SHOCKS, FRICTIONS, AND BUSINESS CYCLES IN A DEVELOPING SUB-SAHARAN AFRICAN ECONOMY

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Thesis submitted to the University of Nottingham for the degree of Doctor of Philosophy

December 2010

Abstract

This thesis examines the sources of business cycle fluctuations in a developing Sub-Saharan African economy. We develop an open economy dynamic stochastic general equilibrium model (DSGE), which is log-linearized, calibrated, and estimated with Bayesian techniques using South Africa macroeconomic data. The model incorporates various features such as external habit formation, internal investment adjustment cost, variable capacity utilization, domestically produced goods prices and wages stickiness, incomplete exchange rate pass-through, and financial accelerator. The DSGE model also integrates seven orthogonal structural shocks: a financial market shock that affects both the premium on the assets held by households and the foreign interest rate, a cost-push shock, a productivity shock in the domestically produced goods sector, an export demand shock, a terms of trade shock, a government spending shock, and a monetary policy shock. The introduction of those structural shocks allows for an empirical investigation of their effects and contributions to business cycle fluctuations in the South African economy. Simulating the DSGE model and decomposing the forecast error variances of the observable macroeconomic variables, it emerges that the main driving forces of the growth rates of real GDP, consumption, and investment, as well as the trade balance to GDP ratio, are the export demand shock, the government spending shock, the terms of trade shock, and the productivity shock. The productivity, the price mark-up, and the terms of trade shocks drive the aggregate inflation. The financial market shock predominantly impels the domestic interest rate, while the monetary policy shock largely causes the exchange rate fluctuations. Real, nominal, and financial frictions are necessary to capture the dynamic of the South Africa macroeconomic data and critical to explain their volatility and persistence.

Acknowledgements

I am grateful to the School of Economics at the University of Nottingham for hospitality and research support during the three academic years of my Ph.D. research.

Many people have contributed to the achievement of this research in various ways. First and foremost, I am in special debt to my supervisors Michael Bleaney and Mark Roberts from whom I received valuable advice underlying much of the ideas found here. Working with them has always been a source of inspiration, and their work has sharpened my understanding of modern economic research and deepened my fascination to macroeconomics.

Many other professors and PhD students at the School of Economics have helped me to improve my research with specific comments, or through discussions over the years of my Ph.D.

I owe a special thank to my family from which I have received a constant affection and generosity during those years far from home, and for his indefectible support to this Ph.D. research from day one.

I am also thankful to a non-exhaustive list of friends for their valuable counsel and support.

I am solely responsible for remaining errors.

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Introduction

This research examines the sources of business cycle fluctuations in a developing Sub-Saharan African economy. The main objective is to give a theoretical and an empirical account of the fundamental driving forces that lead key macroeconomic variables to evolve along patterns marked by peaks and troughs in Sub-Saharan Africa. For empirical application purposes, we choose the South African economy as it provides macroeconomic series at a quarterly frequency and over a long horizon of time.

Over the recent decades, the evolution of macroeconomic variables in many Sub-Saharan African economies has been far from steady. A part the low long-term growth rate observed in many of those economies, the cyclical components of numerous macroeconomic variables display markedly large fluctuations, and to some extent, exhibit erratic movements. How one should view and explain those macroeconomic fluctuations? What disturbances are relevant in a developing Sub-Saharan African economy? These questions still require further answers, as is likely, the conceptual framework appropriate for their investigation.

The effort to understand business cycle fluctuations and the dynamic correlations amongst macroeconomic variables in industrialized economies has led to the development of dynamic stochastic general equilibrium (DSGE) models. That analytical framework has emerged as the workhorse for the analysis of economic fluctuations and their implications for macroeconomic policies and welfare as well. It has also become the backbone of medium scale models adopted by the majority of the central banks in the developed world.

Dynamic stochastic general equilibrium models could also help to explain the economic fluctuations in developing Sub-Saharan African economies. Those tools could provide the theoretical underpinnings needed to conceptualize appropriate macroeconomic policy responses to exogenous shocks, while uncover the most relevant source of economic fluctuations in those economies. This research's additionally defining goal is to extend the dynamic stochastic general equilibrium framework to developing economies, as well as to underscore the premises of such analytical framework in Sub-Saharan African economies.

Our philosophy is conceptually straightforward. A developing Sub-Saharan African economy is an economic system with a stationary equilibrium (also known as a steady state) or a balanced growth path. Exogenous stochastic shocks hit that economic system, cause its departure from the steady state equilibrium or the balanced growth path, and generate fluctuations of macroeconomic variables. In addition, the economic system encloses real, nominal, and financial frictions, which slowdown the economic adjustment as the economy returns to its steady state or its balanced growth path after a departure triggered by an exogenous shock. This philosophy underscores the importance of exogenous shocks as sources of economic fluctuations in developing Sub-Saharan African economies; an interpretation of economic fluctuations that remains in line with the consensual view of modern macroeconomics.

The analytical framework underlining this philosophy of business cycles in Sub-Saharan African economies has a core structure of the new generation of New Keynesian dynamic stochastic general equilibrium models. From a methodological point of view, the bulk of New Keynesian dynamic stochastic general equilibrium models developed over the past decades could cover and investigate a large number of issues in various economies,

including developing Sub-Saharan African economies. Despite the cross-countries difference in living standard, various assumptions[1] that constitute the hallmark of the New-Keynesian apparatus are realistic for developing economies. An infinitely lived representative agent that maximises the utility from consumption and leisure subject to an intertemporal budget constraint, a large number of firms with access to an identical technology subject to exogenous shock, monopolistic competition where private agents set the prices of goods in order to maximize their objectives, nominal rigidities that constraint the frequency with which firms adjust the prices of the goods and services they sell, or workers the wages at which they supply labour, are theoretical assumptions likely to characterize a Sub-Saharan African economy as well.

In addition to the New-Keynesian distinguishing traits mentioned above, the analytical framework of this research seeks a more realistic picture of a developing sub-Saharan African economy. It gives a specific accent to structural characteristics susceptible to make such an economy more vulnerable to exogenous shocks. Thus, by enhancing the basic features associated with a developing sub-Saharan African economy, this research's analytical framework is more likely to minimize the conflict between theoretical predictions and empirical evidence, or between normative implications and policy practice.

We therefore introduce in this thesis' theoretical dynamic stochastic general equilibrium model two striking structural features, profoundly discernible, and noticeably in many Sub-Saharan African economies. First, the model embeds financial frictions that hinder investment financing and amplify the effect of interest rate and exchange rate fluctuations on borrowers' real net worth positions, the balance sheets, and the real macroeconomic

[1] Some of these assumptions are drawn from the Real Business Cycles model (Kydland and Prescott 1982), particularly, the infinitely lived representative household that maximises its utility and a large number of firms with access to an identical technology subject to exogenous random shocks. However, the canonical version of the New Keynesian model (See for example chapter 5 in Walsh, 2003 for more details) does not include the capital accumulation.

equilibrium (Krugman 1999, Aghion, Bacchetta, and Benerjee 2000). Second, it incorporates the exchange rate pass-through since the speed at which exchange rate shocks feed into the domestic price level seems to be higher in developing economies than in industrial economies (Calvo and Reinhard 2002, Choudhri and Hakura 2006, and Devereux and Yetman 2005).

While those two features are not specifically restricted to developing Sub-Saharan African economies, as they could feature economies in other regions, the sensitivity analysis conducted in this research (see chapter 4) shows that they are necessary to capture the empirical dynamic of the South Africa macroeconomic data. Other distortions crucial for developing economies are excluded from this framework. Amongst the most important, the "economic dualism" (Lewis 1954), which refers to the partition of a single economy into two sectors that seem at a very dissimilar level of development; one sector is generally capital intensive and exports its entire output, while the other is labour intensive and merely supplies the domestic market, and an abundant unskilled labour force generally engenders a "reservoir of labour force". Simplifying the analytical framework is the prime reason of excluding those distortions, and to some extent, the desirability of a framework that could explain data from the specific case study rather than a more generic framework that falls short in that dimension.

A third characteristic unveiled by the estimated DSGE model is perhaps the most distinctive trait of developing Sub-Saharan economies. In fact, exogenous shocks tend to have larger magnitude in developing economies, at least in the specific case of the South African economy, which data are used for the Bayesian estimation. This stylized fact is

particularly characterized by higher standard deviations to exogenous shocks in developing Sub-Saharan African economies.

The dynamic stochastic general equilibrium model that constitutes the cornerstone of this research also incorporates additional features necessary to capture the empirical persistence in macroeconomic data. More specifically, external habit formation in consumption, internal investment adjustment costs, and variable capital utilization rate are introduced to capture the sluggish response of consumption, investment, and rental rate to exogenous shocks.

The model also introduces some theoretical ideas that the empirical analysis tends to confirm. First, it shows that the foreign interest rate (for the South African economy) and the premium on assets held by domestic households are both driven by an identical financial market shock. Second, it establishes that the trade credit contracted in advance by foreign households in order to finance their consumption of import goods also affects the export dynamic of the small open economy. Introducing the trade credit into the export dynamic appears useful to capture the export persistence and explain the role of financial disturbances in export fluctuations.

Seven orthogonal structural shocks drive the economic system away from the steady state or the balanced growth path. We seek the contributions to business cycle fluctuations of a cost-push shock, a productivity shock, an export demand shock, a terms of trade shock, a government spending shock, a monetary policy shock, and a financial market shock that affects both the premium on the assets held by households and the foreign interest rate. While this selection of exogenous shocks could be subjective, it has the virtue of enclosing relevant shocks that empirically affect most of developing economies.

In general, dynamic stochastic general equilibrium models do not have clear analytical solutions[2]; to find out the insights our model conveys about economic fluctuations in Sub-Saharan African economies, we rely on computational methods. Those computational methods require approximating with perturbation methods, solving, and simulating the theoretical dynamic stochastic general equilibrium model around the steady state or the balanced growth path in order to produce artificial data. The artificial data describe the responses of economic variables to exogenous shocks. A collection of statistics, also known as theoretical moments, could summarize the contributions of exogenous shocks to economic fluctuations. Since structural shocks are orthogonal[3], it is possible to decompose unambiguously the forecast error variance of each macroeconomic variable into components that exclusively reflect the variability attributed to each specific shock. The fraction of the variable's forecast error variance exclusively explained by a particular exogenous shock determines the relative importance of that shock in explaining the fluctuations of the macroeconomic variable of interest.

The Real Business Cycles literature (see Cooley 1995) has established that methodology of summarizing business cycles by a collection of statistics calculated from artificial data. Nowadays, four major empirical methods exist to generate artificial data from a theoretical dynamic stochastic general equilibrium model: the calibration, the limited-information method, the maximum likelihood based estimation, and the Bayesian estimation.

In this research, we combine two of these empirical methodologies. First, we use the South Africa macroeconomic data to calibrate the DSGE model's parameters that are linked to the steady state values of endogenous variables as they might exhibit convergence

[2] Usually, the optimality conditions involve a system of high order non-linear difference equations and expectations that are proved hard to solve mathematically.
[3] At least as it emerges from the estimated variance-covariance matrix of structural shocks which is diagonal.

problems. To achieve that goal, we solve for those structural parameters using the non-linear optimality conditions evaluated at the steady state. Structural parameters to be calibrated are reversed engineer as functions of endogenous variables evaluated at the steady state and sample long-run averages are subsequently used to compute the values of those parameters. Second, we infer the values of the remaining structural parameters by estimating the linearized DSGE model with Bayesian techniques using the South Africa macroeconomic data transformed into mean-zero covariance stationary stochastic processes. Performing the statistical inference with mean-zero covariance stationary time series, which describe macroeconomic variables expressed as percentage deviations from the steady state or the balanced growth path, aims at establishing a symmetric of treatment and a correspondence between the theoretical model's variables and what is being measured by the actual data. Compare with the standard uses of Structural VARs that either combine short run and long run restrictions, or implement sign restrictions to identify solely structural shocks; the estimation of the DSGE model offers a fully-fledged structural approach to pin down structural parameters as well as identify exogenous shocks in a theoretically consistent manner.

The use of log linear approximation in explaining business cycles fluctuations in developing Sub Saharan African economies requires additional explanation. Arguably, Sub Saharan African economies could be far away from their steady state equilibrium or their balanced growth path, a conjecture that a large number of frictions in those economies may reasonably support. If that is the case, results generated by linear approximation methods may be inaccurate since their accuracy depends on the distance from the steady state or the balanced growth path and the degree of non-linearity in the model subject to the

approximation. In this context, non-linear methods, such as projection methods, value-function iterations, and policy-function iterations are naturally the most appropriate[4]. Non-linear methods are beyond the scope of this research. However, it is worth noting that a main objective of the business cycle literature is to determine whether models capable of capturing salient features of economic growth can also account for the patterns of business cycle activity. Under this objective, the specification of a DSGE model is subject to the constraint that it must successfully characterize the steady state or the balanced growth path and growth model's properties. One of those properties is the convergence (see Solow 1956), which stipulates that countries far away from the steady state grow relative faster than those closer. As a corollary, macroeconomic variables of the former are likely to display trend. To take into account that eventuality, we build both cyclical behaviour and trend into the theoretical DSGE model using the labour-augmenting deterministic growth rate in the economy, and then, eliminate trends from the model and actual data in the same fashion using the differencing technique.

The calibrated-estimated dynamic stochastic general equilibrium model is subsequently used to address key macroeconomic issues relevant for the South African economy and Sub Saharan African economies as well. First, we conduct a sensitivity analysis to determine the frictions that are important in explaining the dynamic of the macroeconomic data in the South African economy. Then, we perform a sequence of simulations to generate artificial data that describe the economy adjustment to exogenous shocks. We explore the economy volatility and analyse the persistence in macroeconomic data by computing

[4]Alternatives within non-linear methods, the researcher could take say a second-order (or higher order) approximations around the efficient or the distorted steady-state.

theoretical moments from the model's artificial data and comparing those moments with empirical moments calculated from the actual data.

To answer the fundamental question of this research: what are the sources of business cycle fluctuations in a developing Sub-Saharan African economy? We decompose the forecast error variances of the observable macroeconomic variables at various horizons that characterize the short, medium, and long term. Several results of our analysis are worth highlighting. The fluctuations in the growth rates of the real GDP, consumption and investment, the trade balance to GDP ratio are primarily driven by the productivity shock, the export demand shock, the terms of trade shock, and the government spending shock. The importance of those shocks varies with the horizons (short run, medium run, or long run). The productivity shock, the price mark-up shock, and the terms of trade shock drive the aggregate inflation. The financial market shock predominantly explains movements in the domestic interest rate, while the monetary policy shock largely drives the exchange rate. The estimated Dynamic stochastic general equilibrium model also replicates many variances and cross-covariances between observable macroeconomic variables.

Having discussed the objectives of this research and its analytical framework, this introduction ends with a brief description of this thesis organization.

The thesis is organized into four chapters. Chapter 1 briefly discusses some empirical evidence on business cycle fluctuations in Sub-Saharan African economies; a specific accent is given to the South African economy, our case study. Chapter 2 develops the theoretical dynamic stochastic general equilibrium model that tailors the unified framework of this thesis. Chapter 3 presents the calibration and the estimation of the DSGE model using South Africa macroeconomic data, and then, it reports the model evaluation that discusses the

DSGE model ability to replicate the moments generate by actual data. Chapter 4 reports the results from the DSGE model applications and uses to address key business cycle questions for the South African economy. We investigate three key issues. First, we assess the importance of frictions in South African business cycles, then we explore the macroeconomic adjustment to exogenous shocks, and finally we decompose the forecast error variance to identify the main driving forces of key macroeconomic variables.

Chapter 1

Empirical evidence on business cycle fluctuations in Sub-Saharan African economies

This chapter summarizes some key stylized facts related to business cycle fluctuations in the Sub-Saharan African region. The main objective is to give a brief empirical account of some short run and long run stylized facts observed in the Sub-Saharan African region and motivate the theoretical and empirical analysis that follows in the next chapters. In general, empirical studies in Sub-Saharan African economies are constrained by data availability. Many Sub-Saharan African countries mostly provide annual data beginning in 1960, and sometime, several observations are missing. In face of such a constraint, it becomes difficult to produce in numerous Sub-Saharan African economies a comprehensive empirical analysis of business cycle fluctuations, which in general requires high frequency data over a long time horizon.

Although the data availability constraint renders difficult the conduct of a widespread business cycle analysis in many Sub-Saharan African countries, recurrent stylized facts could emerge from a cross-country analysis of annual data. Moreover, given that applied macroeconomic studies are much relevant when guided by the need to shed light on empirical facts, a much focus could be on a specific economy in the Sub-Saharan Africa, the South Africa economy. For empirical application purposes, the South African economy provides macroeconomic series at a quarterly frequency over a long horizon of time; it is consequently a suitable case for applying the theoretical model developed in this thesis.

As specific cases are better understood when embedded in their regional context, this chapter commences with a brief overview of the growth cycle in the Sub-Saharan Africa as a whole. The idea is to present the regional growth rate and illustrate the fact that large swings characterize its patterns, and that, economic growth performances in the region are marked by high volatility and instability. While this remains a pure descriptive presentation, it could help to forge the intuition that the volatility of growth rates is much higher in the Sub-Saharan Africa; hence, exogenous shocks that hit the economies in the region might have higher magnitudes. The second section is devoted to the South African economy and presents the long run ratios that characterize its steady state equilibrium and describes some short run co-movements observed amongst its key macroeconomic variables.

1.1-Sub-Saharan African economies

Over the recent decades, economic growth in many Sub-Saharan African economies has been far from steady. A part the low long-term economic growth rate observed in many of those economies, growth paths display markedly large fluctuations, and to some extent, exhibit erratic movements. To illustrate that stylized fact at the aggregate level, figure 1.1 plots in percentage four series of growth rates calculated from different measures of the (aggregate) Sub Saharan Africa GDP. In a descending order, its depicts the annual growth rate of the purchasing power parity (PPP) GDP, the annual growth rate of the PPP GDP per capita, the real GDP growth rate (as traditionally measured from the national account), and the real GDP per capita growth rate. Computation of the PPP GDP growth rates are based on the aggregate Sub-Saharan Africa PPP GDP, whilst the Sub Saharan real GDP growth rate (respectively real per capita growth rate) are weighted averages of individual countries real

growth rates (respectively per capita growth rates). Weights are countries shares in the cumulative Sub Saharan Africa PPP GDP. Variables are at annual frequency and cover the period 1970-2006.

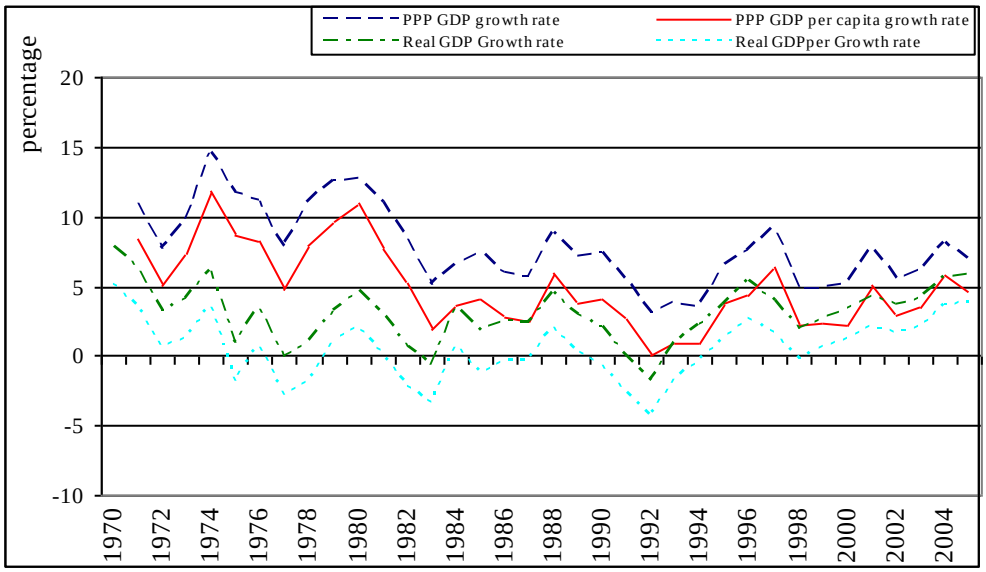


Figure 1.1: Sub Saharan African average growth rates.
Source: Author calculations using data from IMF World Economic Outlook database.

The first observation is an extreme volatility of the regional average growth rates; this observation seems independent of the variables or the method used to compute the regional growth rate. The Sub-Saharan African growth rate usually climbs steadily, reaches a peak, and then falls sharply. This repetitive movement is reflected by successive up and down fluctuations in the regional growth rates with the lowest point located approximately in 1992. The amplitude between some successive peaks and troughs could reach 5 to 10%.

In the 2000s decade, the Sub-Saharan African real GDP growth rate appears to be on an increasingly trend after a decade of a declining trend, and it is rising slowly. The regional average economic growth rate continues a gradual progression, from the level of 3.5% in 2002, it moved to 4.1% in 2003, then 5.6% 2004 and it stood at the level of 5.3% in 2006.

Large swings tend to be less frequent although the average regional growth rate remains below its highest historical level.

In addition to the extreme volatility, a high instability also seems to characterize the real GDP growth rate in the region albeit more countries record higher growth rates and the average growth rate is climbing. Episodes of sustainable growth rates are rare; period of economic expansion are short lived as the growth rate tends to fall into a new trough every four years. Structural breaks are also frequent in Sub Saharan Africa. Put in other words, the factors (economic management, structural policies, and exogenous disturbances) that theory and other empirical analyses suggest have an important impact on growth shift frequently in the region. Particularly, structural factors such as institutions remain weak. Next, we present some long run and short run stylized facts observed in the South African economy.

1.2- South African economy

In this section, we present the background of the South African economy, describe its steady state equilibrium, and characterize some of its business cycles stylized facts. We use data from the South African Reserve Bank database[5] covering the period 1960:1-2008:4 and we focus on the evolution of real variables such as the real GDP, consumption, investment, government spending, exports, and imports. An interest is also devoted to nominal variables like inflation, exchange rate, and nominal interest rate. South Africa adopted the inflation targeting in late 1990's and has a policy related interest rate since then. Because that policy rate spans over a short period, the (money market) short-term interest rate serves as a proxy of the domestic interest rate controlled by the central bank.

[5] These data can be downloaded at http://www.reservebank.co.za.

1.2.1 Macroeconomic background

As illustrated on figure 1.2, South Africa enjoys a sustainable growth period which witnesses real GDP moving gradually through an increasingly path. Real GDP growth has strengthened in the recent decades reflecting the upward trend in exports and foreign direct investment. The average annual long term growth of the real GDP is at about 3.136 percent.

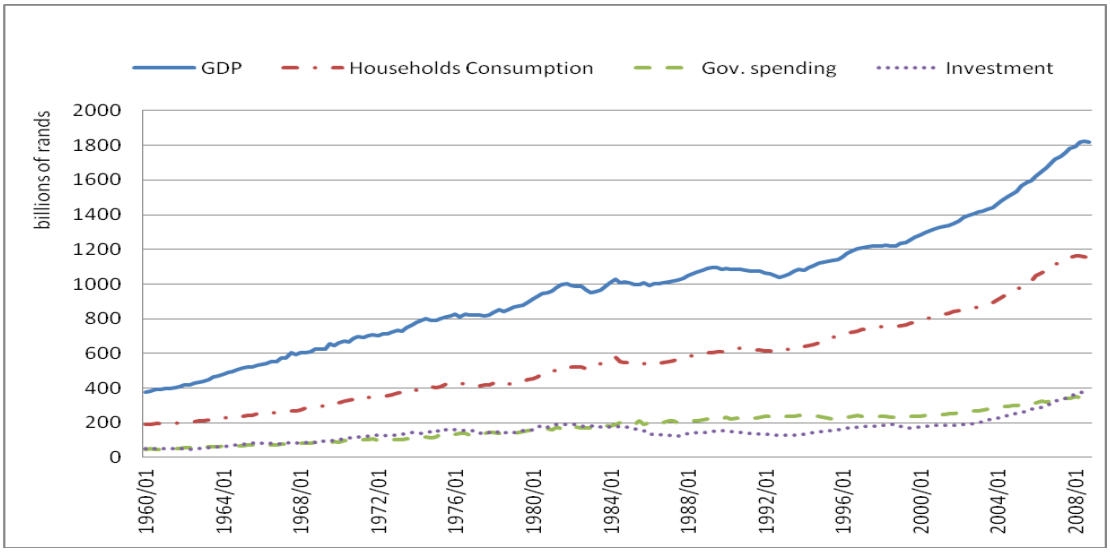


Figure 1.2: South African economy real GDP, households' consumption, government spending, and investment
Data Source: South African Reserve Bank database.

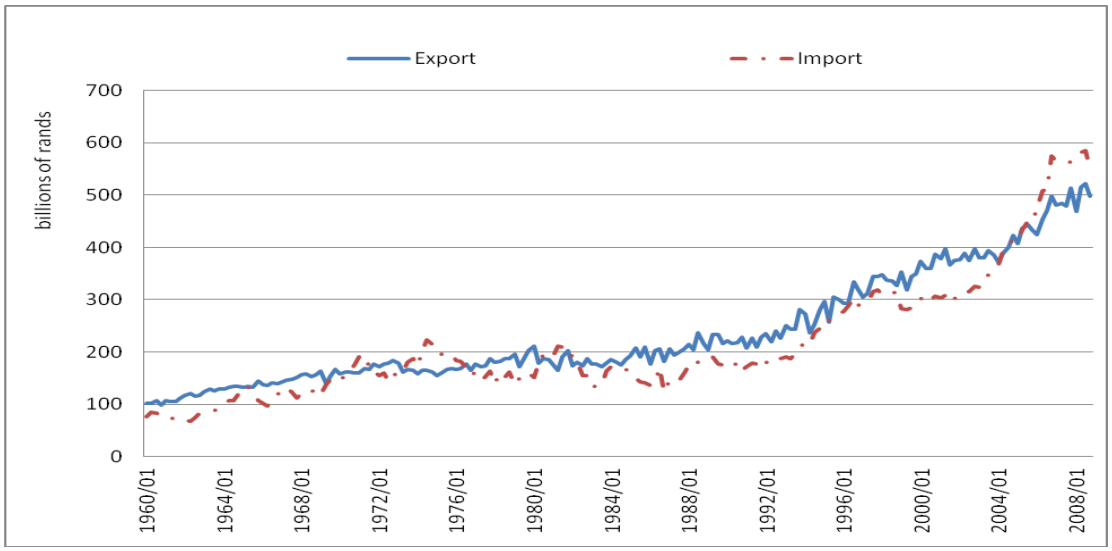


Figure 1.3: South African economy exports and imports
Data source: South African Reserve Bank database.

In addition, figure 1.2 shows that a growing private consumption is supporting the domestic absorption and the economic activity. Investment increases slightly and recent public and private investment projects are shifting its trend upward. Figure 1.3 indicates that exports recovered after a long period of stagnation and are growing faster, though they have become more volatile. Furthermore, imports mostly constituted of intermediate and capital goods are on an increasing path.

Twelve month average annual inflation rate is estimated at about 8.71 percent. As indicated on figure 1.4, South Africa has experienced a period of high inflation and high nominal interest rate prior to the adoption of the inflation targeting regime in the late 1990s. The exchange rate was more stable in the 1960s and 1970s, but it has depreciated substantially in the last decade reflecting developments in the domestic financial markets.

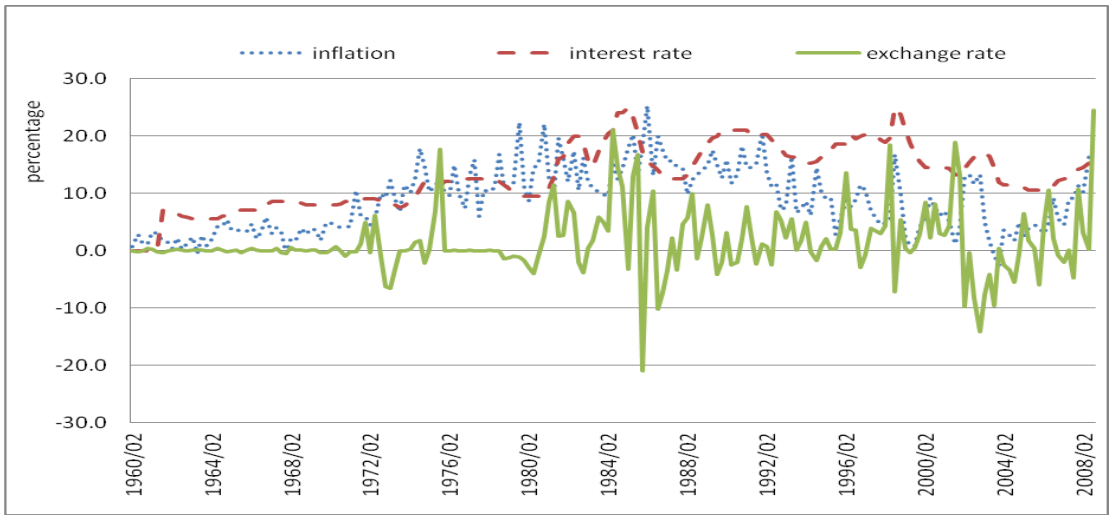


Figure 1.4: South African economy inflation, interest rate, and exchange rate
Data source: South African Reserve Bank database. Note: the figure depicts the changes in the rand per US dollar exchange rate

Figure 1.5 shows that the overall trade balance to GDP ratio is within the range -10 to 10 percent. South Africa has registered a trade balance surplus most of the time, though, episodes of trade balance deficit were observed in the earlier 1970s and 1980s, and in the late

2000s. The real effective exchange rate, defined as the relative price of domestically produced goods in term of foreign goods, appears to be highly volatile. In addition, sharp peaks and deep troughs characterize its path suggesting the importance of terms of trade shocks and their significance for the trade balance fluctuations[6]. The trade balance to GDP ratio and the real exchange rate also exhibit a strong correlation.

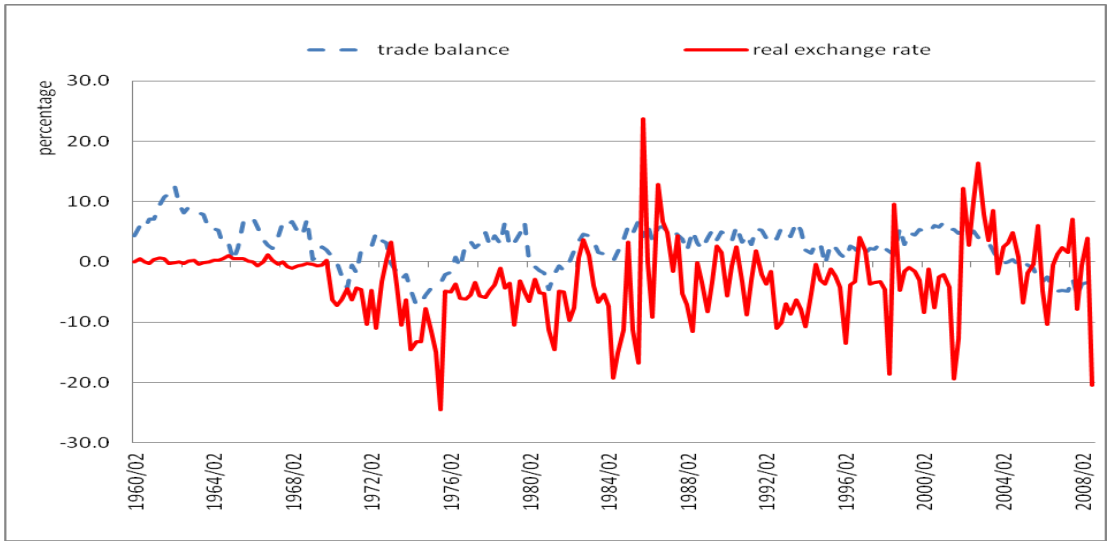


Figure 1.5: South African economy trade balance to GDP ratio and real exchange rate
Source: Author calculations using data from the South African Reserve Bank database. Note: the figure depicts the changes in the real effective exchange rate; an increase represents an appreciation.

1.2.2 Some long run stylized facts

The long run stylized facts are described through the averages long run ratios that serve as proxies to judge the numerical steady state of the South African economy and some of the basic empirical correlations observed amongst its key macroeconomic variables at the long run. The steady state ratios are evaluated using the sample averages. We use the entire sample period to compute those averages. The variables are quarterly seasonally adjusted at constant 2005 prices. Table 1 reports the South African economy key steady state ratios.

[6] The application of the DSGE model confirms this empirical observation. See the trade balance forecast error decomposition in chapter 4.

Table 1.1: long-run averages and ratios[7]

Ratio	Sample long-run average	Definition
$\frac{C_{ss}}{Y_{ss}}$	0.517	steady state ratio of household consumption to output
$\frac{C_{ss}^{e}}{Y_{ss}}$	0.051	steady state ratio of entrepreneur consumption to output
$\frac{I_{ss}}{Y_{ss}}$	0.158	steady state ratio of investment to output
$\frac{G_{ss}}{Y_{ss}}$	0.183	steady state ratio of government spending to output
$\frac{X_{ss}}{Y_{ss}}$	0.241	steady state ratio of exports to output
$\frac{Y_{Mss}}{Y_{ss}}$	0.221	steady state ratio of imports to output
$\frac{A_{ss}}{Y_{ss}}$	0.910	steady state ratio of domestic absorption to output
$\frac{D_{ss}}{Y_{ss}}$	0.115	steady state ratio of households' debt denominated in foreign currency to output
D_{ss}	0.027	steady state households debt denominated in foreign currency (trillions of US dollars)
π_{*}	1.087	steady state annual gross inflation rate
g	1.008	quarterly gross long run growth rate

Source: Author calculations using data from the South African Reserve Bank database available at http://www.reservebank.co.za.

On average, the household consumption is approximately half of the output, while the entrepreneur consumption is about five percent of the output. The investment is about 16 percent of the output and government spending is much higher at about 18 percent. The exports are on average approximately a quarter of the output whilst imports are lower at about 22 percent, which implies that the South African economy might generate a trade balance surplus at the steady state. However, the current account is on deficit as the economy seeks external financing at the steady state.

Table 1.2 reports some long run basic correlations amongst key macro economic variables; the numbers in parenthesis correspond to the associated t-statistic. We filtered the data using the Hodrick-Prescott filter[8] to eliminate the short run volatility and then calculated the correlations amongst the trend components of the variables. There are strong correlations

[7] GDP, investment, government spending, exports, and imports are the measures provide by the national accounts with the GDP denoted here output. The private consumption in national accounts sums the household consumption and entrepreneur consumption with the latter measures by the consumption of the socio-professional category "professional". The domestic absorption sums the private consumption, the investment, the government spending. Household debts (in domestic and foreign currencies) are provided by the national accounts.

[8] In the Hodrick-Prescott filter, the parameter λ equals to the ratio of the volatility of the cycle component over the volatility of the trend. In developing economies, the trend is generally more volatile than in developed economies (see Aguiar and Gopinath, 2007), which requires a value of λ lower than the 1600 calibrated (for quarterly frequency data) in the US economy. However, given the smooth path of South Africa GDP (see figure 1.2) the trend is stable in this economy (the standard deviation of quarterly growth rate is 0.011), we take $\lambda = 1600$.

amongst real variables (GDP, household consumption, investment, exports, and imports) in the long run; the coefficients of correlation vary between 0.9-1 and the relationship is almost one to one between consumption and real GDP. This empirical evidence suggests that long run trends of South Africa real variables are strongly correlated.

Table 1.2: sample correlations

	Real GDP	**Cons.**	**Invest.**	**Export**	**import**	**inflation**	**Interest rate**	**Exchange rate**
Real GDP	1.000							
Consumption	0.996	1.000						
	(153.42)	-----						
Investment	0.923	0.908	1.000					
	(32.93)	(29.77)	-----					
Export	0.951	0.971	0.875	1.000				
	(42.37)	(55.30)	(24.78)	-----				
Import	0.919	0.935	0.930	0.971	1.000			
	(32.06)	(35.99)	(34.67)	(55.87)	-----			
Inflation	0.212	0.139	0.249	-0.087	-0.070	1.000		
	(2.97)	(1.93)	(3.53)	(-1.20)	(-0.97)	-----		
Interest rate	0.593	0.566	0.377	0.399	0.272	0.624	1.000	
	(10.10)	(9.41)	(5.58)	(5.96)	(3.87)	(10.94)	-----	
Exchange rate	0.893	0.922	0.744	0.964	0.882	-0.189	0.411	1.000
	(27.25)	(32.56)	(15.29)	(49.72)	(25.69)	(-2.63)	(6.176)	-----

The long run correlation between inflation and exchange rate is loose but negative. There is a strong positive correlation between the nominal interest rate and the inflation suggesting that the Fisher's effect may be at play. The long run correlation between output and inflation is weak but positive, which might suggest that demand shocks could play an important role in long run output fluctuations[9]. A prominent role to supply shocks in long run output fluctuations would have generated a negative correlation between output and inflation.

1.2.1 Some short run stylized facts

Conventionally, the focal point of the business cycles literature is on short run fluctuations. One key objective is to determine whether models capable of capturing salient

[9] This assumption is basically confirmed by the forecast error variance decomposition, see chapter 4 for details.

features of economic growth can also account for the patterns of business cycle fluctuations. Under this objective, the specification of the model is subject to the constraint that it must successfully characterize the steady state or the balanced growth path. In general, not always, business cycles fluctuations are analysed in the neighbouring of the steady state or the balanced growth path. This usually requires a particular treatment of variables, which are generally expressed as deviations from the steady state or the balanced growth path.

The empirical methodology in business cycles literature requires treating symmetrically the artificial data from the model and the actual data. Since we seek to analyse business cycles with a linearized DSGE model, trends should be removed from the South Africa macroeconomic variables. This is achieved through the first difference technique[10]. Figure 1.5 depicts the Log difference of real GDP, consumption, investment and exchange rate along side with the trade balance to GDP ratio, the inflation and the nominal interest rate.

In general, "cyclical components" of macroeconomic variables display marked fluctuations in the South African economy. Fluctuations in real variables were particularly wider at the beginning of the sample period with substantially large amplitudes between peaks and troughs. There are high frequency fluctuations in the inflation, which tends to move on a path characterized by large swings. The exchange rate has become more volatile in the last three decades.

To assess the persistence and co-movements amongst macroeconomic variables in the short run, we compute at 20 quarters horizon period the VAR-based Autocorrelation

[10] In general trend removal and cycles isolation techniques are various. There are three leading approaches for removing trend from macroeconomic time series: detrending using a linear trend, differencing, and filtering (the most common used filters are the Hodrick-Prescott (H-P) filter and the Band Pass (B-P) filter) (see Canova 2007, or Dejong and Dave 2007). Here, the presentation and analysis of data obtained through differencing technique aims primarily at establishing a symmetric of treatment between what the actual data measure and what is explained by the model. Latter in chapter 3, which lays down the ground for an empirical application of the DSGE model developed in chapter 2, we show that differencing actual time series could easily be connected to the model variables through a well defined measurement equation system.

functions (ACF) [11]. The unrestricted VAR is estimated on the Log difference of the real GDP $(\Delta LnGDP_t)$, (households) consumption (ΔLnC_t), investment (ΔLnI_t), and exchange rate (ΔLnS_t); the trade balance to GDP ratio (TB_t / GDP_t), the inflation (π_t), and the nominal interest rate (i_t). The data sample covers the period 1960:2-2008:4. The VAR satisfies the stability condition since all its roots lie inside the unit circle. The maximum number of lags, three, introduced in the VAR model was determined using the sequential modified likelihood ratio test, and the Akaike and Schwarz based lower maximum likelihood criteria (see appendix 1). The Portmanteau autocorrelation test, the normality test, and the White heteroskedasticity test performed on the residuals also confirm this VAR representation[12].

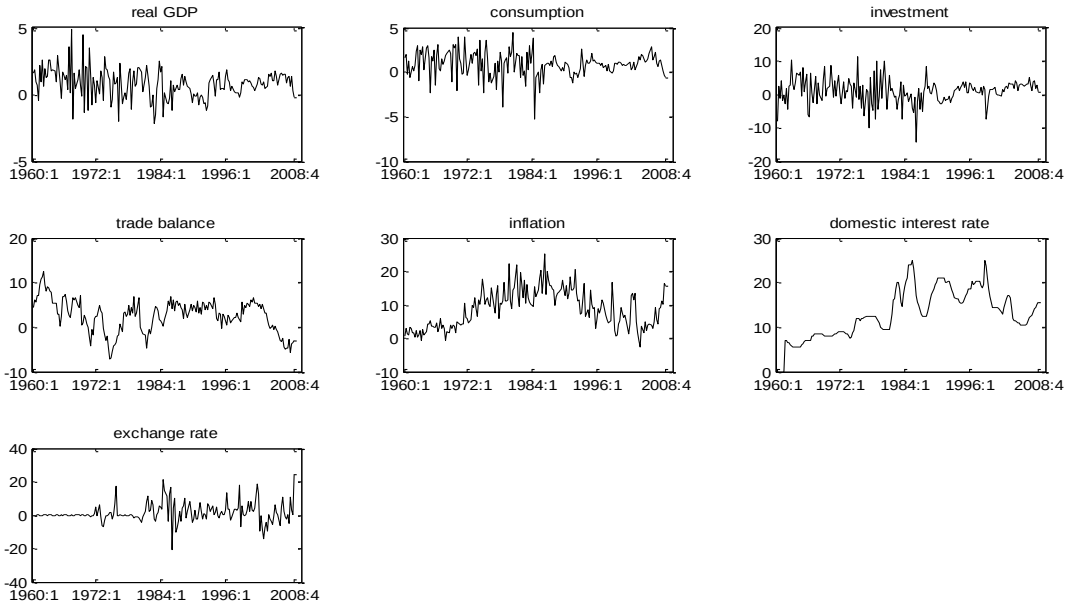
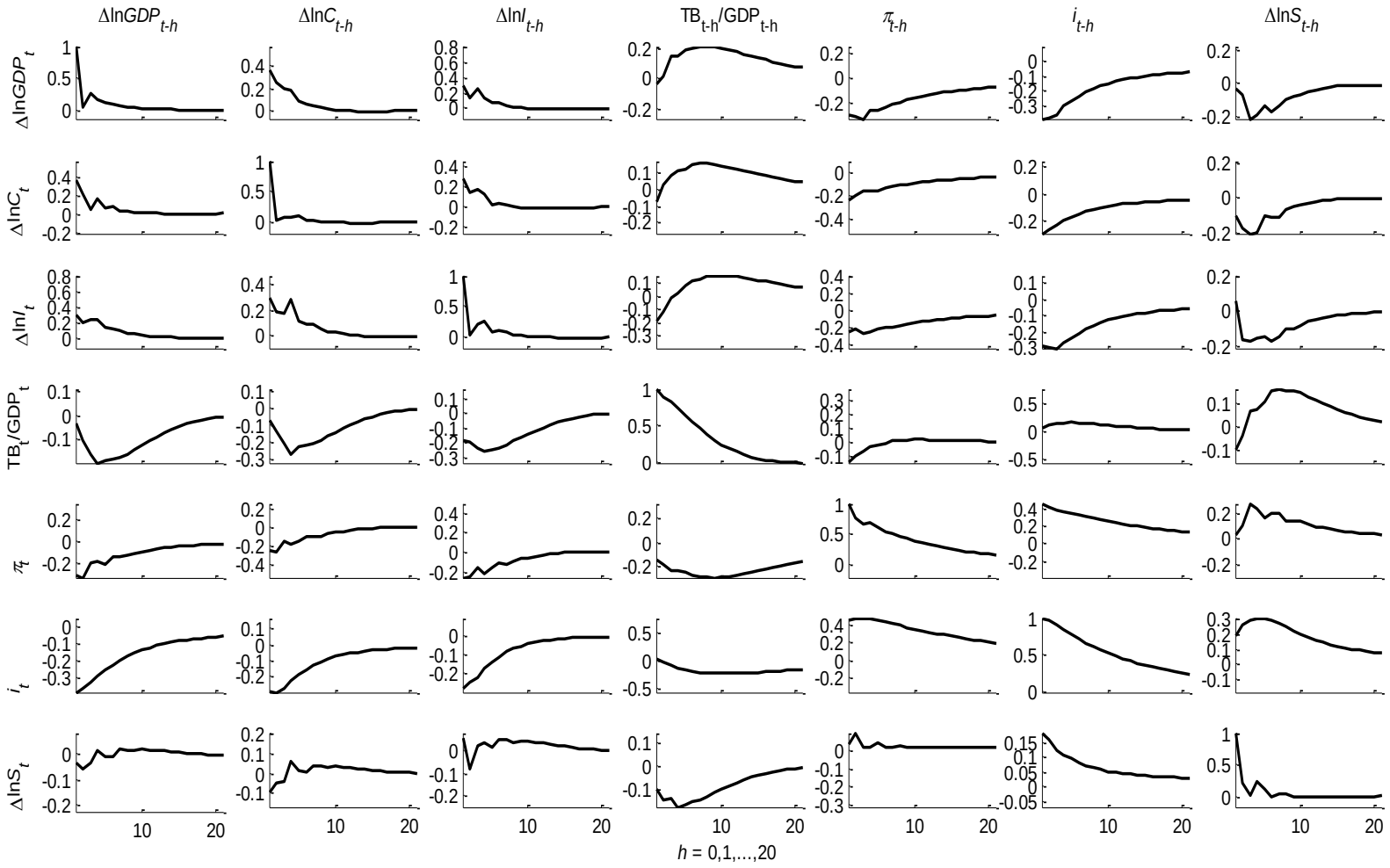


Figure 1.6: "Cyclical components" of South Africa key macroeconomic variables
Source: Author calculations using data from the South African Reserve Bank database.

[11] Note that a vector autoregressive model VAR (p) specified for an *m x 1* vector has a companion form $z_t = Az_{t-1} + e_t$. The s^{th} -order covariance matrix is given by $\Gamma(s) = A^s\Gamma(0)$, where $\Gamma(0) = E(z_t z_t')$ is the contemporaneous variance-covariance matrix of z_t, which satisfies $\Gamma(0) = A\Gamma(0)A' + \Sigma_e$, and to which the solution is given by $vec[\Gamma(0)] = [I - A \otimes A]^{-1} vec[\Sigma_e]$, where $\Sigma_e = E(z_t z_t')$ (see Hamilton, 1994). The correlations are obtained by normalizing the variances and cross-covariances by the variances.

[12] Traditionally researchers validate DSGE models by comparing the model-based variances and covariances with those in the data; we present the results of that exercise in the third section of chapter 3 devoted to the model evaluation.

Figure 1.7 VAR-based cross-correlations between South Africa key macroeconomic variables



Legend: On this figure, $\Delta LnGDP_t$ stands for the Log difference of the real GDP, ΔLnC_t the Log difference of household consumption, ΔLnI_t the Log difference of investment, TB_t / GDP_t the trade balance to GDP ratio, π_t the inflation rate, i_t the nominal interest rate, and ΔLnS_t the Log difference of nominal exchange rate.

Figure 1.7 presents the autocorrelations functions (ACF) that assess the persistence and co-movements of South Africa macroeconomic variables. In general, South Africa macroeconomic variables display persistence and co-movements in a very short run period.

The empirical cross-correlations suggest a much higher degree of persistence in the actual data at the first and second quarter horizon. In general, growth rates of real variables, and particularly the real GDP growth rate, tend to display a low persistence. The persistence in consumption and investment (growth rates) is also lower, while the trade balance exhibits a much higher persistence.

Nominal variables, namely, the aggregate inflation and the interest rate exhibit the highest degree of persistence. The aggregate inflation is serially correlated over the 20 quarters horizon. The strong persistence demonstrated by the aggregate inflation indicates that large shocks can persist and influence the volatility forecast of inflation for several periods.

