Theoretical analysis of a membrane-based cross-flow liquid desiccant system

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Abstract: Liquid desiccant air dehumidification has become one of the most widely used dehumidification technologies with advantages of high efficiency, no liquid condensate droplets and capability of energy storage. In this paper a cross-flow mathematical model is developed for a single layer membrane unit. The governing equations are solved iteratively by finite difference method. The performance analysis is carried out for a small-scale membrane-based dehumidification module consisting of 8 air channels and 8 solution channels. The influences of main design parameters on system effectiveness are evaluated. These include air flow rate (NTU), solution to air mass flow rate ratio (m*) and solution inlet temperature and concentration. It is revealed that higher sensible and latent effectiveness can be achieved with larger NTU and m*. Increasing solution concentration can also improve the dehumidification effect.

Keywords: Dehumidification, Liquid desiccant, Membrane-based

1 Introduction

The liquid desiccant dehumidification has advantage in terms of low grade thermal energy input for solving energy crisis problem and providing precise internal environmental control [1]. For example, solar energy and waste heat can be adopted. By avoiding condensation in solid surface, liquid desiccant also help prevent epidemic respiratory disease and improve indoor air quality. By avoiding condensation in solid surface, liquid desiccant system could prevent epidemic respiratory disease and improve indoor air quality [2]. The system has high dehumidification capability and controls the environment without extra energy consumptions in ‘overcooling’ and ‘re-heating’ [3]. In conventional direct-contact system, two packed beds are employed as a dehumidifier and a regenerator. This configuration brings a problem of carry-over where some desiccant solution droplets are carried out by the process air, which causes corrosion in the ducting system [4]. To overcome the carry-over problem, the selectively permeable membranes are used to the liquid desiccant dehumidification to form a new kind liquid desiccant dehumidification system so called membrane-based liquid desiccant dehumidification system [5]. Membrane mainly acts as a selective barrier to prevent cross-over of contaminants and liquids.

Many studies on membrane based desiccant dehumidification system have been carried out. The common study method is experimental analysis since it reflects real performance of the system. The experimental method has disadvantages in high cost, time consuming and difficult parameter control. In order to gain comprehensive understanding of the system performance, some mathematical models are developed, which have advantages in quickly providing analysis results and assessing different parameter effects.

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In this study, a three-dimensional steady-state mathematical model for a cross flow flat plat membrane-based dehumidifier is developed and the performance of the system is investigated under different operating conditions through simulation. The influences of main parameters, such as air temperature and humidity, solution temperature and concentration, air to liquid ratio, number of heat transfer unit (NTU) etc. are investigated. The established model can be used for the system performance prediction and optimization in future.

2 Method
This study aims to numerically analyze the system performance under various operating conditions. Air and desiccant solution flow alternatively in a flat-plate membrane-based unit in cross flow configuration. The air and solution channels are separated by porous hydrophilic membrane, which allows vapour diffusion but prevents solution carry-over. The numerical model is developed based on Fan’s model [6] [7]. NTU method is used to analyze the system performance under various operating conditions. Infinite differential method is applied to solve governing equations of heat and mass balances. The mathematical model is coded in Matlab for simulation.

2.1 Mathematical model
The schematic of three-dimensional mathematical model is shown in Figure 1. Each unit consists of one air channel and two solution channels. The air and solution channels are separated by a semi-permeable membrane which allows heat and vapour transfer but prevents liquid penetration. The geometry properties of the membranes are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Geometric properties of the unit</th>
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<tbody>
<tr>
<td>Unit length L (m)</td>
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<tr>
<td>Unit height H (m)</td>
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<tr>
<td>Air channel thickness (m)</td>
</tr>
<tr>
<td>Solution channel thickness (m)</td>
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</tbody>
</table>

Figure 1 Geometry of cross-flow liquid desiccant unit
Other main assumptions made in mathematical modelling are concluded below:
1. Heat and mass transfer only occur normal to the flow direction (i.e. in z direction) and axial diffusion is neglected [8].
2. Both air and solution streams are assumed to be laminar since in most practical applications Reynolds numbers are usually much less than 2300 [9].
3. Both fluid flows are Newtonian with constant thermophysical properties (density, thermal conductivity, viscosity and specific heat capacity).
4. Both fluids are fully developed and the entrance effects are neglected, but developing both thermally and in concentration.
5. The heat released during vapour condensation is added to the solution side.

