Robust control of flow separation over a pitching aerofoil using plasma actuators

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Abstract: We address the problem of controlling the unsteady flow separation over an aerofoil, using plasma actuators. Despite the complexity of the dynamics of interest, we show how the problem of controlling flow separation can be formulated as a simple output regulation problem, so that a simple control strategy may be used. A robust multivariable feedback control is designed and tested in a configuration with two actuator/sensor pairs. Accurate numerical simulations of incompressible flows on a pitching NACA 0012 at Reynolds Re = 20,000 are performed in order to illustrate the effectiveness of the proposed approach. Robust, fast flow reattachment is achieved, along with both stabilisation and increase/reduction of the lift/drag, respectively. The control system shows good dynamic performances, as the angle of attack is varied. The chosen output can be experimentally measured by appropriate sensors and the extension of the proposed approach to 3D configurations is straightforward.

Keywords: Flow control, Robust control, Flow separation, Plasma actuators, Boundary layer control

1. INTRODUCTION

Closed-loop flow control is aimed at altering a natural flow state into a more desirable state, which is chosen depending on control objectives. Crucial examples are: manipulation of flow separation, drag reduction, noise suppression, stall prevention etc. Within this context, the incorporation of control theory into many open problems in fluid mechanics presents a host of new opportunities, with a wide range of applications in disparate fields (e.g. gas turbines, aircraft, as well as ground and marine vehicles). The control input is usually an electric signal, which has to be converted to a physical quantity by means of an actuator. A new and original technology using non-thermal surface plasmas has witnessed a significant growth in interest in recent years (see, for example, Choi et al., 2011; Corke et al., 2007; Feng et al., 2015), as they: have no moving parts; exhibit an extremely fast time-response; are characterised by low mass and low input power. These surface dielectric barrier discharge (DBD) actuators are used to accelerate the near-wall flow, thus modifying the velocity profile within the boundary layer.

In this paper, we focus on the robust feedback control of the flow separation. In most flow control applications the objective is to suppress the separation bubble, as it is responsible for both a loss of the lift and an increase of the drag and it might lead to stall conditions. However, the coupled neutrally-charged fluid and plasma dynamics are not trivial to control: neither the analytical model, which results in a system of nonlinear Partial Differential Equations (PDEs), nor the high-dimensional discretised dynamics, are suitable for control design purposes. Furthermore, the dependence of the dynamical properties on the both the unknown flow and geometry parameters is highly nonlinear. Therefore, it is very difficult to obtain an accurate control-oriented model, which allows for the design of effective adaptive controllers. On the other hand, we show that the problem of controlling flow separation along the aerofoil can be formulated as a simple output regulation problem, so that a simple control strategy may be used. Recent works on feedback flow separation control using plasma actuators include Benard and Moreau (2009), where a slope-seeking algorithm is proposed to obtain maximum time-averaged lift, which is measured by a three-component balance. Cho and Shyy (2011) proposed a retrospective cost adaptive algorithm to minimize the variation of the aerodynamic lift. However, the latter, which is the chosen output in both Benard and Moreau (2009) and Cho and Shyy (2011), cannot be measured in real-time in practical flow control applications.

Our objective is to solve the problem of directly controlling the unsteady flow separation using real-time velocity measurements, which are available in realistic applications (see, for example, Hanson et al., 2014; Segawa et al., 2010). We propose this flow separation problem as a practical application of the new theoretical results in Marino and Tomei (2015). In particular, we extend the simple SISO (Single-Input-Single-Output) robust output regulator presented in Marino and Tomei (2015) to MIMO (Multi-Input-Multi-Output) configurations. The aim of this paper is to show how, despite the high complexity of the system, a very simple robust output regulator is sufficient to effectively suppress the flow separation along an aerofoil, using DBD plasma actuators. Accurate numerical simulations of flows past a pitching NACA 0012 at Reynolds Re = 20,000 are performed in order to test the control effectiveness, in the presence of complex nonlinear dynamics, which are neglected.
in the control design. Robust performances, with respect to both parameter variations (e.g. geometry of the domain and Reynolds number) and model uncertainties, are achieved.

