Enhancing Electromagnetic Launch Capability: Design of a motor system to launch large UAV’s from small naval platforms

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Synopsis
Unmanned Aerial Vehicles (UAVs) play a critical and increasing role in modern combat, giving significantly enhanced surveillance, communication and strike capabilities. There are currently a large range of UAV’s available, but in general the more capable the UAV, the larger and heavier it becomes. The largest UAVs such as the Naval MQ-4C Triton are currently operated exclusively from land-based runways. The BAE systems “UXV Combatant” is designed as a drone mothership, able to deploy large numbers of UAV’s and including two 50tn flight decks for launch and recovery systems. In order for the UXV combatant to deploy large naval drones such as the Triton, a significant electromagnetic launch system will be needed. This work looks at design and modelling of an electromagnetic launch system capable of launching a Triton sized large UAV at high acceleration from a small naval platform. The work considers both the UXV combatant design, and the use of the rear flight deck of the existing Type 45 design. The work consists of outlining the launch requirements of the drone, initial scoping of system power requirements and selection of launch topology, analytical modelling of the system launch cycle and Finite Element Analysis to verify the chosen motor system design. The results of this work show that an electromagnetic linear motor based system capable of launching large naval drones is feasible from a Type 45 scale launch platform, either configured as a drone carrier or by using the existing flight deck.

Keywords: UAV, Electromagnetic Launch, UXV Combatant, EML

1. Introduction

1.1. Advances in electromagnetic launch

In recent years, Electromagnetic Launch (EML) of aircraft has gone from a theoretical concept with limited and largely unsuccessful practical applications, for example the “Electropult” system of the 1940’s, as detailed in (Proverbs, 2011), to technical proving and full scale testing with individual machines, such as the EMCAT system (GE Power Conversion, 2013). The technology has finally arrived at a point where a practical system is currently undergoing sea trials as part of a carrier system (US Navy, 2015), replacing the long established steam catapult as the prime means of aircraft assisted launch. The main drivers of the move to electromagnetic launch are the significant limitations of steam catapults (Doyle, 1995), including:

Weight and size: The US navy Catapult Assisted Take-Off Barrier Arrested Recovery (CATOBAR) steam catapult systems has a volume of 1133 m² and a weight of 486 metric tons, most of which is required to be at or near flight deck level. This gives significant challenges in terms of ship dynamics, as well as taking up operationally valuable space.

Control: Steam catapults operate without effective feedback control, giving large and unpredictable transient forces during launch that may cause significant airframe stress and damage. Compensating for this force variability necessitates that extra force must be supplied for an average launch in order to ensure that a minimum safe take-off velocity will be achieved for all potential transient conditions.

Author’s Biographies
Tom Cox is an Assistant Professor in Electrical Machines at the University of Nottingham. He previously spent several years in industry, principally focussed on electromagnetic launch systems for civil and defence application. His areas of interest include linear machines for high speed electromagnetic launch of aircraft, induction energy conversion and electric drive systems.

Tao Yang is an Assistant Professor with the University of Nottingham. His currently research covers aircraft and marine electrical power systems, system integration, renewable energy and electric motor drives.

Chris Gerada is Professor of Electrical Machines and the Royal Academy of Engineering Chair of Electrical Machines at the University of Nottingham. His principal research interest lies in electromagnetic energy conversion in electrical machines and drives, focusing mainly on more-electric transport and distributed energy generation.
**Low adaptability:** Steam catapults are designed for a specific operational load, and as they lack effective closed loop control, it is challenging to vary the thrust level in order to launch significantly lighter payloads such as Unmanned Aerial Vehicles (UAVs).

**Limited energy:** The operational energy limit of the present CATOBAR system is around 95 MJ, while modern aircraft such as the F35 would require 120 MJ or more. To launch the F35 with a steam catapult system would require an even larger and heavier system than employed in the current generation of carriers.

**Low efficiency:** The efficiency of a steam piston launch system is only around 4-6%. To produce 120 MJ for an F35 launch, a steam piston system would need 2-3 GJ of energy, translating to a 44-67 MW continuous supply over a 45s sortie time, approximately equivalent to the total generation capacity of a Type 45 destroyer.

**Fault tolerance:** Steam catapults have very limited fault tolerance, as they cannot easily be modularised for operational redundancy. They are also maintenance intensive and have low availability.

