# Musical Cryptography: 

# Codes, Ciphers, Form, and Function 

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## ABOUT THIS THESIS

This thesis investigates the conception, historical development, and function of musical cryptography. Musicologists such as David Løberg Code, Eric Sams, Roy Howat, and Helen Elizabeth Rudeforth will provide the framework for cipher and code-based study, while Johannes Kepler, Mihail Cocos, and Shawn Fowers provide the framework for study of the relationship between music and mathematics. By clarifying the origins of codification and examining the musical applications of cryptographic practice, this thesis hopes to establish the significance of musical cryptology, not as supplementary to other analyses, but as an autonomous branch of musicology.

## The Researcher and the Researched

The concept of musical cryptography was first introduced to me during a Music in $20^{\text {th }}$ Century Britain lecture in 2021. Discussing Elgar's Enigma Variations, the lecturer jovially remarked that the piece comprised a true enigma, the clues to which were outlined by Elgar in a cryptic programme note. I was fascinated by the notion that music could be used as a vessel for encryption and following this encounter, absorbed any and all information I could on the subject.

In the subsequent three years of formal and informal research, I have observed two consistent shortcomings in much of the current scholarship. Firstly, while musicologists like Howat and Rudeforth do engage in cryptographic analysis, it is rarely the focus of their articles, instead being used to underpin other analyses. Secondly, papers that do make cryptography the focal point (like Eric Sams' work on Schumann) are never long-form studies, instead appearing as articles or columns that handle very focused moments of method analysis. As a result, musical cryptography as a practice is yet to be contextualised functionally, in terms of its purpose, or historically, in terms of its development and origins.

In order to address the gaps left by these publications, it is important to acknowledge that this thesis will feel abnormally broad in scope for a paper of this size. This is especially true of the first half which hopes to acclimatise the reader to complex concepts and terminology that are vital to any significant study of the topic.

## Research Questions

My research questions draw from the information, knowledge, and expertise in musical cryptographic methodology from David Løberg Code, Eric Sams, and the many contributors to cryptography forums and blogs whose online anonymity means I am unable to personally acknowledge them. That being said, I am continually in awe of the remarkable online communities born of a shared passion for cryptography, and feel very fortunate to have interacted with them while developing this paper.

The central research questions are as follows. What are the origins of musical cryptography? What are the different encryption techniques and how do they affect musical output? Finally, why did composers choose to encrypt their music, what was the function?

## The Chapters

The paper is outlined with four chapters in mind. Chapter one defines cryptography, introduces the three fundamental modes of musical encryption, and explores the ancient origins of musical cryptography as a practice. This chapter also addresses the academic avoidance of musical codes and examines three musical codification case studies. The second chapter examines the various forms of musical ciphers. Establishing five differing construction techniques, the chapter identifies multiple examples of each and evaluates the strength of the encryption, and coherence of the resulting music. Chapter three steers away from cryptographic form, focusing on the function of musical encryption. Analysing three large case studies, the examples are evaluated in the final chapter. The fourth chapter also establishes musical cryptography within the wider musicological landscape.

## CHAPTER 1: WHAT IS MUSICAL CRYPTOGRAPHY?

Deriving from the Greek 'kryptos’ (крилтós, meaning 'hidden'), cryptography ('hidden writing') is the practice though which written communications are disguised. While many subcategories of cryptography exist, each fall into one of three overarching groups: steganography, ciphers, and codes. Musical cryptography is the application of these encryption techniques to any stage of musical practice from composition to performance. For each term, I will define the technique and then contextualise it musically with one example.

### 1.1.1 STEGANOGRAPHY

Steganography is best understood as the concealment of the act of communication, rather than the message it conveys; for instance, spelling out a word in the clouds of a landscape painting, or using invisible ink at the foot of a letter. In both instances, the 'hidden writing' has not been altered, only the context in which it appears. As such, solving a steganographic encryption is as simple as noticing it, though this is far easier said than done.

Musical adaptions of steganographic techniques are predominately visual. ${ }^{1}$ Cryptographers disguise correspondences through the appropriation of naturally occurring visual changes present in a musical score, such as the visual contours of melodic passages, dynamic markings, stem lengths, or slurring. Aspects most commonly changing are best for steganographic encryption; more information can be codified at a faster rate. An ineffective component would be the clef; while capable of visual alteration, it changes infrequently and by too small a degree for significant encryption to take place.

At Figure 1.1, the melody from Stephen Foster's 'My Old Kentucky Home' is used as a vessel for steganographic encryption. Using a method devised in 1605 by Francis Bacon, the music appears completely ordinary. ${ }^{2}$ However, closer inspection reveals small breaks in the stems of the notes. Isolating and rotating the stems reveals a set of dots and dashes, similar to morse code, and translatable as 'enemy advancing'. ${ }^{3}$ In this instance, steganography is used as the carrier for a separate encryption method. While later examples will demonstrate pure steganographic encryption, this pairing of steganography and a second

[^0]cipher is often the case when applied to music; the rigidity of musical notation means that visual manipulation can easily undermine musical coherence, alerting codebreakers to a possible encryption.


Figure 1.1. My Old Kentucky Home with Steganographic encryption (New York Public Library, Manuscripts and Archives Division 1916)

### 1.1.2 CIPHERS

Ciphers are algorithmic encryption methods. Having identified a word to encrypt, each letter is substituted in accordance with another set of letters or symbols. The cipher is the specific way these letters are substituted. For example, an alphabetic cipher may shift the alphabet two letters on: A becomes C, B becomes D, and so on, with Z wrapping round to B . In this example, the word 'tea' would become ' vgc '. ${ }^{4}$ As illustrated, the original text - often referred to as the 'plaintext' - is transformed to a nonsensical set of symbols far removed from the original. This encrypted set of symbols is known as the 'ciphertext'. Solving an enciphered message requires knowledge of the cipher and ciphertext together. In possession of these parts, the ciphertext can be reverse-engineered through the cipher, revealing the plaintext.

These complexities hold true with musical application. Beginning with the plaintext, each letter is substituted by a predetermined musical note, rhythm, or set of notes through the cipher. These substituted notes can then be notated in the order that spells out the plaintext. To solve a cipher of this variety, one must possess the cipher and encrypted musical notation.

Dating back to 1602, Porta's cipher is a perfect introduction to this approach (Figure 1.2). ${ }^{5}$ The cipher is in two halves, rising in stepwise semibreves from $a$ to $m$, and falling in stepwise minims from $n$ to $z$. Each pitch is repeated twice with two different substituted letters, for instance, $h$ and $q$ share the same pitch but have different note values. This method halves the intervallic range of the cipher, meaning the substituted melody has a better chance of sounding like convincing music by avoiding too many unnatural leaps.

[^1]

Figure 1.2. Porta Musical Cipher (Giambattista della Porta, De Furtivis Literarum Notis, Vulgo de Ziferis Libri V, 1602)

To illustrate the substitution process, the 'tea' plaintext has been elongated to 'teatime' and substituted through Porta's cipher (Figure 1.3). The ungainly motif has an intervallic range of an eleventh and no stepwise motion. Consequentially, a large volume of later analysis will focus on the musical validity of encrypted melodies; there appears to be some connection between the strength of a cipher and the musicality of the resulting melodies.


Figure 1.3. 'Teatime' plaintext substituted using Porta's musical cipher (Benjamin Thompson, 2023)

### 1.1.3 Codes

Codes are more arbitrarily symbolic than literal in their concealment of a communication. As with ciphers, a word or phrase is converted to another form. However, this secondary form can be anything and won't necessarily have a discernible method of conversion from the plaintext. ${ }^{6}$ To return to the 'tea' plaintext, an encoded ciphertext could be 'twelve', 'pineapple', 'celeste', 'pretzel', or indeed any word, emotion, sound, or symbol that has been assigned to be code for 'tea'.

Historically, musical codes seem to be the prevailing cryptographic approach. Unrestricted by a need for direct plaintext to ciphertext correlation, their versatility through symbolic representation has made them popular amongst composers; pieces can be charged

[^2]with meaning without the creation of a cipher or sourcing of the local invisible-ink dealer. As with enciphering, the process of musical encoding can be as simple or complex as the composer deems appropriate. Beginning with the plaintext, an element of musical composition is written as the ciphertext. From this point onwards, all appearances of the symbolic motif are representative of the plaintext. It could reasonably be suggested that programmatic music has undergone codification; the music is symbolic of something larger and only understood by those in possession of the code (programme). However, while the process may be similar, programmatic music is hardly of the enigmatic degree we are after.

A better example comes from Mervyn Cooke who convincingly suggests that Benjamin Britten employed musical codes to emphasise the thematic material of his operas. ${ }^{7}$ First recognising Britten's 'fondness for witty or ironic musical gestures', Cooke builds his case from the pitch pun, 'Sometimes B\#, never Bb, but always Bh!' (sometimes be sharp, never be flat, but always be natural). ${ }^{8}$ From whimsical quip to dramatic inspiration, Cooke suggests that in some contexts Britten adopted the phonetic interpretation as 'be flat' and 'be natural'. Bb (the key of military bugles) symbolises being 'weighed down by discipline, duty and conformity', with $B \natural$ representing the pursuit of 'freedom and dreams, and to remain faithful to one's private desires'. ${ }^{9}$ Figure 1.4 depicts a short passage from Britten's Billy Budd (1951) in which an authoritative, disciplinary voice exclaims 'Give orders to dismiss!' while both the tonal and thematic centres of the excerpt are Bb (be flat). Cooke goes on to provide thirteen more instances of the B-flat/B-natural tonal symbolism in varying complexities, all in related dramatic contexts.


Figure 1.4. Benjamin Britten's musical code at its simplest (Mervyn Cooke, 'Be Flat or Be Natural? 2013)

[^3]
### 1.2 Origins of Musical Cryptography: Music and Mathematics

As outlined by David Løberg Code in 'Can Musical Encryption be Both?', musical ciphers began to appear 'in the mid- $15^{\text {th }}$ century', with the earliest example 'attributed to Brother Nicholas Philip...in $1436{ }^{\prime 10}$. At present, musicologists like Eric Sams take Philip's cipher to be the beginning of the cryptographic timeline, documenting cryptographic history through a series of meticulously devised ciphers that culminate in his seminal work on the ciphers of Robert Schumann. ${ }^{11}$ Curiously, I have observed that like Sams', most publications choose to omit any analysis of musical codes, possibly due to their comparative simplicity, but more likely due to their relatively unresearched origins. With reference to ancient philosophy from a diverse array of cultures, this chapter examines the relationship between music and mathematics, exploring the proposition that musical codification began developing almost two thousand years before the oldest musical cipher was notated by Brother Nicholas Philip in 1436.

### 1.2.1 Signal Music

Before attempting to digest the dogmas of ancient philosophers, it is important to identify that we already have evidence that these venerable cultures were engaging with codification (albeit to a simple degree) in the form of signal music.

Signal Music unambiguously refers to music that is composed for the delivery of a message only comprehensible to those who know the code. Used most notably during conflicts, brass and percussion instruments would pierce the clamour of ancient battle to deliver tactical instruction. From 500BCE, the Persians made use of kettle drums to organise cavalry and naval groups. Trumpets, horns, and drums could be found on ancient Greek and Roman battlefields. During the reign of Alexander the Great (336-323 BCE), 'trumpets and fifes...were used to control the phalanx of his army'. ${ }^{12}$ Jumping forward to 1300-1521, the Aztecs made use of conch shells and drums to incite military formations. Even as far back as Genghis Khan, Mongol cavalry carried compact bugles for complex communication.

[^4]
### 1.2.2 Pythagorean Tuning

As evident through observing the overtones produced by all naturally occurring notes, music is fundamentally constructed through a series of proportional mathematical relationships. Grecian origins of this ideology are told through an anecdote of Pythagoras' exploration of harmony. Documented two hundred years after Pythagoras' death by the Greek philosopher, Nicomachus, the tale is re-told best by Flora R. Levin (1967):

One day [Pythagoras]...walked by a smithy and, by divine chance, heard the hammers beating out iron on the anvil and mixedly giving off sounds which were most harmonious with one another, except for one combination. He recognised in these sounds the consonance of the octave, the fifth, and the fourth. But he perceived that the interval between the fourth and the fifth was dissonant in itself but was otherwise complementary to the greater of these two consonances. Delighted, therefore, he ran into the smithy and found by various experiments that the difference of sound arose from the weight of the hammers, but not from the force of the blows, nor from the shapes of the hammers, nor from the alteration of the iron being forged. ${ }^{13}$

The Pythagorean philosophers, Philolaus and Archytas, were among the first to express musical scales in this ratioed manner. Observing that the pitch of a note is inverse to the length of the string that produces it, they proposed that intervallic differences between any two pitches could be expressed as simple numerical ratios: the octave as $2: 1$, the fifth as $3: 2$, and the fourth as $4: 3$. Using these three ratioed intervals, a traditional major scale is constructed with Pythagorean tuning. Beginning with C, the fifth degree of the scale (G) is calculated by multiplying the frequency of C by the ratio for a fifth $(261 \mathrm{~Hz} \mathrm{x} 3 / 2=392 \mathrm{~Hz})$. Having established G, the second degree of the scale (D) is calculated by dividing the frequency of G by the ratio for a fourth $(392 \mathrm{~Hz} \div 4 / 3=294 \mathrm{~Hz}$ ). This process of multiplying by a fifth and dividing by a fourth is repeated until the scale is complete ${ }^{*}$.

While Pythagorean tuning was eventually abandoned at the start of the sixteenth century, its development is key in any discussion of musical symbolism; the Pythagoreans discovered that musical intervals were codified expressions of mathematical ratios.

[^5]
### 1.2.3 Celestial Mechanics

Referring back to Philolaus and Archytas, it was recognised that the pitch of note is inverse to the length of the string that produced it. By the same logic, it was proposed that the Sun, Moon, and planets must emit a unique 'hum' ${ }^{14}$ based on their orbital revolution, the larger the orbit, the lower the note. This theory is known as the Harmony of the Spheres or musica universalis.

In a discussion of the Harmony of the Spheres, Julyan Cartwright provides a detailed account of historical attempts to ratify the Pythagorean supposition. ${ }^{15}$ Of these endeavours, it is the seventeenth-century German astronomer and mathematician, Johannes Kepler, whose efforts to unravel the universe stand out. In congruence with Grecian theory, Kepler believed in a 'creator' whose existence was evident through the explicit connections between geometry, astronomy, and music. Citing the ratios found between pitches in a Pythagorean scale, Kepler hoped to observe the same ratios in the movements of celestial bodies. However, the universe is a complicated place; orbits are eccentric, the sun is off-centred, and planets are constantly changing speeds. This concerned Kepler, whose exploration hoped to provide evidence of the divine; why had the creator not chosen a simpler cosmic design?

With the omission of Neptune and Uranus (discovered 200 years later), Kepler began by establishing the maximum (at perihelion) and minimum (at aphelion) angular speeds of each planet's orbital revolution. Condensing these figures to inter-planetary ratios, he found that when comparing velocities, convincing mathematical harmonies were created. The larger the eccentricities (the difference between the maximum and minimum orbital speed), the greater the production of pitches. For instance, 'Earth's maximum and minimum speeds...are a ratio of roughly $16: 15$, or that of a semitone, whereas Venus' orbit is nearly circular, and therefore only produces a single note. Mercury has the largest eccentricity and interval, a minor tenth, or ratio of $12: 5$ '. ${ }^{16}$ Kepler was able to explain the frantic, temperamental complexity of the cosmos in a way that 'appealed to [his] beliefs in a heavenly creator'. That is to say, 'a simple universe would [have produced]...boring music'. ${ }^{17}$

[^6]His research spans a series of five books, concluding with his celestial choir: Mercury our solitary soprano, Venus and Earth our altos, Mars the tenor, and Saturn and Jupiter our basses (Figure 1.5). 'The heavenly motions are nothing but a continuous song for several voices, a music which through discordant tensions, through syncopations and cadenzas, progresses towards certain pre-designed six-voice cadences and thereby sets landmarks in the immeasurable flow of time'. ${ }^{18}$


Figure 1.5. Page from Johannes Kepler's Harmonices Mundi - The Scales of the Six known Planets and Moon. ${ }^{19}$

While not overt, it is evident that discussions of music ascribe to the ideology that even at a monophonic level, it bears a deep-rooted, interconnected, and representational association with its environment. In around 400 BCE, the Pythagoreans proved that musical notes are underpinned by mathematical ratios, the very same ratios that Kepler would identify over two thousand years later in the constitution of our solar system. Since then, Mihail Cocos and Shawn Fowers have developed this notion of musical representation, devising an evaluative system whereby scales and modes can be expressed as star-like shapes, indicative of harmonic potential (Figure 1.6). ${ }^{20}$ Codification is defined as the adoption of one idea symbolically representative of another. This chapter provides evidence that music has always been naturally codified with mathematics, representational of the natural world.

[^7]


Figure 1.6. Geometric depiction of the harmonic potential of A major (left) and Japanese Insen (right). (Mihail Cocos, Shawn Fowers. International Journal of Applied Sciences and Technology Vol. 1 No. 6)

Supporting this idea of symbolism, Kepler remarked of his own findings that 'harmonic order is only mimicked by man but has origin in the alignment of the heavenly bodies ${ }^{21}$. Despite being a comment of musical inclination, this mimicry is equally evident in places of worship; ancient Greek temples were constructed 'follow[ing] the laws of numbers, that is, the particular proportions giving a sense of harmony and beauty through all the levels of creation'. ${ }^{22}$ Acknowledging these natural numerical codes and their relationship with music suggests that musical codification (at an observatory level) predates the first musical cipher and extends the history of musical cryptography back by almost two-thousand years.

### 1.3 Mathematical Codification: Case Studies

The following case studies illustrate compositional use of the mathematical codification discussed previously. As covered in later chapters, musical codification is not just used to symbolise numbers; the focus on mathematics in this chapter is to compound the previous conclusion that mathematical codification is the oldest and most persistent form of musical cryptography. Composers have long been codifying music with allusions to numbers of personal and universal significance.

