$

Proof:

When p is less than or equal to $E(v| H)$: [6] is nonnegative because max (p_U, p_L) is no higher than p[52](see [6] ′). According to [7], the first term of [6]′ is higher than that of

[52] We have mentioned in section 2.4.2.1 that p is higher than $E(v)$.

[5]′. Because the second terms of both [6]′and [5]′ are identical, [6]′is always higher than [5]′ .

Thus the price that sets [6] equal to zero is no lower than $E(v|\ H)$. According to Part I of Appendix 2.A, at the price such that the first term of [6] is zero, say, price p', the first term of [5] is negative. Since the second terms of [5] and [6] are identical and negative, [5] is smaller than [6] and [6] is smaller than zero at p'. Recall that both [5] and [6] strictly decrease in *p*, so that we can conclude that [6] lies above [5] everywhere when the price is below p' (regardless of the rate at which profits decrease as the price increases), which includes the price that sets [6] equal 0. Hence [5] is negative when [6] is zero.

So an H investor has an incentive to bid at a higher price until the expected profit from absorbing all shares, given by [6], becomes zero. However, at that price the profit of sharing shares with other H investors ([5]) is negative and thus the strategy cannot be an equilibrium one. □

Appendix 2.B:

Derivation of the slope value when each u investor's demand function is equal to

$$r_u - \frac{\sigma(p - p_0)}{1-k}.$$

The total demand *above* the reservation price p_0 is 1:

$$(1-k)r_u + Nd^a(p_0, HorL) = 1$$

Solving this equation we get $d^a(p_0,\text{HorL}) = \dfrac{1-(1-k)r_u}{N}$

The residual supply for each informed player at price *p* is:

$$1-(1-k)r_u-(N-1)\frac{1-(1-k)r_u}{N}+N\sigma(p-p_0)$$
$$=\frac{1-(1-k)r_u}{N}+N\sigma(p-p_0)$$

An H investor wants to maximize her expected profit $\Pi(p)$ by choosing the price p:

$$\Pi(p)=\left[E(v\mid H)-p\right]\left[\frac{1-(1-k)r_u}{N}+N\sigma(p-p_0)\right]$$

The first derivative with respect to p is:

$$\frac{\partial\Pi(p)}{\partial p}=N\sigma\left[E(v\mid H)-p\right]-\left[\frac{1-(1-k)r_u}{N}+N\sigma(p-p_0)\right]$$

which is decreasing in p. So it is negative at all prices higher than p_0 if it is negative at p_0, i.e.

$$\sigma\leq\frac{1-(1-k)r_u}{N^2\left(E(v\mid H)-p_0\right)}$$ □

Appendix 2.C:

Part I:

The lowest start-bidding price to support the tacit collusion equilibrium

Suppose all the investors start bidding from price p (i.e., no bid is placed above p). Then if an H investor tries to absorb the entire supply by bidding at a higher price, the market price would be raised to p from the market price p_m. To make it unprofitable to do so, the gain from deviation should be no higher than that from following the demand function [8]:

$$E(v\mid H)-p\leq\left[E(v\mid H)-p_m\right]\frac{1}{N+1}$$

which requires $p\geq\frac{NE(v\mid H)+p_m}{N+1}$

We denote the right hand side as $\bar{p}$. When the demand function is vertical, investors should start bidding at least at that height. If this condition is satisfied, the U investor and each L investor also do not have an incentive to deviate by absorbing the entire shares at p (which requires that p is no lower than $\frac{NE(v) + p_m}{N+1}$ and $\frac{NE(v \mid L) + p_m}{N+1}$, respectively).

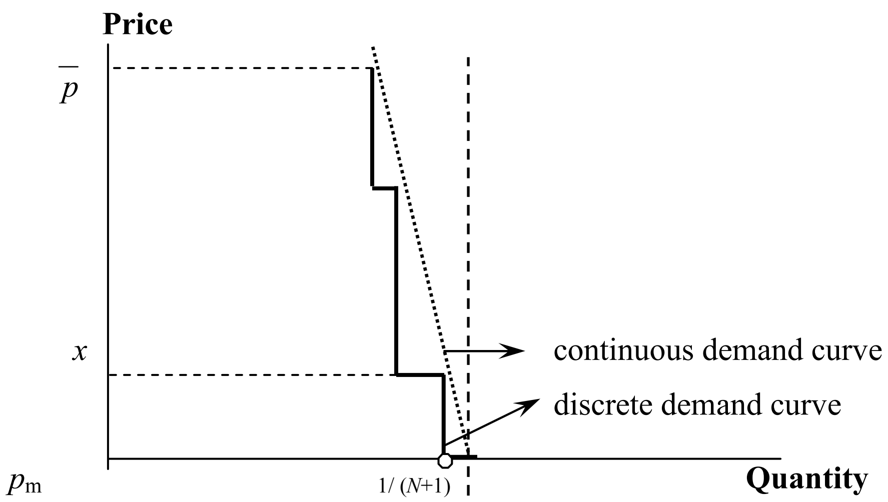
Part II:

Conditions on the slope of the demand function in the discrete case

For each investor, the residual supply above p_m in the continuous case is $\frac{1}{N+1}$[53]. In the discrete case, however, since demand gaps exist between any two discrete prices, the expected residual supply above p_m becomes $\frac{1}{N+1}$ plus the expected residual demand gap from the other N bidders (see the bold curve in Figure 2.C1). Since the probability that no bid is placed between p_m and a price (slightly) higher than p_m, say, x, is positive, with positive probability an investor can increase her allocation by bidding more at a price slightly higher than p_m, with the market price not changing or changing a little. Hence an investor, with positive probability, can improve her profits by deviating unless the total demand of each of the other bidders is $\frac{1}{N+1}$ above p_m, which happens only if the demand function is vertical above p_m, i.e. σ is zero.

[53] Without lose of generality, we assume that the U investor behaves the same way as an informed investor.

Figure 2.C1 Residual Supply under Discrete Demand Curve

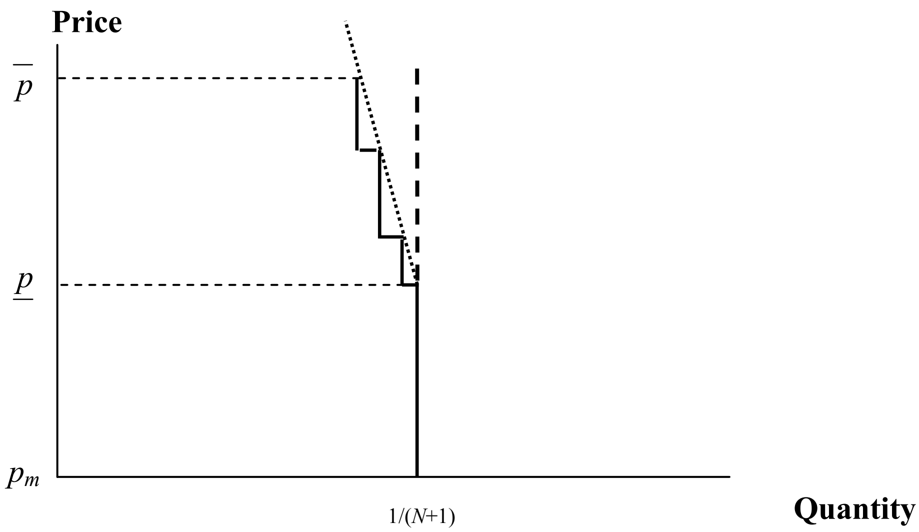


Thus in the discrete case, the demand curve should be vertical above the market price under the tacit collusion equilibrium. However, it needn't be vertical in the whole price range above p_m. In other words, the step-bids demand function may exist above some price higher than p_m. Suppose each investor keep the demand as $\frac{1}{N+1}$ between p_m and some price $\underline{p} \geq p_m$, and allow the demand function above $\underline{p}$ to have a positive slope σ (see the bold curve in Figure 2.C2). An investor can increase her allocation only if she raises the market price to at least $\underline{p}$. Part I of Appendix 2.C has shown that all bidders have to start bidding from price $\bar{p}$, so the maximum possible extra allocation that one investor can obtain from the other investors' demand gap is $N\sigma(\bar{p} - \underline{p})$, which happens when all the other investors place no bid between $\underline{p}$ and $\bar{p}$. To make it unprofitable to deviate, the maximum expected payoff from deviation should not exceed the one that corresponds when the market price is equal to p_m:

$$\left(E(v \mid H) - \underline{p}\right)\left[\frac{1}{N+1} + N\sigma(\overline{p} - \underline{p})\right] \leq \left(E(v \mid H) - p_m\right)\frac{1}{N+1}$$

Which requires that $\sigma \leq \dfrac{\underline{p} - p_m}{N(N+1)\left(E(v \mid h) - \underline{p}\right)\left(\overline{p} - \underline{p}\right)}$.

Figure 2.C2 Discrete Demand Curve: Positive slope above $\underline{p}$



The above argument provides only an upper-bound for the value of the slope. The equilibrium value will depend on each investor's beliefs about the expectations of other investors. For example, consider the following case of symmetric beliefs.

Suppose each informed investor and the U uninformed investor have the same belief that the expected demand gap of any other investor above market price p_m is $\int_{p_m}^{\overline{p}} \sigma(x - p_m)\mathbf{Pr}(x)dx$. Then the total expected residual supply above p_m faced by an investor is:

$$\frac{1}{N+1} + N\int_{p_m}^{\overline{p}} \sigma(x - p_m)\mathbf{Pr}(x)dx \qquad \textbf{[24]}$$

Where $Pr(x)$ is the probability that x is the lowest price at which an investor places a bid above p_m. $Pr(x)$ is positive only if $x > p_m$ (whatever the distribution of bids is). So

[24] equals $\frac{1}{N+1}$ only if the demand curve is vertical, i.e., σ=0. Otherwise, with positive probability an H investor can improve her allocation and hence her payoff, by bidding more than $\frac{1}{N+1}$ at a price slightly higher than p_m.

Hence in the discrete case, the demand curve should be vertical above the market price under the tacit collusion equilibrium. However, it needn't be vertical in the whole price range above p_m. Suppose each investor keeps the demand at $\frac{1}{N+1}$ between p_m and some price $\underline{p} \geq p_m$, while allow the demand function above $\underline{p}$ have a positive slope σ (see the bold curve in Figure 2.C2). An investor can increase her allocation only by raising the market price to at least $\underline{p}$. To make it unprofitable to deviate, for any price $p \geq \underline{p}$ that this investor may choose, the expected payoff at that price should not exceed the corresponding payoff obtained under the demand schedule [8].

$$(E(v \mid H) - p)\left[\frac{1}{N+1} + N\sigma(p - \underline{p}) + N\int_{p}^{\bar{p}} \sigma(x - \underline{p})\Pr(x)dx\right] \leq (E(v \mid H) - p_m)\frac{1}{N+1},$$

where $p \in [\underline{p}, \bar{p}]$

hence $\sigma \leq \dfrac{p - p_m}{N(N+1)(E(v \mid h) - p)(p - \underline{p} + A)}$

where $A = \int_{p}^{\bar{p}} (x - \underline{p})\Pr(x)dx$, decreases in both p and $\underline{p}$.

The right hand side of the condition increases with $\underline{p}$. So the condition can be satisfied for any $\underline{p} \geq p_m$ given that it is satisfied at $\underline{p} = p_m$, i.e.:

$$\sigma \leq \frac{1}{N(N+1)\left(E(v \mid h) - p\right)\left(1 + \frac{A}{p - p_m}\right)}$$

And because the right hand side increases with p, for any $p \geq \underline{p}$, the condition can be satisfied if

$$\sigma \leq \frac{1}{N(N+1)\left(E(v \mid h) - \underline{p}\right)\left(1 + \frac{A}{\underline{p} - p_m}\right)}$$ □

Appendix 2.D:

Derivation of the value of the slope of the demand function for prices below p_n in an EH type equilibrium.

With the demand function $\min\left[\frac{1}{n} - \sigma(p - p_n), \frac{1}{n-1}\right]$ for $p \in (v_{n-1}, p_n)$, the residual supply for an H investor at p is:

$$1 - (n-1)\left[\frac{1}{n} - \sigma(p - p_n)\right] = \frac{1}{n} + (n-1)\sigma(p - p_n)$$

And the corresponding profit is:

$$\Pi(p, H) = (v_n - p)[\frac{1}{n} + (n-1)\sigma(p - p_n)]$$

The first derivative is given by

$$\frac{\partial \Pi(p, H)}{\partial p^-} = \sigma(n-1)(v_n - 2p + p_n) - \frac{1}{n}$$

which decreases in p. Thus if $\frac{\partial \Pi(p, H)}{\partial p^-}$ is nonnegative at p_n then $\frac{\partial \Pi(p, H)}{\partial p^-} \geq 0$ at any other prices in (v_{n-1}, p_n). This requires that $\sigma \geq \frac{1}{n(n-1)(v_n - p_n)}$. □

Appendix 2.E:

Derive the condition that must be satisfied so that it is unprofitable to lower the price to p_{n-2} if it is unprofitable to lower price to p_{n-1}.

If it is unprofitable for an investor who has observed signal *s*, which is H or L or U, to deviate by lowering the price from p_n to p_{n-1} for any $n \in [1,N]$, then the following relationship must hold:

$$(v_n - p_n)d^a(p_n, s) \geq (v_n - p_{n-1})r_n^a(p_{n-1}, s) \Rightarrow \frac{v_n - p_n}{v_n - p_{n-1}} \geq \frac{r_n^a(p_{n-1}, s)}{d^a(p_n, s)} \qquad \textbf{[25]}$$

where $r_n^a(p_{n-1}, s)$ is the cumulated residual supply above price p_{n-1} for an investor with signal *s*, when there are *n* high signals ($n>0$).

We should also have $(v_{n-1} - p_{n-1})d^a(p_{n-1}, s) \geq (v_{n-1} - p_{n-2})r_{n-1}^a(p_{n-2}, s)$ where $n>1$. Adding $(v_n - v_{n-1})d^a(p_{n-1}, s)$ on both sides of the second inequality we have:

$$(v_n - p_{n-1})d^a(p_{n-1}, s) \geq (v_{n-1} - p_{n-2})r_{n-1}^a(p_{n-2}, s) + (v_n - v_{n-1})d^a(p_{n-1}, s)$$

Because $d^a(p_{n-1}, s) \geq r_{n-1}^a(p_{n-2}, s)$, if the second inequality is satisfied,

$$(v_n - p_{n-1})d^a(p_{n-1}, s) \geq (v_n - p_{n-2})r_{n-1}^a(p_{n-2}, s)$$

So

$$\frac{v_n - p_{n-1}}{v_n - p_{n-2}} \geq \frac{r_{n-1}^a(p_{n-2}, s)}{d^a(p_{n-1}, s)} \qquad \textbf{[26]}$$

The product of [25] and [26] implies that $\frac{v_n - p_n}{v_n - p_{n-2}} \geq \frac{r_n^a(p_{n-1}, s)r_{n-1}^a(p_{n-2}, s)}{d^a(p_n, s)d^a(p_{n-1}, s)}$

If

$$r_n^a(p_{n-1}, s)r_{n-1}^a(p_{n-2}, s) \geq d^a(p_{n-1}, s)r_n^a(p_{n-2}, s), \qquad \textbf{[27]}$$

then we have

$$(v_n - p_n)d^a(p_n, s) \geq (v_n - p_{n-2})r_n^a(p_{n-2}, s)$$

which implies that it is unprofitable to lower the price further to p_{n-2}.

Hence if all investors' demand functions satisfy [27], we only need to check if it is profitable to lower the price p_n down to the neighbour price p_{n-1}.

We can derive the residual supplies from the general market clearing condition:

$$Nd^a(p_n, L) + nc^a(p_n) + d^a(p_n, U) = 1$$

Then [27] is reduced to

$$c^a(p_{n-1})[d^a(p_{n-2}, L) - c^a(p_{n-2})] \le c^a(p_{n-2})d^a(p_{n-1}, L) \qquad \textbf{[27a]}$$

for both H and L investors; and

$$c^a(p_{n-1})[d^a(p_{n-2}, U) - c^a(p_{n-2})] \le c^a(p_{n-2})d^a(p_{n-1}, U) \qquad \textbf{[27b]}$$

for the U investor.

In the first example provided for EHL, $d^a(p_{n-2}, L) = d^a(p_{n-1}, L) = K$ and $c^a(p_{n-1}) = \frac{1-NK}{n-1}$, so $\frac{(1-NK)^2}{(n-1)(n-2)} \ge K(\frac{1-NK}{n-1} - \frac{1-NK}{n-2})$ and [27a] is satisfied. So an investor would not try to reduce price further if it is unprofitable to reduce the price from p_n to p_{n-1}.

The same method and results apply for all the other examples we have provided for EH, EHL, EHU and EHLU. (The proof is trivial and thus is ignored).

The method to derive the condition that must be satisfied so that it is unprofitable to raise the price to p_{n+2} if it is unprofitable to lower the price to p_{n+1} is the same as above.

If we have $(v_n - p_n)d^a(p_n, s) \ge (v_n - p_{n+1})r_n(p_{n+1}, s)$ for any $n \in$ [0,N-1]

and also $(v_{n+1} - p_{n+1})d^a(p_{n+1}, s) \ge (v_{n+1} - p_{n+2})r_{n+1}(p_{n+2}, s)$ for any $n \in$ [0,N-2],

by subtracting $(v_{n+1} - v_n)d^a(p_{n+1}, s)$ from both sides of the second equation we have:

$$(v_n - p_{n+1})d^a(p_{n+1}, s) \geq (v_n - p_{n+2})r_{n+1}^a(p_{n+2}, s).$$

Thus $\dfrac{v_n - p_n}{v_n - p_{n+2}} \geq \dfrac{r_n^a(p_{n+1}, s)r_{n-1}^a(p_{n+2}, s)}{d^a(p_n, s)d^a(p_{n+1}, s)}$, and so if

$$r_n^a(p_{n+1}, s)r_{n+1}^a(p_{n+2}, s) \geq d^a(p_{n+1}, s)r_n^a(p_{n+2}, s) \qquad \textbf{[28]}$$

is satisfied, it is unprofitable to raise further the price to p_{n+2} even if p_{n+2} is lower than v_n.. The condition is reduced to

$$c^a(p_{n+1})[d^a(p_{n+2}, L) + c^a(p_{n+2})] \geq c^a(p_{n+2})d^a(p_{n+1}, L)$$

for both H and L investors; and

$$c^a(p_{n+1})[d^a(p_{n+2}, U) + c^a(p_{n+2})] \geq c^a(p_{n+2})d^a(p_{n+1}, U)$$

for the U investor.

All the examples that we have provided for EH, EHL, EHU and EHLU satisfy the above conditions. □

Chapter Three:

Uniform Price Auctions and Fixed Price Offerings in IPO: An Experimental Comparison

Chapter 3:

Uniform Price Auctions and Fixed Price Offerings in IPO: An Experimental Comparison

3.1. Introduction

Underpricing is a common feature of almost every offering format in IPOs (Ibbotson, 1975); nevertheless, the performance of different IPO methods varies greatly across countries. For example, an average underpricing rate of 34.7 percent is reported for fixed price offerings in Singapore according to the data from 38 issues that took place from 1973 to 1987 (Koh and Walter, 1989). For the US market where Bookbuilding is used for almost all IPOs, Ritter and Welch (2002) report an average underpricing rate of 18.8% for a sample of 6249 IPO issued during 1980 to 2001. In contrast, in countries where uniform price auctions are used, the extent of underpricing seems to be much less. For instance, the average underpricing level is 4.5 percent from December 1993 to December 1996 for auctions in Israel (Kandel et al., 1999). In Argentina during the 1990's, the shares were even overpriced.[1] Although second price auctions with private values can elicit bidders' true values, this nice property generally does not apply to uniform price auctions where bidders face multiunit demand. On the contrary, because bidders pay the same price for all units won, they

[1] However, in Argentina the overpricing in IPO auctions has lead to the collapse of the stock market (Sherman, 2002).

have an incentive to shade their demand so that the market price could be lowered should their bids be marginal. It has been shown that demand reduction is inevitable in uniform price auctions when bidders have multiunit demand (Krishna, 2002). In the extreme case, bidders behave the same regardless of their expected market values of goods and as a result their behaviours do not provide useful information for eliciting market values. Sherman (2002) argues that Bookbuilding is better than auctions in the sense that it can generate more information. In some countries such as Japan, virtually all issuers select Bookbuilding rather than an auction though the former engages higher commission fees and issue costs (Kutsuna and Smith, 2001).[2]

While the popularity of IPOs has declined for a while after the internet bubble, comparisons of optimality among different designs have received attention both at the theoretical and the empirical levels in recent years (E.g., Loughran et al. 1994, Derrien and Womack, 2001, etc.). Revenue is not the only criterion in evaluating IPO methods. Efficiency, that means assigning the shares to those who value them more, may be more important from the social welfare point of view. As the real value of shares in an IPO is the same for all investors, efficiency is not a problem in comparing different methods. However, because investors have different *expected* values when placing bids, a concept called partial *allocational efficiency*, which says that the bidder who values the good the most (before the real value is revealed) receives the largest share, has been introduced by Wang and Zender (2002). In addition, an ideal IPO mechanism should be able to reflect the real value of shares, not be volatile in pricing and should guarantee full subscription. The empirical comparisons based on such criteria are very sensitive to the chosen sample. For instance, for the French

[2] Japan introduced Bookbuilding as an alternative method of IPO to auction, which had been required since 1989. The auction format in Japan is a discriminatory auction in the first stage where each bidder pays her own bid price. In the second stage, the final offering price is determined for the shares left according to the weighted average bid

stock market where three issuing mechanisms (a modified uniform price auction, Bookbuilding, and fixed price) are used, thus, a unique arena to compare their performance is offered, Derrien and Womack (2001) find that the auction mechanism is associated with less underpricing and lower variance of underpricing. They also suggest that the auction procedure's ability to incorporate more information from recent market conditions into the IPO price is an important reason. However, Sherman (2002) shows a completely opposite result in her paper based on an international study. It is difficult, if it is possible at all, to find an environment where different IPO methods have been used at the same time for a long enough period. These considerations have motivated us to design an experiment to compare various IPO formats under a controlled environment.

On the other hand, we are also interested in bidders' behaviour especially in uniform price auctions. Although the rules of an uniform price auction are straightforward, letting the market decide the market clearing price, investors have more incentives and opportunities to bid strategically because their behaviour influences both the market price and their allocation. This has attracted researchers who are interested not only in the outcomes of auctions per se, but also in the study of bidders' strategic behaviour as well. In chapter two, we have found multiple equilibria in uniform price auctions and would like to examine how subjects really behave in practice. Because it is the equilibrium most likely to happen that matters for auctioneers, in practice we should try to avoid the unrealistic equilibria to over-influence policy makers' decision, though we need to bear in mind the potential existence of such equilibria and introduce some rules to prevent them to be achieved if they can lead to low revenue or inefficient outcomes.

prices in state one (after December 1992, underwriters and issuers are free to set the final offering prices without abiding by the weighted average prices).

