Self-Tuning Resonant Control of a 7-Leg Back-to-Back Converter for Interfacing Variable Speed Generators to 4-Wire Loads

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Abstract — This paper considers the control of a 7-leg back-to-back Voltage Source Inverter (VSI) arrangement feeding a 4-wire load from a 3-phase Permanent Magnet Synchronous Generator (PMSG) operating at variable speed. The PMSG is controlled using a sensorless Model Reference Adaptive System (MRAS) to obtain the rotor position angle. The 7-leg converter is regulated using Resonant Controllers (RCs) at the load side and self-tuning resonant controllers at the generator side. The control system is augmented by a feed-forward compensation algorithm which improves the dynamic performance during transients. Experimental results, obtained from a prototype, are presented and discussed.

Index Terms— AC-AC Power Conversion, Power generation control, Converters, Variable speed generation.

I. INTRODUCTION

Variable speed operation of generation systems has several advantages which are well reported in the literature. For instance more energy capture in wind generators [1]; higher efficiency of diesel engines, which can be operated at the optimum power/fuel consumption ratio [2]; less stress in the mechanical components; smaller portable generation systems [3]; etc. To connect a 4-wire load (e.g. an off-grid residential load) to a 3-phase variable-speed generator, several power converter topologies are feasible [4]–[8]. For instance a conventional 3-leg back-to-back voltage source converter connected to a Δ-Y transformer can be used. The star-connected secondary of the transformer is then used to allow the circulation of zero sequence current through the load. However, this is a bulky solution with a low power density.

Another alternative is to use a conventional back-to-back converter with the neutral point of the load connected to the middle of a split capacitor bank in the dc-link. The main problem of this approach is that relatively large capacitors are required to minimise the ripple [8]. A different topology is presented in [7], where a 4-leg matrix converter is proposed to feed the output load. This is a good solution if a compact generation system with a high power density is required. However there are also some problems with the matrix topology which have been reported in the literature. For instance matrix converters do not have boost capability [4]; therefore low speed operation of the generator is not feasible if constant load voltage is required. Another problem is produced when the generation system feeds unbalanced/non-linear loads. In this case, because of the lack of a dc capacitor bank, pulsations in the instantaneous output power produce harmonic distortion in the input current [6].

In this paper the application of a 7-leg back-to-back voltage source PWM converter is proposed with a 3-leg Machine-Side Converter (MSC) and a 4-leg Load-Side Converter (LSC) as shown in Fig. 1. Both converters are controlled using space vector modulation algorithms and RCs. RCs have been selected in this application because they have several advantages in 4-wire applications. For example they provide the capability to control zero sequence currents and voltages [5]–[7] (which do not exist in signals obtained by conventional α-β and d-q transformations) and they also allow straightforward implementation of power factor control at the generator side (as discussed in Section IIIA). Additionally, RCs provide a simple approach to eliminate waveform distortion using cascade controllers [9]–[14].

Fig. 1. Proposed 7-leg variable speed generation system.
this paper a feed-forward compensation algorithm is proposed (see Fig. 1) which is more conveniently implemented when both sides of the 7-leg converter are controlled using resonant controllers.

In this paper the LSC output is controlled to operate at a constant $\omega_L$, corresponding to an electrical frequency of 50Hz. Therefore the resonant controllers of the LSC are tuned to operate at fixed frequencies and they have to regulate the positive, negative and zero sequence load-voltage components. When non-linear loads are fed by the LSC, waveform distortion has to be also eliminated using RCs. In the MSC side the generator output frequency varies with rotational speed. Therefore a resonant controller is proposed which has a frequency adaptive (self-tuning) structure, designed to obtain a good dynamic performance over the whole operating range.

In Fig. 1 a speed demand calculation block is required to adaptively change the speed according to some control law, e.g. to reduce the fuel consumption in a diesel-based generation system, to increase the performance of a micro-hydro system, or to improve the energy capture in a wind energy conversion system. The control system proposed in this work has been designed to operate across the whole speed range and the calculation/regulation of the optimal rotational speed, is considered outside the scope of this paper.

The contribution of this work can be summarised as follows:

- To the best of our knowledge this is the first paper where a variable speed generator is interfaced by a 7-leg back-to-back VSI to a 4-wire load. The power converter topology presented in this paper can be applied to variable speed diesel systems [15], low voltage micro-grids [16][17], [18], wind-diesel hybrid systems[19], utility power supplies [3], etc. In general this topology can be used in any application where a variable speed 3φ generator has to be interfaced with a 4-wire load/grid.

- A new methodology for the design of a self-tuning RC, capable of operating over a wide frequency range, is presented. The RC is designed in the z-domain, to avoid the problems related with the bilinear transform or other discretisation methods [20]. This design methodology can be advantageously used for grid connected power converters, [21], [22], droop-controlled converters for micro-grids and variable speed machines [23], [24]. The design methodology presented in this paper is certainly superior to that conventionally used to implement the “PR controller” reported in the literature [25].