2. PROBLEM STATEMENT AND OBJECTIVES

In this paper, we address the practical problem of robustly controlling the unsteady flow separation over an aerosolid, using the plasma actuators’ voltage as the control inputs and realistically available real-time velocity measurements as the control outputs. In particular, we aim to formulate and solve the flow separation problem, i.e. to make

\[ \partial_t u_i(x,t) + \nabla \cdot (\mathbf{u} u_i(x,t)) = \mathbf{f}_i(x,t), \quad x \in \Omega, \quad 0 < t \leq T, \]

as a simple output regulation problem, i.e. to make the measured outputs

\[ y_i(t) = u_i(x_t), \]

for \( i = 1, \ldots, n_s, \) Here: \( u \) is the time-dependent flow velocity vector; \( x \) and \( x_t \) denote the spatial coordinates and the \( i \)-th sensor location, respectively; \( \Gamma_y \) represents the aerosolid boundary; \( n \) and \( \tau \) are the normal and tangent unit vectors to \( \Gamma_y \), respectively; \( n_s \) is the number of sensors. Our objective is to design a simple robust output feedback control, along with suitable reference signals \( \gamma \), in order to suppress the flow separation over the aerosolid in different scenarios, depending on uncertain parameters, e.g., Reynolds number \( Re \) and angle of attack \( \beta \). To this end, we assume there exist suitable configurations of actuators and sensors, along with suitable reference gains \( \epsilon_i \) for the outputs \( y_i(t) \), which guarantee that, given a certain range for both \( Re \) and \( \beta \), the solution of the output regulation problem (2) implies the solution of the flow separation problem (1). This is formalised by the following assumption.

Assumption 1. For any \( \delta > 0 \) there exist some reference gains \( \epsilon_i > 0 \), a \( T_{\epsilon} > 0 \) and a \( T_{\beta} \geq \max_{i=1,\ldots,n_s} T_{\epsilon} \), such that, if \( y_i(t) > \epsilon_i \) for all \( t > T_{\epsilon}, i = 1, \ldots, n_s, \) then \( \partial_t u_i(t,x) \rvert_{\partial \Omega} > -\delta \) for all \( t > T_{\epsilon}, \) \( Re \in [Re_{\epsilon},Re_{\beta}], \beta \in [\beta_{\epsilon},\beta_{\beta}] \).

3. FLOW MODEL

Let \( \Omega \) be an open bounded domain in \( \mathbb{R}^2 \) and let \( T > 0 \) denote the final time. The flow of an incompressible viscous Newtonian fluid can be described by the non-dimensionalised Navier-Stokes equations, which are derived from the conservation of mass and momentum, namely,

\[ \partial_t \mathbf{u} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{\nu}{Re} \Delta \mathbf{u} + \mathbf{f} \quad \text{in} \quad (0,T) \times \Omega, \]

\[ \mathbf{u}(0,x) = \mathbf{u}_0(x) \quad \text{in} \quad (0,T) \times \Omega, \]

with initial condition

\[ \mathbf{u}(0,x) = \mathbf{u}_0(x) \quad \text{in} \quad (0,T) \times \Omega, \]

and boundary conditions

\[ \mathbf{u}(t,x) = \mathbf{g}(t,x) \quad \text{on} \quad \Gamma_{in}, \]

\[ \mathbf{u}(t,x) = \mathbf{0} \quad \text{on} \quad \Gamma_{out}, \]

\[ \left( \frac{1}{Re} \nabla \mathbf{u} - p \mathbf{I} \right) \mathbf{n} = 0 \quad \text{on} \quad \Gamma_{out}. \]

Here: \( x \in \Omega; \mathbf{n} \) denotes the unit outward normal vector on \( \partial \Omega = \Gamma_{in} \cup \Gamma_{out} \cup \Gamma_{out}; \Gamma_{in}, \Gamma_{out} \) denote the inflow and outflow (aerodynamic) boundaries, respectively; \( \mathbf{u} : [0,T] \times \Omega \rightarrow \mathbb{R}^2 \) is the velocity vector; \( p : [0,T] \times \Omega \rightarrow \mathbb{R} \) is the pressure; \( \mathbf{g} : [0,T] \times \Omega \rightarrow \mathbb{R}^2 \) is a sufficiently smooth function denoting the inflow boundary condition; \( I \in \mathbb{R}^{2 \times 2} \) is the identity matrix; \( Re = \frac{\rho U_{inf} c}{\mu} \) is the Reynolds number; \( \mu \) is the fluid viscosity (in Pa·s); \( \rho \) is the fluid density (in kg/m³); \( \mu = 0.1 \) m is the chord length; \( \mathbf{f} : [0,T] \times \Omega \rightarrow \mathbb{R}^2 \) is the total body force field, which depends on the control inputs. The latter can be expressed as