In this study, we report an experiment for comparing the fixed price offering (also known as public offer) and the uniform price auction. Fixed price offerings probably are the simplest formats among all IPO methods. Although in recent years they have been less popular in some countries relatively to the past, they continue to be the main format used in countries that rely on retail investors and are widely used in conjunction with other formats like Bookbuilding[3]. The uniform price auction is now used in Israel and is allowed in some countries such as Finland and Hongkong (Sherman, 2002). A company WR Hambrecht + Co. uses the uniform price auction "OpenIPO" to conduct online IPOs. In the UK, uniform price auctions were frequently used in IPOs until the mid-1980s then have fallen out of favour since 1986, for some unclear reasons (Jenkinson and Ljungqvist, 2001). Uniform price auctions have received particular attention recently after the internet giant Google had decided to use an online uniform price auction for its IPO and conducted its offering successfully.

In the past, most experiments on auctions focused on *symmetric private value* auctions (Cox, Roberson, and Smith 1982, Kagel, Harstad, and Levin 1987, etc.) and *pure common value* auctions (Kagel and Levin 1992). In most of these types of experiments there is a single unit object for sale. In recent years, the study of auctions has been expanded to new areas that include topics such as multi-unit auctions, asymmetric and interdependent valuations that result in a more realistic design. For example, Abbink et al. (2002) model a bidder's valuation as the sum of the *common value* component and the *private value* component in their study for the British 3G auction (*multi-unit*). Kirchkamp and Moldovanu (2001) find that an English auction yields a more efficient allocation than other standard auctions in experiments on

[3] The countries where the fixed price offering is the most popular method for IPO include India, Mexico, Singapore, etc.

auctions with a single unit under *asymmetric interdependent values,* which is in accord with what the theory predicts (Maskin 1992; Krishna, 2002). Engelmann and Grimm (2003) conduct experiments on *multi-unit* auctions with *private values* to compare the optimal strategy and performance of different auction mechanisms. The theoretical prediction of demand reduction behaviour is supported by experimental studies (Engelmann and Grimm, 2003; Kagel and Levin, 2001) and field experiment studies (List and Riley, 2000). However, in the experiment with two bidders who each have two units demand and private information, though demand reduction is observed, bids for low valued units is higher than the equilibrium prediction of zero (Porter and Vragov, 2003). The experiment conducted by Sade, Schnitzlein and Zender (2004) finds little evidence of collusive behaviour in uniform price auctions even when communication is allowed. The combination of multi-unit, common value, inter-dependent values as well as asymmetry in information make IPO auctions nearly reach the limits of auction theory (Krishna, 2002). To our knowledge none of the existing studies tackles an environment with all these components as in this experiment.

The rest of the chapter is organized as follows. In section 3.2 we briefly review the theoretical literature on uniform price auctions and fixed price offerings. In section 3.3 we describe the model that our experiment is based on, its theoretical predictions, and the design and conduct of the experiment. Results are reported in section 3.4 and we summarize and conclude in section 3.5.

3.2. Theoretical Background

3.2.1 Uniform Price Auctions

In uniform price auctions, the market price is at the level that clears the market. All bidders who win pay the same price. In multi-unit uniform auctions with private values, it has been shown that truthful bidding on the first unit is a weakly dominant strategy. Engelmann and Grimm (2003) have also shown that when there are 2 units for sale and bidders have constant marginal value, truthful revelation on both units, and full demand reduction on the second unit are both equilibria.

However, in IPO auctions instead of having independent private values, investors only have estimated values based on the information, usually modelled as signals, they observe. Higher estimated values tend to coincide with a higher value of shares but the relationship is not perfect. The derivation of equilibrium is a very delicate problem in a multi-object common value auction.

As we have discussed in detail in chapter two, many researchers point out that there exists a tacit collusion equilibrium in perfectly divisible goods (or share auctions) which yields a significantly lower sale price (see e.g., Back and Zender, 1993; Biais and Faugeron-Crouzet, 2002; Maxwell, 1983; Wilson, 1979; etc). Because this type of auction is subject to manipulation by the bidders and the seller obtains no advantages from increased competition when the number of bidders increases, the collusive strategies, even though they are non-cooperative equilibria, are self-enforcing in uniform-price divisible-good auctions. By submitting steep demand curves, bidders' inframarginal bids are costless in a uniform-price auction. A tacit collusion demand function is also provided for the model we use in the experiment, which we will explain in detail in the next section. Because this equilibrium fails in generating high revenues for sellers, uniform price auction seems less attractive for auctioneers. However, this collusion prediction of the IPO model relies on the assumption that the investors believe that the other investors also follow the same strategy, and the equilibrium strategy requires that each bidder submit a steep demand function. Even if one bidder submits "crazy" bids for example by mistake, or chooses to follow other strategies, it is very likely that the other buyers will suffer losses. With a larger number of bidders, the equilibrium condition requires the demand function even steeper; in addition, there is a higher chance that one or several bidders do not follow the collusive strategy, thus the risk of following the tacit collusion strategy also grows. This makes the strategy less attractive, and in turn breaks down investors' beliefs about other bidders' choices. Hence this kind of equilibrium is not only difficult to achieve as it requires coordinated behaviour among bidders, but also very risky for players who choose to collude.

Does then any other equilibrium exist in the uniform price auction? Wang and Zender (2002) demonstrate that in perfectly divisible goods auctions when bidders possess asymmetric information, there exist equilibria that are characterised by both the winners' curse consideration (in the sense that the bidders who have higher expected valuations bid more aggressively) and strategic aspects (in the sense that all bidders bid strategically by demand reduction). As a result the bidder who has the highest expected value of the good receives the largest share. In Chapter two, we find a continuum of equilibria in uniform price IPO auctions in which market prices increase with market values thus revealing some information of the market values. Unlike the previous studies, our equilibrium predictions apply to auctions with discrete demand function such as the model used in our experiment.

Two other considerations relevant for uniform price auctions are the free rider problem and the Winner's Curse. Since investors do not necessarily pay the price they bid, it is possible that some bidders bid at high prices just to make sure they obtain some allocation. These bidders are regarded as information free riders since their bids do not reveal any information about the market value. On the other hand, a frequently observed phenomenon in common value auctions is the Winner's Curse, i.e. bidders who have the highest expected value of good win, but if they do not take this into consideration and shade their bids accordingly, they usually pay more than the real value of good and suffer a loss.

3.2.2 Fixed Price Offerings

In fixed price offerings, market prices are determined before the sale of shares. Shares are randomly rationed or prorated among all the bidders if they are oversubscribed.

In financial markets, institutional investors have an informational advantage and more experience than retail investors. They increase the demand when they have higher expected valuations regarding the market value of shares. Retail investors, on the contrary, do not have such information and thus cannot submit their demand contingent on such a valuation. Thus when the value of shares is at the lowest level, the total demand is also at its lowest level and retail investors are allocated the largest proportion. To induce uninformed investors to participate so that the shares can be fully subscribed at any possible valuation, this winner's curse problem has to be taken into account when setting the fixed market price. As a consequence the market price has to be below the expected market value of the shares (Rock, 1986). Test results from issues in Singapore's stock market are consistent with the theoretical implications of Rock's model (Koh and Walter, 1989). An empirical study (Faugeron-Crouzet, Ginglinger and Vijayraghavan, 2000) shows that in French IPO markets, fixed price offering is chosen by firms if they know a lot of information about the market value of shares and try to signal high values of shares by a price discount.

3.3. The Experiment

3.3.1 The Model

The model that we have used in our experiment follows Biais and Faugeron-Crouzet (2002). In their paper, the authors compare four main IPO formats that are widely used, namely Bookbuilding, fixed price offerings, uniform price auction and *Offre a Prix Minimum*, a modification of the uniform price auction used in the French market. Their model provides a convenient environment for our experiment to compare the performances of various IPO designs.

In this model, there are S units of shares for sale in the market. The seller and the investors are assumed to be risk neutral. Each of the N informed investors who represent an institutional investor observes a signal reflecting the value of the shares. The signal can take a high value with probability π, or a low value with the complementary probability. The market value of shares, v, increases with the number of high value signals n.

There are a large number of uninformed investors who do not receive signals, representing the retail investors.

Each informed investor has enough money to buy all the shares, while the uninformed investors in total can only buy up to $S(1-k)$ shares, where $0 \leq k \leq 1$.Each informed investor can bid for up to Q ($Q>S$) units. This is reasonable since in reality, bidders may demand more in order to increase their allocation of shares, as over-subscription is frequently (if not always) observed in IPOs. Investors who bid more than S must pay a cost of c, which “can be thought of as the cost of immobilizing funds during the period of the IPO” (Biais and Faugeron-Crouzet, 2002, P18). Technically, this cost is used to distinguish the equilibrium strategies between the investors who observe high signals and those who observe low signals in the fixed price offerings.

Hence this is a multi-unit common value auction with asymmetric and incomplete information.

3.3.2 Experimental Design

Slight modifications have been made to simplify the game. One modification is that in the uniform price auction treatment, we use the highest losing price as the market-clearing price instead of the lowest winning price. This will not change the results of the theory. [4] Moreover, since in reality (as well as in our experiment) there are lots of bids submitted, the highest losing price and the lowest winning price are the same in most cases. The parameters (N, k, π, S, v, c, Q) and the formula of the market value are chosen following the principle of simplicity. The value of the parameters used are shown in Table 3.1.(The instructions are provided in Appendix 3.A).

Table 3.1 Experimental Parameters

N	k	π	S	Q	c	v
3	0.2	0.5	100,000	150,000	5,000	1 + number of high signals

3.3.2.1 The Market

Subjects are randomly divided into groups of four by the computer at the beginning of each session. The members of groups do not change during each session and groups are completely independent.

[4] For example, when the lowest winning price issued, since the market price is the highest price where all the shares are sold out, instead placing S shares at a price, players can do better if they place S-1 at that price and 1 share at a lower price. If we use the highest losing price, the equivalent strategy requires that they place S at that price.

In each group, there is one uninformed buyer[5] and three informed buyers (N=3). Each of the informed buyers receives a signal that represents either a *high value* or a *low value* with equal probabilities (π=0.5). The *true value* of each unit of the good is set equal to one plus the number of high signals received in the group. Hence an investor receiving a low value signal (an L investor) knows that the value must be 1, 2 or 3, and an investor who receives a high value signal (an H investor) should know that the true value must be 2, 3 or 4. The uninformed buyers do not observe a signal so they only know that the true value is 1 or 2 or 3 or 4. So the information they have is noisier. The roles of subjects (informed or uninformed) remain fixed across rounds.

The value can be seen as the resale price in the secondary market, where according to the *Efficient Markets Hypothesis*, prices take account of all available information in organized markets. Fourteen market value sequences are randomly predetermined. For each group in one treatment there is a group facing the same market values across rounds in the other treatment, so that we have fourteen independent markets that are comparable across treatments.

3.3.2.2 Decisions

In the uniform price auction treatment (noted as uniform treatment hereafter), subjects are asked to submit up to six bids, each of which is a price-quantity pair consisting of the price they would like to bid and the quantity they bid for at that price. The lowest acceptable price is zero (this is equivalent to a reservation price of zero, which implies that no artificial restriction on the price level is imposed). We want to infer bidders' demand function from the multiple price-quantity pairs. Since it is impractical (and unrealistic) for bidders to submit demand in continuous prices, and we have to keep

[5] The uninformed investors as a whole are regarded as one investor in the paper, so in our experiments one subject represents the uninformed investors in each group.

the balance between flexibility and simplicity, we allow at most six pairs to be submitted. [6] The upper bound of the accepted price is set at six (implicitly) in order to prevent subjects from submitting unreasonably high bids because of misunderstanding especially at the beginning of the experiment, which might have led them to bankruptcy. The ceiling on bids is higher than the highest possible market value because although bidders do not have to pay the prices that they bid, they may bid at high prices to make sure that they obtain some allocation.

In the fixed price offerings treatment (noted as fixed treatment hereafter), subjects need to submit a quantity that they would like to bid for at the predetermined market price.

In both treatments subjects are asked to repeat their choices 20 times (we call each repetition a *round*).

3.3.2.3 Price and Allocation Rules

The computer calculates the market price and the allocation after all the bidders submit decisions.

In the uniform treatment, the market price is the highest price where demand exceeds supply. Bids placed above the market price are fully filled, and then the amount left is prorated among bids placed at the market price. Bids placed below the market price are ignored. If demand is less than or equal to supply, the market price is determined as the reservation price of zero.

[6] At least we wished to offer one position for each possible value (1,2,3,4), one for the price under the tacit collusion prediction (zero), and one for bids from "information free-riders". More pairs would be too confusing and complicated for subjects. Few bidders have used all six pairs to submit their bids. The percentage of each possible number of pairs used by subjects in our experiments is as follows.

Number of Pairs	0	1	2	3	4	5	6
Percentage	0	0.22	0.28	0.27	0.12	0.05	0.06

In the fixed treatment, the market price is predetermined at 1.94, which is the highest possible price to guarantee fully subscriptions according to the theoretical solution provided by Biais and Faugeron-Crouzet (2002)[7] . Shares are prorated among winning buyers.

3.3.2.4 Earnings

Prices and earnings of players are described in points. The point earnings that each subject earns equal the market value minus the market price, times the number of units allocated. Each buyer who bids for more than 100,000 units needs to pay a cost (c) of 5,000 points.

At the outside of the experiment, subjects are endowed some points on their account to start with. The endowments are listed in table 3.2.

Table 3.2 Endowments

	Uniform	Fixed
Each informed buyer	350,000 (£14)	100,000 (£4)
Each uninformed buyer	400,000 (£16)	150,000 (£6)

Since uninformed buyers have an information disadvantage in the experiment, and the quantity they are allowed to bid for is less than the informed buyers, we give

[7] To make all shares could be fully subscribed even when there were no high signal observed, the highest predetermined market price p is set at the level such that the expected profit of an L investor is zero: $\sum_{h=0}^{N-1} \pi_h \tau_h (v_h - p) = 0$ so $p = \frac{\sum_{h=0}^{N-1} \pi_h \tau_h v_h}{\sum_{h=0}^{N-1} \pi_h \tau_h}$, where h represents the number of high signals, π_h is the probability of h out of N - 1 buyers observe high signals, v_h is the market value when h high signals are observed, and τ_h is the execution rate of a buyer who observes a low signal when there are h buyers observe high signals. (See Biais and Faugeron-Crouzet, 2002, P31).

them a higher endowment to compensate for their disadvantage. In the fixed price treatment, the predetermined market price is even lower than the expected value of a buyer who has a low signal, so subjects rarely have a chance to lose points. Furthermore, they only need to decide on the quantity, which is a much simpler decision than the decisions in the uniform price auction. In fact, the experiment on the fixed price auction proceeded much faster than the uniform one. We decided to give subjects higher endowments in the uniform treatment to avoid bankruptcies in case subjects fall prey to the winner's curse, and to account for the longer duration of play in the more complex auction environment. Since the size of the endowment does not play a crucial role in experiment, the differences among the endowments should not have influenced subjects' behaviour.

We set the exchange rate between points and pounds at 250 points for one penny. The numbers in brackets in Table 3.2 show the endowments in pounds.

If all the subjects follow the equilibrium strategies described in the paper, each buyer could earn up to fifty pounds in the uniform treatment besides the endowments. Each informed and uninformed buyer would earn an additional 10.66 and 6.77 pounds respectively in the fixed treatment. Both of the earnings are considerable compared with the endowments. So we believe that subjects have sufficient motivation to take their decisions seriously.

3.3.2.5 Information

To help subjects understand the market rules, we have included a *test* phase after reading the instructions. Subjects had to pass this test before entering the real experiment. Right answers were shown on their screens when they answered incorrectly, and then they were given an unlimited number of chances to get it right.

In addition, in the uniform price auction there was a *help* menu describing an example of the market rules on each terminal, which could be retrieved at any time during the session. The numbers used in the questions and the example were different from those in the real experiment. We believe that such a procedure was helpful for the subjects to understand the experiment. This is confirmed by the questionnaires that the subjects filled in after the experiment.

Subjects were given information about the market price, the realization of the market value, the allocation and point earnings of each member in their groups, the total demand at the market price, their own signals and decisions of that round, and their own point earnings. Previous results could be retrieved at any time from the *history* menu.

3.3.3 Equilibrium Prediction

3.3.3.1 Predictions for the Uniform Treatment

Biais and Faugeron-Crouzet derive a tacit collusion demand function for the uniform price auction. According to this function, the demand of each investor at price p is given by $\frac{S}{N+1} - \sigma(p - p_0)$, where p_0 is the reservation price, and the slope of the demand function should satisfy $\sigma \leq \frac{S}{N(N+1)(E(v \mid H) - p_0)}$ (Biais and Faugeron-Crouzet, 2002, P19). The implication is that to get a higher allocation an investor has to raise the market price to a higher level so that she can absorb the residual supply from the other investors; when the slope of the demand function is steep, the gain from an increase in the allocation cannot offset the cost from the price increase thus no one can benefit from deviation. As a result, each bidder obtains $\frac{1}{N+1}$ of the supply at the reservation price. They suggest that the highest possible reservation price in this case equals $E(v|L)$, the expected value of an L investor[8]. According to the settings of our experiment, under this equilibrium prediction each bidder obtains 25,000 units at price zero, with a slope no higher than $\frac{25{,}000}{9}$. In fact, any price between zero and two, the expected value of an L investor, could be the market price in a Nash Equilibrium, but the one resulting in the market price being equal to the reservation price weakly dominates the others.

However, in both experiments and in reality, it is not practical for investors to submit a continuous demand function, even if they are allowed to do so. [9] Then if the slope is not zero, i.e. if the demand function is not vertical, investors always have a chance to improve their expected payoffs by trying to absorb the possible demand gap between the discrete prices. So the tacit collusion equilibrium has to be adjusted for the discrete demand case. In the previous chapter, we have shown that in the discrete case there exist multiple equilibria resulting in the market price being equal to the reservation price. In this set of equilibria, each bidder starts bidding from a price no lower than $\frac{NE(v \mid H) + p_0}{N+1}$ (which is 2.25 for our experiment). The demand function is vertical above the reservation price of zero, and the slope σ above the price $\underline{p}$ below which the demand curve is vertical should satisfy $\sigma \le \frac{S(\underline{p} - p_0)}{N(N+1)\left(E(v \mid h) - \underline{p}\right)\left(p - \underline{p}\right)}$ (see Proposition 2 in Chapter two) [10]. If this equilibrium could be achieved in the experiment, in the symmetric case we should

---

[8] $E(v|L) = \sum_{h=0}^{2} \pi_h v_h = 0.5^2 \times 1 + 2 \times 0.5^2 \times 2 + 0.5^2 \times 3 = 2$ .Similarly, $E(v|H) = \sum_{h=0}^{2} \pi_h v_{h+1} = 3$.

[9] In practice, bidders seem to use a relatively small number of price-quantity pairs in bidding even if they are allowed to submit as many pairs as they can (Wang and Zender, 2002, Bikhchandani and Huang, 1993).

observe that each bidder bids no more than a quarter of the supply (25,000 units) above the reservation price of zero (or the resulting market price which is no higher than 2, if the weakly dominated strategies were played); if asymmetry is applied, we should not observe the demand to exceed supply at a price higher than zero (or 2). So the market price could not exceed zero (or $E(v|L)$) under this equilibrium.

In chapter two we have also shown that some other equilibrium sets exist in this model. Among all the equilibria, the upper bound strategy for each H player is placing $\frac{S}{n-1}$ at the price v_n for every n > 1(see chapter two, 2.4.2.3) [11]. In the equilibrium EH, all shares are allocated to H investors if only there exists at least one high signal. This could be achieved if each H investor bids for $\frac{S}{n}$ units *above* price v_n and bids for $\frac{S}{n-1}$ *at* any price p_n between v_{n-1} and v_n. The resulting market price then is p_n. In this equilibrium, L investors and the uninformed investor only place bids at prices that are no higher than v_0. Further price decreasing relies on more demand reduction. To push the market price lower than v_{n-1}, H investors have to lose some shares to L or uninformed investors. Then we would have the sets of equilibria in which H investors share the market with L investors (EHL), or with uninformed investors (EHU), or with both of them (EHLU). In all these equilibria, market price increases with market value except in the extreme case of the tacit collusion equilibrium, where the market price does not reflect any information from signals received by the bidders. Furthermore, as shown in chapter 2, the information asymmetry among bidders and the inverse relationship between market value and the investors' allocation make a flat

[10] For the settings in our experiments, the right hand side of the inequality equals $\frac{25,000\,(\underline{p}-p_0)}{3(3-\underline{p})(2.25-\underline{p})}$, where p_0 equals zero. However, if investors play the weakly dominated strategies, which lead to a market price between zero and 2 (the expected value of an L investor), p_0 is the resulting market price.

demand function not a part of the equilibrium solution in this model. Thus we do not expect subjects to submit flat demand functions in our experiment. The Bidders' demand function should be downward sloping. The predictions are summarized in table 3.3.

Table 3.3 Summary of Predictions[12]

Equilibrium predictions of the uniform price auction			
Strategy		**Price**	**Feature**
Tacit Collusion	Symmetric case: Each bidder bids for 1/4 at prices ≥ 2.25 Asymmetric case: Bidders in total bid for 1 at prices ≥ 2.25	[0,2]	Bidders submit demand functions regardless of signals; same price for different value.
EH	H: bids for 1/n above v_n, and increases bids up to1/(n-1) at $p_n \in [v_{n-1}, v_n]$ (for $n \geq 1$): L, U: bids at $p \in [0, 1]$ ($v_0 = 1$)	$[v_{n-1}, v_n]$ if $n \geq 1$; [0,1] if $n = 0$	Hs absorb all the shares; Price increases with value.
EHL	H: bids less than 1/n at v_n L: bids less than H U: bids in [0,1]	$[v_{n-1}, v_n)$[13]	H and L share the market; H bids more aggressively than L; Price increases with value.
EHU	H: bids less than 1/n at v_n L: bids in [0,1].	$[v_{n-1}, v_n]$	H and U share the market; Price increases with value.
EHLU	H: bids less than1/n at v_n L: bids less than H (but positive).[14]	$[0, v_n)$	All investor participate; H bids more aggressively than L; Price increases with value.

[11] Here v_n means the maker value of shares when there are n informed investors who observe high value signals.

[12] In the table H, L and U represent an H, L investor and uninformed investors as a whole respectively. All the demand functions are non-increasing in price.

[13] Upper bound: $v_n - \frac{d(p_{n-1}, L)}{d(p_n, L) + c(p_n)}(v_n - p_{n-1})$, see chapter two, 2.4.2.4 -I.

[14] Here we use a strict concept of EHLU, all bidders bid positive amount, and H investors demand strictly higher than L bidders. If we relax the restrictions all the other equilibria can be nested in EHLU (specifically, the equilibrium becomes the tacit collusion equilibrium if H and L bidders bid the same amount; if both L and uninformed bidders bid zero, it becomes EH; if L or uninformed investors bid zero, EHLU becomes EHU or EHL.