- A novel feed-forward compensation algorithm is analysed and presented. The feed-forward term compensates the perturbations produced by fast variations of an unbalanced linear/non-linear load on the dc-link voltage. This feed forward compensation method can be used in other applications where high dynamic response (in the presence of power oscillations produced by unbalanced signals) is required. For instance in conventional 3-leg back to back converters, [1] and even single phase systems.

- Small signal models are presented, describing the dynamics of the dc-link, power balance equation, dynamics of the PMSG, etc. These small signal models consider the effect of non-linear loads and can be used to design the controllers using conventional linear control tools. The linearised models presented in this work can be extended to other applications where power converters are used to feed non-linear unbalanced loads.

The remainder of the paper is organised as follows. Section II briefly discusses the sensorless control system; In Section III the self-tuning resonant control system is analysed; the load-side resonant control is very briefly presented in Section IV. Section V discusses the feed-forward algorithm and Section VI presents results from an experimental prototype. Finally, Section VII discusses the conclusions from the work. Parameters of the experimental rig are presented in the Appendix.

II. SENSORLESS CONTROL

Later, in Section III, it is shown that for the implementation of the self-tuning resonant controller, the rotor position ($\theta_r$) and rotational speed ($\omega_L$) of the Permanent Magnet Synchronous Generator (PMSG) are required. In this work $\theta_r$ and $\omega_L$ are estimated using a sensorless Model Reference Adaptive System (MRAS) observer. Such systems have been extensively discussed in the literature before [26]–[29] so only a brief discussion is provided here for completeness.

The MRAS observer is based on a reference model and an adaptive model [28], [29], [31]. The reference model is obtained as:

$$\psi_s = f(v_s - R_s i_s)dt \quad (1)$$

Where $\psi_s$ is the stator flux, $v_s$ is the stator voltage, $i_s$ is the stator current and $R_s$ is the stator resistance (=0.2Ω).

Unlike motors, PMSG are not expected to operate at very low rotational speeds. As stated in Section III.C it is assumed in this work that the PMSG is operating between 500rpm-2000rpm (0.25 $\omega_L$ to $\omega_L$). Therefore, even at 500rpm, the voltage $v_s$ is relatively large compared to the small voltage drop variations produced by changes in $R_s$ with temperature (see (11)).

For instance if the PMSG winding temperature varies from 20°C to the maximum value of 140°C, the stator resistance will change from $R_s$=0.2Ω to $R_s$=0.294Ω (assuming a temperature coefficient $\alpha$=0.00393Ω/°C for the copper windings). Therefore the resistive voltage drop will change from 3V to 4.4V at rated current (i.e. $AV$=1.4V). Hence even for the extreme case of rated current, minimum speed and the maximum allowable temperature rise, the change in resistive voltage drop is less than 5% of the phase voltage. Moreover, as reported in [31], for permanent magnet machines the stability of MRAS-based sensorless control loops is not compromised by stator resistance variation.

The adaptive model is obtained using:

$$\hat{\psi}_s = L_s i_s + \psi_m e^{j \theta_r} \quad (2)$$

Where the superscript "*" indicates an estimated variable. In (2) $L_s$ is the stator inductance and $\psi_m = \psi_m e^{j \theta_r}$ is the estimated rotor flux. A smooth air-gap permanent magnet machine is used in this work (i.e. $L_s$=L_d=L_m).

The error between the stator flux estimated by the adaptive
model and that obtained from (1) is defined as:
\[ \varepsilon = |\psi_s \otimes \hat{\psi}_s| - |\psi_{sto} \otimes \hat{\psi}_{sto}| = |\psi_s| |\hat{\psi}_s| \sin(\theta) \] (3)

In (3) the symbol \( \otimes \) represents cross-product and \( \theta \) is the phase angle between the vectors \( \psi_s \) and \( \hat{\psi}_s \). The advantages of using cross product for the calculation of \( \varepsilon \) are discussed in [32].

Unlike induction machines, in a PMSG there is no slip velocity (i.e. \( \omega_{slip}=0 \)) and the rotational speed is equal to the stator electrical frequency. Therefore the speed can be correctly estimated from the frequency of the electrical signals, even if the stator resistance is affected by temperature variations. This is also concluded by inspecting (2)-(3) and a rigorous mathematic demonstration is presented in [31].

However because the error \( \varepsilon \) is defined as the cross product between \( \psi_s \) and \( \hat{\psi}_s \) (see (3)), the rotor position angle \( \theta_r \) can be incorrectly estimated if the phase angle of the stator flux \( \psi_s \) (obtained from (1)) is affected by large stator resistance variations. Nevertheless, as discussed before, for the speed operating range the variation in the stator resistance voltage drop, due to temperature effects, is relatively small compared with the PMSG internal voltage (see Fig. 2a). Moreover, in this work the generating system is designed to operate with a power factor close to unity. Hence, the position error in \( \psi_s \) is further reduced considering that \( \psi_s \) and \( -R_s i_s \) (see (3)) have almost identical phase. This is depicted in Fig. 2a, where the calculation of the vectors \( \psi_s - R_s i_s \) and \( \psi_s - (R_s + \Delta R_s) i_s \) is shown. If the angle \( \phi \neq 0 \) (corresponding to close to unity power factor operation of the PMSG), then the phase shift between both vectors is also zero. Notice that Fig. 2a is not drawn to scale and the voltage drops \( R_s i_s \) and \( \Delta R_s i_s \) have been magnified in that figure.