### 1.3.1 Tool \& Claude Debussy: The Fibonacci Sequence

While the term 'Golden Ratio' was coined in the 1800s, the value was recognised in Ancient Greece and documented in Euclid's Elements (circa. 300 BCE). Expressed by Luca Paciolo to be a 'divine proportion', ${ }^{23}$ the 'Golden Ratio' is observable across all of nature, from plants and animals to weather structures and star systems. The value, 1.618, was established in accordance with the Fibonacci Sequence, a numerical series recorded by Leanardo Pisano

[^8]Bogollo in Liver Abaci (1202) whereby each number in the sequence is the sum of its previous two values, for example, $0+1=1$ and $1+1=2$. The sequence $(0,1,1,2,3,5,8,13,21$, etc.) can also be expressed graphically (Figure 1.7).


Figure 1.7. Fibonacci spiral formed by the first nine values of the sequence.
When a curved line is drawn from opposite corners of each square, a Fibonacci spiral is created (Figure 1.7). The spiral bears considerable resemblance to the Golden Ratio, a relationship used by the progressive metal band, Tool in their 2001 album, Lateralus.

Adopting a codified approach to composition, the titular track was devised in observation of the Golden Ratio; Tool let its values dictate fundamental decisions in the composition of 'Lateralus', beginning with the time signature. The track is split between the signatures of $9 / 8,8 / 8$, and $7 / 8$, their upper digits combining to form the number 987 , which is the sixteenth value of the Fibonacci sequence. ${ }^{24}$ In a similar vein, the lyrics of the track adhere (without fail) to the values of the sequence, never jumping a figure:


Figure 1.8. Verse I notation - 'Lateralus' by Tool
Counting each syllable, Tool use the rests between lyric sets to indicate a return to zero (Figure 1.8). Counting the syllables between the rests results in a short ascent and descent of part of the Fibonacci sequence; depicted in the table below, the values climb to 8 before working their way back down to 3 .

[^9]| LYRICS SPLIT BY A REST | SYLLABLES |
| :---: | :---: |
| Black | 1 |
| Then | 1 |
| White are | 2 |
| All I see | 3 |
| In my infancy | 5 |
| Red and yellow that came to be | 8 |
| Reaching out to me | 5 |
| Lets me see | 3 |

The final instance of numerological encoding in 'Lateralus' comes with a horological analysis of lyric placement; the first word, "black", is sung exactly 1 minute and 37 seconds into the track after a lengthy guitar introduction. Using a time to decimal calculator, 1 minute and 37 seconds (1:37) is expressed as 1.62 , a mere 0.002 out from the Golden Ratio. ${ }^{25}$

Similarly to Tool, Claude Debussy is thought to have used the Golden Ratio in relation to the proportionality of his music. By comparing the length of thematic sections, Roy Howat proposes that Debussy utilised the Golden ratio in his compositions. Howat goes on to suggest that these intricate proportional systems account for both the precise nature of the music's unorthodox forms and for the difficulty in defining them in more familiar terms. ${ }^{26}$

Debussy's 'Golden Section' (Figure 1.9) is such that 'the ratio of the shorter portion to the longer portion equals the ratio of the longer portion to the entire length ${ }^{27}$, or $\frac{b}{a}=$ $\frac{a}{a+b},{ }^{28}$ Howat stresses that this GS is simply a proportional expression of the Golden Ratio, like that depicted in Figure 1.7. Referring to 'Mouvement' and 'Cloches a travers les feuilles' from Debussy's piano Images (1905 \& 1907), Howat identifies two instances where the

[^10]musical apex coincides with the end of the GS. In 'Mouvement', the GS aligns with bars 109110, where the piece swells to 'fff' (Figure 1.10), and in 'Cloches a travers les feuilles', the GS appears in the second half of bar 31, as the music reaches its only 'ff' apex (Figure 1.11). These accents are 'given maximum force' through Debussy's deliberate dynamic shaping; 'both pieces begin and end quietly'. ${ }^{29}$


Figure 1.9. Graphical depiction of Debussy's Golden Section, Debussy in Proportion: A musical analysis, Roy Howat (1983)


Figure 1.10. 'Mouvement', bars 109-110 (Claude Debussy, Images Serie I Mvt. III, 1908)


Figure 1.11. 'Cloches a travers les feuilles', bar 31 (Claude Debussy, Images Serie II Mvt. I, 1908)

[^11]
### 1.3.2 Alban Berg: ‘23’

According to Hans Heinz Stuckenschmidt, 'no Frenchman was closer to [Alban] Berg than Debussy, ...[who] showed a high regard [for Berg's music] in private conversation'. ${ }^{30}$ Such a sentiment is believed to be shared by Berg, though 'recognition of Debussy...would have been considered improper in the Schoenberg circle, ${ }^{31}$ Stuckenschmidt proposes a convincing musical quotation in the last of Berg's Vier Lieder (Op. 2), where two bars from the fourth of Debussy's Six epigraphes antiques are cited. Figure 1.12 depicts Debussy's descending chordal motif that glides from G to Eb with a bassline that rises a fourth and falls a fifth sequentially.


Figure 1.12. 'Pour la Danseuse aus Crotales' piano reduction (Claude Debussy, Six epigraphes antiques Mvt. IV, 1914-15)

Figure 1.13 illustrates Berg's direct quotation of Debussy's descending motif; the piano part moves identically through the same sequence with each chord 'consisting of a tritone with a perfect fourth above it'. ${ }^{32}$


Figure 1.13. 'Warm Die Lüfte’ (Alban Berg, Vier Lieder, Mvt. 4, 1910)

[^12]Knowing that Debussy employed mathematical codification and Berg was a great admirer of his work, it makes sense that such encoding may be present in Berg's own works. Combining lyricism with the twelve-tone technique, Alban Berg's Violin Concerto (1935) is a prime example of number symbolism by means of encoding. Already harbouring numerological beliefs in the significance of the number 23, Berg was fascinated by the theories of Wilhelm Fliess, a writer who (similarly to the Ancient Greeks) believed in the worldly truths of numerological interconnectivity: 'all life is controlled by a periodic rhythm through a mechanism that is exactly the same for human beings, for animals, and for plants; a mechanism that informs the hour of our birth with the same certainty as that of our death'. ${ }^{33}$ Fliess' research was expounded in two books published in the early nineteen-hundreds: Der Ablauf des Lebens (The Rhythm of Life, 1906) and Vom Leben und Tod (Of Life and Death, 1909). Through the numerical analysis of planetary motion, menstruation, and many other natural cycles, Fleiss concluded that life was 'governed by two constants: the numbers 28 , associated with women, and 23 with men,.$^{34}$ As with Tool and Debussy, it was of significance to Berg that Fleiss accentuated that this was not an 'invention', it was a 'discovery' detected by him in nature. Berg's obsession with the value had been validated; as far as he was concerned, 23 had ascended personal significance and been compounded in the realms of numerological importance, central to the natural order of things.

The Violin Concerto displays many instances of codification in alignment with Fleiss' governing numbers. Dedicated 'to the memory of an angel', Berg's requiem-esque work was inspired by the tragic death of Manon Gropius at the age of eighteen. In addition to Fleiss' significant numbers (23 and 28), the work reflects the significance of the number 10. The value is widely agreed to represent Hanna Fuchs, a lady with whom Berg engaged in an affair, well-documented in the fourteen letters of 'rapturous professions of love' sent between them. ${ }^{35}$ In true codified fashion, the link between the plaintext (Hanna) and code (10) is delightfully arbitrary. While never explained, the final of Berg's written correspondences with Hanna do at least refer to the code: 'it is full of our numbers 10 and 23 and our initials H F A B ${ }^{36}$

[^13]The majority of allusions to 23 appear in Part II of the concerto. The movement consists of $\underline{23} 0$ bars, the opening tempo is ' $=69^{\prime}$ or $3 \times 23$, and a significant rhythmic motif (Hauptrhythmus) enters at bar 23, where Berg's musical monogram (Bb-A-G-E) is first heard. In regard to Hanna's number, Berg composes a 10-bar introduction and then unconventionally signposts it, 'Introduction (10 Bars)', a declaration of focus. Furthermore, Fleiss' number for 'female' representation appears regularly in calculation sketches along the margins of the original concerto manuscript. ${ }^{37}$ The calculations are re-written clearly below. Each resolve to 28, although there is conjecture as to Berg's methodology in reaching the value; there doesn't appear to be any mathematical consistency to the sums.

| 132 | 210 | 16 |
| :---: | :---: | :---: |
| 175 | 175 | 13 |
| $57=28$ | 35 | 28 |
|  | 28 |  |

As with 23, the number 28 can be identified within the concerto, as pointed out by Douglas Jarman:

The metronome markings of the Andante and Allegretto which form Part I, for example, are [crotchet] $=56(2 \times 28)$ and $112(4 \times 28)$ respectively; the bridge passage of the Andante begins at bar 28 ; the 'Tempo primo' which marks the beginning of the codetta starts at bar $84(3 \times 28)$ and the 'ritmico' figuration of the Allegretto is first introduced at bar $140(5 \times 28) .{ }^{38}$

### 1.3.3 Lili Boulanger: '13'

Also engaging with numbers of personal value, the Lili Boulanger found symbolic value in the number 13. Boulanger was writing at a time when 'personal systems of composition [were] relatively inaccessible to public understanding or imitation'. ${ }^{39}$ As such, it was only through Boulanger's self-documentation of a 'lucky number' that cryptographic analysis of her music took place. It should be noted that for most, the number 13 is considered unlucky; triskaidekaphobia is a recognised superstition regarding the value. Through interviews with Boulanger's closest friend, Miki Pire, it was discovered that Lili seemed to identify with the number 13; there are 13 letters in her name, and her initials

[^14]closely resemble the digits. Having acknowledged triskaidekaphobia, it could be suggested that 'adopting 13 as her lucky number...represented laughter in the face of fate for Lili, who was extremely sick all her life' ${ }^{40}$ Aware of her frailty, Boulanger focused on composition, winning the First Grand Prize in composition at the Prix de Rome with her cantata, Faust et Helene. Alluding to her symbolic number, the work implements a 13-measured ostinato bass illustrating the 'shades of the victims in the Trojan War'. ${ }^{41}$ She began composing at the age of 13, achieved the Prix de Rome in 1913 exactly seven weeks before her twentieth birthday (20-7 = 13), and was one of 13 competing contestants that year - coincidences which are unlikely to have escaped her. This is supported by her subsequent actions; deciding to destroy 13 of her early compositions and compose music for several biblical texts of which all contain the ' 13 ' figure: Psalms 130, 131, 137, and 1 Corinthians 13.

In her longest and perhaps most significant work, Clairières dans le ciel, Boulanger composed a cycle of 13 songs set to the poetry of Francis Jammes. Describing a collection of emotions endangered by memories of an amorous relationship's demise, Lili composed for only 13 of the 24 poems. This has led researchers to believe that Lili could be relaying her own personal experience of unhappiness in love, or simply identifying with the titular character 'Elle', whose name is a homophone of Lili's initial, 'L'. Supporting this notion, Elle is first introduced as being 'decked with flowers of plants whose stems love to sprout in the water', a clear reference to water lilies.

This analysis of Clairières dans le ciel begins with song 12 as the piece makes use of large, thirteenth chords. Depicting a lonely man with memories of his lost love, the song is dominated by the pitch D. Supported by an accompanying D-A-D-A-D figure, 'Je' is sung on a D , and the single closing bass note from the piano is a D . It has been suggested that the lonely man is represented by this pitch. ${ }^{42}$ When taking a 'pitch inventory of the nonchromatic vocal line' across the full song, all the notes of a D-minor scale are present apart from $\mathrm{Bb} .{ }^{43}$ Significantly, Elle is also absent from the twelfth song. Returning to our original supposition, if Elle is representative of Lili and Elle is missing from this song, then we can also attribute the missing Bb to be symbolic of Lili; Bb for Boulanger. Lili chose the alexandrine meter of the French text, which is comprised of three, 13-syllable lines followed

[^15]by a twelve-syllable line built over a 'species of half-cadence on a dominant seventh chord in G , ${ }^{44}$ While the sound is desperate to resolve to C , Lili plunges back into D minor to coincide with bar 13, perhaps symbolic of a collision between the lonely man and her. Moments later, Lili's initials are symbolised when the text falls synchronously to 'Elle' as the melody arrives at $\mathrm{Bb}(\mathrm{LB})$ for the first time in the song. Most pertinently, if D is a representation of Lili's lost love, and Bb of Lili Boulanger, the most significant numerological observation is that those pitches have an intervallic range of a $13^{\text {th }}$.

Having analysed examples from Tool, Claude Debussy, Alban Berg, and Lili Boulanger, I feel it can be stated that the musical codification of significant numbers appears to be a consistent cryptographic practice. Just as Kepler used the mathematics of astronomical bodies to construct his 'celestial choir', both Tool and Debussy adopted the Fibonacci sequence to devise intricate proportional systems that informed their music's structure. Alban Berg encoded the 'natural governing numbers' ${ }^{55}$ ( $23 \& 28$ ) of Wilhelm Fliess by aligning them with key musical moments in his work. And Lili Boulanger utilised the number 13 to compose tonally rich cluster chords, highlight moment of personal significance, and possibly as a declaration of strength 'in the face of fate, ${ }^{46}$. While these observations are all suppositions, there can only be minimal conjecture; Maynard James Keenan (Tool frontman) definitively speaks of Lateralus' encryption, Boulanger's friends and family refer to her connection with the number 13, and while not overt in the Violin Concerto, Berg has revealed hidden programmes in other works like the Lyric Suite. ${ }^{47}$ Ultimately, I feel that the discovery of musical cryptography's true age and its generational endurance should reframe musical codes from academically omissible, to the precursor to musical ciphers and the recognised origin of musical cryptography.

[^16]
## CHAPTER 2: FORMING A MUSICAL CIPHER

In light of this reframing of musical codification, this chapter will examine the various encryption techniques used in the construction of musical ciphers. Through this evaluation, I hope to categorise and group differing approaches, establish a rough timeline of developing techniques, and identify the most effective cipher form.

### 2.1.1 An Issue of Numbers: Nuances of The Musical Cipher

Before commencing any analysis, there are a handful of cipher-related nuances that the reader should be aware of. Firstly, the fundamental problem with cipher construction is the disparity between the number of distinctly named musical notes (12), and the number of letters in the alphabet (26); letters cannot be substituted for notes at a ratio of 1:1.

Secondly, in German notation, the English $B b$ is termed ' $B$ ' and $B 4$ is known as ' $H$ '. Schoenberg states in Theory of Harmony (1911) that while B $\mathfrak{G}$ and Bb are no longer considered distinct scalic degrees, German notation has retained the B and H naming system. ${ }^{48}$ Additionally, while accidentals are notated with traditional sharp and flat symbols, spoken and written references are modified to a single syllable word: '-is' replaces 'sharp', and '-es' replaces 'flat'. Rather than express F\# as 'F sharp', German nomenclature would pronounce it 'Fis'. This provides a cryptographer the means to phonetically encrypt a new letter; Eb is 'Es' which audibly resembles ' S '. These helpful nuances provide cryptographers with two extra substitutable letters.

### 2.2 Cipher Techniques: The Big Five

Prior to David Løberg Code's pioneering article, 'Can Musical Encryption be Both? ${ }^{49}$, academia had not attempted to categorise the various approaches to musical encryption. In 2022, Løberg Code devised a set of five classifications that all musical ciphers adhered to: Steganographic, Monophonic, Diatonic, Chromatic, and Compound Motivic. In this chapter, I will outline the encryption process of each approach, identify a couple of examples, and substitute the plaintext, 'teatime' to evaluate cipher efficacy (the strength of the encryption) and musicality (the quality of the resulting cipher-melody). In musical applications, the conspicuity of an enciphered musical passage depends on the cryptographer's knowledge of

[^17]compositional convention; unorthodox harmony, odd intervallic leaps, and questionable enharmonics all gesture towards a degree of cryptographic interference.

### 2.2.1 STEGANOGRAPHIC

Where most approaches strive for illegibility, steganography strives for invisibility; to disappear within the payload. Successful encryptions require the cryptographer to have a scrupulous understanding of the chosen payload's construction. Without this knowledge, a payload will appear inconsistent, drawing attention rather than existing innocuously. ${ }^{50}$

The 'Conceal yourself' cipher is a stand-alone steganographic encryption dating back to the seventeenth century. The anecdote goes that following the Royalist's defeat to Oliver Cromwell's Parliamentarians at The Battle of Worcester, Charles Stuart (later Charles II) was forced to retreat to Boscobel where he evaded capture by hiding in an oak-tree. While he was hiding, an unnamed lady delivered to him a scrap of folded parchment on which was scribbled two taves of musical notation (Figure 2.1). The notation does not appear musically inconsistent; the intervallic range is typical, the key is consistent, and there is believable variation in note length. As with the earlier 'My Old Kentucky Home' example, the note stems are encrypted. However, where 'My Old Kentucky Home' hides a separate morse code-like cipher in stem gaps, the 'Conceal yourself' cipher is entirely steganographic. ${ }^{51}$


Figure 2.1. Steganographic message delivered to Charles Stuart (later Charles II) after The Battle of Worcester that reads 'Conceal yourself, your foes look for you' ( $17^{\text {th }}$ Century, Western Manuscripts, The British Library, Add MS 89288)

[^18]The enciphered message is revealed through a series of precise folds. Facing the manuscript, the intended recipient would fold the parchment twice: firstly, along the second lowest line on the bass clef stave, sending the lower portion backwards so the bass-clef stems are separated from their note-heads; and secondly, between the two staves, bringing the bottom half forwards until the previously folded bass-clef stems align below the treble clef stems. Having aligned the treble and bass-clef stems, the hidden message, 'Conceal yourself, your foes look for you' is revealed (Figure 2.2). In this case, steganography is used in isolation of another cryptographic approach.


Figure 2.2. Charles Stuart (later Charles II) steganographic cipher 'Conceal yourself, your foes look for you' revealed after folding parchment. ( $19^{\text {th }}$ Century, Western Manuscripts, The British Library, Add MS 45850, f. 68)

Whether for the time-consuming process or high level of intricate detailing required, steganography is this pure form is a rare occurrence. However, a lack of analysable examples does not render the approach irrelevant to a conversation of cipher techniques. ${ }^{52}$ The steganographic consideration of payload coherence is imperative to all four of the other cipher forms; while a strong substitution cipher can be impenetrable, a seamless blend into its payload can render it invisible.