3.3.3.2 Predictions for the fixed treatment

For the fixed price offering, Biais and Faugeron-Crouzet (2002) have shown that there exists an equilibrium where H investors can be distinguished from L investors. According to the corresponding equilibrium strategy, each H investor bids for Q, each L investor bids for S, and the uninformed investors' aggregate bid is (1-k) S shares. Since uninformed investors are assumed not be able to absorb all the shares, in order to guarantee full subscription even when high value signals are not observed at all, the highest possible market price has to be set such that the expected profit of an L investor is zero. In addition, as investors' demand increases with market value, an L investor obtains the highest allocation when the market price is the lowest. This "Winner's Curse" problem has to be taken into account when setting the price and as a result the market price has to be set smaller than the expected value of an L investor. So uninformed investors still face the winners curse problem in the sense that they are allocated the most when the value of shares is at the lowest level. However, they obtain a positive expected payoff because of the way that the market price is determined. However, this equilibrium is subject to a cost condition. The cost that an informed investor needs to pay should she demand more than S is located in a range such that either an L or an H investor does not have incentive to pretend to be other type of investors in equilibrium. Given that the others follow the equilibrium strategy, it is best for all types of bidders to follow the equilibrium strategies as well.[15]. If the cost were set beyond this range, then the informed investors would behave the same by bidding either Q (if the cost is set smaller than the lower bound) or S (if the cost is

[15] The cost should locate between $S\sum_{h=0}^{N-1}\pi_h(v_h - p)\frac{Q}{S[(1-k)+N-(h+1)]+(h+1)Q}$ and, $S\sum_{h=0}^{N-1}\pi_h(v_{h+1} - p)[\frac{Q}{S[(1-k)+N-(h+1)]+(h+1)Q} - \frac{S}{S[(1-k)+N-h]+hQ}]$ where p is the predetermined market price (See Biais and Faugeron-Crouzet, 2002, P32). Under the settings of our experiment, the lower and upper bounds are about 240 and 8212 respectively so 5000 locates in this range.

set higher than the upper bound). Then an L investor would obtain the same allocation regardless of the number of high signals. As a consequence, the fixed market price could be set equal to $E(v|L)$. Hence the seller could improve her revenue by adjusting informed investors' cost, if it is practicable.[16] On the other hand, if uninformed investors have enough funds to buy all the shares, the market price could be higher, and making the uninformed investors' expected payoff equal to zero is enough. Nevertheless, we keep the settings in our experiment consistent with the equilibrium shown by Biais and Faugeron-Crouzet. The cost is set such that H and L investors bid for different amounts in the equilibrium, and the predetermined market price is set at a level (1.94, according to the formula in footnote 7) such that an L investor gains a zero expected profit. According to their prediction, in equilibrium in each group, the H, the L and the uninformed investor each bids for 150,000, 100,000 and 80,000 units respectively. Uninformed investors are allocated the most when the value of shares is at the lowest (i.e.1).

So according to the paper by Biais and Faugeron-Crouzet (2002), the fixed price offering should generate more proceeds for the seller in our experimental settings, and investors with H signals should get a higher allocation in fixed price offerings.

3.3.4. Conduct of the Experiment

The experiment consists of ten sessions conducted at the *University of Nottingham* in March 2004[17]. One hundred sixteen subjects participated in the experiment. We recruited them from a mailing directory that comprises of undergraduate students from the entire university, who have indicated their willingness to be paid volunteers from the CeDEx website.

---

[16] This can easily seen from footnote 7, the highest possible price equals $E(v|L)$ when the execution rate τ be constant.

We have 28 valid independent groups in total, 14 groups for each treatment[18]. Fourteen samples of signals were generated randomly by the computer, given the condition that the probability of drawing either a high or a low signal is 50 percent in each sample. So in each sample the number of high or low signals is 30 and the average market value is 2.5, i.e. the expected market value. Thus for each group in each treatment, there exists a corresponding group with the same market values across rounds in the other treatment.

The experiment lasted for about one hour for the fixed treatment, and about one and a half hour for the uniform treatment, including both the time for reading instructions and the test phase.

At the end of each session, subjects were paid in cash in private. The average payment in pounds was £16.02, [19]which is considerably higher than the students' regular wage. The highest payment was 45.34 pounds, and the lowest payment was 1.18 pounds (This subject obtained three pounds, the minimum payment we promised when recruiting subjects). The standard deviation of earnings in the uniform treatment was 7.87, much larger than 3.94 the standard deviation in the fixed treatment.

3.4. Results

Since subjects that were in the same group interacted with each other, their decisions are inevitably influenced by each other. So we cannot conduct statistic tests based on each subject's behaviour. The independent observations that we can use are the

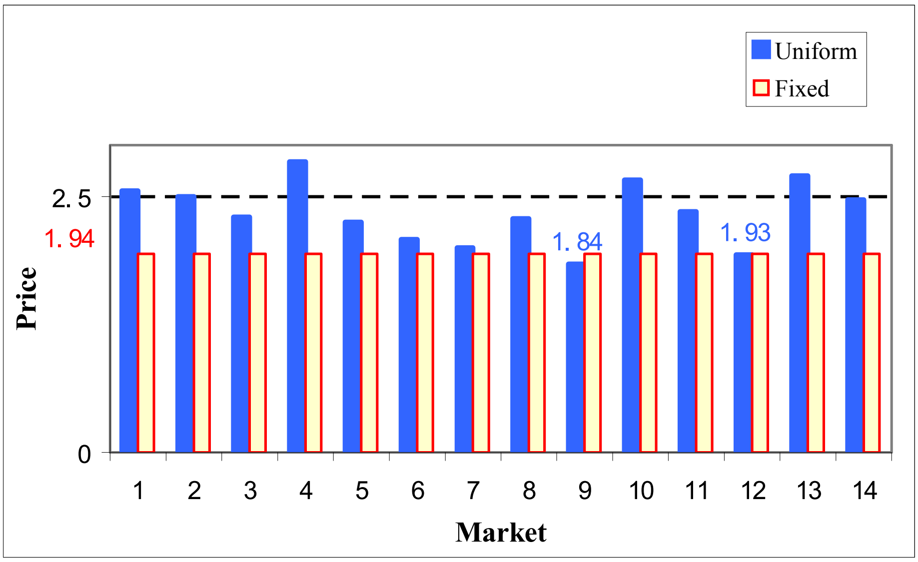
[17] The experiments were computerized and conducted with the help of the software Visual Basic 6.
[18] There was one bidder who went to bankruptcy in round 15 in the uniform price auction treatment. Since he was not able to continue in the experiment and the computer took his role following the pre-designed strategy, we didn't include the data of this group in our results. In another session, the computer stopped working at the last round after all but one subject had submitted their decisions. In that case we calculated their payments by hand. The subjects who failed to submit decisions gave us her decision record for the round right after this before she realized what happened to the program. We believe that this has not influenced the subjects' behaviour so we have used the data from this group.
[19] At the time of experiment, the exchange rate between Pound Sterling and US Dollar is about 1: 1.82.

groups' results. Because for each of the fourteen independent samples of market value used in the experiment, both uniform price and fixed price treatments have been conducted, we can compare whether the performances are different across treatments or whether one treatment is superior to the other especially in terms of revenues. As there exist different equilibria in the uniform treatment, we are also interested in examining which prediction fits the data the best.

3.4.1. Market Price, Seller's Revenue and Buyers' Earnings

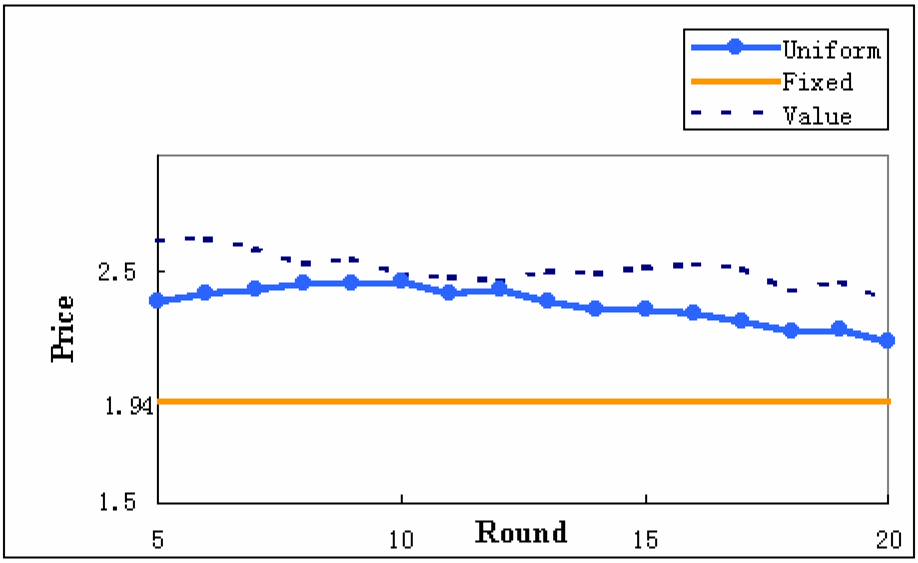
The average market price in the uniform treatment falls below the predetermined fixed market price (as well as price 2) in only 2 out of 14 markets, and only slightly. Figure 3.1 plots the average prices of the two treatments for all 14 samples of signals, in each of which the two treatments have the same market values. According to the Sign test, the market prices in the uniform price auction is significantly higher than that in the fixed price auction (p =0.0065, one-tailed).

Figure 3.1 Average Price across Markets



We are also interested to see if the seller's revenues change over rounds. Figure 3.2 plots the average price (over 14 markets) for each round. Since the shares were fully subscribed in all 20 rounds on average (for each round, we calculate the average demand of that round of all 14 markets), in the figure of seller's revenue, there are only scale differences as shown in Figure 3.2. To make the relationship clearer, we use the moving average of prices over five rounds to smoothen the curve. The dashed line at the top describes the moving average of market values. The plot of the moving average market prices for the uniform treatment lies between the market value line and the fixed price line, which indicates that on average bidders obtain a positive profit, but they earn less than in the fixed treatment.

Figure 3.2 Moving Average Price over Five Rounds



As the average value of shares is the same for each market, bidders' payoff is, on average, higher in the fixed price offerings. This holds for both uninformed bidders and informed bidders with either high or low signals. In the uniform treatment, we cannot reject the hypothesis that the uninformed investors obtain zero profits (p=0.6257, Wilcoxon signed-ranks test[20]). The L investors' average payoff is significantly negative (it is negative in 12 out of 14 markets; p = 0.004, one-tailed). In the fixed treatment, L investors gain profits statistically undistinguishable from zero (p=0.2676), while H investors get positive payoffs in all markets, as do uninformed investors[21], which is in accord with the prediction of the model. H investors gain higher profits in the fixed than in the uniform treatment (p=0.0001, one-tailed).

Result 1: The average market price in the uniform treatment is significantly higher than that in the fixed treatment both across markets and across rounds. The seller generates higher revenues and investors obtain lower payoffs in the uniform than in the fixed treatment.

The average market prices across rounds are plotted in Figure 3.3. We find that the trend lines of the average market values and the uniform prices are nearly parallel. This suggests that there may exist a positive correlation between the market prices and the market values. As we cannot test this among markets because the expected market value is identical for each market, we have to check this within each of the markets.

However, since subjects in the same market are very likely to influence each other, the observations of each round within a group are not independent. Thus we have to use an alternative way to test this relationship. First we calculate the Spearman rank-order correlation coefficient r_s between the value and the market price for each market. If there is no relationship between them, the signs of the correlation coefficients should be random. As Table 3.4 shows, there is only one negative sign

[20] Unless otherwise specified, the test statistic used in this chapter is Wilcoxon signed-ranks test (two-tailed).
[21] In one group, the uninformed bidder bids for zero units throughout the session thus obtained zero earning besides the endowment. All the other uninformed bidders gain positive payoffs.

among the 14 outcomes. So we conclude that there exists a positive correlation between the market price and the market value (p=0.0009, one-tailed sign test).

Figure 3.3 Average Market Price across Rounds

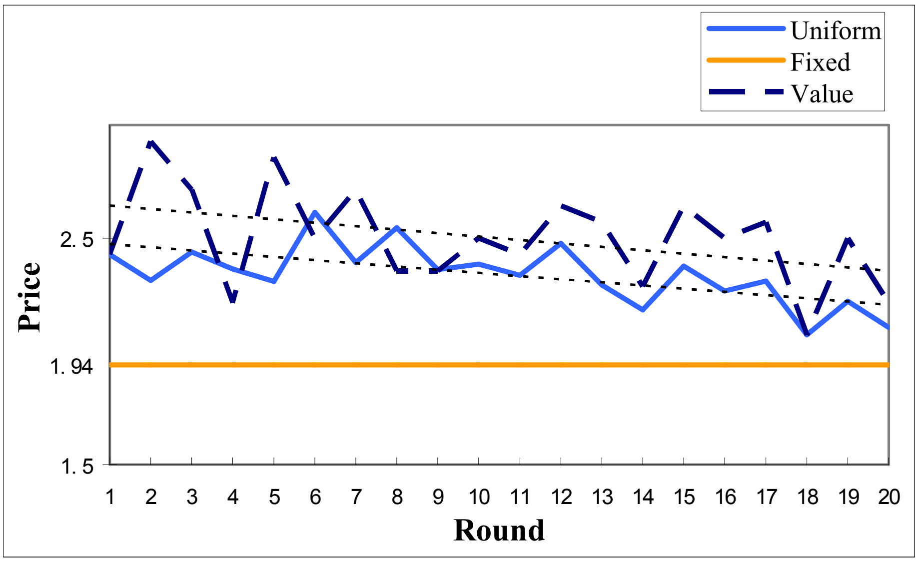


Table 3.4 Spearman rank-order correlation coefficients (value over price)

Market	1	2	3	4	5	6	7	8	9	10	11	12	13	14
r_s	0.35	0.36	0.44	0.21	0.05	0.62	-0.35	0.38	0.04	0.33	0.09	0.42	0.04	0.24

The price is less than value in 131 out of 280 rounds, and equals value in 60 rounds. The average market price in the uniform treatment is no higher than 2.5, i.e. the expected value of one unit share[22]. Though we did not set a reservation price, the resulting market price is higher than the highest possible reservation price suggested by the model, which is the L investor's expected value 2 (p=0.0009, one tailed). In addition, we find that in four occasions the price exceeds value during the first ten rounds, but only once during the last ten rounds (overpriced at a level of only 0.002), indicating that bidders have learnt to shade their bids as the experiment goes on.

[22] If the alternative hypothesis is that price different from value, we are unable to reject the hypothesis that the average market prices are not significantly different from 2.5, the expected (unit) value of shares (p=0.0803).

Result 2: There exists a positive relationship between market price and market value in the uniform treatment. The average market price is no higher than the expected market value but higher than an L investor's expected value.

Results 1 and 2 imply that the uniform price auction is superior to the fixed price offering in terms of revenue maximization and market value revelation in our experiment.

3.4.2 Strategies

Because there are multiple equilibria in this game in the uniform price auction, and some of them can lead to similar market prices, we take a closer look at the strategies used by bidders.

We first examine if the tacit collusion equilibrium has been achieved in the experiment. According to the equilibrium prediction provided by Biais and Faugeron-Crouzet (2002), the resulting equilibrium market price is zero. Among all 280 rounds the market price was zero in only four cases. All of them happened in the first three rounds, and the total demands were less than the quantity for sale. This is more likely because of risk avoidance rather than collusion, since it seems that bidders tried to learn at a low cost what would happen in the market by submitting low demands to avoid losses. As soon as they understood what they were doing, their demands were increased.

If we include the weakly dominated tacit collusive Nash equilibria, the market price under the tacit collusion equilibrium in the uniform treatment should lie between

However, if the alternative hypothesis is that price is lower than value, we can reject the hypothesis that price equals value and accept the alternative hypothesis at 5% level (p= 0.04, one-tailed).

zero and two. However, in no group bidders obtain the same allocation. So no tacit collusion equilibrium under the symmetry assumption has been achieved in the experiment. There was one subject who bid for 25,000 units from round 8 until the session was over, but none of her competitors have cooperated with her. Many subjects tried to obtain more than a quarter of the entire shares.

We then look for behaviour consistent with the tacit collusion equilibrium but where bidders demand different amounts. As different strategies can lead to the same market price in this game, that should satisfy the following conditions. First, the resulting market price should be no more than 2. Second, there were some bidders who only bid at low prices to avoid obtaining any allocation at high prices. In order to distinct from these risk avoidance behaviours, in a tacit collusion equilibrium a subject should bid from a sufficient high price, at 2.25 as we have proved in chapter two. Third, the demand curve above the market price should be steep. The main property of the tacit collusion equilibrium is that bidders behave in the same way regardless their signals. So if the strategies in a round satisfy the above three conditions, we then take a closer look at subjects' behaviour across rounds, especially the rounds with different signals before and after the "nominated" round. However, although we relax the assumptions and allow a broader price range, we fail to find even one group that has achieved collusion in any of the markets. Some collusion-like behaviour failed either because there was at least one buyer in the group that did not cooperate, or because their strategy was not self-enforcing (the demand function was not steep, or did not involve a high enough price, i.e. 2.25). Subjects tend to change their behaviour with a change in signals. In particular, bidding more at higher prices when receiving high signals rather than low signals, though they are the same players receiving different signals (see the average cumulated demand functions of different type bidders in Appendix 3.B).

Result 3: No tacit collusion equilibrium has been achieved in the uniform treatment.

The reason that no tacit collusion equilibrium has been achieved may be the existence of a large set of equilibria. It is unlikely that all players in a group choose the same extreme strategy in this set. Because of this difficulty and the steep demand function required in the tacit collusion equilibrium, it is risky for subjects to choose this strategy. This makes the strategy even less attractive.

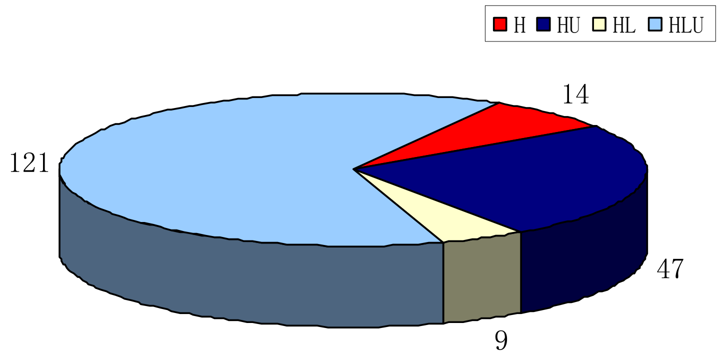
The equilibrium where H investors absorb all shares is the other extreme case of the equilibrium set. If the L and the uninformed investors participate, H investors can easily suffer a loss especially when bidding above the lower bound of EH (i.e., bidding $1/n$ at a prices above v_n but no higher than v_{n+1}). Hence if the above analysis is in the right direction, there should be few cases in that the EH equilibrium is achieved in the experiment as well.

In all the 14 markets each type of subjects obtain a positive allocation. We then count the numbers of rounds in which only H bidders, H and L bidders, H and the uninformed bidders, and all type bidders obtain allocation respectively. In 191 out of 280 rounds, the market price is no higher than the market value, among which all type bidders share the market 63.4% of the time, much higher than when only H, H and L, and H and uninformed bidders obtain an allocation, which are 7.3%, 4.7% and 24.6% of the time respectively. The frequencies of different distributions of allocation are plotted in Figure 3.4.[23] The equilibria EH, EHL and EHU predict that the type of

[23] When the market value is 4, there is no L bidder so the allocation to "L" bidders is certainly zero. If we deduct these cases, the number of rounds that only H bidders obtain allocation is reduced to 5, and that H and uninformed

bidders who are excluded from the market place bids no higher than v_0, 1. Among all the bids submitted by the uninformed bidders, only 3.2% are placed at prices that are less than or equal to 1. The percentage for L bidders is 7.9%. Hence the results seem closer to the equilibrium EHLU rather than the others.

Figure 3.4 Frequencies (rounds) of Different Allocation Distributions



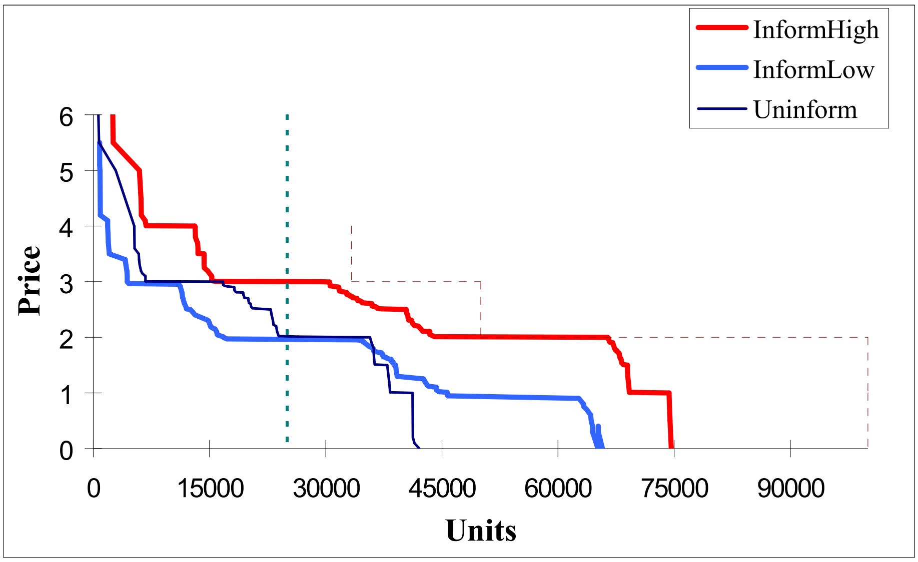
We check the strategies further by examining the average aggregate demand function of each type of bidders, which are plotted in Figure 3.5. In the figure the dotted vertical line hits the horizontal axis at one quarter of the total shares. Though L and uninformed investors bid for less than a quarter at price 2 (the $E(v|L)$), the H investor's demand curve crosses this line at the price that equals to $E(v|H)$. This again confirms that no tacit collusion equilibrium has been achieved. The demand curve of the H investors shifts upward relative to that of the L investors, indicating bidders behave according to their signals, namely bidding more at higher prices when

investors share the market becomes 24. Except that the chances that all type bidders share the market is even larger, the distribution of allocation remains similar if including the rounds where prices exceed value (whether count in the rounds when market value is 4 or not). In chapter two we have shown that market price should be lower than market value in equilibrium if L investors participate. The times of allocation to HL and HLU become 4 and 78 if deducting the rounds in which price equals value. However, the distribution of allocation changes in neither of these cases.

receiving high value signals. The two demand curves look very similar except one is above the other. The demand function of the H investors thus can be seen as that of the L investors plus an extra amount at any price levels. This strategy reflects the winners' curse feature of common value auctions, in the sense that investors with higher valuations bid more aggressively. In addition, the downward sloping demand function implies the existence of demand reduction. Although demand reduction is observed, the aggregate demand curve is generally flat around the market prices (1.8405 to 2.838). This property is similar to Kandel, Sarig and Wohl's (1999) empirical findings using data from the market for IPOs in Israel. The demand curves have a common feature that they are steep at higher prices and turn flat at lower prices. We also observe there exist mass points along the demand curves at the price levels that are equal to bidders' expected market values. Below the price that equals her expected market value, an L subject bids even more than an uninformed bidder. The reason is that subjects bid less at higher prices to avoid paying too much, and bid more at lower prices to enlarge their allocation, which is safe from a bidder's point of view especially when her bids are no higher than her expected value. We also observe some bidders rarely place bids above their expected values. (See Appendix 3.B for the plots of the average demand curves of each type of bidders in each market).