If the PMSG is utilised at an operating point where the effect of the stator resistance variation is no longer negligible, then the implementation of on-line identification methods could be required. For instance the PMSG parameter identification method reported in [16] based on adaptive observers, or the sliding-mode observer proposed in [33]. Alternatively some of the methods proposed for stator resistance identification in induction machines, e.g. the P-based MRAS observer reported in [34] could be modified for \( R_s \) identification in PMSGs.

The MRAS observer used in this work is shown in Fig. 2b. To avoid the drift produced by integrating dc signals, the reference stator flux \( \psi_s \) is calculated using a band-pass filter instead of a pure integrator. The cross-product is calculated using the \( \alpha-\beta \) components of (1)-(2). In Fig. 2b a PI controller is used to drive the error of (3) to zero, by adjusting the position of the magnetic flux \( \psi_m \).

In most of the applications related to variable speed generation of electrical energy (e.g. wind energy systems, diesel generation, etc.) the changes in rotational speed are relatively slow, due to the inertia of the prime-mover. Therefore to design the PI controller of Fig. 2b, a simplified small-signal model can be used, similar to that discussed in [26].

Using the parameters of the PMSG and experimental rig (see the appendix), the MRAS has been designed for a bandwidth of about 20Hz.

III. SELF-TUNING RESONANT CONTROL OF THE MACHINE-SIDE CONVERTER

A. Proposed Control System for the MSC.

The position angle \( \hat{\theta}_r \) is estimated from the MRAS observer of Fig. 2. Because \( \hat{\theta}_r \) corresponds to the flux vector \( \psi_m \) angle, (see Fig. 3), the position of the PMSG machine internal voltage \( \hat{\theta}_M \) is estimated as:
\[ \hat{\theta}_M = \hat{\theta}_r + \frac{\pi}{2} \] (4)

Figure 3 shows the proposed control system for the MSC. A PI controller, whose output is the current \( i_p \), regulates the dc-link voltage of the back-to-back converter. An additional term from a feed-forward compensation algorithm (see \( i_p \) in Fig. 3) can be used to improve the dynamic performance of the dc-link voltage. This is further discussed in Section V.

In order to operate the generator with unity displacement factor, the reference currents required are:
\[ i_a^* = i_M \cos(\hat{\theta}_M) \]
\[ i_b^* = i_M \sin(\hat{\theta}_M) \] (5)

It is also possible to introduce a phase shift angle \( \theta_{ph} \) between the voltage and stator current of the PMSG. For instance three alternatives to obtain \( \theta_{ph} \) have been presented in the literature [27], [35] (see the phasor diagram of Fig. 4). Option 1 operates the PMSG at unity power factor, maximising the power transfer from the PMSG to the load [27], with the MSC providing the reactive power required by the inductance \( L_r \). Option 3 operates the MSC at unity power factor with the
required phase angle $\theta_{ph}$ calculated by setting $v_M \approx \omega_r \psi_m$ yielding:

$$\theta_{ph} \approx -\sin^{-1}\left(\frac{v_c}{\psi_m} |i_M|\right) \tag{6}$$

Option 2 attempts to reduce saturation and obtain a compromise between the converter rating and the generator rating [35], by locating the current mid-way between the voltage vectors $v_M$ and $v_c$ with $\theta_{ph}$ set to half the value of (6). The control system proposed in this work could be used to implement any of these power factor control strategies.

### B. Design of the dc-link Control System.

As discussed before, the machine stator current magnitude is controlled using a PI controller augmented by a feed-forward compensation term (see $i_f$ in Fig. 3). The design of the PI controller is discussed below. The feed-forward algorithm is discussed in Section V.

Neglecting the MSC losses, the power supplied by the PMSG is equal to that supplied by the MSC to the dc-link. Therefore, the following expression can be written:

$$E_{dc}i_{dc,in} = k_{ab}(v_{Ma}i_{Ma} + v_{Mb}i_{Mb}) \tag{7}$$

Where $i_{dc,in}$ is the dc current on the generator side and $k_{ab}$ is dependent on the $\alpha$-$\beta$ transformation being used. Using the angle $\theta_{ph}$, then (7) can be written as:

$$E_{dc}i_{dc,in} = k_{ab} i_M v_M \cos(\theta_{ph}) \tag{8}$$

where $v_M$ and $i_M$ are the magnitude of the generator voltage and current vector respectively. For a PMSG $v_M \approx \psi_m \omega_r$. Therefore, the dc link current $i_{dc,in}$ is obtained as:

$$i_{dc,in} = k_{ab} \frac{\omega_r \psi_m}{E_{dc}} i_M \cos(\theta_{ph}) \tag{9}$$