\[ \mathbf{f}(t,x) = \frac{c}{\rho U_{inf}^2} f_i(t,x), f_j(t,x) = \frac{c}{\rho U_{inf}^2} \sum_{j=1,...,n_a} f_j^0(t,x), \]

where \( f_i, f_j \) are the streamwise and normal component (in N/m²) and \( f_j^0 \) is the single force distribution of the \( j \)-th actuator (in N/m³). All the above listed functions are assumed to be sufficiently smooth. The wall-tangential velocity, evaluated at the selected sensor location \( x_t \),

\[ y_i(t) = u_i(x_t) = \tau(x_t) \cdot u_i(x_t), \]

where \( \tau \) denotes the tangent unit vector, is chosen as the measured output. Several models for the DBD actuator force have been proposed (see, for example, (Corke et al., 2007) for a detailed review). Here, we select a modified version of the recent model proposed by Yang and Chung (2015), which has shown a good agreement with the experimental data. The model is characterised by an exponential dependence on the spatial coordinates and, in particular, the force is modelled by a Rayleigh distribution (see Yang and Chung, 2015); thereby,

\[ f_j^0(t,x) = f_j^0(t,x) = \frac{\nu}{Re} \sum_{j=1,...,n_a} f_j^0(t,x) \]

\[ = f_j^0(t) \ln \frac{\nu}{Re} \left( \frac{\nu}{Re} \right) \]

\[ \text{for} \quad j = 1, ..., n_a, \text{where,} \quad f_j^0(t) = k_n \nu \ln \frac{\nu}{Re} \ln \frac{\nu}{Re}, \quad \nu \in \mathbb{R}^n \text{is the total plasma force;} \quad V_j^0(t) = \mathbf{v} \text{is the amplitude variation of the operation voltage (in kV);} \quad v_j(t) = V_j^0(t)/V_m \text{is the corresponding non-dimensionalised voltage input, scaled by} \quad V_m; \quad f_j^0(t) \text{is the tangential and normal components, with respect to the aerosolid, of the force density, respectively;} \quad \lambda_j^0, \lambda_j^0 \geq 0 \text{are related to} \quad x = (x,y) \text{by a coordinate transformation and respectively refer to the tangential and normal components, relative to the geometry, in the reference frame centred in} \quad x_a \text{(see figure 1).} \]

The parameters

\( 2.1 \) Tangential force density \( f_j^0 \) and reference frames.

\( A_j^0 = 1.6, \sigma_j^0 = 1.9, k_n^0 = 52000 \ln(2) / A_j^0 \lambda_j^0, \) for \( j = 1, ..., n_a, \) are chosen as in Yang and Chung (2015), where this model has been compared with particle image velocimetry (PIV) data, whilst, for the sake of simplicity, a simple linear dependence of the body force on the applied peak-to-peak voltage is assumed here.

A high-order, nonlinear state-space system can be obtained by spatially discretising the system of nonlinear PDEs (3), (5), (4), (7), thus yielding a nonlinear system of \( n \) ODES,

\[ \begin{cases} E \mathbf{x} = \mathbf{F}(\mathbf{x}) + \mathbf{G} \mathbf{v} \quad \text{in} \quad (0,T), \\ y = H \mathbf{x} \quad \text{in} \quad (0,T), \end{cases} \]

with initial conditions \( \mathbf{x}(0) = \mathbf{x}_0, \) where: \( \mathbf{x} = \mathbf{x}(t) : \mathbb{R} \rightarrow \mathbb{R}^n \) is the state vector representing the evolution in time of the nodal values of the flow fields; \( x = dx/dt, E \in \mathbb{R}^{2n \times n} \) is related to the mass matrix; \( \mathbf{F} : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a sufficiently
smooth nonlinear function; $\mathbf{G} \in \mathbb{R}^{\eta \times \eta}$ is the input matrix representing the nodal values of the time-independent part of the body force; $\mathbf{v} = \mathbf{v}(t) : \mathbb{R} \to \mathbb{R}^{\eta}$ is the input vector; $\mathbf{y} = \mathbf{y}(t) : \mathbb{R} \to \mathbb{R}^{\eta}$ is the chosen output vector; $\mathbf{H} \in \mathbb{R}^{\eta \times \eta}$ is the output matrix representing the space-discretisation of (7). In particular, we discretise system (3), (5), (4), (7), using $\chi$navis, a general-purpose, second order, finite volume, multiblock, unsteady Reynolds averaged Navier-Stokes equations (uRaNSe) based solver, developed at CNR-INSEAN. For the sake of conciseness, details of the numerical solver are not given here, the interested reader is addressed to Di Mascio et al. (2009); Broglia et al. (2014).