The real GDP growth rate is correlated with real and nominal variables in the leads and lags. The correlation between (the growth rates of) real GDP and consumption is strong in the very short run horizon. To some extent, the cross-correlation between real GDP and consumption (growth rates) could echo the similarity that emerges from their impulse responses to exogenous shocks (see chapter 4). The correlation between real GDP growth rate and inflation is negative and appears to be important in the short run; the real GDP growth rate is also correlated with the interest rate and the exchange rate. The VAR-based cross-correlations functions show that the real GDP growth rate is negatively correlated with those two variables at the lags and leads. In general, the cross-correlations between nominal variables and real variables tend to be different from zero. However, the cross-correlations between the trade balance to GDP ratio and nominal variables, but the exchange rate, are closest to zero.

1.4-Conclusion

This short chapter had briefly presented some long run and short run stylized facts in developing Sub-Saharan African economies, stylized facts that motivate a much profound and deeper business cycle analysis in the region. In Sub-Saharan Africa, economic growth rates are extremely volatile, while growth rate paths tend to be erratic and unstable. The data availability constraint limits a widespread business cycle analysis in many Sub-Saharan African economies; however, a much focus could be on a specific economy in the Sub-Saharan Africa, the South African economy, which provides macroeconomic series at a quarterly frequency over a long horizon of time and is consequently a suitable case study.

In the long run, real variables show strong correlations in the South African economy and share a common trend. In the short run, consumption, investment, or inflation display persistence, those macroeconomic variables also exhibit strong co-movements. The effort to understand business cycles fluctuations and the dynamic correlations amongst macroeconomic variables in industrialized economies has led to the development of dynamic stochastic general equilibrium models. The next chapter conceptualizes such analytical framework to analyse in a much more detail the South Africa data and explain the sources of its economic fluctuations. Such a model could reasonably include features that capture short run persistence in consumption, investment, wages, and prices. The following chapters show that the developed dynamic stochastic general equilibrium model fit the South Africa data quite well.

Appendix 1: The VAR statistics

Endogenous variables in the VAR: Log difference of the real GDP $(\Delta LnGDP_t)$, Log difference of the (households) consumption (ΔLnC_t), Log difference of the investment (ΔLnI_t), the trade balance to GDP ratio (TB_t / GDP_t), the inflation (π_t), the nominal interest rate (i_t), and Log difference of the exchange rate (ΔLnS_t).

Sample 1960:2-2008:4

VAR stability test: Roots of Characteristic Polynomial, lag specification 3

Root	Modulus
0.941338	0.941338
0.877319	0.877319
0.832293 + 0.119690i	0.840855
0.832293 - 0.119690i	0.840855
-0.248355 + 0.618701i	0.666687
-0.248355 - 0.618701i	0.666687
-0.165148 - 0.580823i	0.603845
-0.165148 + 0.580823i	0.603845
0.570740 - 0.174042i	0.596686
0.570740 + 0.174042i	0.596686
-0.334179 + 0.441580i	0.553777
-0.334179 - 0.441580i	0.553777
-0.489839 + 0.083234i	0.496860
-0.489839 - 0.083234i	0.496860
0.184008 - 0.402541i	0.442604
0.184008 + 0.402541i	0.442604
-0.250137 + 0.341469i	0.423285
-0.250137 - 0.341469i	0.423285
0.040326 + 0.355739i	0.358017
0.040326 - 0.355739i	0.358017
-0.203336	0.203336

No root lies outside the unit circle.
VAR satisfies the stability condition.

VAR lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-3255.120	NA	3347895.	34.88898	35.00993	34.93799
1	-2707.684	1048.031	16209.43	29.55812	30.52573*	29.95020*
2	-2628.783	145.1445	11797.01*	29.23832*	31.05258	29.97346
3	-2587.537	72.78788*	12884.39	29.32125	31.98216	30.39945
4	-2556.483	52.47602	15767.51	29.51319	33.02075	30.93445
5	-2519.856	59.15190	18295.90	29.64552	33.99974	31.40985
6	-2494.793	38.59880	24221.69	29.90153	35.10241	32.00893
7	-2456.852	55.59384	28223.26	30.01980	36.06733	32.47026
8	-2421.464	49.20181	34211.20	30.16539	37.05957	32.95892

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Chapter 2
A Dynamic stochastic general equilibrium model for a developing economy

This chapter develops a simple macroeconomic model to analyse the sources of business cycle fluctuations in a developing Sub-Saharan African economy. The dynamic stochastic general equilibrium (DSGE) model built is specialised towards a small open economy. It describes a small open economy that produces a tradable good, which is consumed domestically and exported on world markets at an exogenous price since the small open economy is price taker. The model emphasizes structural characteristics that may make a developing economy more vulnerable to exogenous shocks.

The first emphasis is on financial frictions that hinder investment financing in developing economies. As forcefully documented by a widespread literature on the "credit channel of monetary policy"; Carlstrom and Fuerst (1997); Bernanke, Gertler and Gilchrist (1999); Cespedes, Chang and Velasco (2002a, b); Cook (2004); Cook and Devereux (2006); and Devereux, Lane and Xu (2006); financial frictions or financial market imperfections affect the real macroeconomic equilibrium and the transmission mechanism through which exogenous shocks channel to the real economy. Specifically, Krugman (1999), Aghion, Bacchetta and Benerjee (2001) argue that foreign interest rate and exchange rate fluctuations have large impacts on borrowers' real net worth positions in developing countries, and through the balance sheet constraint that affects investment spending, have larger effects on the real macroeconomic equilibrium than in industrial economies. To understand financial frictions in a developing Sub-

Saharan African economy, we analyze the "financial accelerator" in such an economy by exploring the role of collateral constraint associated with investment financing and quantitatively assess the dynamic properties of the economy under this feature.

A second relevant feature for a developing economy is the exchange rate pass-through, or the speed at which exchange rate shocks feed into the domestic price level. Calvo and Reinhard (2002), Choudhri and Hakura (2006), and Devereux and Yetman (2005) document that exchange rate shocks tend to feed into the aggregate inflation at a much faster pace in developing economies than industrial economies. Engel (1999) pushes the idea further and provides substantive evidence that deviations from the law of one price determine the real exchange rate in developing economies. The DSGE model built in this chapter takes these findings into account and introduces an incomplete exchange rate pass-through to capture the underlying implication of such a feature to the dynamic process a developing Sub-Saharan African economy follows as it adjusts to exogenous shocks.

This chapter's dynamic stochastic general equilibrium (DSGE) model also embodies a large number of real and nominal frictions common to the new Keynesian DSGE literature and necessary to capture the empirical persistence exhibited by macroeconomic data. Furthermore, the model introduces a larger number of structural shocks to evaluate their relative contributions to business cycle fluctuations in developing Sub-Saharan African economies.

As in McCallum and Nelson (1999), Fuhrer (2000a, b), the external habit formation in consumption is integrated to capture the empirical persistence in consumption. The model also incorporates a variable capital utilization rate, which tends to smooth the adjustment of the rental rate in response to variation in output. Investment is subject to internal adjustment costs that create a hump-shaped response of the aggregate demand. Following Christiano,

Eichenbaum and Evans (2005); Smets and Wouters (2003, 2007); Adolfson, Laséen, Lindé and Villani (2007); and Christiano, Motto and Rostagno (2007); these costs are expressed as a function of the change in the investment, rather than the level as is frequently done in the literature. This specification of the investment adjustment costs results from the assumption that costs of adjusting the capital stock are mostly expressed in term of consumption goods in developing economies. Domestic prices and wages exhibit nominal rigidities with sticky prices and wages that adjust according to a Calvo (1983) mechanism. A partial indexation is introduced on the prices and wages that are unable to reoptimize following Smets and Wouters (2003, 2007); or Christiano, Eichenbaum and Evans (2005). The indexation generates a general dynamic inflation equation that depends on the past and future inflation. The relative importance of these frictions in explaining business cycle fluctuations in the specific case of the South African economy is empirically assessed in chapter 4 through the sensitivity analysis. Since the DSGE model is estimated with Bayesian techniques, the marginal likelihood can be interpreted as a summary statistic of in-sample model goodness, which serves to compare different models and judge the benefit of a model complexity. Practically, we examine the contribution of each of the frictions to the marginal likelihood by estimating the model under different assumptions regarding parameters related to frictions and determining whether those parameters impose penalties to the model likelihood.

Seven structural shocks are introduced to various structural equations. A productivity shock is associated with the production function and a "cost-push shock" with the firm's mark up in the goods production sector. A financial-market shock originates in the integrated domestic and foreign bonds markets and affects both the premium on the return to debt denominated on domestic currency and the foreign interest rate. Two additional shocks arise

from the external sector: the terms of trade shock and the export demand shock. Finally, the model integrates two policies shocks, a monetary policy shock as an innovation to the monetary authority's instrument, and a government spending shock impinging the aggregate demand.

Six major agents operate within the home economy: households, goods production firms, capital good producing firms, entrepreneurs, importing firms, and the government. Households consume and supply labour to producing firms in return to wages. Producing firms use the capital services rented out by entrepreneurs and the labour supplied by households and entrepreneurs to produce domestic goods consumed locally and exported on world market. Capital producing firms produce and sell capital goods to entrepreneurs on competitive markets. Entrepreneurs manage the production of producing firms and finance the acquisition of capital goods that yield the capital services used in production. Entrepreneurs must borrow from foreign lenders, and due to imperfections in capital markets, entrepreneurs are subject to collateral constraints in investment financing such that their demand of capital goods depends upon their net worth positions. There are two types of importing firms. One type of firm imports a homogenous good from the world market and turns it into multiple differentiated goods through a "differentiating technology" or brand naming. Another type of firm buys and packages those differentiated imported goods into a final imported good, and then sells it to households and capital good producing firms on domestically competitive good markets. The government sets the monetary and fiscal policies of the economy.

The rest of this chapter derives the equilibrium relations based on the optimizing behaviours of those agents. Section 1 describes the households sector. Section 2 develops the equilibrium of good producing firms. Section 3 discusses the capital good firms sector. Section 4 presents the entrepreneur sector. Section 5 sets out the government policies and section 6

depicts the external sector. Fourth appendices elaborate the details on households' consumption, entrepreneur external finance premium, balance of payments, and model log-linearization.

2.1- Households

A continuum of households distributed on the unit interval populates the economy. The representative household has preferences for consumption and leisure, and it maximises an intertemporal utility function in form:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t - Z_t, H_t\right), \tag{2.1}$$

where $U\left(C_t - Z_t, H_t\right) = \frac{\left(C_t - hC_{t-1}\right)^{1-\phi}}{1-\phi} - \frac{H_t^{1+\eta}}{1+\eta}$.

E_0 indicates the mathematical expectations operator conditional on the information set available at time 0. The discount factor β is subject to the constraint that $0 < \beta < 1$. The instantaneous utility function is additively separable in consumption and leisure, and the function $U(\bullet)$ is strictly concave and continuously differentiable. C_t stands for the representative household's composite consumption index. Z_t denotes the external habit formation in consumption level, and Z_t captures the persistence in consumption (see, e.g. Abel 1990, McCallum and Nelson 1999 and Fuhrer 2000a, b). Z_t is unaffected by any household current decision and is proportional to the aggregate past consumption level[13]. We define $Z_t = hC_{t-1}$ where $0 < h < 1$.

[13] In this model, the formulation of the habit formation depends upon a lagged aggregate consumption. This approach is known as the "catching up with the Joneses" effect (Abel, 1990). Catching up with the Joneses exists "if the others consume more today, you yourself will experience a higher marginal utility from an additional unit of consumption in the future" (Ljungqvist and Uhlig, 2000 p. 356). Gali (1994) explores an

The household total available time is normalized to one and allocated between work and leisure. The household's leisure is given by $l_t = 1 - H_t$, where H_t denotes the household hours worked. The parameter ϕ is the coefficient of the relative risk aversion or the inverse of the elasticity of intertemporal substitution in consumption, while the parameter η is the inverse of the elasticity of work effort with respect to the real wage.

The household's composite consumption index C_t is a nested constant elasticity of substitution (CES), which combines the household's consumption of domestically (home) produced goods C_{Nt} and imported (foreign traded) goods C_{Mt},

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \quad \gamma \in (0,1) \text{ and } \theta > 0. \tag{2.2}$$

γ is the share of domestically produced goods in the consumer price index (CPI) P_t, and θ is the elasticity of intratemporal substitution between home produced goods and imported goods.

Households can hold interest rate bearing debt B_t denominated in domestic currency at an exogenous gross interest rate $\varepsilon_t^B(1+i_t)$. As in Smets and Wouters (2007), ε_t^B is an exogenous premium on the gross return to debt denominated in domestic currency versus the risk free interest rate i_t controlled by the central bank. The exogenous premium characterizes the risk premium that households require to hold the one period government bonds, or it reflects the aggregate imperfections and inefficiencies into domestic financial markets. These aggregate imperfections and inefficiencies in the domestic financial market arise from asymmetric information (adverse selection, moral hazard, monitoring costs, agency costs)[14]. They may affect the nature of financial contracts households receive, widen the wedge between the cost of

alternative approach in which the utility function captures the effect of "keeping up with the Joneses". In that formulation, the household's preferences depend on the current consumption level.

[14] See Walsh (2003) chapter 7 for more details.

internal and external financing, or induce a rationing equilibrium whereby only a limited number of agents have access to external financing. The household risk premium is treated as a stochastic process that follows a first-order autoregressive process given by

$$\ln \varepsilon_t^B = (1-\rho_B)\ln \varepsilon^B + \rho_B \ln \varepsilon_{t-1}^B + \eta_t^B, \tag{2.3}$$

where $0 \le \rho_B < 1$, and the structural financial-market-shock η_t^B is a serially uncorrelated, independently, and identically distributed such that $\eta_t^B \sim N\left(0, \sigma_B^2\right)$.

Households also have access to international bonds markets. They can borrow or lend an amount D_t denominated in foreign currency at a fixed interest rate i_t^*, which is the foreign interest rate. Borrowing and lending in foreign currency are subject to small portfolio adjustment costs measured in term of the composite consumption goods. These portfolio adjustment costs in real terms are represented by:

$$C^D(D_t) = \frac{1}{2}\psi_D\left(D_t - \overline{D}\right)^2, \tag{2.4}$$

while $\overline{D}$ is the household's stationary level of net foreign (real) debt and ψ_D is the portfolio adjustment cost parameter. As discussed in Schmitt-Grohe´ and Uribe (2003), and emphasized by Devereux, Lane and Xu (2006), Benigno and Thoenissen (2007), portfolio adjustment costs on foreign borrowing and lending eliminate the unit root on net foreign assets and ensure their convergence to an exogenous steady state level $\overline{D}$. The portfolio adjustment costs implement the transversality condition of the households' foreign currency denominated debt.

Since the domestic bonds market and the international bonds market are integrated, the structural financial-market-shock drives the (gross) foreign interest rate, which is assumed to follow an ARMA (1, 1) stochastic process given by

$$\ln\left(1+i_t^*\right) = (1-\rho_R)\ln\left(1+i_{ss}^*\right) + \rho_R \ln\left(1+i_{t-1}^*\right) + \eta_t^B - \mu_R \eta_{t-1}^B \tag{2.5}$$

The MA term captures the frequency of the financial-market-shock on the foreign interest rate.

Households own monopolistically competitive firms and receive all the profits Π_t. Capital goods producing firms operate on competitive markets, their profits are zero. A government lump sum taxes (or subsidizes) T_t is levied (or allocated) to households each period t. The flow of real incomes the representative household receives from supplying labour to intermediate goods firms is given by $W_{Ht}H_t$, where W_{Ht} is the real wage desired by households. The representative household's real budget constraint is given by

$$C_t + \frac{B_t}{P_t} + S_t D_t + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2$$

$$\leq W_{Ht}H_t - T_t + \prod_t + \varepsilon_{t-1}^B\left(1+i_{t-1}\right)\frac{B_{t-1}}{P_t} + \left(1+i_{t-1}^*\right)S_t D_{t-1}. \qquad (2.6)$$

Here S_t is the nominal exchange rate defined as the price of the foreign currency unit in term of the domestic currency.

Equation (2.6) expresses a conventional budget constraint, cash expenditure on consumption, new lending on domestic and/or international financial markets, and portfolio adjustment costs must not exceed cash revenue from labour supply, transfer from the government, profits from monopolistically competitive firms, and repayment from last period loans.

The representative household solves its decision problem sequentially in two stages. First, regardless of the level of the composite consumption C_t, the household optimally purchases the combination of domestically produced goods C_{Nt} and imported goods C_{Mt} that minimizes the cost of achieving the level C_t. Second, the household chooses consumption C_t, domestic currency denominated debt holding B_t, foreign currency denominated debt holding D_t,

and leisure that maximize (2.1) subject to (2.6). We assume that the stochastic process ε_t^B is known at the time the household solves its optimal plan.

Dealing first with the problem of minimizing the cost of buying C_t, the consumer's demand function for domestically produced goods C_{Nt} is given by[15]

$$C_{Nt} = \gamma\left(\frac{P_{Nt}}{P_t}\right)^{-\theta} C_t, \tag{2.7}$$

where P_{Nt} is the price of the domestically produced goods. The demand for imported goods is expressed as

$$C_{Mt} = (1-\gamma)\left(\frac{P_{Mt}}{P_t}\right)^{-\theta} C_t, \tag{2.8}$$

where P_{Mt} denotes the price of the imported goods. The consumer price index is given by

$$P_t = \left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.9}$$

The second stage of the household's decision problem consists of maximizing the intertemporal utility function (2.1) subject to the resource constraint (2.6). Hence, the household solves the following optimization problem:

$$Max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t - hC_{t-1}\right)^{1-\phi}}{1-\phi} - \frac{H_t^{1+\eta}}{1+\eta}\right],$$

subject to the budget constraint:

$$C_t + \frac{B_t}{P_t} + S_t D_t + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2$$

$$\leq W_{Ht} H_t - T_t + \prod_t + \varepsilon_{t-1}^B \left(1 + i_{t-1}\right)\frac{B_{t-1}}{P_t} + \left(1 + i_{t-1}^*\right) S_t D_{t-1}.$$

[15] Appendix 2 associated to this chapter derives the demand equations for the domestically produced goods and imported goods.

The Lagrangian associated with this household's decision problem is given by the expression

$$L_t = E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\left(C_{t+s} - hC_{t+s-1}\right)^{1-\phi}}{1-\phi} - \frac{H_{t+s}^{1+\eta}}{1+\eta} + \Xi_{t+s} \left(\begin{array}{r} W_{Ht+s} H_{t+s} - T_{t+s} + \Pi_{t+s} + \varepsilon_{t+s-1}^{B} \left(1+i_{t+s-1}\right) \frac{B_{t+s-1}}{P_{t+s}} + \left(1+i_{t+s-1}^{*}\right) S_{t+s} D_{t+s-1} \\ - C_{t+s} - \frac{B_{t+s}}{P_{t+s}} - S_{t+s} D_{t+s} - \frac{\psi_D}{2} \left(D_{t+s} - \overline{D}\right)^2 \end{array} \right) \right],$$

where Ξ_t is the Lagrangian multiplier.

The first order necessary condition with respect to consumption is $\left(C_t - hC_{t-1}\right)^{-\phi} - \Xi_t = 0$. And the first order necessary condition with respect to bonds denominated in domestic currency is given by

$$-\Xi_t \left(\frac{1}{P_t}\right) + \beta E_t \left[\varepsilon_t^B \left(1+i_t\right) \left(\frac{1}{P_{t+1}}\right) \Xi_{t+1} \right] = 0.$$

Combining these two equations, we obtain the household's optimal consumption-saving allocation given by:

$$\left(C_t - hC_{t-1}\right)^{-\phi} = \beta E_t \left[\varepsilon_t^B (1+i_t) \frac{P_t}{P_{t+1}} \left(C_{t+1} - hC_t\right)^{-\phi} \right]. \tag{2.10}$$

Equation (2.10) is a classical Euler equation for the optimal intertemporal allocation of consumption and saving. The dynamic of the household's aggregate consumption explicitly depends on external habit formation. When there is no external habit formation in consumption, hence $h = 0$, equation (2.10) reduces to the traditional forward-looking consumption equation (see for example Walsh 2003, equation (5.67) page 244, or equation (10.37) in Wickens (2008) page 245). With external habit formation in household preferences, the percentage deviation of consumption[16] from its steady state level depends on a weighted average of the past and future consumption.

[16] See linearized form in the appendix 5, equation (A4.4).

The real interest rate also affects the consumption. The interest rate elasticity of consumption depends on both the intertemporal elasticity of substitution and the habit persistence parameter. A low degree of persistence tends to increase the marginal impact of the real interest rate on consumption at a given elasticity of substitution. The last factor that affects the consumption is the premium ε_t^B, which corresponds to the wedge between the risk free interest rate controlled by the central bank and the return on assets held by the households. The sensitivity of consumption to the premium ε_t^B is equal to the interest rate elasticity of consumption. Thus, a positive shock to that wedge increases the return on assets held by the households and reduces the consumption. It is worth observing that the exogenous risk premium induces on consumption similar qualitative and quantitative implications as the preference shock (see Smets and Wouters, 2003)[17].

The first order necessary condition with respect to bonds denominated in foreign currency is given by

$$-\Xi_t\left[S_t+\psi_D\left(D_t-\overline{D}\right)\right]+\beta E_t\left[\Xi_{t+1}\left(1+i_t^*\right)S_{t+1}\right]=0\,.$$

From the first order necessary condition with respect to bonds denominated in domestic currency we can obtain the relation $E_t\left(\frac{\Xi_t}{\Xi_{t+1}}\right)=\beta E_t\left(\varepsilon_t^B\left(1+i_t\right)\frac{P_t}{P_{t+1}}\right)$, which one can use to simplify the optimal choice of bonds denominated in foreign currency and obtains

$$E_t\left[\varepsilon_t^B(1+i_t)\frac{P_t}{P_{t+1}}\left(1+\frac{\psi_D}{S_t}\left(D_t-\overline{D}\right)\right)\right]=E_t\left[(1+i_t^*)\frac{S_{t+1}}{S_t}\right]. \qquad (2.11)$$

Equation (2.11) provides the interest rate parity condition for households' choice of domestic currency denominated debt and foreign currency denominated debt. In the

[17] One more precision, Smets and Wouters (2003) introduce a preference shock to the household utility function such that both the present value and the expected value of the preference shock affect consumption. So, their model allows for the combining effect of the present and the expected value of the preference disturbance on consumption.

equilibrium, both domestic currency denominated debt and foreign currency denominated debt must have the same expected return.

Equation (2.11) differs from the standard uncovered interest parity condition (see Wickens 2008, equation (11.37) page 281) on international bond market in two major points. First, it integrates a stochastic process ε_t^B as an entire component. Second, it takes into account the effect of portfolio adjustment costs on the interest rate differential. The latter affect the parity condition through the factor $\left(1+\frac{\psi_D}{S_t}\left(D_{t+1}-\overline{D}\right)\right)$, which introduces a wedge in the interest rate differential and therefore constitutes a finance premium. For comparison purpose, the factor $\left(1+\frac{\psi_D}{S_t}\left(D_{t+1}-\overline{D}\right)\right)$ has the same properties as the cost function multiplicatively introduced to the gross foreign interest rate in Benigno and Thoenissen (2007) [18]. Particularly, it equals to unity when the net foreign asset matches its steady state level, and it is a differentiable and decreasing function near $\overline{D}$. Thus, when the foreign debt is greater (less) than its stationary level, the foreign lender exacts and international premium (discount). In general, one can follow Benigno and Thoenissen (2007), or Benigno (2001), who rationalize portfolio adjustment costs by assuming the presence of foreign-owned intermediaries in the foreign debt market who apply a spread over the risk-free foreign interest rate when borrowing or lending to home agents in foreign currency.

Equation (2.11) also provides some helpful insights on international bond market, specifically on the integration between domestic and foreign bonds markets. The stochastic process, induced by aggregate financial imperfections on the domestic credit market, not only translates to a wedge between the risk free (domestic) interest rate controlled by the central bank

[18] Note that Benigno and Thoenissen (2007) express the domestic and foreign bonds on a discounted basis.

and the return on assets held by the households, but also it widens the interest rate differential between home and abroad. A positive financial market shock then raises the expected real interest rate differential between the domestic economy and the rest of the world economy, which increases capital inflows and appreciates the domestic currency (decrease in S_t). A solution for the spot exchange rate can be derived by performing a forward-looking iteration of equation (2.11). That forward-looking solution shows that the exchange rate responds instantaneously and negatively to expected premium shocks and all new information about expected future real interest rate differentials. Eventually, the initial appreciation of the domestic currency following a financial market shock induces its future expected depreciation (increase in $\hat{S}_{t+1}$) as the equilibrium in the international household debt market needs to be restored.

Before presenting the wage dynamic and the households' decision rule on labour supply, more details on the labour market are needed. Households supply their homogenous labour to intermediate labour unions, which differentiate the labour and set wages according to a Calvo (1983) mechanism. The households' labour H_t used by intermediate goods producers is a composite given by

$$H_t = \left[\int_0^1 H_{jt}^{\frac{1}{1+\varepsilon^w}} dj \right]^{1+\varepsilon^w} . \qquad (2.12)$$

There are "labour packers" who buy labour from the labour unions, package H_t, and resell it to intermediate goods producers. Labour packers operate in a perfectly competitive labour market in which they maximize their profits $W_t H_t - \int_0^1 W_{jt} H_{jt} dj$ subject to the composite

labour equation (2.12). The first order condition from the labour parkers' optimization problem yields the labour demand

$$H_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\frac{1+\varepsilon^w}{\varepsilon^w}} H_t . \tag{2.13}$$

Combining this equality with the zero profit condition of the labour parkers gives an expression for the wage cost for the intermediate goods producers

$$W_t = \left[\int_0^1 W_{jt}^{-\frac{1}{\varepsilon^w}} dj \right]^{-\varepsilon^w} . \tag{2.14}$$

The labour unions are intermediates between the households and the labour packers; they have market power, and the can choose the wage subject to the labour demand equation (2.13). The markup above the marginal disutility is distributed to the households. The profits Π_t in the households' budget constraint, equation (2.6), also contain the dividends from the labour unions distributed to the households.

The first order necessary condition for households' labour supply is given by:

$$-H_t^{\eta} + \left(C_t - hC_{t-1}\right)^{-\phi} W_{Ht} = 0 . \tag{2.15}$$

The intratemporal optimality condition (2.15) establishes the labour-leisure trade-off. The marginal utility of leisure equals the marginal utility of consumption times the real wage desired by households. In others words, at the optimum, households set their labour supply by equating the marginal rate of substitution between leisure and consumption to their desired real wage. A high degree of habit persistence will tend to increase the impact of consumption on hours worked given the elasticity of intertemporal substitution and the elasticity of the work effort with respect to the real wage.

Labour unions take this marginal rate of substitution as the cost of labour service in the negotiations with the labour packers. The labour unions are subject to nominal rigidities; each period, they can adjust wages following a Calvo (1983) mechanism of adjustment with probability $1-\xi_w$. For those labour unions that can set wages optimally, the problem consists of choosing $\tilde{W}_{jt}$ that maximizes the labour incomes in all states of nature they are stuck with that wage in the future. For the remaining ξ_w labour unions that cannot adjust wage at a period $t+s$, wages increase at the labour augmenting deterministic growth rate g and the weighted average of the steady state (gross) inflation π_* and the last period inflation such that their wage is given by $W_{jt+s} = g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w} \pi_*^{1-l_w} \tilde{W}_{jt}$. l_w is the degree of wage indexation in the small open economy.

The maximization problem by optimizing labour unions is given by

$$\underset{\tilde{W}_{jt}}{Max}\, E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \frac{P_t \Xi_{t+s}}{P_{t+s}\Xi_t} \left[g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w} \pi_*^{1-l_w} \tilde{W}_{jt} - W_{Ht+s} \right] H_{jt+s}$$

subject to the labour demand

$$H_{jt+s} = \left(\frac{g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w} \pi_*^{1-l_w} \tilde{W}_{jt}}{W_{t+s}} \right)^{-\frac{1+\varepsilon^w}{\varepsilon^w}} H_{t+s}.$$

The first order necessary condition for this optimization problem is given by

$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \frac{P_t \Xi_{t+s}}{P_{t+s}\Xi_t} H_{jt+s} \left[\left(\frac{1+\varepsilon^w}{\varepsilon^w}\right) W_{Ht+s} - \frac{1}{\varepsilon^w} g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w} \pi_*^{1-l_w} \tilde{W}_{jt} \right] = 0$, which yields the solution

$$\tilde{W}_{jt} = \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \frac{P_t \Xi_{t+s}}{P_{t+s} \Xi_t} H_{jt+s} \frac{1+\varepsilon^w}{\varepsilon^w} W_{Ht+s}}{E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \frac{P_t \Xi_{t+s}}{P_{t+s} \Xi_t} H_{jt+s} \frac{1}{\varepsilon^w} g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w} \pi_*^{1-l_w}} . \tag{2.16}$$

Each period t, the $1-\xi_w$ optimizing labour unions choose the wage $\tilde{W}_{jt}$; the remaining fraction ξ_w adjusts wage to $g\left(\frac{P_{t-1}}{P_{t-2}}\right)^{l_w} \pi_*^{1-l_w} W_{t-1}$. The aggregate wage expression is then given by

$$W_t^{-\frac{1}{\varepsilon^w}} = \xi_w \left(g\left(\frac{P_{t-1}}{P_{t-2}}\right)^{l_w} \pi_*^{1-l_w} W_{t-1} \right)^{-\frac{1}{\varepsilon^w}} + (1-\xi_w) \tilde{W}_t^{-\frac{1}{\varepsilon^w}} . \tag{2.17}$$

Equations (2.7), (2.8), (2.9), (2.10), and (2.11) determine the household's decision rules on consumption quantities, consumer price index, and debts. Equations (2.16) and (2.17) give the household wage dynamic. The theorem of the envelope $\Xi_t = U_c(C_t - Z_t, H_t) = V_a(a_t)$, which sets the equality between the marginal utility of consumption Ξ_t and the marginal value of the household's financial wealth, and the transversality conditions $\lim_{t\to\infty} \beta^t \Xi_t B_t = 0$, complete the household's equilibrium. In the next section, we describe the goods production sector.

2.2- Goods production firms

As in Chari, Kehoe and McGrattan (2000) closed economy; final goods producers operating in a competitive goods market and monopolistic competition amongst intermediate goods producers characterize the goods production sector. This imperfectly competitive market structure, with intermediate goods producers producing differentiated goods, permits the

introduction of nominal rigidities in the form of price stickiness while motivates an important role to monetary policy.

Imperfect competition might generate an inefficiently low equilibrium output, multiple equilibria (Ball and Romer 1991, Rotemberg and Woodford 1995), or aggregate demand externalities (Blanchard and Kiyotaki 1987). Nevertheless, imperfect competition does not permit monetary policy to be neutral in the short-run. If all firms are able to adjust their prices one-time, monetary policy shocks will induce proportional changes in all prices without effects on the real equilibrium.

To add price stickiness in this model, we assume that intermediate goods producers engage into a multi-periods staggered price setting process that follows a Calvo (1983) mechanism of adjustment. In that process, a random fraction of intermediate goods firms of size $1-\xi_n$ optimize their prices each period t. The remaining ξ_n of all intermediate goods firms that cannot optimize prices indexes prices according to a scheme that depends on the past inflation.

Our starting point in this section explores the equilibrium conditions of the final goods producer, and then follows the intermediate goods producers' decision problem.

2.2.1 The final goods producer

The final good Y_t is a composite good produced by final goods producers. More specifically, Y_t is a continuum of intermediate goods Y_{it} buy on the market and package according to an aggregator function Ψ. The general form of Ψ is that of the flexible aggregator in Kimball (1995) neo-monetarist model,

$$1 = \int_0^1 \psi\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right) di \ . \tag{2.18}$$

The index *i* identifies the intermediate good producer that produces Y_{it}; ε_t^P is a stochastic process that channels exogenous shocks to the aggregator function Ψ. The general aggregator function Ψ satisfies the conditions $\Psi(1)=1$. In addition, Ψ is an increasing function ($\Psi'(a)>0$), strictly concave ($\Psi''(a)<0$), and symmetric on intermediate goods such that the final good Y_t is determined in a symmetric fashion.

Shocks to the general aggregator function change the elasticity of the demand for Y_{it} and the markup of the producer *i*. The conventional interpretation of these shocks is that of a "cost-push shocks" (Clarida, Gali and Getler 1999; Smets and Wouters 2003, 2007) to the inflation dynamic. To clarify that concept of cost-push shock and explicitly elucidate the transmission channels through which those shocks affect the inflation and generate economic fluctuations, we follow two steps. First, we solve the final goods producers' problem, and then we use the subsequent decision rules to analyze the relations between ε_t^P and both the desired mark up and the relative prices.

2.2.1.1 The final goods producers' problem

Final goods producers operate into a competitive goods market. Their wealth-maximization problem consists of maximizing the profits given by

$$P_{Nt}Y_t - \int_0^1 P_{it}Y_{it}di\,, \tag{2.19}$$

where the maximization is carried over (Y_t, Y_{it}) and is subject to the constraints imposed by (2.18). P_{Nt} is the final goods price, and P_{it} is the intermediate good *i*'s price.

The solution to the final goods producers' problem is generally obtained by using the Lagrangian optimization method. Let λ_t be the Lagrangian multiplier associated with the

constraint (2.18), the Lagrangian for the final good firm's maximization problem is characterized by

$$L_t = P_{Nt}Y_t - \int_0^1 P_{it}Y_{it}di + \lambda_t\left[\int_0^1 \Psi\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)di - 1\right].$$

The first order necessary conditions with respect to Y_t and Y_{it} are the following

$\frac{\partial L_t}{\partial Y_t} = P_{Nt} + \lambda_t \int_0^1 \Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)\left(\frac{-Y_{it}}{Y_t^2}\right)di = 0$, which can be rewritten as

$$P_{Nt} = \frac{\lambda_t}{Y_t}\int_0^1 \Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)\left(\frac{Y_{it}}{Y_t}\right)di; \tag{2.20}$$

and $\frac{\partial L_t}{\partial Y_{it}} = -P_{it} + \frac{\lambda_t}{Y_t}\Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right) = 0$, which is equivalent to

$$P_{it} = \frac{\lambda_t}{Y_t}\Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right). \tag{2.21}$$

We can divide side by side equations (2.20) and (2.21), and we obtain

$$\frac{P_{Nt}}{P_{it}} = \frac{\int_0^1 \Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)\left(\frac{Y_{it}}{Y_t}\right)di}{\Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)}.$$

After some straightforward arrangements, this result takes the following simplified form

$\Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right) = \frac{P_{it}}{P_{Nt}}\int_0^1 \Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)\left(\frac{Y_{it}}{Y_t}\right)di$. The demand curve for the input Y_{it} is given by the expression

$$Y_{it} = Y_t\Psi'^{-1}\left[\frac{P_{it}}{P_{Nt}}\int_0^1 \Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)\left(\frac{Y_{it}}{Y_t}\right)di\right]. \tag{2.22}$$

As in Kimball (1995), the properties of the aggregator function ensure that the demand for the intermediate good Y_{it} is a decreasing function of the relative price[19] $\frac{P_{Nt}}{P_{it}}$. The competitive

[19] To prove that the demand for the intermediate good is a decreasing function on the relative price; first, recall that $\Psi''(x) < 0$

market structure for the final goods firms imposes a zero profit condition to the expression (2.19); that condition helps to derive the aggregate price P_{Nt}, which has the expression

$$P_{Nt} = \int_0^1 P_{it} \Psi'^{-1} \left[\frac{P_{it}}{P_{Nt}} \int_0^1 \Psi' \left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P \right) \left(\frac{Y_{it}}{Y_t} \right) di \right] di \,. \tag{2.23}$$

Several aggregator functions are used in the literature. The original functional forms, tracing back to Dixit and Stiglitz (1977), give emphasis to a constant elasticity of substitution between varieties. Recent developments in the DSGE literature tend to advocate a time varying elasticity of substitution through which exogenous shocks impinge into the aggregate function. For example, Christiano, Motto and Rostagno (2007); Smets and Wouters (2003, 2007); Del Negro, Shorfheide, Smets and Wouters (2007); and Adolfson, Laséen, Lindé and Villani (2007) propose the functional form $\Psi(x) = (x)^{\frac{1}{1+\varepsilon_t^P}}$. In this case, the final good is given by the relation

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{1+\varepsilon_t^P}} di \right]^{1+\varepsilon_t^P} . \tag{2.24}$$

ε_t^P characterizes the stochastic process through which shocks affect the aggregator function ψ. It embodies the time varying elasticity of the demand curve facing by the supplier i and is assumed to follow an ARMA (1, 1) process given by

$$\ln \varepsilon_t^P = (1-\rho_P) \ln \varepsilon^P + \rho_P \ln \varepsilon_{t-1}^P + \eta_t^P - \mu_P \eta_{t-1}^P , \tag{2.25}$$

where the "cost-push shock" η_t^P is identically and independently distributed such that $\eta_t^P \sim N(0, \sigma_P)$. As in Smets and Wouters (2007), the inclusion of a MA term in the ε_t^P process is designed to capture the high frequency fluctuations of the domestic good inflation.

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implies Ψ' is an invertible function, and Ψ'^{-1} denotes that inverse function. Second, for any positive value x one can easily write $\Psi'\left(\Psi'^{-1}(x)\right) = x$ .The derivative of this last equality yields $\Psi''\left(\Psi'^{-1}(x)\right)\left(\Psi'^{-1}(x)\right)' = 1$, which implies $\left(\Psi'^{-1}(x)\right)' = \frac{1}{\Psi''\left(\Psi'^{-1}(x)\right)}$. As $\Psi''(x) < 0$, we have $\left(\Psi'^{-1}(x)\right)' < 0$. In order words, the derivative of the function Ψ'^{-1} (which is denoted $\left(\Psi'^{-1}(x)\right)' < 0$) has a negative sign. That property notably makes Ψ'^{-1} a decreasing function on its arguments, hence, a decreasing function on the relative price.

Likewise, the demand curve for the intermediate good i, determined from the equation (2.22) of the final good producer's profit maximization problem, is given by

$$Y_{it} = \left(\frac{P_{it}}{P_{Nt}} \right)^{-\frac{1+\varepsilon_t^P}{\varepsilon_t^P}} Y_t . \tag{2.26}$$

The aggregate price index, as specified by (1.23), is given by:

$$P_{Nt} = \left[\int_0^1 P_{it}^{-\frac{1}{\varepsilon_t^P}} di \right]^{-\varepsilon_t^P} . \tag{2.27}$$

Final goods producers sell Y_t on a domestically competitive goods market to households, entrepreneurs, government, and capital producing firms; they also export a fraction of the final good on world markets at an exogenous price. The demand for domestically produced goods is given by

$$Y_t = \gamma \left(\frac{P_{Nt}}{P_t} \right)^{-\theta} \left[C_t + C_t^e + I_t + G_t + \frac{\psi_D}{2} \left(D_t - \overline{D} \right)^2 \right] + X_t , \tag{2.28}$$

where C_t^e is the entrepreneur's consumption, I_t is the domestic gross investment, G_t is the government spending, and X_t is the small economy total exports. Next, we briefly show how shocks to the aggregator function are relevant sources business cycles fluctuations.

2.2.1.2 Shocks to the aggregator function and business cycles

Shocks to the aggregator function affect the intermediate good firm's demand curve's elasticity and desired mark up. The inverse of the demand curve's elasticity is

$$\frac{1}{\theta_N \left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P \right)} = -Y_{it} \frac{d \ln(P_{it})}{dY_{it}} . \tag{2.29}$$

Equation (2.20) can be used to evaluate the right side of expression (2.29). That evaluation made, its substitution in equation (2.29) gives the expression

$$\frac{1}{\theta_N\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)} = -\frac{Y_{it}}{Y_t}\frac{\Psi''\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)}{\Psi'\left(\frac{Y_{it}}{Y_t}, \varepsilon_t^P\right)}. \tag{2.30}$$

Let $x = \frac{Y_{it}}{Y_t}$ be the intermediate good firm *i*'s relative output, the demand curve's elasticity is

$$\theta_N(x, \varepsilon_t^P) = -\frac{\Psi'\left(x, \varepsilon_t^P\right)}{x\Psi''\left(x, \varepsilon_t^P\right)}. \tag{2.31}$$

The properties of the function Ψ ($\Psi'(a) > 0$ and $\Psi''(a) < 0$) imply that the demand curve's elasticity is a positive function of the relative output, or respectively, a negative function of the relative price. Equation (2.31) forcefully shows that shocks to the aggregator function Ψ impinge on the demand curve's elasticity.

The stochastic process ε_t^P, thought shocks impinging through that process, mostly matter for two reasons. First, the desired mark up $\mu_N(x, \varepsilon_t^P) = \frac{\theta_N(x, \varepsilon_t^P)}{\theta_N(x, \varepsilon_t^P) - 1}$ applied by the intermediate good supplier *i* in calculating his desired price depends on the relative output and the stochastic process ε_t^P. Shocks to the aggregator function trigger mark up shocks that propagate to the domestically produced good firm's prices. As a result, domestic good inflation fluctuates in response to those mark-up shocks, which are indeed "cost-push shocks" (Clarida, Gali and Getler, 1999).

Second, as documented in Kimball (1995), the elasticity relates movements in relative output $\frac{Y_{it}}{Y_t}$ to movements in the relative price $\frac{P_{Nt}}{P_{it}}$. This can be observed from the log-

linearization of equation (2.20) around the steady state equilibrium where $Y_{it} = Y_t$ and $P_{it} = P_{Nt}$. The linearized version of equation (2.20) is

$$\hat{Y}_{it} - \hat{Y}_t = -\theta_N^* \left(\hat{P}_{it} - \hat{P}_{Nt} \right), \tag{2.32}$$

where a hat denotes the percentage deviation of a variable around its steady state value, and θ_N^* is the demand curve's elasticity evaluated at the steady state[20].

The implications of this approximation are the following. Shocks to the aggregator function, which also induce a time varying mark-up and demand curve's elasticity, generate output fluctuations, and consequently, they cause business cycles fluctuations. Next we present the intermediate good firm decision rules.

2.2.2 The intermediate good producer

An intermediate good firm produces Y_{it} using the technology of production:

$$Y_{it} = \varepsilon_t^Y K_{it}^{s\alpha} \left(g^t L_{it} \right)^{1-\alpha} - \Phi g^t, \text{ where } 0 < \alpha < 1. \tag{2.33}$$

ε_t^Y is the total factor productivity, which evolves according to a first order autoregressive process specified by

$$\ln \varepsilon_t^Y = (1 - \rho_Y) \ln \varepsilon^Y + \rho_Y \ln \varepsilon_{t-1}^Y + \eta_t^Y, \tag{2.34}$$

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[20] Note that if the subscript "ss" denotes the steady state value of a variable, a function $y_t = f(x_t)$ can be log-linearized around the steady state equilibrium as follows. First, the function is rewritten using the natural logarithmic function "ln" as $\ln(y_t) = \ln\left(f(e^{\ln(x_t)}) \right)$, where $x_t = e^{\ln(x_t)}$. Then, the log-linearization is performed to obtain the expression $\ln(y_t) \approx \ln\left(f(x_{ss})\right) + \frac{f'(x_{ss})x_{ss}}{f(x_{ss})}\left(\ln(x_t) - \ln(x_{ss})\right)$. The latter expression can be rewritten as $\ln\left(\frac{y_t}{y^{ss}}\right) \approx \frac{f'(x^{ss})x^{ss}}{f(x^{ss})}\left(\ln\left(\frac{x_t}{x^{ss}}\right)\right)$ using $\ln(y_{ss}) = \ln\left(f(x_{ss})\right)$. When it is recognized that $y_t = \left(1 + \hat{y}_t\right) y_{ss}$, where $\hat{y}_t$ is the percentage deviation of y_t around its steady state value y_{ss}, and that the following approximation $\ln\left(1 + \hat{y}_t\right) \approx \hat{y}_t$ holds; the log-linearized expression of the function $f(x_t)$ in term of percentage deviations around the steady state is $\hat{y}_y \approx \frac{f'(x_{ss})x_{ss}}{f(x_{ss})}\hat{x}_t$, where $\frac{f'(x_{ss})x_{ss}}{f(x_{ss})}$ is the elasticity of the function $f(x_t)$ with respect to the variable x_t.

where $0 \leq \rho_Y < 0$, and the structural productivity shock η_t^Y is identically and independently distributed, $\eta_t^Y \sim N(0, \sigma_Y)$.

Φ is the fixed cost of production, which ensures that profits are zero at the steady state. g denotes the labour-augmenting deterministic growth rate of the economy and requires to render non-stationary optimality conditions stationary prior linearization (appendix 5). L_{it} is the aggregate labour input. K_{it}^s is the capital effectively utilized by the intermediate good firm and rented out by entrepreneurs on a competitive market at a real rental rate R_{Kt}. K_{it}^s is proportional to the installed capital in the firm, the relation $K_{it}^s = u_t K_{it-1}$, where u_t is the economy capital utilization rate and K_{it-1} the firm's installed capital, connects the two quantities.

The formulation of the effective capital follows Smets and Wouters (2003, 2007). We assume that capital depreciates at a constant rate ∂, which is independent of the capital utilization rate. An alternative formulation of the effective utilization of the capital is that of Baxter and Farr (2005), or King and Rebelo (2000). The later links the current effective capital to the stock of capital K_{it} and the current capital utilization rate.

The introduction of the capital utilization rate allows for variable capital utilization. The literature built upon the applications of DSGE models to developed economies gives some insights on the role played by the capital utilization rate in business cycles. Burnside, Eichenbaum and Rebelo (1995) provide some evidence supporting that the flow of capital services increases during expansions; on the contrary, small flows of capital services characterize recessions. More importantly, variable capital utilization in dynamic equilibrium models help to eliminate counterfactual correlations that could occur otherwise. As an example, in Greenwood, Hercowitz and Huffman (1988)'s model, which characterizes the effects of new

investment productivity shocks, the incorporation of the capital utilization rate eliminates the opposite movement between consumption and investment that might otherwise occur.

Baxter and Farr (2005) forcefully show two improvements accomplished by the incorporation of the variable capital utilization in an open economy dynamic stochastic equilibrium model. First, the variable capital utilization reduces the volatility of the productivity shock necessary to match the second moments computed from artificial data generated by the model with those calculated from actual data. Second, the variable capital utilisation improves the DSGE model's ability to capture the correlations in actual data and provide realistic co-movements of international business cycles.

In addition, the variable capital utilisation in a small open developing economy DSGE model smoothes the macroeconomic adjustment to shocks. It enhances the economy capacity to respond to exogenous shocks in a market structure characterized by frictions and imperfect information. Hence, producers can vary the utilization rate in response to productivity shocks rather than change the quantities of inputs used in the production.

As in Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), or Devereux, Lane and Xu (2006), the labour input L_{it} aggregates households and entrepreneurs employments. Following these authors, we assume that entrepreneurs supplement their income by working in addition to operating firms. Furthermore, entrepreneurs supply their labour inelastically, and total entrepreneurial labour is normalized to unity. The total labour input is given by

$$L_{it} = H_{it}^{\Omega} H_{it}^{e^{1-\Omega}} \text{ , where } 0 < \Omega < 1. \tag{2.35}$$

H_{it} is the intermediate good firm's demand for households' labour, and H_{it}^{e} is the firm's entrepreneurial labour. The production function can be rewritten as follows

$$Y_{it} = \varepsilon_t^Y K_{it}^{s\alpha} \left(g^t H_{it}^{\ \Omega} H_{it}^{e1-\Omega} \right)^{1-\alpha} - g^t \Phi . \tag{2.36}$$

Next, we present the intermediate goods firms' decision problem on production factors, and then follow their price setting problem.

2.2.2.1 The intermediate goods firms' decision problem

Each period t, the intermediate good firm minimizes its total costs subject to the constraint of its technology of production. The intermediate good firm's real cost minimization problem is given by

$$\underset{K_{it}^s, H_{it}, H_{it}^e}{Min} R_{Kt} K_{it}^s + W_t H_{it} + W_t^e H_{it}^e ,$$

subject to the production technology

$$Y_{it} = \varepsilon_t^Y K_{it}^{s\alpha} \left(g^t H_{it}^{\ \Omega} H_{it}^{e1-\Omega} \right)^{1-\alpha} - g^t \Phi .$$

R_{Kt} is the real rental rate on capital services, W_t is the households real wage, and W_t^e is the entrepreneurs real wage. The labour and capital services markets are perfectly competitive; the real rental rate and wages are exogenous for each infinitesimally small intermediate good firm.