Linearising the system about an operating point indicated by the subscript “0” yields:

$$\Delta i_{dc,in} = k_{ab} \psi_m \cos(\theta_{ph}) \left[ \frac{\omega_{r0}}{E_{dco}} \Delta i_M + \frac{i_{m0}}{E_{dco}} \Delta \omega_r - \frac{\omega_{r0} i_{m0}}{E_{dco}} \Delta E_{dc} \right] \tag{10}$$

As discussed before, in this work it is assumed that the PMSG speed changes slowly, therefore the term $\Delta \omega_r$ can be neglected when the dynamics of the current $i_M$ are considered. Moreover the variation $\Delta i_{dc,in}$ produced by $\Delta E_{dc}$ (last term in (10), is compensated by an identical variation in $\Delta i_{dc,out}$ at the LSC side. This is due to the fact that the LSC is operating with constant power output and the load voltage is regulated with a fast dynamic response. Therefore neglecting $\Delta \omega_r, \Delta E_{dc}$ the current $\Delta i_{dc}$ circulating through the dc link capacitors is obtained as:

$$\Delta i_{dc} = k_{ab} \cos(\theta_{ph}) \frac{\omega_{r0} \psi_m}{E_{dco}} \Delta i_M \tag{11}$$

The transfer function of (11) and the small signal model of Fig. 5 can be used for the design of the dc-link PI controller. Notice that the gain of the controller is a function of $\omega_r$. This allows the system to remain tuned in spite of speed variations (see (11)). Moreover, even if the relatively small losses of the system (not considered in (7)) affect the gain of (11), linear control tools can be used to design a robust PI controller whose performance is little affected by small variations of this gain.

Notice that in (9-11) it is assumed that the feed-forward current ($\Delta i_f$) is an external perturbation. Therefore it can be considered that $\Delta i_f = \Delta i_M$ because the closed loop poles of the dc link voltage control system are not affected by the feed-forward compensation algorithm.

### C. Generator-Side Resonant Control System.

Resonant Controllers are based on the internal model principle and they can be used in control systems with sinusoidal reference signals [5]–[7], [9], [10], [25], [36]–[38]. One of the advantages is that a single RC per phase can be used to regulate the positive, negative and zero sequence signals at the load-side converter. In this application RCs are used to regulate the stator current in the PMSG and the voltage of the load fed by the LSC.

Resonant controllers have been discussed in the literature however RCs are generally used in applications where variations in the resonant frequency are small, e.g. for grid-connected converters [10], [25], [36]. However, in the proposed system, the PMSG can operate over a wide speed range (e.g. 500rpm to ±2000rpm). Therefore, the coefficients of the resonant control system have to be adjusted according to the stator frequency variation, in order to operate with a suitable bandwidth and phase margin over the whole speed range.
range. This type of controller is usually called a “self-tuning” resonant controller in the literature [23, 24, 39, 40]. To the best of our knowledge, [39] is the only paper where a self-tuning RC is designed and experimentally tested for a system where operation over a wide frequency range is considered. However, in contrast to the approach proposed here, the methodology reported in [39] proposes to locate the resonant poles in a position where the PMSG stator currents cannot be regulated with low or zero steady state error. In this paper, digital design in the z-plane is proposed, avoiding the problems associated with the conventional implementation of resonant controllers based on discretisation methods [20].

The stator current of the PMSG is obtained from:

$$v_M = R_s i_M + L_s \frac{di_M}{dt} + v_c$$

(12)

where $v_c$ is the MSC voltage vector. Using (11)-(12) the control loop shown in Fig. 6 is proposed where the reference current vector ($i_M^*$) is derived from (5). The MSC voltage vector $v_c$ is obtained at the output of a self-tuning resonant controller (block labelled “RC” in Fig. 6) whose transfer function is:

$$RC(z) = K_r \frac{(z - r(\hat{\omega}_r) e^{j\hat{\omega}_r T_s})(z - r(\hat{\omega}_r) e^{-j\hat{\omega}_r T_s})}{(z - e^{j\hat{\omega}_r T_s})(z - e^{-j\hat{\omega}_r T_s})}$$

(13)

Notice that in (13), $\hat{\omega}_r$ is the rotational speed (in electrical rad/s) estimated by the MRAS observer. As demonstrated in [31], tracking of the rotational speed by an MRAS observer in a PMSG is not affected by inaccurate identification of the machine parameters, i.e. $R_s$, $L_s$, therefore if the rotational speed changes relatively slowly, the resonant controller of (13) is tuned to the correct frequency even if the machine parameters change. In Fig. 6 the SVM and MSC is represented as a zero order hold (see block labelled “ZOH”) and a delay of one sampling period.

