On-the-Job Search, Precationary Savings and the Progressivity of Income Taxes

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Abstract

This paper analyzes the effects of income taxation on the behaviors of workers and firms, and the equilibrium distribution of workers across income and wealth. The model extends the strand of literature that integrates an equilibrium model of job search into an incomplete market model of consumption and savings. It thus incorporates firm heterogeneity, endogenous wage offer of firms, as well as endogenous job search effort and savings of workers. The effects of income taxes ripple through the search and savings behaviors of workers, and the consequent response of the wage offer, which leads to additional changes in the search and savings of workers. In the simulations across the different tax schemes, a higher tax progressivity is shown to depress the motive of job search and precautionary savings and the wage offer of firms, leading to the lower average wage rates and the higher interest rate.

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1 Introduction

Income taxation is a key means by which governments earn tax revenues, redistribute income and mitigate risks and also affect the economic activities of workers and firms. Yet, on the other side, taxation distorts incentives and undermines economic efficiency. In order to evaluate the effects of a tax policy, the model should be able to account for the changes of the economic agents' behavior in response to the changes in the tax scheme and the consequent effects in equilibrium labor distribution and prices. In this paper, I therefore study the effects of income taxes using the general equilibrium model where endogenously determined job search and savings of workers and the response of heterogeneous firms affect the equilibrium stationary distribution of workers. Specifically, I analyze how the labor market outcomes such as job search effort, savings, interest rate, wages and unemployment rate are affected when the income tax progressivity increases, or the average income tax rates are adjusted.

The key channel of the effects of taxes in my model is the interaction between wealth accumulation and job search in a general equilibrium setting. The model thus extends the pioneering work of Lise (2013) in the strand of literature that integrates an equilibrium model of job search into an incomplete market model of consumption and savings. Search includes both off- and on-the-job search, and workers are ex-ante identical. Workers choose their search effort considering the structure of the wage ladder, and a desire for consumption smoothing in the face of the wage ladder is associated with savings and wealth accumulation. Through their decisions on search and savings, workers become ex-post heterogeneous. Firms then respond in their wage offers given the optimal search and savings of workers, which additionally changes the workers' behavior. To see the effects of income taxes on this mechanism, I introduce the tax function of Heathcote et al. (2017) who represent the tax scheme as two parameters: the tax progressivity and the average tax rates. Changing the parameters in the tax function, we could obtain the labor market equilibrium variables and the distribution of workers across wages and assets, estimating the effects of the changes in the tax progressivity or the average tax rates. Yet, to fully analyze the effects of tax policies, the model should be extended in future work in twofold: Labor demand in this model is now exogenously given by one, and we can endogenize it by introducing firms' entry and exit. Additionally, we could incorporate an ex-ante heterogeneity of workers as well to capture the role of worker heterogeneity in labor market matching and the distribution of workers across income and wealth.

While this paper mainly focuses on the model presentation, I provide simple simulations on three different tax schemes. We first set the baseline model and compare the equilibrium with those under the two more progressive taxation systems. First reform is to have a higher tax progressivity, and we assume the baseline economy has the tax progressivity as in the US, and then choose the higher tax progressivity in the UK. We then also compare with the case where the tax progressivity increases the same as the first reform, but the average tax rates are adjusted to have the same tax revenues. The results show that the higher tax progressivity reduces both the optimal search effort and the optimal savings, while the increase of the average tax burden can offset the decrease in the optimal savings and search effort. Yet, these exercises are solved based on the preliminary calibration values, and thus they may need to be analyzed with more rigorous calibration in future work.

The paper proceeds as follows. Section 2 presents the model and characterizes the equilibrium. In section 3, we set the model's calibration in which the values are mostly taken from the literature. We then solve for the equilibrium numerically using the algorithm in Achdou et al. (2020), and show the results of two tax reform simulations. Section 4 mentions the future work to be done in order to conduct the policy analysis completely. Finally, section 5 concludes, and the appendices contain the derivations of the equations.

2 Literature Review

2.1 Literature on On-the-Job Search with Precautionary Savings

The model in this paper extends the model in Lise (2013), building on the strand of literature that integrates job search into models of precautionary savings in the style of Bewley-Hugget-Aiyagari. Both the parts integrated are key models of the macroeconomics: (i) The incomeplete-markets model of Bewley (1977), Hugget (1993), and Aiyagari (1994), and (ii) The labor market search model departed from Diamond (1981), Moretensen (1982), and Pissarides (2000). In the model of (i), workers choose their consumption and savings, facing the labor income risks and a borrowing limit. They cannot fully insure against the risks due to the incomplete markets. Yet, the labor market is frictionless, and therefore it can have only the exogenous job separation when considering the unemployment. On the other hand, the model of (ii) does not consider the wealth of workers. In this type of models, workers and firms search each other in the various ways of search, and workers are assumed to use all the labor income for consumption at every period. Consequently, they only take into account the felicity from current consumption and search effort and the value of expected returns from work, not having the precautionary motive.

Krusell, Mukoyama, and Şahin (2010) and Lise (2013) are the pioneering papers in this strand of research. Krusell et al. (2010) extends the Aiyagari (1994) model with endogenous job finding rate which is determined in the labor market with Diamond-Mortensen-Pissarides structure. This means that in the labor market, firms post vacancies and match with workers according to the matching function, and the wages are set through Nash bargaining. Using the structure, Krusell et al. (2010) showed first that the share of surpluses that workers derive from the match depends on their wealth. Because the model of Krusell et al. (2010) only allows the job search to only unemployed workers, the employed workers become savers, and the unemployed workers dissave to smooth consumption, becoming borrowers. Thus, the consumption smoothing is achieved by two types of workers with extreme motives of job search and savings. The unemployed workers use their ability to search for jobs and form the expectation on future income, and hence borrow to hit the borrowing constraint. On the other hand, the precautionary motive of the employed workers is strengthened so that the workers at the top of wealth distribution prefer to save even more, binding the upper bound of assets. Since only a fraction of workers (unemployed workers) could search jobs, the role of wage ladder mechanism as a self-insurance can be underestimated and the equilibrium outcome is rather close to Aiyagari (1994) model without job search.

Lise (2013) incorporates on-the-job search into a partial equilibrium Hugget (1993) type model. Similar to Krusell et al. (2010), risk averse workers choose search effort and can borrow or save using a risk free asset. Yet, workers can search both on- and off-the job in Lise (2013)'s model, so as to account for large job-to-job transitions observed in the data. Workers' search and savings behaviors are determined by the asymmetric wage ladder mechanism, where there is an asymmetry between climbing and falling off the ladder. The wage increases are generated in increments through the on-the-job search process, whereas the wage decrease happens sharply from the current wages to the lowest rung of ladder due to the exogenous job loss shock. Because of the asymmetry, workers' savings and search behaviors differ across the joint distribution of earnings and wealth. This implies that the workers in low paying jobs prefer to dissave tapping the expectation of wage growth, whereas the workers in high paying jobs choose to save to insure against the possibility of job loss. Since the model assumes the ex-ante homogeneous workers, the authors use the sample consisting soley of white males participating at the US economy to estimate the parameters in the model. The model shows the good fit with the aggregate distribution of wages, consumption, and wealth, as well as the employment and wage dynamics in the data.

Yet, there exist the critiques that it is not appropriate for the policy analysis, since Lise (2013)'s model is in a partial equilibrium. In the partial equilibrium setting, workers choose their optimal savings and search effort given the fixed wage offer distribution of firms. In this case, the effect of policy ripples only through the reduction/increment of the labor income. The expected return of the given search effort level (before policies such as taxes or deductions are implemented) remains the same. This indicates that the partial equilibrium model cannot capture the additional changes in optimal search effort and its consequent effects on equilibrium labor distribution.

Similar to my model, Chaumont and Shi (2020) develop the model that integrates on-the-job search into Hugget (1993) type model in a general equilibrium setting. Yet, the wage determination, search technology, and the algorithm solving equilibrium are different from my model. The model of Chaumont and Shi (2020) endogenizes the wage offer in the way that firms create vacancies for the submarkets which are indexed by the wage offer and the potential workers' wealth. On the worker's side, workers with high wealth prefer to wait for the high-wage offer, decumulating their wealth if they fail to form matches, whereas workers with low wealth tend to choose the low-wage offer to be hired fast and accumulate wealth fast. Firms, therfore, prefer to hire the wealthiest possible worker available in the submarket, since higher wealth implies a lower job finding rate. Additionally, the authors assume the search technology of unemployed workers and employed workers to be different. Unemployed workers meet the firm's wage offer with probability one, while they accept or reject the offer. On the other hand, employed workers' meeting probability is estimated as the job-to-job transition rate in the data. From the assumption of the heterogeneous search technology, the wage dispersion is obtained sufficiently large as much as that in the data.

2.2 Literature on On-the-Job Search and Equilibrium Taxes

This paper is closely related to existing literature on quantitative evaluation of income tax policies. In reality, income tax system has a very complex structure of various taxes and deductions that have different conditions and limits. In the related literature, instead of introducing such a complex structure in the model, researchers assume and estimate a tax function, focusing on the actual tax burden of agents. Developments in general equilibrium heterogeneous-agents models enable researchers to address the policy analysis. The Aiyagari-Bewley-Hugget model quantitatively analyzes the impacts of taxes on precautionary savings and intensive and extensive labor supply margins, yielding a cross-sectional wealth distribution as an equilibrium. There have been several approaches to introduce the frictional labor market that extend upon the standard Aiyagari-Bewley-Hugget model. This is so that the models can incorporate both impacts on the search intensity of workers and the response of firms (with regards to their wage offer distribution) by endogenizing them.