4. MIMO FEEDBACK CONTROL PROBLEM

A Balanced Dynamic Mode Decomposition (BDMD) linear model (see Pasquale et al., 2016) approximating system (9), for given Reynolds number $Re$ and angle of attack $\beta$, can be obtained in the following form:

$$
\begin{cases}
\dot{\xi} = A\xi + Bv, \quad \xi(0) = \xi_0, \\
y = C\xi,
\end{cases}
$$

(10)

where: $\xi : \mathbb{R} \to \mathbb{R}^{\eta}$ is the reduced-order state vector; $A \in \mathbb{R}^{\eta \times \eta}$ is a low-order linear operator approximating the nonlinear dynamics, whose eigenvalues belong to the open left half of the complex plane; $B \in \mathbb{R}^{\eta \times \eta}$ is the input matrix; $C \in \mathbb{R}^{\eta \times \eta}$ is the output matrix. Let: $I_s$ be the $r \times r$ identity matrix and $P(s) = C(sI - A)^{-1}B$, whose poles have all negative real part, be the open-loop $n_s \times n_s$ transfer function matrix of system (10). Define $P_d(s) = \frac{n_{s+1}(s)}{n_{s+1}(s)}$ and let $v^*, y^*$ and $\xi^*$ be the references for the input, output and state, respectively. Denoting $\hat{\xi} = \xi - \xi^*$ and $\eta = -v^* : \mathbb{R} \to \mathbb{R}^{\eta}$, the error dynamics are given by

$$
\begin{cases}
\dot{\hat{\xi}} = A\hat{\xi} + B(v + \eta), \\
\dot{\eta} = 0, \quad \eta(0) = \eta_0, \\
\dot{\hat{y}} = C\hat{\xi},
\end{cases}
$$

(11)

so that the control problem can be formulated as a disturbance rejection problem, where the reference input $v^* = -\eta$ can be viewed as a disturbance vector, which matches the control input $v$ (see Marino and Tomei, 2015). We aim at modifying the results in Marino and Tomei (2015) for our MIMO system (10), in the case of equal number of sensors $n_s$ and actuators $n_a$, i.e., $n_p = n_a = n_s \geq 1$, where $n_p$ denotes the number of actuator/sensor pairs. Here, $\chi^{(i)}_a$ and $\chi^{(i)}_s$ denote the position of the $j$-th actuator and the $i$-th sensor with respect to the chord length, respectively (see figure 1). To this end, we assume the following.

Assumption 2. Given any $i \in \{1, \ldots, n_p\}$, there exist some positive constants $\epsilon, \epsilon_0, \epsilon_1$ and $\epsilon_2$, such that the coefficients of $A, B, C$, belong to their corresponding compact sets $[a_{\underline{l}} - \epsilon a_{\underline{l}}, a_{\underline{l}} + \epsilon a_{\underline{l}}], [b_{\underline{l}} - \epsilon b_{\underline{l}}, b_{\underline{l}} + \epsilon b_{\underline{l}}], [c_{\underline{l}} - \epsilon c_{\underline{l}}, c_{\underline{l}} + \epsilon c_{\underline{l}}]$, for any $k, l = 1, \ldots, r$, and are such that $P_d(0)$ does not change sign, for any $Re \in \mathbb{R}_{Re} = [\mathbb{R}_{Em}, \mathbb{R}_{EM}], \beta \in \mathbb{R}_\beta = [\beta_m, \beta_M]$, where $\mathbb{R}_{Em}$ and $\mathbb{R}_{EM}$ denote the minimum and maximum Reynolds numbers, respectively, and $\beta_m$ and $\beta_M$ denote the minimum and maximum angles of attack, respectively.

