Fixed vs countercyclical LTV ratio:

The effectiveness of macroprudential policy for a small open economy

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This dissertation is presented in part fulfilment of the requirement for the completion of an MRes in the School of Economics, University of Nottingham. The work is the sole responsibility of the candidate.

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Abstract

In this paper, I develop a dynamic stochastic general equilibrium (DSGE) model for a small open economy. The model includes a housing market and aims to study the effectiveness of a macroprudential policy, namely the loan to value (LTV) ratio, in taming the financial and business cycles. This paper compares the implementation of LTV under fixed and countercyclical rules. A countercyclical LTV rule is more effective for a closed economy in taming both the financial and business cycles. However, for a small open economy, while a countercyclical LTV rule is more effective in taming the financial cycle, the impact on the business cycle is indifferent under both LTV rules. This phenomenon is mainly explainable by the final good producers' ability to change their composition between domestic and imported goods in their productions. This reflects the expenditure switching channel between foreign and domestic goods. I also explore the benefit of targeting the nominal exchange rate by the central bank or augmenting the countercyclical LTV rule by including the movement in the nominal exchange rate. There seems to be a policy trade-off between achieving financial and output stabilisation that the central bank needs to consider when the nominal exchange rate is included in the monetary policy or countercyclical LTV rules. In both cases, more stabilisation in output is achieved at the expense of more volatility in the financial cycle.

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Contents

List of Tables

List of Figures

1 Introduction

Monetary policy, typically targeting the short-term nominal interest rate, has been a vital tool deployed by central banks to achieve their output and inflation stabilisation mandates. In particular, monetary policy is a critical countercyclical tool in preserving macroeconomic stability. The short-term nominal interest rate will be adjusted accordingly, depending on where the economy is currently in the business cycle stage. More accommodative monetary policy is warranted when inflation is below a central bank's target, which typically coincides to when output is below its potential growth path or when the output gap is negative. Beyond the short-term nominal interest rate as a conventional tool for monetary policy, there is also a broader application of unconventional monetary policy tools in advanced and emerging market economies, especially after the Covid-19 pandemic crisis. However, the motivation for deploying unconventional monetary policy tools by emerging market central banks varies across countries. In addition to conventional and unconventional monetary policy tools, many central banks, especially in emerging markets, also have macroprudential policies or instruments in their policy arsenal. The sole purpose of deploying macroprudential policies by emerging market central banks is to achieve financial stability. Delivering financial stability, in many cases, is one of emerging market central banks' critical objectives that they need to achieve, together with output and inflation stabilisation mandates.

The two intermediate objectives of macroprudential policies are to increase the financial system's resilience and to constrain financial booms, as suggested by Borio (2014). Macroprudential policies are not blunt tools like nominal interest rates and are often deployed as a more targeted approach to prevent potential systemic risks from building up in the future. The significance of macroprudential policies to tame the financial cycle is greater if the drivers and properties that characterise the financial cycle are sufficiently different compared to the drivers and properties that generally represent the regular business cycle. The underlying differences between the financial and business cycles make the monetary policy an imperfect instrument for dealing with the financial cycle (Rünstler, 2016). There is a variety of macroprudential policies that central banks have introduced to achieve their financial stability mandate. The policies are designed and tailored toward achieving specific objectives, including preventing the housing market bubble and reducing the reliance on external financing. External financing could amplify the domestic financial cycle and affect debt repayment capacity as domestic agents are highly exposed to the movement of the nominal exchange rate. The indirect consequences of these policies are, among others, reducing concentration risks by banks, controlling speculative activities, and managing excessive risk-taking activities by households, firms, and financial institutions. This, in turn, will reduce the amplification mechanism of the financial cycle, such as the financial accelerator mechanism.

Specifically related to the housing market, the standard macroprudential instruments that central banks and other government agencies have deployed are the loan-to-value (LTV) ratio and the real property gain tax (RPGT). In managing house prices, a central bank may impose a lower LTV ratio on the house buyers, limiting the maximum amount of housing loans that the buyers can get from commercial banks. The objective of RPGT is also similar, where a higher RPGT will make it less profitable for house buyers to sell their houses, which will indirectly tame demand for houses. Central banks also use specific instruments to promote financial stability. For example, the Central Bank of Malaysia decides to shorten the maximum tenure for housing loans as a tool to increase the monthly repayment for borrowers. Meanwhile, the central bank of the Philippines imposes a maximum lending threshold to the real estate sector of no more than 20% of an individual bank's total loan portfolio (BIS, 2017). The main objective of these instruments introduced in the housing market is to dampen the financial cycle and promote financial stability. These instruments are typically imposed on the second or third property onward as a tool to tame speculative activities in the housing market, but at the same time not to tighten the lending standards for the first time house buyers or house occupiers. Furthermore, according to a BIS (2017) report, a study by the Hong Kong Monetary Authority shows that the LTV ratio has strengthened banks' resilience to property shocks, even if they had a limited impact on house prices themselves. This is achieved by reducing banks' concentration risk, which is essential to promoting financial stability.

This paper focuses on the effectiveness of the LTV ratio for a small open economy as a mechanism to introduce borrowing constraints for households to accumulate houses. The borrowing constraints act as a tool to tame the financial cycle which will help central banks achieve their mandates of promoting financial stability. In this paper, financial stability mainly refers to the magnitude of deviation of credit growth from its steady state path. Significant credit growth expansion will usually coincide and be followed by boom and bust episodes in the business cycle. Mian, Sufi, and Verner (2017) using panel data for advanced economies suggested that a one-standard-deviation increase in household debt to GDP (equivalent to 6.2 percentage points) in a 3-year interval leads to a 2.1% decline in GDP over the following three years. Similarly, Dell'ariccia et al. (2012) observed that real GDP growth during the credit up-cycle exceeds the rate observed in standard years by roughly two percentage points, on average. In particular, credit expansion and house prices have been identified as two critical financial cycle variables that policymakers need to monitor closely. Empirical evidence suggests that many financial crises have been preceded by credit and housing booms (Jordà, Schularick, and Taylor, 2016). Similarly, Claessens, Kose, and Terrones (2012) argued that recessions associated with financial disruption episodes, notably house and equity price busts, are often longer and more profound than other recession. This is not entirely surprising, given that the lion's share of households' indebtedness is typically mainly for housing loans. On the flip side, large proportions of commercial banks' assets are in the form of housing loans.

This paper describes a small open economy with a housing sector for a dynamic stochastic general equilibrium (DSGE) model. The core of this model is adopted from Iacoviello (2005) and Adolfson et al.

(2007), where Adolfson et al. (2007) extend the closed economy DSGE model of Christiano, Eichenbaum, and Evans (2005) to an open economy setup. To the best of my knowledge, this is the first attempt to set up a model combining the housing sector as in Iacoviello (2005) with a small open economy with a flexible exchange rate regime as in Christiano, Trabandt, and Walentin (2011). I believe the closest paper to this was by Vitola and Ajevskis (2011), but with a small open economy pursuing a fixed exchange rate strategy. The extension of this model to a small open economy from a closed economy setup by Iacoviello (2005) is to study the impact of macroprudential policies on financial and business cycles. The small open economy model implies that any development in the domestic economy will not affect the rest of the world. The small open economy model allows me to explore the propagation of domestic and external shocks through the trade and exchange rate channels. First, I show that the countercyclical LTV rule, as opposed to the fixed LTV rule, is more effective in promoting financial stability. The result is consistent with a closed economy. However, unlike a closed economy, the impact of the countercyclical rule in stabilising the business cycle is not as effective, as the exchange rate movement induces the expenditure switching channel. For instance, the appreciation in the exchange rate following a monetary policy shock will lead to higher imports, leading to greater volatility in domestic production. Second, I evaluate the effectiveness of including the nominal exchange rate as another target variable in the central bank's monetary policy and countercyclical LTV rule. In both cases, more stabilisation in output is achieved at the expense of more volatility in the financial cycle.

The remainder of the paper is organised as follows. The next section provides a literature review on the issues related to macroprudential policies and briefly discusses the DSGE model development for a small open economy. Section 3 gives a detailed explanation of the foundation and specification of the model. Section 4 explains the estimation of the model, and section 5 introduces a small extension from the basic model explained in section 3. The final section, 6, concludes this paper's findings and provides suggestions for future work.

2 Literature review

Many central banks also have financial stability mandates they need to deliver. This mandate comes together with inflation and output stabilisation responsibility that central banks typically need to achieve. Initially, the broad adoption of macroprudential policies by emerging market central banks was mainly to prevent vulnerabilities from building up again, as part of the lesson learned by the policymakers when financial crises hit the emerging economies in Asia and Latin America. However, in the aftermath of the GFC, there has been significant development in using macroprudential policies to promote financial stability in both emerging markets and advanced economies (Akinci and Olmstead-Rumsey, 2018). Given the nature of macroprudential policies that are more targeted in dealing with specific issues, in the past decades, central banks have deployed a wide array of instruments (Villar, 2017). The objectives of the instruments, however, remain the same, to enhance the financial system's resilience and to constrain financial booms. Table 1 provides a list of standard macroprudential policies implemented across emerging market economies.

Table 1: Instruments in macroprudential frameworks

Instrument	Countries example
Countercyclical capital buffer	China, Israel and Korea
Dynamic provisions	Argentina, India and Peru
Sectoral capital requirements	Argentina, Brazil, Czech Republic
Countercyclical capital requirements	Poland, Russia and UAE
Margins and haircuts	Czech Republic, Peru and Saudi Arabia
LTV ratios	Hungary, India and South Africa
Debt-to-income ratios	Hong Kong, Malaysia and Philippines
Limits on currency mismacthes	India, Mexico and Thailand
Reserve requirement	Colombia, Indonesia and Turkey

Source: Villar, 2017

The need for macroprudential policies to promote financial stability has been established by many studies that have shown that the financial cycle exhibits different characteristics than the regular business cycle. While monetary policy is effective in dealing with the business cycle, the fundamental differences between financial and business cycles imply that monetary policy may not necessarily be the best instrument to deal with the financial cycle. Many previous studies have concluded that the financial cycle is considerably longer than the business cycle. For example, Claessens, Kose, and Terrones (2012) analysed data for 44 countries for both advanced and emerging markets and concluded that financial cycles tend to be longer, deeper, and sharper than business cycles. Likewise, Drehmann, Borio, and Tsatsaronis (2012) used data from 7 advanced countries and found that the financial cycle and the business cycle are different phenomena, where the latter typically takes eight years to complete its entire cycle, but the former lasts, on average, around 16 years. Similarly, Rünstler and Vlekke (2016) used data for the U.S. and the five largest economies in Europe and found that pronounced cycles in credit and house prices had a length of 12 to 18 years in most cases, which is much longer than the traditional business cycle (3 to 8 years). The length of credit and house price cycles is closely tied to the housing market structure, where the financial cycles are longer and more extensive for countries with high rates of private home ownership.

Over the past years, many studies have attempted to estimate the importance and implications of macroprudential and monetary policies in constraining the financial cycle. Of significance, there is a growing interest in theoretical and empirical research on the role of macroprudential policies in smoothing financial cycles by mitigating growth in credit and housing markets. Iacoviello (2005) used a DSGE framework for a closed economy to study the impact of housing collateral and its interaction between asset prices and economic activity. He found that allowing the monetary authority to respond to asset prices yields negligible gains in terms of output and inflation stabilisation. Iacoviello and Neri (2010) used a DSGE framework for the U.S. economy to find that the housing sector market spillovers to the broader economy are non-negligible and have become more critical over time. Rubio and Carrasco-Gallego (2015) developed a two-country and two-sector monetary union DSGE model with housing. One country calibrates to the Spanish data, and another represents the rest of the European monetary union. The authors found that a countercyclical LTV rule that mainly responds to house prices would have mitigated the credit boom in Spain. More recently, using an estimated DSGE model tailored to Sweden, Chen and Columba (2016) found that the demand side macroprudential measures are more effective in curbing household debt ratios than monetary policy, and are less costly in terms of foregone consumption. In response to the surge in house prices in New Zealand, Funke, Kirkby, and Mihaylovski (2018) developed a DSGE model to study the impact of LTV restrictions. They found that LTV restrictions reduce house prices with negligible effects on consumer prices, suggesting macroprudential policy can be used without derailing monetary policy. Equally important, they also found that macroprudential policy is a valuable addition to monetary policy, and the two do not need to be coordinated. Rabanal (2018) developed a small open economy DSGE model with a housing sector and calibrated it with Hong Kong data to evaluate the effectiveness of macroprudential policies in controlling appreciation in house prices as well as reducing household leverage. He founds that without the macroprudential policies, house prices in Hong Kong would have been 10.5 percent higher, and the household credit to GDP ratio would have been 14 percent higher.

There is also a growing body of empirical research based on panel data to study the effectiveness of macroprudential policies. Cerutti, Claessens, and Laeven (2015) used a sample of 119 countries and found that macroprudential policies are generally associated with reductions in the growth rate in credit. However, they also suggest that the effect is weaker in more developed and financially open economies, and can impact growth in house prices. Based on data from 57 countries, Akinci and Olmstead-Rumsey (2018) suggested that macroprudential tightening has been associated with lower bank credit growth, weaker housing credit growth, and more muted house prices appreciation. They also concluded that macroprudential policies that curb house price appreciation are more effective in an environment where bank finance is significant. Choi et al. (2018) used a sample of multiple countries to examine the effectiveness of coordinated macroprudential policies in dampening the incidence of widespread banking crises across countries. They found that coordinated implementation of macroprudential policies between major trading partners can help to reduce the risks of widespread banking crises. However, this positive effect may take some time to materialise. Meanwhile, Belkhir et al. (2020) used a sample that covers more than 100 countries over the 2000-2017 period to examine the impact of macroprudential policies on the likelihood of systemic banking crises. They concluded that macroprudential policy has a positive net effect on financial stability by reducing the probability of systemic banking crises. However, they also showed that macroprudential policies have a destabilising indirect impact, which works through the depressing of economic growth.

The core models developed for a small open economy in DSGE literature are mostly adopted either from Gali and Monacelli (2005) or Adolfson et al. (2007) and Christiano, Eichenbaum, and Evans (2005). Here are some examples of the small open economy models that can be traced back to either Gali and Monacelli (2005) or Adolfson et al. (2007) and Christiano, Eichenbaum, and Evans (2005). A recent IMF working paper by Adrian et al. (2020), where the authors develop a quantitative model for the integrated policy framework, is built based on Gali and Monacelli (2005) as the foundation for their model. A compact open economy DSGE model for Switzerland developed by Rudolf and Zurlinden (2014) as one of the tools used for policy analysis and forecasting at the Swiss National Bank is also based on Gali and Monacelli (2005). On the other hand, an estimated DSGE model for Romania developed by Copaciu, Nalban, and Bulete (2015) for the National Bank of Romania is based on work by Adolfson et al. (2007) and Christiano, Eichenbaum, and Evans (2005). In this model, the authors capture the contractionary impact following a domestic currency depreciation through the balance sheet channel, which more than offsets the expansionary effect from the trade channel. A DSGE model developed to study the optimal management of capital flows in a small open economy model with financial frictions and multiple policy instruments by Lama and Medina (2020) is also based on Adolfson et al. (2007) and Christiano, Eichenbaum, and Evans (2005).