Bagger, Moen and Vejlin (2017) study the impact of taxes on search choices of workers who are risk neutral and heterogeneous in their productivity. The role of firms is simply set in that they just open vacancies and offer wages to all workers as much as the output that may be created by the match. The workers accept the offer whenever the offered wage is higher than their current wage. Thus, taxes do not affect the wage formation or the job finding rate of other workers, meaning that externalities are only associated with the choice of search intensity. In the model of Bagger et. al. (2017), the social planner solves the optimal taxation problem by choosing the optimal net of tax income vector that maximises the social welfare which is determined by the weighted sum of the net present lifetime income of unemployed workers. Then the taxes are set as the output lessened by the net of tax wages. Solving the optimal taxation problem leads to the planner balancing between two countervailing effects: a distributional effect and an incentive effect. First, the distributional effect is caused by the concave welfare function. A positive distribution effect makes the planner want to redistribute from high-paid to low-paid jobs. Second, the incentive effect considers the efficiency loss of taxes. The higher the rung is, the more fraction of workers that are affected in terms of their search intensity. To limit the distortion on value creation, the planner wants to impose lower taxes for jobs at higher rungs. Therefore, efficiency concerns put a limit on how much can be reallocated. The authors simulated the model using Danish data and compared the labor market outcomes of no tax, poll tax, proportional tax, and optimal tax cases. They found that in the optimal taxation, search choices are less distorted than proportional tax and higher paid workers earn more net of tax income than poll tax, leading to the largest social welfare among the three tax cases.

Sleet and Yazici (2017) highlight a profit channel of income and profit taxes, thereby modifying the optimal tax formulas with worker heterogeneity and on-the-job search. Unlike Bagger et. al. (2017), the effect of taxes ripples through the firms' contribution to the search process. There are sub-markets determined by talents that workers are heterogeneous in, and firms sort themselves across the sub-markets. Then, they choose how much to extract from worker output as profits, and the variation of extractiveness among firms creates job ladders within the talent markets. In the model, the policymaker problem maximises expected utility of payoffs subject to budget constraints. Solving the problem, higher marginal tax rates yield the effects on distribution of profit-per-worker offers and on the threshold of talents active in the labor market (in addition to the standard channel of workers' intensive margin that reduces output). The threshold of talent is raised, and hence the maximal profit-per-worker offer decreases to retain marginal workers, poaching workers from lower paying firms. This also reduces the profit-per-worker offer of such firms. Thus, the profit channel refers to the additional effect of taxes on firm profits being squeezed and being redistributed to workers. The impacts of the profit channel are twofold. First, within each talent market, workers at the lower rungs of the job ladder benefit more from profit squeeze. Second, profits are redistributed from low talent markets to high talent markets, since high-talented workers are those who produce more output and pay more rent to firms. Due to higher welfare weights for lower-paid jobs, optimal tax is determined by the balance of the former and latter forces. Therefore, the direction of optimal marginal tax rates is not clear. Using US data, the authors show that under the Affine tax which allows constant marginal tax rates and lump sum transfers, the optimal tax rates become lower compared to the standard public finance model.

Bagger, Hejlesen, Sumiya and Vejlin (2018) provide the model closely related to this paper. They analyze the effect of labor income taxation using the on-the-job search model with two-sided heterogeneity, endogeneized job search effort, and equilibrium wage formation. The main difference is that their model assumes complete markets in which workers fully insure their income risk by trading a set of contingent claims of consumption. That is, workers are heterogeneous only in their ability and type-a worker has the identical consumption at time t, independent of current labor income and labor market status. Therefore, the effects of income taxation are derived by the changes in the workers' search efforts and its impacts on the wage offer distribution of firms, whereas my paper takes into account the channel of precautionary savings and asset accumulation. The other difference is that firm's wage policy is conditional on worker ability, meaning the directed search of firms. To evaluate the tax reforms in Denmark, Bagger et. al. (2018) assume the progressive tax function with three brackets and estimate the parameters to represent the increases in marginal rates between brackets. They found Danish income tax reforms improved the equilibrium allocation of labor, showing the decrease in the steady state unemployment rate and the increase in the steady state labor income. Also, they provide the optimal taxation parameters that maximises the steady state aggregate utility of workers.

On the other hand, Heathcote, Storesletten, and Violante (2017) develop an equilibrium model that finds the optimal degree of tax progressivity. They estimate a tax function based on a simple parametric model using the gross and disposable income of workers. In this functional form, the tax progressivity is represented by a single parameter in the index. Their specification differs from the other traditional specifications such as the Affine taxes which are simple but have constant marginal rates, or very complex tax functions with a number of parameters to estimate. The model has incomplete markets where workers with idiosyncratic productivity shocks choose how many hours they work and invest in their skills. Unlike the literature mentioned above, this model assumes a frictionless labor market, and therefore there is no search behavior of workers and firms. The optimal tax progressivity is then chosen such that balances out the trade-off between the equality concerns and the effects on incentives to work or invest in skills.

3 Model

3.1 The Environment

Time is continuous and the future is dicounted at rate ρ. The model basically extends Aiyagari-Bewley-Hugget model to endogeneous labor supply and demand.

Two-sided Heterogeneity

On the one side, there is a continuum (a unit measure) of workers who are ex-ante identical, but become endogenously heterogeneous ex post in their assets $a \in [\underline{a}, \overline{a}]$ and wages $w \in [\underline{w}, \overline{w}]$. Ex-ante omogenous workers are subject to idiosyncratic labor income risks with the lowest income state being interpreted as the home production of unemployed workers. Because the income risks are only partially insurable by a risk-free asset, workers' assets and wages evolve alongside the histories of search outcomes. On the other side of the market, firms are heterogeneous in their productivity $z \in [b, \overline{z}]$ where b is home production value. Productivity z is drawn from the exogenous distribution $\Gamma(z)$, which is normalized to have $\underline{z} = b$, because firms with $z < b$ should offer the wages $w < b$ that workers will reject surely.

Worker Preferences

Workers derive utility from consumption of a composite good c and disutility from job search effort σ. The preferences are represented by the Constant Relative Risk Aversion (CRRA) utility function, and the power function for disutility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{and} \quad \psi(\sigma) = \frac{\mu}{\eta}\sigma^{\eta}$$

where $\gamma > 0$ is the coefficient of relative risk aversion, $\eta > 1$ is the elasticity of search costs, and $\mu > 0$ is a scaling parameter. As shown above, they are continuously differentiable with $u'(\cdot) > 0, u''(\cdot) < 0, \psi'(\cdot) > 0$,and $\psi''(\cdot) > 0$. As we shall see below, the convexity of the search cost function and the concavity of the utility function influence the direction of the effects of taxes on worker's behavior.

Wage Ladder

In the frictional labor market, workers and firms should exert effort to form productive matches and have the parts of the surplus of matches. Workers search both off- and on-the-job. A worker who is unemployed with assets a and home production income b exerts job search effort $\sigma(a, b)$. The job finding rate is described by a Poisson arrival process, and the arrival rate depends positively on the search intensity: $\lambda\sigma(a, b)$, where λ is the exogeneous job finding rate per unit of search effort. Similarly for the employed workers with assets a and a labor income w exerts $\sigma(a, w)$, and receives a job offer at rate $\lambda\sigma(a, w)$. It implies that the unemployed and employed workers share the same search technology.

A job offer is drawn from the wage offer distribution $F(w)$ with support $[\underline{w}, \overline{w}]$. $F(w)$ is determined by the firms posting a single wage w according to their productivity z prior to meeting a potential worker. Therefore, the wage offer is endogeneous and is an equilibrium object. When a worker accepts the wage offer, then the match is formed and the surplus is produced and shared between a worker and a firm immediately. Matches are separated in one of two ways. First, a worker could be laid off exogeneously at the job destruction rate δ. Then the worker becomes unemployed with home production value b. Second, a worker may quit his job to move to another firm offering higher wages as a result of on-the-job search. This endogeneously occurs at rate $\lambda\sigma(a,w)\overline{F}(w)$,where $\overline{F}(w) = 1 - F(w)$.

Taxation

We introduce income taxation imposed on the sum of the labor income and interest income. We denote the pre-tax income of an individual as $y = w + ra$. Then, we set the tax revenues as $T(y)$ at every pretax income level y. The log-linear form of $T(y)$ is borrowed from the literature (e.g. Benabou, 2002; Heathcote et. al., 2017) as follows:

$$T(y) = y - \epsilon y^{1-\tau}, 0 \leq \tau < 1 \tag{1}$$

where τ reflects the degree of tax progressivity, and ϵ shifts the tax function and controls the average tax rates.[1] $T(y)$ is continuously differentiable with $0 \leq \tau'(y) < 1$. Then, the disposable income y' is given as follows:

$$y' = \epsilon y^{1-\tau}$$

Taking logs on both sides of equation (1) yields:

$$log(y') = log\epsilon + (1-\tau)logy$$

$1-\tau$ denotes the elasticity of disposable income to gross income, which means when gross income increases by 1%, disposable income increases by $(1-\tau)$%. Additionally, and most importantly, $1-\tau$ can be described as the ratio of marginal to average tax rates:

$$\frac{1 - T'(w)}{1 - \frac{T(w)}{w}} = 1 - \tau$$

Thus, the tax progressivity is characterized in the sense that the marginal rate $T'(w)$ is larger than the average tax rate $\frac{T(w)}{w}$. $\tau > 0$ means that the marginal tax rates are always larger than the average rates, hence the taxation scheme is progressive, while $\tau < 0$ indicates a regressive taxation scheme. When $\tau = 0$, the marginal rates and average rates are always constant as $1-\epsilon$ for all pretax wage levels w, representing a flat tax. Consequently, the increase in parameter τ indicates a higher progressivity of a tax scheme. In the case of ϵ, it depends negatively on the average rates of taxes.

3.2 Workers

A worker is infinitely lived, and is risk averse. He saves (borrowes) at a risk-free interest rate r in the incomplete markets where he could only partially insure against idiosyncratic employment risk. He has the utility and disutility flows from future consumption $c_t$and future search effort σ_t discounted at rate $\rho \geq 0$ according to

$$E_0 \int_0^{\infty} e^{-\rho t}[u(c_t) - \psi(\sigma_t)]dt$$

A worker is paid a gross income w_t from a match between a worker and a firm, and could consume or save a net of tax income $\epsilon w_t^{1-\tau}$. His wealth takes the form of bonds and evolves as follows:

$$\dot{a}_t = \epsilon(w_t + r_t a_t)^{1-\tau} - c_t$$

The risk-free interest rate r is assumed to be less than the discount rate ρ, so that workers are not too patient to have the precautionary motive of accumulating the assets infinitely. A worker also faces a borrowing limit

[1] According to Heathcote et al. (2017), this tax function form has been used in many public finance literature since Feldstein (1969).

$$a_t \geq \underline{a},$$

where $-\infty < \underline{a} < 0$[2]. Finally, workers' gross labor income evolves stochastically over time. Here, for simplicity, we assume that the home production value of an unemployed worker b is also taxable. The changes in the gross labor income are as follows:

$$dw_t = \begin{cases} dq_{\lambda\sigma} 1(\upsilon(a_t, \tilde{w_t}) \geq \upsilon(a_t, b))[\tilde{w}_t - b] & \text{when unemployed,} \\ dq_{\lambda\sigma} 1(\upsilon(a_t, \tilde{w_T}) \geq \upsilon(a_t, w_t))[\tilde{w}_t - w_t] + dq_\delta[w_t - b)] & \text{when employed,} \end{cases}$$

where wages $\tilde{w} \in [b, \overline{w}]$ is drawn from the wage offer distribution $F(w)$, $dq_{\lambda\sigma} = 1$ if a job offer arrives and 0 otherwise, and $dq_\delta = 1$ if a job is exogeneously destroyed and 0 otherwise.

