Alethic Undecidability Doesn't Solve the Liar

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1 Introduction

Stephen Barker (2014) presents a novel approach to solving semantic paradoxes, including the Liar and its variants and Curry's paradox. His approach is based around the concept of *alethic undecidability*. His approach, if successful, renders futile all attempts to assign semantic properties (*truth*, *falsity*, gap or glut) to the paradoxical sentences, whilst leaving classical logic fully intact. And, according to Barker, even the T-scheme remains valid, for validity is not undermined by undecidable instances.

Barker's approach is innovative and worthy of further consideration, particularly by those of us who aim to find a solution without logical revisionism. As it stands, however, the approach is unsuccessful, as I shall demonstrate below.

2 Barker's Account

Barker takes as his starting point a version of the *truthmaker principle* (2014, 201):

(TM) If a sentence is true (or false), then it is true (or false) in virtue of non-alethic facts.

A *non-alethic fact* is something like a state of affairs not involving the properties *truth* or *falsity*: *that students drink* is one such fact, whereas *that the proposition* ⟨*that students drink*⟩ *is true* is not, since the latter involves the property *being true*.

Truth or falsity requires a sentence to connect, eventually, to non-alethic facts. Barker (2014, 202) offers two examples of this connection failing. Consider the following infinite sequence of sentences:

(R_1) Sentence R_2 is true.

(R_2) Sentence R_3 is true.

(R_3) Sentence R_4 is true.

$\vdots$

Or, consider the *truth-teller* sentence:

(T) Sentence T is true.

In each case, no sentence has the right kind of connection to non-alethic facts. They are *ground-unspecifiable* (Barker 2014, 203). Notice that there's nothing paradoxical about such cases: we can consistently assign them all the value *true* or all the value *false*. But Barker claims, based on (TM), that we should not, for truth or falsity require the kind of connection to non-alethic reality that's missing in each case.

He then argues as follows:

> Ground-unspecifiable sentences are undecidable with respect to their grounding status. Given TM – which says that all alethic properties must be grounded – it follows that such sentences are undecidable with respect to their alethic properties. ... Call this undecidability with respect to alethic properties *alethic-undecidability* or *A-undecidability* for short. (Barker 2014, 206)

As a consequence, for each example sentence above, it is undecidable whether it 'is either true or false, lacks truth and falsity, is both true and false, and so on' (Barker 2014, 206). Alethic undecidability thus implies unassertability: 'we cannot assert, in principle, whether they are true, false, either true or false, neither true nor false, both true and false, and so on' (Barker 2014, 201).

Barker then applies this reasoning to the (strengthened) Liar sentence:

(L) L is not true;

and to the Curry sentence:

(C) If C is true, then $\perp$

(where $\perp$ is any absurd sentence you like). The claim is that both L and C are ground-unspecifiable, hence A-undecidable. Moreover, according to Barker (2014, 208–9), all attempts at 'revenge' fail under this analysis. So, for example:

(R) R is A-undecidable

is ground-unspecifiable, hence A-undecidable (Barker 2014, 208). But this result doesn't imply that R is true, since for any A-undecidable sentence, we're banned from saying whether it 'is either true or false, lacks truth and falsity, is both true and false, and so on' (Barker 2014, 206). That, at least, is Barker's story.

In this paper, I'll demonstrate that Barker's approach is flawed. I'll do so using only A-decidable reasoning from A-decidable premises. That guarantees our conclusions will likewise be A-decidable and hence assertable. That's because valid reasoning (from finite premises, at least) preserves A-decidability. For if the premises are A-decidable, then so is their conjunction; and if the reasoning is valid, then the conditional from the conjoined premises to the conclusion is valid, hence

A-decidable. Since logical proof is one way to decide the truth of a statement, that conditional itself must be A-decidable; and a conditional with an A-decidable antecedent must have an A-decidable consequent. It follows that A-decidable reasoning from A-decidable premises results in A-decidable conclusions.

My case against Barker is in two steps. First, we can in certain cases establish (in an A-decidable way) facts about a sentence's dependencies (§3). I then argue (§4) that reasoning in this way allows us to reinstate the (strengthened) Liar paradox.

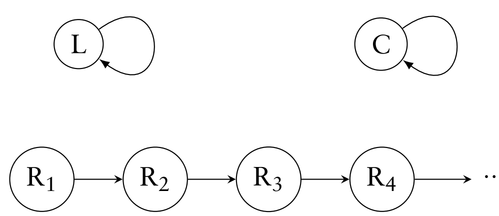
3 Dependency Graphs

We can establish, in an A-decidable way, certain facts about dependency between entities, including sentences like L and C. We can use these facts to build directed *dependency graphs* for the sentences in question.

The idea is simple: when entity e_1 depends on e_2, we put an arrow from the e_1 node to the e_2 node in our graph. For full generality, we would need to capture different senses of dependency. (An entity may depend on another for its *existence*; but in the case of sentences, we often talk about S_1 depending on S_2 for its *truth*.) We could capture different dependency relations in our graphs with different colour arrows.

Full generality would also require us to capture the distinction between full and partial dependency. $(A \wedge B) \vee C$'s truth depends on the truth of A, B and C, but in different ways: C is a full ground, whereas A and B are individually mere partial grounds, but jointly a full ground, for $(A \wedge B) \vee C$. We can track the difference using an AND-OR graph structure, familiar from the technical literature.