3 The model

The model is populated by two types of households, patient and impatient. The two kinds of households work, consume and accumulate houses. Patient households are the unconstrained savers, and impatient households are the constrained borrowers. Patient households have a lower discount rate than impatient households. They can lend their savings to either domestic borrowers or invest in foreign assets. However, impatient households can only borrow from domestic savers to finance their spending or to accumulate houses. Impatient households are constrained borrowers because they need to collateralise their debt repayments. The total amount of loans that impatient households can borrow from patient households is also subject to the LTV rule. The collateralise debt repayment and the LTV rule imply that debt repayment in the subsequent period must be within a specific limit of the expected future value of the current stock of houses owned by impatient households. On the production side, there are three types of firms in the economy. Firms involved either in the production of a domestic homogeneous final good, intermediate goods, or a final consumption good. Firms producing a domestic homogeneous final good and a final consumption good operate in a perfect competition market, while intermediate goods producers operate in a monopolistic competition market. Final consumption good producers require inputs from domestic and foreign firms, but domestic homogeneous final good and intermediate good producers only need inputs from domestic firms. Producers of the domestic homogeneous final goods sell their products to domestic households and also export them to foreign consumers. However, the productions of intermediate and final consumption goods are only for the domestic market. Intermediate firms require labour from both types of households for their output production. There are also a federal government and a central bank in this model. The federal government spending in this model is treated as exogenous. The central bank conducts its monetary policy based on Taylor's rule to bring output and inflation to its targets. The stock of houses in this model is fixed in the aggregate, meaning that any housing market development will be reflected fully in house prices. The open economy model also means that any movements in the nominal exchange rate will have implications on the terms of trade and the balance of payments. Intermediate goods producers are the source of nominal rigidity in this model, where they can re-optimise their prices based on Calvo friction.

3.1 Households

3.1.1 Households - Patient households (unconstrained)

Patient households lend to domestic borrowers and also invest abroad in foreign assets. The maximisation problem of patient households is given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^{\varphi}}{\varphi} \right) \tag{1}$$

where $\frac{1}{\varphi-1}$ is the labour supply elasticity and $\varphi > 0$. $j > 0$ is the weight of housing in the utility function. The budget constraint of patient households is given by

$$S_t A_{t+1}^f + P_t C_t^u + Q_t \left(H_t^u - H_{t-1}^u \right) + B_t^u \leq W_t^u L_t^u + R_{t-1} B_{t-1}^u + S_t [\Phi_{t-1} R_{t-1}^f] A_t^f + Profit_t \tag{2}$$

where B_t^u is the stock of domestic loans extended to domestic borrowers, R_t is the rate of return for domestic savings. P_t is the price of the final consumption good, and C_t^u is the quantity of the final consumption good purchased by patient households. $Profit_t$ captures transfers and profits to patient households, generated by the firms owned by patient households. S_t is the nominal exchange rate. A_t^f is the net stock of foreign assets, R_t^f is the return for foreign investments, and Φ_t is the risk premium of foreign assets returns over foreign risk-free rate assets. Q_t is house prices, and H_t^u is the number of houses owned by patient households. W_t^u is the wage for patient households, and L_t^u is the labour supply of patient households to intermediate producers.

The first-order conditions (FOCs) for this problem are as follows:

$$C_t^u : \frac{1}{P_t C_t^u} = \lambda_t$$

$$C_{t+1}^u : \frac{1}{P_{t+1} C_{t+1}^u} = \lambda_{t+1}$$

$$B_t^u : \lambda_t = \beta \lambda_{t+1} R_t$$

$$H_t^u : \frac{j}{H_t^u} = \lambda_t Q_t - \beta \lambda_{t+1} Q_{t+1}$$

$$L_t^u : (L_t^u)^{\varphi-1} = \lambda_t W_t^u$$

$$A_t^f : \lambda_t S_t = \beta \lambda_{t+1} \frac{S_{t+1} \Phi_t R_t^f}{\pi_{t+1}}$$

The FOCs for the patient households can be characterised by four equations:

$$B_t^u : \frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right) \tag{3}$$

$$w_t^u = (L_t^u)^{\varphi-1} C_t^u \tag{4}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1} \tag{5}$$

$$A_t^f : \frac{S_t}{C_t^u} = \beta E_t \frac{S_{t+1} R_t^f \Phi_t}{\pi_{t+1} C_{t+1}} \tag{6}$$

where $w_t^u = W_t^u / P_t$, $\pi_{t+1} = P_{t+1}/P_t$ and $q_t = Q_t/P_t$ are scaled with final consumption good price. These scaled variables can be considered real variables. The first equation is the standard Euler that defines households' intertemporal decisions for consumption and domestic assets. The second equation is a labour supply equation, where the wage equals the marginal productivity of patient households. The third one is the equation that captures patient households' demand for housing. Similar to the Euler equation for domestic assets, the final equation defines patient households' intertemporal decisions for consumption and foreign assets.

3.1.2 Households - Impatient households (constrained)

Impatient households receive loans from domestic savers and do not have access to international lenders. If their expenditures exceed their incomes, impatient households can only borrow from domestic savers or patient households to finance their consumption and accumulate houses. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained consumers. This assumption ensures that the borrowing constraint is binding in the steady state and that the economy is endogenously split into borrowers and savers. This also implies that the discount factor of impatient households, $\tilde{\beta}$, is smaller than the discount factor of patient households, β. The maximisation problem of impatient households is given by

$$\max E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\ln C_t^c + j \ln H_t^c - \frac{(L_t^c)^{\varphi}}{\varphi} \right) \tag{7}$$

The budget constraint of the borrowers is given by

$$P_t C_t^c + Q_t \left(H_t^c - H_{t-1}^c \right) + R_{t-1} B_{t-1}^c \leq W_t^c L_t^c + B_t^c \tag{8}$$

where W_t^c is the wage for impatient households and L_t^u is the labour supply of impatient households. B_t^c is the total borrowing of impatient households. The budget constraint captures the notion of income, and the level of borrowing must be sufficient to cover consumption, house accumulation and debt repayment. The maximisation problem is also subject to the borrowing constraint where the maximum amount that impatient households can borrow from domestic savers is subject to the LTV rule, where $LTV < 1$. The borrowing constraint is equivalent to a collateral constraint where the current debt repayment must be equal to or less than the future value of the current stock of houses, adjusted for expected future inflation, and also subject to the LTV rule defined as:

$$B_t^c \leq \frac{LTV}{R_t} E_t \pi_{t+1} Q_{t+1} H_t^c \tag{9}$$

The FOCs for this problem are as follows:

$$C_t^c : \frac{1}{P_t C_t^c} = \lambda_t$$

$$C_{t+1}^c : \frac{1}{P_{t+1} C_{t+1}^c} = \lambda_{t+1}$$

$$B_t^c : \lambda_t = \tilde{\beta} \lambda_{t+1} R_t + \upsilon_t$$

$$H_t^c : \frac{j}{H_t^c} = \lambda_t Q_t - \tilde{\beta}\lambda_{t+1}Q_{t+1} - \upsilon_t \frac{LTV}{R_t} E_t \pi_{t+1} Q_{t+1}$$

$$L_t^c : (L_t^c)^{\varphi - 1} = \lambda_t W_t^c$$

λ_t is the Lagrangian multiplier for the budget constraint, and υ_t is the Lagrangian multiplier for the borrowing constraint. υ_t captures the increase in lifetime utility as a result of borrowing R_t dollars. Combining the FOCs and set $\upsilon_t = \lambda_t R_t$, the FOCs for the impatient households can be characterised by three equations:

$$\frac{1}{C_t^c} = \widetilde{\beta} E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^c} \right) + \lambda_t R_t \tag{10}$$

$$w_t^c = (L_t^c)^{\varphi - 1} C_t^c \tag{11}$$

$$\frac{j}{H_t^c} = \frac{1}{C_t^c} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^c} q_{t+1} - \lambda_t LTV E_t \pi_{t+1} Q_{t+1} \tag{12}$$

and replacing $\lambda_t = \frac{\upsilon_t}{R_t} = \frac{\lambda_t - \lambda_{t+1} R_t}{R_t} \rightarrow \lambda_t = \frac{1}{P_t C_t^c R_t} - \frac{1}{P_{t+1} C_{t+1}^c}$, equation (12) becomes

$$\frac{j}{H_t^c} = \frac{1}{C_t^c} \left(q_t - \frac{LTV E_t q_{t+1} \pi_{t+1}}{R_t} \right) - \widetilde{\beta} E_t \frac{1}{C_{t+1}^c} (1 - LTV) q_{t+1} \tag{13}$$

where $w_t^c = W_t^c / P_t$ and $q_t = Q_t / P_t$. The first equation is the standard Euler equation that defines households' intertemporal decisions for consumption and domestic liabilities. The second equation is the labour supply equation for impatient households, where the wage equals the marginal productivity of impatient households. The third one is an equation that captures impatient households' demand for housing, taking into account the additional borrowing constraint faced by them. However, unlike patient households, impatient households do not face intertemporal decisions between consumption and foreign assets due to the restriction imposed on them as they can only borrow from domestic savers.

3.2 Production of the domestic homogeneous final goods

Producers of the domestic homogeneous final goods purchase intermediate goods from intermediate goods producers as inputs and subsequently transform them into domestic homogeneous final goods. A domestic homogeneous final good is produced using the following production function:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{14}$$

where $Y_{i,t}$ denotes intermediate goods, and ε reflects the degree of substitutability between the intermediate goods. I assume that the domestic homogeneous final good is produced by a competitive, representative firm, which takes the price of output, P_t^d, and the price of inputs, $P_{i,t}$ as given. The profit maximisation problem of the domestic homogeneous final good producers is given by:

$$P_t^d Y_t - \int_0^1 P_{i,t} Y_{i,t} dj$$
$$\text{subject to } Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Profit maximisation leads to the following first-order condition

$$Y_{i,t} = Y_t \left(\frac{P_t^d}{P_{i,t}} \right)^{\varepsilon} \tag{15}$$

3.3 Production of intermediate goods

There are a variety of intermediate goods produced domestically. Intermediate goods are required as the only inputs to produce the domestic homogeneous final good. I assume that firms producing intermediate goods do not need capital and only require labour to produce their outputs. The production of each intermediate good, $Y_{i,t}$, mainly needs a supply of labour from both patient and impatient households as follows:

$$Y_{i,t} = A_t \left(L_{i,t}^u \right)^{\gamma} \left(L_{i,t}^c \right)^{(1-\gamma)}$$

where A_t represents the aggregate technology, $L_{i,t}^u$ is labour provided by patient households and $L_{i,t}^c$ is labour supplied by impatient households. γ is the share of patient households required in the production of intermediate goods. γ is treated as static across all intermediate goods producers in this paper.

By substituting (15) into (14), it defines the following relationship between the price level of the domestic homogeneous final good, P_t^d, and the price of intermediate good of i^{th} intermediate producer, $P_{i,t}$ as follows:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$
$$Y_t = \left[\int_0^1 \left(Y_t \left(\frac{P_t^d}{P_{i,t}} \right)^{\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{16}$$
$$\Rightarrow P_t^d = \left(\int_0^1 P_{i,t}^{(1-\varepsilon)} di \right)^{\frac{1}{1-\varepsilon}}$$

3.4 Labour demand by intermediate goods producers

Labour is demanded by intermediate producers as the only inputs required to produce outputs. The homogeneous labour demand for the patient and impatient households by i^{th} intermediate producer is given by the following minimisation problem:

$$\min_{L_t^u, L_t^c} w_t^u L_t^u + w_t^c L_t^c$$

subject to

$$Y_t = A_t \left(L_t^u\right)^{\gamma} \left(L_t^c\right)^{(1-\gamma)}$$

The FOCs for this problem are as follows:

$$L_t^u : w_t^u = X_t \gamma A_t \left(L_t^u\right)^{\gamma-1} \left(L_t^c\right)^{(1-\gamma)} \Rightarrow w_t^u = \gamma X_t \frac{Y_t}{L_t^u} \tag{17}$$

$$L_t^c : w_t^c = X_t (1-\gamma) A_t \left(L_t^u\right)^{\gamma} \left(L_t^c\right)^{-\gamma} \Rightarrow w_t^c = (1-\gamma) X_t \frac{Y_t}{L_t^c} \tag{18}$$

where X_t is the Lagrangian multiplier for the minimisation problem, and it can be interpreted as the nominal marginal cost. When $\gamma < 1$, cost minimisation by the i^{th} intermediate good producer leads it to equate the relative wage of its patient and impatient households inputs to their corresponding relative marginal productivities:

$$\frac{w_t^u}{w_t^c} = \frac{\gamma}{1-\gamma} \frac{L_t^c}{L_t^u}$$

which implies that the ratio of labour demanded for the patient and impatient households will be uniform across intermediate producers, regardless of the price of intermediate good imposed by the i^{th} intermediate producer, $P_{i,t}$.

3.5 Optimal price setting by intermediate goods producers

There are a variety of intermediate goods produced domestically. The intermediate goods are distinguishable from each other. The i^{th} intermediate good is produced by a monopolist, and the i^{th} intermediate good cannot be replicated by other intermediate goods producers. This implies that the i^{th} intermediate has its demand function, and the same goes for other intermediate producers. From (15), the demand curve for i^{th} monopolist can be expressed as:

$$Y_{i,t} = Y_t \left(\frac{P_t^d}{P_{i,t}}\right)^{\varepsilon}$$

The monopolistic structure for the intermediate goods market implies that i^{th} monopolist firm can set its price to maximise its profit. However, price setting by intermediate producers is subject to Calvo frictions. The price stickiness faced by intermediate producers is the only source of nominal rigidity in this model. Calvo frictions indicate that with probability θ, the intermediate good firm cannot re-optimise its price, which is defined as follows:

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases}$$

i^{th} intermediate goods firm's objective to maximise profit subject to Calvo friction is given by:

$$E_t^i \sum_{n=0}^{\infty} \beta^n \lambda_{t+n} \left[P_{i,t+n} Y_{i,t+n} - P_{t+n}^d mc_{t+n} Y_{i,t+n} \right]$$

where λ_{t+n} is the Lagrange multiplier on household budget constraint and $mc_t = \frac{X_t}{P_t^d}$ is the real marginal cost. The left term of the equation, $P_{i,t+n}Y_{i,t+n}$, is the revenue, and the right term, $P_{t+n}^d mc_{t+n} Y_{i,t+n}$, is the total cost for i^{th} intermediate producers. The solution of i^{th} intermediate goods firm's profit maximisation problem will lead to

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t^d} \equiv \frac{K_t}{F_t}$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1}(1-V)\frac{Y_t}{C_t} mc_t + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon} K_{t+1} \tag{19}$$

$$F_t = \frac{Y_t}{C_t} + \beta\theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \tag{20}$$

and set $V = \frac{\varepsilon-1}{\varepsilon}$ to minimise monopoly distortion.

