Information Rigidities and the News-Adjusted Output Gap*

by

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Abstract

A vector-autoregressive model of actual output and expected output obtained from surveys is used to test for information rigidities and to provide a characterisation of output dynamics that accommodates these information structures. News on actual and expected outputs is decomposed to identify innovations understood to have short-lived effects and these are used with the model to derive a ‘news-adjusted output gap’ measure. The approach is applied to US data over 1970q1-2014q2 and the new gap measure is shown to provide a good leading indicator of inflation.

Keywords: Information Rigidities, Survey-based Expectations, Output Gap.

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1 Introduction

Expectation formation and agents’ use of information are recognised as being central to the understanding of output and price dynamics and of macroeconomic policy effects but the nature of their role remains controversial. For example, the influential papers by Mankiw and Reis (2002), Sims (2003) and Woodford (2001) explore the consequences of various forms of information rigidity in rational expectations models. Here, agents are assumed rational but are either slow to take account of macroeconomic information even when it is publicly available (“sticky information models”) or are only able to observe the fundamentals on which decisions are based with error (“noisy information”). In these circumstances, the divergence between belief and reality can generate short-run fluctuations in prices and output that are quite separate from their long-run time-paths. This can have important implications for the conduct of policy since policy will be most effective if it works with, and takes advantage of, agents’ use of information.¹

This paper describes how survey data on expectations can be used to obtain direct measures of the news on current and future output levels as perceived by agents in real time and taking into account the potential for information rigidities. A novel decomposition method is then described to investigate the agents’ use of this new information, separating out that part which is expected to have a permanent effect on output from that part expected to have more short-lived effects. The methods are illustrated using data from the U.S. Survey of Professional Forecasters over the last 45 years and their usefulness is illustrated through the construction of a ‘news-adjusted’ output gap measure that is purged of the effects of output movements that are known to be short-lived. The measure is found to be a good leading indicator of inflation in the US, showing the potential value of taking into account agents’ use of information in the conduct of policy.

The survey responses of professional forecasters have been used recently to examine the nature and extent of information rigidities by Carroll (2003), Andrade and Le Bihan (2013), Coibion and Gorodnichenko (2011, 2012) and Dovern et al. (2014), inter alia. In

¹See for example, Ball et al. (2005) and the recent work by Blanchard et al. (2013), Kulish and Pagan (2013) and Lorenzoni (2010), among others.
those papers, the analysis of forecasters’ responses at the individual level and at an aggregated level provides evidence in favour of both sticky information models and relatively simple noisy information models.\footnote{There is, of course, a long tradition of examining expectation formation processes through the analysis of survey data; see Pesaran and Weale (2003) or Croushore (2010) for reviews.} The information contained in surveys of professional forecasters is exploited again here in this paper to test for the presence of information rigidities through an analysis of the relationships between forecast errors and revisions in forecasts, following the approach of Coibion and Gorodnichenko (2012) but extended in two ways: first, the tests are conducted in the context of a linear VAR analysis of data on actual output and expected output at various forecast horizons so they provide a more information-rich context for the tests than in univariate analyses; and then the tests are carried out in a non-linear extension of the multivariate model accommodating the possibility that agents’ use of information changes according to the state of the business cycle. Moreover, having tested and imposed an appropriate information structure on the multivariate VAR, we can obtain direct measures of the news content contained in the actual and survey output data as perceived by agents in real time and taking into account the information rigidities found in the data. It is these measures of news that are then further exploited to identify forecasters’ beliefs on the long-term and short-term consequences of output innovations and which provide the basis of the decomposition of output innovations into separate meaningful elements distinguished according to agents’ views on the permanence of their effects.\footnote{Thapar (2008) also makes use of direct measures of expectations and timing assumptions to identify economically-meaningful shocks assuming rationality and a Choleski ordering to identify monetary policy shocks. Krane (2008) also uses the patterns of revisions to short-, medium- and long-horizon survey predictions to measure the size and dynamic effects of different types of permanent and transitory shocks.}

Our decomposition of the innovations to the VAR is in the spirit of Blanchard and Quah (1989) in that it assumes output is characterised as a unit root process and identifies a single stochastic trend which drives the permanent changes in actual and expected outputs and the associated Beveridge-Nelson (1981) [BN] trend.\footnote{In a similar vein, Mertens (2016) uses long-run forecasts from models including actual and expected inflation and financial market data to define trend inflation. And Kozicki and Tinsley (2012) use actual and expected inflation to construct long-horizon expected inflation measures.} Blanchard and Quah noted that
the trend derived from the permanent shocks alone will not adequately represent the
trend in a standard business cycle decomposition though as this should accommodate
fluctuations in output caused by short-term, transitory shocks as well as permanent ones.
Practically there are also a variety of ‘policy lags’ between the time a macro problem arises
and the time a policy response takes effect. Failure to incorporate the short-lived effects
into the trend means the associated gap measure will over-react to changes in output
and the size and timing of any implied price pressures, say, will be misjudged. Policy
based on the gap will also over-react to output change and generate unnecessary policy-
induced volatility. The ‘news-adjusted gap’ proposed in this paper addresses this problem
providing a tool for policy makers that works with, and takes advantage of, agents’ use
of information. We illustrate the importance of the news-adjustments in this paper by
comparing our gap measure with other measures known to perform well in explaining
inflationary pressures in the U.S. over the last forty-five years.

The layout of the remainder of the paper is as follows. Section 2 introduces the linear
VAR model that can capture the time series properties of actual output and the direct
measures of output expectations. It also describes the non-linear extension used to ac-
commodate the possibility that these properties could change over time. The section then
describes the restrictions implied by the different forms of information rigidities, motivates
the decomposition of the innovations into permanent and known-to-be-transitory shocks,
and describes how we can obtain output gap measures based on the BN trend output
alone and then adjusted to take into account a news-adjustment. Section 3 describes the
application of the methods to quarterly US data over the period 1970q1-2014q1. Linear
and non-linear versions of the VAR are estimated based on data on actual and output
expectations for up to four quarters ahead and tests on the information structures are
carried out. As we shall see, the ‘noisy information model’ appears to fit the data well
and so we consider in detail the gap measures based on the model incorporating these
restrictions, comparing their properties to those of other popular gap measures both in

Footnote:
The use of survey data helps address any ‘recognition lags’ arising if only backward-looking data is
used to monitor the economy. But time lost in making and implementing decisions, and in their taking
effect, means that there can be considerable delays involved in some policy responses.
statistical terms and in terms of their ability to capture inflationary pressures. Section 4 concludes.

2 Use of Information in VAR Models of Actual and Expected Outputs

2.1 VAR Models and Tests of Information Rigidities

A simple linear VAR model of the joint determination of actual output and direct measures of expected future output assumes that actual output is first-difference stationary, and that expectational errors are stationary. The first of these assumptions is supported by considerable empirical evidence and the latter assumption is consistent with a wide variety of hypotheses on the expectations formation process, including hypotheses in which confidence or optimism can generate their own self-fulfilling (but non-explosive) dynamic or the Rational Expectations (RE) hypothesis, for example. In what follows, the logarithm of actual output at time $t$ is denoted by $y_t$ and the direct survey-based measure of the logarithm of the expectation of output at time $t+1$, as published at time $t$, is denoted by $t^e_{f_{t+1}}$. The stationarity assumptions say explicitly that actual growth, $y_t - y_{t-1}$, and current expectation errors, $y_t - t^{-1} y^e_{f_t}$ are stationary. But, of course, they also imply that expected growth in output, $t^e_{f_{t+1}} - y_t$, is stationary as it can be decomposed into actual output growth, $y_{t+1} - y_t$, and expectational error, $y^e_{f_{t+1}} - y_{t+1}$, both of which are stationary by assumption. Similarly, revisions in expectations, e.g. $t^e_{f_{t+2}} - t^e_{f_{t+1}}$ can be decomposed into two expectational errors $t^e_{f_{t+2}} - y_{t+2}$ and $y_{t+2} - t^e_{f_{t+2}}$ and are also stationary.