### 2.2.2 Monographic

Popularly referred to as musical monograms, the approach identifies all the letters from a word that match existing musical pitch names. The identified letters are then substituted as their matching pitch within a musical motif. Variations of this technique appear in popular musical mnemonic devices; 'FACE' represents the notes that make up the spaces on a treble clef and 'BEAD', the first four flats. While the purpose is not cryptographic, these

[^19]mnemonics illustrate the idea that note names can be used to construct words. A step beyond this would be to identify words that do not conscribe to a musical order as 'bead' and 'face' do. From 'cabbage' to 'decaf', there are over 130 possible word permutations constructible from the seven named pitch classes in the English language, A through to G. Adding in H and S from German notation increases this number to 495 words. This is not to say that all these words are useful; as suggested in the term monogram, the majority of these ciphers are musical realisations of names, the most famous of these belonging to Johann Sebastian Bach. ${ }^{53}$


Figure 2.3. B-A-C-H Motif(Henry E. Dudeney, 'A Musical Enigma' from the 'Perplexities' column, Strand Magazine 63, 1922)

While the arrangement found in Figure 2.3 was never used by JS Bach himself, the signature elegantly demonstrates its composition. Beginning with the stave running from left to right, the central note reads as a Bb . Continuing in clockwise motion, the tenor clef reads as an A , the alto clef as a C , and the second treble clef as a B . With our understanding of German notation, we can deduce that $\mathrm{Bb}-\mathrm{A}-\mathrm{C}-\mathrm{B}$ 白 will be read as $\mathrm{B}, \mathrm{A}, \mathrm{C}, \mathrm{H}$, a motivic representation of the composer's name (below). ${ }^{54}$


The BACH musical motif (above) appears consistently across much of JS Bach's output. To name but a few examples, the fugue from BWV 898, bar 109 of the continuo from

[^20]the Brandenburg Concerto No. 2 (BWV 1047), and a transposed version in the subject from the Sinfonia in F minor (BWV 795). This monographic motif also appears in the works of other famous composers paying homage to Bach. Following the Bach revival in the first half of the $19^{\text {th }}$ century, the motif's popularity soared. From Robert Schumann's Sechs Fugen über den Namen: Bach, Op. 60 (1845) and Johannes Brahms' Fugue in A-flat minor, WoO 8 (1856), to Arnold Schoenberg's Variations for Orchestra, Op. 31 (1926-28) and Anton Webern's String Quartet, Op. 28 (Figure 2.4), there was an influx of his musical motif. (This is not to say that Schoenberg and Webern composed in the style of Bach, just that the inclusion of his motif was as a result of the revival). ${ }^{55}$


Figure 2.4. Anton Webern's String Quartet, Op. 28, tone row, composed of three tetrachords: Bar 1 BACH motif, Bar 2 - BACH motif inversion, Bar 3 - BACH motif retrograde inversion (1937-38). ${ }^{56}$

Monographic ciphers constitute a large majority of musical ciphers, perhaps due to the ease at which they are employed, but more likely due to the ease at which they can be identified. Having proved that (with the inclusion of S and H) there are over 495 word combinations, it would make sense that composers accidently construct words by means of a coincidental monographic cipher. It is impossible to conclusively distinguish between intention and coincidence, but in absence of a composer's allusion to encipherment, the best way lies in its consistency and musical significance. Take for example, Dmitri Shostakovich's Violin Concerto No. 1, Op. 77. His musical monogram, DSCH (D-Eb-C-Bদ) is consistently identifiable in both the orchestral and solo parts of the concerto, regularly coinciding with emotional centres of the work by solidifying the poignance of grand string gestures or reserved moments of textural homophony.

[^21]

Figure 2.5. Dmitri Shostakovich Violin Concerto No. 1, Op. 77, second movement, solo violin part at b35
This fragment (Figure 2.5) from the scherzo second movement depicts such monographic gesturing, transposing, and signposting; the violin states a four-bar octave motif covering $\mathrm{D} \#, \mathrm{E}$,, $\mathrm{C} \#$, and B ( (the DSCH motif transposed up a semitone). This gesture is attributed significance through the accents and fortissimo dynamic marking, a dramatic increase from the mezzo-forte moments before. Although Shostakovich did not document his cipher, its existence is acknowledged by its continued identification from the violin concerto in 1947 to his Fifteenth Symphony in 1970. As with Bach's monogram, a true monographic cipher is often compounded through identification in the works of other composers. Like the Bach Motif, Shostakovich's monogram undergoes the same adoption in works such as Schnittke's Prelude in memory of Dmitri Shostakovich (1997) or Benjamin Britten's Rejoice the Lamb (1943). ${ }^{57}$ While the cryptograms of Shostakovich and Bach are widely recognised, the enciphered output of their contemporaries is decidedly less established. Robert Schumann's Abegg Variations enciphers the eponymous character with the pitches A-Bb-E-G-G. In AC/DC's track, Whole Lotta Rosie, the nickname 'Acka Dacka' is achieved with the pitches A-C-A-D-A-C-A. In his piano work, Sechs Kleine Klavierstücke, Arnold Schoenberg writes his own name with the pitches A-Eb-C-B-Bb-E-G (Figure 2.6). And Edvard Greig enciphers his initials as E, B, G; E for Edvard, B(H) for Hagerup, and G for Greig into the opening phrase of his Piano Sonata in E minor.


Figure 2.6. ‘Aschbeg’, Schoenberg’s Musical Monogram (Arnold Schoenberg, Sechs Kleine Klavierstucke Op. 19, No.1, M.5)

[^22]Composers also employed monographic ciphers to represent the names of others; Rimsky-Korsakov, Borodin, Lyadov, and Glazunov used the cryptogram B-A-F in a quartet to honour Mitrofan Belyayev who died in 1904. In the October 1922 issue of La Revue Musicale, unnamed Fauré enthusiasts devised an ingenious monographic realisation of 'Gabriel Fauré' using note names and solmisation in tandem. This process gifts a cryptographer eight new sounds (Do, Re, Mi, Fa, So, La, Ti, Do) and six new letters (R, M, I, O, L, T). Using solmisation, German notation, and standard note names, 'Gabriel Fauré' was achieved at Figure 2.7. ${ }^{58}$

| Enciphered Notes | G | A | Bb | D | B | E | E | F | A | G | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Deciphered Letters | G | A | B | Re | H | E | Le | F | A | U | Re | E |
| Phonetic Realisation | G | A | B | R | I | E | L | F | A | U | R | E |

Figure 2.7. Anonymous monographic cipher enciphering 'Gabriel Faure’
When using solmisation, the convention is to assume that C corresponds to Do, with the following note and solmisation sounds paired through the octave. With that in mind, while the G to U translation is tenuous, phonetically sounding the deciphered letters (Figure 2.7) gives a codebreaker, GABReHELe / FAUReE (Gabriel Faure). A second example of this solfege adoption can be found in Oliver Knussen's, Flourish with Fireworks (2003). Knussen monographically enciphers LSO (London Symphony Orchestra) and MTT (Michael Tilson Thomas) with the pitches A, Eb, G, E, B, B (Figure 2.8)

| Enciphered Notes | A | Eb | G | E | B | B |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Deciphered Letters | La | Es | So | Mi | Ti | Ti |
| Phonetic Realisation | L | S | O | M | T | T |

Figure 2.8. LSOMTT monographic cipher substitution (Oliver Knussen)
Having established that solmisation sounds correspond to each scalic degree starting with C, reference to the table (Figure 2.8) shows that (in omission of the German realisation of Eb symbolising S), each enciphered note contributes to a phonetic spelling of the plaintext, LSO/MTT.

[^23]
### 2.2.3 DiAtonic

In 1602, Giambattista della Porta documented one of the earliest known diatonic ciphers in his book, De Furtivis Literarum. ${ }^{59}$ As with all diatonic ciphers, Porta's contained only plain pitches with no additional sharps or flats, a choice that would typically restrict a scale to only seven unique note names. To avoid this, diatonic ciphers opt to alter the octave and duration of notes to afford a greater number of unique pitches. The benefit of being able to substitute two or more letters to one note are that the encrypted melodies become impervious to frequency analysis. If there are 26 symbols denoting 26 letters, a codebreaker is working on a 1:1 ratio; if they can work out the substituted letter for one symbol, they know that every time that symbol appears, it is representing that unique letter. Furthermore, every time a letter is correctly deduced, the odds of accurately working out the rest exponentially increase. Frequency analysis can be made infinitely harder if there is more than one letter being mapped to a symbol; working out that a symbol is representative of a letter means very little if that symbol is connoting a different letter later on. Diatonic ciphers engage with this double-mapping.

Depicted in Figure 2.9, Porta's cipher has an eleven-note scalic range, rising from E3 to A4 in semibreves, and descending in minims. ${ }^{60}$ With this duration switch, the cipher is able to uniquely map 22 letters of the alphabet to only 11 pitches, omitting ' j ' and ' $k$ ' for their visual and audible similarities with ' $i$ ' and ' $q$ ', and ' $v$ ' and ' $w$ ' for their similarities to ' $u$ '.


Figure 2.9. Porta Diatonic Cipher (Giambattista della Porta, De Furtivis Literarum Notis, Vulgo de Ziferis Libri V, 1602)

[^24]Possession of the 'teatime' substitution (Figure 2.10) is easily decrypted with Porta's cipher algorithm. Notes are clearly labelled and there are no two letters that share an identical value. Even if a word that included ' $j$ ' or ' $k$ ' had been enciphered, the plaintext would still be decipherable; the word 'joke' would come out as 'ioqe' which whilst not the same, is phonetically and visually similar.


Figure 2.10. 'Teatime' enciphered using Porta's Diatonic Cipher

With a focus on concealment, the issue with these diatonic ciphers lies not in their encryption strength, but rather their lack of musicality. When regarding the 'teatime' tune (Figure 2.10), a musician would note that there is no step-wise motion, an unidiomatic use of note duration, and seemingly no rhythmic cohesion at all. The rhythmic flaws could be overlooked with the addition of bar lines after each note and a minim rest between the A and T, but the weird motion and note duration would surely be flagged as odd. These musical peculiarities are the giveaways that something may be afoot, raising suspicion and increasing the chances that a cipher will be gleaned from an intercepted manuscript.

When analysing the real-world utility of such ciphers, interception is just as damaging as decryption by an unintended recipient. Take for instance the work of Dr. Merryl Goldberg and the Klezmer Conservatory Band. ${ }^{61}$ In 1985, it was exceptionally difficult to leave the Soviet Union. The easiest way out was to be invited; however, invitations could only be sent by those on the outside and had to include a plethora of specific information from full names and family relationships to birth dates and addresses. Under the guise of a travelling music ensemble, the twenty-five year old Goldberg and three bandmates smuggled this valuable information across Soviet lines using encrypted sheet music to carry the details of those within. ${ }^{62}$ While Goldberg has chosen to keep the intricacies of her diatonic substitution cipher a secret, she did impart some brief descriptions in an interview at the RSA Conference (a security event) in 2022. ${ }^{63}$ In addition to traditional diatonic substitution, Goldberg used key-

[^25]signature changes to denote rule sets, small diagrams of buildings disguised as chord symbols, and replaced performance directions with descriptions of transport routes. Goldberg reported that border security paid little attention to the encoded manuscripts as a result of their musical coherence, a compliment that cannot be paid to Porta's cipher. It is imperative that musical ciphers concerned with secrecy also endeavour to achieve musical cohesion. Embarrassingly for Porta, an advanced consideration of musical coherence had already been made some 150 years prior to his cipher. Documented in 1436, Brother Nicholas Philip's cipher displays a shockingly high level of musical thought.

Depicted at Figure 2.11, Philip's maps four short scalic groups, two ascending and two descending. ${ }^{64}$ The first ascent consists of quavers, then descending crotchets, ascending minims, and finally, descending semibreves. These four groups are assigned specific letter selections; the quavers represent $a, e, i, o$, and $u$, the crotchets $b, c, d, f, g$, the minims $k, l, m$, $n, p$, and the semibreves denote $q, r, s, t$, and $z$. Where Porta's cipher assigns the alphabet as it comes, Philip's groups vowels separately. While the reasoning behind this decision is undocumented, I might suggest that Philip's rationale comes from acknowledging the natural interactions between consonants and vowels. Most European languages place vowels between consonants. Assigning quavers to vowels means that enciphered words should produce melodies that blend shorter note durations with longer ones, enriching the musical potential of an encryption. Furthermore, Philip caps the intervallic range at a fifth by designing a 4:1 letter to pitch substitution scheme. ${ }^{65}$


Figure 2.11. Philip Diatonic Cipher (Brother Nicholas Philip, The Sermon Booklets of Friar Nicholas Philip, 1436)

[^26]Observing Figure 2.12, it is clear that this 'teatime' melody makes more musical sense. Assuming the addition of bar lines and rests to amend the rhythm, the melody displays more rhythmic variation, no unconventional leaps, and moves in a convincing (albeit uninteresting) manner.


Figure 2.12. 'Teatime' enciphered using Philip's Diatonic Cipher

It is amusing to note that as a practice, musical cryptography seems to contain a high volume of algorithm stealing between cryptographers. For instance, Porta's system resurfaces through Schwenter (in 1622), Wilkins (in 1641), Kircher (in 1650), and all without acknowledging Porta. While this chapter will not cover all the offending parties, the most notably developed example can be found one hundred years later, in 1772.

Philip Thicknesse's approach is a 'greatest hits' of Porta and Philip's diatonic ciphers (Figure 2.13). ${ }^{66}$ Following 'Porta's familiar model of an extended diatonic scale with two signifying durational values, the approach is expanded through the addition of Philip's more specific alphabetic mapping. Aside from the grouping of vowels from F $\# 4$ to C 5 , the cipher notes appear to have been arbitrarily assigned letters.


Figure 2.13. Thicknesse Diatonic Cipher (Philip Thicknesse, A Treatise on the Art of Decyphering, and of Writing a Cypher. With a Harmonic Alphabet, 1772)

Having applied the 'teatime' plaintext (Figure 2.14), this melody is easily the most convincing of the three; motion contains both idiomatic leaps and stepwise movement, the melodic range is probable, and the phrase is even coherently developed with no ledger-line movement at all. There are three techniques through which this coherence was achieved.

[^27]Firstly, letter-to-note substitutions are paired through a shared regularity; the most commonly used letters are represented by the most commonly notated pitches. 'Vowels and more common consonants (e.g., $t, s, n$ ) occur as ...[crotchets] within the staff; less frequent letters appear as half notes or outside the staff ${ }^{9} .{ }^{67}$ Melodies (like that in Figure $2.14 \& 2.15$ ) should appear crotchet heavy with the occasional minim breaking up the sound. Secondly, is Thicknesse's use of only two note durations, crotchets and minims. From a visual perspective, these classes are easily differentiated; crotchets' note-heads are filled in, and minims are not. Thicknesse seems to take this observation and apply it to all note types, resulting in a situation where crotchet substitutions apply to any visually filled-in note durations (e.g., quavers or semiquavers), and minim substitutions apply to any visually outlined note durations (e.g., semibreves, dotted minims, or breves etc.).


Figure 2.14. 'Teatime' enciphered using Thicknesse Diatonic Cipher
This approach is evident in the example provided by Thicknesse where 'near yonder cops where' is substituted (Figure 2.15). The ' e ' and ' r ' from 'yonder' are substituted as quavers despite the cipher algorithm denoting no such duration. Cryptographers using Thicknesse's cipher are gifted a level of compositional autonomy unobtainable in Porta and Philip's entries; depending on the musical context, cryptographers can choose the note duration most appropriate. Without knowing it, Thicknesse alludes to the earliest known form of cryptographic embedding; a technique whereby a cipher tune is 'woven into a freelycomposed piece, with harmony and meter, by a skilled musician'. ${ }^{68}$ This technique will reoccur frequently throughout the rest of the paper, although in a slightly different form to this. Thicknesse's embedding involved the strategic placement of 'channel-bearing notes', identifiable through their downward stretching stems. Between these notes there can be anything: rests, chords, entire motifs if necessary, as long as the stems travelled upwards, a codebreaker could ignore them.

[^28]

Figure 2.15. Thicknesse Diatonic Cipher (Philip Thicknesse, A Treatise on the Art of Decyphering, and of Writing a Cypher. With a Harmonic Alphabet, 1772)

Unfortunately, many of the shortcomings discussed with Porta and Philip are also relevant here. While the approach finds strength in a non-alphabetic substitution order, the algorithm fails to reach the security levels of Philip's $4: 1$ substitution system. It is difficult to claim that Thicknesse's embedding approach is effective when it relies so heavily on stem direction (steganographic) as there will be visual abnormalities in the score. Finally, while the non-embedded 'teatime' melody is convincing (Figure 2.14), the wrong collection of words (Figure 2.15) can result in a melody full of musically unnatural leaps.

### 2.2.4 Chromatic

Evolving alongside musical convention, the emergence of chromatic ciphers seem to mirror the more contemporary compositional style of the $19^{\text {th }}$ and $20^{\text {th }}$ centuries. Operationally, these ciphers are identical to the diatonic approach; in every case, the alphabet is substituted for an equal number of musical symbols. The differences appear in musical context and simple addition of chromaticism.

While most cryptographic tributes take monographic form, some, like Arthur Honegger's compositional dedication to the French composer, Albert Roussel, utilise more comprehensive ciphers. The short piano work is headed by a 26 note chromatic cipher spanning a thirteen note intervallic range (Figure 2.16). Where the diatonic examples would occasionally double up letters to a single note ( $u$ and w), Honegger provides a unique substitution for every letter of the alphabet. The cipher consists of three scales starting on A4, B4, and Cb5 that combine to cover every pitch between A4 and F6. ${ }^{69}$


Figure 2.16. Honegger Chromatic Cipher (Arthur Honegger, Homage a Albert Roussel, H.69, 1928)

[^29]When the 'teatime' plaintext is applied to Honegger's cipher, the resulting melody is peculiar (Figure 2.17). Given the chromatic nature of the cipher, it would seem an oversight on Honegger's part that no key signature solution is provided; the melody is a confused muddle of accidentals. While it would be easy to add bar lines in, this ease is at the cost of any rhythmic interest at all; the crotchet-centric cipher leaves little room for durational variation.


