ABSTRACT

We perform lens modelling and source reconstruction of Sub-millimetre Array (SMA) data for a sample of 12 strongly lensed galaxies selected at 500 μm in the Herschel Astrophysical Terahertz Large Area Survey (H-ATLAS). A previous analysis of the same data set used a single Sérsic profile to model the light distribution of each background galaxy. Here we model the source brightness distribution with an adaptive pixel scale scheme, extended to work in the Fourier visibility space of interferometry. We also present new SMA observations for seven other candidate lensed galaxies from the H-ATLAS sample. Our derived lens model parameters are in general consistent with previous findings. However, our estimated magnification factors, ranging from 3 to 10, are lower. The discrepancies are observed in particular where the reconstructed source hints at the presence of multiple knots of emission. We define an effective radius of the reconstructed sources based on the area in the source plane where emission is detected above 5σ. We also fit the reconstructed source surface brightness with an elliptical Gaussian model. We derive a median value $r_{\text{eff}} \sim 1.77$ kpc and a median Gaussian full width at half-maximum $\sim 1.47$ kpc. After correction for magnification, our sources have intrinsic star formation rates (SFR) $\sim 900$–$3500 M_{\odot}$ yr$^{-1}$, resulting in a median SFR surface density $\Sigma_{\text{SFR}} \sim 132 M_{\odot}$ yr$^{-1}$ kpc$^{-2}$ (or $\sim 218 M_{\odot}$ yr$^{-1}$ kpc$^{-2}$ for the Gaussian fit). This is consistent with that observed for other star-forming galaxies at similar redshifts, and is significantly below the Eddington limit for a radiation pressure regulated starburst.

Key words: gravitational lensing: strong – instrumentation: interferometers – galaxies: structure.

1 INTRODUCTION

The samples of strongly lensed galaxies generated by wide-area extragalactic surveys performed at sub-millimetre (sub-mm) to millimetre (mm) wavelengths (Negrello et al. 2010, 2017, hereafter N17; Vieira et al. 2013; Wardlow et al. 2013; Planck Collaboration XXVII 2015; Nayyeri et al. 2016), with the Herschel space observatory (Pilbratt et al. 2010), the South Pole Telescope (Carlstrom et al. 2011), and the Planck satellite (Cañameras et al. 2015) provide a unique opportunity to study and understand the physical properties of the most violently star-forming galaxies at redshifts $z > 1$. In fact, the magnification induced by gravitational lensing makes these objects extremely bright and, therefore, excellent targets for...
spectroscopic follow-up observations aimed at probing the physical conditions of the interstellar medium in the distant Universe (e.g. Omont et al. 2011, 2013; Valtchanov et al. 2011; Harris et al. 2012; Lupu et al. 2012; Yang et al. 2016; Oteo et al. 2017a). At the same time, the increase in the angular sizes of the background sources due to lensing allows us to explore the structure and dynamics of distant galaxies down to sub-kpc scales (e.g. Swinbank et al. 2010, 2015; Dye et al. 2015; Rybak et al. 2015).

In order to be able to fully exploit these advantages, it is crucial to reliably reconstruct the background galaxy from the observed lensed images. The process of source reconstruction usually implies an analytic assumption about the surface brightness of the source, for example by adopting Sérsic or Gaussian profiles (e.g. Bolton et al. 2008; Bussmann et al. 2013, hereafter B13; Bussmann et al. 2015; Calanog et al. 2014; Spilker et al. 2016). However, this approach can be risky, particularly for objects with often complex, clumpy, morphologies like those exhibited by sub-mm/mm selected dusty star-forming galaxies (DSFG) when observed at resolutions of tens of milliarcseconds (e.g. Swinbank et al. 2010, 2011; Dye et al. 2015).

Sophisticated lens modelling and source reconstruction techniques have recently been developed to overcome this problem. Wallington, Kochanek & Narayan (1996) introduced the idea of a pixellated background source, where each pixel value is treated as an independent parameter, thus avoiding any assumption on the shape of the source surface brightness distribution. Warren & Dye (2003) showed that with this approach the problem of reconstructing the background source, for a fixed lens mass model, is reduced to the inversion of a matrix. The best-fitting lens model parameters can then be explored via standard Monte Carlo techniques. In order to avoid unphysical solutions, the method introduces a regularization term that forces a certain degree of smoothness in the reconstructed source. The weight assigned to this regularization term is set by the Bayesian analysis (Suyu et al. 2006). Further improvements to the method include pixel sizes adapting to the lens magnification pattern (Dye & Warren 2005; Vegetti & Koopmans 2009; Nightingale & Dye 2015) and non-smooth lens mass models (Vegetti & Koopmans 2009; Hezaveh et al. 2016) in order to detect dark matter sub-structures in the foreground galaxy acting as the lens.

The method has been extensively implemented in the modelling of numerous lensed galaxies observed with instruments such as the Hubble Space Telescope (HST) and the Keck telescope (e.g. Treu & Koopmans 2004; Koopmans et al. 2006; Dye et al. 2008, 2014, 2015; Vegetti et al. 2010). For DSFGs, high-resolution imaging data usable for lens modelling can mainly be achieved by interferometers at the sub-mm/mm wavelengths where these sources are bright. Since the lensing galaxy is usually a massive elliptical, there is virtually no contamination from the lens at those wavelengths. However, an interferometer does not directly measure the surface brightness of the source, but instead it samples its Fourier transform, named the visibility function. As such, the lens modelling of interferometric images needs to be carried out in Fourier space in order to minimize the effect of correlated noise in the image domain and to properly account for the undersampling of the signal in Fourier space, which produces un-physical features in the reconstructed image.

Here, we start from the adaptive source pixel scale method of Nightingale & Dye (2015) and extend it to work directly in the Fourier space to model the Submillimetre Array (SMA) observations of a sample of 12 lensed galaxies discovered in the Herschel Astrophysical Terahertz Large Area Survey (H-ATLAS; Eales et al. 2010); 11 of these sources were previously modelled by B13 assuming a Sérsic profile for the light distribution of the background galaxy. We reassess their findings with our new approach and also present SMA follow-up observations of seven more candidate lensed galaxies from the H-ATLAS (N17), although we attempted lens modelling for only one of them, where multiple images can be resolved in the data.

The paper is organized as follows: Section 2 presents the sample and the SMA observations. Section 3 describes the methodology used for the lens modelling and its application to interferometric data. In Section 4 we present and discuss our findings, with respect to the results of B13 and other results from the literature. Conclusions are summarized in Section 5. Throughout the paper we adopt the Planck13 cosmology (Planck Collaboration XVI 2014), with $H_0 = 67\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, $\Omega_m = 0.32$, $\Omega_{\Lambda} = 0.68$, and assume a Kroupa (2001) initial mass function (IMF).

2 SAMPLE AND SMA DATA

2.1 Sample selection

Our starting point is the sample of candidate lensed galaxies presented in N17 which comprises 80 objects with $F_{500} \geq 100\,\text{mJy}$ extracted from the full H-ATLAS survey. We keep only the sources in that sample with available SMA $870\,\mu\text{m}$ continuum follow-up observations, which are presented in B13 (but see also Negrello et al. 2010; Bussmann et al. 2012). There are 21 in total. We excluded three cluster scale lenses for which the lens modelling is complicated by the need for three or more mass profiles for the foreground objects (HATLASJ114637.9−001132, HATLASJ141351.9−000026, HATLASJ132427.0+284449). We also removed those sources where the multiple images are not fully resolved by the SMA and therefore are not usable for source reconstruction, i.e. HATLASJ090302.9−014127, HATLASJ091304.9−005344, HATLASJ091840.4+023048, HATLASJ113526.2−014606, HATLASJ144556.1−004853, HATLASJ13859.2+292326. Finally we have not considered in our analysis HATLASJ090311.6+003907, also known as SPD81, which has been extensively modelled using high-resolution data from the Atacama Large Millimetre Array (ALMA Partnership et al. 2015; Dye et al. 2015; Hatsukade et al. 2015; Rybak et al. 2015; Swinbank et al. 2015; Tamura et al. 2015; Hezaveh et al. 2016). We have added an extra source to our sample, HATLASJ120127.6−014043, for which we recently obtained new SMA data (see Section 2.2). Therefore, our final sample comprises 12 objects, which are included in Table 1.

