Labour Unionisation Structure, Innovation and Welfare

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We show the effects of cooperation among the labour unions with complementary workers on innovation, consumer surplus and welfare. Although cooperation among the unions reduces wage, it may either increase or decrease the firm’s incentive for innovation, and may also make the consumers and the society worse off by reducing innovation. While cooperation (compared to non-cooperation) among the unions makes the workers better off, it may not make all final goods producers better off. (JEL: D43; J51; L13; O31)

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1 Introduction

What is the effect of different labour unionisation structure on innovation? This issue is gaining popularity in recent years due to the diversity of unionised labour markets across countries. The effects of firm-specific labour unions are compared to that of an industry-wide labour union. Under firm-specific labour unions, a firm deals with a labour union that is associated with that firm, whereas under an industry-wide labour union, the labour union deals with all firms in that industry.

In a patent race model, Haucap and Wey (2004) show that if an industry-wide labour union charges a uniform wage to all firms, the incentive for a labour-saving innovation is higher under an industry-wide labour union. However, if the industry-wide labour union can charge different wages to different firms, the incentive for innovation is higher under firm-specific labour unions. In a model with R&D competition, Calabuig and Gonzalez-Maestre (2002) show that the incentive for a labour-saving innovation is higher under firm-specific labour unions for non-drastic innovations, while the incentive for innovation can be higher under an industry-wide labour union for drastic innovation. Manasakis and Petrakis (2009) show that, under non-cooperative R&D, the incentive for a labour-saving innovation is higher under firm-specific labour unions if knowledge spillover is high, but the incentive for innovation is always higher under firm-specific labour unions under cooperative R&D. They also show that welfare is higher under firm-specific unions than under an industry-wide labour union. Considering an innovating firm and a non-innovating firm, Mukherjee and Pennings (2011) show the implications of technology licensing ex-post innovation and the unions’ preferences for wage compared to employment in determining the effects of the unionisation structure on a labour-saving innovation.

Although the above mentioned papers provide important insights, they all consider perfectly substitutable workers, while it is often found that labour unions exist for workers providing complementary services. For example, as mentioned in Aghadadashli and Wey (2015, p. 667), in Germany, “[…] hospital doctors are mainly represented by the Marburger Bund (a craft union), while the remaining workers are represented by the German Trade Union (Verdi). […] The Deutsche Bahn (the dominant railway operator) must bargain with the German Train Drivers Union (Gewerkschaft Deutscher Lokomotivführer; GDL) and the Railway and Transport Union (Eisenbahn- und Verkehrs-gewerkschaft; EVG). […] the GDL is a craft union, which is complementary to workers represented by the EVG. The former takes care of the train drivers’ employment conditions, and the latter represents the remaining railway workers’ interests. […] Other examples include airlines (where pilots are represented by Vereinigung Cockpit) and airports (where air traffic controllers are organized in the Gewerkschaft der Flugsicherung).” Hence, employers’ are exposed to the unions representing complementary workers.

1 See, e.g., Calmfors and Drifill (1988), Moene and Wallerstein (1997) and Flanagan (1999) for the difference in labour unions with respect to the degree of wage setting centralisation. Decentralised wage setting is often contrasted with centralised wage setting. Under a decentralised wage setting, wages are set between employers and firm-specific unions, while under a centralised wage setting, an industry-wide union negotiates wages with all firms (Haucap and Wey, 2004).
Different workers organised under different labour unions can be observed in other countries also. For example, different types of workers are organised in different unions in Sweden (see, e.g., Kjellberg, 2014, for a detailed discussion on the labour unionisation structure in Sweden).

Uppmann and Müller (2014) also provide evidence for labour unions with complementary workers and different types of wage bargaining, viz., separate wage bargaining, where different labour unions with different types of workers bargain separately with firms, and wage bargaining by an encompassing labour union, where an union bargain with firms for all types of workers. There are also other evidences showing that wages set by the unions can be applicable to the entire industry. For example, as mentioned in Haucap et al. (2001, p. 288), “a common feature of many labor market systems in continental Europe are coverage extension rules. Under these rules, the coverage of collectively negotiated wage contracts can be extended to entire industries through legal means. With coverage extension, some or all employment terms are made generally binding not only for the members of unions and employers’ associations, but for all industry participants. In Germany, for example, collective wage agreements between a union and an employers’ association can be made compulsory even for independent employers through the so-called Allgemeinverbindlicherklärung (AVE), a legal instrument provided for in §5 Tarifvertragsgesetz (TVG). The Ministry of Labor can, on application of either unions or employers’ associations, use an AVE to make some or all terms of a collectively negotiated employment contract generally binding for an entire industry, where otherwise only those unions, employers and employers’ associations that have actually negotiated and signed the contract would be directly bound by it (§3 I TVG).” They also provide evidence for the coverage extension rules in other countries and mention that the number of AVEs increased from 448 in 1975 to 588 in 1998. Venn (2009) provides the evidence of bargaining between the employers and the employees at the industry level for the OECD and selected non-OECD countries. The evidence of industry-wide wage bargaining can be found also in Carluccio et al. (2015) for France and in Lamarche (2013) for Argentina.

Given this background, the purpose of this paper is to show the effects of cooperation among the labour unions of complementary workers on innovation and welfare. In what follows, in a model with an innovating and multiple non-innovating firms, we show in Section 2 that an encompassing labour union of complementary workers may decrease (increase) the incentive for innovation compared to separate labour unions if the technological improvement through innovation is large (small). Hence, our analysis follows the literature considering competition between innovating and non-innovating firms (see, e.g., Mookherjee and Ray, 1991, Gallini, 1992, Ray Chowdhury, 1995, Mukherjee, 2003, Mattoo et al., 2004 and Mukherjee and Pennings, 2004). We show in Appendix A.7 that our result holds even if there are multiple but not all innovators.

Although we consider the input suppliers as labour unions, our analysis is applicable if the input suppliers are profit maximising firms charging linear input prices.
As discussed in the following analysis, the “raising rival’s cost”\(^3\) motive may explain why an encompassing union (compared to separate unions) may either increase or decrease the incentive for innovation. Under the raising rival’s cost motive a firm can take an action that increases the costs of its competitors. Even if that action increases the cost of the concerned firm, the competitors’ cost increase can be significantly more compared to its own cost increase, and on the balance, this action helps the concerned firm to increase its profit by acquiring a larger market share. We show that innovation by a firm may either increase or decrease the marginal costs of the non-innovating firms. If innovation reduces the marginal costs of the non-innovating firms, it benefits the non-innovating firms and may discourage the innovator to innovate a new technology in order to raise the cost of its rival. Although this effect remains under both encompassing and separate unions, the raising rival’s cost motive is stronger under the former than the latter since an encompassing union (compared to separate unions) benefits the innovator from a lower marginal cost by reducing the complements problem discussed below.