A general linear time series representation of any combination of these stationary series would be able to capture the potentially complex interactions between actual and expected outputs and in what follows we work with a simple VAR($p - 1$) representation
of actual and expected output growth:
\[
\begin{bmatrix}
y_t - y_{t-1} \\
ty_{t+1} - y_t \\
ty_{t+2} - ty_{t+1}
\end{bmatrix} = \mathbf{B}_0 - \mathbf{B}_1 \begin{bmatrix}
y_{t-1} - y_{t-2} \\
t^{-1}y_{t-1} - y_{t-1} \\
t^{-1}y_{t+1} - t^{-1}y_{t+1}
\end{bmatrix} - \ldots
\]
\[
... - \mathbf{B}_{p-1} \begin{bmatrix}
y_{t+p-1} - y_{t-p} \\
t^{-p+1}y_{t-p+2} - y_{t-p+1} \\
t^{-p+1}y_{t-p+3} - t^{-p+1}y_{t-p+2}
\end{bmatrix} + \begin{bmatrix}
\epsilon_{0t} \\
\epsilon_{1t} \\
\epsilon_{2t}
\end{bmatrix}
\]
(2.1)

where, for illustrative purposes, we assume here that survey measure are available for one- and two-periods ahead expectations only. Here, \(\mathbf{B}_0\) is a a 3 \(\times\) 1 vector of intercepts and the \(\{\mathbf{B}_j\}, j = 1, \ldots, p - 1\), are 3 \(\times\) 3 matrices of parameters. Actual output growth at time \(t\) and the growth in output expected to occur in times \(t+1\) and \(t+2\) are driven by disturbances \(\epsilon_{0t}\), \(\epsilon_{1t}\) and \(\epsilon_{2t}\). The \(\epsilon_{0t}\) represents “news on output growth in time \(t\) becoming available at time \(t\)”, while \(\epsilon_{ht}\) is “news on output growth expected in time \(t+h\) becoming available at time \(t\)” for \(h = 1, 2\). These innovations are unpredictable based on information dated at time \(t - 1\) and earlier.

As elaborated in the Appendix, the model in (2.1) can be written in a variety of forms, including as a \(p\)-th order VAR in the levels vector \(\mathbf{z}_t = (y_t, ty_{t+1}, ty_{t+2})'\) or as a cointegrating VAR describing \(\Delta \mathbf{z}_t\):
\[
\Delta \mathbf{z}_t = \mathbf{a} + \Pi \mathbf{z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta \mathbf{z}_{t-j} + \mathbf{u}_t,
\]
(2.2)

where the error terms of \(\mathbf{u}_t\) are interpreted as “news on the successive output levels” with \(\mathbf{u}_t = (\epsilon_{0t}, \eta_{1t}, \eta_{2t})' = (\epsilon_{0t}, (\epsilon_{0t} + \epsilon_{1t}), (\epsilon_{0t} + \epsilon_{1t} + \epsilon_{2t}))'\). The model can also be written, through recursive substitution of (2.2), as the moving average representation
\[
\Delta \mathbf{z}_t = \mathbf{g} + \mathbf{C}(L)\mathbf{u}_t
\]
(2.3)

where \(\mathbf{C}(L) = \sum_{j=0}^{\infty} \mathbf{C}_j(L^j)\), and \(L\) is the lag-operator. The parameters in \(\Pi, \Gamma_j\) and \(\mathbf{C}(L)\) are functions of the parameters of the model in (2.1) and the stationarity assumptions

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6 In the empirical work, we include survey expectations upto one year ahead and model the data with a 5-variable VAR.

7 It is worth emphasising that all the terms on the left-hand-side of (2.1) other than \(y_{t-1}\) are dated at \(t\) and that, for example, \(ty_{t+1} - y_t\) is a “quasi difference” since \(ty_{t+1} - y_t \neq \Delta ty_{t+1} (= ty_{t+1} - t^{-1}y_{t+1})\).

[5]
underlying that model translate into restrictions on the parameters of the cointegrating VAR and the moving average representation. Specifically, \( \Pi \) and \( C(1) = \sum_{i=0}^{\infty} C_i \) take the forms

\[
\Pi = \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22} \\
k_{31} & k_{32}
\end{bmatrix}
\begin{bmatrix}
1 & -1 & 0 \\
1 & 0 & -1
\end{bmatrix}, \quad \text{and} \quad
C(1) = \begin{bmatrix}
k_4 & k_5 & k_6 \\
k_4 & k_5 & k_6 \\
k_4 & k_5 & k_6
\end{bmatrix}
\]

(2.4)

for scalars \( k_{ij}, (i = 1, 2, 3, j = 1, 2) \), \( k_4 \), \( k_5 \) and \( k_6 \). All of these forms will provide an equivalent statistical characterisation of the data. They capture the potentially complicated dynamic interactions between the actual and expected output series but are restricted to reflect the underlying stationarity assumptions that ensure the series, while each growing according to a unit root process, are tied together over the long run.

2.1.1 The implications of particular information structures

In testing for the presence of particular information structures, we need to distinguish between the measures of expected output published in the surveys as discussed above and the RE forecasts of the variable based on all the available information. In what follows, we denote the full-information RE (FIRE) forecast with a ‘*’ superscript: e.g. \( t-1 y_t^* = E[y_t \mid - t-1] \), where \( E[.\] \) is the mathematical expectations operator and \(- t \) is the information available to all agents at time \( t \). Rationality implies that \( y_t = t-1 y_t^* + \epsilon_0 t \) and \( t y_t^* + 1 = t-1 y_t^* + \eta_1 t \) so that the FIRE errors reflect directly the news that becomes available at time \( t \), uncorrelated with information dated at \( t - 1 \) or earlier which is fully captured by the RE forecast.

If we assume expectations are formed rationally and there are no information rigidities, the survey responses will reflect FIRE forecasts for every respondent so that \( t-1 y_t^* = t-1 y_t^* \) and \( t-1 y_{t+1}^* = t-1 y_{t+1}^* \). The model in (2.1) can accommodate this assumption by imposing the restrictions that

\[
B_1 = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
* & * & *
\end{bmatrix} \quad \text{and} \quad
B_j = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
* & * & *
\end{bmatrix}
\]

(2.5)

for \( j = 2, ..., p - 1 \).
The $\varepsilon_{0t}$ and $\eta_{ht}$ defined in (2.1) and subject to the restrictions (2.5) are direct measures of the news on current and one-period-ahead-expected output as perceived by the FIRE survey respondents. In the absence of a direct measure of $ry_{t+3}$ in this illustrative example, no restrictions are imposed by the FIRE assumption on the third rows of the B matrices in (2.5) and the model can capture growth dynamics freely in this equation. However, the $6 \times (p-1)$ zero and unit restrictions imposed in the first two rows reflects the high degree of structure imposed on the system dynamics by the FIRE assumption.

When there is rationality with sticky information, it is typically assumed that agents update their information each period with probability $1 - \lambda$ so that the average forecast of $y_t$ published in a survey at $t-1$ consists of a weighted average of the RE forecasts over the past; i.e. $t-1y_{t+h} = (1 - \lambda)\sum_{j=0}^{\infty} \lambda^j t-1-jy_{t+h}$ for any $h \geq 0$. This structure implies that particularly simple relationships exist between survey-based expectational errors and survey revisions over time; namely,

\begin{equation}
(y_t - t-1 y_t^e) = \frac{\lambda}{1 - \lambda} (t-1y_t^e - t-2 y_t^e) + \varepsilon_{0t} \tag{2.6}
\end{equation}

and

\begin{equation}
(ty_{t+1}^e - t-1 y_{t+1}^e) = \frac{\lambda}{1 - \lambda} (t-1y_{t+1}^e - t-2 y_{t+1}^e) + \eta_{ht}. \tag{2.7}
\end{equation}

Although expectations are formed rationally, expectational errors and revisions in the surveys contain systematic content here because of the influence of the forecasters who have not updated their information. The restrictions implied by the sticky-information RE (SIRE) assumptions of (2.6) and (2.7) can again be readily accommodated within the model in (2.1), allowing for an additional estimated parameter to be estimated in the first two rows of the B matrices compared to the FIRE restrictions in (2.5). Or, indeed, a generalisation would be to estimate two distinct additional parameters compared to the FIRE specification on the grounds that forecasters might update information that is relevant for the one-step-ahead forecast more frequently than that relevant for two-step-ahead forecasts.

Expectational errors in surveys will also be found to contain systematic content in a noisy information model in which agents do not observe the variable of interest directly.

[7]
but observe a noisy indicator of the variable instead. This is because forecasters, knowing the signal they receive is imperfect, discount some part of the news that arrives on the variable in each period. If the variable of interest displays a degree of autocorrelation, this means that expectational errors and revisions are also related over time. Coibion and Gorodnichenko (2012) note that, in the particular case where there is a single variable of interest and this follows an AR1 process, the ‘rationality with noisy information’ (NIRE) model implies exactly the same restrictions as those implied by the SIRE model in (2.6) and (2.7). More generally though, where there is more than one variable under consideration and the autocorrelation pattern is more complicated, expectational errors and time-$t$ revisions in the expectations of each variable can depend on past revisions in all the variables involved. Of course, this can still be readily accommodated within the model of (2.1) although this is a much less restrictive model than the FIRE or SIRE models, allowing $2 \times (p - 1)$ parameters to be freely estimated in each of the first two rows of the $\mathbf{B}$ matrices.