**Steam subsystems:** Steam as a subsystem is not readily available on the newest generation of largely electric drive ships.

Aircraft Electromagnetic Launch systems have the potential to significantly improve on steam in all of the above areas. Size and weight can be significantly decreased (Lu and Ma, 2011) when compared to the steam catapult. Effective control of the output power will allow for the effective launch of any payload within the power envelope of the launch system. This is particularly relevant for UAV launch given the huge variation in potential payload mass.

The efficiency of EML is significantly higher than that of steam catapult systems, with the efficiency of large Linear Induction Motor (LIM) and Linear Synchronous Motor (LSM) systems for EML potentially around 85-90 % or more (Bertola and Cox, 2016). This will give a huge reduction in power supply requirements and lead to significant reductions in system size and weight in comparison to the steam catapult. Decentralization of the power supply and conditioning system will allow more flexibility of positioning inside the ship, reducing the challenges of integration.

Linear motors and drive systems for EML can be subdivided into smaller motor and drive modules that allow for easy and quick maintenance and replacement, and permit limited failure or damage without critically compromising the overall launch. Linear motors produce thrust without contact and so have high reliability and low maintenance requirements, even when operated in adverse conditions.

### 1.2. Developments in Unmanned Aerial Vehicle (UAV) Technology

In parallel to the development of EML systems, developments in UAV technology have led to a new form of aerial vehicle for combat, surveillance and communication purposes that is capable of fulfilling or supporting many of the roles previously requiring manned aircraft (US Department of Defence, 2011). These UAVs can range from small drones under 1kg for visual range operations such as the USMC WASP, to 16,000 kg aircraft capable of significant surveillance and combat operations with a range of 8,200 nautical miles, such as the Global Hawk based MQ-4C Triton (Northrop Grumman, 2013).

Some work has been undertaken into the launch of UAVs, but due to the huge variation in UAV masses between 1-16,000 kg, a range of solutions have been developed for particular areas of application (Fahlstrom and Gleason, 2012). These range from manual, hand thrown launch of the smallest UAVs to mid-scale hydraulic and pneumatic systems, while the largest UAVs are currently limited to using conventional runways for take-off and landing.

Electromagnetic launch of UAVs has been proposed for UAV systems, with some references in literature. Reck (2003) uses a basic analytical method for sizing of a synchronous motor system for EML of UAV’s, but the method is not further developed or verified. In reference (Tan and Kou, 2011), sensorless control of a UAV EML system is investigated, but the work focusses on the control method exclusively. Other work (Yang and Wang, 2010) considers the development of a superconducting LIM system for UAV launch, but in a limited form with fixed frequency, variable voltage supply that leads to poor power system performance and limited efficiency. Finally Yu and Jun (2010) simulate various control strategies for small UAV EML systems using a relatively simple system model.

The Converteam/GE Power Conversion Electromagnetic Kinetic Integrated Technology (EMKIT) demonstrator described in (Foster, Lewis and Thomson, 2006) successfully developed a system capable of launching UAV’s of around 500 kg to 50 m/s, and was also used to support the development of the full scale fighter aircraft launch electromagnetic catapult (EMCAT) system.
1.3. The Type 45 UXV combatant drone ship concept

A concept vessel has been developed that may act as a catalyst to successfully combine these two technologies, EML and UAVs. The UXV combatant (BAE Systems, 2007) is a concept drone mothership developed by BAE systems, conceived from the Type 45 destroyer to be the first vessel to act primarily as a carrier exclusively for UAVs. The UXV combatant would be able to deploy large numbers of UAV’s and would include two 50m flight decks for launch and recovery systems. This form of vessel could offer significant benefits, with the ability to address a wide range of operational needs through the deployment of a significant UAV presence from a single vessel.

In order to achieve a system capable of deploying a wide variety of military UAVs, particularly the heavier aircraft with significant reconnaissance and combat capabilities from a fixed and limited take-off length, some form of EML system must be considered. An EML system would allow use of a wide range of UAVs to meet operational requirements.

1.4. The Type 45 flight deck EML concept

An alternative proposed by the author is the modification of the existing Type 45 layout to accommodate UAV launch. The current design has a ~29 m flight deck on the rear of the vessel, currently used for helicopter operations. Adding an EML system would significantly improve combat capability through allowing the launch of fixed wing UAVs able to meet a significantly greater range of operational requirements. This could range from the launch of smaller UAVs suitable for shorter range operations and recovery on vessel, to the largest of UAVs such as the Triton that have the range to return to a land based facility post mission (as recovery at sea of this scale of aircraft is unproven on any vessel short of a full carrier).