We also show that although an encompassing union reduces wage compared to separate unions, it may make the consumers and the society worse off by reducing innovation. While an encompassing union makes the workers better off compared to separate unions, it may not make all final goods producers better off.

Our paper contributes to the literature following Horn and Wolinsky (1988), Shapiro (2000), Mukherjee and Pennings (2011), Upmann and Müller (2014) and Aghadadashli and Wey (2015). Shapiro (2000) shows that while choosing the prices of complementary inputs non-cooperatively, input suppliers do not internalise the negative external effects of their pricing on other complementary input suppliers’ revenues, thus creating the complements problem.\(^4\) Cooperation among the complementary input suppliers solves the complements problem and reduces the input prices, thus increasing the profits of the input suppliers and making the consumers better off by reducing the prices of the final goods. We show that although cooperation among the complementary input suppliers (labour unions in our case) reduces the input price (wage in our analysis) by solving the complements problem, it may create a negative impact by reducing innovation. If the latter effect dominates the former, cooperation among the complementary workers may not benefit the consumers and the society.

Our framework is similar to the ‘no technology licensing’ case of Mukherjee and Pennings (2011) with the exception that we consider complementary workers instead of substitutable workers. It follows from Mukherjee and Pennings (2011) that, in the absence of technology licensing, a final goods producer’s incentive for innovation is higher under cooperation among the unions of substitutable workers. In contrast, cooperation among the unions in our analysis may either increase or decrease innovation. This difference is attributable to the different effects of cooperation among the labour unions on wage.

\(^3\) See Williamson (1968), Salop and Scheffman (1983 and 1987) and Haucap et al. (2001) for some earlier papers explaining the raising rival’s cost effect.

\(^4\) This is also called “royalty stacking” (Gilbert and Katz, 2011).
Horn and Wolinsky (1988), Upmann and Müller (2014) and Aghadadashli and Wey (2015) consider the effects of cooperation among labour unions with complementary workers. Horn and Wolinsky (1988) show that the workers are better off (worse off) under cooperation among the unions if the workers are substitutable (complements). In contrast, we show that complementary workers can be better off under cooperation. This difference occurs, since unlike them but like other papers mentioned above, we allow the firms to determine workers after wage determination. Further, unlike Horn and Wolinsky (1988), we consider product-market competition among the final goods producers and show the effects of cooperation among the unions on innovation and welfare, which is our main focus.

Both Aghadadashli and Wey (2015) and Upmann and Müller (2014) consider that a firm and two labour unions bargain over wage and employment, thus considering “efficient bargaining” model, and show that an encompassing labour union makes complementary workers worse off compare to separate unions. In contrast, like all other papers mentioned above, we consider a “right-to-manage” model where the unions (or bargaining between the firms and the unions) determine wage and the firms determine employment.\(^5\) Hence, the bargaining structure considered in our paper is different from theirs. Further, unlike them, we consider product-market competition and the effects through innovation. We show that an encompassing labour union makes the complementary workers better off. We also show the effects of different unionisation structure on innovation and welfare, which is our main focus.

The remainder of the paper is organised as follows. Section 2 describes the model and derives the results. Section 3 discusses the implications of some of our assumptions. Section 4 concludes. We relegate many mathematical details in the Appendix.

## 2 The model and the results

Assume that there are one innovating firm (firm 1) and \((n – 1)\) non-innovating firms (firms \(2, ..., n\)), where \(n \geq 2\). These firms compete like Cournot oligopolists with homogeneous goods, facing the inverse market demand function \(P = 1 – q\), where \(P\) is price and \(q\) is the total output. Production of the final goods requires two types of workers, \(x\) and \(y\), which are unionised. We assume that the workers \(x\) and \(y\) are perfect complements. The reservation wages of all workers are \(c\), which are normalised to zero for simplicity. We consider a “right-to-manage” model of labour unions where the unions determine the wages and the firms hire workers according to their requirements.

Like Horn and Wolinsky (1988), we consider two types of labour unions: First, separate labour unions, where union \(X\) (resp. under union \(Y\)) organises workers of type \(x\) (resp. type \(y\)) and the unions determine wages \(w_x\) and \(w_y\) simultaneously to maximise their own utilities. Second, an encompassing labour union, where a single union organises both types of workers and determines wages \(w_x\) and \(w_y\)\(^5\)

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\(^5\) See, Layard et al. (1991) for arguments in favour of the right-to-manage models.
simultaneously to maximise the total utilities of all workers. We assume that an encompassing union can charge different wages for different types of workers but it charges the same wage to different firms for the same type of worker. Charging the same wage to different firms for the same type of workers is in line with many previous works such as Haucap and Wey (2004), Manasakis and Petrakis (2009) and Mukherjee and Pennings (2011), and can be motivated by the empirical evidence cited in the introduction for industry-wide wage bargaining.

To start with, we assume that all firms require one unit of each type of worker to produce one unit of the final good. However, the innovating firm, firm 1, can invest \( k \) amount in R&D to reduce its labour coefficients for both workers to \( s \), where \( s < 1 \). Hence, to show our results in the simplest way, we consider that innovation is not biased towards any type of worker and creates a neutral technological progress.\(^6\) Thus, innovation increases labour productivities in firm 1 from \( l \) to \( 1/s \). It is intuitive that if \( s \) is very large, the non-innovating firm would go out of the market. In order to ensure that all final goods producers always hire workers, we assume that \( 2/(n+2) < s \) (see Appendix A.1 for details). Hence, we consider that \( s \in (2/(n+2),1) \). We consider the binary choice for firm 1’s R&D decision for analytical convenience. As we discuss below, our main result holds even if firm 1 chooses the extent of technological improvement through R&D.

We consider the following game. Conditional on the unionisation structure, at stage 1, firm 1 decides whether or not to innovate. At stage 2, wages are determined by the unions. At stage 3, the final goods producers (firms \( 1,...,n \)) determine their outputs simultaneously, and the profits are realised. We solve the game through backward induction.