Figure 2.17. 'Teatime' enciphered using Honegger Chromatic Cipher
Figure 2.18 depicts the opening encrypted theme from Homage a Albert Roussel (1928). Emphasised by accents, the tune undergoes a small degree of chordal embedding, although this somewhat undermines the cipher given the lack of clarity as to which note within the chord is enciphered. Honegger appears to have taken creative liberties with certain substitutions to improve the musicality of the tune; the distinction between ' $e$ ' and ' $r$ ' is blurred when both are substituted as Eq in the second bar. He also seems to have disregarded the cipher's dependence on octave registers to differentiate certain letter pairs, completely undermining the use of a cipher in the first place. The resulting music is a complete departure from his normal compositional style, a huge indication to a codebreaker that something is being hidden. While Honegger's cipher is technically functional, the relentless chromaticism and requirement to leap renders its musical façade useless, evident in the fact that he consistently alters the algorithm to suit his compositional style.


Figure 2.18. 'Albert Roussel' Melody (Arthur Honegger, Homage a Albert Roussel, H.69, 1928)
The second chromatic cipher that warrants individual analysis was devised by Olivier Messiaen. While the algorithm boasts no great innovation, its composition does; it is 'quite likely the only musical cipher that was truly developed to embody the stylistic and aesthetic
goals of the composer'. ${ }^{70}$ Mapping all 26 letters of the alphabet to unique musical symbols, the substituted notes vary through pitch, octave, accidentals, rhythm, slurs, and clef, resulting in a visually striking cipher algorithm (Figure 2.19). The intervallic range is enormous, spanning 31 notes (from D 2 to $\mathrm{F} \# 6$ ) and there is no key signature. ${ }^{71}$


Figure 2.19. Messiaen Chromatic Cipher (Olivier Messiaen, Méditations sur le Mystère de la Sainte Trinité, 1969)

One would be forgiven for thinking this cipher incapable of achieving musical coherence; indeed, substituting 'teatime' produces a bizarre set of rhythms and tones (Figure 2.20). However, the cipher was not devised to produce music generally, it was designed to imitate Messiaen. ${ }^{72}$ From rhythmic patterns to melodic progressions, the chosen pitches were assigned with the artificial production of his style in mind. Taking Messiaen's often atonal and wildly avant-garde compositional approach (post 1950) into consideration, the serialist 'teatime' melody is idiomatic of his style, no more likely to attract attention than any of his other works.


Figure 2.20. 'Teatime' enciphered using Messiaen Chromatic Cipher

[^30]
### 2.2.5 Compound Motivic

When considered in relation to their period of activity, both diatonic and chromatic ciphers operate as a reflection of music from that period; where diatonic enciphering appears in the music of late baroque, classical, and romantic epochs, the chromatic variations seem to have emerged during the atonal experimentalism of the avant-garde. While applications of the technique vary, compound-motivic enciphering operates with just one notable difference to diatonic and chromatic ciphers; letters are substituted at a motivic level, rather than for a single note.

First appearing circa 1600, the technique's earliest form relates back to the notion of universal interconnectivity, with reference to the names of angels (Figure 2.21). Known as a Polybius square, the cipher appears as a $5 \times 5$ grid with solmisation pitches as the coordinate labels (Ut, Re, Mi, Fa, Sol; the first five degrees of a major scale). ${ }^{73}$ The twenty-five spaces within the grid are filled with the names of angels, of which the third letter of each name functions as the plaintext, every letter of the alphabet is present apart from ' j ' which is once again represented by ' i ' for their visual similarity. ${ }^{74}$ For example, 'Gabriel' is representative of the letter 'B'. Devised and documented by Friedrich vo Ottingen-Wallerstein around 400 years ago, the algorithm dictates that each plaintext letter be substituted for two notes (Figure 2.21). The duration and octave of these notes are down to the composer; using solfège notes means that the only predetermined musical factor is the note's pitch within a scale. ${ }^{75}$


Figure 2.21. Ottingen-Wallerstein Compound Motivic Cipher (Friedrich von Ottingen-Wallerstein, Steganographia comitis, ca. 1600)

[^31]Having established the key in which the cipher will go (in this case, C major), each letter of the 'teatime' plaintext is substituted using the two intersecting solmisation pitches at Figure 2.21. Taking ' $A$ ' for instance, one must first identify the corresponding angel name. In this case, the name is name is 'Staniel', and the two corresponding solfege pitches are 'FA' (fourth degree) and 'UT' (first degree). Applied to C major, 'FA' becomes F, and 'UT' becomes C. The 'teatime' example at Figure 2.22 is simply an exemplar of pitch; as mentioned, cryptographers are encouraged to alter the duration and octave of notes to aid musicality.


Figure 2.22. 'Teatime' enciphered using Ottingen-Wallerstein Compound Motivic Cipher

An exploration of compound motivic potential was undertaken in the following 150 years. Spearheaded by the likes of Johann Balthasar Friderici (1665), cryptographic design gained new perspectives on the construction of motifs. ${ }^{76}$ Friderici devised a set of three new and functionally unrelated approaches, beginning with minor deviations from Ottingen Wallerstein's two-note technique (Figure 2.23), devising a multi letter to motif triadic cipher (Figure 2.24), and ending in the construction of an entirely rhythmic system (Figure 2.25). ${ }^{77}$


Figure 2.23. Encrypted message from Johann Balthasar Friderici based on the Ottingen-Wallerstein cipher (Friderici, 1665)

[^32]

Figure 2.24. Friderici Triadic Compound Motivic Cipher (Friderici, 1665)


Figure 2.25. Friderici Rhythmic Compound Motivic Cipher (Friderici, 1665)
Disregarding Friderici's first cipher (Figure 2.23) in acknowledgment of its many similarities with Ottingen-Wallerstein's angelic algorithm, this brief analysis will begin with his Triadic cipher (Figure 2.24). Immediately, there are a couple of visual observations distinct from Ottingen-Wallerstein's Polybius square. Firstly, there are only eight substitutable motifs, and each is assigned three alphabetic letters, numbered 1,2 , and 3 . The
substitution system is humorously comparable to that of texting on an old mobile phone; without the use of 'qwerty' keyboards, the alphabet was assigned to the 10 dialling numbers, three letters at a time ( $1=\mathrm{ABC}, 2=\mathrm{DEF}$ etc.). To select a ' C ', 1 was pressed three times, and Friderici's compound motivic cipher functions in much the same way, with the only difference being that each repetition is recorded and notated: to encipher a C, the corresponding motif is played three times. Its labelling as triadic is simply down to the chordal nature of the intervals, but while such composition ensures tonality, it is at the cost of melodic interest; encrypted melodies risk being bloated with triple repetitions of motifs and the consistent re-appearance of pitches with little variation.

The third cipher (Figure 2.25) strides the furthest in regard to autonomy. Entirely rhythmic, the system foregoes pitch dictation in favour of control over note duration and rhythm. Presented in a manner reminiscent of the diatonic and chromatic approaches, the cipher provides 24 , three-note rhythmic motifs for all alphabetic letters in omission of ' j ' and ' $v$ ' (as now appears standard practice). ${ }^{78}$ Having produced an encrypted rhythmic melody, the cryptographer can alter the pitches to carve out a melody, guaranteeing musical cohesion and a certain degree of consistency in the compositional output of the composer.

It is highly likely that Wolfgang Amadeus Mozart wrote a compound motivic cipher. Dating back to 1787, the manuscript is in Mozart's hand and appears to encrypt the name 'Franziska', perhaps the daughter of botanist, chemist, and doctor, Nikolaus Joseph von Jacquin. ${ }^{79}$ Mozart would spend most Wednesday evenings as a guest in the Jacquin household, teaching composition to Nikolaus' son, Gottfried, and piano to his daughter, Franziska. Caroline Pichler, another of Mozart's piano students, recounts that Mozart would often chat and joke with Franziska while scholarly topics were discussed in Nikolaus' salon. Pilcher writes in her autobiography, 'Denkwürdigkeiten aus meinem Leben' (Memorable events of my life) that 'Franziska played the piano exceedingly well, she was one of Mozart's best students, and he wrote the Trio with Clarinet for her'. ${ }^{80}$ With this fond relationship in

[^33]mind, it would appear that Mozart used her name to produce an exemplar cipher melody at the foot of the fragment. ${ }^{81}$

Mozart's cipher is the first to operate in musical excess; for each letter of the alphabet, two, unique, two-bar motifs are composed (Figure 2.26). The method by which a cryptographer should determine which of the two motifs to substitute is not included in the cipher. Hideo Noguchi proposes that motivic selection was either left to chance (like a coin toss) or simply at the compositional preference of the cryptographer. ${ }^{82}$ This negates any threat of decryption via frequency analysis; an ' $A$ ' could appear in two different motivic forms throughout a cipher-melody. However, of equal importance are the musical benefits garnered from providing a choice; one motif may be more musically appropriate than another, improving the coherence and security as a biproduct. ${ }^{83}$


Figure 2.26. Mozart Compound Motivic Cipher (Wolfgang Amadeus Mozart, Music manuscript MS 25301, Bibliotheque Nationale de France, 1787)

With Mozart's inclusion of a standardised introduction and conclusion to provide a distinct beginning and end to the melody, Figure 2.27 depicts 'teatime' as substituted using Mozart's cipher. While the rest of the letters are represented by their first variant, the opening ' $T$ ' has been substituted with its second variant. Disregarding for a moment the cryptographic

[^34]content, this melody is shockingly articulate. In omission of the minor $10^{\text {th }}$ and tritone leaps found at the end of bars 4 and 10, the inter-motivic transitions are both reasonable and musically compelling, almost appearing composed. To achieve this compositional quality, Mozart ensures that each motif could feasibly link to each other, constructing a compositional web that ensures cryptographic strength whilst maintaining the integrity of his style, similarly to Messiaen. To provide a sense of scale to the enormity of Mozart's cipher, if I limited the plaintext length to 2-6 letters and did not include any repeats, there would still be 23,251,631 possible permutations. While many of these would fail to produce 'words', the cipher was composed in such a way that the melody would still be musically legible. For instance, the melody at Figure 2.28 enciphers the randomly selected letters, 'klqp', while remaining coherent.


Figure 2.27. 'Teatime' enciphered using the Mozart Compound Motivic Cipher


Figure 2.28. 'klqp' enciphered using the Mozart Compound Motivic Cipher
Approaching the $19^{\text {th }}$ century, compound motivic ciphers seem to have reached operational maturity; the design systems vary less and less with a more centralised approach beginning to be solidified. Johann Bucking's 1804 entry follows much the same formula (albeit with single-bar motifs) as Mozart's; a collection of unique musical melodies used to
represent 24 letters of the alphabet (Figure 2.29). ${ }^{84}$ From the rather primitive letter-to-note encryptions employed in diatonic and chromatic ciphers to the more ambitious endeavours presented by Mozart and Bucking, the development of ciphers, while shorter than that of codes, certainly exhibits a larger degree of evolution. From the $15^{\text {th }}$ to $20^{\text {th }}$ century, we are able to categorise four distinct developmental stages of musical cryptography: monographic, diatonic, chromatic, and compound motivic.


Figure 2.29. Bucking Compound Motivic Cipher (Johann Bucking, Answeisung zur geheimen Correspondenz systematisch entworfen, 1804)

Having analysed the encryption process, efficacy, and musical cohesion of a multitude of ciphers from each of Løberg Code's categories, it seems clear to me that the compound motivic approach is the closest to achieving the mutual success of both components. While successful in terms of musical coherence, the monographic ciphers of Schoenberg, Webern, and Shostakovich are barely complex enough to be considered encrypted. At the other extreme, the diatonic and chromatic ciphers of Philip, Thicknesse, Honegger, and Porta excel in terms of cipher strength, but unlike Messiaen's, take little consideration of contemporaneous musical convention. The compound motivic ciphers of Bucking, Mozart, and Ottingen-Wallerstein manage to maintain a balance between music and encryption through the retention of a miniscule level of compositional autonomy. Control over just one or two bars of music appears to alleviate that feeling of the aleatory that is

[^35]present in the other approaches. Finally, it is worth noting that Messiaen's chromatic cipher is the only non-compound motivic approach that finds success in encryption strength and musicality. When one considers this in conjuncture with autonomy of compound motivic ciphers, it can be concluded that to create an effective musical cipher, the most important consideration is to reflect the music of the era in which you are composing.

## CHAPTER 3: FUNCTION

In the previous two chapters, I have grappled with the various forms that musical cryptography can take. From the symbolic codification of musical structure and harmony to the diverse array of cipher approaches, this paper has covered most (if not all) modes of musical encryption. As one would expect, the previous two chapters have been written with the assumption that the function of musical cryptography is to conceal. In the following two chapters, I will explore the notion that with regards to music, encryption is undertaken for reasons other than (or in addition to) concealment.

That in mind, this chapter will present cryptographic work from three composers who I believe engage with encryption for three distinct reasons: theology, ostentation, and catharsis. Making use of the techniques and concepts covered in the previous two chapters, the monograms of Johann Sebastian Bach will be examined for evidence of theological function, the elaborate ciphers of Edward Elgar will be examined for evidence of ostentatious function, and the musical codes and monographic cipher of Pyotr Ilyich Tchaikovsky will be examined for evidence of cathartic function. Following this dense analysis, an informed evaluation of the purpose of musical encryption will be undertaken in the final chapter of this paper ('Conclusion’).

### 3.1 Johann Sebastian Bach: Theology

Christoph Wolff heralded J. S. Bach as a 'creator of works of musical science' (2000). ${ }^{85}$ It is with this sentiment in mind that we return to mathematics, focusing on the religious implications of interconnectivity being evidential of a creator. While the Roman Catholic Mass in B Minor (BWV 232) is often credited as his greatest work, Bach was in fact a devout Lutheran. This is supported by the dedications to 'God Alone’ ('Soli Deo Gloria') that appear beneath many of his works. That being said, the 'social upheaval of the $18{ }^{\text {th }}$ century' makes

[^36]defining Bach's faith difficult; 'the Age of Enlightenment with its...liberal thoughts and conquests of science clashed with religion,,${ }^{86}$ prompting scholarship to question whether the hundreds of cantatas were composed out of faith, occupation, or simply to provide for his 20 children. ${ }^{87}$ It is this juxtaposition between 'musical scien[tist]' and seemingly pious composer that this chapter will explore.

### 3.1.1 Bach Motif As Evidence of Creation

The numerological writings of musical theorist, Andreas Werkmeister were known to the young JS Bach. Indeed, his paper on equal temperament, Musikalische Temperatur (1691), informed Bach's The Well-Tempered Clavier (1722/42). Considering Bach's awareness of Musikalische Temperatur, it is reasonable to suggest that he was familiar with more of Werckmeister's papers. Published four years prior to Musikalische Temperatur, his paper on mathematical music, Musicae Mathematicae Hodegus Curiosus (1687) may have introduced Bach to both numerology, and also an interconnective ideology reminiscent of Johannes Kepler's Musica Universalis. ${ }^{88}$ Referring back to chapter 1, Werkmeister develops Kepler's proposition that the universal persistence of mathematics is evidence of divine creation. It follows that Bach's mathematical approach to composition honoured this creator, satisfying his characterisation as both 'musical scientist' and religious composer.

Using letter-to-number rationalisation as an exemplar ( $a=1, b=2, c=3$, etc.), the crux of Werckmeister's research was the idea that all components of the world could be rationalised as numbers. Beginning with words most significant to himself, Bach may have engaged in letter-to-number rationalisation using his family name, Bach, which translates to $2,1,3,8$. Curiously, these are the correctly ordered digits that comprise the majority of his birthday: 21/03/85 (21 March 1685). Not only had he identified interconnectivity between two sets of numbers unique to him, but these numbers were also beyond his control, universally predetermined.

That Bach noticed this numerological coincidence is evident in the pertinence of other recurring numbers/puzzles throughout his life. Found on Bach's most famous portrait (painted by Elias Gottlob Haussmann in 1746), he is depicted as holding a puzzling manuscript upside down so as onlookers can read the music (Figures $3.1 \& 3.2$ ).

[^37]Depicting Bach's Canon triplex a 6 (BWV 1076), Friedrich Smend (1950) found that there were four basic approaches that splintered into 480 solutions thanks to inversions, mirrored retrogrades, and clef exchanges. ${ }^{89}$ As a biproduct of such in-depth analyses, Smend noticed the prevalence of the number 14 during this portrait analysis. In discussion of Smend's findings, Rumsey summarises that in regard to non-musical allusions to the number, 'there were 14 buttons on Bach's coat in the original portrait, exactly 14 buttons on the second, and Bach appeared to have waited to be the $14^{\text {th }}$ member of the Mizler Society ${ }^{90}$ (a group founded by Lorenz Mizler for those interested in exploring the connections between science and music). In regard to the canon in Bach's hands, 'there are 14 notes between the repeat barlines in the top two voices of the canon, the three canons of the first basic form occur as canons at the $4^{\text {th }}, 5^{\text {th }}$, and $5^{\text {th }}$ respectively, adding up to 14 , and the three canons of the third basic form occur as canons at the $2^{\text {nd }}, 3^{\text {rd }}$, and $9^{\text {th }}$, adding up to 14 again. ${ }^{91}$


Figure 3.1. [LEFT] Portrait of Johann Sebastian Bach (Elias Gottlob Haussmann, 1746).
Figure 3.2. [RIGHT] Enlarged view of Manuscript (Elias Gottlob Haussmann, 1746)

[^38]The significance of 14 is most efficiently demonstrated with a return to Bach being rationalised as $2,1,3,8$ which combine to give $\underline{14}$. As such, the number links to his musical monogram ('Bach motif'); the most famous instance of the motif occurs without transposition or retrograde at bar 193 of the $14^{\text {th }}$ contrapuntus in Die Kunst der Fuge (The Art of Fugue, BWV 1080). While the Motif appears consistently through much of his compositional output, examples of the greatest quality and volume appear in Die Kunst der Fuge through four permutations (Figure 3.3).


Figure 3.3. Bach Motif and its Permutations (Cory Hall, The Bach Scholar, 2017)
In 'Contrapunctus 8', a retrograde motif can be heard in the alto line across bars 4041. Appearing twice in quick succession, the motif is transposed to begin on $G$ and $F$ natural with the two variants overlapping across the second variant's ' C ' substitution, $\mathrm{F} \#$. Also note that while each letter is assigned one quaver, the letter ' C ' substitution is played three times. Four bars later, the motif appears for the first time at original pitch. Rhythmically identical to the previous appearance, the retrograde permutation lies across bars 44-45 in the soprano line, hesitating once more on the ' C ' substitution (this time pitched at a C natural). In 'Contrapunctus 11 ', the motif makes its first appearance facing forward at original pitch. Heard in the alto line, the instigating Bb is situated on the crotchet upbeat to bar 40, heralding the quaver rhythm with a familiar three-quaver hesitation at letter ' C '. As mentioned in the discussion of 14 having significance to Bach, the final instance occurs between bars 193-195 of the incomplete 'Contrapunctus 14 '; the Bach Motif sounds at original pitch with each note substituted as a minim. It is the overt nature of this example that warrants such focus; not only through its elongated duration, but also in its position as the final subject, harmonically isolated in the tenor line.