[8]

Note that the restrictions implied by FIRE, SIRE or NIRE do not alter the interpretation of $\varepsilon^t_0$ and $\eta^t_1$ in (2.1) as being the news arriving at $t$ on output at $t$ and on expected output at $t + 1$. However, the imposition of the restrictions - assuming they are valid - could have a substantial impact on the measurement of the news in estimation. Empirically, the news content of the observed series is measured by the residuals from the VAR model but, in practice, these residuals reflect the parameter uncertainty arising in estimation as well as the arrival of new information on the variables. If the parameter estimates are unbiased, the residuals will still provide unbiased measures of the news. But the imposition of the restrictions implied by the specified information structures will reduce the measurement errors associated with parameter uncertainty if the structures are valid. This could be important in producing our ‘news-adjusted’ gap measures which rely on identifying survey respondents’ beliefs on the permanence of the effects of different parts of the news content on current and future outputs.
2.1.2 A non-linear extension to accommodate state-dependence

Coibion and Gorodnichenko (2012) find evidence to suggest that agents update information more quickly during recessions. A relatively simple generalisation of the linear VAR model of (2.1) that can accommodate state-dependencies of this sort is given by

\[
\Delta z_t = a + \Pi z_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + \tilde{\Pi} \left[ I_R(-t) \ast z_{t-1} \right] + \sum_{j=1}^{p-1} \tilde{\Gamma}_j \left[ I_R(-t) \ast \Delta z_{t-j} \right] + u_t ,
\]

where \( I_R(-t) \) is an indicator function taking the value 1 when the economy is in recession and zero otherwise, and recession is defined by the simultaneous occurrence of some specified events based on the available information which, here, includes all current and past values of \( z_t \). The specification in (2.8) allows for changes in regime so that, if information rigidities of the SIRE or NIRE form exist, the model can capture differential speeds of collecting information in ‘normal’ and ‘recessionary’ times through the parameters in \( \tilde{\Pi} \) and \( \tilde{\Gamma}_j \). Examples of the definition of recession that we might consider here include when output lies below its previous peak, say, or when output falls in two consecutive quarters. As we discuss below, the non-linearity in (2.8) introduces some complexity in the measurement of the BN trend and associated gaps. But the steady-state output measures and the measures of the news content of the series will be improved by working with this extended model if there is time-variation in the rate of information collection and, hence, in the model parameters.

2.2 Multivariate BN Trends

The time-\( t \) measure of the BN trend of a variable describes the infinite horizon forecast of the variable obtained at \( t \) having abstracted from deterministic growth; i.e., for an \( n \times 1 \) first-difference stationary vector process \( z_t \), the BN trends \( z_t \) are defined by

\[
z_t = \lim_{h \to \infty} E[z_{t+h} \mid \cdot, t] - g h
\]

where \( g \), the element of deterministic growth, is a vector of constants. In our context, the trends can be thought of as comprising the current observed value of the actual and
expected output series plus all forecastable future changes in these series, abstracting from
the dynamics of the paths taken to obtain these levels. While the BN trend is a statistical
concept, its forward-looking nature means that it matches closely with the economic idea
of the “steady-state” output level.

As Garratt, Robertson and Wright (2006) point out, any arbitrary partitioning of \( z_t \)
into permanent and transitory components, \( z_t = z^{P}_t + z^{T}_t \) will have the property that
the infinite horizon forecast of the transitory component is zero while the infinite horizon
forecast of any permanent component converges on the BN trend; i.e.

\[
\lim_{h \to \infty} E[z^{T}_{t+h} | - t] = 0 \quad \text{and} \quad \lim_{h \to \infty} E[z^{P}_{t+h} | - t] = \bar{z}_t.
\] (2.10)

Many of the various alternative measures of trends and cycles provided in the literature,
and below, effectively represent alternative methods of characterising the dynamic path
of the permanent component to the BN steady state therefore.\(^8\)

In the linear multivariate moving average representation of (2.3), the BN trend can be
expressed as

\[
\Delta z_t = g + C(1)u_t
\] (2.11)

so the trends are correlated random walks with the change in the trends reflecting the
accumulated future effects of the system shock \( u_t \). Given the structure of the \( C(1) \) in
(2.4) imposed by the initial stationarity assumptions on output growth and expectational
errors, (2.11) shows the steady-state value of all three series in \( z_t \) is the same, denoted \( \bar{z}_t \),
where \( \Delta \bar{z}_t = g + q_t \), driven by the single stochastic term

\[
q_t = k_4 \varepsilon_{0t} + k_5 \eta_{1t} + k_6 \eta_{2t}.
\] (2.12)

Empirically, the BN trend can be obtained analytically in this linear case using the resid-
uals and the parameters of \( C(1) \) from an estimated version of (2.1).

The definition of the BN trend in (2.9) is also applicable to the non-linear represen-
tation of (2.8) where the VAR is extended to accommodate potential state-dependence.
The trend’s measurement is not as straightforward as in the linear case however, given

\(^{8}\)See also Kiley’s (2013) discussion of alternative output gap concepts.
the difficulty in computing the infinite horizon forecasts in non-linear models. Here the
dynamic and ultimate effects of shocks depend on the initial output position, the size
of the shocks and other contingent factors so that the BN trend depends on the entire
evolution of all possible future output paths as well as past realisations. In the non-linear
case, the conditional expectation is evaluated by integrating over all of these potential
paths and this renders the BN trend analytically intractable. However, as noted by Clar-
ida and Taylor (2003), it can be obtained relatively easily through simulation, replacing
the conditional expectation with the mean of the $k$-step-ahead forecast obtained from $M$
simulated futures.\footnote{Here, $k$ is chosen to be sufficiently long for the forecast to settle to the deterministic trend so that
it approximates the infinite horizon outcome, and $M$ is chosen to be sufficiently large for the simulation
average to converge to the conditional expectation. We use $k = 50$ and $M = 1000$ in our empirical work
below.} In this, each simulated future accommodates the non-linear feedbacks
from output-outcome to recession-definition to model-specification to output-outcome and
so on so that, if the number of simulations is large enough, we obtain an explicit empirical
description of all the possible future paths that output could take.

### 2.3 A ‘News-Adjusted’ Output Gap Measure

The residuals from an estimated version of (2.1) or (2.8) - estimated unrestrictedly or
subject to NIRE, SIRE or FIRE restrictions - provide measures of the news arriving on
current, one-period-ahead-expected and two-period-ahead-expected output. But they also
implicitly provide information on the extent to which the news on current period output
are expected to persist or to be reversed and, in the latter case, whether the reversal
will be immediate or more prolonged. This could be important for policy makers as the
inflationary pressures signalled by a rise in the gap between current and steady-state
output levels are likely to prompt a more moderate responses if it is understood that the
rise is the outcome of a very short-lived event. This suggests using the residuals from the
model, which provide a direct insight on the ‘news’ on current output and the way that
translates to expected output over the coming quarters, to identify shocks whose effects
are more or less long-lived.
For the purpose of exposition, consider again the simple three variable linear system of (2.1), where we have direct measures of expected output at $t + 2$. Here, assuming there is a single permanent shock which has a persistent effect on output levels, we can identify two staged transitory shocks, namely: a shock that has a direct effect on output on impact only, $s_{0t}$; and a shock that effects output directly for at least one further period and possibly more, $s_{1t}$. News arriving at time $t$ on output at $t$ can be decomposed into the separate elements relating to the permanent shock and the two staged transitory shocks:

$$\varepsilon_{0t} = \beta_0 q_t + \lambda_0 s_{1t} + s_{0t}. \quad (2.13)$$

News arriving at $t$ on expected output in $t + 1$ reflects the effects of $q_t$ and $s_{1t}$ but excludes a direct effect from $s_{0t}$:

$$\eta_{1t} = \beta_1 q_t + s_{1t}. \quad (2.14)$$

By (2.12), news on expected output at $t + 2$ is defined by:

$$\eta_{2t} = -\frac{k_4}{k_6} \varepsilon_{0t} - \frac{k_5}{k_6} \eta_{1t} + \frac{1}{k_6} q_t, \quad (2.15)$$

deproviding three equations in the three shocks $s_{0t}$, $s_{1t}$ and $q_t$. Assuming the staged structural shocks are independent of each other, the $\beta$ and $\lambda$ coefficients can be estimated through simple regressions using the residuals from the estimated VECM model explaining $\Delta z_t$, (2.2), and the $s_{1t}$ and $s_{0t}$ are obtained as the residuals from the first two subsidiary regressions above (estimated in the reverse order to the way they are presented).