2.1 Separate labour unions

If the separate unions \( X \) and \( Y \) charge \( w_x \) and \( w_y \) as wages for workers \( x \) and \( y \) respectively, firm 1 and the \( i \)th firm, \( i=2,...,n \), determine their outputs by maximising the following expressions respectively:

\[
\text{Max}_{q_1} \left[ 1 - q - t(w_x + w_y) \right] q_1 - f
\]

\[
\text{Max}_{q_i} \left[ 1 - q - (w_x + w_y) \right] q_i,
\]

where, \( t = 1 \) and \( f = 0 \) under no innovation by firm 1, and \( t = s \) and \( f = k \) under innovation by firm 1.

The equilibrium outputs of firms 1 and the \( i \)th firm, \( i=2,...,n \), can be found as

\[
q_i = \frac{1 - nt(w_x + w_y) + (n-1)(w_x + w_y)}{n+1} \quad \text{and} \quad q_1 = \frac{1 - 2(w_x + w_y) + t(w_x + w_y)}{n+1}.
\]

It is clear from (1) that a lower wage decreases the output of firm 1, i.e., \( q_1 \), if \( t < (n-1)/n \). Since, in our analysis, the profit of firm 1 is equal to \( (q_1^2 - f) \), a lower wage decreases the profit of firm 1 for \( t < (n-1)/n \).

\(^6\) We discuss the implications of this assumption in Section 3.
The demand for workers faced by unions $X$ and $Y$ are $q_x = q_y = t q_1 + \sum_{i=2}^{n} q_i$.

Unions $X$ and $Y$ determine their wages by maximising the following expressions:

$$\max_{w_i} w_i(t q_1 + \sum_{i=2}^{n} q_i)$$

$$\max_{w_i} w_i(t q_1 + \sum_{i=2}^{n} q_i).$$

The equilibrium wages are

$$w^{nc}_x = w^{nc}_y = \frac{n-1+t}{6(t-1)+3n(2-2t+t^2)}.$$  

If $t < 1$, the equilibrium wages fall as the number of firms (i.e., $n$) increases. If $t < 1$, more firms increase the elasticity of demand for workers and reduce the equilibrium wages.

We obtain from (1) and (2) that the equilibrium outputs of firm 1 and firm $i$, $i=2,\ldots,n$, are

$$q^{nc}_i = \frac{-4(1-t) + 2n^2(1-t) + n(2-2t+t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]}$$

and

$$q^{nc}_i = \frac{-2(1-t^2) + n(2-4t+3t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]}$$

respectively. The total equilibrium outputs are

$$q^{nc} = q^{nc}_1 + \sum_{i=2}^{n} q^{nc}_i = \frac{-2n(1-t) - 2(1-t)^2 + n^2(4-6t+3t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]}.$$  

The equilibrium profits of firm 1 and firm $i$, $i=2,\ldots,n$, are $\pi^{nc}_1 = (q^{nc}_1)^2 - f$ and $\pi^{nc}_2 = (q^{nc}_i)^2$ respectively.

**LEMMA 1** Under separate labour unions, firm 1 innovates for

$$k < \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 - \frac{1}{9(n+1)^2} \equiv k^{nc}.$$ 

**PROOF** See Appendix A.2. Q.E.D.

The expression $k^{nc}$ shows firm 1’s maximum willingness to pay for innovation under separate unions. Innovation increases firm 1’s product-market profit compared to no innovation. However, innovation also imposes a cost on firm 1. Hence, firm 1 innovates if the cost of innovation is not very high.

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7 We get that $\frac{\partial w^{nc}_x}{\partial n} = \frac{\partial w^{nc}_y}{\partial n} = \frac{-2(1-t)(1-t)t}{3[-2(1-t) + n(2-2t+t^2)]^2} < 0$. 

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2.2 *An encompassing labour union*

If the encompassing union charges \( w_x \) and \( w_y \) as wages for workers \( x \) and \( y \) respectively, the equilibrium outputs of firms 1 and 2 are given by (1) and the demand for workers is given by \( q_x = q_y = tq_i + \sum_{i=2}^{n} q_i \). The wages are determined by maximising the following expression:

\[
\text{Max}(w_x + w_y)(tq_i + \sum_{i=2}^{n} q_i).
\]

The equilibrium wages are

\[
w^{e}_i = \frac{n-1+t}{-8(1-t)+4n(2-2t+t^2)}.
\]

Like separate unions, we get under an encompassing union that, if \( t < 1 \), as the number of firms (i.e., \( n \)) increases, the elasticity of demand for workers increases and the equilibrium wages fall.\(^8\)

We obtain from (1) and (3) that the equilibrium outputs of firm 1 and firm \( i, i=2,...,n \), are

\[
q^{e}_1 = \frac{-3(1-t) + n^2(1-t) + n(2-2t+t^2)}{(n+1)[-4(1-t) + 2n(2-2t+t^2)]}
\]

and

\[
q^{e}_i = \frac{-(2-t+t^2) + n(2-3t+2t^2)}{(n+1)[-4(1-t) + 2n(2-2t+t^2)]}
\]

respectively. The total equilibrium outputs are

\[
q^{e} = q^{e}_1 + \sum_{i=2}^{n} q^{e}_i = \frac{-2n(1-t) - (1-t)^2 + n^2(3-4t + 2t^2)}{(n+1)[-4(1-t) + 2n(2-2t+t^2)]}.
\]

The equilibrium profits of firm 1 and firm \( i, i=2,...,n \), are \( \pi^{e}_1 = \left(q^{e}_1\right)^2 - f \) and \( \pi^{e}_i = \left(q^{e}_i\right)^2 \) respectively.

**Lemma 2** Under an encompassing labour union, firm 1 innovates if

\[
k < \frac{-3(1-s) + n^2(1-s) + n(2-2s+s^2)}{(n+1)[-4(1-s) + 2n(2-2s+s^2)]}^2 - \frac{1}{4(n+1)^2} \equiv k^c.
\]

**Proof** See Appendix A.3. Q.E.D.

The expression \( k^c \) shows firm 1’s maximum willingness to pay for innovation under an encompassing labour union. The intuition for Lemma 2 is similar to that of Lemma 1.

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\(^8\) We get that \( \frac{\partial w^{e}_i}{\partial n} = \frac{\partial w^{e}_i}{\partial n} = \frac{-(2-t)(1-t)t}{4[-2(1-t) + n(2-2t+t^2)]^2} < 0 \).
2.3 Comparison between separate and encompassing labour unions

2.3.1 The effects of an encompassing labour union on wage and the final goods producers

The comparison of (2) and (3) gives the following result immediately.