2.2 SMA data

The SMA data used here have been presented in B13 (but see also Negrello et al. (2010)). They were obtained as part of a large proposal carried out over several semesters using different array configurations from compact (COM) to very-extended (VEX), reaching a spatial resolution of ~0.5 arcsec, with a typical integration time of one to two hours on-source, per configuration. We refer the reader to B13 for details concerning the data reduction.

Between 2016 December and 2017 March we carried out new SMA continuum observations at $870\,\mu\text{m}$ of a further seven candidate lensed galaxies from the N17 sample (proposal ID: 2016B-S003 PI: Negrello). These targets were selected for having a reliable optical/near-IR counterpart with colours and redshift inconsistent with those derived from the Herschel/SPIRE photometry. Therefore they are very likely to be lensing events, where the lens is clearly detected in the optical/near-IR. They are listed in Table 1,
and shown in Fig. 1. Since we were awarded B grade tracks, not all of the observations were executed. Thus while all seven sources were observed in COM configuration, only for three did we also obtain data in the extended (EXT) configuration.

Observations of HATLASJ120127.6−014043, HATLASJ120319.1−011253, and HATLASJ121301.5−004922 were obtained in COM configuration (maximum baselines ~77 m) on 2016 December 29. The weather was very good and stable, with a mean atmospheric opacity $\tau_{225\,\text{GHz}} = 0.06$ (translating to 1 mm precipitable water vapour). All eight antennas participated, with 6 GHz of continuum bandwidth per sideband in each of two polarizations (for the equivalent of 24 GHz total continuum bandwidth). The central frequency of the observations was 344 GHz (870 μm). The target observations were interleaved over a roughly four hour rising to transit period, resulting in 40 min of on-source integration time for three targets (HATLASJ134158.5+292833 only received 30 min). Gain calibration was performed using observations of the nearby radio source 3C286, while bandpass and absolute flux scale were determined using observations of Callisto. Imaging the visibility data produced a synthesized beam with FWHM ~ 2 arcsec, and all four targets were detected with high confidence, with image rms values of 1.5–1.7 mJy beam$^{-1}$. HATLASJ120127.6−014043, HATLASJ120319.1−011253, and HATLASJ121301.5−004922 were also observed in the EXT configuration (maximum baselines 220 m) on 2017 March 29. The weather was excellent and fairly stable, with a mean $\tau_{225\,\text{GHz}}$ of 0.04 rising to 0.05 (translating to 0.65–0.8 mm precipitable water vapour). All eight antennas participated, now with 8 GHz of continuum bandwidth per sideband in each of two polarizations (for the equivalent of 32 GHz total continuum bandwidth); however, on one antenna only one of the two receivers was operational, resulting in a small loss of signal-to-noise ratio (SNR: ~3 percent). The mean frequency of the observations was 344 GHz (870 μm). The target observations were interleaved over a roughly six hour mostly rising transit period, resulting in 75 to 84 min of on-source integration time for each target. Bandpass and phase calibration observations were of 3C273, and the absolute flux scale was determined using observations of Gansymede. These extended configuration data were then imaged jointly with the compact configuration data from 2016 December 29. For each source, the synthesized resolution is roughly 1.0 × 0.8 arcsec$^2$, and the rms in the combined data maps ranges from 800 to 900 μJy beam$^{-1}$. 

Table 1. List of H-ATLAS lensed galaxies with SMA imaging data selected for the lens modelling and source reconstruction. Most are taken from Bussmann et al. (2013), excluding group/cluster scale lenses and sources which are not clearly resolved into multiple images by the SMA. The list also includes candidate lensed galaxies from N17 for which we have obtained new SMA observations. However only one of them is clearly resolved into multiple images because of the limited resolution achieved and therefore only this object, HATLAS J121301.5−004922, is considered for the lens modelling. Reading from left to right, columns following the identifier are: redshifts of the lens and of the background galaxy (from N17; when no spectroscopic redshift is available the photometric one is provided instead, in italic), SPIRE/Herschel flux densities at 250, 350, and 500 μm (from N17), flux density from the SMA, array configuration of the observations performed with the SMA (SUB=sub-compact, COMP=compact, EXT=extended, VEX=very extended).

<table>
<thead>
<tr>
<th>H-ATLAS IAU name</th>
<th>$z_{\text{opt}}$</th>
<th>$z_{\text{sub-mm}}$</th>
<th>$F_{250}$ (mJy)</th>
<th>$F_{350}$ (mJy)</th>
<th>$F_{500}$ (mJy)</th>
<th>$F_{\text{SMA}}$ (mJy)</th>
<th>SMA array configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>HATLASJ03051.0+013225</td>
<td>0.6261±0.0002</td>
<td>3.634</td>
<td>248.5 ± 7.5</td>
<td>305.3 ± 8.1</td>
<td>269.1 ± 8.7</td>
<td>76.6 ± 2.0</td>
<td>COM+EXT</td>
</tr>
<tr>
<td>HATLASJ085538.9+015537</td>
<td>2.0925</td>
<td>396.4 ± 7.6</td>
<td>367.9 ± 8.2</td>
<td>228.2 ± 8.9</td>
<td>50.6 ± 2.6</td>
<td>COM+EXT+VEX</td>
<td></td>
</tr>
<tr>
<td>HATLASJ090074.0−004200</td>
<td>0.6219</td>
<td>477.6 ± 7.3</td>
<td>327.9 ± 8.2</td>
<td>170.6 ± 8.5</td>
<td>20.3 ± 1.8</td>
<td>COM+EXT</td>
<td></td>
</tr>
<tr>
<td>HATLASJ101043.0−000322</td>
<td>0.793</td>
<td>1.786</td>
<td>420.8 ± 6.5</td>
<td>370.5 ± 7.4</td>
<td>221.4 ± 7.8</td>
<td>24.4 ± 1.8</td>
<td>COM+EXT+VEX</td>
</tr>
<tr>
<td>HATLASJ121513.5+261457</td>
<td>3.675</td>
<td>157.9 ± 7.5</td>
<td>202.3 ± 8.2</td>
<td>206.8 ± 8.5</td>
<td>64.5 ± 3.4</td>
<td>COM+EXT</td>
<td></td>
</tr>
<tr>
<td>HATLASJ122532.4+233527</td>
<td>0.2551</td>
<td>3.565</td>
<td>209.3 ± 7.3</td>
<td>288.5 ± 8.2</td>
<td>264.0 ± 8.5</td>
<td>85.5 ± 5.6</td>
<td>COM+EXT</td>
</tr>
<tr>
<td>HATLASJ131230.6+334410</td>
<td>0.8786</td>
<td>2.951</td>
<td>190.6 ± 7.3</td>
<td>281.4 ± 8.2</td>
<td>278.5 ± 9.0</td>
<td>483 ± 2.1</td>
<td>EXT</td>
</tr>
<tr>
<td>HATLASJ132308.4+245900</td>
<td>0.4276</td>
<td>3.1112</td>
<td>271.2 ± 7.2</td>
<td>278.2 ± 8.1</td>
<td>203.5 ± 8.5</td>
<td>49.5 ± 3.4</td>
<td>COM+EXT</td>
</tr>
<tr>
<td>HATLASJ133649.9+291800</td>
<td>2.2024</td>
<td>294.1 ± 6.7</td>
<td>286.0 ± 7.6</td>
<td>194.1 ± 8.2</td>
<td>37.6 ± 6.6</td>
<td>SUB+EXT+VEX</td>
<td></td>
</tr>
<tr>
<td>HATLASJ134429.4+303034</td>
<td>0.6721</td>
<td>2.3010</td>
<td>462.0 ± 7.4</td>
<td>647.5 ± 8.6</td>
<td>343.3 ± 8.7</td>
<td>55.4 ± 2.9</td>
<td>COM+EXT+VEX</td>
</tr>
<tr>
<td>HATLASJ142413.9+022303</td>
<td>0.595</td>
<td>4.243</td>
<td>112.2 ± 7.3</td>
<td>182.2 ± 8.2</td>
<td>193.3 ± 8.5</td>
<td>101.6 ± 7.4</td>
<td>COM+EXT+VEX</td>
</tr>
</tbody>
</table>
There is evidence of extended structure in several of our new targets, even from the COM data alone (e.g. HATLASJ133038.2+255128 and HATLASJ133846.5+255054); however only in HATLASJ120127.6−014043, which benefits from EXT data, are the typical multiple images of a lensing events clearly detected and resolved.

