The Investigation of the Market Disequilibrium in the Stock Market

Jin Suk PARK

Thesis submitted to the University of Nottingham for the degree of Doctor of Philosophy

February 2013

Abstract

This thesis investigated stock market disequilibrium focusing on two topics: the impact of multiple market makers on the market disequilibrium at the market microstructure level, and the detection of the long-run market disequilibrium in the context of bubbles and the changes in transition probabilities.

The multiple market makers increased the resilience of price rather than improving its efficiency when a multiple market maker system (the NASDAQ) was compared with a single market maker system (the NYSE) in terms of lowering non-stationarity and raising predictability. On the other hand, the volatility modelling of intraday data showed that market maker's under-estimation (higher-than-estimated size of return) increased volatility while over-estimation decreased it. Also, intraday seasonality in mean and volatility was confirmed, but leverage effects were denied in the GJR-GARCH-type models.

The evidence of price bubbles in the Indian markets (1987-2008) and positive duration dependence in negative runs in the Korean market (1990-2008) were revealed using the duration dependence tests. The unconditional transition probabilities that a positive or negative run continues or ends were mostly significantly different from 0.5. On the other hand, the structural break based duration dependence test was devised to detect the changes in the transition probabilities between the market (dis)equilibrium. The NASDAQ and the Indian market showed positive duration dependence in positive runs and the Korean market displayed it in negative runs. In other words, the transition probability in those markets increases as a price run between structural breaks lengthened.

Acknowledgements

I am deeply grateful for the wise academic guidance of my supervisors, Doctor Robert Young and Doctor Rodion Skovoroda. Their constructive advice and considerable encouragement were crucial in finalising this research. Also, I owe a debt of gratitude to them in being patient with my shortcomings and supporting me to develop my skills and knowledge.

Without the unconditional love of my wife and parents, I would not have dared to begin and complete the study for a PhD. They have always been on my side and backed me to fulfil my desires in anything I choose to pursue. I am greatly indebted to all of the family members.

If it were not for Professor Shahid Ebrahim, I would not have come to Nottingham. I appreciate his advice on the early development of this research. Thanks to all of my friends who cheered me up throughout the study and my colleagues at NUBS for their support and feedback. I was also grateful to receive valuable comments from two examiners. Last of all, thank you to those who are not here anymore for the wonderful memories you gave me.

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Chapter 1

Introduction

1. Introduction

Market equilibrium in financial markets occurs where the quantity of financial asset demanded is equal to the quantity supplied (Mishkin, 2008), or simply where supply equals demand. The price at market equilibrium is called the equilibrium price, or the market clearing price as all market participants are satisfied (Mankiw, 2006). Although the financial markets tend to move toward and stay at the market equilibrium, the market disequilibrium can occur any time when there is excess demand or supply. It can be caused by the shift of supply or demand for exogenous reasons to the market that may correspond to the changes in the long-term fundamental prices. In addition, random market fluctuation or market frictions can be responsible for the market disequilibrium.

The disequilibrium can temporarily exist until the original equilibrium is restored or during the transition period from the old to the new equilibrium, but can be extended long-term. This may be a natural phenomenon in the markets. However, some disequilibrium can significantly affect market participants as well as non-participants. For example, long-term market disequilibrium due to excess demand in the stock market, like price bubbles, can have a severe adverse impact on not only listed firms and stock investors but also the wealth of the general public and the overall economy. Note that the definition of a price 'bubble' in this thesis follows the descriptive pattern of Shiller (2005) and the technical definition of Blanchard (1979) as reviewed in Section 3.1.1, which is accompanied by a crash.

This thesis investigates the market disequilibrium in stock markets. However, it is a vast area of study in finance and economics that cannot be covered in one piece of research. Therefore, the thesis particularly examines two main topics.

First, the impact of a different market making system on the market disequilibrium will be investigated. Market makers in the stock market are subject to adverse information against the informed traders and the possibility of market failure from excess inventory position due to the uncertainty of market orders. However, even symmetric information shocks and the perfect knowledge of equilibrium price may not prevent or quickly remove the disequilibrium because of market frictions in market making; for example, a market maker's inability or intentional deviation of his prices from the equilibrium. Another source of market frictions, which was rarely covered in the literature, is the number of market makers in the stock market. In particular, a market making system with a multiple market maker may result in different outcomes compared with a system with a single market maker in terms of price.

Second, the detection of market disequilibrium will be examined in the context of bubbles. Since there is commonly a high level of fluctuations of supply and demand in the financial market, it is usually difficult to distinguish the market disequilibrium. However, a stock price bubble is one of the most significant examples of market disequilibrium and its detection may be essential to mitigate its potential harm. Although many detection methods for price bubbles have been devised, their empirical applications are not yet complete. Regarding this, transition probabilities between market (dis)equilibrium and their changes will be estimated. Although not every transition is related to a bubble, the changes between bull-bear markets can be part of a bubble. Also, these probabilities may vary depending on the duration of a specific type of market. On the other hand, a structural break at which a different price generating process arises may be relevant to a life-cycle of bubbles or more generally a transition to different market (dis)equilibrium.

In summary, the thesis will cover two main research questions:

(1) Whether and what impact the number of market makers has on the market disequilibrium at the market microstructure

(2) How price bubbles are detected and when/where they exist, and how their transition probabilities and changes are estimated.

The structure of the thesis follows the above research topics and questions. Chapter 2 investigates the impact of multiple market makers on the market disequilibrium and provides empirical evidence. Chapter 3 provides new evidence of price bubbles and the duration dependence of price runs, and investigates how to estimate transition probability and its changes using duration dependence and structural breaks. Chapter 4 summaries all the findings and concludes.

Chapter 2

The impact of multiple market makers on the market disequilibrium at the market microstructure level

2. The impact of multiple market makers on the market disequilibrium at the market microstructure level

At the market microstructure level, market making can be a source of market friction that may cause the market disequilibrium. Some examples are: adverse information between informed traders and market makers, inventory control of market makers and strategic interaction among market participants. Adverse information makes a market maker slowly follow the market equilibrium because he can only gradually retract information from incoming orders, and thus it lengthens the market disequilibrium. Inventory control leads to a market maker's intentional disequilibrium behaviour to restore his preferred inventory level. Strategic interactions can bring various results in terms of the market disequilibrium.

On the other hand, different market microstructures in the stock market can affect this. For instance, two hypotheses can be formed depending on the different impact of the number of market makers. The increased number of market makers may amplify the market disequilibrium since it increases their aggregate power of controlling prices and absorbing demand/supply shocks than the case of a single market maker. This can be called 'the increased resilience hypothesis'. Conversely, multiple market makers for one firm may decrease the size and time length of the disequilibrium because it increases the informational efficiency of the market due to the higher degree of competition. It can be defined as 'the improved efficiency hypothesis'.

Despite the earlier research on theories and some more recent evidence, extensive empirical research on the impact of multiple market makers has not been conducted. This chapter analyses the impact of multiple market makers on price to identify a better supported hypothesis, and also investigates different theories of market making. It adopts intraday return data for empirical analysis. The chapter is structured as follows. Section 2.1 reviews the related literature of market making. Section 2.2 presents a simple market making model which clarifies empirical expectations about the impact of multiple market makers under different hypotheses. Section 2.3 provides empirical evidence and Section 2.4 discusses the findings and concludes.

2.1. Literature review

Market microstructure is the study of the investigation of the economic forces affecting trades and prices, and direct implementation of economic theory to the actual working of markets (Biais et al., 2005). It also studies how specific trading mechanisms affect the price formation process and generate the disequilibrium, unlike other asset pricing studies that focus on the properties of the equilibrium (O'Hara, 1997). The essential idea of market microstructure is that market frictions can impose transaction (or trade) prices not equal to the equilibrium prices or the fundamental prices (Bradfield, 1979, Madhavan, 2000) that represent all discounted future benefits of owning assets in the efficient market. Their empirical study focuses on intraday data.

One of the widely-researched sources of market frictions in market microstructure is market making. Early researchers of market microstructure did not consider the price impact of the market making mechanism (Demsetz, 1968) as market makers are regarded as passive liquidity suppliers against order imbalance. However, they soon realised the price impact of market makers when they control their inventory, which is generally referred to as stock inventory like in Amihud and Mendelson (1980)'s study, but cash inventory can be included as in Garman (1976)'s. For example, in one of the first literature pieces of such research, Garman (1976) emphasized that a market maker must relate his activities to inventory so as not to incur a failure in market making. In doing so, the market maker affects prices and causes a departure from their equilibrium.

Later, the focus was to some degree moved on to the impact of adverse information between a market maker and traders on price patterns (Glosten and Milgrom, 1985, Kyle, 1985, Easley and O'Hara, 1987). More recent developments such as in Madhavan (2000) and Cao et al. (2006) combine two approaches into pricing models where a market maker's decision is affected by both inventory and adverse information.

It is essential to depict who market makers are and how they work in real markets before moving on to the detailed review of the literature on market making. The literature often focuses on the New York Stock Exchange (O'Hara, 1997). On the NYSE, each stock is allocated to one market maker (a designated market maker or formerly a specialist) who is the contact point between sellers and buyers. He is responsible for fairly and orderly market making as well as liquidity supply to the market. The market makers are employees of the NYSE member firms. There are more than 400 market makers operating in the exchange and they may be responsible for more than one listed firm (NYSE, 2003). They are physically located at a trading post on a trading floor and execute more than 95% of orders, mainly electronically via SuperDOT, an order routing system (NYSE, 2006).

In terms of total volume, the designated single market makers participated in 9.3% of market trades with their inventory, and 78.8% of their trading volume stabilised price movement on average between 2001 and 2010 (NYSE, 2010). Specialist stabilisation is when specialists provide liquidity in a way to reduce market volatility. Conversely, when specialists trade for other reasons such as active inventory control, it reduces stabilisation rates (Bacidore and Sofianos, 2002). Designated market makers do not see incoming orders before setting quote prices unlike specialists, and thus they have supposedly utilised only public information on orders since 2011 (NYSE, 2009, NYSE, 2011).

On the other hand, on the NASDAQ, several market makers compete over one stock and their entry and exit are relatively easy. The market makers attempt to post the best prices for their customers. This keeps transaction costs low and makes price discovery easier (Huang, 2002). Their bid-ask prices are on the screen of the NASDAQ trading system called SelectNet. All market making is electronically conducted without a physical floor.

On the London Stock Exchange (LSE), the presence of market makers is important for providing liquidity to low trading volume stocks (Neal, 1992) and is known to raise liquidity compared to stocks without designated market makers (Venkataraman and Waisburd, 2007). Competing market makers quote bid-ask prices using a quotation system called the SEAQ. On the other hand, other financial markets such as foreign exchange and government bond markets may have dealers that simultaneously function as market makers (Lyons, 1995, Bjønnes and Rime, 2000, Gravelle, 2005).

Most of the market participants such as private investors who can put limit orders for specified prices, dealers and brokers can be regarded as competing liquidity suppliers (Cohen et al., 1981, Biais et al., 2005). Then, they can be interpreted as broad-term market makers whose main responsibility is to supply liquidity. This increases the number of total market makers in a market and thus may generalise the results from market microstructure literature.

A market maker works in financial intermediation and consequently causes the cost of immediacy (or intermediary) that impacts prices. The cost of immediacy consists of inventory, information and order processing costs (Stoll, 1978), and it also represents the costs caused by market frictions. Order processing cost is essential in market clearing (O'Hara, 1997), that is, a market maker is compensated for his services by this cost. Thus, related research on market making on prices is naturally divided into two areas (Hasbrouck, 1988): (1) inventory/position control of market makers and (2) adverse information between informed traders and uninformed market makers. Although the development of the theories began without the existence of the electronic trading system, they are relevant in explaining the price change with it at the market microstructure level (Freyre-Sanders et al., 2004)

The inventory control model was originally suggested by Garman (1976). He argued that, when a market maker simply processes orders without considering the level of his inventory, his inventory is going to be depleted subject to a stochastic flow of orders, and thus a market failure is caused. Garman's model also showed that the market maker's bid-ask spread cannot completely eliminate the possible failure. Stoll (1978) further argued that spread can accommodate order processing and information cost, but not inventory cost. Therefore, the market maker must control inventory by lowering his prices after purchase and raising them after sales in order to induce future transactions to equilibrate inventory (Stoll, 1989). Otherwise, a larger inventory position on aggregate will reduce the total liquidity provided by market makers (Comerton-Forde et al., 2010).

A simple representation of an inventory control model is:

$$P_t = E[P_t^*] - f(I_t)$$

where P is a market maker's quote prices as the mid prices of bid-ask prices and $E[P_t^*]$ is the equilibrium price at time t; $f(I_t)$ is the function of the market maker's net inventory (I). In this model, a market maker changes his quotes depending on his net inventory. For example, in the case of a long position i.e., $f(I_t)>0$, he lowers the prices to attract buyers; conversely, he raises them if he is in a short position. A market maker cannot have clear knowledge about the equilibrium price. He may only guess it since, for example, he can evaluate the imbalance of his limit order book (Bradfield, 1979, Bradfield, 1982).

Garman's (1976) original net inventory function depends on the cumulative number of net orders, in which the arrival rates buy and sell orders are assumed to follow Poisson distribution. Prices are set by a market maker's balancing action to prevent market failure. However, the market maker sets the prices only one time in Garman's and also the later Stoll's models, thus they did not consider the cost of conveying inventory over time (O'Hara, 1997).

When time period is extended over one period, the accumulated net inventory position from market opening plays a main role:

$$I_t = \sum_{s=0}^{t-1} I_s$$

where s is a time index.

In Amihud and Mendelson's (1980) multi-period model, a function of the discrepancy between stochastic market demand and supply is used as a net inventory function. A profit-maximising market maker's pricing depends on inventory that is exogenously bound to exclude a possible failure. Ho and Stoll's (1981) utility-maximising market maker's pricing also depends on inventory, but uncertainty about inventory, wealth variances and transaction size is only reflected in spread size not its placement.

On the other hand, as implied in Bradfield (1979) and Amihud and Mendelson (1980), the market maker may have a preferred level of inventory (I^*). The market maker moves his quote prices away from the equilibrium price as the differential between current inventory level and the target level become larger. Then, $f(I_t)$ can be further specified as $\beta(I_t-I^*)$ where β is a positive coefficient of the deviation from the target level (Hasbrouck, 1988). This target can be a range of inventory positions (Bradfield, 1979, Bradfield, 1982).

O'Hara and Oldfield (1986) further included overnight markets between each trading period where all trades settle and a market maker closes his position. Their model was extended to infinite horizon by repeating a trading period, which resembles real markets more. The market maker's price decision is affected by uncertainty about carrying-over inventory and trading profits (π), which is partly decided by random prices at overnight market (P^v). A similar approach was suggested by Zabel (1981). A brief representation of their model is:

$$P_t = E[P_t^* + f(I_{t+1}) + \pi(P_t^v)]$$

Other factors suggested in inventory control models are: a market maker's base wealth (Ho and Stoll, 1981, Stoll, 1978), remaining time before market closure (Ho and Stoll, 1981), institutional feature (Calamia, 1999) and trading practices (Menyah and Paudyal, 2000), economic factors and cross-listing (Hansch, 2004). However, these inventory control models do not explicitly consider the fundamental value of the asset (O'Hara, 1997) as it relies on the exogeneity of supply and demand. This is subject to criticism by the information models later.

Inventory models were supported by empirical evidence presented by Barnea (1974), Amihud and Mendelson (1980), Ho and Stoll (1981), Ho and Macris (1984) and Madhavan and Smidt (1993) among many others. They showed that the inventory position affects price determination and the prices are a decreasing function of inventory. Hasbrouck and Sofianos (1993) and Hansch et al. (1998) confirmed the mean reversion of a relative inventory position supporting the existence of preferred inventory level.

Although this departure from the preferred level can be prolonged (Hasbrouck, 1991, Hasbrouck and Sofianos, 1993), if the possible shift of the preferred level is considered, the departure is not as long as previously thought (Madhavan and Smidt, 1993). That is because mean reversion becomes stronger as relative inventory position moves farther away from preferred level by inventory control (Hansch et al., 1998). Madhavan and Sofianos (1998) also discovered that market makers more actively participate in sell (buy) side trading when they are deeper in a long (short) position.

Naik and Yadav (2003) added that inventory affects the pricing decision regardless of part of it being hedged within the same intermediary. Hendershott (2007) recently presented empirical evidence of inventory control from 1994-2004 NYSE data. Meanwhile, Snell and Tonks (1998), Hansch and Naik (1998) and Hansch (2004) discovered a strong inventory control in the UK data.

One of the weaknesses of the inventory control model is the assumption of stochastic order flows that also represent exogenous supply and demand. It disregards the possibility of informed trading, the existence of fundamental values and the changes of future asset values. Thus, it creates difficulty in both connecting future values to market making decisions and modelling long-term market behaviour (Hasbrouck, 1988, O'Hara, 1997). Consequently, information-based market making models, namely adverse information models emerged. Adverse (or asymmetric) information models were first presented by Glosten and Milgrom (1985), Kyle (1985) and Easley and O'Hara (1987) based on Begehot's (1971) idea of information cost.

Adverse information models dropped the assumption that both sides of trading have the same set of information and instead supposed that some of the traders (the informed traders) have superior private information. Adverse information likely incurs gains to informed traders and losses to uninformed traders when they trade with each other. A market maker is commonly assumed to be uninformed although adverse information can include informational advantage to a market maker (Logue, 1975). Being uninformed himself, a market maker also loses from trading with the informed traders. This adverse information will persist as long as price does not fully reflect private information.

However, adverse information can lead to possible non-trading as uninformed traders do not have strong motives to keep participating in trading. This is because they have no chance to revert their losses under adverse information. It is called Milgrom's no trade theorem (Hasbrouck, 2007). To overcome this, it is usually assumed that the uninformed investors trade for liquidity by exogenous means (Glosten and Milgrom, 1985). This can be also done by asserting the unspecified private values like diversification or risk exposure needs for traders, which generate trading behaviour (Hasbrouck, 2007).

On the other hand, whenever the informed traders attempt to exploit their information advantages, it is possible for their private information to be instantly revealed to the uninformed market maker and other traders. This is because trades are informative unlike in typical inventory models (Hasbrouck, 2007). This instant revelation prevents the informed traders' motives to gather any information, which contradicts adverse information itself. Therefore, limited transaction size, learning framework or uncertain timing of trades is commonly introduced to prevent the instant revelation (Glosten and Milgrom, 1985). In these models, it is assumed that the market eventually reaches full-information prices and clears the market (O'Hara, 1997).

In terms of market making, he will adjust his price as private information is conveyed to him through market transactions like limit orders or block trades. He can even win back the loss from trading with the uninformed traders. This market making behaviour alone can explain the changes in prices or spreads without relying on inventory controlling behaviour regardless of the assumption of risk preference or competition of market makers. The impact of private information was not explicitly considered in inventory control models (Hasbrouck, 1988, O'Hara, 1997).

In adverse information models, when informed traders observe price differentials, they put in corresponding orders. For example, if the informed equilibrium price is higher than a market maker's quote price, traders put in buy orders. Then, as buy orders arrive, the market maker revises his quotes upward because he knows the possibility of the order being information-motivated (Glosten and Harris, 1988). By doing so, he can win back the loss later from trading with the uninformed.

A simple representation of adverse information is:

$$P_t = E[P_t^*] - g(J_t)$$

or equivalently,

$$J_t = g^{-1}(E[P_t^*] - P_t)$$

where $g(J_t)$ is the function of the price impact of net trades (J) by the informed traders, which represents information asymmetry. In this model, transaction prices can be adverse information prices and the equilibrium prices reflect full-information prices (Madhavan, 2002).

The examples include the Copeland and Calai model (1983), where a monopolistic market maker attempts to maximise profits from trading with both informed and uninformed traders by working out the optimal bid-ask prices. However, one period setup with fixed probabilities of receiving buy or sell orders cannot represent a market maker's incentive to change the prices related to true prices. For this, it is essential that incoming orders in a dynamic setup contain informational content (Glosten and Milgrom, 1985, O'Hara, 1997), which a market maker can use for his own profits. In Glosten and Milgrom's (1985) model, a market maker revising his information set against incoming orders creates uncorrelated price changes. It supports a semi-strong form of market efficiency, but still maintains the connection between current transaction data and future prices. Easley and O'Hara (1987) further considered the possibility of non-occurrence of new information.

Kyle (1985), Easley and O'Hara (1987) and Glosten and Harris (1988) considered how the informed traders would act if they can submit an order of different size. They showed that the informed would prefer a larger transaction size to quickly exploit profit opportunity. They also argued that trading volume can be used as a proxy for the amount of private information conveyed to market makers, or in other words, the quality of information (Blume et al., 1994). In a similar context, block trades (Easley and O'Hara, 1987), order imbalance, price history, time interval between trades and sequence of trade can consist of private information which is able to affect the prices. This is because a market maker is able to use one or some of them to revise his quote prices and thus past trades have an impact on current market price (O'Hara, 1997). In the meantime, if the competitive setup is added, the market maker is expected to more fully and immediately adjust the prices (Kyle, 1985), thus improve market efficiency.

One advantage of the information model is that it can model quotes as a market maker's belief which is separate from transaction prices (O'Hara, 1997). However, the information approach has an inherent difficulty of giving enough incentives for market participants including market makers to participate in trades because any information or belief does not guarantee actual payoffs that actually make them stay in the market.

Subsequently, strategic elements are considered in a market maker's revising process against adverse information to provide more incentives to market participants, particularly the informed. Strategic consideration can provide different outcomes from where information alone affects market participants pricing decisions. For example, the informed traders can delay order timing and change transaction size to fully utilise the information they possess. Kyle (1985) focused on the situation that one informed trader strategically and rationally exploits both the distribution and realisation of uninformed traders' net orders and a market maker's pricing rule that considers aggregate net order. In this model, the informed trader can properly hide his information and maximise expected profits by changing his trade size according to the variance of uninformed trades. The market maker's motivation is the responsibility of market making and the uninformed participates in trading due to liquidity concern. These elements are not commonly modelled in inventory control models (Hasbrouck, 2007).

In the meantime, the market maker's linear optimal pricing is represented by:

$$P_t = E[P_t^*] + k(\sigma_U^2) \times (x_U + J_t)$$

where k is the function of the variances, σ_U^2, of net trades by the uninformed (U) and x_U is their realised net trades. Kyle (1984) also argued that when this model is extended into multi-periods, information will be gradually revealed onto prices, but they are still uninformative to the uninformed traders. A similar argument is suggested by Back (1992) in continuous trade setup.

The assumption of multiple informed traders enables competition between them. If strategic elements are not considered, the competition can reveal the information quickly rather than slowly (Holden and Subrahmanyam, 1992). Then, it can drive return to information to zero, and thus removes any incentive to collect information (O'Hara, 1997). However, the revelation process can still be slow if two informed traders differ in the amount of private information and place orders at different times (Foster and Viswanathan, 1994). Holden and Subrahmanyam (1992) actually adopted an auction setup and said that weak competition between traders increases the time of revealing private information. A similar outcome is expected when informed traders are risk averse, which reduces overall market liquidity (Subrahmanyam, 1991). In any case of strategic elements between multiple informed traders being considered, the pay-off to the informed by strategic motivation should be high enough to overcome quick information exposure by competition.

Uninformed traders can adopt trading strategies against the informed or among themselves. For example, it is possible that one of two groups of uninformed traders strategically infer the timing of trades to minimise possible loss (Admati and Pfleiderer, 1988, Foster and Viswanathan, 1990). Uninformed traders facing the informed ones may better strategically split orders (Chordia and Subrahmanyam, 2004). Competition between uninformed traders as liquidity suppliers can direct some of the traders to submit limit orders (Viswanathan and Wang, 2002) and possibly to different market makers (Biais et al., 2000). Meanwhile, the uninformed can be risk-averse hedgers whose trade decisions depend on the number of the informed (Spiegel and Subrahmanyam, 1992).

The adverse information model is also supported by empirical evidence. The existence of the informed traders was supported by Meulbroek's (1992) and Cornell and Sirri's (1992) evidence of abnormal returns from illegal insider trading. Hasbrouck's (1988) early empirical model of quotes shows how to calculate the amount of private information using the trade innovation over the projected demand on public information. He also argued that adverse information generates a permanent shift of prices.

Huang and Stoll (1994) provided the evidence for the persistent impacts of informed transaction. Holthausen (1990) and Keim and Madhavan (1996) assumed large block trades were driven by private information and proved that the price impacts of these trades were permanent. More recently, Dufour and Engle (2000) and Chung et al. (2005) showed that the shorter trade interval, which can mean the higher probability of the trades conveying information, had the larger price impact. Then, private information would more likely explain price volatility than public information (French and Roll, 1986).

On the other hand, adverse information research did not usually investigate public information for its price impacts since public information was supposed not to create adverse information among market participants as they symmetrically know its existence. Instead, research on public information focused on the size of total bid-ask spread and the analysis of its components (Krinsky and Jason, 1996, Voetmann, 2008). Public information can be separately considered as a case of certain adverse information in the market compared with a case of uncertain adverse information caused by private information as Easley and O'Hara (1987) briefly argued. When public and private information co-exist, it is usually assumed that the increase of public information makes the markets more efficient and prices more informative and thus the advantages of informed trading decreases (Kyle, 1984).

In the meantime, it is possible to add back inventory control to adverse information models. The added inventory control provides enough incentives of active price adjustment to market makers under adverse information (Madhavan and Smidt, 1993, Madhavan, 2000, Biais et al., 2005). This is possible because inventory can be interpreted as a type of information privately known to a market maker (Cao et al., 2006) or non-trade information that causes market friction (Hasbrouck, 1991).

Madhavan and Smidt (1993) was one of the first who devised a model to empirically test the importance of two effects. They argued that these types of market makers are also active investors who exploit short-term profitable opportunities using incoming order imbalance while maintaining their long-term inventory target. They also presented a general representation of a combined model (Madhavan, 2000), in which a market maker sets prices as:

$$P_t = E[P_t^* \mid J_t] - f(I_t)$$

or using a market maker's inventory target, I^*:

$$P_t = E[P_t^* \mid J_t] - \beta(I_t - I^*)$$

where β is the coefficient of the inventory effect on the prices. Then, the behaviour of prices transaction to transaction can be represented as:

$$\Delta P_t = -\Delta I_t + \left(\frac{\alpha\sigma_t}{2} + 2\right)\Delta J_t$$

where α is the degree of information asymmetry and σ_t is the standard deviation of P_t^* as the degree of value uncertainty. It says that, as information asymmetry worsens or uncertainty about the true value of the asset goes up, the prices rise. In the meantime, as the market maker accumulates a long position, the prices decrease. On the other hand, if he always takes the opposite position to all orders of customers, ΔI_t can be simply replaced by $-J_{t-1}$. A similar idea was modelled by Stoll (2003) and Biais et al. (2005).

Inventory control and adverse information cause different price impacts. Without the costs related to them, there is only an order processing cost, and then the price tends to bounce between bid and ask prices (Roll, 1984). However, each inventory and information effect can produce more complicated and dissimilar price patterns although they can produce a similar order imbalance (Chordia et al., 2002). According to Hasbrouck (1988), trade price revisions are negatively correlated in the inventory control model, but impacts are only transient because the deviation of inventory is not related to the future values of the asset. This can create mean-reversion of prices. Conversely, price revisions by adverse information are not serially correlated but impacts are persistent because of the changes in future values. However, if a certain limitation is put on the reflection of asymmetric information such as transaction size, it can lead to positive correlation by adverse information (Calamia, 1999).

Madhavan and Smidt (1993) argued that their empirical evidence using quote prices revealed that a market maker's quote revision is negatively related to his transactions and positively related to information delivered through impending orders. This result supports their specification above. There are other mixed results as well. Hasbrouck (1988) found out that negative autocorrelation exists in trade prices of low-volume stocks, but trades have persistent impacts on quote revisions for all stocks. That is, inventory control has an impact on smaller stocks while asymmetric information shocks are present.

The empirical evidence was not conclusive and suggests that both may be in effect. Manaster and Mann (1996) and Madhavan (2000) reached the same conclusion. This mixed result is possibly because lagged effects reside in market making (Hasbrouck, 1991). In particular, the total duration of the effects of inventory control may be longer than those of adverse information (Hasbrouck and Sofianos, 1993). For example, price adjustment caused by large block trades, interpreted as private information, completes within as small as three trades (Holthausen et al., 1990), which is much faster than price adjustment by inventory control that can last several days (Hansch et al., 1998). Also, preferred inventory position itself, for which empirical tests aim, may deviate in the long-run (Madhavan and Smidt, 1993). Consequently, all of these effects may cause difficulties in separating two effects, or both effects may be intrinsically difficult to distinguish.

In the NASDAQ and the LSE, more than one market maker operates for each traded stock of a firm unlike the NYSE. For example, the NASDAQ has around 14 market makers per stock. The impact of multiple market makers on market making behaviour and price movement may differ and deliver more implications to market microstructure research. For example, in a set up of adverse information, Glosten and Milgrom (1985) show that if perfect competition exists between market makers, they all know each other's prior information and optimal prices. Their price adjustments will be instant and essentially the same. Then, they end up with zero economic profits. That is, market making does not affect price patterns. On the other hand, if multiple market makers are subject to a certain heterogeneous inventory and information structure or strategically interact with each other as seen later, it is expected that different price adjustment patterns and positive profits emerge. This type of strategic element was not considered in pure inventory control models even with multiple market makers (Hasbrouck, 2007).

Ho and Stoll (1983) first introduced two market makers trading two different stocks with market traders in a one-period inventory control model in which inter-dealer trading is allowed. Each market maker attempts to maximise his utility based on inventory, cash and initial wealth. The market makers keep their prices more strongly than a single market maker because they are ready to trade more shares. In this model, each market maker's price adjustment does not depend on the other's inventory level since they do not strategically consider what the competitor does although his inventory level consists of two stocks. On the other hand, Biais (1993) suggested strategically competing market makers where they are able to observe others' quoted prices and assess inventory positions. He anticipates that the market makers' differences in risk aversion to inventory lead to a positive relationship between the number of market makers and the volatility of asset price. This is opposite to Ho and Stoll's expectations.

Likewise, there are two conflicting lines of study regarding the impact of multiple market makers on price and volatility. First, a multiple market maker system can limit price adjustment (the increased resilience hypothesis). More market makers can increase their ability to keep the price (Ho and Stoll, 1983). Also, they are more willing to maintain a profitable price even under adverse information. For example, Bernhardt and Hughson (1997) showed that informed traders put split orders to more than one market maker to keep their advantage longer. However, it enables the market makers to maintain less-competitive prices. That may create positive economic profits for them unlike where only a single market maker exists. Then, it generates resilience in price movement and reduces volatility.

Also, it is possible that multiple market makers' information processing and inventory control contain strategic elements that prevent instant price adjustment, for example by setting more attractive prices than when they encounter only the problem of adverse information (Ángeles de Frutos and Manzano, 2005). This disequilibrium behaviour happens because they attempt to infer information by attracting more orders, which also shows their collective power of price setting. On the other hand, dealers acting as market makers also trade with each other in the interdealer market (Hasbrouck, 2007). This trading is used either to control their inventory position (Ho and Stoll, 1983) or to increase their profits at other market makers' quotes. They would exploit each other's quotes through interdealer trading until it cannot improve their inventory positions (Ho and Stoll, 1983). The inter-dealer trading in this case possibly slows down price adjustment.

The possibility that one market maker has superior information to the others was investigated in more recent models. For example, Calcagno and Lovo (2006) argued if one market maker knows the true value of the asset, he can gain positive profits by slowly revealing the information in quotes and trades. They also presented interview results arguing that real market makers know the existence of leading quote setters.

Second, if multiple market makers indeed make price discovery easier due to competition (Holden and Subrahmanyam, 1992, O'Hara, 1997, Huang, 2002), prices become more informationally efficient (the improved efficiency hypothesis). The market makers' risk aversion to inventory (Biais, 1993) may be responsible for this faster infusion of information shocks into the price process. Biais et al. (2000) supposed that both informed traders and uninformed multiple market makers were risk averse but to a different degree while the market makers still consider inventory level. Then, he showed that the market prices in his model depend on informed prices, the degrees of risk aversion, inventory level and the number of market makers. In particular, as more market makers operate, price deviation caused by adverse information is increased.

Also, strategic games among them (Meyer and Saley, 2003) can produce quick price movement to the equilibirum. Similarly, the facilitation of competition using electronic communication networks increased market efficiency compared with non-computer based market making (Huang, 2002). When inter-dealer trading is permitted, for example conducted via the Inter-Dealer Broker (IDB) system in the NASDAQ (Gravelle, 2005), it may bring more competitive outcomes.

Viswanathan and Wang (2004) allowed active inter-trading between market makers following a customer order in a multi-stage trading model. Increasing competition in their model drives profits to the market makers to zero. Even if one of the market makers is informed, one-sided information is able to still produce a fair game when two market makers perform the repeated game of trading both risk-free and risky assets (Meyer and Saley, 2003). Multiple market makers theoretically produced the higher volatility of returns than a single marker maker system in a sequential trade model (Biais, 1993). Then, the prices of multiple market makers have relatively weaker predictability as uncertainty goes up than those with a single market maker. On the other hand, Flood et al. (1999) proved improved efficiency in an experiment with multiple market makers. This improvement of market efficiency can be represented by reduced spread size (Mayhew, 2002).

The history of market making theories shows that the initial focus of research moved from stochastic orders and inventory control to adverse information. Then, the subsequent research was combined into a setup which assumed that market participants' behaviours including market makers contained strategic elements. There were dissimilar expectations regarding the impacts of multiple market makers.

2.2. The impact of multiple market makers in the neoclassical market making model

This section constructs a simple market making model that illustrates two hypotheses of the price implication under a multiple market maker system. The basic setup of the model is that a representative market maker adjusts his prices responding to external shocks while considering excessive inventory position. Shocks represent the potential changes in the equilibrium price and they may be random shocks or come from the informed traders.

The market in this model trades one identical type of financial asset which has real value. Sellers and buyers submit orders to single or multiple market makers.

The market traders are homogenous, that is, their individual variables like income and wealth are identical. A market maker has the power of price setting but is bound by the responsibility of clearing incoming orders. He is uninformed about the changes in the equilibrium price brought by information shocks.

There is adverse information between market traders and a market maker. The market traders are informed in the sense that they can find the equilibrium price and its changes collectively. In other words, they possess collective knowledge of the market-clearing equilibrium prices where they have no excess demand. The shocks to the equilibrium price exogenously arise. Using this superiority, they are able to discover any discrepancy between a market maker's prices and the equilibrium prices, and then place market orders for arbitrage profits. In this sense, they are arbitrageurs.

Market making and trading behaviour are modelled in a discrete time framework. This is because quote and trade prices are discrete-valued in nature e.g. multiples of tick size (Tsay, 2005) and most series in market microstructure consist of discrete events that may randomly happen in continuous time (Hasbrouck, 2007) due to non-trading. Therefore, although it may be feasible to treat the data series as a continuous variable realised at regular discrete intervals for some purposes (O'Hara, 1997, Hasbrouck, 2007), the choice between a discrete and continuous time framework is not critical in this study.

A multiple period model is employed because a one-period model has difficulty in approximating a market maker's incentive of changing prices (O'Hara, 1997). The number of trading periods is finite and a market maker is well aware of the possibility that he must close his position at a certain point in the future. Thus, he has intention at least to return to his preferred inventory as close as possible. It is assumed that his preferred inventory is a squared position. A market maker's acquired position is carried over to the next trading period, but overnight markets (O'Hara and Oldfield, 1986) that can settle transactions and accumulate information are not considered.

The other technical assumptions are: no commission, taxes and other transaction costs. Also, a market maker does not pay any explicit cost of holding long or short positions although he may incur loss when closing this position.

Now, suppose the monopolistic market maker is faced with an information shock, his price at time t (P_t) is different from the new unknown equilibrium ($E[P_t^*]$) by the amount of mis-estimation of ζ_t that is essentially the error term representing price differential from the equilibrium prices.

Equation 1

$$P_t = E[P_t^*] + \zeta_t$$

where ζ_t is a random variable identically and independently distributed with zero mean and constant variance. $E[P_t^*]$ is uncorrelated, but left as unspecified for now.

Suppose the demand by informed traders follows a simple neoclassic demand function at any time. That is, their demand at time t (c_t) is proportional to the difference (ζ_t) between the market makers price and the new equilibrium. Let α' be the positive coefficient.

Equation 2

$$c_t = -\alpha' \times \zeta_t = -\alpha'(P_t - E[P_t^*])$$

or

$$P_t = -\frac{1}{\alpha'} c_t + E[P_t^*]$$

It is a linear demand function with the slope of $-(1/\alpha')$. Subsequently, the market maker's inventory position at the next period (I_{t+1}) changes by the demand:

Equation 3

$$-(I_{t+1} - I_t) = c_t$$

As an uninformed market maker processes market orders, his excess inventory position is accumulated.

It can be also assumed that a market maker's price adjustment becomes stronger as the change in inventory position gets larger. Then, it is represented by a simple linear function of inventory control.

Equation 4

$$P_{t+1} - P_t = -\beta'(I_{t+1} - I_t) = \beta' c_t$$

where β' is another positive coefficient.

Then, the trader demand and the market maker's interim inventory control can be combined by substituting the trader demand for c_t in the above.

Equation 5

$$P_{t+1} - P_t = \beta' c_t = \beta'[-\alpha'(P_t - E[P_t^*])]$$

or

Equation 6

$$P_{t+1} = \gamma' E[P_t^*] + (1-\gamma')P_t$$

where γ' = α'β'

This is the market maker's adjusted price that is a weighted average between the equilibrium price and his current price. The price generating process will be a weighted average between the process of the equilibrium price and his past price. The randomness of the market maker's pricing is generated from the uncertainty of the equilibrium price.

Now, the empirical expectations of the impact of multiple market makers can be derived. If a multiple market maker system brings in price resilience (the increased resilience hypothesis), their prices will be less dependent on customer demand. That is, β' will be close to 0 in a multiple market maker system in Equation 4. That will lead to a smaller γ' and larger (1- γ') in Equation 6. The market maker's price will show less random deviation and stronger positive autocorrelation than a single market maker system. Their prices will more distantly drift away from the equilibrium price.

On the other hand, if multiple market makers increase efficiency (the improved efficiency hypothesis), β' will be close to 1 as they reflect the changes in customer demand more and then γ' will be increased. Consequently, the price will show larger random movement but move closer to the equilibrium price than a single market maker system. If the equilibrium price follows a random walk, the market maker's price will change similarly to a random walk.