The relationships between the VECM residuals in $u_t$ and the structural shocks $w_t = (s_{0t}, s_{1t}, q_t)'$ are summarised by

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
k_4 & k_5 & k_6
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{0t} \\
\eta_{1t} \\
\eta_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
1 & \lambda_0 & \beta_0 \\
0 & 1 & \beta_1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
s_{0t} \\
s_{1t} \\
q_t
\end{bmatrix};
$$

that is

$$u_t = Qw_t.$$
where \( Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k_4 & k_5 & k_6 \end{bmatrix} \) and \( \mathbf{w}_t = (s_{0t}, s_{1t}, q_t)' \). Hence, we can rewrite (2.3) as

\[
\Delta z_t = \mathbf{g} + C(L)\mathbf{u}_t = \mathbf{g} + C(L)QQ^{-1}\mathbf{u}_t = \mathbf{g} + \tilde{C}(L)\mathbf{w}_t \tag{2.16}
\]

where \( \tilde{C}(L) = C(L)Q \). This is an alternative linear MA representation for \( \Delta z_t \) in which the shocks have a structural interpretation. Clearly, \( \tilde{C}(1)\mathbf{w}_t = C(1)Q\mathbf{w}_t = C(1)\mathbf{u}_t \) so that the output series are, of course, driven by the same single stochastic shock, \( q_t \), in the long run and the steady-state measure provided by the BN trend remains unchanged.

We have argued that policy might be best informed by a ‘news-adjusted’ output gap measure in which the effects of short-lived innovations are excluded from the gap. To see how this is obtained, note that, using (2.3) and (2.11), the deviation of output from its steady-state level \( \bar{\pi}_t = y_t - \bar{y}_t \) can be found from the first row of

\[
\mathbf{z}_t - \bar{\mathbf{z}}_t = \mathbf{C}^\dagger(L)\mathbf{u}_t = \tilde{\mathbf{C}}^\dagger(L)\mathbf{w}_t
\]

where \( \mathbf{C}^\dagger(L) = \sum_{j=0}^{\infty} \mathbf{C}^\dagger_j L^j \), \( \mathbf{C}^\dagger_j = -\sum_{i=j+1}^{\infty} \mathbf{C}_i \), and \( \tilde{\mathbf{C}}^\dagger(L) = \mathbf{C}^\dagger(L)Q \). In this illustrative example, there is only one short-lived shock, namely \( s_{0t} \), because \( s_{1t} \) is transitory but there is no limit on how long its effects impact on output. If we assume that the ‘news-adjusted’ trend output level should accommodate the influence of the short-lived staged disturbance, then it is defined by

\[
\tilde{y}_{0t} = \bar{y}_t + \tilde{\mathbf{C}}^\dagger_{11}(L)s_{0t} \tag{2.17}
\]

where the \( \tilde{\mathbf{C}}^\dagger_{ij} \) are the \( i, j^{th} \) elements of \( \tilde{\mathbf{C}}^\dagger \). The ‘news adjusted’ output gap, defined by the difference between the actual and news-adjusted levels of output, \( \tilde{x}_{0t} = y_t - \tilde{y}_{0t} \), will then be purged of the effects of the short-lived transitory disturbances. Clearly, if there are survey data on expectations on longer horizons (up to \( h \) say), then \( h-1 \) short-lived shocks can be identified and \( h-1 \) corresponding gap measures obtained purged of the effects of
the short-lived shocks which have progressively longer but time-limited effects on output. As noted previously, the choice of which of these to use depends on the decision-making context and any lag between decisions being made and taking effect.

This decomposition, and news-adjustment, can also be applied in the non-linear case accommodating state-dependence given in (2.8). Of course, given the non-linearities involved, the exact relationship of (2.12) no longer holds in this case, but a linear approximation can be obtained regressing $q_t$ on the observed residuals to obtain estimates of parameters corresponding to $k_4$, $k_5$ and $k_6$. Assuming the residuals from this approximate relationship are orthogonal to $s_{0t}$ and $s_{1t}$, the transitional shocks can still be identified from regressions of the form in (2.13) and (2.14). These can then be used to obtain a news-adjusted gap measure through (2.17) now taking into account that $\hat{C}_{11}^d(L)$ changes over time to accommodate the state-dependence built into (2.8).

3 Information Rigidities and News-Adjusted Output Gap Measures in the US

This section provides tests of the presence of information rigidities and estimates of the steady-state and news-adjusted output gap measures defined above based on US data over the period 1970q1-2014q1. The analysis is based on actual output series and on expected future output at the one-, two-, three- and four-period ahead horizons obtained from the Survey of Professional Forecasters; i.e. we use $z_t = (y_t, t_y_{t+1}^e, t_y_{t+2}^e, t_y_{t+3}^e, t_y_{t+4}^e)$ in a five-variable system corresponding to the illustrative model of (2.1)-(2.3) with the tests of information rigidities and news adjustments suitably modified. A description of the data, their sources and the transformations used are provided in the Data Appendix.

It is worth noting that considerable attention has been paid to the use of real-time data in the study of output gaps - see the discussion in Garratt et al. (2008) for example - and one advantage of the BN trend and the news-adjusted gaps discussed above is that the measures are expressed in terms of currently available data and are based on survey data which are real-time by nature. Having said this, in the empirical work below, we make use of the most recently published vintage of data to measure actual output, aligning the expectations series with this in a way that still maintains internal consistency with the SPF.
as explained in the Data Appendix. This means we can compare our derived measures with those found elsewhere in the literature. However, to check that this assumption does not influence the results, the analysis below was also carried out using the SPF respondents’ reported measure of contemporaneous output $\tau y_t$ as the measure of actual output. This alternative analysis gave very similar results to those using the final vintage measure, providing reassurance that the measures taken to maintain internal consistency were appropriate and that the results and gap measures are robust to our choice of measure of actual output.

Table 1 and Figure 1 illustrate the nature of the actual and expected data series. Table 1 shows the means of the actual and expected quarterly growths for the various forecast horizons are all very similar, at around 0.7% per quarter. The expectations series display considerable conservatism though, with the standard deviation of the one-quarter-ahead expected growth series half of that of actual output and with more conservatism shown as the forecast horizon grows. This conservatism is also highlighted by the relatively small range between minimum and maximum values of the expected series compared to the actual growth series and their relatively smooth evolution over time, as captured by the high autocorrelations. These features are illustrated in Figure 1 which demonstrates how the average growths calculated over longer horizons move together with the actual growths over time, but reflect also a tendency towards the mean as the survey respondents expect that the effects of shocks will be offset over the year ahead.

### 3.1 The Linear Multivariate VAR and Tests of Information Rigidities

The first part of our empirical work estimates our linear multivariate model of actual and expected outputs and tests for the presence of informational rigidities. The empirical counterpart of the VECM model in equation (2.2) was estimated for the five variables in $z_t$ with a lag order of two. The underlying assumptions that actual and expected outputs are difference-stationary but (pairwise) cointegrated with vector $(1, -1)'$ were tested and shown to hold.$^{10}$ The multivariate model is simple in form but is complex in the sense that $^{10}$Details of the tests on the order of integration for the variables and those for the choice of lag order in the VAR are available from the authors on request.
each of the equations of the system explaining the five terms in $\Delta z_t$ includes two lags of all five variables plus feedback from the five cointegrating vectors plus intercepts, making 80 parameters in total. In addition, in order to accommodate the events of the financial crisis and earlier extreme shocks to growth, we also experimented with the inclusion of dummy variables which take the value of unity in outlying observations (zero otherwise). These were identified as being those for which residuals from an unrestricted regression lie more than three standard deviations from zero. In the event, we included six dummies for the periods 1971q1, 1978q2, 1980q2, 1981q1, 1982q1 and 2008q4.\footnote{For consistency, the deterministic effect of the dummies are included in the estimated BN trend and their effect are taken into account in the second-stage regressions identifying the short-lived shocks.}

The model obtained in this way is able to capture sophisticated dynamic interactions and we do indeed find large and statistically significant feedbacks among the actual and expected future output measures, including statistically significant coefficients on the estimated (loading) coefficients on the cointegrating terms in each of the five equations in our VECM system.\footnote{Diagnostic statistics show the equations fit the data well, explaining between 54% and 39% of the variation in the actual and expected growth series, and that there are no serious problems of serial correlation, non-normality and heteroskedasticity in the residuals. Details available on request.} Figure 2 provides an illustration of the dynamic properties of the system, plotting the Generalised Impulse Response of the five series to a one standard error shock to actual output. These responses show the effect of the specified shock on impact, taking into account the shocks to the other variables that are typically observed at the same time, and the resultant dynamic adjustments. The figure shows that an unexpected increase in actual output is typically associated with the expectation of a further rise over the next quarters (as the survey responses experience a larger rise on impact) and that convergence of the various series to their common path takes some three or four years to work through. Further, the estimated model captures the relative conservatism in the expectations data: the responses show that the expectations series rise more slowly than the actual series over the first year following the shock, and that the increases in the actual output series observed over this time are ultimately partially offset as the actual series converges to the expected output series over the subsequent three years. This pattern gives some credence to the idea that the expectations series contain useful information.
on innovations to actual outputs which are known to be short-lived and whose effects are ultimately reversed.