The measured 870 μm flux density for each source is reported in Table 1. It was computed by adding up the signal inside a customized aperture that encompasses the source emission. The quoted uncertainties correspond to the rms variation of the primary-beam corrected signal measured within the same aperture in 100 random positions inside the region defined by the primary beam of the instrument.

As explained in Section 3, our lens modelling and source reconstruction are performed on the SMA data by adopting a natural weighting scheme. The SMA dirty images obtained with this scheme are shown in the left panels of Fig. 2.

3 LENS MODELLING AND SOURCE RECONSTRUCTION

In order to perform the lens modelling and to reconstruct the intrinsic morphology of the background galaxy, we follow the Regularized Semi-linear Inversion (SLI) method introduced by Warren & Dye (2003), which assumes a pixelated source brightness distribution. It also introduces a regularization term to control the level of smoothness of the reconstructed source. The method was improved by Suyu et al. (2006) using Bayesian analysis to determine the optimal weight of the regularization term and by Nightingale & Dye (2015) with the introduction of a source pixelization that adapts to the lens model magnification. Here we adopt all these improvements and extend the method to deal with interferometric data.

We provide below a summary of the SLI method, but we refer the reader to Warren & Dye (2003) for more details.

3.1 The adaptive semi-linear inversion method

The image plane (IP) and the source plane (SP), i.e. the planes orthogonal to the line-of-sight of the observer to the lens containing the lensed images and the background source, respectively, are grided into pixels whose values represent the surface brightness counts. In the IP, the pixel values are described by an array of elements $d_j$, with $j = 1, ..., J$, and associated statistical uncertainty $\sigma_j$, while in the SP the unknown surface brightness counts are represented by the array of elements $s_i$, with $i = 1, ..., I$. For a fixed lens mass model, the IP is mapped to the SP by a unique rectangular matrix $f_{ij}$. The matrix contains information on the lensing potential, via the deflection angles, and on the smearing of the images due to convolution with a given point spread function (PSF). In practice, the element $f_{ij}$ corresponds to the surface brightness of the $j$th pixel in the lensed and PSF-convolved image of source pixel $i$ held at unit...
Figure 2. Results of the lens modelling and source reconstruction. From left to right: input SMA image (created using a natural weighting scheme); minimum $\chi^2$ image; residuals obtained by first subtracting the observed visibilities with the model ones and then transforming back to the real space; image obtained by lensing the reconstructed SP using the best-fitting lens model; the reconstructed background source with contours at 3 $\sigma$ (black curve) and 5 $\sigma$ (white curve). The caustics and the critical lines are shown in brown (in the second and fourth panels from left) and in red (in the right panel), respectively. The white hatched ellipse in the bottom left corner of the leftmost panels represents the SMA synthesized beam.
Figure 2 – continued
surface brightness. The vector, $S$, of elements $s_j$ that best reproduces the observed IP is found by minimizing the merit function

$$G = \frac{1}{2} \chi^2 = \frac{1}{2} \frac{1}{\sigma_j} \sum_{j=1}^{l} \left( \frac{\sum_{i=1}^{j} s_i f_{ij} - d_i}{\sigma_j} \right)^2 .$$

(1)

It is easy to show that the solution to the problem satisfies the matrix equation

$$F \cdot S = D .$$

(2)

where $D$ is the array of elements $D_i = \sum_{j=1}^{l} (f_{ij} d_i)/\sigma_j^2$ and $F$ is a symmetric matrix of elements $F_{ik} = \sum_{j=1}^{l} (f_{ij} f_{ik})/\sigma_j^2$. Therefore, the most likely solution for the source surface brightness counts can be obtained via a matrix inversion

$$S = F^{-1} D .$$

(3)

However, in this form, the method may produce unphysical results. In fact, each pixel in the SP behaves independently from the others and, therefore, the reconstructed source brightness profile may show severe discontinuities and pixel-to-pixel variations due to the noise in the image being modelled. In order to overcome this problem a prior on the parameters $s_j$ is assumed, in the form of a regularization term, $E_{\text{reg}}$, which is added to the merit function in equation (1). This forces a smooth variation in the value of nearby pixels in the SP:

$$G_\lambda = \frac{1}{2} \chi^2 + \lambda E_{\text{reg}} = \frac{1}{2} \chi^2 + \lambda \frac{1}{2} S^T H S ,$$

(4)

where $\lambda$ is a regularization constant, which controls the strength of the regularization, and $H$ is the regularisation matrix. We have chosen a form for the regularization term $E_{\text{reg}}$ that preserves the matrix formalism [see equation (9)]. The minimum of the merit function in equation (4) satisfies the condition

$$[F + \lambda H] \cdot S = D .$$

(5)

and, therefore, can still be derived via a matrix inversion

$$S = [F + \lambda H]^{-1} D .$$

(6)

The presence of the regularization term ensures the existence of a physical solution for any sensible regularization scheme.

The value of the regularization constant is found by maximizing the Bayesian evidence $1$ $\epsilon$ (Suyu et al. 2006)

$$2 \ln \{ \epsilon(\lambda) \} = -G_\lambda(S) - \ln(\det(F + \lambda H))$$

$$+ \ln(\det(\lambda H)) - \sum_{j=1}^{l} \ln(2\pi \sigma^2_j) ,$$

(7)

$S$ representing here the set of $s_j$ values obtained from equation (6) for a given $\lambda$.

The errors on the reconstructed source surface brightness distribution, for a fixed mass model, are given by the diagonal terms of the covariance matrix (Warren & Dye 2003):

$$\sigma^2_{ij} = \sum_{j=1}^{l} \sigma_j^2 \frac{\partial s_i}{\partial d_j} \frac{\partial s_i}{\partial d_j} = R_{ik} - \lambda \sum_{j=1}^{l} R_{ij}[R[H]_{ik}] ,$$

(8)

where $R = [F + \lambda H]^{-1}$. We use this expression to draw SNR contours in the reconstructed SP in Fig. 2 for the best-fitting lens model.

Equations (6)–(8) allow us to derive the SP solution for a fixed lens mass model. However, the parameters that best describe the mass distribution of the lens are also to be determined. This is achieved by exploring the lens parameter space and computing each time the evidence in equation (8) marginalized over $\lambda$, i.e.

$$\epsilon = \int f(\epsilon) P(\lambda) d\lambda ,$$

where $P(\lambda)$ is the probability distribution of the values of the regularization constant for a given lens model.

The best-fitting values of the lens model parameters are those that maximize $\epsilon$. We follow Suyu et al. (2006) by approximating $P(\lambda)$ with a delta function centred around the value $\lambda$ that maximizes equation (8), so that $\epsilon \approx \epsilon(\lambda)$.

The pixels in the SP that are closer to the lens caustics are multiply imaged over different regions in the IP, and therefore benefit from better constraints during the source reconstruction process, compared to pixels located further away from the same lines. As a consequence, the noise in the reconstructed source brightness distribution significantly varies across the SP. At the same time, in highly magnified regions of the SP the information on the source properties at sub-pixel scales is not fully exploited. In order to overcome this issue we follow the adaptive SP pixelisation scheme proposed by Nightingale & Dye (2015). For a fixed mass model the IP pixel centres are traced back to the SP and a k-means clustering algorithm $2$ is used to group them and to define new pixel centres in the SP. These centres are then used to generate Voronoi cells, mainly for visualization purposes. Within this adaptive pixelization scheme we use a gradient regularization term defined as:

$$E_{\text{reg}} = \sum_{i=1}^{N_s} \sum_{k=1}^{N_v(i)} (s_i - s_k)^2 ,$$

(9)

where $N_v(i)$ are the count members of the set of Voronoi cells that share at least one vertex with the $i$th pixel.

### 3.2 Modelling in the $uv$-plane

We extend the adaptive SLI formalism to deal with images of lensed galaxies produced by interferometers.