Therefore, by comparing the properties of price movements in two market making systems, the more preferred hypothesis can be found. In sum, smaller volatility and stronger autocorrelation with multiple market makers prefers the increased resilience hypothesis while larger volatility and weaker autocorrelation prefers the improved efficiency hypothesis. This is consistent with the literature.

Although the price generating process of market makers depends on how the equilibrium price is specified, one simple (and unrealistic) specification can provide the time series structure of a market maker's prices. That is, it can be assumed that $E[P_t^*]$ does not change over time i.e. $E[P_t^*]=P^*$. Then, Equation 6 becomes:

$$P_{t+1} = \gamma' P^* + (1-\gamma')P_t$$

It is essentially a difference equation and can be solved by forward successive substitution as follows:

$$\begin{aligned} P_t &= \gamma' P^* + (1-\gamma')P_{t-1} \\ &= \gamma' P^*[1+(1-\gamma')+(1-\gamma')^2+\ldots+(1-\gamma')^{t-1}]+(1-\gamma')^t P_0 \\ &= \sum_{j=0}^{t-1}(1-\gamma')^j \gamma' P^* + (1-\gamma')^t P_0 \end{aligned}$$

where $t < \infty$

This price pattern is equivalent to the pattern expected from exponential smoothing which is known to be regarded as an ARIMA (0,1,1) model with MA coefficient (1- γ') (Brooks, 2008). (1- γ') will be larger in a multiple market maker system under the increased resilience hypothesis and be smaller under the improved efficiency hypothesis. This is an interesting property to test, but is based on an unrealistic assumption.

2.3. Empirical analysis

Theoretical concepts in market microstructure such as adverse information cannot be directly observed nor easily quantified. It makes the comparisons of empirical expectations between different market making systems inherently difficult (Andersen et al., 2001). Therefore, indirect empirical evidence must be used to find support for the hypotheses about the impact of multiple market makers and for the theories presented in the literature review.

This section will look for such evidence by examining empirical data in the following three points. First, this section examines which hypothesis better matches with empirical data: improved efficiency or increased resilience. The related literature was reviewed in Section 2.2 for two hypotheses and the simple neoclassical market making model illustrated how those empirical expectations are contained in the model. Additionally, based on the increased resilience hypothesis, multiple market makers are expected to lower non-stationarity because of price resilience. However, the improved efficiency increases non-stationarity due to quick and persistent price adjustment assuming independent shocks. Predictability is conversely affected. Table 1 summaries the empirical implications where H is for higher values and L is for lower values. Note that these hypotheses are based on adverse information and strategic interaction models.

	Increased resilience		Improved efficiency	
	Single	Multiple	Single	Multiple
Volatility	H	L	L	H
Non-stationarity	H	L	L	H
Predictability	L	H	H	L
(+)Autocorrelation	L	H	H	L

Table 1 The increased resilience hypothesis and the improved efficiency hypothesis

Second, this section also checks which market making theory, inventory control or adverse information, is broadly supported by empirical findings. Different market making models may expect varying degrees of stationarity. For example, the price processes in the inventory control model are likely to be stationary due to its mean reversion feature (Hasbrouck, 1988) while those in the asymmetric information model will have stronger non-stationarity.

Also, they may produce different types of autocorrelation in price increments. According to Hasbrouck (1988), Madhavan and Smidt (1993) and Calamia

(1999), common inventory control models argue that inventory control produces the negative autocorrelations of increments and the mean reversion of prices, but asymmetric information incurs positive or no correlation in increments because of slow or instant release of information. These empirical properties of two market making models are summarised in Table 2. However, these two criteria are only suggestive because two effects may be mixed in the data and the use of averaged series over many days probably blurs any difference.

	Inventory control model	Adverse information model
Non-stationarity	Weaker	stronger
Sign of autocorrelation	(-)	(+) or none

Table 2 Empirical properties of market making models

Last, time series structures and volatility models are investigated and compared between systems. Therefore, in terms of the areas of investigation, five empirical areas are examined to spot the impact of multiple market makers: descriptive statistics, stationarity in terms of the existence of unit roots, predictability using variance ratio tests, time series structure based on autocorrelation and partial autocorrelation functions, and volatility and GARCH modelling.

The following sub-sections will basically examine the dataset as a whole at the same time compare two sample groups in terms of the characteristics of trade prices. The two sample datasets of share prices are from different, but similar in many aspects, market microstructures: a single market maker system of the NYSE and a multiple market maker system of the NASDAQ. The empirical analysis in the remainder of the section begins with descriptive statistics, and continues to stationarity, general predictability, time series structure, and volatility modelling.

2.3.1. Data

For the empirical analysis, 15 firms that have a single designated market maker (Group A) and 34 firms that have multiple market makers (Group B) are selected. These firms are selected among US S&P SmallCap 600 index because a market maker's activities are stronger in trading of the shares of small firms. This index covers the listed firms in which market capitalisation is between $300mil and $1.4bil (Standard&Poor's, 2010). All the selected firms have the names beginning with either I or T to give randomness in the selection process. All the firms in Group A are listed on the NYSE and all the firms in Group B are listed on the NASDAQ. The list of the firms in Group A are in Table 3 and the list of the firms in Group B are in Table 4.

Number	Name	Code
1	Ion Geophysical Corporation	IO
2	Inland Real Estate Corporation	IRC
3	Intermec Inc	IN
4	Invacare Corporation	IVC
5	Investment Technology Group Inc.	ITG
6	Tanger Factory Outlet Centers	SKT
7	Teledyne Technologies Inc	TDY
8	Tetra Technologies Inc	TTI
9	The Dolan Company	DM
10	Toro Co	TTC
11	Tredegar Corp	TG
12	TreeHouse Foods Inc	THS
13	Triumph Group Inc	TGI
14	TrueBlue Inc	TBI
15	Tyler Technologies Inc	TYL

Table 3 The list of the firms in Group A

Num.	Name	Code	Num.	Name	Code
1	ICU Medical Inc	ICUI	18	THQ Inc	THQI
2	II-VI Inc	IIVI	19	TTM Technologies Inc	TTMI
3	IPC The Hospitalist Co Inc	IPCM	20	Take-Two Interactive Software	TTWO
4	Iconix Brand Group	ICON	21	Taleo Corp A	TLEO
5	Independent Bank Cp (MA)	INDB	22	Tekelec Inc	TKLC
6	Infinity Property & Casualty Ins Corp	IPCC	23	Tessera Technologies	TSRA
7	Infospace Inc	INSP	24	Tetra Tech Inc	TTEK
8	Insight Enterprises Inc	NSIT	25	Texas Capital Bancshares	TCBI
9	Insituform Technologies Inc	INSU	26	Texas Industries Inc	TXI
10	Integra Lifesciences Hldg	IART	27	Texas Roadhouse	TXRH
11	Integral Systems Inc	ISYS	28	Tompkins Financial Corporation	TMP
12	Interactive Intelligence Inc	ININ	29	Tower Group	TWGP
13	Interface Inc A	IFSIA	30	Tradestation Group Inc	TRAD
14	Interval Leisure Group	IILG	31	Triquint Semiconductor	TQNT
15	Intevac Inc	IVAC	32	True Religion Apparel Inc	TRLG
16	Teletech Holdings Inc	TTEC	33	Trustco Bank Corp (NY)	TRST
17	The Ensign Group	ENSG	34	Tuesday Morning Corp	TUES

Table 4 The list of the firms in Group B

The sample period is one month between 1 May 2010 and 31 May 2010. Trading days are 20 days from 3 May 2010 to 28 May 2010 that covers 4 weeks from Monday to Friday. Trading hours are from 0930 to 1600. The number of observations for each firm is 1,560. The original data set of per-transaction prices was downloaded from the TAQ database on the Wharton Research Data Services (WRDS). This dataset was then converted into 5-minute average prices to prevent the non-synchronicity issue with time. The conversion was completed on Excel using AVERAGEIFS functions.

As seen in the following Figure 1 and Figure 2, the overall price pattern of S&P SmallCap 600 index in the sample period is not largely different from that of S&P 500 index that represent larger firms in the US stock markets in this sample period.

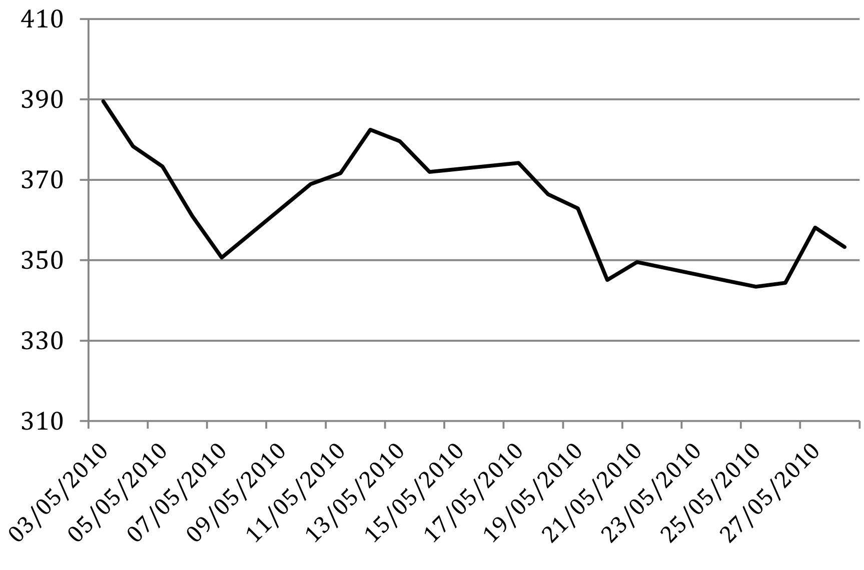

Figure 1 S&P SmallCap 600 Index from 03/05/2010 to 28/05/2010

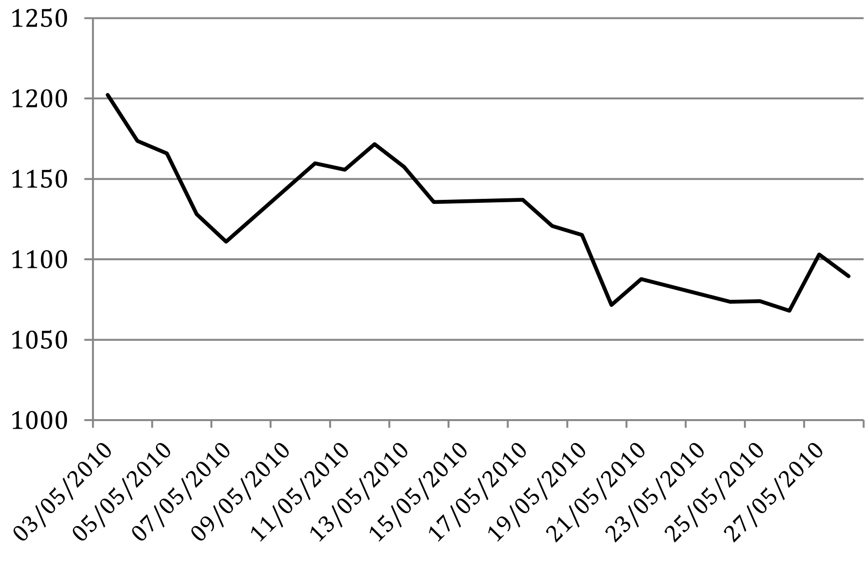

Figure 2 S&P 500 Index from 03/05/2010 to 28/05/2010

2.3.2. Descriptive statistics

The mean and standard deviation of price, log price (the logarithm of price), and log return series of Group A and Group B are presented in Table 5 alongside the average skewness, kurtosis and the Jarque-Bera (JB) statistics of log returns. The descriptive statistics are produced using the Excel Analysis Toolpak add-on.

The average mean of log return is negative as expected from the price pattern in Figure 1. Remembering that this is 5-min data, the absolute value of the return is only around 0.005%. N_A is the number of Group A firms in the sample and N_B is that of Group B firms.

	Group A N_A=15	Group B N_B=34	Difference-of-means t test (p-value)
Price			
Mean	26.2298	19.6758	0.2411
S.D	1.1051	0.8275	0.2899
Log price			
Mean	3.0133	2.8177	0.3752
S.D	0.04487	0.0471	0.6713
Log returns			
Mean	-0.000046	-0.000060	0.3679
S.D	0.0035	0.0039	0.2070
Skewness	0.4874	0.4315	0.9204
Kurtosis	44.5765	60.8539	0.2119
Jarque-Bera	167190.1447	445191.1915	0.2343

Table 5 Descriptive statistics of the share prices and log returns

Both sample probability distributions of the intraday 5-minute trade returns of the firms in Group A and B on average show positive skewness. Only 4 firms in Group A and 9 firms in Group B have negative skewness. This is consistent with Amihud and Mendelson's (1987) earlier research on the daily returns of the NYSE, but different from the returns distributions of typical daily returns (Mills and Markellos, 2008) and 15-per-day intraday returns (Markellos et al., 2003). In terms of kurtosis, both show leptokurtic distribution which is characterised by fat-tailed and highly-peaked distribution that are consistent with daily data (Mills and Markellos, 2008).

The difference between Group A and Group B are analysed using a difference-of-means t test (a unpaired t test), which compares the means of two groups of samples for difference using t statistic assuming unequal variances (Sullivan, 2011):

$$t = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{\frac{\hat{\sigma}_A^2}{N_A} + \frac{\hat{\sigma}_B^2}{N_B}}}$$

where X_A is price or log return of Group A and X_B is those of Group B. $\bar{X}$ is the sample mean and $\hat{\sigma}^2$ is the estimated variance. N_A and N_B are the sample sizes of each group. The test results between Group A and B are summarised in the rightmost column in Table 5.

It seems that the multiple market maker system brings in the higher volatility, thus supports the improved efficiency hypothesis. However, the difference is not statistically significant. Meanwhile, none of the individual null hypotheses of no difference in two means, standard deviations, skewness and kurtosis are rejected.

Additionally, the normality of the return series is tested and compared. The Jarque-Bera (JB) test is a test for normality. Its statistic is calculated using skewness (S) and kurtosis (K) of a series as:

$$JB = N_i \left[\frac{S^2}{6} + \frac{(K-3)^2}{24} \right]$$

which follows χ^2 distribution with 2 degrees of freedom and N_i is the number of firms in the group. All of the individual test results reject the normality of both sample and benchmark firms. That is common in financial data due to extreme values or volatility clustering (Brooks, 2008). If the non-normality persists in the error of the regression models, it possibly affects the validity of the statistical tests. A difference-of-means test shows that there is no statistically significant difference between the JB statistics of two samples. Therefore, it can be concluded that no statistical difference between two groups was found in terms of descriptive statistics.

2.3.3. The comparison of stationarity with unit roots

The first non-descriptive characteristics that will be compared between Group A and Group B are the existence of a unit root in price series. If a series contains a unit root, it characterises a non-stationary process which can also defy the validity of statistical tests and cause spurious regression, but possibly direct the subsequent time series modelling. Then, any shock to a variable of interest can be interpreted as a permanent shock (Campbell et al., 1997), which may be due to some shocks coming from adverse information in market making. If the market is efficient, it may result in instant and full price adjustment to the new equilibrium and thus increase non-stationarity of price. On the other hand, if most of the shocks are temporary, the series will be mean-reverting and will not contain a unit root. Inventory control may be dominant in this case.

In this section, an Augmented Dickey-Fuller (ADF) test (Gujarati and Porter, 2009) is first used for a unit root test. This test can control the effects of autocorrelation of the original error terms (i.e. autoregressive structure of the increments) on a unit root test that cannot be addressed in the Dickey-Fuller test. The specification for ADF tests for price series using ρ lags (Brooks, 2008) is:

$$\Delta P_t = \psi P_{t-1} + \sum_{i=1}^{\rho} \alpha_i \Delta P_{t-i} + \varepsilon_t$$

where α and ψ are coefficients. The null hypothesis is that ψ=0. That is, the original non-differenced price series has a unit root. The alternative is ψ<0, which is a property of a stationary series. If the null cannot be rejected, the original series is:

$$P_t = P_{t-1} + \sum_{i=1}^{\rho} \alpha_i \Delta P_{t-i} + \varepsilon_t$$

The proper selection of ρ is important in testing the unit root of the price series of Group A and B. The maximum number of lag (ρ) is set at 12, which represents 1 hour lag in real time. Although there is not an obvious choice of ρ in higher frequency financial data (Brooks, 2008) unlike quarterly and monthly economic data, the maximum lag of 1 hour seems reasonable in intraday data. Then, for each firm data, the optimum number of lags (ρ) is selected based on the smallest Schwarz Information Criteria (SIC, also known as Bayesian Information Criteria) as in Gujarati (2009).

Table 6 shows p-value and the optimal number of ρ from the ADF tests for the price series of each firm. In all cases, the null hypothesis cannot be rejected at the 5% significance level. Thus, all series seem to have a unit root. All the time series analyses in this section are conducted using Eviews statistical software unless otherwise stated. Log price series have almost identical results.

Price			Log price		
Firms	p-value	Lag (ρ)	Firms	p-value	Lag (ρ)
IO	0.2371	1	IO	0.2411	1
IRC	0.5295	4	IRC	0.5436	4
IN	0.5654	2	IN	0.5561	2
ITG	0.4749	2	ITG	0.4725	2
IVC	0.6480	2	IVC	0.6566	2
DM	0.0531	1	DM	0.0554	1
SKT	0.1711	0	SKT	0.1584	0
TDY	0.4573	2	TDY	0.4028	1
TTI	0.3354	1	TTI	0.3587	1
TTC	0.1580	0	TTC	0.1627	0
TG	0.1376	0	TG	0.1318	0
THS	0.2009	3	THS	0.1962	3
TGI	0.4640	1	TGI	0.4612	1
TBI	0.8631	2	TBI	0.8753	2
TYL	0.5088	1	TYL	0.5112	1

Table 6 ADF unit root tests of the prices and log prices of Group A

Meanwhile, the panel unit root test can give an overall idea about the existence of a unit root of all the series in each group. First, the Fisher-type unit root test (Baltagi, 2005) is conducted using p-values from the individual ADF tests. Its test statistic, p_{PANEL}, is known to follow χ^2 distribution where i is each individual time series:

$$p_{PANEL} = -2\sum_{i=1}^{N} \ln p_{ADF,i}$$

The results cannot reject the null of all unit root processes with unit roots with a test statistic of 34.8446 and p-value of 0.2483. The null hypotheses of the log price series are not rejected with a p-value of 0.2420. They support the existence of unit roots.

Next, another panel unit root test, the Im, Pesaran and Shin test (Baltagi, 2005) tests the null hypothesis that each series in the panel has a unit root against the alternative hypothesis that only some series have unit roots. This test is essentially based on the average of individual t statistics of ADF tests.

$$\overline{t} = \frac{1}{N}\sum_{i}^{N} t_{ADF,i}$$

where $t_{ADF,i}$ is the value of the ADF statistic of firm i.

The test statistic of the panel of Group A is -1.3687 and its p-value is 0.0850. At the 5% significance level, the null hypothesis cannot be rejected, which further supports the finding in the ADF tests. Log price series provided the same results with a p-value of 0.0860.

Table 7 presents the results of ADF tests for the prices of Group B. Compared with Group A, a lower number of series contain a unit root. At the 5% significance level, 5 are rejected (14.7% of Group A) and this is increased to 9 cases (26.5%) at the 10% level. In Table 8, the ADF test results of log prices show a slightly fewer number of rejections. 4 and 7 series are rejected at the 5% and 10% significance level, respectively.

Price					
Firms	p-value	Lag	Firms	p-value	Lag
ENSG	0.0406	2	THQI	0.3117	3
IART	0.0182	2	TKLC	0.1232	1
ICON	0.2968	1	TLEO	0.0729	2
ICUI	0.5997	2	TMP	0.3865	1
IFSIA	0.4642	2	TQNT	0.0997	2
IILG	0.2116	2	TRAD	0.5513	2
IIVI	0.2623	4	TRLG	0.0608	1
INDB	0.3143	0	TRST	0.7534	3
ININ	0.3806	0	TSRA	0.3365	3
INSP	0.1821	1	TTEK	0.3902	2
INSU	0.6603	2	TTEC	0.7802	2
IPCC	0.0439	0	TTMI	0.5393	1
IPCM	0.2444	1	TTWO	0.4358	1
ISYS	0.6621	0	TUES	0.0358	2
IVAC	0.7245	4	TXI	0.2103	0
NSIT	0.1919	1	TWGP	0.3881	0
TCBI	0.1335	2	TXRH	0.0130	1

Table 7 ADF unit root tests of the prices of Group B

Log price					
Firms	p-value	Lag	Firms	p-value	Lag
ENSG	0.0941	4	THQI	0.3366	3
IART	0.0307	2	TKLC	0.1392	1
ICON	0.2908	1	TLEO	0.0688	2
ICUI	0.6053	2	TMP	0.3745	1
IFSIA	0.4705	2	TQNT	0.1029	2
IILG	0.2199	2	TRAD	0.5723	2
IIVI	0.2619	4	TRLG	0.0616	1
INDB	0.3147	0	TRST	0.8258	5
ININ	0.3750	0	TSRA	0.3601	3
INSP	0.2124	1	TTEC	0.8278	2
INSU	0.6671	2	TTEK	0.3924	2
IPCC	0.0453	0	TTMI	0.5368	1
IPCM	0.2460	1	TTWO	0.4094	1
ISYS	0.7063	0	TUES	0.0334	2
IVAC	0.7378	4	TWGP	0.3858	0
NSIT	0.1968	1	TXI	0.1950	0
TCBI	0.1353	2	TXRH	0.0129	1

Table 8 ADF unit root tests of the log prices of Group B

The test statistic from the Fisher-type unit root test of panel data is 104.98 and the p-value is 0.0027. In the case of log prices, they are 101.175 and 0.0056, respectively. These results reject the null hypothesis of all series having a unit root for the alternative hypothesis that some of them do not have unit roots. An Im, Pesaran and Shin test also rejects the same null hypothesis with a statistic value of -3.5227 and p-value of 0.0002 in the price series and -3.2754 and 0.0005 in the log price series.

These (panel) unit root tests revealed a clear difference between Group A and Group B. The price series of Group A firms have unit roots, so they may be subject more strongly to adverse information. Meanwhile, some of the price series of Group B firms do not have unit roots, and their difference is statistically confirmed.

Also, it can be concluded that multiple market makers may create price stationarity to some degree compared with a single market maker case. In other words, the time series of the share price of the firms under multiple market makers (Group B) are more likely to be stationary with a mean reverting property. This difference supports the increased resilience hypothesis while a generally high level of non-stationarity in both groups may support the dominance of adverse information shocks in the sample.

In the meantime, the above results of the unit root tests give evidence for unit roots in most of the price series. The use of the differenced series eliminates the unit root (Brooks, 2008) before proceeding with any time series modelling. In finance research, the convention is actually to use log return series, that is, the differenced series of the natural logarithm of prices. The log return series removes unit roots from a unit root series just like simple differencing. Also, it has some other advantages over the differenced series since it can approximate relative changes well in the given series for small changes (Gujarati and Porter, 2009), which is also scale-free and easily compounded by addition i.e. additive. However, the use of return series may remove some structures in price series and introduce others.

On the other hand, the log return series of all firms in Group A and Group B are additionally tested for unit roots using ADF, Fisher-type and Im, Perasan and Shin tests. All the test results of individual series and panel data reject the null hypothesis with a p-value of 0.000. Thus, it supports that unit roots do not exist in the log return series.

However, it may require further testing for how price or returns are correlated. Even if the null hypothesis of the unit root tests for a price series is not rejected, it cannot be concluded that the increments, e.g. log returns, do not have correlations or predictability in general. The reason for this is that unit root tests cannot dispute the predictability of movement (Campbell et al., 1997) originating from correlated errors. Note that optimal lag in unit root tests being larger than 0 in price series implies the autoregressive structure in the differenced series, or equivalently that of log return series.

2.3.4. The analysis of predictability using variance ratio tests

When the log price follows a random walk, or equivalently log returns are identically and independently distributed with zero mean and constant variance, the variance of return increments over a specific time period has a linear relationship with the time gap between the first and last observations of the period (Lo and MacKinlay, 1988, Campbell et al., 1997). A variance ratio test is based on the following statistics to test for the departure from random walks.

$$VR(q) = \frac{\mathrm{var}(r_t + r_{t-1} + \ldots + r_{t-q+1})}{q\,\mathrm{var}(r_t)}$$

where r_t is a log return and q is a time gap. This is equal to one when the variable follows a random walk. In practice, the standardised variance ratio statistic (Campbell et al., 1997) is used, which is asymptotically normally distributed with zero mean and unit variance. Since this test uses a random walk as the basis for the null hypothesis, the rejection of the null hypothesis implies predictability like an autoregressive structure (except random walk). Note that earlier unit root tests focus on the existence of unit roots only, so they can contain an autoregressive structure of the increments in the null hypothesis of unit roots.

The original test assumes identically and independently distributed increments. However, this linear relationship can hold where the increments are only independent or uncorrelated (Campbell et al., 1997). In this sense, this variance ratio test is able to detect a general departure from a random walk (Mills and Markellos, 2008) and allows for heteroskedasticity which is found in autoregressive conditional heteroskedasticity (ARCH) processes (Lo and MacKinlay, 1989). Meanwhile, there exists the joint test of the null hypotheses for all time gaps of interest. The Chow-Denning max|z| statistic is commonly used for this purpose, which utilises the maximum absolute value of individual test statistics (Chow and Denning, 1993, Mills and Markellos, 2008, Charles, 2009)

The inventory control model inherently has the property of mean reversion, and thus the variance ratio tests on the data that are strongly affected by inventory control may be more likely to reject the null. On the other hand, if the data series experiences a large number of the independent adverse information shocks, the variance ratio tests cannot reject the null. Only when external information shocks are correlated in some way or price adjustment to new information is slow, can the null be rejected. On the other hand, multiple market makers may generate higher predictability under the increased resilience hypothesis and lower predictability under the improved efficiency hypothesis assuming independent shocks.

Table 9 presents the variance ratios of the log return series of Group A and p-values for the tests for the null hypothesis of random walks. To be precise, the variance tests used here allow for heteroskedasticity, so the null hypothesis is that a return process follows a random walk with uncorrelated increments. That is, the increments can be non-independent and non-identical. 4 different time gaps are selected 4 (20 minutes), 12 (1 hour), 24 (2 hours) and 48 (4 hours). Additionally, the results of the Chow-Denning test, joint significance test, of all 4 variance ratios are provided (Chow and Denning, 1993). Note that the presented variance ratios are not directly used for the tests. The max |z| statistics and p-values need to be calculated using the heteroskedasticity-robust variance ratio test statistics by Lo (1989).

The null hypotheses are rejected in all cases. The results display that all log price processes of Group A firms are not random walks. The joint test results also confirm them. On the other hand, as the time gap becomes larger, the values of variance ratios decrease away from the value of 1. However, in terms of p-values of the variance ratio tests, there is a slight tendency of less-likely rejection of the null hypothesis although it does not lead to non-rejection of the null in any case.

Gap	4		12		24		48		Joint test	
	Variance ratio	p-value	Variance ratio	p-value	Variance ratio	p-value	Variance ratio	p-value	max|z| Statistic	p value
IO	0.3278	0.0000	0.0989	0.0000	0.0509	0.0001	0.0270	0.0009	4.7793	0.0000
IRC	0.3092	0.0000	0.0904	0.0000	0.0485	0.0001	0.0256	0.0009	5.4248	0.0000
IN	0.2849	0.0001	0.0853	0.0004	0.0451	0.0017	0.0235	0.0062	3.8268	0.0005
ITG	0.2445	0.0000	0.0710	0.0000	0.0397	0.0000	0.0207	0.0002	5.8241	0.0000
IVC	0.2808	0.0001	0.0901	0.0004	0.0456	0.0012	0.0245	0.0037	3.9059	0.0004
DM	0.3368	0.0000	0.1045	0.0000	0.0541	0.0001	0.0293	0.0017	5.6786	0.0000
SKT	0.2681	0.0000	0.0832	0.0000	0.0454	0.0002	0.0232	0.0013	4.7556	0.0000
TDY	0.3109	0.0000	0.0907	0.0000	0.0495	0.0000	0.0267	0.0005	5.4544	0.0000
TTI	0.3122	0.0000	0.0972	0.0000	0.0504	0.0000	0.0265	0.0004	4.8915	0.0000
TTC	0.2748	0.0124	0.0855	0.0122	0.0454	0.0147	0.0238	0.0184	2.5058	0.0480
TG	0.2844	0.0000	0.0835	0.0000	0.0450	0.0000	0.0235	0.0001	6.5280	0.0000
THS	0.3070	0.0000	0.0951	0.0000	0.0495	0.0001	0.0259	0.0007	4.8961	0.0000
TGI	0.2904	0.0000	0.0899	0.0000	0.0463	0.0001	0.0252	0.0007	4.8961	0.0000
TBI	0.3013	0.0000	0.0972	0.0000	0.0477	0.0001	0.0254	0.0009	4.7553	0.0000
TYL	0.2920	0.0000	0.0894	0.0000	0.0466	0.0000	0.0241	0.0005	5.1146	0.0000
Avg.	0.2950	0.0008	0.0901	0.0009	0.0473	0.0012	0.0250	0.0025	4.8824	0.0033
S.D.	0.02374		0.00806		0.00336		0.00203			

Table 9 Variance ratio tests: Group A

Seemingly similar results are obtained from the variance ratio tests for Group B. Again, the null hypotheses in all individual and joint tests are rejected for predictability except 2 firms. In log returns series of TCBI, the null is not rejected individually or jointly, and that of TLEO shows insignificance in the joint test. Similar to Group A, the values of variance ratios become smaller but the p-value increases as the gap between observations goes up.

Gap	4		12		24		48		Joint test	
	Variance ratio	p-value	Variance ratio	p-value	Variance ratio	p-value	Variance ratio	p-value	max|z| Statistic	p value
ENSG	0.2941	0.0000	0.0899	0.0000	0.0445	0.0000	0.0234	0.0002	7.3493	0.0000
IART	0.2569	0.0005	0.0902	0.0115	0.0481	0.0243	0.0253	0.0402	3.4974	0.0019
ICON	0.3250	0.0000	0.0967	0.0000	0.0495	0.0002	0.0260	0.0031	4.8017	0.0000
ICUI	0.2487	0.0032	0.0791	0.0049	0.0399	0.0074	0.0210	0.0119	2.9483	0.0127
IFSIA	0.2469	0.0136	0.0830	0.0482	0.0439	0.0632	0.0217	0.0763	2.4681	0.0532
IILG	0.2755	0.0000	0.0870	0.0000	0.0465	0.0000	0.0241	0.0008	7.0927	0.0000
IIVI	0.3101	0.0000	0.0911	0.0000	0.0467	0.0000	0.0240	0.0006	6.3588	0.0000
INDB	0.2747	0.0000	0.0823	0.0000	0.0445	0.0002	0.0229	0.0013	4.7730	0.0000
ININ	0.2609	0.0000	0.0868	0.0000	0.0447	0.0000	0.0232	0.0002	5.6144	0.0000
INSP	0.2930	0.0000	0.1033	0.0001	0.0530	0.0005	0.0273	0.0022	4.3484	0.0001
INSU	0.2580	0.0000	0.0755	0.0009	0.0432	0.0092	0.0214	0.0275	4.7940	0.0000
IPCC	0.2251	0.0000	0.0752	0.0000	0.0421	0.0000	0.0215	0.0000	4.3558	0.0001
IPCM	0.2941	0.0000	0.0926	0.0001	0.0492	0.0007	0.0256	0.0023	4.4699	0.0000
ISYS	0.2511	0.0000	0.0877	0.0000	0.0443	0.0000	0.0219	0.0002	5.6406	0.0000
IVAC	0.3149	0.0002	0.0965	0.0007	0.0484	0.0020	0.0255	0.0052	3.6982	0.0009
NSIT	0.2713	0.0000	0.0855	0.0001	0.0478	0.0003	0.0248	0.0013	4.3843	0.0000
TCBI	0.1967	0.0734	0.0705	0.1321	0.0381	0.1515	0.0191	0.1637	1.7906	0.2627
THQI	0.2697	0.0006	0.0858	0.0070	0.0488	0.0234	0.0249	0.0548	3.4406	0.0023
TKLC	0.3185	0.0465	0.0993	0.0364	0.0532	0.0409	0.0276	0.0466	2.0926	0.1378
TLEO	0.2811	0.0000	0.0904	0.0000	0.0466	0.0006	0.0242	0.0054	4.5640	0.0000
TMP	0.2863	0.0000	0.0954	0.0000	0.0493	0.0000	0.0252	0.0001	6.7592	0.0000
TQNT	0.2940	0.0000	0.0890	0.0001	0.0470	0.0004	0.0247	0.0016	4.4304	0.0000
TRAD	0.2748	0.0001	0.0880	0.0002	0.0460	0.0008	0.0232	0.0033	3.9346	0.0003
TRLG	0.2881	0.0000	0.0897	0.0000	0.0449	0.0001	0.0241	0.0007	4.6527	0.0000
TRST	0.2836	0.0000	0.0828	0.0000	0.0448	0.0001	0.0237	0.0008	4.8815	0.0000
TSRA	0.2877	0.0000	0.0877	0.0000	0.0470	0.0001	0.0251	0.0005	5.3710	0.0000
TTEC	0.2638	0.0007	0.0871	0.0020	0.0463	0.0041	0.0230	0.0078	3.4069	0.0026
TTEK	0.2888	0.0000	0.0887	0.0000	0.0467	0.0003	0.0240	0.0022	4.6959	0.0000
TTMI	0.3275	0.0000	0.1024	0.0001	0.0545	0.0007	0.0285	0.0036	4.1321	0.0001
TTWO	0.2680	0.0048	0.0884	0.0070	0.0449	0.0106	0.0240	0.0164	2.8188	0.0191
TUES	0.2904	0.0000	0.0934	0.0000	0.0471	0.0000	0.0259	0.0005	5.8937	0.0000
TWGP	0.2624	0.0017	0.0897	0.0032	0.0443	0.0062	0.0229	0.0139	3.1364	0.0068
TXI	0.2664	0.0035	0.0835	0.0041	0.0438	0.0060	0.0233	0.0096	2.9231	0.0138
TXRH	0.3114	0.0001	0.0905	0.0002	0.0483	0.0005	0.0258	0.0017	3.8366	0.0005
Avg.	0.2782	0.0044	0.0884	0.0076	0.0464	0.0104	0.0241	0.0149	4.3928	0.0151
S.D	0.0277		0.0071		0.0034		0.0020		1.3538	

Table 10 Variance ratio tests: Group B

The increased resilience hypothesis is supported when comparing the variance ratios of two groups, which are known to follow normal distribution (Campbell et al., 1997), using difference-of-means t tests in Table 11. A statistically significant difference at the 5% significance level (**) is found at gap 4. Group B firms have lower variance ratios. It may mean that the returns of Group B have higher predictability within a 20 minute gap. However, it is not greatly meaningful since the results of individual tests are the same. Meanwhile, the return series of Group A have higher variance ratios overall, which means the price series are closer to random walks. However, this is not statistically significant.

Variance ratio	Gap 4	Gap 12	Gap 24	Gap 48
Group A	0.2950	0.0901	0.0473	0.0250
Group B	0.2782	0.0884	0.0464	0.0241
t statistic	2.0395**	2.0639	2.0518	2.0555
p-value	0.0382	0.4763	0.3955	0.1593

Table 11 t tests for differences in variance ratios between Group A and B

In sum, the variance ratio tests confirm that almost all price series are not random walk, thus it implies predictable time series structure in means and variances. The earlier results already suggested the presence of unit roots in price series. Then, it is probable that mean and variance processes have a predictable structure like autocorrelation, an ARMA and a GARCH (generalised autoregressive conditional heteroskedasticity) structures. Meanwhile, the difference between two groups may support the increased resilience hypothesis at least for a shorter interval.

2.3.5. The analysis of autocorrelation and partial autocorrelation functions and the time series structures

It was expected that inventory control produces negative autocorrelation and asymmetric information generates positive autocorrelation (Table 2). In the meantime, each market making model may imply a different autoregressive and moving average (ARMA) structure in time series data of price or return series. For example, inventory control models support the more significant ARMA terms in those time series data more than adverse information models do, but adverse information models can also explain such a structure when there are correlated shocks or the accommodation of shocks is not instant. The results in Section 2.3.3 already revealed the general existence of the autoregressive (AR) structure in both differenced series. That means the time series models of the differenced series of log prices, i.e. log return series, may be an AR model. However, a more formal approach of identifying the time series structures are necessary for further discussion.

This section investigates autocorrelation functions (ACF) and partial autocorrelation functions (PACF) to examine the characteristics of the intraday return series and to find implications for the market making models. Then, it identifies an ARMA structure using information criteria. At the same time, the impact of the presence of multiple market makers is also examined to see if any hypothesis in Table 1 is particularly supported. The order of the ARMA structure of the return series is also identified and compared.

ACFs and PACFs are essential in identifying a dynamic structure of a time series model which is based on the Box-Jenkins method (Hamilton, 1994). For ACFs, the estimated autocorrelations at lag ρ of time series data are calculated by solving a system of Yule-Walker equations that are obtained by dividing the autocovariance between the non-lagged value and the ρth lagged value by the variance of the series (Brooks, 2008, Mills and Markellos, 2008). The partial autocorrelations (PACF) between the current observation and an observation ρ periods ago are obtained by estimating the k^{th} AR coefficient of an AR(k) model that controls for correlations at intermediate lags (Mills and Markellos, 2008). When they are plotted or tabulated against lags, autocorrelation functions and partial autocorrelation functions are obtained.

Based on the significance of lags in the ADF test results of unit roots in Table 6 and Table 7, autocorrelations and partial autocorrelations are calculated up to 4^{th} lags. The calculated values of autocorrelation and partial autocorrelations are presented in Table 12. Their significance is indicated by '*' which is attached if the autocorrelation and partial autocorrelation values exceed the upper and lower bounds. The bounds at the 5% significance level are calculated by:

$$\pm\frac{1.96}{\sqrt{N}}$$

where N is the number of observations. If the autocorrelations and partial autocorrelations at a specific lag are larger than this value, they are significantly different from zero at the 5% significance level. When a sample size is 1,559 (e.g. lag 1), the bounds are ±0.0496. Meanwhile, '†' in the table indicates that the autocorrelations are larger than ±0.1 and '‡' is for the values over ±0.2. They are intended to simply clarify the strength of autocorrelations and partial autocorrelations.