### 3.1.2 Soli Deo Gloria

Supporting the idea that Bach's encryptions were employed to give evidence of his faith, Hans Heinrich Eggebrecht imagines Bach to be professing, 'I am identified with the tonic, and it is my desire to reach it ... Like you I am human. I am in need of salvation; I am certain in the hope of salvation, and have been saved by grace, ${ }^{92}$ Eggebrecht's sentiment is in response to the motif being used as the final subject in 'Contrapunctus 14'. He suggests that its unaltered nature is a putative expression of Lutheran thought; that having reached the final contrapunctus, Bach offers himself in purest musical form.
'Soli Deo Gloria' (SDG) is a dedication that appears at the foot of many composers' works to signify creation for the sake of God. Used by Bach, Joseph Haydn, George Frideric Handel, and Christoph Graupner, the doctrine demands the exclusion of pride for the just glorification of God. While Bach's use of this dedication is widely acknowledged, I have found that most papers gloss over its use without providing an accurate or verifiable depiction. Many incorrectly attribute the autograph depicted at Figure 3.4 to Bach, when it in fact comes from the holographic manuscript of Christoph Graupner's cantata, Sorget nicht fur den andern Morgen (1726).


Figure 3.4. 'Soli Deo Gloria’ (Christoph Graupner, Sorget nicht fur den andern Morgen, GWV 1156)
Having examined many of Bach's autograph manuscripts, I have come across only two convincing 'Soli Deo Gloria' dedications that appear to be in his hand. The first in the, Matthauspassion BWV 244 (Figure 3.5), and the second in the Mass in B Minor BWV 232 (Figure 3.6).

[^39]

Figure 3.5. [LEFT] Bach 'Soli Deo Gloria' (Johann Sebastian Bach, Matthauspassion BWV 244, Staatsbibliothek Berlin, 1736-40)

Figure 3.6. [RIGHT] Bach 'Soli Deo Gloria' (Johann Sebastian Bach, Great Mass in B Minor BWV 232, Staatsbibliothek Berlin, 1733-49 )

From a graphological standpoint, it is clear that these SDG signatures (Figure 3.5 \& 3.6) are written by the same person; not only is there a consistent calligraphic slant, the ' S ' remains disconnected with ' $D$ ' and ' $G$ ' holding a cursive link across both signatures. In addition to this, there is an identically positioned full-stop following the initialism, perhaps an instinctive physical accentuation of the phrase's importance. These dedications bear great similarity to writing confirmed to be in Bach's hand (Figure 3.7). Having drawn attention to the recurring full stop, one may notice that the ' S ' at the head of both SDG signatures matches the slant and quill pressure of the ' S ' in the heading, Sanctus (Figure 3.7).


Figure 3.7. Sanctus heading from the B Minor Mass (Johann Sebastian Bach, Great Mass in B Minor BWV 232, Staatsbibliothek Berlin, 1733-49)

Bach did not write out the dedication as many of his contemporaries did, instead opting to reduce the phrase to its initials, SDG. Considering his proclivity for encryption and the monographic potential of SDG being substituted (via German notation) as $\mathrm{Eb}, \mathrm{D} \natural$, and G曰, The Art of Fugue was re-examined with this new monographic cipher in mind. As with
the Bach Motif, the presently theoretical SDG encryption is first expanded to illustrate its inversion, retrograde, and retrograde inversion permutations (Figure 3.8).


Figure 3.8. SDG Motif and its Permutations (Cory Hall, The Bach Scholar, 2017)

The search for possible SDG encryptions begins by revisiting the three contrapuncti already established to involve monographic ciphers ( 8,11 , and 14); Bach clearly had encryption on his mind during these compositional moments. When engaging with the specific bar ranges that contain the Bach Motif, the volume of possible SDG substitutions was high with multiple instances of all four of its permutations identifiable in the score (Figure 3.9).

Now visualising the 'Bach Motif' in transposed retrograde across bars 40 and 41, an analysis of the soprano line illuminates two 'SDG Motifs' spanning the same space: the first, a modified variant falling by a semitone before rising a perfect fourth, and the second, an original variant transposed down a perfect fourth. A similar pattern emerges across bars 44 and 45 , this time with the 'SDG Motif' sounding beneath the Bach Motif in the bass line. Although Bach's monogram appears in retrograde, it is important to note that as the $44^{\text {th }}$ bar transitions into the $45^{\text {th }}$, both the SDG and Bach motifs can be heard at original pitch for the first and same time. Their interconnection is also identifiable through equal note durations, both lasting exactly three crotchets. The significance of bar 44 is heightened by numerological considerations. If the name ' BACH ' adds up to 14 , and the initialism 'SDG' adds up to 30 , then Bach $+\mathrm{SDG}=44$; the bar in Contrapunctus 8 where the two motifs are heard at their original pitch for the first time.


Figure 3.9. SDG/BACH Motifs and their Permutations (Johann Sebastian Bach, The Art of Fugue BWV 1080, Contrapunctus 8, B39-46, 1749, Ed. Cory Hall 2017)

Arriving at 'Contrapunctus 11', the Bach Motif is visualised four times; the aforementioned alto example at original pitch on the upbeat to bar 90, and three transposed instances in the alto and tenor lines moving through to bar 91. This passage contains the first example of an inverted SDG motif, transposed to $A$ and $C \neq$, and played directly beneath the Bach Motif on the bass line again.


Figure 3.10. SDG/BACH Motifs and their Permutations (Johann Sebastian Bach, The Art of Fugue BWV 1080, Contrapunctus 11, B89-93, 1749, Ed. Cory Hall 2017)

Returning to Contrapunctus 14, Hall annotates two more Bach motifs in addition to the instance at bar 193 (Figure 3.11). ${ }^{93}$ With the exception of the second motif being transposed up a perfect fifth, the first and third examples appear at their original pitch. Unlike the previous two fragments (Figures 3.9 and 3.10) the three SDG motifs do not sound alongside the Bach motifs, instead appearing transitionally across the $41 / 2$ bar gap approaching the third Bach Motif. On the upbeat to bar 198, the first SDG motif is heard as an untransposed inversion starting on $\mathrm{D} \#$, with the first notes of the second and third instances forming themselves a modified SDG motif; $\mathrm{D} \#$ falls a tone to $\mathrm{C} \#$, before rising a perfect fourth to F .


Figure 3.11. SDG/BACH Motifs and their Permutations (Johann Sebastian Bach, The Art of Fugue BWV 1080, Contrapunctus 14, B193-203, 1749, Ed. Cory Hall 2017)

I might suggest that Bach devised this four-bar development through SDG permutations to symbolise his own salvation, from a transposed Bach Motif beginning at bar 195, to an unaltered Bach Motif beginning at bar 201. As Eggebrecht said of Bach, 'I am identified with the tonic, and it is my desire to reach it ... Like you I am human. I am in need of salvation; I am certain in the hope of salvation, and have been saved by grace', ${ }^{94}$ Bach illustrates that sentiment here, his signature made pure through the musical representation of ‘God Alone’ (SDG Motif).

It is important to note that the SDG Motif does not appear to be tied to the use of the Bach Motif. While previous fragments may indicate the contrary, the first subject of

[^40]'Contrapunctus 8 ' is comprised of both modified and original SDG motifs. Furthermore, the opening six notes of 'Contrapunctus 10 ' consist of another two variants, one inverted and one transposed. Indeed, the entire fugue of 'Contrapunctus 10 ' is based on these two permutations. The motif is not a rarity; variants are identifiable in both sacred and secular works from the Coffee Cantata (BWV 211) to the second beat of the first bar of the Concerto for Two Violins (BWV 1043) in transposed retrograde inversion.

### 3.2 Edward Elgar: Ostentation

Studying Elgar in relation to cryptology is certainly not novel; his interest in the subject has long been an open secret. However, such examples as the 'Dorabella Cipher' and Enigma Variations require solutions that operate at a level of complexity as yet unexplored in this paper. Elgar contradicts the nature of secret-writing, frequently referencing his encryptions in programme notes, interviews, letters, and memoirs. The Allegretto for piano and violin (1888) was composed using the monographic cipher, GEDGE representing the surname of two pupils from his time teaching in Great Malvern. His daughter's name, Carice, is an amalgamation of his wife's names, Caroline and Alice. And the epithet 'Satanford' can be found monographically within the 'Demon's Chorus' from The Dream of Gerontius - a subtle jab at Charles Villiers Stanford. While warranting their own analysis, none of these examples were 'discovered' like the SDG motif; Elgar has alluded to them all.

### 3.2.1 Dorabella Cipher

In July 1897, Elgar travelled to Wolverhampton, visiting the Pennys. Despite the trip's brevity, a friendship developed between Edward and Dora Penny, to whom Elgar dedicated the tenth movement of his Enigma Variations. Upon his return to Great Malvern, Alice Elgar (Edward's mother) sent a letter of thanks, beside which Edward placed a small note addressed to 'Miss Penny'. Known now as the 'Dorabella Cipher' (Figure 3.12), this seemingly inconsequential note has left a thorn in the side of cryptographers for almost 100 years. Consisting of 87 small glyphs, the cipher-text is spread across three lines with 24 unique symbols, each containing 1,2 , or 3 small hoops. The glyphs are oriented on one of eight different axes, best visualised as the eight points on a compass.

$$
\begin{aligned}
& \text { ThC14. } 97
\end{aligned}
$$

Figure 3.12. Facsimile of the Dorabella Cipher (Edward Elgar/Dora Penny, Letter to Dora Penny, 1937)
Having received Elgar's note in 1897, Dora Penny was unable to decipher the message and set it aside for almost fifty years, after which it was reproduced in her memoir, Edward Elgar: Memories of a Variation (1947). ${ }^{95}$ The original fragment is yet to be found, understandably raising questions of authenticity. Elgar is known to have used the cipher on three other occasions: first, along the left-hand margin of a Liszt programme from 1886 (Figure 3.13); second, sketched onto an undated card with the word 'cryptogram' etched at its centre (Figure 3.14); and third, in one of Elgar's notebooks that seems to detail the supposed alphabetic cipher key (Figure 3.15).


Figure 3.13. The Liszt Fragment with magnified and rotated cipher beneath (Edward Elgar, 1886)

[^41]

Figure 3．14．The Cryptogram Card（Edward Elgar circa 1840－1900）


Figure 3．15．Cipher pages from Elgar＇s Notebook with＇Marco Elgar＇translation added（Edward Elgar circa 1840－1900）

Figure 3.15 may appear strange；having just professed the Dorabella Cipher as unbreakable，we now learn that a cipher－key exists with correct decryptions of＇Do you go to London？＇and＇Marco Elgar＇（Elgar＇s dog）translated alongside it．However，decrypting the Dorabella Cipher with this key poses more questions than it answers：the resulting plaintext is a muddle that fails to construct words of any meaning（Figure 3．16）．Similar nonsensical plaintexts are produced when the Liszt fragment and Cryptogram Card glyphs are decrypted．

# BLTACEIARWUNISNFNNELLHSYWYDUO 

## INIEYARQATNNTEDMINUNEHOMSYRRYUO

## TOEHO'TSHGDOTNEHMOSALDOEADYA

Figure 3.16. Dorabella Cipher decrypted using the key from Elgar's notebook.
Having identified this peculiarity, solutions branch out into one of two approaches, alphabetic or musical. By far the more popular route, alphabetic entries seek to establish an algorithm that makes lexical sense of Elgar's glyphs. However, as Sams discovered, the frequency and position of the symbols cause one to be 'daunted by the fear that the solution itself may well be odd, even nonsensical'. ${ }^{96}$ Having exhausted frequency counts, contact charts, and brute force, Sams established the value of a few commonly occurring glyphs. Appealing to Elgarian oddities, Sams proposed that the remaining glyphs be substituted phonetically, making use of Greek letters (such as pi $[\pi]=$ ' $P$ '), solfège, and sounds as frequently used by Elgar in correspondence with friends: 'bung yirds' = young birds, or 'warbling wigorously in Worcester wunce a week' ${ }^{97}$ A decidedly unusual table, Sams' cipher-key (Figure 3.17) produces the plaintext:
'STARTS: LARKS! IT'S CHAOTIC, BUT A CLOACK OBSCURES MY NEW LETTERS A, B (alpha, beta?) BELOW: I WON THE DARK MAKES E. E. SIGH WHEN YOU ARE TOO LONG GONE.


Figure 3.17. Eric Sams' Cipher-Key for the Dorabella Cipher (Eric Sams, 'Elgar's letter to Dorabella', The Musical Times, 1970)