**PROPOSITION 1** Wages are higher under separate labour unions than under an encompassing labour union, irrespective of firm 1’s R&D decision.

Since the workers are complements, an encompassing labour union reduces wages compared to separate unions, thus solving the complements problem and reducing the marginal costs of all final goods producers compared to separate labour unions.

Since the encompassing labour union reduces the marginal costs of all final goods producers compared to separate unions, it is immediate that if firm 1 does not innovate (i.e., \( t = 1 \)), it increases the profits of all final goods producers compared to separate unions. However, as we will show below, this may not be the case if firm 1 innovates.

**PROPOSITION 2** If firm 1 innovates, an encompassing labour union decreases (increases) the profit of firm 1 compared to separate labour unions for \( s \in (2/(n+2), (n-1)/n) \) (\( s \in ((n-1)/n, 1) \)) but it increases the profits of other final goods producers.

**PROOF** See Appendix A.4. Q.E.D.

Although an encompassing union (compared to separate unions) reduces the marginal costs of all final goods producers, the marginal cost saving is higher for the non-innovators compared to the innovator. It follows from the discussion made after (1) that a lower wage reduces the output and profit of firm 1 if \( s < (n-1)/n \). Hence, if the technological difference between the innovator and the non-innovators is large (i.e., \( s \in (2/(n+2), (n-1)/n) \)), the marginal cost saving under an encompassing union (compared to separate unions) is significantly higher for the non-innovators compared to the innovator, thus reducing the profit of the innovator under an encompassing union.

2.3.2 The effect of an encompassing labour union on innovation

**PROPOSITION 3** If \( s \in (2/(n+2), s') \) (\( s \in (s', 1) \), where \( s' = (3 + 4n - 7n^2 + \sqrt{9 - 58n^2 + 49n^4}) / 4n \), an encompassing labour union decreases (increases) firm 1’s incentive for innovation compared to separate labour unions for \( k \in (k^c, k^{nc}) \) (\( k \in (k^{nc}, k^c) \)).
The reason for the above result is as follows. Firm 1’s incentive for innovation depends on the difference in its profit between innovation and no innovation. On one hand, an encompassing union (compared to separate unions) tends to reduce firm 1’s incentive for innovation by increasing its profit under no innovation. On the other hand, an encompassing union (compared to separate unions) tends to increase (decrease) firm 1’s incentive for innovation by increasing (decreasing) its profit under innovation if its technological improvement through innovation is small (large). It follows from the discussion made after (1) that firm 1’s profit reduces (increases) with a lower wage if \( s < (>)(n-1)/n \). Hence, if firm 1’s technological improvement through innovation is large, both the above-mentioned effects reduce firm 1’s incentive for innovation under an encompassing union compared to separate unions. However, if firm 1’s technological improvement through innovation is small, the above-mentioned second effect can dominate the first effect, and an encompassing union can increase firm 1’s incentive for innovation.

The above discussion suggests that the “raising rival’s cost” motive is behind the result shown in Proposition 3. Innovation by firm 1 reduces its marginal cost but it may either increase or decrease the marginal costs of the non-innovating firms. If \( s \) is sufficiently small (i.e., \( s < s^* \)), innovation by firm 1 reduces the marginal costs of the non-innovating firms, implying that firm 1 cannot capture the entire benefit from innovation, which benefits also the non-innovating firms. Hence, the raising rival’s cost motive may discourage firm 1 from innovating if \( s \) is sufficiently small. Although this effect remains under both encompassing and separate unions, the effect is stronger under the former than the latter unionisation structure since an encompassing union benefits firm 1 from a lower marginal cost compared to separate unions. Hence, firm 1’s motive for raising the rival’s cost is higher under an encompassing union than under separate unions, and its incentive for innovation is lower under an encompassing union than under separate unions if \( s \) is sufficiently small.

The above argument suggests that competition in the product market may play an important role for the innovation reducing effect of an encompassing union. In other words, an encompassing union may not reduce firm 1’s incentive for innovation compared to separate unions if firm 1 is a monopolist producer of the product. We show that this is indeed the case. It is easy to check that if \( n = 1, s^* < 2/(n+2) \), implying that the range \( (2/(n+2), s^*) \) is empty. Hence, an encompassing union does not reduce firm 1’s incentive for innovation compared to separate unions if firm 1 is a monopolist producer of the product.

In the above analysis, we have considered an innovator and \((n-1)\) non-innovators to show the innovation reducing effect of an encompassing union in the simplest way. We show in Appendix A.7 that this result holds if there are multiple but not all innovators. If all firms innovate under an encompassing union,
we show in Appendix A.7 that the encompassing union does not reduce innovation compared to separate unions. This happens since the “raising rival’s cost” effect mentioned above is absent in this situation.

We have done our analysis under the assumption that the unions set the same wage to different firms, which gets significant support from the empirical evidence. If the unions set different wages to different firms, the incentive for innovation is higher under an encompassing union than under separate unions, again due to the absence of the “raising rival’s cost” effect.

2.3.3 The effect of an encompassing labour union on the total output

It follows from Propositions 1 and 3 that an encompassing union has two opposing effects on the total outputs. On one hand, it tends to reduce wages, and on the other hand, it may reduce innovation by firm 1. The following proposition shows that, depending on the extent of firm 1’s technological improvement through R&D, an encompassing union may either increase or decrease the total outputs produced by all firms, thus may have an ambiguous effect on consumer surplus.

**Proposition 4** Assume that $s \in (2/(n+2),s^*)$ and $k \in (k^*,k^{nc})$. An encompassing labour union decreases (increases) the total outputs of the final goods producers compared to separate labour unions for $s \in (2/(n+2),s^{**})$ ($s \in (s^{**},s^*)$), where

$$2/(n+2) < s^{**} = \frac{(-4n+3n^2 - \sqrt{3n^2 + n^4})}{(-4 + 3n^2)} < s^*.$$ 

**Proof** See Appendix A.6. Q.E.D.

Proposition 4 suggests that although an encompassing labour union creates the beneficial wage effect, which solves the complements problem, its adverse effect on firm 1’s innovation may dominate the beneficial wage effect, thus reducing the total outputs of the final goods producers under an encompassing union compared to separate unions. Since consumer surplus in our analysis is $q^2/2$, the above result implies that an encompassing labour union makes the consumers worse off compared to separate unions if the technological improvement through R&D is large.