An interferometer correlates the signals of an astrophysical source collected by an array of antennas to produce a visibility function $V(u, v)$, that is the Fourier transform of the source surface brightness $I(x, y)$ sampled at a number of locations in the Fourier space, or $uv$-plane:

$$V(u, v) = \int \int A(x, y) I(x, y) e^{-2\pi i (ux + vy)} dxdy$$

(10)

where $A$ is the effective collecting area of each antenna, i.e. the primary beam. Because of the incomplete sampling of the $uv$-plane the image of the astrophysical source obtained by Fourier transforming the visibility function will be affected by artefacts, such as side-lobes, and by correlated noise. Therefore, a proper source reconstruction performed on interferometric data should be carried out directly in the $uv$-plane.

We define the merit function using the visibility function $3$

$$G_s = \frac{1}{2} \sum_{u,v} \left| \frac{V_{\text{model}}(u, v) - V_{\text{obs}}(u, v)}{\sigma(u, v)} \right|^2 + \frac{1}{2} S^T H S$$

(10)

$2$ This is slightly different than the h-means clustering scheme adopted by Nightingale & Dye (2015), though the same adopted in Dye et al. (2017).

$3$ Besides the presence of the regularization term, this definition of the merit function is exactly as in B13.

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$1$ Assuming a flat prior on $\log \lambda$.

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**H-ATLAS: modelling of lensed galaxies**
\[ G_j = \frac{1}{2} \sum_{i, \nu} \left( \frac{V_{\text{model}}(u, \nu) - V_{\text{obs}}(u, \nu)}{\sigma(u, \nu)} \right)^2 + \lambda \frac{1}{2} S^T H S. \]  

where \( N_{\text{vis}} \) is the number of observed visibilities \( V_{\text{obs}} = V_{\text{vis}} + iV_{\text{vis}}, \) while \( \sigma^2(u, \nu) = \sigma_{\text{real}}(u, \nu)^2 + \sigma_{\text{imag}}(u, \nu)^2, \) with \( \sigma_{\text{real}} \) and \( \sigma_{\text{imag}} \) representing the uncertainties on the real and imaginary parts of \( V_{\text{obs}}, \) respectively. With this definition of the merit function we are assuming a natural weighting scheme for the visibilities in our lens modelling. 

Following the formalism of equation (1), we can introduce a rectangular matrix of complex elements \( \hat{f}_{jk} = \hat{f}_{jk}^r + i \hat{f}_{jk}^i, \) with \( k = 1, ..., N_{\text{vis}} \) and \( j = 1, ..., N, N, \) being the number of pixels in the SP. The term \( \hat{f}_{jk} \) provides the Fourier transform of a source pixel of unit surface brightness at the \( j \)th visibility point in the \( u, v \)-plane, calculated at the location of the \( j \)th visibility point in the\( u, v \)-plane. 

The effect of the primary beam is also accounted for in calculating \( \hat{f}_{jk}. \) Therefore, equation (11) can be re-written as  

\[ G_j = \frac{1}{2} \sum_{i, \nu} \left( \frac{V_{\text{model}}(u, \nu) - V_{\text{obs}}(u, \nu)}{\sigma(u, \nu)} \right)^2 + \lambda \frac{1}{2} S^T H S. \]  

In deriving this expression we have assumed that \( S \) is an array of real values, as it describes a surface brightness.

The set of \( s \) values that best reproduces the observed IP can then be derived as in Equation (6):  

\[ S = [\hat{F} + \lambda H]^{-1} \hat{D}, \]  

with the new matrices \( \hat{F} \) and \( \hat{D} \) defined as follows:  

\[ \hat{F}_{jk} = \sum_{l=1}^{N_{\text{vis}}} \hat{f}_{jk}^r \hat{f}_{jl}^r + \hat{f}_{jk}^i \hat{f}_{jl}^i \]  

\[ \hat{D}_{ij} = \sum_{l=1}^{N_{\text{vis}}} \hat{f}_{jl}^r V_{\text{obs},l}^r + \hat{f}_{jl}^i V_{\text{obs},l}^i \sigma_i^2. \]  

The computation of the regularization constant is exactly as in equation (7) with \( G_j \), \( F \), and \( \sigma \) replaced by the corresponding quantities defined in this section.

3.3 Lens model

In order to compare our findings with the results presented in B13, we model the mass distribution of the lenses as a Singular Isothermal Ellipsoid (SIE; Kormann, Schneider & Bartelmann 1994), i.e. we assume a density profile of the form \( \rho \propto r^{-\alpha}, r \) being the elliptical radius. Our choice of a SIE over a more generic power-law profile, \( \rho \propto r^{-\alpha}, \) is also motivated by the results of the modelling of other lensing systems from literature (e.g. Barnabè et al. 2009; Dye et al. 2014, 2015, 2017), which show that \( \alpha \sim 2, \) and by the need of keeping to a minimum the number of free parameters. In fact, the resolution of the SMA data analysed here is a factor of three to four times worse than the one provided by the optical and near-infrared imaging data — mainly from the HST — used in the aforementioned literature.

However it is important to point out that a degeneracy between different lens model profiles can lead to biased estimates of the source size and magnification. In fact, as first discussed by Falco, Gorenstein & Shapiro (1985), a particular rescaling of the density profile of the lens, together with an isotropic scaling of the SP coordinates, produces exactly the same observed image positions and flux ratios (but different time delays). This is known as the mass-sheet transformation (MST) and represents a special case of the more general source-position transformation described by Schneider & Sluse (2014). Schneider & Sluse (2013) showed that the MST is formally broken by assuming a power-law model for the mass distribution of the lens, although there is no physical reason why the true lens profile should have such an analytic form. Furthermore, the power-law model is also affected by the \( \sigma - q - \alpha \) degeneracy between the lens mass (expressed in terms of the 1D velocity dispersion \( \sigma \)), the axis ratio \( (q) \), and the slope \( (\alpha) \). In fact, as discussed in Nightingale & Dye (2015), different combinations of these three parameters produce identical solutions in the IP, but geometrically scaled solutions in the SP, thus affecting the measurement of the source size and magnification. However, the same author also showed that the use of a randomly initialized adaptive grid (the same adopted in this work), with a fixed number of degree-of-freedom, removes the biases associated with this degeneracy. We will test our assumption of a SIE profile in a future paper using available HST and ALMA data, by comparing the lens modelling results obtained for \( \alpha = 2 \) with those derived for a generic power-law model (Negrello et al., in preparation).

The SIE profile is described by five parameters: the displacement of the lens centroid, \( \Delta x_L \) and \( \Delta y_L, \) with respect to the centre of the image, the Einstein radius, \( \theta_E, \) the minor-to-major axis ratio, \( q_L, \) the orientation of the semi-major axis, \( \theta_1, \) measured counter-clockwise from west. For simplicity we do not include an external shear unless it is needed to improve the modelling. In that case, its effect is described by two additional parameters: the shear strength, \( \gamma, \) and the shear angle, \( \theta_\gamma, \) also measured counter-clockwise from west, thus raising the total number of free parameters from 5 to 7.

3.4 Implementation

The lens parameter space is explored using MULTINEST (Feroz & Hobson 2008; Feroz, Hobson & Bridges 2009), a Monte Carlo technique implementing the nested sampling described in Skilling (2006). Flat priors are adopted for the lens model, within the range: 0.1 arcsec \( \leq \theta_E \leq 3.0 \) arcsec; 0 \( \leq \theta_L < 180^\circ; \) 0.2 \( \leq q_L < 1.0; \) \(-0.5 \) arcsec \( \leq \Delta x_L \leq 0.5 \) arcsec; \(-0.5 \) arcsec \( \leq \Delta y_L \leq 0.5 \) arcsec; 0 \( \leq \gamma \leq 0.3; \) 0 \( \leq \theta_\gamma < 180^\circ. \) In order to lighten the computational effort, a mask is applied to the IP pixels, keeping only those relevant, i.e. containing the lensed image, with minimum background sky. These are then traced back to the SP where they define the area used for the source reconstruction.