Assumption 3. The $n_p$ pairs are numbered so that $\chi^{(i)}_a < \chi^{(i)}_s < \chi^{(i+1)}_a < \chi^{(i+1)}_s$, for $i = 1, \ldots, n_p - 1$.

Assumption 4. The distance $|\chi^{(i+1)} - \chi^{(i)}|$ is sufficiently large and, thus, the transfer function between the output $y_i$ and the input $v_j$ is such that $P_j(s) = 0$, for any $i < j$. In particular, Assumption 4 implies that the dynamics of the outputs do not depend on the inputs of the actuators that are located further downstream, so that, by virtue of Assumption 3, $P(s)$ is a lower triangular matrix. Similarly to Marino et al. (2015), the control problem becomes to design suitable feedback laws $v_j(t)$ for system (10), based on the real-time measurements $y_i(t)$, in order to robustly regulate the latter to given reference regions, as in (2). The key objective is to design $v$ such that the closed-loop trajectories of system (10) are guaranteed to evolve within some “safe” invariant set in different scenarios, depending on uncertain parameters. To this aim, on the basis of the recent results in Marino and Tomei (2015), we design a robust output regulator guaranteeing exponential convergence of the regulation error: it only requires the system to have a non-zero steady-state gain of known sign.

4.1 Control Algorithm

We translate the initial control objective (2) into the following: $y_i(t) \in \Omega_c = [\epsilon_m, \epsilon_M]$, where $\epsilon_m$ and $\epsilon_M$ are chosen positive constants. In particular, the lower bound for the output reference can be chosen in order to guarantee any $a$ priori fixed requirement, such as, in the present application, the suppression of the separation bubble over the aerofoil; the upper bound can be chosen in order to limit the power consumption. Therefore, the control problem (similarly to Marino et al., 2015) becomes to design $v$ such that the chosen controlled output $y$ belongs to a “safe” compact set $\Omega_c = \Omega_{c_1} \times \Omega_{c_2} \times \ldots \times \Omega_{c_n}$. To this aim, the reference outputs $y_i^{(c)}$ are chosen as

$$
y_i^{(c)}(t) = \begin{cases}
\epsilon_m, & \text{if } y_i(t) < \epsilon_m, \\
y_i(t), & \text{if } y_i(t) \in \Omega_c, \\
\epsilon_M, & \text{if } y_i(t) > \epsilon_M.
\end{cases}
$$

(12)

The resulting control algorithm reads

$$
\begin{cases}
\dot{\hat{y}}_i = k_i \text{sign}(P_d(0))\hat{y}_i, \\
\hat{y}_0(0) = \hat{y}_0, \\
\hat{y}_i(0) = -\hat{y}_i,
\end{cases}
$$

(13)

for $i = 1, \ldots, n_p$, where $\hat{y}_i = y_i - y_i^{(c)}$. The overall control algorithm (13), (12) depends on: the measured outputs $y_i$; the bounded reference $y_i^{(c)}$; the sign of the diagonal elements $P_d(0)$; the positive design parameters $k_i, \epsilon_m, \epsilon_M$. Note that, when $\epsilon_m = \epsilon_M = \epsilon_M$, for $i = 1, \ldots, n_p$, the control algorithm (13), (12) reduces to an output regulator with a constant output reference.

Assumption 5. The positive control gains $k_i > 0$ are chosen so that the dynamics of the $i$-th input $v_i$, which is related to the pair $(\chi^{(i)}_a, \chi^{(i)}_s)$, are much faster than the ones of the $i + 1$-th input $v_{i+1}$, which is related to the pair $(\chi^{(i+1)}_a, \chi^{(i+1)}_s)$, for any $i = 1, \ldots, n_p$.

Assumption 5 implies a time-scale separation between the actuator/sensor pairs, so that $v_j$ act as constant inputs for the dynamics of $y_i$, for any $i > j$. Furthermore, by virtue of Assumptions 3, 4, the dynamics of $y_i$ do not depend on $v_j$, for any $i < j$. This is physically reasonable since, given a suitable reference set, the solution of the regulation problem for the $i$-th pair implies that the flow separation might occur only downstream of $\chi^{(i)}_a$. Thus, the upstream outputs are not affected by the downstream inputs.