[^42]```
MESSAGE
\begin{tabular}{|l|l|l|l|l|l|l|l|l|:|c|l|l|l|l|l|l|l|l|l|l|l|l|l|}
\hline\(S\) & \(T\) & \(A\) & \(R\) & \(T\) & \(S\) & \(L\) & \(A R K S\) & 1 & \(T\) & \(S\) & \(C H\) & \(A\) & \(O\) & \(T\) & \(I\) & \(C\) & \(B\) & \(U\) & \(T\) & \(A\) & \(C\) & \(L\) \\
\hline
\end{tabular}
```









Figure 3.18. Eric Sams' Dorabella Cipher plaintext table (Eric Sams, 'Elgar's letter to Dorabella', The Musical Times, 1970)

Tim Roberts disputes Sams on the grounds that his solution contains too many letters; the Dorabella Cipher contains 87 glyphs and Sams' decryption is made up of 109 letters. Roberts' simple $1: 1$ alphabetic cipher-key was devised in response to the query: 'why would Elgar...send such a [complex cipher] to a young lady whom he had met on only a few occasions, and who apparently had no interest in puzzles or ciphers? ${ }^{98}$ Looking back over the limited correspondence between Elgar and Dora Penny, Roberts identifies a letter on the subject of gardening that could function as a cipher-key. While tenuous, applying the Dorabella Cipher does produces a semi-coherent plaintext relating to the theme of gardening:
'P.S. NOW DROOP BEIGE WEEDS SET IN IT - PURE IDIOCY - ONE ENTIRE BED! LUIGI CCIBUNUD LOVINGLY TUNED LIUTO STIDIO TWO.’

## PSNOWDROCPBEIGEWEEDSSETINITBU

 REIDIOCYONEENDTIREBEDLUIGICCIBU
## NUDLVNGLYTUNEDLIUTOSTUDOTWO

Figure 3.19. Roberts’ Dorabella Cipher decryption (Tim Roberts, Solving the Dorabella Cipher, 2012)

As with Sams' solution, Roberts has taken some creative liberties in reaching his plaintext from the decryption (Figure 3.19); the ' c ' in 'now drocp' has been changed to an ' $o$ ', the ' $b$ ' in 'bure idiocy' has been altered to a ' $p$ ', and 'luvngly' has been quite generously

[^43]attributed as 'lovingly'. It is safe to say that the plausibility of both Sams' and Roberts' solutions rely heavily on Elgar's use of phonetics in written correspondence (e.g. 'bung yirds'). As two of the more coherent alphabetic propositions, neither are wildly convincing; the sheer difference between the two propositions means at least one of them has got it catastrophically wrong.

As one may expect, musical solutions have gained traction with the every-growing incoherence of new alphabetic propositions. Indeed, these musical suggestions seem to stand in good stead; the 24 glyphs match the 24 notes in a two octave chromatic scale, the Liszt fragment (Figure 3.13) contains 18 glyphs positioned next to an 18 note melody, and I might suggest that the glyphs are reminiscent of shorthand musical notation used by the likes of Bach, a fellow musical cryptographer. However, the most complex proposition belongs to Javier Atance who suggests the construction of a compass-like cipher-key (similar to that found in Elgar's notebook, Figure 3.15) whereby the 8 orientations denote the 8 notes in an octave, and the three hooped glyph variants represent the accidentals: $b=$ one hoop, $\mathfrak{a}=$ two hoops, and \# = three hoops (Figure 3.20).


Figure 3.20. Speculative construction of Atance's Musical Cipher Key (Benjamin Thompson, 2023)

Having been unable to find Atance's original key or research, Figure 3.20 depicts my own construction of the cipher-key using written acknowledgments from Robert Padgett and Nick Pelling as reference, ${ }^{99}$ it is therefore unlikely that my substitution of the accidentals and pitch degrees matches Atance's. Furthermore, the design requires repetition of one scale degree. Alphabetically, I chose 'A', although Atance will likely have identified a different letter. That being said, the general idea is preserved.


Figure 3.21. Plaintext melody using speculative Atance Musical Cipher Key (Benjamin Thompson, 2023)

While the sporadic use of accidentals is musically peculiar (Figure 3.21), substitutive alterations to the cipher-key will produce a variety of melodies, one of which one will be Atance's proposed solution. While the plaintext melody contains no rhythmic variation, this might suggest that Elgar expected the melody to be recognised by Dora Penny through pitch alone. Referring to the Liszt fragment and their resemblance to Bach's shorthand organ tablature of his Fantasia in C minor (Figure 3.22), it seems clear that the glyphs must bear some relation to shorthand musical notation.

[^44]

Figure 3.22. JS Bach Shorthand Organ Tablature (Johann Sebastian Bach, Fantasia in C minor, BWV $1121,1728-30)$

Of course, the Dorabella Cipher could in fact have been a joke, a gag that grew out of hand when Elgar refused to tell Dora the truth. The glyphs found on the Liszt fragment and Cryptogram Card may well be genuine attempts at encryption by Elgar, but there is no evidence to suggest that these scribbles were functional. ${ }^{100}$ I propose that the Dorabella Cipher was a note written in the spur of the moment, following a suggestion from Alice Elgar that Edward add to the letter of thanks she was sending the Pennys. Chuckling to himself, perhaps, Elgar plausibly scribbled the same, meaningless symbols that appear on the Liszt fragment, before signing and handing it to Alice to be sent. Supported by the consistent disagreement that permeates all proposed solutions, the impossibility of the Dorabella Cipher is summarised best by Nick Pelling in his community blog dedicated to solving the Cipher, 'Cipher Mysteries': 'the mystery... is neither a whodunnit (because Elgar signed and dated it), nor a howdunnit (because it seems that we already have the cipher-key), but more like a

[^45]whodunwhat - though we can apparently decipher its text, we don't know what it means, or even how to try and read it'. ${ }^{101}$

### 3.2.2 Enigma Variations - A 'Dark Saying'

In June 1899, Elgar's Variations on an Original Theme, Op. 36, was premiered at St. James’ Hall in London. The performance of the piece, later popularly known as the Enigma Variations, was accompanied by a cryptic programme note written by Elgar himself:

It is true that I have sketched for their amusement and mine, the idiosyncrasies of fourteen of my friends, not necessarily musicians; but this is a personal matter, and need not have been mentioned publicly. The Variations should stand simply as a "piece" of music. The Enigma I will not explain, its "dark saying" must be left unguessed, and I warn you that the connexion between the Variations and the Theme is often of the slightest texture; further, through and over the whole set another and larger theme "goes" but is not played ... So the principal Theme never appears, even as in some late dramas e.g., Maeterlinck's L'Intruse and Les sept Princesses, the chief character is never on the stage. ${ }^{102}$

If the programme is to be believed, Elgar is suggesting that the theme from the Enigma Variations functions as counterpoint to another 'principal theme' - a melody he would later describe to Troyte Griffith (portrayed in Variation VII) as 'so well known...it is extraordinary no one has spotted it'. Though Elgar took the solution to his grave, interviewers and biographers were able to glean some key additional information in the 34 years between the work's publication and Elgar's passing. In addition to the programme note, the interview with The Musical Times (1900), ${ }^{103}$ Robert J. Buckley's Elgar biography (1905), ${ }^{104}$ and descriptive pianola notes (1929) form the four primary sources used most frequently in any analysis of the Enigma Variations. Examining these references, Robert Padgett has complied a series of six conditions that Enigma solutions must follow to be considered valid. ${ }^{105}$

[^46]1. 'The Enigma Theme is counterpoint to the Principal Theme.'
2. 'The Principal Theme is not heard.'
3. 'The Principal Theme is famous.'
4. 'Fragments of the Principal Theme are present in the Variations.'
5. 'The Principal Theme is a melody that can be played "through and over" the whole set of variations including the entire Enigma Theme.'
6. 'The Enigma Theme comprises measures 1 through to 19.'

The work was dedicated by Elgar 'to my friends pictured within', each identified by their initials. Referring to Figure 3.24, this naming system is broken on five occasions: Variation VI (Ysobel), Variation VII (Troyte), Variation IX (Nimrod), Variation X (Dorabella), and Variation XIII (Romanza ***). 'Ysobel', 'Troyte', and 'Dorabella’ are simply nicknames and warrant little analytical attention; however, 'Nimrod' and 'Romanza ***' are far more interesting. Surely the most famous variation, 'Nimrod' was composed for Elgar's close friend, August Johannes Jaeger, who wrote a letter encouraging Elgar to persist with composition, citing Beethoven's perseverance in the face of similar self-doubt. Jaeger was German, and in German, Jaeger translates as 'hunter'. Elgar titled Jaeger's variation 'Nimrod' after the 'mighty hunter' mentioned in the Bible (Genesis: 10). Furthermore, the second movement of Beethoven's Pathétique Sonata (Piano Sonata No. 8 in C Minor, Op.13) bears great harmonic resemblance to 'Nimrod', with some even proposing it as the Enigma melody.


Figure 3.23. Enigma Theme below the melody from Beethoven's Pathétique Mvt. II (Edward Elgar, Variations on a Theme, 1899, and Ludwig van Beethoven, Pathetique Sonata, Op.13, 1798)

| Variation number | Heading | Interpretation | Cue (first bar of each variation) | Bars | Ends fermata or attacta |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Theme | Enigma |  |  | 1-17 | bars 18-19 are a link |
| I | (C.A.E.) | Caroline Alice Elgar, the composer's wife | 2 | 20-40 | fermata |
| II | (H.D.S-P.) | Hew David Steuart-Powell, amateur pianist | 5 | 41-96 | fermata |
| III | (R.B.T.) | Richard Baxter Townshend, scholar, author, eccentric | 8 | 97-131 | fermata |
| IV | (W.M.B.) | William Meath Baker, 'squire' of Hasfield Court | 11 | 132-163 | fermata |
| V | (R.P.A.) | Richard Penrose Arnold, son of Matthew Arnold | 15 | 164-187 | attacca |
| VI | (Ysobel) | Isabel Fitton, amateur viola player | 19 | 188-209 | fermata |
| VII | (Troyte) | Arthur Troyte Griffith, artist and architect | 23 | 210-280 | fermata |
| VIII | (W.N.) | Winifred Norbury, secretary, Worcestershire Philharmonic Society | 30 | 281-307 | attacca |
| IX | (Nimrod) | August Johannes Jaeger, of Novello's | 33 | 308-350 | fermata |
| X | (Dorabella) Intermezzo | Dora Penny (later Mrs Richard Powell) | 38 | 351-424 | fermata |
| XI | (G.R.S.) | George Robertson Sinclair, organist at Hereford, owner of Dan, a bulldog | 47 | 425-464 | fermata |
| XII | (B.G.N.) | Basil Nevinson, amateur cellist | 52 | 465-492 | attacca |
| XIII | $\begin{aligned} & (* * *) \\ & \text { Romanza } \end{aligned}$ | Lady Mary Lygon (later Trefusis), of Madresfield Court | 55 | 493-543 | fermata |
| XIV | $\begin{aligned} & \text { (E.D.U.) } \\ & \text { Finale } \end{aligned}$ | Edu=Elgar himself | 61 (to 83) | 544-780 | fine |

Figure 3.24. Variation No., Initialism, Interpretation, and Cue Bars of each variation (Julian Rushton, Elgar: ‘Enigma’ Variations, Cambridge University Press, 1999)

While its cryptic presentation divides opinion, the thirteenth variation, 'Romanza ***' is widely believed to represent Lady Mary Lygon, a dear friend and promoter of Elgar's music. In addition to early sketches of Variation 13 being titled 'L.M.L', Elgar explains the lack of an initialism to be as a consequence of the 'pretty lady' being on a sea-voyage at the time of composition. ${ }^{106}$ This is supported by Elgar's inclusion of musical quotations from Mendelssohn's Calm Sea and Prosperous Voyage, with the noisy, rhythmic motifs imitating boat's engines.

Despite Elgar's reference to the puzzle, it was in Jaeger's hand that the word 'Enigma' first appears above the theme on the holographic manuscript. This is not to suggest that the enigma follows such trickery as that proposed in discussion of the Dorabella Cipher; one was a private letter designed to be read by a friend, and the other is a well-documented, large scale musical work that Elgar is on record to have enciphered. Depicted in Figure 3.25, the theme is dark in nature, indecisively swaying between major and minor tonalities, unsettled and vast in intervallic range. The enigma is decidedly simple; above this theme, 'another [well known] theme goes but is not played' ${ }^{107}$ - what is this famous theme? Could it be Mendelsohn, who we have already identified as being quoted in Variation XIII (satisfying Padgett's fourth condition), or Robert Schumann, who Elgar is known to have cryptographically idolised; many of his ciphers are reworkings of those devised by Schumann?

[^47]

Figure 3.25. Piano reduction of the 'Theme' from 'Enigma Variations' (Edward Elgar, Variations on a Theme, 1899)

Dora Penny's husband, Richard C. Powell, proposed a solution which is interesting for a number of reasons. First, it is the oldest detailed proposition I have come across, published in 1934 but written a decade and a half earlier in 1920. Powell begins by establishing a few key details in regard to the 'friends depicted within'. It is highly unlikely that the enigmatic tune was 'drawn from the classics [western canon] or the church'; the majority of friends depicted in the variations were amateur musicians of different religions and backgrounds. Powell was drawn towards traditional songs; tunes that everyone of all musical and theological backgrounds would know of. In search of a traditional song well known to Englishmen, and with thematic roots in friendship, one need look no further than 'Auld Lang Syne'.

At first glance, ‘Auld Lang Syne’ seems implausible as a source; while sharing a G tonal centre, it is in G major while the enigma theme is in the minor. Powell observes that altering the tune to a minor tonality has very little effect on its melodic content; the harmonic spelling is unchanged thus the melody appears identical on the page. The enigma theme oscillates between major and minor passages in accordance with its A-B-A structure; it is G minor until bar 7 (A section), G major until bar 11 ( B section), and then returns G minor until the end (A section repeated). Given that 'Auld Lang Syne' is in an A-B structure, Powell manipulates its tonality to mirror that of the enigma; the A section is played in G minor (rather than G major), the B section is in its unaltered G major, and then the A section is repeated in G minor.

Note that the 'Auld Lang Syne' melody has been written in double time, appearing in $2 / 4$ over the $4 / 4$ enigma theme (Figure 3.26). Powell's solution is often criticised as without this doubling, the solution is completely incoherent. However, I would argue that this adjustment is entirely reasonable; the enigma theme's tempo is notated at crotchet $=60$, so while written at double speed, 'Auld Lang Syne' would sound at regular speed.


Figure 3.26. Piano reduction of the 'Theme' from 'Enigma Variations' with 'Auld Lang Syne' superimposed above (Edward Elgar, Variations on a Theme, 1899)

The resulting music is in parts convincing, especially the aligned $B$ sections starting at bar 7 which last four bars, feeling as though they were composed in tandem. Aside from the questionable tonal and tempo alterations made to 'Auld Land Syne', it is the mismatched melody lengths that strike me as a more apparent shortcoming; the A section of 'Auld Lang Syne' stops after four bars while the enigma theme continues for another two. Powell suggests that Elgar 'found it impossible to hide Auld Lang Syne...[when] he made all three sections of his theme the right length, and so... extend[ed] the first section and its repetition'. This reasoning is weak, though I am not sure he need justify the two-bar gaps; Padgett's sixth
condition is that 'the enigma theme [should] comprise measures 1 through to 19 ', ${ }^{108}$ not that the theme be played consistently throughout. Unfortunately for Powell, Elgar flatly denied that 'Auld Lang Syne' was the hidden melody despite Dora's persistence in suggesting it to him. Though there is little evidence to support this suggestion, I might propose that in line with his character, Elgar denied 'Auld Lang Syne' as the enigma theme through irritation that Powell had cracked it.

Of the hundreds of proposed musical solutions, few make it as far as Powell's. In 1976, Theodore van Houten proposed ‘Rule, Britannia!' as a solution following his observation that the first five scale degrees of the Enigma Theme are identical to those found in the chorus where 'never, never, never' is sung. ${ }^{109} \mathrm{He}$ suggests that 'the principal theme never appears', should actually be read, 'the principal theme, never, appears', signposting the chorus of ‘Rule, Britannia!' In 1977, James M. Nosworthy proposed the enigma to be related to the slow movement of Mozart's Prague Symphony (Symphony No. 38 in D major, K. 504)..$^{110} \mathrm{He}$ does not suggest that the Mozart functions as a primary melody, just that Elgar was influenced by it; the first five scale degrees are the same in both works, the Prague Symphony was played at the 1988 premiere of the variations, and Elgar is 'well-documented to have love[d]... Mozart'. ${ }^{111}$ In 1999, Patrick Turner (rather bravely) proposed 'Twinkle, Twinkle Little Star' as the enigma theme; it compellingly addresses Elgar's 'dark saying' and is certainly 'well-known'. ${ }^{12}$ However, its substitution alongside the theme is disastrous; while descriptively valid, the awkward musical fit and nonsensical counterpoint render it useless in application. Denis Stevens writes, 'if you have a friend who prides [them]self on [their] musical knowledge, try this one: "tell me the solution of Elgar's Enigma - and no wild guesses like "God Save the Queen", or a Mozart symphony, a Brahms sextet, or (heaven forfend) "Twinkle, Twinkle, Little Star"". ${ }^{113}$

Despite Dora Penny (Powell) stating categorically that 'Elgar told me personally more than once that the enigma concerned another tune', ${ }^{114}$ the next solution proposes that the Enigma is not a tune, but rather the mathematical ratio for the circumference of any circle to

[^48]its diameter, pi. Charles and Matthew Santa suggest that 'it is the concept of the circle that brings together $p i$, the structure of the variations, and the dedication'. ${ }^{115}$ Indeed, Elgar is known to have used to term 'circle' in conjuncture with his friends. Writing to Mrs. Sidney Colvin in 1911, Elgar pens, 'our little life's circle' and 'the charmed circle' in reference to his friends, Pietro d'Alba and George Bernard Shaw. In conversation with Clare Wortley (ten years before the variations were published), Elgar was introduced to the phrase 'a little circle of constant friends, ${ }^{116}$ Elgar recognised a level of synonymity between his friendship group and a circle, an awareness that may have been heightened in 1897 (the year before his composition of the variations) when the infamous Indiana Pi Bill entered the news.