As suggested in N15, a nuisance in lens modelling algorithms is the existence of unrealistic solutions, occupying significant regions of the parameter space where the Monte Carlo method gets stuck. In general these local minima of the evidence correspond to a reconstructed SP that resembles a demagnified version of the observed
Table 2. Results of the modelling for the lens mass distribution, for which a SIE profile is assumed. The parameters of the model are: the normalization of the profile, expressed in terms of the Einstein radius ($\theta_E$); the rotation angle ($\theta_\text{L}$); measured counter-clockwise from west; the minor-to-major axis ratio ($q_L$); the position of the lens centroid from the centre of the images in Fig. 2; the shear strength ($\gamma$), and the shear angle ($\theta_\gamma$; counter-clockwise from west).

Table 3. Lens modelling results: source properties. Magnifications, $\mu_{3\sigma}$ and $\mu_{5\sigma}$, are evaluated as the ratio between the total flux density of the region in the SP with SNR \( \geq 3 \) and SNR \( \geq 5 \), respectively, and the total flux density of the corresponding region in the IP. $A_{\text{dust},3\sigma}$ and $A_{\text{dust},5\sigma}$ are the areas of the regions with SNR \( \geq 3 \) and SNR \( \geq 5 \) in the SP, while $r_{\text{eff},3\sigma}$ and $r_{\text{eff},5\sigma}$, are the radius of a circle with area equal to $A_{\text{dust},3\sigma}$ and $A_{\text{dust},5\sigma}$, respectively. FHWMs are the values of the FWHM of the major and minor axis length obtained from the Gaussian fit to the reconstructed source surface brightness, while $\text{FWHM}_{\text{maj}} = \sqrt{\text{FWHM}_{\text{maj}} \times \text{FWHM}_{\text{min}}}$.

### Results and Discussion

The best-fitting values of the lens model parameters are reported in Table 2, while the results of the source reconstruction are shown in Fig. 2. The first panel on the left is the SMA dirty image, generated by adopting a natural weighting scheme. The second and the third panels from the left show the reconstructed IP and the residuals, respectively. The latter are derived by subtracting the model visibilities from the observed ones and then imaging the differences. The panel on the right shows the reconstructed source with contours at $3\sigma$ (black curve) and $5\sigma$ (white curves), while the second panel from the right shows the image obtained by assuming the best-fitting lens model and performing the gravitational lensing directly on the reconstructed source. The lensed image obtained in this way is unaffected by the sampling of the $uv$-plane and can thus help to recognize in the SMA dirty image those features that are really associated with the emission from the background galaxy.

The estimated magnification factors, $\mu_{3\sigma}$ and $\mu_{5\sigma}$, are listed in Table 3 for the two adopted values of the SNRs in the SP, i.e. SNR \( \geq 3 \) and SNR \( \geq 5 \), respectively. The area, $A_{\text{dust}}$, of the regions in the SP used to compute the magnification factors is also listed in the same table together with the corresponding effective radius, $r_{\text{eff}}$. 

---

**Table 3**

<table>
<thead>
<tr>
<th>IAU name</th>
<th>$\theta_\text{E}$ (arcsec)</th>
<th>$\theta_\text{L}$ (°)</th>
<th>$q_L$</th>
<th>$\Delta q_L$ (arcsec)</th>
<th>$\Delta q_L$ (arcsec)</th>
<th>$\gamma$</th>
<th>$\theta_\gamma$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HATLASJ10383051.0+013225</td>
<td>0.31 ± 0.03</td>
<td>38.5 ± 7.5</td>
<td>0.33 ± 0.07</td>
<td>-0.49 ± 0.04</td>
<td>+0.07 ± 0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HATLASJ085358.9+015537</td>
<td>0.58 ± 0.05</td>
<td>172.6 ± 16.8</td>
<td>0.82 ± 0.08</td>
<td>+0.18 ± 0.03</td>
<td>-0.63 ± 0.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HATLASJ090740.0−004200</td>
<td>0.65 ± 0.02</td>
<td>143.7 ± 7.0</td>
<td>0.75 ± 0.07</td>
<td>-0.09 ± 0.02</td>
<td>-0.06 ± 0.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HATLASJ091043.1−000321</td>
<td>0.91 ± 0.03</td>
<td>112.9 ± 10.2</td>
<td>0.62 ± 0.09</td>
<td>0.00 ± 0.07</td>
<td>+0.33 ± 0.05</td>
<td>0.20 ± 0.05</td>
<td>76.0 ± 12.0</td>
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<tr>
<td>HATLASJ1120127.6−014043</td>
<td>0.82 ± 0.04</td>
<td>169.0 ± 6.7</td>
<td>0.58 ± 0.09</td>
<td>+0.06 ± 0.06</td>
<td>+2.00 ± 0.05</td>
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<td>-</td>
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<tr>
<td>HATLASJ1225135.4+261457</td>
<td>1.10 ± 0.02</td>
<td>280.0 ± 2.5</td>
<td>0.51 ± 0.06</td>
<td>-0.23 ± 0.05</td>
<td>+0.39 ± 0.04</td>
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<tr>
<td>HATLASJ125632.7+233625</td>
<td>0.69 ± 0.03</td>
<td>246.4 ± 7.4</td>
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<tr>
<td>HATLASJ132630.1+334410</td>
<td>1.76 ± 0.05</td>
<td>194.9 ± 9.0</td>
<td>0.62 ± 0.08</td>
<td>-0.49 ± 0.10</td>
<td>+0.67 ± 0.10</td>
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<tr>
<td>HATLASJ133008.4+245900</td>
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<td>172.1 ± 2.2</td>
<td>0.51 ± 0.03</td>
<td>-1.54 ± 0.08</td>
<td>+0.95 ± 0.04</td>
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<tr>
<td>HATLASJ133649.9+291801</td>
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<td>38.5 ± 4.3</td>
<td>0.53 ± 0.12</td>
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<td>+0.20 ± 0.04</td>
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</tr>
<tr>
<td>HATLASJ134429.4+303036</td>
<td>0.96 ± 0.01</td>
<td>82.7 ± 1.5</td>
<td>0.53 ± 0.07</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HATLASJ142413.9+022303</td>
<td>0.98 ± 0.02</td>
<td>91.0 ± 4.9</td>
<td>0.79 ± 0.04</td>
<td>+1.09 ± 0.03</td>
<td>+0.33 ± 0.04</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 3. Comparison between our results (blue error bars) and those of B13 (red error bars) for the parameters of the SIE lens mass model: Einstein radius, $\theta_\text{E}$, rotation angle, $\theta_\text{L}$, and minor-to-major axis ratio, $q_\text{L}$. Two data points are plotted whenever two lenses are employed in lens modelling (HATLASJ083051.0+013225, HATLASJ142413.9+022303).

The latter is defined as the radius of a circle of area equal to $A_{\text{dust}}$. We note that, despite the difference in the value of the area in the two cases, the derived magnification factors are consistent with each other. In fact, as the area decreases when increasing the SNR from 3 to 5, the centre of the selected region, in general, moves away from the caustic, where the magnification is higher. The two effects tend to compensate each other, thus reducing the change in the total magnification. Below we discuss our findings with respect to the results of B13 and other results from the literature.

4.1 Lens parameters

Fig. 3 compares our estimates of the lens mass model parameters with those of B13. In general we find good agreement, although there are some exceptions (e.g. HATLASJ133008.4+245900), particularly when multiple lenses are involved in the modelling, i.e. for HATLASJ083051.0+013225 and HATLASJ142413.9+022303. We briefly discuss each case individually. 

HATLASJ083051.0+013225: This is a relatively complex system (see fig. 1 of N17), with two foreground objects at different redshifts (B17) revealed at 1.1 $\mu$m and 2.2 $\mu$m by observations with HST/WFC3 (N17; Negrello et al., in preparation) and Keck/AO (Calanog et al. 2014), respectively. However, the same data show some elongated structure north of the two lenses, which may be associated with the background galaxy, although this is still unclear due to the apparent lack of counter-images (a detailed lens modelling of this system performed on ALMA+HST+Keck data is currently ongoing; Negrello et al., in preparation). In our modelling we have assumed that the two lenses are at the same redshift, consistently with the treatment by B13. However, compared to B13, we derive an Einstein radius that is higher for one lens (0.57 versus 0.43 arcsec) and lower for the other one (0.31 versus 0.39 arcsec). The discrepancy is likely due to the complexity of the system, which may induce degeneracies among the model parameters; however it is worth mentioning that while we keep the position of both lenses as free parameters, B13 fixed the position of the second lens with...
H-ATLAS: modelling of lensed galaxies

This system was observed with Keck/NIRC2 in the K$_\lambda$ band (Calanog et al. 2014). The background galaxy is detected in the near-IR in the form of a ring-like structure that was modelled by Calanog et al. assuming an SIE model for the lens and a Sérsic profile for the background source surface brightness. Our modelling of the SMA data gives results for the lens mass model consistent with those of Calanog et al., both indicating an almost spherical lens. B13 also find a nearly spherical lens ($q_L \approx 0.94$) but with a different rotation angle ($\theta_L \approx 160^\circ$ versus $\theta_L \approx 62^\circ$), even though the discrepancy is less than 3$\sigma$ once considered the higher confidence interval consequence of a spherical lens.