We needn't go into the details here, however, as the cases we're interested in – L, C and the R$_i$s – involve simple 1-1 dependencies. Their graphs are as follows:



We may interpret an edge $A \longrightarrow B$ as saying that A's truth-value depends on B's truth-value; or alternatively, as saying that the fact *that* A depends on (or obtains in virtue of) the fact *that* B (obtaining). For our purposes, it doesn't matter.

Let's focus on the finite graph for L for the time being. It's clear that the process of associating a sentence like L with its graph is algorithmically decidable, given the facts (i) that the only referring term in L is 'L'; and (ii) that 'L' refers only to L. Fact (i) concerns syntax only and so is clearly A-decidable. Fact (ii) is about reference. It would be absurd to suggest that the reference-facts are A-undecidable in these cases. For that would imply that we can't assert that L is self-referential and hence that the set-up to the paradoxes cannot even be stated. Putting these points together, the association of L with its graph is A-decidable. (Similarly for C and its graph.)

Given that these graphs are determinate, finite structures, we can clearly make determinate statements about them. We can say, for example, that both L's and C's graph consists of a single node and that neither graph has a leaf node. And from such facts, we can infer further facts about L's and C's dependencies: that L does not depend on C, for example. And crucially, we infer such facts from A-decidable premises using only A-decidable reasoning, hence our inferred conclusions are A-decidable.

One class of facts we can determine, using this kind of reasoning, concern whether given nodes correspond to alethic or to non-alethic facts. Since nodes are sentences, this amounts to a syntactic check for the predicates 'is true' and 'is false'. Clearly, that check is A-decidable. (And even if we thought the class of alethic predicates was open-ended, or otherwise A-undecidable, it's clear that 'is true' and 'is false' are alethic predicates. So we clearly have a sufficient condition for a node's representing an alethic fact.) This allows us to infer, A-decidably, that L's graph's sole node states an alethic fact and hence that no node in the graph states a non-alethic fact. Such inferences will be crucial in the argument below.

It isn't so clear whether this kind of reasoning applies to the infinite graph associated with R_1. It isn't in general algorithmically decidable whether such graphs contain such-and-such nodes. (A tree automaton search for such nodes may never terminate.) But now consider how the regress case is presented. We give its first few cases, R_1, R_2, R_3 explicitly; and then we say '...' or 'and so on'. By this, we mean to say that the case consists of infinitely many sentences, one for each $n \in \mathbb{N}$, of the form:

(R_n) Sentence R_{n+1} is true.

If that is the implicit content of the specification of the case, then we have a *general* premise from which to reason. We infer that each R_n depends only on R_{n+1} and that there is no last R_n. From this, it follows mathematically (and hence A-decidably) that R_1's graph has certain features: in particular, that each node states an alethic condition and hence that there is no branch in the graph terminating in a non-alethic node. This kind of reasoning allows us to reason about infinite cases, including Yablo's paradox.

4 The Problem

The worry for Barker's proposal goes like this. L's graph contains no non-alethic nodes. All its nodes are alethic nodes, expressing alethic facts. So L's dependencies involve only alethic facts. But by (TM), no alethic fact grounds any sentence's truth. So L has no alethic ground. It is ungrounded. So by (TM), it is not true. This conclusion was inferred from A-decidable premises using only A-decidable reasoning and so it A-decidable that L is not true. We can then reason (A-decidably) in the usual way to a contradiction.

Barker (2014, 4–5) considers a related argument along these lines and argues that it does not work: 'to infer ... that [L] is ungrounded from that fact is to forget the infinite deferment of grounding' (Barker 2014, 5), involving 'an infinite series of interpretative stages' (Barker 2014, 6). In the analogous case of the truth-teller, T, he argues that

> ... at each stage, no grounding condition is specified. But there is always a further interpretative stage after any given stage. ... Since T at no stage can be held up as the source of T's failing to be grounded, we cannot say that T is ungrounded on the basis that no sentence at any interpretive stage describes a non-alethic condition. (Barker 2014, 6)

Barker's reasoning here is faulty. We can infer general conclusions about T's dependencies. The reasoning via T's graph allows us to infer (A-decidably) that no node in T's graph states a non-alethic condition and hence (A-decidably) that T does not depend on any non-alethic fact. We don't need to enter into Barker's infinite sequence of interpretive stages: we need only the premises that (i) 'T' refers (only) to T and (ii) T contains 'is true' and hence states an alethic fact. That is sufficient information to build the dependency graph and conclude that T does not depend on any non-alethic fact and hence is ungrounded.

Exactly the same goes for L and C, with the usual paradoxical consequences. Given that (i) 'L' refers (only) to L and (ii) L contains 'is true' and hence states an alethic fact, we can build L's dependency graph and determine, A-decidably, that it contains only alethic nodes. So, A-decidably, L is not grounded by any non-alethic fact and hence, by (TM), is not true. It is A-decidable that L is not true. (Exactly the same reasoning applies to C.) Then L is assertible and apt for inclusion in standard logical reasoning. But that reasoning quickly leads to absurdity: we can assert the Liar equivalence (L is true iff it is not true), from which it follows that L is both true and not true. The paradox has not been blocked.

5 Conclusion

I've argued that we can, in certain cases, draw clear, A-decidable conclusions about a sentence's alethic dependencies. From clear facts about a sentence's syntax and reference of its terms, we can build dependency graphs (§3) and reason about a sentence's dependencies. In particular, we can determine, A-decidably, that L is ungrounded, hence not true; but this quickly results in absurdity. So, A-decidably, we must reject Barker's proposal.

References

Barker, S. 2014. Semantic paradox and alethic undecidability. *Analysis* 74: 201–9.