In chapter two we have shown that the upper bound of the equilibrium set is bidding $\frac{1}{n-1}$ at v_n for every n >1. In the experiment few bids are observed above this range. The dotted step-like curve in Figure 3.5 is the lower bound of the equilibrium set EH (i.e., bidding $\frac{1}{n}$ at v_n). We find the average demand function of H bidders located even below this lower bound. This again confirms that the equilibrium EH in general has not been realised in this experiment. H investors have to lose some shares to L and uninformed investors to avoid the market price being raised too high. We have seen in result 1 that in the uniform treatment L investors gain negative average payoffs, hence the informed investors would have been better off had they not participated when observing low value signals. However, their participation as well as the uninformed bidders' does influence the H investors' behaviour. Because in most cases where prices are lower than market values all types of bidders obtain some allocation, and bidders' demand increases with their expected value which results in the market prices being positively related to the market value (result 2), the equilibrium set EHLU captures more characteristics of the subjects' strategies played in this experiment than the other equilibria.

Figure 3.5 Average Aggregated Demand Functions



Result 4: The bidder's strategy in the uniform treatment is closest to the equilibrium set EHLU, where all bidders participate, H bidders bid more aggressively than L and uninformed bidders, and the market price increases with the market value.

We also observe some information free rider problem in the sense that there are bids placed at prices higher than the highest possible market value. However, this does not cause losses because the amount of such bids is very small. Bidders keep the demand curve steep at such prices carefully to avoid pushing the market price too high. As Figure 3.5 shows, the demand curves of all types of bidders are steep at prices higher than 4, the highest possible market price. In the experiment, there is no overpricing when the market value is 4. However, the existence of the information free rider problem may influence other subjects' behaviour, who may be more cautious in choosing strategies from the set of equilibria in order to avoid the potential risk inherent in certain strategies such as the tacit collusion strategy and the EH strategies (except the lower bound).

At last we check if bidders submit flat demand functions in the experiment, which we have shown is not an equilibrium strategy. A flat demand function is defined as bidding all shares at one price. So if an informed player bids for 100,000 (or more) or an uninformed player bids for 80,000 units we regard it as a flat demand function. 12 out of 56 players have submitted 51 flat demand functions among the 1120 bids during the experiment, which is less than 5%. Among them seven bidders submit flat demand functions no more than three times; two bidders bid for the same amounts (S and Q respectively) at the same prices (2 and 1 respectively) repeatedly, and it looks more like they were trying to get some allocation at a low cost rather than competing to get a higher allocation; the remaining three bidders submit flat demand functions (in 4, 6 and 9 rounds respectively) at prices no higher than the expected value given the observed signal and bid at a higher price when receiving high signals. However, only in one group and only in two rounds, there are two different pairs of players submitting flat demand functions. Otherwise in no market two bidders submit flat demand functions at the same time. So we do not observe Bertrand competition in this experiment.

3.4.3 Price Volatility

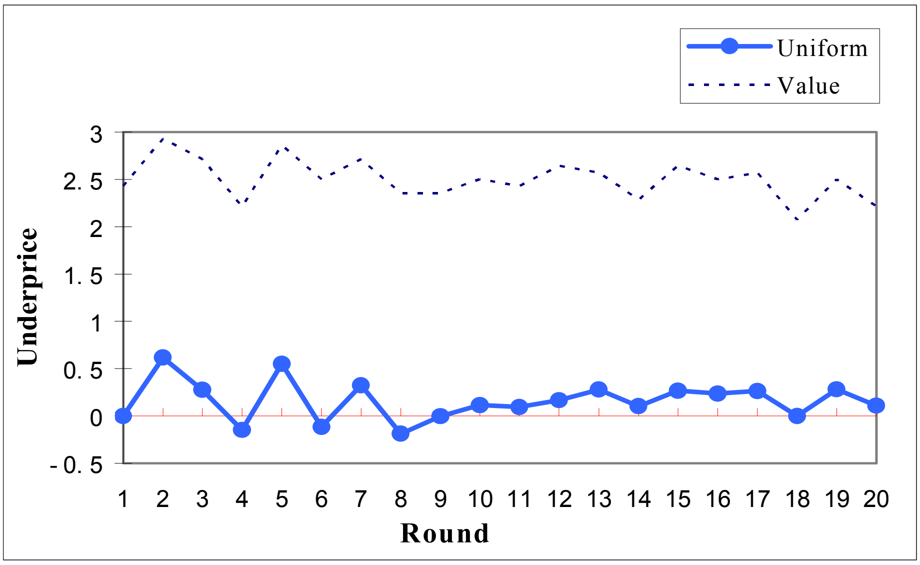
The price volatility over time is also an important factor to take into account when choosing an auction format. For the uniform treatment, we find that although on average the shares are underpriced, the level of underpricing varies with the market value. There is no overpricing when the market value is 4 (when all informed investors receive high signals), and no underpricing when the value is 1 (when no one receives a high signal). It is more likely that underpricing exists when the market value is high (5.04% overpriced, for value equals 3 or 4), and more likely overpricing is observed when the market value is low (13.47% underpriced, for value equals 1 or 2), as Table 3.5 indicates.

Table 3.5 Times of Being Over- or Underpriced at Different Market Values

Value	Number of		
	price > value	price = value	price < value
1	25	6	0
2	57	34	19
3	7	18	82
4	0	2	30

In Figure 3.6 we plot out the underpricing level in the uniform treatment against the market values across rounds.[24] The movements of the underpricing level is in accordance with that of the market value, which suggests that there is a positive relationship between the market value and the underpricing level, that is higher market values are related to more severe underpricing in the uniform treatment. The Spearman rank-order correlation coefficients between the market value and the underpricing level are positive for all 14 markets (See Appendix 3.C, Table 3.C1 for the detailed coefficients).

Figure 3.6 Average Underpricing across Rounds

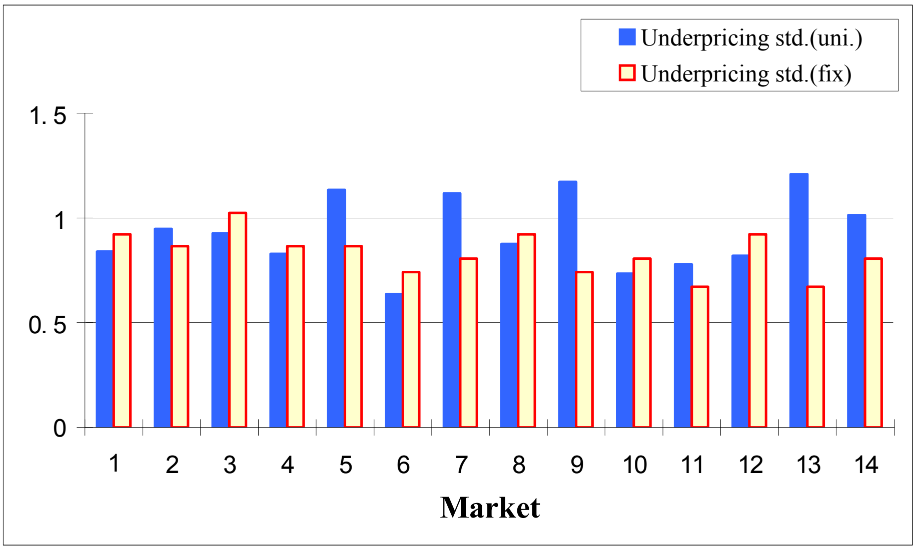


Though bidders submit their bids according to their expected market value, not all the information is revealed by the prices, and more demand reduction is engaged for higher market value. This is consistent with the characteristic of the example that we provided for the equilibrium set EHLU in chapter two.

To check the volatility within a market, we calculate the standard deviation of the underpricing level for both treatments[25]. As shown in Figure 3.7, it seems that prices are more volatile in the uniform treatment. However, the standard deviation in the uniform treatment is not significantly different from that in the fixed treatment (p=0.1175, Wilcoxon rank sum test).

[24] The underpricing level in the fixed treatment is just the market values minus the fixed price 1.94. We do not plot it out in the figure as it is just a downward shift of the market value curve.
[25] In the fixed treatment, the market price is fixed but the market value changes over rounds. So the underpricing level in the fixed treatment is naturally volatile.

Figure 3.7 Standard Deviation of Underpricing across Markets



Result 5: In the uniform treatment, there is a positive relationship between the degree of underpricing and the market value, and the price becomes less volatile as subjects are getting more experience. The price volatilities in both treatments are not significantly different.

3.4.4 Allocational Efficiency

In both treatments, H investors obtain a higher allocation than both the L subjects (in all the markets) and the uninformed bidders (in all the markets of the fixed treatment; and in 12 out of 14 markets in the uniform treatment, p=0.007, one-tailed). There is no significant difference in terms of the allocation of shares between L investors and uninformed bidders (p=0.6698 in the uniform treatment and 0.2676 in the fixed treatment). Investors with higher valuations (H investors) get the highest allocation so *partial allocational efficiency* (Wang and Zender, 2002) is qualitatively achieved. In the uniform treatment, this is either because bidders with H values reduce their demand in order to pay less for the shares won, and thus have to lose some units to bidders with lower expected valuations, or because the participation of L investors forces H investors to reduce their demand in order to avoid unprofitable prices. We compare the H investors' allocation rate between the two treatments and fail to find significant difference between them (Wilcoxon rank sum test, p=0.9634. See the allocation rate of both treatments in Appendix 3.D). Thus we are not able to tell which method is more efficient in terms of allocational efficiency.

Result 6: Partial allocational efficiency is achieved in both treatments.

3.4.5 Demand in the Fixed Treatment

In the fixed treatment, there is oversubscription on average (both across markets and across rounds) as we have pointed out in section 3.4.1.[26]

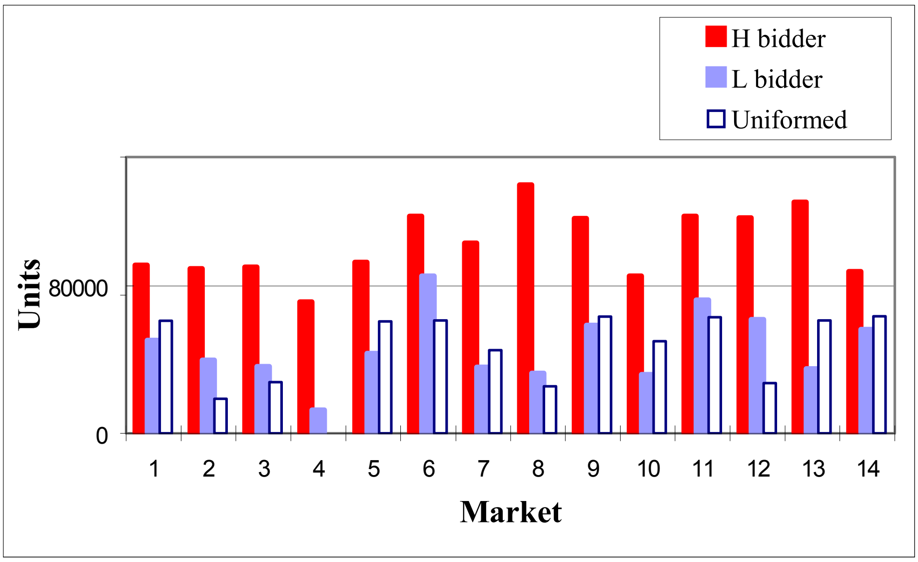
The average demand of both uninformed and informed investors falls below the equilibrium prediction in all 14 markets (Figure 3.8)[27]. Though there is no evidence of following the equilibrium strategy during the early rounds, some players learnt extremely fast. From Figure 3.9 we can see that H investors and uninformed bidders increase their demand across rounds, especially at the beginning of the experiment. The demand of L investors, however, remains stable across rounds, though by definition of the market price, the expected earnings are equal to zero and L investors make no difference when bidding zero or 100,000 units. H investors bid less than the optimal strategy (bidding for 150,000, the highest accepted demand) on average, which may be due to loss aversion. Once an investor observes a high signal, he should

[26] There was one uninformed bidder who always submitted a zero unit demand and this led to undersubscription in ten rounds. Except this, only in three rounds demand was below supply.

know that he would not lose anything (the market value then is at least 2 which is higher than the market price 1.94). Then to gain a higher payoff, an H investor should try to increase her allocation as much as possible. However, if the loss from paying the cost counts more for them than the gain of points in terms of utility, they may bid less than the equilibrium prediction. Since the shares for sale were always oversubscribed, this strategy variation did not make much difference for the seller.

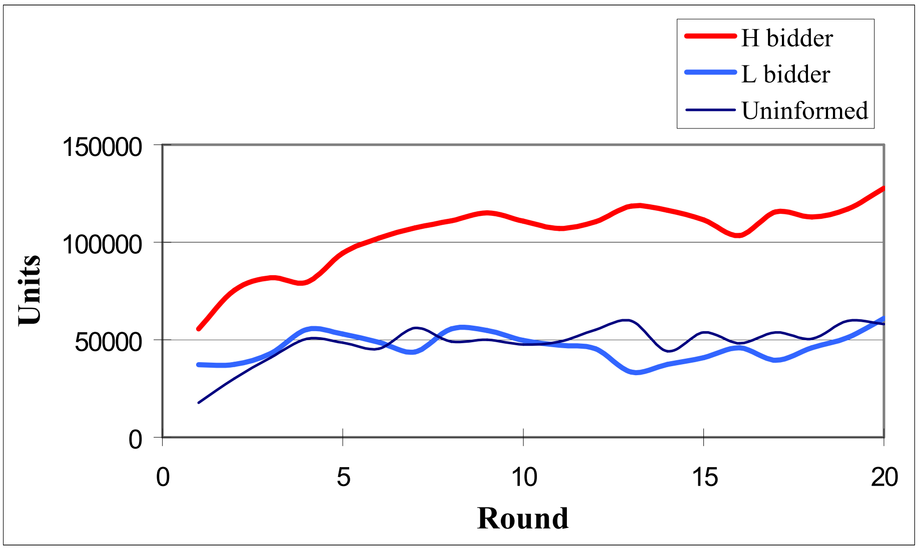
Result 7: In the fixed treatment, shares are oversubscripted. However, on average all types of bidders bid less than the equilibrium prediction.

Figure 3.8 Average Demands of Different Type Subjects (Fixed Treatment) across Markets



[27] This is still the case even if we use the data of the last 10 rounds only except in group 14 the uninformed investor bid fro 80,000 from round 8 until the end of the experiment. And there were more H investors bid for 150,000 in the last half of the experiment.

Figure 3.9 Average Demands of Different Type Subjects (Fixed Treatment) across Rounds



H bidders always bid more than L bidders in all the 14 markets at the expected value of an H investor (3) and at most prices. So demand is positive related with the expected market value, consistent with the empirical findings of the positive correlation between oversubscription levels and first-day returns (Cornelli and Goldreich, 2002). All the Spearman rank-order correlation coefficients between market value and bidders' demand are positive in the fixed treatment. However, we fail to find the same correlation in the uniform treatment (see Appendix 3.C, Table 3.C2). This again confirms that the demand in the uniform price auction is more stable, because demand has two effects in that auction, namely to raise the allocation and to raise the market price.

Another prediction of the fixed price offerings is that uninformed investors face a winners' curse problem, in the sense that they are allocated more shares as the value gets lower. Since H investors demand more than L investors, this prediction is not surprising. The Spearman rank-order correlation coefficients between uninformed bidders' allocation and the market value are negative in 12 out of 13 markets (excluding the one where the uninformed bidder always bid zero units), which is consistent with the prediction (p=0.002 by one-tailed Sign test). The same result happens in the uniform price auction, where in 13 out of 14 markets the correlation coefficients are negative (p=0.001, one-tailed Sign test)(See Appendix 3.C, Table 3.C3 for the values of the coefficients).

Result 8: Subjects' demand is positively related with the expected market value in the fixed treatment, and thus the winner's curse phenomenon is evident. The winner's curse property is also observed in the uniform treatment.

3.5. Summary and Concluding Remarks

We report the results of an experiment that compares the performances of the uniform price auctions and the fixed price offerings. There are three main findings of this study.

First, in our experiment, the market price in the uniform treatment is higher than the highest possible price in fixed price offerings. As a result, a seller receives more revenue while bidders obtain lower payoffs in the uniform treatment. So the uniform price auction is superior to the fixed price offerings in terms of revenues in our experiment. In addition, the market price in the uniform treatment is well above the expected market value of an investor with a low signal and no higher than the expected market value. Thus though it seems that underpricing still exists, the degree of underpricing is within a relatively narrow range.

Second, there is no evidence to support the tacit collusion equilibrium that has been emphasised by Biais and Faugeron-Crouzet and many other researchers. On the contrary, subjects submit demand functions increasing with their signals and consequently the market price is positively related to the market value. The demand functions in the uniform price auctions in our experiment include both a winner's curse aspect, in the sense that bidders with higher expected valuations bid more aggressively and a demand reduction aspect, in the form of a downward sloping demand function. According to this strategy, the price does reflect the market value up to some extent, but not all information is revealed by the market price. Bidders adjust demand with signals to keep a balance between their allocation and the market price so that their expected profits are maximized. As a result the price adjusts with market value partly and therefore underpricing is inevitable.

Thirdly, the results show that the equilibrium set in which all types of bidders receive an allocation fits the data the best. The characteristics of that equilibrium is that all types of bidders participate in the market, an H investor bids for a higher amount than an L investor at all prices and thus the market price increases with market value and an H investor obtains a larger allocation than an L investor. All of these are consistent with the results in our experiment.

Because of the existence of a large equilibrium set, choosing extreme strategies in the set is risky for bidders when other bidders are likely to choose other strategies. Hence although the tacit collusion equilibrium is theoretically interesting, it should not be overemphasized for practical purpose.

However, the existence of multiple equilibria including those that lead to low revenues provides potential uncertainty for the performance of uniform price auctions. Introducing rules to prevent choosing such strategies by investors should be taken into account by sellers who consider the uniform price auction method. Finally, the consideration of equilibrium in mixed strategies is also a potentially interesting direction for future researches on uniform price auctions.

Appendix 3.A Instructions (This is the instructions for the uniform price auction. The different parts for the fixed price offerings are described in the brackets.)

Thank you for coming to the experiment. This session is part of an experiment in the economics of decision-making. During the session, please do not talk or communicate with any of the other participants. If you have a question, please raise your hand and one of us will come to your desk to answer it.

Description of the experiment

Each person in this room plays the role of a buyer. You are randomly paired with three other buyers in a group (the same three throughout the session). One buyer of your group is an uninformed buyer, and the other three are informed buyers. The status of each of you as an informed or uninformed buyer remains the same in the session. You will not know which of the participants are in your group. The session consists of 20 rounds, each of which is structured as follows.

The computer takes the role of the seller. The number of units for sale is 100,000. At the beginning of the round, each informed buyer receives a signal revealing the value of the goods. The probability of observing either a *high value* or a *low value* signal is 50%. The uninformed buyer does not receive a signal. No one knows the other buyers' signals.

The value of each unit of the goods equals one plus the number of *high value*

signals received in your group.

The decision you will make is to submit up to 6 bids. Each bid is a price-quantity pair consisting of the price you would like to bid and the number of units you bid for at that price. Prices cannot be negative. (The decision you are asked to make is to submit the number of units you would like to bid for at the predetermined market price.) The uninformed buyer can bid up to 80,000 units. Each of the informed buyers can bid up to 150,000 units.

When all the buyers submit the decisions, the seller will decide the market price and allocate the goods. The **Price and allocation rules** are as follows.

If the total quantity bid for in your group is less than or equal to the quantity for sale (i.e.100,000 units), the market price will be zero. Each of you will be allocated the number of units you have submitted.

If the total quantity bid for in your group exceeds the quantity for sale (100,000 units), the goods will be allotted from the highest bid price to lower prices until all the units are allocated. The market price will be set as the highest bid price where the cumulated units bid for exceed the quantity for sale. The quantity you have bid for *above* the market price will be fully allocated, those *below* the market price will be ignored, and those *at* the market price will be allocated proportionately to fully sell the goods (always rounded to integer).

Allocation rule

When all the buyers submit the decisions, the seller will allocate the goods.

If the total quantity bid for in your group is less than or equal to the quantity for sale

(100,000 units), you will be allocated the number of units you have submitted.

If the total quantity bid for exceeds the quantity for sale (100,000 units), the units of goods will be allocated proportionally among you and your competitors (always rounded to integer).

Your payment

At the beginning of the experiment, there are 350,000 points in each informed buyer's account, and 400,000 points in the uninformed buyer's account.(there are 100,000 points in each informed buyer's account, and 150,000 points in the uninformed buyer's account.) In each of the 20 rounds you can earn or lose points, which will be credited or debited to your account.

Point earnings (or losses) in each round, which are depending on both your and your competitors' decisions, equal the per unit value minus the market price, multiplied by the number of units allocated to you. If you bid more than 100,000 units, you need to pay a cost of 5000 points.

At the end of the session, the total points on your account will be converted into pounds and be paid to you in cash. The exchange rate will be 250 points = 1p. The more points you have, the more money you will earn.

In our research, we are interested in what will happen if everybody tries to earn as many points as possible. Making unreasonable decisions may make your account balance negative. If you go bankrupt, you cannot continue participating in this experiment and will be paid only three pounds. By understanding the instructions and making decisions carefully, you may earn a considerable amount of money.

Information you will receive

At the end of each round, you will be informed the results of that round including the units and the points obtained by you and your competitors, the total cumulated units placed at the market price, the per unit value and market price of the goods, and your total point earnings. The results will be displayed for 30 seconds before the new round begins. However, results of previous rounds can be checked from the "History" menu at any time during the session.