Log returns	ACF				PACF			
	lag1	lag2	lag3	lag4	lag1	lag2	lag3	lag4
IO	0.166†	-0.011	-0.055*	-0.091*	0.166†	-0.039	-0.048	-0.076*
IRC	0.115†	-0.055*	-0.089*	-0.091*	0.115†	-0.070*	-0.076*	-0.077*
IN	0.075*	-0.065*	-0.020	-0.056*	0.075*	-0.071*	-0.009	-0.059*
ITG	-0.066*	-0.068*	0.001	-0.042	-0.066*	-0.073*	-0.009	-0.048
IVC	0.093*	-0.081*	0.049	-0.018	0.093*	-0.091*	0.067*	-0.038
DM	0.215‡	0.031	-0.003	-0.057*	0.215‡	-0.015	-0.007	-0.057*
SKT	0.030	-0.053*	-0.072*	-0.038	0.030	-0.054*	-0.069*	-0.037
TDY	0.134†	-0.053*	-0.057*	-0.076*	0.134†	-0.073*	-0.040	-0.068*
TTI	0.149†	-0.005	-0.088*	-0.062*	0.149†	-0.028	-0.085*	-0.037
TTC	0.043	-0.007	-0.035	-0.049	0.043	-0.009	-0.034	-0.046
TGI	0.102†	-0.009	-0.018	-0.040	0.102†	-0.020	-0.015	-0.037
THS	0.131†	-0.087*	-0.110†	-0.063*	0.131†	-0.106†	-0.086*	-0.046
TG	0.055*	-0.035	-0.041	-0.073*	0.055*	-0.038	-0.037	-0.071*
TBI	0.130†	-0.066*	-0.018	-0.045	0.130†	-0.084*	0.002	-0.050*
TYL	0.089*	-0.009	-0.041	-0.062*	0.089*	-0.017	-0.039	-0.056*
5% c.v.	0.0497	0.0497	0.0497	0.0497	0.0496	0.0497	0.0497	0.0497
Average	0.097*	-0.038	-0.040	-0.058*	0.097*	-0.052*	-0.032	-0.054*

Table 12 ACF and PACF of Group A

Except one firm (TTC), which is 6.67% of all sample firms, all other firms have statistically significant autocorrelations at several lags. Although an individual correlogram is not presented, which is commonly used for the informal identification procedure in Box-Jenkins as a time series modelling (Hamilton, 1994), the above ACF and PACF show that most of the return series have an ARMA structure. The averages of the group are presented in the last row where '*' indicates significance at the 5% significance level. This average pattern shows a significant positive correlation at lag 1 and negative correlation at the following lags that are marginally significant or insignificant. Ljung-Box Q statistics for lags 1 to 4 are separately reported in Table 14.

Log returns	ACF				PACF			
	lag1	lag2	lag3	lag4	lag1	lag2	lag3	lag4
ENSG	0.068*	-0.077*	-0.049	-0.095*	0.068*	-0.082*	-0.039	-0.096*
IART	0.109†	-0.199†	0.013	0.087*	0.109†	-0.214‡	0.067*	0.036
ICON	0.154†	-0.032	-0.077*	-0.098*	0.154†	-0.057*	-0.065*	-0.080*
ICUI	-0.064*	-0.074*	-0.013	-0.056*	-0.064*	-0.079*	-0.023	-0.065*
IFSIA	-0.001	-0.242‡	0.007	0.015	-0.001	-0.242‡	0.007	-0.046
IILG	0.074*	-0.074*	-0.022	-0.020	0.074*	-0.080*	-0.011	-0.023
IIVI	0.087*	-0.010	-0.044	-0.129†	0.087*	-0.018	-0.042	-0.123†
INDB	0.037	-0.059*	-0.074*	-0.054*	0.037	-0.061*	-0.069*	-0.053*
ININ	0.044	0.008	-0.037	0.006	0.044	0.006	-0.037	0.009
INSP	0.188†	0.052*	0.036	0.050*	0.188†	0.018	0.024	0.040
INSU	-0.017	-0.126†	0.040	-0.047	-0.017	-0.127†	0.036	-0.063*
IPCC	-0.037	-0.025	0.027	0.068*	-0.037	-0.026	0.025	0.069*
IPCM	0.125†	-0.042	-0.023	-0.027	0.125†	-0.058*	-0.010	-0.025
ISYS	0.023	-0.048	-0.004	0.008	0.023	-0.048	-0.002	0.005
IVAC	0.100*	0.005	-0.046	-0.130†	0.100*	-0.005	-0.046	-0.122†
NSIT	0.086*	0.000	-0.065*	0.010	0.086*	-0.007	-0.065*	0.022
TCBI	-0.150†	-0.245‡	0.080*	0.098*	-0.150†	-0.273‡	-0.008	0.050*
THQI	0.107†	-0.104†	0.053*	0.038	0.107†	-0.117†	0.080*	0.010
TKLC	0.186†	0.076*	-0.034	-0.032	0.186†	0.043	-0.058*	-0.020
TLEO	0.091*	-0.072*	-0.032	-0.019	0.091*	-0.081*	-0.018	-0.020
TMP	0.132†	-0.042	-0.055*	0.010	0.132†	-0.060*	-0.042	0.021
TQNT	0.103†	-0.083*	-0.042	-0.053*	0.103†	-0.094*	-0.024	-0.054*
TRAD	0.061*	-0.079*	-0.043	-0.030	0.061*	-0.083*	-0.033	-0.032
TRLG	0.068*	-0.024	-0.051*	-0.071*	0.068*	-0.028	-0.047	-0.066*
TRST	0.065*	-0.093*	-0.094*	-0.059*	0.065*	-0.098*	-0.082*	-0.058*
TSRA	0.081*	-0.051*	-0.099*	-0.055*	0.081*	-0.058*	-0.091*	-0.043
TTEC	0.065*	-0.084*	-0.074*	0.017	0.065*	-0.089*	-0.063*	0.019
TTEK	0.093*	-0.065*	-0.014	-0.045	0.093*	-0.074*	-0.001	-0.049
TTMI	0.209‡	-0.024	-0.008	-0.035	0.209‡	-0.071*	0.012	-0.039
TTWO	0.072*	0.008	0.043	0.008	0.072*	0.003	0.042	0.002
TUES	0.122†	-0.080*	-0.072*	-0.017	0.122†	-0.096*	-0.051*	-0.009
TWGP	0.037	0.004	-0.036	-0.009	0.037	0.003	-0.036	-0.006
TXI	0.032	-0.037	-0.048	-0.029	0.032	-0.038	-0.045	-0.028
TXRH	0.125†	-0.023	-0.029	-0.086*	0.125†	-0.039	-0.021	-0.082*
5% c.v.	0.0496	0.0497	0.0497	0.0497	0.04964	0.04966	0.04967	0.04969
Average	0.073*	-0.058*	-0.026	-0.023	0.073*	-0.069*	-0.022	-0.027

Table 13 ACF and PACF of Group B

The ACF and PACF's of Group B (Table 13) present similar results to the case of Group A. Only 4 firms (11.76%) do not show any autocorrelation or partial autocorrelation at all lags. The remaining firms have potential ARMA structures. On the other hand, all of the autocorrelation values at specific lags are significant at the 5% significance level.

Table 14 summarises the values of Q statistics in the Ljuing-Box test for autocorrelation up to 4 lags for the firms in Group A and B. The Ljung-Box Q statistic, which is actually a modified Box-Pierce statistic, is based on the estimated autocorrelations and is known to follow χ^2 distribution with ρ (the number of lags) degrees of freedom. It is calculated as:

$$Q_{LB} = N(N+2)\sum_{k=1}^{\rho}\left(\frac{\hat{\phi}_{\rho}^{2}}{N-k}\right)$$

where $\hat{\phi}_{\rho}$ is the estimated value of autocorrelation at lag ρ. The results of Ljung-Box tests re-confirm that there are autocorrelations at least up to lag 4 in most of the firms in Group A and B.

Group A			Group B					
	Q statistic	p-value		Q statistic	p-value		Q statistic	p-value
DM	78.844*	0.000	ENSG	34.462*	0.000	THQI	41.602*	0.000
IN	20.766*	0.000	IART	92.802*	0.000	TKLC	66.734*	0.000
IO	60.863*	0.000	ICON	63.141*	0.000	TLEO	23.166*	0.000
IRC	50.885*	0.000	ICUI	20.179*	0.000	TMP	34.747*	0.000
ITG	16.793*	0.002	IFSIA	91.601*	0.000	TQNT	34.311*	0.000
IVC	28.048*	0.000	IILG	18.598*	0.001	TRAD	19.834*	0.001
SKT	16.272*	0.003	IIVI	40.972*	0.000	TRLG	20.070*	0.000
TBI	36.779*	0.000	INDB	20.672*	0.000	TRST	39.327*	0.000
TDY	46.652*	0.000	ININ	5.243	0.263	TSRA	34.376*	0.000
TG	17.649*	0.001	INSP	65.453*	0.000	TTEC	26.609*	0.000
TGI	24.396*	0.000	INSU	31.369*	0.000	TTEK	23.505*	0.000
THS	64.297*	0.000	IPCC	11.352*	0.023	TTMI	71.015*	0.000
TTC	8.5756	0.073	IPCM	29.133*	0.000	TTWO	11.221*	0.024
TTI	52.788*	0.000	ISYS	4.559	0.336	TUES	41.503*	0.000
TYL	21.213*	0.000	IVAC	45.338*	0.000	TWGP	4.346	0.361
			NSIT	18.286*	0.001	TXI	8.658	0.070
			TCBI	153.940*	0.000	TXRH	38.233*	0.000

Table 14 Ljung-Box Q statistics of Group A and Group B

On the other hand, the pattern of converting sign of autocorrelations is observed in most of the firms. The specific pattern of conversion is similar between the two groups in Table 12 and Table 13. At the first lag, the strongest positive autocorrelations are observed, and then weaker negative autocorrelations appear at the larger lags. The percentage of the firms that show this pattern is different. 80% of the Group A firms and 47.06% of the Group B firms display the pattern, respectively.

In addition, the comparison between the degree of autocorrelation of Group A and Group B is made using the data in Table 12 and Table 13. The following Table 15) presents the average autocorrelations, and t statistics and p-values of difference-of-means t tests for each specific lag. It is discovered that the two groups show a significant difference in both ACF and PACF at the 4th lag.

Positive correlations at lag 1 do not show any difference in strength between the two groups, so this does not particularly support either hypothesis. On the other hand, the returns of Group A have negative autocorrelations stronger than Group B at lag 4. If negative autocorrelations at lag 2, 3 and 4 are caused by inventory control, for example, this may show that a single market maker's inventory control takes a longer time than multiple market makers, which can mean a single market maker's weaker power. That may support the increased resilience hypothesis, but there can be another explanation for this like intraday-level overreaction.

ACF	Lag 1	Lag 2	Lag 3	Lag 4
Group A average	0.097*	-0.038	-0.040	-0.058*
Group B average	0.073*	-0.058*	-0.026	-0.023
t statistics	1.167	1.307	-1.063	-3.232*
p-value	0.253	0.198	0.297	0.002
PACF	Lag 1	Lag 2	Lag 3	Lag 4
Group A average	0.095*	-0.052*	-0.032	-0.054*
Group B average	0.073*	-0.069*	-0.022	-0.027
t statistics	1.167	1.133	-0.861	-3.003*
p-value	0.253	0.263	0.397	0.004

Table 15 Comparison of autocorrelation between Group A and Group B

Finally, the time series structures are identified for both sample and benchmark firms. The estimation results of ARMA models up to lag 4 are compared and the best fit model is selected based on the lowest value of Schwarz Information Criteria (SIC). The results of identification are presented in the following two tables where S.D is standard deviation.

	AR	MA
IO	0	1
IRC	2	1
IN	0	1
ITG	1	1
IVC	2	2
DM	1	0
SKT	2	2
TDY	0	1
TTI	0	1
TTC	0	0
TGI	0	1
THS	2	1
TG	0	0
TBI	0	1
TYL	0	1
Avg.	0.6667	0.9333
S.D	0.8997	0.5936

Table 16 Time series structure of the return series of Group A

Two firms (TTC and TG, 13.3% of all sample firms) in Group A do not contain an ARMA structure in their return series in Table 16. The return series of five firms (ININ, ISYS, TRLG, TWGP and TXI, 14.7%) in Group B shows no ARMA structure in Table 19. The average AR and MA orders are around 1. The results of two groups look similar, and the separate difference-of-means t tests do not show any statistically significant difference in the number of orders at the 5% level. p values for the difference in the order of AR structure is 0.2407 and for MA structure 0.6695 in the two-tailed tests. On average, there is no difference in ARMA structure between the two groups.

	AR	MA
ENSG	0	4
IART	2	0
ICON	0	1
ICUI	1	1
IFSIA	0	2
IILG	2	0
IIVI	4	0
INDB	2	1
ININ	0	0
INSP	1	0
INSU	0	5
IPCC	0	0
IPCM	0	1
ISYS	0	0
IVAC	4	0
NSIT	1	0
TCBI	2	0
THQI	0	2
TKLC	1	0
TLEO	2	0
TMP	0	1
TQNT	2	0
TRAD	2	0
TRLG	0	0
TRST	2	1
TSRA	1	2
TTEC	2	2
TTEK	0	1
TTMI	0	1
TTWO	1	0
TUES	1	2
TWGP	0	0
TXI	0	0
TXRH	2	1
Avg.	1.0294	0.8235
S.D	1.1411	1.1927

Table 17 Time series structure of the return series of Group B

On the other hand, the neo-classical market making model (Section 2.2) can be directly estimated and tested for the hypotheses of the impact of a multiple market maker. The following model is estimated.

$$\Delta P_t = \alpha + \varepsilon_t + \gamma\varepsilon_{t-1}$$

where α is constant, γ is the coefficient of the MA term and γ = (1- γ') in Section 2.2.

The purpose of this estimation is to check the difference in the value of the coefficient γ of the MA term and to find the support for a specific hypothesis only. Thus, the MA(1) model above does not have any importance in the time series structure of the data. The estimation results, in particular the coefficient values and standard errors of the MA term only, are presented in Table 18 and Table 19.

	Coefficient	S.E
IO	0.1843***	0.0249
IRC	0.1351***	0.0251
IN	0.0845***	0.0252
ITG	-0.0741***	0.0253
IVC	0.1210***	0.0251
DM	0.2182***	0.0247
SKT	0.0345	0.0253
TDY	0.1599***	0.0250
TTI	0.1586***	0.0250
TTC	0.0404	0.0253
TGI	0.1047***	0.0252
THS	0.1539***	0.0251
TG	0.0601**	0.0253
TBI	0.1634***	0.0250
TYL	0.0897***	0.0253
Avg.	0.1089	0.0251
S.D	0.0735	0.0002

Table 18 The coefficient of MA term in the simple neoclassical market making model: Group A

These results may support the improved efficiency hypothesis under multiple market makers. The coefficient value of the MA term in a multiple market maker system (0.0796) appears to be smaller than that in a single market maker system (0.1089). That is, a smaller proportion of past error affects the pricing of multiple market makers. However, the difference is not statistically significant when tested by the separate difference-of-means t test at the 5% level with a p value of 0.2366 in the two tailed test. Thus, no evidence of the difference is found. Remember that this model is based on an unrealistic assumption that the equilibrium price does not change in intraday data.

	Coefficient	S.E
ENSG	0.0826***	0.0252
IART	0.1956***	0.0249
ICON	0.1415***	0.0250
ICUI	-0.0778***	0.0253
IFSIA	-0.0053	0.0253
IILG	0.0894***	0.0253
IIVI	0.0925***	0.0252
INDB	0.0423*	0.0254
ININ	0.0421*	0.0253
INSP	0.1818***	0.0249
INSU	-0.0151	0.0253
IPCC	-0.0373	0.0253
IPCM	0.1357***	0.0251
ISYS	0.0260	0.0253
IVAC	0.1048***	0.0252
NSIT	0.0925***	0.0252
TCBI	-0.2583***	0.0244
THQI	0.1406***	0.0251
TKLC	0.1585***	0.0250
TLEO	0.1108***	0.0252
TMP	0.1397***	0.0251
TQNT	0.1315***	0.0251
TRAD	0.0739***	0.0253
TRLG	0.0851***	0.0253
TRST	0.0826***	0.0253
TSRA	0.0905***	0.0252
TTEC	0.0763***	0.0253
TTEK	0.1149***	0.0252
TTMI	0.2346***	0.0247
TTWO	0.0793***	0.0253
TUES	0.1522***	0.0251
TWGP	0.0346	0.0253
TXI	0.0329	0.0253
TXRH	0.1343***	0.0251
Avg.	0.0796	0.0252
S.D	0.0890	0.0002

Table 19 The coefficient of MA term in the simple neoclassical market making model: Group B

In sum, the analysis of autocorrelation and time series structure revealed stronger negative autocorrelation in larger lags in a simple market making system. This may support the stronger inventory control in a single market maker system. On the other hand, no statistically significant evidence was found regarding the preference between two hypotheses of the impact of multiple market makers.

2.3.6. Volatility and GARCH modelling

The implicative evidence provided so far was mostly based on linear structural models like a unit root process, a random walk and ARMA processes. However, as Campbell et al (1997) and Brooks (2008) argued, these linear models do not capture non-linearity in volatility such as volatility clustering, excessive leptokurtosis and leverage effects as asymmetric volatility change between positive and negative news (French et al., 1987, Ghysels et al., 2005). GARCH (Generalised Autoregressive Conditional Heteroscedasticity) models are the most popular choice to address this type of non-linearity (Brooks, 2008).

In the dataset in this chapter, strong leptokurtosis was revealed in Section 2.3.2 and possible volatility clustering was implied from the discovered strong correlation and predictability of return series in Sections 2.3.4 and 2.3.5. Also, leverage effects (Brooks, 2008) and intraday seasonality in volatility (Mills and Markellos, 2008) are known to commonly exist in financial data. Thus, it is natural to examine non-linearity and asymmetry in intraday data in the identical dataset using GARCH modelling.

The GARCH models (Bollerslev, 1986) are to represent volatility processes. In particular, it models conditional variances (σ_t^2) that are the expected values of the squares of the errors (ε_t^2) given the past values of the errors in mean processes. This also addresses heteroskedasticity of the errors (non-constant or time-varying variances) in mean processes (Mills and Markellos, 2008).

Two effects, GARCH and ARCH (Autoregressive Conditional Heteroscedasticity), are fundamental in the GARCH models. The former allows the effects of own lag of the conditional variances (σ^2_{t-s}) and the latter additionally allows the conditional variances of the errors to be affected by a lagged value of the squared errors (ε^2_{t-s}).

A GARCH(1,1) formulation has one lag of each GARCH and ARCH effect.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2$$

where α and β are coefficients. This specification is generalised into GARCH(p,q):

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \ldots + \alpha_p \sigma_{t-p}^2 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_q \varepsilon_{t-q}^2$$

In essence, the GARCH models are required for empirical regularities like volatility clustering in the above. From a theoretical perspective, the market making models for volatility need to support the existence of such regularities. However, some of the theoretical structural mean models only provide the qualitative patterns of mean price or returns but do not provide the anticipated pattern of volatility.

In the early market microstructure literature, Roll (1984) first adopted the idea of intraday price volatility being created by the existence of a bid-ask spread. That is, the alternation of the trade prices between bid and ask prices can generate excess volatility. However, it does not bring any implications for the volatility of the returns or its clustering. On the other hand, the inventory control model may generate volatility clustering while market makers adjust their prices and adverse information models may rely on the clustering of information shocks.

However, it is more reasonable to separately analyse the empirical volatility model of market making in the empirical analysis because the GARCH model can be used in conjunction with any mean model that is adopted based on empirical analysis or theoretical considerations.

The objectives in this section are to reveal the characteristics of GARCH and ARCH effects in intraday data and to investigate the impact of multiple market makers. In addition, the leverage effects and the intraday seasonality are estimated for significance. One new factor, a market maker's mis-estimation, is tested for explaining power for return volatility. For the remaining part of this section, the GARCH model for intraday data is built and compared between two sub-datasets (Group A and B).

First of all, it is necessary to check the return series of interest contains general non-linearity. The BDS independence tests (Brock et al., 1996) investigate the data with the null hypothesis of independent and identical distribution against time-varying non-linearity in mean or non-linearity in volatility. This is the most commonly used test for general non-linearity and gives support for the use of non-linear models (Campbell et al., 1997) like GARCH.

The BDS independence tests were conducted on the log return series of all firms in Group A and Group B in the same dataset in the earlier sections. All results rejected the null hypothesis at the 5% level for the alternative hypothesis of non-linearity.

The choice of a mean model is important. To be precise, each ACF and PACF of individual series need to be identified as in Section 2.3.5, but as one of the objectives of this analysis is comparison, one general but parsimonious model is chosen. On the other hand, the use of absolute or squared returns is common practice in GARCH modelling (Markellos et al., 2003) as it avoids some non-negativity of coefficients that may lead to negative volatility (Brooks, 2008). Absolute returns are occasionally preferred since it provides less sensitive volatility measures to outliers (Andersen et al., 2001). For absolute return series, the ARMA(1,1) structure is suggested by Engle (2000) and Markellos et al. (2003) as a reasonable approximation for GARCH modelling.

On the other hand, the specifications of non-absolute (nominal) return series was investigated in Section 2.3.5. In this case, ACF and PACF up to 4 lags were presented and individual ARMA structures were identified. Although a global consensus structure was not obtained, the average identified orders of them indicate that ARMA(1,1) is a reasonable parsimonious choice. Also, the brief investigation of the ACFs and PACFs of absolute return series (not reported) supports ARMA(1,1). Thus, this will be used as the mean model throughout the rest of the section.

Subsequently, the specification of the GARCH model is required. The first building block of the modelling is a GARCH(1,1) specification since this is a popular choice in intraday or daily GARCH modelling (Baillie and Bollerslev, 1992, Franses and van Dijk, 1996, Andersen et al., 2001). Consequently, an ARMA-GARCH specification is:

$$|r_t| = a_0 + a_1 |r_{t-1}| + \varepsilon_t + b_1 \varepsilon_{t-1}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2$$

where a and b are AR and MA coefficients, respectively, and $\varepsilon_t \sim \text{i.i.d.}(0,\sigma^2_t)$

In addition, there are other factors to consider, which are unique in volatility in intraday absolute return data. One of the notable findings in the market microstructure literature was the intraday seasonality of volatility pattern.

Specifically, Wood (1985), McInish (1990), Foster (1990), Andersen (1997) and Tsay (2005) showed that a U-shaped pattern emerged during a trading day in the NYSE where one market maker operates. Similar patterns were observed in dealership markets such as the UK stock market (Abhyankar et al., 1997) and the Greek stock market (Markellos et al., 2003).

Following French's finding (1986) that adverse information is the main driver of trading-day volatility, Madhavan et al. (1997) argued that the U-shaped pattern arises due to information flows during a trading day. Information asymmetry and uncertainty about fundamentals decreases over a trading day while transaction cost increases. From the perspective of market making, Brock (1992) explained that transactional demands are higher at the opening and closing of the market, which is supported by a similar pattern in trading volume. In the meantime, Kalev (2004) showed the relationship between news and stock return volatility. The news has a positive impact on volatility of stock returns even after controlling for volume effect and opening volatility. Easley and O'Hara (1992) argued that any type of new information affects volatility because it causes more frequent trading. Similar results are suggested by Engle (2000) and Manganelli (2005).

To accommodate the empirical intraday volatility pattern into the GARCH models, a trading day can be divided into several intraday periods using dummy variables i.e. seasonally adjusted. For the analysis in this section, one trading day becomes 6 intraday periods: 09:30–10:35, 10:35-11:40, 11:40-12:45, 12:45-13:50, 13:50-14:55 and 14:55-16:00. Each period is 65 minutes, that is 13 observations in 5-minute average data. In a GARCH model, they are represented by 5 dummies where the values of all dummies are 0 for the first intraday period. Then, D2=1 for the second period and otherwise it is 0, and so on. In the meantime, the same 5 dummies (D2, …, D6) are used to check whether there are the same patterns in the intraday absolute returns. The use of these dummies are supported by the finding of Harris (1986) who discovered excessive positive returns within the first 45 minutes after the stock market opens, except Monday mornings.

On the other hand, the leverage effects are also known to affect intraday return volatility (Bollerslev et al., 2006). That is, a negative shock increases volatility. However, the literature on daily or long-term data provides contradictory evidence regarding the exact relationship: return shocks and volatility are either negatively (Nelson, 1991, Glosten et al., 1993) or positively related (French et al., 1987, Ghysels et al., 2005). To model these effects, another qualitative variable, which represents whether past information shock is positive (good news) or negative (bad news), is required. However, the residuals from the mean model of absolute returns cannot properly represent this since they do not contain information about the signs. Therefore, a separate estimation of the auxiliary mean model of non-absolute returns with the same specification is necessary to obtain residuals for this purpose.

The auxiliary regression model of the ARMA(1,1) specification is:

Equation 7

$$r_t = a_0' + a_1' r_{t-1} + \hat{\varepsilon}_t + b_1' \hat{\varepsilon}_{t-1} + c_1' D2_t + c_2' D3_t + c_3' D4_t + c_4' D5_t + c_5' D6_t$$

where $\hat{\varepsilon}_t$ is the errors in the auxiliary regression model. a', b' and c' are coefficients.

Subsequently, a new dummy (DN_t) is constructed based on the sign of previous residuals ($\hat{\hat{\varepsilon}}_{t-1}$) from this auxiliary model. The value of DN is 1 for negative residuals and 0 for positive residuals. Then, the dummy is added to the original GARCH model after multiplying it by the square of the previous absolute residuals ($DN_t \times |\hat{\hat{\varepsilon}}_{t-1}|$), which is essentially a slope dummy to accommodate the effect of the size and sign of news shocks. This is a similar approach to the typical GJR (or threshold) GARCH models (Glosten et al., 1993), but the difference is that the residuals from a separate auxiliary mean model are used in the absolute return GARCH model.

In the meantime, this section adds a new variable that is specific in the context of market making in intraday data. That is, a market maker's mis-estimation of information shocks in the previous time period is considered as another factor in this analysis. The market maker's mis-estimation can be categorised into two scenarios. The first is the over-estimation of both positive and negative shocks. The second is the underestimation of them. It means either lower estimation than positive shocks or higher estimation than negative shocks. This includes wrong-directional estimation cases. For example, a marker maker estimates a shock will be negative, which turns out to be positive. An alternative interpretation could exist without referring to market making. The first case of a market maker's over-estimation is equivalent to the case of over-estimated shocks in size in terms of investor pricing. Meanwhile, the second case is under-estimated in size or adverse-directional shocks.

Another qualitative variable is necessary for this category. The dummy (DM_t) for this is constructed as follows using the auxiliary regression model (Equation 7) that represents a market maker's estimation. As the size of the shock was considered in the dummy DN, this dummy DM only incorporates whether a market maker's mis-estimation is over-estimation (when DM=1) or under-estimation (when DM=0) in the previous period. It is essentially an intercept dummy. The values for the dummy are assigned in Table 20.

	Sign of the returns	Sign of the residuals	Product	DM
Over-estimation	+	-	-	1
	-	+	-	1
Under-estimation	+	+	+	0
	-	-	+	0

Table 20 Dummy for mis-estimation of the size of information shocks

This idea looks similar to the use of the magnitude and sign of errors in the EGARCH model (Nelson, 1991, St. Pierre, 1998), but this dummy is constructed based on both signs of returns and errors and the EGARCH model cannot generally distinguish the type of mis-estimation. Also, this does not require the error to be distributed as non-normal distribution (St. Pierre, 1998).

In total, 7 dummies are constructed to address the asymmetries of information shocks in the ARMA-GARCH model: the first five (D2, …, D6) are for intraday volatility pattern. The next one (DN) is for the sign and the size of the information shock and the last one (DM) is for mis-estimation of information shocks. The first five are also used for intraday seasonal adjustment.

Finally, the ARCH-GARCH formulation becomes:

Equation 8

$$|r_t| = a_0 + a_1 |r_{t-1}| + \varepsilon_t + b_1\varepsilon_{t-1} + c_1 D2_t + c_2 D3_t + c_3 D4_t + c_4 D5_t + c_5 D6_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1\sigma_{t-1}^2 + \beta_1\varepsilon_{t-1}^2 + d_1 D2_t + d_2 D3_t + d_3 D4_t + d_4 D5_t + d_5 D6_t + d_6\hat{\varepsilon}_{t-1}^2 DN_{t-1} + d_7 DM_{t-1}$$

where c and d are the coefficients for dummies.

The maximum likelihood estimation method is used to estimate the values of the GARCH model parameters for Group A and B. Table 21 shows the *averages* of individual coefficient values, standard errors (S.E.) and p-values of the firms in Group A. These results are only effective in showing the overall pattern of significance. Note that lnL is the maximum log likelihood values. One, two and three '*' indicate the significance level at 1, 5 and 10%, respectively.

Mean model	Coefficient	S.E.	p-value	
Constant	0.003430	0.000406	0.0002	***
AR(1)	0.314013	1.567686	0.6217	
MA(1)	-0.252843	1.571894	0.6320	
D2	-0.001438	0.000402	0.0417	**
D3	-0.001736	0.000431	0.0238	**
D4	-0.001939	0.000444	0.0641	*
D5	-0.001929	0.000466	0.0662	*
D6	-0.001987	0.000612	0.0312	**
Volatility model	Coefficient	S.E.	p-value	
Constant	0.000009	0.000001	0.0374	**
ARCH(1)	0.130917	0.030882	0.0726	*
GARCH(1)	0.582856	0.037654	0.0362	**
D2	-0.000007	0.000001	0.0068	***
D3	-0.000007	0.000001	0.0039	***
D4	-0.000007	0.000001	0.0922	*
D5	-0.000007	0.000001	0.0089	***
D6	-0.000003	0.000001	0.4018	
DN	0.033979	0.022792	0.5996	
DM	-0.000003	0.000000	0.0138	**
R^2	0.0945	lnL	7437.5214	
Adjusted R^2	0.0905	SIC	-9.4687	

Table 21 The averages of coefficients, standard errors and p-values of the GARCH models: Group A

The ARMA structure on average does not explain the variation of mean returns well, but dummies are all significant in the mean model. It supports the existence of intraday patterns in returns. In particular, absolute returns on the first trading period of the day are significantly higher than the rest. Normality is rejected in all residual series.

In the estimation results of the GARCH model, ARCH and GARCH terms are on average significant in explaining the variation of the volatility. Their coefficient values are positive which suggests clustering. Meanwhile, the first 4 of the dummies (D2, …, D5) are significant and have negative coefficient values. The insignificance of D6 means its coefficient value is not significantly different from that of the first intraday period. In sum, the first and the last periods have significantly higher volatility than the other intraday period. It confirms that the intraday U-shaped volatility patterns are in the dataset in this empirical section.

DN has a positive coefficient value, which may mean negative information shock or news increases return volatility. However, the coefficient of DN is not statistically different from zero. That is, whether return in the previous period is positive or negative does not affect the volatility. This is comparable with the insignificance in Campbell and Hentschel (1991), but different from other leverage effect literature.

Instead, the newly added dummy variable, DM, is significant and its coefficient value is negative. It indicates that a market maker's overestimation in the previous period reduces the volatility of (absolute) returns. In the meantime, a market maker's underestimation (or wrong-directional) estimation increases the volatility. There may be a behavioural reason for this. For example, a market maker is more likely to be surprised by higher-than-estimated size of return movements, and it increases the volatility of his price decision. It should be also noted that the impact of mis-estimation can be linked to the leverage effects or the separate impact from the sign and magnitude of errors.

On the other hand, the average values of the same parameters for the firms in Group B are presented in the next table (Table 22). The overall results are fairly similar to those of the firms in Group A. The individual significance and sign of the coefficients of the variables are identical. It seems that the dummies in the mean models look more strongly significant than those of Group A while the dummies in the volatility models are more weakly significant. However, their differences are not statistically significant when the difference-of-means t tests are conducted on the series of individual coefficients of the dummies between the two groups.

Mean model	Coefficient	S.E.	p-value	
Constant	0.003767	0.000511	0.0000	***
AR(1)	0.429143	1.854610	0.4795	
MA(1)	-0.348651	1.860372	0.5049	
D2	-0.001679	0.000491	0.0226	**
D3	-0.002063	0.000509	0.0015	***
D4	-0.002267	0.000527	0.0029	***
D5	-0.002234	0.000554	0.0077	***
D6	-0.002111	0.000748	0.0572	*
Volatility model	Coefficient	S.E.	p-value	
Constant	0.000011	0.000001	0.0833	*
ARCH(1)	0.165160	0.034280	0.0580	*
GARCH(1)	0.499850	0.030722	0.0218	**
D2	-0.000009	0.000001	0.0749	*
D3	-0.000009	0.000001	0.0347	**
D4	-0.000009	0.000001	0.0688	*
D5	-0.000008	0.000001	0.0551	*
D6	-0.000005	0.000001	0.2662	
DN	0.055773	0.025052	0.4280	
DM	-0.000003	0.000000	0.0723	*
R^2	0.0987	lnL	7256.1427	
Adjusted R^2	0.0947	SIC	-9.2357	

Table 22 The averages of coefficients, standard errors and p-values of the GARCH models: Group B

In sum, this section confirms the findings in the previous literature that stock returns and volatility display specific intraday patterns. In particular, absolute returns are higher early on in a trading day and the volatility of absolute returns is higher in the first and last period of the day which supports U-shaped patterns. However, leverage effects are not present in the current dataset at least. In other words, the nature of news does not asymmetrically affect the volatility of the following day. On the other hand, a market maker's over- or under-estimation in the past period affects return volatility. Over-estimation reduces the volatility in the next period while under-estimation increases it.

On the other hand, not much statistically significant difference was observed in the estimation results of the return and volatility models between Group A and B. Thus, it can be concluded that the GARCH structure and observed empirical regularities do not vary between two market making systems. That is, the impact of multiple maker makers is not present when a volatility process is considered.

2.4. Discussion and conclusions

In summary, the impact of multiple market makers was found to relatively support the increased resilience hypothesis rather than the improved efficiency hypothesis. In other words, multiple market makers have stronger power than a single market maker in keeping their prices against shocks that may come from adverse information and maintaining the disequilibrium prices to extract more information. Also, the strategic interactions between market makers can slow down price adjustment to the equilibrium price. Therefore, it can be concluded that the increased number of market makers increases the market disequilibrium.

This is supported by lower non-stationarity in price and higher predictability in return under a multiple market maker system than a single market maker system. However, the investigation of volatility and other descriptive statistics, autocorrelation, time series structure and the volatility models did not reveal any significant difference.

There can be an alternative explanation for higher resilience in a multiple market maker system regarding causality. The study in this chapter compared the NASDAQ with the NYSE and used the small-size sample firms chosen from the S&P SmallCap 600 index. The firms listed on the NASDAQ are relatively smaller and newer than the ones in the NYSE. That is, they are more likely to be subject to higher market inefficiency originating from lack of liquidity, adverse information or other market frictions in stock trading. At the same time, these may raise resilience in empirical data. Then, multiple market makers may have been allowed to increase the market efficiency of the NASDAQ by supplying more liquidity in a competitive environment. In other words, expected increase in resilience may have caused the assignment of more than one market maker.

On the other hand, in terms of support for market making models, strong presence of unit roots in general may prefer the adverse information models over the inventory models. However, the positive autocorrelation of small lags and the negative autocorrelation of large lags may exemplify the co-existence of both forces in market making as in Madhavan (2000) and Biais et al. (2005). In particular, it may be possible that stronger impact of adverse information instantly after a shock is followed by weaker and prolonged effects of inventory control afterwards. That may be because a market maker's inventory control may be not easily distinguishable due to the small size of price adjustment and thus hidden within the influence of adverse information. If this is the case, a sequential trade model (Hasbrouck, 2007) can be a theoretical alternative. For example, Fleming's (1999) two-stage adjustment process to public news in the bond market can create comparable patterns.

Otherwise, it can be alternatively explained by a general pattern of 'market overreaction' discovered by De Bondt (1985) at a market microstructure level: a market sometimes irrationally overreacts to news and then prices slowly mean-revert over time. In terms of time scale, it seems that initial overreaction lasts about 5 minutes and mean-reverting adjustment is for an additional 5 to 15 minutes. However, this evidence may be due to sampling frequency, if shorter frequency data like 1 minute or longer frequency data is used, the discovered autocorrelation pattern may disappear.

The empirical analysis of the GARCH models showed a market maker's mis-estimation, specifically whether it was over-estimation or under-estimation, asymmetrically affecting the volatility in the next period. Specifically, over-estimation reduces it, but under-estimation increases it. For example, when higher-than-estimated return in absolute size (i.e. under-estimated shocks) is revealed in the market, it increases the volatility in the following period. This is possibly because a market maker or investors tend to overreact to the surprise by excessively adjusting their prices to correct their previous under-estimation. Consequently, it raises volatility.

In the meantime, intraday seasonality existed but the leverage effects did not in the dataset. The dummy variable of mis-estimation may have absorbed the explaining power of the leverage effects. That is, whether shocks are over- or under-estimated is more significant in explaning volatility than the sign of shock.

The limitations of the empirical analysis in this chapter need to be addressed. The first limitation is the non-normality of the return series and the residuals from the regression models. As commonly found in finance literature and intraday data (Hausmann et al., 1992), the normality tests for the return series and the residuals in mean and volatility models mostly rejected the null hypothesis of normality (the results were not separately reported). This non-normality is caused by strong excess kurtosis as reported in Table 5, which may come from outliers (Brooks, 2008). Partly, the kurtosis can be reduced by modelling a GARCH structure (Bai et al., 2001) that may be in the return volatility series, but this was not the case in this chapter. Also, outliers can be removed by introducing dummy variables, but it will delete useful information represented by such outliers (Brooks, 2008). It may subsequently invalidate the results of normal distribution based statistical tests. However, sample size in the intraday analysis is usually large as is the dataset in this analysis (1,560 for each firm), therefore non-normality does not cause serious consequences in testing because of the Central Limit Theorem (Brooks, 2008).

Another related limitation is the small number of firms in Group A (15). In particular, for the use of the difference-of-mean t test, the sample must be a random sample and its population must be normally distributed or the number of the sample must be larger than or equal to 30 to satisfy the Central Limit Theorem (Sullivan, 2011). The first requirement of random sampling can be met by the random choice of the first two letters of the sample firms (letter I and T). The second requirement needs to be met by the additional assumption of the tested property being normally distributed since the sample size is less than 30.