The controller of (13) has two poles located in the unit circle (see Fig. 7) and two zeroes, relatively close to the poles, used to increase the damping coefficient of the closed loop system. In (13), $T_s$ is the sampling time, $K_r$ is the controller gain and $r(\hat{\omega}_r)$ is the distance from the controller zeroes to the origin. For variable speed operation of the PMSG, the poles of (13) are moved along the unit circle in order to track, with zero steady-state error, the reference currents of (5) (see $\Delta\hat{\omega}_r$ in Fig. 7).

In this work the values of $K_r$ and $r(\hat{\omega}_r)$ have been tuned using Bode diagrams and Evans’ root locus at different operating points. For instance in Fig. 8 the open loop Bode diagram, considering operation of the PMSG at $\approx 2000$ rpm, is shown. The control system has been designed to obtain a phase margin of $\approx 60^\circ$ at this operating point with a current control system bandwidth of $\approx 60-65$ Hz.

From the analytical and experimental work, and considering the parameters of the experimental system presented in the appendix, it has been concluded that for most of the operating range an almost fixed value of $r(\hat{\omega}_r) \approx 0.95$ produces a good dynamic response. However for a relatively low speed (close to 500 rpm), the plant pole is closer to the poles and zeroes of the RC (see Fig. 7) and the value of $r(\hat{\omega}_r)$ has to be changed in order to maintain a good dynamic performance. This approach is simple to implement and produces a good result considering that the PMSG acceleration is relatively slow. A small look-up table or similar implementation methodology can be used to obtain the value of $r(\hat{\omega}_r)$.

Expanding (13), the z-plane transfer function is obtained as:

$$RC(z) = K_r \frac{(z^2 - 2r(\hat{\omega}_r) \cos(\hat{\omega}_r T_s) z + r(\hat{\omega}_r)^2)}{(z^2 - 2r(\hat{\omega}_r) \cos(\hat{\omega}_r T_s) z + 1)}$$

(14)

Using (14) the self-tuning resonant controller can be implemented in real time using a Digital Signal Processor (DSP).

IV. CONTROL OF THE LOAD SIDE CONVERTER

In order to feed a stand-alone load and provide a path for the circulation of zero sequence currents, the LSC has 4 legs at the output (see Fig. 9). Resonant controllers are used to regulate the load phase-to-neutral voltages ($v_{us}, v_{bs}, v_{cs}$). It is assumed that the output frequency is constant; therefore, the LSC resonant controllers do not require frequency adaptation. The approach is essentially the same as that discussed in [5]–[7] for a 4-leg matrix converter so only a brief treatment is given here.

Assuming a resistive load, the transfer function relating the phase to neutral voltage of the load to that at the output of the LSC is:

Fig. 6. Resonant control system for the generator side converter.

Fig. 7. Poles and zeroes of the resonant controller.

Fig. 8. Open loop bode diagram for operation at 2000rpm.
where $V_{an}$ is the load voltage and $V_{oa}$ is the output voltage of the LSC; $R_{Lc}$ is the load resistance and $C_f$, $L_f$ are the capacitance and inductance of the second order output filter respectively. A resistive load has been assumed in (15), however the control system presented in this work can be used with both leading and lagging power factor loads. Substituting $R_{Lb}$ and $R_{dc}$ for $R_{Lc}$, transfer functions similar to (15) are obtained for the voltages $V_{bn}$ and $V_{oa}$ in terms of $i_{db}$ and $i_{oc}$.

The resonant control system is designed for the worst case operating point, i.e. when there is no load connected to the output of the filter, and the transfer function of (15) is:

$$\frac{V_{an}(s)}{V_{oa}(s)} = \frac{R_{Lc}}{s^2R_{Lc}C_fL_f+sL_f+R_{Lc}}$$  \hspace{1cm} (15)

in this case the damping coefficient of the second order system is $\zeta=0$ and the poles of (16) are located on the $j\omega$ axis. Using (16) the control system shown in Fig. 10 is designed and implemented.

For the control system shown in Fig. 10, only one RC per phase is used. However if the output load is strongly non-linear, multiple resonant controllers could be required to supply voltages with low harmonic distortion to the load.

In the experimental work discussed in this paper, the control of the LSC is realised using a single RC per phase when the 4-leg front-end converter is feeding linear loads. For loads with strong non-linear behaviour three controllers per phase are implemented for the regulation of the load voltage (see Section VI). A full discussion of the issues related to the implementation of multiple order fixed-frequency resonant controllers, is considered outside the scope of this paper and the interested reader is referred elsewhere [5], [7], [36].

V. FEED-FORWARD COMPENSATION ALGORITHM

The PMSG stator current control system is augmented with a feed-forward compensation term (see $i_{ff}$ in Fig. 3) improving the dynamic response of the system when fast variations in the load fed by the LSC are produced. The feed-forward algorithm is based on input/output power balancing.