The Lagrangian associated to this minimization problem is given by the expression

$$L_t = R_{Kt} K_{it}^s + W_t H_{it} + W_t^e H_{it}^e + \Theta_{it} \left(Y_{it} - \varepsilon_t^Y K_{it}^{s\alpha} \left(g^t H_{it}^{\ \Omega} H_{it}^{e1-\Omega} \right)^{1-\alpha} + g^t \Phi \right),$$

where Θ_{it} is the period t Lagrangian multiplier.

The first order necessary conditions with respect to K_{it}^s, H_{it}, and H_{it}^e are the following:

$$R_{Kt} = \alpha g^{(1-\alpha)t} \Theta_{it} \varepsilon_t^Y K_{it}^{s\alpha-1} \left(H_{it}^{\ \Omega} H_{it}^{e1-\Omega} \right)^{1-\alpha} , \tag{2.37}$$

$$W_t = \Omega(1-\alpha) g^{(1-\alpha)t} \Theta_{it} \varepsilon_t^Y K_{it}^{s\alpha} \left(H_{it}^{\ \Omega} H_{it}^{e1-\Omega} \right)^{-\alpha} H_{it}^{\ \Omega-1} H_{it}^{e1-\Omega} , \tag{2.38}$$

$$W_t^e = (1-\Omega)(1-\alpha)g^{(1-\alpha)t}\Theta_{it}\varepsilon_t^Y K_{it}^{s\alpha}\left(H_{it}^{\ \Omega}H_{it}^{e^{1-\Omega}}\right)^{-\alpha} H_{it}^{\ \Omega}H_{it}^{e^{-\Omega}}. \quad (2.39)$$

The next developments use these first order necessary conditions to determine the firm's real marginal cost.

To eliminate the Lagrangian multiplier from the two first-order conditions, we divide equations (2.37) and (2.38) side by side and obtain the expression:

$\frac{R_{Kt}}{W_t} = \frac{\alpha}{\Omega(1-\alpha)}\frac{H_{it}}{K_{it}^s}$, which can be rewritten as

$$H_{it} = \frac{\Omega(1-\alpha)}{\alpha}\frac{R_{Kt}}{W_t}K_{it}^s. \quad (2.40)$$

Idem, dividing equations (2.38) and (2.39) side by side to eliminate the Lagrangian multiplier gives

$$\frac{W_t}{W_t^e} = \frac{\Omega}{1-\Omega}\frac{H_{it}^e}{H_{it}},$$

or

$$H_{it}^e = \frac{1-\Omega}{\Omega}\frac{W_t}{W_t^e}H_{it}. \quad (2.41)$$

Using expression (2.41) to substitute for the entrepreneurial labour in the production function, we obtain

$$Y_{it} + g^t\Phi = \varepsilon_t^Y K_{it}^{s\alpha}\left(g^t H_{it}\left(\frac{1-\Omega}{\Omega}\frac{W_t}{W_t^e}\right)^{1-\Omega}\right)^{1-\alpha}, \quad \text{or}$$

$$Y_{it} + g^t\Phi = \varepsilon_t^Y g^{(1-\alpha)t} K_{it}^{s\alpha} H_{it}^{\ 1-\alpha}\left(\frac{1-\Omega}{\Omega}\frac{W_t}{W_t^e}\right)^{(1-\alpha)(1-\Omega)}.$$

Next, introducing into this latest expression the households labour obtained in equation (2.40) gives

$$Y_{it} + g^t\Phi = \varepsilon_t^Y g^{(1-\alpha)t} K_{it}^s \left(\frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} \right)^{1-\alpha} \left(\frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} \right)^{(1-\alpha)(1-\Omega)} .$$

Now, we draw an expression for the intermediate good firm *i*'s demand for capital, which is written as

$$K_{it}^s = \frac{\left(Y_{it} + g^t\Phi\right)}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} \right)^{\alpha-1} \left(\frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} \right)^{-(1-\alpha)(1-\Omega)} .$$

We can use this expression of the firm's demand for capital to substitute K_{it}^s in equation (2.40) and obtain the firm's demand for households labour

$$H_{it} = \frac{\left(Y_{it} + g^t\Phi\right)}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} \right)^{\alpha} \left(\frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} \right)^{-(1-\alpha)(1-\Omega)} ,$$

which in turn is used in equation (2.41) to get the firm's demand for entrepreneurs labour,

$$H_{it}^e = \frac{\left(Y_{it} + g^t\Phi\right)}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} \right)^{\alpha} \left(\frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} \right)^{1-(1-\alpha)(1-\Omega)} .$$

Substituting factors demands into the relation $R_{Kt}K_{it}^s + W_t H_{it} + W_t^e H_{it}^e$, which gives the intermediate good firm's real total cost, leads to the expression

$$\frac{\left(Y_{it} + g^t\Phi\right)}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} \right)^{\alpha} \left(\frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} \right)^{-(1-\alpha)(1-\Omega)} \left(W_t \frac{1}{\Omega(1-\alpha)} \right).$$

After some arrangements, the intermediate good firm's real total cost is given by

$$\frac{1}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{R_{Kt}}{\alpha} \right)^{\alpha} \left(\frac{W_t}{\Omega(1-\alpha)} \right)^{\Omega(1-\alpha)} \left(\frac{W_t^e}{(1-\Omega)(1-\alpha)} \right)^{(1-\alpha)(1-\Omega)} \left(Y_{it} + g^t\Phi\right), \qquad (2.42)$$

the real marginal unit cost is expressed by

$$MC_t = \frac{1}{\varepsilon_t^Y g^{(1-\alpha)t}} \left(\frac{R_{Kt}}{\alpha} \right)^{\alpha} \left(\frac{W_t}{\Omega(1-\alpha)} \right)^{\Omega(1-\alpha)} \left(\frac{W_t^e}{(1-\Omega)(1-\alpha)} \right)^{(1-\alpha)(1-\Omega)} . \qquad (2.43)$$

The real marginal cost is independent on the intermediate good producer's characteristics. It depends exclusively on the input prices and the production function parameters. Since real wages, rental rates, and technology are identical for all intermediate goods firms; and at any period t, the real marginal costs are the same across intermediate goods firms. The equilibrium is symmetric.

In this symmetric equilibrium, all intermediate goods firms choose the same labour inputs and capital services. Therefore, the capital-labour ratio is also identical across intermediate goods firms, and firms produce at the same level. We can drop the index i from the relations (2.40) and (2.41), and rewrite these expressions as follows:

$$H_t = \frac{\Omega(1-\alpha)}{\alpha} \frac{R_{Kt}}{W_t} K_t^s, \tag{2.44}$$

and

$$H_t^e = \frac{1-\Omega}{\Omega} \frac{W_t}{W_t^e} H_t. \tag{2.45}$$

Equations (2.44) and (2.45) determine the intermediate goods firms' equilibrium decision rules on production factors. The real marginal cost equation (2.43) and the production function (2.36) evaluated at the symmetric equilibrium are two other intermediate goods firms' decision rules. The rest of the intermediate goods firms' decision rules are derived from the price-setting problem.

2.2.2.2 The intermediate goods firms' price setting problem

Monopolistic competition provides intermediate goods firms with market power. Following a Calvo (1983) mechanism, a fraction $1-\xi_n$ of intermediate goods firms are randomly selected to set their prices optimally each period. This happens after these firms have

received a "price signal" (Christiano, Eichenbaum and Evans 2005; Smets and Wouters 2003, 2007). The remaining ξ_n fraction of intermediate goods firms that are unable to optimize prices adjust their prices mechanically according to a rule of thumb described in Smets and Wouters (2007) or Del Negro, Shorfheide, Smets and Wouters (2007). That rule of thumb implies increase prices by a weighted average of the last period domestically produced goods gross inflation rate ($\pi_{Nt-1} = P_{Nt-1}/P_{Nt-2}$) and the steady state gross inflation π_*. Under that rule of thumb, the ξ_n firms adjust their time t prices to: $P_{it} = \left(\frac{P_{Nt-1}}{P_{Nt-2}}\right)^{l_n} \pi_*^{1-l_n} P_{it-1}$. The parameter $l_n \in [0,1]$ is the degree of indexation to the past inflation in the production sector.

An intermediate good firm able to set price optimally to $\tilde{P}_{it}$ in period t and has not been able to reset it optimally since will have a price of

$$P_{it+s} = \left(\frac{P_{Nt}}{P_{Nt-1}}\frac{P_{Nt+1}}{P_{Nt}}.....\frac{P_{Nt+s-1}}{P_{Nt+s-2}}\right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it}$$

$$= \prod_{k=1}^{s}\left(\frac{P_{Nt+k-1}}{P_{Nt+k-2}}\right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it} = \left(\prod_{k=1}^{s} \pi_{Nt+k-1}^{l_n} \pi_*^{1-l_n}\right)\tilde{P}_{it} \quad \text{in period } t+s.$$

This price expression simplifies to

$$P_{it+s} = \left(\frac{P_{Nt+s-1}}{P_{Nt-1}}\right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it}. \tag{2.46}$$

A price setting intermediate good firm, which sets price in period t, chooses $\tilde{P}_{it}$ to maximize the discounted present value of the firm's profits through the horizon over which the price $\tilde{P}_{it}$ is in effect. This is, it chooses $\tilde{P}_{it}$ that maximizes

$$E_t \sum_{s=0}^{\infty} \xi_n^s \Delta_{t,s} \left[\left(\prod_{k=1}^{s} \pi_{Nt+k-1}^{l_n} \pi_*^{1-l_n}\right)\tilde{P}_{it} Y_{it} - R_{Kt+s} K_{it+s}^s - W_{t+s} H_{it+s} - W_{t+s}^e H_{it+s}^e\right], \tag{2.47}$$

subject to the demand constraint

$$Y_{it+s} = \left(\frac{\left(\prod_{k=1}^{s} \pi_{Nt+k-1}^{l_n} \pi_*^{1-l_n} \right) \tilde{P}_{it}}{P_{Nt+s}} \right)^{-\frac{1+\varepsilon_t^P}{\varepsilon_t^P}} Y_{t+s},$$

where $\Delta_{t,s}$ is the discount factor.

Since entrepreneurs are individuals in the household sector and households ultimately own all intermediate goods firms, $\Delta_{t,s} = \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$, which is the discount factor of shareholders-households.

Equation (2.42) shows that the intermediate goods firms' real total cost is equal to $MC_{t+s}\left(Y_{it+s} + \Phi\right)$ in each period $t+s$. Substituting the expressions of the discount factor, the total real cost, and the demand into the expression (2.47); intermediate goods firms optimizing prices in period t choose $\tilde{P}_{it}$ that solves:

$$\underset{\tilde{P}_{it}}{Max} E_t \sum_{s=0}^{\infty} \xi_n^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left\{ \left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it} \left(\frac{\left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{Nt}(i)}{P_{Nt+s}} \right)^{-\frac{1+\varepsilon_t^P}{\varepsilon_t^P}} Y_{t+s} \right.$$

$$\left. - P_{Nt+s} MC_{t+s} \left(\left(\frac{\left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it}}{P_{Nt+s}} \right)^{-\frac{1+\varepsilon_t^P}{\varepsilon_t^P}} Y_{t+s} + \Phi \right) \right\}.$$

The firm's price setting behaviour maximizes the difference between the discounted streams of the firms' expected income and cost. The first order necessary condition from this profit maximization is

$$0 = E_t \sum_{s=0}^{\infty} \xi_n^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{it} \left[\left(1 - \frac{1+\varepsilon_{t+s}^P}{\varepsilon_{t+s}^P} \right) \left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it} + \left(\frac{1+\varepsilon_{t+s}^P}{\varepsilon_{t+s}^P} \right) P_{Nt+s} MC_{t+s} \right],$$

or

$$0 = E_t \sum_{s=0}^{\infty} \xi_n^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{Y_{it}}{\varepsilon_{t+s}^P} \left[\left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n} \tilde{P}_{it} - \left(1 + \varepsilon_{t+s}^P\right) P_{Nt+s} MC_{t+s} \right].$$

The pricing rule that emerges is given by:

$$\tilde{P}_{it} = \frac{E_t \sum_{s=0}^{\infty} \left(\beta \xi_n\right)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{it+s} \frac{1 + \varepsilon_{t+s}^P}{\varepsilon_{t+s}^P} P_{Nt+s} MC_{t+s}}{E_t \sum_{s=0}^{\infty} \left(\beta \xi_n\right)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{Y_{it+s}}{\varepsilon_{t+s}^P} \left(\frac{P_{Nt+s-1}}{P_{Nt-1}} \right)^{l_n} \pi_*^{1-l_n}}. \tag{2.48}$$

The fraction ξ_n of intermediate goods firms unable to set prices optimally in period t had a price P_{Nt-1} at time $t-1$. The rule of thumb, applied by these firms, implies updating their period t prices up to $\left(\frac{P_{Nt-1}}{P_{Nt-2}} \right)^{l_n} \pi_*^{1-l_n} P_{Nt-1}$. The price-setting rule of the final good firm, given in equation (2.27), results to an aggregate domestically good price that evolves according to the law of motion

$$P_{Nt}^{-\frac{1}{\varepsilon_t^P}} = \xi_n \left(\left(\frac{P_{Nt-1}}{P_{Nt-2}} \right)^{l_n} \pi_*^{1-l_n} P_{Nt-1} \right)^{-\frac{1}{\varepsilon_t^P}} + \left(1 - \xi_n\right) \tilde{P}_t^{-\frac{1}{\varepsilon_t^P}}. \tag{2.49}$$

Equation (2.48) which establishes the intermediate goods firms' pricing rule and equation (2.49) that gives the final goods firms' pricing rule are the two equations that determine the dynamic properties of the domestically produced goods prices inflation around the steady state. If the degree of partial indexation is set to zero ($l_n = 0$), the inflation dynamic in the domestic goods production sector becomes similar to the traditional forward-looking Phillips curve. The degree of prices indexation determines the backward looking component of the domestically produced goods inflation process.

The speed of adjustment of the domestically produced goods prices to the desired mark-up depends on the degree of price stickiness(ξ_n), the curvature of the Kimball's aggregator in the domestically produced goods market, and the steady state mark-up which in the equilibrium is a function of the share of the fixed costs in production. If flexible prices are restored in the goods production sector, equation (2.48) reduces to the monopolistically standard condition that goods prices are set as a mark up of the real marginal cost. In addition, assuming that all prices are indexed to the lagged or the steady state inflation rate leads to a vertical Phillips curve in the long run. Next, we present the capital goods sector.

2.3- The capital goods sector

The capital goods sector produces the physical capital goods use in the economy. The capital stock evolves according to the law of motion

$$K_t = (1-\partial)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t, \tag{2.50}$$

where ∂ is the depreciation rate of the capital, I_t is the economy gross investment, and K_{t-1} is the installed capital of the sector. $S(\bullet)$ is a convex adjustment cost function, which at the steady state satisfies the properties $S(\bullet) = 0$ and $S'(\bullet) = 0$.

Each period t, capital producers combine the installed capital K_{t-1} and the purchased investment I_t to produce the new installed capital K_t. This transformation is done according to the technology given by the law of motion (2.50). The new installed capital K_t is available at

the beginning of period $t+1$, and it is sold on a competitive market to entrepreneurs, who rent out the induced capital services K_{t+1}^{s} to all productive units in the domestic economy.

As indicated previously, the capital utilisation rate u_t links the capital services K_t^s to the installed capital goods K_{t-1} . Capital producing firms choose the optimal capital utilization rate in the economy. This optimization process allows capital good producing firms to "recover" the costs associated with the variable capital utilization in this economy[21]. These costs, measured in term of consumption goods, are equal to $J(u_t)K_{t-1}$. As in Christiano, Eichenbaum and Evans (2005), or in Smets and Wouters (2003, 2007), cost of adjusting the capital utilisation rate are zero at the steady state, which implies $J(1)=0$.

The capital producing firms minimize the real cost of producing the new installed capital K_t. The firm's real cost minimization problem is recursive because of the convex adjustment cost function $S(\bullet)$ that models the cost of adjusting the capital stock as a function of change in the investment. Ultimately, all firms belong to individuals in households, and the firm discounts rate is $\Delta_{t,s}=\dfrac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$, which is the discount rate of shareholders-households. The capital producing firm's real cost minimization problem is formulated as follows:

$$\underset{I_t,K_t,u_t}{Min}\; E_t \sum_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t}\left(I_{t+s}+R_{Kt+s}u_{t+s}K_{t+s-1}-J\left(u_{t+s}\right)K_{t+s-1}\right),$$

subject to the constraint of the capital law of motion given by equation (2.50). The Lagrangian associated with this optimization problem is given by:

[21] The recovering of those costs associated with the capital utilization rate might be viewed as the benefit from recycling used capital.

$$L_t = E_t \sum_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t} \left[\begin{array}{c} I_{t+s} + R_{Kt+s} u_{t+s} K_{t+s-1} - J(u_{t+s}) K_{t+s-1} \\ + Q_{t+s} \left(K_{t+s} - (1-\partial) K_{t+s-1} - \left(1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right) I_{t+s} \right) \end{array} \right].$$

The Lagrangian multiplier associated with this real cost minimization problem is Q_t. Because of the competitive capital market structure, Q_t is the market value of capital in term of consumption goods, and Q_t conceptually denotes the economy "Tobin's Q"[22]. The first order necessary condition with respect to the new installed capital goods is given by:

$$Q_t - \beta E_t \frac{\Xi_{t+1}}{\Xi_t} \left(R_{Kt+1} u_{t+1} - J(u_{t+1}) + (1-\partial) Q_{t+1} \right) = 0 . \tag{2.51}$$

Equation (2.51) expresses the value of the installed capital as a function of its expected value net to the depreciation and the expected real return. The expected real return is measured by the real rental rate times the capital utilization rate minus the adjustment cost.

The first order necessary condition with respect to the investment I_t is the following:

$$1 - Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} \right) \right] - \beta E_t \left[\frac{\Xi_{t+1}}{\Xi_t} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] = 0 . \tag{2.52}$$

Capital adjustment cost, measured as a function of change in the investment, introduces an investment dynamic that depends on both past and future investments. In addition, as calculated from the first order necessary condition with respect to bonds denominated in domestic currency, $E_t \left(\frac{\Xi_t}{\Xi_{t+1}} \right) = \beta E_t \left(\varepsilon_t^B (1 + i_t) \frac{P_t}{P_{t+1}} \right)$, which implies that the financial-market-shock affects the Tobin's Q (market value of the capital) and the investment level as well. The effects of these shocks on the investment and the market value of the capital operate in the same

[22] Note that the Lagrange Multiplier is the *shadow price* or *shadow value of the constraint.* Since the marginal rate of transformation from the previously installed capital (after it has depreciated by $1-\partial$) to the new capital is exactly unity, the price of the new and used capital must be the same.

fashion as the so-called net-worth shocks illustrated in Bernanke, Gertler and Gilchrist (1999). The effects of the financial-market-shock on investment and Tobin's Q differentiate that shock from the discount factor shock (preference shock), which only affect the consumption Euler equation.

The first order necessary condition with respect to capital utilization rate is given by

$$R_{Kt} = J'(u_t). \tag{2.53}$$

Equation (2.53) establishes the equality between the cost of higher utilization rate and the rental price of capital services.

Equations (2.51), (2.52), and (2.53) represent the equilibrium decision rules of the capital producing firm. The first observation is that the investment dynamic and the optimal choice of capital utilization rate are identical to those in Smets and Wouters (2003, 2007) closed economy models, which allows households to own capital and make decisions on investment, capital stock, and capital utilization rate. Transferring capital good's ownership from the household to a distinctive component called entrepreneurs does not distort the dynamics of investment; this dynamic seems to depend primarily on the capital good market structure rather than the ownership of the capital stock. The dynamic of the shadow price of capital Q_t is also similar; an expected increase in the real return rate of capital, *ceteris paribus*, boosts the demand for capital goods, which in turn raises the market value of capital.

This setting assumes that capital producers use the capital services to produce physical capital goods. Christiano, Motto and Rostogno (2007) adopt a different perspective. In their model, capital producers purchase the installed capital goods, which they combine with the current period investment to produce the new capital stock ready for use in the next period. Entrepreneurs subsequently purchase the capital goods, rent out to goods producing firms, and

choose the capital utilization rate. Those alternative assumptions eventually yield a different equilibrium condition for the Tobin's Q. Next, we describe the entrepreneurial sector.

2.4- The entrepreneurial sector

Entrepreneurs rent capital services to producing firms on a competitive capital services market; they possess the technology necessary to transform the physical capital goods into capital services. Entrepreneurs need to purchase the physical capital goods first from capital producers. To some extent, entrepreneurs partly finance the acquisition of physical capital goods from their own resources. Nevertheless, they must borrow from foreign lenders the remaining amount necessary to cover the total capital goods expenditure and acquire the property rights. In general, foreign loans provide the foreign currency needed by the developing economy to purchase capital goods, which contain substantial quantities of imported goods.

In this section, we describe the entrepreneurial behaviour and the external finance premium associated with investment financing in developing economies. The external finance premium arises from asymmetric information in the relationship between foreign lenders and domestic entrepreneurs. In modelling the entrepreneurial behaviour, we retain the setting of Devereux, Lane and Xu (2006), which follows the Bernanke, Gertler and Gilchrist (1999) model for a closed economy.

2.4.1 The financial accelerator mechanism

At the end of each period, a continuum of entrepreneurs indexed by *i* demand fund to finance the purchase of a new capital K_{it}. Entrepreneurs use that new capital to rent out capital services to good producing firms at period $t+1$. A portion of the desired capital good is financed

through the entrepreneur net worth NW_{it}, which represents entrepreneurs accumulated wealth over the past periods. The remaining value of capital, $Q_t K_{it} - NW_{it}$, must be financed by external borrowing from foreign lender on international finance markets. The entrepreneur debt denominated in foreign currency is given by

$$D_{it}^{e} = \frac{Q_t K_{it} - NW_{it}}{S_t}. \tag{2.54}$$

The investment project is subject to an idiosyncratic productivity disturbance ω_i. We assume ω_i to be independently and identically distributed across entrepreneurs and time, and ω_i follows a log normal distribution with mean $-\frac{\sigma_\omega^2}{2}$ and variance σ_ω^2 such that $E_t(\omega_i) = 1$. The probability distribution function of ω_i is denoted $f(\omega_i)$, and its distribution function is $F(\omega_i)$. The gross return of an invested unit of domestic currency is $\omega_i R_{t+1}^{e}$ where the entrepreneur's gross return is R_t^{e}. The investment return is $\omega_i R_{t+1}^{e} Q_t K_{it}$, and the associated expected return is given by $E_t\left(R_{t+1}^{e} Q_t K_{it}\right)$.

The entrepreneur observes ω_i costlessly and has private information on the outcome of the investment. The foreign lender must pay some monitoring costs, denoted $C(\omega_{t+1})$, to observe the actual outcome of the investment project. This asymmetric information is known as "costly state verification" (Townsend, 1979). For convenience, $C(\omega_{t+1})$ is positive, proportional to the investment pay offs, and less than the investment expected return.

This financial environment was considered long ago by Townsend (1979), Gale and Hellwig (1985), and Williamson (1986). These authors show that the "optimal contract" is a debt contract whereby the lender requires a threshold $\overline{\omega}$ of the productivity disturbance. If the idiosyncratic productivity disturbance $\omega_i > \overline{\omega}$, the foreign lender does not monitor the project

and receives a fixed payment $\overline{\omega} R^e_{t+1} Q_t K_{it}$ from the entrepreneur; on the contrary, if $\omega_i < \overline{\omega}$, the foreign lender monitors the investment project and takes the entire proceeds net of monitoring costs. The entrepreneur receives nothing, and eventually, he becomes bankrupt. The contract is optimal in the sense that it produces a better outcome than any lending arrangement in which the foreign lender takes a fraction of the entrepreneurs declared return. In this case, the entrepreneur would have a strong incentive to report lower outcomes.

Next, we derive the external finance premium. Since ω_i is independently and identically distributed across entrepreneurs, every entrepreneur faces the same optimal financial contract. The equilibrium is symmetric, and we can drop the subscript. We assume both the entrepreneur and the foreign lender to be risk neutral.

The entrepreneur expected return from the investment project is given by

$$R^e_{t+1} Q_t K_t \left[\int_{\overline{\omega}_t}^{\infty} \omega f(\omega) d\omega - \overline{\omega}_{t+1} \int_{\overline{\omega}_t}^{\infty} f(\omega) d\omega \right]$$

$$= R^e_{t+1} Q_t K_t A(\overline{\omega}_t),$$

where $A(\overline{\omega}_t) = \left[\int_{\overline{\omega}_t}^{\infty} \omega f(\omega) d\omega - \overline{\omega}_t \int_{\overline{\omega}_t}^{\infty} f(\omega) d\omega \right]$ is the aggregate fraction of the return that receives the entrepreneur.

The foreign lender expected return is given by

$$R^e_{t+1} Q_t K_t \left[\overline{\omega}_t \int_{\overline{\omega}_t}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\overline{\omega}_t} \omega f(\omega) d\omega \right]$$

$= R^e_{t+1} Q_t K_t B(\overline{\omega}_t)$, where $B(\overline{\omega}_t) = \left[\overline{\omega}_t \int_{\overline{\omega}_t}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\overline{\omega}_t} \omega f(\omega) d\omega \right]$ is the aggregate fraction of return net to monitoring cost, which receives the foreign lender. The aggregate fraction of monitoring cost is expressed by $C(\overline{\omega}_t) = \mu \int_0^{\overline{\omega}_t} \omega f(\omega) d\omega$. The following equality

$A(\overline{\omega}_t)+B(\overline{\omega}_t)=1-C(\overline{\omega}_t)$ holds. If the monitoring cost is set to zero, that relation implies $A'(\overline{\omega}_t)=-B'(\overline{\omega}_t)$.

Risk neutral foreign lenders require a return at less equal to the world opportunities cost $1+i_t^*$, and their participation constraint in term of domestic currency is expressed by:

$$\frac{R_{t+1}^e Q_t K_t B(\overline{\omega}_t)}{S_{t+1}} = (1+i_{t+1}^*)\frac{\left(Q_t K_t - NW_t\right)}{S_t}. \tag{2.55}$$

The optimal contract imposes the entrepreneur to choose the physical capital good K_t and the threshold $\overline{\omega}_t$ that maximize his expected return subject to the participation constraint of the foreign lender. This maximization problem is summarized by:

$$\underset{K_t,\overline{\omega}_t}{Max}\, E_t\left(R_{t+1}^e Q_t K_t A(\overline{\omega}_t)\right),$$

subject to the participation constraint

$$\frac{R_{t+1}^e Q_t K_t B(\overline{\omega}_t)}{S_{t+1}} = (1+i_{t+1}^*)\frac{\left(Q_t K_t - NW_t\right)}{S_t}.$$

In the aggregate, the entrepreneur and the foreign lender face an uncertainty on the exchange rate that will prevail when the loan will be repaid in foreign currency. As in Devereux, Lane and Xu (2006), we assume that the entrepreneur will bear all the aggregate risk due to the exchange rate uncertainty. The return R_{t+1}^e and the threshold $\overline{\omega}_t$ are state contingent on the exchange rate realization, which implies a participation constraint that holds at all state. The Lagrangian L_t^e associated to the entrepreneur maximization problem is given by the relation

$$L_t^e = E_t\left\{\left(R_{t+1}^e Q_t K_t A(\overline{\omega}_t)\right) - \lambda_t^e\left(\frac{R_{t+1}^e Q_t K_t B(\overline{\omega}_t)}{S_{t+1}} - \frac{(1+i_{t+1}^*)\left(Q_t K_t - NW_t\right)}{S_t}\right)\right\},$$

where λ_t^e is the Lagrangian multiplier. The external finance premium results from the combination of the first order necessary conditions with respect to capital and productivity disturbance threshold, the relation is given by[23]:

$$E_t\left[R_{t+1}^e\left(B(\overline{\omega}_t)-\frac{B'(\overline{\omega}_t)A(\overline{\omega}_t)}{A'(\overline{\omega}_t)}\right)\right]=E_t\left((1+i_{t+1}^*)\frac{S_{t+1}}{S_t}\right). \tag{2.56}$$

Equation (2.56) gives the equilibrium condition of financing capital through external borrowing. At the optimum, the optimal contract equals the investment expected return to the opportunity cost of the funds borrowed. Eventually, in the absence of monitoring costs, in other words, when there is perfect information in the international capital market, the equilibrium condition (2.56) reduces to the form $E_t\left(R_{t+1}^e\right)=E_t\left((1+i_{t+1}^*)\frac{S_{t+1}}{S_t}\right)$, which is the standard textbook Interest Rate Parity condition (see e.g. Blanchard, 2005, page 389).

Costly state verification in capital financing widens the gap between the opportunity cost of fund borrowed abroad and the entrepreneur expected return on domestic investment. The asymmetric information between entrepreneur and foreign lender creates a return gap, which imposes the condition $E_t\left(R_{t+1}^e\right)>E_t\left((1+i_{t+1}^*)\frac{S_{t+1}}{S_t}\right)$. The external finance premium $\wp(\overline{\omega}_{t+1})$ that fills this return gap is given by

$$\wp(\overline{\omega}_{t+1})=\frac{1}{E_t\left[B(\overline{\omega}_{t+1})-\frac{B'(\overline{\omega}_{t+1})A(\overline{\omega}_{t+1})}{A'(\overline{\omega}_{t+1})}\right]}.$$

Equations (2.55) and (2.56) can be used to show that the external finance premium is an increasing function of the leverage ratio $\frac{Q_tK_t}{NW_t}$ (see Bernanke, Gertler and Gilchrist, 1999). A

[23] Appendix 3 associated with this chapter shows the development that yields this equation.

decrease in the entrepreneur net worth, triggered for instance by a depreciation of the nominal exchange rate, raises the leverage ratio and the external finance premium. Subsequently, the cost of capital increases, and the demand for physical capital goods falls. This mechanism is called the "financial accelerator".

A monetary policy contraction that aims at slowing down the economy if it induces a sizeable exchange rate appreciation will rather increase the entrepreneur's net worth and trigger the opposite mechanism. If the policy substantially appreciates the currency such that external finance premium falls drastically, there will be a further increase on the capital goods demand and an output expansion. In this way, the financial accelerator contributes to counter and reverse the expected effect of a monetary policy contraction.

In summary, the most relevant point from the "financial accelerator" mechanism (Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1999) is that, financial frictions amplify the effects of exogenous shocks. Hence, financial frictions create new propagation mechanism of business cycle fluctuations and channels that alter the dynamic properties of the economy. These additional channels modify the propagation mechanism of others shocks originate outside the financial sector. To complete the analysis of the entrepreneur behaviour, we determine the entrepreneur's decision rules on net worth, consumption, and gross return. We also describe his budget constraint.

2.4.2 Entrepreneur's decision rules on consumption, net worth and real return

As in Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) or Devereux, Lane and Xu (2006), the entrepreneurs are in a constant need of fund. Entrepreneurs die at a

rate $1-\nu$, and entrepreneurs who die consume their return of capital. The entrepreneurs' aggregate real consumption is given by

$$C_t^e = (1-\nu)R_t^e Q_{t-1}K_{t-1}A(\overline{\omega}_{t-1}). \tag{2.57}$$

The law of motion for the entrepreneur's real net worth is

$$NW_t = \nu R_t^e Q_{t-1}K_{t-1}A(\overline{\omega}_{t-1}) + W_t^e. \tag{2.58}$$

To elucidate the exchange rate effect on entrepreneur net worth, we make preliminary transformations. Using the relation $A(\overline{\omega}_{t-1}) + B(\overline{\omega}_{t-1}) = 1 - C(\overline{\omega}_{t-1})$ to substitute for $A(\overline{\omega}_{t-1})$ in equation (2.58), the entrepreneur real net worth is rewritten as

$$NW_t = \nu(1-C(\overline{\omega}_{t-1}))R_t^e Q_{t-1}K_{t-1} - \nu R_t^e Q_{t-1}K_{t-1}B(\overline{\omega}_{t-1}) + W_t^e. \tag{2.59}$$

The participation constraint equation (2.54), written at time $t-1$, imposes the equality:

$$R_t^e Q_{t-1}K_{t-1}B(\overline{\omega}_{t-1}) = (1+i_t^*)\frac{S_t}{S_{t-1}}\left(Q_{t-1}K_{t-1} - NW_{t-1}\right).$$

The right side of this equality is used to substitute for $R_t^e Q_{t-1}K_{t-1}B(\overline{\omega}_{t-1})$ in (2.59), and we obtain

$$NW_t = \nu(1-C(\overline{\omega}_{t-1}))R_t^e Q_{t-1}K_{t-1} - \nu(1+i_t^*)\frac{S_t}{S_{t-1}}\left(Q_{t-1}K_{t-1} - NW_{t-1}\right) + W_t^e. \tag{2.60}$$

Equation (2.60) shows a negative relationship between the entrepreneur's real net worth and the exchange rate. This implies, exchange rate depreciations[24] worsen the entrepreneur's real net worth, raise the leverage ratio and the external finance premium. In the case of a fixed exchange rate, this mechanism suggests that devaluation is likely to be procyclical if the balance sheet effect induces enough contraction on investment that offsets the expansion the real exchange rate creates on export (Frankel, 2005).

[24] Here an increase in S_t since the exchange is defined as the price of the foreign currency in term of the domestic currency.

An identical development leads to an expression for the entrepreneur's consumption written as follows

$$C_t^e = (1-\nu)\left(1-C\left(\overline{\omega}_{t-1}\right)\right)R_t^e Q_{t-1}K_{t-1} - (1-\nu)(1+i_t^*)\frac{S_t}{S_{t-1}}\left(Q_{t-1}K_{t-1} - NW_{t-1}\right). \quad (2.61)$$

Equations (2.60) and (2.61) can be summed side by side; the outcome forms the entrepreneur's real budget constraint.

$$C_t^e + NW_t = \left(1-C\left(\overline{\omega}_{t-1}\right)\right)R_t^e Q_{t-1}K_{t-1} - (1+i_t^*)\frac{S_t}{S_{t-1}}\left(Q_{t-1}K_{t-1} - NW_{t-1}\right) + W_t^e .$$

When it is recognized that equation (2.54) corresponds to $NW_t = Q_t K_t - S_t D_t^e$ in a symmetric equilibrium, and that the same equation expresses $D_t^e = \frac{\left(Q_{t-1}K_{t-1} - NW_{t-1}\right)}{S_{t-1}}$ at time t-1, the entrepreneur's real budget constraint is rewritten as

$$C_t^e + Q_t K_t + (1+i_t^*)S_t D_{t-1}^e = W_t^e + \left(1-C\left(\overline{\omega}_{t-1}\right)\right)R_t^e Q_{t-1}K_{t-1} + S_t D_t^e . \quad (2.62)$$

The entrepreneur's budget constraint is a conventional budget constraint. Entrepreneur's cash expenditure on consumption, plus purchase of capital goods and repayment of the last period debts must equal entrepreneur's labour income, plus his return on the installed physical capital goods net to monitoring cost plus new foreign borrowing.

The entrepreneur's gross rate of return equals the rental returns on capital services he earns from all producing firms in the economy plus the market value of the non-depreciated capital, both deflated by the current capital price. The relation is given by

$$R_t^e = \frac{R_{Kt}u_t + R_{Kt}u_t + (1-\partial)Q_t}{Q_{t-1}} . \quad (2.63)$$

The capital utilization rate smoothes the rental payment entrepreneurs receive from intermediate good firms and capital good producing firms.

To summarize, the participation constraint (2.55), the equilibrium condition of financing capital acquisition (2.56), the entrepreneur consumption (2.57), the entrepreneur's real net worth (2.58), the entrepreneur's budget constraint (2.62), and the entrepreneur's gross rate of return (2.63) describe the entrepreneur's equilibrium.

In the next sections, we first characterize the government fiscal and monetary policies. Then, we determine the market equilibrium conditions associated to this dynamic stochastic general equilibrium model.

2.5- Government policies

The government sets the monetary and fiscal policies in the economy. The monetary policy authority adjusts the short run interest rate, and the government's budget constraint describes the public finance.

The monetary policy instrument follows an interest rate rule in the form of Ortiz and Sturzenegger (2007)[25]:

$$R_t = R_{t-1}^{\rho_R}\left[R^{ss}\pi_*\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}\left(\frac{\frac{S_t}{S_{t-1}}}{\Delta S_{ss}}\right)^{\psi_3}\right]^{1-\rho_R} e^{\eta_t^R}, \tag{2.64}$$

where $R_t = 1+i_t$, and R^{ss} is the steady state real interest rate. The parameter ρ_R captures the degree of interest rate smoothing, while the parameters ψ_1, ψ_2, and ψ_3 are Taylor's coefficients on inflation, output, and exchange rate fluctuations respectively. We assume the monetary policy shock η_t^R independently and identically distributed according to a normal distribution, $\eta_t^R \sim N(0,\sigma_R)$.

[25] Ortiz and Sturzenegger (2007) use the steady state output as the potential output level.

The monetary policy rule follows a generalized Taylor (1993) rule. Monetary authority responds to exogenous shocks that hit the domestic economy by a gradual adjustment of the policy-controlled interest rate to three major goals. First, monetary authority responds to the deviation of the inflation rate from its target π_*, which coincides to the steady state (gross) inflation rate. Second, the monetary policy instrument adjusts to the output gap, which is defined, following Taylor (1993), as the difference between the actual output Y_t and the natural output $Y_t^* = Y_{ss}e^{\bar{g}t}$ where the quarterly trend growth rate to real GDP is $\bar{g} = 100(g-1)$. Third, the exchange rate fluctuations are an integral component of the monetary policy reaction function. Consistent with the DSGE literature, the steady state exchange rate is set to one.

The monetary policy rule's parameters conform to the Taylor principle. That is, monetary authority responds greater than one-for-one to inflation. The economy then has unique, stationary, and rational expectation equilibrium.

Monetary policy authorities in developing economies, particularly in Sub-Saharan African economies, do not necessarily characterize their policy by a Taylor rule. However, Mohanty and Klau (2004) provide substantive evidence that in some emerging economies, including South Africa, a Taylor rule as designed in equation (2.64) can quite well describe monetary policy operating procedures.

The government budget constraint is of the form

$$G_t + \varepsilon_{t-1}^{B}(1+i_{t-1})\frac{B_{t-1}}{P_t} = \frac{B_t}{P_t} + T_t, \tag{2.65}$$

where T_t are real lump-sum taxes (subsidies) levied (allocated) from (to) the representative household. We assume that government spending, expressed as a percentage deviation from the

steady state equilibrium and weighted by the steady state ratio of government spending to output, $\hat{\varepsilon}_t^G = \frac{G^{ss}}{Y^{ss}}\hat{G}_t$, follows an autoregressive process given by

$$\hat{\varepsilon}_t^G = \rho_G \hat{\varepsilon}_{t-1}^G + \eta_t^G . \tag{2.66}$$

The coefficient of persistence is constrained by $0 \le \rho_G < 1$, and the zero mean structural government spending shock η_t^G is serially uncorrelated, independently, and identically distributed such that $\eta_t^G \sim N(0, \sigma_G^2)$. The following section presents the external sector.

2.6- The external sector

The external sector characterizes the exports dynamic, the incomplete exchange rate pass-through, and the aggregate balance of payments.

2.6.1 The exports dynamic

Final goods producers export domestically produced goods on foreign competitive goods markets at an exogenous price since the small open economy is price taker. Domestically produced goods are purchased by foreign households. We assume that foreign households' aggregate consumption C_t^* is described by a CES function that combines home produced and import goods in the same fashion as equation (2.2):

$$C_t^* = \left[\gamma_1^{\frac{1}{\theta_1}} X_t^{\frac{\theta_1 - 1}{\theta_1}} + (1-\gamma_1)^{\frac{1}{\theta_1}} C_{Nt}^{*\frac{\theta_1 - 1}{\theta_1}} \right]^{\frac{\theta_1}{\theta_1 - 1}},$$

where γ_1 is the share of the home economy's exports in the rest of the world consumer price index P_t^*, and θ_1 is the elasticity of substitution between the home economy's exports and the foreign produced goods.

Likewise, foreign households determine their demands for goods by minimizing their consumption expenditure. We allow for trade credit in the foreign economy and assume that a fraction τ of the home economy's exports must be financed in advance through loans contracted by foreign households from foreign financial intermediaries. The cost minimization problem is identical to that of the domestic households, and the export demand is given by the relation

$$X_t = \gamma_1 \left(\frac{P_{Xt}\left(1+\tau i^*_{t-1}\right)}{P^*_t} \right)^{-\theta_1} C^*_t ,$$

where P_{Xt} is the foreign currency price of exports. We assume that the law of one price holds in the export sector. The local currency price setting is imposed to exporters such that final goods producers set the price they charge for the exports in the foreign currency. This implies $P_{Nt} = S_t P_{Xt}$, which allows for rewriting the export demand as

$$X_t = \gamma_1 \left(\frac{P_{Nt}}{S_t P^*_t}\left(1+\tau i^*_{t-1}\right) \right)^{-\theta_1} C^*_t . \tag{2.67}$$

The quantity $\frac{P_{Nt}}{S_t P^*_{Nt}}$, where P^*_{Nt} is the price of foreign produced goods, denotes the real exchange rate[26]. The rest of the world's aggregate consumption C^*_t is exogenous for the small open economy, and its path is assumed to evolve along a stochastic log-linear autoregressive process given by

$$\ln C^*_t = (1-\rho_X)\ln C^*_{ss} + \rho_X \ln C^*_{t-1} + \eta^X_t , \tag{2.68}$$

where $0 \le \rho_X < 1$, and the structural export demand shock η^X_t is identically and independently distributed, $\eta^X_t \sim N(0, \sigma_Y)$. Next, we describe the incomplete exchange rate pass-through.

[26] More precisely, this small open economy's real exchange rate could be approximated by $\frac{P_{Nt}}{S_t P^*_t}$ since its share in the world trade is too small. The next footnote explains how this assumption implies $P^*_{Nt} \approx P^*_t$.

2.6.2 Local currency pricing of import goods

To account for the incomplete exchange rate pass-through in this model, we distinguish between the world market price of imported goods and their price in the domestic economy. To implement this distinction, we allow for imperfect competition and pricing to market in the domestic economy. Furthermore, we assume that the law of one price does not hold in the imported goods sector such that there is a difference between the imported goods price in the domestic market P_{Mt} and the price at the dockside $S_t P_t^*$ [27].

The imported goods sector consists of two types of importing firms. One type of firms buys a homogenous good on the world market at a price P_t^*, and it turns that good into multiple differentiated goods through a "differentiating technology" or brand naming. Another type of firm buys these differentiated imported goods, packages them into a final imported goods, and sells the final imported good to households, entrepreneurs, capital goods producing firms, and government in a domestically competitive goods market.

Differentiated good importing firms follow a Calvo (1983) price-setting rule; those firms are able to set their prices if they receive a price change signal. In general, intermediate imported good firms face a random probability $1-\xi_m$ to set their prices each period t. With a probability ξ_m, a firm does not reset price and mechanically adjusts prices according to a rule of thumb, which is identical to that of domestically produced good firms. All these ξ_m firms adjust their time t prices to $P_{Mt}(i) = (\pi_{Mt-1})^{l_m} (\pi_*)^{1-l_m} P_{Mt-1}(i)$, where $\pi_{Mt-1} = P_{Mt-1}/P_{Mt-2}$ is the previous period imported goods gross inflation and π_* is the steady state gross inflation rate.

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[27] Let P_{Nt}^* be the price in foreign currency of goods producing abroad (the rest of the world GDP deflator), following equation (1.8), the rest of the world consumption price index P_t^* is given by $P_t^{*1-\theta_1} = \left[\gamma_1 \left(P_{Xt}\left(1+\pi_t^*\right)\right)^{1-\theta_1} + (1-\gamma_1) P_{Nt}^{*1-\theta_1}\right]$. Since the share of the home economy's exports in the rest of the world consumption basket is too small $\gamma_1 \approx 0$, and we could approximate $P_{Nt}^* \approx P_t^*$.

The profit maximization problem by price-setting imported goods firms is identical to that of the intermediate domestically producing good firms. Nonetheless, we assume no shocks to the intermediate good firm's demand curve elasticity, and the period $t+s$ imported goods aggregator is defined as a constant elasticity of substitution (CES) bundle $Y_{Mt+s}=\left[\int_0^1 Y_{Mt+s}(i)^{\frac{\theta_m-1}{\theta_m}}di\right]^{\frac{\theta_m}{\theta_m-1}}$, where $\theta_m>1$. The demand for intermediate imported goods is given by $Y_{Mt+s}(i)=\left(\frac{\left(\frac{P_{Mt+s-1}}{P_{Mt-1}}\right)^{l_m}\pi_*^{1-l_m}\tilde{P}_{Mt}(i)}{P_{Mt+s}}\right)^{-\theta_m}Y_{Mt+s}$ each period $t+s$.

The intermediate imported good firm's nominal marginal cost is $S_{t+s}P^*_{t+s}$. The real marginal cost $MC_{Mt+s}=\frac{S_{t+s}P^*_{t+s}}{P_{Mt+s}}$ is up to the log linearization the difference between the nominal marginal cost and the average price. The real marginal cost is independent of the firm's characteristics. This relation between the imported firm's marginal cost and the foreign price of imported goods shows that all exogenous shocks that might affect the latter propagate to the domestic inflation. Exchange rate shocks feed immediately to the imported good firm's marginal cost, but sticky imported goods prices tend to slow the speed at which local imported prices adjust to the desired mark-up. As a result, exchange rate shocks feed to the domestic inflation at a lower speed or with delays. This feature characterizes an incomplete exchange rate pass-through. If all imported good prices are flexible ($\xi_m=0$), the standard condition that imported goods prices are a constant mark-up of the marginal cost holds, and there is full exchange rate pass-through.

Intermediate imported goods firms that can set prices in period t choose the price $\widetilde{P}_{Mt}(i)$ that solve $\underset{P_{Mt}(i)}{Max}\, E_t \sum_{s=0}^{\infty} \xi_m^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left(\frac{\left(\frac{P_{Mt+s-1}}{P_{Mt-1}} \right)^{l_m} \pi_*^{1-l_m} \widetilde{P}_{Mt}(i)}{P_{Mt+s}} \right)^{-\theta_m} Y_{Mt+s} \left[\left(\frac{P_{Mt+s-1}}{P_{Mt-1}} \right)^{l_m} \pi_*^{1-l_m} \widetilde{P}_{Mt}(i) - P_{Mt+s} MC_{Mt+s} \right]$.

The first-order necessary condition associated with that problem is given by:

$$0 = E_t \sum_{s=0}^{\infty} \xi_m^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{Mt+s}(i) \left[(1-\theta_m) \left(\frac{P_{Mt+s-1}}{P_{Mt-1}} \right)^{l_m} \pi_*^{1-l_m} \widetilde{P}_{Mt}(i) + \theta_m P_{Mt+s} MC_{Mt+s} \right].$$

As a result, an intermediate imported good firm that can set its price optimally in period t will choose $\widetilde{P}_{Mt}(i)$ that respects the price-setting rule

$$\widetilde{P}_{Mt}(i) = \frac{\theta_m}{\theta_m - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_m)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{Mt+s}(i) P_{Mt+s} MC_{Mt+s}}{E_t \sum_{s=0}^{\infty} (\beta \xi_m)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{Mt+s}(i) \left(\frac{P_{Mt+s-1}}{P_{Mt-1}} \right)^{l_m} \pi_*^{1-l_m}}, \tag{2.69}$$

where $\frac{\theta_m}{\theta_m - 1} > 1$ represents the gross markup the intermediate imported goods firm applies over the ratio of the discounted stream of total nominal cost divided by the discounted stream of real output.