Then workers' consumption-saving decisions can be summarized by a Hamilton-Jacobi-Bellman (HJB) equation. We should set up the HJB equations for the unemployed and the employed separately, using the value of unemployment and the value of employment respectively. However, from Lise (2013), it is known that $\tilde{w}(a) = b$ is obtained by substituting each HJB equation into the reservation wage definition $\upsilon(a, \tilde{w}(a)) = \upsilon^u(a)$, where $\upsilon^u(a)$ denotes the value of being unemployed with assets a. This uses the assumption that the search technology is identical for the unemployed and the employed. The result implies that the reservation wage for an unemployed worker is constant as b, independently of assets. Consequently, unemployment could be interpreted as the lowest rung of the job ladder, and therefore the HJB equations for the unemployed and the employed could be represented as a single HJB equation for all workers.[3] For simplicity, I omit the subscripts.

$$\rho\upsilon(a, w) = \max_{c \in [0, a - \underline{a}], \sigma \geq 0} \Bigg\langle u(c) + \lambda\sigma \int \max\left(\upsilon(a, \tilde{w}) - \upsilon(a, w), 0\right) dF(\tilde{w}) \\ + \delta\left(\upsilon(a, b) - \upsilon(a, w)\right) + \upsilon_a(a, w)\left(\epsilon(w + ra)^{1-\tau} - c\right) - \psi(\sigma)\Bigg\rangle \tag{2}$$

The flow value of being (un)employed with assets a and wages w is given by the felicity from consumption c and search effort σ plus the expected change in the value of (un)employment. The value of (un)employment changes in expectation in one of three ways. First, it increases by the product of the job ffer arrival rate depending on the search effort and the expected net gain of receiving a wage offer $\tilde{w}$. Second, it decreases by the product of the job destruction rate and the net loss of being laid off and losing the current wage w. If the worker is currently unemployed, there is no income risk by unemployment, and therefore net loss would be zero. Finally, due to the asset accumulation, the value of (un)employment changes as much as the product of the savings policy function and the marginal value of assets.

The first order conditions (FOCs) are given by:

$$u'(c) = \upsilon_a(a, w) \qquad \text{and,} \qquad \psi'(\sigma) = \lambda \int \max\left(\upsilon(a, \tilde{w}) - \upsilon(a, w), 0\right) dF(\tilde{w}) \tag{3}$$

Lemma 1.The value function in the HJB equation is strictly increasing in assets and income.
Proof. See Appendix. A.2.

Lemma 1 states that HJB is strictly incresing in assets and income and therefore optimal acceptance decision is to accept any income higher than currently being paid.

Taking the derivative of HJB with respect to w using the Envelope theorem, and substituting $\upsilon_w(a, w)$ into FOC on search effort yields the equation for optimal search effort.

[2] Following Aiyagari (1994), we impose the "natrual borrowing limit". In the steady state, the natural borrowing limit would be $a_t \geq -\frac{\epsilon w^{1-\tau}}{r}$.

[3] See Appendix A.1 for more details.

$$\psi'(\sigma) = \lambda \int_{w}^{\overline{w}} \frac{u'(c)\epsilon(1-\tau)(\tilde{w}+ra)^{-\tau} + u''(c)c_w[\epsilon(w+ra)^{1-\tau} - c]}{\rho + \delta + \lambda\sigma\overline{F}(\tilde{w})} \overline{F}(\tilde{w}) d\tilde{w}$$

The optimal search effort balances the marginal cost with the marginal expected return. Workers have differential search behavior across their assets and income: more assets dampen the precautionary motive of savings which in turn reduces the desire for a higher income, and higher wages resultantly decrease the probability of being reallocated to an even higher rung of job ladder. Additionally, the convexity of the search cost function and the concavity of the utility function also affect the optimal search effort: the optimal search effort is inversely related to the convexity of the search cost function and positively related to the concavity of the utility function. Then, introducing income taxation distorts the choice of optimal search effort. First, receiving a disposable income net of taxes reduces available permanent income, which therefore decreases the optimal consumption c, leading to a higher $u'(c)$ and more negative $u''(c)$. Second, in the numerator, τ-related terms multiplied by $u'(c)$ and $u''(c)$have the opposite impacts, reducing the absolute values of the two terms. The intuition is that the expected return of the search effort decreases overall, but the decrease is partially offset by the curvature of the utility function. The more progressive the tax scheme is, the larger these effects might be. Lastly, income taxation and the changes in the worker's search behavior also induce the corresponding responses of firms on the wage offer distribution $F(w)$, which will be analyzed below.

Similarly, taking the derivative of HJB with respect to a, applying the Envelope theorem, and substituting the FOCs into the differentiated HJB, the optimal consumption growth for the steady state is characterized as follows:

$$\frac{\dot{c}}{c} = \frac{1}{\gamma}\left(\epsilon(1-\tau)(ra+w)^{-\tau}r - \rho - \lambda\sigma\left(\overline{F}(w) - \int_{w}^{\overline{w}} \frac{u'(c(a,\tilde{w}))}{u'(c(a,w))} dF(\tilde{w})\right) + \delta\left(\frac{u'(c(a,b))}{u'(c(a,w))}\right)\right)$$

The equation accounts for the savings behavior of workers across the different rungs of a wage ladder. First, before income taxation ($\tau = 0, \epsilon = 1$), $r - \rho$ represents the relative importance of time preferences to the interest rate. Due to the identical time preferences among workers, this term is constant across workers' assets and income. Second, the $\lambda\sigma$-related term determines the effects of the expected return of on-the-job search. The higher the current wages, the lower the chance of experiencing a wage increase through job transition. The marginal utility of consumption is also lower with higher current wages, because the optimal consumption increases along the amount of wages. Therefore, being at a higher rung leads to the higher optimal growth of consumption, implying more savings. Third, the δ-related term affects the precautionary savings of workers. The more distant from the value of home production b the current wages are, the higher the marginal utility rate. This strengthens the motive for precautionary savings. Then, introducing income taxation affects the optimal savings. First, the interest income at each period under income taxation is smaller than risk-free interest rate r, which means ($\epsilon(1-\tau)(ra+w)^{-\tau}r < r$). This lowers the optimal growth of consumption, reducing savings. Second, the $\lambda\sigma$-related term is also influenced by the changes in the wage offer distribution $F(w)$ and the marginal utility from consumption. The marginal utility increases due to decrease in consumption, and as we shall see, given the wages w, the income taxes decrease $\overline{F}(w)$. Third, the δ-related term also changes due to the changes in the marginal utility.

3.3 Asset Market

The equilibrium of the asset market is a steady-state joint distribution of assets and income, which we denote $g(a,w)$ and an equilibrium interest rate that is pinned down by the level of aggregate savings. The Kolmogorov-Forward equations that solve the steady-state joint distribution of wealth and income are given by

for the unemployed

$$\frac{\partial}{\partial a}\left(s(a,b)g(a,b)\right) = -\lambda\sigma(a,b)\left(1-F(b)\right)g(a,b) + \delta\int_{b^+}^{\overline{w}} g(a,w')dw' \tag{4}$$

for the employed

$$\begin{aligned}\frac{\partial}{\partial a}\left(s(a,w)g(a,w)\right) = &-\left(\delta + \lambda\sigma(a,w)\left(1-F(w)\right)\right)g(a,w) + \lambda f(w)\int_{b^+}^{w}\sigma(a,w')g(a,w')dw' \\ &+ \lambda f(w)\sigma(a,b)g(a,b)\end{aligned} \tag{5}$$

[4]The interest rate r is pinned down by the requirement that, in a stationary equilibrium, the aggregate savings are fixed as zero:

$$\begin{aligned}AS(r) &\equiv \int_{\underline{a}}^{\infty}\int_{b}^{\overline{w}} a g(a,w)dwda \\ &= 0\end{aligned} \tag{6}$$

This implies that the equilibrium interest rate is such that balances out the demand and supply of assets to smooth consumption.

3.4 Firms

The present discounted value of a firm of productivity z, hiring a worker of wage w with asset level a is given by the Bellman equation below.

$$\rho J(a,w,z) = z - w + J_a(a,w,z)s(a,w) - \left(\delta + \lambda\sigma(a,w)\overline{F}(w)\right)J(a,w,z) \tag{7}$$

The flow value of hiring a worker of assets a with wages w is given by the sum of the output of the match z minus the wages w and the expected chnges in the value of a filled job. The value of a job changes in expectation in one of three ways. First, a job is separated exogenously at a rate of δ. Second, the workers of one firm can be poached by another firm offering higher wages. The possibility of loss positively depends on the intensity of on-the-job search of the worker. When the job separation or poaching occurs, the match is dissolved and produces zero output, therefore rendering zero profits for the current firm. Finally, the asset accumulation of workers also affects the value of a filled job. Since more assets lessen the optimal search effort of workers, the probability of a job transition decreases and a worker stays longer in the current firm. Arranging the Bellman equation yields:

$$\left(\rho + \delta + \lambda\sigma(a,w)\overline{F}(w)\right)J(a,w,z) = z - w + J_a(a,w,z)s(a,w)$$

When hiring a worker, firms sample potential workers who vary in assets and wages from a sampling distribution with joint cdf given by

$$\Phi(a,w) := \frac{1}{S}\int^{a}\left(\sigma(a',b)g(a',b) + \int_{b^+}^{w}\sigma(a',w')g(a',w')dw'\right)da'.$$

Where S is the aggregate measure of searchers

$$S := \int^{\overline{a}}\left(\sigma(a',b)g(a',b) + \int_{b^+}^{\overline{w}}\sigma(a',w')g(a',w')dw'\right)da'$$

[4] See Appendix A.3 for the derivation.