Assuming that the load voltage is well regulated and balanced, the instantaneous LSC output power is calculated as:

$$P_{out} = Re(k_{ap}V_p^L_{oc})$$  \hspace{1cm} (17)

where the superscript “c” is the complex conjugate operator, $V_p$ is the load voltage vector (see $V_{ain}$, $V_{bin}$, $V_{cen}$ in Fig. 9) and $i_p$ is the LSC output current vector. Expanding (17) $P_{out}$ is obtained as:

$$P_{out} = Re[k_{ap}V_p^L \sum(k_i e^{j(\omega t+\theta_i)} + \sum h_i e^{-j(\omega t+\theta_h)})^c]$$  \hspace{1cm} (18)

which can be rewritten as:

$$P_{out} = Re[k_{ap}V_p^L \sum(k_i e^{j((1-k)\omega t+\theta_k)} + \sum h_i e^{j((h+1)\omega t+\theta_h)})]$$  \hspace{1cm} (19)

With $k, h \geq 1$. In (18-19) the index “$k$” is used to denote the positive sequence LSC output currents, and “$h$” is used for the negative sequence components. It is assumed that $(\theta_k, \theta_h)$ are arbitrary phase angles.

Neglecting the losses, the power balance in the 7-leg converter can be written as:

$$Re(k_{ap}V_p^M e^{j\omega c t} i_{hd}) = -\frac{1}{C} \frac{dE_{dc}}{dt} + Re[k_{ap}V_p^L \sum(k_i e^{j((1-k)\omega t+\theta_k)} + \sum h_i e^{j((h+1)\omega t+\theta_h)})]$$  \hspace{1cm} (20)

where the term at the left hand side of (20) is the power supplied by the PMSG and the term $(1/2)C(dE_{dc}/dt)$ is the instantaneous power absorbed or supplied by the dc-link capacitor bank $C$.

From (20) a feed-forward term can be calculated in order to improve the regulation of the dc-link voltage $E_{dc}$. However, it is relatively simple to demonstrate that the instantaneous power absorbed/supplied by the dc-link capacitance $C$ cannot be driven to zero without producing harmonic distortion in the PMSG stator current when the LSC feeds an unbalanced non-linear load. Therefore some voltage variation has to be allowed in $E_{dc}$ which can be obtained using:

$$E_{dc} \frac{dE_{dc}}{dt} = -\frac{1}{C} Re[k_{ap}V_p^L \sum(k_i e^{j((1-k)\omega t+\theta_k)} + \sum h_i e^{j((h+1)\omega t+\theta_h)})] \quad (k > 1)$$  \hspace{1cm} (21)

Eq. (21) can be useful for designing the dc-link capacitor bank considering the expected load characteristics.

Harmonic distortion of the stator currents are not produced when the power generated by the PMSG is balanced with the dc instantaneous power produced by the positive sequence of the fundamental load current. Therefore replacing $V_p \approx \psi_m \omega r$ a feed-forward compensation current can be obtained calculating the term $\bar{i}_{ff}$ in (20) as:

$$\bar{i}_{ff} = \frac{P_{avg}}{k_{ap}\psi_m \omega r \cos(\theta_p)}$$  \hspace{1cm} (22)
where \( P_{avg} \) is the dc component (i.e. obtained using \( k=1, \ i_0=0 \) in (21)) of the load power.

The proposed feed-forward control system is shown in Fig. 11. The power \( P_{avg} \) is calculated using:

\[
    P_{avg} = \frac{(v_{an}i_{oa} + v_{bn}i_{ob} + v_{cn}i_{oc})}{\prod_m (s^2 + m^2\omega_e^2)}
\]

The term at the right of (23) represents a cascade of notch filters tuned at \( m\omega_e \), where \( \omega_e \) is the output frequency. If the load is unbalanced but linear, only one notch filter is required tuned at \( 2\omega_e \). If the output is non-linear, additional notch filters are required to eliminate the power pulsations due to the current distortion.

Fig. 11 shows the implementation of the feed-forward compensation term. The output power is calculated using measurements of the phase to neutral load voltages and LSC output currents. From these measurements the average power \( P_{avg} \) is calculated using (23). Because of simplicity only one notch filter is shown in Fig. 11. An additional low-pass filter is used to eliminate the harmonics produced by the switching of the IGBT devices.

Interconnection of the control systems, discussed in the previous sections, is shown in Fig. 12.

VI. EXPERIMENTAL WORK

The control system of Fig. 12 has been experimentally implemented (see Fig. 13) using a DSP based control board and an FPGA, the latter providing the switching signals for the 14 IGBT gate drivers. Data acquisition uses 20 Analogue to Digital (ADC) channels of 14bits, 1\( \mu s \) conversion time each, interfaced to the DSP. Additionally two digital oscilloscopes, operating simultaneously in single shot mode (with sampling frequencies of 5MHz) have been used in some of the experimental tests to store the current and voltages of the input and output side of the 7-leg converter. Hall-effect transducers are used to measure the input currents, input voltages and output load voltages. A switching frequency of 10kHz has been used to implement the SVM algorithms.