Table 2 reports the results of the tests of the restrictions imposed on the VAR according to the structures implied by the FIRE, SIRE and NIRE assumptions as described in (2.5)-(2.7). Working with the survey data reporting expectations up to four periods ahead, we impose zero or unity restrictions on all of the parameters in the first four equations of our five variable VAR according to the FIRE assumption, representing 14 restrictions in each equation. The final column in Table 2 shows the F-test associated with these restrictions to be strongly rejected in every case.

In the absence of data on five-period-ahead survey forecasts, the SIRE assumptions translate to restrictions on the first three of the equations only. In this case, one lagged revision is included in each equation, meaning a single parameter is estimated in each, accommodating the possibility that the parameter differs across equations because agents update their information with different frequencies depending on the forecast horizon. The thirteen restrictions implied for each equation are again strongly rejected in every case. In contrast, the restrictions implied by the NIRE model for the same three equations appear to be much more consistent with the data. This model allows revisions in the expectations of all variables to enter into each equation, implying eight restrictions are imposed on each of the first three equations of our VAR. In this case, we find no evidence to reject at the 5% level in two equations and only a marginal rejection in the third. We take this to provide some support for the NIRE model, and the presence of informational rigidities and we focus on the trends and gap measures derived from the linear NIRE model in the next section.

3.2 News, Trends and the Output Gap from the Linear NIRE Model

The residuals from the linear VECM incorporating the NIRE restrictions can be used with the estimated model parameters to construct the permanent shocks ($q_t$) and the steady-state, BN output trend ($\bar{y}_t$) and associated output gap measure ($\pi_t$) as in (2.11)
and (2.12). The news arriving at time $t$ can also be decomposed to show the influence of the permanent shocks and, in the case of our five-variable linear VAR, four staged transitory shocks $s_{it} i = 0, \ldots, 3$. Here $s_{0t}$ is assumed to have an effect on output on impact only, $s_{1t}$ and $s_{2t}$ affect output for one and two further periods respectively, and $s_{3t}$ affects output for at least three further periods and possibly more. The identification of these transitory shocks is achieved through four regressions of the form in (2.13)-(2.15) and allows us to construct ‘news-adjusted’ output gap measures in which we purge the steady-state measure of the effect of the short-lived shocks as in (2.17). The news-adjusted series are denoted $\tilde{x}_{it}$ where $i = 0, 1, 2$ depending on how many of the short-lived shocks are taken into account.

Figure 3 plots the estimated short-lived shocks over the sample. The shocks mainly lie in the interval $\pm 0.4\%$ although there are some as large as $+0.8\%$ and $-0.6\%$. The shocks are strikingly smaller over the second half of the sample than over the first half reflecting the reduced output volatility during the Great Moderation. As noted earlier, the decision on which of the short-lived shocks should be purged from the gap depends on the policy decisions to be made and the length of any ‘policy lags’ involved. It is an empirical issue on which of $\tau_t$, $\tilde{x}_{0t}$, $\tilde{x}_{1t}$, or $\tilde{x}_{2t}$ might be more appropriate in any particular decision context therefore and Table 3 provides summary statistics for all the gap measures. The correlations between the steady-state gap $\tau_t$ and the adjusted gap measures $\tilde{x}_{0t}$, $\tilde{x}_{1t}$, and $\tilde{x}_{2t}$ are 0.96, 0.84 and 0.75 respectively, demonstrating a strong similarity between the measures but also reflecting the fact that the adjustments are reasonably large in places.

Table 3 also compares these NIRE steady state and news-adjusted gap measures with four other regularly-used gap measures: a gap based on marginal costs, $x_t^{MC}$; the measure produced by the Congressional Budget Office (CBO), $x_t^{CBO}$; a gap obtained using a simple linear trend, $x_t^{LT}$; and a gap obtained applying the Hodrick-Prescott (HP) smoother to

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13To be clear, these shocks, trends and gaps are based on a model which incorporates the NIRE restrictions and, in principle, one might use a superscript ‘NI’ to distinguish these measures from those that could be obtained from a model incorporating the FIRE restrictions, with superscript ‘FI’ say. Given that we focus only on the NI model in what follows, the superscript is omitted.

14Batini and Nelson’s (2001) review concludes that it takes over a year before monetary policy actions have an effect on inflation, for example, while fiscal policy lags are likely to be even more prolonged.
the output series, $x_i^{HP}$. A marginal cost measure of the gap has been shown by Gali and Gertler (1999) [GG], Gali, Gertler and Lopez-Salido (2001, 2005) [GGL] and others as being particularly relevant for capturing inflationary pressures.\(^{15}\) In this paper, as explained in the Data Appendix, we use the marginal cost measure suggested by McAdam and Willman (2013) which allows for capital–labor factor substitution and non-neutral technical change and which performs well in New Keynesian Phillips curve estimates explaining US inflation. The CBO series is the Office’s 2014q1 estimate of the maximum level of sustainable output achievable in each period based around a neoclassical production function and calculated levels of factor inputs (see CBO, 2001, for detail of the estimation methods employed). The gap based on the linear and HP trends are standard detrended measures found in the literature (the latter calculated using a smoothing parameter of 1600).

The summary statistics of Table 3, and the plots of Figure 4, show that, in terms of the means, standard deviations and minimum and maximum values of the series, the size of the four NIRE gap measures are broadly in line with the alternatives found in the literature.\(^{16}\) The plots show relatively persistent dynamics in the NIRE gaps, with first-order autocorrelation coefficients in the range 0.72-0.75, also broadly similar to the corresponding statistics for the other gaps in Table 3. This is an interesting finding that contrasts with gap estimates based on BN trends obtained in univariate exercises which typically find that much of the variation in output is variation in trend and that the gap is small and noisy. (See Morley et al., 2003, for further discussion.)

The table also shows there is a broad consensus on the size and timing of the cycles based on the three NIRE gap measures and on the marginal cost measure, with significantly positive correlations existing between these four measures and agreement on the

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\(^{15}\)These papers note that, under certain conditions on the form of nominal rigidities and the nature of capital accumulation, there is a proportional relationship between the natural output gap measure derived in a micro-founded DSGE model and the deviation of marginal cost from its steady-state and this latter can be well approximated by demeaned average unit labour costs.

\(^{16}\)The sample is restricted here to 1969q4-20087q4 which is the period for which we have McAdam and Willman (2013)’s carefully-constructed marginal cost gap measures. It means that the statistics relate to the ‘normal’ period before the financial crisis. This is true also for the analysis of inflation in the sub-section below.
sign of the gap significantly in excess of the 50% that would be achieved at random across all the measures in most cases. This is in stark contrast to the differences that exist between these five measures and the gap measures based on the statistical ‘smoothing’ algorithms underlying the linear trend, CBO and HP definitions of the gap. The correlations between $x_t^{LT}$, $x_t^{CBO}$ and $x_t^{HP}$ themselves are high, averaging at 0.77, and the agreement on the sign of these gap is in the region of 69%. But the correlations between the first group of gap measures and the latter group are mainly negative, some of them significantly so, and the proportion of times in which there is agreement on the sign of the gap is never significantly greater than 50%. The features are illustrated in Figure 4 which plot the $x_t$, $\bar{x}_t$ and $x_t^{MC}$ series to show the relatively strong similarities between the first group of gaps in Figure 4a, and then $\bar{x}_t$ and the gaps based on smoothing to show the difference between the two sets of gaps in in Figure 4b.