4.2 Stability Analysis

The main result of this section, which extends the results obtained in Marino and Tomei (2015) to MIMO systems of the form (10), is summarised in the following theorem.
Theorem 1. Consider the closed-loop system (10), (12), (13). Assume that $P_k(0) \neq 0$ with known sign. Then, for any initial condition $(\epsilon_n, \eta_n, \hat{y}_0)$, there exist sufficiently small $k_j > 0$, such that the regulation error $\bar{y} = y(t) - y^*(t)$ and the control input error $v(t) - v^*(t)$ exponentially tend to zero, as $t$ tends to infinity, for any $0 < k_j \leq k_j^*, i = 1, \ldots, n_p$.

Proof. a) Case $\epsilon_j = \epsilon_m = \epsilon M$. System (11) can be rewritten as $\dot{Y}(s) = P(s)(V(s) + \eta)$. Define 
$$Q_n(s) = 1 + k, P_n(s) \left( \frac{\text{sign}(P(0))}{s} \right) = n_Q(s),$$
which represents the closed-loop transfer function of the first pair $i = 1$, i.e., by Assumption 3, the most upstream pair. By the root locus, for sufficiently small $k_i > 0$, $r$ zeros of $Q_n(s)$ are sufficiently close to the $r$ poles of $P_n(s)$ and, therefore, they have negative real part. The remaining branch of the root locus starts from $0$ in the $s$-plane with angle $\pi$, so that also the remaining zeros of $Q_n(s)$ have negative real part. The time-scale separation between each pair implies that the stability of the closed-loop system is determined only by the zeros of the transfer function (14), which have negative real part for sufficiently small $k_i, i = 1, \ldots, n_p$.

b) Case $\epsilon_m < \epsilon_m$. Let $\bar{y} = v - v^* = \eta - \bar{y}$ and $\bar{x} = [\bar{\xi}, \bar{\eta}]^T$. Define $K = \text{diag}(k_1 \text{sign}(P_n(0)), \ldots, k_p, \text{sign}(P_{np}(0)))$. The closed-loop error dynamics can be written as
$$\dot{\bar{x}} = A \bar{x} + B \bar{v},$$
$$\bar{y} = [C, 0] \bar{x}.$$ The characteristic polynomial of the closed-loop matrix $A_k$ can be computed as
$$p_{A_k}(s) = \text{det}(s I_{n_p} - A_k) = \text{det}(s I_n - A) K C I_{n_p} = \text{det}(s I_n - A) \text{det}(s I_{n_p} + KC(s I_n - A)^{-1} B) = n_Q(s) \cdots n_Q(s),$$
where $I_{n_p}$ is the $n_p \times n_p$ identity matrix. Therefore, $A_k$ is Hurwitz, as its eigenvalues coincide with roots of $n_Q(s)$, and have negative real part for any sufficiently small $k_i, i = 1, \ldots, n_p$. Thus, there exist two symmetric, positive definite matrices $P$ and $Q$ satisfying the Lyapunov equation: $PA_k A_k^T P = Q$. Consider the candidate Lyapunov function $V(t) = \bar{x}(t)^T P \bar{x}(t)$, satisfying
$$\alpha_1 \|\bar{x}(t)\|^2 \leq V(t) \leq \alpha_2 \|\bar{x}(t)\|^2,$$
where $\alpha_1, \alpha_2 > 0$ are positive constants. The time derivative of $V(t)$, along the trajectories of the closed-loop system satisfies the following inequality:
$$\dot{V} \leq -\alpha_1 \|\bar{x}(t)\|^2 - \alpha_2 \|V(t)\|^2,$$
thus implying the closed-loop boundedness and the exponential convergence to zero of both the regulation error $\bar{y}(t)$ and the control input error $v(t) - v^*(t)$, as $t$ tends to infinity.

