Resource Logics with a Diminishing Resource

Extended Abstract

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ABSTRACT
Model-checking resource logics with production and consumption of resources is a computationally hard and often undecidable problem. We show that it is more feasible under the assumption that there is at least one diminishing resource, that is, a resource which is consumed by every action.

KEYWORDS
Model-checking; resources

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1 INTRODUCTION
There has been a considerable amount of work on resource logics interpreted over structures where agents’ actions produce and consume resources, for example [2, 3, 6–9, 12–14, 17–19]. There exists also a large body of related work on reachability and non-termination problems in energy games and games on vector addition systems with state [1, 11, 15, 16, 21]. The resource logics considered in this paper are extensions of the Alternating Time Temporal Logic (ATL), [10]. For ATL under imperfect information and with perfect recall uniform strategies, ATL-R, the model-checking problem is undecidable for three or more agents [20]. It is however decidable in the case of bounded strategies [23].

In this paper we introduce a special kind of models for resource logics satisfying a restriction that one of the resources is always consumed by each action. This is a very natural setting that occurs in many verification problems. One obvious example of such a resource is time. Other examples include systems where agents have a non-rechargeable battery and where all actions consume energy, e.g., nodes in a wireless sensor network; and systems where agents have a store of propellant that cannot be replenished during the course of a mission and all actions of interest involve manoeuvring, e.g., a constellation of satellites. We call this special resource that is consumed by every action a diminishing resource.

We study RB ± ATL⁰ and RB ± ATL⁰˞, diminishing resource versions of Resource-Bounded Alternating Time Temporal Logic (RB±ATL) [5]. The model-checking problem for RB±ATL is known to be 2EXPTIME-complete [6], while RB ± ATL⁰ model-checking is in PSPACE if resource bounds are written in unary. In the case of RB ± ATL⁰˞, the result of [23] does not apply immediately because the bound is not fixed in advance, but its model checking problem is decidable in EXPSPACE given encoding in unary. We also study RAL⁰, a diminishing resource version of Resource Agent Logic (RAL) [13]. Decidability of RAL⁰ follows from the result on the decidability of RAL on bounded models [13], but the PSPACE upper bound (for unary encoding) is new.

2 RB ± ATL⁰
The syntax of RB ± ATL⁰ is defined relative to the following sets: \( Agt = \{a_1, \ldots, a_n\} \) is a set of \( n \) agents, \( Res = \{res_1, \ldots, res_r\} \) is a set of \( r \) resource types, \( \Pi \) is a set of propositions, and \( B = \{Res^{agt}\} \) is a set of resource bounds (resource allocations to agents). Elements of \( B \) are vectors of length \( n \) where each element is a vector of length \( r \). We will denote by \( B_A \) (for \( A \subseteq Agt \)) the set of possible resource allocations to agents in \( A \). Formulas of RB ± ATL⁰ are defined by:

\[ \phi, \psi ::= p | \neg \phi | \phi \lor \psi | \langle\langle A^b \rangle\rangle \phi | \langle\langle A^b \rangle\rangle \psi U \langle\langle A^b \rangle\rangle \psi | \langle\langle A^b \rangle\rangle \psi R \psi \]

where \( p \in \Pi \), \( A \subseteq Agt \), and \( b \in B_A \). \( \langle\langle A^b \rangle\rangle \phi \) means that a coalition \( A \) can ensure that the next state satisfies \( \phi \) under resource bound \( b \). \( \langle\langle A^b \rangle\rangle \phi U \langle\langle A^b \rangle\rangle \psi \) means that \( A \) has a strategy to enforce \( \psi \) while maintaining the truth of \( \phi \) and the cost of this strategy is at most \( b \). \( \langle\langle A^b \rangle\rangle \phi R \langle\langle A^b \rangle\rangle \psi \) means that \( A \) has a strategy to maintain \( \psi \) until and including the time when \( \phi \) becomes true, or to maintain \( \psi \) forever if \( \phi \) never becomes true, and the cost of this strategy is at most \( b \).