The potential for this system is vast. The ability to launch aircraft with significant intelligence gathering and combat capability from this size of mobile and non-specialist platform has not been seen since the era of seaplanes, and would allow the direct deployment of UAVs at short notice, exactly where they are needed.

2. Defining launch requirements

One of the key steps in the development of this system will be to define the EML requirements needed for a UAV launch system based on the potential payloads. A significant advantage of EML systems in this respect is that due to the controllability of electromagnetic systems, an EML system will allow for the launch of any aircraft within its power envelope. To put it another way, if designed to successfully launch a payload at a certain mass and velocity, any payload with an equal or lower mass and velocity can also be launched with proper control. This allows us to design the system for a single payload at the maximum required capability of the system, knowing that all aircraft with lower requirements can also be launched. From the above, it has been decided to use the MQ-4C Triton as our baseline aircraft. The Triton is a Northrop Grumman naval Intelligence, Surveillance and Reconnaissance (ISR) UAV, based on the Global Hawk design. As such, it is likely the largest and heaviest aircraft (short of manned fighters) that would potentially be deployed from a marine platform.

From (BAE Systems, 2007), the required take-off length for the UXV platform is 50 m and the maximum take-off weight is 14,630 kg (Northrop Grumman, 2013). The take-off speed can then be defined by following the process outlined in (Goraji and Friedlychengicz, 2004) as 52 m/s.

For the Type 45, the flight deck is around 29 m, so we will assume around 25 m available for a launch system. This allows us to develop the following launch system specifications for both cases (drone carrier, and rear flight deck mounted EML).
Table 1: Launch system requirements for MQ-4C Triton electromagnetic launch from a Type 45

<table>
<thead>
<tr>
<th></th>
<th>Type 45 UXV Combatant</th>
<th>Type 45 flight deck</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take-off Distance</td>
<td>50</td>
<td>25</td>
<td>m</td>
</tr>
<tr>
<td>Take-off Velocity</td>
<td>52</td>
<td>52</td>
<td>m/s</td>
</tr>
<tr>
<td>Max Take-off weight</td>
<td>14630</td>
<td>14630</td>
<td>kg</td>
</tr>
<tr>
<td>Energy</td>
<td>19.8</td>
<td>19.8</td>
<td>MJ</td>
</tr>
<tr>
<td>Required time</td>
<td>1.92</td>
<td>0.96</td>
<td>s</td>
</tr>
<tr>
<td>Required Acceleration</td>
<td>27.0</td>
<td>54.1</td>
<td>m/s²</td>
</tr>
<tr>
<td>Total Required Force</td>
<td>396</td>
<td>791</td>
<td>kN</td>
</tr>
<tr>
<td>Engine thrust</td>
<td>40</td>
<td>40</td>
<td>kN</td>
</tr>
<tr>
<td>Full Thrust Requirement</td>
<td>356</td>
<td>751</td>
<td>kN</td>
</tr>
<tr>
<td>Aircraft length</td>
<td>14.5</td>
<td>14.5</td>
<td>m</td>
</tr>
<tr>
<td>Active Motor Length</td>
<td>5</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>Force/m of stator</td>
<td>71.1</td>
<td>150.2</td>
<td>kN/m</td>
</tr>
<tr>
<td>Force/m/side</td>
<td>35.6</td>
<td>75.1</td>
<td>kN/m/side</td>
</tr>
</tbody>
</table>

From Table 1, the requirements for our EML system have been established. Simple dynamic equations allow us to calculate the required force given the take-off speed and distance, and the aircraft thrust is added based on the peak output of the engine (a Rolls-Royce AE 3007 turbofan). From this, the full thrust requirements are computed. With an active motor length estimated based on the targeted force density and a reasonable mechanical length relative to the launch, Force per m length of stator can be computed. We can assume that we will use a double sided stator configuration, standard practice in EML systems to minimise moving mass, eliminate attractive forces between stator and mover and increase thrust capability. Under this assumption, we arrive at a figure of 35.6 kN/m/side for the UXV combatant system, and 75.1 kN/m/side for the Type 45 flight deck variant. Detail of the method used to develop this data is given in Appendix A.