From the optimal price setting as per equation (16), we can also express the Calvo equation as

$$P_t^d = \left((1-\theta)\tilde{P}_t^{1-\varepsilon} + \theta (P_{t-1}^d)^1 - \varepsilon \right)^{\frac{1}{1-\varepsilon}}$$

and divide by P_t^d to get

$$1 = \left((1-\theta)\tilde{p}_t^{1-\varepsilon} + \theta \left(\frac{1}{\bar{\pi}_t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

where $\tilde{p}_t$ is the relative price of the marginal price setter. It follows then,

$$\tilde{p}_t = \left[\frac{1 - \theta\,(\bar{\pi}_t)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

3.6 Production of final domestic consumption goods

Final consumption goods purchased by both types of households are produced by a representative, competitive firm given by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}}(C_t^d)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_t^m)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{22}$$

where C_t is the final consumption good. The production of final consumption goods consists of inputs from domestic input and imported goods. The domestic input is represented by the domestic homogeneous final good, C_t^d. C_t^m represents goods imported from abroad required to produce the final consumption good. η is the elasticity of substitution between foreign and domestic homogeneous final goods. α is the share of foreign goods required to produce final consumption goods.

The profit maximisation problem by representative firms producing final consumption good is given by:

$$\begin{aligned} &\max_{P_t} P_t C_t - P_t^m C_t^m - P_t^d C_t^d \\ &\text{subject to} \quad C_t = \left[(1-\alpha)^{\frac{1}{\eta}}(C_t^d)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_t^m)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

where P_t is the price of final consumption good, P_t^m is the price of imported goods, and P_t^d is the price of domestic homogeneous output goods. The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_t^m : P_t \frac{dC_t}{dC_t^m} = P_t^m \to C_t^m = \alpha \left(\frac{P_t}{P_t^m}\right)^{\eta} C_t \tag{23}$$

$$C_t^d : P_t \frac{dC_t}{dC_t^d} = P_t^d \to C_t^d = (1-\alpha) \left(\frac{P_t}{P_t^d}\right)^{\eta} C_t \tag{24}$$

Substituting the demand functions of (23) and (24) into the production function (22) gives

$$P_t = \left[(1-\alpha)(P_t^d)^{1-\eta} + \alpha\,(P_t^m)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

which becomes

$$p_t = \left[(1-\alpha) + \alpha\,(p_t^m)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{25}$$

where $p_t \equiv \frac{P_t}{P_t^d}$ and $p_t^m \equiv \frac{P_t^m}{P_t^d}$. p_t is also known as the final consumption price index.

3.7 Real exchange rate and inflation

The real exchange rate is defined by:

$$p_t^m = \frac{P_t}{P_t}\frac{P_t^m}{P_t^d} = \frac{P_t}{P_t^d}\frac{P_t^m}{P_t} \Rightarrow p_t^m = p_t \times reals_t \tag{26}$$

where $reals_t \equiv \frac{P_t^m}{P_t} \equiv \frac{S_t P_t^f}{P_t}$ is the real exchange rate. This equation governs the relationship between domestic and imported goods prices, where the real exchange rate, $reals_t$, equals the price of imported goods in local currency, $S_t P_t^f$, relative to the final consumption good price, P_t. This implies either a decrease in final consumption good price, P_t, or depreciation in the nominal exchange rate, S_t, or higher foreign goods prices in foreign currency unit, P_t^f, will lead to a weakness in the real exchange rate.

From the price index as per equation (25), we can define the consumption good inflation and homogeneous good inflation growth rates as follows:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{P_t^d p_t}{P_{t-1}^d p_{t-1}} = \bar{\pi}_t \left[\frac{(1-\alpha) + \alpha\,(p_t^m)^{1-\eta}}{(1-\alpha) + \alpha\left(p_{t-1}^m\right)^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{27}$$

3.8 Exports

Homogeneous final goods produced by the domestic homogeneous final goods producers are also being exported out to foreign countries. The total demand by foreigners for domestic exports takes on the following form:

$$EX_t = \left(\frac{P_t^x}{P_t^f}\right)^{-\eta_f} Y_t^f = (p_t^x)^{-\eta_f}\, Y_t^f$$

where Y_t^f is foreign output, P_t^f is foreign currency price of foreign goods and P_t^X is an index of export prices in foreign currency. η_f is the elasticity of demand of foreign consumers for domestic exports. The effective terms of trade that capture movement of the relative prices of export prices and foreign homogeneous goods prices is given by:

$$p_t^x = \frac{P_t^x}{P_t^f}$$

3.9 Rate of depreciation and inflation

Competition between domestic homogeneous final goods producers and foreign producers will lead to the index of export prices in a domestic currency equal to the marginal cost. This leads the index of export prices in a domestic currency to equal the price of the domestic homogeneous final good, defined as:

$$S_t P_t^x = P_t^d$$

By dividing both sides by P_t^d, the relationship becomes

$$
\begin{aligned}
1 &= \frac{S_t P_t^x}{P_t^d} = \frac{P_t S_t P_t^f P_t^x}{P_t^d P_t P_t^f} \\
&= p_t reals_t p_t^x
\end{aligned}
\tag{28}
$$

and the growth rates of the real exchange rate, nominal exchange rate and foreign inflation are given by:

$$
\frac{reals_t}{reals_{t-1}} = s_t \frac{\pi_t^f}{\pi_t}, s_t \equiv \frac{S_t}{S_{t-1}}, \pi_t^f \equiv \frac{P_t^f}{P_{t-1}^f} \tag{29}
$$

3.10 Homogeneous goods market clearing

Let Y_t^* denote the unweighted integral of gross output across intermediate good producers:

$$
\begin{aligned}
Y_t^* &\equiv \int_0^1 Y_{i,t} di \\
&= \int_0^1 A_t \left(L_{i,t}^u\right)^\gamma \left(L_{i,t}^c\right)^{1-\gamma} di \\
&= A_t \left(L_t^u\right)^\gamma \left(L_t^c\right)^{(1-\gamma)}
\end{aligned}
$$

An alternative representation of Y_t^* makes use of the demand curve, as per equation (15):

$$
Y_t^* = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = Y_t P_t^\varepsilon \int_0^1 P_{i,t}^{-\varepsilon} di = Y_t P_t^\varepsilon (P_t^*)^{-\varepsilon}
$$

This leads to

$$
Y_t = p_t^* Y_t^* \Rightarrow Y_t = p_t^* A_t \left(L_t^u\right)^\gamma \left(L_t^c\right)^{(1-\gamma)}
$$

where $p_t^* = \left(\frac{P_t^*}{P_t}\right)^\varepsilon$. Here, $P_t^* \leq 1$ denotes Yun's (1996) measure of the output lost due to price dispersion. From equation (21), p_t^* can be defined as

$$
\begin{aligned}
p_t^* &= \int_0^1 P_{i,t}^{-\varepsilon} di \\
&= \left[(1-\theta)\tilde{p}_t^{-\varepsilon} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*}\right]^{-1} \\
&= \left[(1-\theta)\left(\frac{1-\theta\bar{\pi}_t^{(\varepsilon-1)}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \frac{\bar{\pi}_t^\varepsilon}{p_{t-1}^*}\right]^{-1}
\end{aligned}
\tag{30}
$$

Clearing in the domestic homogeneous goods market leads to the production of domestic homogeneous goods, Y_t, equals to the absorption of domestic homogeneous goods, defined as:

$$\begin{aligned} Y_t &= (C_t^u)^d + (C_t^c)^d + EX_t + G_t \\ &= C_t^d + EX_t + G_t \\ &= (1-\alpha)\,(p_t)^{\eta}\,C_t + EX_t + G_t \end{aligned}$$

where total consumption of the domestic homogeneous goods by patients and impatient households, $C_t = C_t^u + C_t^c$, equals to $(1-\alpha)(p_t)^{\eta}C_t$. G_t represents spending by the government. EX_t is exports of domestic homogeneous goods. Substituting out in previous expression for $Y_t = A_t p_t^*(L_t^u)^{\gamma}(L_t^c)^{1-\gamma}$, the relationship is given by:

$$A_t p_t^*(L_t^u)^{\gamma}(L_t^c)^{1-\gamma} = (1-\alpha)\,(p_t)^{\eta}\,C_t + EX_t + G_t \tag{31}$$

3.11 Balance of Payments

Equality of international flows relating to trade in goods and in financial assets is defined by:

$$S_t A_{t+1}^f + \text{expenses on imports}_t = \text{receipts from exports}_t + S_t[\Phi_{t-1}R_{t-1}^f]A_t^f$$

where $S_t A_{t+1}^f$ defines the acquisition of new net foreign assets in domestic currency units and $S_t[\Phi_{t-1}R_{t-1}^f]A_t^f$ captures receipts from existing stock of net foreign assets in domestic currency units. A_t^f is the net stock of foreign assets at the beginning period, t. A positive A_t^f reflects net foreign assets held by domestic residents. On the contrary, a negative A_t^f reflects net foreign liabilities held by domestic residents.

Exports and imports in domestic currency are given by:

- expenses on $\text{imports}_t = S_t P_t^f \alpha \left(\frac{P_t}{P_t^m}\right)^{\eta} C_t$
- receipts from $\text{exports}_t = S_t P_t^x EX_t$

Incorporating the expression for exports and imports in domestic currency unit, the equation for the BoP can be defined as:

$$S_t A_{t+1}^f + S_t P_t^f \alpha \left(\frac{P_t}{P_t^m}\right)^{\eta} C_t = S_t P_t^x EX_t + S_t[\Phi_{t-1}R_{t-1}^f]A_t^f$$

and dividing by P_t^d on both sides

$$\frac{S_t A_{t+1}^f}{P_t^d} + \frac{S_t P_t^f}{P_t^d}\alpha \left(\frac{P_t}{P_t^m}\right)^{\eta} C_t = \frac{S_t P_t^x}{P_t^d} EX_t + \frac{S_t R_{t-1}^f \Phi_{t-1} A_t^f}{P_t^d}$$

the final expression for BoP is given by:

$$a_t^f + p_t^m \alpha \left(\frac{p_t}{p_t^m}\right)^\eta C_t = p_t reals_t p_t^x EX_t + \frac{s_t R_{t-1}^f \Phi_{t-1} a_{t-1}^f}{\bar{\pi}_t} \tag{32}$$

where $a_t^f = \frac{S_t A_{t+1}^f}{P_t^d}$. And the previous definitions are $p_t \equiv \frac{P_t}{P_t^d}$, $p_t^m \equiv \frac{P_t^m}{P_t^d}$, $\frac{P_t^m}{P_t} \equiv \frac{S_t P_t^f}{P_t}$ and $S_t P_t^x = P_t^d$. Intuitively, a_t^f can be interpreted as a value of net foreign assets in terms of domestic homogeneous goods.

3.12 Risk Term

The risk term that governs the movement of the nominal exchange rate is given by:

$$\Phi_t = \Phi\left(a_t^f, R_t^f, R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(R_t - R_t^f - \left(R - R^f\right)\right) + \tilde{\phi}_t\right) \tag{33}$$

where $\tilde{\phi}_a > 0$, $\tilde{\phi}_s > 0$ and $\tilde{\phi}_t \backsim$ mean zero, iid. $\tilde{\phi}_a > 0$ implies if $a_t^f > \bar{a}$, then the return on foreign assets is low, and will lead to a reduction in net foreign assets, a_t^f, held by domestic residents. On the other hand, if $a_t^f < \bar{a}$ then the return on foreign assets is high and will lead to the increase in net foreign assets. $\tilde{\phi}_t$ captures the possibility that there is a shock to the required return on domestic assets. The term $\tilde{\phi}_s > 0$ is required to adjust for the standard uncovered interest parity (UIP) relationship. The adjustment will make a higher domestic nominal interest rate leading to an increase in the rate of return required for people to hold domestic assets. Under the standard UIP, which implies $\tilde{\phi}_s = 0$, it means that a higher domestic nominal interest rate and expected appreciation of the currency represents a double-boost to the return on domestic assets. However, the scenario where investors increase their domestic asset holding due to higher domestic nominal interest rates is not observed empirically. Therefore, the modification of the UIP condition allows for a negative correlation between the risk premium and the expected change in the nominal exchange rate, and it will increase the persistence in the real exchange rate, which has an empirical advantage compared with the standard UIP specification (Adolfson et al., 2007).

3.13 Monetary policy - Taylor rule

The central bank conducts policy according to a Taylor-type rule. The central bank reacts to changes in aggregate inflation and the growth rate of output as follows:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} E_t\left[\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_{t+1}}{Y}\right)^{r_y}\right]^{1-\rho_R} \varepsilon_{R,t} \tag{34}$$

where ρ_R is a smoothing parameter, where a bigger ρ_R means changes in nominal interest rate will become more persistent. r_π is the weight given to minimise the expected deviation of inflation from its target, and it satisfies the Taylor Principle when $r_\pi > 1$. r_y is the weight given to minimise the expected deviation of output from its target.

3.14 Government spending

The model assumes that the government also purchases domestic homogeneous final good. The government, however, does not spend its money on imported goods. The process of the government spending in this model is defined as:

$$\log(G_t) = (1 - \rho_g)\eta_g \log Y + \rho_g \log(G_{t-1}) + \varepsilon_{G_t} \tag{35}$$

where ρ_g reflects the persistence of policy-induced changes in the fiscal policy and η_g captures the size or ratio of the government spending to domestic production.

3.15 Housing market

The total quantity of houses is fixed in this model. The total units of houses available for patient and impatient households to own is given by:

$$H_t^u + H_t^c = 1 \tag{36}$$

In this model, any developments in the housing market will not alter domestic output directly, as domestic production only accounts for the production of domestic homogeneous final good. However, any developments in the housing market, which will affect house prices, still could indirectly affect domestic output through a knock-on effect on consumption.

3.16 Equilibrium equations

1. Total labour supply

   Total labour supply in the economy equals aggregation of labour supply from patient and impatient households.

   $$L_t = L_t^u + L_t^c$$

2. Total consumption

   Total domestic consumption in the economy equals aggregation of consumption from patient and impatient households.

   $$C_t = C_t^u + C_t^c$$

3. Total savings and borrowing in the domestic economy

Aggregate lending from patient households is equal to aggregate borrowing of impatient households.

$$b_t^c = b_t^u$$

4. Aggregate inflation, domestic homogeneous inflation and inflation target

   In equilibrium, aggregate inflation equals domestic homogeneous inflation and the inflation target by the central bank.

$$\pi = \bar{\pi} = \pi^T$$

3.17 Foreign sector and risk term

The model assumes that foreign inflation, output and interest rate are exogenously given. The risk term shock is also exogenously given in this model. All the exogenous processes for the foreign sector and the risk term follow an AR(1) process given by:

1. Foreign inflation

$$\log(\pi_t^f) = 0.95 \log(\pi_{t-1}^f) + \varepsilon_{\pi_t^f}$$

2. Foreign output

$$\log(Y_t^f) = 0.95 \log(Y_{t-1}^f) + \varepsilon_{Y_t^f}$$

3. Foreign interest rate

$$\log(R_t^f) = 0.95 \log(R_{t-1}^f) + \varepsilon_{R_t^f}$$

4. Risk term

$$\log(\Phi_t) = 0.95 \log(\Phi_{t-1}) + \varepsilon_{\Phi_t}$$

4 Model parameters and analysis

The time period for the model is a quarter. I calibrate the full model by using parameters that have already been estimated by Iacoviello (2005) and Christiano, Trabandt, and Walentin (2011). Christiano, Trabandt, and Walentin (2011) estimate their model using Swedish data, where the sample period covers the first quarter of 1995 until the third quarter of 2010. In solving the model, all the equations are log-linearised around the steady state, and the perturbation methods are only valid in a neighbourhood of the steady state. The departure from the steady state will result in the deterioration in the quality of the linear approximation.