The language is interpreted on the following structures:

Definition 2.1. A resource-bounded concurrent game structure with diminishing resource (RB-CGS⁰) is a tuple \( M = (Agt, Res, S, \Pi, \pi, Act, d, c, b) \) where:

- \( Agt, Res \) and \( \Pi \) are as above; the first resource type in \( Res \) is the distinguished diminishing resource;
- \( S \) is a non-empty finite set of states;
- \( \pi : \Pi \rightarrow \phi(S) \) is a truth assignment that associates each \( p \in \Pi \) with a subset of states where it is true;
- \( Act \) is a non-empty set of actions;
- \( d : S \times Agt \rightarrow \phi(Act) \setminus \{\emptyset\} \) is a function that assigns to each \( s \in S \) a non-empty set of actions available to each agent \( a \in Agt \);
- \( c : S \times Act \rightarrow Z^r \) is a partial function that maps a state \( s \) and an action \( \sigma \) to a vector of integers, where a positive (negative) integer in position \( i \) indicates consumption (production) of resource \( r_i \) by the action. The first position in the vector is always at most \( -1 \).
- \( \delta : S \times Act^{\left|Agt\right|} \rightarrow S \) is a partial function that maps every \( s \in S \) and \( \sigma \in d(s, a_1) \times \cdots \times d(s, a_n) \) to a state resulting from executing \( \sigma \) in \( s \).
In what follows, we use the usual point-wise notation for vector comparison and addition, and, given a function \( f \) returning a vector, we denote by \( f_1 \) the function that returns the \( i \)-th component of the vector returned by \( f \). Given an RB-CSG\( ^8 \) and a state \( s \in S \), a joint action by a coalition \( A \subseteq Agt \) is a tuple \( \sigma = (\sigma_a)_{a \in A} \) such that \( \sigma_a \in \{d, a\} \). The set of all joint actions for \( A \) at state \( s \) is denoted by \( D_A(s) \). Given a joint action by \( Aqt \), \( \sigma \in D_A(s) \), \( \sigma_A \) denotes the joint action executed by \( A \) as part of \( \sigma \): \( \sigma_A = (\sigma_a)_{a \in A} \). The set of all possible outcomes of a joint action \( \sigma \in D_A(s) \) at state \( s \) is:

\[
\text{out}(s, \sigma) = \{s' | \exists \sigma' \in D_A(s) : \sigma = \sigma' \land s' = \delta(s, \sigma')\}.
\]

A strategy for a coalition \( A \subseteq Agt \) in an RB-CSG\( ^8 \) is a mapping \( F_A : S^+ \rightarrow \text{Act}^{|A|} \) such that, for every \( \lambda \in S^+ \), \( F_A(\lambda) \in D_A(\lambda[|\lambda|]) \). A computation \( \lambda \) is consistent with a strategy \( F_A \) iff, for all \( i \leq 1 \leq |\lambda| \), \( \lambda[i+1] \in \text{out}(\lambda[i], F_A(\lambda[1:|\lambda|])) \). We denote by \( \text{out}(s, F_A) \) the set of all computations \( \lambda \) starting from state \( s \) that are consistent with \( F_A \). Given a bound \( b \in B \), a computation \( \lambda \in \text{out}(s, F_A) \), and \( \lambda = (\lambda[i])_{i \\in I} \). The set of all joint actions for \( s \) that are compatible with \( F_A \) for every \( i \geq 0 \), for every \( a \in A \), \( b - \sum_{i=0}^{\lambda[i]} c(F_A(\lambda[0,|\lambda|])) \geq c(F_A(\lambda[0,|\lambda|])) \).

A computation \( \lambda \) is \( b \)-maximal for a strategy \( F_A \) if it cannot be extended further while remaining \( b \)-consistent. The set of all \( b \)-maximal computations starting from state \( s \) that are \( b \)-consistent with \( F_A \) is denoted by \( \text{out}(s, F_A) \).

Given an RB-CSG\( ^8 \) and a state \( s \) of the truth of an RB-CSG\( ^8 \) formula \( \phi \) with respect to \( M \) and \( s \) is defined as follows (omitting the cases for \( \land \) and \( \rightarrow \)):

\[
M, s \models \langle A \rangle^8 \phi \quad \text{iff} \quad \exists \text{ strategy } F_A \text{ such that for all } b-\text{maximal } \lambda \in \text{out}(s, F_A, b), |\lambda| \geq 2 \text{ and } M, \lambda[2] \models \phi.
\]

\[
M, s \models \langle A \rangle^8 \phi \Upsilon \psi \quad \text{iff} \quad \exists \text{ strategy } F_A \text{ such that for all } b-\text{maximal } \lambda \in \text{out}(s, F_A, b), \exists i \text{ such that } 1 \leq i \leq |\lambda|: M, \lambda[i] \models \psi \land M, \lambda[j] \models \phi \quad \text{for all } j \in \{1, \ldots, i-1\}.
\]