Proposition 3 shows that an encompassing union increases firm 1’s incentive for innovation for $s \in (s^*,1)$ and $k^{nc} < k < k^c$. In this situation, the total outputs of the final goods producers are lower under “separate labour unions with no R&D by firm 1” compared to that of under “an encompassing labour union with R&D by firm 1”, implying that consumer surplus is higher under an encompassing union than under separate unions. The positive wage effect as well as the positive innovation effect helps to reduce the marginal costs of final goods production, thus making the consumers better off under an encompassing union.
We have considered a binary choice for firm 1’s R&D decision (i.e., firm 1 innovates or doesn’t innovate). As a result, firm 1 may not innovate under an encompassing union if the technological improvement through R&D is not small. However, no innovation by firm 1 under an encompassing union is an extreme situation and is the artefact of the binary choice. If firm 1 could choose the extent of technological improvement through R&D, say, by investing $F(s) = s^2 / 2$ to reduce its labour coefficient by $s$, it would innovate under an encompassing union but the extent of technological improvement could be lower under an encompassing union compared to separate unions. Hence, even if firm 1’s R&D decision is not a binary choice and firm 1 can choose the extent of technological improvement, the adverse effect of an encompassing union on firm 1’s innovation remains, which, in turn, may also make the consumers worse off under an encompassing union compared to separate unions. The consideration of a binary choice for firm 1’s R&D decision helps us to prove our point in the simplest way.

2.3.4 The effect of an encompassing labour union on the union utilities

So far, we have done the analysis under separate and encompassing unions. However, it is important to see whether the workers have the incentive to form an encompassing union.

The utilities of the unions are

$$\pi_{i}^{nc} = \pi_{y}^{nc} = \frac{(n-1+t)^2}{9(n+1)[-2(1-t) + n(2-2t + t^2)]}$$

and

$$\pi_{i}^{c} = \pi_{y}^{c} = \frac{(n-1+t)^2}{8(n+1)[-2(1-t) + n(2-2t + t^2)]}$$

under separate unions and an encompassing union respectively. It is immediate that if firm 1 either innovates or does not innovate irrespective of the unionisation structure, the utilities of the unions are higher under an encompassing union than under separate unions.

Now consider the situation where firm 1 innovates under separate unions but it does not innovate under an encompassing union, which occurs for $s \in (2/(n+2), s^*)$ and $k^c < k < k^{nc}$. In this situation, the utilities of the unions are

$$\pi_{i}^{nc,rd} = \pi_{y}^{nc,rd} = \frac{(n-1+s)^2}{9(n+1)[-2(1-s) + n(2-2s + s^2)]}$$

and

$$\pi_{i}^{c,rd} = \pi_{y}^{c,rd} = \frac{n}{8(n+1)}$$

under separate unions and an encompassing union respectively. Straightforward comparison shows that the union utilities are higher under an encompassing union than under separate unions in this situation.

\[10\] The intuition follows from Marjit and Mukherjee (2008), which shows in a different context that an input price reduction may either increase or decrease investment in innovation.
Finally, consider the case where firm 1 innovates under an encompassing union but it does not innovate under separate unions, which occurs for $s \in (s^*, 1)$ and $k^{nc} < k < k^c$. In this situation, the utilities of the unions are

$$\pi_{x, nd}^{nc, nd} = \pi_{y, nd}^{nc, nd} = \frac{n}{9(n+1)}$$

and

$$\pi_{x, rd}^{c, rd} = \pi_{y, rd}^{c, rd} = \frac{(n-1+s)^3}{8(n+1)[-2(1-s) + n(2-2s + s^2)]}$$

under separate unions and an encompassing union respectively. We get that the union utilities are higher under an encompassing union than under separate unions in this situation.

The following proposition is immediate from the above discussion.

**Proposition 5** An encompassing labour union increases the utilities of the unions compared to separate labour unions, irrespective of its effect on innovation by firm 1.

### 2.3.5 The effect of an encompassing labour union on social welfare

Finally, we want to see the effects of an encompassing union on social welfare, which is the sum of the union utilities, the net profits of the final goods producers and consumer surplus. If firm 1 either innovates or does not innovate irrespective of the unionisation structure, welfare is higher under an encompassing union compared to separate unions. Given the technology level, the lower wage under an encompassing union compared to separate unions helps to create a higher welfare under the former than the latter unionisation structure.

Now consider the case where firm 1 innovates only under separate unions, i.e., when $s \in (2/(n+2), s^*)$ and $k^{nc} < k < k^c$. We will see that an encompassing union may reduce welfare in this situation. Due to the complicated welfare expression, we will consider two numerical examples to show that whether an encompassing union reduces welfare in this situation depends on the product-market competition, given by $n$.

Assume that $n=2$, $s \in (1/2, s^* = (-17 + \sqrt{561})/8)$ and $k^c < k < k^{nc}$. We get in this situation that welfare is higher under “an encompassing labour union and no innovation” than under “separate labour unions and innovation” even if we consider maximum welfare under separate unions, which occurs at the cost of innovation $k^c$. However, if we consider that $n=20$, $s \in (1/11, s^* = (-2717 + \sqrt{7816809})/80)$ and $k^c < k < k^{nc}$, we get that welfare in this situation is lower (higher) under “an encompassing labour union and no innovation” than under “separate labour unions and innovation” for $s \in (1/11, 13/100)$ ( $s \in (13/100, (-2717 + \sqrt{7816809})/80)$ ) when we consider minimum welfare under separate unions, which occurs at $k^{nc}$. 

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The reason for the above result is as follows. If an encompassing union reduces innovation, it creates two opposing effects on welfare. On one hand, it tends to increase welfare by reducing wage. On the other hand, it tends to reduce welfare by reducing innovation. We have seen that more firms reduce the equilibrium wage. Since more firms create significantly lower wage under separate unions, further benefit from an encompassing union due to a lower wage is not significant if the product-market competition is significant. Hence, if the number of firms and the technological improvement through innovation are large, the loss from an encompassing union due to a lower innovation dominates the gain created by the encompassing union through lower wage, thus creating lower welfare under “an encompassing labour union and no innovation” compared to “separate labour unions and innovation”.

However, if the product market is very much concentrated (i.e., \( n \) is small), the gain from an encompassing union due to a lower wage is significant to outweigh the negative effect of an encompassing union on innovation. In this situation, an encompassing union increases welfare even if it reduces innovation.