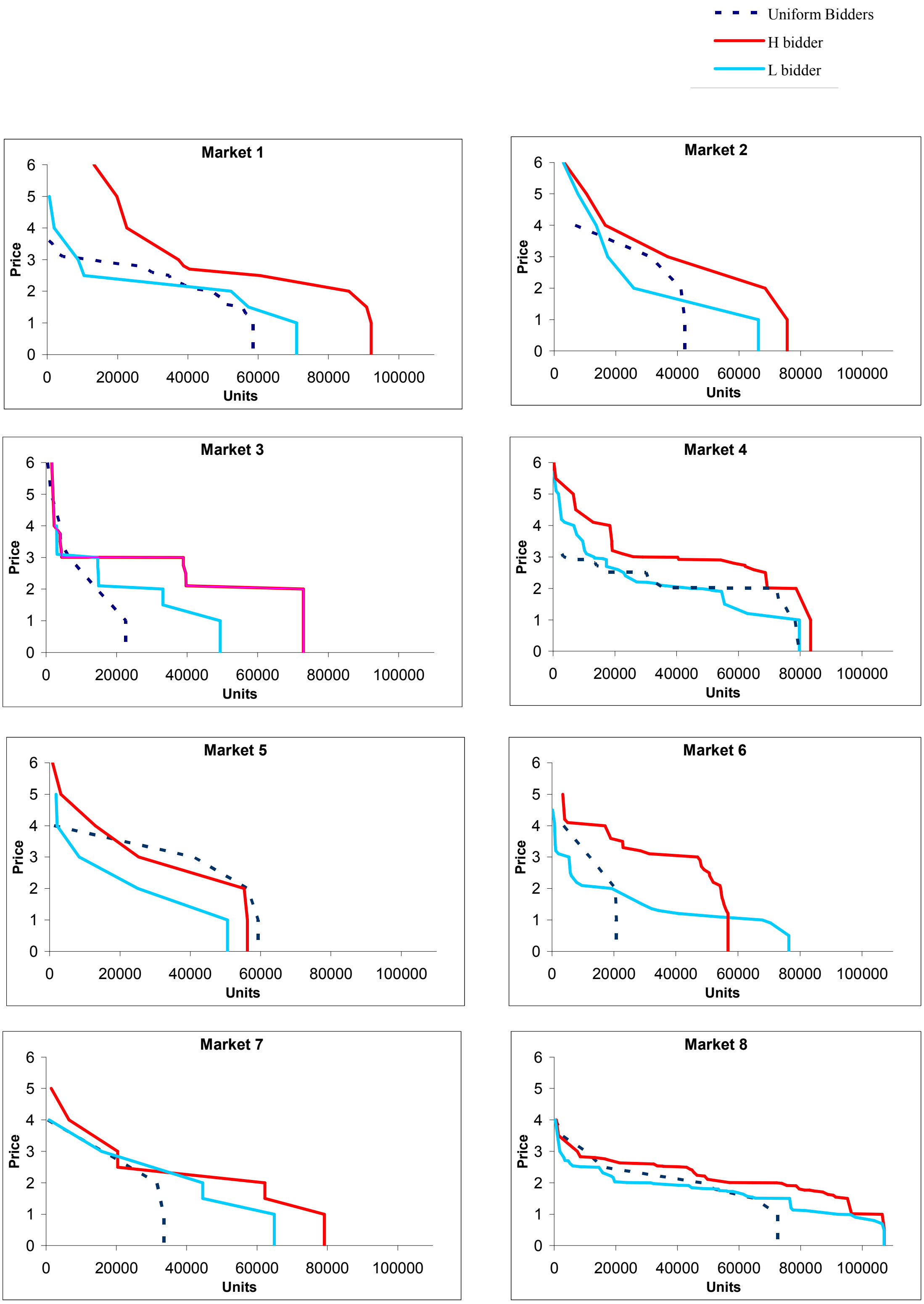
You can also check the description of the experiment on this page from the "Help" menu (gives an example of the market rules) and the "Session Information" menu (explains other session information) on your screen.

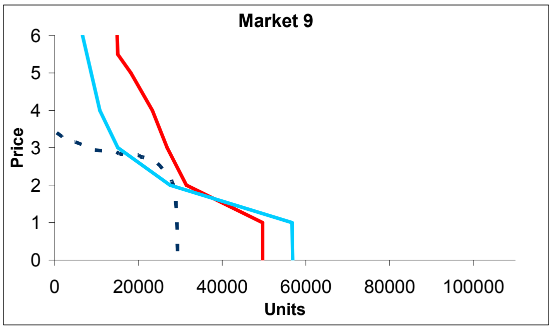
The formal session will begin after you correctly answer some questions from your computer. The numbers in these questions as well as those in the Help menu do not relate to the numbers of the real experiment. (The numbers of the questions do not relate to the numbers of the real experiment.)

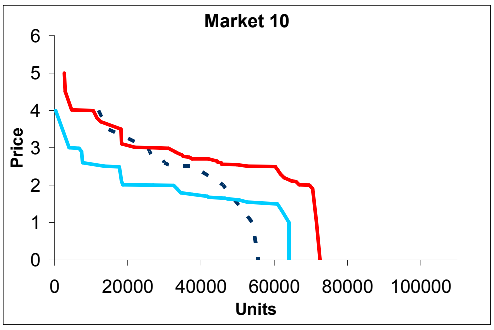
Now please press the "Start" button and follow the prompts on your screen.

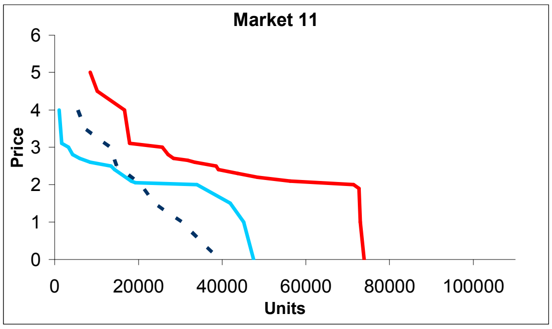
Appendix 3.B

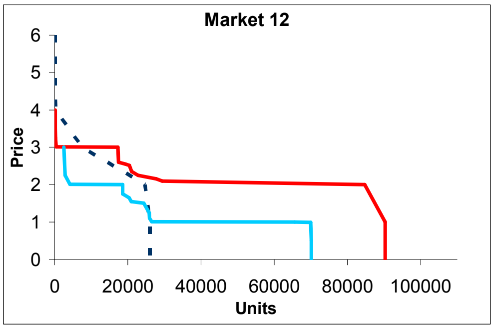
Figure 3.B Average Demand Functions

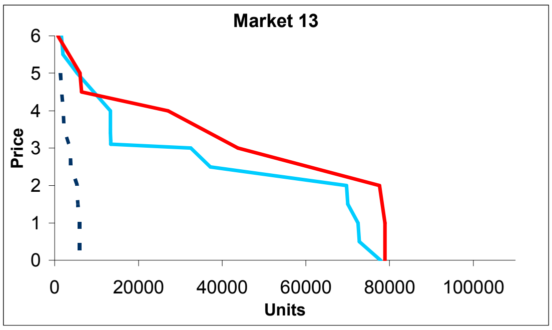


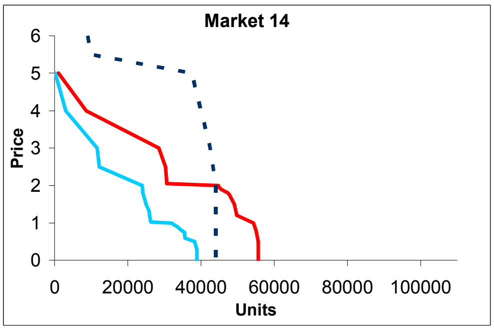

Market 9
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units


Market 10
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units


Market 11
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units


Market 12
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units


Market 13
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units


Market 14
Price
0
1
2
3
4
5
6
0
20000
40000
60000
80000
100000
Units

Appendix 3.C Spearman Rank-order Correlation Coefficients

Table 3.C1: Underpricing Level and Market Value, Uniform Treatment

Markets	r_S	Markets	r_S
1	0.9463	8	0.9248
2	0.5536	9	0.7013
3	0.7797	10	0.8866
4	0.8459	11	0.7425
5	0.6873	12	0.9650
6	0.4090	13	0.5350
7	0.8859	14	0.6329

Table 3.C2: Market Value and Bidders' Demand

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14
rs -- fixed	0.49	0.58	0.67	0.49	0.46	0.09	0.43	0.76	0.54	0.38	0.04	0.58	0.66	0.67
rs--uniform	0.01	0.18	0.05	-0.29	-0.08	0.03	-0.32	0.24	-0.07	-0.25	0.29	0.49	-0.03	0.34

Table 3.C3: Market Value and Uninformed Investor's Allocation

Speaman Rank-order correlation coefficient between market value and uninformed investors' allocat	
Fixed price	Uniform price
-0.40	-0.28
-0.65	-0.06
n.a.	-0.20
-0.22	-0.01
-0.46	-0.66
-0.11	-0.66
-0.45	-0.39
-0.23	-0.13
-0.47	-0.34
-0.12	-0.40
-0.05	-0.23
-0.34	-0.22
-0.58	*0.11*
0.01	-0.09

Appendix 3.D

Table 3.D: H investors' Allocation Rates

Uniform	Fixed
0.564	0.479
0.452	0.560
0.611	0.561
0.611	0.658
0.412	0.466
0.560	0.568
0.479	0.676
0.548	0.518
0.426	0.555
0.578	0.510
0.623	0.514
0.620	0.553
0.605	0.594
0.373	0.446

Appendix 3.E: Screenshots

Figure 3.E1: Screenshot of the Uniform Treatment

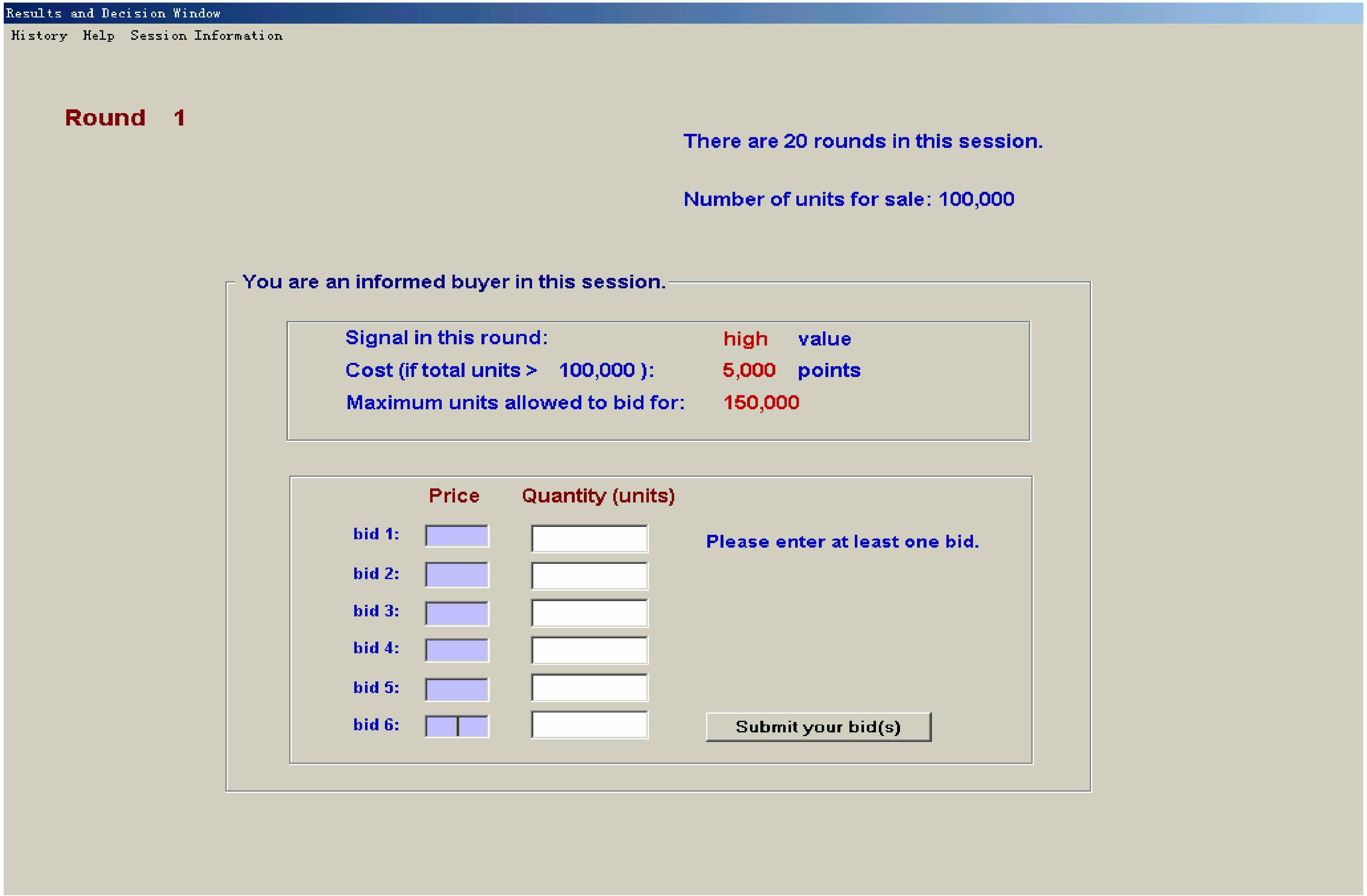
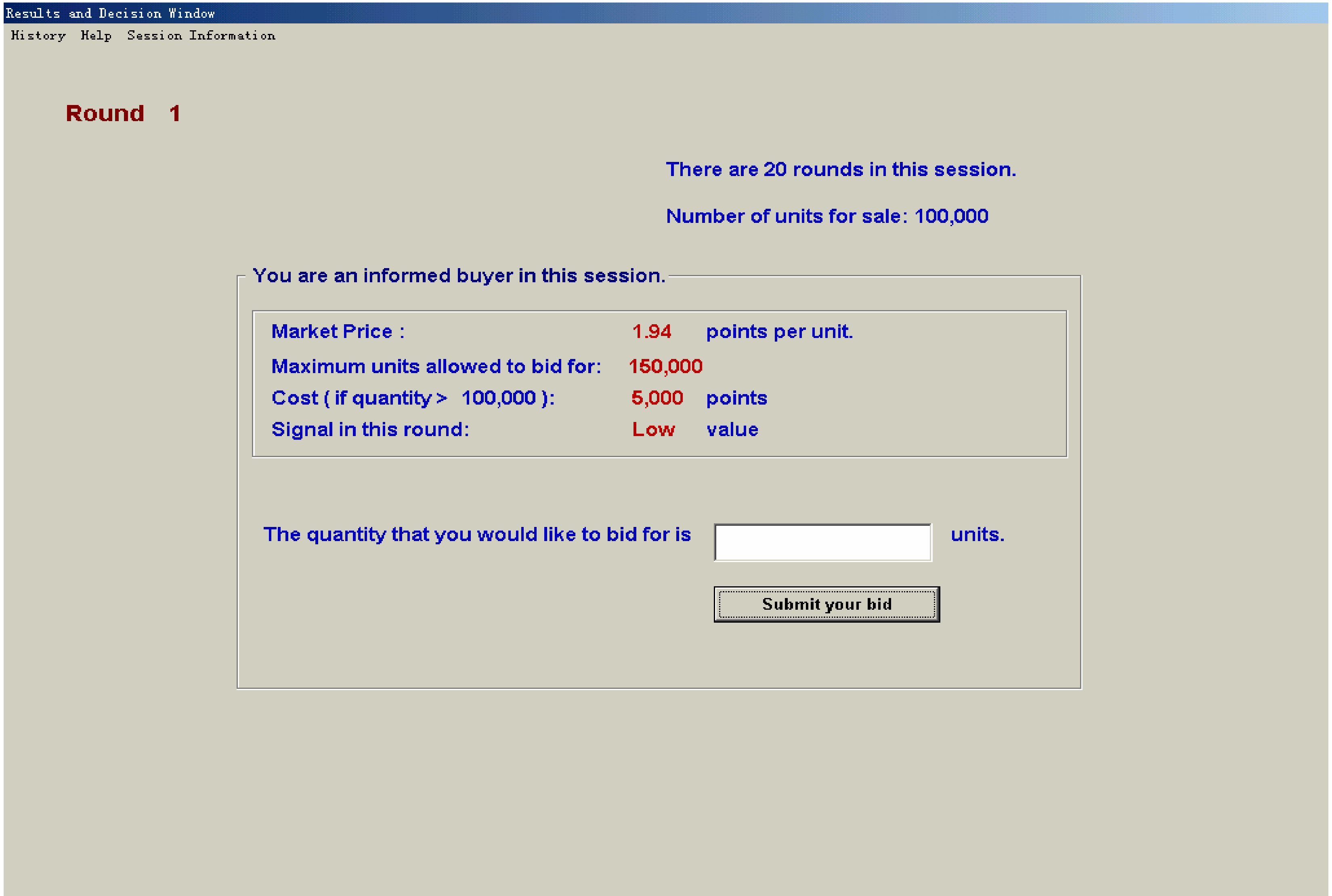


Figure 3.E2: Screenshot of the Fixed Treatment



Chapter Four:

Enlargement and the Balance of Power: An Experimental Study

Chapter 4:

Enlargement and the Balance of Power:

An Experimental Study*

4.1 Introduction

Many important collective bodies make decisions by weighted majority voting: different members control different numbers of votes. Examples are the Electoral College in the United States, the International Monetary Fund, and the Council of Ministers in the European Union.

Different voting weights typically reflect differences in sizes of populations or resources of the states. However, it is well known that the voting power of a state may be very imperfectly correlated with its voting weight (see e.g. Banzhaf, 1965). For example, a state controlling 51% of the votes has all the power under simple majority voting. The imperfect correlation between weights and power makes institutional design more complex. In order to analyze voting power a number of power indices have been proposed, the most important of which are the Shapley (1953) value or Shapley-Shubik (1964) index, and the Banzhaf (1965) index. These indices often predict unintended consequences of institutional change. For example, in the context of changes in the IMF voting rules, Dreyer and Schotter (1980) claim that there is "a

* This chapter is based on joint work with Maria Montero and Martin Sefton.

noticeable discrepancy between what one would think the consequences of the voting changes would be and what they actually are”.

A particular instance of counterintuitive effects of institutional change is the paradox of new members (Brams and Affuso, 1976). Brams and Affuso show that when a new member is added to a voting body the power indices of some original members may increase, even if the original members’ weights and the decision rule remain unchanged. The reason is that the addition of the new member may present some of the existing members but not others with greater opportunities to form winning coalitions.

Brams and Affuso’s examples are not merely theoretical curiosities. For EU enlargements it has been viewed as desirable that “the existing balance of power between the Member States … be broadly preserved” (Enlargement of the Community: Transitional Period and Institutional implications, 1978; cited in Brams and Affuso, 1985); between 1973 and 1986 the voting weights of existing members and the percentage of votes needed for a majority have been kept constant in enlargements of the EU Council of Ministers (see e.g. Felsenthal and Machover, 2001, table I). However, formal power analysis suggests the existing balance of power was *not* preserved in any enlargement: every enlargement increased either the Shapley value or Banzhaf index for some existing member state (Brams and Affuso, 1985).[1]

Of course, whether the enlargements have actually increased an original member state’s voting power is more difficult to ascertain. A natural empirical measure of voting power is the proportion of the benefits from legislation that accrue to a voter, but in order to use this one would need information on the material consequences of enlargement of the voting body. Two difficulties immediately present themselves.

First, changes in the membership of the voting body are usually accompanied by other changes in the legislative process, so that it may be difficult to disentangle the effect of enlargement from changes in these other factors. Second, the material consequences of membership may be difficult to measure, and also be influenced by numerous other factors. For example, even for the case of the EU, where weights and decision rules have remained stable across enlargements, it is difficult to measure the benefits of membership for a particular member, let alone how these benefits have changed due, *ceteris paribus*, to enlargement.[2]

These difficulties with uncovering empirical relationships between voting structure and voting power have led to a heavy reliance on assumptions about the relevant power index for formal analysis. The objective of this chapter is to use laboratory experiments to empirically examine the relation between the distribution of voting weights and voting power. In our experiment participants propose and vote on how to distribute a fixed budget among themselves.[3] This approach allows us to use a natural empirical measure of voting power -- the average share of the budget realized by a voter -- and relate it to voting weight, decision rule, and the composition of the voting body. In particular we study three treatments, corresponding to the examples in Brams and Affuso (1976), and examine how the balance of power between the original parties is affected by the addition of a new member. We find significant discrepancies between empirical measures of voting power and standard power indices. However, in a comparative static sense the theoretical predictions of Brams and Affuso are borne out. In particular, the empirical voting power of an existing

[1] The paradox has also theoretically occurred in the US Electoral College (see Brams and Affuso, 1976) and a weak version of it has been claimed to be common in the Dutch parliament (see van Deemen and Rusinowska, 2003).
[2] There have been attempts to correlate voting weights and net financial transfers in the EU, but these are qualified by the substantial difficulties with obtaining accurate net transfer data: see Baldwin (1994).
[3] Thus we focus on purely distributive policies.

member increases with the addition of a new member, thus the paradox of new members is observed.

4.2 Three Voting Games

Our three treatments correspond to the examples used in Brams and Affuso (1976). In all treatments there is a 'strong' player with three votes and two 'weak' players with two votes each. In our first treatment these three players comprise the voting body, and five votes are needed to pass a proposal. Here, the strong player has veto power, since no proposal can be passed without her votes, and so we refer to this as our VETO treatment. Our second treatment is identical except that only four votes are needed to pass a proposal. As a consequence, no player has a strategic advantage over the others, since any two members have enough votes to pass a proposal. This is reflected by all sophisticated measures of voting power, which assign equal power to each player, and thus we refer to this as our SYMMETRIC treatment. In our third treatment five votes are needed to pass a proposal, and there is an additional member with a single vote: we refer to this as our ENLARGED treatment.

In all treatments proposals specify how to divide a budget of 120 points. Table 4.1 contains predicted voting powers for each treatment. The predicted voting power of a player is the number of points they can expect to gain; the predictions are based on the core (which, when empty, is replaced by Schmeidler's (1969) extension, the Nucleolus), the two best-known power indices (Shapley and Banzhaf) and a naïve power index that assigns payoffs proportionally to the voting weights.[4]

Table 4.1. Summary Predictions

VETO Treatment

		Voting Power			
	Votes*	Core/ Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	120	80	72	51
Player 2	2	0	20	24	34
Player 3	2	0	20	24	34

* 5 votes required to pass a proposal

SYMMETRIC Treatment

		Voting Power			
	Votes*	Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	40	40	40	51
Player 2	2	40	40	40	34
Player 3	2	40	40	40	34

* 4 votes required to pass a proposal

ENLARGED Treatment

		Voting Power			
	Votes*	Nucleolus	Shapley	Banzhaf	Proportional
Player 1	3	40	50	50	45
Player 2	2	40	30	30	30
Player 3	2	40	30	30	30
Player 4	1	0	10	10	15

* 5 votes required to pass a proposal

[4] Conceptually the Shapley value seems the most appropriate power index for distributive decisions (Roth, 1977a, 1977b; Laruelle and Valenciano, 2005). However, there are some arguments in favor of the Banzhaf index (see Felsenthal and Machover, 1998, p.174 for a discussion), and the Nucleolus (see Montero, 2005).

Relative to VETO, ENLARGED allows us to test whether the paradox of new members occurs when the total number of votes required to pass a proposal is held constant. As with any enlargement, the naïve, proportional, power index predicts all the original members lose voting power and so the paradox will not occur. However, the other measures predict that the weak players' voting power will increase. This is because the addition of the new member eliminates the strong player's veto power, and so the other original members no longer depend on the strong player.[5]

Relative to SYMMETRIC, ENLARGED allows us to test the paradox of new members when a simple majority of votes is required to pass a proposal. Here, the effect on the original members is less intuitive. Intuitively, the weak original members are worse-off after enlargement: they no longer have enough votes to enforce a proposal on their own, and now need the cooperation of the new member. The large player becomes the most powerful in relative terms, but it is not clear whether his power should increase in absolute terms; this is indeed the case according to the Banzhaf and Shapley measures. The Nucleolus makes the extreme prediction that the existing members are unaffected.

4.3 The Experiment

4.3.1 Design and Procedures

The experiment comprised twelve sessions conducted at the University of Nottingham using subjects recruited by e-mail from a university-wide pool of undergraduate students. Four sessions were conducted with each treatment, and involved either 12

[5] For this type of enlargement, where a player loses veto power, it is intuitive that other original members will be empowered. However, this is not a general result. Suppose four voters have 4, 2, 1, and 1 votes respectively, and 7 votes are required to pass a proposal (so that the two larger players have veto power). If a fifth voter with 4 votes is

subjects (VETO and SYMMETRIC treatments) or 16 subjects (ENLARGED treatment) per session.[6] Thus, 160 subjects participated in total.