4.1 Calibration

There are 19 parameters that need to be calibrated altogether in this model. The discount factor for patient households, β, is set to 0.99 so that the annual interest rate is 4% in a steady state. The discount factor for impatient households, $\tilde{\beta}$, is set to 0.98. The weight of housing on the householdsâ utility function, j, is set to 0.1. The households labour supply elasticity is set to $\varphi - 1 = 0.01$. The LTV ratio is set to 0.90. The labour income share of patient households, γ, is set to 0.6. The elasticity of demand for domestic intermediate goods by domestic homogeneous final goods producers, φ is set to 6. The steady state gross inflation target is set to 1.005 or 2% annually. The share of imported goods in the production of final consumption goods, α, is set to 0.6, consistent with Christiano et al. (2011). The elasticity of substitution between foreign and domestic homogeneous final goods in the production of final consumption goods is set to 5. For the risk term, I used $\tilde{\phi}_a = 0.03$ and $\tilde{\phi}_s = 0.7$. For the Taylor-rule parameters, r_π and r_y are set to 1.5 and 0.15 respectively. ρ_R is set to 0.90, as found widely in the literature. The probability of not changing prices, θ, is set to 0.75, implying that prices change every four quarters on average. The government expenditure share of GDP, η_g, is set at 0.30. The full calibrated values are displayed in Table 2.

Table 2: Calibrated parameters

Parameter	Value	Description
A	1	Aggregate technology
α	0.6	Share of imported goods in final consumption
β	0.99	Savers' discount factor
$\tilde{\beta}$	0.98	Borrowers' discount factor
ε	6	Elasticity of demand for domestic intermediate goods
η	5	Elasticity of substitution between foreign and domestic final goods
η_f	1.5	Elasticity of demand for exports
η_g	0.3	Government expenditure share of GDP
γ	0.6	Savers' labor income share
j	0.1	Weight of housing on the utility function
LTV	0.9	Loan-to-value ratio
π^T	1.005	Steady state gross inflation target
$\tilde{\phi}_a$	0.03	Weight of net foreign assets in risk term
$\tilde{\phi}_s$	0.7	Weight of interest rate differential in risk term
φ	1.01	$\varphi - 1$ is the labor-supply elasticity
r_π	1.5	Taylor Rule, inflation parameter
r_y	0.15	Taylor Rule, output parameter
ρ_R	0.9	Taylor Rule, interest rate smoothing parameter
θ	0.75	Calvo parameter, probability of intermediate good firms cannot set prices

4.2 Shocks

In total, the model has seven exogenous stochastic variables, consisting of four domestic and three foreign shocks. Compared to a closed economy model, a small open economy model allows us to shock the model with the risk term and foreign shocks. The shock on risk terms will enable me to study the impact on key variables as a result of the change in the relative riskiness of foreign assets compared to domestic assets. In addition, shocks in the foreign variables provide information on how they will propagate to the domestic economy and produce the optimal policy response by the central bank.

The domestic shocks are as follows:

1. Inflation target shock, π^T
2. Government spending shock, G
3. Monetary policy shock, ε_R
4. Risk term shock, $\tilde{\phi}$

The foreign shocks are as follows:

1. Foreign inflation shock, π^f
2. Foreign output shock, Y^f
3. Foreign interest rate shock, R^f

I can shock any of these seven exogenous stochastic variables in the model to generate the impulse response functions. Some of the key results are discussed in subsection 4.3.

4.3 Impulse response functions

The impact of borrowing constraints and the LTV ratio on key macroeconomics variables is discussed here. I start the discussion by comparing the effect of borrowing constraints, with and without the LTV ratio, in taming the financial and business cycles. Removing the LTV implies that impatient households will only face the collateralise debt repayment. Without the LTV, borrowing constraint and equation (13) need to be adjusted as follows:

$$B_t^c \leq \frac{E_t \pi_{t+1} Q_{t+1} H_t^c}{R_t} \tag{37}$$

$$\frac{j}{H_t^c} = \frac{1}{C_t^c} \left(q_t - \frac{E_t q_{t+1} \pi_{t+1}}{R_t} \right) \tag{38}$$

Subsequently, I compare the impact of borrowing constraints and the LTV ratio in managing business and financial cycles by altering the LTV rule. The LTV ratio can be either under fixed or countercyclical LTV rules. The fixed LTV rule is simply setting the borrowing constraint imposed on impatient households unchanged at $LTV = 0.9$, regardless of where the economy is currently in the financial cycle stage. Alternatively, I make the LTV rule to be countercyclical and more responsive to the financial cycle by following this simple rule:

$$LTV = 0.9 - 0.5(b_t^c - b_{t-1}^c)$$

where it says that the LTV ratio will be tightened if credit growth expands positively, and the LTV ratio will be relaxed if credit growth contracts. This means that the LTV ratio will become more expansionary during the credit growth upcycle and more contractionary during the credit growth downcycle. This analysis allows us to understand better the LTV ratio's effectiveness under different circumstances. Furthermore, for us to have a better understanding of how the different LTV rules affect the economy following the extension of the original model by Iacoviello (2005) for a closed economy, the discussion on the impact of different LTV rules also includes the comparison between open and closed economy. This will help us to see whether the effectiveness of LTV rules may change as we move from a closed to an open economy model. The detailed specifications for the open economy as described above and the exact specifications are applied for the closed economy, except it became much simpler without nominal and real exchange rates, external trade and cross-border financial flows.

4.3.1 Response to monetary policy shock - Figures 1 to 8

1. Open economy: Fixed LTV vs Without LTV (Figure 1)

   The impact on domestic output and credit growth is more significant without the LTV rule. The impulse response functions of all variables are more muted with the LTV ratio, as this successfully mitigates large fluctuations caused by the monetary policy shock. This underscores the effectiveness of the LTV ratio in constraining the financial cycle.

2. Open economy: Fixed LTV vs Countercyclical LTV (Figure 2)

   The impact on domestic output is similar under both LTV rules. However, consumption for the patient and impatient households, credit growth and house prices are more impacted under the fixed LTV rule. This implies that, under the countercyclical LTV rule, the producers of final consumption goods change their composition between domestic and imported goods in their production by using more imported goods, as suggested by a smaller contraction in imports. Higher imports offset the smaller contraction in consumption by both households, resulting in the decline of domestic output in the countercyclical LTV rule, almost perfectly similar to the fixed LTV rule. This simply reflects the expenditure switching channel as imported goods become cheaper, at least temporarily, thanks to an appreciation in nominal and real exchange rates

3. Closed economy: Fixed LTV vs Countercyclical LTV (Figure 3)

   The initial contraction in domestic output is smaller under the countercyclical LTV rule. Similarly, all other variables demonstrate a more muted impact under the countercyclical LTV rule which is more effective in managing the business cycle and financial stability than the fixed LTV rule. Without trade and movement in the exchange rate, the impact of monetary policy shock is more potent and fully reflected in the decline in consumption of both households without the expenditure switching channel in place.

4. Fixed LTV: Open vs Closed economy (Figure 4)

   The initial magnitude of contraction in domestic output is similar for an open and closed economy, but the subsequent recovery in output is faster for a closed economy. However, the impact on house prices and consumption for patient households is more severe for a closed economy. On the contrary, consumption for impatient households experiences a larger fall for the open economy. Credit growth registers a bigger decline for the open economy, as demand for houses by impatient households falls more dramatically compared to the closed economy. The increase in consumption for patient households despite the contraction in domestic output may be well explained by the expenditure switching channel. Final consumption goods producers will use more imported goods in their productions as they become cheaper relative to domestic homogeneous final goods due to the appreciation in the exchange rate.

5. Countercyclical LTV: Open vs Closed economy (Figure 5)

   The initial contraction in domestic output is larger for an open economy compared to a closed economy. The subsequent recovery for an open economy is also weaker as opposed to a closed economy. However, the impact on house prices and consumption for patient households is more severe for a closed economy. Similarly, credit growth registers a bigger decline for the open economy, as demand for houses by impatient households falls more dramatically compared to the closed economy. Consistent with the fixed LTV case as per Figure 3, this again can be explained by the expenditure switching channel.

6. Fixed LTV: Open ($\alpha \approx 0$) vs Closed economy (Figure 6)

   In order to close the expenditure switching channel under the fixed LTV rule, I have set the share of imported goods, α, in the production of final consumption goods to be equal to 0.01% as opposed to 60% under the baseline model. By doing this, we should expect all variables to respond in a similar fashion for both open and closed economies. As expected, once I shut down the expenditure switching channel, the magnitude of contraction in domestic output and other variables for the open and closed economy is very similar.

7. Countercyclical LTV: Open ($\alpha \approx 0$) vs Closed economy (Figure 7)
   As per case 6, in order to close the expenditure switching channel under the countercyclical LTV rule, I have set the share of imported goods, α, in the production of final consumption goods to be equal to 0.01% as opposed to 60% under the baseline model. As expected, once we mute the expenditure switching channel, the magnitude of contraction in domestic output and other variables for an open and closed economy is very similar.

8. Countercyclical LTV: Open ($\eta = 0.1, 5, \text{and} \, 9$) vs Closed economy (Figure 8)
   To better understand the expenditure switching channel, I have set the elasticity of substitution between foreign and domestic final goods, η, to 0.1 and 9 from 5 in the baseline model. Setting $\eta = 0.1$, the degree of substitutability between foreign and domestic final goods is smaller. In this environment, the expenditure switching channel will become less significant, and the results between open and closed economies are expected to be closer. This is an alternative to studying the expenditure switching channel, instead of bringing the share of imported goods, α, in the production of final consumption goods to be equal to 0.01% compared to 60% under the baseline model, as per Figures 6 and 7. On the other hand, setting $\eta = 9$, makes the degree of substitutability between foreign and domestic final goods larger. The appreciation in the exchange rate will lead to higher import of foreign final goods. As expected, the magnitude of contraction in domestic output and other variables for an open and closed economy is very similar when $\eta = 0.1$, and it becomes more significant when $\eta = 9$, as it increases the role of the expenditure switching channel.

The main results suggest that the LTV ratio is effective in taming the financial cycle for a small open economy. The result is consistent with a closed economy. However, the impact of the LTV ratio in smoothing the business cycle for a small open economy is highly dependable on the size of trade openness and elasticity of substitution between domestic and imported goods in the production of final consumption goods. Between the fixed and countercyclical LTV rules, the latter is more effective in ensuring key variables experience smaller deviations from their steady state levels. More importantly, from a financial stability point of view, credit growth registers a more muted impact under the countercyclical LTV rule. This, in turn, will lead to a more manageable financial cycle, which is fundamental to achieving financial stability.

Figure 1: Open economy: Fixed LTV vs Without LTV - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

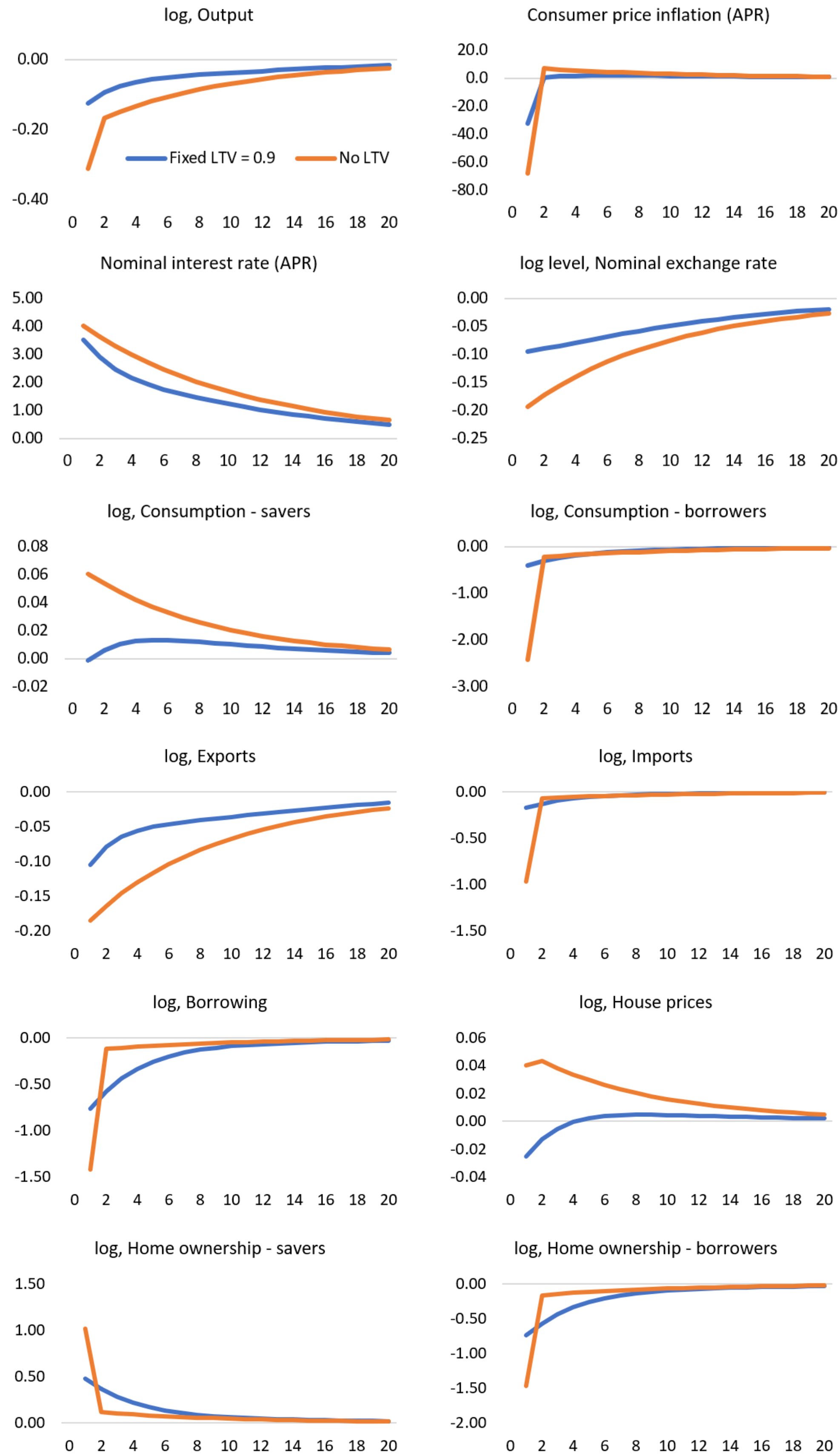


Figure 2: Open economy: Fixed LTV vs Countercyclical LTV - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