For the experimental tests a Control Techniques, 4kW, 2000rpm, 8 pole PMSG with surface mounted magnets is used. This PMSG supplies a sinusoidal voltage waveform with a Total Harmonic Distortion (THD) of less than 1.1% (the PMSG voltage waveforms are shown in Fig 13a). The

![Fig. 11. Feed-forward compensation system.](image)

![Fig. 12. Proposed control system for the 7-leg converter.](image)

parameters of the PMSG are given in Table I at the appendix. The prime mover is a 2 pole, 2910rpm, 5kW cage machine. A commercial inverter is used to drive the cage machine using V/F control. The machines are shown in Fig. 13c. The position encoder has not been used to implement the control strategies.

A 7-leg power converter has been designed and implemented for the experimental validation of the proposed control system. The PCB is shown in Fig. 13b. Each leg has been implemented using a 1200V, 35A dual IGBT switch Infineon BSM35GB120DN2 device. The experimental system is controlled using a Texas Instruments TMS320C6713 DSP. A daughter board with an ACTEL FPGA is used to implement the PWM generation and for interfacing the A/D, D/A converters to the DSP. A 3\( \phi \) resistor bank with resistor taps of 10.7\( \Omega \) and 14.7\( \Omega \) is used for the load. An electronic relay controlled using one of the D/A output channels is used to implement the load-step variations presented in this section.

Figs. 14-15 show the control system performance for an unbalanced load-step. Before the load-step the system is operating with a dc-link voltage \( E_{dc}=325V \), a load voltage of 115V peak, \( \omega_e=1650rpm \) and a balanced LSC output current of 5.3A (rms). At \( t=42\)ms additional resistors are connected increasing the current to 12.4A (rms) in two of the phases. The output currents \( i_a, \ i_b \) and \( i_c \) are shown in Fig. 14a. The neutral current, with the presence of zero sequence components, is shown in Fig. 14b. Finally Fig. 14c shows the load voltage which has a small perturbation when the load impact is applied; this is eliminated in less than 5ms (a quarter of a cycle) by the load-side RCs.

Fig. 15 shows more experimental results corresponding to the unbalanced load step of Fig. 14. Fig. 15a shows the 3\( \phi \) output power. Before the unbalanced load-step, the power is
1.27 kW without oscillations. After the load step the mean output power is about 2 kW with a relatively large 100 Hz component. Using a digital notch filter tuned at 100 Hz and a first-order digital low pass filter tuned at 50 Hz (see Figs. 11-12 and (23)), the feed-forward current $i_{ff}$ is calculated by the DSP using (22)-(23) and depicted in Fig. 15b. As discussed in Section V, this current is fed-forward to the stator current control system, to improve the dynamic response of the dc-link voltage regulation.

In Fig. 15c the dc-link voltage is shown. With feed-forward compensation, the dip is only $\approx 6 V$. Notice that the dc-link has a 100 Hz oscillation after the unbalanced load step is applied. As discussed in Section V the controller has not been designed to compensate the ac ripple in the dc-link voltage, to avoid distortion in the PMSG stator currents.

Notice that Fig. 15c has a different time scale. In Figs. 15a and 15b signals internally calculated by the DSP are shown. Fig. 15c shows a dc voltage signal which is captured by the digital scope.

Fig. 16a shows the PMSG stator current corresponding to the test depicted in Figs. 14-15. The input current has a frequency of about 110 Hz with virtually no distortion. For this case the current is regulated with $\theta_{ph} = 0$ (see Fig. 3). Figs. 16b and 16c show the $\alpha$-$\beta$ tracking error of the self-tuning resonant control system. For the whole test the tracking error is low, with a peak below 0.75 A produced when the current $i_{ff}$ has a fast variation from $i_{ff} = 12 A$ to $i_{ff} \approx 18 A$ (peak current) in $t = 0.5 s$.

The control system of Fig. 12 has been tested considering a relatively fast ramp speed variation. For this test the experimental results are shown using the effective (rms) current of each phase. This methodology is preferred because
of the problems associated with displaying \(d-q\) currents under unbalanced operation. Moreover, the zero sequence components are not reflected in the \(d-q\) signals.

The LSC (rms) output currents are calculated using a digital implementation of:

\[
i_{\text{rms}} = \sqrt{\frac{1}{T} \int i^2(t) \, dt}
\]  

(24)

A low pass filter is used to calculate the root mean square value of (24).

Fig. 17 shows results for a speed ramp variation between \(\omega_0 \approx 800\text{rpm}\) to \(\approx 1500\text{rpm}\). At \(t=2\text{s}\) an unbalanced load step is applied to two of the phases and disconnected at \(t=7.3\text{s}\). Fig. 17a shows the estimated rotational speed \(\hat{\omega}_r\) and the reference speed \(\omega_r^*\) sent to the commercial inverter. In Figs. 17b-17e the LSC output currents \(i_a\), \(i_b\) and \(i_c\) are shown. Notice that the effective current is constant in phase \(c\) and that the unbalanced load-step variations are applied to phases \(b\) and \(c\). Finally Fig. 17d shows the zero sequence components circulating through the \(4^{th}\) leg, used as a neutral connection in this application.