Each period, all $1-\xi_m$ intermediate import goods firms that can set prices have the same markup and identical price $\widetilde{P}_{Mt}(i)$. In addition, ξ_m intermediate import goods firms adjust prices to the last period import good price P_{Mt-1}. The aggregate import goods price's law of motion is defined by:

$$P_{Mt}^{1-\theta_m} = \xi_m \left(\left(\frac{P_{Mt-1}}{P_{Mti2}} \right)^{l_m} \pi_*^{1-l_m} P_{Mt-1} \right)^{1-\theta_m} + (1-\xi_m) \widetilde{P}_{Mt}^{1-\theta_m}. \tag{2.70}$$

Equations (2.69) and (2.70), which give the pricing rules of the intermediate import goods firms and the final import goods firms respectively, are the two equations that define the import goods inflation dynamic.

Movement of the export price P_{Nt} relative to the import price at the dockside $S_t P_t^*$ characterize the terms of trade fluctuations in this small open economy. Thus, the terms of trade are given by the relation $\varepsilon_t^T = \frac{P_{Nt}}{S_t P_t^*}$. Since the rest of the world consumption price index P_t^* is exogenous to the small open economy and is assumed to follow an exogenous stochastic process; the terms of trade ε_t^T also follows a stochastic process that is assumed to be an ARMA(1,1) given by

$$\ln \varepsilon_t^T = (1-\rho_T)\ln \varepsilon^T + \rho_T \ln \varepsilon_{t-1}^T + \eta_t^T + \mu_T \eta_{t-1}^T, \tag{2.71}$$

where $0 \le \rho_T < 1$. The structural terms of trade shock η_t^T is normally, identically, and independently distributed such that $\eta_t^T \sim N(0, \sigma_T)$. The MA term is designed to capture the high frequency fluctuations of the import and export prices, specifically their component related to commodities prices[28].The balance of payments identity closes the model.

2.6.3 The balance of payments identity

The economy's aggregate balance of payment is obtained by adding the budget constraints of the households, entrepreneurs, and government. This yields the relation[29]:

$$C_t + C_t^e + I_t + G_t + S_t D_t + (1+i_t^*) S_t D_{t-1}^e + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2 + C(\overline{\omega}_{t-1}) R_t^e Q_{t-1} K_{t-1}$$

[28] Empirically we investigated various specifications for the terms of trade process, including the first order log-linear autoregressive process, the data clearly prefer an ARMA (1,1) as the marginal likelihood is greater under that specification.
[29] The derivation of this equilibrium condition is detailed in the appendix 4 associated with this chapter.

$$= Y_t + \Pi_{Mt} + J(u_t)K_{t-1} + (1+i^*_{t-1})S_t D_{t-1} + S_t D^e_t \,. \tag{2.72}$$

At the equilibrium, total expenditures must equal total receipts. Total expenditures include households and entrepreneurs consumption, investment, total repayment of foreign debts, bond adjustment costs, and the loan monitoring cost. Total receipts comprise the economy output, new borrowing, profits from the import goods sector, and the capital adjustment cost "recovered" by capital producing firms. The later ends up as additional resources to the economy. The import goods sector's real profit is given by $\Pi_{Mt} = \left(\frac{P_{Mt} - S_t P^*_t}{P_t}\right) Y_{Mt}$, and the import demand is defined by the relation

$$Y_{Mt} = (1-\gamma)\left(\frac{P_{Mt}}{P_t}\right)^{-\theta}\left(C_t + C^e_t + I_t + G_t + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2\right), \tag{2.73}$$

which is a fraction of the domestic absorption.

2.7- Conclusion

This chapter had characterized a developing economy environment. The equilibrium is a collection of 28 sequences of allocation, which the variables are listed below.

Sector	variables name	symbol
Households		
	consumption of domestically produced goods	C_{Nt}
	consumption of import goods	C_{Mt}
	consumer price index	P_t
	aggregate consumption	C_t
	debt in foreign currency	D_t
	households real wage	W_t
Goods production firms		
	production	Y_t
	return on capital services	R_{Kt}
	households labour demand	H_t
	entrepreneurs real wage	W_t^e
	marginal cost	MC_t
	domestically produced goods prices	P_{Nt}
Capital goods firms		
	capital	K_t
	Tobin's Q	Q_t
	investment	I_t
	capital utilization rate	u_t
Entrepreneur sector		
	labour supply	H_t^e
	debt in foreign currency	D_t^e
	investment idiosyncratic productivity threshold	$\overline{\omega}_t$
	consumption	C_t^e
	net worth	NW_t
	return	R_t^e
Monetary authority		
	central bank interest rate	i_t
External sector		
	exports	X_t
	import goods price	P_{Mt}
	firm marginal cost	MC_{Mt}
	import goods demand	Y_{Mt}
	exchange rate	S_t

The 28 economy non-linear optimality conditions that define the decisions that dictate economic agents' behaviours in their decision-making are listed below[30]:

Households

1- consumption of domestically produced goods

$$C_{Nt} = \gamma\left(\frac{P_{Nt}}{P_t}\right)^{-\theta} C_t, \tag{2.7}$$

2- consumption of import goods

$$C_{Mt} = (1-\gamma)\left(\frac{P_{Mt}}{P_t}\right)^{-\theta} C_t, \tag{2.8}$$

3- aggregate consumer price index

$$P_t = \left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{2.9}$$

4- optimal consumption-saving allocation

$$\left(C_t - hC_{t-1}\right)^{-\phi} = \beta E_t\left[\varepsilon_t^B (1+i_t)\frac{P_t}{P_{t+1}}\left(C_{t+1} - hC_t\right)^{-\phi}\right], \tag{2.10}$$

5- arbitrage condition on international financial market

$$E_t\left[\varepsilon_t^B (1+i_t)\frac{P_t}{P_{t+1}}\left(1+\frac{\psi_D}{S_t}\left(D_t - \overline{D}\right)\right)\right] = E_t\left[(1+i_t^*)\frac{S_{t+1}}{S_t}\right], \tag{2.11}$$

6- households' wages setting equations

intermediate labour unions

$$\tilde{W}_{jt} = \frac{E_t\sum_{s=0}^{\infty}\left(\beta\xi_w\right)^s \frac{P_t\Xi_{t+s}}{P_{t+s}\Xi_t} H_{jt+s}\frac{1+\varepsilon^w}{\varepsilon^w} W_{Ht+s}}{E_t\sum_{s=0}^{\infty}\left(\beta\xi_w\right)^s \frac{P_t\Xi_{t+s}}{P_{t+s}\Xi_t} H_{jt+s}\frac{1}{\varepsilon^w} g\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{l_w}\pi_*^{1-l_w}}, \tag{2.16}$$

[30] To help the reader traces back those non-linear optimality conditions, each equation is followed by its reference number in the text.

labour packers

$$W_t^{-\frac{1}{\varepsilon^w}} = \xi_w \left(g\left(\frac{P_{t-1}}{P_{t-2}}\right)^{l_w} \pi_*^{1-l_w} W_{t-1} \right)^{-\frac{1}{\varepsilon^w}} + (1-\xi_w)\tilde{W}_t^{-\frac{1}{\varepsilon^w}}; \quad (2.17)$$

Goods producing firms

7- demand of domestically produced goods

$$Y_t = \gamma\left(\frac{P_{Nt}}{P_t}\right)^{-\theta}\left[C_t + C_t^e + I_t + G_t + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2\right] + X_t, \quad (2.28)$$

8- production function

$$Y_t = \varepsilon_t^Y \left(u_t K_{t-1}\right)^\alpha \left(g^t H_t^{\ \Omega} H_t^{e^{1-\Omega}}\right)^{1-\alpha} - g^t \Phi, \quad (2.36)$$

9- capital-labour ratio

$$H_t = \frac{\Omega(1-\alpha)}{\alpha}\frac{R_{Kt}}{W_t} u_t K_{t-1}, \quad (2.44)$$

10- labour ratio

$$H_t^e = \frac{1-\Omega}{\Omega}\frac{W_t}{W_t^e} H_t, \quad (2.45)$$

11- marginal cost

$$MC_t = \frac{1}{\varepsilon_t^Y}\left(\frac{R_{Kt}}{\alpha}\right)^\alpha \left(\frac{W_t}{\Omega(1-\alpha)}\right)^{\Omega(1-\alpha)} \left(\frac{W_t^e}{(1-\Omega)(1-\alpha)}\right)^{(1-\alpha)(1-\Omega)}, \quad (2.43)$$

12- domestically produced good prices setting equations

intermediate goods firms

$$\tilde{P}_{it} = \frac{E_t \sum_{s=0}^{\infty} (\beta\xi_n)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{it+s} \frac{1+\varepsilon_{t+s}^P}{\varepsilon_{t+s}^P} P_{Nt+s} MC_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\xi_n)^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{Y_{it+s}}{\varepsilon_{t+s}^P}\left(\frac{P_{Nt+s-1}}{P_{Nt-1}}\right)^{l_n} \pi_*^{1-l_n}}, \quad (2.48)$$

final goods firm

$$P_{Nt}^{-\frac{1}{\varepsilon_t^P}} = \xi_n \left(\left(\frac{P_{Nt-1}}{P_{Nt-2}} \right)^{l_n} \pi_*^{1-l_n} P_{Nt-1} \right)^{-\frac{1}{\varepsilon_t^P}} + (1-\xi_n) \tilde{P}_t^{-\frac{1}{\varepsilon_t^P}}; \quad (2.49)$$

Capital producing firms

13- capital law of motion

$$K_t = (1-\partial) K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}} \right) \right) I_t, \quad (2.50)$$

14- Tobin's Q

$$Q_t - \beta E_t \frac{\Xi_{t+1}}{\Xi_t} \left(R_{Kt+1} u_{t+1} - J(u_{t+1}) + (1-\partial) Q_{t+1} \right) = 0, \quad (2.51)$$

15- investment

$$1 - Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \left(\frac{I_t}{I_{t-1}} \right) \right] - \beta E_t \left[\frac{\Xi_{t+1}}{\Xi_t} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] = 0, \quad (2.52)$$

16- capital utilization rate

$$R_{Kt} = J'(u_t); \quad (2.53)$$

Entrepreneurs

17- labour supply

$$H_t^e = 1,$$

18- participation constraint of the foreign lender

$$\frac{R_{t+1}^e Q_t K_t B(\overline{\omega}_t)}{S_{t+1}} = (1 + i_{t+1}^*) D_t^e, \quad (2.55)$$

19- external finance premium

$$E_t\left[R^e_{t+1}\left(B(\overline{\omega}_t)-\frac{B'(\overline{\omega}_t)A(\overline{\omega}_t)}{A'(\overline{\omega}_t)}\right)\right]=E_t\left((1+i^*_{t+1})\frac{S_{t+1}}{S_t}\right), \tag{2.56}$$

20- consumption

$$C^e_t=(1-\nu)R^e_tQ_{t-1}K_{t-1}A(\overline{\omega}_{t-1}), \tag{2.57}$$

21- net worth

$$NW_t=\nu R^e_tQ_{t-1}K_{t-1}A(\overline{\omega}_{t-1})+W^e_t, \tag{2.58}$$

22- return

$$R^e_t=\frac{R_{Kt}u_t+R_{Kt}u_t+(1-\partial)Q_t}{Q_{t-1}}; \tag{2.63}$$

Monetary authority

23- $$R_t=R^{\rho R}_{t-1}\left[R^{ss}\pi_*\left(\frac{\pi_t}{\pi_*}\right)^{\psi 1}\left(\frac{Y_t}{Y_{ss}}\right)^{\psi 2}\left(\frac{\frac{S_t}{S_{t-1}}}{\Delta S_{ss}}\right)^{\psi 3}\right]^{1-\rho_R}e^{\eta t^R}; \tag{2.64}$$

External sector

24- export goods demand

$$X_t=\gamma_1\left(\frac{P_{Nt}}{S_tP^*_t}\left(1+\tau i^*_{t-1}\right)\right)^{-\theta_1}C^*_t, \tag{2.67}$$

25- import goods prices setting equation (incomplete exchange rate pass-through)

intermediate import goods firms price

$$\tilde{P}_{Mt}(i)=\frac{\theta_m}{\theta_m-1}\frac{E_t\sum_{s=0}^{\infty}(\beta\xi_m)^s\frac{\Xi_{t+s}P_t}{\Xi_tP_{t+s}}Y_{Mt+s}(i)P_{Mt+s}MC_{Mt+s}}{E_t\sum_{s=0}^{\infty}(\beta\xi_m)^s\frac{\Xi_{t+s}P_t}{\Xi_tP_{t+s}}Y_{Mt+s}(i)\left(\frac{P_{Mt+s-1}}{P_{Mt-1}}\right)^{l_m}\pi_*^{1-l_m}}, \tag{2.69}$$

final import goods price

$$P_{Mt}^{1-\theta_m} = \xi_m \left(\left(\frac{P_{Mt-1}}{P_{Mt-2}} \right)^{l_m} \pi_*^{1-l_m} P_{Mt-1} \right)^{1-\theta_m} + (1-\xi_m) \tilde{P}_{Mt}^{1-\theta_m}, \tag{2.70}$$

26- intermediate import goods firms' marginal cost

$$MC_{Mt} = \frac{S_t P_t^*}{P_{Mt}},$$

27- import goods demand

$$Y_{Mt} = (1-\gamma) \left(\frac{P_{Mt}}{P_t} \right)^{-\theta} \left(C_t + C_t^e + I_t + G_t + \frac{\psi_D}{2} \left(D_t - \overline{D} \right)^2 \right), \tag{2.73}$$

28- balance of payment

$$C_t + C_t^e + I_t + G_t + S_t D_t + \left(1 + i_t^*\right) S_t D_{t-1}^e + \frac{\psi_D}{2} \left(D_t - \overline{D} \right)^2 + C(\overline{\omega}_{t-1}) R_t^e Q_{t-1} K_{t-1}$$

$$= Y_t + \Pi_{Mt} + J(u_t) K_{t-1} + \left(1 + i_{t-1}^*\right) S_t D_{t-1} + S_t D_t^e . \tag{2.72}$$

The uncertainty economic agents confront in decision-making is characterized by seven exogenous processes introduced to the dynamic stochastic general equilibrium model. There processes are listed in the following table.

Exogenous processes	**symbol**
exogenous premium	ε_t^B
exogenous process in the aggregator function	ε_t^P
total factor productivity	ε_t^Y
government spending	$\hat{\varepsilon}_t^G$
foreign interest rate	i_t^*
terms of trade	ε_t^T
rest of the world aggregate consumption	C_t^*

The processes associated to those exogenous variables are summarized below:

1- the exogenous premium

$$\ln \varepsilon_t^B = (1-\rho_B) \ln \varepsilon^B + \rho_B \ln \varepsilon_{t-1}^B + \eta_t^B, \tag{2.3}$$

2- the foreign interest rate

$$\ln\left(1+i_t^*\right)=(1-\rho_R)\ln\left(1+i_{ss}^*\right)+\rho_R\ln\left(1+i_{t-1}^*\right)+\eta_t^B-\mu_R\eta_{t-1}^B, \tag{2.5}$$

3- the exogenous process to the aggregator function

$$\ln\varepsilon_t^P=(1-\rho_P)\ln\varepsilon^P+\rho_P\ln\varepsilon_{t-1}^P+\eta_t^P-\mu_P\eta_{t-1}^P, \tag{2.25}$$

4- the total factor productivity

$$\ln\varepsilon_t^Y=(1-\rho_Y)\ln\varepsilon^Y+\rho_Y\ln\varepsilon_{t-1}^Y+\eta_t^Y, \tag{2.34}$$

5- the government spending

$$\hat{\varepsilon}_t^G=\rho_G\hat{\varepsilon}_{t-1}^G+\eta_t^G, \tag{2.66}$$

6- the rest of the world's aggregate consumption

$$\ln C_t^*=\left(1-\rho_X\right)\ln C_{ss}^*+\rho_X\ln C_{t-1}^*+\eta_t^X, \tag{2.68}$$

7- and the terms of trade

$$\ln\varepsilon_t^T=\left(1-\rho_T\right)\ln\varepsilon^T+\rho_T\ln\varepsilon_{t-1}^T+\eta_t^T+\mu_T\eta_t^T. \tag{2.71}$$

Seven exogenous shocks drive the economy away from its stationary state (or equivalently the balanced growth path) and trigger business cycles fluctuations. These shocks are:

Exogenous shocks	**symbol**
financial-markets (bonds markets) shock	η_t^B
cost-push shock	η_t^P
productivity shock	η_t^Y
terms of trade shock	η_t^T
export demand shock	η_t^X
government spending shock	η_t^G
monetary policy shock	η_t^R

Finally, the economy features a number of real and nominal frictions susceptible to slow down the adjustment process to exogenous shocks, while increasing the persistence of macroeconomic variables. In the next chapter, the non-linear optimality conditions listed above are used to calibrate the DSGE model such that it mimics the South African economy as closely as possible alongside its long run properties. Appendix 4 presents the log-linearization version of this dynamic stochastic general equilibrium model estimated with Bayesian techniques using South Africa key macroeconomic variables.

Appendix 2: The consumer's demands for domestically produced goods C_{Nt} and import goods C_{Mt}.

The consumer's consumption is a composite constant elasticity of substitution (CES) function index given by

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \gamma \in (0,1) \text{ and } \theta > 0, \quad \text{(A1.1)}$$

where C_{Nt} denotes the household's consumption level of home produced goods and C_{Mt} his consumption level of imported (foreign produced) goods.

At the first stage of his optimization, the household chooses the combination of domestically (or home) produced goods and imported goods that minimizes the cost of achieving any level of a composite consumption index C_t defined by a CES function (A1.1).

To determine the division of his consumption between C_{Nt} and C_{Mt}, the household minimizes his consumption expenditures $P_{Nt}C_{Nt} + P_{Mt}C_{Mt}$ subject to the constraint (A1.1). P_{Nt} is the aggregate price level of home produced goods C_{Nt}, and P_{Mt} represents the aggregate price level of imported goods C_{Mt}.

This corresponds to the minimization problem formulated by

$$\underset{C_{Nt}, C_{Mt}}{Min} P_{Nt}C_{Nt} + P_{Mt}C_{Mt},$$

subject to the constraint

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Let λ_{ct} be the Lagrangian multiplier associated to the constraint (A1.1). The Lagrangian is given by

$$\ell_t = P_{Nt}C_{Nt} + P_{Mt}C_{Mt} + \lambda_{ct}\left\{C_t - \left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\right\}.$$

The first order necessary condition with respect to C_{Nt} is

$$P_{Nt} - \lambda_{ct}\left(\tfrac{\theta}{\theta-1}\right)\left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}}\left(\tfrac{\theta-1}{\theta}\right)\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{-1}{\theta}} = 0,$$

which is equivalent to

$$P_{Nt} = \lambda_{ct}\left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} \gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{-1}{\theta}}. \tag{A1.2}$$

The first order necessary condition with respect to C_{Mt} is the following

$$P_{Mt} = \lambda_{ct}\left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{-1}{\theta}}. \tag{A1.3}$$

We can divide side-by-side equations (A1.2) and (A1.3) and obtain the result

$$\frac{P_{Nt}}{P_{Mt}} = \frac{\gamma^{\frac{1}{\theta}}}{(1-\gamma)^{\frac{1}{\theta}}} \frac{C_{Nt}^{\frac{-1}{\theta}}}{C_{Mt}^{\frac{-1}{\theta}}}.$$

After some rearrangements this yields the following relation between C_{Nt} and C_{Mt};

$$C_{Nt} = \frac{\gamma}{1-\gamma}\left(\frac{P_{Nt}}{P_{Mt}}\right)^{-\theta} C_{Mt}. \tag{A1.4}$$

Equation (A1.4) shows that θ is the elasticity of substitution between domestically produced goods C_{Nt} and imported goods C_{Mt}, and that the household's preferences are homothetic. This means the relative demand C_{Nt}/C_{Mt} depends only on the relative price P_{Nt}/P_{Mt}.

To determine the consumer's price index P_t, one needs to observe that the consumption-based price index P_t measures the minimum expenditure such that the households consumption index C_t equals unit given P_{Nt} and P_{Mt}. In term of the consumer's decision problem, P_t

represents the marginal increase in the total consumption expenditure when the household raises its consumption by a unit. This implies $P_t = \lambda_{ct}$, which indicates a perfect equality between the aggregate consumer's price and the Lagrangian multiplier associated to the household constraint (A1.1).

To obtain the Lagrangian multiplier λ_{ct}, we introduce the expression (A1.4) into the equality (A1.4) to eliminate for C_{Nt} and derived the expression of λ_{ct}. This yields the following development

$$P_{Mt} = \lambda_{ct}\left[\gamma^{\frac{1}{\theta}} C_{Nt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{-1}{\theta}}$$

$$= \lambda_{ct}\left[\gamma^{\frac{1}{\theta}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{\theta-1}{\theta}}\left(\frac{P_{Nt}}{P_{Mt}}\right)^{1-\theta} C_{Mt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{-1}{\theta}}$$

$$= \lambda_{ct}\left[\frac{\gamma^{\frac{1}{\theta}}\gamma^{\frac{\theta-1}{\theta}} P_{Nt}^{1-\theta} C_{Mt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}(1-\gamma)^{\frac{\theta-1}{\theta}} P_{Mt}^{1-\theta} C_{Mt}^{\frac{\theta-1}{\theta}}}{(1-\gamma)^{\frac{\theta-1}{\theta}} P_{Mt}^{1-\theta}}\right]^{\frac{1}{\theta-1}} (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{-1}{\theta}}$$

$$= \lambda_{ct}\left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta}\right]^{\frac{1}{\theta-1}} \frac{1}{P_{Mt}^{-1}}.$$

The final result is given by

$$1 = \lambda_{ct}\left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta}\right]^{\frac{1}{\theta-1}},$$

which we can rewrite as

$$P_t = \left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \qquad \text{(A1.5)}$$

Now, the demand for imported goods is found by substituting (A1.4) into the expression of the composite consumption index (A1.1).

$$C_t = \left[\gamma^{\frac{1}{\theta}}\left(\frac{\gamma}{1-\gamma}\right)^{\frac{\theta-1}{\theta}}\left(\frac{P_{Nt}}{P_{Mt}}\right)^{1-\theta} C_{Mt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Mt}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

$$= \left[\frac{\gamma^{\frac{1}{\theta}} \gamma^{\frac{\theta-1}{\theta}} P_{Nt}^{1-\theta} + (1-\gamma)^{\frac{1}{\theta}} (1-\gamma)^{\frac{\theta-1}{\theta}} P_{Mt}^{1-\theta}}{(1-\gamma)^{\frac{\theta-1}{\theta}} P_{Mt}^{1-\theta}} \right]^{\frac{\theta}{\theta-1}} C_{Mt}$$

$$= \left[\gamma P_{Nt}^{1-\theta} + (1-\gamma) P_{Mt}^{1-\theta} \right]^{\frac{\theta}{\theta-1}} \frac{1}{(1-\gamma) P_{Mt}^{-\theta}} C_{Mt} .$$

When it is recognized that the expression in bracket is $P_t^{-\theta}$, the solution for the imported goods demand is given by

$$C_{Mt} = (1-\gamma) \left(\frac{P_{Mt}}{P_t} \right)^{-\theta} C_t . \qquad \text{(A1.6)}$$

A straightforward introduction of (A1.6) in (A1.4) rules out C_{Mt} and provides the expression for the demand of domestically produced goods.

$$C_{Nt} = \gamma \left(\frac{P_{Nt}}{P_t} \right)^{-\theta} C_t . \qquad \text{(A1.7)}$$

Equation (A1.7) (respectively (A1.6)) establishes the proportionality of the demand for domestically produced goods (respectively imported goods) to the consumption index C_t. The coefficient of proportionality is equal to a fraction γ (respectively $1-\gamma$) of an isoelastic function that depends upon the ratio of domestically produced goods price P_{Nt} (respectively imported goods prices P_{Mt}) to the price of the composite consumption index P_t.

Appendix 3: Deriving the entrepreneur's external finance premium

The external finance premium is derived from the first order necessary conditions with respect to capital and productivity disturbance threshold. The first order necessary condition with respect to the capital good K_t is given by

$$E_t\left(R^e_{t+1}A(\overline{\omega}_t)\right)= E_t\lambda^e_t\left(\frac{R^e_{t+1}B(\overline{\omega}_t)}{S_{t+1}}-\frac{(1+i^*_{t+1})}{S_t}\right), \tag{A2.1}$$

and the first order necessary condition with respect to the threshold $\overline{\omega}_t$ is

$$E_t\left(A'(\overline{\omega}_t)\right)= E_t\left(\lambda^e_t\frac{B'(\overline{\omega}_t)}{S_{t+1}}\right), \text{ or } \lambda^e_t = E_t\left(\frac{A'(\overline{\omega}_t)S_{t+1}}{B'(\overline{\omega}_t)}\right). \tag{A2.2}$$

Expression (A2.2) can be used to substitute for λ^e_t in equation (A2.2), and that substitution yields

$$E_t\left(R^e_{t+1}A(\overline{\omega}_t)\right)= E_t\left(\frac{A'(\overline{\omega}_t)S_{t+1}}{B'(\overline{\omega}_t)}\left(\frac{R^e_{t+1}B(\overline{\omega}_t)}{S_{t+1}}-\frac{(1+i^*_{t+1})}{S_t}\right)\right).$$

After some straightforward rearrangements, the latest expression is rewritten as

$$E_t\left[R^e_{t+1}\left(\frac{A'(\overline{\omega}_t)B(\overline{\omega}_t)}{B'(\overline{\omega}_t)}-A(\overline{\omega}_t)\right)\right]= E_t\left((1+i^*_{t+1})\frac{S_{t+1}}{S_t}\frac{A'(\overline{\omega}_t)}{B'(\overline{\omega}_t)}\right).$$

If we multiply both sides by $\frac{B'(\overline{\omega}_t)}{A'(\overline{\omega}_t)}$, we obtain

$$E_t\left[R^e_{t+1}\left(B(\overline{\omega}_t)-\frac{B'(\overline{\omega}_t)A(\overline{\omega}_t)}{A'(\overline{\omega}_t)}\right)\right]= E_t\left((1+i^*_{t+1})\frac{S_{t+1}}{S_t}\right). \tag{A2.3}$$

Appendix 4: The aggregate balance of payment identity

The consolidation of the households budget constraint (2.5), the entrepreneurs budget constraint (2.55), and the government budget constraint (2.58) yields

$$C_t + C_t^e + G_t + S_t D_t + \left(1 + i_t^*\right) S_t D_t^e + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2 + Q_t K_{t-1}$$

$$= W_t H_t + \Pi_t + \left(1 + i_{t-1}^*\right) S_t D_{t-1} + W_t^e \qquad \text{(A3.1)}$$

$$+ \left(1 - C\left(\overline{\omega}_{t-1}\right)\right) R_t^e Q_{t-1} K_{t-1} + S_t D_t^e .$$

The capital accumulation is given by $K_t = (1-\partial)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$, multiplying both side by Q_t gives the expression $Q_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t = Q_t K_t - \left(1-\partial\right) Q_t K_{t-1}$. (A3.2)

The zero capital producing firm profit implies $Q_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t = I_t - R_{Kt} u_t K_{t-1} - J\left(u_t\right) K_{t-1} = 0$;

substituting $Q_t\left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$ using (A3.2) yields

$$Q_t K_t = \left(1-\partial\right) Q_t K_{t-1} + I_t + R_{Kt} u_t K_{t-1} - J\left(u_t\right) K_{t-1} . \qquad \text{(A3.3)}$$

The household profit aggregates import goods firms profits and domestically produced goods firms' profit: $\Pi_t = \Pi_{Mt} + \Pi_{Nt}$. Domestically produced goods firm's profits are given by $\Pi_{Nt} = Y_t - R_{Kt} u_t K_{t-1} - W_t H_t - W_t^e$ (entrepreneurial labour supply is normalized to one: $H_t^e = 1$). Thus, total profits can be rewritten as

$$\Pi_t = Y_t - R_{Kt} u_t K_{t-1} - W_t H_t - W_t^e + \Pi_{Mt} . \qquad \text{(A3.4)}$$

From the entrepreneur return equation we have $R_t^e Q_{t-1} = R_{Kt} u_t + R_{Kt} u_t + (1-\partial) Q_t$, multiplying both sides by K_{t-1} yields

$$R_t^e Q_{t-1} K_{t-1} = R_{Kt} u_t K_{t-1} + R_{Kt} u_t K_{t-1} + (1-\partial) Q_t K_{t-1}. \quad \text{(A3.5)}$$

Now in the relation (A3.1), we can substitute for $Q_t K_t$ using (A3.3), Π_t using (A3.4), and $R_t^e Q_{t-1} K_{t-1}$ using (A3.5); these substitutions yield the aggregate balance of payments

$$C_t + C_t^e + I_t + G_t + S_t D_t + (1+i_t^*) S_t D_{t-1}^e + \frac{\psi_D}{2}\left(D_t - \overline{D}\right)^2 + C(\overline{\omega}_{t-1}) R_t^e Q_{t-1} K_{t-1}$$

$$= Y_t + \Pi_{Mt} + J(u_t) K_{t-1} + (1+i_{t-1}^*) S_t D_{t-1} + S_t D_t^e. \quad \text{(A3.6)}$$

The import goods sector's real profit is given by $\Pi_{Mt} = \left(\frac{P_{Mt} - S_t P_t^*}{P_t}\right) Y_{Mt}$.

Appendix 5: The linearized DSGE model

Before its log-linearization, the DSGE model is initially detrended with the deterministic trend g. For instance, the capital becomes $\overline{K}_t = \frac{K_t}{g^t}$, the real wage $\overline{W}_t = \frac{W_t}{g^t}$, the consumer marginal utility $\overline{\Xi}_t = g^{-\phi}\Xi_t$. Detrended real variables can be considered as stationary processes that have a well defined steady state.

To obtain the linear approximation of the DSGE model, we apply the log-linearization methodology put forward by Campbell (1994) and which Uhlig (1999) extensively popularises. According to the basic principle of that methodology[31], a (detrented) variable $\overline{X}_t$ can be expressed around its steady state X_{ss} as $\overline{X}_t = X_{ss}e^{\hat{X}_t} = X_{ss}\left(1+\hat{X}_t\right)$. The quantity $\hat{X}_t = \log\left(\frac{\overline{X}_t}{X_{ss}}\right)$ denotes the percentage deviation of the (detrended) variable $\overline{X}_t$ around its steady state value. By applying this simple principle to all economic agents' decision rules and equilibrium conditions, we obtain a system of linear equations. That linear system characterizes the dynamic of the economy in term of small deviations around the steady state (or equivalently the balanced growth path). This appendix presents, for each decision rule, the final equation of the log-linearization. We begin with the households sector.

Households

Six equations determine the household equilibrium. Equations (2.7) and (2.8) determine the demand for domestically produced goods and the demand for import goods respectively. Equation (2.9) gives the consumer price index. Equation (2.10) defines the optimal choice on

[31] Recall that, the first order Taylor series approximation of the exponential function is $e^x \approx 1+x$, and that of the logarithm function is $\log(1+x_t) = x_t$.

the aggregate consumption. Equation (2.11) represents the uncovered interest rate parity condition, which characterises the household's optimal arbitrage between domestic currency denominated debt and foreign currency denominated debt. Equations (2.16) and (2.17) establish the wage dynamic.

The approximation of the household's demand for domestically produced goods is

$$\hat{C}_{Nt} = -\theta\left(\hat{P}_{Nt} - \hat{P}_t\right) + \hat{C}_t . \tag{A4.1}$$

The household's demand for import goods is closely parallel and it is expressed by

$$\hat{C}_{Mt} = -\theta\left(\hat{P}_{Mt} - \hat{P}_t\right) + \hat{C}_t . \tag{A4.2}$$

The percentage deviations from the steady state of the household's consumption of domestically produced goods, as well as its consumption of import goods, depend upon the prices differential and the aggregate consumption both expressed as percentage deviations from the stationary state.

The consumer price index is approximated by:

$$\hat{P}_t = \gamma\left(\frac{P_{Nss}}{P_{ss}}\right)^{1-\theta} \hat{P}_{Nt} + (1-\gamma)\left(\frac{P_{Mss}}{P_{ss}}\right)^{1-\theta} \hat{P}_{Mt} .$$

Taking the first difference, the economy aggregate inflation rate is given by

$$\hat{\pi}_t = \gamma\left(\frac{P_{Nss}}{P_{ss}}\right)^{1-\theta} \hat{\pi}_{Nt} + (1-\gamma)\left(\frac{P_{Mss}}{P_{ss}}\right)^{1-\theta} \hat{\pi}_{Mt}, \text{ where } \hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} . \tag{A4.3}$$

Equation (A4.3) expresses the aggregate inflation rate as a weighted average of the domestically produced goods inflation rate and the imported goods inflation rate.

The household's aggregate consumption equation, with external habit formation, is given by

$$\hat{C}_t = \frac{h/g}{1+h/g}\hat{C}_{t-1} + \frac{1}{1+h/g}E_t\hat{C}_{t+1} - \frac{1-h/g}{(1+h/g)\phi}E_t\left[\left(\frac{i_{ss}}{1+i_{ss}}\right)\hat{i}_t - \hat{\pi}_{t+1} + \hat{\varepsilon}_t^B\right]. \quad \text{(A4.4)}$$

The arbitrage condition for the household's optimal choice of domestic currency denominated debt and foreign currency denominated debt yields the uncovered interest parity condition for household's debt:

$$\psi_D\left(\frac{\overline{D}}{S_{ss}}\right)\hat{D}_t = E_t\left[\left(\frac{i^*_{ss}}{1+i^*_{ss}}\right)\hat{i}^*_t - \left(\frac{i_{ss}}{1+i_{ss}}\right)\hat{i}_t + \hat{\pi}_{t+1} + \hat{S}_{t+1} - \hat{S}_t - \hat{\varepsilon}_t^B\right]. \quad \text{(A4.5)}$$

The household's debt denominated on foreign currency depends positively on the expected real return differential on integrated financial markets. It also depends positively on the foreign currency expected rate of appreciation and negatively on the exogenous premium on the return to bonds.

The real wage equation is given by:

$$\hat{W}_t = \frac{1}{1+\beta g^{1-\phi}}\hat{W}_{t-1} + \frac{\beta g^{1-\phi}}{1+\beta g^{1-\phi}}\hat{W}_{t+1} + \frac{\beta g^{1-\phi}}{1+\beta g^{1-\phi}}\hat{\pi}_{t+1} - \frac{1+\beta g^{1-\phi}l_w}{1+\beta g^{1-\phi}}\hat{\pi}_t + \frac{l_w}{1+\beta g^{1-\phi}}\hat{\pi}_{t-1}$$

$$- \frac{1}{1+\beta g^{1-\phi}}\frac{(1-\xi_w)(1-\beta g^{1-\phi}\xi_w)}{\left(1+\frac{(1+\varepsilon^w)\eta}{\varepsilon^w}\right)\xi_w}\left(\hat{W}_t - \eta\hat{H}_t - \frac{\phi}{1-h/g}\left(\hat{C}_t - (h/g)\hat{C}_{t-1}\right)\right). \quad \text{(A4.6)}$$

The real wage is a function of expected and past real wages and the expected, current, and past inflation rates, where the relative weights depend on the degree of wage indexation.

Goods production firms

The dynamic of the good firm is characterized by the demand for domestically produced goods, the production function, the capital-labour ratio, the marginal cost, and inflation.

The demand for domestically produced goods around the steady state is given by

$$\hat{Y}_t = \gamma\left(\frac{P_{Nss}}{P_{ss}}\right)^{-\theta}\left(\frac{A_{ss}}{Y_{ss}}\right)\left[-\theta\left(\hat{P}_{Nt}-\hat{P}_t\right)+\hat{A}_t\right]+\left(\frac{X_{ss}}{Y_{ss}}\right)\hat{X}_t, \tag{A4.7}$$

where $A_{ss} = C_{ss} + C^e_{ss} + I_{ss} + G_{ss}$ is the steady state domestic absorption. The absorption is approximated around the steady state by $\hat{A}_t = \left(\frac{Y_{ss}}{A_{ss}}\right)\left[\left(\frac{C_{ss}}{Y_{ss}}\right)\hat{C} + \left(\frac{C^e_{ss}}{Y_{ss}}\right)\hat{C}^e_t + \left(\frac{I_{ss}}{Y_{ss}}\right)\hat{I}_t + \hat{\varepsilon}^G_t\right]$,

where $\frac{C_{ss}}{Y_{ss}}, \frac{C^e_{ss}}{Y_{ss}}, \frac{I_{ss}}{Y_{ss}}, \frac{X_{ss}}{Y_{ss}}$, and $\frac{A_{ss}}{Y_{ss}}$ are respectively the steady state ratios to domestic output of household consumption, entrepreneur consumption, investment, exports, and domestic absorption.

In a symmetric equilibrium, the capital services are approximated by $\hat{K}^s_t = \hat{u}_t + \hat{K}_t$. The labour input around the steady state is a linear function of the percentage deviations from the steady state of the households' hours worked and entrepreneur's hours worked: $\hat{L}_t = \Omega\hat{H}_t + (1-\Omega)\hat{H}^e_t$. Since we assume that entrepreneurs supply their labour inelastically, and furthermore we had normalized the entrepreneurial labour to unity, $\hat{H}^e_t$ equals zero around the steady state equilibrium. Labour inputs deviations from the steady state reduce to households' hours worked deviations.

The domestic output around the steady state is a linear function of the percentage deviations of the capital utilization rate, the capital stock, the households' labour supply (recall that entrepreneurs supply their labor inelastically), and the total factor productivity

$$\hat{Y}_t = \left(1+\frac{\Phi}{Y_{ss}}\right)\left[\alpha\left(\hat{u}_t + \hat{K}_{t-1}\right)+(1-\alpha)\Omega\hat{H}_t + \hat{\varepsilon}^Y_t\right]. \tag{A4.8}$$

The deviations from the steady state of the entrepreneur wage are equal to the sum of the deviations around the steady state of the household's wage and hours worked

$$\hat{W}_t^e = \hat{W}_t + \hat{H}_t \,. \tag{A4.9}$$

The short run dynamic around the steady state of the intermediate good firm's capital-labour ratio is a positive function of the household's real wage and depends negatively on the rental rate of capital services exacted from the good production sector. This relation is written in this form

$$\hat{H}_t = \hat{R}_{Kt} - \hat{W}_t + \hat{u}_t + \hat{K}_{t-1} \,. \tag{A4.10}$$

A constant return to scale intermediate firm's technology of production requires that the deviations of the firm's real marginal cost around the steady state equilibrium be a linear function of deviations of factor prices around their steady state values, minus the total factor productivity. The contribution of the deviation of each factor price is proportional to the share of output allocated to that factor. The relation is the following:

$$\hat{MC}_t = \alpha\hat{R}_{Kt} + \Omega(1-\alpha)\hat{W}_t + (1-\Omega)(1-\alpha)\hat{W}_t^e - \hat{\varepsilon}_t^Y \,. \tag{A4.11}$$

In a monopolistic competitive goods market, domestic good prices stickiness as in Calvo (1983) and partial indexation to lagged inflation of prices that cannot be reset optimally impose a sluggish adjustment to the desired mark-up. It follows that, the domestic good inflation rate dynamic is characterized by a New-Keynesian Phillips curve given by

$$\hat{\pi}_{Nt} = \frac{l_n}{1+\beta g^{1-\phi} l_n}\hat{\pi}_{Nt-1} + \frac{\beta g^{1-\phi}}{1+\beta g^{1-\phi} l_n} E_t\hat{\pi}_{Nt+1}$$

$$+ \frac{1}{1+\beta g^{1-\phi} l_n}\frac{(1-\xi_n)(1-\beta g^{1-\phi}\xi_n)}{\xi_n}\left(\hat{MC}_{Nt} + \frac{\varepsilon^P}{1+\varepsilon^P}\hat{\varepsilon}_t^P\right). \tag{A4.12}$$

Domestically produced good inflation (GDP deflator inflation rate) depends positively on the past inflation, the expected inflation, the firm's real marginal cost, and the good production sector mark-up prices shock.

Capital goods firms

The capital goods sector determines the Tobin's Q, the investment, the rental rate of capital, and the capital stock dynamics. The dynamic of the Tobin's Q (market value of capital) is related to its expected future value and the expected rental rate on the capital services. The Tobin's Q also depends negatively on the real interest rate and the exogenous premium in the domestic bond market. The relation is given by

$$\hat{Q}_t = \frac{1-\partial}{R_{Kss}+1-\partial} E_t \hat{Q}_{t+1} + \frac{R_{Kss}}{R_{Kss}+1-\partial} E_t \hat{R}_{Kt+1} - E_t \left[\left(\frac{i_{ss}}{1+i_{ss}} \right) \hat{i}_t - \hat{\pi}_{t+1} + \hat{\varepsilon}_t^B \right]. \tag{A4.13}$$

The dynamic of the investment obtained from the log-linearization of the investment Euler equation is given by

$$\hat{I}_t = \frac{1}{1+\beta g^{1-\phi}} \hat{I}_{t-1} + \frac{\beta g^{1-\phi}}{1+\beta g^{1-\phi}} E_t \hat{I}_{t+1} + \frac{1}{\chi g^2 \left(1+\beta g^{1-\phi}\right)} \hat{Q}_t, \tag{A4.14}$$

where $\chi = S''(1)$ is the steady state elasticity of the capital adjustment cost function. As in Christiano, Eichenbaum and Evans (2005), or Smets and Wouters (2003, 2007), an increase in the elasticity of the cost of adjusting capital lowers the sensitivity of the current investment to the real value of the sector's capital stock.

The capital utilization rate is related to the rental rate of capital paid by the firm, the relation is given by

$$\hat{u}_t = \left(\frac{1-\zeta_u}{\zeta_u} \right) \hat{R}_{Kt}, \tag{A4.15}$$

where ζ_u is a positive function of the elasticity of the marginal cost of adjusting the capital utilization rate $\left(\frac{J''(1)}{J'(1)} \right)$ normalized to be between zero and one.

The capital accumulation law of motion is specified by the expression

$$\hat{K}_t = \frac{1-\partial}{g}\hat{K}_{t-1} + \left(1 - \frac{1-\partial}{g}\right)\hat{I}_t . \tag{A4.16}$$

Entrepreneur sector

Turning to the entrepreneur sector, the entrepreneur borrowing (from foreign lender) is described by the equation

$$E_t\left[\hat{D}_t^e + \left(\frac{i_{ss}^*}{1+i_{ss}^*}\right)\hat{i}_{t+1}^* - \hat{R}_{t+1}^e + \hat{S}_{t+1}\right] = E_t\left[\hat{Q}_t + \hat{K}_t + \left(\zeta_l \frac{\varpi_{ss}}{B(\varpi_{ss})}\right)\hat{\varpi}_t\right], \tag{A4.17}$$

where $\zeta_l = B'(\varpi_{ss})$ is the steady state marginal share of the project's return devoted to the foreign lender and $\frac{B'(\overline{\omega}_{ss})}{B(\overline{\omega}_{ss})}\overline{\omega}_{ss}$ is the steady state elasticity of the aggregate share of the investment's project return devoted to the foreign lender as well. $\overline{\omega}$ is the unconditional mean associated with the investment's project idiosyncratic productivity shock and normalized to one at the steady state. The entrepreneur cost of borrowing over the period $(t, t+1)$ must equal the market value of the capital augmented by a disturbance term, which is proportional to the investment project idiosyncratic productivity disturbance threshold. The coefficient of proportionality is the steady state elasticity of the aggregate fraction of return received by the foreign lender.

The optimal debt contract specifies an investment productivity threshold that determines the premium required by the foreign lender to participate. The linear expression of the finance premium is determined by the equation

$$E_t\left(\hat{R}_{t+1}^e - \zeta_P\hat{\overline{\omega}}_t\right) = E_t\left(\left(\frac{i_{ss}^*}{1+i_{ss}^*}\right)\hat{i}_{t+1}^* + \hat{S}_{t+1} - \hat{S}_t\right), \tag{A4.18}$$

where $\zeta_P = \frac{\wp'(\overline{\omega}_{ss})}{\wp(\overline{\omega}_{ss})}\overline{\omega}_{ss}$ is the steady state elasticity of the external finance premium function, which determines the steady state risk spread.

The entrepreneur consumption depends positively on the entrepreneur's current gross rate of return, the market value of the current stock of capital, the capital stock, and a disturbance term. The disturbance term is a combination of the idiosyncratic productivity distubance and the steady state elasticity of the aggregate share of the investment project's return allocated to the entrepreneur. The relation is given by

$$\hat{C}_t^e = \hat{R}_t^e + \hat{Q}_{t-1} + \hat{K}_{t-1} + \zeta_e \left(\frac{\varpi_{ss}}{A(\varpi_{ss})} \right) \overline{\omega}_{t-1}, \tag{A4.19}$$

where $\zeta_e = A'(\varpi_{ss})$ is the steady state marginal share of the project's return devoted to the entrepreneur and $\frac{A'(\overline{\omega}_{ss})}{A(\overline{\omega}_{ss})}\overline{\omega}_{ss}$ is the steady state elasticity of the aggregate share of the investment project's return allocated to the entrepreneur.

The entrepreneur's net worth is approximated around the steady state as a weighted average of the entrepreneur consumption and real wage. The relation is given by

$$\hat{NW}_t = \frac{\nu}{1-\nu} \left(\frac{C_{ss}^e}{NW_{ss}} \right) \hat{C}_t^e + \left(\frac{W_{ss}^e}{NW_{ss}} \right) \hat{W}_t^e . \tag{A4.20}$$

The entrepreneur's gross return depends on the rental payment on capital services received from producing firms and the expected capital gain associated to the current stock of capital. The expression is given by

$$\hat{Q}_{t-1} + \hat{R}_t^e = 2 \left(\frac{R_{Kss}}{R_{ss}^e} \right) \left(\hat{R}_{Kt} + \hat{u}_t \right) + \left(\frac{1-\partial}{R_{ss}^e} \right) \hat{Q}_t . \tag{A4.21}$$

Monetary authority

The monetary policy reaction function around the steady state equilibrium is described by the equation

$$\left(\frac{i_{ss}}{1+i_{ss}}\right)\hat{i}_t = \rho_R\left(\frac{i_{ss}}{1+i_{ss}}\right)\hat{i}_{t-1} + (1-\rho_R)\left[\psi_1\hat{\pi}_t + \psi_2\hat{Y}_t + \psi_3\left(\hat{S}_t - \hat{S}_{t-1}\right)\right] + \eta_t^R . \quad \text{(A4.22)}$$

External sector

The export demand depends upon the relative price between the home economy and the rest of the world, the exchange rate, the foreign interest rate, and the rest of world aggregate consumption. The export demand is given by

$$\hat{X}_t = -\theta_1\left(\hat{P}_{Nt} - \hat{S}_t - \hat{P}_t^* + \tau\frac{i_{ss}^*}{1+\pi_{ss}^*}\hat{i}_{t-1}^*\right) + \hat{C}_t^* . \quad \text{(A4.23)}$$

Prices stickiness as in Calvo (1983) and partial indexation to lagged inflation as in Smets and Wouters (2007) for import goods prices that cannot re-optimize imply that import goods prices adjust sluggishly to the desired mark-up. The dynamic of the import goods inflation rate is given by

$$\hat{\pi}_{Mt} = \frac{l_m}{1+\beta g^{1-\phi} l_m}\hat{\pi}_{Mt-1} + \frac{\beta g^{1-\phi}}{1+\beta g^{1-\phi} l_m}E_t\hat{\pi}_{Mt+1} + \frac{1}{1+\beta g^{1-\phi} l_m}\frac{(1-\xi_m)\left(1-\beta g^{1-\phi}\xi_m\right)}{\xi_m}\hat{MC}_{Mt} . \quad \text{(A4.24)}$$

The import good firm's marginal cost depends negatively on the domestic prices of import goods, but positively on the exchange rate and the foreign prices of import goods,

$$\hat{MC}_{Mt} = \hat{S}_t + \hat{P}_t^* - \hat{P}_{Mt} . \quad \text{(A4.25)}$$

The approximation of the equilibrium conditions involves the macroeconomic identities. The log-linearization of the aggregate balance of payment yields the following relation:

$$\left(\frac{C_{ss}}{Y_{ss}}\right)\hat{C}_t + \left(\frac{C_{ss}^e}{Y_{ss}}\right)\hat{C}_t^e + \left(\frac{I_{ss}}{Y_{ss}}\right)\hat{I}_t + \hat{\varepsilon}_t^G + \left(\frac{D_{ss}}{Y_{ss}}\right)\left(\hat{S}_t + \hat{D}_t\right)$$

$$+\left(1+i_{ss}^{*}\right)\left(\frac{D_{ss}^{e}}{gY_{ss}}\right)\left[\left(\frac{i_{ss}^{*}}{1+i_{ss}^{*}}\right)\hat{i}_{t}^{*}+\hat{S}_{t}+\hat{D}_{t-1}^{e}\right]+C\left(\varpi_{ss}\right)R_{ss}^{e}\left(\frac{I_{ss}}{\left(g-1+\partial\right)Y_{ss}}\right)\left[\zeta_{c}\left(\frac{\varpi_{ss}}{C\left(\varpi_{ss}\right)}\right)\hat{\varpi}_{t-1}+\hat{R}_{t}^{e}+\hat{Q}_{t-1}+\hat{K}_{t-1}\right]$$

$$=\hat{Y}_{t}+\left(\frac{\Pi_{Mss}}{Y_{ss}}\right)\hat{\Pi}_{Mt}+R_{Kss}\left(\frac{gI_{ss}}{\left(g-1+\partial\right)Y_{ss}}\right)\hat{u}_{t}+\left(1+i_{ss}^{*}\right)\left(\frac{D_{ss}}{gY_{ss}}\right)\left[\left(\frac{i_{ss}^{*}}{1+i_{ss}^{*}}\right)\hat{i}_{t}^{*}+\hat{S}_{t}+\hat{D}_{t-1}\right]$$

$$+\left(\frac{D_{ss}^{e}}{Y_{ss}}\right)\left(\hat{S}_{t}+\hat{D}_{t}^{e}\right), \tag{A4.26}$$

where $\zeta_{c}=C\left(\varpi_{ss}\right)$ is the steady state marginal share of the project's return devoted to the monitoring cost and $\zeta_{c}=\frac{C'\left(\varpi\right)}{C\left(\varpi\right)}\varpi$ is the steady state elasticity of the aggregate share of the investment project's return allocated to monitoring cost.