Firms offer wages conditional on productivity ex ante of meeting, and the optimal wage maximizes the following Bellman equation

$$\Pi(z) = \arg\max_{w} \left\langle \int J(a, w, z)\Phi_a(a, w)da \right\rangle \tag{8}$$

The first order condition satisfies

$$\int J(a, w, z)\Phi_{aw}(a, w)da = -\int J_w(a, w, z)\Phi_a(a, w)da, \tag{9}$$

where,

$$\Phi_a(a, w) = \frac{1}{S}\left(\sigma(a, b)g(a, b) + \int_{b^+}^{w} \sigma(a, w')g(a, w')dw'\right) \qquad \Phi_{aw}(a, w) = \frac{1}{S}\sigma(a, w)g(a, w)$$

A firm's wage policy is the best response to the worker's savings and search behaviours, as well as the expected loss of exogenous layoffs and other firms' wage offers drawn from $F(w)$. Thus, a firm should consider the trade-off between the profits after paying wages and the expected loss from worker turnover when choosing how much to offer to a potential worker. Then, firms with low productivity offer low wages, and firms with high productivity offer high wages, implying $w'(z) > 0$.

Introducing income taxation affects the firm's wage offer through changes in workers' savings and search behaviours, as well as other firms' wage policies. Progressive taxes reduce the workers' asset target levels and optimal search efforts, which lessens worker turnover. Therefore, workers have a lower probability of being poached, because their on-the-job search is less active than it would be with the absence of taxes. Consider the firm with very low productivity paying wages just above the home production value b. Facing income taxation, the firm would offer wages even closer to b, since there is less probability for workers to meet with another firm offering higher wages. This affects the wage offer of firms with higher productivity trying to poach from a firm with lower productivity. This mechanism additionally lowers the wage offer value. In sum, the presence of progressive taxes causes the support of the pretax distribution of wages to shift downwards and become more compressed.

3.5 Government and Optimal Income Taxation

The government obtains the income tax revenue, and spends on government purchases, G, which do not directly enter into the worker's utility. We assume that government has the balanced government budget. The tax revenue is determined by the tax progressivity parameter τ and the average tax rate parameter ϵ and the budget constraint is given by:

$$\int_{\underline{a}}^{\overline{a}} \int_{b}^{\overline{w}} \left(w + ra - \epsilon(w + ra)^{1-\tau}\right) g(a, w)dwda = G$$

4 Simulation

4.1 Calibration

To simulate the model in the benchmark economy, I borrow the calibrated values of parameters from the existing literature. I note here that the model calibration described in this section is preliminary, thus some of the parameters need to be adjusted to match the target moment calculated using the microdata in future work. The model is calibrated to a yearly frequency. The parameters encompass the felicity function, job search process, and the tax system.

We first have the coefficient of relative risk aversion γ. According to Attanasio (1999), the coefficient of relative risk aversion lies in between 1 and 2 in most of the existing literature. We then set the γ at two as in Lise (2013). For disutility of search, we assume search costs are quadratic with $\eta = 2$ and $\mu = 1$. We then have the parameters for job search process: job contact rate λ, and job destruction rate δ. Here the parameters are fixed at exogenous rate, and they should match the labor market target moments. We borrow the values of $\lambda = 0.6$ and $\delta = 0.1$ from Lise (2013) who estimates the moments for the white male sample of the US labor market. The parameters are the same across the unemployed and the employed workers, which means that the model assumes the identical search technology for off- and on-the-job search. For a firm heterogeneity in productivity z, we assume the beta distribution $\Gamma(z) = B(\frac{z}{\beta_0}; \beta_1, \beta_2)$ with $\beta_0 = 0.5$, $\beta_1 = 2$, $\beta_2 = 5$.

Finally, we set the parameters for the tax system. As seen in section 2, the government imposes progressive taxes on the sum of labor income and interest income of workers. Heathcote et al. (2017) estimated the income tax progressivity in the US economy as 0.181, but this estimate is based on the income taxes on households, not on the individual workers. Thus, the tax progressivity parameters are taken from Holter et al. (2019) who estimate the tax progressivity parameter based on taxes imposed on average earnings of employed individuals over the period of 2000 - 2007. The tax progressivity parameter τ for the US economy (0.138) and the UK economy (0.200) are borrowed to analyze the countries with less/more progressive taxation systems, respectively. The average tax controls ϵ are calculated to match the UK tax-to-GDP rate (0.330).

Table 1: Parameters

Parameter	Target
$\rho = 0.05$	time preference (standard in macro calibration)
$\gamma = 2$	relative risk aversion (standard in macro calibration)
$\mu = 1$	search costs scale (Lise, 2013)
$\eta = 2$	elasticity of search costs w.r.t. effort (Lise, 2013)
$\lambda = 0.6$	job contact rate (Lise, 2013)
$\delta = 0.1$	job destruction rate (Lise, 2013)
$\beta_0 = 0.5,\ \beta_1 = 2,\ \beta_2 = 5$	firm productivity distribution, $\Gamma(z) = B(\frac{z}{\beta_0}; \beta_1, \beta_2)$
$\tau = \begin{cases} 0.138 \\ 0.200 \end{cases}$	progressivity of income tax (Holter et al., 2019)
$\epsilon = 0.751$	average level of income tax

4.2 A Stationary Equilibrium

As shown in Section 2, workers' search and consumption-saving decisions are characterized by a HJB equation, and firms' wage offer decision is determined by the Bellman equation that maximizes the expected profits from hiring a worker. Then, the joint distribution of workers across income and wealth is summarized with another differential equation: a KF equation. Finally, the interest rate r is pinned down in equilibrium by the fact that bonds are in zero net supply. A stationary equilibrium is therefore represented by the following system of the differential equations (2), (4), and (5), together with (3), (8), and (9), and the equilibrium relationship (6). To solve the equilibrium numerically, I use the finite difference algorithm developed in Achdou et al. (2020) for solving the HJB equation of workers and the wage determination of firms.[5]

4.3 Simulation Results

In this section, we simulate the model under the three tax systems. First, we set the benchmark economy with low progressivity ($\tau = 0.138$) and the average tax rates ($\epsilon = 0.752$) calculated to match the UK average tax rates. We then compare the results with the steady state after the progressive tax reform to a higher tax progressivity (as in the UK). Finally, we additionally compare them to the case where the government chooses the higher tax progressivity, but increases the level of taxes so that the total tax revenue does not change.

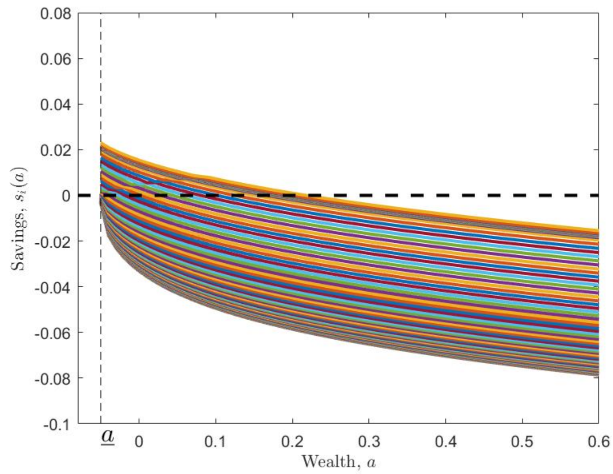
4.3.1 The effects of the progressive tax reform

In this simulation, we increase the tax progressivity parameter τ from 0.138 into 0.200, and then solve for the new steady state under the progressive tax reform. The results show how workers' search and savings patterns change and therefore how the equilibrium price rates in the model are affected. The increase in the tax progressivity reduces the relative slope of the disposable income for the workers with a higher income compared to that for the workers with a lower income, imposing more burden to the higher income workers.

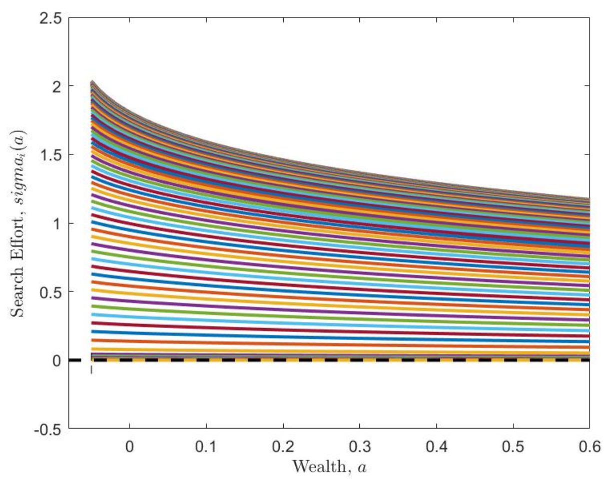
[5] See the technical details in Appendix. B.