HATLASJ090740.0--004200: This is one of the first five lensed galaxies discovered in H-ATLAS (Negrello et al. 2010), and is also known as SDP.9. High-resolution observations at different wavelengths are available for this system, from the near-IR with HST/WFC3 (Negrello et al. 2014), to sub-mm with NOEMA (Oteo et al. 2017a) or 1.1 mm with ALMA (Wong et al. 2017), to the X-ray band with Chandra (Massardi et al. 2017). The results of our lens modelling of the SMA data are consistent with those obtained by other groups at different wavelengths (e.g. Dye et al. 2014; Massardi et al. 2017). However, B13 found a significantly lower lens axis-ratio compared to our estimate ($q_L = 0.50$ versus $q_L = 0.75$).

HATLASJ010431.3--000321: This is SDP.11, another of the first five lensed galaxies discovered in H-ATLAS (Negrello et al. 2010). HST/WFC3 imaging data at 1.1 $\mu$m reveal an elongated Einstein ring (Negrello et al. 2014), hinting to the effect of an external shear possibly associated with a nearby edge-on galaxy. In fact, Dye et al. (2014) introduced an external shear in their lens modelling of this system, which they constrained to have strength $\gamma \sim 0.23$. We also account for an external shear in our analysis. Our results are consistent with those of Dye et al. They also agree with the $\textit{Einstein}$ radius estimated by B13, although our lens is significantly more elongated and has a higher rotation angle. It is worth noticing, though, that B13 does not introduce an external shear in their analysis, which may explain the difference in the derived lens axial ratio.

HATLASJ20127.5--014043: This is the H-ATLAS source that we have confirmed to be lensed with the new SMA data. It is the only object in our sample for which we still lack a spectroscopic measure of the redshift of the background galaxy. The redshift estimated from the Herschel/SPIRE photometry is $z_{\text{sub-mm}} = 3.80 \pm 0.58$. The reconstructed source is resolved into two knots of emission, separated by $\sim 3.5$ kpc.

HATLASJ25135.4+261457: The estimated $\textit{Einstein}$ radius is slightly higher than reported by B13 ($\theta_E = 1.10 \pm 0.02$ arcsec versus $\theta_E = 1.02 \pm 0.03$ arcsec) while the rotation angle of the lens is smaller ($\theta_L = 28 \pm 2.5$ versus $\theta_L = 38 \pm 1^\circ$). The reconstructed source is quite elongated, extending in the SW to NE direction, with a shape that deviates from a perfect ellipse. This might suggest that, at the scale probed by the SMA observations, the source comprises two partially blended components. This morphology is not accounted for by a single elliptical Sérsic profile, which may explain the observed discrepancies with the results of B13.

HATLASJ25632.7+233625: For this system we find a lens that is more elongated compared to the value derived by B13 ($q_L = 0.54 \pm 0.09$ versus $q_L = 0.69 \pm 0.03$). The reconstructed source morphology has a triangular shape which may bias the results on the lens parameters when the modelling is performed under the assumption of a single elliptical Sérsic profile, as in B13.

HATLASJ132630.1+334410: The background galaxy is lensed into two images, separated by $\sim 3.5$ arcsec, none of them resembling an arc. This suggests that the source is not lying on top of the tangential caustic, but away from it, although still inside the radial caustic to account for the presence of two images. As revealed by HST/WFC3 observations (see N17, fig. 3), the lens is located close to the southermmost lensed image. The lack of extended structures, like arcs or rings, makes the lens modelling more prone to degeneracies. Despite that, we find a good agreement with the results of B13.

HATLASJ133008.4+245900: Besides the lens modelling performed by B13 on SMA data, this system was also analysed by Calanog et al. (2014) using Keck/AO $K_{\lambda}$-band observations, where the background galaxy is detected. The configuration of the multiple images is similar in the near-IR and in the sub-mm suggesting that the stellar and dust emission are co-spatial. We derive an $\textit{Einstein}$ radius $\theta_E = 1.03$ arcsec, higher than B13’s result ($\theta_E = 0.88$ arcsec). Our estimate is instead in agreement with the finding of Calanog et al. (2014) and Negrello et al. (in preparation; based on HST/WFC3 imaging data). Interestingly, the reconstructed background source is very elongated. This is due to the presence of two partially blended knots of emission, a main one extending across the tangential caustic and a second, fainter one located just off the fold of the caustic. This is another example where the assumption that the source is represented by a single Sérsic profile, made by B13, is probably affecting the estimated lens model parameters.

HATLASJ133649.9+291801: This is the single-lens system in our sample with the smallest $\textit{Einstein}$ radius, $\theta_E = 0.4$ arcsec. Our lens modelling gives results consistent with those of B13.

HATLASJ134429.4+303036: This is the 500 $\mu$m brightest lensed galaxy in the entire N17 sample. The observed lensed images indicate a typical cusp configuration, similar to what was observed in the well-studied lensed galaxy SDP.81 (e.g. Dye et al. 2015), where the background galaxy lies on the fold of the tangential caustic. According to our modelling the lens is significantly elongated ($q_L = 0.53$) in the north–south direction, consistent with what is indicated by available HST/WFC3 imaging data for the light distribution of the foreground galaxy (see fig. 3 of N17). We estimate a higher $\textit{Einstein}$ radius than the one reported by B13, although the two results are still consistent within 2$\sigma$.

HATLASJ142413.9+022303: This source – a 500 $\mu$m ‘riser’ – was first presented in Cox et al. (2011) while the lens modelling, based on SMA data, was performed in Bussmann et al. (2012). Observations carried out with HST/WFC3 and Keck/AO (Calanog et al. 2014) revealed two foreground galaxies, separated by $\sim 0.3$ arcsec, although only one currently has a spectroscopic redshift, $z = 0.595$ (B13). No emission from the background galaxy is detected in the near-IR. B13 modelled the system using two SIE profiles. We attempted the same but found no significant improvement in the results compared to the case of a single SIE mass distribution, which we have adopted here. We find $\theta_E = 0.97$ arcsec, consistent with the value derived from the lens modelling of ALMA data performed...
Figure 4. Magnification profiles of the reconstructed sources. The magnification factor, $\mu$, is evaluated in steps of SNR in the SP, from two up to the maximum and shown as a function of the effective radius of the area defined by the SP pixels with SNR above the adopted steps. The squares mark the values of the magnification calculated for SNR = 3 (outermost; light blue square) and SNR = 5 (innermost; dark blue square). The red point is the magnification factor estimated by B13. We have placed it at a radius corresponding to $2 \times r_{\text{half}}$, as this is the radius of the region in the SP used by B13 to compute the magnification.

For HATLASJ142313.9+022303, the point of B13 is located outside the plotted region, at $r_{\text{eff}} \sim 7 \, \text{kpc}$.

by Dye et al. (2017), which also assumed a single SIE profile. On the other hand, B13 obtained $\theta_{\text{E,1}} = 0.57$ arcsec and $\theta_{\text{E,2}} = 0.40$ arcsec for the two lenses. In this case the comparison with the B13 results is not straightforward. It is also important to note, as shown in Fig. 2 [see also Dye et al. (2017)], that the background source has a complex extended morphology, which cannot be recovered by a single Sérsic profile. Bussmann et al. (2012) modelled this system assuming two Sérsic profiles for the background galaxy but their results, particularly for the position of the second knot of emission in the SP, disagree with ours and with the findings of Dye et al. (2017).