At the steady state, the import good sector's profits are non-zero and all import goods prices have been adjusted and set as a mark-up of the firm marginal cost at the steady state. This implies $P_{Mss}=\frac{\theta_{m}}{\theta_{m}-1}S_{ss}P_{ss}^{*}$, and the steady state ratio of the import sector's profit to output is defined by $\frac{\Pi_{Mss}}{Y_{ss}}=\frac{1}{\theta_{m}-1}\left(\frac{S_{ss}P_{ss}^{*}}{P_{ss}}\right)\left(\frac{Y_{Mss}}{Y_{ss}}\right)$. The import sector real profit is approximated around the steady state by the relation $\hat{\Pi}_{Mt}=\left(\theta_{m}-1\right)\left[\left(\frac{P_{Mss}}{S_{ss}P_{ss}^{*}}\right)\hat{P}_{Mt}-\hat{S}_{t}-\hat{P}_{t}^{*}\right]-\hat{P}_{t}+\hat{Y}_{Mt}$.

Following the approximation of the domestically produced goods demand defined in equation (A4.9), the import goods demand is given by

$$\hat{Y}_{Mt}=-\theta\left(\hat{P}_{Mt}-\hat{P}_{t}\right)+\left(\frac{Y_{ss}}{A_{ss}}\right)\left[\left(\frac{C_{ss}}{Y_{ss}}\right)\hat{C}_{t}+\left(\frac{C_{ss}^{e}}{Y_{ss}}\right)\hat{C}_{t}^{e}+\left(\frac{I_{ss}}{Y_{ss}}\right)\hat{I}_{t}+\hat{\varepsilon}_{t}^{G}\right]. \tag{A4.27}$$

The approximation of the premium ε_{t}^{B} is given by the first-order autoregressive process with an IID-Normal error term $\hat{\varepsilon}_{t}^{B}=\rho_{B}\hat{\varepsilon}_{t-1}^{B}+\eta_{t}^{B}$. (A4.28)

The foreign interest rate is exogenous for the domestic economy, and it follows an ARMA (1.1) with an IID-Normal innovation

$$\left(\frac{i^*_{ss}}{1+i^*_{ss}}\right)\hat{i}^*_t = \rho_{R^*}\left(\frac{i^*_{ss}}{1+i^*_{ss}}\right)\hat{i}^*_{t-1} + \eta^B_t - \mu_R\eta^B_{t-1}. \quad \text{(A4.29)}$$

Domestically produced goods prices mark-up shock follows an ARMA (1,1) process where η^P_t is an IID-Normal mark-up prices shock $\hat{\varepsilon}^P_t = \rho_P\varepsilon^P_{t-1} + \eta^P_t - \mu_P\eta^P_{t-1}$. (A4.30)

The total factor productivity follows an autoregressive process with an IID-Normal error term $\hat{\varepsilon}^Y_t = \rho_Y\hat{\varepsilon}^Y_{t-1} + \eta^Y_t$. (A4.31)

The approximation of the rest of the world consumption is given by a first order autoregressive process with an IID-Normal error term $\hat{C}^*_t = \rho_X\hat{C}^*_{t-1} + \eta^X_t$. (A4.32)

The approximation of the terms of trade ε^T_t is given by the following ARMA (1,1) process with an IID-Normal error term $\hat{\varepsilon}^T_t = \rho_T\hat{\varepsilon}^T_{t-1} + \eta^T_t + \mu_T\eta^T_{t-1}$, this relation is rewritten as

$$\left(\hat{P}_{Nt} - \hat{P}^*_t - \hat{S}_t\right) = \rho_T\left(\hat{P}_{Nt-1} - \hat{P}^*_{t-1} - \hat{S}_{t-1}\right) + \eta^T_t + \mu_T\eta^T_{t-1}. \quad \text{(A4.33)}$$

The approximation of government spending is given by a first order autoregressive process with an IID-Normal error term $\hat{\varepsilon}^G_t = \rho_G\hat{\varepsilon}^G_{t-1} + \eta^G_t$. (A4.34)

The linear system, characterized by equations (A4.1) to (A4.27), determines the dynamic properties of the 27 endogenous variables listed below[32]. The seven exogenous processes are characterized by equations (A4.28) to (A4.34). The system of linear equations (A4.1) to (A4.34) determines the linearized model used to estimate the structural parameters.

[32] Note that the normalization of the entrepreneurial labour to unit rules out that variable in the log-linearization, which reduced the number of endogenous variables from 28 in the non linear system to 27 in the linear rational expectation system.

Sector	variables name	symbol
Households		
	consumption of domestically produced goods	$\hat{C}_{Nt}$
	consumption of import goods	$\hat{C}_{Mt}$
	consumer price index or inflation	$\hat{P}_t \quad \hat{\pi}_t$
	aggregate consumption	$\hat{C}_t$
	debt in foreign currency	$\hat{D}_t$
	Households real wage	$\hat{W}_t$
Good production firms		
	production	$\hat{Y}$
	return on capital services	$\hat{R}_{Kt}$
	demand for households labour	$\hat{H}_t$
	entrepreneurs real wage	$\hat{W}_t^e$
	marginal cost	$\hat{MC}_t$
	domestically produced goods prices or inflation	$\hat{P}_{Nt} \quad \hat{\pi}_{Nt}$
Capital goods firms		
	capital	$\hat{K}_t$
	Tobin's Q	$\hat{Q}_t$
	investment	$\hat{I}_t$
	capital utilization rate	$\hat{u}_t$
Entrepreneur sector		
	debt in foreign currency	$\hat{D}_t^e$
	investment idiosyncratic productivity threshold	$\hat{\bar{\omega}}_t$
	consumption	$\hat{C}_t^e$
	net worth	$\hat{NW}_t$
	return	$\hat{R}_t^e$
Monetary authority		
	central bank interest rate	$\hat{i}_t$
External sector		
	exports	$\hat{X}_t$
	import goods price or inflation	$\hat{P}_{Mt} \quad \hat{\pi}_{Mt}$
	firm marginal cost	$\hat{MC}_{Mt}$
	import goods demand	$\hat{Y}_{Mt}$
	exchange rate	$\hat{S}_t$

Chapter 3

Calibration, estimation, and evaluation of the DSGE model

The linearized dynamic stochastic general equilibrium model describes the deviations of the endogenous variables as linear equations. Coefficients associated with those linear equations involve structural parameters and steady state ratios. Structural parameters are a collection of technological parameters, preference parameters, and auxiliary or nuisance parameters that include standard deviations of exogenous shocks and persistence of disturbances. Steady state ratios are determined by functions of macroeconomic variables evaluated at the steady state. These steady state ratios appear on equilibrium conditions such as the balance of payments, or they are embodied in some linear decision rules such as the entrepreneur's net worth decision rule.

In this chapter, we determine the values of the structural parameters that consistently describe a developing Sub-Saharan African economy along its long-run and short-run dimensions. Since each Sub-Saharan African economy might have its own set of consistent parameter values, a tedious task will be to attempt at assessing the values of structural parameters for each economy. To avoid that unnecessary lengthy and tiresome task, we rather focus on a specific case study, the South African economy. In an empirical application of DSGE models, the South African economy offers some interesting advantages. First, it presents a diversified economic activity and an integrated economic structure. Second, and unlike other Sub-Saharan African economies, reliable data are available and provide information on that

economy at a quarterly frequency. The task of determining consistent values of structural parameters to this economy is achieved through two major steps.

First, we focus on calibrating the parameters that are functions of steady state ratios. Those parameters are weakly identified by variables expressed as deviations from the steady state and therefore cannot be pinned down consistently by estimating the linearized DSGE model with detrended data. The calibration aims at accurately depicting the structure of a developing economy, hence the South African economy, along its long-run properties. This implies determining a set of parameter values consistent with the steady state ratios presented in the first chapter. To achieve that goal, we solve for the structural parameters using the non-linear decision rules evaluated at the steady state. Structural parameters to calibrate are therefore reversed engineer as functions of endogenous variables evaluated at the steady state. Sample long-run averages are subsequently used to compute the values of those parameters.

Second, we infer the values of the remaining structural parameters by estimating the linearized DSGE model using the South Africa macroeconomic data transformed into mean-zero covariance stationary stochastic processes. Performing the statistical inference with mean-zero covariance stationary variables, which describe macroeconomic variables expressed as percentage deviations from the steady state or the balanced growth path, aims at establishing a symmetric of treatment and a correspondence between the theoretical model's variables and what measure the actual data.

Methods to estimate DSGE models are various and generally fall into the category of limited information methods or that of full information methods. Generalized method of moments (GMM), simulated method of moments (SMM), and indirect-inference methods are

examples of limited information methods; maximum likelihood and Bayesian estimation are full information based methods.

We estimate the linearized DSGE model with Bayesian techniques. The estimation requires using the Kalman filter algorithm to form the maximum likelihood function of the observed data and maximizing the posterior distribution through numerical methods. The non-existence of a clear analytical solution to the maximization of the posterior distribution motivates the use of numerical methods. Once the posterior mode and the hessian at the mode have been computed, we rely on a Monte-Carlo Markov-Chain technique, hence, the Metropolis-Hastings algorithm, to construct draws from the posterior distribution and get its complete picture. To test the stability of the sample and monitor the convergence of the posterior distribution to its target distribution, we calculate convergence diagnostic as described in Brooks and Gelman (1998) and compare between and within moments of multiple chains.

Bayesian estimation requires the formation of prior distributions to the structural parameters, which are combined with the likelihood function of the observed data to form the posterior density. This approach allows once to formalize the prior information on structural parameters coming from either microeconometric studies or previous macroeconomic studies, and therefore establishes the link with previous business cycles studies. In this research, we form our prior distributions upon a multiple strategies. First, we use a training sample to build up prior information on structural parameters for which we had little information. Second, we exploit empirical results from macroeconomic studies on developed and developing economies to form prior information on the structural parameters linked to the rest of the world.

Once consistent values of parameters have been obtained, a model evaluation, which constitutes the last step of this chapter, is implemented. The purpose is the assessment of the

parameterised DSGE model ability to capture the empirical dynamic and stochastic of South Africa data. We therefore conduct a posterior predictive analysis where the artificial data generated by the calibrated-estimated DSGE model are compared to the actual data. More specifically, we compare the empirical and the model-based variances and cross-covariances using Vector Autoregressions (VARs) based Autocovariances functions.

3.1-Model calibration

This section calibrates the DSGE model's parameters that are related to the long run values of the endogenous variables. Those parameters could not be pinned down properly by estimating the linearized model. From a methodological viewpoint, the calibration exercise, pioneered by Kydland and Prescott (1982), parameterized a structural model to address a quantitative question. The quantitative question of interest generally falls into one of the following two headings: the assessment of the theoretical implications of changes in policy, or the assessment of a model ability to capture the salient features of the actual economy, which is for instance a purpose in this business cycles analysis. In this empirical application, the calibration exercise uses the South Africa macroeconomic data briefly presented in the first chapter. The DSGE model economy developed in the second chapter is calibrated such that its mimics the South African economy as closely as possible alongside its long run properties or dimensions.

3.1.1 Calibrated structural parameters

Reversing to the notation used in chapter 2, the discount factor β, the share of domestically produced goods in the CPI γ, and the elasticity of substitution between

domestically produced goods and import goods θ are the parameters calibrated in the household sector. We also set the steady state value of the parameter ε^{w} that captures the markup in the wage setting equation since convergence is not easily achieved by attempting to estimate that parameter. In the goods and capital production sectors, we calibrate α the share of capital in the domestic output, the depreciation rate of capital ∂, and Ω the share of household's labour in the effective labour. The entrepreneurs' aggregate saving rate ν is calibrated in the entrepreneur sector. We calibrate these parameters using a South Africa data sample covering the period 1960:01-2008:04.

We begin by calibrating the household sector parameters and the capital depreciation rate. Equation (2.10) evaluated at the steady state yields $\frac{1}{\beta} = \frac{1+i_{ss}}{\pi_{*}}$, or $\beta = \frac{\pi_{*}}{1+i_{ss}}$. The sample long run average of the nominal central bank interest rate is 13.43% on an annual basis or 3.36% on a quarterly basis. The sample long run average of the gross inflation rate is 8.71% on an annual basis or 2.18% on a quarterly basis. These values imply a subjective discount factor of $\beta = \frac{1.0871}{1.1343} = 0.958$ on an annual basis, or $\beta = \frac{1.0218}{1.0336} = 0.989$ on a quarterly basis. The investment Euler equation implies $Q_{ss} = 1$ at the steady state. Turning to the Tobin's Q equation evaluated at the steady state, the following relation holds $1 = \beta\left(R_{Kss} + 1 - \partial\right)$, which implies $\frac{1}{\beta} = R_{Kss} + 1 - \partial$ or $\partial = 1 + R_{kss} - \frac{1}{\beta}$. The sample long run average of the rental rate of capital services, as measured by the real lending rate on loans for purchasing capital goods, is 9.16% on an annual basis or 2.29% on a quarterly basis, which implies a capital depreciation rate of $\partial = 1 + 0.023 - \frac{1}{0.989} = 0.011$ on a quarterly basis.

The demand for domestically produced goods, equation (2.28), evaluated at the steady state gives $Y_{ss} = \gamma\left(\frac{P_{Nss}}{P_{SS}}\right)^{-\theta} A_{ss} + X_{ss}$, where $A_{ss} = C_{ss} + C_{ss}^{e} + I_{ss} + G_{ss}$ is the steady state domestic absorption. This expression is rearranged as follows $\frac{Y_{ss} - X_{ss}}{\gamma A_{ss}} = \left(\frac{P_{Nss}}{P_{ss}}\right)^{-\theta}$, and taking the logarithm on both side yields

$$\log\left(\frac{Y_{ss} - X_{ss}}{\gamma A_{ss}}\right) = -\theta \log\left(\frac{P_{Nss}}{P_{ss}}\right). \tag{3.1}$$

In a similar development, the import goods demand, equation (2.73), yields at the steady state the relation

$$\log\left(\frac{Y_{Mss}}{(1-\gamma)A_{ss}}\right) = -\theta \log\left(\frac{P_{Mss}}{P_{ss}}\right). \tag{3.2}$$

Divide (3.1) and (3.2) side by side gives the expression

$$\frac{\log\left(\frac{Y_{ss} - X_{ss}}{\gamma A_{ss}}\right)}{\log\left(\frac{Y_{Mss}}{(1-\gamma)A_{ss}}\right)} = \frac{\log\left(\frac{P_{Nss}}{P_{ss}}\right)}{\log\left(\frac{P_{Mss}}{P_{ss}}\right)}\ . \tag{3.3}$$

The right side of expression (3.3), let denote it $a_0 = \frac{\log\left(\frac{P_{Nss}}{P_{ss}}\right)}{\log\left(\frac{P_{Mss}}{P_{ss}}\right)}$, is straightforwardly evaluated from the sample data. The sample long run average of the domestically produced goods prices index (GDP deflator index) is $P_{Nss} = 56.987$. The sample long run average of the import goods prices index is $P_{Mss} = 43.951$, and the long run average of the consumer price index

is $P_{ss} = 41.188$ in that same sample[33]. These values imply $a_0 = \frac{\log(1.384)}{\log(1.067)} = 4.999$. The expression (3.3) could therefore be rewritten as $\log\left(\frac{Y_{ss} - X_{ss}}{\gamma A_{ss}}\right) = a_0 \log\left(\frac{Y_{Mss}}{(1-\gamma)A_{ss}}\right)$, or $\log\left(\frac{Y_{ss} - X_{ss}}{\gamma A_{ss}}\right) = \log\left(\frac{Y_{Mss}}{(1-\gamma)A_{ss}}\right)^{a_0}$. This last expression could be re-arranged as an equation in γ, which is given by the following expression:

$$\left(\frac{Y_{ss} - X_{ss}}{A_{ss}}\right)(1-\gamma)^{a_0} - \gamma\left(\frac{Y_{Mss}}{A_{ss}}\right)^{a_0} = 0. \qquad (3.4)$$

The sample long run average of the GDP is 992.298 billions of rands, the sample long run average of the absorption is 902.582 billions of rands, the sample long run average of import goods is 219.565 billions of rands, and the sample long run average of exports is 238.669 billions of rands[34]. These measures implies the following values $\frac{Y_{Mss}}{A_{ss}} = \frac{219.565}{902.582} = 0.243$ and $\frac{Y_{ss} - X_{ss}}{A_{ss}} = \frac{992.298 - 238.669}{902.582} = 0.835$. The equation $\frac{(1-\gamma)^{-4.999}}{\gamma} - 0.00102 = 0$ corresponds to these sample ratios. We use numerical methods[35] to solve for that equation and obtain a value $\gamma = 0.749$. Equation (3.1) then implies $\theta = -\frac{\log\left(\frac{Y_{Mss}}{(1-\gamma)A_{ss}}\right)}{\log\left(\frac{P_{Mss}}{P_{ss}}\right)} = -\frac{\log(0.969)}{\log(1.067)} = 0.492$ using the value of γ given above.

[33] Note that the South Africa consumer price index was recently rebased to year 2008, but this calibration uses the consumer prices index 2000 = 100 since it shares the same base year with the index of domestically produced goods prices (GDP deflator). We also rebase the index of import goods prices to 2000 as the series provided by the South African Reserve Bank is 2005=100. The three indexes therefore have the same base, which eliminates on the price ratios any eventual effect dues to the difference in the base.

[34] Recall that real sector data are quarterly at constant 2005 prices and seasonally adjusted.

[35] We solve equation (3.4) in two steps using the Matlab non-linear solver fsolve.m. First we write an M-file that computes equation (3.4) for any value of γ; then, we invoke the fsolve.m optimization routine. The optimization routine applies the Gauss-Newton algorithm to find a zero.

Next, we undertake the calibration of Ω the share of the household's labour in the effective labour and that of α the share of capital in the domestic output. Equation (2.45) evaluated at the steady state gives $W_{ss}^{e}H_{ss}^{e}=\frac{1-\Omega}{\Omega}W_{ss}H_{ss}$; where $W_{ss}H_{ss}$ is the steady state income households receive from labour supply, and $W_{ss}^{e}H_{ss}^{e}$ is the entrepreneur steady state labour income. This relation yields $\Omega=\frac{W_{ss}H_{ss}}{W_{ss}H_{ss}+W_{ss}^{e}H_{ss}^{e}}$. The sample long run average of the households labour income is 520.170 billions of rands, and the sample long run average of the entrepreneurs labour income is 24.549 billions of rands. Thus, the share of household labour supply in the total labour input is given by $\Omega=\frac{520.170}{520.170+24.549}=0.955$ in this sample.

The capital to labour ratio, equation (2.44), gives at the steady state the relation

$$W_{ss}H_{ss}=\frac{\Omega(1-\alpha)}{\alpha}R_{Kss}u_{ss}K_{ss}. \tag{3.5}$$

At the steady state $u_{ss}=1$; in addition, the steady state capital accumulation, evaluated from the capital accumulation law of motion (2.50), is given by $K_{ss}=\frac{I_{ss}}{\partial}$. These equalities allow for rewriting equation (3.5) as $W_{ss}H_{ss}=\frac{\Omega(1-\alpha)}{\alpha}R_{Kss}\frac{I_{ss}}{\partial}$, which implies $\alpha=\frac{\Omega R_{Kss}I_{ss}}{\delta W_{ss}H_{ss}+\Omega R_{Kss}I_{ss}}$.

The sample long run average of the investment is 157.205 billions of rands, which implies, in combination with the previously mentioned long run averages for $W_{ss}H_{ss}$, $W_{ss}^{e}H_{ss}^{e}$, and R_{kss}, a value of $\alpha=\frac{0.955*0.023*157.205}{0.011*520.170+0.955*0.023*157.205}=0.368$.

In the next step, we calibrate the entrepreneur saving rate ν. This parameter could be solved from equation (2.57) that determines the entrepreneur consumption, but we first need the

steady state aggregate fraction of return received by the entrepreneur. Ingredients to achieve this preliminary step are the steady state share of the investment's project return received by the foreign lender and the steady state share of return allocated to monitoring cost.

Equation (2.55) that determines the foreign lender's participation constraint in the investment project yields the expression $R^e_{ss}K_{ss}B(\varpi_{ss}) = (1+i^*_{ss})(Q_{ss}K_{ss} - NW_{ss})$ at the steady state. This relation could be rearranged to obtain the steady state share of the investment's project return received by the foreign lender as $B(\varpi_{ss}) = \left(\frac{1+i^*_{ss}}{R^e_{ss}}\right)\left(1-\frac{NW_{ss}}{Q_{ss}K_{ss}}\right)$, where $\frac{NW_{ss}}{Q_{ss}K_{ss}} = \frac{\partial NW_{ss}}{I_{ss}}$ is the inverse of the steady state leverage ratio (recall that $Q_{ss}=1$ and $\partial K_{ss} = I_{ss}$). The sample long run average of the foreign interest rate over the period 1960:01-2008:04, as measured by the United Kingdom clearing bank lending rate, is 8.302% on an annual basis, or 2.075% on a quarterly basis. Equation (2.63) at the steady state gives $R^e_{ss} = 2R_{Kss} + 1 - \partial$. Given the sample average rental rate of 2.29% and the capital depreciation rate of 0.011, both on a quarterly basis, the entrepreneur's gross return is $R^e_{ss} = 2*0.0229 + 1 - 0.011 = 1.034$ on a quarterly basis. Aron and Muellbauer (2004, 2006) provide estimates for (entrepreneurs) households' balance sheet for South Africa. The (entrepreneurs) household's net worth is evaluated at about 3,308 billions of rands in 2005. The sample long run average of investment is 157.205 billions of rands; given the capital depreciation rate of 0.01136, the steady state stock of capital is around 13,832 billions of rands. The leverage ratio is $\frac{Q_{ss}K_{ss}}{NW_{ss}} = \frac{13{,}832}{3{,}308} = 4.18$. These quantities yield a steady state value of $B(\varpi_{ss}) = \left(\frac{1.021}{1.034}\right)\left(1-\frac{1}{4.18}\right) = 0.751$ to the share of investment return allocated to the foreign lender.

To compute the steady state aggregate fraction of monitoring cost, we use the balance of payments identity evaluated at the steady state and we obtain

$$C_{ss} + C_{ss}^e + (1-\partial) I_{ss} + G_{ss} + i_{ss}^* D_{ss}^e + C(\varpi_{ss}) R_{ss}^e \left(\frac{I_{ss}}{\partial} \right)$$

$$= Y_{ss} + \Pi_{Mss} + i_{ss}^* D_{ss},$$

which implies $C(\varpi_{ss}) = \frac{\partial Y_{ss}}{R_{ss}^e I_{ss}} \left[1 + \frac{\Pi_{Mss}}{Y_{ss}} + i_{ss}^* \frac{D_{ss}}{Y_{ss}} - \left(\frac{C_{ss}}{Y_{ss}} + \frac{C_{ss}^e}{Y_{ss}} + (1-\partial) \frac{I_{ss}}{Y_{ss}} + \frac{G_{ss}}{Y_{ss}} + i_{ss}^* \frac{D_{ss}^e}{Y_{ss}} \right) \right]$.

The steady state ratio of the import goods sector's profit to output is $\frac{\Pi_{Mss}}{Y_{ss}} = 0.009$ for a ratio $\frac{Y_{Mss}}{Y_{ss}} = 0.221$; the other sample's long run ratios are $\frac{D_{ss}}{Y_{ss}} = 0.115$, $\frac{C_{ss}}{Y_{ss}} = 0.517$, $\frac{C_{ss}^e}{Y_{ss}} = 0.051$, $\frac{I_{ss}}{Y_{ss}} = 0.158$, $\frac{G_{ss}}{Y_{ss}} = 0.183$, and $\frac{D_{ss}^e}{Y_{ss}} = 0.121$. These steady state ratios imply a steady state aggregate fraction of monitoring cost $C(\varpi_{ss}) = 0.007$.

At the steady state, the equality $A(\varpi_{ss}) + B(\varpi_{ss}) = 1 - C(\varpi_{ss})$ holds. Thus, the steady state share of return received by the entrepreneur is given by $A(\varpi_{ss}) = 1 - B(\varpi_{ss}) - C(\varpi_{ss})$, which corresponds to a value $A(\varpi_{ss}) = 1 - 0.751 - 0.007 = 0.242$ given the values of $B(\varpi_{ss})$ and $C(\varpi_{ss})$ calculated earlier. Given this value of $A(\varpi_{ss})$; equation (2.57) evaluated at the steady state gives $C_{ss}^e = (1-\nu) R_{ss}^e K_{ss} A(\varpi_{ss})$, or $\nu = 1 - \frac{\partial}{R_{ss}^e A(\varpi_{ss})} \left(\frac{C_{ss}^e}{I_{ss}} \right)$. The sample gives a long-run average $\frac{C_{ss}^e}{I_{ss}} = 0.323$, and given the other parameter values or long run averages calculated above, the entrepreneur saving rate is $\nu = 1 - \frac{0.011}{1.034 * 0.751} * 0.323 = 0.985$.

To conclude the calibration exercise, we set the steady state value of the curvature parameters of the aggregator function in the labour market (ε^w) to 1.01 since efforts to pin down that parameter by estimating the model does not induce a convergent optimizing process[36]. The elasticity of substitution between varieties in the import goods sector is set to 11 such that the steady state mark up in that sector is 10%. The calibrated structural parameters are summarized on table 3.1.

Table 3.1: Parameters calibrated[37]

Parameter	Calibrated value	Definition
α	0.368	share of capital in the production of domestic goods
β	0.989	household's discount factor
∂	0.011	capital depreciation rate
θ	0.492	elasticity of substitution between domestically produced and import goods
γ	0.749	share of domestically produced goods in the CPI
Ω	0.955	share of household labour in effective labour
ν	0.985	aggregate entrepreneur saving rate
ε^w	1.01	curvature parameter in the labour market
θ_m	11	elasticity of substitution between varieties in the import goods sector
γ_1	0.00074	share of import goods (South Africa exports) in the rest of the world CPI

In the next section, we use the above calibrated parameters to compute the entrepreneurs' steady state ratios that appear in the linear net worth equation (A4.20).

3.1.2 The entrepreneur sector's steady state ratios

We calculate two steady state ratios in the entrepreneur sector; the ratio of the entrepreneur consumption to net worth $\frac{C^e_{ss}}{NW_{ss}}$, and the entrepreneur wage to net worth ratio $\frac{W^e_{ss}}{NW_{ss}}$. At the steady state, the entrepreneur consumption is given by the

[36] We use various values to calibrate this parameter and the value 1.01 has emerged as the once that produces the best marginal likelihood for the estimated DSGE model.

[37] In the section "3.2.2 Prior distributions of the estimated parameters", we explain in detail the calibration of the parameter γ_1 as it relates to our formulation of prior information on τ, the share of export financed by trade credit.

relation $C_{ss}^{e}=(1-\nu)R_{ss}^{e}K_{ss}A(\overline{\omega}_{ss})$, which can be rewritten as $\frac{C_{ss}^{e}}{1-\nu}=R_{ss}^{e}K_{ss}A(\overline{\omega}_{ss})$. This equality is used in equation (2.58) to rewrite the entrepreneur real net worth as $NW_{ss}=\frac{\nu}{1-\nu}C_{ss}^{e}+W_{ss}$, which yields the relation

$$1=\frac{\nu}{1-\nu}\left(\frac{C_{ss}^{e}}{NW_{ss}}\right)+\left(\frac{W_{ss}}{NW_{ss}}\right). \tag{3.6}$$

We can therefore determine $\frac{W_{ss}^{e}}{NW_{ss}}$ and use equation (3.6) to compute the value of $\frac{C_{ss}^{e}}{NW_{ss}}$.

The foreign lender participation constraint is written at the steady state as $R_{ss}^{e}K_{ss}B(\overline{\omega}_{ss})=(1+i_{ss}^{*})(K_{ss}-NW_{ss})$, this equality is re-arranged in the manner that reflects the steady state entrepreneur net worth

$$NW_{ss}=K_{ss}\left[1-\left(\frac{R_{ss}^{e}}{1+i_{ss}^{*}}\right)B(\overline{\omega}_{ss})\right]. \tag{3.7}$$

The intermediate good firm's demand for entrepreneur labour, equation (2.39), evaluated at the steady state gives the expression $W_{ss}^{e}=(1-\alpha)(1-\Omega)g^{(1-\alpha)}\Theta_{ss}\frac{Y_{ss}+\Phi}{H_{ss}^{e}}$; with the entrepreneur steady state labour supply normalized to unit, this relation becomes

$$W_{ss}^{e}=(1-\alpha)(1-\Omega)g^{(1-\alpha)}\Theta_{ss}(Y_{ss}+\Phi). \tag{3.8}$$

The steady state ratio $\frac{W_{ss}^{e}}{NW_{ss}}$ is obtained by dividing (3.8) and (3.7) side by side; this gives

$$\frac{W_{ss}^{e}}{NW_{ss}}=\left(1+\frac{\Phi}{Y_{ss}}\right)\frac{(1-\alpha)(1-\Omega)}{\left[1-\left(\frac{R_{ss}^{e}}{1+i_{ss}^{*}}\right)B(\overline{\omega}_{ss})\right]}g^{(1-\alpha)}\Theta_{ss}\left(\frac{Y_{ss}}{K_{ss}}\right). \tag{3.8}$$

Note that the rental rate of capital services, equation (2.37), evaluated at the steady state yields $R_{Kss} = \alpha g^{(1-\alpha)}\Theta_{ss}\left(1+\frac{\Phi}{Y_{ss}}\right)\left(\frac{Y_{ss}}{K_{ss}}\right)$, which implies $\frac{Y_{ss}}{K_{ss}} = \frac{R_{Kss}}{\alpha g^{(1-\alpha)}\Theta_{ss}\left(1+\frac{\Phi}{Y_{ss}}\right)}$. This expression is used to substitute for $\frac{Y_{ss}}{K_{ss}}$ in equation (3.8), and we obtain $\frac{W^e_{ss}}{NW_{ss}} = \frac{(1-\alpha)(1-\Omega)}{\alpha}\frac{R_{Kss}}{\left[1-\left(\frac{R^e_{ss}}{1+i^*_{ss}}\right)B(\varpi_{ss})\right]}$. This steady state ratio is easily computes since it involves the previously calibrated parameters and the long-run averages. Thus, we have $\frac{W^e_{ss}}{NW_{ss}} = \frac{(1-0.368)*(1-0.955)*0.023}{0.368*\left(1-\left(\frac{1.034}{1.021}\right)*0.751\right)} = 0.007$. Reversing to the relation (3.6); the expression $\frac{C^e_{ss}}{NW_{ss}} = \frac{1-\nu}{\nu}\left(1-\frac{W^e_{ss}}{NW_{ss}}\right)$ holds, and $\frac{C^e_{ss}}{NW_{ss}} = \frac{1-0.985}{0.985}(1-0.007) = 0.015$. The steady state ratios consistent to the calibrated parameters and characterizing the long-run properties of the South African economy are summarized on table 3.2.

Table 3.2: Steady state ratios and sample long-run averages

Ratio	Samples long-run average	Definition
$\frac{C_{ss}}{Y_{ss}}$	0.517	steady state ratio of household consumption to output
$\frac{C^e_{ss}}{Y_{ss}}$	0.051	steady state ratio of entrepreneur consumption to output
$\frac{I_{ss}}{Y_{ss}}$	0.158	steady state ratio of investment to output
$\frac{G_{ss}}{Y_{ss}}$	0.183	steady state ratio of government spending to output
$\frac{X_{ss}}{Y_{ss}}$	0.241	steady state ratio of exports to output
$\frac{Y_{Mss}}{Y_{ss}}$	0.221	steady state ratio of imports to output
$\frac{A_{ss}}{Y_{ss}}$	0.910	steady state ratio of domestic absorption to output
$\frac{D_{ss}}{Y_{ss}}$	0.115	steady state ratio of households' debt denominated in foreign currency to output
$\frac{D^e_{ss}}{Y_{ss}}$	0.121	steady state ratio of entrepreneurs' debt denominated in foreign currency to output

$\frac{C_{ss}^{e}}{NW_{ss}}$	0.015	steady state ratio of entrepreneur consumption to real net worth
$\frac{W_{ss}^{e}}{NW_{ss}}$	0.007	steady state ratio of entrepreneur wage to real net worth
$\frac{P_{Nss}}{P_{ss}}$	1.384	steady state ratio of domestically produced goods prices to CPI
$\frac{P_{Mss}}{P_{ss}}$	1.067	steady state ratio of imports goods price to CPI
$\frac{P_{ss}^{*}}{P_{ss}}$	1.545	steady state ratio of the rest of the world CPI to domestic economy CPI
$\frac{\Pi_{Mss}}{Y_{ss}}$	0.009	steady state ratio of import goods sector's profits to output
$\frac{X_{ss}}{C_{ss}^{*}}$	0.001	steady state ratio of the home economy's export to the rest of the world consumption
$A(\varpi_{ss})$	0.242	steady state share of the investment project's return devoted to entrepreneurs
$B(\varpi_{ss})$	0.751	steady state share of the investment project's return devoted to foreign lenders
$C(\varpi_{SS})$	0.007	steady state share of the investment project's return devoted to monitoring cost
D_{ss}	0.027	steady state households debt denominated in foreign currency (in trillions of US dollars)
R_{Kss}	0.023	steady state rental rate on capital
R_{ss}^{e}	1.034	steady state entrepreneur return
i_{ss}^{*}	0.021	steady state rest of the world interest rate
S_{ss}	1	steady state exchange rate
π_{*}	1.087	steady state annual gross inflation rate
g	1.008	quarterly gross long run growth rate

The next section describes the estimation methodology and presents the main estimation results.

3.2-Model estimation

In this section, we first describe the estimation methodology applied in this research, and then, we present the main estimation results.

3.2.1 Estimation methodology

There are various ways of estimating the structural parameters of a linearized DSGE model. In the literature, the most commonly used methods belong to one of the following main categories: limited information methods or full information methods.

Limited information methods are statistical inference based methodology, which formally accounts the uncertainty associated with model parameterizations. These methods use

a collection of actual statistics as targets to estimate the structural parameters of DSGE models. Estimating dynamic models by such methods allows the researcher to perform the tests of hypothesis that decide on the significance of parameter estimates. Prominent examples of limited information methods are generalized method of moments (GMM), simulated method of moments (SMM), and indirect-inference methods.

Maximum likelihood and Bayesian estimation are full information based methods. The maximum likelihood method generally consists of four steps. The first step solves for a reduced form state equation in the predetermined variables the DSGE model expressed in term of stationary variables and written as a linear rational expectations (LRE) system in the form $\Gamma_0(\Theta)\overline{X}_t = \Gamma_1(\Theta)\overline{X}_{t-1} + \Gamma_\varepsilon(\Theta)\in_t + \Gamma_\eta(\Theta)\eta_t$, where $\overline{X}_t$ denotes the vector of endogenous variables, $\in_t$ stacks the innovations of the exogenous processes, and η_t is composed of rational expectation forecast errors. The matrices Γ characterize the dynamic of the system and the vector Θ collects the structural parameters. Generally, the solution is in the form $\overline{X}_t = \Lambda_1(\Theta)\overline{X}_{t-1} + \Lambda_2(\Theta)\in_t$, and it usually serves to determine the policy function. Several techniques can be applied to solve a LRE system and find its solution. The Blanchard and Kahn (1980) method, the Anderson and Moore (1985) algorithm, the Sims (2002) method, the Klein (2000) method, and Uhlig (1999) method emerge as the most prominent in applied macroeconomics literature.

In the second step, the researcher casts the linearized model into a state space specification. This involves augmenting the state equation in the predetermined variables with an observation equation that links observable macroeconomic variables to predetermined variables. This observation equation, also known as measurement equation, usually takes the form: $Y_t = H\overline{X}_{t-1} + A(\Theta)$ or $Y_t = H\overline{X}_{t-1} + A(\Theta) + u_t$, where Y_t is the vector of observable

variables, $A(\Theta)$ captures the mean of Y_t which is related to the structural parameters, and u_t denotes the measurement errors. The existence of two alternatives implies that the researcher needs to take a stand on the measurement errors that enter the observation equations.

The third step entails the use of the Kalman filter to form the likelihood function of the observed data. The final step involves the estimation of the structural parameters by maximizing the likelihood function. The maximum likelihood method implements the statistical inference in the spirit of the classical econometrics (see Ireland 2001).

Alternative within the full information based methods, Bayesian estimation techniques (see, Geweke 1999; Del Negro, Schorfheide, Smets, and Wouters 2007; Lubik and Schorfheide 2005, 2007; Smets and Wouters 2003, 2007) combine in the forth step the likelihood function with the prior distribution of the structural parameters in order to form the posterior distribution function. The prior distribution, denoted $p(\Theta)$, summarizes the prior information on structural parameters coming from either microeconometric studies, previous macroeconomic studies, or a training sample. The likelihood function, denoted $p(Y_T / \Theta, M_i)$, where Y_T is the matrix of observed data and M_i stands for the model, summarizes the sample information conditional on the model M_i. The Bayes' theorem defines the posterior distribution of the parameters as $p(\Theta / Y_T, M_i) = \frac{p(\Theta) p(Y_T / \Theta, M_i)}{p(Y_T / M_i)}$. The denominator $p(Y_T / M_i) = \int p(\Theta, Y_T / M_i) d\Theta$ terms the marginal data density[38] conditional on the model M_i.

The Bayesian approach to this problem uses the idea of a loss function $L(\Theta, \hat{\Theta})$, which captures the loss incurred between the true value Θ of the parameter and its estimate $\hat{\Theta}$. The

[38] In Bayesian econometrics, the marginal data density is a measure of model fit as it could assess the goodness of the in-sample and allows for models comparison. Note that this marginal density is also the integral of the numerator.

Bayes estimator seeks the value that minimizes the expected value of the loss function, where the expectation is taken over the posterior distribution of Θ. In other words, the Bayes estimator seeks $\hat{\Theta}$ that minimizes $E_t\left[L\left(\Theta,\hat{\Theta}\right)\right]=\int L\left(\Theta,\hat{\Theta}\right)p\left(\Theta/\mathrm{Y}_T,\mathrm{M}_i\right)d\Theta$. Rarely, there is an analytical solution to this optimization problem, consequently, researchers generally estimate structural parameters through numerical methods and subsequently use a Monte-Carlo Markov-Chain sampling method, which could be for example the Gibbs Sampler algorithm or the Metropolis-Hastings algorithm, to obtain the complete picture of the posterior distribution.

We estimate the linearized DSGE model presented in appendix 4 with Bayesian techniques. To estimate the model, we use quarterly South Africa data for the period 1960:2-2008:4. The period 1960:2-1980:1 serves as a training sample[39] to form our prior information on structural parameters and exogenous processes. The remaining period[40] 1980:2-2008:8 is used for the inference. The macroeconomic variables that form the vector of observable variables are the Log difference of the real GDP, real (households) consumption and real investment, the trade balance to GDP ratio, the Log difference of the CPI, the "prime rate" which is the South African Reserve Bank policy rate, and the Log difference of the nominal exchange rate.

To obtain the linearized version of the DSGE model, we have defined a detrended endogenous variable X_t as $X_t = X_{ss}e^{\hat{X}_t}$ where X_{ss} is the (detrended) endogenous variable steady state (and defined the balanced growth path when combines with the trend growth rate) and $\hat{X}_t$ is its deviation from the steady state (or from the balanced growth path). To establish the correspondence between what is explained by the model and what the data actually measure, an

[39] With the training sample, we take advantage of the Bayesian updating; we combine the likelihood function from the training sample with a relative uninformative prior to yield a first stage posterior distribution which provides information to form prior on structural parameters for the remainder of the sample. The uninformative prior is defined with a large variance and the mean of zero.

[40] In this remaining period, the first 15 observations, covering the period 1980:2-1983:4, serve as a pre-sample such that the model is estimated over 100 observations corresponding to the period 1984:1-2008:4.

observable (real sector's) variable $Xobs_t$ is defined as $Xobs_t = \overline{X}obs_t e^{\hat{X}_t}$, where $\overline{X}obs_t$ denotes the trend component (balanced growth path) and $\hat{X}_t$ the deviation from that trend (cyclical component). The trend component is characterized by $\overline{X}obs_t = X_{ss}e^{\bar{g}t}$ where $\bar{g}$ is the quarterly long run growth rate. Thus, the observable variable $Xobs_t$ is defined by $Xobs_t = X_{ss}e^{\bar{g}t}e^{\hat{X}_t}$. As the theoretical DSGE model characterizes endogenous real variables as sharing a common trend component, the balanced growth path, symmetry dictates the removal of a common trend from all observable real variables (GDP, consumption, investment, export, imports…).

Taking the logarithm on both sides of the observable variable's definition we obtain $Log(Xobs_t) = Log(X_{ss}) + \bar{g}t + \hat{X}_t$. Trend removal is achieved through differencing, and the first difference of the endogenous observable variable X_t is given by:

$$\begin{aligned} dLog(Xobs_t) &= Log(Xobs_t) - Log(Xobs_{t-1}) = (1-L)Log(Xobs_t) \\ &= Log(X_{ss}) + \bar{g}t + \hat{X}_t - Log(X_{ss}) - \bar{g}(t-1) - \hat{X}_{t-1} \\ &= \hat{X}_t - \hat{X}_{t-1} + \bar{g}\,. \end{aligned}$$

L is the conventional lagged operator. We apply this differencing technique to obtain the measurement equation associated to the Log difference of real GDP, real consumption, and real investment. Before given the complete measurement equation system, a brief description of the trade balance to GDP ratio measurement equation is necessary.

The trade balance (TB_t) to GDP ratio is defined as $\frac{TB_t}{GDP_t} = \frac{Xobs_t - Yobs_{Mt}}{Yobs_t}$, where the observable macroeconomic time series $Xobs_t$ is the exports, $Yobs_{Mt}$ is the imports, and $Yobs_t$ is the GDP. This equality could be rewritten as $\frac{TB_t}{GDP_t} = \frac{Xobs_t - Yobs_{Mt}}{Yobs_t} = \frac{X_{ss}e^{\bar{g}t}e^{\hat{X}_t} - Y_{Mss}e^{\bar{g}t}e^{\hat{Y}_{Mt}}}{Y_{ss}e^{\bar{g}t}e^{\hat{Y}_t}}$

$= \frac{X_{ss}}{Y_{ss}} e^{\hat{X}_t - \hat{Y}_t} - \frac{Y_{Mss}}{Y_{ss}} e^{\hat{Y}_{Mt} - \hat{Y}_t}$. Since the exponential function is approximated by $e^x \approx 1 + x$; the trade balance to GDP ratio reduces to $\frac{TB_t}{GDP_t} = \frac{X_{ss}}{Y_{ss}}\left(1 + \hat{X}_t - \hat{Y}_t\right) - \frac{Y_{Mss}}{Y_{ss}}\left(1 + \hat{Y}_{Mt} - \hat{Y}_t\right)$, which we finally put in the form $\frac{TB_t}{GDP_t} = \frac{X_{ss}}{Y_{ss}} - \frac{Y_{Mss}}{Y_{ss}} + \frac{X_{ss}}{Y_{ss}}\left(\hat{X}_t - \hat{Y}_t\right) - \frac{Y_{Mss}}{Y_{ss}}\left(\hat{Y}_{Mt} - \hat{Y}_t\right)$. We differentiate the CPI and the nominal exchange rate to obtain respectively the inflation and the currency rate of appreciation. The measurement equation system[41] associated with this estimation is then given by:

$$\begin{bmatrix} dLogGDP_t \\ dLogC_t \\ dLogI_t \\ \frac{TB_t}{GDP_t} \\ dLogCPI_t \\ i_t \\ dLogS_t \end{bmatrix} = \begin{bmatrix} \bar{g} \\ \bar{g} \\ \bar{g} \\ 100\left(\frac{X_{ss}}{Y_{ss}} - \frac{Y_{Mss}}{Y_{ss}}\right) \\ \bar{\pi} \\ i_{ss} \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{Y}_t - \hat{Y}_{t-1} \\ \hat{C}_t - \hat{C}_{t-1} \\ \hat{I}_t - \hat{I}_{t-1} \\ \frac{X_{ss}}{Y_{ss}}\left(\hat{X}_t - \hat{Y}_t\right) - \frac{Y_{Mss}}{Y_{ss}}\left(\hat{Y}_{Mt} - \hat{Y}_t\right) \\ 4\hat{\pi}_t \\ 4\hat{i}_t \\ \hat{S}_t - \hat{S}_{t-1} \end{bmatrix}.$$

The data are in percentage (100 times Log difference), $\bar{g} = 100(g - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment, exports, and imports; $\bar{\pi} = 100(\pi_* - 1)$ is the annual steady state inflation rate, and i_{ss} is the annual steady state interest rate (in percentage). The inflation rate and the interest rate are annualized; the use of annualized inflation and interest rate series makes the highly parameterized algorithm more stable.

To estimate the linearized DSGE model, we first maximize the log posterior distribution function, which combines the prior distribution[42] on the parameters with the likelihood function of the data obtained through the Kalman filter. Once the mode and the Hessian at the mode have

[41] For the measurement equation associated with the domestic interest rate, we could write $1 + i_t = (1 + i_{ss})e^{\hat{i}_t}$. Taking the logarithm on each side yields $Log(1 + i_t) = Log(1 + i_{ss}) + \hat{i}_t$; given that $\log(1 + x_t) \approx x_t$ we obtain $i_t = i_{ss} + \hat{i}_t$, which is later annualized and expressed in percentage.