In summary, then, the proposed gap measures based on the (tested and accepted) NIRE model have reasonable statistical properties comparable to those of many gap measure found in the literature. Although based on a purely statistical analysis of the actual and expected output series, the gaps’ time series properties are quite distinct from those of other statistically-based series and are instead more closely related to the marginal cost gap measure which has been found previously to help in explaining inflation. Of course, both the NIRE gap measures (being based on the BN trend) and the marginal cost measure (accommodating cost pressures and changing utilisation rates) incorporate a forward-looking element on future output trends that make this more likely to capture inflationary pressures and we investigate this possibility below.

3.3 Measuring the Output Gap during Recessions

We have noted previously that the costs and benefits of collecting information (and hence the parameters of the model underlying our gap measures) may differ at different stages of the business cycle and the extended VAR model of (2.8) might be required to accommodate this state-dependence. To investigate this possibility, we repeated the exercise described above but using the extended form and defining recession as occurring when output falls
below its previous peak,

\[ I^{\text{PEAK}}_R(t) = \begin{cases} 
1 & \text{if } y_t < \max_{i=1,2,...}(y_{t-i}) \\
0 & \text{otherwise}
\end{cases} \]

or when output drops for two consecutive quarters,

\[ I^{\text{DROP}}_R(t) = \begin{cases} 
1 & \text{if } \Delta y_t < 0 \text{ and } \Delta y_{t-1} < 0 \\
0 & \text{otherwise}
\end{cases} \]

The results of the tests on information rigidities and for the presence of state-dependence are provided, for the below-previous-peak case, in Table 4. These show that, when the model is extended to accommodate the state-dependence, there is even stronger evidence to reject the SIRE restrictions than there was in the linear case, while the NIRE restrictions are more readily accepted than previously. If we impose the information structures (ignoring the evidence in the SIRE case), we find strong evidence for non-linearities in the SIRE case and some evidence on non-linearities in the NIRE case, with the tests in two equations just significant at the 5% level. These results again support the NIRE assumptions, then, and suggest that the extent of the information rigidities might change during recessions when output falls below its previous peak.\(^{17}\)

Figure 5 and Table 5 provide details of the steady-state and news-adjusted output gap measures obtained using the non-linear NIRE model, denoted \( \pi^N_t, \hat{z}^N_t, \hat{x}^N_{1t}, \) and \( \hat{x}^N_{2t} \). The results show that these measures have, for the most part, broadly similar characteristics to those obtained from the linear model.\(^{18}\) For example, the correlation between \( \pi_t \) and \( \pi^N_t \) is 0.85 and the measures based on the non-linear model are again closer in character to the marginal cost output gap measure \( x^M_t \) than to the measures based on smoothed trends, \( x^L_t, x^CBO_t \) and \( x^HP_t \). However, the measures based on the non-linear model do differ from those based on the linear model in an interesting and

\(^{17}\)Qualitatively similar results are obtained when recession is defined by two consecutive quarters of negative growth. Of course, this definition covers a subset of the observations defined as recession by the below-previous-peak definition.

\(^{18}\)Of course, the methods used in their computation are entirely different though, with the BN trend from the linear model obtained analytically and that from the non-linear model obtained through simulation.
potentially important way. Figure 5 illustrates this well, highlighting the various periods of recession experienced over the sample. The figure shows that, in both the linear and non-linear case, the gap measures based on the BN trend tend to be positive during the early stages of a below-previous-peak recession as output falls but by less than the forward-looking trend measure.\(^{19}\) Interestingly though, the positive gap is larger for the gaps based on the linear model than for those based on the non-linear model, especially during the recessions of the early 1980’s, the early 2000’s and the financial crisis. This is because the long-term consequences of a downturn are found to be smaller in the non-linear model than the linear model. If the non-linearity we have found in our analysis is due to more rapid information collection during recessions, this provides further support for the view that information rigidities play an important role in business cycle dynamics. Certainly the results suggest that the gap measures used by policy-makers should accommodate this possibility.

3.4 The Output Gap as a Forward Indicator of Inflation

One area in which output gap measures are frequently used is in explaining and forecasting inflation and an obvious first step in investigating the usefulness of our gap measures in capturing inflationary pressures is to consider some simple correlations between the measures and inflation at different leads and lags. In what follows, inflation is measured by the change in (the logarithm of) the GDP deflator, denoted \(\pi_t\), and the relationship between the various gap measures dated at \(t\) and inflation dated at \(t + k\), \(k = -8, ..., +8\), are illustrated through the dynamic cross-correlations provided in Figure 6.

As a point of reference, Figure 6(e) shows that the gap measure based on the linear trend \(x_t^{LT}\) - chosen to exemplify the properties of the gap measures based on smoothed trends - has neither contemporaneous nor any useful leading indicator properties for inflation (and indeed has negative correlations at \(k > 0\)). The marginal cost gap measure \(x_t^{MC}\) shown in Figure 6(d) is better, with a significantly positive correlation contemporaneously

\(^{19}\)As noted previously, this is in stark contrast to the gap measures based on the smoothed output trends which take large negative values at these times.
and for one quarter ahead, although the strong positive correlations with lagged inflation suggest the measure is more backward-looking than forward-looking. However, Figures 6(a)-6(c) show that the gap measures based on the BN trend would all usefully serve as a leading indicator for inflation. Figure 6(a) relates to $\bar{\tau}_t$, the steady-state gap from the linear NIRE model, and shows that this variable is significantly positively correlated with inflation contemporaneously and up to four quarters ahead. Figure 6(b) relates to $\tilde{x}_{0t}$, based on $\bar{\tau}_t$ but with one short-lived shock purged from the measure, and again shows strong positive correlations with current and future inflation, with the correlogram shifting a little to the right (i.e. showing positive correlations with inflation at longer horizons) compared to that for $\bar{\tau}_t$. And, finally, Figure 6(c) relates to $\tilde{x}_{0t}^{NL}$ based on the steady-state gap from the non-linear NIRE model and with the effects of one short-lived shock purged from this gap measure. This shows a still stronger set of positive correlations with future inflation, taking values in the region of 0.35 contemporaneously and one quarter ahead and being significant up to six quarters ahead.

4 Conclusions

The recent interest in the role of information rigidities in macrodynamics has focused attention once more on the way in which beliefs and expectations are formed and the importance of ensuring that macropolicy works with, and takes advantage of, agents’ use of information. The tests on the time series properties of the US actual output and expected output data described in the paper provides no evidence with which to reject rationality in expectation formation but acknowledges that there may be systematic content in the expectational errors found in the survey data due to agents’ interpretation of noisy information. The results also suggest that agents’ use of information may change according to the economy’s position in the business cycle. This means that care needs to be taken to measure the news content contained in the actual and expected series by applying the appropriate information structure in our multivariate VAR model of the output series. Having done this, we have suggested a procedure with which to decompose

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20 With $n = 153$, statistical significance at the 5% level is obtained with a correlation in excess of ±0.15.

[23]
the innovations in the output series in a way that reflects forecasters’ beliefs on how short-lived or long-lived different influences will be. The ‘news-adjusted output gap’ measures that we derive for the US using this decomposition, which abstract from the effects of the most short-lived influences, have similar time series properties to gap measures based on estimates of firms’ real marginal costs even though they are based on a relatively simple time series representation of the output series alone (compared to the more data-intensive and complicated structural modelling underlying the marginal cost measure of McAdam and Willman (2013), for example). Most importantly, the news-adjusted gap measures, and particularly those based on the extended non-linear model, serve as robust and informative leading indicators for inflationary pressures. The gap measures therefore provide useful tools which have a straightforward economic interpretation, which can be estimated easily and which can be readily applied to formulate policy which does indeed work with, and take advantage of, agents’ use of information.
Appendix: Alternative Statistical Representations for Actual and Expected Output