Other important design considerations are the sortie rate (how often an aircraft launch is required). This will govern the allowable current density, as cumulative temperature increases in both stator and rotor should be accounted for. It will also govern the system power supply requirements as the shorter the time between launches, the greater the continuous power that must be drawn from the ship network. For the Type 45 flight deck type configuration, sortie rate is not a great concern as the number of aircraft (particularly large aircraft that would require significant launch energy) available for launch would likely be limited to a single aircraft, as is the case with the current Type 45’s carrying a single rotary wing aircraft. In the case of the UXV combatant, multiple sequential launches must be considered, and so this will become much more limiting. A good baseline would be the designed sortie rate of the EMALS system, 45 s (Doyle, 1995).

The power system must also be considered. The Type 45 has an installed generation capacity of around 48 MW, with a 4.1 kV distribution grid (Hodge, 2008). Although theoretically the above energy levels could be drawn directly from the shipboard grid, in practice an energy storage subsystem would be used to avoid severe effects on power network stability. The use of energy storage also means that the actual impact on the distribution grid of an EML system over a 45 s charging time would likely be low, and would be unlikely to require additional generation.
capacity. From the above, we will assume a maximum voltage of up to 4 kV at the motors, allowing for a highly
efficient Medium Voltage marine converter system such as (GE Power Conversion – MV7000) with reported
efficiencies of up to 99%. This would be used to supply a variable voltage and frequency to the linear motors in
order to control the launch effectively. We will also keep the converter fundamental frequency within ~200 Hz.
While this can be significantly higher, particularly if employing modern SiC based devices, this relatively low
frequency is achievable by commercially available systems, suitable for deployment at the high power levels
required.

3. Machine design

The system design will be based around a LIM type system. While academic work has looked at various
machine types including synchronous AC and DC forms (Lu and Ma, 2011), LIM systems are the only type that
have been developed and proven at this power scale. Further, In a large scale application such as this, the inherent
disadvantages of the LIM (such as slip losses) are minimised, and due to the high overload capability LIMs remain
very competitive with synchronous designs (Bertola and Cox, 2016).

Following the general design principle of maximising pole pitch to give optimal LIM performance (Laithwaite,
1966), the pole pitch was set at 0.3 m, giving a maximum synchronous speed of 55 m/s at 91.5 Hz, more than
adequate for a 52 m/s max velocity. A 6-pole LIM was designed with 5 slots per pole and phase and a 20 mm slot
pitch, giving a total LIM length of 2.04 m, with 2.1 m between adjacent stator blocks. The individual stator section
length was limited for reasons of manufacturability and modularity. As the machine was designed for Medium
Voltage up to 4 kV, a slot fill of 0.4 was assumed, with 50 mm deep slots and a formed diamond-type winding
suitable for MV insulation. Core width was chosen as 0.5 m to keep the deck side mass of the system as low as
practical. Airgap clearance length was chosen as 7 mm, a reasonable assumption (Lu and Ma, 2011) given the
considerable flex in a ship under adverse weather conditions. The machine parameters above are summarised in
Table 2.

The machine was designed with a parallel delta connection, with all 6-phase bands in parallel. This
configuration was shown in (Eastham and Cox, 2011) to eliminate the significant force transients experienced on
transition between machines in series connected operation.

The final design was based on the Type 45 flight deck launch specification, a much more demanding design
envelope. A system that would meet these requirements will also meet the less demanding requirements of the
UXV system launch, as long as the higher sortie rate is accounted for.

Table 2: Machine design parameters

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Target speed</td>
<td>52 m/s</td>
<td>Synchronous speed</td>
</tr>
<tr>
<td></td>
<td>Pole pitch</td>
<td>0.3 m</td>
<td>Distance between stators</td>
</tr>
<tr>
<td></td>
<td>Peak frequency</td>
<td>91.5 Hz</td>
<td>Slot depth</td>
</tr>
<tr>
<td></td>
<td>Poles</td>
<td>6</td>
<td>Core width</td>
</tr>
<tr>
<td></td>
<td>Slot pitch</td>
<td>0.02 m</td>
<td>Mechanical airgap/side</td>
</tr>
<tr>
<td></td>
<td>Slots per pole and phase</td>
<td>5</td>
<td>Slot fill factor</td>
</tr>
</tbody>
</table>
4. Performance simulation