[42] We assume that parameters are prior independent from each other, which implies a joint prior distribution equals to the product of marginal prior distributions. This assumption is widely used in applied DSGE literature.

been computed, we use the Metropolis-Hasting algorithm with 2,000,000 draws (in two parallel chains) to obtain a complete picture of the posterior distribution and evaluate the marginal likelihood of the model. We tune to 0.2 the scale for the jumping distribution in the Metropolis-Hasting such that the acceptation rates are closed to 30% (36.75% in the first chain and 33.05% in the second chain). The first half of the draws is disregarded. To test the stability of the sample and monitor the convergence of the posterior distribution, we perform the convergence diagnostics in the spirit of Brook and Gelman (1998) and compare the between and within moments of multiple chains (see appendix 6 at the end of the chapter). Before presenting the estimation results, we give an overview of our assumptions regarding the prior distributions.

3.2.2 Prior distributions of the estimated parameters

The first three columns of table 3.3 summarize our assumptions regarding the prior distributions of the 42 estimated parameters. Those assumptions were formalized by estimating the model with the training sample. All the standard errors of the structural shocks (innovations of the exogenous processes) are assumed to follow an Inverse-gamma distribution with two degrees of freedom. The Inverse-gamma probability distribution function ensures that the variances of the shocks are positive and estimated over a rather large domain. The precise mean for these prior distributions are based on estimation outcomes and trial using the training sample[43]. The persistent coefficient of each AR(1) describing an exogenous process is Beta distributed with a mean 0.85 and a standard error 0.1. Since the Beta distribution covers the 0-1 range, the autoregressive parameter of each exogenous process then falls between 0 and 1, which ensures the stationarity of the estimated DSGE model.

[43] We investigated the alternative in which the prior distributions of the standard deviations of the structural shocks are harmonized as much as possible. This corresponds to a loose prior where all standard errors of the structural shocks are assumed to follow an inverse-gamma distribution with the same mean 0.10 and two degrees of freedom. The log marginal likelihood (Laplace approximation) from that estimation strategy is -1575.66 while a training sample based prior distribution yields a log marginal likelihood (Laplace approximation) of -1467.37. The data obviously prefer the prior assumptions from the training sample.

We also assume a Beta distribution for the MA parameters in the processes of the foreign interest rate, the domestic price mark-up, and the terms of trade. As the training sample suggests a mean around 0.7 for the MA coefficients in the domestic price mark-up and the terms of trade processes, we retain for those two parameters a Beta distribution with a mean 0.7 and a standard deviation of 0.2. Information from the training sample rather supports a Beta distribution with a mean around 0.5 and a standard deviation of 0.2 for the MA coefficients in the foreign interest rate process. The quarterly trend growth rate follows a Gamma distribution with a mean 0.784 percent, which corresponds to the sample average quarterly growth rate of the real GDP. The standard deviation is 0.011, which also corresponds to the sample standard deviation of the quarterly real GDP growth rate. The steady state inflation also follows a Gamma distribution with a mean 8.712 percent, the sample annual average, and a standard deviation of 0.056 which is the sample standard deviation of the inflation rate as well.

Prior distributions on technology and utility parameters are basically formed using the information gained from the training sample. The coefficient of the relative risk aversion is Normal distributed with a mean 2.1 and a standard error of 0.375. Similarly, the inverse of the elasticity of the work effort with respect to the real wage is assumed to be Normal around a mean of 2.5 with a standard deviation of 0.75. The habit formation parameter is Beta distributed with a mean of 0.57 and a standard deviation 0.1. The fixed cost parameter in the good production sector and the portfolio adjustment cost parameter are assumed to be Normal distributed. The mean of the fixed cost parameter[44] is set at 1.5 and the standard deviation is 0.125; the portfolio adjustment cost parameter fluctuates around 0.001 with a standard deviation of 0.05. The elasticity of the capital utilization cost function follows a Beta distribution that has

[44] Note that we estimate $\Phi_c = 1 + \frac{\Phi}{Y_{ss}}$, which is one plus the fixed cost as defined in the theoretical DSGE model (see second chapter) to steady state output.

a mean of 0.5 and a standard deviation of 0.15 as indicated by the training sample. The Beta distribution ensures that the capital adjustment cost remains within the range 0-1.

For the elasticity of the cost of adjusting investment, we also take as a starting point the values suggested by the estimation performed with the training sample. Thus, this parameter is Normal distributed with a mean 1.48 and a standard deviation 1.5. The rationality of this prior assumption might be that the cost of adjusting investment (and therefore the capital) could be lower in this developing economy relatively to the observed level in advanced economies.

Turning to the parameters that define the financial accelerator in the entrepreneur sector, we first recall that the shares parameters $A(\overline{\omega}_{ss})$, $B(\overline{\omega}_{ss})$, and $C(\overline{\omega}_{ss})$ of the investment project's return as defined in chapter 2 are (portions of) a cumulative distribution function (cdf). This implies that their associated derivatives $A'(\overline{\omega}_{ss})$, $B'(\overline{\omega}_{ss})$, and $C'(\overline{\omega}_{ss})$ characterize (portions of) a probability distribution function (pdf), which restricts them to be within the 0-1 range. To conform to that restriction, the parameters $\zeta_e = A'(\overline{\omega}_{ss})$, $\zeta_l = B'(\overline{\omega}_{ss})$, and $\zeta_c = C'(\overline{\omega}_{ss})$ are assumed to be Beta distributed with a mean 0.5, 0.65 and 0.45 respectively as suggested by the training sample. A standard deviation of 0.15 is associated with each of those Beta distributions. The parameter of the external finance premium is Normal distributed with a mean 3.75 and a standard deviation 0.5.

The training sample also provides good insights on the price and wage setting parameters. The Calvo parameters in the domestically produced goods and the import goods sectors are assumed to follow Beta distributions with mean 0.57, suggesting an average length of price contract around half a year (7 months); the standard deviation is at about 0.05 so that the domain covers a reasonable range of parameters values. The training sample suggests a much higher mean to the Calvo probability for wage, thus, this parameter is Beta distributed

with a mean 0.75 and a standard deviation of 0.05. The price indexation parameters in the domestically produced goods sector and in the import goods sector are assumed to fluctuate around 0.4, while the wage indexation parameter fluctuates around 0.5. The standard deviation to those Beta distributions is 0.15.

We draw the prior information describing the monetary policy rule from the standard Taylor rule. The long run reaction to inflation, output gap, and currency depreciation are described by Normal distributions with mean 1.5, 0.125, and 1.112 respectively. The standard deviations associated with these Normal distributions are 0.25, 0.05, and 0.05 respectively. The persistence of the monetary policy, described by the coefficient of the lagged interest rate, is assumed to be Beta distributed with a mean 0.75 and a standard deviation 0.15.

In the external sector, the rest of the world's elasticity of substitution between home produced goods and import goods θ_1 and the share of export financed through trade credit τ are both estimated. The two parameters appear in the export dynamic equation (2.67). We form our prior information on those parameters such that their prior distributions are consistent with the volatility and the persistence observed in key macroeconomics variables of the world largest economies[45]. Lubik and Schorfheide (2005) estimated a two-country DSGE model with Bayesian techniques using quarterly data of the United States and Euro area. They found a value of 0.43 to the intratemporal substitution elasticity between home and foreign consumption goods in these two largest economies. Thus, we assume the parameter θ_1 follows a Gamma distribution with a mean 0.43 and a standard deviation 0.1. In addition, Lubik and Schorfheide (2005) found a share of import in the consumer price index at about 0.13 indicating a home bias

[45] Note that the parameters $\gamma_1, \theta_1,$ and τ only appear in the export dynamic equation, which implies two degrees of freedom in solving for $\gamma_1, \theta_1,$ and τ using the export equation evaluated at steady state. The corollary is that the three parameters should not be estimated all together; once two of those parameters are known, the third could be identified using the export equation evaluated at the steady state. We choose to estimate θ_1 and τ since there is a priori much more uncertainty about their values. Trade data almost provide an accurate value for γ_1.

toward domestically produced goods in the world largest economies. Nonetheless, it is worth noting that the total share of developing Sub-Saharan African economies in the world trade is lower and by far below 13%. To obtain a consistent calibrated value of γ_1 (the share of South Africa exports in the rest of the world CPI), we weight the Lubik and Schorfheide (2005)'s estimate of import goods share in the CPI indexes of the United States and Euro Area by the South Africa's share in the trade between the European Union[46] and the rest of the world. The long run average over the period 1960:01-2008:04 of the total European Union imports from the rest of the world is about 353.576 billions of dollars. The long run average of total European Union imports from South Africa is 2.017 billions of dollars over the same period. Given these sample long run averages, the calibrated value of the share of South Africa export in the rest of the world CPI is given by $\gamma_1 = \frac{2.017}{353.576} 0.13 = 0.00074$.

Thus, given these values of θ_1 and γ_1 as indicated above, and given the steady state values of the domestically produced goods prices, the rest of the world CPI index, the rest of the world interest rate, and the ratio $\frac{X_{ss}}{C^*_{ss}}$ of the South Africa exports to the rest of the world aggregate consumption; a sample mean for the parameter τ could be calculated using the export dynamic equation (2.67). The sample long run averages of the South Africa exports and the world's aggregate consumption yield the ratio $\frac{X_{ss}}{C^*_{ss}} = 0.001$. Thus, the induced sample mean of τ is about 0.495. In addition, the sample volatility associated with that same ratio implies a standard deviation of τ at about 0.1. Accordingly, we assume the share of exports financed by trade credit τ follows a Beta distribution with a mean 0.495 and a standard deviation 0.1.

[46] We use the trade data from the European Union since detailed trade data for the Euro area are not available and it is therefore difficult to identify trade flows between South Africa and the Euro area.

Table 3.3: Prior and posterior distributions of estimated parameters

Parameter		Prior distribution			Estimated maximum posterior		Posterior distribution MH		
		Shape	Mean	St. error	Mode	St. error (Hessian)	5%	Mean	95%
ϕ	coefficient of relative risk aversion	Normal	2.100	0.375	2.2450	0.2497	1.8110	2.2789	2.7389
η	inverse of the elasticity of work effort with respect to real wage	Normal	2.500	0.750	3.1555	0.4058	2.5028	3.0144	3.5324
h	habit formation in consumption	Beta	0.570	0.100	0.2960	0.0396	0.1888	0.2786	0.3693
Φ_c	(one plus) fixed cost parameter in goods production sector	Normal	1.500	0.125	1.4934	0.0584	1.2515	1.4411	1.6267
ψ_D	portfolio adjustment cost parameter	Normal	0.001	0.050	$2.6\text{x}10^{-5}$	$1.7\text{x}10^{-5}$	0.0000	0.0000	0.0001
l_n	degree of prices indexation in the good production sector	Beta	0.400	0.150	0.4663	0.1023	0.2528	0.4556	0.6571
ξ_n	Calvo parameter in the goods production sector	Beta	0.570	0.050	0.5709	0.0138	0.4494	0.5361	0.5973
l_m	degree of prices indexation in the import goods sector	Beta	0.400	0.150	0.4737	0.1017	0.2638	0.5083	0.7551
ξ_m	Calvo parameter in the import goods sector	Beta	0.570	0.050	0.5992	0.0405	0.5532	0.6285	0.7033
l_w	degree of wage indexation	Beta	0.500	0.150	0.3443	0.0512	0.1628	0.2975	0.4305
ξ_w	Calvo parameter for wage	Beta	0.750	0.050	0.8242	0.0200	0.7748	0.8100	0.8478
χ	steady state elasticity of the investment adjustment cost	Normal	1.480	1.500	1.4415	0.1106	0.6005	0.8897	1.1695
ζ_c	steady state marginal share of the project's return devoted to monitoring cost	Beta	0.450	0.150	0.4367	0.1270	0.3178	0.4933	0.6685
ζ_e	steady state marginal share of the project's return devoted to entrepreneur	Beta	0.500	0.150	0.4991	0.0927	0.2567	0.3954	0.5344
ζ_l	steady state marginal share of the project's return devoted to foreign lender	Beta	0.650	0.150	0.6928	0.0646	0.6120	0.6994	0.7829
ζ_p	steady state elasticity of the external finance premium	Normal	3.750	0.500	3.7507	0.4817	3.2050	3.7827	4.3456
ζ_u	steady state elasticity of the marginal cost of adjusting capital utilization rate	Beta	0.500	0.150	0.1202	0.0278	0.3238	0.4416	0.5657
τ	fraction of export financed by trade credit	Beta	0.495	0.150	0.4963	0.1085	0.2500	0.4977	0.7410
ρ_R	degree of central bank's interest rate smoothing	Beta	0.750	0.150	0.9672	0.0075	0.9582	0.9741	0.9894
ψ_1	Taylor coefficient of inflation	Normal	1.500	0.250	1.1893	0.0372	1.1282	1.2096	1.2843
ψ_2	Taylor coefficient of output gap	Normal	0.125	0.050	$1.02\text{x}10^{-5}$	0.0001	0.0000	0.0003	0.0005
ψ_3	Taylor coefficient of currency depreciation	Normal	1.112	0.050	1.1372	0.0163	1.1336	1.1825	1.2274

$\bar{g}$	quarterly trend growth rate to real GDP	Gamma	0.784	0.011	0.7796	0.0109	0.7596	0.7776	0.7959
$\bar{\pi}$	annual steady-state inflation rate	Gamma	8.712	0.056	8.7247	0.0560	8.6352	8.7276	8.8210
θ_1	rest of the world elasticity of substitution between home and import goods	Gamma	0.430	0.100	0.4999	0.0471	0.3604	0.5005	0.6463
ρ_B	persistence coefficient of the premium	Beta	0.850	0.100	0.8876	0.0199	0.8596	0.9440	0.9492
ρ_{i^*}	persistence coefficient of the foreign interest rate	Beta	0.850	0.100	0.8340	0.0509	0.6793	0.8120	0.9416
ρ_P	persistence coefficient of the domestic prices mark-up	Beta	0.850	0.100	0.9230	0.0167	0.9579	0.9730	0.9896
ρ_Y	persistence coefficient of the technology	Beta	0.850	0.100	0.9998	0.0003	0.9991	0.9995	0.9999
ρ_X	persistence coefficient of the rest of the world consumption	Beta	0.850	0.100	0.8892	0.0071	0.7980	0.8409	0.8814
ρ_T	persistence coefficient of the terms of trade	Beta	0.850	0.100	0.9804	0.0163	0.9687	0.9794	0.9909
ρ_G	persistence coefficient of the government spending	Beta	0.850	0.100	0.9023	0.0079	0.8549	0.8823	0.9096
μ_{i^*}	MA coefficient of the foreign interest rate	Beta	0.500	0.200	0.5497	0.0964	0.1334	0.4500	0.7440
μ_P	MA coefficient of the domestic prices mark-up	Beta	0.700	0.200	0.8429	0.0283	0.6017	0.7728	0.9019
μ_T	MA coefficient of the terms of trade	Beta	0.700	0.200	0.9404	0.0912	0.5622	0.7787	0.9991
σ_B	Standard deviation of the financial markets shock	Inverse gamma	0.100	2	0.0430	0.0115	0.0262	0.0523	0.0780
σ_P	Standard deviation of the mark-up prices shock	Inverse gamma	0.650	2	0.8786	0.1389	0.8346	1.0264	1.2151
σ_Y	Standard deviation of the technology shock	Inverse gamma	0.900	2	0.8052	0.1438	0.5524	0.7816	1.0012
σ_X	Standard deviation of the export demand shock	Inverse gamma	3.500	2	4.9441	0.1241	4.5004	5.0944	5.6996
σ_T	Standard deviation of the terms of trade shock	Inverse gamma	1.350	2	1.8452	0.3079	1.9340	2.7701	3.6374
σ_G	Standard deviation of the government spending shock	Inverse gamma	1.250	2	1.9038	0.2284	1.7314	2.0074	2.2800
σ_R	Standard deviation of the monetary policy shock	Inverse gamma	0.150	2	0.2733	0.0662	0.0886	0.2242	0.3600
	Log marginal likelihood: Laplace approximation				-1467.3666				
	Modified harmonic mean				-1403.4053				

Note: For the inverse gamma distribution, the mode and the degree of freedom are reported. The posterior distribution is obtained using the Metropolis-Hastings (MH) algorithm.
Two options are used to evaluate the marginal likelihood; the Laplace approximation, which is based on the Normal distribution, and the modified harmonic mean following Geweke (1998).
All estimations are done with Dynare version 4.1.1 available at (http://www.cpremap.cnrs.fr/dynare). The corresponding model file is presented in appendix 7 at the end of the thesis.

3.2.3 Posterior estimates of the parameters

Table 3.3 also shows two set of results regarding the parameter estimates. The first set presents the posterior mode, which is obtained by numerical optimization on the log posterior density with respect to the parameters, and the approximated standard errors based on the corresponding Hessian. The second set of results reports the mean along with the 5th and 95th percentile of the posterior distribution of the parameters obtained through the Metropolis-Hasting algorithm. The latter is based on 2,000,000 draws. The convergence diagnostic test performed using the "multivariate diagnostic", depicted in figure 3.1, shows that the convergence is reached (see also univariate diagnostics in appendix 6 at the end of the chapter).

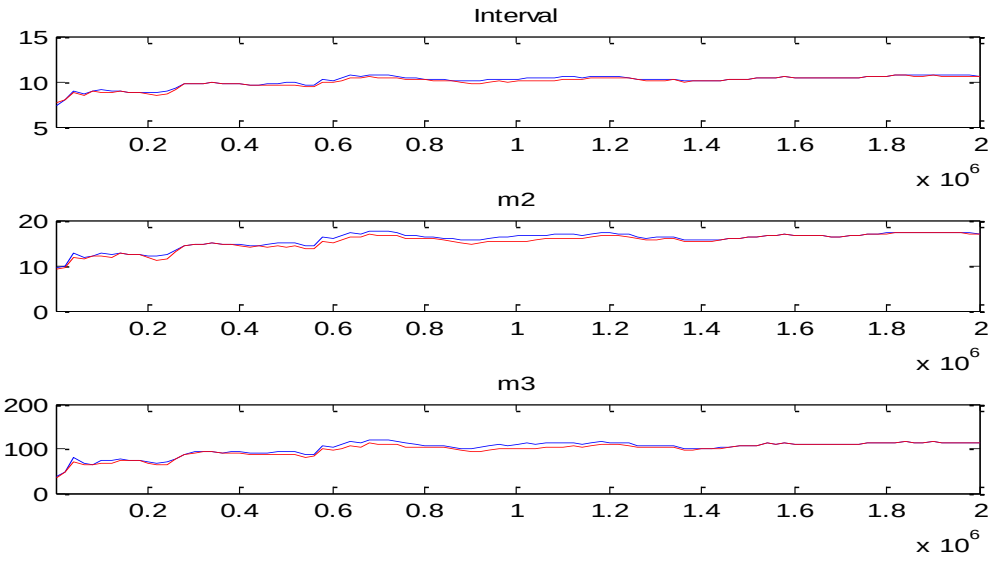


Figure 3.1: multivariate convergence diagnostic.

Figure 3.2 summarizes the estimation results visually by plotting the prior distribution in dashed line, the posterior distribution in thick line, and the mode from maximization into a vertical broken line. Overall, the data appear to be very informative on the structural parameters as well as the parameters that defined the stochastic processes for the exogenous disturbances.

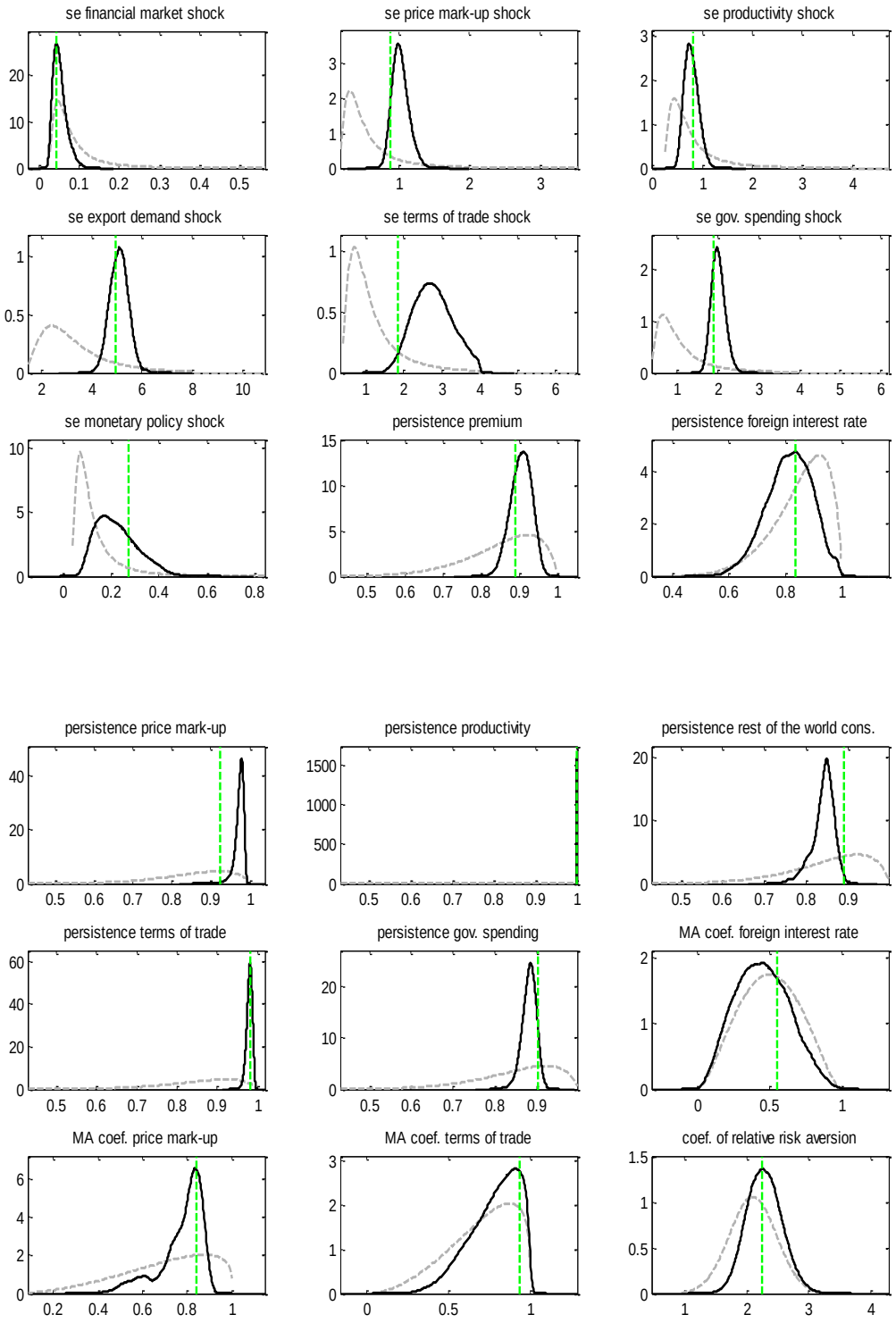


Figure 3.2A: estimated parameters' prior and posterior distributions.

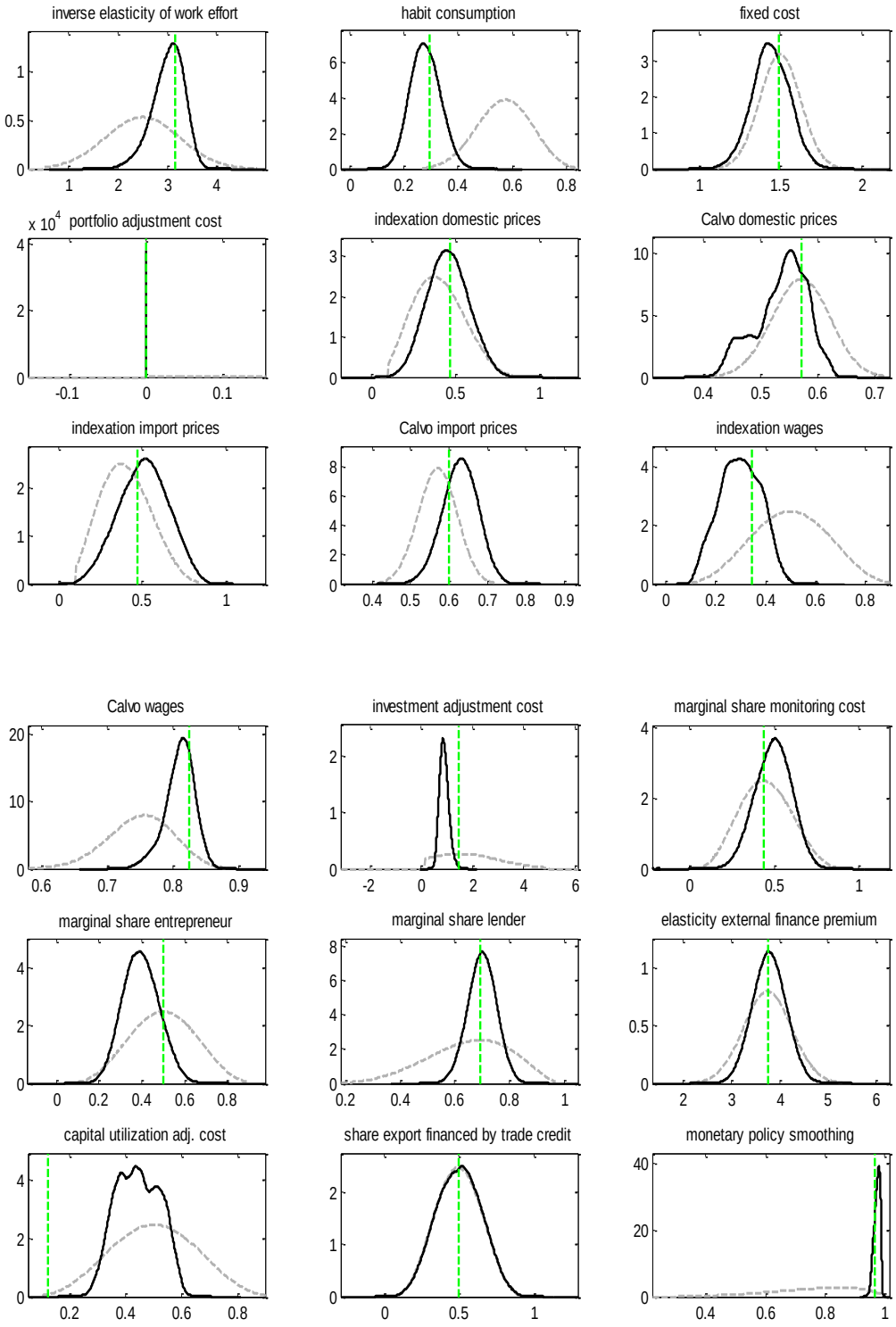


Figure 3.2B: estimated parameters' prior and posterior distributions.

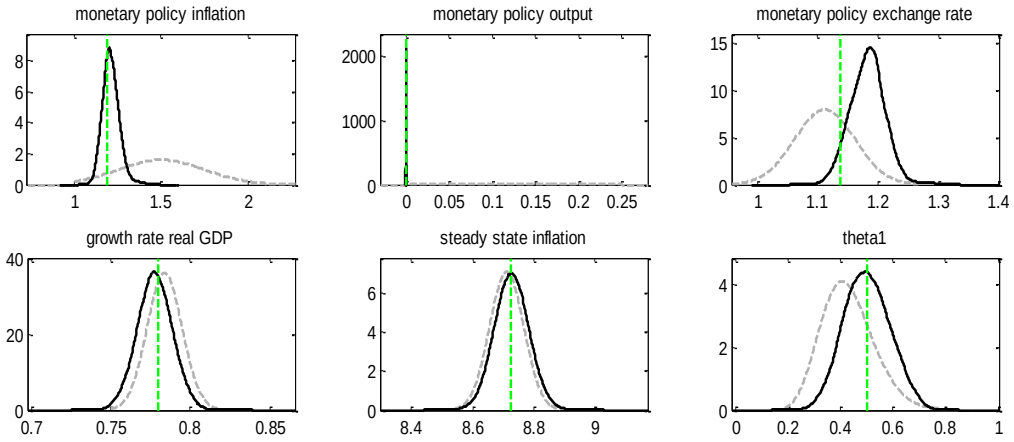


Figure 3.2C: estimated parameters' prior and posterior distributions [47].

Overall, structural parameters are estimated to be significantly different from zero at a step size of 5% but the portfolio adjustment cost parameter and the output coefficient in the monetary policy rule. This suggests that the transversality condition on the household's foreign currency denominated debt might be loose and that the South African Reserve Bank gives very little weight to output gap when setting its policy rate. The standard deviations of all structural shocks are also estimated significantly different from zero. However, although the financial market shock and the monetary policy shock are significantly different from zero at a test size of 5%, they seem to play a limited role in explaining real variables fluctuations. This point is much elaborated in the forecast error variance decomposition discussed in the next chapter.

Focusing first on the structural shocks that drive the model, the posterior means of the standard deviations of structural shocks are substantially larger in this Sub-Saharan African

[47] Theta1 represents the rest of the world elasticity of substitution between home and imports goods.

economy. For example, the mean of the standard deviation of the technology shock is estimated at about 0.7816, which is higher than the 0.519 standard deviation of the stationary technology shock estimated for the Euro area by Adolfson et al. (2007). The magnitudes of other structural shocks, specifically the government spending shock, or the mark-up price shock, are also substantially larger than estimates for developed economies (see Smets and Wouters 2003, 2007 for estimates in the Euro area and the US). Exogenous shocks arising from the rest of the world, the terms of trade shock and the export demand shock, are in term of the magnitude of standard deviation the largest in this economy. The terms of trade shock appears quite important with the mean of its standard error estimated at about 2.7701. The mean of the export demand shock at about 5.0944 is also big, reflecting the high volatility exhibited by actual exports. The monetary policy shock estimated at about 0.2242 is quite closer to common estimate for the United States. It is worth noting the lower magnitude of the financial market shock that characterizes a simultaneous shock to the domestic premium and the foreign interest rate and which has a standard deviation lowers than the 0.23 estimate in Smets and Wouters (2007) for the US.

The fact that standard deviations of exogenous shocks are generally larger in this developing Sub-Saharan African economy is independent from the prior assumptions. As a test of the sensitivity of the estimation results to the prior assumptions, we estimate the model with a loose prior in which the prior of the stochastic processes are harmonized as much as possible. All standard errors of structural shocks are assumed to follow an Inverse-gamma distribution with a mean of 0.10 and two degrees of freedom. The estimates of the standard error of the structural shocks are closer and fall within the confidence interval presented in table 3.3. Nonetheless, the log marginal likelihood under that estimation strategy is lower than the log marginal likelihood obtained under the training sample based prior distributions.

Autoregressive parameters (persistence) of the exogenous processes are estimated to have posterior means that lay between 0.8120 (for the foreign interest rate) and 0.9995 for the total factor productivity. The productivity and the terms of trade processes are the most persistent with AR(1) coefficients closest to one. The posterior means of the MA coefficients associated with the exogenous processes in the aggregator function and the terms of trade are estimated to be higher than the mean assumed in their prior distributions. However, the posterior mean of the MA coefficient in the foreign interest rate is lower than its prior mean.

The trend growth is estimated to be around 0.7776, which is pretty close to the average growth rate of real GDP over the sample. The posterior mean of the steady state inflation at about 8.7276 is closest to the sample average and the share of the exports financed by trade credit is around the sample average with an estimated posterior mean of 0.4977. The mean of the rest of the world's elasticity of substitution between home produced goods and import goods θ_1 is estimated at about 0.5005, somewhat greater than Lubik and Schorfheide (2005) estimate of 0.43 for the United States and Euro area. However, Lubik and Schorfheide (2005) estimate falls within the confidence interval associated with θ_1.

Turning to the structural parameters that define the behavior of the domestic household, the coefficient of relative risk aversion is greater than 1, which implies domestic preferences different from the log preferences and from the findings of Casares (2001) for the euro area. The external habit stock is estimated to be about 30 percent of the past consumption, which is somewhat smaller than the estimates reported in Smets and Wouters (2003, 2007) for the euro area or the United States. The estimate posterior mean of η is around 3.0144, implying a greater inverse of the elasticity of work effort with respect to real wage in this Sub-Saharan African economy relatively to those estimated for developed economies.

Focusing on the six parameters characterizing the degree of price and wage stickiness, the posterior mean of the indexation parameter in the domestically produced goods sector is estimated around 0.4556 and greater than the mean of its prior distribution. This implies a weight on lagged inflation in the domestic good inflation rate at about 0.315. The indexation in the import goods sector is higher at about 0.5083, which implies a greater weight to the lagged import inflation around 0.339. There is a relative low degree of Calvo prices stickiness. The average duration of domestically produced good price contracts is estimated to be around 2.156 quarters, while the average duration of the price contracts in the import goods sector is about 2.692 quarters. Prices adjust at a high speed in this developing economy and exchange rate shocks feed into the domestic price level at a much faster pace than in developed economies. There is, however, a considerable degree of Calvo wage stickiness. The average duration of wage contracts is estimated around 5.263 quarters, which is somewhat longer than a year. The degree of wage indexation 0.2975 is estimated to be much lower than the 0.5 prior mean.

The posterior mean of the investment adjustment cost parameter at about 0.8897 is smaller than the one reported in Smets and Wouters (2007) for the United States. The estimated posterior mean of the fixed cost parameter at about 1.4411 is closer to estimates usually reported for developed economies. In general, parameters characterizing the entrepreneurial behaviour are estimated to be very close to the means assumed in their prior distributions.

Finally, our estimation delivers reasonable parameters for the long run reaction function of the monetary authorities. The estimates imply that in the long run the response of the interest rate to inflation is greater than one, thereby satisfying the Taylor principle. The response to output is, however, negligible and insignificant. Finally, the monetary authorities respond to exchange rate fluctuations and there is a considerable degree of interest rate smoothing.

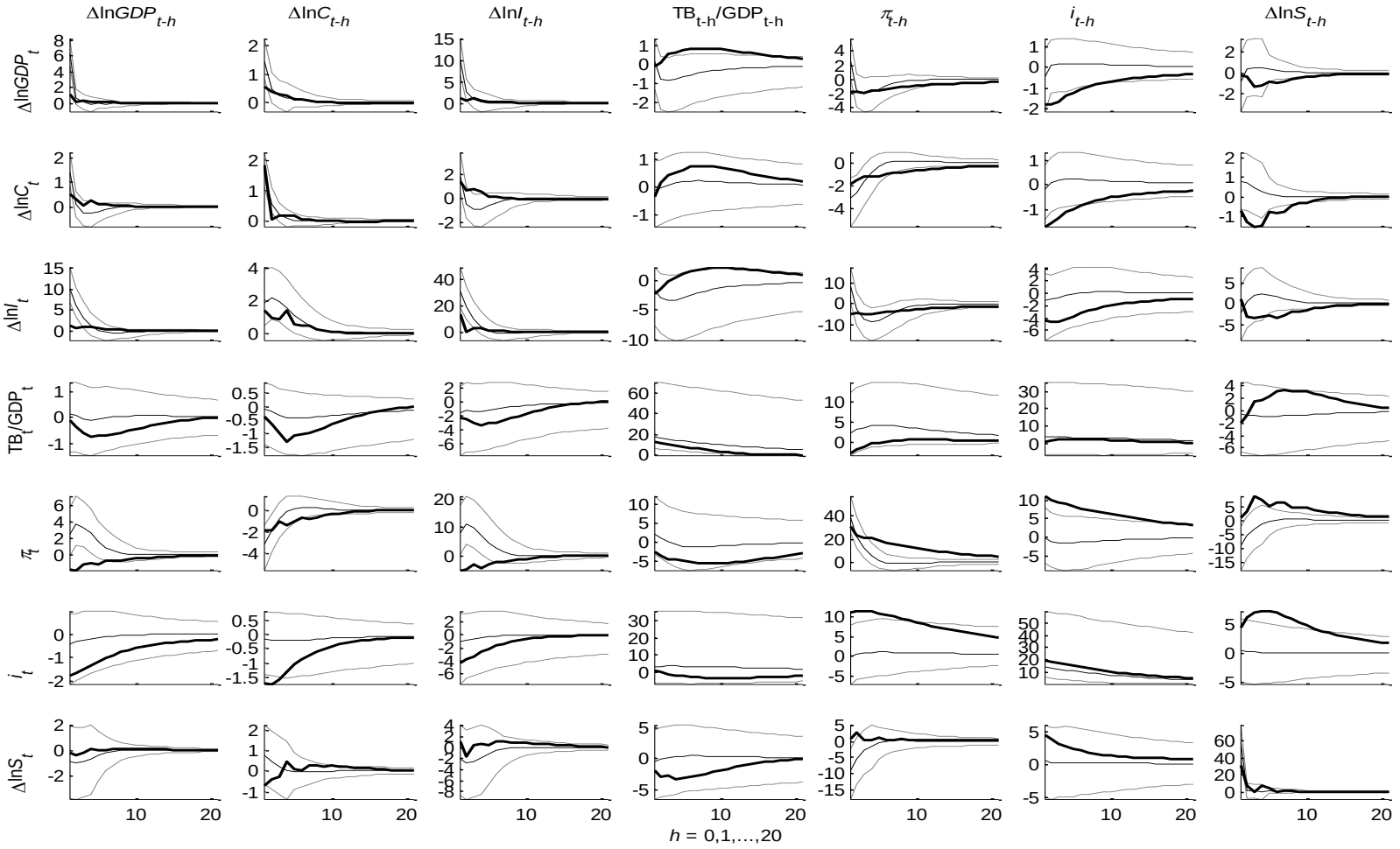
3-Model evaluation

The discussion in the previous section has shown that the DSGE model is able to deliver reasonable and significant parameters for the South African economy. An interesting exercise would be to confront the performance of the calibrated-estimated DSGE model with that of non-theoretical VAR models estimated on the same dataset. We conduct this exercise in this section.

Traditionally, researchers validate DSGE models by comparing model-based variances and covariances with those in the data. We consequently compute the cross-covariances between the seven observable data series implied by the calibrated-estimated DSGE model and compare those with the empirical cross-covariances. The empirical cross-covariances are based on an unrestricted VAR(3) model (that includes the Log difference of the real GDP, households' consumption, investment, and exchange rate; the trade balance to GDP ratio, the inflation, and the nominal interest rate) estimated with an uninformative prior on over the full sample period 1960:2–2008:4. For consistency, the model-based cross-covariances are also calculated by estimating a VAR(3) on more than 20,000 random samples of 195 observations generated from the DSGE model[48]. Figure 3.3 summarizes the results of this exercise. We report the median vector autocovariance function in percentage in the DSGE model (thin line) along with the 2.5 and 97.5 percentiles (dotted lines) for the subset of the variables included in the VAR. The thick line gives the empirical cross-covariances based on the VAR(3) models estimated on the observed actual data.

[48] To compute the vector autocovariance functions from the DSGE model, we keep the last 31.88% draws in each chain of the Metropolis-Hastings simulations (burn-in 68.12%) and determine the sample of unique parameter combinations, then we draw 5% of the sample size of unique parameter combinations, this is equivalent to 21,538 unique parameter combinations from the posterior distribution; and for each parameter draw, we simulate an artificial data set of the same length as the South Africa actual data series (195 observations). Then, we use the 21,538 data sets to estimate vector autocovariance functions using exactly the same VAR specification as was applied on the actual data (see chapter 1).

Figure 3.3 Autocovariance functions: Thick solid line - VAR, Thin solid line - Model median, Dashed lines - confidence interval



Legend: On this figure, $\Delta LnGDP_t$ stands for the Log difference of the real GDP, ΔLnC_t the Log difference of household consumption, ΔLnI_t the Log difference of investment, TB_t / GDP_t the trade balance to GDP ratio, π_t the inflation rate, i_t the nominal interest rate, and ΔLnS_t the Log difference of nominal exchange rate.

In general, the empirical autocovariance functions fall within the error bands, suggesting that the model is able to mimic the cross-covariances in the data. Furthermore, posterior predictive intervals for the vector autocovariance functions turn out to be relatively narrow in this model, which indicates a small amount of uncertainty surrounding the model-based cross-covariances. From figure 3.3, we observe that the realized volatility and persistence of most observed variables are well captured by the model. The DSGE model's probability intervals cover the variance in the data for all variables except the investment, which turn out to be too high in the model relative to the data. Though, the DSGE model also overestimates the variability of the real GDP growth rate. The variance of domestic inflation is very well captured by the model, however, at the medium run, the autocovariance of the domestic inflation moves outside the error bands. One reason of the higher persistence in actual inflation rate relatively to the model generated inflation rate could be the specification of the model's annual inflation rate. The inflation rate in the actual data is annual rate, which by definition is more persistent than the quarterly inflation rate, while the observed inflation rate generated by the model, even though annualized, is primarily linked to the quarterly inflation rate by the measurement equation[49]. The posterior predictive median volatility of the exchange rate of appreciation is very close to the data estimate, the model also matches its persistence.

Looking at the off-diagonal cross-covariances, we observe a number of cross-covariances where the discrepancies between the model-based cross-covariances and the empirical ones are somewhat larger. More specifically, the cross-covariances with the inflation rate do not seem to be fully satisfactory. The model seems to have problems fitting the very short run negative cross-covariances between current real GDP growth and future inflation, and

[49] We note that Adolfson et al. (2007) also report a discrepancy between the inflation model based cross-covariance and its empirical counterpart, and they underline the fact that their DSGE model cannot typically generate a persistent sequence of inflation as in the data during the sample.

that between current investment and future inflation. Furthermore, the model has a very hard time replicating the joint behaviour of the current exchange rate of appreciation and future domestic inflation. The empirical autocovariance function falls outside the error band in the short run even though the model based and the empirical cross-covariances have identical shapes. In the data, current depreciation leads to higher future inflation, whereas the model predicts lower future inflation. In fact, exchange rate depreciation in the model boosts exports which triggers domestic supply side effects that significantly expand real GDP leading to a fall in future domestically produced goods prices and afterwards into future domestic inflation.

4-Conclusion

This chapter has prepared the ground for an empirical application of the DSGE model developed in the second chapter. Two methods were used to fit the DSGE model to the South African economy. First, parameters that are functions of steady state ratios and that are weakly identified when the model is estimated with observed variables expressed as deviation from the steady state or the balanced growth path were calibrated. The calibration yielded a set of parameters consistent with the South African economy long-run properties and steady state ratios. Second, the remaining structural parameters were inferred by estimating the linearized DSGE model using the South Africa macroeconomic data transformed into mean-zero covariance stationary stochastic process. Overall, the DSGE model fit the South Africa data quite well, exogenous shocks and structural parameters are estimated to be significantly different from zero at a step size of 5%, but the portfolio adjustment cost parameter and the coefficient to output in the monetary policy rule. The developing economy is confronting

exogenous shocks of larger magnitudes than generally estimated in advanced economies. There is a relative low degree of Calvo prices stickiness, but a considerable degree of Calvo wage stickiness.

The model evaluation shows that the empirical variances and covariances, in general, fall within the error band. The posterior predictive intervals for the vector autocovariance functions are also tight in the model. Although the model has some difficulties in replicating the joint behaviour of the current exchange rate and future domestic inflation, or that of the current interest rate and future domestic inflation, which suggest rooms for improvements in some of its aspect; we assess that the DSGE model does a good job in replicating conventional statistics for measuring the fit of a model. In the next chapter, we undertake three applications of the DSGE model, the sensitivity analysis to assess the empirical importance of frictions, the impulse response analysis to study the economy adjustment to exogenous shocks, and the forecast error variance decomposition to identify the driving forces of the observable macroeconomic variables.

Appendix 6: Monitoring the convergence of the posterior distribution to its target distribution

To assess the stability of the parameters and monitor the convergence of the posterior distribution to its target distribution, we compute convergence diagnostics as described in Brook and Gelman (1998) and compare the between and within moments of the two chains. The results from those diagnostics are depicted on figures A6.1 to A6.7. Three measures are reported for each estimated parameter: "interval" is constructed from an 80% confidence interval around the mean, "m2" is a measure of the variance, and "m3" is based on the third moments. The red and blues lines on the charts represent those specific measures both within and between chains. In general, results within iterations of Metropolis-Hastings simulations are similar, and the results between the two chains are very close. We obtain convergence and relative stability in all measures of the parameter moments with 2,000,000 draws of the Metropolis-Hastings albeit slight differences might be exhibited by some measures related to "marginal share monitoring cost" and "marginal share entrepreneur" parameters.