In some cases, this assumption is not required. For example, the average log return of a group of firms is approximately normally distributed by being a linear function of approximately normally distributed average log returns of an individual firm; the latter approximation is by the central limit theorem based on the large number of observations (1,560) for each firm (Gujarati and Porter, 2009). Otherwise, normality assumption is required for this t test if non-average values are tested. For instance, when the values of skewness of two groups are compared, the distributions of the values can be simply assumed away as normal, but not much is known about their exact population distributions.

The assumption of independent information shocks used in theoretical derivation of two hypotheses (increased resilience and improved efficiency) can be another limitation in this analysis. The earlier comparisons between the expected outcomes of a single market maker and multiple market maker system were based on external information shocks being independent of each other. Also, the empirical implications of two market making models for the time series properties of return series were derived from independent information shock. The independence assumption is common practice in modelling market making behaviour (Hasbrouck, 2007), but in reality this assumption may not hold as the results of empirical analysis like volatility clustering suggest (Campbell et al., 1997, Brooks, 2008).

The GARCH models used in the last sub-section of the empirical analysis have some known inherent weaknesses. The parameter values of the GARCH models may not stay constant over time (Mills and Markellos, 2008) or they vary depending on optimisation options, initial values and the choice of statistical packages (Brooks et al., 2003). In terms of GARCH modelling in this chapter, the contemporaneous effects from the volatility to mean return are not considered unlike some approaches using the GARCH-M models (Cheung and Ng, 1992, Blair et al., 2002). Also, the GARCH model in this chapter relied on a GJR GARCH type specification with an added dummy for mis-estimation for possible asymmetries, but exponential GARCH (EGARCH) can be alternatives.

There are factors that were not considered in this chapter. First, Diamond's (1987) research showed that short selling constraint on traders in fact slowed the price adjustment process, in particular under a negative information shock. Then, in terms of mis-estimation in market making, under-estimation of positive shocks and over-estimation of negative shocks are more constrained than the other opposite cases. Also, it may lengthen inventory control. This factor was not investigated as it may have a symmetric impact to single and multiple market makers.

Second, trades may not always happen in a specific time period in financial markets. No trade itself can inform about the underlying value of the financial assets (O'Hara, 1997) or at least about the existence of information (Easley and O'Hara, 1992). Also, non-trading can cause a bias in measuring statistics such as autocorrelation at a lower frequency (Campbell et al., 1997). The dataset in this chapter is based on the highly frequent time interval and does not have non-trading periods, so it may not cause an issue. On the other hand, there is a different approach that focuses on the property or non-trading period such as in the autoregressive conditional duration (ACD) (Tsay, 2005).

Third, the impacts of limit orders and institutional block trades (O'Hara, 1997) on market making behaviour were not modelled, but they can be regarded as a type of general shock to a market maker. If the limit order book is open to market traders or the details about institutional block trades are instantly revealed, it is just other public information.

Last, trading volume was not analysed as it is more likely the consequences of adverse information in the market since stronger adverse information and public information are known to lead to greater trading volume (Kim and Verrecchia, 1991). However, trading volume can be another source of information to market makers and traders.

This chapter focused on the level of price and volatility and did not examine the size of spread although spread can be variable in market making. The spread decomposition issue is one of the important topics in market microstructure (Glosten and Harris, 1988): a market maker may attempt to compensate for both information and inventory risks by increasing spread size while not changing the mid-prices. As a result, spread size is positively related to the impact of private information (Hasbrouck, 1991). Multiple market makers can generate a different impact on spread size compared with a single market maker. This can be considered for future research.

Another point for future research is a market without designated market makers. Without market makers, market traders can behave like market makers (Biais et al., 2005). That is, the market makers' role as liquidity or immediacy suppliers can be mimicked by traders' limit orders (O'Hara and Oldfield, 1986) and their other role as inventory-controlling dealers can be imitated by traders facing counterparty orders. However, the impact of this market microstructure will be probably different from the markets with market makers.

In addition, the causality between increased resilience and multiple market maker can be analysed in the future research. One of the possible approaches will be the analysis of the impact of the change in the number of market makers allocated for a particular stock. Since the entry and exit to market making is relatively easier in the NASDAQ, the number of market makers is expected to fluctuate over time. Then, it is possible to investigate its impact on the resilience of stock price movement. Moreover, the lead-lag relationship between the number of market makers and the change in resilience can be empirically analysed.

On the other hand, the following list of factors can be examined as part of a theoretical approach in the future research: the proportion of informed traders, the probability of such traders being informed, and the precision of private information, the accuracy or ability of the market maker and the liquidity of the market for an asset. Meanwhile, an empirical approach using limited dependent variable models such as ordered probit models (Tsay, 2005) can be considered.

Chapter 3

Detecting market disequilibrium in the context of bubbles and estimating transition probability using duration dependence

3. Detecting market disequilibrium in the context of bubbles and estimating transition probability using duration dependence

The market is in disequilibrium if supply and demand does not match. The market disequilibrium may be temporal and random, and then it may finish within a short spell that does not have any significance. However, if certain underlying market forces drive the disequilibrium, fairly one-sided long-term price movement to the new equilibrium will be observed. Such a price movement can be justified by the changes in price fundamentals or by employing a price generating process that represents the underlying forces. Otherwise, it can be identified as a bubble. On the other hand, without specifying the price fundamental, a run of one-sided price, possibly driven by the market disequilibrium, can be simply categorised as a bubble if it is followed by relatively short and strong reverse movement (McQueen and Thorley, 1994). Then, the detection of long-term market disequilibrium can be regarded as empirically equivalent to the detection of price bubbles.

Despite their long history, stock price bubbles remain among the most poorly understood and mysterious of economic phenomena, although bubbles and their respective crashes have long been studied (Rosser, 2000). As reviewed later, many different theories have been suggested to explain them. However, there is substantial disagreement over the reasons for bubble initiation and the processes of bubble developments. Even the existence of a bubble itself in a particular period is often re-examined and sometimes rejected (Garber, 2001, Siegel, 2003).

Most of the bubble models are related to the present value model (PVM) and focus on an investor's pricing decisions in the financial markets. In this sense, they are more micro-economic. The literature in finance can be further divided into two categories. The main difference is the belief in rational expectation and the efficient market hypothesis (EMH). Both predominantly utilise the present value model at least as benchmarks, in which the current stock price must be equal to the discounted future stream of dividend, cash flow or all future payoffs, i.e. 'fundamental price'. However, there is a non-PVM based method that does not depend on the fundamental price.

This chapter investigates bubble theories, and then focuses on the detection of stock price bubbles as one type of long-term market disequilibrium and continues to the estimation of transition probabilities. This chapter adopts a test method that can detect not only bubbles but also some other types of market disequilibrium. One of the non-PVM (present value model) based detection methods, the duration dependence test, is employed for its ability to detect negative bubbles and different market disequilibrium. It is based on the concepts of (price) runs and duration. 'Price run' is defined as a run of continuous abnormal returns of the equal sign and 'duration' is its time length. Another type of price run will be developed later in this chapter.

In detail, the chapter is organised as follows. Section 3.1 reviews past literature on bubble theories in finance in addition to the empirical models and evidence. Section 3.2 analyses price runs and presents the results of duration dependence tests for price bubbles on a range of market data. This section also estimates the unconditional transition probabilities of a run growth. Section 3.3 introduces structural breaks as another method of obtaining price runs and investigates its implications for duration dependence and transition probability between the market (dis)equilibrium. Section 3.4 concludes and discusses the research in this chapter.

3.1. Stock price bubbles and detection

There is a wide range of bubble theories and empirical methods to detect the existence of a bubble. The theories of bubbles vary depending on how each theory defines a bubble and explains its generating mechanism and a life-cycle. This section reviews the theories about the market-level price bubbles and the empirical methods of bubble detection.

3.1.1. Stock price bubbles

The stock price bubbles are the price bubbles created by the internal or external forces in the stock market. The main force is mostly how the investors price the stocks in the market. There are two main theories: rational and irrational bubbles. But also theories based on disciplines other than economic and financial exist.

Strict followers of the EMH, namely standard neoclassical theorists, argued that it was not possible for any type of bubble to arise because stock prices always reflect fundamentals by instantly incorporating all relevant information. They argued a bubble phenomenon may be explained by a non-bubble rational expectation model (Meltzer, 2005). From this perspective there is nothing called a bubble, as all price movements are explained by the EMH.

However, 'rational bubble' theorists prefer to use a bubble term. They believed rational deviation from fundamentals could arise while rational traders still quickly assimilated information. In this sense, a rational bubble is defined as "rational deviation of the price from the fundamental value given the assumption of rational behaviour and expectation" (Blanchard, 1979). It basically exists because any rational expectation of finding a buyer could push the price of even a non-profitable firm's shares. It is also called 'the-castle-in-the-air' theory (Malkiel, 2003, Blanchard, 2006).

The important point in this rational bubble argument is how to define fundamentals since rational bubbles usually arise from the same mechanism of pricing. Fundamentals are typically determined by the present value of expected future dividends or cash flows using a 'present value model (PVM)' or 'rational valuation formula (RVF)' (Cuthbertson and Nitzsche, 2004), which is the adoption of a two-period Euler equation to infinite time horizon. It was generally thought that the fundamental should be the unique discounted value (Rosser, 2000) as solved in the following:

The Euler equation for two periods is $P_t = \delta E_t[D_{t+1} + P_{t+1}]$ where P is the price of a stock, D is a dividend (or cash flow), expectations (E) are conditional on the information set and δ is the discount factor where δ = 1/(1+κ) and κ = required return and 0<κ <1. It is usually assumed that all agents are risk-neutral and they use the same discount factor and information set. This can be solved by repeated forward-looking substitution to infinite horizon as a difference equation. Then, the general solution (see also Section 3.1.2) is:

Equation 9

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}] + \lim_{n\to\infty}(1+\kappa)^{-n} E_t[P_{t+n}]$$

Basically, it has infinite solutions. However, assume the transversality condition ($\lim_{n\to\infty}[(1+\kappa)^{-n} E_t[P_{t+n}]] = 0$) or a terminal condition for finite horizon (Adam and Szafarz, 1992), and the solution will be unique:

Equation 10

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}] \quad \text{(forward solution)}$$

It is a well-known stock pricing equation, 'the present value model (PVM)', and is commonly regarded to represent the fundamental price (P^f) of the stock.

However, it is well-known that the Euler equation itself did not rule out the possibility that the price may contain an explosive bubble as a part of the solution (Blanchard, 1979, Cuthbertson and Nitzsche, 2004). This possibility has been discussed in much literature (Gourieroux et al., 1982, Diba and Grossman, 1987, Adam and Szafarz, 1992, Salge, 1997, LeRoy, 2004) and can be briefed as follows.

Now, suppose the transversality condition does not hold, the last term of Equation 9 can be interpreted as the bubble term:

$$B_t = \lim_{n\to\infty}(1+\kappa)^{-n} E_t[P_{t+n}]$$

It can be another solution to the original Euler equation. Precisely, the bubble term is a complementary solution and the fundamental is the particular solution of the Euler equation.

To generalise, now suppose that the price has a bubble term B_t such that:

Equation 11

$$P_t = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}] + B_t$$

At time t+1, it becomes:

$$P_{t+1} = \sum_{i=1}^{\infty} \delta^i E_{t+1}[D_{t+1+i}] + B_{t+1}$$

By taking the expectation of time t and using the iterated expectation:

$$E_t[P_{t+1}] = \sum_{i=1}^{\infty} \delta^i E_t[D_{t+1+i}] + E_t[B_{t+1}]$$

Substitute this back into the original Euler equation:

$$\begin{aligned} P_t &= \sum_{i=1}^{\infty} \delta^i E_t[D_{t+i}] + \delta E_t[B_{t+1}] \\ &= P_t^f + \delta E_t[B_{t+1}] \end{aligned}$$

Then, it can be clearly seen that a bubble solution is allowable.

In addition, the comparison of this result to Equation 11 shows that $B_t = \delta E_t[B_{t+1}]$ or equivalently $E_t[B_{t+1}] = (1+\kappa)B_t$. In other words, in this simple model, the bubble process is a martingale and grows at the rate of required return (κ). Also, note that, since the bubble term incorporates a deterministic and a stochastic part (ε_t, the error term) like $B_{t+1} = (1+\kappa)B_t + \varepsilon_{t+1}$, either or both of the parts can be responsible for exploding features. It is also difficult to distinguish the exact proportion of the bubble in the prices.

Therefore, given the rational behaviour of investors, a price bubble can exist. The main motive behind the rational bubbles is that the self-fulfilling (rational) expectation of price changes drives the actual price changes. However, it must be kept in mind that rational bubbles were specifically defined in the context of a model (Adam and Szafarz, 1992), and without such a model it was impossible to define market fundamentals and a bubble (Flood and Garber, 1980). On the other hand, it can be known that rational bubbles only occur in an infinite horizon. This is because knowing the bubble is zero at the last date of finite horizon, by backward induction, the bubble should be zero at the beginning. Moreover, if a firm pays no dividend, the prices of its shares consist of only bubbles. Negative bubbles are not possible.

Some notable rational bubble theories were Blanchard's (1979) simple probabilistic bubble, Froot and Obstfeld's (1991) intrinsic bubble, Friedman and Aoki (1986) and Adam and Szafarz's (1992) information bubble, and Tirole's

(1985) bubbles from overlapping generations. Evans and Honkapohja (1992) generalised the possible rise of rational bubbles from ARMA processes.

Blanchard (1979) developing Brock's (1974) work argued that deterministic bubbles were rather counterintuitive, although the bubble term has stochastic components. He suggested a 'simple probabilistic model' built on the probabilities of continuing and bursting the rational bubble. The bubble term B_{t+1} in his model is:

Equation 12

$B_{t+1} = (1+\kappa)B_t \times \frac{1}{\pi_B} + \varepsilon_{t+1}$ with a probability of π_B (the bubble continues)

$B_{t+1} = \varepsilon_t$ with a probability of (1- π_B) (the bubble bursts)

In this model, if the bubble sustains, the actual return grows at a higher rate than κ compensating for the risk of the bubble burst. When the probability π increases, the bubble term develops at a decreasing exponential rate. The average duration of the bubble during *i* periods is $(1-\pi_B)^{-i}$ (Blanchard and Kahn, 1980, Camerer, 1989). As Gourieroux et al. (1982) pointed out, there could be in fact infinite solutions of this type since the probability of bursting could be a function of time or have a variety of distribution functions. However, this type of bubble may be more likely to exist in inefficient markets (Weil, 1987).

On the other hand, a simple probabilistic bubble grows independently of fundamentals. To address this, Froot and Obstfeld (1991) first developed a fundamental (or dividend)-dependent bubble model. In their continuous time set up, a solution to the Euler equation is:

$$P_t = \sum_{s=t}^{\infty} e^{-\kappa(s-t+1)} E_t[D_s] + e^{-\kappa} E_t[B_{t+1}] = P_t^f + B_t$$

where s and t are time index.

Now, suppose that log dividend, $\ln D_t$ is generated by the geometric martingale:

$\ln D_{t+1} = \mu + \ln D_t + \varepsilon_{t+1}$ where μ is a constant and $\varepsilon_t \sim N(0, \sigma^2)$.

Then, it can be shown that the forward equation on the fundamental is:

$$P_t^f = (e^{\kappa} - e^{\mu+\sigma^2/2})^{-1} D_t$$

or simply,

$$P_t^f = \gamma D_t$$

which is proportional to dividends.

Then, define the bubble function which also depends on the dividends, D_t:

$$B(D_t) = cD^{\lambda}$$

where c is an arbitrary constant and λ is the positive root of

$$\lambda^2\sigma^2 / 2 + \lambda\mu - \kappa = 0$$

Note that $B(D_t)$ also satisfies a martingale process. Finally, the final bubble solution to the Euler equation is:

$$P_t = P_t^f + B(D_t) = \gamma D_t + cD^{\lambda}$$

It contains a bubble term which is dependent on dividends. Likewise, any arbitrary linear stochastic process of dividends can be transformed into a non-linear process and become the solution. This type of bubble is 'an intrinsic bubble'. Also, an intrinsic bubble can additionally depend on time.

On the other hand, these rational bubble models are met with several challenges. One of them was that without such a model it is impossible to both define market fundamentals and isolate the trajectories characteristic of a bubble (Flood and Garber, 1980). That is, the contribution of hypothetical bubbles to asset prices is not directly distinguishable from that of market fundamentals (Diba and Grossman, 1988b, Meltzer, 2005). Simply, it is difficult to conclude whether a particular price movement contained a bubble due to observational equivalence. Moreover, the definitions of the fundamental and a bubble are not highly agreeable. For example, with slightly richer specifications, bubble equilibriums are technically indistinguishable from fundamentals equilibriums (Hamilton and Whiteman, 1985). Then, as Flood and Garber (1980) argued, supposedly bubbles may not actually be bubbles since they could be explained by changes in the fundamentals or the unobserved fundamentals to researchers.

Then, researchers can no longer assert whether a particular event is a bubble or not.

Nonetheless, this ambiguity of the rational bubble theories gave some modelling advantages. For example, bubbles can be defined as a deviation of any bubble solution from the fundamental solution. Also, either a deterministic or stochastic part can be modelled to be consistent with the popular understanding of speculative bubbles (Gourieroux et al., 1982, Adam and Szafarz, 1992). Therefore, it becomes rather difficult to rule out the possibility of rational bubbles because the misspecification of the model can be the likely explanation and then alternative market fundamentals or bubbles can be suggested (Flood and Hodrick, 1990, van Norden, 1996).

On the other hand, it was argued that the theoretical foundations of rational bubble theories are not sound enough to generate the bubbles. Rosser (2000) summarised that a rise of rational bubbles was only possible when some of Tirole's 'no bubble conditions' loosen. Tirole (1982) had argued that it was not possible for bubbles to arise when a finite number of risk-averse and infinitely-lived agents with common beliefs traded a finite number of assets with real returns in discrete time periods, and particularly if they adopted a backward induction in their expectation. Thus, the usage of risk neutral, loving or infinite finitely-lived agents, failure to respond to common information or infinite dimensional commodity was required to induce rational bubbles. Some of them do not fit well with the understandings of the EMH. However, it was argued that even in the theoretical sequential market that satisfies Tirole's no bubble condition, a bubble could arise if investors are faced with the constraints on debt accumulation (Kocherlakota, 1992).

Meanwhile, Allen et al. (1993) also suggested that the necessary conditions for the existence of the bubbles were short-sale constraints, private information or no common knowledge on trades. For example, short-sale constraints make the market less efficient in reacting to negative news (Bris et al., 2007) and may induce a price bubble; although, it is not responsible for some bubbles in the stock market since investors could have used options to go short (Temin and Voth, 2004, Battalio and Schultz, 2006). Also, infinite patience or technological pathology may need to be assumed for a bubble to arise in the equilibrium in the rational expectation framework (Bewley, 1972, Prescott and Lucas, 1972, Gilles, 1989, Gilles and LeRoy, 1992). The above factors may not go well with at least certain forms of rational investors or market efficiency.

'Irrational bubble' theories explained stock price bubbles without relying on investor rationality and market efficiency. Instead, they considered psychological, social, and structural factors as more important causes of bubbles. For example, mass psychology of the irrational agents may be behind bubble creation (Baker and Wurgler, 2006). Otherwise, the traders were assumed to be not fully rational, but they did optimise with expectations that were biased in some way. Since irrationality can come in many different forms, the irrational bubble theories do not share the same model or fundamentals, unlike the Euler equation of the rational bubble theories.

However, their description of a life-cycle of bubbles has common characteristics. Investors are irrationally attracted to investment profits by precipitating factors and a positive feedback mechanism boosts prices, but finally leads to the final crash (Shiller, 2005). Rosser (2000) illustrated the factors for bubble initiation: "The trigger was a critical accumulation of general confidence and enthusiasm without any specific displacement from any fundamentals". Similarly, it can be initiated by exaggerated expectation about new technology (Meltzer, 2005). Shiller (2005) described subsequent bubble development by saying that "news of price increase spurs investor enthusiasm, which is spread by psychological contagion … through envy of others' successes and … gambler's excitement". Essentially, the bubbles are driven by euphoric optimism about the future upward movement of price. Wrongly-timed monetary policy such as credit expansion or a fraud scheme can add to this movement. Similarly, Eatwell et al. (1987) illustrated the reason for bubble development as the expectations of further rises attracting more buyers who are interested in speculative profits. However, after passing the period of distress, the burst stage finally emerges because of tightened credit, disclosed fraud, negative displacement of market fundamental or a random shock (Shiller, 2000).

On the other hand, irrational bubbles were sometimes defined as excess price movement compared to fundamentals. It is similar to rational bubbles but the bubble part is explained without the dependence of rational expectation. For example, Garber (2001) suggested an irrational bubble as part of a price movement that cannot be explained by fundamentals. Rosser (2000) described it as prices that do not equal fundamentals for a certain period of time apart from their random movements. In sum, irrational bubbles can be described as an entire or part of abnormal price movement that cannot be explained by ordinary rational (bubble) theories but by psychology and other behaviours of traders.

Researchers of irrational bubbles challenged the rational expectation and the EMH, on which the rational bubble theory was based. Shiller's (1981) famous 'excess volatility' argument raised a serious challenge to the EMH. The forward solution of the EMH implies that the variance of the forecast (the price) should be less than (or equal to) the variance of the variable being forecast (Kleidon, 1986). Empirical data generally showed the opposite. That is, the movement of stock price was too volatile to be explained by market fundamentals and new information. Thus, it appeared to be quite decisive evidence against the EMH.

However, Marsh and Merton (1986) and Gilles and LeRoy (1991) showed that the above conclusion is not statistically clear and the dispute over excess volatility seems to be mainly about statistical issues (Shiller, 2000). Also, the supporters of rational bubbles insisted that the same bubble phenomena can be rationalised by the decisions of rational investors or by small sample bias (Blanchard and Watson, 1982, West, 1988). Likewise, it was argued that many irrational theories have the alternatives of rational explanation (Rosser, 2000).

Meanwhile, there is evidence for the inefficient market and irrationality of investors. For instance, market anomalies imply that the EMH did not hold at least for a specific time period. Many of them like the January effect, the weekend effect, the closed-end fund, the winner's curse and the small firm effect were discovered in the stock market, e.g., Cooper et al. (2006); although, some of these anomalies disappear when known to the public (Cuthbertson and Nitzsche, 2004). Furthermore, some even argued that belief in rationality is responsible for bubbles and crashes (Ball, 2009). Experiments in a laboratory generally supported irrationality (Caginalp et al., 2000). Therefore, it might be more reasonable to simply admit the elements of irrationality in explaining most large historical bubbles (Blanchard, 1979).

Subsequently, the researchers further developed irrational bubble theories about motivations and processes behind the irrational bubble. Some examples of them are: greater fool (Roll, 1986), fads (Camerer, 1989), overreaction (De Bondt and Thaler, 1985), and underreaction (Bernard and Thomas, 1990), of which details follows later. More irrational bubble theories are summarised in a detailed survey article by Hirshleifer (2001). These theories relied on mass psychology of the irrational agents to explain price movements. Furthermore, those mechanisms can be mutually reinforced and combined to generate a variety of sub-theories. These theories appeared to be rather more divergent than rational bubble theories because they share only a typical psychological pattern of a life-cycle of a bubble.

First, 'greater fool' theory was originally based on the common phrase, 'Devil takes the hindmost', which is also the title of the book on the history of bubbles by Chancellor (1999). Roll (1986) developed this idea into modern finance by arguing that bidding firms were infected by over-optimism about their decisions and simply paid too much for a target firm. Then, 'the greater fool' who valued the target the unreasonably highest obtained it. The greater fool theory of the stock price bubbles explains that investors expect to find other investors (greater fools) who believe the bubble will last longer. Even after investors discover the presence of the bubble, they try to ride and take advantage of it (Fisher and Statman, 2002). Hence, the bubble keeps expanding. However, the unluckiest investor will be stuck with the assets with no buyer left in the market. Then, the bubble bursts.

The greater fool theory is called the 'winner's curse' theory in common auction settings (Thaler, 1988), and the 'hot potato' theory is the other name for it. More broadly, contrary to the fundamentals-based pricing theory, the greater fool theory refers to the other tradition of asset pricing, in which an asset is worth whatever another investor will pay for it (Malkiel, 2003). This kind of bubble can be caused by biased optimism among a finite number of traders (Camerer, 1989, Rosser, 2000). Although bidders are supposed not to participate in this winner's curse if they are rational, there is evidence of the existence of such curses in several field surveys and experiments (Cox and Isaac, 1984, Thaler, 1988).

On the other hand, Abreu and Brunnermeier (2003) rationalised this behaviour. A price bubble grows until rational arbitrageurs collectively decide to attack it. Brunnermeier and Nagel (2004) suggested that the behaviour of hedge funds during the dot-com bubble was a good example of this. Conversely, Shleifer and Vishny (1997) insisted that it showed they failed in arbitrage as it was conducted by a small number of professionals. Other studies are also mixed in their results about rationality (Goeree and Offerman, 2002).

Second, the 'fads' theory emphasised that social or psychological forces create a type of contagious fad or fashion in financial markets just like in commodity markets or political belief (Shiller et al., 1984, Camerer, 1989). This theory was largely developed by Shiller and also called 'social contagion' (Mishkin, 2008). Shiller suggested 12 possible factors affecting asset prices, including structural, social and psychological factors (Shiller, 2000).

Similar to the rational bubble theory, fads can be represented in a simple way by adding the fads factor to the fundamentals of the present value model (West, 1988, Camerer, 1989):

$$P_t = \sum_{i=1}^{\infty} \delta_i E_t[D_{t+i}] + F_t$$
$$F_{t+1} = C \times F_t + \varepsilon_t$$

where F is fads and C is the rate of decay of fads.

If C is (1+κ), the fad grows just like the rational bubble does. However, when C is less than 1, the expected return on the fads part will be less than κ. That is, investors should sell them on this occasion and the fads will disappear. Consequently, an equilibrium bubble price is not reached, unlike rational bubbles. In particular, Camerer (1989) suggested that there are three possible kinds of fads: fads in utility of holding assets, $F(D_t)$, fads in belief about future fundamentals, $F(E[D_{t+1}])$, and fads in expected returns, $F(\kappa)$. Similar theories are 'herding' or 'epidemics' theories in the financial markets (Kirman, 1993, Dass et al., 2008).

Third, 'overreaction' theory is also able to explain a price bubble. In a psychology study, people are known to overreact to unexpected and dramatic events. Using this, Debondt and Thaler (1985) argued that stock markets initially overreacted to the change of fundamentals (or the revelation of related information) and then slowly reverted to their mean. They also discovered empirical evidence that the portfolios of prior losers outperformed the portfolio of prior winners over as long as five years. This result attracted supporters like Barsky and de Long (1993), Campbell and Kyle (1993), Dreman and Lufkin (2000) and Franklin et al. (2006), as well as critics such as Zarowin (1990), Clare and Thomas (1995), Zhang (2006), and Daniel and Titman (2006) in disputing both theories and evidence.

Fourth, 'underreaction' theory is suggested, which may be responsible for part of a bubble life-cycle. Bernard and Thomas (1990), Abarbanell and Bernard (1992) and Michaely and Thaler (1995) found out that investors in fact underreact to the announcement of earnings or omission of cash dividends. That is, investors reacted too slowly and insufficiently, unlike the predictions by both the overreaction theory and the EMH. Then, prices continue to drift in the same direction for a while (Michaely and Thaler, 1995). Dreman and Lufkin (2000) argued that overreaction and underreaction may share the same psychological root.

Fifth, the 'overconfidence' of investors in their own ability or self-deception can lead to an irrational bias in stock or general asset pricing. This overconfidence theory supposes that financial investors tend to believe that their ability is better than that of other human beings in addition to their other psychological biases such as heuristic simplification and emotional loss of control (Hirshleifer, 2001). For instance, their overconfidence makes them put more weight on their own information and pay higher than their own valuation of fundamentals, expecting to take advantage of the mistakes of others (Scheinkman and Xiong, 2003). In Gervais and Odean's (2001) model, investors learn about their abilities from success and failures, but put too much confidence on their success especially in the early stage of their career.

On the other hand, the overconfident traders are not always losers in the market. Since they buy and sell more aggressively than other types of traders, they can earn more returns than rational traders (Hirshleifer and Luo, 2001) or take benefits from the competition among all types of informed traders (Kyle and Wang, 1997). Their overconfidence subsequently affects market price and volatility, and produces a price bubble and crash. On the other hand, overconfident market participants appear to overinvest in costly information acquisition (Ko and Huang, 2007), which can improve market efficiency contrary to typical expectations in behavioural finance.

Sixth, trader's 'myopia' occurs when traders give only a short-run consideration of the future, contrary to the expectation of the EMH and the PVM. It is presumably another bias to cause a price bubble. As in his 'no bubble theorem', Tirole (1982) argued that a rational expectation with a fully dynamic optimisation did not allow a bubble in a sequential trading stock market if traders do not have myopia. Costly arbitrage in long-term assets and managerial interests in short-term profits drive investors to become myopic (Shleifer and Vishny, 1990) and they irrationally push up the prices without justification by the fundamentals.

Seventh, the 'regret aversion' theory focuses on the development of bubbles. Investors in this theory do not worry about absolute gains or losses, but they are concerned about the amount of regret they would have when they choose one investment over other alternatives (Hirshleifer, 2001). As a bubble grows by any reason, their regret of not buying a stock becomes stronger and it amplifies the bubble. Similarly, a bubble can develop from relative wealth concern (DeMarzo et al., 2008). It is argued that, even if investors suspect a bubble, they do not instantly sell stocks because they do not want to lag behind in wealth against the other traders if the bubble continues to grow further.

Last, there exist other theories that are about the behavioural biases in asset pricing which may be responsible for market inefficiency such as momentum and a price bubble. The 'house money' effect (Thaler and Johnson, 1990) focuses on the effects of prior gains or losses. For example, when investors gain from previous trades, they tend to take more risks. In the case of prior losses, they are tempted even by the offer of the possibility of breaking even. In the meantime, 'loss aversion' refers to the phenomenon that investors are almost twice more sensitive to loss than gain. Benartzi and Thaler (1995) argued that it is responsible for the equity premium. Both theories explain the momentum in the growth of stock price bubbles. Campbell and Cochrane (1999) captured another psychological bias, 'habit formation': repetition of a stimulus diminishes the perception of the stimulus and responses to it. The theory adds the slow-moving external habit of utility maximising agents to asset pricing models in the explanation of stock price movements. On the other hand, Keynes's 'beauty contest' idea (Keynes, 1936), which originally adopted rational agents and was later formalised by Angeletos (2010) among others, was employed to support irrational bubbles using asymmetric information (Shiller, 2000, Brunnermeier, 2001).

In addition, practitioners in the market are also well aware of the irrationality of investors causing market disturbance (Thaler, 1999). For instance, a renowned trader, George Soros, introduced the concept of 'reflexivity', emphasising that a market participant's bias in connecting thinking and reality—particularly a negative feedback loop in fundamentals—between prices and price perception, may lead to dynamic disequilibrium of the market (Soros, 2003). Meanwhile, it is even argued that a stock price bubble is only an example of a bubble phenomenon discoverable everywhere in human society (Gisler and Sornette, 2010).

In reality, however, researchers cannot precisely know whether a price movement is driven by rationality or irrationality of the market investors and which actual pricing models are used by the market participants (Meltzer, 2005). Therefore, it may be a more reasonable approach to assume that there are both types of rational and irrational investors in the markets. However, the price impact of this coexistence may be even more difficult to analyse in theory or in practice.

Two schools of the bubble theories have different views about the coexistence of rational and irrational traders (Cespa and Vives, 2009), although this is partly a compromising approach to incorporate both approaches. The main difference is in whether this coexistence of rational and irrational investors temporarily or permanently reduces the degree of rationality in an overall market.

For instance, researchers supporting rational expectation and the EMH occasionally employed the concept of noise traders to explain an irrational and temporary departure from rational prices, which introduced a certain bias into the market. This is allowed in the rational expectation framework because the EMH only requires a sufficient number of rational investors to recover the fundamental prices (Cuthbertson 2004). However, rational bubble theories conclude that rational prices will prevail in the end.

Some irrational bubble theories also used the existence of rational investors in their development of argument, e.g., Hirshleifer and Luo (2001). Nonetheless, their outcomes from this coexistence were a persistent departure from fundamentals. The reason for this is that less-than-rational investors can survive in a competitive market with rational investors (Russell and Thaler, 1985, De Long et al., 1991). Furthermore, Kogan (2006) developed this view by showing that the price impact by irrational investors was not relevant to their survival. They may earn an abnormal return, even higher than rational investors (Hirshleifer et al., 2006). In this context of coexistence, stock price bubbles are sometimes defined as the difference between prices as they are and what they would be without noise traders (LeRoy, 2004).

In the meantime, this persistent deviation of prices can be explained by assuming biased rational investors who admit the possible existence of noise traders. For example, the myopia theory in the above can explain informational inefficiency caused by risk averse but myopic rational investors and their coexistence with noise traders. The myopic traders respond to noise trader risk by incorporating the degree of noise trading into their expected returns (De Long et al., 1990, Kogan et al., 2006) or consumption risk (Loewenstein and Wallard, 2006). Then, even when the rational short-horizon traders are privately informed, they buy an asset only if subsequent arbitrageurs are likely to drive up the price. This leads to herding on the same information (Froot et al., 1992) and not utilising all the information (Dow and Gorton, 1994). It results in market inefficiency and possibly a price bubble and a crash.

Meanwhile, there are completely different approaches to explain stock price bubbles by importing theories from mathematics, physics and other disciplines e.g. Rosser (2000) and Arthur (1999). The examples are catastrophe, chaos and complexity theories. Technical analysis of asset trading examines price bubbles in their own context.

'Catastrophe theory' was established by René Thom to explain gradually changing forces (Zeeman, 1977), which can produce sudden effects in natural sciences. Zeeman (1977) expanded the theory into financial economics. In his application of the theory to stock bubbles, the interaction between fundamentalists and chartists in a three-dimensional surface with an additional index axis initiates a slow recovering conversion from the bear market to the bull market. As encouraged chartists in the earlier bull markets increasingly participate in trading, overvaluation eventually occurs and arbitrageurs' departures give way to a sudden crash. However, Zeeman's expansion attracted criticism (Rosser, 2000) such that excessive reliance on qualitative methods and strong mathematical assumptions restrict the theory better suited for a mode of thought.

'Chaos theory' is a non-linear deterministic process which 'looks' random. The little change in initial conditions and parameter values generates extreme fluctuation in final outcomes. The chaos theory is better at explaining large movements such as stock price bubbles and crashes that occur at greater frequency than the expectations in the stochastic theory of normal distributions. However, this theory may be just a technique to discover better-fit models, and at the same time it requires a large number of samples to acquire predictability or to detect chaos (Hsieh, 1991, De Grauwe et al., 1993, Cuthbertson and Nitzsche, 2004).

'Complexity' theory comprises more general non-linear and process-oriented dynamics than the previous two theories (Rosser, 2000). A related theory, 'Cybernetics', which is a close relative to complexity theory, applied Forrester's system dynamics approach (Forrester, 1991) to financial economics. In complex or cybernetic systems, multiple elements including human agents with strategy and expectations interact with each other. If agents choose a more accurate hypothesis among multiple market hypotheses including rational expectations, their parameter values are slowly updated and homogenous rational expectation dominates. Conversely, when they are updated faster, complex regimes, bubbles and crashes and other anomalies are observed (Arthur, 1999). In essence, this theory argues that a temporal out-of-equilibrium state can exist when a system contains non-linearity of positive feedbacks or increasing returns (Arthur, 1995, Arthur, 1999) that resemble bubbles. One disadvantage of this approach is its reliance on computer simulation to demonstrate results (Rosser, 2000).

'Fractal theory' was occasionally employed to describe a general movement of stock prices including bubbles and crashes (Mandelbrot et al., 1997, Mandelbrot, 1999). In this fractal theory, stock price movements can be decomposed on smaller but essentially identical parts. This theory also provides a theoretical basis for a certain technical analysis.

Practitioners in financial markets, mainly chartists among traders, developed their own way of distinguishing the phases of price bubbles, that is part of 'technical analysis'. They use historical price data to pick up a turning point of trends in prices to maximise their payoff from trading (Kirkpatrick and Dahlquist, 2006), but they do not consider any finance or economic theories for this. Although it is widely used among so-called technicians or chartists, it is not strictly accepted as academic research although they can be combined (Brown and Jennings, 1989, Bettman et al., 2009).

For the examples of technical analysis for a pricing bubble and burst, the chartists use mathematical techniques such as a simple or exponential moving average and stochastic oscillators to tell when the market trend is reversed. Without referring to any price fundamental, they sometimes actually 'read' the price and other charts to infer what is behind price movements. For instance, they impose certain patterns such as a cup and saucer, a double bottom, a head-and-shoulder and a breakaway gap onto the charts and guess the next movement (Kirkpatrick and Dahlquist, 2006). There are various different techniques and their main concern is investment decisions, particularly the timing of trading.

3.1.2. Detection of a price bubble

How to statistically detect a price bubble is another issue different from explaining the causes and mechanisms of price bubbles. Market-level and economy-level price bubble theories provide either theoretical or descriptive patterns of bubble development. In the case of theoretical patterns of both bubble and non-bubble prices being presented, those mathematical movements, which are commonly based on the PVM, can be used to build empirical tests for bubble detection. Otherwise, like in most of the cases of the irrational price bubble theories and the economy-level bubble theories, the characteristics of descriptive price patterns expected from the models can be tested.

The basis for many types of bubble detection methods is the PVM. The stock prices are presumed to be decided by this, which is derived from the Euler equation. The original PVM states that the stock prices are the sum of all discounted future expected dividends. Suppose P is the stock price of a firm, iterative expectation and successive substitutions transform the Euler equation:

$$P_t = \left(\frac{1}{1+\kappa}\right) E_t[P_{t+1} + D_{t+1}]$$

to the PVM:

$$\begin{aligned} P_t &= \left(\frac{1}{1+\kappa}\right) E_t[D_{t+1}] + \left(\frac{1}{1+\kappa}\right)^2 E_t[E_{t+1}[D_{t+2}]] + \dots \\ &= \sum_{i=1}^{\infty} \left(\left(\frac{1}{1+\kappa}\right)^i E_t[D_{t+i}] \right) \end{aligned}$$

where κ is a discount rate or the rate of required return ($0<\kappa<1$), D is a dividend (or other price fundamentals) and t is a time subscript. All expectations denoted by E are rational expectations.