From the calculated motor parameters above, the predicted performance of the above machine was calculated from a Steinmetz equivalent circuit, following a method derived from (Laithwaite, 1966) and detailed in Appendix 1. Specific motor parameters were calculated from the above dimensions, using standard formulae for inductances and resistances of rotary machines with diamond wound coils as detailed in Say (1983). The full secondary resistance was modelled from 2D parameters using the method of (Russell and Norsworthy, 1958). The method of using a simplified equivalent circuit to simulate the performance of the machine is supported by (Cox, Eastham, 2007), where it was shown that the behaviour of a short rotor linear machine can be approximated by the performance of its rotating counterpart at a number of rotor poles equal to or greater than 6. This approximate equivalent circuit method was used for initial design, with the behaviour of the final machine design verified using more accurate modelling methods, as described later in this work.

This method was used to study the individual motor performance of the system, given that full electromagnetic finite element analysis would be used for the final outcome. Its torque speed profile is shown in Figure 1, and Table 3 shows the key results at the peak velocity point of 52m/s and 3,500Vrms. With 2.1m between adjacent stator blocks, the thrust/m of stator must be 79 kN/m, so our new target point is 158 kN from the modelling work.

From Table 3, it can be seen that the thrust per m length of this system meets the requirements outlined in Table 1 for Type 45 flight deck UAV launch. It can be seen that efficiency and power factor are reasonably good, comparing favourably to (Lu and Ma, 2011) and current density is within allowable limits for a very short term rated machine such as this (Bertola and Cox, 2016).

![Figure 1: Torque-speed characteristic for the UAV launch motor at 3,500 V, 91.5 Hz](image)

**Table 3: Key motor power characteristics at 52 m/s and 91.5 Hz.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust</td>
<td>167,600 N</td>
</tr>
<tr>
<td>Efficiency</td>
<td>91.3 %</td>
</tr>
<tr>
<td>Voltage</td>
<td>3,500 V&lt;sub&gt;RMS&lt;/sub&gt;</td>
</tr>
<tr>
<td>PF</td>
<td>0.83</td>
</tr>
<tr>
<td>Current</td>
<td>1,900 A&lt;sub&gt;RMS&lt;/sub&gt;</td>
</tr>
<tr>
<td>Input VA</td>
<td>11.5 MVA</td>
</tr>
<tr>
<td>Output Power</td>
<td>8.68 MW</td>
</tr>
<tr>
<td>Current Density</td>
<td>36 A/mm&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
</tbody>
</table>
Using this design, a transient launch simulation for the system was run with 2D Finite Element Analysis (FEA) based on the single stator model. A single stator pair in the double sided model configuration with a reaction plate is shown in Figure 2. This modelling uses additional resistance and inductance in the primary circuit to represent the resistance and inductance of the stator winding end turns. The conductive sheet secondary material resistance is modified using a factor (Russell and Norsworthy, 1958) to represent the full current path in the rotor. The machine was modelled with nonlinear iron conditions, Figure 3, as saturation is a critical factor in high force short rated machines such as this. The simulation works on the basis of a fixed supply current equal to that found in the simulation of Table 3, assuming that the converter will maintain a constant supply current. This is reasonable, as the previous simulation work has shown that this current can be produced without exceeding our voltage limit. The simulation below controls velocity and frequency to ensure that slip remains within the design boundary. From this, the developed thrust should meet or exceed our requirement.

From the FEA analysis, the thrust force and voltage (for constant current operation) were plotted in Figures 4 and 5 respectively.

Figure 4 shows that the developed thrust meets or exceeds our requirements at all parts of the launch (the 158kN thrust requirement is indicated on the plot). Figure 5 indicates that we stay within the converter voltage limits set previously, just reaching the limit of 4 kV$_{RMS}$ at the peak power and take-off speed.
From the above, we can summarise the overall launch system power requirements as shown in Table 4. It should be noted that these requirements assume stators are switched on only while needed and do not include the extra losses and reactive power due to stator switching. At high speeds, this may have a reasonably significant impact, as for optimum performance stators require activation before they are required to produce thrust in order
to ensure the field is fully built up. This impact is generally in the form of reduced power factor, so while it will increase the required converter rating it will generally have a much smaller effect on active power required from the supply.