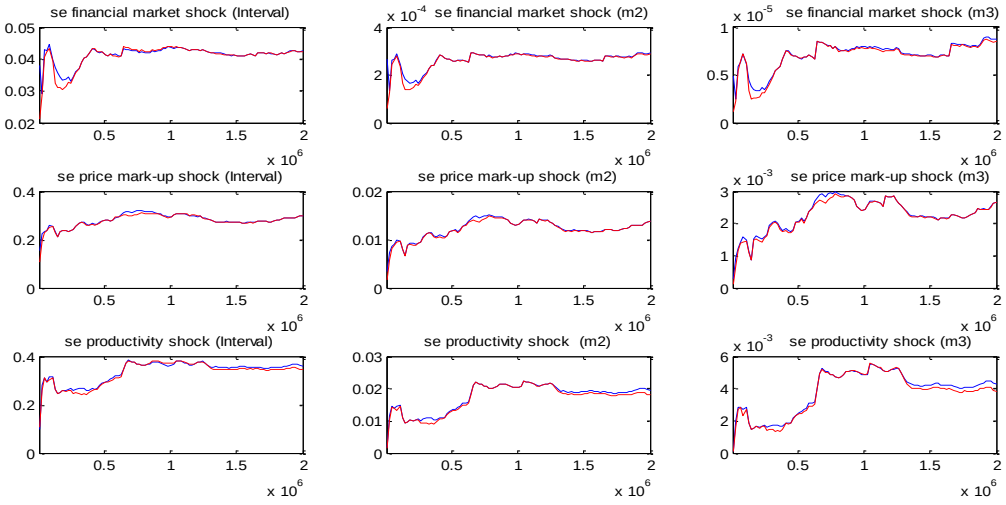


Figure A6.1: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

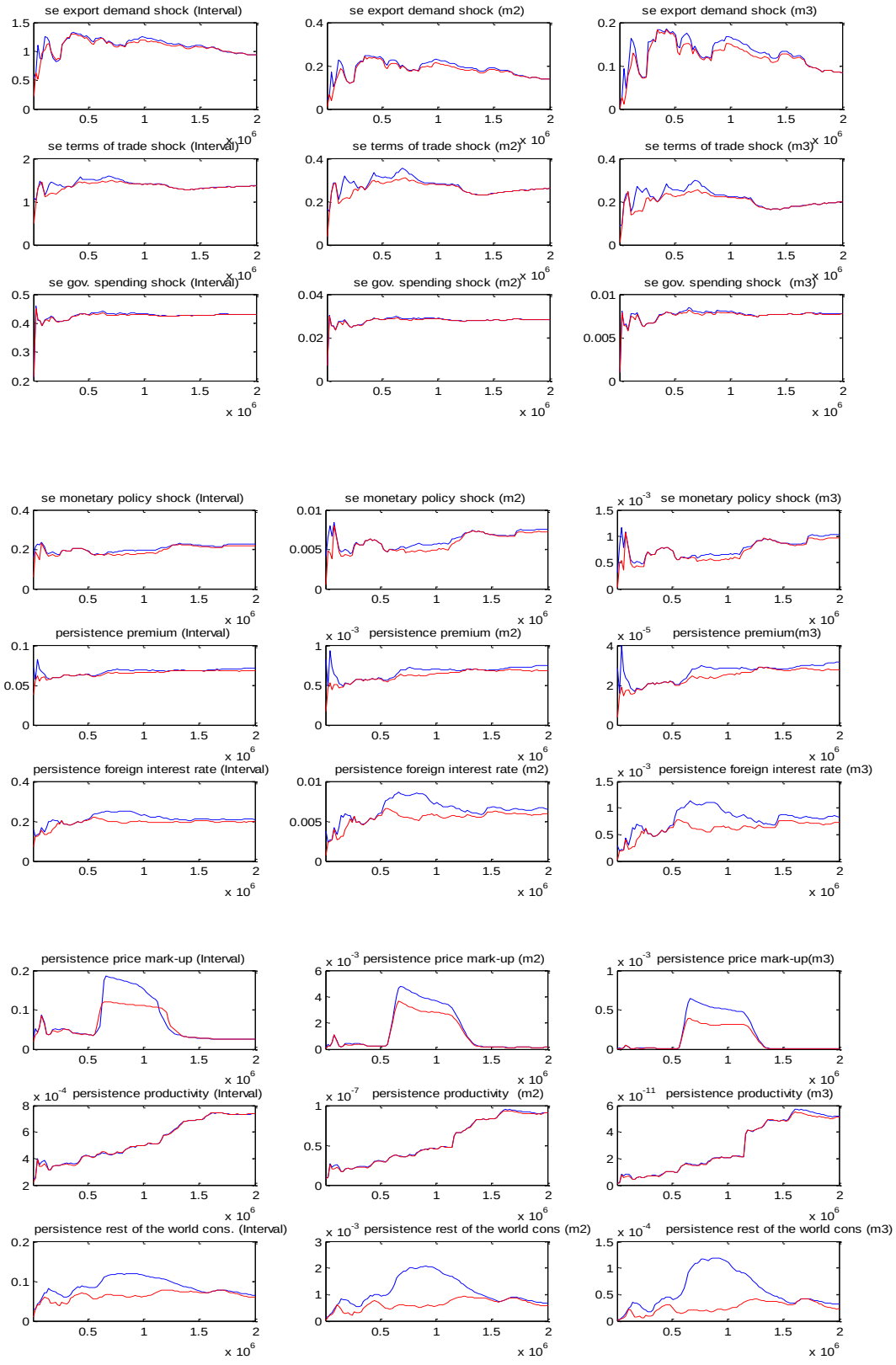


Figure A6.2: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

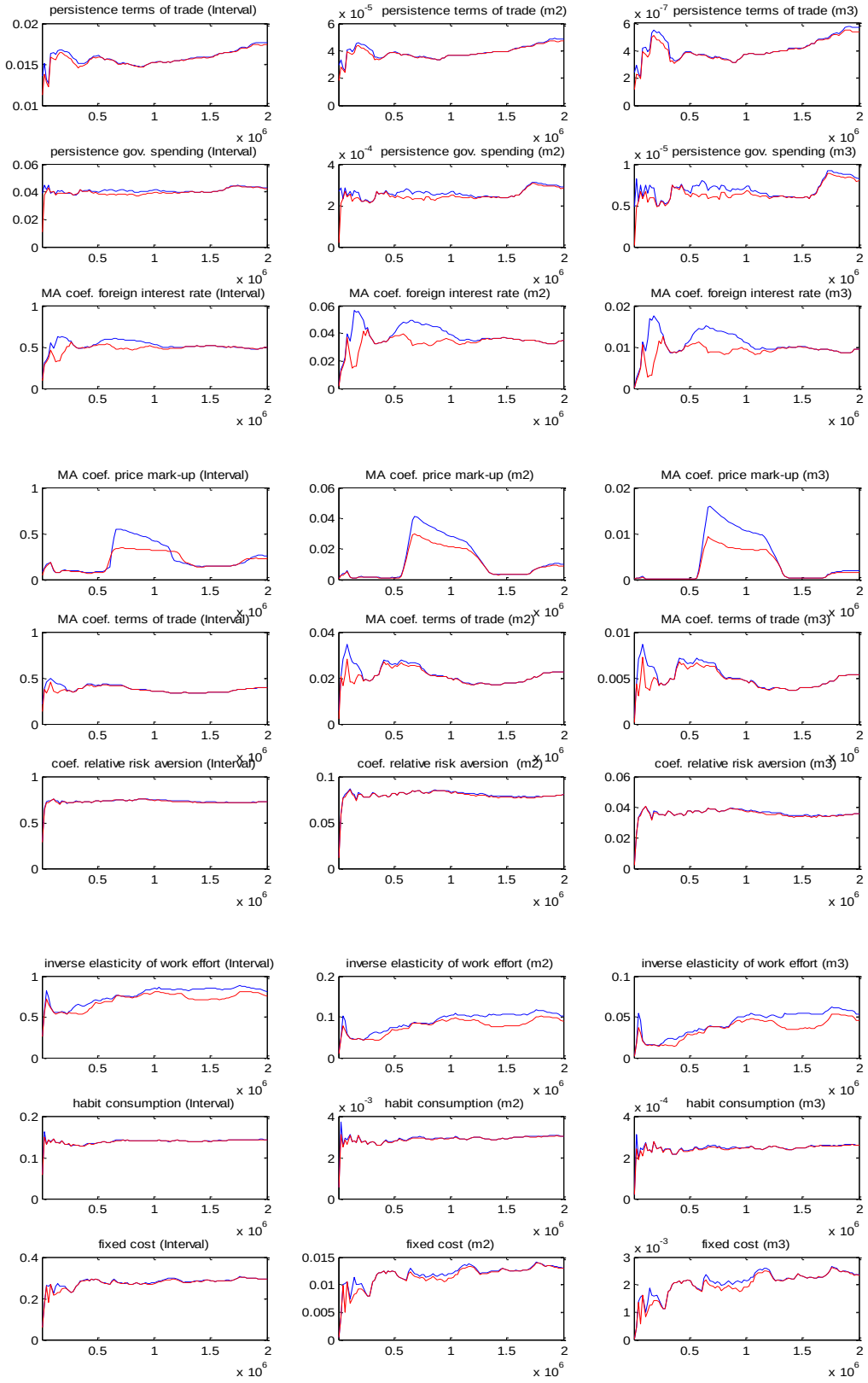


Figure A6.3: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

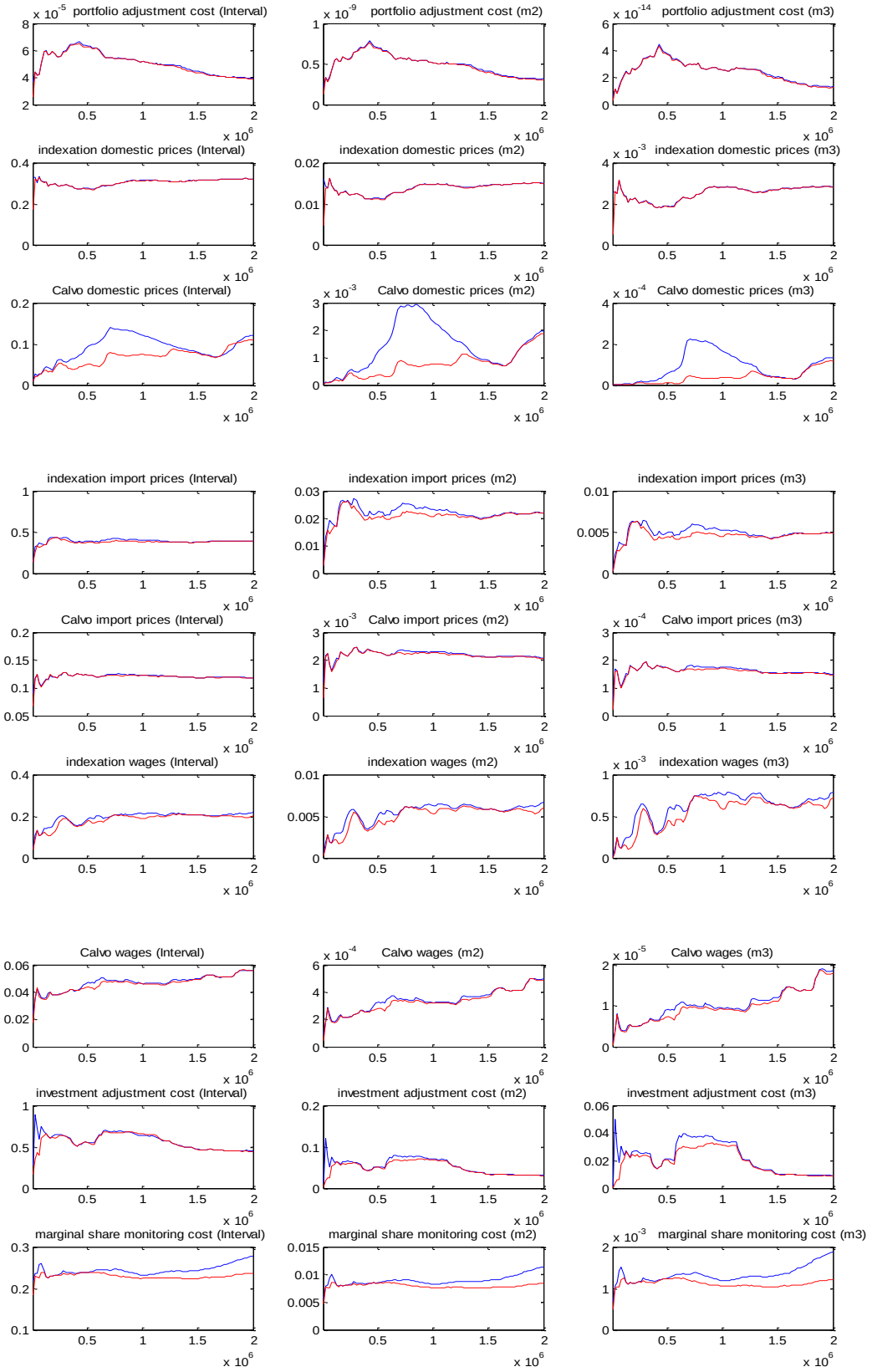


Figure A6.4: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

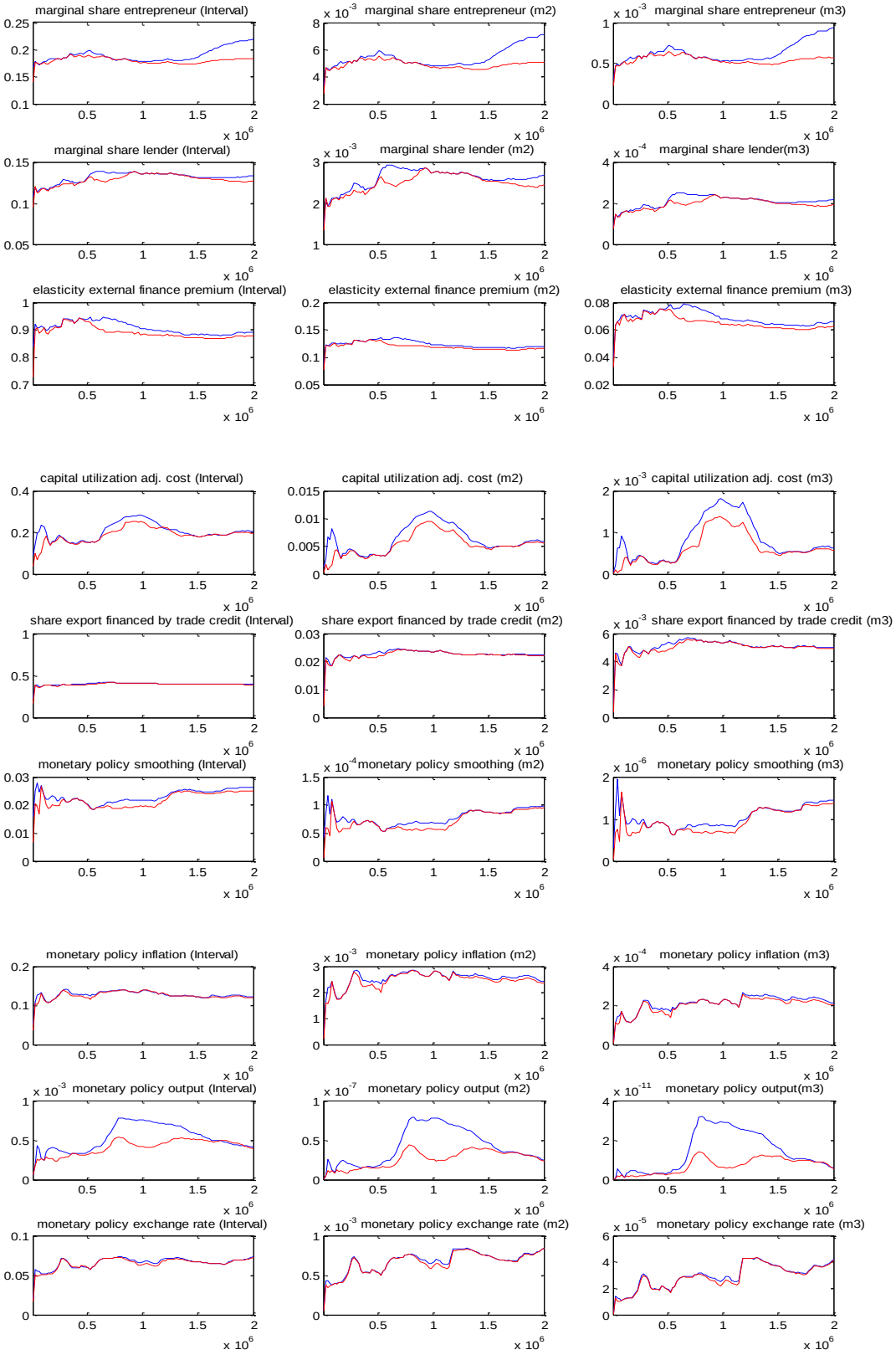


Figure A6.5: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

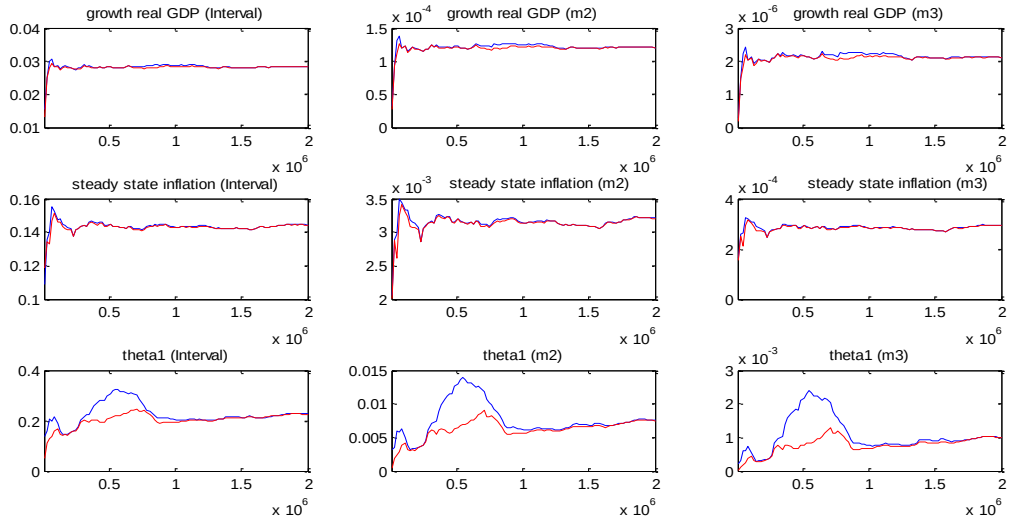


Figure A6.6: Monte Carlo Markov Chains univariate diagnostics (Brook and Gelman, 1998)

Measures that show slight differences might indicate signs of weak convergence. While these convergence issues may be explained by some structural breaks in the data, it is common in the literature to fix the values of parameters that may exhibit convergence problems. Alternatively, one could also reduce the size of the model and eliminate the features for which related parameters might show sign of weak convergence. Nonetheless, this last option is less appealing since many features are needed to fit the DSGE models to the South Africa data (see Chapter 4).

Chapter 4
Applications of the DSGE model: Business cycles in the South African economy

This chapter presents the main results from our analysis of business cycle fluctuations in the South African economy. To address the key questions of economic fluctuations in this Sub-Saharan African economy, our philosophy conceptualizes that economy as a dynamic system with a stationary equilibrium known as a steady state or a balanced growth path. Exogenous shocks hit that system, trigger its departure from the stationary equilibrium, and generate fluctuations of macroeconomic variables. The economic system embodies financial, nominal, and real frictions that define its dynamic properties and shape its adjustment back to the steady state. From this philosophy, exogenous stochastic shocks cause business cycle fluctuations in the South African economy, the macroeconomic adjustment as well as the persistence in macroeconomic data depend upon financial, nominal, and real frictions.

The above conceptualization of the source of business cycle fluctuations in the South African economy lays down the foundations for an application of the DSGE model developed and estimated (using South Africa macroeconomic data) in the previous chapters. This empirical use of the DSGE model allows us to address keys macroeconomic issues and provide substantive answers to fundamental business cycles questions investigated in this research. Chapter 2 had provided the theoretical underpinnings of the structural shocks likely to generate economic fluctuations in a developing economy such as the South African economy. The

estimation undertaken in chapter 3 had identified the statistical significance of those exogenous shocks. In this chapter, we focus on determining the exogenous shocks that drive key macroeconomic variables. More importantly, we seek the contribution of each exogenous shock to the volatility of observable macroeconomic variables so we can assess whether a particular shock is, or is not, a significant driving force for an observable macroeconomic variable. The contributions of exogenous shocks to the volatility of a macroeconomic variable could be evaluated from artificial data generated by simulating the theoretical DSGE model. Since structural shocks are orthogonal, at least as it emerges from the estimated variance-covariance matrix of exogenous shocks, which is diagonal, it is possible to decompose unambiguously the forecast error variance of each macroeconomic variable into components that exclusively reflect the variability attributed to a specific shock. The relative contribution of an exogenous shock into the forecast error variance of a specific macroeconomic variable determines the virtual importance of that shock in explaining the fluctuations of that variable of interest. Exogenous shocks with the highest contributions into the forecast error variance of a macroeconomic variable of interest are its main driving forces, or its main sources of fluctuations. This exercise is known as the forecast error variance decomposition, and it aims at answering business cycle questions through a collection of statistics. Summarizing business cycles fluctuations by a collection of statistics, measured from artificial data generated by simulating a theoretical DSGE model, is an approach popularized by the real business cycle literature (see Cooley 1995). It has been applied extensively in the estimated DSGE literature.

Before presenting the forecast error variance decomposition, we discuss two important applications of the estimated DSGE model. First, we report the results from the sensitivity analysis, which intends to assess the empirical importance of financial, nominal, and real

frictions embedded in the model. As mentioned earlier, the marginal likelihood is a measure of in-sample goodness in Bayesian econometrics; it constitutes therefore a summary statistic of the model complexity. The sensitivity analysis aims at examining the contribution of each of the frictions to the marginal likelihood of the DSGE model. The first section presents the results from that analysis for the South African economy. The second section reports the impulse response analysis. In that section, we describe the macroeconomic adjustment to exogenous shocks and analyse the dynamic properties of the South African economy. The final section then reports the variance decomposition and unveils the short-term driving forces of observable macroeconomic variables.

1- The sensitivity analysis: empirical importance of frictions

The DSGE model constructed in the second chapter introduces a large number of financial, real, and nominal frictions. The financial accelerator characterizes the main financial friction. The exchange rate pass-through, the domestic price and wage rigidities, and the price and wage indexation represent the nominal frictions. The external habit formation in consumption, the internal investment adjustment cost, the fixed cost, and the variable capital utilization characterize the real frictions. The fundamental question that emerges from such an incorporation of a large number of frictions is the following: which frictions are empirically essential in explaining the dynamic of the South Africa data.

In this section we use the DSGE model to answer that question and examine the contribution of each of the frictions to the marginal likelihood of the estimated DSGE model. Since the DSGE model is estimated with Bayesian techniques, the marginal likelihood can be interpreted as a summary statistic of in-sample goodness, which serves to compare different

specifications of the model, and then, constitutes a criterion to judge the benefit of a model complexity. Practically, we re-estimate the model under a different assumption for each parameter related to a friction, the assumption is generally formulated to weaken the friction or cancel out its effect; we then compare the obtained marginal likelihood with that of the benchmark model presented in chapter 3. The expectation is that weakening a friction necessary to capture the dynamic of the data is costly to the marginal likelihood of the estimated DSGE model.

Table 4.1 reports the results of this sensitivity analysis, it presents the estimated mode of the parameters as well as the marginal likelihood when each frictions (exchange rate pass-through, domestic price and wage rigidities, price and wage indexation, external habit formation, internal investment adjustment cost, fixed cost, variable capital utilization rate) is drastically reduced one at a time. For the financial frictions, we compare the estimation results from the model without entrepreneur versus those from the benchmark model, which includes an entrepreneur sector.

As indicated in table 4.1, removing the entrepreneurial sector and setting up the model such that the capital is directly accumulated by households is costly in terms of the deterioration of the marginal likelihood. The marginal likelihood falls significantly by about 39.69. In addition, there is an increase into exogenous shock arising from the rest of the world. The export demand shock increases to 5.74 and remains significant at the test size of 5%; the terms of trade shock also remains significant and its standard deviation slightly increases to 2.36. The most important observation on the structural parameters side is that the portfolio adjustment cost parameter, which is statistically insignificant at a test size of 5% in the benchmark model, now turns out to be significant at that same test size. This implies on international financial markets a

tranversality condition for optimizing households that effectively depends on portfolio adjustment cost such that households' borrowing actually converges to a stationary level.

The deterioration of the marginal likelihood suggests that introducing financial frictions, hence the financial accelerator, in a DSGE model designed to analyze business cycle fluctuations in the South African economy is helpful to capture the empirical dynamic of the macroeconomic data. In addition, for this small open economy, a financial environment in which households directly accumulate capital is not necessary less stringent than the one where entrepreneurs operate and could borrow abroad albeit subject to a balance sheet constraint. An entrepreneurial sector engenders a frictional access to external financing of capital accumulation, which permits the developing economy to finance its investment demand and weather exogenous shocks. In fact, as one could observe from the impulse response analysis in the model without entrepreneur (see figure 4.8 to figure 4.14), responses to some exogenous shock have larger amplitudes when the entrepreneur sector has been withdrawn. Hence, shutting off the external financing channel to the small developing economy makes financial stresses severe and amplifies business cycle fluctuations.

Nominal frictions are also essential to capture the empirical dynamic of the South Africa data, though some are relatively less important. Reducing the exchange rate pass-through to a near complete exchange rate pass-through slightly lowers the marginal likelihood. Structural parameters are not notably affected although the standard deviation to the terms of trade shock falls a bit. A lower degree of price indexation in the import goods sector or in the domestic goods production sector tends to be relatively costly to the marginal likelihood. Nevertheless, not all indexation is very important, restricting the wage indexation parameter closest to zero

induces a small improvement in the marginal likelihood. This may indicate that wages that cannot optimally adjust actually stay constant.

Reducing the degree of domestically produced goods price stickiness to a Calvo probability of 0.05 is costly in terms of the deterioration of the marginal likelihood. The marginal likelihood falls by approximately 94.26. The degree of price indexation in the sector drastically increases, while the Calvo wage probability is greater relatively to the benchmark model. Lowering the wage stickiness also deteriorates the marginal likelihood while reduces the inverse of the elasticity of work effort with respect to real wage.

Turning to the real frictions, the external habit formation in consumption, the internal investment adjustment cost, the fixed cost, and the variable capital utilization are all significantly important in explaining the fluctuations of South Africa macroeconomic data. In each case, the marginal likelihood falls slightly. Reducing to a low level (about 1.04) the fixed cost in the domestic production function deteriorates the marginal likelihood by about 6. A lower external habit formation in consumption is also costly and deteriorates the marginal likelihood greater than a drastic diminution of the investment adjustment cost, which appears necessary to capture the hump-shaped endogenous dynamic of the South African investment. Contrary to Smets and Wouters (2007)'s finding for the United States economy, the introduction of the variable capital utilization rate matters to explain the dynamic of South Africa macroeconomic data and business cycles. In fact, lowering the variable capital utilization rate is costly in terms of the deterioration of the marginal likelihood, which falls by about 9.35. Contrary to the discussion in King and Rebelo (2000) or Baxter and Farr (2005) (see chapter 2) the standard deviation of the productivity shock does not increase with lower variable capital utilization rate; however, the magnitudes of shocks arising from the external sector do.

Table 4.1: Testing the empirical importance of the financial, nominal, and real frictions embedded in the DSGE model

		Benchmark DSGE model	No entrepreneur DSGE model	Near Complete exchange rate pass-through	Low import price indexation	Low domestic price stickiness	Low domestic price indexation	Low wage stickiness	Low wage indexation	Low habit formation
				$\xi_m = 0.05$	$l_m = 0.2$	$\xi_n = 0.05$	$l_n = 0.05$	$\xi_W = 0.15$	$l_w = 0.2$	$h = 0.1$
Log marginal likelihood (Laplace approximation)		-1467.37	-1507.06	-1471.46	-1470.78	-1561.63	-1468.67	-1478.87	-1461.09	-1476.80
Parameter										
ϕ	coefficient of relative risk aversion	2.2450	2.7904	2.1733	2.2482	1.7156	2.2775	2.0518	2.3712	2.3111
η	inverse of the elasticity of work effort with respect to real wage	3.1555	3.8115	3.1822	3.1405	3.5236	3.1366	1.0858	2.8371	2.9495
h	habit formation in consumption	0.2960	0.2938	0.2782	0.2922	0.2295	0.2931	0.2514	0.3029	0.1000
Φ_c	(one plus) fixed cost parameter in goods production sector	1.4934	1.4276	1.4891	1.4920	1.3467	1.4867	1.6042	1.4532	1.4965
ψ_D	portfolio adjustment cost parameter	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
l_n	degree of price indexation in the good production sector	0.4663	0.1452	0.4078	0.4562	0.7109	0.0500	0.2605	0.4869	0.4678
ξ_n	Calvo parameter in the goods production sector	0.5709	0.6549	0.5637	0.5707	0.0500	0.5676	0.6195	0.5905	0.5607
l_m	degree of price indexation in the import goods sector	0.4737	0.2998	0.3874	0.2000	0.4824	0.4529	0.4029	0.5410	0.4436
ξ_m	Calvo parameter in the import goods sector	0.5992	0.6335	0.0500	0.5968	0.6070	0.5878	0.6165	0.6099	0.5925
l_w	degree of wage indexation	0.3443	0.4613	0.3656	0.3523	0.5152	0.3550	0.4867	0.2000	0.3783
ξ_w	Calvo parameter for wage	0.8242	0.6912	0.8227	0.8248	0.8522	0.8240	0.0500	0.8198	0.8260
χ	steady state elasticity of the investment adjustment cost	1.4415	1.6780	1.3908	1.5134	1.3396	1.5587	0.5857	0.9201	1.3752
ζ_c	marginal steady state share of … monitoring cost	0.4367	n.a.	0.4373	0.4369	0.4378	0.4367	0.3767	0.4373	0.4366
ζ_e	marginal steady state share of … entrepreneur	0.4991	n.a.	0.4989	0.4993	0.4997	0.4997	0.5023	0.5001	0.4991
ζ_l	marginal steady state share of … foreign lender	0.6928	n.a.	0.6923	0.6922	0.6914	0.6923	0.6926	0.6922	0.6926
ζ_p	steady state elasticity of the external finance premium	3.7507	n.a.	3.7518	3.7508	3.7485	3.7504	3.7450	3.7502	3.7519
ζ_u	steady state elasticity of the marginal cost of adj. cap. utilization rate	0.1202	0.4877	0.0553	0.0988	0.1004	0.0975	0.6468	0.2598	0.1042
τ	fraction of export financed by trade credit	0.4963	0.4888	0.4975	0.4972	0.4988	0.4959	0.4909	0.4890	0.4964
ρ_R	degree of central bank's interest rate smoothing	0.9672	0.9840	0.9751	0.9700	0.9837	0.9687	0.9936	0.9840	0.9675
ψ_1	Taylor coefficient of inflation	1.1893	1.0000	1.1862	1.1894	1.2015	1.1732	1.5475	1.2299	1.1907
ψ_2	Taylor coefficient of output gap	0.0000	0.0026	0.0003	0.0000	0.0018	0.0000	0.0001	0.0000	0.0000
ψ_3	Taylor coefficient of currency depreciation	1.1372	1.2053	1.1302	1.1347	1.1333	1.1361	1.1421	1.1219	1.1388
$\bar{g}$	quarterly trend growth rate to real GDP	0.7796	0.7831	0.7786	0.7795	0.7788	0.7796	0.7801	0.7800	0.7796
$\bar{\pi}$	quarterly steady-state inflation rate	8.7247	8.7180	8.7260	8.7247	8.7229	8.7245	8.7314	8.7261	8.7249
θ_1	rest of the world elasticity of substitution home and import goods	0.4999	0.8126	0.5687	0.5033	0.4382	0.4830	0.4733	0.4945	0.4962

ρ_B	persistence coefficient of the premium	0.8876	0.9908	0.9020	0.8912	0.9018	0.8927	0.9676	0.7848	0.8895
ρ_{i^*}	persistence coefficient of the foreign interest rate	0.8340	0.9934	0.8373	0.8346	0.8335	0.8357	0.8873	0.9431	0.8289
ρ_P	persistence coefficient of the domestic prices mark-up	0.9230	0.8114	0.8556	0.8903	0.9722	0.7943	0.9612	0.9626	0.9135
ρ_Y	persistence coefficient of the technology	0.9998	0.9857	0.9999	0.9998	0.9999	0.9998	0.9961	0.9999	0.9997
ρ_X	persistence coefficient of the rest of the world consumption	0.8892	0.8684	0.8987	0.8938	0.8781	0.8990	0.7896	0.8847	0.8889
ρ_T	persistence coefficient of the terms of trade	0.9804	0.9994	0.9872	0.9818	0.9988	0.9832	0.9919	0.9831	0.9809
ρ_G	persistence coefficient of the government spending	0.9023	0.9665	0.9048	0.9054	0.8895	0.9088	0.8245	0.8967	0.9013
μ_{i^*}	MA coefficient of the foreign interest rate	0.5497	0.2744	0.5467	0.5523	0.3652	0.5462	0.4948	0.2814	0.5176
μ_P	MA coefficient of the domestic prices mark-up	0.8429	0.3961	0.7047	0.7886	0.1562	0.4399	0.6368	0.8639	0.8381
μ_T	MA coefficient of the terms of trade	0.9404	0.9145	0.9226	0.9402	0.9487	0.9377	0.9249	0.9563	0.9366
σ_B	Standard deviation of the financial market shock	0.0430	0.0479	0.0474	0.0449	0.0805	0.0434	0.0454	0.0728	0.0433
σ_P	Standard deviation of the mark-up price shock	0.8786	0.7233	0.7563	0.8380	4.0000	0.8000	0.8612	0.9075	0.8838
σ_Y	Standard deviation of the technology shock	0.8052	0.9644	0.7604	0.8068	0.5214	0.7883	1.2702	0.8243	0.6771
σ_X	Standard deviation of the export demand shock	4.9441	5.7352	4.9467	4.9550	5.1431	5.0081	5.2419	5.0026	4.9421
σ_T	Standard deviation of the terms of trade shock	1.8452	2.3562	1.0825	1.6964	3.3578	1.7205	1.5065	2.4047	1.6409
σ_G	Standard deviation of the government spending shock	1.9038	1.3233	1.9001	1.8946	1.9290	1.8903	1.4643	1.9184	1.9108
σ_R	Standard deviation of the monetary policy shock	0.2733	0.1400	0.2065	0.2493	0.1385	0.2599	0.0554	0.1321	0.2709

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Table 4.1 (continued): Testing the empirical importance of the financial, nominal, and real frictions embedded in the DSGE model

		Benchmark DSGE model	Low investment adjustment cost	Low fixed cost	Low elasticity of capital utilization rate
			$\chi = 0.1$	$\Phi_c = 1.04$	$\zeta_u = 0.50$
Log marginal likelihood (Laplace approximation)		-1467.37	-1474.91	-1473.35	-1476.72
Parameter					
ϕ	coefficient of relative risk aversion	2.2450	1.7164	2.2561	2.2661
η	inverse of the elasticity of work effort with respect to real wage	3.1555	3.6882	3.1550	3.2259
h	habit formation in consumption	0.2960	0.2589	0.2779	0.2831
Φ_c	(one plus) fixed cost parameter in goods production sector	1.4934	1.4887	1.04	1.4375
ψ_D	portfolio adjustment cost parameter	0.0000	0.0000	0.0000	0.0000
l_n	degree of price indexation in the good production sector	0.4663	0.2228	0.4388	0.4525
ξ_n	Calvo parameter in the goods production sector	0.5709	0.4420	0.5940	0.5880
l_m	degree of price indexation in the import goods sector	0.4737	0.2718	0.4681	0.4856
ξ_m	Calvo parameter in the import goods sector	0.5992	0.5958	0.6106	0.6197
l_w	degree of wage indexation	0.3443	0.1293	0.3249	0.3597
ξ_w	Calvo parameter for wage	0.8242	0.8063	0.8349	0.7934
χ	steady state elasticity of the investment adjustment cost	1.4415	0.1000	1.0367	0.8972
ζ_c	marginal steady state share of … monitoring cost	0.4367	0.4337	0.4367	0.4366
ζ_e	marginal steady state share of … entrepreneur	0.4991	0.5008	0.5013	0.5014
ζ_l	marginal steady state share of … foreign lender	0.6928	0.6922	0.6921	0.6920
ζ_p	steady state elasticity of the external finance premium	3.7507	3.7534	3.7482	3.7499
ζ_u	steady state elasticity of the marginal cost of adj. cap. utilization rate	0.1202	0.4807	0.4585	0.5000
τ	fraction of export financed by trade credit	0.4963	0.4940	0.4973	0.4973
ρ_R	degree of central bank's interest rate smoothing	0.9672	0.9198	0.9815	0.9816
ψ_1	Taylor coefficient of inflation	1.1893	1.1068	1.1767	1.1637
ψ_2	Taylor coefficient of output gap	0.0000	0.0001	0.0000	0.0000
ψ_3	Taylor coefficient of currency depreciation	1.1372	1.1393	1.1388	1.1361
$\bar{g}$	quarterly trend growth rate to real GDP	0.7796	0.7747	0.7781	0.7783
$\bar{\pi}$	quarterly steady-state inflation rate	8.7247	8.7298	8.7269	8.7260
θ_1	rest of the world elasticity of substitution home and import goods	0.4999	0.5028	0.5125	0.5379
ρ_B	persistence coefficient of the premium	0.8876	0.9049	0.9317	0.9318
ρ_{i^*}	persistence coefficient of the foreign interest rate	0.8340	0.9534	0.8651	0.8677
ρ_P	persistence coefficient of the domestic prices mark-up	0.9230	0.9786	0.9823	0.9851
ρ_Y	persistence coefficient of the technology	0.9998	0.9999	0.9991	0.9994
ρ_X	persistence coefficient of the rest of the world consumption	0.8892	0.7927	0.8721	0.8565
ρ_T	persistence coefficient of the terms of trade	0.9804	0.9763	0.9875	0.9826
ρ_G	persistence coefficient of the government spending	0.9023	0.8153	0.8900	0.8844
μ_{i^*}	MA coefficient of the foreign interest rate	0.5497	0.6043	0.4871	0.4897
μ_P	MA coefficient of the domestic prices mark-up	0.8429	0.6520	0.7871	0.7870
μ_T	MA coefficient of the terms of trade	0.9404	0.8918	0.9432	0.9422
σ_B	Standard deviation of the financial market shock	0.0430	0.4440	0.0446	0.0449
σ_P	Standard deviation of the mark-up price shock	0.8786	1.4018	0.8524	0.9090

σ_Y	Standard deviation of the technology shock	0.8052	0.4940	0.8653	0.7346
σ_X	Standard deviation of the export demand shock	4.9441	4.9970	5.0160	5.0628
σ_T	Standard deviation of the terms of trade shock	1.8452	1.1730	1.8396	2.1261
σ_G	Standard deviation of the government spending shock	1.9038	2.2300	1.9611	1.9634
σ_R	Standard deviation of the monetary policy shock	0.2733	0.6675	0.1542	0.1533

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In sum, the sensitivity analysis shows that financial, real, and nominal frictions are relevant to capture the empirical dynamic of the South African economy. In addition, it illustrates that the estimated parameters are relatively robust to changes in the frictions one by one.

2- The impulse response analysis

In this section, we present the responses of the South African economy to various exogenous shocks and compare its quantitative results to the conventional wisdom built upon estimated DSGE models on developed economies data. The impulse response analysis also helps to examine the dynamic properties of the model, check its stability, and identify the variables that display complex and interesting dynamic, for example, by undershooting or overshooting their steady state values. The simulation generates the Bayesian impulse responses of the observable macroeconomic variables following a one-off temporary change into each exogenous shock. The magnitude of each shock is set to a one (positive) unit standard deviation of the exogenous shock. The figures presented in this section report, for the real GDP, consumption, and investment the percentage deviations of their respective growth rates from the balanced growth path[50]. The trade balance to GDP ratio, the inflation, and the domestic interest rate are in term of percentage deviations from the steady state level, while the currency

[50] Note that since we are interested in observable variables, denomination real GDP, consumption, and investment afterward refer to real GDP growth, consumption growth, and investment growth.

appreciation rate[51] depicts the exchange rate. We begin with the effects of the financial market shock.

2.1 Impulse responses to a financial market shock

Figure 4.1 illustrates the estimated impulse responses to a one standard deviation financial market shock. A positive financial market shock increases through its premium shock effect the return on assets held by households leading to a fall into consumption as theoretically predicted in the second chapter. Investment also declines as anticipated.

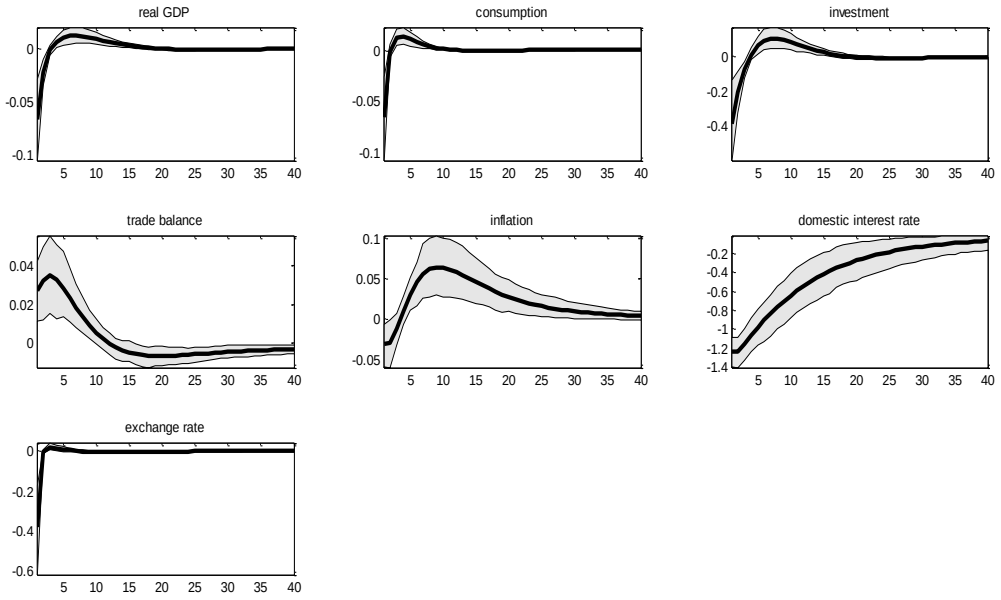

Figure 4.1: estimated impulse responses to a one (positive) unit standard deviation of financial market shock.
Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

The initial fall into consumption, at about 0.06 percent, is by far lesser than the 0.4 percent decline undergone by the investment, which is therefore more sensitive to the premium shock. In addition, the decrease in the domestic absorption depresses the real GDP by about

[51] Note that a positive value represents depreciation since the nominal exchange rate is defined as the price of the foreign currency unit in term of the domestic currency.

0.06 percent, which contrasts the empirical evidence from the US economy (see Smets and Wouters, 2007), or those from the Euro area (see Adolfson et al., 2007), where the initial output's expansion in response to a premium shock is roughly within the range 0.1 - 0.5 percent. The contraction of the domestic absorption also induces an initial trade balance surplus at about 0.03 percent of the GDP.

After a slight decline, the aggregate inflation moves upward, and inflationary pressures push the aggregate inflation above its stationary level. Inflation rises gradually reaching a peak of 0.05 percent. A high persistence characterizes the inflation path, which also displays a wider interval band than the real GDP. The premium shock forces an initial decline in the domestic interest rate at about 120 basis points, partially to respond to the initial deflation and moderately to restore the equilibrium condition imposed by the uncovered interest rate parity condition on the integrated financial markets.

The domestic interest rate and the exchange rate both adjust; the exchange initially appreciates by 0.04 percent. The unanticipated rise in the domestic premium first leads to an exchange rate appreciation, this immediate but temporary fall in the nominal exchange rate increases expected exchange rate depreciation cushioning the nominal interest rate from the full effect of the premium shock.

The economy recovers progressively, real GDP, consumption, and investment return gradually to the balance growth path, which they slightly overshoot.

2.2 Impulse responses to a price mark-up shock

Figure 4.2 shows that the aggregate inflation picks up instantaneously in response to a price mark-up shock; the initial increase of inflation above steady state level is approximately 4 percent. The inflation's response seems quite similar to the stylized fact from the euro area (see

Smets and Wouters, 2003) albeit the amplitude is substantially higher than the 1 percent reported for that economy.

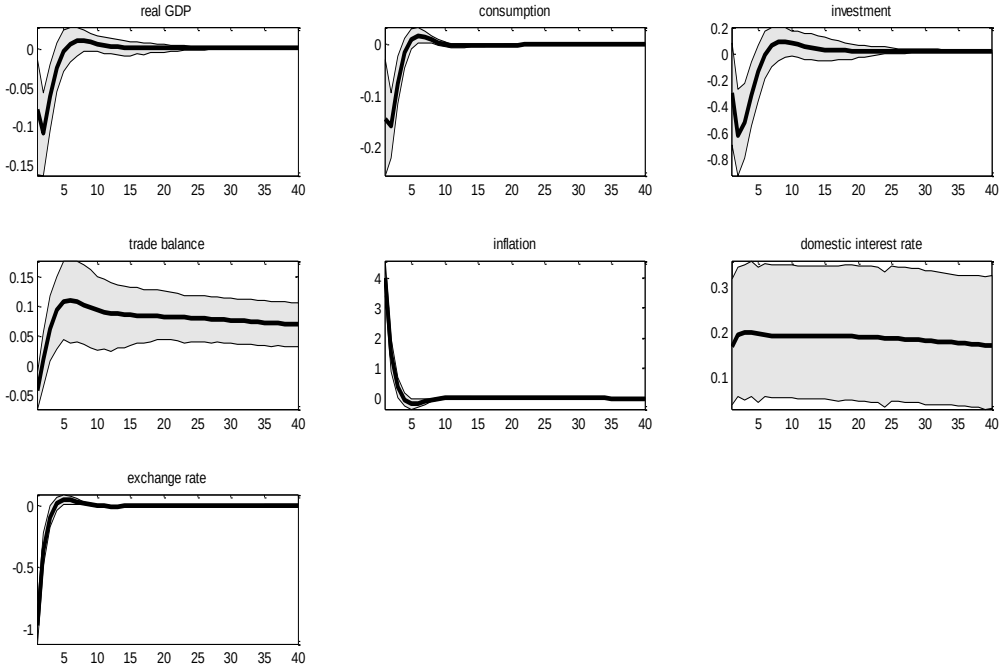

Figure 4.2: estimated impulse responses to a one (positive) unit standard deviation of cost-push shock.
Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

The mark-up price shock also induces on real GDP, consumption, and investment qualitative effects analogous to those reported on the euro area (see Smets and Wouters, 2003); the quantitative impacts are also comparable. Real GDP falls to a trough at about 0.1 percent, consumption 0.15 percent, and investment decline further by 0.6 percent. The trade balance improves gradually after an initial deficit at about 0.05 percent of the GDP.

The economy experiences a stagflation in which inflation and economic stagnation occur simultaneously and remain unchecked in the short term. The mark-up shock then creates a trade-off between inflation and output gap stabilization. Because of the Taylor's principle,

which requires a much greater adjustment of the domestic interest rate than the aggregate inflation, and an appreciation of domestic currency lower than the inflation rate, the monetary policy authority strongly and permanently adjusts the domestic interest rate upward in response to the persistent mark-up price shock. Domestic interest rate increases first by about two basis points; nonetheless, the confidence interval is wide unveiling a large uncertainty. Since the real GDP and the aggregate inflation return rapidly to the balanced growth path or the steady state equilibrium, the trade-off vanishes in the medium term.

2.3 Impulse responses to a productivity shock

Figure 4.3 shows that the responses of the real GDP, consumption, investment, inflation, and domestic interest rate to a productivity shock are in line with the Real Business Cycle model (RBC) predictions. The qualitative effects of this shock on real GDP, inflation, and interest rate are also very similar to the empirical evidence from developed economies such as the Euro area or the United States (see Smets and Wouters 2003, 2007). In fact, a positive productivity shock initially expands output by about 0.3 percent, while consumption increases further by approximately 1 percent. As GDP and consumption return to the balanced growth path, their growth rates decline, moderately for the GDP because of the highly persistent total factor productivity process. The investment response follows a hump-shaped pattern, which distinguishes that impulse response from those of the real GDP and consumption. The hump-shaped path reaches a peak at about 0.7 percent. Quantitatively, the real GDP, consumption, and investment amplitude are also closed to those reported in the Euro area or the United States (see Smets and Wouters 2003, 2007).

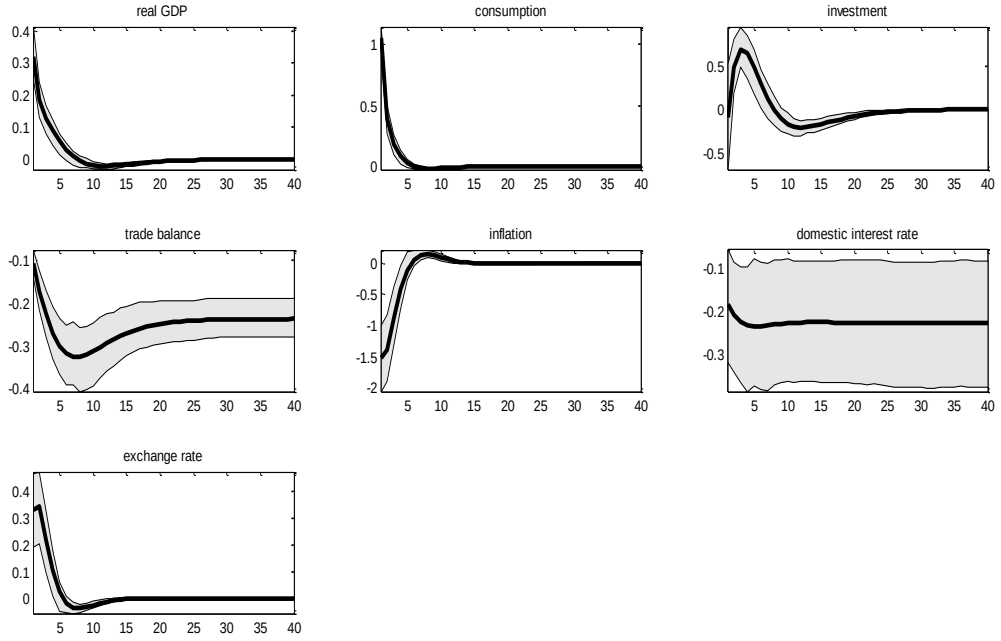

Figure 4.3: estimated impulse responses to a one (positive) unit standard deviation of productivity shock.
Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

Due to the rise in productivity, the intermediate good firm marginal cost falls and the aggregate inflation initially goes below steady state by about 1.5 percent. The monetary policy authority reduces the domestic interest rate to offset the effect of a declining marginal cost on the inflation. However, that action is moderated, since the persistent productivity shock induces a permanent trade balance deficit at about 0.25 percent of the GDP, which requires an exchange rate depreciation that pushes up the domestic interest rate. Domestic interest rate tends to fall permanently below the stationary level by about 0.22 percent, mostly because of a strong interest rate smoothing. Although the productivity process is persistent, real GDP growth returns to the balanced growth path after 10 quarters, consumption, and investment also adjust in the medium term.