Santa and Santa propose that the Enigma theme contains two separate encipherments of Pi. Given that the theme is in G minor, G is assumed to represent the number 1. Assigning numbers to the scale degrees of the opening four pitches gives the values, 3-1-4-2 or 'the decimal approximation of Pi '. The second musical representation comes as a result of Elgar's comment that 'the drop of the seventh should be observed in bars 3 and 4'. Santa and Santa note that there are 'two drops of the seventh', and 'exactly 11 notes [played] before'. To rephrase their point, eleven notes times two sevenths, or $11 \times 2 / 7=22 / 7$ ', 'the fractional representation of $P{ }^{\prime} .{ }^{117}$

scale degrees: 3-1-4-2 $(3.142=P i)$
Figure 3.27. Musical representations of Pi in the Enigma Theme. (Santa and Santa, Solving Elgar's Enigma, 2010)

Given that the enigma theme reappears consistently across all of the variations, it can be stated that the Pi solution, whilst not a tune, can definitely be classed as a 'theme that "goes" but is not played'. ${ }^{118}$ Referring to Dora Penny's reports that Elgar would often 'play all sorts of amusing things; old songs, [and] nursery rhymes', ${ }^{119}$ Santa and Santa propose that the missing 'dark saying' can be found in the nursery rhyme, 'Sing a Song of Sixpence'. The lyrics 'black' and 'pie' in the lines 'four and twenty blackbirds, baked in a pie' fulfil this 'dark' requirement whilst providing evidence that $P i$ is the enigma theme. I find this hard to

[^49]believe, but admit that I enjoy the suggestion that 'theme' might be taken in its literary sense. Rationalising 'theme' in this manner opens the door to a variety of possible solutions that are far more codified that encrypted, an encouragement to attribute meaning to the enigma, not glean meaning from it.

Elgar quotes a Mendelsohn fragment in Variation XIII (Romanza '****').
Consequentially, Padgett suggests that Elgar's hidden theme may appear in a comparably quoted context within one of Mendelsohn's symphonies as a basis for a set of variations. Identifying the four instances of quotation in Variation XIII, Padgett assumes that the covert theme should appear in the fourth movement of said symphony. Such a melody can be found in the final movement of Mendelsohn's Fifth Symphony, where the hymn, Ein Feste Burg (Martin Luther, 1529) is stated and followed by a series of variations. Expanding the hymn to its full title, Ein Feste Burg ist unser Gott (A Mighty Fortress is Our God), Padgett used frequency analysis to establish that there are six unique bass and melody notes. Applying the alphabet to a $6 \times 6$ grid, the melody and bass notes function as x and y coordinates on a Polybius square. Having worked out the corresponding letter frequencies of Ein Feste Burg ist unser Gott, Padgett assigned the most commonly occurring instances to each other. These pairings have been filled out on the Polybius Square Cipher at Figure 3.28.


Figure. 3.28. Padgett's Polybius Square (Robert Padgett, Elgar's Enigma: A Musical Polybius Box Cipher, 2019)

To refer back to Sams, it was understood that Elgar enjoyed using phonetic spellings in letters and messages to close friends. With this in mind, each of the plaintext decryptions for bars 1-6 can be rationalised as phonetic spellings of words. GSUS $=$ Jesus, GRTS $=$ the Latin word Gratis, INOU = 'I know you', BETR = 'Better', TENI = the Aramaic word 'Teni',
and FETE $=$ the English word 'Fete' (Figure 3.29). Employing the use of four different languages, Padgett suggests that Elgar has constructed an acrostic anagram of his name; English, Latin, German, $\underline{\text { Aramaic }}=$ ELGAr (Elgar). Most impressively however, the plaintext solution, GSUSGRTSINOUBETRTENIFETE is a perfect anagram of the proposed hidden theme, EIN FESTE BURG IST UNSER GOTT (Ein feste Burg ist Unser Gott).

| Measure | Melody | Bass | Plaintext Solution |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Rest | G | G <br> S <br> U <br> S | $\text { Rhythmic Palindrome } 1$ |
|  | B-flat | (G) |  |  |
|  | G | (G) |  |  |
|  | C | A |  |  |
|  | A | (A) |  |  |
| 2 | Rest | B-flat |  |  |
|  | D | (B-flat) | G |  |
|  | B-flat | C | R |  |
|  | A | (C) | T |  |
|  | C | (C) | S |  |
| $3$ | Rest | D |  |  |
|  | B-flat | (D) | I |  |
|  | D | (D) | N |  |
|  | G | E-flat | O |  |
|  | A | (E-flat) | U |  |
| 4 | Rest | B |  |  |
|  | F | B | B |  |
|  | G | C | E |  |
|  | A | (C) | T |  |
|  | B-flat | (C) | R |  |
| $5$ | Rest | C-sharp |  | $\begin{gathered} \text { 冗 } \\ \text { 元 } \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |
|  | A | (C-sharp) | T |  |
|  | G | (C-sharp) | E |  |
|  | D | D | N |  |
|  | B-flat | (D) | I |  |
| 6 | Rest | E-flat |  |  |
|  | B-flat | E-flat | F |  |
|  | G | C | E |  |
|  | A | C | T |  |
|  | G | C | E |  |

Figure 3.29. Padgett's Decryption Table (Robert Padgett, Elgar's Enigma: A Musical Polybius Box Cipher, 2019)

### 3.3 Pyotr Ilyich Tchaikovsky: Catharsis

Similarly to Elgar, Tchaikovsky engaged with cryptography away from music, using literary codes in diaries and letters to express the innermost aspects of his character. As stated by his brother, Modest, Tchaikovsky wrote diary entries consistently from 'the end of the 1870s to the later 1880 s and intermittently into the early 1890 s , ${ }^{120} \mathrm{He}$ would burn the majority of these in 1891, with only a few entries from 1884 surviving. Covertly detailing Tchaikovsky's self-hatred as a result of his various homosexual activities, he writes ' $Z$ is really tormenting me. May God forgive me such despicable feelings!', and 'tormented tonight not by Z itself,

[^50]but by the torment that exists within me'. ${ }^{121}$ David Brown suggests that the symbols, ' X ' and ' Y ' are representative of Tchaikovsky's 'homosexual drives'. ${ }^{122}$ Anthony Holden develops this theory, stating that ' $Z$ would appear to represent the sex-drive..., and far less frequent $X$ [Tchaikovsky's] guilt at his methods of relieving it'. ${ }^{123}$

Tchaikovsky's poems are full of literary codes. Alexander Poznansky alludes to the 'sexual undercurrents [that] subtly affect', Landyshi ('Lilies of the Valley'). ${ }^{124}$ The title is linguistically masculine, meaning that all references to 'yearning' and 'longing' are governed by the masculine pronoun. Poznansky suggests that 'it is not a "she" but a "he" that, like a flowering wine, warms and intoxicates me, like music, takes my breath away, and like a flame of love, suffuses my burning cheeks'. ${ }^{125}$ This sentiment certainly adds poignance to the brevity of the poem; 'it is likely that Tchaikovsky saw the lovely flower as a symbol of a young man's fleeting beauty'. ${ }^{126}$ These themes of brevity, longing, lust, and loneliness are regularly codified in all of Tchaikovsky's song output.

### 3.3.1 Fate Motif

Rudeforth examines Tchaikovsky's potential musical encryptions, focusing specifically on the theme of fate and its reflection in his music. Discussing the opera, Eugene Onegin, Rudeforth presents a six-note motif that 'acquired its association with fate' through the accompanying lyrics sung by Tatiana: 'Who are you: my angel and my keeper, or are you an insidious tempter?' (Figure 3.30). ${ }^{127}$ Resurfacing later in the opera, the same six-note melodic shape can be heard as Lensky muses upon his own fate in the 'farewell aria', shortly followed by his death in the 'coming day' (Figure 3.31).


Figure 3.30. Fate motif, Tatiana (Pyotr Ilyich Tchaikovsky, Eugene Onegin, 1878)

[^51]

Figure 3.31. Fate motif, Lensky (Pyotr Ilyich Tchaikovsky, Eugene Onegin, 1878)
As Lensky's passing in Eugene Onegin suggests, the Fate motif is also synonymous with themes of death. Tchaikovsky's opera, The Queen of Spades (1890) first ties the two ideas together when the motif accompanies the lyrics; 'you will die when a third man, driven on by despair, will strive to tear from your heart the secret of the three cards!' (Figure 3.32).


Figure 3.32. Fate motif, 'Three Card’ Theme (Pyotr Ilyich Tchaikovsky, The Queen of Spades, 1890)
This notion of fate and death play a key role in his Sixth Symphony, the premiere of which was conducted by Tchaikovsky nine days before his own passing. The motif culminates in the final movement, forming the central melodic theme (Figure 3.33). Temirkanov links the descending six-note motif with the 'traditional death knell sounded in Russia..., when somebody died, the local church bells would ring out in a pattern...from highest note by step to the lowest'. ${ }^{128}$ Temirkanov goes on to describe the finale of the Sixth Symphony as 'reeking of death, as its descending passages reach literally towards the grave'. Depicted in Figure 3.34, the Fate motif can be heard in the upper cello line.


Figure 3.33. Fate motif, 'Symphony No.6', Opening 4-bars of the Finale derived from interplay between violin parts (Pyotr Ilyich Tchaikovsky, Symphony No. 6 in B minor, 1893)

[^52]

Figure 3.34. Fate motif, 'Symphony No.6', last 11-bars of the Cello \& Bass parts in the final movement. (Pyotr Ilyich Tchaikovsky, Symphony No. 6 in B minor, 1893)

These examples make up but a few of the numerous instances Tchaikovsky employs the motif. The validity of this this musical code is evident through the contextual consistency in which it appears; when the motif sounds alongside a text, 'the text invariably speaks of a fateful situation'. ${ }^{129}$ The same can be said of passages that do not involve text; Tchaikovsky's orchestration of the motif has a dependably sorrowful nature that while versatile, rarely fails to elicit a tone of melancholy. This code feels decidedly personal, perhaps a musical carnation of the ' $\mathrm{X} \& \mathrm{Y}$ ' literary code, born of Tchaikovsky's tragic 'self-hatred'.

### 3.3.2 Artot Contour

Tchaikovsky is also said to have experimented with monographic ciphers. Known as the 'Artot Contour', the short motif is suggested to encipher the name of the soprano, Desiree Artot, with whom Tchaikovsky became acquainted in 1868. It is clear from letters to Modest that he took a liking to the soprano: 'I have been devoting all my free time to one person...whom I love very, very much'. ${ }^{130}$ Rudeforth alludes to this infatuation through a

[^53]consistent monographic cipher that appears in the symphonic poem Fatum (1868), the First Piano Concerto (1874-5), and the Third Symphony (1875).

The contour appears in two forms: first, as a six-note monographic cipher using English, French, and German notation to spell 'Desiree A' (Figure 3.35), and second, as a two note motif spanning a diminished fourth (from Db to A ) to give the initialism ' D A' (Figure 3.36). Having already examined German notation, a French scale (starting on C) is notated as such: 'do, re, mi, fa, sol, la, si'. With that in mind, the six-note cipher is
 (English ' E '), and A (English 'A').


Figure 3.35. [Above] Artot Contour substituted in full (Pyotr Ilyich Tchaikovsky)
Figure 3.36. [Below] Artot Contour as an initialism (Pyotr Ilyich Tchaikovsky)

Despite Brown's statement that the 'cipher-generated motifs...[are] far too inconclusive...to be included in the main text', ${ }^{131}$ the example presented in the First Piano Concerto is decidedly convincing (Figure 3.38). However, beginning with the symphonic poem, Fatum, Brown proposes that the opening melodic idea played by the strings in bar 1, is a transposition of the final four notes of the Artot contour, B-D-E-A, retaining its shape as, A-C-D-G (Figure 3.37).

[^54]

Figure 3.37. Transposed Artot Contour (Pyotr Ilyich Tchaikovsky, Fatum, 1868)
Brown's tentativeness with this example is understandable; given that this is the opening of the work, Tchaikovsky could have just notated the cipher at pitch, writing the piece in A minor rather than G minor. Furthermore, the contour is missing its first two values. Moving to the First Piano Concerto, the contour is used as a harmonic progression that unites the opening passage with the beginning of the second subject. The theme starts with the enciphered initialism, Db to A ( (Des. A) before progressing though a melody that bears great resemblance to the long-form cipher (Figure 3.38).


Figure 3.38. Artot contour at the start of the second subject (Pyotr Ilyich Tchaikovsky, Piano Concerto No. 1, 1875-75)

The Artot contour, whilst feasible, lacks that cutting evidential edge that is so important to the validity of monographic encryptions. Brown goes on to suggest several more monographic ciphers, though his tone is so tentative he need not bother. ${ }^{132}$ When examining Tchaikovsky with a view to unveiling encryptions, I am certain that the most fruitful analysis will come with a focus on codes, not ciphers.

[^55]
## CHAPTER 4: CONCLUSION

In light of the examination of Bach, Elgar, and Tchaikovsky's cryptographs, I feel it is clear that their rationales for encryption are different. Looking first to Bach, I believe that the utilisation of monographic ciphers was intended to deepen his theological beliefs. From the recitation of Buddhist chants and Christian hymns to Islamic prayer and the improvisational framework of the Hindu raga, religious ceremony has been long been defined by its musical content. I propose that by engaging with musical encryption, Bach's music was not only religiously dedicated, but also had religion at the heart of the compositional process. Bach's SDG Motif is evidence of this; while examination of his autographed scores produced religious dedications in the form of S.D.G. initialisms, further analysis surfaced a consistent monographic cipher within the music, elevating the work from being in dedication to God, to being symbolic of God. This proposition is developed when considered in conjuncture with Kepler's Musica Universalis, or more specifically, the notion that music 'has [its] origins in the alignment of the heavenly bodies', with 'harmonic order... only [ever] mimicked by man' ${ }^{133}$ It can be suggested that by engaging with musical cryptography, composers were treating music as a vessel for prayer.

I have attributed the rationale of ostentation to composers like Elgar, for whom 'secret writing' cannot have been the objective. I might add that the term should not be considered distasteful and is only used in regard to the showiness with which Elgar alludes to his encryptions. As covered in the examination of Elgar, cryptography of this variety is fundamentally juxtaposing, on the one hand there is a borderline impenetrable, ultra-complex cipher, and on the other hand you have the composer speaking openly of its existence. Returning to the Enigma Variations, the public are told from the very beginning that something is afoot. The clue-like nature of Elgar's programme note (1899) would indicate that this supposed cipher was always devised as more of a puzzle, something to be discussed and solved, rather than buried in the music. Whether he enjoyed observing the logic of proposed decryptions, or simply devised the enigma as a publicity boost for his first largescale work, I am certain that without the cryptic programme note and post-premiere 'Enigma' labelling, the encryption would never have been noticed; it is far too complex and on far too broad a scale. I propose that the content of these ostentatious encryptions is rarely of real significance; the focus is on the process and not the result. Perhaps this is the reason Elgar

[^56]never revealed the enigma; the encrypted message was likely of no importance, and he knew the public would get far more from a never-ending line of proposed decryptions, than they would ever get from a definitive solution. Thus Elgar's Enigma is yet to be solved, and will likely remain so indefinitely.

I believe that Tchaikovsky's utilisation of musical codes and ciphers functions as a form of catharsis, confiding in his music that which he could not say. Illustrating the shame and 'self-hatred' he felt as a closeted homosexual in his letters and diary entries, I propose that Tchaikovsky used musical codes like the Fate Motif to express his uncensored self.

While I do not think that the early, programmatic iterations found in Eugene Onegin and The Queen of Spades are indicative of such 'self-hatred', its utilisation in his Sixth Symphony does suggest some level of cryptographic catharsis. David Brown remarks that the work is often interpreted as depicting Tchaikovsky's struggles with his sexuality, a central theme of his life that he may well have associated with the negative aspects of fate, just as Tatiana describes 'an insidious tempter ${ }^{134}$ alongside the Fate Motif in Eugene Onegin.

### 4.1 A Compositional Device

As observed in the progression from diatonic/chromatic ciphers to compound motivic, with no requirement to improve the strength of ciphers, cryptographic development will naturally shift to musical coherence. As the melodic potential of encryptions improved, usage became more compositionally novel than functional. For instance, while their reasons differ, neither Bach, Tchaikovsky, nor Elgar were engaging with musical cryptography for the purpose of secrecy. Whether to express emotion, engender intrigue, or heighten sacred dedications, in this relatively modern context, I believe musical cryptography is best defined as a compositional device.

Defined as a set of rules or guidelines to which a composer must adhere, there are an abundance of compositional devices that would at first appear to limit a composer's autonomy. From Bach's fugues and Fux's species counterpoint to Schoenberg's tone rows and surrealist aleatoricism, it is my proposition that academia should recognise musical cryptography as the latest inductee into this clique of regulatory compositional devices. Curiously, while these techniques posture to restrict creativity, it is often the case that working within a set of rules can make composition far easier; if the cipher dictates pitch,

[^57]then more focus can be placed on rhythm and orchestration. If the cipher dictates rhythm, then more focus can be put into the melodic structure. Take for instance, the enormous D-flat major love theme from Alban Berg's opera, Lulu (1937). The melody appears utterly spontaneous, but is in fact pre-determined by both a 12 -tone row and an early example of rhythmic serialism. Referring to this self-regulation, George Perle writes, that 'number magic was for Berg a necessary catalyst for creative inspiration ${ }^{135}$. It should be noted that this compositional stimulus functions in much the same way as cryptography which Berg also utilised.

This paper contributes to the scholarly field of Musical Cryptology in three ways. Firstly, as the only long-form study, this paper is the first to combine the collective expertise of leading musical cryptologists to establish a central document outlining all facets of the field. In addition to this, this paper's novel focus on musical codes and cipher-melodies has facilitated the introduction of a plethora of new encryptions, including discovery of Bach's SDG Motif. Finally, through the consideration of cryptological development, I believe that this paper has managed to successfully define musical cryptography within the contemporary musicological landscape as a compositional device.