Unlike rational bubble theories, the terminal price is now supposed to be zero, i.e. the transversality condition holds ($\lim_{n\to\infty}[(1+\kappa)^{-n} E_t[P_{t+n}]] = 0$). Additionally suppose both dividends and discount rates are constant over time. Then, the price at t is simply:

$$P_t = \left(\frac{1}{1+\kappa}\right)\left[1+\frac{1}{1+\kappa}+\left(\frac{1}{1+\kappa}\right)^2+\ldots\right]\times D = \frac{D}{\kappa}$$

That is, stock prices do not change as long as dividends and discount rates are constant. Meanwhile, depending on how the assumptions are relaxed, there are different types of the PVM. For example, when dividends (D_t) and discount rates (κ_t) are time-varying (Barsky and de Long, 1993, Yogo, 2006), the Euler equation becomes a more general form of the PVM:

$$\begin{aligned} P_t &= E_t\left[\left(\frac{1}{1+\kappa_{t+1}}\right)D_{t+1}\right] + E_t\left[E_{t+1}\left[\left(\frac{1}{1+\kappa_{t+1}}\right)\left(\frac{1}{1+\kappa_{t+2}}\right)D_{t+2}\right]\right]+\ldots \\ &= E_t\left[\sum_{i=1}^{\infty}\left[\prod_{j=1}^{i}\left(\frac{1}{1+\kappa_{t+j}}\right)\right]D_{t+i}\right] \end{aligned}$$

In the meantime, it can be argued that either the assumption of investors using constant discount rates and dividends all the time or foreseeing their changes up to infinity is fairly unrealistic. Thus, it can be supposed that investors anticipate one period ahead of a discount rate and dividend, and calculate the stock prices based on them. In this setup, both dividends and discount rates are changeable each period subject to external shocks while they are not completely time-constant or time-varying. Consequently, the PVM becomes slightly simpler:

$$P_t = E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+\kappa_{t+1}}\right)^i \times D_{t+1}\right]$$

This version of the PVM is adopted in the next chapter as well.

In general, the PVM provides a good starting point to test for bubbles because if the PVM holds in efficient markets, an excessive movement of prices can be a temporary bubble. In a similar context, a price pattern can be tested against a specific type of rational bubble since the rational bubbles can be built on the PVM.

The first of this type of test are the 'variance bounds tests' (Shiller, 1981). They were initially designed for verifying the efficient market hypothesis, specifically the validity of the present value model (West, 1988, Salge, 1997). The reason for this is that violation of the variance bounds is known to support the excess volatility of stock prices and dismiss the EMH. The original tests were devised by Shiller (1981) and LeRoy and Porter (1981). The tests did not specify any specific bubble process but emphasised the increased volatility by possible presence of bubbles and utilised potential differences in variance between ex-ante actual price and ex-post rational price (or perfect foresight price).

Ex-ante price, or actual market price built on expected dividends assuming time-varying dividends and discounted at the constant risk-free rate r_f is:

$$P_t = \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+r_f}\right)^i E_t[D_{t+i}]\right]$$

and ex-post rational price which is calculated after all dividends are realised is:

$$P_t^* = \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+r_f}\right)^i D_{t+i}\right]$$

Since ex-post price includes the summation of forecast error, the relationship between two prices is expressed as:

$$P_t^* = P_t + \sum_{i=1}^{\infty}\left(\frac{1}{1+r_f}\right)^i \varepsilon_{t+i}$$

Empirically, ex-post price is on average equal to ex-ante price when zero-mean error (ε) is assumed, but the variance of ex-post prices is:

$$\mathrm{var}(P_t^*) = \mathrm{var}(P_t) + \frac{1/(1+r_f)^2}{1-1/(1+r_f)^2}\mathrm{var}(\varepsilon_t)$$

$$\mathrm{var}(P_t^*) \geq \mathrm{var}(P_t)$$

Then, it must be larger than or equal to the variance of ex-ante price. In other words, the variance of the forecast (P_t) should be smaller than or equal to the variance of the variable being forecasted (P_t^*).

However, the existence of rational bubbles tends to increase the variances of the stock price as Blanchard and Watson (1982) and Tirole (1985) suggested. The above variance bound is not likely to hold with bubbles. Note that equality instead of inequality in variance bounds is usually adopted for statistical tests (LeRoy and Porter, 1981).

On the other hand, the rejection of the variance bound does not necessarily mean the existence of rational bubbles. Mankiw et al. (1985) and Flood and Hodrick (1986) showed that the same variance bound can be derived even in the presence of bubbles. Moreover, these tests have the joint hypothesis problem. Since they combine the assumptions of rational expectation, risk neutrality, a constant discount rate and a stationarity of dividend process, it is difficult to interpret the results. In addition, Marsh and Merton (1986) showed that the tests were highly sensitive to non-stationarity of price and dividend.

Campbell and Shiller (1988) revised the test using the log linear approximation of dividend/price ratio and they showed that an unexplained portion in variance remained even after relaxing the assumption of constant discount rate, which may be due to bubbles. However, this was not clear evidence of bubbles since Cochrane (1992) showed a time-varying discount rate can actually explain the remaining variance without resorting to bubbles.

Therefore, although the variance bounds test may reject the null hypothesis of the efficient market (Shiller, 1981, Salge, 1997), this test cannot purposely detect bubbles since it is difficult to accept the alternative hypothesis of the existence of a bubble due to the joint hypothesis problem. It may be the case that the violation of the bounds only reveals that there is something other than what the present value model can explain.

On the other hand, 'West's two-step tests' are the first of their kind to explicitly test a bubble in the alternative hypothesis. His idea is to consider two sets of consistent estimates, of which only one set is affected by the existence of a bubble (West, 1987); the estimated parameters from the equation can be compared to possibly bubble-infected parameters from the actual price-dividend relationship. Also, using sequential steps of testing, this test is devised to tackle the joint hypothesis problem.

In the first step of the test, the Euler equation is estimated by the instrument variable (IV) method and the risk-free rate r_f is obtained from the results. The r_f is not affected by the existence of bubbles. In addition, a dividend process is specified and estimated by the OLS method to obtain the estimates of necessary parameters, for example, ϕ when the dividend follows autoregressive (AR) of order one (no constant):

$$D_t = \phi D_{t+1} + \varepsilon_t$$

In the second step, using the obtained estimates, the parameters of the fundamental price are calculated. For example, the fundamental price (P^f) can be calculated when D is generated by the AR(1) process as follows.

$$P_t^f = \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+r_f}\right)^i E_t[D_{t+1}]\right] = \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+r_f}\right)^i E_t[\phi D_t + \varepsilon_t]\right] = \sum_{i=1}^{\infty}\left[\left(\frac{1}{1+r_f}\right)^i \phi D_t\right]$$

Simplify this:

$$P_t^f = \frac{(\phi/1+r_f)}{1-(\phi/1+r_f)} D_t = \overline{\beta} D_t$$

Then, the value of parameter $\overline{\beta}$, which is not influenced by bubbles, is obtained.

On the other hand, actual price process P_t is regressed on D_t such that

$$P_t = \hat{\beta} D_t + B_t$$

Another estimated value of parameter $\hat{\beta}$ can be obtained. In the absence of a bubble (B_t=0), the value of parameter $\hat{\beta}$ should be the same as that of parameter $\overline{\beta}$. However, since actual price process can be contaminated by bubbles, these two parameters are different in the presence of bubbles. In other words, West's test aims to find statistical significance in the difference between two parameter values. It uses the Hausman coefficient test. Flood and Hodrick (1990) incorporated more complex ARIMA processes into the test.

West's own results showed that there was a bubble in the US stock market from 1871 to 1981 and Meese (1986) also found a bubble in the foreign exchange market using the same methods. Casella (1989) identified a macroeconomic price level bubble in Germany after World War I. However, using slightly modified test statistics, Dezbakhsh and Demirguc-Kunt (1990) did not discover any evidence of bubbles using the same data as West.

To conduct West's test, the dividend process should be correctly specified based on the results of model specification tests. However, Dezbakhsh and Demirguc-Kunt (1990) questioned the validity of model specification tests, particularly about that of the Hausman test. Casella (1989) discovered the sensitivity of the test to the choice of instrument variables. In addition, Flood et al. (1994) pointed out that although the Euler equation held for two consecutive periods or for any two periods, a statistical error accumulated as the time gap of the two periods widened. Then, it consequently reduced the power of the test.

'The cointegration test' was invented by Hamilton and Whiteman (1985), and developed by Hamilton (1986) and Diba and Grossman (1988a). It is based on the effect of rational bubbles on cointegration between the stock price and dividend and their individual integration.

The fundamental price of the PVM in the above using the general discount rate κ is:

$$P_t^f = \sum_{i=1}^{\infty} \left(\frac{1}{1+\kappa} \right)^i E_t[D_{t+1}]$$

However, the actual price which may include a bubble is:

$$P_t = P_t^f + B_t$$

In the case of no bubbles (B_t =0), if dividends follow a stationary process or integrated process of order n, the price process (P_t) should be stationary or integrated of the same order. However, when a bubble exists (B_t >0), cointegration would not hold because the bubble process itself is non-stationary by definition. It can be seen as follows:

Suppose a rational bubble follows an explosive process:

$$E_t[B_{t+1}] = (1+\kappa)B_t \quad \text{or} \quad B_{t+1} - (1+\kappa)B_t = \varepsilon_t$$

where ε is zero-mean stochastic error.

Using a lag operator, L, it becomes at time t:

$$(1-(1+\kappa)L)B_t = \varepsilon_t$$

When the bubble process is differenced at the first order,

$$(1-(1+\kappa)L)(1-L)B_t = (1-L)\varepsilon_t$$

It can be noticed that $(1\text{-}L)B_t$ is stationary, so original B_t is not generated by a stationary ARMA process (Diba and Grossman, 1988b) and this is true for the n^{th} differenced bubble process. As a result, the cointegration relationship between actual prices and fundamental price based on dividends can be used as a bubble test. Any cointegration test under the null hypothesis of cointegration of order (1,1) can be used for testing for a rational bubble as most financial data are I(1). For example, Diba and Grossman (1988a) adopted the Dickey-Fuller test for individual integration and Bhargava's (1986) test for cointegration between them.

However, when the integration/cointegration based tests reject the null hypothesis, it may merely indicate there is something non-stationary in the price process. It may simply be a bubble, but it can be anything else (Gurkaynak, 2008). Therefore, they argued that the tests would only prove the non-existence of a rational bubble when it cannot reject the null hypothesis.

In addition, as Meese (1986) and Evans (1991) pointed out, cointegration tests cannot detect any other kind of bubble than non-stop exploding bubbles. For example, a periodically collapsing bubble is rather stationary and not easily detected by the cointegration test. Furthermore, Gurkaynak (2008) argued that the reliability of cointegration tests is also in question as different tests sometimes produced opposite results.

'Partial sum of residuals test' was designed by Wu and Xiao (2002). It is a test for rational bubbles to overcome the weakness of the integration and cointegration based tests. Their approach is unique in the sense that they developed a statistic based on the partial sum of residuals. Their test begins from the regression of the logarithm of price on the logarithm of dividend such that

$$\ln P_t = \alpha + \beta \ln D_t + \varepsilon_t$$

where α and β are intercept and slope coefficients.

They argued that in the absence of a rational bubble (b_t= lnB_t, included within ε_t), the order of magnitude of the partial sum process up to time k,

$$\sum_{t=1}^{k} \varepsilon_t$$

should be proportional to $k^{1/2}$

However, if a bubble does exist, even with a positive probability of collapse, the absolute value of a rational bubble by definition will grow since:

$$\left|E_t[b_{t+1}]\right| > \left|b_t\right|$$

Also, the variance of b_t and consequently that of the error term ε_t will increase exponentially with time. Hence, the partial sum diverged to infinity. They suggested the following test statistic

$$\underline{R} = \max_{k-1,\ldots,n} \frac{k}{\hat{w}_{\varepsilon.\ln D}\sqrt{n}} \left| \frac{1}{k}\sum_{t=1}^{k} \tilde{\varepsilon}_t^{+} - \frac{1}{n}\sum_{t=1}^{n} \tilde{\varepsilon}_t^{+} \right|$$

where $\tilde{\varepsilon}_t^{+}$ is from non-parametric estimation of the residuals and $\hat{w}^2_{\varepsilon.\ln D}$ is the non-parametric long-run variance estimator of ε_t from lnD_t. The details of the calculation were given in Wu and Xiao (2002). They found weak evidence of the US market bubbles, but relatively strong evidence from the Hong Kong market from 1974 to 1998.

Hall et al. (1999) developed 'regime switching bubble tests' by incorporating the collapsing bubbles of Evans (1991) into a regime switching model. In Evans's model, a bubble stochastically grows at the average rate of (1+κ) under one regime, but once reaching threshold $\widehat{B}$, the regime transits and the bubble grows faster with a positive probability of burst. Their two regimes are:

$$(1)\ B_{t+1} = (1+\kappa)B_t \times v_{t+1} \quad (B_t \le \widehat{B})$$

$$(2)\ B_{t+1} = B_0 + \frac{1}{\pi}\left[(1+\kappa)\theta_{t+1}B_t - \theta_{t+1}\delta\right]\ (B_t > \widehat{B})$$

where v is a random factor in bubble size, B_0 is the initial size of a bubble, θ is a binary variable indicating the survival of a bubble,

$$prob(\theta_{t+1} = 1) = \pi_B,\ prob(\theta_{t+1} = 0) = 1 - \pi_B,\ \text{and } E_t(v_{t+1}) = 1,$$

That is, over the threshold value, the bubble survives with a probability of π_B and bursts with a probability of $1-\pi_B$. After a burst, the bubble term reduces to B_0.

This system of regimes is equivalent to Evans's (1991) collapsing bubble model. Then, Markov regime switching Augmented Dickey-Fuller (ADF) tests were conducted to detect a bubble. On the other hand, van Norden (1996) applied a regime switching model to Blanchard's (1979) probabilistic rational bubble model. He assumed regime switching bubbles implied the testable coefficient restriction on innovations. By estimating the model and conducting a Wald test, he found weak evidence for bubbles. However, the regime-switching-based tests sometimes have serious size distortion (van Norden, 1996) and the choice of process affected their outcomes (van Norden and Vigfusson, 1998).

Meanwhile, it is possible to conduct 'direct bubble tests', which are bubble tests using the specifications of bubble processes. Previous tests did not directly estimate the bubble term and its coefficients. The tests indirectly utilised the variances of prices, the coefficients of dividends and residuals, and integration and cointegration between them. Flood and Garber (1980) and Flood et al. (1984) employed a deterministic bubble process for a direct test for bubbles at price level. Although their test is not originally designed for stock price, it gives a good implication for similar tests. Their model starts from a monetary inflation model. Money demand is:

$$m_t - p_t = \alpha + \beta(INF_{t+1}) + \varepsilon_t$$

where p is price and m is exogenous money stock both in logarithm, INF is inflation specified by an autoregressive function of growth rates in the money stock, α and β are the coefficients and ε is an error term.

The general solution of the model is:

$$p_t = \left[m_t - \gamma + \frac{\beta}{\beta-1} \sum_{i=0}^{\infty} E_t \left[(m_{t+i+1} - m_{t+i}) - (\varepsilon_{t+i+1} + \varepsilon_{t+1}) \right] \left(\frac{\beta-1}{\beta} \right)^i - \varepsilon_t \right] + \beta \left(\frac{\beta-1}{\beta} \right)^t A_o$$

The last term is a deterministic bubble term with an arbitrary constant, A_0.

Their idea was that with proper estimation procedures, A_0 can be estimated and the test for bubbles can be conducted under the null hypothesis of A_0=0. They carried out three-step estimation: money stock on past money stock, money demand on inflation, and then the final estimation of A_0 after the substitution of the previous results into the rearranged general solution. They found the evidence of a bubble of German hyperinflation in the 1920s, but strong correlation among α, β and A_0.

Froot and Obstfeld's (1991) intrinsic bubble theory (Section 3.1.1) is another example of deterministic bubbles and can be converted to a bubble test. Their 'intrinsic bubble test' utilised the PVM in continuous time to derive a price process with a deterministic rational bubble:

$P_t = \gamma D_t + c D_t^{\lambda} + \varepsilon_t$ where is $\gamma = (e^{\kappa} - e^{\mu + \sigma^2/2})^{-1}$

The first term is the fundamental price (P_t^f) and the second term is the bubble process where μ and σ are mean and standard deviation of error (ε). λ is the positive root of $\lambda^2 \sigma^2 / 2 + \lambda \mu - \kappa = 0$.

Then, divide the equation by D_t to obtain:

$$\frac{P_t}{D_t} = \gamma + c D_t^{\lambda-1} + \iota_t$$

where $\iota_t = \varepsilon_t / D_t$

This equation is estimated to test for rational bubbles. If a bubble exists, c must be significant since it shows a non-linear relationship between prices and dividends. They examined 1900-1988 US S&P index data and concluded that the intrinsic bubble existed. However, they admitted their results may give weak evidence of bubbles because the rejection of the null hypothesis merely shows a non-linear relationship between price and dividend process (Froot and Obstfeld, 1991). Moreover, the existence of intrinsic bubbles can be dismissed by adopting a Markov regime switching dividend process (Driffill and Sola, 1998).

Some bubble tests are not dependent on the PVM. They rely solely on residuals from any pricing model, so they can be used to test for rational, irrational bubbles, and economy-level bubbles. They are more flexible in this sense but more ambiguous in specifying the pricing model of the fundamental.

Hurst's (1951) 'rescaled range tests' originally intended to test for long-term dependence or persistence were first imported into finance by Mandelbrot (1972) and generalised by Lo (1991). The test has a potential for bubble tests since a bubble may generate strong persistence in residuals from the fundamental prices. Hurst's test begins with calculation of range *R(n)* and scale factor *S(n,q)* from residual series, e_t, and its mean $\bar{e}$ of *n* samples.

$$R(n) = \left[\max_{1\le k\le n} \sum_{j=1}^{k} (e_j - \bar{e}) - \min_{1\le k\le n} \sum_{j=1}^{k} (e_j - \bar{e}) \right]$$

and

$$S(n,q) = \sqrt{n}\sqrt{\hat{\sigma}^2 \left[1 + 2\sum_{i=1}^{q} \left(1 - \frac{i}{q+1} \right) \hat{\rho}_i \right]}$$

where $\hat{\sigma}^2$ is the sample variance, $\hat{\rho}_i$ is estimated autocorrelation, q is a period of short-term dependence.

When q is assumed to be zero, *S(n,p)* becomes a scaled standard deviation, $S(n) = \sqrt{n} \times \hat{\sigma}$ (Mandelbrot, 1972). Hurst showed that the ratio of *R(n)/S(n)*, 'R/S statistic', is proportional to *n*H, where H is ½ in the case of a random walk and H is larger than ½ when persistence exists in the data. In the case of non-zero q (Lo, 1991), the null of short-term dependence can be tested. An example of the application in bubble tests comes from Ahmed et al. (2006). They calculated Hurst's H for Chinese stock markets in the 1990s and found some evidence of persistence in residuals. However, this test only distinguished the existence of persistence not that of specific bubbles.

'BDS independence tests' are devised to test for long-term dependence and non-linearity in residuals. They can be used to find price bubbles in any well-specified pricing model (Brock et al., 1996). They constructed the tests based on Grassberger and Procaccia's (1983) correlation integral ($\widehat{c}$), which is calculated from T observations of residuals e_1, ..., e_T and n-histories (or embedding dimension): e_{t-n+1}, e_{t-n+2}, ..., e_t for a chosen tolerance distance k.

$$\widehat{c}_n(k) = \lim_{T\to\infty} \widehat{c}_{n,T}(k) = \lim_{T\to\infty} \frac{\sum_{s=1}^{T}\sum_{t=s}^{T} K_{st}}{T(T-1)/2}$$

where K_{st} is a closeness indicator and has a value of 1 if $\max_{i=0,\dots,n-1}|e_i - e_s| < k$ and otherwise 0.

Then, the BDS test statistic is calculated using two correlation integrals with different length of history:

$$BDS = \sqrt{T}\,\frac{\widehat{c}_{n,T}(k) - \widehat{c}_{1,T}(k)^n}{\hat{\sigma}_{n,T}(k)}$$

The statistic follows asymptotically normal under the null of independent and identical distribution (i.i.d) since $\widehat{c}_n(k) = \widehat{c}_1(k)^n$ for any n. Due to a known small sample bias, the sample size is recommended to be over 500. By rejecting the null, the residuals are expected to have unexplained long-term dependence and non-linearity, but the alternative hypothesis is not specified. For example, it is also used to give some indication for ARCH effects (Brooks, 2008). Thus, it could simply be the evidence of a time-varying conditional variance (Hsieh, 1991, Campbell et al., 1997).

'Duration dependence tests' are brought by McQueen and Thorley (1994) into the study of stock price bubbles. Duration dependence under a bubble is the phenomenon of the probability of the end of the run (i.e. a crash) becoming lower as abnormal runs (a bubble) extend, but the size of the crash is larger as a result. Abnormal runs can be counted using any fundamental pricing model. For example, time series based models can be employed for this purpose. Meanwhile, McQueen and Thorley (1994) used Fama-French 3 factor model (Fama and French, 1989) as a fundamental. The original idea of the tests came from Durland and McCurdy (1994)'s work. Note that duration used in this chapter is different from bond or equity duration (Leibowitz, 1986), which indicates the price sensitivity of bonds or equities to interest rates.

Let f be the probability distributions of the length of a run I ending at a particular length i and its cumulative counterpart (F) is the probability of the run not extending over the length of i.

$$f_i = prob(I = i)$$
$$F_i = prob(I < i)$$

Then, a hazard rate is defined as:

$$h_i = \frac{f_i}{1 - F_i}$$

This must be decreasing when duration dependence exists possibly due to a bubble.

Using the logistic hazard function, the rate can be specified as:

$$h_i = \frac{1}{1 + e^{-(\alpha + \beta \ln i)}}$$

The coefficients α and β can be estimated using logit estimation. Then, a likelihood ratio test can be conducted on the value of β. If a price bubble does not exist, the hazard function should be constant even if i increases. That is, β=0 and h=1. Otherwise, if there is negative duration dependence (i.e. β<0 and 0<h<1), a bubble may exist. McQueen and Thorley (1994) found that positive runs have negative duration dependence while negative runs do not have such dependence using NYSE monthly return data between 1927 and 1991. Recent evidence of a bubble was also discovered (Lunde and Timmermann, 2004, Chen and Shen, 2007, Zhang, 2008). Further details about this bubble test follow in Section 3.2.

There are also other types of bubble tests. They mostly utilise the statistical attributes of bubbles and crashes. 'Runs and tail tests' are similar to the idea of the duration dependence tests. Blanchard and Watson (1982) suggested that bubbles produce positive runs in innovations, but crashes are responsible for a fat tail distribution due to being negative outliers. They used data from the gold market between 1975 and 1981, and discovered evidence of bubbles. However, these attributes are not unique in bubbles and are often associated with fundamentals (McQueen and Thorley, 1994) as they are quite common in financial data (Taylor, 2005).

Evans (1986) developed a 'median test' for rational speculative bubbles in foreign exchange markets. He expected that bubbles may produce non-zero median of data and recognised a bubble in US-UK exchange rates in the early 1980s. Nonetheless, he admitted that his method had difficulties in distinguishing speculative bubbles from asymmetric fundamentals or irrationality of markets.

In the business cycle literature, there exists a widely accepted rule to decide the 'turning points' of bull and bear markets, and subsequently bubbles and crashes. It is not actually a statistical test but a formal rule devised by the National Bureau of Economic Research (NBER), and academics utilise the variants of the rule for their research; a peak/trough is identified as a point higher/lower than all points six months before and after. A cycle must last 15 months from a peak to a trough. Each phase must span at least 5 months (Gonzalez et al., 2005).

Technical analysts have their own 'technical tests' to attempt to recognise deterministic and predictable trends or bubbles in charts. For example, Dow theory insists stock markets consist of three main trends: major, intermediate and short-run, and each trend has three phases of accumulation, change and distribution (Reilly and Brown, 2003). However, the theory depends on arbitrary decisions on qualitative price patterns. Meanwhile, Elliot wave theory suggests that the basic movement of stock price has identifiable repetition of up and down movements similar to Fibonacci series or mathematical fractal geometry (Mandelbrot, 1999). For example, there are five ups and three downs in a bull market and the opposite in a bear market. However, most academics criticise the theory as more like an art form and dependent on subjective judgement (Mandelbrot, 1999).

3.2. The duration dependence test for price bubbles

This section aims to discover the evidence for price bubbles in more recent data spanning subsets of 1979-2008 from 5 stock markets using mainly one of the statistical tests for stock price bubble, the duration dependence test. As briefly reviewed in Section 3.1.2, the duration dependence test is a non-PVM based test for pricing bubbles. One of the advantages is its independence from the PVM-based price fundamentals. Instead, it can adopt any pricing model which is frequently used on time series econometric analysis. In other words, this test has more generality over the other tests since the test is applicable even without the data of the price fundamental like dividend data which are sometimes not reliable or available in some countries like China. Although it also means that the results of the test rely on the choice of pricing models, as long as the choice is reasonable, the findings in the duration dependence tests bring good insights into the price movement in the sample data.

This section utilises several univariate time series models including ARMA or ARMA-GARCH models used as price generating processes or proxies for price fundamentals following earlier research (McQueen and Thorley, 1994, Chan et al., 1998, Lunde and Timmermann, 2004, Zhang, 2008). Then, the duration dependence tests are conducted and the test results are investigated for further implications for the expansion of the duration dependence test.

3.2.1. Methodology

The first step of the duration dependence tests for a bubble on financial data is to retrieve the duration data. It first requires estimating the mean model and obtaining its residuals. Then, all the duration of runs are recorded based on the 'sign' of the residuals. In other words, the number with the same sign as them is counted until the sign is reverted, and then recorded as a positive or negative run. For example, suppose the estimation of the GJR-GARCH(1,1) model provides the following residuals between the 11^{th} and the 20^{th} observations. They are converted to the runs where the numbers in the last two rows are duration i.e. length of run.

week	10	11	12	13	14	15	16	17	18	19	20	21
residual	...	+0.008	+0.006	+0.001	-0.002	-0.004	-0.003	+0.001	-0.006	+0.005	+0.002	...
sign	-	+	+	+	-	-	-	+	-	+	+	-
(+) run	...			3				1			2	...
(-) run	...						3		1			...

Table 23 Example: converting residuals to positive and negative runs

These positive and negative runs can be considered as spells of overperfoming and underperforming markets, or simply bull and bear markets.

Using this retrieved duration data, it is actually possible to estimate the 'unconditional' probability that a run grows and ends, that is, π_R and (1-π_R) by supposing a specific probability distribution of a run. This is a different proposition from the following duration dependence test that relies on hazard rates and the probability that a bubble grows and crashes (π_B and 1-π_B).

The theoretical background of estimation of unconditional π_R requires the main assumption that duration is a random variable with geometric distribution which is based on binominal distribution of run continues and ends. The histogram of durations from one of the datasets in the study, US S&P500 index (1978-2008) displays a typical pattern of geometric distribution.

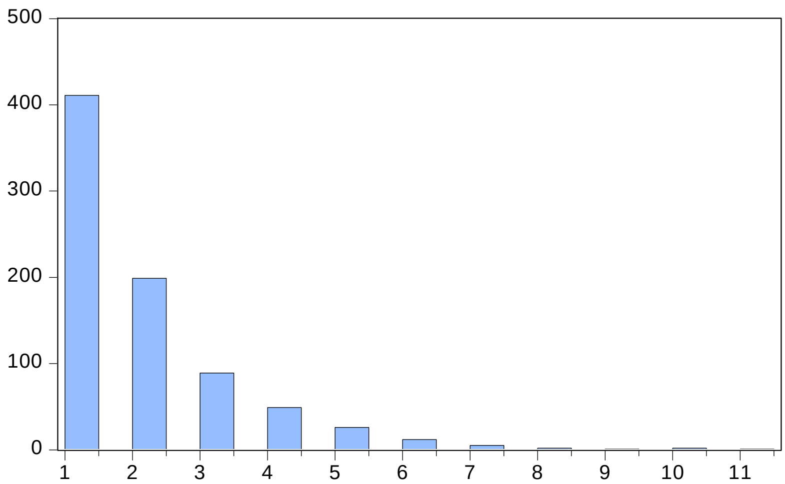

Figure 3 The histogram of duration series: S&P500

Now suppose the number of time periods of a run growing before it ends, i.e. the duration of a run is *d* (i in hazard rates). Then, it can be derived from the general results of geometric distribution (Jordan and Smith, 2008) that the probability that a run ends after d periods of growth is equivalent to that of having one failure after k (=d-1) successes in k+1 trials, which is known to follow geometric distribution:

$$prob(X = k) = prob(D_R = (d - 1)) = \pi_R^k (1 - \pi_R) \quad \text{where k=0,1,2,3,...}$$

where X is a random variable representing the number of successes before failure and D_R is an equivalent random variable representing the duration before a run ends. Note that geometric distribution is a generalisation of Bernoulli distribution and the special case of negative binomial distribution of continuous random variables and Pascal distribution of discrete random variable (Jordan and Smith, 2008).

It is also known that the expected value of the number of successes before failure is:

$$E(X) = \frac{\pi_R}{1 - \pi_R}$$

Then, the expected duration before the end of a run is:

$$E(D_R) = E(X) + 1$$

Meanwhile, the variance of the number of successes before failure is equal to that of the duration before the end:

$$\operatorname{var}(X) = \operatorname{var}(D_R) = \frac{\pi_R}{(1 - \pi_R)^2}$$

The likelihood function of a failure or a run end is:

$$L(p_e \mid k_i) = (1 - p_e)^{k_1} p_e \times (1 - p_e)^{k_2} \times \cdots \times p_e (1 - p_e)^{k_N} p_e = (1 - p_e)^{\sum_1^N k_i} \left(p_e\right)^N$$

where p_e=(1-π_R), where N is the total number of runs and k_1, ..., k_N are the observed values of k (=d-1) for each of N runs.

Then, the corresponding log likelihood function is obtained:

$$\ln L(p_e \mid k_i) = \left(\sum_1^N k_i \right) \ln(1 - p_e) + N \ln p_e$$

The first order condition is that the first derivative of the above function is zero:

$$\frac{\partial \ln L(p_e \mid k_i)}{\partial p_e} = -\frac{\sum_1^N k_i}{(1-p_e)} + \frac{N}{p_e} = 0$$

Solve this for p_e, and the maximum likelihood estimator of p_e is obtained.

$$\hat{p}_e = \frac{N}{N + \sum_1^N k_i}$$

This is the estimated probability that a run ends. Also, since $p_e = 1 - \pi_R$, the estimator of π_R, i.e., the probability of run growth, is easily retrieved by:

$$\hat{\pi}_R = 1 - \hat{p}_e = 1 - \frac{N}{N + \sum_1^N k_i} = \frac{\sum_1^N k_i}{N + \sum_1^N k_i}$$

Now, it is represented in terms of durations (d):

$$\hat{\pi}_R = \frac{\sum_1^N k_i}{N + \sum_1^N k_i} = \frac{\sum_1^N (d_i - 1)}{N + \sum_1^N (d_i - 1)} = \frac{\sum_1^N d_i - N}{\sum_1^N d_i}$$

On the other hand, the same analysis can be repeated on the duration of negative runs, and then the probabilities of negative run continues or ends are obtained. In other words, the unconditional probabilities from under-performing markets to over-performing markets are estimated.

However, it is more realistic to allow for the possibility that these probabilities may change, in particular conditional on the length of duration. This is the main idea of the duration dependence test that argues that this probability decreases as a run grows in length if there is a bubble. Then, it can be thought that the unconditional probabilities are the special case of conditional probabilities, specifically with the assumption of constant hazard rate. However, two approaches use different assumptions about the distribution of duration.

The essential idea of the duration dependence test for rational price bubbles is supported by the theoretical results using the PVM-based bubble theory although the duration dependence test itself does not use the PVM. To be precise, the simple probabilistic bubble model of Blanchard and Watson (1982) provides the theoretical background to the test.

The simple probabilistic model in Section 3.1.1 can be slightly more generalised by adding a small positive initial bubble term, B_0, in a two-period model (Bollerslev and Hodrick, 1992, McQueen and Thorley, 1994).

Suppose a bubble grows at the rate of κ without probabilistic collapse.

$$E(B_{t+1}) = (1+\kappa)B_t$$

By introducing a simple probabilistic collapse with a probability of ($1-\pi_B$).

$$E(B_{t+1}) = \pi_B[\frac{1}{\pi_B}(1+\kappa)B_t] + (1-\pi_B)[0]\left(=(1+\kappa)B_t\right)$$

Although a bubble on average grows at the same rate, the possibility of probabilistic collapse is compensated by a faster growth rate when a bubble grows.

Add an initial bubble B_0 and rearrange:

$$E(B_{t+1}) = \pi_B[\frac{1}{\pi_B}(1+\kappa)B_t - \frac{1}{\pi_B}\left(1-\pi_B\right)[B_0]] + \left(1-\pi_B\right)[B_0]\left(=(1+\kappa)B_t\right)$$

Now it can be seen that:

(1) If a price bubble continues with a probability of π_B., the size of a bubble at t+1 is:

$$B_{t+1} = \frac{1}{\pi_B}(1+\kappa)B_t - \frac{(1-\pi_B)}{\pi_B}B_0 + \varepsilon_{t+1}$$

(2) If the bubble bursts with a probability of ($1-\pi B$), it goes back to its initial status:

$$B_{t+1} = B_0 + \varepsilon_{t+1}$$

where ε is the unexpected changes or the errors in the bubble process.

On the other hand, the unexpected changes in the fundamental price process (η) can be defined as:

$$\eta_{t+1} = P_{t+1} + D_{t+1} - (1+\kappa)P_t$$

where D_t is dividend at time t.

This is derived from the two-period present value formula or the dividend discount model as follows.

$$\begin{aligned}
P_t &= \frac{1}{1+\kappa}(P_{t+1} + D_{t+1}) + \varepsilon_t \\
P_{t+1} + D_{t+1} &= (1+\kappa)P_t - (1+\kappa)\varepsilon_t \\
P_{t+1} + D_{t+1} - (1+\kappa)P_t &= -(1+\kappa)\varepsilon_t \\
P_{t+1} + D_{t+1} - (1+\kappa)P_t &= \eta_t
\end{aligned}$$

The errors (ε) in the bubble process are stochastically decided.

(1) The unexpected difference in bubble growth if the bubbles continues,

$$\begin{aligned}
\varepsilon_{t+1} &= B_{t+1} - (1+\kappa)B_t \\
&= \frac{(1-\pi_B)}{\pi_B}\left[(1+\kappa)B_t - B_0\right]
\end{aligned}$$

with a probability of π_B and

(2) The unexpected difference between an actually collapsed bubble and an expected growing bubble if the bubble ends,

$$\varepsilon_{t+1} = -(1+\kappa)B_t + B_0$$

with a probability of $(1-\pi_B)$.

Then, the total unexpected changes (ε^T) are either:

(1)

$$\begin{aligned}
\varepsilon_{t+1}^T &= \eta_{t+1} + \varepsilon_{t+1} \\
&= \eta_{t+1} + \frac{(1-\pi_B)}{\pi_B}\left[(1+\kappa)B_t - B_0\right]
\end{aligned}$$

with a probability of π_B or

(2)

$$\varepsilon_{t+1}^{T} = \eta_{t+1} + [-(1+\kappa)B_t + B_0]$$

with a probability of (1-π_B).

π_B is commonly assumed to be larger than 1/2 because of the nature of bubbles, and then the errors of the bubble process are negatively skewed even with zero mean, unlike symmetric probability of the unexpected changes in the fundamental prices. Consequently, the overall process has negative skewness from this.

Now suppose the cumulative distribution function of the errors in the bubble process is G and the probability distribution function is g, then the probability of the total unexpected changes being negative is (McQueen and Thorley, 1994):

$$prob(\varepsilon_{t+1}^{T} < 0) = \pi_B G\left[-\frac{(1-\pi_B)}{\pi_B}\left[(1+\kappa)B_t - B_0\right]\right] + (1-\pi_B)G\left[(1+\kappa)B_t - B_0\right]$$

It can be differentiated with respect to B_t to investigate the effect of the growing bubble to the probability of price collapsing.

$$\frac{\partial prob(\varepsilon_{t+1}^{T} < 0)}{\partial B_t} = -(1-\pi_B)(1+\kappa)\left[g\left[-\frac{(1-\pi_B)}{\pi_B}\left[(1+\kappa)B_t - B_0\right]\right] - g\left[(1+\kappa)B_t - B_0\right]\right]$$

The above derivative is negative as long as π_B is greater than 1/2. This is due to the first term in the square bracket being larger than the second because the former is closer to the mean and has a higher probability of happening. Another assumption of a bell-shaped probability function is required for this i.e. the probability of a specific value happening decreases as it moves farther away from the mean.

These theoretical results support the empirical expectations in Section 3.1.2 by showing that there is less chance of bubble bursts as the price bubble grows. That is, negative duration dependence may be discovered in the market data under the presence of a bubble.

On the other hand, the empirical test of duration dependence considers whether a hazard rate increases or decreases when the runs are extended. The hazard rate is the probability of a run ending after lasting a specific time period. This is formally represented as follows: suppose f is the probability distribution function of the run (I) ending after the lasting time period i. Then, the cumulative distribution function F is the probability the run ends before i.

$$f_i = prob(I = i)$$
$$F_i = prob(I < i)$$

Then, a (population) hazard rate is:

$$h_i = \frac{f_i}{1 - F_i}$$

This is expected to decrease if a (positive) bubble exists. That is, there is negative duration dependence. For statistical inference of the hazard rate from the sample, its functional form needs to be specified; this section utilises the logistic hazard functions of McQueen and Thorley (1994)'s original study:

$$h_i = \frac{1}{1 + e^{-(\alpha + \beta \ln i)}}$$

The duration dependence tests have been employed in the empirical research to detect price bubbles or other types of duration dependence. After McQueen and Thorley (1994)'s initial development of the test in the field of finance, there was a line of following research. For example, Maheu and McCurdy (2000) discovered the evidence of bubbles in monthly CRSP (Center for Research in Security Prices) value-weighted index on the NYSE between 1802 and 1995. Lunde and Timmermann (2004) applied the test on 110 years of daily US S&P500 index series (1885-1995). They used the residuals from an ARMA-GARCH-based model and revealed that positive duration dependence exists in bear markets, but no evidence of bubbles in bull markets. Yuhn (2010) discovered a bubble in the US stock market from the data between 2000 and 2007.