Finally, consider the case where firm 1 innovates only under an encompassing union, which occurs for \( s \in (s^*, 1) \) and \( k^c > k < k^e \). We get in this situation that welfare is higher under “an encompassing labour union and innovation” than under “separate labour unions and no innovation” even if we consider minimum welfare under an encompassing union, which occurs at the cost of innovation \( k^e \). The positive effects of both lower wage and innovation following an encompassing union are responsible for this result.

The following result summarises the above discussion.

**Proposition 6** If an encompassing labour union reduces innovation compared to separate labour unions, social welfare may be lower under “an encompassing labour union and no innovation” than under “separate labour unions and innovation” if the product market is sufficiently competitive and the technological improvement through R&D is large.

An encompassing union solves the complements problem but it may reduce innovation by firm 1. As mentioned above, this trade-off is responsible for the above result. Since the benefit from an encompassing union due to a lower wage depends on the number of final goods producers, product-market competition plays an important role for the above result.

As already mentioned, no innovation under an encompassing union is the artefact of the binary choice for firm 1’s R&D decision and we consider this binary choice to prove our point in the simplest way. If we consider a non-binary choice for firm 1’s R&D decision and firm 1 can choose the extent of technological improvement, firm 1 would innovate under an encompassing union but the extent of technological improvement chosen by firm 1 could be lower under an encompassing union compared to separate unions. Hence, even if we allow firm 1 to choose the extent of technological improvement, an encompassing union creates the adverse effect on innovation, which, in turn, may reduce social welfare compared to separate unions.
3 Discussion

We now discuss the implications of some of our assumptions.

Like the related literature mentioned in the introduction, we have considered that labour is the only factor of production. This helped us to compare and contrast our results with that of the extant literature in a simplest way by showing the trade-off created by the complements problem and the raising rival’s cost effect on innovation. However, one can extend the analysis by incorporating non-labour factors of production. The raising rival’s cost effect shown in our analysis would remain in this extended model with labour and non-labour factors of production if the innovation is labour saving but that effect would not occur if innovation saves the non-labour factors of production. If the production process involves both labour and non-labour factors of production, firm may also have a choice regarding the type of innovation, i.e., labour saving and/or non-labour saving, depending on the labour unionisation structure. We leave this issue for future research.

To show the trade-off created by the complements problem and the raising rival’s cost effect on innovation in a simplest way, we assume that innovation allows the innovator to reduce the use of different types of labours in the same way. Hence, the innovation is not biased towards any type of worker and creates a neutral technological progress. This makes the analysis simple by creating symmetric behaviour of different labour unions. However, it is needless to say that if innovation reduces the use of different types of labours differently, our main result showing the adverse effects of an encompassing labour union on innovation and welfare remain.

Finally, to show the adverse effects of an encompassing labour union on innovation and welfare, we considered that the unions have full bargaining power in determining wages. However, it is easy to understand that if the firms have bargaining power, it will reduce wages and would affect the equilibrium outputs and profits, yet the trade-off created by the complements problem and the raising rival’s cost effect remain. As long as the unions have the bargaining power, the complements problem creates a lower wage under an encompassing union compared to separate unions. Hence, under no innovation, the profit of firm 1 is higher under an encompassing union compared to separate unions, reducing firm 1’s benefit from innovation under an encompassing union. On the other hand, under innovation, firm 1’s profit is lower (higher) under an encompassing union compared to separate unions if \( s < (>) (n-1)/n \), reducing (increasing) firm 1’s benefit from innovation under an encompassing union for \( s < (>) (n-1)/n \). These are similar to the effects discussed after Proposition 3. Hence, even if there is bargaining between the firms and the unions, an encompassing union reduces (but may increase) the incentive for innovation compared to separate unions if the technological improvement through innovation is large (small).
4 Conclusion

While the firms use complementary workers in reality, the existing literature examining the effects of the labour unionisation structure on innovation considered only substitutable workers and did not pay attention to complementary workers. This paper fills this gap in the literature.

We show that cooperation among the labour unions (or an encompassing labour union) of complementary workers may either increase or decrease a final goods producer’s incentive for innovation compared to non-cooperation among the labour unions (or separate labour unions). Although cooperation among the unions solves the complements problem, it may have an adverse effect on the final goods producer’s technological improvement. We show that the adverse effect on the technological improvement may dominate the beneficial wage effect of cooperation among the unions, thus making the consumers and the society worse off. While cooperation among the unions makes the workers better off, it may not make all final goods producers better off. Thus, our results provide new insights to the literature on labour unionisation structure and innovation, and suggest that whether the workers are substitutes or complements are important factors to consider.

Appendix

A.1 The restriction on $s$ to ensure that all final goods producers always hire workers

We show here that if the final goods producers differ in terms of their technologies, all final goods producers hire workers under separate and encompassing unions if $2/(n+2) < s$. If the unions want to charge the wage in a way so that it is not profitable for all firms to hire workers at that wage, it is easy to understand that the unions can prevent the non-innovating firms from hiring workers but cannot prevent only the innovating firm from hiring workers. This happens since the outputs of the innovating firm are always positive whenever the outputs of the non-innovating firms are positive, and the unions cannot charge a wage that will induce only the non-innovating firms to hire workers.

Separate labour unions: First, consider the case of separate labour unions and the equilibrium with symmetric wages. If the unions want to provide workers only to the technologically superior final goods producer (i.e., to firm 1, which innovates a new technology and creates technological difference between the final goods producers), wages need to be such that it is not profitable for the technologically inferior non-innovating firms to hire workers. If the unions provide workers to firm 1 only, the demand for workers is $q_x = q_y = s[1 - s(w_x + w_y)]/2$ and the equilibrium wages are $w_x^{mc,m} = w_y^{mc,m} = 1/3s$. The outputs of the non-innovating firms are zero at these wages if $s \leq 4/5$. If $4/5 < s$, the equilibrium wages need
to be $\hat{w}_x = \hat{w}_y = 1/2(2 - s)$ to prevent the non-innovating firms from hiring workers. Since $\hat{w}_x = \hat{w}_y = 1/2(2 - s)$ is the constrained wage, it is immediate that the equilibrium union utilities are lower from charging the wage $\hat{w}_x = \hat{w}_y = 1/2(2 - s)$ than from charging the wage $\nu_x^{nc,m} = \nu_y^{nc,m} = 1/3s$.