Figure 1: Results of the Baseline Model ($\tau = 0.138$)

(a) Savings Behavior



(b) Search Behavior



(c) Densities

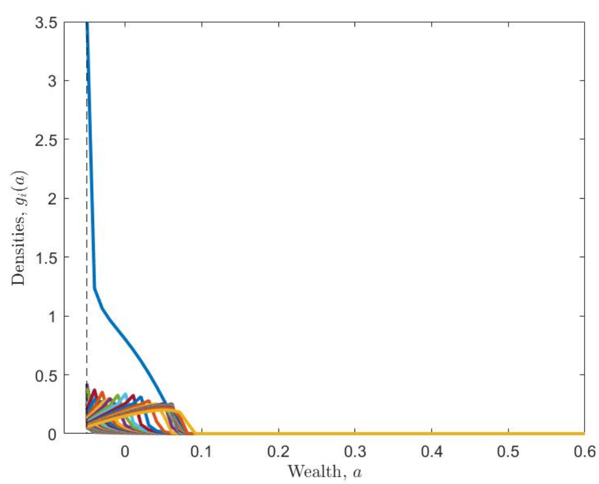
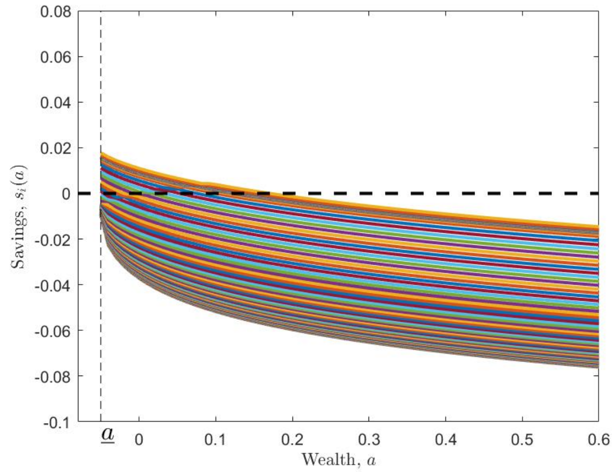
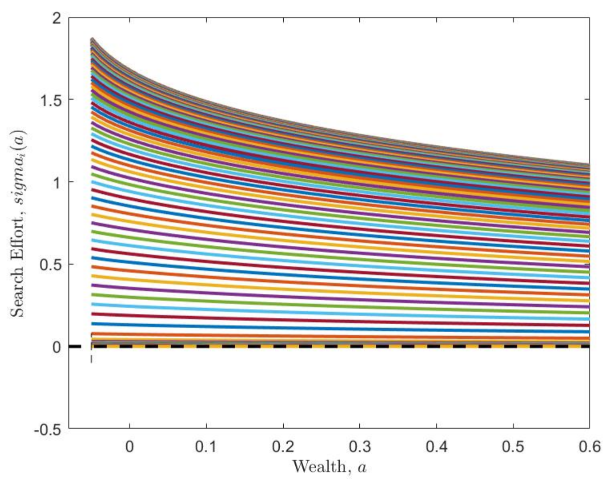


Figure 2: Results of the Progressive Tax Reform ($\tau = 0.200$)

(a) Savings Behavior



(b) Search Behavior



(c) Densities

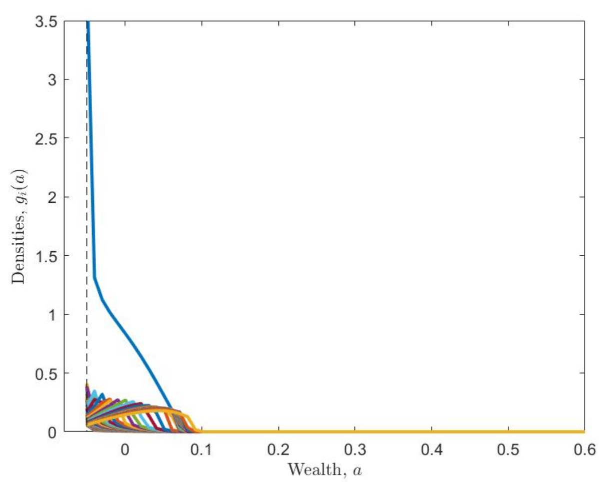


Figure 1 and figure 2 show the search and savings behaviors, and the stationary wealth distribution under the two different tax schemes $\tau = 0.138$ and $\tau = 0.2$ respectively. To make the average tax rates equal, controls ϵ is adjusted from 0.752 when $\tau = 0.138$, to 0.712 when $\tau = 0.2$. Across the panels, the policy functions conditional on assets a of the workers receiving wages w are represented. In panels (a) and (b) of both figure 1 and figure 2, both the savings and search effort policies are decreasing in assets. We also find that the unemployed workers choose the highest search effort given the equal assets, while they dissave most, hitting the borrowing constraint in panel (c). As the wages w increase, the optimal search efforts decrease and the precautionary savings increase, showing the zero search effort and the highest savings behavior at the highest rung of wage ladder. On the other hand, the steady-state distribution of workers are represented in panel (c). The density of unemployed workers $g(a, b)$ explodes as $a \to \underline{a}$ and therefore there is a spike in the density $g(a, b)$ at $\underline{a}$.

Comparing the panel (a)'s of the figure 1 and figure 2, when the tax progressivity becomes higher, savings and borrowings of workers both shrink. This is because the presence of the progressive taxes dampens the motive of consumption smoothing. Panel (b)'s of figure 1 and figure 2 clearly show that the higher tax progressivity distorts the search effort more, reducing workers' search intensity, and therefore leading to less active job search of workers.

Table 2: The Effects of the Progressive Tax Reform

	r	$E(w)$	$E(\sigma)$	$E(c)$	u (%)
$\tau = 0.138$	0.026	0.154	0.50	0.150	11.4
$\tau = 0.200$	0.031	0.152	0.48	0.153	12.1

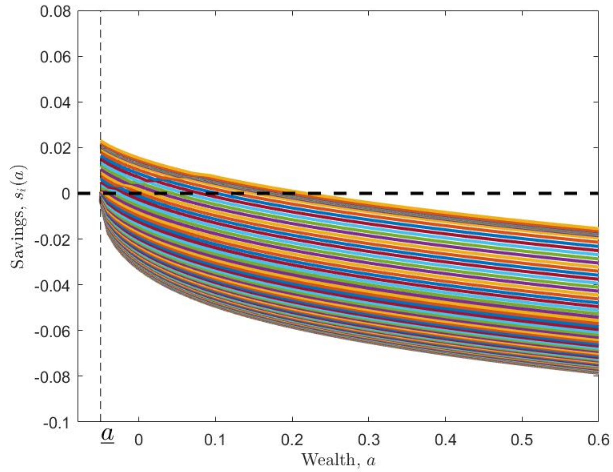
Table 2 describes the equilibrium labor market outcomes under the different tax schemes. The more progressive the tax scheme is, the more increases in equilibrium interest rate are, due to the weakened motive of the precautionary savings. The average search effort decreases, and therefore the average wage offer decreases. Since the search intensity decreases, workers would remain in the status of unemployment longer than they would do in the less progressivity case. Consequently, this leads to the higher equilibrium unemployment rate. Average consumption increases as the motive of savings is weakened.

4.3.2 The effects of the revenue-neutral progressive tax reform

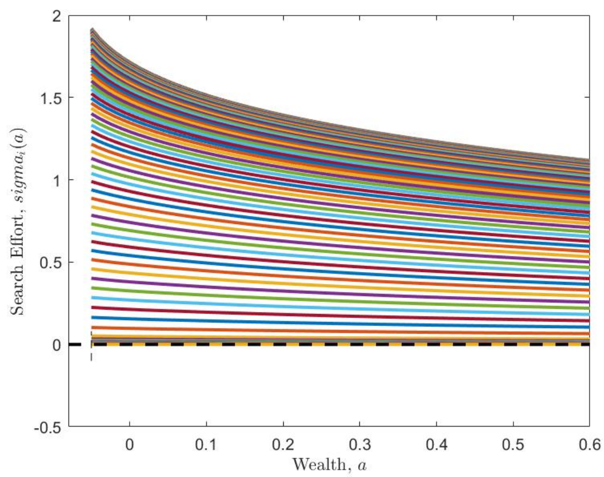
In the simulation of section 1.3.1., the higher tax progressivity reduces the average wages and savings policies, leading to slower asset accumulation. Consequently, the gross income $ra + w$ of workers also becomes smaller, and therefore the tax revenue of the government also decreases. In this section, we simulate the model adjusting the average tax rate so as to have the equal tax revenue in both the taxation schemes. We thus simulate the model with the more progressive scheme ($\tau = 0.200$) with ϵ adjusted into 0.668, which is smaller than that of the progressive tax reform in the previous section.

Figure 3: Results of the Revenue-neutral Progressive Tax Reform ($\tau = 0.200$)

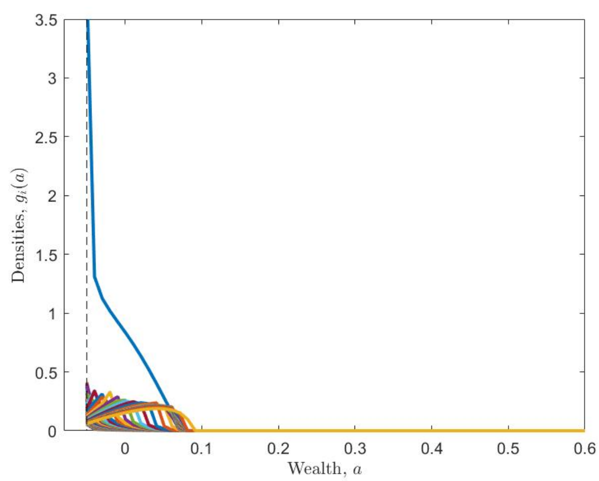
(a) Savings Behavior



(b) Search Behavior



(c) Densities



Comparing figure 3 with figure 1 of the baseline calibration, in panel (a), the savings policy function seems almost the same under the baseline and the revenue-neutral tax scheme. The higher average tax rates thus offset the decrease in precautionary motive of savings. Yet, the distribution of workers across wealth is compressed downward, since the average tax rates of workers increase to maintain the same tax revenues as in the baseline. In panel (b), due to the higher tax progressivity, the optimal search efforts decrease, but the decrease is smaller than the progressive tax reform without the change in the average tax rate.

Table 3: The Effects of the Revenue-neutral Progressive Tax Reform

	r	$E(w)$	$E(\sigma)$	$E(c)$	u
Baseline	0.026	0.154	0.50	0.150	11.4
Progressive Reform	0.031	0.152	0.48	0.153	12.1
Revenue-neutral Progressive Reform	0.033	0.153	0.49	0.148	11.9

Table 3 compares the equilibrium labor market outcomes of the three tax schemes: the baseline, progressive reform and revenue-neutral progressive reform. Compared to the baseline model, the revenue-neutral reform increases the interest rate and the unemployment rate, and lowers the average wages and average search effort, since the higher tax progressivity distorts the search effort and savings decision. Yet, the average consumption decreases, because the higher average tax rates reduce the disposable income of workers.