Mokhtar et al. (2006) used simple return models and found a bubble in the Malaysian stock market before and after the 1997 Asian crisis. Chen and Shen (2007) extensively examined more recent but less frequent monthly data of Asia-Pacific stock markets and AR-based models. They discovered positive duration dependence commonly existed in stock indices. For instance, Singaporean and Taiwanese stock markets showed positive duration dependences in both bull and bear markets between 1970 and 2004 while Korean, Japanese and Hong Kong stock markets displayed positive duration dependence in bear markets only. On the other hand, Zhang (2008) discovered price bubbles in two weekly indices of Chinese stock markets, Shanghai Composite and Shenzhen Composite from 1991 to 2001. Bhaduri (2009) did not discover bubbles in the Indian market data between 1990 and 2007 based on AR models.

In this study, the duration dependence test will be conducted in the following steps. First, using the duration data retrieved as described above, the number of a run with specific duration *i* (N_i) is recorded, and separated into two groups of positive and negative runs. Then, the number of runs with duration greater than i is also calculated (M_i). Subsequently, the sample hazard rates for each duration i are calculated as:

$$\hat{h}_i = \frac{N_i}{N_i + M_i}$$

Then, two tables, each representing positive and negative runs, are constructed. For instance, the table for positive runs from the above table is:

Duration	N_i	M_i	$\hat{h}_i$
1	1	2	0.333
2	1	1	0.500
3	1	0	1.000

Table 24 Example: positive runs and sample hazard rates

Finally, suppose the logistic hazard function is used for the functional form of the hazard function:

$$h_i = \frac{1}{1 + e^{-(\alpha + \beta \ln i)}}$$

The coefficients α and β are estimated using logit estimation which maximises the log likelihood function (McQueen and Thorley, 1994):

$$\ln L(\alpha,\beta \mid N_i, M_i, h_i) = \sum_{i=1}^{\infty} N_i \ln h_i + M_i \ln(1-h_i)$$

Subsequently, a likelihood ratio (LR) test can test for a restriction on the value of β using the LR statistic (Verbeek, 2004):

$$LR = 2[\ln L_{UR} - \ln L_R] \sim \chi^2_{(1)}$$

where $\ln L_{UR}$ is the log likelihood value of the unrestricted model and $\ln L_R$ is that of the restricted model. In particular, the significance of β i.e. β=0 is tested in the duration dependence test for price bubbles. If significant, the sign of β is essential in interpreting whether the data shows positive or negative duration dependence. Price bubbles are expected to display negative duration dependence in positive runs.

3.2.2. Data

To examine the existence of bubbles, a total of five stock price indices are selected and investigated by the duration dependence tests. First, the Standard & Poor's (S&P) 500 index is selected because of its representativeness of the two largest US stock exchanges, the NYSE and the NASDAQ. The index consists of 500 large-cap firms in two stock markets. Second, NASDAQ 100 index represents relatively smaller firms in the growing industry than the S&P500. It may be more strongly subject to overall market sentiment. This index contains the largest 100 firms listed on the NASDAQ. Third, another stock index from a developed country, FTSE100 of the London Stock Exchange is chosen to reflect a possible geographical difference. Fourth, the addition of Indian BSE100 index is for developing countries, which characterizes the five regional stock markets in India. Last, Korean KOSPI200 index is chosen as Korea experiences dramatic economic development as well as crisis. Also, Indian and Korean indices have been rarely tested for duration dependence.

Weekly price data is employed for the analysis. Specifically, Wednesday close-indices are chosen instead of average weekly prices not to remove the time series properties of the original series. Weekly data can avoid excessive noise in daily data. Also, it can more easily maintain enough observations compared with monthly or quarterly datasets since duration dependence tests convert price or return observations into the count of positive and negative runs.

For S&P500, the sample period is 30 years between 03/01/1979 and 31/12/2008. The total number of observations is 1,566. NASDAQ 100 has slightly fewer observations, 1,357 ranging from 05/01/1983 and 31/12/2008 (26 years). UK FTSE100 has the same sample period as S&P500. The sample of BSE100 index contains the data between 07/01/1987 and 31/12/2008 which consists of 1,200 observations in 22 years. The sample for the Korean KOSPI200 has the least number of observations, which is 992 covering 03/01/1990 to 31/12/2008 (19 years). The difference in the length of the sample periods reflects the limited data availability in some markets.

All data is obtained from Datastream online database. The patterns of log prices (natural logarithm of the raw index) are displayed in the following figures where the x axis represents year and the y axis is for log prices. The scale of the y axis is adjusted for each index. Either log price or log return series will be used depending on the nature of the tests or investigations in the following sections.

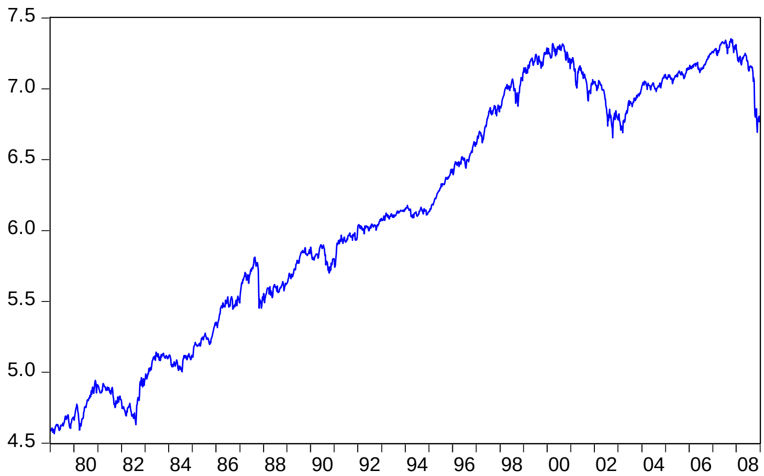

Figure 4 S&P500 from 03/01/1979 to 31/12/2008

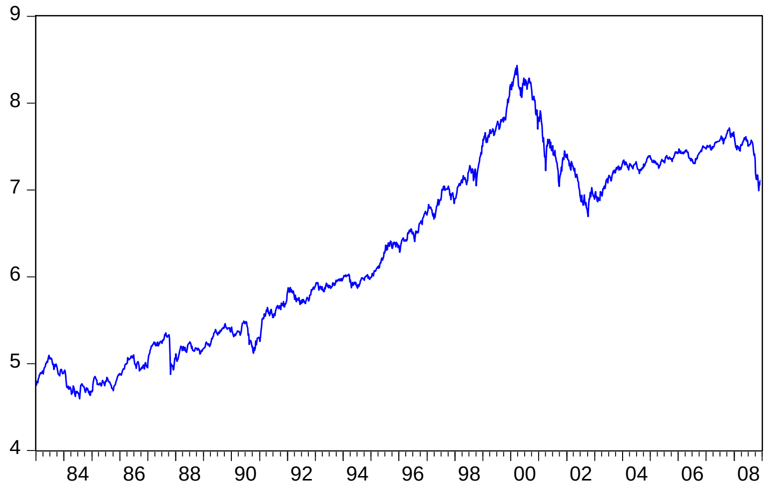

Figure 5 NASDAQ100 from 05/01/1983 to 31/12/2008

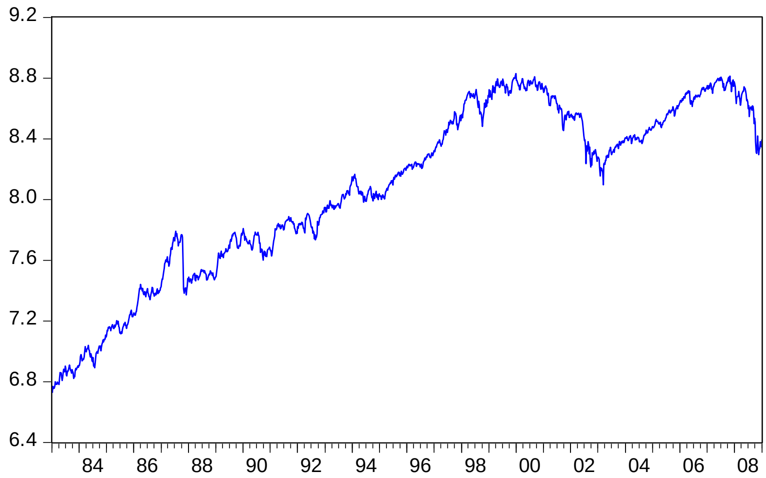

Figure 6 FTSE100 from 03/01/1979 to 31/12/2008

The movements of the US and UK indices (Figure 4, Figure 5, and Figure 6) show the descriptive patterns of several bubbles and crashes. For example, 1987 Black Monday, 2000 dotcom bubble and 2007-08 subprime crisis can be all observed. Regarding the sample periods of 30 years, the patterns of a bubble and crash rarely happened. Thus, it may imply negative duration dependence.

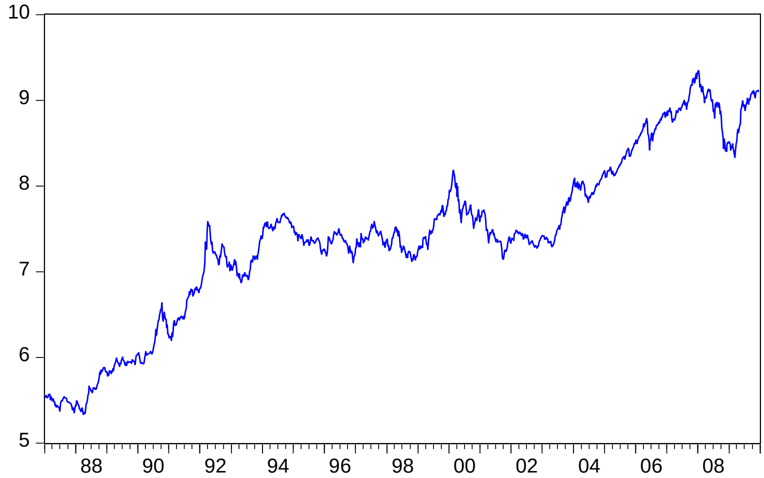

Figure 7 BSE100 from 07/01/1987 to 31/12/2008

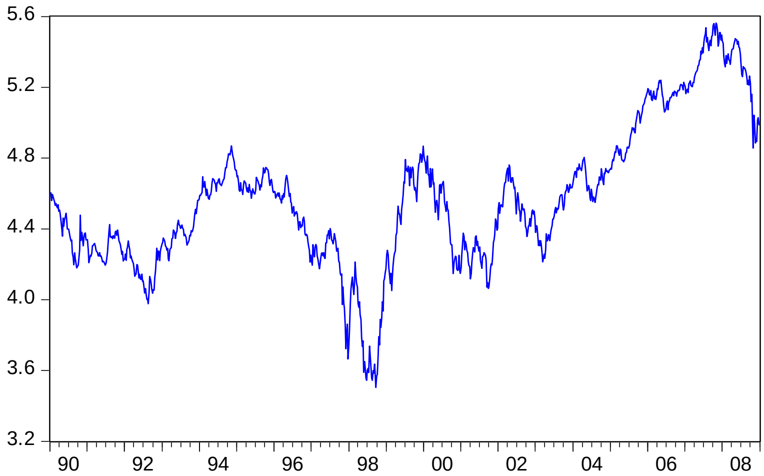

Figure 8 KOSPI200 from 03/01/1990 to 31/12/2008

The remaining two indices also display the patterns of bubbles and crashes throughout the sample period. Notably, the Korean stock market was severely affected by the 1997 East Asian crisis, which was not observed in other stock indices. Like the first three indices, negative duration dependence can be expected from rarely happened bubbles and crashes in the overall build-up of price, but empirical evidence needs to be explored.

The next table (Table 25) summarises the descriptive statistics of all five stock indices and their returns. In those log return series, 4 out of 5 indices show negative skewness and all indices display leptokurtosis. They is descriptive evidence for the presence of long-term building of price bubbles (positive returns) and short-term crashes (negative returns). Their distributions all depart from normality as usually observed in financial data, which commonly contain outliers and volatility clustering, as Jarque-Bera (JB) statistics represent. Otherwise stated, all the empirical analysis is conducted using Eviews statistical package in this section.

Log price	US S&P 500	US NASDAQ100	UK FTSE100	INDIA BSE100	Korea KOSPI200
observations	1,566	1,357	1,566	1,200	992
Mean	651.846	943.038	3693.704	2521.658	105.195
Maximum	1562.470	4596.800	6835.910	11449.950	260.420
Minimum	96.280	98.960	834.300	207.870	33.260
S.D.	469.003	876.472	1773.486	2397.565	46.947

Log return	US S&P 500	US NASDAQ100	UK FTSE100	INDIA BSE100	Korea KOSPI200
Mean	0.001	0.002	0.001	0.003	0.000
Maximum	0.102	0.184	0.136	0.248	0.196
Minimum	-0.167	-0.218	-0.178	-0.178	-0.196
S.D.	0.022	0.037	0.023	0.041	0.043
Skewness	-0.752	-0.578	-0.726	-0.072	0.034
Kurtosis	8.520	6.954	9.599	5.968	5.297
Jarque-Bera	2134.131	959.038	2580.010	441.059	218.138

Table 25 Descriptive statistics of five sample indices

3.2.3. Return models

In the duration dependence tests, the focus is on the duration of runs of abnormal positive returns and negative returns. Thus, the choice of models for returns may be important as the return model provides residuals that are later converted to runs for actual duration dependence tests. As reviewed earlier, the common choice is ARMA or ARMA-GARCH based time series models. US S&P 500 index is first employed to obtain general knowledge about the series and the procedure is repeated for all the other indices.

Econometric properties of log return series (r_t) are examined first. The ACF and PACF of the log return series of S&P500 (Table 26) display that the series is not much different from white noises as their values and Q statistics suggest. That is, an ARMA structure is not present and the price generating process is a random walk. Note that if '^' is attached in the table, it means the value of ACF or PACF is significant at the 5% significance level. Otherwise, it is not significant.

Meanwhile, although the Augmented Dickey-Fuller (ADF) unit root tests of prices accept the null hypothesis with the t statistic of -1.1037 with 7 lags that is selected by the smallest Akaike Information Criteria (AIC), that of the return series reject the null hypothesis of unit roots at the 5% significance level with the t statistic of -40.202 with no lag. That may indicate the return generating process is a white noise (ε_t) or the (log) price generating process is a random walk:

$$r_t = \varepsilon_t$$

Autocorrelation	Partial Correlation		ACF	PACF	Q-Stat	p-value
|	|	1	-0.018	-0.018	0.5037	0.478
|	|	2	-0.016	-0.016	0.9034	0.637
|	|	3	0.023	0.023	1.7622	0.623
|	|	4	-0.010	-0.009	1.9141	0.752
|	|	5	0.021	0.021	2.6009	0.761
|	|	6	0.037	0.037	4.7249	0.580
|	|	7	-0.056	-0.053	9.6048	0.212
|	|	8	-0.031	-0.033	11.096	0.196
|	|	9	0.039	0.035	13.462	0.143
|	|	10	-0.000	0.003	13.462	0.199
|	|	11	0.008	0.008	13.565	0.258
|	|	12	-0.026	-0.027	14.632	0.262

Table 26 ACF and PACF of log returns: S&P500

However, mean return values in the descriptive statistics imply that log return series may contain at least a constant term. The preliminary regression of the constant return model of S&P500 (Table 27) displays the possible presence of a trend in price or a constant in return. The ACF and PACF of the residuals from the constant return model are the same as Table 26. r_t is returns, a_0 is constant and ε_t is the errors.

$$r_t = a_0 + \varepsilon_t$$

Variable	Coefficient	S.D	t-Statistic	p-value
Constant(a_0)	0.00142	0.00056	2.5186	0.0119
SIC	-4.7633			

Table 27 Estimation results of the constant return model: S&P500

However, BDS independence test (Brock et al., 1996) confirms that there exists non-linearity in both residuals of the models. The null hypotheses of independent and identically distributed residuals are rejected with p-values of 0.000 for all 2 to 6 dimensions. These results propose a GARCH structure in the volatility of return processes although ARMA structure does not reside in the return process. Therefore, GARCH terms (volatility models) are added into the return model as follows. Note that, as in Chapter 2, GARCH(1,1) may be a reasonable choice in financial data (Baillie and Bollerslev, 1992, Franses and van Dijk, 1996, Andersen et al., 2001) as a white noise model is too simple and volatility clustering is known to occur in financial data (Lunde and Timmermann, 2004). GARCH(1,1) specification is:

$$
\begin{aligned}
r_t &= a_0 + \varepsilon_t \\
\sigma_t^2 &= \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2
\end{aligned}
$$

On the other hand, the duration dependence tests in the literature revealed the asymmetry in the runs of positive or negative abnormal returns. Thus, if the leverage effect is incorporated in the volatility model, it may be able to reduce the significance of duration dependence. Then, it is appealing to add a dummy variable (DUM), which represents the leverage effect, to the volatility process and test for whether it makes a difference in duration dependence. As a result, the model specification is:

$$
\begin{aligned}
r_t &= a_0 + \varepsilon_t \\
\sigma_t^2 &= \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2 + c_1 \mathrm{DUM}_{t-1} \varepsilon_{t-1}^2
\end{aligned}
$$

where c_1 is the coefficient of the dummy. The value of DUM_t is 1 if ε_t is negative and 0 otherwise. This is essentially a GJR-GARCH(1,1) model (Glosten et al., 1993).

The return models that have been so far considered are: white noise, constant mean return, constant mean GARCH(1,1) and constant mean GJR-GARCH(1,1) models. Except for the white noise model, which can be excluded by the BDS test results, the remaining 3 models are estimated on the S&P500 return samples. The estimation results of the models are in Table 28. In the case of S&P500, the leverage effects are present in the data. Possibly for the same reason, the GJR-GARCH(1.1) model is preferred in terms of the significance of individual coefficients and the Schwartz information criteria (SIC). Although the mean model is identical, different a_0 estimates can be produced due to the nature of the maximum likelihood estimation method that is practically based on a numerical maximisation procedure. This may lead to different p values and results of the following duration dependence tests.

	Coefficient	S.D	t-statistic	p-value
Constant return model				
constant in mean	0.0014	0.0006	2.5186	0.0119
SIC	-4.7633			
GARCH(1,1) model				
	Coefficient	S.D	t-statistic	p-value
constant in mean	0.0021	0.0005	4.1592	0.0000
constant in variance	0.0000	0.0000	4.4249	0.0000
ARCH	0.1339	0.0131	10.2152	0.0000
GARCH	0.8424	0.0184	45.8087	0.0000
SIC	-4.9316			
GJR-GARCH(1,1) model				
	Coefficient	S.D	t-statistic	p-value
constant in mean	0.0013	0.0005	2.6211	0.0088
constant in variance	0.0000	0.0000	5.5673	0.0000
ARCH	0.0247	0.0158	1.5592	0.1189
GARCH	0.8068	0.0216	37.2961	0.0000
DUM(leverage)	0.2289	0.0228	10.0535	0.0000
SIC	-4.9588			

Table 28 Estimation results of the return models: S&P500

The next four tables present the estimation results of the remaining four indices. Their ARMA structures, which are implied by the ACF and PACF, are quite close to those of S&P500 and the BDS independence tests using 2 to 6 dimensions are all rejected for non-linearity in these indices just like S&P500. Thus, the individual procedure to obtain the return models for each index is not separately presented. Instead, the identical three return models are estimated for each of four datasets: NASDAQ100, FTSE100, BSE100 and KOSPI200.

Constant return model				
	Coefficient	S.D	t-statistic	p-value
constant in mean	0.0017	0.0010	1.7470	0.0809
SIC	-3.7736			
GARCH(1,1) model				
	Coefficient	S.D	t-statistic	p-value
constant in mean	0.0030	0.0008	3.5961	0.0003
constant in variance	0.0000	0.0000	5.1501	0.0000
ARCH	0.1108	0.0134	8.2605	0.0000
GARCH	0.8656	0.0168	51.5194	0.0000
SIC	-3.9955			
GJR-GARCH(1,1) model				
	Coefficient	S.D	t-statistic	p-value
constant in mean	0.0022	0.0008	2.6940	0.0071
constant in variance	0.0001	0.0000	6.0494	0.0000
ARCH	0.0432	0.0190	2.2735	0.0230
GARCH	0.8295	0.0194	42.7783	0.0000
DUM(leverage)	0.1777	0.0222	8.0196	0.0000
SIC	-4.0146			

Table 29 Estimation results of the return models: NASDAQ100

First, the empirical results of the NASDAQ in Table 29 above are not much different from S&P500 except that the ARCH term still has significance with the added dummy for leverage effects.

Constant return model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0012	0.0006	1.9494	0.0515
SIC	-4.6786			
GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0021	0.0005	3.9967	0.0001
constant in variance	0.0000	0.0000	4.7125	0.0000
ARCH	0.1215	0.0161	7.5406	0.0000
GARCH	0.8444	0.0163	51.8104	0.0000
SIC	-4.8342			
GJR-GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0014	0.0006	2.5970	0.0094
constant in variance	0.0000	0.0000	5.2688	0.0000
ARCH	0.0199	0.0226	0.8796	0.3791
GARCH	0.8043	0.0244	32.9775	0.0000
DUM(leverage)	0.1957	0.0346	5.6586	0.0000
SIC	-4.8497			

Table 30 Estimation results of the return models: FTSE100

Next, in FTSE100 results in Table 30, the significance of the ARCH term drops dramatically as the leverage dummy is included in the model although the overall results about the fitness of models are similar to S&P500.

Constant return model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0030	0.0012	2.5500	0.0109
SIC	-3.5596			
GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0028	0.0010	2.8976	0.0038
constant in variance	0.0001	0.0000	4.6722	0.0000
ARCH	0.1969	0.0219	8.9697	0.0000
GARCH	0.7465	0.0270	27.6520	0.0000
SIC	-3.6945			
GJR-GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0028	0.0010	2.7524	0.0059
constant in variance	0.0001	0.0000	4.6675	0.0000
ARCH	0.1962	0.0223	8.7905	0.0000
GARCH	0.7459	0.0283	26.3750	0.0000
DUM(leverage)	0.0022	0.0310	0.0715	0.9430
SIC	-3.7098			

Table 31 Estimation results of the return models: BSE100

Next, the main difference of the Indian index data (BSE100) is that the coefficient of the leverage effect dummy is not statistically significant. In other words, the volatility is not affected by the leverage effects. It may indicate that leveraged investors in the Indian market do not need to care about deleverage or not own a tool for it. But the GJR-GARCH specification is a slightly better fit than the others in terms of the SIC.

Constant return model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0004	0.0014	0.2811	0.7787
SIC	-3.4473			
GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0018	0.0011	1.6955	0.0900
constant in variance	0.00003	0.00001	2.3682	0.0179
ARCH	0.1040	0.0177	5.8879	0.0000
GARCH	0.8850	0.0190	46.5599	0.0000
SIC	-3.6491			
GJR-GARCH(1,1) model				
	Coefficient	S.D	t-Statistic	p-value
constant in mean	0.0010	0.0011	0.9283	0.3532
constant in variance	0.00003	0.00001	2.7148	0.0066
ARCH	0.0516	0.0176	2.9343	0.0033
GARCH	0.9013	0.0174	51.7363	0.0000
DUM(leverage)	0.0700	0.0216	3.2391	0.0012
SIC	-3.6506			

Table 32 Estimation results of the return models: KOSPI200

Last, in the Korean data, all ARCH and GARCH terms and the dummy are significant in the volatility model similar to NASDAQ200 (Table 32). However, one noticeable difference from the other indices data is that the constant in the return model tends to be not significant. It also shows that there may not be a significant trend in this price series.

As a whole, the GJR-GARCH(1,1) models performed better than other less sophisticated models. Thus, this model is employed as the main return model for counting the runs of the residuals. Also, the constant return model chosen as a benchmark to check the performance of simpler models is able to meet that of more complicated models in the duration dependence tests.

3.2.4. The estimation of unconditional probabilities that a run continues

The unconditional probabilities that a run continues and ends can be estimated before investigating the duration dependence tests with conditional probabilities. This analysis can be considered as the analysis with constant hazard rate in duration dependence, but it actually uses a different assumption about the distribution of duration. Meanwhile, the same set of the retrieved duration data is employed as in the following duration dependence test. Only the constant-mean GJR-GARCH model is used in this analysis unlike the following duration dependence tests later.

The table below shows the duration data from the S&P500 index (1979-2008) which contains the only required data for the estimation of unconditional probabilities.

Positive runs		Negative runs	
Duration	N_i	Duration	N_i
1	187	1	224
2	108	2	91
3	45	3	44
4	23	4	26
5	20	5	6
6	9	6	3
7	3	7	2
8	1	8	1
9	1	9	0
10	1	10	1
11	1		

Table 33 Duration data - S&P500 with the GJR-GARCH model

The probability that a run continues (π_R) is estimated from the maximum likelihood estimator using the duration data from the positive runs:

$$\hat{\pi}_R = \frac{\sum_1^N d_i - N}{\sum_1^N d_i}$$

N is the count of the observed runs that is equal to the summation of the figures in the second column. In this case, it is 399 (=187+108+…+1+1). Σd_i is the summation of all the counted durations. From the above table, it is calculated as the sum of the products of duration and N_i. That is 843 (=1x187 + 2x108 +…+ 10x1 +11x11). Then, the estimated probability of run growth is:

$$\hat{\pi}_R = \frac{843-399}{843} = 0.5267$$

and the estimated probability that a run ends is:

$$1-\hat{\pi}_R = 1-0.5267 = 0.4733$$

Subsequently, it can be expected that the estimated mean duration of a positive run is:

$$\hat{\hat{D}} = \frac{\hat{\pi}_R}{1-\hat{\pi}_R} + 1 = \frac{1}{1-\hat{\pi}_R} = \frac{1}{0.4733} = 2.1128$$

In other words, a positive run is likely to last for 2.11 weeks. Although the sample standard deviation is not highly meaningful under geometric distribution, it is:

$$\hat{\sigma}(D) = \sqrt{\frac{\pi_R}{(1-\pi_R)^2}} = \sqrt{\frac{0.5267}{(1-0.5267)^2}} = 1.5333$$

On the other hand, the negative runs can be used to estimate the probability that a bear market persists. From the rightmost two columns in the above table, the estimated π_R is:

$$\hat{\pi}_R = \frac{722-444}{722} = 0.4488$$

$$1-\hat{\pi}_R = 1-0.4488 = 0.5512$$

$$\hat{\hat{D}} = \frac{1}{0.5512} = 1.8141$$

$$\hat{\sigma}(D) = \sqrt{\frac{0.4488}{(1-0.4488)^2}} = 1.2152$$

The negative run in the given US data has the probability of 0.45 to continue, and it is likely to persist around 1.81 weeks.

In the meantime, all five stock index data are inspected using the same estimation method for any difference in the probabilities of run continues and ends across different stock markets. Two return models, the constant return and the GJR-GARCH model are employed to calculate abnormal returns which are subsequently converted into the duration data. The estimation results are summarised in the next table where S.D is sample standard deviation (= $\hat{\sigma}(D)$)

Index		Positive runs		Negative runs	
		GJR-GARCH	constant return	GJR-GARCH	constant return
US	$\hat{\pi}_R$	0.5267	0.5186	0.4488	0.4507
S&P500	$1-\hat{\pi}_R$	0.4733	0.4814	0.5512	0.5493
	duration	2.1128	2.0771	1.8141	1.8204
	S.D	1.5333	1.4958	1.2152	1.2221
US	$\hat{\pi}_R$	0.5184	0.5238	0.4753	0.4720
NASDAQ100	$1-\hat{\pi}_R$	0.4816	0.4762	0.5247	0.5280
	duration	2.0762	2.1000	1.9059	1.8938
	S.D	1.4948	1.5199	1.3140	1.3010
UK	$\hat{\pi}_R$	0.5273	0.5327	0.4743	0.4741
FTSE100	$1-\hat{\pi}_R$	0.4727	0.4673	0.5257	0.5259
	duration	2.1154	2.1399	1.9021	1.9015
	S.D	1.5361	1.5618	1.3099	1.3093
INDIA	$\hat{\pi}_R$	0.5669	0.5633	0.5142	0.5150
BSE100	$1-\hat{\pi}_R$	0.4331	0.4367	0.4858	0.4850
	duration	2.3091	2.2899	2.0584	2.0618
	S.D	1.7386	1.7186	1.4760	1.4796
KOREA	$\hat{\pi}_R$	0.5324	0.5430	0.5605	0.5547
KOSPI200	$1-\hat{\pi}_R$	0.4676	0.4570	0.4395	0.4453
	duration	2.1384	2.1883	2.2756	2.2455
	S.D	1.5602	1.6126	1.7037	1.6724

Table 34 Estimated probabilities that a run continues and ends

The probably of positive run persistence ranges from 0.52 to 0.57, and it is almost identical across the five stock indices despite the difference in their geographical location, development status and the sample period. Their estimated durations are slightly over 2 weeks. The choice of a return model did not generate any statistically significant impact by the difference in mean test. On the other hand, the negative runs have slightly lower probability of continuing which is between 0.45 and 0.56 than positive runs except in the case of the Korean stock market data. The difference of mean probability between positive and negative runs is statistically significant only at 10%. The lower probability leads to mostly shorter duration of negative runs than positive runs. On the other hand, standard deviation of duration ranges from 1.22 to 1.74.

Binomial tests for significance are conducted on the results with the GJR-GARCH models using normal approximation (Siegel, 1956) as the probabilities are close to 0.5 and the sample size is large. Under the null hypothesis of the equal probability that a run continues and ends (π_R=0.5), z statistics are calculated by:

$$z = \frac{n\hat{\pi}_R[+,-]0.5 - n\pi_R}{\sqrt{n\pi_R(1-\pi_R)}}$$

where n is the number of observations, [+,-]0.5 is adjustment for normal approximation: if $n\hat{\pi}_R$ is larger than $n\pi_R$, its value of -0.5 and if $n\hat{\pi}_R$ is smaller than $n\pi_R$, its value of +0.5. Note that there is one less observation for each index.

The z statistics and p-values are presented in the following table. The null hypothesis is rejected for most of the sample indices at the 5% significance level. Positive runs in the UK market show marginal significance at the 10% level. The only exception is the negative runs in the Indian market; the probabilities that a run continues or ends are not significantly different from 0.5.

Index		Positive runs	Negative runs
US	$\hat{\pi}_R$	0.5267 **	0.4488 ***
S&P500	z statistic	2.0865	-4.0244
(n=1,564)	p-value	0.0158	0.0000
US	$\hat{\pi}_R$	0.5184 *	0.4753 **
NASDAQ100	z statistic	1.3275	-1.7913
(n=1,355)	p-value	0.0735	0.0322
UK	$\hat{\pi}_R$	0.5273 **	0.4743 **
FTSE100	z statistic	2.1340	-2.0075
(n=1,564)	p-value	0.0158	0.0202
INDIA	$\hat{\pi}_R$	0.5669 ***	0.5142
BSE100	z statistic	4.6022	0.9541
(n=1,198)	p-value	0.0000	0.1469
KOREA	$\hat{\pi}_R$	0.5324 **	0.5605 ***
KOSPI200	z statistic	2.0071	3.7754
(n=990)	p-value	0.0202	0.0000

Table 35 The results of binomial tests for significance from 0.5

In summary, using the duration data, the probabilities of positive runs of abnormal returns reverting to negative runs or vice versa have been empirically estimated and compared across five indices with two return models. Those probabilities are relatively similar across different market samples, but positive runs have slightly higher probability to continue than negative runs. The estimation method in this section may provide another perspective of investigating a return series. Instead of relying on logistic hazard function or other related functions like Weibull, the geometric distribution of duration can be directly employed to estimate the underlying probabilities that a run continues and ends. All estimated probabilities are different from 0.5 except that of negative runs in the Indian market index.

It should be noted that the estimated probabilities in this section are unconditional probabilities which are based on the geometric distribution of duration as a random variable. They were implicitly assumed to be constant in a specific market over the sample period. However, they may change as the duration of the run increases as assumed in the duration dependence tests.

On the other hand, the result may provide a rule of thumb for detecting a price bubble. As the estimated mean duration suggests, any type of a run of abnormal returns that is greater than 2 weeks is less likely to happen. Then, considering 2 standard deviations, a run of positive abnormal returns over 5 weeks can be employed as criteria to distinguish a possible market price bubble or an imminent crash. That is, when the duration of a run is larger than 5 i.e. equal to 6 weeks or greater, the run can be categorised as a price bubble. Out of 30 years of S&P500 data, 16 occasions of positive price bubble and 7 occasions of negative bubbles are identified with this rule. The number of occurrences of positive and negative bubbles of all five market indices is presented in the following table along with per year occurrence.

Index	Positive bubbles	Negative bubbles
US	16	7
S&P500	0.53/year	0.23/year
US	20	7
NASDAQ100	0.77/year	0.27/year
UK	16	7
FTSE100	0.53/year	0.23/year
INDIA	24	6
BSE100	1.09/year	0.27/year
KOREA	10	8
KOSPI200	0.53/year	0.42/year

Table 36 Bubble counts by the rule of thumb and their averages per year

In all cases, the number of positive bubbles is much larger than negative ones. It is not highly consistent with the findings that the probability that the runs of positive abnormal returns continues is only slightly greater than that of the runs of negative abnormal returns. Specifically, NASDAQ and BSE indices are subject to larger occurrence of positive bubbles, which may indicate market inefficiency while KOSPI seems to show only a little difference. Note that this counting only depends on the length of run.

3.2.5. The results of duration dependence tests

Two return models, constant return and GJR-GARCH(1,1) are estimated on the five sample market indices, and then the runs and their duration are counted. The next table presents the run counts and the sample hazard rates based on the GJR-GARCH(1,1) model on US S&P 500 index. They are calculated on an Excel spreadsheet using the residuals from Eviews software. Also, the table contains the values of the coefficients α and β, their standard deviation (S.D) and the p-values of Wald tests along with the LR statistics and its p-values, which are calculated on Eviews.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	187	212	0.4687	1	224	174	0.5628
2	108	104	0.5094	2	91	83	0.5230
3	45	59	0.4327	3	44	39	0.5301
4	23	36	0.3898	4	26	13	0.6667
5	20	16	0.5556	5	6	7	0.4615
6	9	7	0.5625	6	3	4	0.4286
7	3	4	0.4286	7	2	2	0.5000
8	1	3	0.2500	8	1	1	0.5000
9	1	2	0.3333	9	0	1	0.0000
10	1	1	0.5000	10	1	0	1.0000
11	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.1041	0.2500	0.6772	α	0.2336	0.28298	0.4091
β	-0.0092	0.2287	0.9681	β	-0.0692	0.26816	0.7965
LR statistic	0.0064	p-value	0.9364	LR statistic	0.2501	p-value	0.6170

Table 37 The results of duration dependence test: S&P500 and the GJR-GARCH(1,1) model

LR statistics show that the value of β is not statistically different from 0. That is, there was not a price bubble in S&P500 index between 1979 and 2008. Also, there is no indication of the existence of negative bubbles, which can be defined as negative duration dependence in negative runs. The other forms of duration dependence are not observed in both types of runs.

The results are comparable with the constant return model on the same dataset in the next table (Table 38). Although they share the same mean model, some differences are observed. The number of counted runs is slightly different and the value of β in positive runs becomes positive, but yet not statistically different from zero. Thus, the conclusion about the existence of bubbles does not change although LR statistics are slightly higher under the constant return model.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	189	213	0.4701	1	226	175	0.5636
2	111	102	0.5211	2	90	85	0.5143
3	47	55	0.4608	3	45	40	0.5294
4	21	34	0.3818	4	26	14	0.6500
5	19	15	0.5588	5	7	7	0.5000
6	9	6	0.6000	6	3	4	0.4286
7	3	3	0.5000	7	2	2	0.5000
8	1	2	0.3333	8	1	1	0.5000
9	0	2	0.0000	9	0	1	0.0000
10	1	1	0.5000	10	1	0	1.0000
11	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0882	0.1998	0.6591	α	0.2304	0.2847	0.4185
β	0.0259	0.1940	0.8936	β	-0.0735	0.2780	0.7914
LR statistic	0.0487	p-value	0.8254	LR statistic	0.2874	p-value	0.5919

Table 38 The results of duration dependence test: S&P500 and the constant return model

US NASDAQ stock index is expected to bring similar results since the sample period is similar to US S&P500 and the price patterns resemble each other. In the results of the test in Table 39, β is not statistically significant although it is negative in both types of runs. The duration of both positive and negative runs is not related to the hazard rates. That is, there is no sign of positive or negative price bubbles.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	165	176	0.4839	1	183	157	0.5382
2	94	82	0.5341	2	74	83	0.4713
3	34	48	0.4146	3	44	39	0.5301
4	20	28	0.4167	4	24	15	0.6154
5	8	20	0.2857	5	8	7	0.5333
6	12	8	0.6000	6	2	5	0.2857
7	6	2	0.7500	7	3	2	0.6000
8	0	2	0.0000	8	2	0	1.0000
9	1	1	0.5000				
10	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0300	0.1726	0.8620	α	0.1035	0.1856	0.5769
β	-0.0799	0.1602	0.6182	β	-0.0100	0.2226	0.9643
LR statistic	0.4071	p-value	0.5235	LR statistic	0.0048	p-value	0.9445

Table 39 The results of duration dependence test: NASDAQ100 and the GJR-GARCH(1,1) model

The results of the constant return model on the same NASDAQ100 (Table 40) bring a similar conclusion with only slightly higher LR statistics. That is comparable with the results on the S&P500 index in Table 37 and Table 38.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	163	177	0.4794	1	184	155	0.5428
2	94	83	0.5311	2	73	82	0.4710
3	34	49	0.4096	3	44	38	0.5366
4	19	30	0.3878	4	24	14	0.6316
5	9	21	0.3000	5	7	7	0.5000
6	12	9	0.5714	6	2	5	0.2857
7	7	2	0.7778	7	3	2	0.6000
8	0	2	0.0000	8	2	0	1.0000
9	1	1	0.5000				
10	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0474	0.1662	0.7755	α	0.1211	0.1807	0.5028
β	-0.0865	0.1545	0.5758	β	-0.0190	0.2125	0.9288
LR statistic	0.4886	p-value	0.4845	LR statistic	0.0174	p-value	0.8952

Table 40 The results of duration dependence test: NASDAQ100 and the constant return model

Therefore, it can be concluded that the US stock market may not contain price bubbles in the sample periods or, at least in two market indices during 1983-2008.