Fig. 18 shows additional signals corresponding to the ramp speed variation test shown in Fig. 17. Fig. 18a shows the magnitude of the PMSG \(d-q\) stator current. Because the system is feeding a constant load at the output, the PMSG power current is proportional to \(1/\omega_0\). Fig. 18b shows the instantaneous power measured at the output. As explained before, this power is filtered and the feed-forward compensating current is obtained using (22)-(23). Fig. 18c shows the phase to neutral load voltage. Because of the boost capability of the MSC the load voltage can be regulated to a value which is higher than the internal voltage (a phase voltage of \(\approx 40\text{V}\) at \(800\text{rpm}\)). This is an advantage compared to previous implementations (see [5]). Notice that the load voltage is well regulated with a dip and an overshoot of less than \(5\text{V}\). Finally Fig 18d shows the dc-link voltage. For this test the dc-link voltage is well regulated \((E_{dc}^* = 300\text{V})\) with a dip and overshoot of less than \(19\text{V}\) (\(\approx 6\%\) of \(E_{dc}\)).

The performance of the feed-forward compensation algorithm, for the regulation of the dc-link voltage, is shown in Fig. 19. The system is operating with a rotational speed of
750rpm, the dc-link voltage is regulated to 200V and a 3ϕ balanced load-step is applied. Fig. 19a shows the output power variation from 500W to 1250W. Fig. 19b shows the response of the dc link control system without the feed-forward term of (22). The nominal voltage is 200V and the dip and the overshoot are about 25V for a dc-link capacitor bank of about 1600µF. When the feed-forward term is included (Fig. 19c) the dip and the overshoot are reduced to 15V and the settling time (considering a 2% band) is reduced from ≈90ms to ≈20ms for the same power step.

In Fig. 20 the performance of the proposed control system, for a non-linear load-step, is shown. The system is operating with a balanced load of about 1300W, 1650rpm, when at t=58ms a non-linear load composed of a 14.7Ω resistor, in series with a rectifier diode, is applied to phase α. For this case additional resonant controllers, tuned to eliminate dc signals and third order harmonics are implemented in the LSC control system to regulate the load voltage. The implementation of high order resonant controllers is discussed in [5], [9], [36], [37].

Fig. 20a shows the LSC output currents $i_a$, $i_b$, $i_c$. Before the non-linear step, the current in phase-a has negligible distortion. After the non-linear load step the current in phase-a is increased with a noticeable dc component whose magnitude is ≈32% with respect to the fundamental. Moreover, after the step, the second and the fourth harmonics are also present in $i_a$ with magnitudes of ≈11% and 3% respectively. Fig. 20b shows the zero sequence current components (produced by the non-linear load) that circulate in the neutral leg.

Fig. 20c shows the stator current which is well regulated with little distortion. Finally the load voltage is shown in Fig. 20d. As shown in this graphic, the load voltage is well regulated and the effects of the non-linear step are negligible. Moreover the high order resonant controller reduces the distortion in the load voltage.

VII. CONCLUSIONS

A control method for a 7-leg back-to-back voltage source inverter has been presented. It is based on resonant controllers and a feed-forward compensation term. A frequency adaptive control system for the regulation of the PMSG stator current has been presented and experimentally validated. A control system topology for the regulation of the dc link voltage, avoiding distortion in the generator current, has been analysed in this work. A feed-forward compensation algorithm has been proposed that effectively improves the dynamic performance of the dc-link voltage control.

The proposed control system has been tested considering balanced, unbalanced and non-linear load operating under variable/fixed rotational speed. The results have shown the good performance achieved with the proposed control methodology.

APPENDIX

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETER OF THE PMSG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal rotational speed</td>
<td>2000rpm</td>
</tr>
<tr>
<td>Nominal Power</td>
<td>4kW</td>
</tr>
<tr>
<td>Maximum Rotational Speed</td>
<td>2800rpm</td>
</tr>
<tr>
<td>Torque constant</td>
<td>1.4Nm/A</td>
</tr>
<tr>
<td>Nominal Torque</td>
<td>20Nm</td>
</tr>
<tr>
<td>Voltage Constant</td>
<td>85.5V/Krpm</td>
</tr>
</tbody>
</table>

(line voltage)
Voltage (line voltage)

Stator Inductance

4.7 mH

Stator Resistance

0.2 Ω

Output Waveform

Sinusoidal (THD<1.1%)

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PARAMETER OF THE EXPERIMENTAL SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching Frequency</td>
<td>10 kHz</td>
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<tr>
<td>Output Filter Inductance</td>
<td>Lf = 2.5 mH</td>
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<tr>
<td>Output Filter Capacitance</td>
<td>C = 40 μF</td>
</tr>
<tr>
<td>dc-link capacitance</td>
<td>1600 μF</td>
</tr>
</tbody>
</table>

REFERENCES


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