According to our findings, in single-lens systems the use of an analytic model for the source surface brightness does not bias the results on the SIE lens parameters as long as the background galaxy is not partially resolved into multiple knots of emission. A way to overcome this problem would be to test the robustness of the results...
by adding a second source during the fitting procedure. However, the drawback of this approach is the increase in the number of free parameters, and, therefore, the increased risk of degeneracies in the final solution. We conceived our SLI method to overcome this problem and we recommend it in the modelling of lensed galaxies.\footnote{The codes used here are available upon request but will be made soon available via GitHub and at the webpage http://www.mattianegrello.com.}

### 4.2 Source magnifications and sizes

Fig. 4 shows, for each source, how the value of the magnification varies with the size of the region in the SP, as defined by the SNR of the pixels and here expressed in terms of \( r_{\text{eff}} \). The values of \( \mu_{3\sigma} \) and \( \mu_{5\sigma} \) are shown at the corresponding effective radii (light and dark blue squares, respectively), together with the magnification factor estimated by B13 (red dots). The latter is placed at a radius \( r_{\text{eff}} = 2 \times r_{\text{halo}} \), where \( r_{\text{halo}} \) is the mean half-light radius of the S\'{e}rsic profile used by B13 to model the source surface brightness. It is calculated as \( r_{\text{halo}} = a_s \sqrt{1 - \epsilon} \), with \( a_s \) and \( \epsilon \) being the half-light semi-major axis length and ellipticity of the S\'{e}rsic profile provided by B13. B13 computed the magnification factor for an elliptical aperture in the SP with semi-major axis length equal to \( 2 \times a_s \). It is easy to show that the area of this region is exactly equal to \( \pi \times (2 \times r_{\text{halo}})^2 \).

In general, we find lower values of the magnification factor compared to B13. Discrepancies are to be expected for systems like HATLASJ083501.0+013225 and HATLASJ142413.9+022303, where the best-fitting lens model parameters differ significantly from those of B13. However, a similar explanation may also be applied to systems like HATLASJ085358.9+015537, HATLASJ091043.0−000322, and HATLASJ134429.4+303034. In HATLASJ125135.3+261457, HATLASJ25632.4+233626, and HATLASJ133008.4+245900 the reconstructed source morphology is indicative of the presence of two partially blended components. The complexity of the source is not recovered by a single S\'{e}rsic profile and a significant fraction of the source emission lies beyond the region defined by B13 to compute \( \mu \). As a consequence their magnification factor is higher than our estimate. HATLASJ090740.0−004200, HATLASJ12630.1+334410, and HATLASJ133649.9+291800 are the only systems where our findings are quite consistent with those of B13 for both the source size and the magnification.

In Fig. 5 we show the effective radius of the dust emitting region in DSFGs at \( 1.5 \lesssim z \lesssim 5 \) from the literature, as a function of their infrared luminosity \( L_{\text{IR}} \), integrated over the rest-frame wavelength range 8–1000 \( \mu \text{m} \). Most of these estimates are obtained from ALMA continuum observations by fitting an elliptical Gaussian model to the source surface brightness. The value of \( r_{\text{eff}} \) reported in the figure is the geometric mean of the values of the FWHM of the minor and major axis lengths, unless otherwise specified. We provide below a brief description of the source samples presented in Fig. 5.

Simpson et al. (2015) carried out ALMA follow-up observations at 870 \( \mu \text{m} \), with \( \sim 0.3 \text{arcsec} \) resolution, of 52 DSFGs selected from the SCUBA-2 Cosmology Legacy Survey (S2CLS). They provide the median value of the FWHM of the major axis for the sub-sample of 23 DSFGs detected at more than \( 10 \sigma \) in the ALMA maps: \( \text{FWHM}_{\text{major}} = 2.4 \pm 0.2 \text{kpc} \). The median infrared luminosity of the same sub-sample is \( L_{\text{IR}} = (5.7 \pm 0.7) \times 10^{12} \text{L}_\odot \).

These are the values we show in Fig. 5 (triangular symbols), bearing in mind that we have no information on the ellipticity of the sources to correct for. Therefore, when comparing with other data sets, the Simpson et al. point should be considered as an upper limit.

Bussmann et al. (2015) have presented ALMA 870 \( \mu \text{m} \) imaging resolution, of 29 DSFGs from the Herschel Multi-tiered Extragalactic Survey (HerMES; Oliver et al. 2012). The sample includes both lensed and unlensed objects. Lens modelling is carried out assuming an elliptical Gaussian. The un-lensed galaxies are also modelled with an elliptical Gaussian. Their results are shown in Fig. 5 (square symbols), with FWHM = \( 2 \times r_s \) where \( r_s \) is the geometric mean of the semi-axes, as reported in their table 3. The infrared luminosity of the lensed samples has been corrected for the magnification. We only show the sources in their sample that are not resolved into multiple components as no redshift and infrared luminosity are available for the individual components. This reduces their sample to nine objects: eight strongly lensed and one un-lensed.

Ikarashi et al. (2015) have exploited ALMA 1.1 \( \mu \text{m} \) continuum observations to measure the size of a sample of 13 AzTEC-selected DSFGs with \( z_{\text{phot}} \sim 3–6 \) and \( L_{\text{IR}} \sim 2–6 \times 10^{12} \text{L}_\odot \). They fit the data in the \( uv \)-plane assuming a symmetrical Gaussian. In Fig. 5 we show their findings as FWHM = \( 2 \times R_\epsilon \), (hexagon symbols), where \( R_\epsilon \) is the value they quote in their table 1 for the half-width at half-maximum of the symmetric Gaussian profile. Their 1.1 \( \mu \text{m} \) flux densities have been rescaled to 870 \( \mu \text{m} \) by multiplying them by a factor of 1.5 (see Oteo et al. 2017b). For most of the sources in the Ikarashi et al. sample the redshift is loosely constrained, with only lower limits provided. Therefore we only consider here two...
sources in their sample with an accurate photometric redshift, i.e. ASXDF1100.027.1 and ASXDF1100.230.1.

Hodge et al. (2016) used high-resolution (0.16 arcsec) ALMA 870 μm continuum observations of a sample of 16 DSFGs with 1 ≤ z ≤ 5 and LIR ∼ 4 × 10^{12} L⊙ from the LABOCA Extended Chandra Deep Field South (ECDFS) sub-mm survey (LESS; Hodge et al. 2013; Karim et al. 2013) to investigate their size and morphology. Their results are represented by the diamond symbols in Fig. 5.

Oteo et al. (2016, 2017) have performed ALMA 870 μm continuum observations, at ~0.12 arcsec resolution, of 44 ultrared DSFGs (i.e. with Herschel/SPIRE colours: F_{250μm}/F_{500μm} > 1.5 and F_{500μm}/F_{850μm} > 1). They confirmed a significant number of lensed galaxies, which we do not consider here because no lens modelling results are available for them yet. We only consider unlensed objects for which Oteo et al. provide a photometric or a spectroscopic redshift (Oteo et al. 2016, 2017b) have performed ALMA 870 μm continuum observations of a sample of 16 DSFGs with 1 ≤ z ≤ 5 and LIR ∼ 4 × 10^{12} L⊙ from the LABOCA Extended Chandra Deep Field South (ECDFS) sub-mm survey (LESS; Hodge et al. 2013; Karim et al. 2013) to investigate their size and morphology. Their results are represented by the diamond symbols in Fig. 5.

In order to compare with the data from literature we also fit our reconstructed source surface brightness using an elliptical Gaussian model. The derived values of the FWHM along the major and the minor axis of the ellipse are reported in Table 3 together with their geometric mean FWHM$_{m}$ = $\sqrt{FWHM_{maj} \times FWHM_{min}}$. However, we warn the reader that the use of a single Gaussian profile to model the observed surface brightness of DSFGs could bias the inferred sizes because of the clumpy nature of these galaxies, as partially revealed by our SMA data. In fact, we find that the values of FWHM$_{m}$ are systematically lower than those of r$_{eff,5\sigma}$ and r$_{eff,3\sigma}$, as demonstrated in Fig. 6.