Next, turning to the UK stock market which is geographically different but with a similar level of development, the UK FTSE100 index (Table 41) does not show the evidence for a price bubble since β is not statistically significant.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	162	176	0.4793	1	181	156	0.5371
2	84	92	0.4773	2	77	79	0.4936
3	41	51	0.4457	3	40	39	0.5063
4	23	28	0.4510	4	22	17	0.5641
5	12	16	0.4286	5	10	7	0.5882
6	9	7	0.5625	6	3	4	0.4286
7	2	5	0.2857	7	2	2	0.5000
8	4	1	0.8000	8	2	0	1.0000
9	0	1	0.0000				
10	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0909	0.7877	0.9081	A	0.1110	0.2633	0.6733
β	-0.0330	0.4895	0.9463	B	-0.0170	0.3209	0.9577
LR statistic	0.0693	p-value	0.7923	LR statistic	0.0140	p-value	0.9058

Table 41 The results of duration dependence test: FTSE100 and the GJR-GARCH(1,1) model

The test results with the constant return model are also presented in the following table. Little discrepancy is spotted in terms of counted duration and the significance of the coefficients. In both test results, the price bubbles in the UK stock index are not discovered.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	162	174	0.4821	1	179	156	0.5343
2	80	94	0.4598	2	78	78	0.5000
3	42	52	0.4468	3	39	39	0.5000
4	22	30	0.4231	4	23	16	0.5897
5	14	16	0.4667	5	9	7	0.5625
6	8	8	0.5000	6	3	4	0.4286
7	2	6	0.2500	7	2	2	0.5000
8	4	2	0.6667	8	2	0	1.0000
9	1	1	0.5000				
10	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0895	0.5796	0.8772	α	0.1045	0.2985	0.7263
β	-0.0731	0.3806	0.8477	β	-0.0016	0.3209	0.9960
LR statistic	0.3534	p-value	0.5522	LR statistic	0.0001	p-value	0.9911

Table 42 The results of duration dependence test: FTSE100 and the constant return model

On the other hand, the duration dependence test on the Indian market (BSE100) does show the existence of positive price bubbles in the data, between 1987 and 2008. The coefficient value of β is negative and significantly different from zero at the 5% level. In other words, the duration of abnormal runs has a negative relationship with the hazard rates, or if positive runs of abnormal returns continued in the Indian stock market, they were less likely to end. Then, it can be said price bubbles emerged.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	137	138	0.4982	1	129	145	0.4708
2	52	86	0.3768	2	68	77	0.4690
3	36	50	0.4186	3	39	38	0.5065
4	17	33	0.3400	4	19	19	0.5000
5	9	24	0.2727	5	13	6	0.6842
6	9	15	0.3750	6	2	4	0.3333
7	7	8	0.4667	7	3	1	0.7500
8	4	4	0.5000	8	1	0	1.0000
9	3	1	0.7500				
10	0	1	0.0000				
11	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
A	-0.0890	0.1461	0.5423	α	-0.1546	0.3091	0.6170
B	-0.2799**	0.1534	0.0681	β	0.1908	0.2849	0.5030
LR statistic	5.2836	p-value	0.0215	LR statistic	1.5931	p-value	0.2069

Table 43 The results of duration dependence test: BSE100 and the GJR-GARCH(1,1) model

The constant return model also contains the evidence for the existence of bubbles, but to a slightly lower degree. The p-value of β is just over 0.05.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	137	139	0.4964	1	130	145	0.4727
2	53	86	0.3813	2	67	78	0.4621
3	36	50	0.4186	3	39	39	0.5000
4	16	34	0.3200	4	20	19	0.5128
5	11	23	0.3235	5	13	6	0.6842
6	9	14	0.3913	6	2	4	0.3333
7	7	7	0.5000	7	3	1	0.7500
8	4	3	0.5714	8	1	0	1.0000
9	3	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.1011	0.1402	0.4706	α	-0.1542	0.2603	0.5537
β	-0.2424*	0.1474	0.1001	β	0.1830	0.2633	0.4871
LR statistic	3.8166	p-value	0.0507	LR statistic	1.4770	p-value	0.2242

Table 44 The results of duration dependence test: BSE100 and the constant return model

Last, the Korean stock market index (KOSPI200) with the GJR-GARCH model does not display any possible existence of a price bubble as β is not significant although the LR statistic is higher than those of the US and the UK indices. Meanwhile, the Korean data shows positive duration dependence in negative runs. That is, as negative runs lengthen, they are more likely to end. However, it does not match with either concept of positive or negative bubbles.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	113	111	0.5045	1	90	135	0.4000
2	50	61	0.4505	2	54	81	0.4000
3	23	38	0.3770	3	43	38	0.5309
4	17	21	0.4474	4	20	18	0.5263
5	11	10	0.5238	5	10	8	0.5556
6	5	5	0.5000	6	4	4	0.5000
7	2	3	0.4000	7	2	2	0.5000
8	0	3	0.0000	8	1	1	0.5000
9	1	2	0.3333	9	1	0	1.0000
10	1	1	0.5000				
11	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0178	0.4128	0.9657	α	-0.4544	0.3076	0.1396
β	-0.1938	0.3351	0.5631	β	0.3652**	0.3957	0.3561
LR statistic	1.7434	p-value	0.1867	LR statistic	5.5431	p-value	0.0186

Table 45 The results of duration dependence test: KOSPI200 and the GJR-GARCH(1,1) model

The test results using the constant return model (Table 46) do not differ from the above results. Positive duration dependence is discovered but a price bubble does not reside in the dataset that ranges from 1990 to 2008.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	110	113	0.4933	1	94	130	0.4196
2	49	64	0.4336	2	51	79	0.3923
3	23	41	0.3594	3	42	37	0.5316
4	18	23	0.4390	4	19	18	0.5135
5	13	10	0.5652	5	10	8	0.5556
6	5	5	0.5000	6	4	4	0.5000
7	2	3	0.4000	7	2	2	0.5000
8	0	3	0.0000	8	1	1	0.5000
9	1	2	0.3333	9	1	0	1.0000
10	1	1	0.5000				
11	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0774	0.3003	0.7966	α	-0.3864	0.2522	0.1255
β	-0.1610	0.2513	0.5218	β	0.2914*	0.3369	0.3872
LR statistic	1.2292	p-value	0.2676	LR statistic	3.5103	p-value	0.0610

Table 46 The results of duration dependence test: KOSPI200 and the constant return model

The next five tables present the results of the duration dependence tests on each of the same five market indices but using monthly data instead of weekly data. This is a robustness test for the discovered duration dependence. Only simpler constant return models are used for the tests this time.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	49	47	0.5104	1	54	42	0.5625
2	22	25	0.4681	2	21	21	0.5000
3	17	8	0.6800	3	10	11	0.4762
4	5	3	0.6250	4	8	3	0.7273
5	2	1	0.6667	5	1	2	0.3333
6	0	1	0.0000	6	1	1	0.5000
7	1	0	1.0000	7	0	1	0.0000
				8	0	1	0.0000
				9	0	1	0.0000
				10	1	0	1.0000
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.0171	0.5227	0.9738	α	0.2388	1.2291	0.8459
β	0.3177	0.6285	0.6132	β	-0.1972	0.8730	0.8213
LR statistic	1.1870	p-value	0.2759	LR statistic	0.5742	p-value	0.4486

Table 47 The results of duration dependence test: monthly S&P500 and the constant return model

Monthly S&P500 index does not show any sign of bubble or duration dependence. The following NASDAQ100 and FTSE100 indices produce the same conclusions.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	37	39	0.4868	1	39	36	0.5200
2	13	26	0.3333	2	15	21	0.4167
3	15	11	0.5769	3	13	8	0.6190
4	5	6	0.4545	4	3	5	0.3750
5	4	2	0.6667	5	3	2	0.6000
6	0	2	0.0000	6	2	0	1.0000
7	0	2	0.0000				
8	1	1	0.5000				
9	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.1459	0.3585	0.6840	A	-0.0105	0.4513	0.9814
β	-0.0210	0.3881	0.9568	B	0.1068	0.5175	0.8366
LR statistic	0.0066	p-value	0.9352	LR statistic	0.1233	p-value	0.7255

Table 48 The results of duration dependence test: monthly NASDAQ100 and the constant return model

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	36	40	0.4737	1	36	39	0.4800
2	19	21	0.4750	2	17	22	0.4359
3	10	11	0.4762	3	14	8	0.6364
4	3	8	0.2727	4	5	3	0.6250
5	4	4	0.5000	5	1	2	0.3333
6	2	2	0.5000	6	1	1	0.5000
7	2	0	1.0000	7	1	0	1.0000
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.1242	1.9224	0.9485	A	-0.1394	0.6271	0.8241
β	0.0011	1.2900	0.9993	B	0.2882	0.7172	0.6878
LR statistic	0.0000	p-value	0.9968	LR statistic	0.8966	p-value	0.3437

Table 49 The results of duration dependence test: monthly FTSE100 and the constant return model

The next table contains the results from the BSE monthly index. Similar to the weekly data, monthly data also provides the evidence for a price bubble in the sample period. Negative runs do not have any duration dependence.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	32	31	0.5079	1	30	33	0.4762
2	16	15	0.5161	2	14	19	0.4242
3	6	9	0.4000	3	6	13	0.3158
4	4	5	0.4444	4	9	4	0.6923
5	0	5	0.0000	5	2	2	0.5000
6	1	4	0.2000	6	1	1	0.5000
7	1	3	0.2500	7	1	0	1.0000
8	1	2	0.3333				
9	1	1	0.5000				
10	0	1	0.0000				
11	0	1	0.0000				
12	0	1	0.0000				
13	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	0.1114	0.5333	0.8345	α	-0.1993	0.5418	0.7130
β	-0.4934*	0.4255	0.2462	β	0.1202	0.4882	0.8055
LR statistic	4.3226	p-value	0.0376	LR statistic	0.1585	p-value	0.6906

Table 50 The results of duration dependence test: monthly BSE100 and the constant return model

In the Korean market index, previously discovered negative duration dependence disappears, and there is no sign of bubble and duration dependence in both types of runs.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
1	20	33	0.3774	1	28	26	0.5185
2	18	15	0.5455	2	13	13	0.5000
3	7	8	0.4667	3	5	8	0.3846
4	3	5	0.3750	4	3	5	0.3750
5	1	4	0.2000	5	3	2	0.6000
6	3	1	0.7500	6	2	0	1.0000
7	0	1	0.0000				
8	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.3793	0.4510	0.4004	α	0.0256	1.2807	0.9841
β	0.2475	0.4482	0.5807	β	-0.0502	0.9596	0.9583
LR statistic	0.6467	p-value	0.4213	LR statistic	0.0220	p-value	0.8822

Table 51 The results of duration dependence test: monthly KOSPI200 and the constant return model

The next table (Table 52) summarises all the findings from the duration dependence tests on the five weekly stock indices in the above section. '*' attached to p-values indicates statistical significance at the 10% level and '**' indicates statistical significance at the 5% level.

		Positive runs		Negative runs	
Index		GJR-GARCH	constant return	GJR-GARCH	constant return
US	α	-0.1041	-0.0882	0.2336	0.2304
S&P500	β	-0.0092	0.0259	-0.0692	-0.0735
	LR	0.0064	0.0487	0.2501	0.2874
	p-value	0.9364	0.8254	0.6170	0.5919
US	α	-0.0300	-0.0474	0.1035	0.1211
NASDAQ	β	-0.0799	-0.0865	-0.0100	-0.0190
100	LR	0.4071	0.4886	0.0048	0.0174
	p-value	0.5235	0.4845	0.9445	0.8952
UK	α	-0.0909	-0.0895	0.1110	0.1045
FTSE100	β	-0.0330	-0.0731	-0.0170	-0.0016
	LR	0.0693	0.3534	0.0140	0.0001
	p-value	0.7923	0.5522	0.9058	0.9911
INDIA	α	-0.0890	-0.1011	-0.1546	-0.1542
BSE100	β	-0.2799**	-0.2424*	0.1908	0.1830
	LR	5.2836	3.8166	1.5931	1.4770
	p-value	0.0215	0.0507	0.2069	0.2242
KOREA	α	-0.0178	-0.0774	-0.4544	-0.3864
KOSPI200	β	-0.1938	-0.1610	0.3652**	0.2914*
	LR	1.7434	1.2292	5.5431	3.5103
	p-value	0.1867	0.2676	0.0186	0.0610

Table 52 Summary: the duration dependence tests on the weekly market indices

As another robustness test, the BDS independence tests (reviewed in Section 3.1.2), which are also used as bubble tests, are conducted on the residuals from GJR-GARCH(1,1) models on the same dataset using 1 to 6 histories (or embedding dimension).

US S&P500	US NASDAQ100	UK FTSE100	INDIA BSE100	KOREA KOSPI100
Non bubble rejected	Non bubble rejected	Non bubble rejected	Non bubble rejected	Non bubble rejected

Table 53 BDS independence test for price bubbles

All the null hypotheses of non bubble all rejected. This could mean the existence of bubbles even after controlling GARCH effects. However, since the null hypothesis is technically independent and with identical distribution of errors, the rejection can simply indicate some degree of remaining non-linearity in residuals.

The summary table of the duration dependence tests on monthly returns are presented below.

Index		Positive runs	Negative runs
US	A	-0.0171	0.2388
S&P500	B	0.3177	-0.1972
	LR	1.1870	0.5742
	p-value	0.2759	0.4486
US	α	-0.1459	-0.0105
NASDAQ100	β	-0.0210	0.1068
	LR	0.0066	0.1233
	p-value	0.9352	0.7255
UK	α	-0.1242	-0.1394
FTSE100	β	0.0011	0.2882
	LR	0.0000	0.8966
	p-value	0.9968	0.3437
INDIA	α	0.1114	-0.1993
BSE100	β	-0.4934 **	0.1202
	LR	4.3226	0.1585
	p-value	0.0376	0.6906
KOREA	α	-0.3793	0.0256
KOSPI200	β	0.2475	-0.0502
	LR	0.6467	0.0220
	p-value	0.4213	0.8822

Table 54 Summary: the duration dependence tests on the monthly market indices

3.2.6. Discussion

The main discovery from the results of the duration dependence tests was that the Indian market in the sample period between 1987 and 2008 had price bubbles, which tend to grow further as their duration increases. This is in contrast to the research by Bhaduri (2009) who did not discover the bubbles on the data between 1990 and 2007. The main differences are the choice of normal return model and the market index used. He adopted a AR(4) specification without considering GARCH effects and used the BSE SENSEX index which is constructed from the share price of 30 large firms on the Bombay Stock Exchange. It may be because the GJR-GARCH model on the BSE100 index in this study revealed the bubbles which may have resided in the other regional Indian markets. It is also possible that the inclusion of the data from 2008 when a sub-prime crisis severely hit the market made an impact on the results.

Another reason for this bubble may be the Indian market being under-developed and thus it has stronger irrationality than developed markets like in the US and the UK. All three indices from the US and the UK in this section did not contain bubbles or other types of duration dependence. In addition, as seen in the research on the US data (Maheu and McCurdy, 2000, Lunde and Timmermann, 2004), excluding the decades of historical price data at the early stage of market development, previously-discovered bubbles disappeared. Moreover, even in the same geographical region, price bubbles were not discovered in the developed countries like Japan and Hong Kong (Chen and Shen, 2007), but appeared in developing markets such as Malaysia and China (Mokhtar et al., 2006, Zhang, 2008). However, using only recent data revealed the bubbles in the index in the developed market in the US S&P500 between 2000 and 2007 (Yuhn et al., 2010). This shows that the recent sub-prime crisis was indeed a crash of a strong bubble.

Irrationality or inefficiency may remain in the developed markets as well, although not in terms of (positive) price bubbles but regarding recoverability from the downturns in the stock markets. Positive duration dependence in negative runs could be evidence of it. It was discovered in a relatively developed market like the Korean stock market in this section, and also in the US and other Asian developed markets in the previous literature (Lunde and Timmermann, 2004, Chen and Shen, 2007). Positive duration dependence in these cases means that negative runs tend to end earlier as their duration lengthens. This may represent irrationally quick recovery from market underperformance.

On the other hand, negative duration dependence may not only be generated by irrationality. It can arise from rational bubbles as theoretically supported in Section 3.2.1. Thus, it may be difficult to make a conclusion about rationality or irrationality of the markets using the duration dependence tests. This is because the tests are designed to detect specific characteristics of the price patterns that may or may not depend on rational expectation of investors despite the theoretical origins of the tests.

In terms of the choice of return models, it seems that the constant return model and the constant mean GJR-GARCH model produced a slight difference in the number of durations counted and the values of the LR statistics although the concusion about duration dependence did not differ. Thus, the choice between different volatility models if they are similar in specification may not be critical. However, it is expected that different mean models provide relatively different results as it may affect more strongly on counted duration. Also, Harman and Zuelke (2004) pointed out that the choices of data frequency or the functional forms of hazard rates like between log logistic and Weibull functions may be able to affect the outcomes.

The summary table of the results of weekly indices (Table 54) can provide some implications for the choice of frequency. The difference between weekly and monthly data (Table 54) in the dataset is marginal in this section. The price bubble in the Indian market was discovered in both cases and no bubble was found in three US and UK indices. The main difference is that negative duration dependence did not reside in the Korean monthly index.

Another issue in using the duration dependence test is the loss of information during the process of converting abnormal returns into positive and negative runs. This is because the converting process is technically an approximation although it is what other pricing models similarly do. To be precise, some of the information in prices or returns is lost such as how strong the price or return movement in each run is and how those runs are connected with each other. They may be part of the time series structure of price movement or, more generally, any type of true information or structure about price determination, if any. What remain after the conversion are the lengths and the type of runs.

Also, it should be noted that this study checked only one type of price bubbles which is based on the transition probabilities between positive and negative runs. That is, the duration dependence test is able to discover a bubble that grows with a decreaing rate of collapsing given a sample period. However, it may be possible that the market was influenced by another type of bubble. As reviewed in the previous chapter, a bubble can be defined as deviation from the fundamentals like a dividend stream. If it grows without affecting the probabilities of collapsing, the duration dependence test cannot detect it. Conversely, it is also possible that the detected bubbles by the duration dependence test may be not a bubble in terms of other definitions. For instance, it may be possibly a wrong interpretation of historical price pattern.

3.3. The detection of the changes in transition probability using the structural break based duration dependence test

This section extends the duration dependence test for price bubbles in the previous section by incorporating structural breaks instead of market upturns and downturns. This newly devised 'structural break based duration dependence test' essentially shows how the probability that the current data generating process (or its parameter values) persists changes as it lasts longer. If a specific market (dis)equilibrium is represented by a set of parameter values, a change in those values can be interpreted as a 'transition' between the market (dis)equilibrium. Then, 'transition probability' can be the probability of the change in paramenter values or equivalently that of a new structural break. Subsequently, 'the changes in transition probability' can be tested using duration dependence of a current set of parameter values. Also, this test can be a supplementary test to the original duration dependence test. In this context, the transition probability is a hazard rate in the duration dependence tests.

The original test was designed to detect price bubbles and other types of duration dependence while assuming no change of a data generating process (i.e. the underlying process of returns) and its parameter values. It shows how the probability that the market turns the tide changes when the market stays in the bull or the bear markets longer.

Permanent shocks to a data generating process can affect its parameter values. They are defined as 'structural breaks' (Verbeek, 2004). In terms of the transitions in market dynamics, the structural breaks can roughly mean any of three changes: the equilibrium to the disequilibrium, the disequilibrium to the equilibrium and the disequilibrium to a different type of disequilibrium. This is a simplification of the dynamic market equilibrium without specifying the models of the equilibrium price. It is an ad hoc approach for the interpretation of the test results only. Structural breaks technically mean the changes in the parameter values of a data generating process or its specification, which may exactly correspond to three types of transition between the (dis)equilibrium. The reason for this use is that structural breaks are easier to detect than the fundamentally unobservable equilibrium prices and they can approximate (the changes in price processes). However, a price process still needs to be estimated for empirical analysis.

One of the simple methods to detect structural breaks is to find the significant changes in the coefficients in time series models. This is possible because pre- and post structural break periods are probably governed by different dynamics and can be empirically detected as different parameter values of a data generating processes or a new process. Then, 'runs' can be re-defined as sample observations between those breaks, which share the same parameter values of the data generating process. 'Duration' is still the time length of a run. Subsequently, the techniques of duration dependence tests using structural breaks can be applied to find the duration dependence of the probability of those changes in parameter values.

If structural breaks randomly happen e.g. following binomial distribution of occurrence, the sample would not show any type of duration dependence given the probability that duration follows geometric distribution that leads to no duration dependence. However, if the occurrences of structural breaks are significantly and systematically affected by market forces, it may generate duration dependence and the distribution of duration deviates from geometric distribution.

The interpretation of the significant duration dependence should be different from that in the original duration dependence test. Negative duration dependence now means that as a run continues there is less likelihood of a new structural break (i.e. new set of parameter values or new process). In other words, the transition probability between the market (dis)equilibrium reduces as the market stays longer in (dis)equilibrium. Unlike the original test, the discovery of negative duration dependence is not directly or clearly linked to a price bubble. On the other hand, positive duration dependence implies that structural breaks are more likely to come in as a run lengthens. That is, the market is more likely to transit to new (dis)equilibrium as it stays longer in (dis)equilibrium.

Little research has been done on examining duration data constructed from structural breaks for the duration dependence tests although many models are suggested as duration models (Tsay, 2005). Traditional research on a structural break focuses on statistically revealing unknown dates of breakpoints, as reviewed later. A similar empirical approach is the regime-switching model (Hamilton, 1989). It estimates the probability of switching among the limited number of regimes. In this context, this structural breaks based duration dependence test examines whether the changes in the probability that a regime (i.e. a run) ends is related to the duration of the regime. In this test, the number of regimes is not limited, but cannot specify which regime comes next.

In the following parts of this section, it is investigated how to empirically find multiple structural breaks (and thus runs) and how to convert them into the series of duration data. Specifically, the findings of structural break literature are reviewed to develop the method of retrieving price runs from a primary time series of prices using linear piecewise regression. Then, new data of alternatively-defined durations are calculated and adopted for the structural break based duration dependence tests. Also, the forecasting ability of duration and related variables are tested.

3.3.1. Structural breaks

A 'structural break' is defined as the permanent change in the parameter values in the time series model (Verbeek, 2004) or broadly the change in mean value, volatility, or the autoregressive relation (Brooks, 2008). It may be due to the effects of microeconomic events, economic policies or other market forces, but randomness can create such breaks. Unlike a regime switch (or shift), a structural break is permanent and not reverting (Brooks, 2008). In summary, a permanent change in a parameter vector is a structural break (Clements et al., 2006). As introduced, this section focuses on the changes in the parameter values of a mean model, which are caused by non-reverting permanent shocks to a price process.

If the date of a structural break is known a priori, a Chow breakpoint test (Chow, 1960), a type of F test, can be conducted to investigate its significance over two sub-samples (Gujarati and Porter, 2009). If the date is unknown, the F statistics of all possible break dates can be used to date and test for a structural break (Verbeek, 2004). In addition, Andrews (1993) provided several test methods for one unknown break based on Wald, Lagrange and likelihood tests by Quandt (1960).

Bai and Perron (1998) generalised dating and testing methods for multiple unknown breakpoints. Their setup of a pure structural change model in terms of price (P) is:

$$\mathbf{P_t} = \mathbf{x_t'}\boldsymbol{\beta}_\mathbf{j} + \boldsymbol{\varepsilon}_\mathbf{t}$$

where x_t is a vector of explanatory variables, and β and ε are the vectors of the coefficients and the error terms, respectively.

Then, suppose $T_1,\dots,T_m$ are the breakpoints in T total observations and j is the index of structural segments or runs divided by the breaks (j=1,…,m+1). For each m, the locations of breakpoints are estimated by the following algorithm:

$$T_1,\dots,T_m = \arg\min_{T_1\dots T_m}[RSS(T_1,\dots,T_m)]$$

where arg min is the argument of minimum and RSS is the residual sum of squares of the above regression model. Optimal number and locations of m are obtained at the lowest value of the information criteria (Bai and Perron, 2003).

Their method provides the algorithm to pick the number and locations of multiple structural breaks. Also, it is a more parsimonious approach to analyse a univariate time series of mean values than the other later methods. Moreover, the method itself contains a test for a break. It will be further detailed in the relevant section.

On the other hand, more sophisticated test methods were also suggested. Bai and Perron (1998) presented a sup F test for no structural break against m breaks and a likelihood ratio test for l breaks against l+1 breaks. This was further developed by allowing stochastic parameters of trend and structural changes in systems like VAR models (Hansen, 2000, Hansen, 2003, Hungnes, 2010). Elliott (2003) suggested a statistic that allows for random, serially correlated or clustered breaks.

CUSUM (cumulative sum) tests were originally designed to test the specification of linear models using accumulated errors (Brown et al., 1975). They are also available for a test for structural change (Stock, 1994). For example, Kramer (1988) showed they are valid in an autoregressive model, although it did not contain power against zero-mean regressors (Clements et al., 2006). Meanwhile, Kuan (1995)'s general fluctuation test is used to identify a structural break by interpreting increasing fluctuation as evidence.

3.3.2. Duration dependence tests based on structural breaks

As introduced, a new applied duration dependence test is devised to find whether the transition probability between the market (dis)equilibrium depends on the time length of the market staying in (dis)equilibrium. It uses structural breaks as breakpoints to separate price runs for duration dependence tests instead of using the changes in the sign of abnormal returns from a mean model. In this sense, the new test may be an alternative to the original duration dependence tests since structural breaks, which were empirically observed in many previous studies e.g. Rapach and Wohar (2006), may invalidate the original test.

The method of detecting structural breaks employed in this section is a simple version of Bai and Perron's (1998) method since it is the parsimonious approach depending on simple linear piecewise regression of prices. This assumes that a data generating process is linear in time which can roughly approximate any non-linear process. Subsequently, the durations are calculated and used for the duration dependence test.

The basis of Bai and Perron's method is a piecewise regression model that splits data at pre-defined break dates and regresses a linear time trend model on each sub-sample (Brooks, 2008). For example, assuming two breaks of the original process occur at time 30 and 60 of the whole sample period (t=0 to 90), the piecewise regression models of price (P) on time (t) for each sub-sample are:

$$P_t = \alpha_1 + \beta_1 \times t + \varepsilon_{1,t} \quad (0 \le \mathrm{t} < 30)$$
$$P_t = \alpha_2 + \beta_2 \times t + \varepsilon_{2,t} \quad (30 \le \mathrm{t} < 60)$$
$$P_t = \alpha_3 + \beta_3 \times t + \varepsilon_{3,t} \quad (60 \le \mathrm{t} \le 90)$$

Since each sub-sample now has fewer observations, there is a loss of information (Brooks, 2008). However, when sub-sample size is large enough to provide a regression model of good fit, it is not of great concern although some more sophisticated model may provide better in-sample fitness. Meanwhile, each linear price segment is able to represent a corresponding time trend in that particular time period. Thus, one time variable is enough to characterize each trend (Zeileis et al., 2002). It is also a popular method of fitting non-linearity (Campbell et al., 1997) as the model can represent a non-linear behaviour of data while each sub-model maintains linearity.

In most of the cases, the breakpoints where structural breaks occur are unknown. Therefore, the matter of where to break the price series must be first dealt with (Campbell et al., 1997). For this, the aim of Bai and Perron (1998)'s method is to find the global minimisers of the objective function as explained above:

$$T_1, \ldots, T_m = \arg \min_{T \cdots T_m} [RSS(T_1, \ldots, T_m)]$$

This minimisation can be accomplished using a grid search. First of all, using one breakpoint (m=1), piecewise linear regressions are repeated on all possible sets of sub-samples. At each repetition, a different position of the breakpoints is chosen. For example, if the pre-specified minimum length of a segment is 20 in a sample of 200 observations, observation 20 is first chosen as a potential breakpoint and a piecewise regression is conducted for two sub-samples: 1-20 and 21-200. Then, the next potential breakpoint, observation 21 is chosen, and two sub-samples, 1-21 and 22-200, are regressed based on the same model. Then, it continues until observation 180 is used.

This whole procedure is repeated for all possible breakpoints, e.g. from one (m=1) to the maximum number of breakpoints (m=9), until the best locations of breakpoints for each number of breakpoints are finally identified. The case of no break is also estimated. The number of operations required is of order $O(T^m)$ which is the number of ordered list of T^m items.

Finally, the best set of number and locations of breakpoints ('optimal breakpoints') is selected by comparing information criteria like Bayesian Information Criteria (BIC) as in Bai and Perron (1998). Information criteria are employed because it is possible to minimise RSS by simply adding more breaks. Optimal price runs are linear price segments between those retrieved optimal breakpoints.

The burden of calculation can be minimised by the dynamic programming algorithm (Bai and Perron, 2003, Zeileis et al., 2003). The optimal breakpoints are obtained by solving the recursive problem:

$$RSS(T_{m,T}) = \min_{mk \le i \le T-k} [RSS(T_{m-1,i}) + RSS(i+1,T)]$$

where $RSS(T_{m,T})$ is the residual sum of squares with optimal m breakpoints using T observations, RSS(i+1,T) is the residual sum of squares obtained by applying least-squares to a price run (i+1 to T), and i is the last breakpoint. The idea is to find the optimal previous partner for each breakpoint i (Zeileis et al., 2003).

Additionally, the minimum length of a price run can be restricted as in the example. It is commonly specified as the minimum proportion (h) of the sample size (T) e.g. h=2%. Then, the maximum number of breaks (k) is also decided by k=(T/hT)-1. For example, when the minimum size is 2% of a total 1000 observations, the maximum number of breaks is 49 excluding the first and last observations.

The decision of the minimum duration (h) of a price run for the algorithm of finding optimal breakpoints is also important although h is arbitrarily chosen for meaningful economic reasons. If the length is not correctly chosen, some of the breakpoints will be missed out or appear at wrong locations. The minimum duration of a run should be chosen to be smaller than the reasonably shortest distance between two breaks of true price processes. Otherwise, the duration data and the following duration dependence test may provide less reliable results. However, true price generating processes or even exact locations of breaks are usually unknown in economics or finance data. The simple solution would be to choose as small a length as possible, but at the same time, it must be meaningfully long in terms of economic implications and to reduce computation burden. For example, one week gap between two breaks is not only of no importance in long-term weekly price data but also drastically increases analysis time.

Once identifying the structural breakpoints, the durations of runs between the identified structural breaks can be easily counted. The original duration dependence tests for the bubble were required to separate runs into the groups of positive and negative runs. The same can be done by checking the sign of the slope of each price run.

One methodological difference of this approach from the original test is that run data is able to include continual runs with the same sign. For example, it incorporates the possibility of a stronger positive run being followed by a weaker positive run while they are counted as separate runs. That is consistent with the objective of investigating probabilities regarding the transitions between the market (dis)equilibrium.

3.3.3. Empirical analysis

The empirical analysis of the actual market data is conducted as follows. The data for the empirical analysis used in this section is identical to the weekly price data used in Section 3.2.2. That is, 5 market indices from 4 different markets. The data is transformed into the logarithm. US S&P500 weekly index between 1979 and 2008 (1,566 observations) will be first examined and the other four indices are subsequently investigated. 8 weeks are selected as the minimum duration (h) in detecting structural breaks. This expects the maximum of around 6 structural breaks a year in weekly data. In the sample of 1,566 observations, 195 breaks are possible at most. The R statistical package is employed for the analysis of structural breaks, and then Eviews is used thereafter.

The normal market movement of the price needs to be controlled similar to the original duration dependence test where a normal return model is adopted to calculate the runs of abnormal returns. This section utilises a constant return model over the whole sample period for its simplicity. The constant return model generated almost identical results to other GARCH-based models in the previous duration dependence tests. Equivalently, for the linear piecewise regression models of price in this section, price data is de-trended. It is possible that a de-trending method may have an impact on the detection of breakpoints. However, the study by Canova (1999) confirmed the robustness of the identified breaking points in the sample regardless of a de-trending method. As long as one method is used as the only de-trending rule, it does not affect the discovered breakpoints.

The next task is to find out the optimal breakpoints and their positions as explained in the relevant section above. In the samples of S&P500, the maximum possible number of breaks is 195. The best positions of breakpoints for each case of 0 to 195 breaks were first detected based on the lowest RSS. Then, out of 196 sets of the best locations for each possible number of breaks, one set of breakpoints with the lowest BIC is picked up. As a result, optimal breakpoints and their locations are identified in the data. Figure 9 shows the values of BICs and RSSs under 70 breakpoints where the x axis is the number of observations.

Table 55 detailed the values between 61 and 70. As the BIC is the lowest at 68, the number of optimal breakpoints in the US S&P500 data is 68. That also means that the price series can be best approximated by 69 linear price runs.

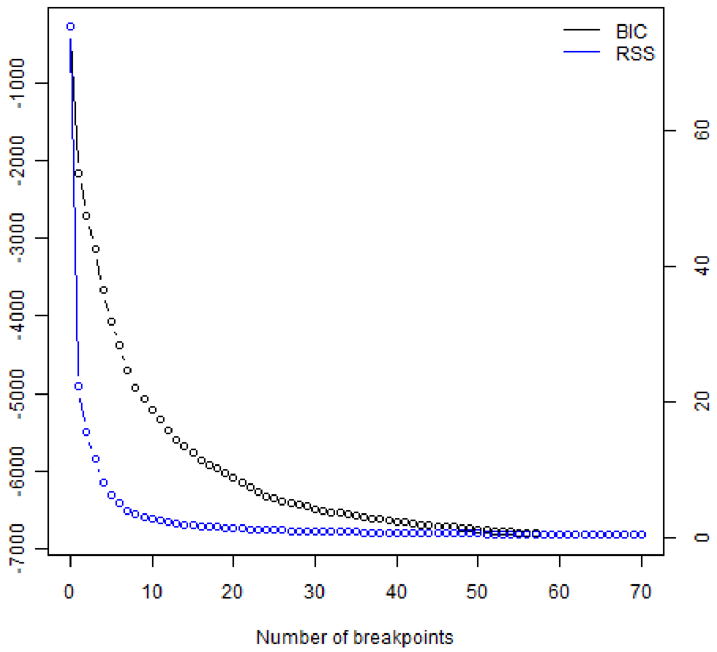

Figure 9 BIC and RSS at all optimal breakpoints under 70

m	BIC	RSS
61	-6813.3403	0.4935
62	-6814.8244	0.4862
63	-6816.3256	0.4790
64	-6817.4590	0.4719
65	-6818.0505	0.4651
66	-6818.4628	0.4585
67	-6818.3955	0.4521
68	-6818.5014	0.4457
69	-6818.3442	0.4395
70	-6817.6990	0.4336

Table 55 BIC and RSS between 61 to 70 breakpoints

Finally, the duration of run data is obtained from structural breaks (Table 56). The maximum duration is 70 and the average duration is 22.70 which are longer than the original duration data. It reflects the nature of the much rarer occurrence of structural breaks than the reversion of the sign of abnormal returns. This is also partly due to the restriction on the minimum length of duration (8). The duration data is positively skewed, platykurtic and non-normal as expected from geometric distribution.

Mean	22.6957
Median	18
S.D	13.6730
Skewness	1.1777
Kurtosis	1.0017
Observations	1566
Runs	69

Table 56 Descriptive statistics of the duration data from structural breaks

The duration data is relatively more dispersed than the original duration data, so it needs to be grouped before conducting the duration dependence test. The size of interval (i.e. bin) for grouping is 7; that is intended for the second bin to contain the mean similar to the case of the original duration dependence test. The duration data is also categorised into two sub-groups according to the sign of slope.

The procedure for the duration dependence test is identical to Section 3.2.5. Both types of the counts of runs (N_i and M_i) are recorded and their sample hazard rates are calculated. The details of the procedure were explained in the same section. The test results of the duration data of US S&P 500 index is in the following Table 57:

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
8 – 14	13	31	0.2955	8 – 14	15	10	0.6000
15 – 21	6	25	0.1935	15 – 21	5	5	0.5000
22 – 28	9	16	0.3600	22 – 28	4	5	0.4444
29 – 35	4	12	0.2500	29 – 35	0	1	0.0000
36 – 42	5	7	0.4167	36 – 42	0	1	0.0000
43 – 49	5	2	0.7143	43 - 49	1	0	1.0000
50 – 56	1	1	0.5000				
57 – 63	0	1	0.0000				
64 – 70	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-1.1475	0.3312	0.0005	α	0.3841	6.50292	0.9529
β	0.4532	0.3118	0.1460	β	-0.5544	3.93172	0.8879
LR statistic	2.5033	p-value	0.1136	LR statistic	0.2616	p-value	0.6090

Table 57 The results of structural break based duration dependence tests: S&P500

In S&P500 index data, there is no evidence of negative or positive duration dependence in both positive run and negative run data. This also supports that it is likely that the structural breaks in S&P500 randomly occurred. No discovery of duration dependence is compatible with the original results, but the interpretation is that there is no relationship between the duration of the market staying in one (dis)equilibrium and the probability that it transits to the new (dis)equilibrium. Simply, it means that the length of a run is not related to the occurrence of the next structural break.

Now, the analysis is extended to cover the rest of the market indices. First, NASDAQ index (Table 58) is revealed to have 56 structural breaks. The duration data shows no indication of negative duration dependence, but it contains the evidence of positive dependence duration in positive runs, which was not discovered in the early duration data. As price runs are extended, they have higher probability of ending the positive runs i.e. meeting a new structural break and moving to other (dis)equilibrium. It may be because market forces rather than randomness strongly worked in the market. The result is supported by the fact that the non-monotonic sample distribution of duration fairly deviates away from geometric distribution.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
8 - 14	4	25	0.1379	8 - 14	9	19	0.3214
15 - 21	9	16	0.3600	15 - 21	7	12	0.3684
22 - 28	4	12	0.2500	22 - 28	8	11	0.4211
29 - 35	4	8	0.3333	29 - 35	3	1	0.7500
36 - 42	3	5	0.3750	36 - 42	1	0	1.0000
43 - 49	2	3	0.4000				
50 - 56	2	1	0.6667				
57 - 63	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-1.6349	0.7419	0.0275	α	-0.8674	1.2147	0.4752
β	0.8463**	0.7870	0.2822	β	0.7305	1.1502	0.5253
LR statistic	5.5894	p-value	0.0181	LR statistic	2.2897	p-value	0.1302

Table 58 The results of structural break based duration dependence tests: NASDAQ100

In the UK FTSE100 index data, 77 breaks were identified. It does not display negative and positive durations in both positive and negative runs (Table 59).