Compared to the progressive reform without the change in the average tax rate, the optimal search effort slightly increases, because, given the fixed tax progressivity, the higher average tax rates reduces disposable income, and hence to smooth consumption, workers strengthen the search intensity. This therefore increases the possibility of the employed workers meeting other firms offering higher wages, and the average wage offer should increase as well. Hence the employment rate is in between the baseline and the progressive reform, since the unemployed workers will remain unemployed longer than the progressive reform, but shorter than the baseline.

5 Future Work

As mentioned above, we should ensure a more rigorous calibration of the model for further research. In order to match the target parameters (such as δ, μ,and η), recalibration is necessary. Additionally and crucially, the model will be adapted twofold: First, the firms' free entry condition will be introduced to endogenize the labor demand in the model. Second, an ex-ante worker heterogeneity would be introduced to capture the additional role of workers in the wage ladder mechanism, and account for the larger wage dispersion.

5.1 Firm Entry and Exit

In the paper's model, all firms in the economy search for workers, and offer wages conditional on their productivity $z \in [b, \overline{z}]$ drawn from the exogenous distribution $\Gamma(z)$.Workers then choose their search effort to meet the vacancies, considering the rate at which workers meet vacancies, which is exogenously given as the parameter λ.

This paper remains confident in the likelihood that firm entry and exit is the future of work. Essentially, the model will incorporate a firm's decision to post a vacancy, obtaining the meeting function depending on the search effort of workers and the vacancies of firms. Given the cost of posting a vacancy and the expected return from a potential worker, firms choose whether to post a vacancy or not, and how much to offer, according to the worker's productivity $z \in [z_0, \overline{z}]$. The free entry condition will then ensure that a potential entrant firm must be indifferent about posting a vacancy. In this respect, labor demand can be endogenized and represented by the total number of vacancies and meetings of workers and firms. In this search process, firms consider the rate at which they contact a potential worker q. In the model, the rates at which firms contact workers (q) and which workers meet firms (λ), and the productivity threshold (z_0) are the equilibrium objects.

With the free entry condition, we can consider the effects of profit taxation whose distortionary effects ripple through the firms' entry and exit, and the derived effects on firms' wage policies. The optimal taxes are then chosen as the value such that achieves the efficient match in the labor market. If the vacancies are too many relative to the search effort of workers, raising the taxes on profits will reduce the number of firms entering and therefore raise q. In the case that this were reversed, raising the taxes on workers' income will raise λ.In this way, the optimal taxation will be associated with how much burden should be imposed on each economic agent.

5.2 Worker Heterogeneity

We should furthermore extend the model to incorporate an ex-ante worker heterogeneity. In this paper, we assume that workers are ex-ante identical, but become ex-post heterogeneous as in the model of Lise (2013). Consequently, the model attributes all of the wage dispersion to the labor income risk. Yet, in reality, some workers just earn persistently higher than other workers due to their ability. Reflecting the worker heterogeneity in ability will change the savings and search patterns of workers, in addition to the wage offer of firms across the ability distribution of workers. For example, the worker with low wealth and low ability may be willing to accept even lower wages than in this paper, and the worker with high wealth and high ability may accept a wage offer even higher than in this paper. Hence, it is expected that the wage and wealth distribution will be widened across workers.

6 Conclusion

This paper has analyzed the effects of income taxes in a search equilibrium model with risk-averse workers who accumulate wealth to smooth consumption in the incomplete markets. Workers search both off and on the job. Workers optimally choose their search effort and savings according to their income and wealth, accumulating or decumulating their wealth. The optimal search and savings decrease in wealth. Firms offer wages conditional on productivity ex ante of meeting, solving the profit maximizing problem. As a result of the behaviors of workers and firms in the frictional labor market, ex-ante identical workers become heterogeneous endogenously. The presence and the changes in the income taxation system then distort the interaction between wealth accumulation and job search, and thus affect the equilibrium allocation of labor across income and wealth. From the literature, we borrow the functional form of taxes that explains the tax scheme with two parameters: the tax progressivity and average tax rates. This paper then has analyzed the effects of the increase of the tax progressivity. In simulation, we show that a higher tax progressivity reduces the optimal search effort due to the decrease in the expected return of on-the-job search, and also reduces the savings due to the weakened motive of consumption smoothing.

There are two extensions of this model to be done in future. First, labor demand should be endogenized in the model. This therefore needs the firms' decision on whether to post a vacancy or not to be solved in the model. This will enable to find the efficient match that balances out the total number of search and the total number of vacancies. Second, an ex-ante worker heterogeneity can be introduced, while in this model we assume the homogenous workers in their ability, and emphasize how they become heterogeneous in their income and wealth as the result of the labor market outcomes. This will ensure that the model has a better fit to the data.

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A Appendix

A.1 Derivation of HJB equation

We derive the HJB equations of the unemployed and the employed from a discrete-time equation with time periods of length Δ_t and then taking the limit as $\Delta_t \to 0$, and then show that the HJB equations of the unemployed and the employed can be written as the single equation.

$$\upsilon(a,w,t) = \max_{c\in[0,a-\underline{a}],\sigma\geq 0} \Big\langle \left[u\left(c(a,w,t)\right) - \psi(\sigma(a,w,t)\right]\Delta_t$$

$$+\ \frac{1}{1+\rho\Delta_t}[\lambda\sigma\Delta_t \int \max\{\upsilon(a+\Delta_a,\tilde{w},t+\Delta_t),\ \upsilon(a+\Delta_a,w,t+\Delta_t)\}dF(\tilde{w})$$

$$+\delta\Delta_t U(a+\Delta_a,t+\Delta_t)\ +\ (1-\lambda\sigma\Delta_t-\delta\Delta_t)\upsilon(a+\Delta_a,\ w,\ t+\Delta_t)\ +\ o(\Delta_t)]\Big\rangle$$

where $o(\Delta_t)$ is the term that goes to zero faster than Δ_t. Mutiplying $1+\rho\Delta_t$ yields

$$\rho\Delta_t\upsilon(a,w,t) = \max_{c\in[0,a-\underline{a}],\sigma\geq 0} \Big\langle (1+\rho\Delta_t)\left[u\left(c(a,w,t)\right) - \psi(\sigma(a,w,t)\right]\Delta_t$$

$$+\ [\lambda\sigma\Delta_t \int \max\{\upsilon(a+\Delta_a,\tilde{w},t+\Delta_t)\ -\ \upsilon(a+\Delta_a,w,t+\Delta_t),\ 0\}dF(\tilde{w})$$

$$+\delta\Delta_t[U(a+\Delta_a,t+\Delta_t)-\upsilon(a+\Delta_a,w,t+\Delta_t)]+[\upsilon(a+\Delta_a,w,t+\Delta_t)-\upsilon(a,w,t)]+o(\Delta_t)]\Big\rangle$$

Dividing by Δ_t and taking the limit as $\Delta_t \to 0$, we obtain

$$\rho\upsilon(a,w,t) = \max_{c\in[0,a-\underline{a}],\sigma\geq 0} \Big\langle (1+\rho\Delta_t)\left[u\left(c(a,w,t)\right) - \psi(\sigma(a,w,t)\right]$$

$$+\ [\lambda\sigma \int \max\{\upsilon(a+\Delta_a,\tilde{w},t+\Delta_t)\ -\ \upsilon(a+\Delta_a,w,t+\Delta_t),\ 0\}dF(\tilde{w})$$

$$+\delta[U(a+\Delta_a,t+\Delta_t)-\upsilon(a+\Delta_a,w,t+\Delta_t)]+\frac{\upsilon(a+\Delta_a,w,t+\Delta_t)\ -\ \upsilon(a,w,t)}{\Delta_t}+\frac{o(\Delta_t)}{\Delta_t}]\Big\rangle$$

Since $lim_{\Delta_t\to 0}\frac{\upsilon(a+\Delta_a,\,w,\,t+\Delta_t)\ -\ \upsilon(a,\,w,\,t)}{\Delta_t} = \upsilon_a\frac{da}{dt}$ and $lim_{\Delta_t\to 0}\frac{o(\Delta_t)}{\Delta_t} = 0$,

$$\rho\upsilon(a,w,t) = \max_{c\in[0,a-\underline{a}],\sigma\geq 0} \Big\langle \left[u\left(c(a,w,t)\right) - \psi(\sigma(a,w,t)\right]$$

$$+\ [\lambda\sigma \int \max\{\upsilon(a,\tilde{w},t)\ -\ \upsilon(a,w,t),\ 0\}dF(\tilde{w})$$

$$+\ \delta[U(a,t)\ -\ \upsilon(a,w,t)]\ +\ \upsilon_a(a,w,t)\left(\epsilon(w+ra)^{1-\tau}-c\right)]\Big\rangle$$

A.2 HJB increasing in w and a

Define asterisk notation as an agent's optimal decision. Given assets a and income w, c^* defines optimal consumption level, σ^* the optimal search effort and W^* the set of all wage offers one will accept. Substitute these optimal decisions and the first order condition for consumption gives the expression

$$\upsilon(a,w) = \frac{1}{\rho+\delta+\lambda\sigma^*}\Big(u(c^*) + \lambda\sigma^*\int_{\tilde{w}\in W^*}\upsilon(a,\tilde{w})dF(\tilde{w}) + \delta\upsilon(a,b) \\ + u'(c^*))\left(ra + \epsilon w^{1-\tau} - c^*\right) - \psi(\sigma^*)\Big).$$

Where for example, c^* is the optimal level of consumption, given a and w, by the envelope condition for arbitrary $\hat{a}$ and $\hat{w}$ we know that

$$\upsilon(\hat{a},\hat{w}) \geq \frac{1}{\rho+\delta+\lambda\sigma^*}\Big(u(c^*) + \lambda\sigma^*\int_{\tilde{w}\in W^*}\upsilon(\hat{a},\tilde{w})dF(\tilde{w}) + \delta\upsilon(\hat{a},b) \\ + u'(c^*))\left(r\hat{a} + \epsilon \hat{w}^{1-\tau} - c^*\right) - \psi(\sigma^*)\Big).$$

Substituting in the expression for $\upsilon(a,w)$ gives

$$\upsilon(\hat{a},\hat{w}) \geq \upsilon(a,w) + \frac{u'(c^*))\left(r(\hat{a}-a) + \epsilon(\hat{w}^{1-\tau} - w^{1-\tau})\right)}{\rho+\delta+\lambda\sigma^*}.$$

Thus for $\hat{a} > a$, $\upsilon(\hat{a},w) > \upsilon(a,w)$ and for $\hat{w} > w$ and $\tau \neq 1$, $\upsilon(a,\hat{w}) > \upsilon(a,w)$.