If the workers are hired only by firm 1 at the wages $\nu_x^{nc,m} = \nu_y^{nc,m} = 1/3s$, which can happen for $s \leq 4/5$, the equilibrium union utilities are $\pi_x^{nc,m} = \pi_y^{nc,m} = 1/18$. We get that, if $2/(n+2) < s$, $\pi_x^{nc,m} = \pi_y^{nc,m} = 1/18$ are lower than $\pi_x^w = \pi_y^w = (n - 1 + s)^2 / 9(n+1)[-2(1-s) + n(2-2s+s^2)]$, which are the union utilities when all final goods producers are provided workers, as considered in the text. Since $\pi_x^{nc,m} = \pi_y^{nc,m} = 1/18$, i.e., the union utilities under the unconstrained wages $\nu_x^{nc,m} = \nu_y^{nc,m} = 1/3s$, are lower than the union utilities from providing workers to all final goods producers, it is immediate that if $4/5 < s$ and the unions charge $\nu_x = \nu_y = 1/2(2 - s)$ to provide workers to only firm 1, the union utilities are lower from providing workers to only firm 1 than from providing workers to all final goods producers. Hence, the separate unions provide workers to all final goods producers for $2/(n+2) < s$, as considered in the text.

**An encompassing labour union:** Now consider an encompassing labour union and the equilibrium with symmetric wages. If the union provides workers to firm 1 only, the demand for workers is $q_x = q_y = s[1 - s(w_x + w_y)]/2$ and the equilibrium wages are $w_x^{cm} = w_y^{cm} = 1/4s$. The outputs of the non-innovating firms are zero at these wages if $s \leq 2/3$. If $2/3 < s$, the equilibrium wages need to be $\nu_x^m = \nu_y^m = 1/2(2 - s)$ to prevent the non-innovating firms from hiring workers. Since $\nu_x^m = \nu_y^m = 1/2(2 - s)$ is the constrained wage, it is immediate that the equilibrium union utility is lower from charging the wage $\nu_x^m = \nu_y^m = 1/2(2 - s)$ than from charging the wage $w_x^{cm} = w_y^{cm} = 1/4s$.

If the workers are hired only by firm 1 at the wages $w_x^{cm} = w_y^{cm} = 1/4s$, which can happen for $s \leq 2/3$, the equilibrium union utilities are $\pi_x^{cm} = \pi_y^{cm} = 1/16$. We get that, if $2/(n+2) < s$, $\pi_x^{cm} = \pi_y^{cm} = 1/16$ are lower than $\pi_x^c = \pi_y^c = (n - 1 + s)^2 / 58(n+1)[-2(1-s) + n(2-2s+s^2)]$, which are the union utilities when all final goods producers are provided workers, as considered in the text. Since $\pi_x^{cm} = \pi_y^{cm} = 1/16$, i.e., the union utilities under the unconstrained wages $w_x^{cm} = w_y^{cm} = 1/4s$, are lower than the union utilities from providing workers to all final goods producers, it is immediate that if $2/3 < s$ and the unions charge $\nu_x = \nu_y = 1/2(2 - s)$ to provide workers to only firm 1, the union utilities
are lower from providing workers to only firm 1 than from providing workers to all final goods producers. Hence, an encompassing labour union provides workers to all final goods producers for \( 2/(n+2) < s \), as considered in the text.

A.2 Proof of Lemma 1

Under separate labour unions, firm 1’s profit under innovation is
\[
\pi_{1}^{nc,rd} = \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 - k,
\]
while its profit under no innovation is \( \pi_{1}^{nc,nrd} = 1/9(n+1)^2 \). Firm 1 innovates if \( \pi_{1}^{nc,rd} > \pi_{1}^{nc,nrd} \), which gives the result. \( Q.E.D. \)

A.3 Proof of Lemma 2

Under an encompassing labour union, firm 1’s profit under innovation is
\[
\pi_{1}^{e,rd} = \left[ \frac{-3(1-s) + n^2(1-s) + n(2-2s+s^2)}{(n+1)[-4(1-s) + 2n(2-2s+s^2)]} \right]^2 - k,
\]
while its profit under no innovation is \( \pi_{1}^{e,nrd} = 1/4(n+1)^2 \). Firm 1 innovates if \( \pi_{1}^{e,rd} > \pi_{1}^{e,nrd} \), which gives the result. \( Q.E.D. \)

A.4 Proof of Proposition 2

If firm 1 innovates under both unionisation structures, its profit is
\[
\pi_{1}^{nc,rd} = \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 - k
\]
under separate labour unions and
\[
\pi_{1}^{e,rd} = \left[ \frac{-3(1-s) + n^2(1-s) + n(2-2s+s^2)}{(n+1)[-4(1-s) + 2n(2-2s+s^2)]} \right]^2 - k
\]
under an encompassing labour union. We get that \( \pi_{1}^{nc} > (\leq) \pi_{1}^{e} \) for \( s \in (2/(n+2),(n-1)/n) \) (\( s \in ((n-1)/n,1) \)).

If firm 1 innovates under separate and encompassing labour unions, the profit of firm \( i, i=2,...,n \), is
\[
\pi_{i}^{nc} = \left[ \frac{-2(1-s^2) + n(2-4s+3s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2
\]
under separate labour unions and
\[
\pi_{i}^{e} = \left[ \frac{-(2-s+s^2) + n(2-3s+2s^2)}{(n+1)[-4(1-s) + 2n(2-2s+s^2)]} \right]^2
\]
under an encompassing labour union. We get that \( \pi_{i}^{nc} < \pi_{i}^{e} \) for \( s \in (2/(n+2),1) \). \( Q.E.D. \)
A.5 Proof of Proposition 3
We obtain that $k^{nc} > (k^c)$ for $s \in (2/(n+2), s^*)$ ($s \in (s^*, 1)$), where 
\[
s^* = (3 + 4n - 7n^2 + \sqrt{9 - 58n^2 + 49n^4})/4n.
\]

If $s \in (2/(n+2), s^*)$ and $k^c < k < k^{nc}$, firm 1 innovates only under separate unions. In this situation, an encompassing union reduces firm 1’s incentive for innovation compared to separate unions. However, the unionisation structure does not affect firm 1’s incentive for innovation if either $k < k^c < k^{nc}$ (where firm 1 innovates irrespective of the unionisation structures) or $k^c < k^{nc} < k$ (where firm 1 does not innovate irrespective of the unionisation structure).

If $s \in (s^*, 1)$ and $k^{nc} < k < k^c$, firm 1 innovates only under an encompassing union. In this situation, an encompassing union increases firm 1’s incentive for innovation compared to separate unions. However, the unionisation structure does not affect firm 1’s incentive for innovation if either $k < k^{nc} < k^c$ (where firm 1 innovates irrespective of the unionisation structure) or $k^{nc} < k^c < k$ (where firm 1 does not innovate irrespective of the unionisation structure). Q.E.D.