The same conclusion was reached in the analysis of the original duration data. It supports no relationship between duration and structural breaks (or the transition between the (dis)equilibrium).

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
8 - 14	18	29	0.3830	8 - 14	11	11	0.5000
15 - 21	10	19	0.3448	15 - 21	5	6	0.4545
22 - 28	8	11	0.4211	22 - 28	2	5	0.2857
29 - 35	6	5	0.5455	29 - 35	4	0	1.0000
36 - 42	3	2	0.6000				
43 - 49	1	1	0.5000				
50 - 56	1	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.6114	0.5865	0.2972	α	-0.1282	1.2253	0.9167
β	0.4119	0.7554	0.5856	β	0.2706	1.0415	0.7950
LR statistic	1.5726	p-value	0.2098	LR statistic	0.2103	p-value	0.6465

Table 59 The results of structural break based duration dependence tests: FTSE100

The Indian market index (BSE100) shows 62 breaks in the sample period and the significant positive duration dependence is discovered in positive runs (Table 60). This evidence is different from the early findings of price bubbles, and implies the increasing transition probability between the (dis)equilibrium when the market stays longer in (dis)equilibrium in bull markets. There is also seemingly strong deviation from geometric distribution in positive runs. Negative runs do not contain duration dependence.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
8 - 14	9	26	0.2571	8 - 14	18	10	0.6429
15 - 21	8	18	0.3077	15 - 21	3	7	0.3000
22 - 28	12	6	0.6667	22 - 28	2	7	0.2222
29 - 35	4	2	0.6667	29 - 35	2	3	0.4000
36 - 42	2	0	1.0000	36 - 42	3	0	1.0000
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-1.2877	0.5974	0.0311	α	0.3456	0.5274	0.5122
β	1.4821***	0.8020	0.0646	β	-0.5962	0.4846	0.2186
LR statistic	10.9241	p-value	0.0009	LR statistic	1.5806	p-value	0.2087

Table 60 The results of structural break based duration dependence tests: BSE100

Last, KOSPI200 has 53 breaks and shows positive duration dependence only in negative runs. Positive runs do not contain any type of duration dependence. Both are the same result as the original tests. It seems that the probability of a structural break occurring becomes higher as a bear market lasts longer. The distribution of negative runs shows fair deviation from geometric distribution.

Positive runs				Negative runs			
Duration	N_i	M_i	h_i	Duration	N_i	M_i	h_i
8 - 14	9	16	0.3600	8 - 14	8	20	0.2857
15 - 21	8	8	0.5000	15 - 21	13	7	0.6500
22 - 28	3	5	0.3750	22 – 28	3	7	0.3000
29 - 35	3	2	0.6000	29 – 35	4	0	1.0000
36 - 42	2	0	1.0000				
Coefficient	Estimate	S.D	p-value	Coefficient	Estimate	S.D	p-value
α	-0.6076	2.6527	0.8188	A	-0.7919	0.7929	0.3180
β	0.7203	2.2252	0.7462	B	1.1950**	0.8159	0.1430
LR statistic	2.0088	p-value	0.1564	LR statistic	4.8321	p-value	0.0279

Table 61 The results of structural break based duration dependence tests: KOSPI200

The summary of the above duration dependence tests are presented in the next table (Table 62). Note the value of α does not have an impact on either type of duration dependence. The differences from the results of the original duration dependence tests are that more positive duration dependence was discovered while no negative duration dependence was found. To be precise, NASDAQ and Indian indices have positive duration dependence in positive runs that were not discovered in the original durations. Meanwhile, the Korean market Index has positive duration dependence in negative runs which was found in the previous tests. S&P500 and FTSE100 also brought the same results of no duration dependence as before.

Additional tests are conducted with pooled data of a total of 320 durations which combined all runs in five market indices into two duration series, one positive run and the other negative runs. This is to investigate the general tendency in the data set using more observations, and the results are presented in the same table. The pooled data shows the existence of positive duration dependence in positive runs and no such dependence in negative runs. In general, extending positive runs may bring the next structural break as a transition between (dis)equilibrium earlier.

Index		Positive runs	Negative runs
US	α	-1.1475	0.3841
S&P500	β	0.4532	-0.5544
	LR	2.5033	0.2616
	p-value	0.1136	0.6090
US	α	-1.6349	-0.8674
NASDAQ100	β	0.8463 **	0.7305
	LR	5.5894	2.2897
	p-value	0.0181	0.1302
UK	α	-0.6114	-0.1282
FTSE100	β	0.4119	0.2706
	LR	1.5726	0.2103
	p-value	0.2098	0.6465
INDIA	α	-1.2877	0.3456
BSE100	β	1.4821 ***	-0.5962
	LR	10.9241	1.5806
	p-value	0.0009	0.2087
KOREA	α	-0.6076	-0.7919
KOSPI200	β	0.7203	1.1950 **
	LR	2.0088	4.8321
	p-value	0.1564	0.0279
Pooled	α	-3.2413	-2.3801
	β	0.2178 ***	0.0797
	LR	7.3904	0.6682
	p-value	0.0066	0.4137

Table 62 The summary of structural break based duration dependence tests

In conclusion, it was discovered that as price runs extend in some of the sample markets, they are more likely to be faced with structural breaks that are represented by the changes in parameter values. In other words, an extending run leads to the higher transition probability between the market (dis)equilibrium. This is particularly dominant in positive runs.

3.3.4. Forecasting duration and direction

On the other hand, the test can tell how likely it is for the market to be met by the next structural break as the run extends. However, unlike the original duration dependence tests, the test in this section cannot tell what will happen after a structural break as many types of runs with different parameter values may follow. This sub-section investigates whether it is possible to predict the changes in the duration of runs and the direction of price movement. The autoregressive model of duration and the price change and duration model (PCD) are examined.

The brief inspection of the duration (d) series of S&P500 index data shows that it may be possible to build a time series model of duration for the purpose of at least predicting the length of the next run. The ACF and PACF of duration series show strong autocorrelation and partial auto correlation in small lags (Table 63) where '^' indicates 5% level significance and '^^' indicates autocorrelation higher than 0.2. Their patterns suggest a certain ARMA structure like AR(1). In the mean time, the existence of a unit root is rejected by the ADF test with no lag. The null hypothesis of the variance ratio test is rejected at the 5% significance level with the max|z| statistic of 2.8348.

Autocorrelation	Partial Correlation		ACF	PACF	Q-Stat	p-value
|^^	|^^	1	0.365	0.365	9.6063	0.002
|^^	|	2	0.251	0.136	14.230	0.001
|	|	3	0.102	-0.032	15.000	0.002
|	|	4	-0.054	-0.131	15.219	0.004
|	|	5	-0.110	-0.078	16.150	0.006
|	|	6	-0.044	0.062	16.301	0.012
|	|	7	-0.189	-0.171	19.118	0.008
|	|	8	-0.064	0.047	19.451	0.013
|	|	9	-0.180	-0.158	22.086	0.009
|	|	10	-0.132	-0.025	23.538	0.009
|	|	11	-0.059	0.028	23.835	0.013
|	|	12	0.000	0.028	23.835	0.021

Table 63 The ACF and PACF of duration: S&P500

Grid search for the ARMA structure based on SIC up to 5th lag found that the AR(1) process has the lowest value of SIC.

Equation 13

$$d_t = \alpha + \beta d_{t-1} + \varepsilon_t$$

where α and β are the coefficients. The estimation result is:

$$\hat{d}_t = \underset{(2.4421)}{22.1865} + \underset{(0.1131)}{0.3681} d_{t-1}$$

R^2 is 0.1382, Adjusted R^2 is 0.1251, DW is 2.0418 and SIC is 8.0170. The standard errors of the coefficients are in brackets and the standard error of regression is 12.7105.The AR term is significant at the 5% level.

The ACF and PACF of the residual series do not contain autocorrelation (Table 64) and the BDS test cannot reject the null hypothesis at the 5% level, but the variance ratio test shows there is still remaining predictability in the series with the max|z| statistic of 3.6961 and the normality is also rejected with a JB statistic of 17.3127. That is, there is a certain amount of predictability in terms of general departure from random walk. In the RESET test, adding the squared fitted values of the AR(1) model is significant with the t statistic of the fitted variable of 2.59 at the 5% level.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
|	|	1	-0.030	-0.030	0.0646	0.799
|	|	2	0.159	0.158	1.8882	0.389
|	|	3	0.024	0.034	1.9310	0.587
|	|	4	-0.042	-0.067	2.0627	0.724
|	|	5	-0.117	-0.134	3.0971	0.685
|	|	6	0.085	0.099	3.6580	0.723
|	|	7	-0.188	-0.146	6.4252	0.491
|	|	8	0.069	0.040	6.7992	0.558
|	|	9	-0.136	-0.109	8.2874	0.505
|	|	10	-0.087	-0.104	8.9025	0.541
|	|	11	-0.063	-0.037	9.2367	0.600
|	|	12	0.063	0.068	9.5731	0.653

Table 64 The ACF and PACF of the residuals from AR(1) of duration: S&P500

The following figure is the actual (red) and the forecasted (blue) values based on the dynamic forecasting using the last 10 observations as out-of-sample. This method uses the forecasted values for the subsequent forecasted values.

It seems that the trend of duration is fairly well predicted. However, R^2 is low (0.1382) and the standard error of regression is high (12.7105), but its Theil inequality coefficient, or U statistic, (0.2516) is lower than that of the benchmark constant duration model (0.2958). That is, the AR(1) model is a better fit in terms of forecasting. Also, root mean squared errors (RMSE) and mean absolute errors (MAE) are smaller in the AR(1) model of duration than the benchmark (12.5018 vs. 14.4153 and 10.9294 vs. 12.2391).

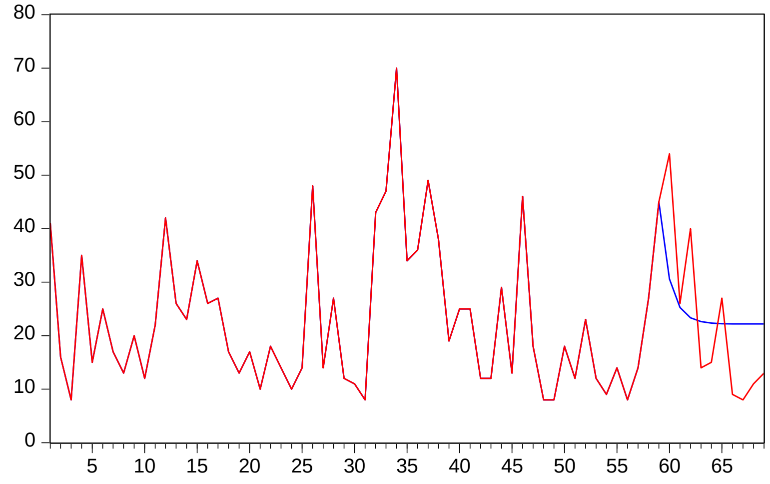

Figure 10 The actual duration (red) vs. forecasted duration (blue): S&P500

The used measures of the accuracy of forecasts are (Brooks, 2008):

Equation 14 Root mean squared error (RMSE)

$$\sqrt{\sum_{t=T+1}^{T+h} (\tilde{y}_t - y)^2 / \tilde{h}}$$

Equation 15 Mean absolute error (MAE)

$$\sum_{t=T+1}^{T+h} |\tilde{y}_t - y| / \tilde{h}$$

Equation 16 Theil inequality coefficient

$$\sqrt{\sum_{t=T+1}^{T+h} (\tilde{y}_t - y)^2 / \tilde{h}} \Bigg/ \left(\sqrt{\sum_{t=T+1}^{T+h} \tilde{y}_t^2 / \tilde{h}} + \sqrt{\sum_{t=T+1}^{T+h} y^2 / \tilde{h}} \right)$$

where $\tilde{h}$ is the number of out-of-sample observations and T is the number of in-sample observations. y is actual values and $\tilde{y}$ is forecasted values.

On the other hand, the estimation and forecasting procedure is repeated for the log series of duration since it can give information about the percentage changes for the percentage changes between the current and past duration. As expected from the nature of log transformation, the informal test of the ACF and PACF and the formal grid search using the SIC indicates the AR(1) model for log duration.

$$\ln d_t = \alpha + \ln \beta d_{t-1} + \varepsilon_t$$

$$\ln d_t = 2.9382 + 0.3524 \ln d_{t-1}$$
$$(0.1002) \quad (0.1139)$$

R^2 is 0.1266, Adjusted R^2 is 0.1134, DW is 2.0326 and SIC is 1.6793. The standard error of regression is 0.5345. The AR term is significant at the 5% level.

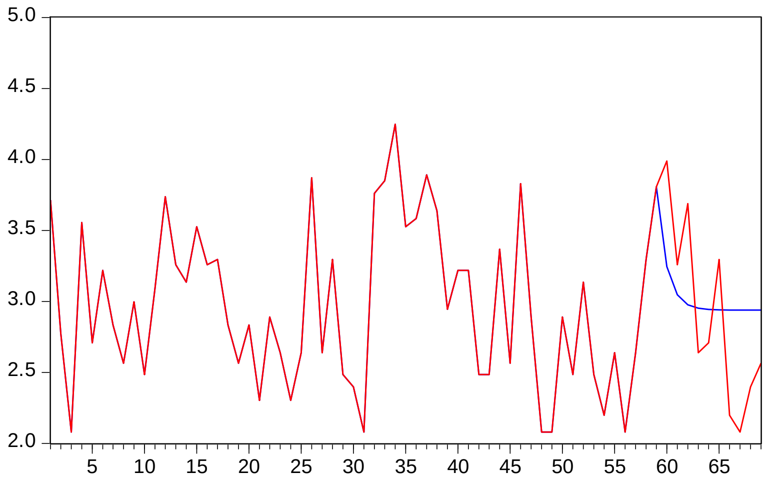

Figure 11 The actual log duration (red) vs. forecasted log duration (blue): S&P500

The MSE, MAE and Theil inequality coefficient are 0.5573, 0.5087 and 0.0939. As a benchmark, the constant log duration model is estimated and forecasted using the same method. The values of the aggregated measures of forecast errors are: 0.6152 0.5562 and 0.1042. It indicates that the AR(1) model performed better.

The above results indicate that there is to some degree forecasting power of the autoregressive (log) duration models where duration data is obtained using structural breaks. However, the duration data is integer data in nature, and is not synchronous with time and its population distribution may be geometric. Therefore, a different approach can be considered to raise the explaining and forecasting power of a duration based time series model. For example, there are discrete variate time series models (McKenzie, 2003), autoregressive conditional duration (ACD) models (Engle and Russell, 1998, Tsay, 2005) or limited dependent variable models like ordered probit models (Brooks, 2008).

A price change and duration (PCD) model was originally devised to estimate the duration between trades and price changes in intraday transactions data by

McCulloch (2001). It models the price changes over irregular time intervals (Tsay, 2005):

For the i^th price change,

$$P_i = P_{i-1} + direct_i \times S_i$$

where *direct* is 1 for positive price changes and -1 for negative changes and S is the size (or height) of the price change as multiples of tick. Since there are trades with no price change in transactions data, duration data is calculated between trades with price changes. This is fairly equivalent to the calculation of duration between structural breaks in the long-term data.

Then, McCulloch (2001) argued that the log duration can be estimated using the following model by a multiple linear regression:

$$\ln d_t = \alpha + \ln \beta_1 d_{t-1} + \beta_2 S_{t-1} + \varepsilon_t$$

This specification can be employed for the duration model in this chapter. The difference is that this model additionally contains the size (S) data. It can be calculated by multiplying the value of the slope coefficient of a run between structural breaks by the corresponding duration.

The estimation results are:

$$\begin{aligned} \ln d_t &= 2.0102 + 0.3459 \ln d_{t-1} - 0.0150 \hat{S}_{t-1} \\ &\quad (0.3647) \quad (0.1143) \qquad\quad (0.0167) \end{aligned}$$

R^2 is 0.1372, Adjusted R^2 is 0.1107, DW is 1.9997 and SIC is 1.7291. The standard error of regression is 0.5353.

The AR term is significant at the 5% level, but the size term is not significant. Although an additional term is added, this model does not explain the variation of log duration better than the model without size term. This is comparable with the result by Tsay (2005) where he used the model on the 1 day intraday transactions data of one firm. The size term was not significant.

However, in terms of forecasting accuracy, it is slightly better: RMSE is 0.5310, MAE is 0.4759 and Theil inequality coefficient is 0.0891.

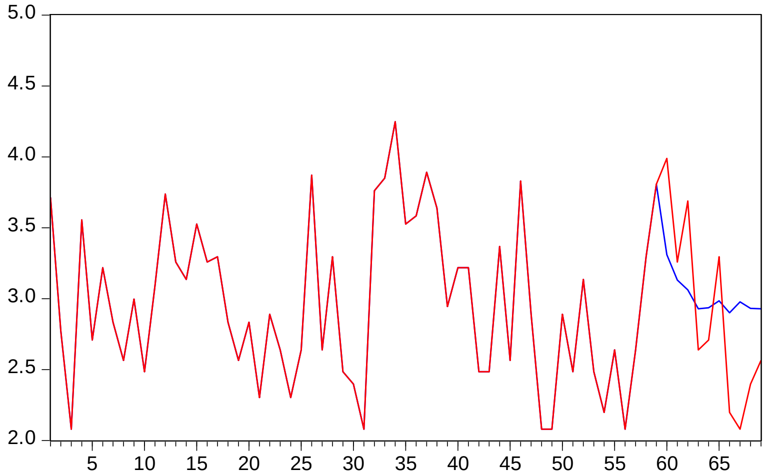

Figure 12 The actual log duration (red) vs. forecasted log duration (blue) with added size variable: S&P500

The structural break based duration dependence model cannot independently predict the next price movement. On the other hand, the PCD model can provide the method of estimating the direction of price movement from the data of duration and direction (McCulloch and Tsay, 2001). Their method can be employed to predict which price movement follows after a structural break. To check the forecasting ability of the PCD, the following model for the direction of price movement is constructed using their specification:

$$direct_i = sign(\mu_i + \sigma_i z)$$

where z is a random variable that follows N(0,1), μ is the mean of *direct* and σ is its standard deviation, and *sign* is the sign of the equation in brackets.

It can be said that the direction of price movement in the PCD model is governed by a normal random variable (Tsay, 2005). The model specifications for the mean and standard deviation are:

$$\mu_i = E[direct_i] = \alpha + \beta_1 direct_{i-1} + \beta_2 \ln d_i$$

$$\ln(\sigma_i) = \beta_3 \,|\, direct_{i-1} + direct_{i-2} + direct_{i-3} + direct_{i-4} \,|$$

Generalised linear regressions are used to estimate the parameter values for two models. The use of 4 lags can mitigate the impact from the frequent reversions of directions. The direction data is obtained by checking the sign of the slope coefficients of price runs in the dataset in this chapter.

The estimation results of the mean model of direction:

Variable	Coefficient	S.E	p-value
Constant	-1.8858	0.5921	0.0014
$direct_{i-1}$	-0.1059	0.1181	0.3698
$\ln d_i$	0.7403	0.2003	0.0002
Mean dependent variable	0.2647	S.D. of dependent variable	0.9715
Sum squared residuals	52.2501	Log likelihood	-87.5642
AIC	2.6637	SIC	2.7616
Deviance statistic	0.8038	Deviance	52.2502
LR statistic	13.6657	Restricted deviance	63.2353
Pearson SSR	52.2502	Prob(LR statistic)	0.0011
Dispersion	0.8038	Pearson statistic	0.8038

Table 65 The estimation results of the mean equation of direction (*direct*): S&P500

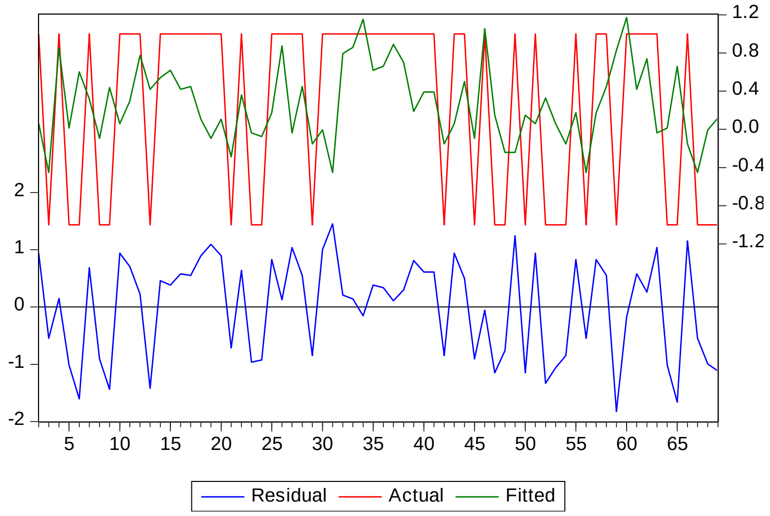

Figure 13 Residual, actual and fitted values of the PCD model of direction

Log duration is significant, but the autoregressive term of direction is not significant. It seems that this model does not have strong explanation power. Then, the fitted values are calculated by converting the fitted mean values to binary direction data (Figure 14). 55% of fitted directions match with the actual directions. Using a binomial test for significance in Section 3.2.4 its z statistic is 0.7034 and p-value is 0.2266. That is, this probability is not statistically different from 0.5 (50%). The forecasting ability is limited and it may be due to a small sample size.

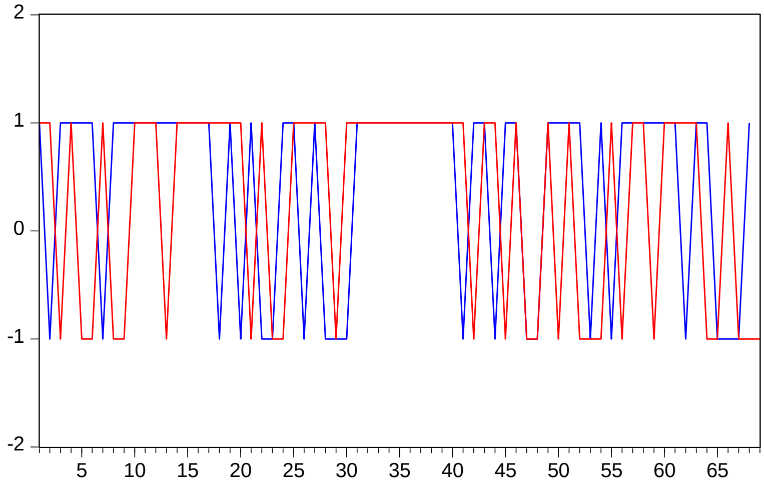

Figure 14 Actual (red) and fitted (blue) values of direction: S&P500

Finally, the forecasted values are plotted along with the actual values in Figure 15. Only 5 directions are correctly forecasted. However, with only 10 observations of binary data as out-of-sample, it may be difficult to distinguish the forecasting power. The PCD model may be a more suitable approach with high-frequency data. Tsay's (2005) model with 194 observations provides a marginally significant past direction term.

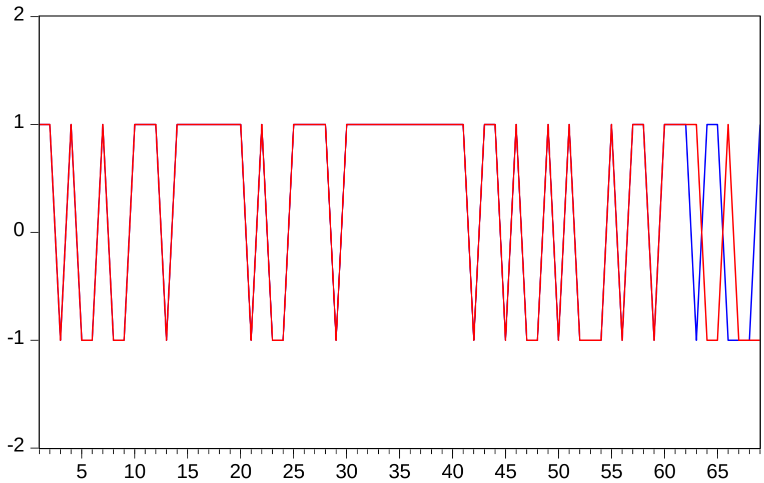

Figure 15 Actual (red) and forecasted (blue) values of direction: S&P500

The log standard deviation equation is also estimated for completion:

$$\ln(\sigma_i) = \underset{(0.0392)}{-0.1489} \,|\, direct_{i-1} + direct_{i-2} + direct_{i-3} + direct_{i-4} \,|$$

	Coefficient		S.E	p-value
β_3	-0.1489		0.0392	0.0001
Mean dependent variable	-0.4273	S.D. dependent variable		0.7237
Sum squared residuals	37.048	Log likelihood		-73.9647
AIC	2.3066	SIC		2.3401
Deviance statistic	0.5789	Deviance		37.0485
Pearson statistic	0.5789	Pearson SSR		37.0485
Dispersion	0.5789			

Table 66 The estimation results of the standard deviation equation

In summary, the forecasting abilities of the autoregressive duration models are based on structural breaks. Although the standard errors of regression seem rather high, it may be possible to predict the next duration after a structural break to some degree. Other duration models like the ACD models (Engle and Russell, 1998) could be tested as alternatives for forecasting.

The PCD model was tested for its forecasting ability for both duration and direction data. Although the results with duration data were slightly better than the autoregressive duration models in terms of forecasting accuracy, the results with direction data are rather disappointing. This is because it basically attempts to predict a binary series with a linear regression method. Similar weakness to the linear probability models may happen in the PCD models such as unbounded prediction (Brooks, 2008). Moreover, only the contemporaneous duration term was significant not the lagged duration variable. However, the validity of the PCD model cannot be easily defied just because of weak predictability of direction. The PCD model contains other regression models for the number of trades with no price change, those with price change and the size of the changes. They can be jointly estimated.

However, in terms of forecasting the direction of the next run, it may be wiser to rely on other limited dependent (or binary) variable models if the concern is only the direction. Otherwise, such forecasting can be dependent on other non-duration based time series models.

3.3.5. Discussion

This section developed an applied duration dependence test based on structural breaks that aims to find the changes in the probability that a run ends by a structural break as the run extends. It was discovered that as a price run is extended, there was positive duration dependence in positive or negative runs in some market indices as in Table 62. In other words, in those markets, the transition probability to another type of market (dis)equilibrium becomes higher as the market stays in (dis)equilibrium longer. In terms of structural breaks, the probability of a new structural break that comes with a new set of parameter values increases as a run with the same parameter values continues.

An advantage of the structural break based duration dependence test can fundamentally accommodate the changes in the parameter values of data generating processes or the processes themselves because of linear approximation. However, it creates another disadvantage in that the test relies on the legitimacy of discovered structural breaks. For example, a structural break can be incorrectly detected if a true price process contains long term swings. To mitigate this issue, it may be possible that instead of a simple constant return model, an autoregressive model is used. In this case, a different detecting method e.g. Stock (1994), may be required. Another caveat is that this test cannot tell whether the market transits from the equilibrium to the disequiblirum or the other way round, or simply it transits between different types of market disequilibrium. It is able to only tell how it is likely to see another transition and how this probability changes. For future research, a new method needs to be added to distinguish different types of structural breaks. For example, by specifying the market equilibrium by stationary price movement. It may become feasible to see if a particular structural break means a transition from or to the market equilbirum.

On the other hand, the autoregressive duration models and the PCD models were tested for the forecasting ability of the next duration or direction of runs. Both the former and the latter models for duration provided some moderatly significant results. However, the latter model for direction may be not a reasonable choice with the duration data from structural breaks in long-term data.

The structural break based duration dependence test can give more insights about long term market dynamics when combined with the results of the original duration dependence test. The new test assumes that data generating processes (or their parameter values) change at structural breaks and approximates them in the longer-term. Run data are essentially the approximated price processes. On the other hand, the original duration dependence test supposes that a data generating process does not change and uses the residuals from the process to generate runs. Its runs are shorter-term based. The difference can be seen in that the length of a run in this section is about 20 weeks on average, but that of the original series is around 2 weeks.

Then, the combined interpretation of the results from two tests can be done. For example, using the same data set, price bubbles (i.e. negative duration dependence in positive runs of abnormal returns) were discovered in the Indian market, and at the same time positive duration dependence in positive runs between structural breaks was revealed. These results can be interpreted as: a run of abnormal returns from a price process was extended with lower probability of ending, but such a price generating process that produced negative duration dependence was more likely to be broken in the long-term as the run of the same price generating process continues. In other words, it can be said that the long-run sustainability of periodic bubble epidemics was weak in the Indian market. Similarly in the Korean market, positive duration dependence was discovered in both the negative runs of abnormal returns and the spells between structural breaks. Then, it can mean that the fast recovery of the Korean market from bear markets was not persistent in the long term. However, it should be noted that a structural break may invalidate the findings using one price generating process.

3.4. Discussion and conclusions

This chapter investigated stock price bubble theories and the detection of them as one type of long-term market disequilibrium. It adopted the duration dependence test for price bubbles and used it to detect bubbles and other types of duration dependence of price runs in the market data that were not thoroughly investigated by the literature that used the same method. Then, the unconditional probability that a price run continues or ends was estimated. In addition, the changes in the probability that a price process meets the next structural break were investigated. That can be interpreted as the change in the transition probability between the market (dis)equilibrium. A new duration dependence test was devised for this.

The findings are summarised as follows. The empirical analysis used the datasets of 5 market indices from various regions. The evidence of bubbles and other duration dependences were discovered in the Indian and the Korean stock indices. The probabilities that a price run continues or ends were estimated based on geometric and binomial distributions. They are significantly different from 0.5. Positive runs of abnormal returns tended to last slightly longer except in the Korean market. Then, the rule of thumb for price bubbles was developed and applied to empirical data to show that some markets are highly prone to develop positive price bubbles than negative ones. Then, structural breaks were introduced to retrieve another type of price run and its duration dependence. Positive duration dependence was discovered in the NASDAQ, the Indian and the Korean stock markets. That can tell that the market was more likely to be faced with a structural break when a price process (or its parameter values) persists longer in the sample data. In other words, it may be more likely to cause a transition to another market (dis)equilibrium.

On the other hand, this chapter applied several statistical tests designed for different financial theories to the same dataset. One issue is that significance results in a separate statistical test may support a different financial theory. Technically, this is because each test adopts a different set of the null and alternative hypotheses from the other tests. For example, Section 3.2 used two tests. One estimated unconditional transition probabilities between positive and negative runs using the geometric distribution of duration while the other estimated conditional probabilities using a logistic hazard function of duration depedence. The significance results in the latter test are able to defy the results of the former tests. From the same perspective, if structural breaks are significantly discovered, it is able to deny the statistical results that do not assume the existence of structural breaks. In this study, the results in Section 3.3 can be evidence agaist the findings in the earlier section. However, statistical tests in nature contain the possibility of commiting errors represented by significance level, so it may be better to present all the results although they are based on different statistical hypotheses and financial theories.

Chapter 4

Conclusions

4. Conclusions

This thesis has investigated the topic of market disequilibrium from two different perspectives. This conclusion chapter summarises all the theoretical and empirical findings in each of the main chapters and briefly explains how they contribute to academics and professionals. Final remarks are added at the end of this chapter.

Chapter 2 aimed to compare two hypotheses about the impact of multiple market makers on intraday price; increased resilience and improved efficiency. It first reviewed past literature about how the impact of multiple market makers was theoretically investigated and what empirical evidence was found. Then, it presented the simple neoclassical market making model that provided more insights into how the market making model can be connected to two empirical hypotheses. The empirical decision criteria were set in Table 1.

In the empirical analysis, the comparison was made between a single market maker system of the NYSE (Group A of the sample firms) and a multiple market maker system of the NASDAQ (Group B). 5-minute intraday price and return series of the firms in S&P SmallCap600 indices were used. Then, the statistical and econometrical characteristics of real market intraday data were examined. In particular, descriptive statistics, non-stationarity of price as the existence of unit roots, the predictability based on variance ratio tests in different gaps, autocorrelations, time series structure and non-linear dependence in mean and variance including GARCH specification are analysed. Also, where appropriate, it examined the empirical evidence for inventory control models and adverse information models.

The empirical analysis revealed that non-stationarity in price was lower and predictability in return was higher with multiple market makers than a single market maker. The differences were statistically significant. These findings support the increased resilience hypothesis. However, both intraday return series showed its distribution was leptokurtic and non-normal. Other descriptive statistics including return volatility did not support either hypothesis. The investigation of autocorrelation, and mean and volatility models did not reveal any significant difference between two market making systems and two empirical hypotheses.

Therefore, it can be concluded that multiple market makers increase the market disequilibrium. That is, they are stronger than a single market maker in terms of their ability to hold the prices against potential adverse information and to keep the disequilibrium prices until more information is revealed. At the same time, their strategic interactions may actually constrain instant price adjustment to the adverse information.

Empirical evidence for market making models was not relatively strong. Only generally strong non-stationarity supports the adverse information models. The sign of autocorrelation, which characterises the difference between adverse information and inventory control models, was mixed at different lags. That is, at 1st lag, the significant positive autocorrelation was found possibly due to adverse information, but the relatively weaker but significant negative autocorrelation was discovered in the following lags because of probably inventory control. On the other hand, the autocorrelation of returns of firms under a single market maker system was significantly higher at lag 4 (the largest lag investigated). It is likely that a single market maker takes longer in inventory control than multiple market makers. It may show the weaker power of a single market maker, which actually somehow supports the increased resilience hypothesis.

Common empirical findings in all groups of firms are as follows. Most price series contained unit roots i.e. they are non-stationary, but the variance ratios confirmed the overall predictability of the return series discovered in price series. That is, the intraday price series are non-stationary but not random walks. The ACFs and PACFs of the return series revealed significant autocorrelation and partial autocorrelation over several lags. The identified time series structure of the return series was, on average, ARMA(1,1).

Volatility was modelled and tested to find the difference of two market making systems and to verify the earlier empirical findings in the previous literature. The ARMA(1,1) - GARCH(1,1) model of absolute returns was constructed along with several dummy variables. They represent well-known leverage effects in volatility and intraday seasonality in both mean and volatility processes. Another new dummy was added to represent a market maker's over and under-estimation of shocks. A seasonally adjusted ARMA(1,1) was used to specify a market maker's estimation. The findings were that the ARMA structure in the GARCH model was not present but the GARCH terms were all significant. Unlike the earlier literature, the leverage effects were not found. Instead, the new dummy was significant and that may show a market maker's overreaction to higher-than-estimated returns.

This chapter provides new evidence for the increased resilience hypothesis which can contribute to the market microstructure literature regarding the price impact of multiple market makers. It is that more market makers can actually increase the market disequilibrium. In addition, new evidence that a market maker's under-estimation may increase volatility was provided. Also, the significance of seasonality in mean and volatility were confirmed, but the presence of leverage effects was denied. On the other hand, it provided many findings about various empirical properties about intraday day price and return data. In addition, it presented the summary characteristics of the identified structures of mean return and volatility models under two different market making systems. They can be used as references for future research.

Chapter 3 first investigated the detecting method of a price bubble as a long-term disequilibrium at the financial market level. It adopted the duration dependence tests as the main method. The duration dependence is observed when the length of a run (duration) of abnormal returns has a positive or negative relationship with the probability that the run ends. Negative duration dependence is consistent with the common concept of stock bubbles and the probabilistic rational bubble model. Meanwhile, positive duration dependence may represent the faster breaks of a price run.

The data used for empirical analysis was 5 different stock index data of the recent 19-30 years. The GJR-GARCH model with constant mean return was identified as the better fit than other simple models, but the leverage effect was not discovered in the Indian stock market. The duration dependence tests on the residuals from this model revealed negative duration dependence in positive runs (i.e. price bubbles) for the Indian market index between 1987 and 2008 and positive duration dependence in negative runs in the Korean market index between 1990 and 2008.

If the probability distribution of duration is supposed to follow a geometric distribution, the unconditional probability that a run continues and ends can be estimated using the maximum likelihood estimation method. The probability of a positive run continuing ranged between 0.52 and 0.57 and that of a negative run was between 0.45 and 0.56. Most of the probabilities are significantly different from 0.5. The estimated duration of both types of run was approximately 2. Then, using this and its standard deviations, the simple rule of thumb was devised to detect historical bubbles. Around twice as many positive bubbles were detected than negative bubbles in the sample markets except Korea.

The concept of structural break was combined with the duration dependence test as a new method of modelling the relationship between the probability that a data generating process or its parameter values significantly change (i.e. structural break) and the duration of the process (i.e. the length of a run). This test is able to accommodate those changes rather easily due to its nature of linear approximation. It can also incorporate non-linearity.

If a price run is roughly interpreted as deviation from or restoration of the equilibrium or the transition between the disequilibrium, the test results may be interpreted regarding the changes in the transition probability between the market (dis)equilibrium. For example, positive duration dependence of price runs shows the probability that the market (dis)equilibrium transits to other market (dis)equilibrium becomes larger as the duration gets longer. The empirical analysis revealed that the NASDAQ and the Indian markets have positive duration dependence in positive runs and the Korean market had it in negative runs. However, the test results cannot be properly used as a tool for prediction although the empirical analysis showed that the series of duration contained a univariate time series structure.

Chapter 3 provided several new methods of investigating market disequilibrium and bubbles and new evidence regarding them. The estimation method of unconditional probability that a price run of abnormal returns continues was obtained. The duration dependence test was extended in the context of structural breaks.

On the other hand, the new evidence of price bubbles and duration dependence was revealed based on the original duration dependence tests. The bubbles in the Indian market were not discovered before and the positive duration dependence in negative runs (i.e. faster recoverability from continuing bear markets) in Korea was also found. Also, structural break based tests discovered several cases of positive duration dependence. These new approaches and evidence can contribute to academics and financial authorities as supplementary tools to analyse price bubbles and market disequilibrium in general.

In conclusion, this thesis has explored a field of market disequilibrium by focusing on two main topics: modelling market disequilibrium at the market microstructure level, and detecting long-run market disequilibrium in the context of stock bubbles and estimating transition probability using duration dependence. A range of concepts, theories and analytic skills were applied to develop models and analyse empirical evidence: market making theories, time series modelling, and asset pricing; duration dependence, structural breaks and macroeconomic mechanisms; probability theory among many others. Despite the use of them, the findings in the thesis probably did not give complete answers. However, they were able to provide new evidence and alternative ways of investigating the topics about market disequilibrium in the stock market. In the meantime, the author will continue to research on those topics in depth in the future.

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