<table>
<thead>
<tr>
<th>Table 4: Overall peak launch system power requirements</th>
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<tbody>
<tr>
<td>Max line-line voltage</td>
</tr>
<tr>
<td>Max line current</td>
</tr>
<tr>
<td>Peak output power</td>
</tr>
<tr>
<td>Efficiency at peak power</td>
</tr>
<tr>
<td>PF at peak power</td>
</tr>
<tr>
<td>MVA at peak power</td>
</tr>
<tr>
<td>Peak consumed power</td>
</tr>
<tr>
<td>Continuous supply for 45 s sortie rate</td>
</tr>
</tbody>
</table>

From Table 4, it can be seen that the power system requirements, while not small, are well within the available capacity of a Type 45, especially when provided through an energy storage system. Assuming a constant rate of input power increase over a launch to the maximum of 43 MW, the constant power requirement of the system over a 45 s sortie time is only 460 kW, which could easily be met from available generation capacity.

5. Conclusions

This work has shown the feasibility of using electromagnetic motors to launch even the largest of UAVs from a small naval platform. This could be applied to a drone ship concept such as the UXV combatant. EML has also been proposed for launch from the rear flight deck of the Type 45 destroyer in order to vastly expand the capabilities of the existing platform, giving the ability for a non-carrier ship to deploy a significant UAV presence at need.

The feasibility of the developed electrical machine has been proven with nonlinear 2DFEA, and a transient simulation was run to verify that the design will meet the thrust requirements of the system while remaining within the design limitations of voltage and current of the Type 45 platform.

The current scope of the work is limited, and does not yet consider the full launch system behaviour including switching and transient effects. Also not considered are the modifications that may be required to the Triton airframe to permit electromagnetic launch. Finally, the full design and specification of the overall power supply, storage and conditioning system still presents significant challenges and should be addressed in more detail to demonstrate viability of the system.

6. References

BAE Systems, 2007. “UXV Combatant”


Northrop Grumman, 2013. NQ-4C Triton Datasheet.


Appendix A: Derivation of launch parameters

The launch parameters and thrusts are developed in Table 1 by the following process, using simple dynamic equations. From a known take-off distance (D) and active motor length (D_a) determined by vessel constraints and design considerations, and a take-off velocity (V), max take-off mass (M) and engine thrust (F_e) determined by the aircraft to be launched, we can determine the following parameters:

- Launch energy (E) = $\frac{1}{2} M V^2$ (1)
- Required time (t) = $2D/V$ s (2)
- Required acceleration (A) = $V^2/(2D)$ m/s² (3)
- Total required force (F_t) = MA N (4)
- Full thrust requirement (F) = $F_t - F_e$ N (5)
- Force/m of stator (F/m) = $F/D_a$ N/m (6)
- Force/m/side (F/m/side) = $F/(2D_a)$ N/m (7)

The results of Table 3 and Figure 1 are derived from a simplified Steinmetz equivalent circuit with per pole parameters calculated using the formulae from Laithwaite (1966), as below.

Primary Resistance ($R_1$) = $4 x (Mlt/(2 x Wc)) x \rho_1 x Ntc^2 x Nsp x Wc / (3 x Sff x Sd x Sw)$ (8)

Where $Mlt$ is the mean length of a coil turn, $Wc$ is the width of the core, $\rho_1$ is the material resistivity of the stator winding, $Ntc$ is the number of turns per coil, $Nsp$ is the number of slots per pole, $Sff$ is the slot fill factor, $Sd$ is the depth of a slot and $Sw$ is the width of a slot.

Secondary Resistance ($R_2$) = $R_1 x \rho_2 x Rnf x (Sff x Sw x Sd / Sp) / (\rho_1 x (Mlt/(2 x Wc)) x Dss)$ (9)

Where $\rho_2$ is the secondary sheet resistivity, $Rnf$ is the Russell and Norsworthy (1958) factor used to account for end ring behaviour, $Sp$ is the slot pitch and $Dss$ is the depth of the sheet secondary.

Magnetizing reactance ($Xm$) = $24 x \mu_0 x Kw f x Pp x Wc x (Ntc x Nsp / 3)^2 / g$ (10)

Where $Kw$ is the winding factor, $f$ is the frequency, $\mu_0$ is the permeability of free space, $Pp$ is the pole pitch and $g$ is the total effective magnetic gap.

Stator leakage reactance ($Xs$) = $8 x \pi x \mu_0 x f x Wc x (Ntc x Kw)^2 x Nsp x Psl / 3$ (11)

Where $Psl$ is the specific slot leakage permeance based on slot geometry.