[^58]
## BIBLIOGRAPHY

Bacon, F. The proficience and advancement of learning divine and humane. Oxford, 1605.

Brockett, Clyde, and Michel Huglo. Odo of Arezzo. Oxford: Oxford University Press, 2001.

Buckley, Robert J. Sir Edward Elgar. London: J. Lane, 1905.
Cartwright, Julyan H., Diego L. González, and Oreste Piro. 'Dynamical Systems, Celestial Mechanics, and Music: Pythagoras Revisited.' The Mathematical Intelligencer 43, no. 1 (2020): 25-39.

Cocos, Mihail, and Shawn Fowers. 'Music by Numbers.' International Journal of Applied Science and Technology 1, no. 6 (2011): 62-27.

Code, David Løberg. 'Can Musical Encryption Be Both? A Survey of Music-Based Ciphers.' Cryptologia 47, no. 4 (2022): 318-64.

Cooke, Mervyn. 'Be Flat or Be Natural? Pitch Symbolism in Britten's Operas.' Essay. In Rethinking Britten, 102-24. New York: Oxford University Press, 2013.

Curry, Robert. Gable, David. and Marshall, Robert Lewis, eds. Variations on the Canon: Essays on Music From Bach to Boulez in Honour of Charles Rosen on His Eightieth Birthday. Rochester, NY: University of Rochester Press, 2008.

Danielou, Alain. Introduction to the Study of Musical Scales. Delhi: Mushiram Manoharlal, 1999.

Daverio, John. Crossing Paths: Schubert, Schumann, and Brahms. Oxford: Oxford University Press, 2002.

David, Brown. Tchaikovsky: A Biographical and Critical Study. London: Victor Gollancz, 1992.

Dopp, Bonnie Jo. 'Numerology and Cryptography in the Music of Lili Boulanger: The Hidden Program in Clairières Dans Le Ciel.' The Musical Quarterly 78, no. 3 (1994): 557-83.

Edwards, Frederick George. 'Edward Elgar.' Musical Times, no. 41 (1900).
Eggebrecht, Hans Heinrich. Bachs Kunst der Fuge: Erscheinung und Deutung. 4th ed. Wilhelmshaven: F. Noetzel, 1998.

Elgar, Edward. Edward Elgar: Letters of a Lifetime. Oxford University Press, 1991.
Fliess, Wilhelm. Vom Leben und vom Tod. Jena: Diederich, 1924.
Floros, Constantin, and Ernest Bernhardt-Kabisch. Alban Berg and Hanna Fuchs: The story of a Love in letters. Bloomington: Indiana University Press, 2008.

Hall, Cory. 'Ground-breaking Discoveries.' The Bach Scholar, 2017. https://www.bachscholar.com/bachs-secret-tempo-code/chapter-1.

Haugan, Edwin L. Anton Von Webern's String Quartet in a Minor (Ca. 1907), M.121: A Reconstruction, Louisiana: Louisiana State University, 1989

Herz, Gerhard. ‘Bach's Religion.' Journal of Renaissance and Baroque Music 1, no. 2 (1946): 124-38.

Honegger, Arthur. 'Homage a Albert Roussel, H.69. Paris: Editions Salabert'. Paris: Editions Salabert. 1928

Houten, Theodore Van. 'The Enigma - A Solution From Holland.' Elgar Society Newsletter, 1976.

Howat, R. Debussy in proportion. Cambridge: Cambridge University Press, 1983.
Jarman, Douglas. ‘Alban Berg, Wilhelm Fliess and the Secret Programme of the Violin Concerto.' The Musical Times 124, no. 1682 (1983): 218-23.

Keppler, Johannes G. Harmonices Mundi. Bologna: Forni, 1969.
Levin, Flora R. 'Nicomachus of Gerasa Manual of Harmonics: Translation and Commentary’ unpublished PhD Diss., Columbia University, 1967

Nectoux, Jean-Michel. Gabriel Fauré: A Musical Life, Cambridge: Cambridge University Press, 1991.

Pacioli, Luca. Divina proportione. Rio de Janeiro: Fundação Biblioteca Nacional, 2008.
Pelling, Nick. ‘Dorabella Cipher.' Cipher Mysteries, 2012. https://ciphermysteries.com/other-ciphers/the-dorabella-cipher.

Perle, George. 'The Music of Lulu: A New Analysis.' Journal of the American Musicological Society 12, no. 2-3 (1959): 185-200.

Philip, Nicholas. ‘The Sermon Booklets of Friar Nicholas Philip'. MS Lat. th.d.I, Bodeleian Library. Oxford, 1436

Pilcher, Karoline. Denkwürdigkeiten aus meinem Leben. Witwe, 1844.
Porta, Giambattista della. De Furtivis Literarum Notis. Vulgo de Ziferis Libri V, 1602.
Powell, Dora M. Penny. Edward Elgar: Memories of a Variation. London: Oxford University Press, 1937.

Poznansky, Alexander. Tchaikovsky: The Quest for the Inner Man. New York: Schirmer Books, 1991.

Roberts, Tim. Solving the Dorabella Cipher. Bundaberg: CQ University, 2012.

Rogan, Joe, and James Maynard Keenan. 'Episode 1326.' The Joe Rogan Experience, June 8, 2018. Video, 5:42. https://www.youtube.com/watch?v=e-DNbAcZo58.

Rosenstiel, Léonie. Nadia Boulanger: A life in music. New York: Norton, 1982.
Rudeforth, Helen Elizabeth. Words, Ideas, and Music: A Study of Tchaikovsky's Last Completed Work. Birmingham: University of Birmingham, 1998.

Rumsey, David. ‘Bach and Numerology: Dry Mathematical Stuff?' Literature and Aesthetics 7 (1997): 143-65.

Sams, Eric. 'Musical Cryptography.' Cryptologia 3, no. 4 (1979): 193-201.
Sams, Eric, and Elgar. 'Elgar's Cipher Letter to Dorabella.' Musical Times 111, no. 1524 (February 1970): 151.

Santa, Charles, and Matthew Santa. 'Solving Elgar's Enigma.' Current Musicology, no. 98 (2010).

Schoenberg, Arnold. Theory of Harmony. London: Faber and Faber, 1978.
Singh, Neelam. 'Mathematics and Music.' International Journal of Scientific Research 5, no. 10 (2016).

Smend, Friedrich. Johann Sebastian Bach bei seinem Namen gerufen: Eine Noteninschrift und ihre Deutung. Kassel: Bärenreiter, 1950.

Stevens, Denis. 'Preserve Harmony.' Newsletter of the Musicians' Company, 2000.
Stuckenschmidt, H. H. 'Debussy or Berg? The Mystery of a Chord Progression.' The Musical Quarterly 51, no. 3 (1965): 453-59.

Thicknesse, Philip. A Treatise on the Art of Decyphering, and of Writing in Cypher: With a Harmonic Alphabet. London: W. Brown, 1772.

Taruskin, Richard. Music in the Late Twentieth Century: The Oxford History of Western Music, Oxford: Oxford University Press, 2010.

Turner, Patrick. Elgar's Enigma Variations: A Centenary Celebration. London: Thames, 1999.

Werckmeister, Andreas. Musicae Mathematicae Hodegus Curiosus. Hildesheim: G. Olms, 1972.

Wolff, Christoph. Johann Sebastian Bach: The Learned Musician. Oxford: Oxford University Press, 2000.


[^0]:    ${ }^{1}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{2}$ Francis Bacon, The proficience and advancement of learning divine and humane (Oxford, 1605)
    ${ }^{3}$ David Løberg Code, ‘Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): $318-364$.

[^1]:    ${ }^{4}$ John Daverio, Crossing Paths: Schubert, Schumann, and Brahms (Oxford: Oxford University Press, 2002)
    ${ }^{5}$ Giambattista della Porta. De Furtivis Literarum Notis (Vulgo de Ziferis Libri V, 1602)

[^2]:    ${ }^{6}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.

[^3]:    ${ }^{7}$ Mervyn Cooke, 'Be Flat or Be Natural? Pitch Symbolism in Britten's Operas', in Philip Rupprecht (ed.), Rethinking Britten (New York: Oxford University Press, 2013), 102-24.
    ${ }^{8}$ Ibid., p. 105.
    ${ }^{9}$ Ibid.

[^4]:    ${ }^{10}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{11}$ Eric Sams, 'Musical Cryptography’, Cryptologia 3:4 (1979) 193 - 201.
    ${ }^{12}$ Christopher H. Sterling, Military Communications: From Ancient Times to the 21 st Century (London: Bloomsbury Publishing, 2008): p. 307.

[^5]:    ${ }^{13}$ Flora R. Levin, 'Nicomachus of Gerasa Manual of Harmonics: Translation and Commentary' (unpublished Ph.D. dissertation, Columbia University, 1967), 28-29.

    * To calculate F, C is multiplied by a fourth: dividing B by a fourth would produce F\#, not F.

[^6]:    ${ }^{14}$ Johannes Kepler, Harmonice Mundi libri V (Linz, 1619)
    ${ }^{15}$ Julyan H. E. Cartwright, Diego L. Gonzalez, Oreste Piro, ‘Dynamical Systems, Celestial Mechanics, and music: Pythagoras revisited, The Mathematical Intelligencer, Vol. 43, No. 1 (2020)
    ${ }^{16}$ Johannes Kepler, Harmonice Mundi libri V (Linz, 1619)
    ${ }^{17}$ Ibid.

[^7]:    ${ }^{18}$ Johannes Kepler, Harmonice Mundi libri V (Linz, 1619).
    ${ }^{19}$ Illustrates the musical pitches capable of production via planetary motion
    ${ }^{20}$ Mihail Cocos, Shawn Fowers, 'Music by Numbers', International Journal of Applied Science and Technology, Vol. 1, No. 6 (2011): 62-67

[^8]:    ${ }^{21}$ Johannes Kepler, Harmonice Mundi libri V (Linz, 1619).
    ${ }^{22}$ Ibid.
    ${ }^{23}$ Luca Pacioli, Divina proportione (Rio de Janeiro: Fundação Biblioteca Nacional, 2008)

[^9]:    ${ }^{24}$ James Maynard Keenan, Joe Rogan, ‘Episode 1887’, The Joe Rogan Experience (2022): Ep. 1887

[^10]:    ${ }^{25}$ Ibid.
    ${ }^{26}$ Roy Howat, Debussy in Proportion: A musical analysis (Cambridge: Cambridge University Press, 1983).
    ${ }^{27}$ Ibid.
    ${ }^{28}$ Ibid.

[^11]:    ${ }^{29}$ Ibid.

[^12]:    ${ }^{30}$ Hans Heinz Stuckenschmidt and Piero Weiss, 'Debussy or Berg? The Mystery of a Chord Progression', The Musical Quarterly, 51:3 (1965): 453-59.
    ${ }^{31}$ Ibid.
    ${ }^{32}$ Ibid.

[^13]:    ${ }^{33}$ Wilhelm Fliess, Vom Leben und vom Tod (E. Diederichs, 1924).
    ${ }^{34}$ Ibid.
    ${ }^{35}$ Constantin Floros, Alban Berg and Hanna Fuchs: The Story of a Love in Letters, trans. E. Bernhardt-Kabisch (Indiana University Press, 2008).
    ${ }^{36}$ Ibid.

[^14]:    ${ }^{37}$ Douglas Jarman, 'Alban Berg, Wilhelm Fliess and the Secret Programme of the Violin Concerto', The Musical Times Vol. 124, No. 1682 (1983): 218-223
    ${ }^{38}$ Ibid.
    ${ }^{39}$ Bonnie Jo Dopp, ‘Numerology and Cryptography in the Music of Lili Boulanger: The Hidden Program in "Clairières dans le ciel"', The Musical Quarterly Vol. 78, No. 3 (1994): 556-583

[^15]:    ${ }^{40}$ Ibid.
    ${ }^{41}$ Leonie Rosenstiel, Nadia Boulanger: A Life in Music (New York: W.W. Norton, 1982).
    ${ }^{42}$ Bonnie Jo Dopp, 'Numerology and Cryptography in the Music of Lili Boulanger: The Hidden Program in "Clairières dans le ciel"', The Musical Quarterly Vol. 78, No. 3 (1994): 556-583
    ${ }^{43}$ Ibid.

[^16]:    ${ }^{44}$ Bonnie Jo Dopp, 'Numerology and Cryptography in the Music of Lili Boulanger: The Hidden Program in "Clairières dans le ciel"', The Musical Quarterly Vol. 78, No. 3 (1994): 556-583
    ${ }^{45}$ Douglas Jarman, 'Alban Berg, Wilhelm Fliess and the Secret Programme of the Violin Concerto', The Musical Times Vol. 124, No. 1682 (1983): 218-223
    ${ }^{46}$ Bonnie Jo Dopp, 'Numerology and Cryptography in the Music of Lili Boulanger: The Hidden Program in "Clairières dans le ciel"', The Musical Quarterly Vol. 78, No. 3 (1994): 556-583
    ${ }^{47}$ George Perle, 'The Secret Programme of the Lyric Suite. 1', The Musical Times, Vol. 118, No. 1614 (1977): 629-632

[^17]:    ${ }^{48}$ Arnold Schoenberg, Theory of Harmony, trans. R. E. Carter (London, Faber and faber, 1983)
    ${ }^{49}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): $318-364$.

[^18]:    ${ }^{50}$ Eric Sams. 'Musical Cryptography.' Cryptologia 3, no. 4 (1979): 193-201.
    ${ }^{51}$ John Daverio, Crossing Paths: Schubert, Schumann, and Brahms (Oxford: Oxford University Press, 2002)

[^19]:    ${ }^{52}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.

[^20]:    ${ }^{53}$ Ibid.
    ${ }^{54}$ Henry E. Dudeney, 'A Musical Enigma' from the 'Perplexities' column, Strand Magazine, 63 (1922)

[^21]:    ${ }^{55}$ Edwin Lyle Haugan, Anton Von Webern's String Quartet in a Minor (Ca. 1907), M.121: A Reconstruction (Louisiana: Louisiana State University, 1989)
    ${ }^{56}$ NB. The inverted and retrograde versions of the 'Bach Motif' are identical. Also, the retrograde inverted motif shares the same intervals as the original motif, thus the third bar could be in retrograde inversion, or just a transposed original.

[^22]:    ${ }^{57}$ Richard Taruskin, Music in the Late Twentieth Century: The Oxford History of Western Music (Oxford: Oxford University Press, 2010)

[^23]:    ${ }^{58}$ Jean-Michel Nectoux, Gabriel Fauré: A Musical Life (Cambridge: Cambridge University Press, 1991)

[^24]:    ${ }^{59}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{60}$ Giambattista della Porta, De Furtivis Literarum Notis, Vulgo de Ziferis Libri V, 1602

[^25]:    ${ }^{61}$ 'How a Musician Used Sheet Music Encryption of Help Soviet Defectors'. Youtube, RSA Conference, 22/08/2022, www.youtube.com/watch?v=1WJQFESWLjg
    ${ }^{62}$ Ibid
    ${ }^{63}$ Ibid

[^26]:    ${ }^{64}$ Brother Nicholas Philip, ‘The Sermon Booklets of Friar Nicholas Philip’. MS Lat. Th.d.I (Oxford: Bodeleian Library, 1436)
    ${ }^{65}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.

[^27]:    ${ }^{66}$ Philip Thicknesse, A Treatise on the Art of Decyphering, and of Writing a Cypher. With a Harmonic Alphabet (1772)

[^28]:    ${ }^{67}$ Ibid.
    ${ }^{68}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers’, Cryptologia 47:4 (2022): 318 - 364.

[^29]:    ${ }^{69}$ Arthur Honegger, 'Homage a Albert Roussel, H.69. Paris: Editions Salabert' (Paris: Editions Salabert. 1928)

[^30]:    ${ }^{70}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{71}$ Olivier Messiaen, 'Meditations sur le mystere de la Sainte Trinite' (Paris: Leduc. 1969)
    ${ }^{72}$ Ibid.

[^31]:    ${ }^{73}$ Friedrich von Ottingen-Wallerstein, Steganographia comitis, ca. 1600
    ${ }^{74}$ NB. It wasn't until 1524 that Gian Giorgio Trissions, an Italian Renaissance grammarian, made a clear distinction between the sounds, ' i ' and ' j '. While the two letters share visual similarity, it is possible that Friedrich Ottingen-Wallerstein's cipher did not yet recognise ' j ' as a separate letter.
    ${ }^{75}$ Friedrich vo Ottingen-Wallerstein, Steganographia comitis, ca. 1600

[^32]:    ${ }^{76}$ Johann Balthasar Friderici. Cryptographia. 1665
    ${ }^{77}$ Ibid

[^33]:    ${ }^{78}$ NB. The Latin ' $V$ ' was the same as ' $U$ '. It wasn't until the mid- $16^{\text {th }}$ century that ' $v$ ' was used to represent the consonant and ' $u$ ', the vowel.
    ${ }^{79}$ David Løberg Code, 'Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{80}$ Karoline Pichler, Denkwürdigkeiten aus meinem Leben (Witwe, 1844)

[^34]:    ${ }^{81}$ David Løberg Code, ‘Can musical encryption be both? A survey of music-based ciphers', Cryptologia 47:4 (2022): 318 - 364.
    ${ }^{82}$ Hideo Noguchi. 'Mozart - Musical Game in C K.516f'. Mitteilungen Der Internationalen Stiftung Mozarteum. 1990
    ${ }^{83}$ Ibid

[^35]:    ${ }^{84}$ Johann Bucking, Answeisung zur geheimen Correspondenz systematisch entworfen, 1804

[^36]:    ${ }^{85}$ Christoph Wolff, Johann Sebastian Bach: The Learned Musician (New York, 2000)

[^37]:    ${ }^{86}$ Gerhard Herz, 'Bach’s Religion', Journal of Renaissance and Baroque Music, Vol. 1, No. 2 (1946): 124-38
    ${ }^{87}$ Ibid.
    ${ }^{88}$ Andreas Werckmeister, Musicae Mathematicae Hodegus Curious (1687)

[^38]:    ${ }^{89}$ Friedrich Smend, Johann Sebastian Bach bei seinem Namen gerufen (Bärenreiter-Verlag, 1950)
    ${ }^{90}$ David Rumsey, 'Bach and Numerology: Dry Mathematical Stuff?', Literature and Aesthetics, Vol. 7 (1997): 143-165
    ${ }^{91}$ Ibid.

[^39]:    ${ }^{92}$ Hans Heinrich Eggebrecht, Bachs Kunst der Fuge: Erscheinung und Deutung (4 ${ }^{\text {th }}$ ed., Noetzel, 1998)

[^40]:    ${ }^{93}$ Cory Hall, ‘Chapter 1: Ground-breaking Discoveries’ (Bach Scholar, 2017), https://www.bachscholar.com/bachs-secret-tempo-code/chapter-1
    ${ }^{94}$ Hans Heinrich Eggebrecht, Bachs Kunst der Fuge: Erscheinung und Deutung (4 ${ }^{\text {th }}$ ed., Noetzel, 1998)

[^41]:    ${ }^{95}$ Dora M. Powell, Edward Elgar: Memories of a Variation (Oxford University, 1947)

[^42]:    ${ }^{96}$ Eric Sams, 'Elgar's Letter to Dorabella', The Musical Times, Vol. 111, No. 1524 (1970): 151-54
    ${ }^{97}$ Ibid.

[^43]:    ${ }^{98}$ Tim Roberts, Solving the Dorabella Cipher (CQ University, Bundaberg, 2012)

[^44]:    ${ }^{99}$ Nick Pelling, 'Dorabella Cipher', Cipher Mysteries (2012) https://ciphermysteries.com/other-ciphers/the-dorabella-cipher

[^45]:    ${ }^{100}$ Robert W. Padgett, ‘Troyte Griffith’s Enigma "Solution" Refuted’, Elgar's Enigma Theme Unmasked (2016) https://enigmathemeunmasked.blogspot.com/2016/06/

[^46]:    ${ }^{101}$ Ibid.
    ${ }^{102}$ Edward Elgar, Variations on an Original Theme Programme Note (London, 1899)
    ${ }^{103}$ Frederick George Edwards, 'Edward Elgar', The Musical Times, No. 41 (1900)
    ${ }^{104}$ Robert John Buckley, Sir Edward Elgar (London/New York, 1905)
    ${ }^{105}$ Robert W. Padgett, ‘Troyte Griffith’s Enigma "Solution" Refuted’, Elgar's Enigma Theme Unmasked (2016) https://enigmathemeunmasked.blogspot.com/2016/06/

[^47]:    ${ }^{106}$ Frederick George Edwards, 'Edward Elgar', The Musical Times, No. 41 (1900)
    ${ }^{107}$ Edward Elgar, Variations on an Original Theme Programme Note (London, 1899)

[^48]:    ${ }^{108}$ Robert W. Padgett, ‘Troyte Griffith’s Enigma "Solution" Refuted', Elgar's Enigma Theme Unmasked (2016) https://enigmathemeunmasked.blogspot.com/2016/06/
    ${ }^{109}$ Theodore van Houten, ‘The Enigma - A Solution from Holland’, Elgar Society Newsletter (1976)
    ${ }^{110}$ James M. Nosworthy, 'Letter', The Daily Telegraph (1977)
    ${ }^{111}$ Edward Elgar, Jerrold N. Moore, Edward Elgar: Letters of a Lifetime (Oxford University Press, 1991)
    ${ }^{112}$ Patrick Turner, Elgar's 'Enigma' Variations - A Centenary Celebration (London: Thames Publishing, 1999)
    ${ }^{113}$ Denis Stevens, 'Preserve Harmony', Newsletter of the Musicians' Company (2000)
    ${ }^{114}$ Dora M. Powell, Edward Elgar: Memories of a Variation (Remploy, 1947)

[^49]:    ${ }^{115}$ Charles R. Santa, Matthew Santa, 'Solving Elgar’s Enigma', Current Musicology, No. 98 (New York, 2010) ${ }^{116}$ Ibid.
    ${ }^{117}$ Ibid.
    ${ }^{118}$ Edward Elgar, Variations on an Original Theme Programme Note (London, 1899)
    ${ }^{119}$ Dora M. Powell, Edward Elgar: Memories of a Variation (Remploy, 1947)

[^50]:    ${ }^{120}$ Helen Elizabeth Rudeforth, Words, Ideas and Music: A Study of Tchaikovsky's Last Completed Work (University of Birmingham, 1998)

[^51]:    ${ }^{121}$ Ibid.
    ${ }^{122}$ David Brown, Tchaikovsky: A Biographical and Critical Study, Vol. 4 (London: Gollancz, 1992)
    ${ }^{123}$ Anthony Holden, Tchaikovsky: A Biography (California, Random House, 1996)
    ${ }^{124}$ Alexander Poznansky, Tchaikovsky: The Quest for the Inner Man (Schirmer Books, 1991)
    ${ }^{125}$ Ibid.
    ${ }^{126}$ Ibid.
    ${ }^{127}$ Helen Elizabeth Rudeforth, Words, Ideas and Music: A Study of Tchaikovsky's Last Completed Work (University of Birmingham, 1998)

[^52]:    ${ }^{128}$ Ibid.

[^53]:    ${ }^{129}$ Ibid.
    ${ }^{130}$ Ibid.

[^54]:    ${ }^{131}$ David Brown, Tchaikovsky: A Biographical and Critical Study, Vol. 4 (London: Gollancz, 1991)

[^55]:    132 Ibid.

[^56]:    ${ }^{133}$ Johannes Kepler, Harmonice Mundi libri V (Linz, 1619)

[^57]:    ${ }^{134}$ Helen Elizabeth Rudeforth, Words, Ideas and Music: A Study of Tchaikovsky's Last Completed Work (University of Birmingham, 1998)

[^58]:    ${ }^{135}$ George Perle, ‘The Music of "Lulu": A New Analysis', Journal of the American Musicological Society, Vol. 12., No. 2/3 (1959): 185-200