2.2 Governing equations
The normalized governing mass and energy balance equations for the solution side are

\[
\frac{dx_{sol}}{dy^*} - (W_{sol,i} - W_{air,i})NTU_m m^*(1 + X_{sol})(\varphi_{air} - \varphi_{sol,mem}) = 0
\] (1)

\[
\frac{d\theta_{sol}}{dy^*} - NTU_m \times H^*(\varphi_{air} - \varphi_{sol,mem}) - NTU \times C^*(\theta_{air} - \theta_{sol,mem}) = 0
\] (2)

The normalized governing mass and energy balance equations for the air side are

\[
\frac{d\varphi_{air}}{dx^*} + 2NTU_m (\varphi_{air} - \varphi_{sol,mem}) = 0
\] (3)

\[
\frac{d\theta_{air}}{dx^*} + 2NTU(\theta_{air} - \theta_{sol,mem}) = 0
\] (4)

The normalized governing mass and energy balance equations for the membrane are

\[
NTU_m (W_{sol,i} - W_{air,i})(\varphi_{air} - \varphi_{sol,mem}) = NTU_{m,mem}\left(\frac{1}{1 + X_{sol}} - \frac{1}{1 + X_{sol,mem}}\right)
\] (5)

\[
NTU (\theta_{air} - \theta_{sol,mem}) + NTU_m H^*(\varphi_{air} - \varphi_{sol,mem}) = NTU_{sol}(\theta_{sol,mem} - \theta_{sol})
\] (6)

Where \(\text{air, sol, mem}\) represent air side, solution side and membrane respectively. \(i\) represents the inlet of air or solution channel. \(W\) is the specific humidity (kg/kg);
\n\(X_{sol}\) is the mass ratio between water and desiccant, which is given by:

\[
X_{sol} = \frac{m_{water}}{m_{water}}
\] (7)

In order to simplify governing equations for simplification, several dimensionless parameters are introduced as follows:

Dimensionless humidity content and temperature:

\[
\varphi = \frac{W - W_{air,i}}{W_{sol,i} - W_{air,i}}, \theta = \frac{T - T_{air,i}}{T_{sol,i} - T_{air,i}}
\] (8)

Dimensionless length:

\[
y^* = \frac{y}{l}, x^* = \frac{x}{w}
\] (9)

Where \(l\) and \(w\) are the length and width of contractor respectively.

The mass flow rate ratio and thermal capacity ratio:

\[
m^* = \frac{\dot{m}_{air}}{\dot{m}_{sol}}, C^* = \frac{(mc_p)_{air}}{(mc_p)_{sol}}
\] (10)
The number of heat transfer unit:

\[ NTU = \frac{UA}{m_{air}c_{p,air}}, NTU_{sol} = \frac{h_{sol}A}{m_{air}c_{p,air}} \]  

(11)

The number of mass transfer unit:

\[ NTU_{m} = \frac{U_{m}A}{m_{air}}, NTU_{m,sol} = \frac{h_{m,sol}A}{m_{air}} \]  

(12)

Where \( U \) (W/m\(^2\)K) is the overall heat transfer coefficient defined by:

\[ U = \left( \frac{1}{h_{air}} + \frac{\delta}{k_{mem}} + \frac{1}{h_{sol}} \right)^{-1} \]  

(13)

\( U_{m} \) (kg/m\(^2\)s) is the overall mass transfer coefficient defined by:

\[ U_{m} = \left( \frac{1}{h_{m,air}} + \frac{\delta}{k_{m,mem}} \right)^{-1} \]  

(14)

\( A \) (m\(^2\)) is the membrane contract area. \( H^{*} \) is the operating factor defined as the ratio between the latent energy difference and sensible energy difference between the air and desiccant solution at the inlet of exchanger, which can be calculated at by:

\[ H^{*} = \frac{W_{sol,i}-W_{air,i}}{T_{sol,i}-T_{air,i}} \frac{h_{fg}}{c_{p,air}} \]  

(15)

Where \( h_{fg} \) (J/kg) is the condensation heat of water.

2.3 Boundary conditions

To solve the above differential equations, boundary conditions are defined. In this case, the inlet condition of cross flow air and solution are:

\[ X_{sol}(y = 0) = X_{sol,i}, T_{sol}(y = 0) = T_{sol,i} \]

\[ W_{air}(x = 0.35) = W_{air,i}, T_{air}(x = 0.35) = T_{air,i} \]

2.4 Discretization and solving procedure

The first-order forward finite difference discretization method is used. The membrane surface is discretized into m*n grids, each grid is a control volume governed by normalized equations. The schematic of discretion is given in Figure 2.
The forward difference scheme is used to obtain information from boundary conditions.

\[
\left( \frac{\partial \emptyset}{\partial x} \right)_{N,M} = \frac{\emptyset_{N+1,M} - \emptyset_{N,M}}{\Delta x}, \quad \left( \frac{\partial \emptyset}{\partial y} \right)_{N,M} = \frac{\emptyset_{N+1,M} - \emptyset_{N,M}}{\Delta y}
\]  

(16)

The algorithm for the dehumidifier used in MatLAB is given in Figure 3.