If actual output growth and expectational errors are both stationary, we can write the Wold representation for actual and expected growth as

\[
\begin{bmatrix}
  y_t - y_{t-1} \\
  t^2y_{t+1} - y_t \\
  t^2y_{t+2} - t^2y_{t+1}
\end{bmatrix} =
\begin{bmatrix}
  \alpha_0 \\
  \alpha_1 \\
  \alpha_2
\end{bmatrix} + A(L)
\begin{bmatrix}
  \varepsilon_{0t} \\
  \varepsilon_{1t} \\
  \varepsilon_{2t}
\end{bmatrix} \quad (4.18)
\]

where \( \alpha = (\alpha_0, \alpha_1, \alpha_2)' \) are the mean growth rates of the actual and expected output series, and the series are driven by \( v_t = (\varepsilon_{0t}, \varepsilon_{1t}, \varepsilon_{2t})' \), a vector of mean zero, stationary innovations, with non-singular covariance matrix \( \Psi = (\psi_{jk}), j, k = 1, 2, 3 \). Assuming \( A^{-1}(L) \) can be approximated by the lag polynomial \( A^{-1}(L) = I + B_1L + .. + B_{p-1}L^{p-1} \), with \( B_0 = A^{-1}(1)\alpha \), we obtain the AR representation given by (2.1) in the text. Hence

\[
\begin{bmatrix}
  y_t \\
  t^2y_{t+1} \\
  t^2y_{t+2}
\end{bmatrix} =
\begin{bmatrix}
  \alpha_0 \\
  \alpha_1 \\
  \alpha_2
\end{bmatrix} + A_1
\begin{bmatrix}
  y_{t-1} \\
  t_{-1}y_{t-1} \\
  t_{-1}y_{t+1}
\end{bmatrix} + A_2
\begin{bmatrix}
  y_{t-2} \\
  t_{-2}y_{t-1} \\
  t_{-2}y_{t+1}
\end{bmatrix} + ...
\]

\[
\begin{bmatrix}
  \varepsilon_{0t} \\
  \varepsilon_{1t} \\
  \varepsilon_{2t}
\end{bmatrix},
\]

where \( a = M_0^{-1}B_0, \Phi_j = M_0^{-1}M_j, j = 1, ..., p, \) and

\[
M_0 = \begin{bmatrix}
1 & 0 & 0 \\
-1 & 1 & 0 \\
0 & -1 & 1
\end{bmatrix}, \quad M_p = B_{p-1} \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

and \( M_j = B_{j-1} \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} - B_j \begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 1 \\
0 & 0 & 0
\end{bmatrix} \) for \( j = 1, ..., p - 1 \). The error terms \( u_t = (\varepsilon_{0t}, \eta_{1t}, \eta_{2t})' \) are defined by \( u_t = M_0^{-1}v_t = (\varepsilon_{0t}, \varepsilon_{0t} + \varepsilon_{1t}, \varepsilon_{0t} + \varepsilon_{1t} + \varepsilon_{2t})' \), and the covariance matrix is denoted \( - = (\sigma_{jk}), j, k = 1, 2, 3 \). The \( \varepsilon_{0t} \) is “news on output level in time \( t \) becoming available at time \( t' \), equivalent to

[25]
news on output growth given that $y_{t-1}$ is known, while $\eta_{ht}$ is the “news on the level of output expected in time $t+h$ becoming available at time $t$”. This incorporates news on output levels at $t$ and on growth expected over the coming period ($\eta_{ht} = \varepsilon_{0t} + \sum_{j=1}^{h} \varepsilon_{jt}$).

Expression (4.19) can be written

$$z_t = g + \Phi_1 z_{t-1} + \Phi_2 z_{t-2} + \ldots + \Phi_p z_{t-p} + u_t$$

(4.19)

where $z_t = (y_t, t\eta_{t+1}, t\eta_{t+2})'$ and this can also provide the VECM representation

$$\Delta z_t = a + \Pi z_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + u_t,$$

(4.20)

where $\Phi_1 = \mathbf{I}_2 + \Pi + \Gamma_1$, $\Phi_i = \Gamma_i - \Gamma_{i-1}$, $i = 2, 3, \ldots, p - 1$, and $\Phi_p = -\Gamma_{p-1}$. Given the form of the $\Phi_i$ described in (4.19), it is easily shown that $\Pi$ takes the form

$$\Pi = \begin{bmatrix} k_{11} + k_{12} & -k_{11} & -k_{12} \\ k_{21} + k_{22} & -k_{21} & -k_{22} \\ k_{31} + k_{32} & -k_{31} & -k_{32} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix},$$

(4.21)

where $k_{ij}, i = 1, 2, 3$ $j = 1, 2$ are scalars dependent on the elements of the $B_j$, $j = 0, 1, \ldots, p - 1$. This form captures the fact that actual and expected output cannot diverge indefinitely by assumption and is incorporated through the inclusion of the disequilibrium terms $y_{t-1} - t-1\eta_t^e$ and $y_{t-1} - t-1\eta_{t+1}^e$ in each of the equations in (4.20).

Alternatively, through recursive substitution of (4.19), we can obtain the moving-average form given by

$$\Delta z_t = g + C(L)u_t,$$

(4.22)

where $C(L) = \sum_{j=0}^{\infty} C_j L^j$, $C_0 = \mathbf{I}$, $C_1 = \Phi_1 - \mathbf{I}_n$, and $C_i = \sum_{j=1}^{i} \Phi_j C_{i-j}$. The presence of the cointegrating relationships between the $y_t$, $t-1\eta_t^e$ and $t-1\eta_{t+1}^e$ imposes restrictions on the parameters of $C(L)$; namely, $\beta'C(1)=0$, as shown in Engle and Granger (1987). Given the form of $\beta'$ in (4.20), $C(1)$ takes the form

$$C(1) = \begin{bmatrix} k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \\ k_4 & k_5 & k_6 \end{bmatrix}$$

(4.22)

for scalars $k_4$, $k_5$ and $k_6$. The BN trend defined by (2.11) shows the steady-state value of all three series in $z_t$ is the same and driven by the stochastic trend $k_4 \varepsilon_{0t} + k_5 \eta_{1t} + k_6 \eta_{2t}$. 

[26]
Data Appendix

The sources and transformations for the data are as follows:

$y_t$: the natural logarithm of US real GDP. Source: St Louis Federal Reserve Economic Database [FRED].

$\tau y_{t+h}, \ h = 1, 2, 3$ and $4$: the natural logarithm of expected $h$ quarter ahead US real GDP reported at time $t$. The series used in the estimation is defined as $\tau y_{t+h} = g_t^h + y_t$ where $g_t^h$ is expected output growth reported in the SPF at $t$, based on expected output in $t + h$ relative to the real-time “nowcast” of current output, $\tau y_{t+h}^{SPF} - \tau y_{t}^{SPF}$. The reported growth is the mean of the survey respondents’ growth expectations as reported by the Philadelphia Fed. Source: Survey of Professional Forecasters at the Philadelphia Fed’s Real Time Data Centre website.

$\pi_t$: inflation, defined as: $400 * (p_t/p_{t-1})$ where $p_t$ is the natural logarithm of the US GDP Price Deflator. Source: FRED.

Table 1: Actual and Expected Output Growths:  
Summary Statistics 1970q1 – 2014q1

<table>
<thead>
<tr>
<th></th>
<th>$y_t - y_{t-1}$</th>
<th>$t\bar{y}_{t+1} - y_t$</th>
<th>$t\bar{y}<em>{t+2} - t\bar{y}</em>{t+1}$</th>
<th>$t\bar{y}<em>{t+3} - t\bar{y}</em>{t+2}$</th>
<th>$t\bar{y}<em>{t+4} - t\bar{y}</em>{t+3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>.0068</td>
<td>.0063</td>
<td>.0069</td>
<td>.0072</td>
<td>.0075</td>
</tr>
<tr>
<td>SD</td>
<td>.0083</td>
<td>.0043</td>
<td>.0033</td>
<td>.0024</td>
<td>.0024</td>
</tr>
<tr>
<td>Min</td>
<td>-.0217</td>
<td>-.0096</td>
<td>-.0060</td>
<td>.0004</td>
<td>.0006</td>
</tr>
<tr>
<td>Max</td>
<td>.0382</td>
<td>.0153</td>
<td>.0155</td>
<td>.0136</td>
<td>.0141</td>
</tr>
<tr>
<td>AR1</td>
<td>.3264</td>
<td>.7920</td>
<td>.8131</td>
<td>.8174</td>
<td>.7352</td>
</tr>
</tbody>
</table>

Notes: The measures relate to actual output growth and expected future output growth at horizons $t + 1$, $t + 2$, $t + 3$ and $t + 4$. Summary statistics refer to the mean, standard deviation, minimum and maximum values, and the first-order serial correlation coefficient respectively.
Table 2: Tests of Information Rigidities in the Linear Model