Let $\xi = \bar{\xi} - \xi$ and $\eta = \bar{\eta} - \eta$. When the output vector belongs to the compact set $\Omega_\epsilon$, we have $\xi = 0, \hat{\xi} = 0, \bar{\eta} = 0$. Thus, for any $t \geq 0$, such that $v(t) \in \Omega_{\epsilon}, \dot{V}(t) \equiv 0$. When the output does not belong to the reference region, there exist three positive constants $\alpha_1, \alpha_2, \alpha_3 > 0$ such that $V(t)$ and its time derivative satisfy (15) and (16), respectively. Therefore, for any $t \geq 0$ such that $y(t) \notin \Omega_{\epsilon}, \dot{V}(t) < 0$ and the distance $d_{\epsilon}(x(t), \Omega_{\epsilon}) \leq \inf_{x \in \Omega_{\epsilon}} \|x - x\| = \sqrt{\|Px\|^2}$ between $x$ and its reference set $\Omega_{\epsilon}$ satisfies $d_{\epsilon}(x(t), \Omega_{\epsilon}) \leq \alpha_3 \|x\|^2 \leq e^{-\alpha_3 t}, \text{where}$ $\alpha = \alpha_3 \alpha_2$. Since $0 \leq \dot{V}(t) \in \mathcal{C}^1$ is lower bounded and its derivative is semi-negative definite, it admits a finite limit (see Courant, 1937, p. 61). Closed-loop boundedness and exponential convergence of $V(t)$ (and, therefore, of $\xi$ and $\bar{\eta}$) to zero are thus guaranteed, according to Barbabals lemma, as $V(t)$ is uniformly continuous. Consequently, $\xi(t)$ converges to a constant reference $\xi \in \Omega_{\epsilon}$ and $v(t)$ converges to a constant value $\bar{v}$, as $t$ tends to infinity. If $\bar{v} \notin \Omega_{\epsilon}$, then $\xi = C \bar{v} = -P(0)v \notin \Omega_{\epsilon}$, which contradicts $\xi \in \Omega_{\epsilon}$. Therefore, $\bar{v} \in \Omega_{\epsilon}$ and the distance $d_{\epsilon}(x(t), \Omega_{\epsilon})$ exponentially tends to zero, as $t$ tends to infinity.

5. SIMULATION RESULTS

Although the unknown theoretical linear model (10) cannot represent an accurate approximation of the actual nonlinear dynamics, we aim to show how the simple control algorithm (12), (13), is sufficient to effectively suppress the separation bubble around the aerofoil in the presence of time-varying angles of attack, using real-time velocity measurements at discrete locations. Only the sign of the steady-state gain, which is assumed to be non-zero, is required to be known. In Pasquale et al. (2016) a BDMD model, yielding a positive steady-state gain of the reduced-order transfer function, has been obtained for 2D flows around a NACA 0012 aerofoil, with angle of attack $\beta = 20^\circ$ and $Re = 1,000$. The sensor was placed at $2\epsilon/5$ and the actuator at $\epsilon/5$. Based on this single reduced-order approximation of the incompressible Navier-Stokes equations (3), (5), (4), (7), we assume, coherently with Assumption 1, a positive sign of the steady-state gains of the transfer function between any pair, i.e., $P_n(0) > 0, i = 1, \ldots, n_p$, if the sensor is close enough to the actuator, i.e., the distance $\Delta_{\epsilon}^{(i)}$ is sufficiently small. This is physically reasonable, as the actuators’ force distribution is directed downstream. The robustness of the proposed control scheme (12), (13), is tested at $Re = 20,000$ in a 2D configuration $C = \{n = 2, n = 2, \epsilon = 0.02, \epsilon = 0.06, \Delta_{\epsilon}^{(i)} = 0.2, \Delta_{\epsilon}^{(i)} = 0.2, \Delta_{\epsilon}^{(i)} = [0.1, 0.15], \Delta_{\epsilon}^{(i)} = [0.05, 0.11]\}$, where the angle of attack is varied within the range $\Delta_{\epsilon}^{(i)} = [5, 25]$. In particular, we consider two different scenarios:

$$\beta_k = -\frac{2n + M}{2n + M} \cos \frac{n(t - t_m)}{t} \leq 0 \leq t_m \leq t \leq t_m + \Delta t, \Delta t > 0, t > t_m + \Delta t \text{ or } t < t_m - \Delta t \text{ or } t < t_m, \Delta t \leq t \leq t_m,$$
where $\Delta_{\epsilon}^{(i)} = -10^6, \Delta_{\epsilon}^{(i)} = 10^6, \beta_0 = 15^6, \Delta_1 = 10^6, t_m = 30, t_m = 50, T = 60, \Delta t = 5 \text{ (see figure 2).}$ In both

Fig. 2. Time-varying angles $\beta_1(t)$ (left) and $\beta_2(t)$ (right).