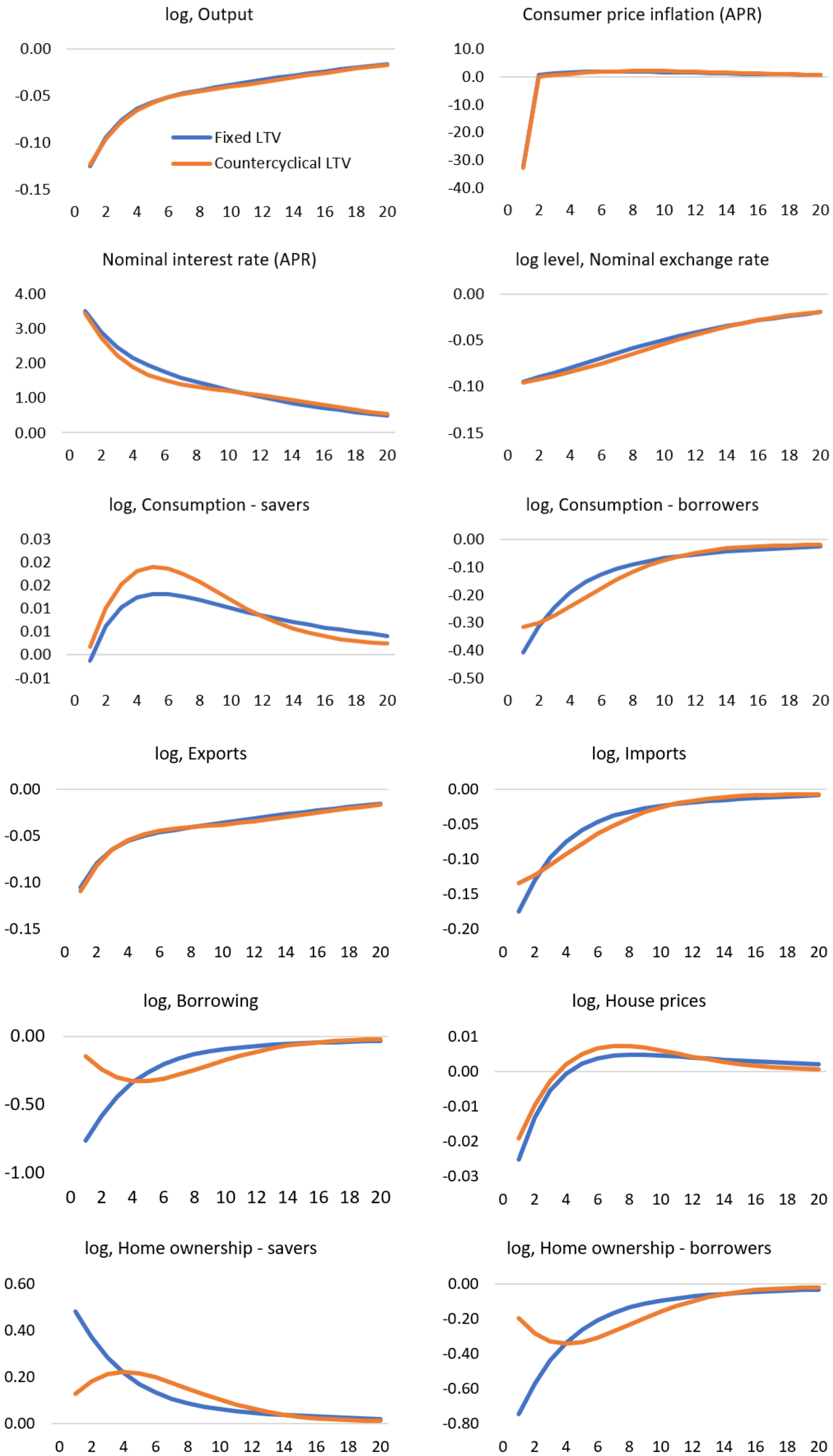


Figure 3: Closed economy: Fixed LTV vs Countercyclical LTV - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

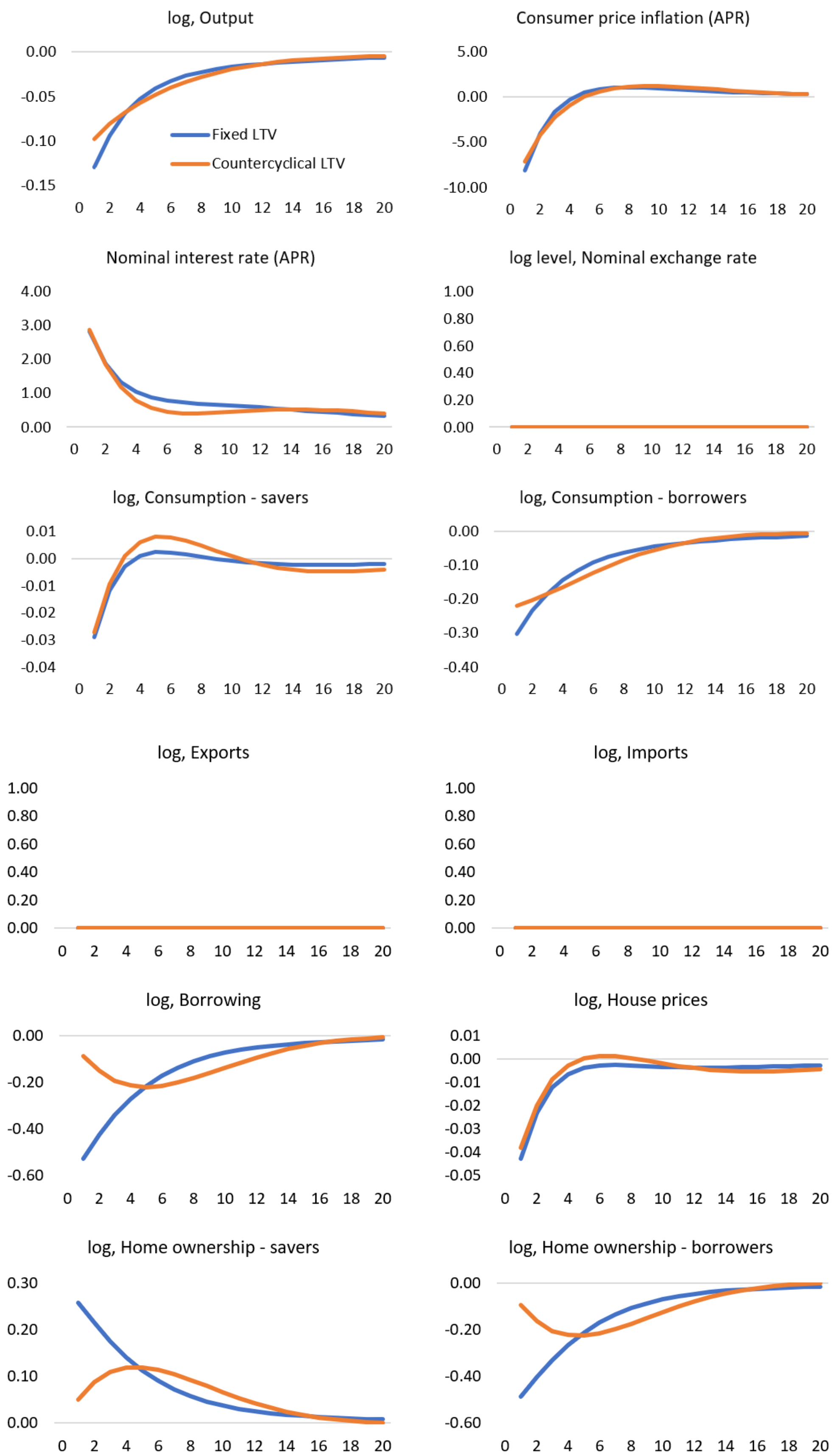


Figure 4: Fixed LTV: Open vs Closed economy - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

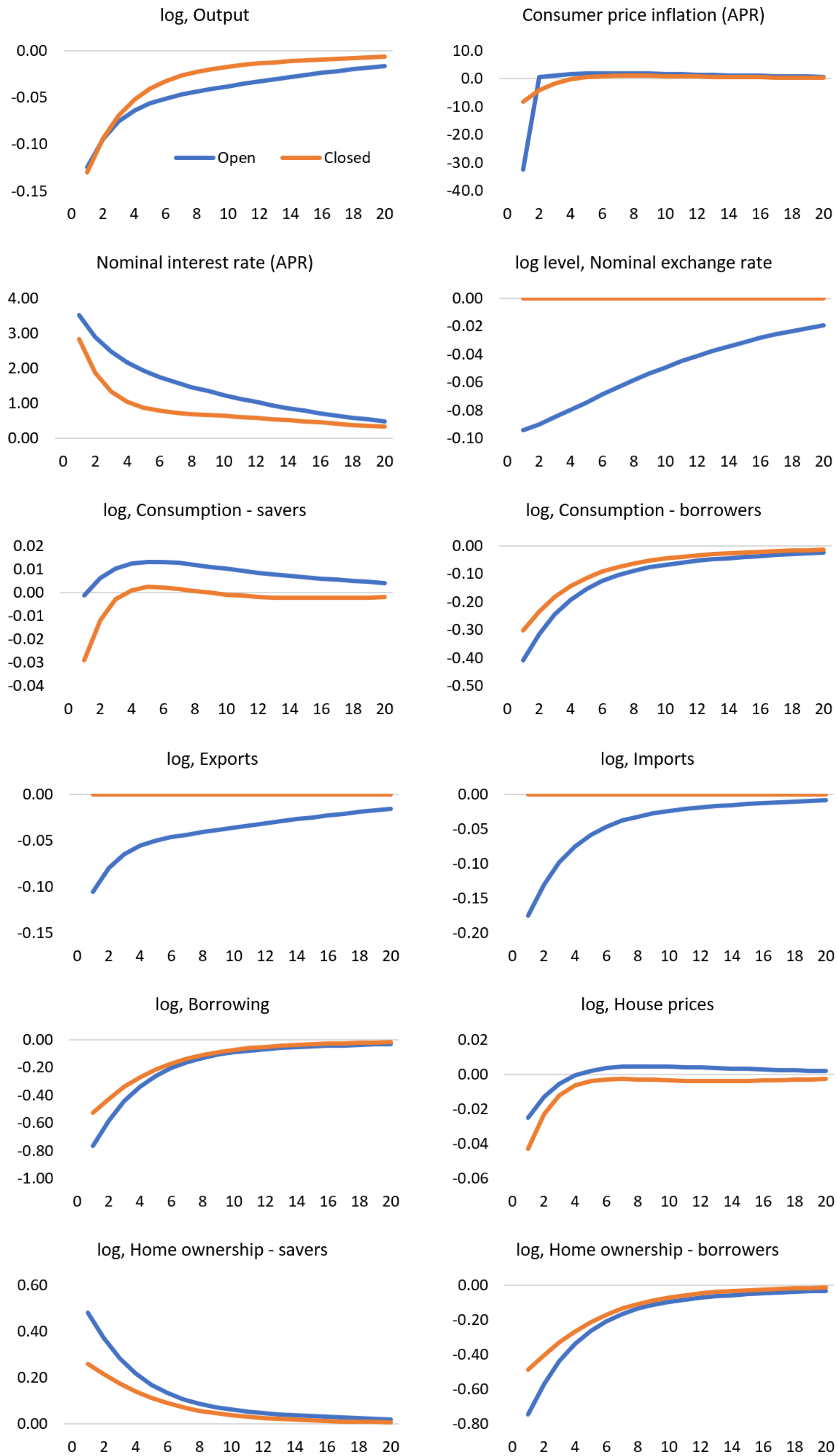


Figure 5: Countercyclical LTV: Open vs Closed economy - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

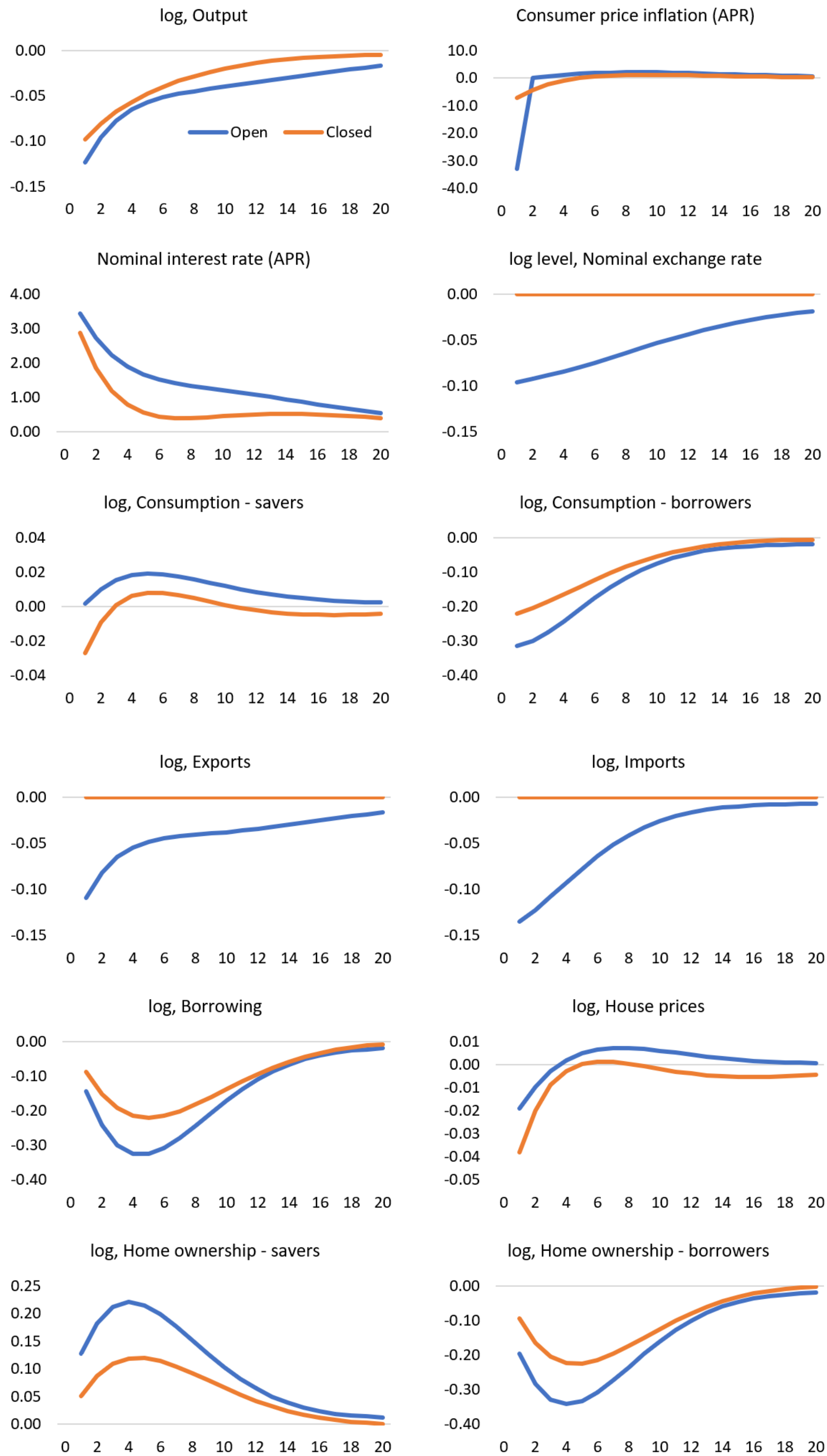


Figure 6: Fixed LTV: Open ($\alpha \approx 0$) vs Closed economy - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