A.3 Joint distribution of assets and income

Forgetting temporarily about income changes, assuming $s(a,w) \leq 0$, the case of $s(a,w) > 0$ is symmetric.

$$\Pr(\tilde{a}_{t+\Delta_t} \leq a) = \underbrace{\Pr(\tilde{a}_t \leq a)}_{\text{already below threshold } a} + \underbrace{\Pr(a \leq \tilde{a}_t \leq a - \Delta_t s(a,w))}_{\text{cross threshold } a} = \Pr(\tilde{a}_t \leq a - \Delta_t s(a,w))$$

So, the probability a worker of wage w has assets $a' \in (a-\Delta_a, a+\Delta_a]$ is given by

$$\Pr(a-\Delta_a < \tilde{a}_{t+\Delta_t} \leq a+\Delta_a) = \Pr(\tilde{a}_t \leq a+\Delta_a - \Delta_t s(a+\Delta_a,w)) - \Pr(\tilde{a}_t \leq a-\Delta_a - \Delta_t s(a-\Delta_a,w))$$

Define the cdf $G(a,w,t)$ as the measure of workers earning less than a wage w and endowed with assets less than a at time t.[6] The cdf $G(a,w,t)$ is defined teh integral of the joint pdf $g(a,w,t)$, such that

$$G(a,w,t) = \int^{w}\int^{a} g(a',w',t)da'dw'.$$

Thus the derivative of $G(a,w,t)$ with respect to w is labelled $G_w(a,w,t)$ and is the measure of workers earning a wage w with assets less than or equal to a. Consider an unemployed worker with income b. The change in the measure of workers with assets $a' \in (a-\Delta_a, a+\Delta_a]$ in a period of time Δ_t is given by the expression below.

$$G_w(a+\Delta_a,b,t+\Delta_t) - G_w(a-\Delta_a,b,t+\Delta_t) = \\ \Big(1-\Delta_t\lambda\sigma(a,b)\Big(1-F(b)\Big)\Big)\Big(G_w(a+\Delta_a-\Delta_t s(a+\Delta_a,b),b,t) - G_w(a-\Delta_a-\Delta_t s(a-\Delta_a,b),b,t)\Big) \\ + \Delta_t\delta\int_{b^+}^{\overline{w}}\left(G_{w'}(a+\Delta_a-\Delta_t s(a+\Delta_a,w'),w',t) - G_{w'}(a-\Delta_a-\Delta_t s(a-\Delta_a,w'),w',t)\right)dw'$$

The first term on the right hand side are those workers who do not find a better paying job and remain in the asset bound. The second term captures workers who were previously employed and are laid off and fall

[6] In steady-state the distribution will be time invariant but it helps with the exposition to include t at this stage.

within the asset boundary. Subtract both sides by $G_w(a+\Delta_a,b,t)-G_w(a-\Delta_a,b,t)$, divide by Δ_t, and take limit as $\Delta_t\to 0$. The left hand side of the expression becomes:

$$\begin{aligned}\text{LHS} &= \lim_{\Delta_t\to 0}\left\langle \frac{G_w(a+\Delta_a,b,t+\Delta_t)-G_w(a+\Delta_a,b,t)}{\Delta_t} - \frac{G_w(a-\Delta_a,b,t+\Delta_t)-G_w(a-\Delta_a,b,t)}{\Delta_t}\right\rangle \\ &= \partial_t G_w(a+\Delta_a,b,t)-\partial_t G_w(a-\Delta_a,b,t) \qquad \text{which in steady-state} \\ &= 0\end{aligned}$$

The first term on the right hand side becomes:

$$\begin{aligned}\text{RHS}_1 &= \lim_{\Delta_t\to 0}\left\langle \frac{G_w(a+\Delta_a-\Delta_t s(a+\Delta_a,b),b,t)-G_w(a+\Delta_a,b,t)}{\Delta_t}\right\rangle \\ &= \lim_{x\to 0}\left\langle \frac{G_w(a+\Delta_a-x,b,t)-G_w(a+\Delta_a,b,t)}{x}\right\rangle s(a+\Delta_a,b) \\ &= -s(a+\Delta_a,b)g(a+\Delta_a,b,t) \qquad \text{given, steady-state} \\ &= -s(a+\Delta_a,b)g(a+\Delta_a,b)\end{aligned}$$

Dropping the t subscripts, the whole expression can be written as

$$\begin{aligned}0 = -s(a{+}\Delta_a,b)g(a+\Delta_a,b) &+ s(a-\Delta_a,b)g(a-\Delta_a,b) \\ &- \lambda\sigma(a,b)\left(1-F(b)\right)\Big(G_w(a+\Delta_a,b)-G_w(a-\Delta_a,b)\Big) \\ &+ \delta\int_{b^+}^{\overline{w}}\left(G_{w'}(a+\Delta_a,w')-G_{w'}(a-\Delta_a,w')\right)dw'\end{aligned}$$

Dividing by $2\Delta_a$ and take limit as $\Delta_a\to 0$.

$$\frac{\partial}{\partial a}\left(s(a,b)g(a,b)\right) = -\lambda\sigma(a,b)\left(1-F(b)\right)g(a,b)+\delta\int_{b^+}^{\overline{w}} g(a,w')dw' \qquad \text{(KF: unemployed)}$$

For $w>b$, the same approach is followed, so

$$\begin{aligned}&G_w(a+\Delta_a,w,t+\Delta_t)-G_w(a-\Delta_a,w,t+\Delta_t) = \\ &\quad \Big(1-\Delta_t\delta-\Delta_t\lambda\sigma(a,w)\Big(1-F(w)\Big)\Big)\Big(G_w(a+\Delta_a-\Delta_t s(a+\Delta_a,w),w,t)-G_w(a-\Delta_a-\Delta_t s(a-\Delta_a,w),w,t)\Big) \\ &\quad + \Delta_t\lambda f(w)\int_{b^+}^{w}\sigma(a,w')\left(G_{w'}(a+\Delta_a-\Delta_t s(a+\Delta_a,w'),w',t)-G_{w'}(a-\Delta_a-\Delta_t s(a-\Delta_a,w'),w',t)\right)dw' \\ &\quad + \Delta_t\lambda f(w)\sigma(a,b)\left(G_w(a+\Delta_a-\Delta_t s(a+\Delta_a,b),b,t)-G_w(a-\Delta_a-\Delta_t s(a-\Delta_a,b),b,t)\right)\end{aligned}$$

Successively, both sides of the above expression are subtracted by $G_w(a+\Delta_a,w,t)-G_w(a-\Delta_a,w,t)$, divided by Δ_t, and the limit taken as $\Delta_t\to 0$.

$$\begin{aligned}0 = -s(a{+}\Delta_a,w)g(a+\Delta_a,w) &+ s(a-\Delta_a,w)g(a-\Delta_a,w) \\ &- \left(\delta+\lambda\sigma(a,w)\left(1-F(w)\right)\right)\Big(G_w(a+\Delta_a,w)-G_w(a-\Delta_a,w)\Big) \\ &+ \lambda f(w)\int_{b^+}^{w}\sigma(a,w')\left(G_{w'}(a+\Delta_a,w')-G_{w'}(a-\Delta_a,w')\right)dw' \\ &+ \lambda f(w)\sigma(a,b)\left(G_w(a+\Delta_a,b)-G_w(a-\Delta_a,b)\right)\end{aligned}$$

Dividing by $2\Delta_a$ and take limit as $\Delta_a\to 0$.

$$\begin{aligned}\frac{\partial}{\partial a}\left(s(a,w)g(a,w)\right) &= -\left(\delta+\lambda\sigma(a,w)\left(1-F(w)\right)\right)g(a,w)+\lambda f(w)\int_{b^+}^{w}\sigma(a,w')g(a,w')dw' \\ &\quad + \lambda f(w)\sigma(a,b)g(a,b) \qquad \text{(KF: employed)}\end{aligned}$$

B Numerical Appendix

B.1 Solving HJB (Implicit 1)

$i = 1, 2...I$, asset grid, $j = 0, 1..J$ wage grid.

$$\frac{\upsilon_{i,j}^{n+1} - \upsilon_{i,j}^{n}}{\Delta} + \rho \upsilon_{i,j}^{n+1} = u(c_{i,j}^{n}) - \psi(\sigma_{i,j}^{n}) + \frac{\upsilon_{i+1,j}^{n+1} - \upsilon_{i,j}^{n+1}}{\Delta a}\left(s_{i,j,F}^{n}\right)^{+} + \frac{\upsilon_{i,j}^{n+1} - \upsilon_{i-1,j}^{n+1}}{\Delta a}\left(s_{i,j,B}^{n}\right)^{-}$$
$$+ \lambda \sigma_{i,j}^{n} \sum_{j'>j} f_{j'}\left(\upsilon_{i,j'}^{n+1} - \upsilon_{i,j}^{n+1}\right) + \delta\left(\upsilon_{i,0}^{n+1} - \upsilon_{i,j}^{n+1}\right)$$

Collecting terms with identical subscripts on the right hand side,

$$\frac{\upsilon_{i,0}^{n+1} - \upsilon_{i,0}^{n}}{\Delta} + \rho \upsilon_{i,0}^{n+1} = u(c_{i,0}^{n}) - \psi(\sigma_{i,0}^{n}) + \upsilon_{i-1,0}^{n+1} x_{i,0} + \upsilon_{i,0}^{n+1} y_{i,0} + \upsilon_{i+1,0}^{n+1} z_{i,0} + \lambda \sigma_{i,0}^{n} \sum_{j'>0} f_{j'} \upsilon_{i+1,j'}^{n+1}$$

and for $j > 0$,

$$\frac{\upsilon_{i,j}^{n+1} - \upsilon_{i,j}^{n}}{\Delta} + \rho \upsilon_{i,j}^{n+1} = u(c_{i,j}^{n}) - \psi(\sigma_{i,j}^{n}) + \delta \upsilon_{i,0}^{n+1} + \upsilon_{i-1,j}^{n+1} x_{i,j} + \upsilon_{i,j}^{n+1} y_{i,j} + \upsilon_{i+1,j}^{n+1} z_{i,j} + \lambda \sigma_{i,j}^{n} \sum_{j'>j} f_{j'} \upsilon_{i+1,j'}^{n+1}$$

$$\begin{aligned} x_{i,j} &= -\frac{\left(s_{i,j,B}^{n}\right)^{-}}{\Delta a} \\ y_{i,j} &= -\frac{\left(s_{i,j,F}^{n}\right)^{+}}{\Delta a} + \frac{\left(s_{i,j,B}^{n}\right)^{-}}{\Delta a} - \lambda \sigma_{i,j}^{n}\left(1 - F_j\right) - \delta \\ z_{i,j} &= \frac{\left(s_{i,j,F}^{n}\right)^{+}}{\Delta a} \end{aligned}$$