All sessions used an identical protocol. Upon arrival, subjects were given a written set of instructions that the experimenter read aloud.[7] Subjects then reviewed the instructions on their computer screens and were allowed to ask questions by raising their hands and speaking to the experimenter in private. Subjects were not allowed to communicate with one another throughout the session, except via the decisions they entered on their terminal.

The decision-making phase of the session then consisted of 10 rounds. At the beginning of each round subjects were assigned to groups of either three or four (depending on treatment). Subjects were not told who of the other people in the room were in their group, and group compositions changed from round to round. In particular, the same set of subjects was never matched together twice. At the beginning of each round subjects were also assigned roles, determining how many votes they controlled, and roles also varied across rounds. In every round subjects entered decisions anonymously, so that it was not possible to build up a reputation across rounds. For statistical reasons, prior to the first round we formed two equally-sized subsets of subjects and then formed groups from within these subsets; no information passed across the two subsets, and so this procedure ensured that each session resulted in two independent observations.

Each round had a random time limit between five and ten minutes for groups to bargain over the division of 120 points. If no agreement were reached before the random deadline, each group member would earn zero points. Bargaining proceeded

added the large players lose veto power, but the other original members are worse-off according to the Shapley and Banzhaf measures and unaffected according to the Nucleolus.

[6] The experiments were computerized and conducted with the help of the software Visual Basic 6.

[7] A copy of the instructions for the VETO treatment can be found in Appendix 4.A.

as follows. Any subject could put a proposal on the table by completing a proposal form on the left side of their screen. Once a proposal was on the table all members of the group would see it on the right side of their screens. Any subject was also able to replace their proposal with another at any time until the round ended. Thus at any time there may be up to three proposals (VETO and SYMMETRIC treatments) or up to four proposals (ENLARGED treatment) on the table. Subjects could indicate which proposals were acceptable or unacceptable, and by indicating a proposal was acceptable they placed their votes in favor of that proposal. The first proposal to receive the required number of votes was enforced, and subjects received the points specified in that proposal.

At the end of the experiment subjects were privately paid according to their accumulated point earnings from all 10 rounds, using an exchange rate of 3p per point (VETO and SYMMETRIC treatments) or 4p per point (ENLARGED treatment). Earnings averaged £12 per subject in all sessions, and ranged from a minimum of £2.12 to a maximum of £20.40.[8] Sessions lasted, on average, 35 minutes, with no session taking longer than 55 minutes.

4.3.2 Results

4.3.2.1 Overview of results

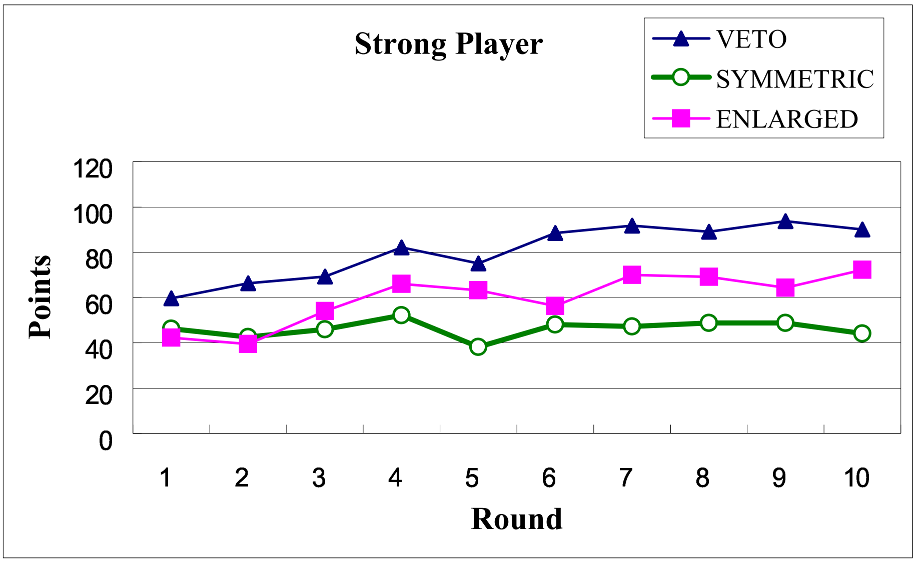
In our design there is a possibility of negotiations breaking down, since each round had a randomly determined time limit between five and ten minutes. In fact, this time restriction did not bind in our experiment. In VETO, where there is a unique core allocation giving the entire budget to the strong player, agreements are quickly reached: only in two of 160 games did bargaining extend beyond five minutes, and

[8] At the time of the experiment the exchange rate was approximately £1 = $1.85.

even then agreements were reached before the random deadline. The games used in SYMMETRIC and ENLARGED have an empty core, and our prior expectations were that subjects would find it more difficult to reach an agreement.[9] As it turned out, an agreement was reached in every game well before there was any danger of the random deadline coming into effect, and the maximum duration of negotiations in these treatments was 2¼ minutes. Thus, there are no disagreement outcomes in any of the 480 games of our experiment.[10]

Figure 4.1 shows how voting power -- measured as average earnings for a given player-role -- develops across rounds for each treatment. As shown in the left panel, the strong player's voting power is greatest in VETO and is smallest in SYMMETRIC. Correspondingly, the weak players have most voting power in SYMMETRIC and least in VETO.[11]

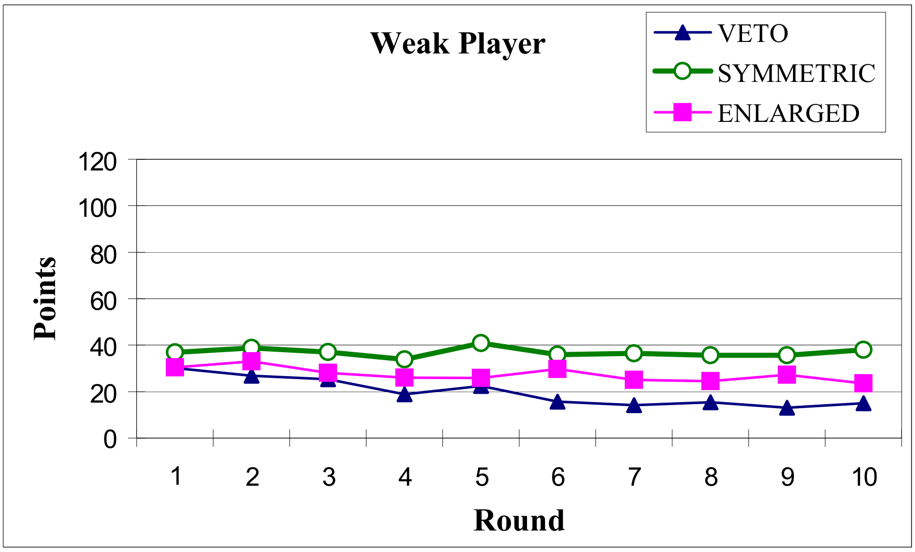
Figure 4.1. Voting Power of Strong and Weak Players



[9] As Kahan and Rapoport (1984, p.67) have argued, "… in a coreless game, some constraints on coalitional activity must be operative in the society, or else it will become ensnared in the endless rounds of successive secessions from proposed [payoff configurations]."

[10] The early agreements in our experiment contrast sharply with results from two-person (Roth *et al.*, 1988, Gächter and Riedl, 2005) and three-person bargaining (Bolton *et al.*, 2003), where there are pronounced deadline effects.

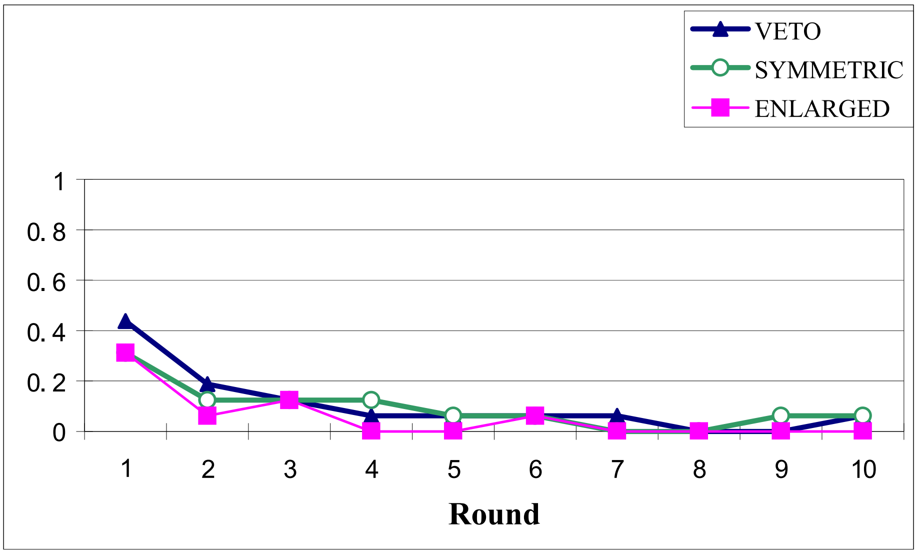
[11] We also noted considerable asymmetry between the earnings of the two weak players in some matching groups. In particular, earnings were significantly higher for player 2 than player 3. However, all the results reported below, with one exception noted later, apply whether the earnings of players 2 and 3 are regarded separately or pooled.



It is also evident in Figure 4.1 that earnings change across rounds, particularly in VETO and ENLARGED. To analyze these dynamics we computed Spearman rank correlation coefficients between the strong player's voting power and round for each matching group. In SYMMETRIC four coefficients are positive and four are negative, and so we cannot reject the null hypothesis that power is equally likely to increase or decrease with experience using a two-sided sign-test (p-value = 1.000). On the other hand, for VETO and ENLARGED all coefficients are positive, and so we can reject the null hypothesis that power is equally likely to increase or decrease with experience (p-value = 0.008). Further analysis of these treatments shows that the significant increase in the strong player's voting power occurs in earlier rounds: in the last four rounds we find no evidence of a relationship between voting power and round (VETO p-value = 0.727; ENLARGED p-value = 1.000). Thus, the strong player's voting power can be described as initially increasing, before stabilizing around 91 in the last four rounds of VETO and around 69 in the last four rounds of ENLARGED. By comparison, in SYMMETRIC the strong player's voting power in the last four rounds is 47.

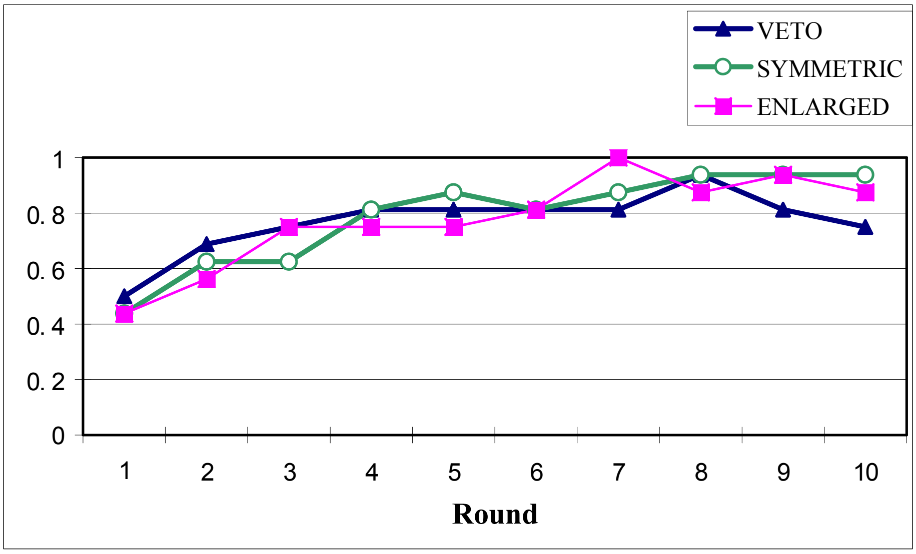
Several patterns in the data suggest the changes in early rounds reflect a learning process as subjects become more familiar with strategic aspects of the game. For example, in the first round agreements to divide the 120 points equally among all members were quite frequent, occurring in 17 of 48 (≈ 35%) groups. For many subjects this must have seemed a natural and acceptable outcome. However, as rounds progressed equal divisions were observed less frequently. In the last round only 2 of 48 groups (≈ 4%) agreed upon an equal division. Figure 4.2 shows that the frequency of equal divisions decreases over rounds in all three treatments.

Figure 4.2. Proportion of Equal Divisions between All Players



Broadly speaking, proposals in which the pie was equally divided tended to be replaced by proposals allocating zero to at least one player. Since in all three treatments a proposal can be implemented without unanimous support, a winning coalition can form that excludes some players, and the members of this coalition maximize their point earnings by allocating zero points to outsiders. In all treatments we observed an increase in the frequency of *minimal winning coalitions* across rounds, as shown in Figure 4.3. Across all three treatments, minimal winning coalitions formed in 22 of 48 groups (≈ 46%) in the first round, compared with 41 of 48 (≈ 85%) in the last round.

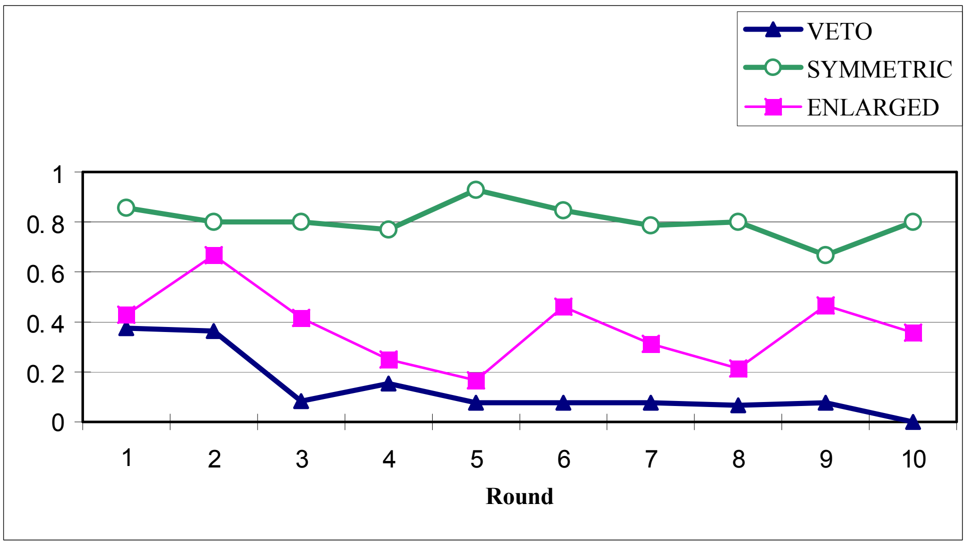
Figure 4.3. Proportion of Minimal Winning Coalitions



In VETO, minimal winning coalitions, which must include the strong player, formed in 123 of 160 (≈ 77%) games overall, and in 53 of 64 (≈ 83%) games in the last four rounds. In SYMMETRIC, minimal winning coalitions, which can be comprised of any two players, formed in 126 of 160 (≈ 79%) games overall, and in 59 of 64 (≈ 92%) games in the last four rounds. In ENLARGED there can be two different types of minimal winning coalition: the strong player with one of the weak players, or the two weak players with the new member. In this treatment, the first type of coalition was much more common. Minimal winning coalitions excluding the strong player formed in only 8 of 160 (5%) games, and in only 1 of 64 (≈ 2%) games in the last four rounds; minimal winning coalitions including the strong player occurred in 116 of 160 (≈ 73%) games, and in 58 of 64 (≈ 91%) games in the last four rounds.[12]

When minimal winning coalitions formed, the budget was often split equally among its members. However, Figure 4.4 shows that the extent to which this occurred varied substantially across treatments. In SYMMETRIC the proportion of 60-60 divisions was rather stable and averaged 80% over all rounds, but in the other two treatments there are fewer cases of equal division within a minimal winning coalition. In ENLARGED, 36% of minimal winning coalitions split the budget evenly between its members. In VETO, where any minimal winning coalition must include the strong player, minimal winning coalitions agreed a 60-60 division 37% of the time in the first round, but this proportion decreased to zero by the last round.

Figure 4.4. Proportion of Minimal Winning Coalitions That Divide Equally

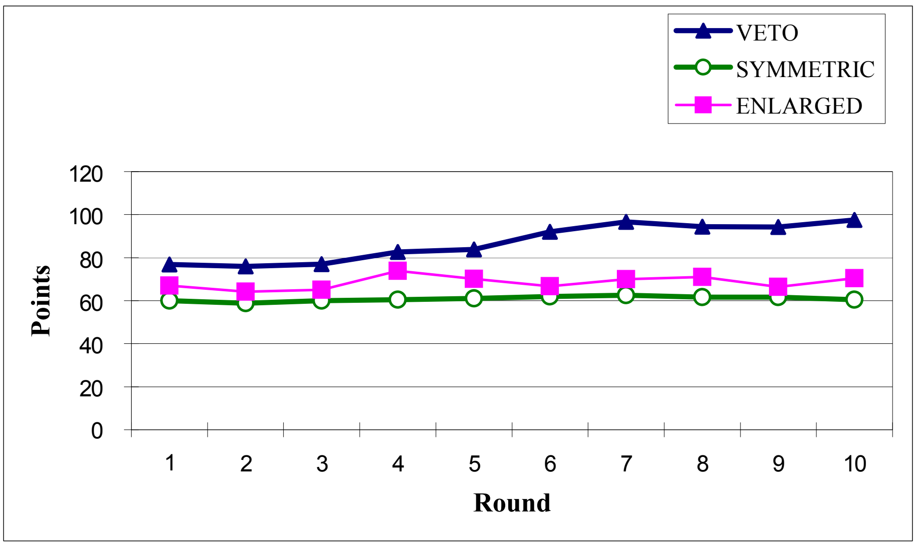


In order to further study departures from equal division within minimal winning coalitions we restrict attention to minimal winning coalitions involving the strong

[12] The finding that some subjects are willing to propose distributions that give zero to another subject, and others are willing to vote for such a proposal, is consistent with other experiments on multi-person bargaining (see Güth and van Damme, 1998, Fréchette *et al*., 2003, Kagel *et al*., 2005, and Okada and Riedl, 2005).

player, and study how the strong player's share varies across rounds and treatments (see Figure 4.5). The set of minimal winning coalitions featuring the strong player is the same in all treatments: she can form a winning coalition with either weak player. However, very different patterns are evident across treatments. In VETO the strong player earned 77 points on average in the first round, compared with 98 points in the last round. Thus, as rounds progressed the strong player was demanding and getting larger shares of the pie. In SYMMETRIC we see a stable pattern: when the strong player is included in a minimal winning coalition she gets about half the pie. In ENLARGED the strong player's earnings within minimal winning coalitions averaged more than 60 points in every matching group, but were nevertheless substantially lower than in VETO, being stable around 69 points. Thus, because the weak players can form a winning coalition with the new member, this threat appears to limit the strong player's ability to extract larger shares of the pie from one of the weak players.

Figure 4.5. Strong Player'S Voting Power within Minimal Winning Coalitions



We find the comparison between the strong player's power in a minimal winning coalition in the SYMMETRIC treatment and her power in the ENLARGED treatment particularly interesting. The strong player's earnings within a minimal winning coalition differ significantly across these two treatments, whether we focus on all rounds (Wilcoxon two-sided p-value = 0.003) or just the last four rounds (Wilcoxon two-sided p-value = 0.004), and this suggests that the amount the strong player gets when she forms a coalition with one of the weak players depends on the alternative coalition opportunities available to the weak players. This contrasts with the predictions of theories of ex post payoff division. Gamson's (1961) theory predicts an ex post payoff division proportional to the voting weights, so that the strong player should get 72 points and the weak player should get 48 points when they form a coalition in both treatments.[13] Other theories of ex post payoff division also fail to capture the significant difference. The Bargaining Set (Davis and Maschler, 1967), Kernel (Davis and Maschler, 1965) and the Aspiration solution concepts (see Bennett, 1983) all predict that the coalition of the strong and one of the weak players divide the 120 points equally in both treatments.

4.3.2.2 Voting power: tests of hypotheses

For formal comparisons of voting power we use non-parametric tests applied to sets of independent observations. We first examine how voting power varies with voting weights within treatments. Table 4.2 presents the strong and weak player's voting power for each treatment.[14]

Table 4.2. Empirical Measures of Voting Power

[13] Fréchette *et al*. (2005) also found that Gamson's law performs badly in a game similar to our SYMMETRIC treatment.

	VETO		SYMMETRIC		ENLARGED	
	all rounds	last 4 rounds	all rounds	last 4 rounds	all rounds	last 4 rounds
strong player	80.6	91.1	46.2	47.2	59.7	68.9
weak player	19.7	14.4	36.9	36.4	27.3	25.1

In VETO the strong player earns more than a weak player in every single matching group, whether we focus on all rounds or just the last four rounds. Thus the voting power of the strong player is significantly greater than that of the weak player (one-sided sign-test p-value = 0.004), as predicted by all the power indices given in Table 4.1. The comparison of strong and weak player earnings in the SYMMETRIC treatment provides the one case in which asymmetries between the two weak players affect our results. We find that the earnings of the strong player are not significantly different from the earnings of player 2 (all rounds: two-sided sign-test p-value = 0.727; last four rounds: two-sided sign-test p-value = 1.000) but are significantly greater than the earnings of player 3 (all rounds: one-sided sign-test p-value = 0.035; last four rounds: one-sided sign-test p-value = 0.035). Averaging over the two weak players, we find that the strong player earns significantly more if we consider all rounds (one-sided sign-test p-value = 0.035), but not if we focus on the last four rounds (one-sided sign test, p-value = 0.363).[15] Finally, for ENLARGED, we find that the strong player earns significantly more than a weak player in every single matching group whether we look at all rounds or just the last four rounds (one-sided sign-test p-

[14] Recall that there are no disagreements. Thus, in VETO and SYMMETRIC the strong player's earnings plus twice the weak player's earnings equals 120 points. In ENLARGED the strong player's earnings plus twice the weak player's earnings is less than 120 points. The residual is the earnings of the new member.

value = 0.004). This last finding is not predicted by the Nucleolus, but is consistent with the other power indices given in Table 4.1.

For each index and treatment we used two-sided sign tests to test the hypothesis that the strong player's voting power is equally likely to be above or below the prediction. None of the indices performed well in this sense. In the case of the Nucleolus it is not surprising that we can reject the hypothesis in VETO[16], since any deviation from the point prediction must be below, but we can also reject the hypothesis in ENLARGED.[17] Likewise, the naïve index is rejected in VETO (p-value = 0.008) and ENLARGED (p-value 0.008). For the case of the Shapley and Banzhaf indices we also reject in ENLARGED (for both indices p-value = 0.008), but their performances in VETO depends on whether we focus on all or just the last four rounds. Using all rounds we cannot reject the null hypothesis for either index (Shapley: p-value = 1.000; Banzhaf: p-value = 0.289).[18] If, however, we focus on the last four rounds we reject the null hypothesis at a 10% significance level for both indices (p-value = 0.070 in both cases).