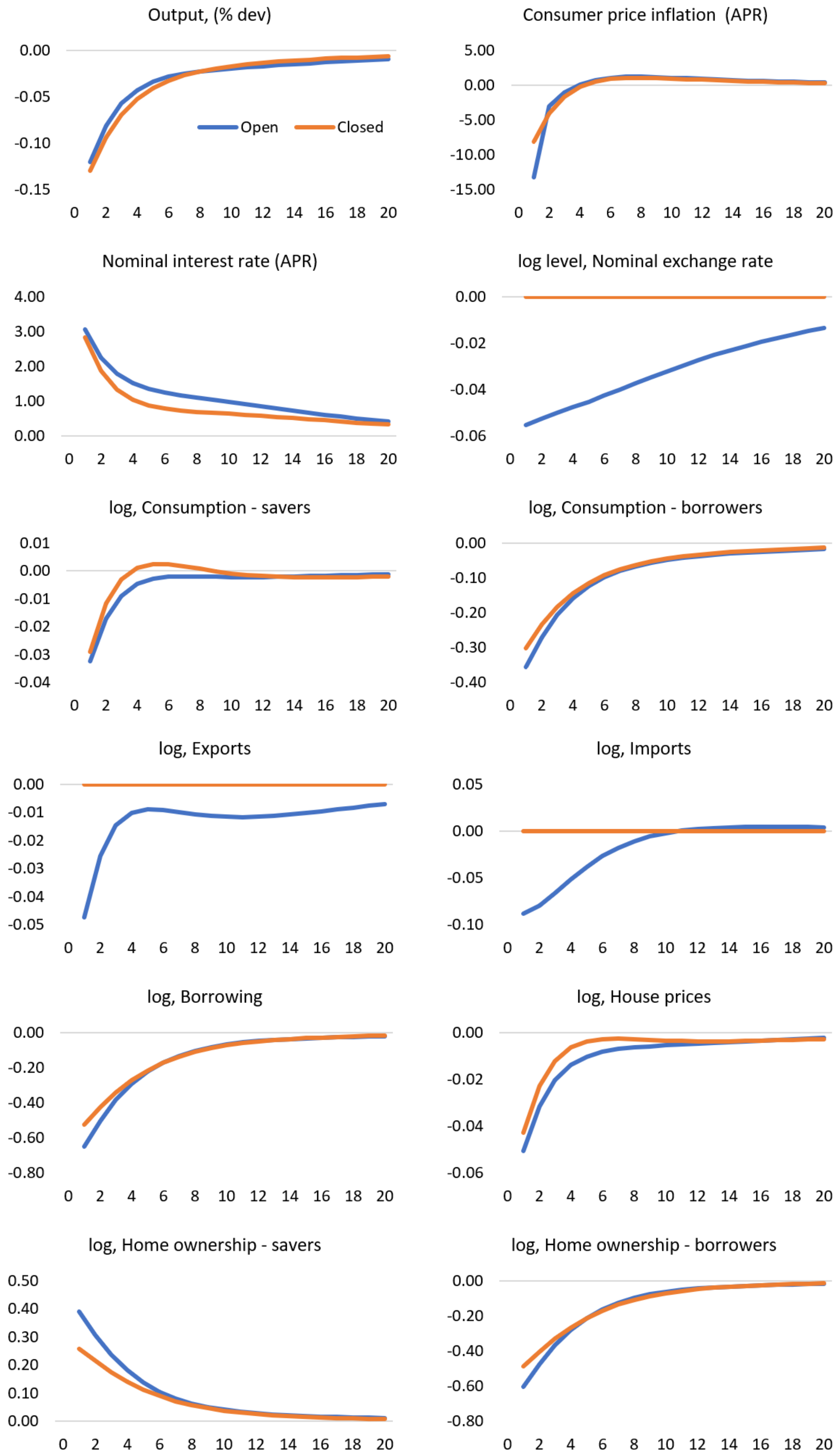


Figure 7: Countercyclical LTV: Open ($\alpha \approx 0$) vs Closed economy - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline

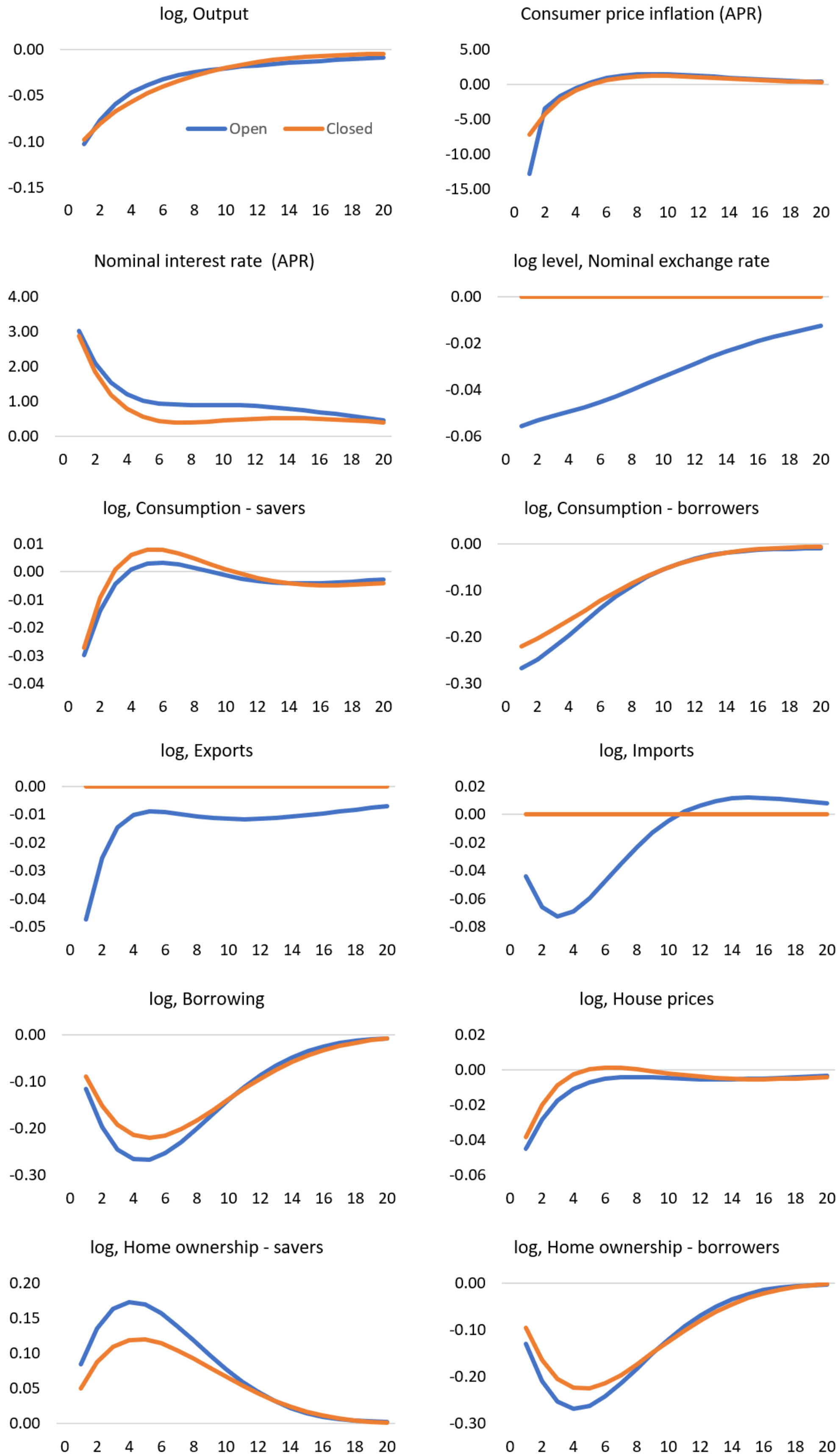
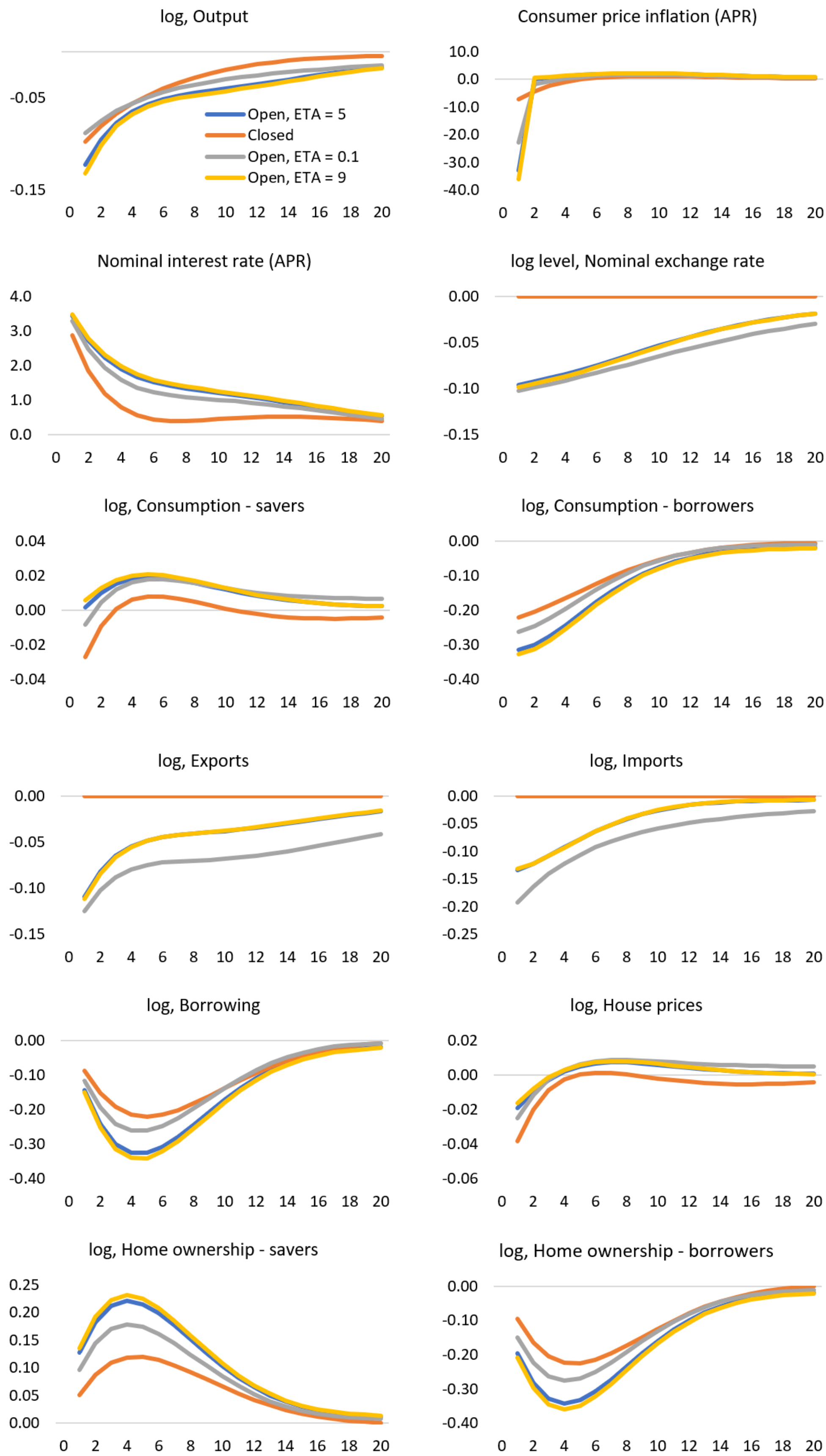


Figure 8: Countercyclical LTV: Open ($\eta = 0.1, 5, \text{and}\, 9$) vs Closed economy - Response to monetary policy shock, $\varepsilon_{R_t} = 0.01$

Coordinate: deviation from the baseline



4.3.2 Response to risk premium shock (Figure 9)

In this section, I want to study the role of the LTV ratio in taming the business and financial cycles following a stationary risk premium shock. The increase in domestic risk premium implies a reduction of domestic and foreign interest in domestic assets in favour of foreign assets. The liquidation of domestic assets and an increase in net foreign assets holding lead to a depreciation in the nominal exchange rate. As a result, output and inflation deviate positively from their steady states, as a weaker exchange rate translates into higher export receipts. Imports also grow positively, but not as strongly as exports, leading to a rise in net export receipts. The smaller increase in imports is also a reflection of expenditure switching by final consumption goods producers as imported goods become more expensive due to the depreciation in the nominal exchange rate. In both fixed and countercyclical LTV rules, domestic output and inflation increase and dissipate at a similar magnitude and pace. However, credit expansion and change in house ownership are more muted under the countercyclical LTV rule. In addition, consumption for impatient households and imports do not increase as much relative to the fixed LTV rule.