<table>
<thead>
<tr>
<th></th>
<th>NI</th>
<th>SI</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ(y_t)</td>
<td>2.059*</td>
<td>2.117*</td>
<td>1.965*</td>
</tr>
<tr>
<td>Δ(y_{t+1})</td>
<td>1.924 (0.060)</td>
<td>8.586* (0.000)</td>
<td>2.097* (0.015)</td>
</tr>
<tr>
<td>Δ(y_{t+2})</td>
<td>1.413 (0.195)</td>
<td>10.871* (0.000)</td>
<td>2.228* (0.009)</td>
</tr>
<tr>
<td>Δ(y_{t+3})</td>
<td>-</td>
<td>-</td>
<td>3.183* (0.000)</td>
</tr>
<tr>
<td>Δ(y_{t+4})</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table reports \(F\)-statistics for restrictions imposed on the five equations of our multivariate VAR under the noisy-information-RE (NI), the sticky-information-RE (SI) and the full-information-RE (FI) hypotheses, with the number of restrictions tested in each equation being equal to 8, 13 and 14, respectively. The statistics in parentheses denote \(p\)-values and ‘*’ indicates significance at 5% level.
Table 3: Output Gap Measures: 1969q4 – 2011q4

<table>
<thead>
<tr>
<th></th>
<th>$\pi_t$</th>
<th>$\tilde{x}_{0t}$</th>
<th>$\tilde{x}_{1t}$</th>
<th>$\tilde{x}_{2t}$</th>
<th>$x_{t}^{MC}$</th>
<th>$x_{t}^{LT}$</th>
<th>$x_{t}^{CBO}$</th>
<th>$x_{t}^{HP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.003</td>
<td>-0.002</td>
<td>0.015</td>
<td>0.004</td>
<td>-0.013</td>
<td>0.007</td>
</tr>
<tr>
<td>SD</td>
<td>0.014</td>
<td>0.016</td>
<td>0.016</td>
<td>0.018</td>
<td>0.023</td>
<td>0.035</td>
<td>0.026</td>
<td>0.016</td>
</tr>
<tr>
<td>Min</td>
<td>-0.042</td>
<td>-0.041</td>
<td>-0.039</td>
<td>-0.038</td>
<td>-0.021</td>
<td>-0.077</td>
<td>-0.083</td>
<td>-0.040</td>
</tr>
<tr>
<td>Max</td>
<td>0.051</td>
<td>0.055</td>
<td>0.054</td>
<td>0.059</td>
<td>0.097</td>
<td>0.075</td>
<td>0.042</td>
<td>0.046</td>
</tr>
<tr>
<td>AR1</td>
<td>0.72</td>
<td>0.75</td>
<td>0.73</td>
<td>0.71</td>
<td>0.83</td>
<td>0.97</td>
<td>0.95</td>
<td>0.88</td>
</tr>
</tbody>
</table>

| $\pi_t$  | 1       | 0.96            | 0.84             | 0.75             | 0.37         | -0.07        | -0.16        | -0.01        |
| $\tilde{x}_{0t}$ | 84.6%   | 1               | 0.89             | 0.82             | 0.32         | -0.04        | -0.08        | 0.07         |
| $\tilde{x}_{1t}$ | 84.0%   | 87.6%           | 1                | 0.87             | 0.19         | -0.02        | -0.03        | 0.09         |
| $\tilde{x}_{2t}$ | 78.7%   | 82.2%           | 87.6%            | 1                | 0.21         | -0.07        | -0.10        | 0.02         |
| $x_{t}^{MC}$ | 59.7%   | 56.2%           | 54.4%            | 55.0%            | 1            | -0.37        | -0.38        | -0.47        |
| $x_{t}^{LT}$ | 50.2%   | 50.3%           | 49.7%            | 43.1%            | 46.7%        | 1            | 0.83         | 0.67         |
| $x_{t}^{CBO}$ | 46.2%   | 49.7%           | 51.5%            | 49.7%            | 34.3%        | 63.9%        | 1            | 0.82         |
| $x_{t}^{HP}$ | 46.7%   | 47.9%           | 48.5%            | 45.6%            | 51.5%        | 79.9%        | 62.7%        | 1            |

Notes: The output gaps measures are: the steady state gap ($\pi_t$), the gap adjusted for instantaneous news ($\tilde{x}_{0t}$), the gap adjusted for instantaneous and one-period ahead news ($\tilde{x}_{1t}$), and the gap adjusted for instantaneous and two-period ahead news ($\tilde{x}_{2t}$) are all based on the linear noisy information RE model. The other gap measures are the marginal cost gap ($x_{t}^{MC}$), linear trend gap ($x_{t}^{LT}$), Congressional Budget Office gap ($x_{t}^{CBO}$) and Hodrick-Prescott gap ($x_{t}^{HP}$). Summary statistics in the upper panel refer to the mean, standard deviation, minimum and maximum values respectively. Figures in the lower panel refer to correlation coefficients and, in italics, the percentage of the sample for which there is agreement that the output gap is positive or negative.
Table 4: Tests of Previous-Peak Recession Effects and Information Rigidities in the Non-Linear Model

<table>
<thead>
<tr>
<th>Information Rigidities</th>
<th>Recession Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NI</strong></td>
<td><strong>SI</strong></td>
</tr>
<tr>
<td>$\Delta y_t$</td>
<td>1.412</td>
</tr>
<tr>
<td></td>
<td>(0.196)</td>
</tr>
<tr>
<td>$\Delta y_{t+1}^e$</td>
<td>1.450</td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
</tr>
<tr>
<td>$\Delta y_{t+2}^e$</td>
<td>1.147</td>
</tr>
<tr>
<td></td>
<td>(0.335)</td>
</tr>
</tbody>
</table>

Notes: Under ‘Information Rigidities’, the table reports F-statistics for restrictions imposed on three of the equations of our multivariate VAR under the noisy-information-RE (NI) and the sticky-information-RE (SI) hypotheses, with the number of restrictions tested in each equation being equal to 8 and 13, respectively. Under ‘Recession Effects’, the table reports F-statistics for tests on the non-linear terms in the equations, assuming the information structures are valid, with the number of restrictions tested in each equation being equal to 3 and 1, respectively. The statistics in parentheses denote p-values and ‘*’ indicates significance at 5% level.
### Table 5: Further Output Gap Measures: 1969q4 – 2011q4

<table>
<thead>
<tr>
<th></th>
<th>$\bar{x}_t$</th>
<th>$\bar{x}_{0t}$</th>
<th>$\bar{x}^{NL}_t$</th>
<th>$\tilde{x}^{NL}_{0t}$</th>
<th>$\tilde{x}^{NL}_{1t}$</th>
<th>$\tilde{x}^{NL}_{2t}$</th>
<th>$x^MC_t$</th>
<th>$x^{LT}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.025</td>
<td>0.015</td>
<td>0.004</td>
</tr>
<tr>
<td>SD</td>
<td>0.014</td>
<td>0.016</td>
<td>0.013</td>
<td>0.013</td>
<td>0.014</td>
<td>0.022</td>
<td>0.023</td>
<td>0.035</td>
</tr>
<tr>
<td>Min</td>
<td>-0.042</td>
<td>-0.041</td>
<td>-0.031</td>
<td>-0.034</td>
<td>-0.029</td>
<td>-0.020</td>
<td>-0.021</td>
<td>-0.077</td>
</tr>
<tr>
<td>Max</td>
<td>0.051</td>
<td>0.055</td>
<td>0.037</td>
<td>0.040</td>
<td>0.043</td>
<td>0.093</td>
<td>0.097</td>
<td>0.075</td>
</tr>
<tr>
<td>AR1</td>
<td>0.72</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.73</td>
<td>0.80</td>
<td>0.83</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 2. The output gaps measures $\bar{x}^{NL}_t$, $\bar{x}^{NL}_{0t}$, $\bar{x}^{NL}_{1t}$ and $\bar{x}^{NL}_{2t}$ are based on the non-linear noisy information RE model.
Figure 1: Output Growth and Average Expected Output Growth

Figure 2: Generalised Impulse Responses for Output and Expected Output
Figure 3: Time Limited Transitory Shocks

-0.008
-0.006
-0.004
-0.002
0
0.002
0.004
0.006
0.008
0.01
s0
s1
s2

Figure 4a: News-adjusted NI and Steady State NI Output Gaps versus Marginal Cost

Figure 4b: News-adjusted NI versus Linear trend and CBO Output Gaps
Figure 5: Linear versus Non-Linear Steady States NI Gaps
Figure 6: Dynamic Cross Correlations

Figure 6a: Steady State NI Output Gap (t), Inflation (t+k)

Figure 6b: News Adjusted NI Output Gap (t), Inflation (t+k)

Figure 6c: Non-Linear News Adjusted NI Output Gap (t), Inflation (t+k)

Figure 6d: Marginal Cost (t), Inflation (t+k)

Figure 6e: Linear Trend Output Gap (t), Inflation (t+k)
References