A.6 Proof of Proposition 4
If $s \in (2/(n+2), s^*)$ and $k \in (k^c, k^{nc})$, firm 1 innovates only under separate unions. The total outputs of the final goods producers under “separate labour unions and innovation by firm 1” and under “an encompassing labour union and no innovation by firm 1” are 
\[
q_{nc, rd}^{nc, rd} = -2n(1-s) - 2(1-s)^2 + n^2(4 - 6s + 3s^2) \over (n+1)[-6(1-s) + 3n(2 - 2s + s^2)]
\]
and 
\[
q_c^{nc, rd} = \frac{n}{2(n+1)}
\]
respectively. We get that $q_{nc, rd}^{nc, rd} > (q_c^{nc, rd}$ if $s \in (2/(n+2), s^*)$ ($s \in (s^*, s^*)$), where 
\[
\frac{2}{n+2} < s^* = \frac{-4n + 3n^2 - \sqrt{3(-n^2 + n^4)}}{-4n + 3n^2} < s^*.
\]
Q.E.D.

A.7 The case of multiple innovators
We show here that an encompassing labour union may reduce innovation compared to separate labour unions even if there are multiple innovators. Since the calculations are straightforward but cumbersome, we skip the mathematical details.

As in the text, we assume that there are $n$ firms in the industry. However, we now assume that all firms can innovate to improve labour productivities.
First, consider the case of separate labour unions. If \((m - 1)\) firms invested in innovation, the \(m\)th firm invests in innovation if its equilibrium net profit\(^{11}\) from innovation (implying that \(m\) number of firms invest in innovation) is higher than its equilibrium profit from no innovation (implying that \((m - 1)\) firms invest in innovation) if

\[
\begin{align*}
k < & \frac{m^2(1-s)^2 + n(3 - 2n(1-s) + 2s) + m(1-s)(3+n+s+ns)^2}{9(1+n)^2(n-m(1-s)(1+m-n(1-s) + s - ms))^2} \\
& \cdot \frac{(m(1-m+n) + 2(m-1)(m-2(1+n))s -(m-1)(-4 + m - 3n)s^2)^2}{9(1+n)^2(m(1-m+n) + 2(m-1)(-1 + m-n)s + (m-1)(2-m+n)s^2)^2} = k^{nc}(m).
\end{align*}
\]

(A1)

We can find that a higher \(m\) corresponds with a lower \(k^{nc}(m)\), implying that if the cost of doing innovation increases, it reduces the number of firms undertaking innovation under separate unions.

Now consider the case of an encompassing labour union. If \((m - 1)\) firms invested in innovation, the \(m\)th firm invests in innovation if

\[
\begin{align*}
k < & \frac{m^2(1-s)^2 + n(-2 - n(1-s) + s) + m(1-s)(2+s+ns)^2}{4(1+n)^2(n-m(1-s)(1+m-n(1-s) + s - ms))^2} \\
& \cdot \frac{(m(1-m+n) + (m-1)(2m-3(1+n))s -(m-1)(-3 + m - 2n)s^2)^2}{4(1+n)^2(m(1-m+n) + 2(m-1)(-1 + m-n)s + (m-1)(2-m+n)s^2)^2} = k^{c}(m).
\end{align*}
\]

(A2)

We can find that a higher \(m\) corresponds with a lower \(k^{c}(m)\), implying that if the cost of doing innovation increases, it reduces the number of firms undertaking innovation under an encompassing union.

Evaluating (A1) and (A2) at \(m=1\) and comparing them gives us Proposition 3. Now we consider other cases.

If \(k < k^{nc}(n)\) and \(k < k^{c}(n)\), all firms innovate under both unionisation structures. However, we get that \(k^{nc}(n) < k^{c}(n)\), suggesting that if the cost of doing innovation is such that all firms innovate under an encompassing union, the incentive for innovation is higher under an encompassing union compared to separate unions. This happens since the “raising rival’s cost” motive, as discussed after Proposition 3, does not work in this situation.

Now consider the case where the costs of doing innovation are not small enough to make innovation by all firms profitable under an encompassing union. Given expressions (A1) and (A2), we cannot compare \(k^{nc}(m)\) and \(k^{c}(m)\) generally. Hence, we use numerical examples to show that the number of innovating firms may be lower under an encompassing union if all firms do not find innovation profitable under an encompassing union.

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\(^{11}\) When determining the equilibrium profits, we have considered the corresponding equilibrium wages.
As an example, consider that \( n = 5 \) and \( m = 2 \). We get that
\[
2(-5184 + 20664s - 37338s^2 + 38151s^3 - 21030s^4 + 2539s^5)
\]
\[
k < k^{nc}(2) = \frac{-4606s^6 - 3112s^7 + 704s^8}{27(72 - 168s + 205s^3 - 124s^3 + 40s^4)^2}
\]
\[
(3456 - 13752s + 25053s^2 - 25893s^3 + 14550s^4 -
\]
\[
k < k^c(2) = \frac{1981s^5 - 3073s^6 + 2152s^7 - 512s^8}{12(72 - 168s + 205s^3 - 124s^3 + 40s^4)^2}.
\]

We plot \( k^c(m = 2, n = 5) - k^{nc}(m = 2, n = 5) \) in Figure 1 and find that
\( k^c(m = 2, n = 5) < k^{nc}(m = 2, n = 5) \) for \( 0.6 < s < 0.9 \) (approx.)\(^{12}\).

![Figure 1: \( k^c(m = 2, n = 5) - k^{nc}(m = 2, n = 5) \)](image)

If \( 0.6 < s < 0.9 \) (approx.) and \( k^c(m = 2, n = 5) < k^{nc}(m = 2, n = 5) \), an encompassing labour union reduces the number of innovating firms compared to separate labour unions.

It is now easy to understand that if an encompassing labour union reduces the number of innovators compared to separate labour unions, it may reduce consumer surplus and welfare than the latter unionisation structure even if it solves the complementary problem.

\(^{12}\) If there are \( k \) innovating and \( (n - k) \) non-innovating firms, in an equilibrium with symmetric wages, the unions provide workers to all firms for \( [2k + n(k - 1)]/ k(2 + n) < s \). Hence, if \( n = 5 \) and \( m = 2 \), the unions provide workers to all firms for \( 0.6 < s \), and we restrict our attention to \( 0.6 < s \).
References


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