4.3.3 Response to technology shock (Figure 10)

In this part, I study the impulse responses on domestic output, credit growth and other key variables following a stationary technology shock. The evolution of a stationary technology shock is defined as follows:

$$\Delta a_t = 0.90 \Delta a_{t-1} + \varepsilon_{\Delta a_t}$$

where $a_t \equiv \log A_t$ and $\Delta a_t = a_t - a_{t-1}$. 0.90 captures the persistence of an initial shock to technology. In order for me to introduce a stationary technology shock in the model, I need to scale H_t^u, H_t^c, C_t^u and C_t^c with A_t. To reflect the changes as a result of the scaling, I need to make some minor adjustments to equations (3), (5), (6), and (13). The new expression for equations (3), (5), (6), and (13) are given by:

$$\frac{1}{c_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} c_{t+1}^u \exp(\Delta a_{t+1})} \right)$$

$$\frac{j}{h_t^u} = \frac{1}{c_t^u} q_t - \beta E_t \frac{1}{c_{t+1}^u \exp(\Delta a_{t+1})} q_{t+1}$$

$$\frac{S_t}{c_t^u} = \beta E_t \frac{S_{t+1} R_t^f \Phi_t}{\pi_{t+1} c_{t+1}^u \exp(\Delta a_{t+1})}$$

$$\frac{j}{h_t^c} = \frac{1}{c_t^c} \left(q_t - \frac{LTV \, E_t q_{t+1} \pi_{t+1}}{R_t} \right) - \widetilde{\beta} E_t \frac{1}{c_{t+1}^c \exp(\Delta a_{t+1})} (1 - LTV) q_{t+1}$$

where $h_t^u = H_t^u/A_t$, $h_t^c = H_t^c/A_t$, $c_t^u = C_t^u/A_t$ and $c_c^u = C_t^c/A_t$. As expected, domestic output and inflation respond positively, where both increase from their steady states following the stationary technology shock. The impulse response functions for domestic output and inflation move in tandem under fixed and countercyclical LTV rules. The positive reaction of domestic output is also attributable to a depreciation in the nominal exchange rate, which benefits exports. As a result of domestic output and inflation deviating positively from their steady states, the central bank needs to tighten its monetary policy, which negatively affects credit growth. In both LTV rules, the initial increase in the nominal interest rate and their trajectories afterwards are similar. However, the impact on credit growth and changes in home ownership is more muted and far smaller under the countercyclical LTV rule. The results suggest that the countercyclical LTV rule is more effective than the fixed LTV rule in taming the financial cycle.

5 Extension

In this section, I study alternatives for the monetary policy regime and the countercyclical LTV rule that the central bank can consider in achieving its financial and output stabilisation mandates. This topic is not new and has been widely debated and discussed in various papers, especially for emerging market economies. In 2019, Agustín Carstens, the General Manager of the BIS, said in one of his public lectures and I quote, "Emerging market economies central banks have, in practice, attached a significant weight to the exchange rate in the conduct of their monetary policy, as reflected for instance in the evolution of FX reserves in emerging market economies over the past two decades". In fact, there are also recent papers that study the effectiveness of targeting the exchange rate instead of the short-term interest rate as the main monetary policy tool like Heipertz, Mihov, and Santacreu (2017) and Mariam et al. (2020)

5.1 An alternative monetary policy regime

I explore the alternative monetary policy regime by including the nominal exchange rate as another target variable by the central bank, on top of inflation and domestic output. The expression for the Taylor rule follows closely the approach taken by Eichenbaum, Johannsen, and Rebelo (2021) and given by:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} E_t \left[\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)^{r_\pi} \left(\frac{Y_{t+1}}{Y}\right)^{r_y} \left(\frac{s_t}{s}\right)^{r_s}\right]^{1-\rho_R} \varepsilon_{R,t} \tag{39}$$

where $r_s > 0$. According to equation (39), the monetary authority raises the nominal interest rate when the nominal exchange rate of domestic currency depreciates.

Figure 9: Response to risk premium shock, $\tilde{\phi}_t = 0.01$ - Open economy: Fixed vs Countercyclical LTV

Coordinate: deviation from the baseline

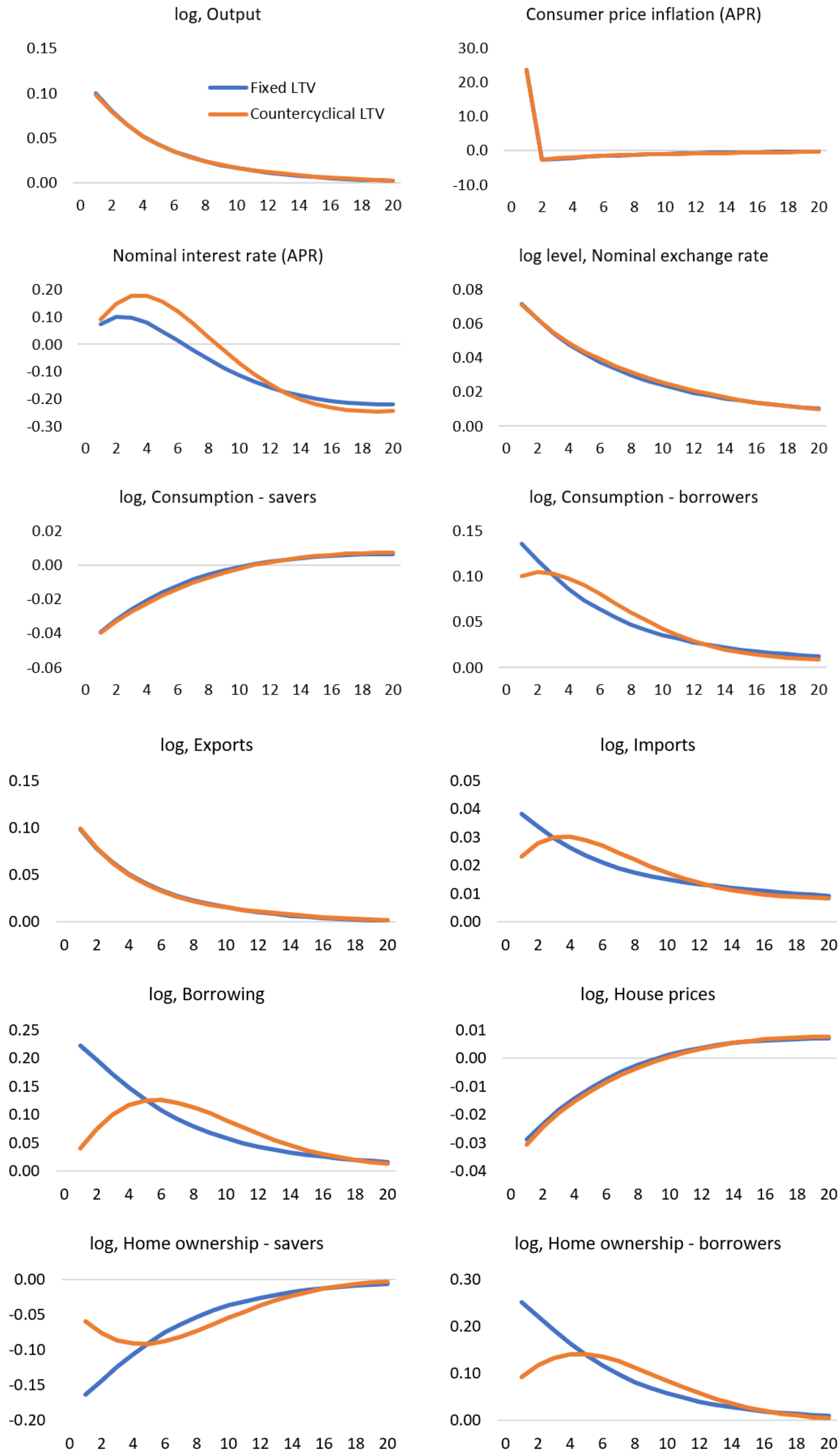
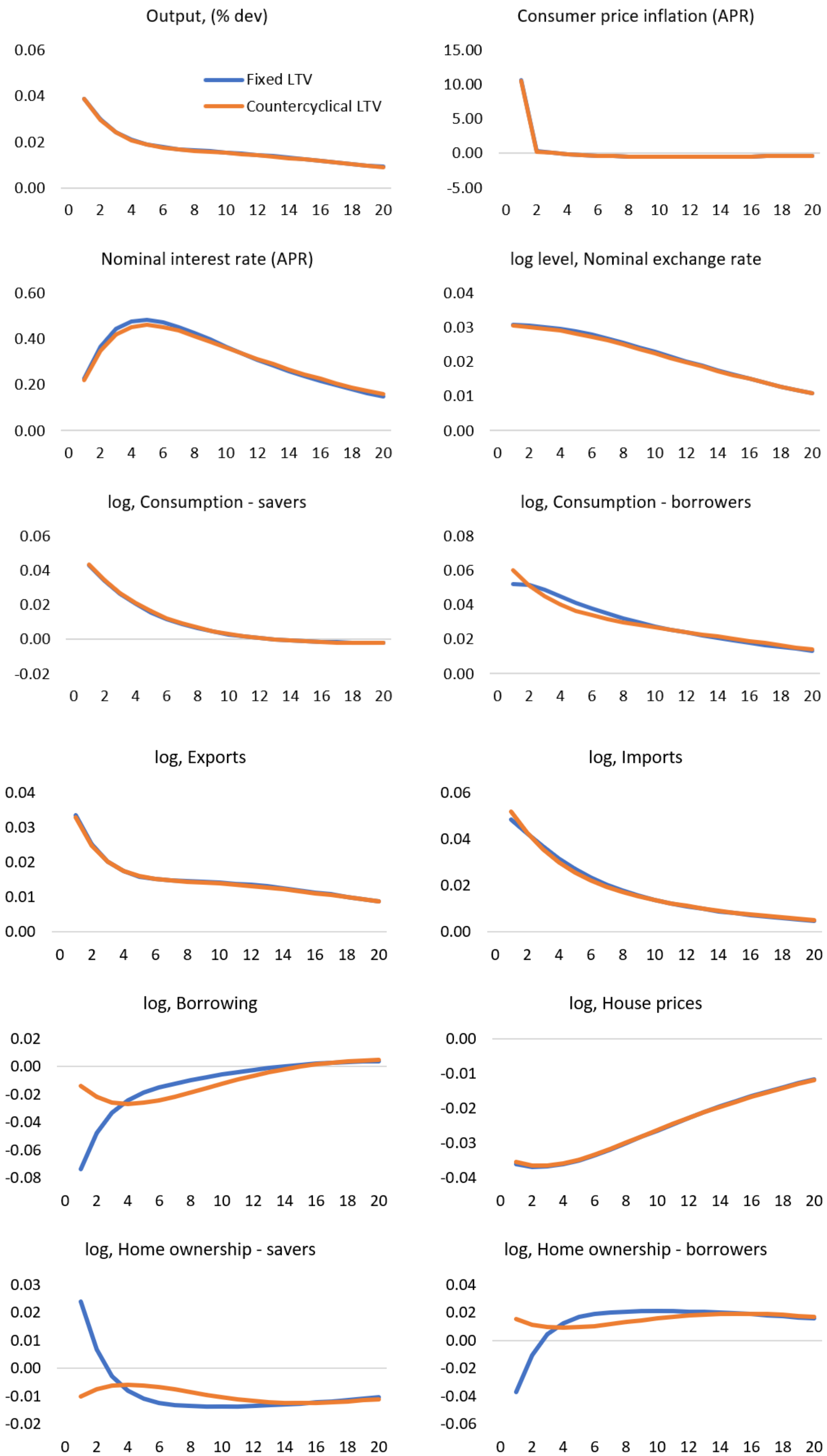


Figure 10: Response to technology shock, $\Delta a_t = 0.01$ - Open economy: Fixed vs Countercyclical LTV

Coordinate: deviation from the baseline



5.1.1 Response to technology shock (Figure 11)

Figure 11 shows that the response to a stationary technology shock results in a more significant increase in nominal interest rate, as the central bank needs to tighten more its monetary policy following a depreciation in domestic currency. The higher nominal interest rate affects the consumption of impatient households more than the standard monetary policy regime. Meanwhile, a more negligible depreciation in the nominal exchange rate leads to a minor export increase. This, in turn, leads to a smaller increase in domestic output. On the contrary, credit growth experiences a larger decline due to a higher nominal interest rate. This suggests that by including the nominal exchange rate as another target variable, the central bank's ability to tame the financial cycle is not as effective as under the baseline model. The result, which holds under the assumption of a frictionless financial market and banking system, may change drastically if I depart from this crucial assumption. This is in the wake of findings by recent studies, which show that depreciation in the exchange rate will weaken the financial cycle through the balance sheets channel. This is especially true for a small open economy with significant exposure to foreign lenders.

The weakness in domestic currency will lead to a drop in the net worth of domestic borrowers or domestic banks, resulting in the tightening of credit demand and supply, which subsequently will affect real economic activities. Under some circumstances, the financial channel dominates the trade channel, leading to a fall in output following a depreciation in the exchange rate (Hofmann et al. (2022). Recent studies have also shown the benefit of targeting the exchange rate. For example, Adrian et al. (2020) included a range of empirically-relevant frictions in their model and found that foreign exchange intervention (FXI) and capital flow management tools (CFMs) improve policy trade-offs under certain conditions. Similarly, Lama and Medina (2020) concluded that FXI and macroprudential policies are tools complementary to the monetary policy that can significantly reduce inflation and output volatility in a scenario of capital outflow.

5.2 An alternative countercyclical LTV rule

Similar to the alternative monetary policy regime, I explore the effectiveness of the countercyclical LTV rule by including the nominal exchange rate as another variable and given by:

$$LTV = 0.9 \times \left(\frac{b_t^c}{b_{t-1}^c}\right)^{-0.5} \times \left(\frac{s_t}{s_{t-1}}\right)^{-0.5} \tag{40}$$

According to equation (40), the countercyclical LTV rule will be tightened when the nominal exchange rate of domestic currency depreciates.

5.2.1 Response to technology shock (Figure 12)

Figure 12 shows that the response to a stationary technology shock results in a bigger decline in borrowing by impatient households, as the countercyclical LTV rule becomes more binding as a result of the weakness in the nominal exchange rate. However, the impact on house prices in both LTV regimes is very similar. The same reaction is observed for other variables, except for the homeownership between patient and impatient households. The impact on homeownership for both patient and impatient households is larger under the augmented countercyclical LTV rule. This suggests that including the nominal exchange rate in the countercyclical LTV rule reduces the central bank's ability to tame the financial cycle.

6 Conclusion

In this paper, I study the role of the LTV ratio in taming the financial and business cycles. I developed a dynamic stochastic general equilibrium (DSGE) model for a small open economy, calibrated with parameters from Iacoviello (2005) and Christiano et al. (2011). To generate the impulse response functions, I shocked the model with multiple exogenous shocks, including a regular monetary policy shock, a stationary technology shock, and a stationary risk premium shock. The results show that the LTV ratio, especially under a countercyclical LTV rule, is more effective in taming the financial cycle. However, I found that the impact of the LTV ratio to smooth the business cycle for a small open economy is not as effective as for a closed economy. The effect of the LTV ratio that is not as effective in smoothing the business cycle for a small open economy is mainly due to the expenditure switching channel. The final goods producers can change the composition of imported goods in their product mix as a consequence of the movement in the nominal exchange rate. I also explored the impact of adding a nominal exchange rate as another variable for the central bank to target in its monetary policy and the countercyclical LTV rule. The results suggest it is more effective in taming the business cycle but not so in stabilising the financial cycle. This result is achieved under the critical assumption of no frictions in the financial market and banking system. Capturing the impact of the movement in the exchange rate on both the trade and financial channels will give a more holistic effect of targeting the exchange rate. In particular, it is critical to assess the impact on borrowers' balance sheets, as it can work as an amplification mechanism of the financial cycle. The scope for extension of this study mainly lies in relaxing the impatient households' budget constraints to allow for foreign borrowing and introducing frictions in the financial market and banking system. The benefit of targeting the exchange rate by the central bank will be better understood once the financial channel is considered.