In matrix notation,

$$\frac{1}{\Delta}\left(\upsilon^{n+1} - \upsilon^{n}\right) + \rho \upsilon^{n+1} = u^{n} - \psi^{n} + \boldsymbol{A}^{n} \upsilon^{n+1}$$

$$\boldsymbol{A}^{n} = \begin{pmatrix} \boldsymbol{D}_0^n & \boldsymbol{U}_{0,1}^n & \boldsymbol{U}_{0,2}^n & \boldsymbol{U}_{0,3}^n & \cdots & \boldsymbol{U}_{0,J}^n \\ \boldsymbol{L} & \boldsymbol{D}_1^n & \boldsymbol{U}_{1,2}^n & \boldsymbol{U}_{1,3}^n & \cdots & \boldsymbol{U}_{1,J}^n \\ \boldsymbol{L} & \boldsymbol{0} & \boldsymbol{D}_2^n & \boldsymbol{U}_{2,3}^n & \cdots & \boldsymbol{U}_{2,J}^n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{L} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{D}_J^n \end{pmatrix} \quad \text{where,} \quad \boldsymbol{D}_j^n = \begin{pmatrix} y_{1,j} & z_{1,j} & 0 & 0 & \cdots & 0 \\ x_{2,j} & y_{2,j} & z_{2,j} & 0 & \cdots & 0 \\ 0 & x_{3,j} & y_{3,j} & z_{3,j} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & x_{I,j} & y_{I,j} \end{pmatrix}$$

$$\boldsymbol{U}_{j,j'}^{n} = \begin{pmatrix} \lambda\sigma_{1,j} f_{j'} & 0 & 0 & \cdots & 0 \\ 0 & \lambda\sigma_{2,j} f_{j'} & 0 & \cdots & 0 \\ 0 & 0 & \lambda\sigma_{3,j} f_{j'} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \lambda\sigma_{I,j} f_{j'} \end{pmatrix} \quad \text{and} \quad \boldsymbol{L} = \begin{pmatrix} \delta & 0 & 0 & \cdots & 0 \\ 0 & \delta & 0 & \cdots & 0 \\ 0 & 0 & \delta & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \delta \end{pmatrix}$$

B.2 Solving HJB (Implicit 2)

From FOC

$$\psi'(\sigma) = \lambda \int \max\left(\upsilon(a, \tilde{w}) - \upsilon(a, w), 0\right) dF(\tilde{w})$$
$$\mu \sigma^{\eta} = \lambda \sigma \int \max\left(\upsilon(a, \tilde{w}) - \upsilon(a, w), 0\right) dF(\tilde{w}) \quad \text{at optimal } \sigma$$

Re-write previous as,

$$\frac{v_{i,j}^{n+1}-v_{i,j}^{n}}{\Delta}+\rho v_{i,j}^{n+1}=u(c_{i,j}^{n})+\left(\frac{\eta-1}{\eta}\right)\mu\sigma^{\eta}+\frac{v_{i+1,j}^{n+1}-v_{i,j}^{n+1}}{\Delta a}\left(s_{i,j,F}^{n}\right)^{+}+\frac{v_{i,j}^{n+1}-v_{i-1,j}^{n+1}}{\Delta a}\left(s_{i,j,B}^{n}\right)^{-}$$
$$+\delta\left(v_{i,0}^{n+1}-v_{i,j}^{n+1}\right)$$

Now, in matrix notation, where $\psi^n=\left(\frac{\eta-1}{\eta}\right)\mu\sigma^{\eta}$

$$\frac{1}{\Delta}\left(v^{n+1}-v^{n}\right)+\rho v^{n+1}=u^{n}+\psi^{n}+\boldsymbol{A}^{n}v^{n+1}$$

and

$$\boldsymbol{A}^{n}=\begin{pmatrix}\boldsymbol{D}_0^n & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \boldsymbol{L} & \boldsymbol{D}_1^n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \boldsymbol{L} & \mathbf{0} & \boldsymbol{D}_2^n & \mathbf{0} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{L} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{D}_J^n\end{pmatrix}$$

B.3 Solving KF Equations

Take (KF:employed) and integrate over $w\in(b,\overline{w}]$

$$\begin{aligned}\int_{b^+}^{\overline{w}}\frac{\partial}{\partial a}\left(s(a,w)g(a,w)\right)dw &= -\delta\int_{b^+}^{\overline{w}}g(a,w)dw-\left[\lambda\left(1-F(w)\right)\int_{b^+}^{w}\sigma(a,w')g(a,w')dw'\right]_{w=b^+}^{w=\overline{w}}\\ &\quad-\left[\lambda(1-F(w))\sigma(a,b)g(a,b)\right]_{w=b^+}^{w=\overline{w}}\\ &=-\delta\int_{b^+}^{\overline{w}}g(a,w)dw+\lambda(1-F(b))\sigma(a,b)g(a,b)\\ &=-\frac{\partial}{\partial a}\left(s(a,b)g(a,b)\right)\end{aligned}$$

Notice, the first line uses the fact that,

$$\frac{\partial}{\partial w}\left((1-F(w))\int_{b^+}^{w}\sigma(a,w')g(a,w')dw'\right)=(1-F(w))\sigma(a,w)g(a,w)-f(w)\int_{b^+}^{w}\sigma(a,w')g(a,w')dw'.$$

Thus putting the Kolmogorov forward equations for the unemployed and all employed together yields the identities below.

$$\frac{\partial}{\partial a}\left(s(a,b)g(a,b)+\int_{b^+}^{\overline{w}}s(a,w)g(a,w)dw\right)=0$$

$$\implies\qquad s(a,b)g(a,b)+\int_{b^+}^{\overline{w}}s(a,w)g(a,w)dw=\mathcal{C}$$

BOUNDED $\mathcal{C}=0$
Returning to equation (KF: unemployed),

$$s(a,b)g_a(a,b)=-\left(s_a(a,b)+\lambda\sigma(a,b)\left(1-F(b)\right)\right)g(a,b)+\delta\int_{b^+}^{\overline{w}}g(a,w')dw'$$

Looking at the final term[7]

$$
\begin{aligned}
\int_{b^+}^{\overline{w}} g(a,w')dw' &= \int_{b^+}^{\overline{w}} \frac{s(a,w')g(a,w')}{s(a,w')} dw' \\
&= -\left[\frac{s(a,b)g(a,b)}{s(a,w')}\right]_{b^+}^{\overline{w}} - \int_{b^+}^{\overline{w}} \frac{s(a,b)g(a,b)}{s(a,w')^2} s_{w'}(a,w')dw' \\
&= -\frac{s(a,b)g(a,b)}{s(a,\overline{w})} + g(a,b) - s(a,b)g(a,b)\int_{b^+}^{\overline{w}} \frac{s_{w'}(a,w')}{s(a,w')^2} dw'
\end{aligned}
$$

Plugging back,

$$
g_a(a,b) = -\left(\frac{s_a(a,b)}{s(a,b)} + \lambda\frac{\sigma(a,b)}{s(a,b)}(1-F(b)) + \delta\frac{s(a,\overline{w})-s(a,b)}{s(a,b)s(a,\overline{w})} - \delta\int_{b^+}^{\overline{w}} \frac{s_{w'}(a,w')}{s(a,w')^2} dw'\right) g(a,b)
$$

B.4 Solving Firm Wage Posting

Grid of $z = \{z_1, z_2, ...z_N\}$ that approximates a continuous support. We look for a solution in which $w'(z) > 0$ and we assume that if a firm draws a type $z = b$ the corresponding wage offer is $w(b) = b$. Since no worker would accept a lower wage offer they opt to hire the worker making zero profit. We start at the lowest value for z and build the corresponding wage grid sequentially.

1. Solve (iteratively) the equation

$$
\left(\rho + \delta + \lambda\sigma(a,w)\overline{F}(w)\right) J(a,w,z) = z - w + J_a(a,w,z)s(a,w)
$$

   At point z with corresponding wage w this gives us a value for $J(a,w,z)$ and $J_a(a,w,z)$ for every value on our asset grid.

2. Guess a value of $w'(z)$.

3. Solve (iteratively) the equation below, using the computed values of $J(a,w,z)$ and $J_a(a,w,z)$ and the guess at $w'(z)$.

$$
\begin{aligned}
\left(\rho + \delta + \lambda\sigma(a,w)\overline{\Gamma}(z)\right) J_w(a,w,z) = &-\left(\lambda\sigma_w(a,w)\overline{\Gamma}(z) - \frac{\lambda\sigma(a,w)\gamma(z)}{w'(z)}\right) J(a,w,z) \\
&- 1 + s(a,w)J_{aw}(a,w,z) + s_w(a,w)J_a(a,w,z)
\end{aligned}
$$

4. Update our guess for $w'(z)$ from the FOC below, using the bisection method.

$$
\int J(a,w,z)\Phi_{aw}(a,w)da = -\int J_w(a,w,z)\Phi_a(a,w)da
$$

   If RHS is too large then that implies $w'(z)$ is too small and vice-versa. Return to step 2 until covergence.

5. Once $w'(z)$ has been solved the next point on the wage grid can be computed as

$$
w_{i+1} = w'(z_i)\left(z_{i+1} - z_i\right)
$$

[7] Using, a quotient rule for integration, see here.

$$
\int \frac{f'(x)}{g(x)}dx = \frac{f(x)}{g(x)} + \int \frac{f(x)g'(x)}{[g(x)]^2}dx.
$$