Brams and Affuso (1976) use the Shapley and Banzhaf power indices to predict that an original member's voting power increases when a new member is admitted to a voting group. We now compare treatments in order to test this prediction. First we consider the comparison between VETO and ENLARGED. The Shapley and Banzhaf indices, and indeed indices based on the core (or its extension), predict that this will increase the voting power of the weak player. Our experiment delivers strong

[15] In this treatment 101/160 (≈ 63%) games result in two-player coalitions who divide the pie 60-60 between themselves. However player 2 was included in such coalitions more than twice as often as player 3.
[16] Kagel *et al*. (2005) obtain a similar result with a structured protocol for negotiations: the share of the veto player is well below the equilibrium prediction.
[17] In every single matching group, regardless of whether we consider all or last four rounds, the strong player earns more than predicted in ENLARGED (p-value = 0.008).
[18] In fact the average earnings of the strong player is remarkably close to the Shapley value. Murnigham and Roth (1977) obtain a similar result.

empirical support for this prediction, since the weak player's earnings are significantly higher in ENLARGED (all rounds: one-sided Wilcoxon test p-value = 0.006; last four rounds: one-sided Wilcoxon test p-value = 0.006).

Next we consider the comparison between treatments SYMMETRIC and ENLARGED. Brams and Affuso predict, on the basis of the Shapley and Banzhaf indices, that the addition of the new player with one vote will increase the strong player's voting power. Interestingly, this prediction does not follow from other indices, such as the Nucleolus. In our experiment, though the new member in the ENLARGED only earns little[19], the strong player earns significantly more in ENLARGED than SYMMETRIC (all rounds: one-sided Wilcoxon test p-value = 0.003; last four rounds: one-sided Wilcoxon test p-value = 0.001). Thus the paradox of enlargement, as predicted by Brams and Affuso, is observed in our experiment.

4.4 Conclusions

Our experiment provides empirical measures of how voting weights and voting rules influence voting power in weighted voting games. Our focus is on the three games discussed in the seminal work of Brams and Affuso (1976). These games illustrate how the enlargement of a voting body may benefit an original member, even if the decision rule and her relative voting weight *vis-à-vis* the other original members are held constant.

In one of the cases studied, a strong player loses her veto power when a new member is added, and this strengthens the power of weaker members. This prediction is shared by sophisticated power indices, and also receives strong empirical support

[19] The new member in ENLARGED earns 5.6 points per round if averaging over all rounds, while averaging over the last four rounds she earns 0.9 points per round.

from our experiment. On the other hand, individual power indices on which this comparative static prediction could be based do not deliver good point predictions. When the strong player is a veto player she gets significantly less than the entire pie (i.e. her core allocation), but in the later rounds of the session she gets significantly more than predicted by the Shapley or Banzhaf indices. In the larger voting body the strong player attains significantly more than predicted by any of the power indices considered.

In the second case studied by Brams and Affuso the prediction that enlargement will benefit an original member is more controversial, since it relies on the particular power indices they use. The Shapley and Banzhaf indices predict that the strong player benefits from the addition of a new member, while the Nucleolus concept predicts no change in voting power. Here our experiment supports Brams and Affuso's comparative static prediction: the paradox of new members is observed in our data.

Our results underscore the important point made by voting theorists: that formal analysis is required in order to accurately predict the effects of changes in voting bodies. While an important part of such analysis should be based on theoretical analysis of the properties of different voting weights and rules, we also argue that empirical methods have an important complementary role. Empirical evidence is particularly valuable when, as in one of the cases we study, alternative solution concepts make different predictions about the effects of institutional changes.

Appendix 4.A

Instructions

Introduction

This is an experiment about group decision-making. There are other people in this room who are also participating in this experiment. You must not talk to them or communicate with them in any way during the experiment. The experiment will take about one hour, and at the end you will be paid in private and in cash. The amount of money you earn will depend on the decisions that you and the other participants make.

In this experiment you will participate in ten rounds. In each round you will be in a group with two other people, but you will not know which of the other people in this room are in your group. The people in your group will change from round to round, and in particular you will never be matched with the same set of two other people twice. The decisions made by you and the other people in your group will determine how many points you earn in that round. At the end of the experiment you will be paid according to your total point earnings from all ten rounds. You will be paid 3p per point.

Description of a round

At the beginning of each round, you will be randomly allocated a subject identification number, either 1, 2, or 3. (Thus, your identification number may change from round to round.) Each person controls a number of votes depending on his or her identification number as follows:

Subject Identification Number	1	2	3
Number of Votes	3	2	2

In each round you and the other people in your group have 120 points to divide. You and the other people in your group can make proposals about how these points are to be divided among the group members. You and the other people in your group can also cast votes in favour of proposals. The first proposal to receive *five* votes will be enforced. When a proposal is enforced the round ends and each person earns the number of points specified in that proposal.

There will be a time limit for each round. This time limit will be some whole number of seconds between 300 and 600, but you will not be informed of the exact time limit. This means that the round could end suddenly at any time between 300 seconds (five minutes) and 600 seconds (ten minutes). If no proposal has been enforced when the round ends, each person in your group will earn zero points in the round.

All rounds will be identical except that your subject number may change from round to round, the other people in your group may change from round to round, and the time limit may change from round to round.

How you make proposals

At the beginning of a round your computer screen will look like the one shown in Figure 4.1. On the left side of the screen there is a form for making proposals. To make a proposal you must specify the number of points that each person in your group will receive. For each person you can type in any whole number between 0 and 120, but the total number of points received by the group members must add up to 120. When you have completed a proposal you click on the "submit" button to submit it.

Your proposal will then appear on the right side of the screen, in the list of "proposals on the table." A sample screen is shown in Figure 4.2, except that the entries marked XXX will be the numbers you entered in your proposal. As long as the round has not already ended, you can amend your proposal by simply completing a new proposal and submitting it. The new proposal will replace the old one.

How proposals are enforced

Once a proposal is on the table, all the people in your group will see it on the right side of their screens. At any time there may be up to three proposals on the table, one submitted by each person in your group. For each proposal there is an "accumulated votes" counter that informs all people in the group of how many votes are currently in favour of the proposal. When a proposal is submitted it automatically receives the votes of the person who submitted it. Thus, during the round your screen might look like the one shown in Figure 4.3 (except that the entries marked XXX will correspond to the decisions made by participants).

For each proposal on the table you can indicate if it is acceptable by clicking on the "acceptable" button next to that proposal. If you do this, the proposal will receive your votes. If you change your mind after indicating that a proposal is acceptable you will be able to withdraw your support by clicking on the "unacceptable" button. The "accumulated votes" counter will change to keep track of how many votes are currently in favour of the proposal.

You can use your votes to support more than one proposal. However, the first proposal to have five votes in its favour will be the one that is enforced, and this proposal will determine how many points you receive.

Ending the session

At the end of round ten your total points from all rounds will be converted to cash at a rate of 3p per point and you will be paid this amount in private and in cash.

Now, please click on the "start" button and begin reviewing the instructions on your screen. If you have any questions raise your hand and a monitor will come to your desk to answer them. When you have finished reading through the screens reviewing the instructions, click on the ready button to indicate that you are ready to begin the decision-making part of the experiment. When everyone in the room is ready, decision-making will begin.

Figure 1

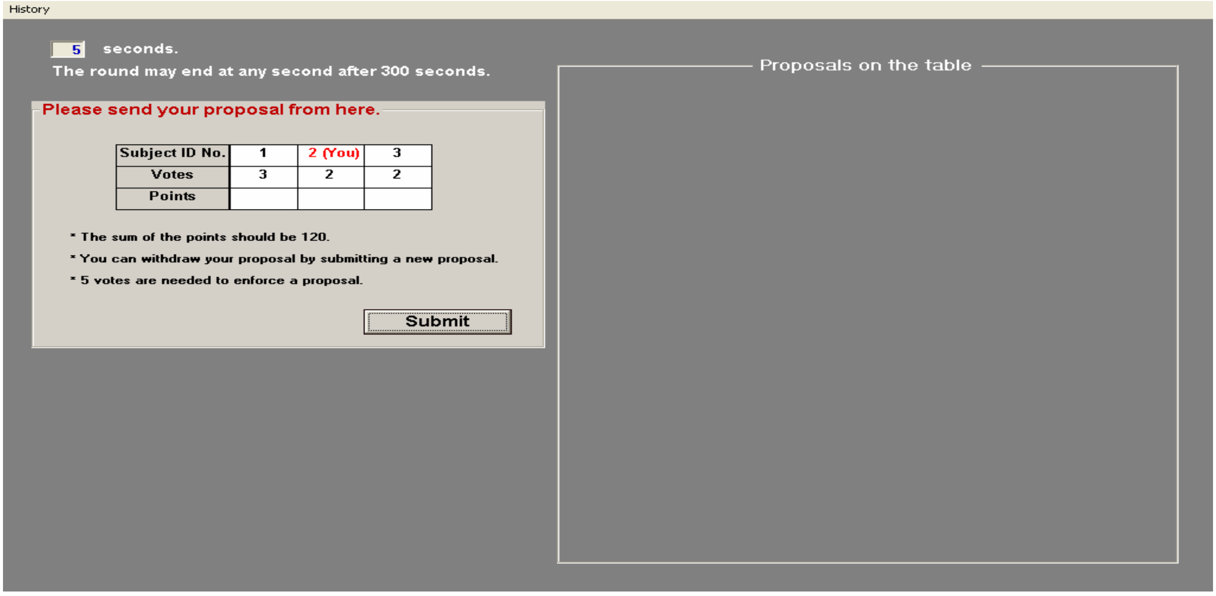


Figure 2

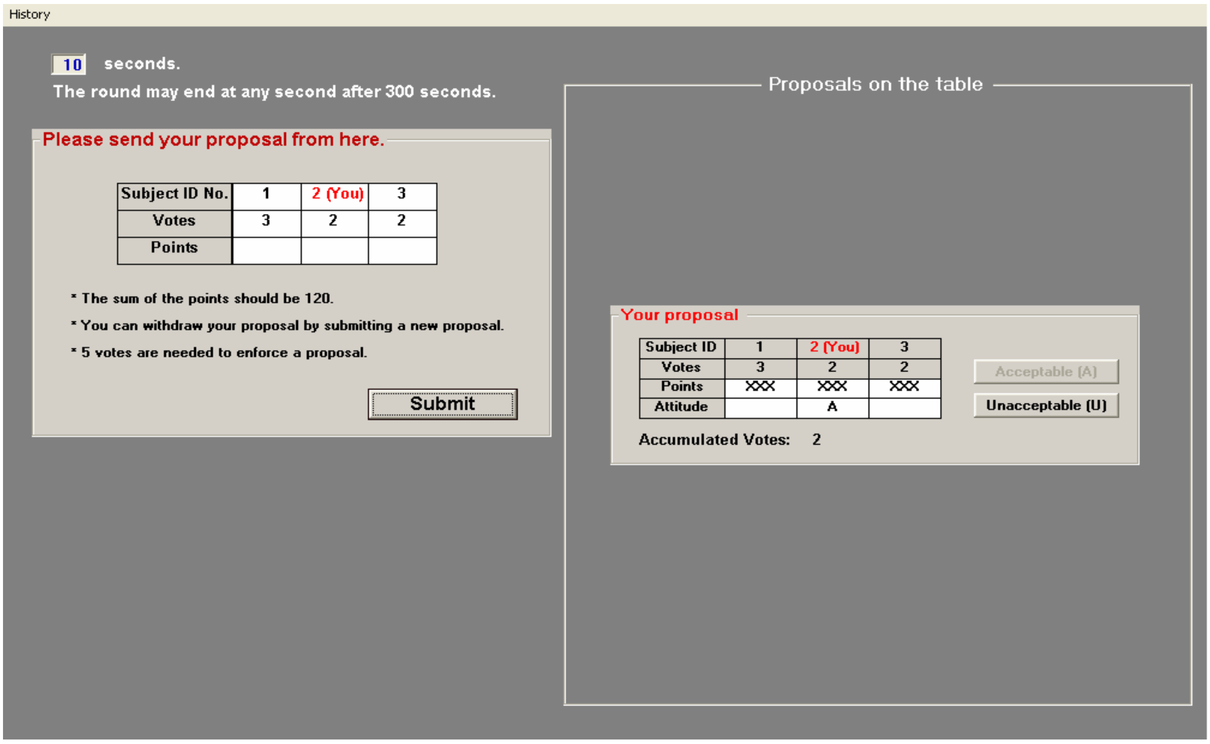
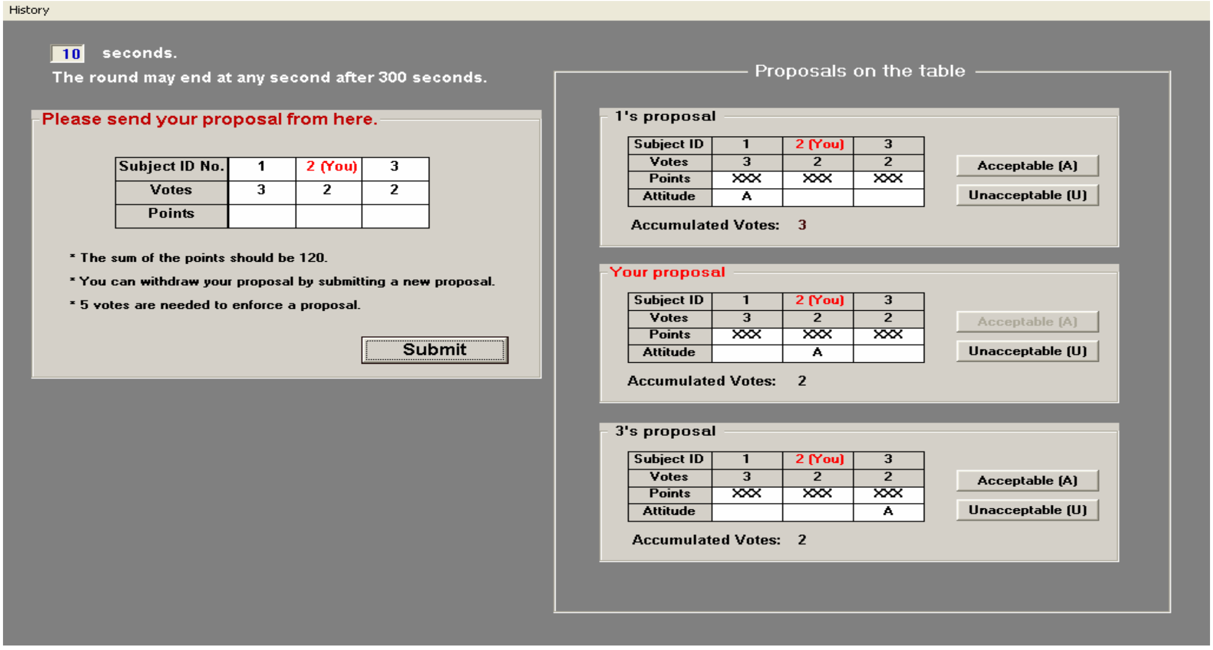


Figure 3


History
10 seconds.
The round may end at any second after 300 seconds.
Please send your proposal from here.
Subject ID No. 1 2 (You) 3
Votes 3 2 2
Points
* The sum of the points should be 120.
* You can withdraw your proposal by submitting a new proposal.
* 5 votes are needed to enforce a proposal.
Submit
Proposals on the table
1's proposal
Subject ID 1 2 (You) 3
Votes 3 2 2
Points XXX XXX XXX
Attitude A
Acceptable (A)
Unacceptable (U)
Accumulated Votes: 3
Your proposal
Subject ID 1 2 (You) 3
Votes 3 2 2
Points XXX XXX XXX
Attitude A
Acceptable (A)
Unacceptable (U)
Accumulated Votes: 2
3's proposal
Subject ID 1 2 (You) 3
Votes 3 2 2
Points XXX XXX XXX
Attitude A
Acceptable (A)
Unacceptable (U)
Accumulated Votes: 2

Chapter Five:

Concluding Remarks

Chapter 5:

Concluding Remarks

In this chapter we briefly summarize the results of the thesis and pay more attention to the limitations of the above studies as well as to possible directions of further research.

5.1 IPOs Mechanisms

5.1.1 Uniform Price Auctions

The study begins with a real-world observation: the existence of an underpricing phenomenon in IPO mechanisms including uniform-price auctions. Wilson (1979), Back and Zender (1993) have shown the existence of a tacit collusion equilibrium in share auctions; Biais and Faugeron-Grouzet (2002) provide a unified model where different IPO mechanisms can be compared, where they also derive a tacit collusion equilibrium solution for uniform price IPO auctions which is in accordance to Wilson/Back and Zender's theory. In designing the experiment we identify a limitation of the theory for predicting the equilibrium in *discrete* uniform price auctions. As discreteness not only exists in laboratory but also in real markets, it is necessary to refine the theory. We generalize the previous theory to discrete format uniform price auctions and then use laboratory experiments to test these predictions.

However, there is no evidence that the tacit collusion equilibrium has ever been achieved in the experiment. Then we go back to theory, and find a broad set of equilibria where the tacit collusion equilibrium is only an extreme case in this set. The new equilibrium set is supported by data from our experiment, and can also be tested by experiments carried out by other researchers. As Engelbrecht-Wiggans, List and Reiley (2005) suggest, we believe this is a manner in which science could ideally proceed.

Our results on uniform price IPO auctions from both the theoretical and the experimental analysis are summarized as follows.

Previous researches have demonstrated that there exist a continuum of collusive equilibria in the continuous case where bidders enjoy a low price because of demand reduction (Ausubel and Cramton,1998; Wang and Zender, 2002; Wilson,1979; etc.). Kremer and Nyborg (2004) argue that the collusive equilibrium of the share auction models of Wilson/Back and Zender do not survive when bidders only make a finite number of bids and instead a Bertrand-like price competition is induced. We show that in uniform price auctions the tacit collusion equilibrium still exists but under a stricter condition, in the sense that the investors' demand curve should be steeper. And unlike Kremer and Nyborg's result, there is no equilibrium under a flat demand function in our model that would be consistent with Bertrand like competition. We find no evidence that the tacit collusion equilibrium has been achieved in our experiments. No Bertrand like competition has been confirmed as well.

Our main finding is the existence of a broad set of equilibria, which nests the tacit collusion equilibrium as one of its extreme cases. In general, this set of equilibria predicts that investors who have higher expected market values bid more aggressively, and as a result the market price increases with market value (except in the tacit collusion equilibrium). In general (except at the upper bound of the equilibrium set) all strategies in this set have the demand reduction feature. Market price can be any level between zero and market value (when there is no reservation price). Investors with high signals are always involved in equilibrium. Other investors can be either present or absent in equilibrium. However, as all investors have symmetric information in the price range between zero and v_0, since the market value is at least v_0, they can behave symmetrically in that range. As a result between zero and v_0 either a flat demand function or a tacit collusion can be an equilibrium. The equilibrium set where all types of investors participate fits our data the best.

As the equilibrium set is broad, bidders have great freedom in choosing their strategies when playing the game. In this thesis only equilibria in pure strategies have been concerned. Further research on equilibrium in mixed strategies may be more helpful in predicting investors' behaviour in the real world.

Except in the care of the tacit collusion equilibrium, the reservation price is set at zero for simplicity. With a positive reservation price, the set of equilibria should be narrower, because more demand reduction is needed to keep investors' expected payoff nonnegative. A formal proof may be necessary because the reservation price is frequently used in real markets.

In the model each informed investor can buy all shares on her own. This assumption simplified the analysis but may not apply to all real markets. It may also be useful to check the equilibrium when relaxing this assumption. Under capacity constraints, collusive equilibrium may be easier to achieve. Experiments can help to find some insights about how serious the problem is (under different capacity constraint levels).

Sellers who consider the uniform price auction method should take into account introducing rules to prevent investors from choosing strategies that lead to low revenues. The performance of uniform price auctions may be improved by modifying its price rule or the allocation rule. The experimental results reported in Chapter 3 show that in the fixed treatment, optimistic bidders bid much more than pessimistic bidders. This is because they know that a higher demand only relates to a higher allocation as the market price is fixed. However, the seller cannot use the information revealed from the demand for adjusting the market price. In the uniform price auction, the market price can be adjusted with demand, but knowing this, bidders tend to lower the market price by demand reduction. The seller, again, does not have much in hand to manage the market price. This suggests a mechanism that may improve sellers' revenue, that is the price can be adjusted according to bidders' bids, but the bottom line is that the seller decides the market price. Since investors with higher expected valuations tend to bid more aggressively, the seller may estimate the real value of the shares after investors submit bids and set the offering price at a reasonable level. The modified auction *Offre a Prix Minimum* used in France has such options and has been shown to have good performance for initial public offerings. This can also explain why some firms choose to set the offering prices lower than the market-clearing prices achieved during the auction process in OpenIPO, the online market for IPOs run by WR Hambrecht + Co. It may also be interesting for auctioneers to look for proper restrictions in order to induce investors to play the equilibrium that generates more revenues for sellers. For example, Kremer and Nyborg (2004) suggest that by modifying the allocation rule sellers can induce investors to compete in price in discrete format uniform price auctions. Strictly speaking, the *uniform rationing rule*, where all winning bids are awarded on a pro rata base regardless whether they are above or at the market price, encourages investors to increase demand at the market price, and this in turn enlarges the demand gap between the market price and a slightly higher price. Thus investors have more incentives to bid higher in order to capture a higher allocation and as a result price competition is induced. This uniform rationing allocation rule is used by the OpenIPO. They also suggest to use a hybrid allocation rule, i.e., a combination (e.g., the average) of the ordinary allocation rule used in Chapter 3 and the uniform rationing rule, when bidders have capacity constraints, so that a bidder is rewarded for being aggressive. Such an allocation rule is used in the Bookbuilding processes. Experiments on testing the performances and features of such institutions will be very suggestive for institutional design and policy making.

5.1.2 Uniform Price Auctions vs. Fixed Price Offerings

Because of the existence of a tacit collusion equilibrium, according to Biais and Faugeron-Crouzet (2002), uniform price auctions generate less revenue than fixed price offerings, under the settings of our experiment. However, the data show that the market price in uniform price auctions is significantly higher than that in fixed price offerings. As all shares are fully subscribed in both treatments, uniform price auctions are superior to fixed price offerings in raising revenue.

However, the predetermined market price in the fixed price offering treatment is determined according to the formula provided by the paper, which is set at a level such that an L investor obtains zero expected payoff in the equilibrium also described in the paper. It will be interesting to test if shares are still fully subscribed under a higher fixed price, for instance, the average market prices in the uniform price auction treatment. If under those prices demand falls below supply, the result that uniform price auctions are superior to fixed price offerings would be more robust.

5.2 Enlargement and Balance of Voting Power

Our experiment provides empirical measures of how voting weights and voting rules influence voting power in weighted voting games. Our focus is on the three games discussed in the seminal work of Brams and Affuso (1976). These games illustrate how the enlargement of a voting body may benefit an original member, even if the decision rule and her relative voting weight *vis-à-vis* the other original members are held constant. The paradox of new members is observed in our data. But individual power indices on which this comparative static prediction could be based do not deliver good point predictions.

Our results underscore the important point made by voting theorists: that formal analysis is required in order to accurately predict the effects of changes in voting bodies. While an important part of such analysis should be based on theoretical analysis of the properties of different voting weights and rules, we also argue that empirical methods have an important complementary role. Empirical evidence is particularly valuable when, as in one of the cases we study, alternative solution concepts make different predictions about the effects of institutional changes.