Figure 11: Response to technology shock, $\Delta a_t = 0.01$ - Fixed LTV: Standard Taylor Rule vs Augmented Taylor Rule

Coordinate: deviation from the baseline

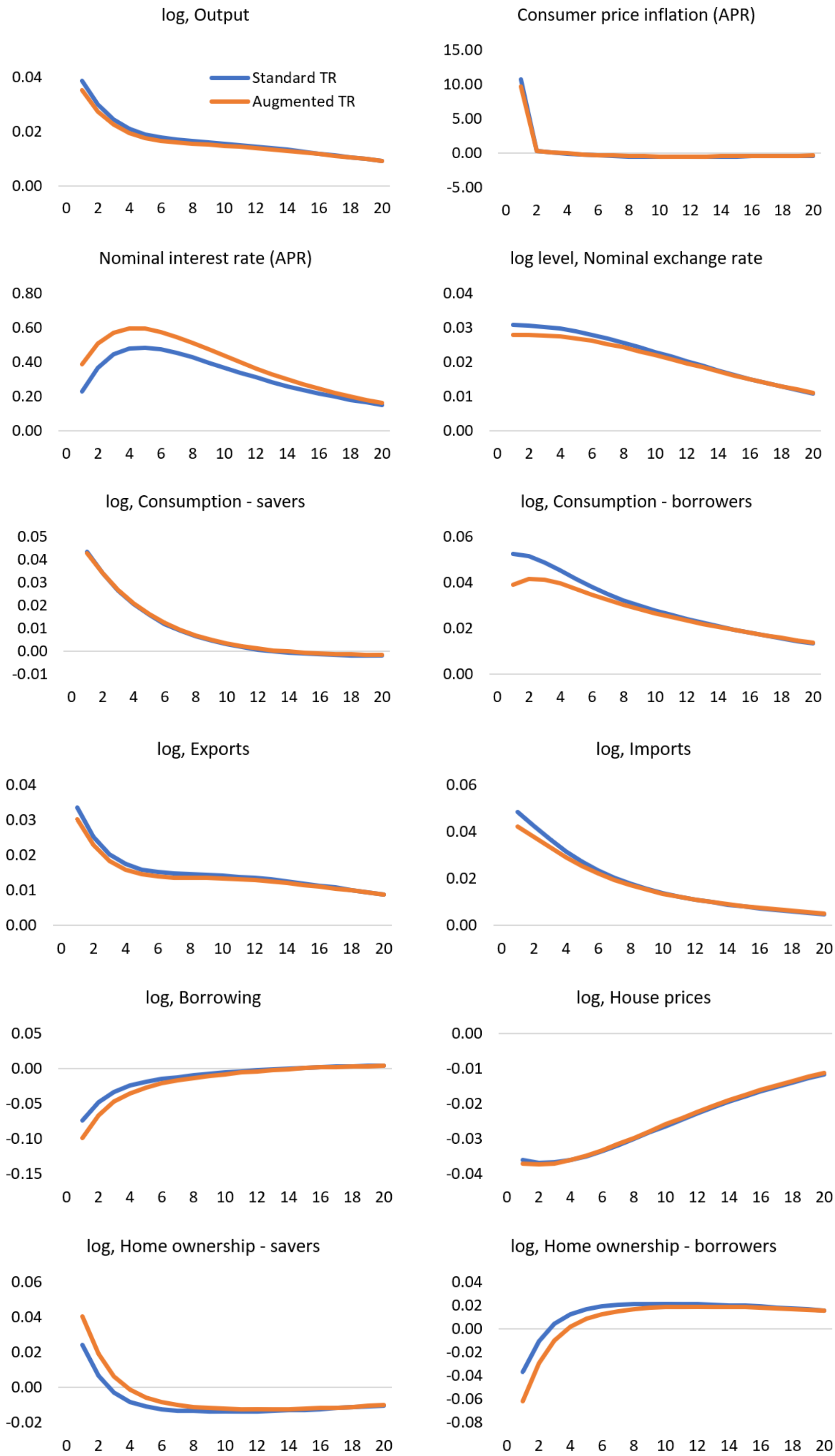
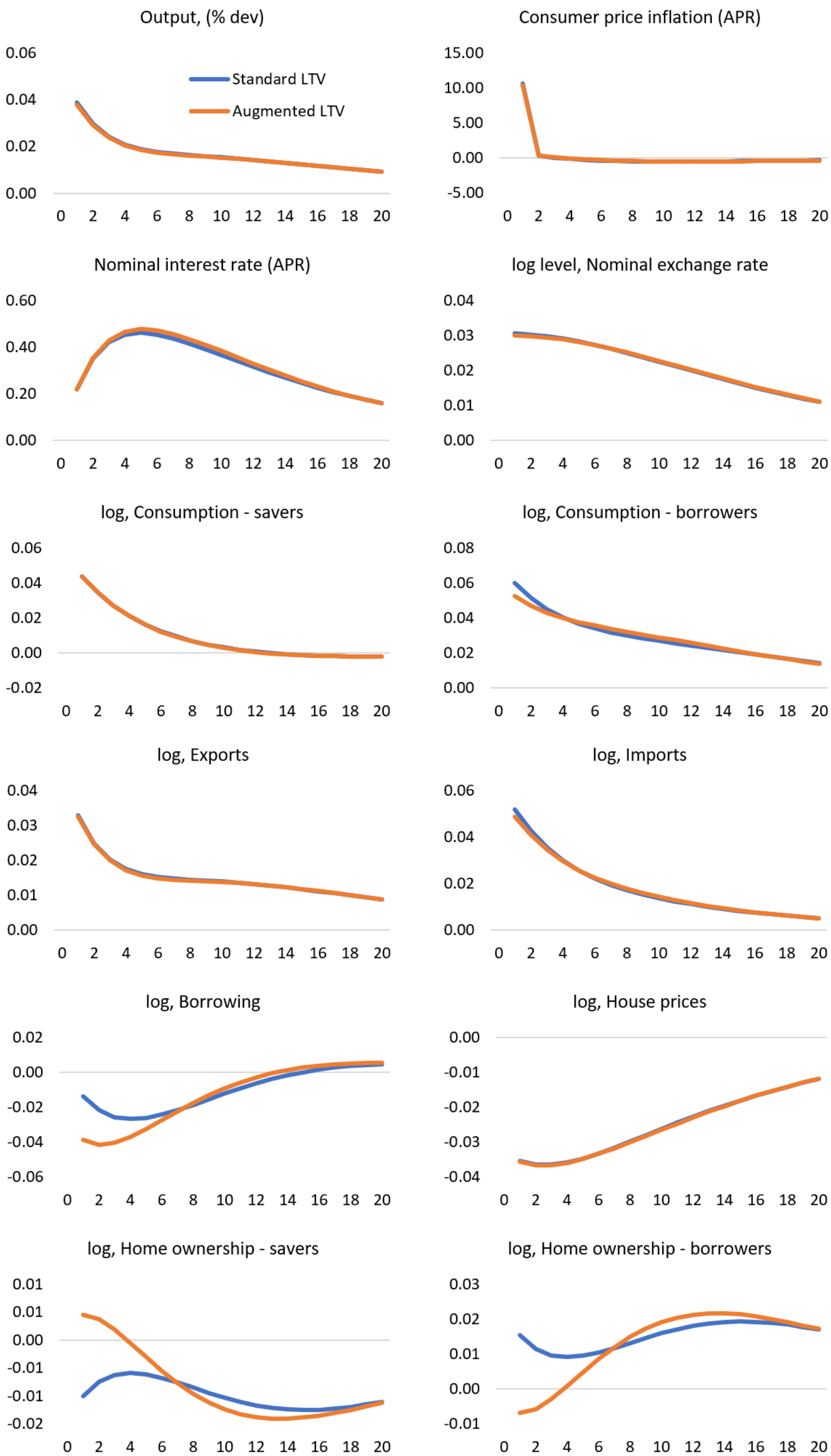


Figure 12: Response to technology shock, $\Delta a_t = 0.01$ - Countercyclical LTV: Baseline LTV vs Augmented LTV Rule

Coordinate: deviation from the baseline



A The model - Dynare code

A.1 Patient households

- Euler equation for patient households

$$\frac{1}{\exp(C_t^u)} = \beta \exp(R_t - \pi_{t+1} - C_{t+1}^u)$$

- Labour supply for patient households

$$\exp(w_t^u) = \exp(L_t^u)^{\varphi-1} \exp(C_t^u)$$

- Housing demand for patient households

$$\frac{j}{\exp(H_t^u)} = \exp(q_t - C_t^u) - \beta \exp(q_{t+1} - C_{t+1}^u)$$

- Euler equation for patient households and risk term

$$\exp(-C_t^u) = \frac{\beta}{\exp(\pi_{t+1} + C_{t+1}^u)} \exp(s_{t+1})\exp(R_t^f)\exp(\Phi_t)$$

A.2 Impatient households

- Budget constraint for impatient households

$$\exp(C_t^c) = \exp(w_t^c + L_t^c) + \exp(b_t^c) - \exp q_t \left(\exp(H_t^c) - \exp(H_{t-1}^c)\right) - \exp(R_{t-1} + b_{t-1}^c - \pi_t)$$

- Borrowing constraint for impatient households

$$b_t^c = \log(LTV) - R_t + \pi_{t+1} + q_{t+1} + H_t^c$$

- Labour supply for impatient households

$$w_t^c = (L_t^c)(\varphi - 1) + C_t^c$$

- Housing demand for impatient households

$$\frac{\exp(j)}{\exp(H_t^c)} = \exp(q_t - C_t^c) - LTV \exp(q_{t+1} + \pi_{t+1} - R_t - C_t^c) - \widetilde{\beta}(1 - LTV) \exp(q_{t+1} - C_{t+1}^c)$$

A.3 Labour demand

- Labour demand for patient households

$$w_t^u = \log(\gamma) + Y_t - L_t^u + X_t$$

- Labour demand for impatient households

$$w_t^c = \log(1-\gamma) + Y_t - L_t^c + X_t$$

A.4 Optimal price setting by intermediate producers

- Equilibrium conditions associated with price setting, K

$$\exp(K_t) = \frac{\varepsilon}{\varepsilon - 1}(1 - V)\exp(Y_t - C_t + mc_t) + \beta\theta\bar{\pi}_{t+1}\varepsilon + (K_{t+1})$$

- Equilibrium conditions associated with price setting, F

$$\exp(F_t) = \exp((Y_t) - (C_t)) + \beta\theta\bar{\pi}_{t+1}(\varepsilon - 1) + (F_{t+1})$$

- Optimal price setting

$$\exp(K_t - F_t) = \left[\frac{1 - \theta \times (\exp(\bar{\pi}_t)(\varepsilon - 1))}{1 - \theta}\right]^{\frac{1}{1-\varepsilon}}$$

A.5 Final good price index

- Price index

$$p_t = [(1-\alpha) + \alpha\exp(p_t^m)(1-\eta)]^{\frac{1}{1-\eta}}$$

A.6 Real exchange rate and inflation

- Domestic and imported goods prices

$$p_t^m = p_t + reals_t$$

- Domestic inflation

$$\exp(\pi_t) = \exp(\bar{\pi}_t)\left[\frac{(1-\alpha) + \alpha\exp(p_t^m)^{1-\eta}}{(1-\alpha) + \alpha\exp(p_{t-1}^m)^{1-\eta}}\right]^{\frac{1}{1-\eta}}$$

A.7 Rate of depreciation and inflation

- Export prices in domestic currency

$$\log(1) = 0 = reals_t + p_t^x + p_t$$

- Real exchange rate

$$\frac{\exp(reals_t)}{\exp(reals_{t-1})} = \exp(s_t)\frac{\exp(\pi_t^f)}{\exp(\pi_t)}$$

A.8 Homogeneous goods market clearing

- P-star

$$\exp(-(p_t^*)) = (1-\theta)\left[\frac{1-\theta \times (\exp(\bar{\pi}_t(\varepsilon-1))}{1-\theta}\right]^{\frac{\varepsilon}{1-\varepsilon}} + \theta \times \exp(\varepsilon\bar{\pi}_t - (p_{t-1}^*)$$

- Domestic output

$$\exp(Y_t) = (1-\alpha)\exp(\eta \times p_t + C_t) + \exp(G_t) + \exp(EX_t)$$

A.9 Balance of payments

- Balance of payments

$$\begin{aligned} & a_t^f + \exp(p_t^m) \times \alpha \times \exp(\eta(p_t - p_t^m) + c_t) = \\ & \exp(p_t + q_t + (1-\eta_f)p_t^x + Y_t^f) + \exp(R_{t-1}^f + \Phi_{t-1}) \times \exp(s_t - \bar{\pi}_t) \times a_{t-1}^f \end{aligned}$$

  where $EX_t = -\eta_f \times p_t^x + Y_t^f$

- Exports in units of domestic homogeneous goods

$$EX_t \times P_t^d = exp_t = -\eta_f \times p_t^x + Y_t^f + reals_t + p_t^x + p_t$$

- Imports in units of domestic homogeneous goods

$$imp_t = p_t^m + \log\alpha + \eta(p_t - p_t^m) + c_t$$

A.10 Risk term

- Risk term

$$\exp(\Phi_t) = \exp\left(-\tilde{\phi}_a\left(a_t^f - \bar{a}\right) - \tilde{\phi}_s\left(\exp(R_t) - \exp(R_t^f) - \left(\exp(R) - \exp(R^f)\right)\right) + \tilde{\phi}_t\right)$$

A.11 Monetary policy - Taylor rule

- Taylor rule

$$(R_t - R) = \rho_R(R_{t-1} - R) + (1 - \rho_R) \times \left[(r_\pi(\pi_{t+1} - \pi^T) + r_y(Y_{t+1} - Y)\right] + \varepsilon_{R,t}$$

A.12 Government spending

- Fiscal rule

$$G_t = (1 - \rho_g)\log(\eta_g)Y + \rho_g G_{t-1} + \varepsilon_{G_t}$$

A.13 Housing market

- The total units of houses available for patient and impatient households

$$\exp(H_t^u) + \exp(H_t^c) = 1$$

A.14 Equilibrium equations

- Total labour supply

$$\exp(L_t) = \exp(L_t^u) + \exp(L_t^c)$$

- Total consumption

$$\exp(C_t) = \exp(C_t^u) + \exp(C_t^c)$$

- Inflation target

$$\exp(\pi_t^T = 0.05\pi_{SS}^T + 0.95\exp(\pi_{t-1}^T) + \varepsilon_{\pi_t^T}$$

A.15 Foreign sector and risk term

- Foreign inflation

$$\pi_t^f = 0.95\pi_{t-1}^f + \varepsilon_{\pi_t^f}$$

- Foreign output

$$Y_t^f = 0.95Y_{t-1}^f + \varepsilon_{Y_t^f}$$

- Foreign interest rate

$$R_t^f = 0.95R_{t-1}^f + \varepsilon_{R_t^f}$$

- Risk term

$$\Phi_t = 0.95\Phi_{t-1} + \varepsilon_{\Phi_t}$$

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