\[
M, s \models \langle A \rangle^8 \phi \bigcirc \psi \quad \text{iff} \quad \exists \text{ strategy } F_A \text{ such that for all } b-\text{maximal } \lambda \in \text{out}(s, F_A, b), \forall i \text{ such that } 1 \leq i \leq |\lambda|: M, \lambda[i] \models \phi \land M, \lambda[j] \models \psi \quad \text{for all } j \in \{1, \ldots, i\}; \text{ or, } M, \lambda[i] \models \psi \quad \text{for all } j \text{ such that } 1 \leq j \leq |\lambda|.
\]

The following theorem is proved by demonstrating a model-checking algorithm for RB-CSG\( ^8 \), see [4]:

**Theorem 2.2.** The model-checking problem for RB-CSG\( ^8 \) is decidable in PSPACE (under unary encoding).

### 3 RB ± ATL\( ^8 \)\_R

In this section, we study RB ± ATL\( ^8 \)\_R, RB ± ATL\( ^8 \) with imperfect information and perfect recall. To model imperfect information, RB-CSG\( ^8 \) are extended with an indistinguishability relation \( \sim_a \) on states, for every agent \( a \). This relation can be lifted to finite sequences of states. Strategies under imperfect information should be uniform: if agent \( a \) is uncertain whether the history so far is \( \lambda \) or \( \lambda' \sim_a \lambda' \), then the strategy for \( a \) should return the same action for both \( \lambda \) and \( \lambda' \). A strategy \( F_A \) for a group of agents \( A \) is uniform if it is uniform for every agent in \( A \). In what follows, we consider strongly uniform strategies [22], that require the existence of a uniform strategy from all indistinguishable states:

\[
M, s \models \langle A \rangle^8 \phi \quad \text{under strong uniformity iff} \quad \text{there exists a uniform strategy } F_A \text{ such that, for all } s', s \text{ where } a \in A, \text{ for all } \lambda \in \text{out}(s', F_A, b), |\lambda| > 1 \text{ and } M, \lambda[2] \models \phi.
\]

The truth definitions for \( \langle A \rangle^8 \psi \Upsilon \psi \) and \( \langle A \rangle^8 \phi \bigcirc \psi \phi \) are also modified to require the existence of a uniform strategy from all states \( s' \) indistinguishable from \( s \) by any \( a \in A \).

**Theorem 3.1.** The model-checking problem for RB ± ATL\( ^8 \)\_R is decidable in EXPSPACE (under unary encoding).

### 4 RAL\( ^8 \)

RAL\( ^8 \) is obtained by modifying the definition of RAL [13] for the diminishing resource setting. The sets \( Agt, Res, \) and \( \Pi \) are as before. An *endowment (function)* \( \eta : Agt \times Res \rightarrow N \) assigns resources to agents: \( \eta_a(r) = \eta(a, r) \) is the amount of resource agent \( a \) has of resource type \( r \). \( En \) denotes the set of all possible endowments. Formulas of RAL\( ^8 \) are defined by:

\[
\phi, \psi : = p | \neg \phi | \psi \land \phi | \langle A \rangle^8_\Pi \psi \land \phi | \langle A \rangle^8_\Pi \phi \land \psi \quad \text{for all } p \in \Pi, A, B \subseteq Agt, \eta \in En,\quad \text{where } \langle A \rangle^8_\Pi \psi \land \phi \quad \text{is defined similarly as in RAL}.
\]

The models of RAL\( ^8 \) are RB-CSG\( ^8 \). Strategies are also defined as for RB ± ATL\( ^8 \). However, to evaluate formulas with a down arrow, such as \( \langle A \rangle^8_\Pi \psi \land \phi \), we need the notion of resource-extended computations. A resource-extended computation \( \lambda \in (S \times En)^+ \) is a sequence over \( S \times En \) such that the restriction to states (the first component), denoted by \( \lambda[S] \), is a path in the underlying model. The projection of \( \lambda \) to the second component is denoted by \( \lambda[En] \). 

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**Theorem 4.1.** The model-checking problem for RAL\( ^8 \) is decidable in EXPSPACE (under unary encoding).
REFERENCES