the scenarios, the controller is activated between $t_0 = 15$ and $t_f = T = 60$. In the first scenario, the initial angle of attack $\beta_0 = 15^6$ is, first, smoothly decreased to $5^6$ and, then, increased to $\beta_0 = 15^6$ again. In the second scenario, the initial angle is, first, smoothly increased to $25^6$ and, then, decreased to $\beta_0 = 15^6$.
again. The output measurements $y_i(t) = \nu(t, x_i, y_i)$ are taken at $y_0 = 0.0005$ above the aerofoil. The computational grid has $N = 127,872$ total volumes and is divided into extremely fine actuator grids (see the right frame of figure 3), a fine C-type inner grid (left sketch of figure 3) and coarser outer grids. The connections between the different grids are handled using an overlapping grid approach. The inner region around the profile has $320 \times 96$ volumes, along the tangent and the normal directions, respectively; the points are clustered towards the wall, where the finest mesh spacing is set equal to $2.1 \times 10^{-4}$. In the near wake region, a block of $128 \times 192$ volumes in the streamwise and vertical directions, respectively, is used in order to correctly characterize the wake time evolution. The chosen control gains are depicted in figures 4, 5 for the scenarios in the near wake region, a block of $128 \times 10 \times 96$ volumes, and actuator’s block (right).

The time histories (blue) and time average (dashed cyan) of the drag and lift coefficients ($C_D = 2F_D/(\rho U_{\infty}^2 c)$ and $C_L = 2F_L/(\rho U_{\infty}^2 c)$, respectively, where $F_D$, $F_L$ denote the total forces per span length) are compared (top figures) with the corresponding time histories (green) and time averaged (dashed red) coefficients for the simulations without actuators: a $75\%$ average drag reduction, for both $\beta_1$ and $\beta_2$, along with a $20\%$, for $\beta_1$, and $50\%$, for $\beta_2$, average lift increase is obtained. When the angle of attack is decreased, $C_D$ becomes negative during the transients because it includes the actuators’ contribution. The inputs show smooth, fast transient performances and the output measurements are robustly regulated to their corresponding reference region, which is shown with a dashed cyan line. The regulation of the outputs $y_i(t)$ to $\Omega_i$ implies the solution of the flow separation problem, as it is shown in figure 6, which depicts the time-averaged tangential velocity computed at the first cell centre node above the aerofoil. The steady-state vorticity contours for $5^\circ$, $15^\circ$ and $25^\circ$, using 10 levels within the range $[-15, 15]$, are compared in figure 7 with the results of the simulation without actuation. The most critical transients, from $15^\circ$ to $25^\circ$, are shown in figure 8 for the scenarios $\beta_2(t)$. The snapshots of vorticity fields show an evident flow reattachment: the proposed robust control has effectively suppressed the separation bubble, as well as the shedding vortices, during both transient and steady-state regimes.

6. CONCLUSIONS

We addressed the practical problem of robustly controlling the unsteady flow separation over an aerofoil, using the plasma actuators’ voltage as the control inputs and realistically available real-time velocity measurements as the control outputs. In particular, under some simplifying assumptions, we formulated the flow separation problem as an output regulation problem and solved the latter by designing a simple MIMO robust feedback control, consisting of $n_p$ SISO regulators. The proposed controller is computationally cheap and only requires a non-zero steady-state gain of known sign, for each actuator/sensor pair. Accurate numerical simulations of flows past a pitching NACA 0012 at Reynolds $Re = 20,000$, for angles of attack between $5^\circ$ and $25^\circ$, are performed in order to test the control effectiveness in the presence of complex dynamics, which are neglected in the control design. Although the proposed controller is simple, as it is based on an integral action, it effectively suppresses the separation bubble along a pitching aerofoil.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from the People Programme (Marie Curie Actions) of the European Unions Seventh Framework Programme (FP7/2007-2013) under REA grant agreement no 608322.

REFERENCES


Fig. 5. Simulation results in the scenario $\beta = \beta_2$.

Fig. 6. Time-averaged tangential velocity.


Fig. 7. Vorticity contours without (left) and with (right) closed-loop control for: $\beta = 5^\circ$ (top); $\beta = 15^\circ$ (middle); $\beta = 25^\circ$ (bottom).

Fig. 8. Vorticity contours for $\beta(t) = \beta_2(t)$ and, from left to right: $t = 30.0, 32.5$ (top); $t = 37.5, 40.0$ (bottom).