In our experiment, the first proposal that receives the required votes is implemented, thus the player who is not included in the proposal cannot get any point. To avoid being excluded from a proposal, subjects rush to make and accept proposals. So although the speed of agreement varied across treatments, agreements happened very quickly. The deadline effect that has been reported by other studies is not evident in ours. It would be interesting to see if more deliberation on the part of the subjects would affect the type of coalition that is formed and the payoff division. With a longer deliberation period, larger coalitions might be more likely to form. And without the pressure of inducing other players to accept her proposal, a subject who has higher voting weighs might be able to bargain a higher proportion for herself. It may also be interesting to test the same questions using different settings like a structured bargaining procedure, or a fixed group setting.

Binmore and Kleperer (2002) argue that auction design should be tailored to the special circumstances. We feel the same way that institutional design is also not "one size fits all". We hope the results from our experiments can contribute to a series of new experiments and help to draw more reliable conclusions " both about what we know and about what we know we don't know"[1] on the topics we have examined.

[1] See Kagel and Roth, 1995, page 23.

Bibliography:

Abbink, Klaus, Brandts, Jordi and Pezanis-Christou, Paul, 2002. "*Auctions for Government Securities: A laboratory Comparison of Uniform, Discriminatory and Spanish Designs*." UFAE and IAE Working Paper No. 551.02.

Abbink, Klaus, Irlenbusch, Bernd, Pezanis-Christou, Paul, Rockenbach, Bettina, Sadrieh, Abdolkarim and Selten, Reinhard, 2002. "An Experimental Test of Design Alternatives for the British 3G/UMTS Auction." *European Economy Review*, 49(2), 505-530.

Alsemgeest, Paul, Noussair, Charles and Olson, Mark, 1998. "Experimental Comparisons of Auctions under Single- and Multi-unit Demand." *Economic Inquiry*, 36(1), 87-97.

Ausubel, Lawrence M., 2002. "*Implications of Auction Theory for New Issues Markets*." Brookings-wharton Papers on Financial Services, 313-343.

Ausubel, Lawrence M., 2004. "An Efficient Ascending-Bid Auction for Multiple *Objects." American Economic Review,* 94(5), 1452-1475.

Ausubel, Lawrence M. and Cramton, Peter, 1998. "*Auctioning Securities.*" Working paper of University of Maryland.

Ausubel, Lawrence M. and Cramton, Peter, 2002. "*Demand Reduction and Inefficiency in Multi-Unit Auctions.*" Working paper of University of Maryland.

Ausubel, Lawrence M. and Cramton, Peter, 2004."Auctioning Many Divisible

Goods." *Journal of the European Economic Association*, 2, 480-493.

Back, Kerry and Zender, Jaime F., 1993. "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment." *The Review of Financial Studies*, 6(4), 733-764.

Back, Kerry and Zender, Jaime F., 2001. "Auctions of Divisible Goods with Endogenous Supply." *Economics Letters*, 73, 29-34.

Baldwin, Richard, 1994. "*Towards an Integrated Europe*." London: CEPR.

Banzhaf, John F., 1965. "Weighted Voting Doesn't Work: a Mathematical Analysis." *Rutgers Law Review*, 19, 317-45.

Beatty, Randolph P. and Ritter, Jay R., 1986. "Investment Banking, Reputation, and the Underpricing of Initial Public Offerings." *Journal of Financial Economics*, 15, 213-232.

Ben-Yashar, Ruth and Milchtaich, Igal, 2005. "*First and Second Best Voting Rules in Committees.*" Bar-Ilan University, Department of Economics, Working Paper 8-03.

Bennett, Elaine, 1983. "The Aspiration Approach to Predicting Coalition Formation and Payoff Distribution in Sidepayment Games." *International Journal of Game Theory*, 12, 1-28.

Benveniste, Lawrence M. and Spindt, Paul A., 1989. "How Investment Bankers Determine the Offer Price and Allocation of New Issues." *Journal of Financial Economics*, 24, 343-361.

Benveniste, Lawrence M. and Wilhelm, William J., 1990. "A Comparative Analysis of IPO Proceeds under Alternative Regulatory Environments." *Journal of*

Financial Economics, 28, 173-207.

Biais, Bruno and Faugeron-Grouzet, Anne M., 2002. "IPO Auction: English, Dutch,…French, and Internet." *Journal of Financial Intermediation*, 11, 9-36.

Biais， Bruno, Bossaerts, Peter and Rochet, Jean-Charles, 2002. "An Optimal IPO Mechanism." *Review of Economic Studies*, 69,117-146.

Bikhchandani, Sushil and Huang, Chi-fu, 1993. "The Economics of Treasury Securities Markets." *Journal of Economic Perspectives*, 7, 117-134.

Binmore, Ken and Klemperer, Paul D., 2002. "The Biggest Auction Ever: the Sale of the British 3G Telecom Licences." *The Economic Journal*, 112, 74-96.

Bolton, Gary E., Kalyan, Chatterjee and McGinn, Kathleen L., 2003. "How Communication Links Influence Coalition Bargaining: A Laboratory Investigation." *Management Science*, 49, 583-598.

Brams, Steven J. and Affuso, Paul J., 1976. "Power and Size: a New Paradox." *Theory and Decision*, 7, 29-56.

Brams, Steven J. and Affuso, Paul J., 1985. "New Paradoxes of Voting Power on the EC Council of Ministers." *Electoral Studies*, 4, 135-39.

Bulow, Jeremy and Klemperer, Paul D., (2002). "Prices and the Winner's Curse." *RAND Journal of Economics*, 33(1), 1-21.

Camerer, Colin F., 2003. "*Behavioral Game Theory: Experiments in Strategic Interaction*." Princeton: Princeton University Press.

Chamley, Christophe P., 2003. "*Rational Herds: Economic Models of Social Learning*." Cambridge: Cambridge University Press.

Chemmanur, Thomas J., 1993. "The pricing of Initial Public Offerings: A Dynamic Model with Information Production." *The Journal of Finance*, 48(1), 285-304.

Comerton-Forde, Carole, Lau, Sie T. and McInish, Thomas H., 2003. "*IPO Madness, Index Rigging, and the Introduction of an Opening and Closing Call: The Case of Singapore*."17-19 December 2003: Australasian Finance and Banking Conference.

Cornelli, Francesca and Goldreich, David, 2002. "*Bookbuilding: How Informative Is the Order Book?*." CEPR discussion paper.

Cox, James C., Roberson, Bruce and Smith, Vernon L., 1982. "Theory and Behavior of Single Object Auctions." In: Vernon, Smith, ed., *Research in Experimental Economics*. Greenwich: JAI Press.

Crowford, Vincent P., 1997. "Theory and Experiment in the Analysis of Strategic Interactions." In: Kreps, David and Wallis, Ken, ed.s, *Advances in Economics and Econometrics: Theory and Applications*, Seventh World Congress, Vol. I, Cambridge, U.K: Cambridge University Press, 206-242. Available at: http://weber.ucsd.edu/%7Evcrawfor/ShortTh&Exp.pdf [Access 27 Sep. 2005].

Davis, Douglas D. and Holt, Charles A., 1993. "*Experimental Economics*." Princeton, New Jersey: Princeton University Press.

Davis, Morton and Maschler, Michael, 1965. "The Kernel of a Cooperative Game." *Naval Research Logistics Quarterly*, 12, 223-259.

Davis, Morton and Maschler, Michael, 1967. "Existence of Stable Payoff Configurations for Cooperative Games," in: Shubik, Martin, ed., *Essays in*

Mathematical Economics in Honor of Oskar Morgenstern. Princeton University Press.

Derrien, Francois and Womack, Kent L., 2001. "Auction vs. Bookbuilding and the Control of Underpricing in Hot IPO Markets." *Review of Financial Studies*,16,31-61.

Dreyer, Jacob S. and Schotter, Andrew, 1980. "Power Relationships in the International Monetary Fund: the Consequences of Quota Changes." *Review of Economics and Statistics,* 62, 97-106.

Engelbrecht-Wiggans, Richard and Kahn, Charles M.,1998. "Multi-unit auctions with uniform prices." *Economic Theory*, 12(2), 227-258.

Engelbrecht-Wiggans, Richard and Kahn, Charles M., 2005. "Low-Revenue Equilibria in Simultaneous Ascending-Bid Auctions." *Management Science*, 51(3), 508 – 518.

Engelbrecht-Wiggans, Richard, List, John A, and Reiley, David H., 2005a. "Demand Reduction in Multiunit Auctions: Evidence from a Sportscard Field Experiment: Reply." *American Economic Review*, 95(1), 472-476.

Engelbrecht-Wiggans, Richard, List, John A, and Reiley, David H., 2005b. "Demand Reduction in Multi-unit Auctions with Varying Numbers of Bidders: Theory and Field Experiments." *International Economic Review*. Forthcoming.

Engelmann, Dirk and Grimm, Veronika. 2003. *"Bidding Behavior in Multi-Unit Auctions: An Experimental Investigation and some Theoretical Insights."* CERGE-EI working paper No. 210.

Faugeron-Crouzer, Anne-Marie, Ginglinger, Edith and Vijayraghavan,Vasumathi, 2002. *"French IPO Returns and Subsequent Security Offerings: Signaling Hypothesis versus Market Feedback Hypothesis."* (September 2002), http://ssrn.com/abstract=486044.

Felsenthal, Dan S. and Machover, Moshé, 1998. "*The Measurement of Voting Power*." Edward Elgar Publishing.

Felsenthal, Dan S. and Machover, Moshé, 2001. "The Treaty of Nice and Qualified Majority Voting." *Social Choice and Welfare*,18, 431-464.

Fréchette, Guillaume R., Kagel, John H. and Lehrer, Steven F., 2003. "Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules." *American Political Science Review*, 97, 221-232.

Fréchette, Guillaume R., Kagel, John H. and Morelli, Massimo, 2005. "Gamson's Law versus Non-Cooperative Bargaining Theory." *Games and Economic Behavior*, 51, 365-390.

Friedman, Daniel and Sunder, Shyam, 1994. "*Experimental Methods: A Primer for Economists."* Cambridge: Cambridge University Press.

Friedman, Milton, 1960. "*A Program for Monetary Stability."* New York, NY: Fordham University Press.

Friedman, Milton, 1991. "How to Sell Government Securities." *Wall Street Journal,* Eastern edition, Aug. 28, 1991.

Gächter, Simon and Riedl, Arno, 2005. "Moral Property Rights in Bargaining with Infeasible Claims." *Management Science*, 51, 249-263.

Gamson, William A., 1961. "A Theory of Coalition Formation." *American Sociological Review*, 26, 373-382.

Goswami, Gautam, Noe, Thomas H. and Rebello, Michael J., 1996. "Collusion in Uniform-price Auction: Experimental Evidence and Implications for Treasury Auctions." *The Review of Financial Studies*, 9(3), 757-785.

Grinblatt, Mark and Hwang, Chuan Y., 1989. "Signalling and the Pricing of New Issues." *The Journal of Finance*, 44(2), 393-420.

Güth, Werner and Damme, Eric van, 1998. "Information, Strategic Behavior, and Fairness in Ultimatum Bargaining: An Experimental Study." *Journal of Mathematical Psychology*, 42, 227-247.

Harbord, David ,Fabra, Natalia, and von der Fehr, Nils-Henrik, 2002. *"Modeling Electricity Auctions*." Game Theory and Information Economics Working Paper Archive at WUSTL.

Hausch, Donald B. and Li, Lode, 1993. "A Common Value Auction Model with Endogenous Entry and Information Acquisition." *Economic Theory*, 3,315-334.

Hoover's online-IPO Central, Available at: www.hoovers.com/ipo/ [Accessed Oct. 2002]

Ibbotson, Roger G., 1975. "Price performance of common stock new issues." *Journal of Financial Economics*, 2, 235-272.

Ibbotson, Roger G. and Sindelar, Jody L., 1988. "Initial Public Offerings." *Journal of Applied Corporate Finance*, 1,37-45.

Jenkinson, Tim and Ljungqvist, Alexander, 2001. "*Going Public, The Theory and Evidence on How Companies Raise Equity Finance*." 2nd ed. Oxford: Oxford University Press.

Kagel John H., Harstad Ronald M.and Levin, Dan, 1987. "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study." *Econometrica,* 55, 1275-1304.

Kagel, John H. and Levin, Dan, 1992. "*Revenue Raising and Information Processing in English Common Value Auctions."* Mimeo, Pittsburgh University.

Kagel, John H. and Levin, Dan, 2001. "Behavior in Multi-unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions." *Econometrica*, 69(2), 413-454.

Kagel, John H., Sung, Hankyoung, and Winter, Eyal, 2005. "Multilateral Bargaining in the Presence of a Veto Player: An Experimental Analysis." Typescript.

Kagel, John H. and Roth, Alvin E., eds., 1995. "*The Handbook of Experimental Economics*. " Princeton: Princeton University Press.

Kahan, James P. and Rapoport, Amnon, 1984. "*Theories of Coalition Formation*." Lawrence Earlbaum Associates.

Kandel, Shmuel, Sarig, Oded and Wohl, Avi, 1999. "The Demand for Stocks: An Analysis of IPO Auctions." *The Review of Financial Studies*, 12, 227-247.

Kirchkamp, Oliver and Moldovanu, Benny, 2001. "*An Experimental Analysis of Auctions with Interdependent Valuations."* Working paper of the University of Mannheim.

Klemperer, Paul D., 2001. "*What Really Matters in Auction Design*." mimeo, Nuffield College, Oxford.

Klemperer, Paul D., 2004. "Google should be aware of the 'Winner's Curse'." *Financial Times*, 3rd, May, 2004, 17.

Klemperer, Paul D and Meyer, Margaret A., 1989. "Supply Function Equilibria in Oligopoly Under Uncertainty." *Econometrica,* 57(6), 1243-1277.

Koh, Francis and Walter, TS, 1989. "A Direct Test of Rock's Model of The Pricing of Unseasoned Issues." *Journal of Financial Economics*, 23, 251-272.

Kremer, Ilan and Nyborg, Kjell, 2004. "Underpricing and Market Power in Uniform Price Auctions." *The Review of Financial Studies*, 17(3), 849-877.

Kreps, David M., 1990. "*A course in Microeconomic Theory*." Essex: Harvester Wheatsheaf.

Krishna, Vijay, 2002. "*Auction Theory*." San Diego, California: Academic Press.

Kutsuna, Kenji and Smith, Richard, 2001. "*Issue Cost and Method of IPO Pricing: Japan's Change from Auction to Book Building*." Unpublished paper, Osaka City University and Claremont Graduate University.

Laruelle, Annick and Valenciano, Federico, 2005. "Bargaining in Committees as an Extension of Nash's Bargaining Theory." *Journal of Economic Theory*, forthcoming.

Levin, Dan, 2005. "Demand Reduction in Multi-Unit Auctions: Evidence from a Sportscard Field Experiment: Comment." *American Economic Review*, 95(1), 467-471.

List, John A., and Reiley, David H., 2000. "Demand Reduction in Multiunit Auctions: Evidence from a Sportscard Field Experiment." *American Economic Review*, 90(4), 961 -972.

Loughran Tim, Ritter, Jay R. and Rydqvist, Kristian,1994. "Initial Public Offerings: International Insights." *Pacific-Basin Finance Journal*, 2, 165-199.

Malvey, Paul F., Archibald, Christine M. and Flynn, Sean T. 1997. "*Uniform-Price Auctions: Evaluation of the Treasury Experience*." US Treasury report.

Manelli, Alejandro M., Sefton, Martin and Wilner,Benjamin S., 1999. "Multi-unit auctions: A Comparison of Static and Dynamic Mechanisms." *Journal of Behavior and Organization*, Forthcoming.

Maskin, Eric,1992. "Auctions and Privatization." In: H.Siebert, ed., *Privalization. Institit fur Weltwirtschaft an der Universitat Kiel*, 115-136.

Maxwell, Charles C., 1983. "*Auctioning Divisible Commodities: A Study of Price Determination*." PhD. Thesis, Harvard University.

McCabe, Kevin A., Rassenti, Stephen J. and Smith, Vernon L., 1990. "Auction Institutional Design: Theory and Behavior of Simultaneous Multiple-Unit Generalizations of the Dutch and English Auction." *The American Economic Review*, 80(5), 1276-1283.

McDonald John G. and Jacquillat Bertrand C., 1974. "Pricing of Initial Equity Issues: The French Sealed-Bid Auction." *The Journal of Business*, 47(1), 37-47.

Michaely, Roni and Womack, Kent L., 1999. "Conflict of Interest and the Credibility of Underwriter Analyst Recommendations." *Review of Financial Studies*, 12,653-686.

Montero, Maria, 2005. "Noncooperative Foundations of the Nucleolus in Majority Games." *Games and Economic Behavior*, forthcoming.

Montero, Maria, Sefton, Martin and Zhang, Ping, 2005. "*Enlargement and the Balance of Power: An Experimental Study.*" University of Nottingham, CeDEx Working paper 2005-08.

Murnighan, Keith J. and Roth, Alvin E., 1977. "The Effects of Communication and Information Availability in an Experimental Study of a Three-Person Game." *Management Science*, 23, 1336-1348.

Natalia, Fabra, 2003. "Tacit Collusion in Repeated Auctions: Uniform Versus Discriminatory." *Journal of Industrial Economics*, 51(3), 271- 294.

Noussair, Charles, 1995. "Equilibria in a Multi-object Uniform Price Sealed Bid Auction with Multi-unit Demands." *Economic Theory*, 5(2), 237-351.

Okada, Akira and Riedl, Arno, 2005. "Inefficiency and Social Exclusion in a Coalition Formation Game: Experimental Evidence." *Games and Economic Behavior*, 50, 278-311.

Orly, Sade, Schnitzlein, Charles and Zender, Jaime F., 2005. "Competition and Cooperation in Divisible Good Auctions: An Experimental Examination." *Review of Financial Studies*, Forthcoming.

Ozcan, Rasim, 2004a. "*The Logistic Function Approach to Discriminatory and Uniform Price Treasury Auctions*." Boston College research paper.

Ozcan, Rasim, 2004b. "*Are Bidders Buddies in Treasury Auctions? The Turkish Case*."

Boston College research paper.

Porter, David and Vragov, Roumen, 2003. *"An Experimental Examination of Demand Reduction in Multi-Unit Version of the Uniform-price, Vickrey, and English Auctions."* George Mason University, ICES Working Paper, 2003.

Rock, Kevin, 1986. "Why New Issues are Underpriced." *Journal of Financial Economics*, 15,197-212.

Roth, Alvin E., 1977a. "The Shapley Value as a von Neumann-Morgenstern Utility." *Econometrica*, 48, 657-664.

Roth, Alvin E., 1977b. "Utility Functions for Simple Games." *Journal of Economic Theory*, 16, 481-489.

Roth, Alvin E., Murnighan, J. Keith and Schoumaker, Francoise, 1988. "The Deadline Effect in Bargaining: Some Experimental Evidence." *American Economic Review*, 78, 155-162.

Sade, Orly, Schnitzlein, Charles and Zender, Jaime F., 2004. "Competition and Cooperation in Divisible Good Auctions: An Experimental Examination." *Review of Financial Studies*, Forthcoming.

Schmeidler, David, 1969. "The Nucleolus of a Characteristic Function Game." *SIAM Journal on Applied Mathematics,* 17,1163-1170.

Shapley, Lloyd S., 1953. "A Value for n-Person Games." in: Kuhn, H. W. and A.W. Tucker (Eds.), *Contributions to the Theory of Games II*, Princeton University Press, 307-317.

Shapley, Lloyd S. and Shubik, Martin, 1954. "A Method for Evaluating the Distribution of Power in a Committee System." *American Political*

ScienceReview, 48, 787-792.

Sherman, Ann E., 2002. "*Global Trends in IPO Methods: Book Building vs. Auctions*." working paper of University of Notre Dame.

Sherman, Ann E. and Titman, Sheridan, 2002. "Building the IPO Order Book: Underpricing and Participation Limits with Costly Information." *Journal of Financial Economics*, 65, 3-29.

Sidney Siegel and Castellan, Jr. N.John, 1988. "*Nonparametric Statistics for The Behavioral Sciences"*. 2nd ed. New York: McGraw-Hill Book Company.

Sloan, Allan, 2004. "IPO's Success Doesn't Justify Google's Price." *The Washington Post*, 24 August 2004, E03.

Smith, Vernon L., 1982. "Microeconomic systems as experimental science." *American Economic Review*, 72(5), 923-955.

Tenorio, Rafael, 1993. "Revenue Equivalence and Bidding Behavior in a Multi-Unit Auction Market: An Empirical Analysis." *The Review of Economics and Statistics*, 75(2), 302-314.

Tenorio, Rafael, 1997. "On Strategic Quantity Bidding in Multiple Unit Auctions." *The Journal of Industrial Economics*, 45(2), 207-217.

Umlauf, Steven R., 1993. "An empirical study of the Mexican Treasury bill auction." *Journal of Financial Economics*, 33(3), 313-340.

van Deemen, Adrian and Rusinowska, Agnieszka, 2003. "Paradoxes of Voting Power in Dutch Politics." *Public Choice*, 115,109-137.

Vickrey, William, 1961. "Counterspeeculation, Auctions, and Competitive Sealed

Tenders." *Journal of Finance*, 16, 8-37.

Vickrey, William, 1962. "Auction and Bidding Games." In: *Recent Advances in Game Theory*. Princeton, New Jersey: Princeton University Press, 15-27.

Vise, David A., 2004. "Investors Greet Google With a $27 Billion Smile Market Value Tops Giants Ford, GM." *The Washington Post*, 20 August 2004, E0.

Wang, James J.D. and Zender, Jaime F., 2002. "Auctioning Divisible Goods." *Economic Theory*, 19, 673-705.

Weber, Robert J., 1983. "Multiple-Object Auction. Auctions." In: Engelbrecht-Wiggans, R., Shubik, M.and Stark RM, ed., *Bidding and Contracting: Uses and Theory*. New York: New York University Press, 165--91.

Welch, Ivo, 1989. "Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings." *The Journal of Finance*, 44(2), 421-449.

Welch, Ivo and Jay, Ritter, 2002. "A Review of IPO Activity, Pricing and Allocations." *Journal of Finance*, 57(4), 1795-1828.

Wilson, Robert, 1979. "Auctions of Shares." *The Quarterly Journal of Economics*, 94, 675-689.

Wolfram, Catherine D., 1998. "Strategic Bidding in a Multiunit Auction: An Empirical Analysis of Bids to Supply Electricity in England and Wales." *The RAND Journal of Economics*, 29(4) , 703-725.

WR Hambrecht + Co., 2002. "*WR Hambrecht + Co. Offering Participation Agreement*." Jun. 2002. Available at: http://www.openipo.com [Accessed 22

Feb. 2004].

WR Hambrecht + Co., 2005. "*WR Hambrecht + Co. Offering Participation Agreement.*" Jul. 2005.Available at: http://www.openipo.com [Accessed 12 Aug. 2005].

Zhang, Ping, 2005. "*Uniform Price Auctions and Fixed Price Offerings in IPO: An Experimental Comparison*." University of Nottingham, CeDEx Working paper 2005-20.