3 Results and discussion

The performance analysis is based on various operating conditions by adopting control variable method. The benchmark testing conditions are defined for simulation process as inlet air temperature of 30°C, specific humidity of 0.01728 kg/kg and inlet solution temperature of 16 °C and solution concentration of 35%. The system dehumidification performance is assessed by the sensitive, latent and total effectiveness, which can be calculated by [10]:

\[
\varepsilon_{\text{sen}} = \frac{T_{\text{air},\text{in}} - T_{\text{air},\text{out}}}{T_{\text{air},\text{in}} - T_{\text{sol},\text{in}}}
\]

(17)

\[
\varepsilon_{\text{lat}} = \frac{W_{\text{air},\text{in}} - W_{\text{air},\text{out}}}{W_{\text{air},\text{in}} - W_{\text{sol},\text{in}}}
\]

(18)

\[
\varepsilon_{\text{tot}} = \frac{\varepsilon_{\text{sen}} + H^*\varepsilon_{\text{lat}}}{1 + H^*}
\]

(19)

3.1 Air flow rate (NTU)

The number of heat transfer unit (NTU) is considered as one controllable variable to study the influence on thermal performance of this unit and predict performance in further application. In this research, the NTU is modified by changing the air mass flow rate, while changing the solution mass flow rate simultaneously to keep the constant mass flow rate ratio.

The influence of NTU on sensible, latent and total effectiveness at \(m^*=0.5\) are given in Figure 4. When the NTU increases from 1 to 3, a strong correlation between the effectiveness and NTU is illustrated. The gradient of change decreases when NTU exceeds 3, and is negligible when NTU rises over 7. This trend reveals that at high NTU the performance improvement is no longer limited by NTU. When NTU

![Figure 4 Effectiveness vs. NTU](image)

![Figure 5 Effectiveness vs. m*](image)
exceeds 7, the sensible effectiveness is approaching 1, which means the membrane layer has little impact on heat transfer. With refer to mass transfer, Figure 4 shows a lower latent effectiveness, which is approximately 93%. Theoretically, the latent effectiveness significantly depends on membrane vapour diffusion resistance, and the latent effectiveness can be raised by reducing vapour diffusion resistance [11]. However, porous membrane has the risk of carry-over. As a result, investigations in the optimization of latent effectiveness considering both diffusion vapour resistance and carry-over problem are required.

3.2 Mass flow rate ratio (m*)
The mass flow rate, defined in Equation 10, is a measurement of relative flow rate of heat and mass exchanging fluids. In this research, m* is modified by changing the solution mass flow rate while maintaining the air mass flow rate to keep the NTU constant. The impact of m* on effectiveness at NTU=4 is shown in Figure 5. Figure 5 reveals that under one NTU, effectiveness can be improved by increasing m*. Specifically, the impact of m* on sensible effectiveness is more significant than that on the latent effectiveness. When m* changes from 0.5 to 3, the sensible effectiveness increases from 0.67 to 0.94, while the latent effectiveness rises from 0.45 to 0.63. Similar to NTU, the gradient of change narrows after m* exceeds 1.5, which means increasing mass flow rate ratio will not improve the system performance.

3.3 Solution inlet temperature and concentration
The solution inlet temperature and concentration have considerable impacts on the system performance due to close relationship among them and the surface vapour pressure. In this study, the solution concentration is in the range of 25% to 45% during which dehumidification effect is valid without the problem of crystallization. The temperature is in the range of 10°C to 20°C, during which the problem of freezing is avoided without compromising the dehumidification.

The impact of solution inlet temperature on effectiveness under $C_{sol} = 40\%$ is given in Figure 6. The impact of solution inlet concentration on effectiveness under $T_{sol} = 16\textdegree C$ is shown in Figure 7. From these figures, it can be concluded that both
temperature and concentration have negligible influence on the sensible effectiveness. It reduces from 0.894 to 0.880 when temperature increases from 10°C to 20°C, while it decreases from 0.895 to 0.886 when concentration increases from 25% to 45%. By contrast, solution temperature and concentration have more considerable effects on sensible effectiveness. The latent effectiveness is negatively related to solution temperature. There is 2% decrease of latent effectiveness when the solution temperature varies from 10°C to 20°C. Comparatively, increasing the solution concentration from 25% to 45% will lead to the increase of sensible effectiveness from 0.67 to 0.71. As a result, reducing the solution inlet temperature and increasing the solution concentration ratio are two effective ways to improve the system dehumidification performance. In real industry, the application of low solution temperature is limited by high energy consumption. Thus improving the solution concentration is a better option.

4. Conclusion
In this study, the performance of a cross-flow membrane-based liquid desiccant cooling and dehumidification unit was investigated through a mathematical model. The performance, indicated by sensible effectiveness and latent effectiveness, is analyzed by assessing various parameters such as NTU, m*, as well as solution inlet temperature and concentration. The method of control variables is used and main parameters are adjusted based on a benchmark model.

The simulation results indicate that higher sensible and latent effectiveness can be achieved by increasing NTU (decreasing air mass flow rate) and m*. However performance improvement is no longer limited by NTU when it exceeds 7. Similarly, for m*, the gradient of change decreases after m* exceeds 1.5, which means increasing mass flow rate ratio will not improve the system performance. The solution inlet temperature and concentration also have significant impacts on the system performance. Sensible effectiveness is insensible to both solution temperature and concentration. Latent effectiveness (dehumidification effect) can be improved by increasing solution concentration. The numerical results can be regarded as the fundamentals for further system optimization design.

References