With this caveat in mind, we show in Fig. 5 the size of the dust emitting region derived from the Gaussian fit to our reconstructed source surface brightness. The infrared luminosity, obtained from a fit to the observed spectral energy distribution (SED; see Section 4.3 for details), has been corrected for lensing by assuming $\mu = \mu_{5\sigma}$. In Fig. 5 the data points are coloured according to their redshift. Most of the objects have a photometric redshift estimate; those with a spectroscopic redshift are highlighted by a black outline. We observe a significant scatter in the distribution of the source sizes, particularly at the lowest luminosities, with values ranging from ≤0.5 kpc to ≥3 kpc. The lack of sources with r$_{eff} ≤ 1$ kpc at LIR ≥ 10^{13} L⊙ is possibly a physical effect. In fact, such luminous sources would have extreme values of the SFR surface brightness, and, therefore, would be quite rare. The absence of z > 3.5 sources with r$_{eff} ≥ 1$ kpc and LIR ≥ 3 × 10^{13} L⊙ is likely due to their lower surface brightness, which makes these objects difficult to resolve in high-resolution imaging data. Based on these considerations it is challenging to draw any conclusion about the dependence of the size on either luminosity or redshift.

The sources in our sample have a median effective radius r$_{eff,5\sigma}$ ∼ 1.77 kpc, rising to r$_{eff,3\sigma}$ ∼ 2.46 kpc if we consider all the pixels in the SP with SNR > 3, while the median FWHM of the Gaussian model is ∼1.47 kpc. These values are consistent with that observed for other DSFGs at similar, or even higher, redshifts.

4.3 Star formation rate surface densities

We derive the star formation rate (SFR) of the sources in our sample from the magnification-corrected IR luminosity, LIR, using the Kennicutt & Evans (2012) relation:

\[
\text{SFR} \left(\text{M}_\odot \text{yr}^{-1}\right) \sim 1.3 \times 10^{-10} \frac{L_{\text{IR}}}{L_{\odot}}. \tag{16}
\]

which assumes a Kroupa IMF. B13 provide an estimate of the total far-infrared (FIR) luminosity, L_{FIR} (integrated over the rest-frame...
Figure 7. Observed far-IR to sub-mm SEDs of the 12 sources (blue error bars; from Herschel and SMA) together with the best-fitting modified blackbody spectrum (red curve; assuming dust emissivity index $\beta = 1.5$). The shaded red area shows the 68 per cent confidence region associated with the best-fitting model. The redshift of the source is reported on the top-right corner of each panel. The redshift is spectroscopic for all the sources but one, i.e. HATLAS J120127.6−014043. This accounts for the significantly larger uncertainty in the fit to the SED of HATLAS J120127.6−014043 compared to the other sources.

The inferred infrared luminosities and SFRs are listed in Table 4 and they have been corrected for the effect of lensing by assuming $\mu = \mu_{3\sigma}$ (see Table 3). To directly compare with B13 we also report, in the same table, the magnification-corrected far-IR luminosity. The dust temperature is not listed in that table because it does not depend on lensing, unless differential magnification is affecting the redshift when the latter is not spectroscopically measured, as in the case of HATLAS J120127.6−014043.
In Table 4, the intrinsic properties of the 12 sources in our sample. The correction for the effect of lensing has been implemented by using the value $\mu = \mu_{509}$ for the magnification as reported in Table 3. The dust temperature, $T_{\text{dust}}$, and the dust luminosities, $L_{\text{dust}}$, are computed by taking the FIR luminosity quoted by B13 and dividing by the source area calculated as $0.014043 \pm 0.015537$ K (Table 3). The correction for the effect of lensing has been implemented by using the value $\Sigma_{\text{1}} = 3482\pm3030$ for the lensing magnification, unless differential magnification is affecting the far-IR to sub-mm photometry. In both Tables 4 and 5, we report the dust luminosity and SFR surface densities, defined as $\Sigma_{\text{dust}} = L_{\text{dust}}/A_{\text{dust}}$ and $\Sigma_{\text{SFR}} = SFR/A_{\text{dust}}$, respectively. Both are corrected for the magnification and computed using the value of $A_{\text{dust}}$ corresponding to the adopted SNR threshold in the SP.

In Table 5, we report the dust luminosity and SFR surface densities, as defined in Table 4, but this time assuming $\mu = \mu_{509}$ and $A_{\text{dust}} = A_{\text{dust},509}$. The dust temperature is not listed here because it does not depend on the magnification, unless differential magnification is affecting the far-IR to sub-mm photometry.

Fig. 8 shows the SFR surface density of the sources in our sample as a function of their infrared luminosity. We find median values $\Sigma_{\text{SFR},\text{FWHM}} = 215 \pm 114\, M_{\odot}\, \text{yr}^{-1}\, \text{kpc}^{-2}$ (dark magenta line) inside the region of radius $r = \text{FWHM}_{\text{in}}$, and $\Sigma_{\text{SFR},\text{50}9} = 132 \pm 69\, M_{\odot}\, \text{yr}^{-1}\, \text{kpc}^{-2}$ (dashed blue line) for the region inside the 50% SNR threshold. The red circles are the findings of B13 for the same sources. We have computed them by using the FIR luminosity quoted by B13 and first converting it into $L_{\text{FIR}}$ (by multiplying $L_{\text{FIR}}$ by a factor of 1.9, as reported in B13) and then into SFR using equation (16). Then, we have divided the SFR by the source area calculated as $A_{\text{dust}} = \pi r_{\text{dust}}^2$. Finally, we have divided the result by 2. In fact, by definition, the region within the circle of radius $r_{\text{dust}}$ contributes only half of the total luminosity (and therefore SFR) of the source. The median value of the SFR surface densities calculated in this way is $\Sigma_{\text{SFR}} \sim 219\, M_{\odot}\, \text{yr}^{-1}\, \text{kpc}^{-2}$ (dotted-dashed red line) which is similar to the estimate inside the region of radius FWHM, although the data points of B13 (red circles) display a much larger scatter than ours. Therefore such an estimate should be taken as an upper limit for the SFR surface brightness of the whole galaxy.

We also show, in the same figure, the median SFR surface density of DSFGs from the Simpson et al. (2015) sample (green triangle). They estimated $\Sigma_{\text{SFR}} \sim 90 \pm 30\, M_{\odot}\, \text{yr}^{-1}\, \text{kpc}^{-2}$, assuming a Salpeter IMF, which decreases to $\Sigma_{\text{SFR}} \sim 67\, M_{\odot}\, \text{yr}^{-1}\, \text{kpc}^{-2}$ if we assume a Kroupa IMF as in equation (16). Their estimate of the SFR surface density is consistent with ours, although their sample has a lower infrared luminosity on average. However their way of calculating
The adopted source reconstruction technique allows us to define a SNR map in the SP. We use it to more robustly define the area of the dust emitting region in the SP and its corresponding magnification, while in B13, the source extension is an arbitrary factor of the half-light radius of the adopted Sersic profile. We report the size of the reconstructed sources in our sample as the radius of a circle that encloses all the SP pixels with SNR > 5 (or SNR > 3). However, for a more straightforward comparison with results in literature, we also quote the value of the FWHM obtained from a Gaussian fit to the reconstructed SP. For almost 50 per cent of our sample the estimated effective radii are larger than 2 × r_{eff,5σ}; i.e. the radius of the region chosen by B13 to represent the source physical size when computing the magnification. As a consequence, we estimate, in general, lower magnification factors than those quoted in B13.

Once corrected for the magnification, our sources still retain very high SFRs ~ 900 - 3500 M_⊙ yr^{-1}. With a median effective radius r_{eff,5σ} ~ 1.77 kpc (r_{eff,3σ} ~ 2.46 kpc) and a median FWHM ~ 1.47 kpc, our sample has a median SFR surface density Σ_{SFR,5σ} ~ 132 M_⊙ yr^{-1} kpc^{-2} (Σ_{SFR,3σ} ~ 78 M_⊙ yr^{-1} kpc^{-2}) or Σ_{SFR,FWHM} ~ 215 M_⊙ yr^{-1} kpc^{-2} from the Gaussian fit). This is consistent with what is observed for other DSFGs at similar red-shifts, but it is only approximately 10 per cent of the limit achievable in a radiation pressure supported starburst galaxy.

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