2.4 Impulse responses to an export demand shock

Figure 4.4 shows that real GDP, consumption, and investment rise following a positive export demand shock. Real GDP undergoes a sharp increase at about 1.5 percent, while investment jumps by 1.4 percent, and consumption moderately increases by approximately 0.2 percent. The higher increase in investment relative to consumption results mostly from the required additional inputs needed to sustain the domestic production, this additional investment absorbs both domestic and import resources.

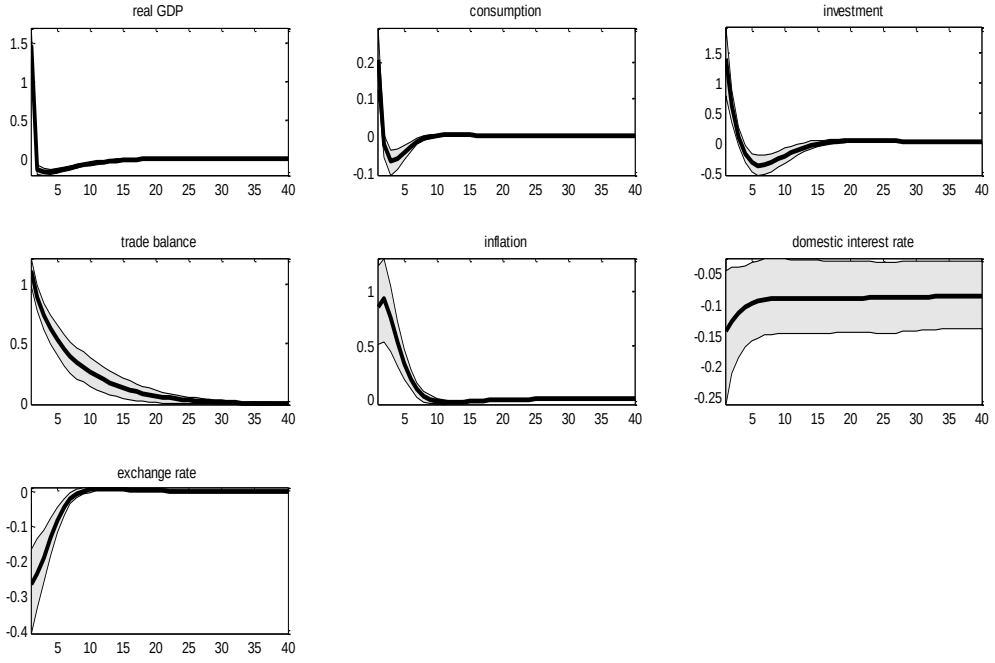

Figure 4.4: estimated impulse responses to a one (positive) unit standard deviation of export demand shock.
Note. The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

The trade balance registers an initial surplus at about 1 percent of the GDP above steady state. As foreign households demand more of domestically produced goods, the aggregate inflation rises to a peak at about 0.8 percent, and the exchange rate appreciates initially by 0.28

percent. The exchange rate appreciation completely offset the monetary authorities response to the rising inflation, as a result, the domestic interest rate falls below its stationary equilibrium by 15 basis points. The initial exchange rate appreciation eventually allows future depreciation; nonetheless, the interest rate smoothing keeps the domestic interest rate persistently below its long run equilibrium to which it returns gradually.

2.5 Impulse responses to a terms of trade shock

As illustrated on figure 4.5, a positive terms of trade shocks induces an initial increase at about 1 percent on the real GDP; consumption rises by 0.2, and the investment grows up substantially by 3 percent. The aggregate inflation rises sharply by approximately 1.5 percent at the beginning, pushes up to a peak of about 1.6 percent, and then decreases slowly toward the steady state inflation rate.

The terms of trade shock also generates a trade balance surplus as it raises the export revenues relatively to the import expenditures. The persistent terms of trade process creates a long lasting positive effect on the trade balance, which begins to reverse to its stationary equilibrium approximately 20 quarters later. The permanent trade balance surplus generated by the persistent term of trade process revolves around an average of 0.6 percent of the GDP.

The concomitant increase of the real GDP and the aggregate inflation above the balanced growth path and the steady state inflation, respectively, requires a positive adjustment of the domestic interest rate. The monetary authority strongly responds to the aggregate inflation by a more than one to one increase in the domestic interest rate (Taylor's principle). The response to the output gap is insignificant; and a third goal, the appreciation rate of the domestic currency, forces downward the interest rate adjustment since it negatively affects the monetary policy instrument.

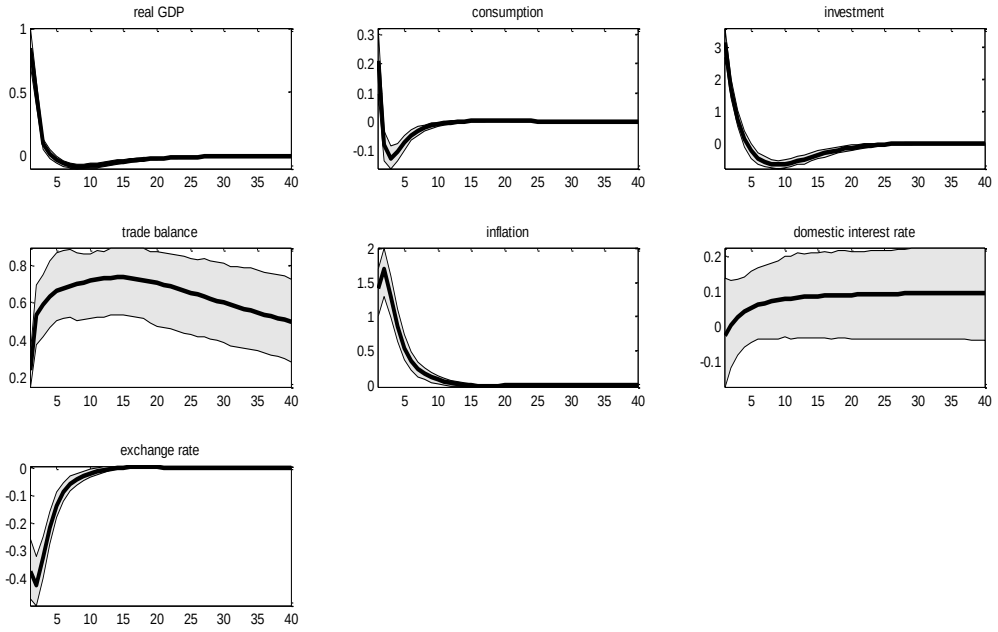

Figure 4.5: estimated impulse responses to a one (positive) unit standard deviation of terms of trade shock.
Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

Although the domestic currency initially appreciates by 0.4 percent, and further appreciation follows, the inflation goal dominates; the domestic interest rate rises steadily up to 10 basis points and starts reversing to its long run equilibrium level in the medium term. Large uncertainties surround, however, trade balance and interest rate responses.

2.6 Impulse responses to a government spending shock

As depicted in figure 4.6, a positive government spending shock increases real GDP by about 1.5 percent. Nonetheless, the effect is short lived since it dies out completely three quarters after. Contrary to the evidence for the Euro area reported in Smets and Wouters (2003), neither the consumption nor the investment are crowding out by an unanticipated increase in

government spending; rather the consumption expands initially by about 0.25 percent, while the investment rises significantly by 2 percent.

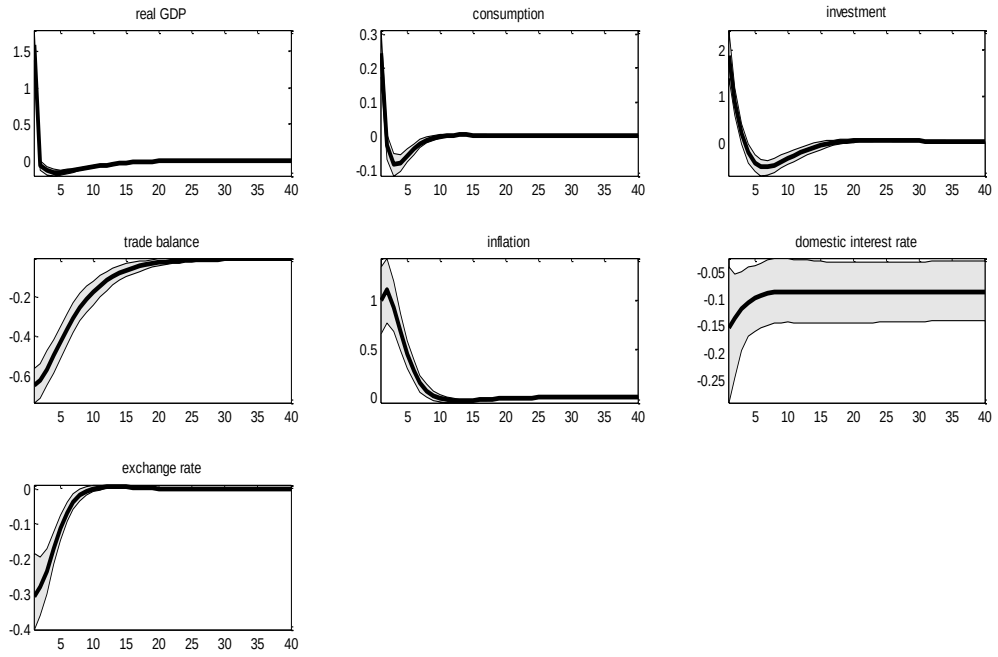

Figure 4.6: estimated impulse responses to a one (positive) unit standard deviation of government spending shock. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

The propagation of government spending shock unveils interesting dynamic properties of the model economy, properties engendered by the frictional access to external financing of capital accumulation. More specifically, without an entrepreneur sector in the model economy, unanticipated government spending does induce a crowding out effect on consumption and investment as well. Figure 4.13 that depicts the impulse responses to government spending shock in a model without entrepreneur forcefully shows that result. In addition, it reports that the initial impact of a government spending shock to the real GDP at about 1 percent is lower than the 1.5 percent observed in the benchmark model.

Returning to the model with entrepreneur, the increase on the domestic absorption puts pressure on prices, and the aggregate inflation initially rises by at about 1 percent. Although the monetary policy rule complies with the Taylor (1993) principle that emphasises inflation stabilization, the domestic interest rate falls below the stationary level since the exchange rate appreciation more than offsets the inflation stabilization.

2.7 Impulse responses to a monetary policy shock

In a closed economy New Keynesian dynamic stochastic general equilibrium model (see Smets and Wouters 2003, 2007), a positive monetary policy shock leads to a rise in both the nominal and the real short-term interest rate. This increase in the real interest rate following a positive monetary policy shock is commonly dubbed the liquidity effect. Moreover, output, consumption, and investment fall through hump-shaped patterns that reflect a certain degree of persistence to a monetary policy shock.

Figure 4.7 illustrates opposite effects in this small open economy model with an entrepreneurial sector that creates a frictional access to the external financing of capital accumulation. In fact, a positive monetary policy shock induces an expansionary effect on output, consumption, and investment; though the increases are very small. A positive monetary policy shock initially expands real GDP and consumption by about 0.0003 percent; investment increases slightly by about 0.0019 percent. In addition, inflation rises and reaches a peak at about 0.004percent. Although these numbers are too small, Bayesian confidence bands show that they are significantly different from zero. This raises the question why monetary policy shock induces different effects in this small open economy.

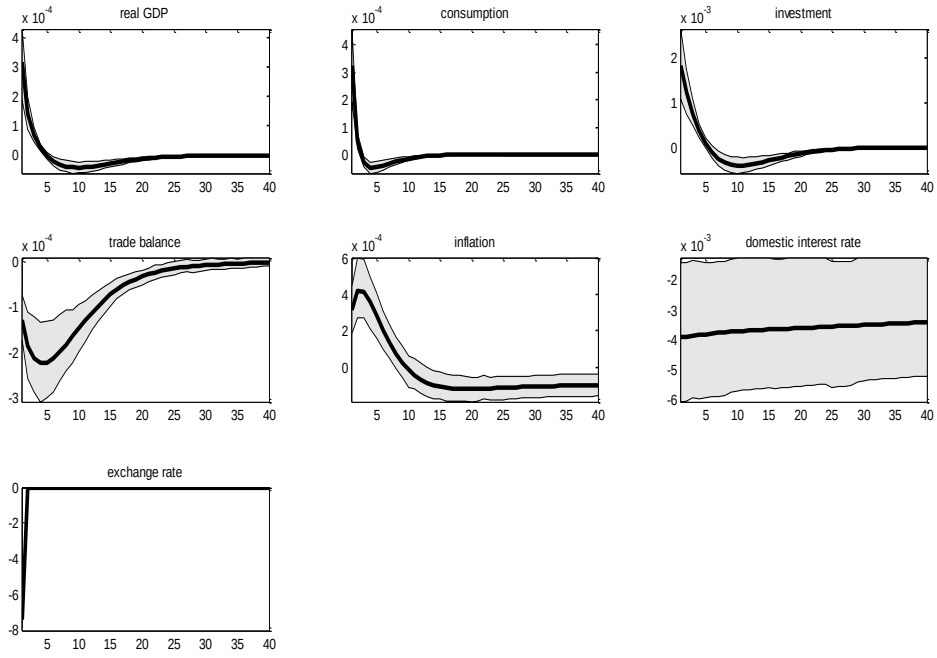

Figure 4.7: estimated impulse responses to a one (positive) unit standard deviation of monetary policy shock.
Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

In fact, the economic and financial environment surrounding monetary policy in this small open economy is quite different from the standard closed economy framework. First, the small open economy has no effect on the foreign interest rate, which it takes as given, and abides to the arbitrage condition imposed by the uncovered interest rate parity (UIP) condition. The UIP condition (see equation 2.11) relates monotonically the domestic interest rate to the domestic currency expected appreciation rate. It requires the nominal exchange rate to appreciate just after a positive monetary policy shock in order to cushion the domestic return (on asset held by the household) from the effect of that exogenous shock. Exchange rate expectations could not adjust instantly as they depends upon the current stock of information that does not yet include new information about exchange rate changing path, and prices are

sticky. Second, and perhaps more importantly, the financial accelerator mechanism influences the propagation of the monetary policy shock. The initial exchange rate appreciation, as one could recall from the theoretical model, increases the entrepreneur net worth, thus, lowers the leverage ratio and the external finance premium, which increases capital goods demand.

One main conclusion from this empirical application of a DSGE model to the South African economy is that financial frictions (the financial accelerator) alter the dynamic properties of the DSGE model, as well as the economy response to a positive monetary policy shock, making that shock expansionary as opposed to the contractionary effect observed in a closed economy. To better illustrates this view; we present on figure 4.14 the estimated impulse responses to one (positive) unit standard deviation of monetary policy shock in a model without entrepreneur. In that case, a positive monetary policy shock contracts real GDP, consumption, and investment. The inflation also falls as conventionally predicted. However, there is no liquidity effect. Gali (2000) made the arguments that the presence of a liquidity effect following a monetary policy shock depends on the persistence of the monetary policy shock. Here, it appears that the goals in the monetary policy rule are also crucial. The exchange rate appreciation following a positive monetary policy shock is sufficient to offset the impact of the Taylor principle.

2.8 Impulse responses in the model without entrepreneur

A brief summary of the main insights gained from the model without entrepreneur concludes this section. Figures 4.8 to 4.14 depict the impulse responses of that model.

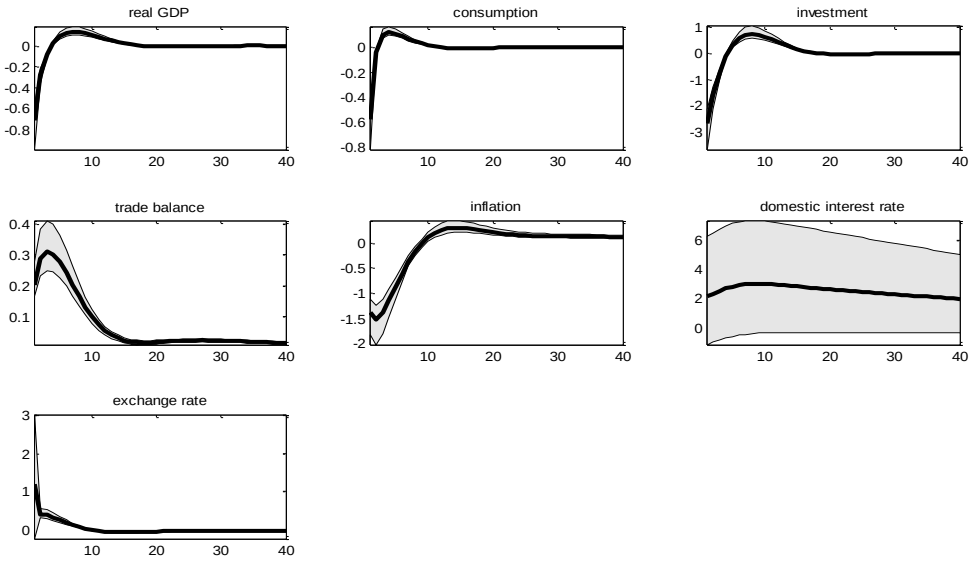

Figure 4.8: estimated impulse responses to a one (positive) unit standard deviation of financial market shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

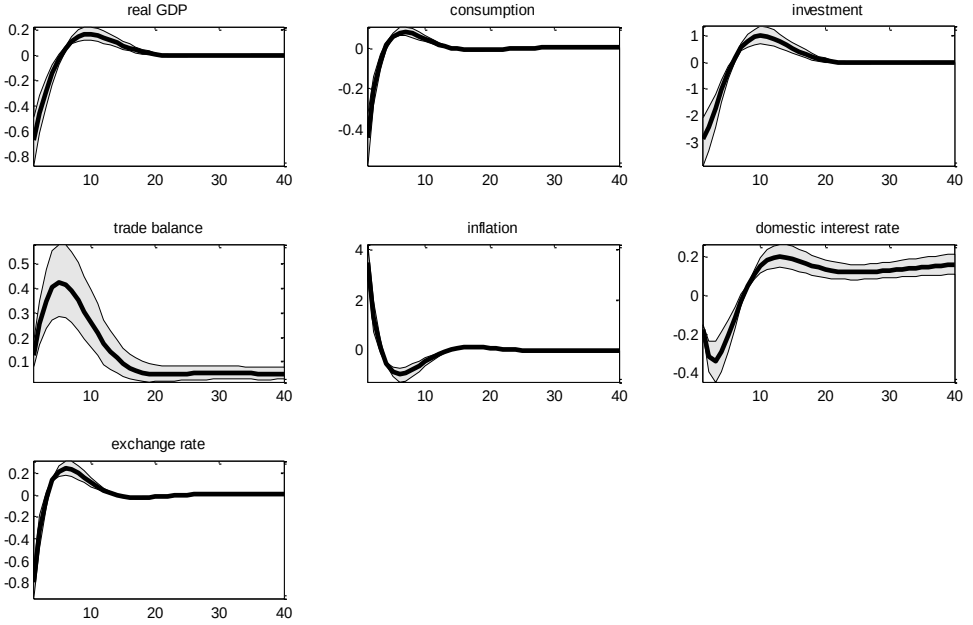

Figure 4.9: estimated impulse responses to one (positive) unit standard deviation of cost-push shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

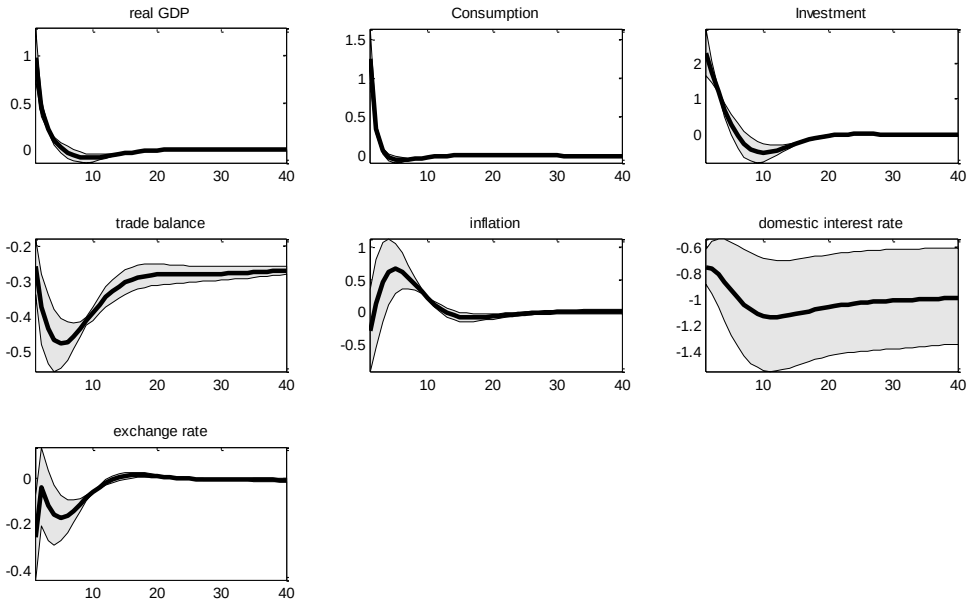

Figure 4.10: estimated impulse responses to one (positive) unit standard deviation of productivity shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

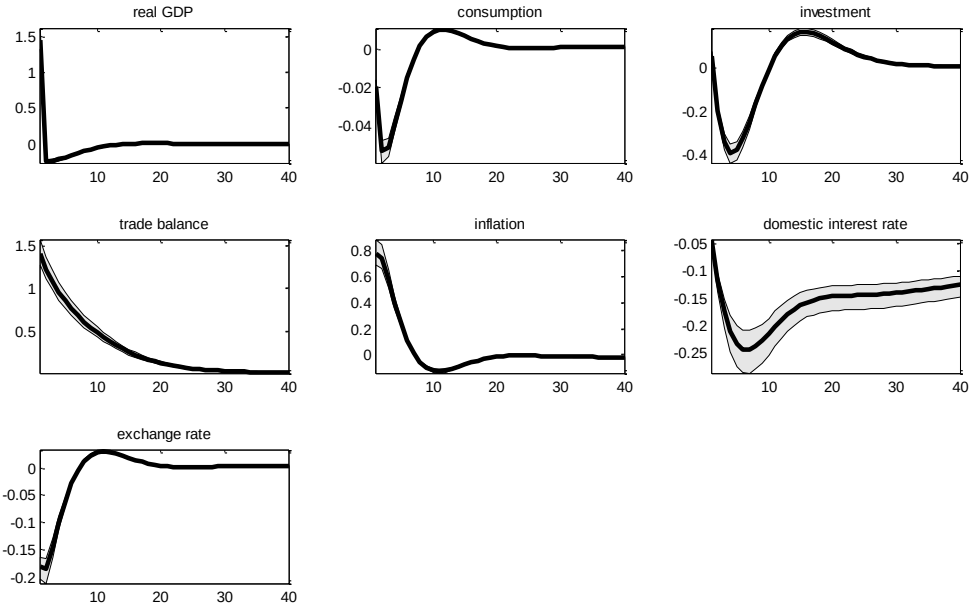

Figure 4.11: estimated impulse responses to one (positive) unit standard deviation of export demand shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

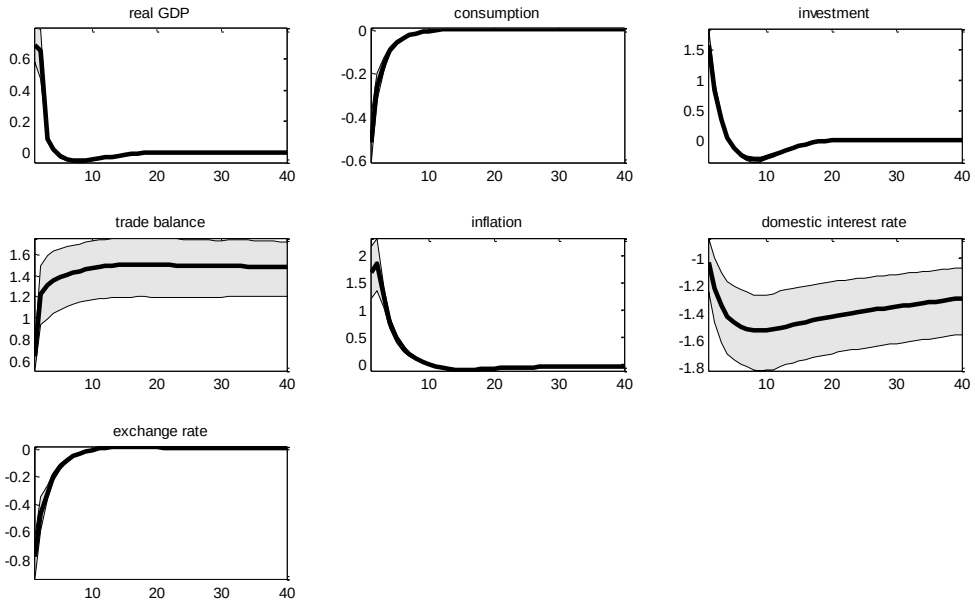

Figure 4.12: estimated impulse responses to a one (positive) unit standard deviation of terms of trade shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

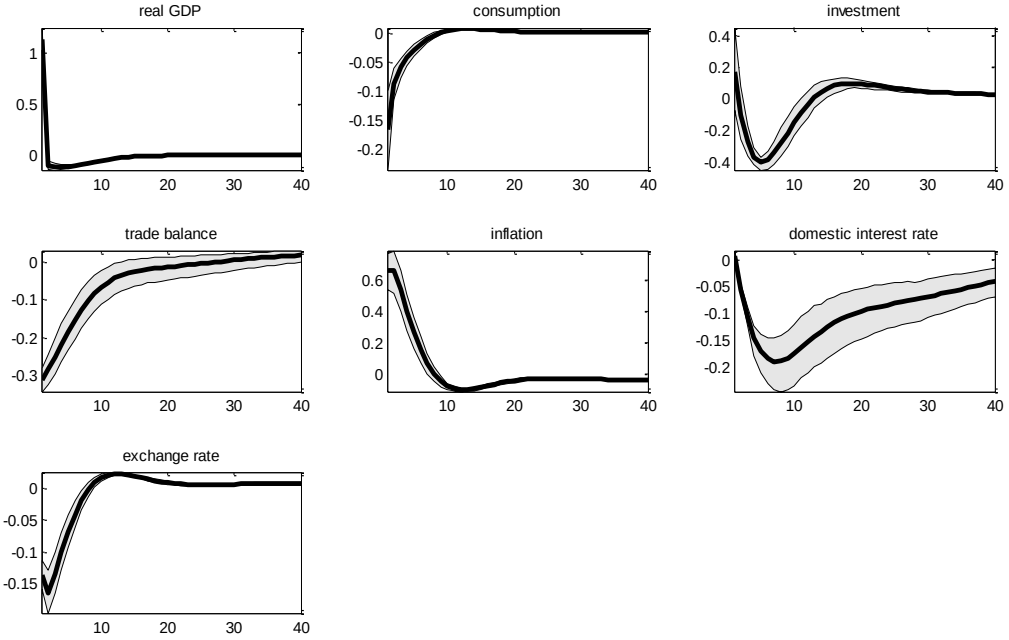

Figure 4.13: estimated impulse responses to a one (positive) unit standard deviation of government spending shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5th and 95th percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

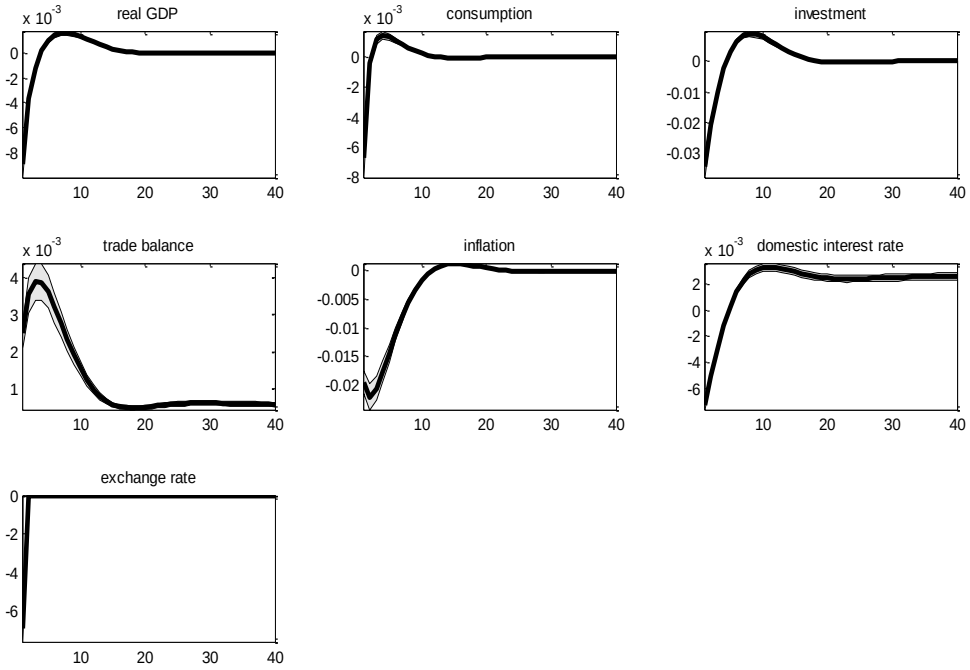

Figure 4.14: estimated impulse responses to a one (positive) unit standard deviation of monetary policy shock in a model without entrepreneur. Note: The figure plots the median (thick line) impulse response together with the 5^{th} and 95^{th} percentiles (grey band). The horizontal axis represents the horizon in quarters, while the vertical axis gives the percentage deviation from the balanced growth path or steady state.

Following a financial market shock, the qualitative effects from the model with entrepreneur and those from the model without entrepreneur are identical for the real GDP, consumption, investment, trade balance, and inflation. However, the amplitudes of the responses increase, specifically for the investment. The domestic interest rate goes up by 200 basis points, while the currency depreciates.

Regarding the cost-push shock, the qualitative effects are similar for all variables but the domestic interest rate, which falls then increases gradually in the model without entrepreneur. The productivity shock still increases real GDP, consumption, and investment; the trade balance also deteriorates. However, aggregate inflation rises, though there is a large uncertainty.

3- The forecast error variance decomposition

In this section, we identify the main driving forces of the South African key macroeconomic variables. We employ the forecast error variance decomposition and account for innovation in the variance of each observable variable at various horizons that could characterize the short run, medium run, and long run. Tables 4.3.1 to 4.3.3 report the forecast error variance decomposition of observable macroeconomic variables at various horizons based on the mean of the model's posterior distribution reported on table 3.3 in Chapter 3. The variables could be divided into two categories: real sector variables (trade balance, growth rates of real GDP, consumption, and investment,) and nominal variables (inflation rate, domestic interest rate, and exchange rate).

In the short run, which is a period within a year, demand shocks, mainly the exogenous government spending shock and the export demand shock, drives movements in the real GDP growth. Together, they account for approximately 82 percent of its forecast error variance at one-year horizon. The remaining real GDP growth fluctuations are partly explained by terms of trade shock, which accounts for 12 to 16 percent, and the productivity shock that explains less than 2 percent. Over time the contribution of the productivity shock slightly increases, though, it remains marginal.

Productivity shock predominantly drives consumption (growth) in the short run and explains approximately 77 percent of its fluctuations. Terms of trade shock accounts for about 12 percent, export demand shock and government spending shock drive the remaining. That consumption fluctuations are mostly driven by productivity shock is not surprising; the calibrated share of domestically produced goods in the household consumer's basket is about 75% and the intratemporal substitution elasticity is about 0.49, those two parameters indicate a

home bias toward domestically produced goods. The terms of trade shock largely explains investment (growth) in the short run and accounts for more than 60 percent of its fluctuations; government spending shock and export demand shock drive the remaining. The important role of terms of trade shock in explaining investment fluctuations could further be justified by the large import content of the South African investment. The export demand shock explains the largest part of trade balance fluctuations in the short run and account for approximately 61 to 74 percent of its forecast error variance, while government spending shock explains the remaining.

Table 4.2.1: Forecast error variance decomposition in the short run horizon

		financial market shock	cost-push shock	productivity shock	export demand shock	terms of trade shock	government spending shock	monetary policy shock
t=1	GDP	0.04	0.04	0.95	41.75	11.98	45.24	0.00
	consumption	0.18	0.00	75.41	5.47	12.14	6.79	0.00
	investment	0.35	0.88	2.67	15.38	60.72	20.01	0.00
	trade balance	0.02	0.13	0.35	73.68	0.54	25.28	0.00
	inflation	0.00	72.80	15.74	1.83	7.43	2.20	0.00
	interest rate	89.08	0.00	1.04	4.80	0.05	5.03	0.00
	exchange rate	0.12	1.58	0.30	0.08	0.15	0.09	97.67
t=2	GDP	0.04	0.12	1.28	39.66	15.94	42.96	0.00
	consumption	0.16	0.61	77.95	4.78	10.56	5.94	0.00
	investment	0.33	1.45	1.85	14.97	61.85	19.55	0.00
	trade balance	0.03	0.07	0.72	68.01	3.38	27.78	0.00
	inflation	0.00	56.01	23.09	3.40	13.36	4.13	0.00
	interest rate	88.98	0.12	2.03	4.18	0.36	4.33	0.00
	exchange rate	0.12	1.73	0.64	0.14	0.39	0.17	96.82
t=4	GDP	0.04	0.14	1.65	39.39	16.22	42.56	0.00
	consumption	0.16	0.67	77.22	4.90	10.98	6.08	0.00
	investment	0.30	1.65	3.20	14.06	62.36	18.43	0.00
	trade balance	0.04	0.12	1.98	61.09	5.91	30.86	0.00
	Inflation	0.00	44.30	26.07	5.51	17.35	6.77	0.00
	interest rate	86.63	0.21	4.56	3.54	1.48	3.58	0.00
	exchange rate	0.12	1.72	0.92	0.24	0.63	0.29	96.08

Turning to the nominal and financial variables, short run fluctuations of the aggregate inflation are mostly a matter of cost-push shock. While the contribution of cost-push shock declines from 73 percent in the first quarter to about 44 percent in the fourth quarter, the portions of the volatility explained by the productivity and the terms of trade shocks relatively increase as the horizon lengthens. Financial market shock predominantly drives domestic interest rate and accounts for more than 86 percent of its short run fluctuations; productivity shock, export demand shock, and government spending shock mostly explain the remaining fraction. Short run fluctuations in the nominal exchange rate are fundamentally driven by monetary policy shock.

Table 4.2.2: Forecast error variance decomposition in the medium run horizon

		financial market shock	cost-push shock	productivity shock	export demand shock	terms of trade shock	government spending shock	monetary policy shock
t=8	GDP	0.04	0.15	1.82	39.42	16.01	42.55	0.00
	consumption	0.17	0.78	75.49	5.27	11.72	6.57	0.00
	investment	0.32	1.67	5.61	14.33	59.30	18.78	0.00
	trade balance	0.04	0.19	5.88	53.31	9.37	31.21	0.00
	inflation	0.00	41.11	24.64	6.65	19.30	8.29	0.00
	interest rate	80.55	0.17	8.97	2.83	4.74	2.74	0.00
	exchange rate	0.12	1.74	0.94	0.31	0.75	0.38	95.76
t=16	GDP	0.05	0.16	1.78	39.06	16.66	42.29	0.00
	consumption	0.17	0.78	75.20	5.30	11.94	6.61	0.00
	investment	0.32	1.51	4.76	14.33	60.05	19.03	0.00
	trade balance	0.03	0.15	11.70	43.13	20.46	24.53	0.00
	inflation	0.02	40.57	24.42	6.68	19.95	8.35	0.00
	interest rate	68.04	0.12	14.42	2.32	13.00	2.10	0.00
	exchange rate	0.12	1.74	0.95	0.31	0.79	0.38	95.72
t=32	GDP	0.05	0.16	1.79	38.94	16.88	42.18	0.00
	consumption	0.17	0.78	75.19	5.30	11.94	6.62	0.00
	investment	0.32	1.46	4.88	14.00	60.72	18.63	0.00
	trade balance	0.03	0.10	14.87	29.53	39.40	16.08	0.00
	Inflation	0.03	40.54	24.41	6.70	19.94	8.37	0.00
	interest rate	49.85	0.09	21.49	2.13	24.65	1.79	0.00
	exchange rate	0.12	1.74	0.95	0.31	0.79	0.38	95.71

In the medium run, exogenous government spending shock and export demand shock still drive movements in the real GDP (growth) and account for approximately 81 percent of its fluctuations. Terms of trade shock explains 17 percent of those fluctuations, while productivity shock accounts for only 1.79 percent. Productivity shock remains the most important driven force of consumption (growth) and accounts for 75 percent of its fluctuations, while the term of trade shock explains most of the remaining volatility. The contribution of the terms of trade shock in investment (growth) volatility increases slightly in 8 to 32 quarters horizon, and at 32 quarters horizon the term of trade shock remains the most important source of investment (growth) fluctuations, which account for 61 percent of its forecast error variance. Government spending shock also drives investment in the medium term and accounts for approximately 19 percent of its fluctuations, while export demand shock explains about 14 percent, and productivity shock barely explains 6 percent each.

The most interesting observation is that the contribution of export demand shock in explaining the trade balance fluctuations declines drastically in the medium run, while the contributions of the term of trade and that of productivity shocks increase sharply. Export demand shock accounts for approximately 30 percent of the trade balance fluctuations at 32 quarters horizon. The portion of the trade balance forecast error variance explained by terms of trade shock reaches 39 percent. The contribution of government spending shock in trade balance fluctuations almost halves in the medium term, while the productivity shock accounts for a increased portion at about 15 percent.

Turning to the nominal and financial variables, the fraction of the inflation's forecast error variance that is explained by cost-push shock declines in the medium term. At 32 quarters horizon, cost-push shock account for about 41 percent of the inflation fluctuations; productivity

shock explains approximately 24 percent, while the contribution of the terms of trade shock had more than doubled and accounts for about 20 percent. Financial market shock impels approximately 50 percent of the domestic interest rate fluctuations in the medium run. However, the importance of the financial market shock in the domestic interest rate fluctuations has diminished drastically in the medium term, while productivity and term of trade shocks have growing roles and account respectively for about 21 percent and 25 percent of the domestic interest rate forecast error variance. Medium run fluctuations in the nominal exchange rate remain driven by monetary policy shock, which explains 96 percent of its fluctuations.

Beyond the short and medium run, surprisingly, exogenous government spending shock and export demand shock remain the most important driven forces of the real GDP growth fluctuations. The two shocks account for approximately 80 percent of its forecast error variance, while terms of trade shock explains 17 percent approximately, and productivity shock accounts for only 1.79 percent. The so-called export led growth strategy and the increasingly persistent government spending might explain the long run predominance of export demand and government spending shock in explaining the real GDP growth fluctuations in the long run.

With the exception of this unconventional result that attributes to demand shocks (export demand shock and government spending shock) most of the long run fluctuations in the real GDP growth, supply shock (productivity shock) predominantly drive real sector's variables in the long run. Productivity shock explains 75 of the consumption (growth) forecast error variance and remains the prime source of consumption (growth) fluctuations, while term of trade shock only accounts for 12 percent of the consumption forecast error variance. Two third of the investment (growth) fluctuations depends on term of trade shock, while 19 percent relates

to government spending shock, and 14 percent to export demand shock; though, productivity shock explains only 5percent and cost-push shock 1.46 percent.

Table 4.2.3: Forecast error variance decomposition in the long run horizon

		financial market shock	cost-push shock	productivity shock	export demand shock	terms of trade shock	government spending shock	monetary policy shock
t=64	GDP	0.05	0.16	1.79	38.94	16.88	42.18	0.00
	consumption	0.17	0.78	75.19	5.30	11.94	6.62	0.00
	investment	0.32	1.46	4.87	14.01	60.69	18.66	0.00
	trade balance	0.02	0.07	20.29	21.56	46.31	11.74	0.00
	inflation	0.03	40.53	24.42	6.70	19.95	8.37	0.00
	interest rate	32.97	0.11	30.30	2.07	32.85	1.70	0.00
	exchange rate	0.12	1.74	0.95	0.31	0.79	0.38	95.71
t=100	GDP	0.05	0.16	1.79	38.94	16.89	42.18	0.00
	consumption	0.17	0.78	75.19	5.30	11.94	6.62	0.00
	investment	0.32	1.46	4.86	14.01	60.69	18.66	0.00
	trade balance	0.02	0.07	26.62	18.63	44.50	10.16	0.00
	inflation	0.03	40.53	24.42	6.70	19.95	8.37	0.00
	interest rate	25.46	0.25	37.88	2.13	32.43	1.84	0.00
	exchange rate	0.12	1.74	0.95	0.31	0.79	0.38	95.71

The trade balance forecast error decomposition confirms the long run importance of term of trade and productivity shock. As opposed to their marginal short run contributions, term of trade shock explains in the long run 45 percent of the trade balance fluctuations and productivity shock account for 27 percent. Export demand shock and government spending shock play a smaller role, they account respectively for 19 percent and 10 percent of the trade balance fluctuations.

In the long run, only 41 percent of inflation fluctuations depends on cost-push shock, though, this represents almost half of the fraction cost-push shock explains in short run inflation fluctuations. The importance of productivity and term of trade shocks is much higher;

productivity shock account for 24 percent of the long run inflation fluctuations, while 20 percent of the inflation forecast error variance depends on term of trade shock.

Shifts in the domestic interest rate forecast error variance provide additional evidence that confirm the long run importance of productivity and term of trade shock. In the very short run, approximately 90 percent of the domestic interest rate fluctuations relate to financial market shock, that proportion drops drastically to only 25 percent in the long run, while a significant increase is observed in the fraction explained by term of trade and productivity shocks. Those shocks explain respectively 32 percent and 38 percent of the domestic interest rate forecast error variance. Monetary policy shock remains the main driven force of the exchange rate fluctuations.

We end this section by an historical decomposition of shocks to the real GDP growth, inflation, and trade balance to GDP ratio, which provides further explanation to the evolution of those variables described earlier in chapter 1. Comparing main sources of recessions, figure 4.15 shows that although export demand and government spending dominate GDP growth fluctuations, productivity shocks contributed to late 1980s and mid 1990s recessions.

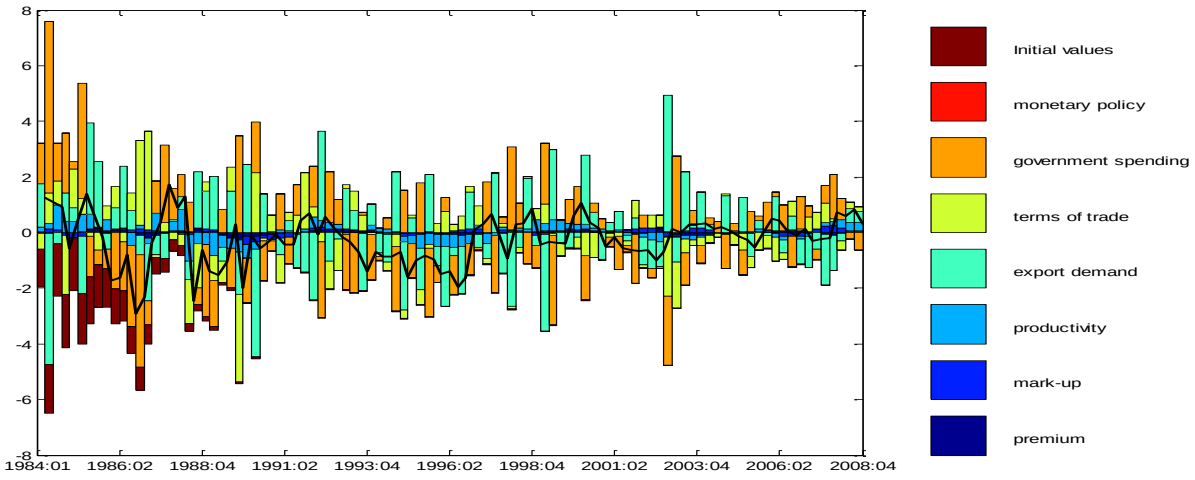

Figure 4.15: Historical decomposition of the quarterly GDP growth (deviations from the trend growth).

Figure 4.16 shows that mark-up shock plays a dominant role in higher inflation episodes in the 1980s and 1990s. However, productivity shock was at the beginning of the late 1980s inflation episode, terms of trade shock as well as government spending shock also contributed. The recent decline in inflation is attributed to mark up shock and productivity shock as well.

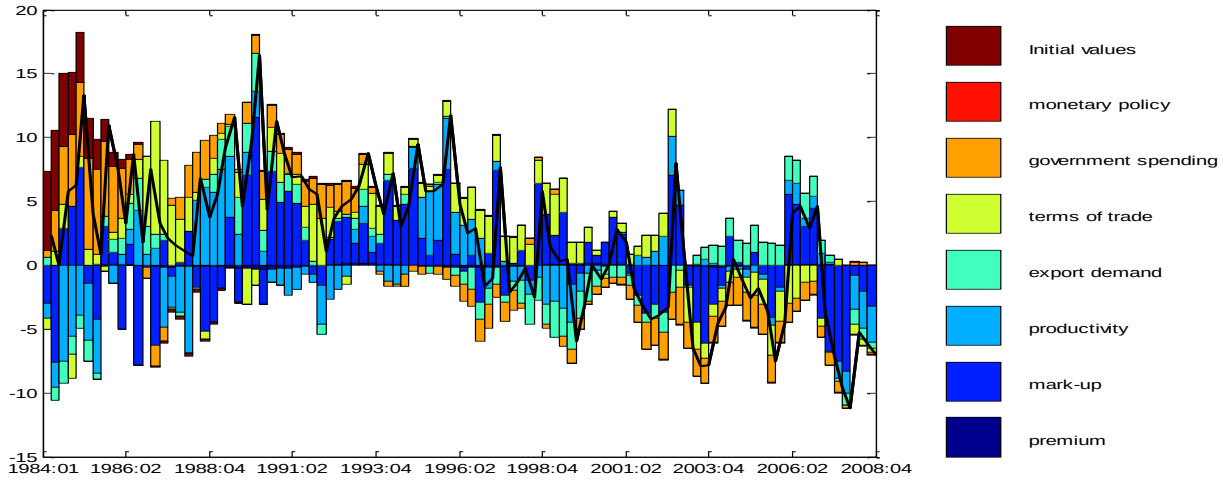

Figure 5.16: Historical decomposition of the inflation

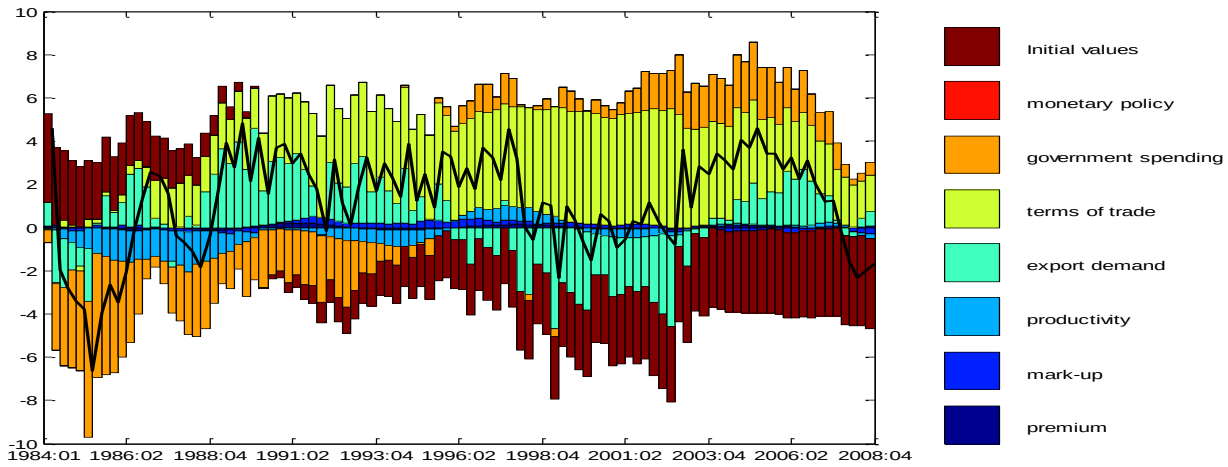

Figure 4.17: Historical decomposition of the trade balance

As illustrates on figure 4.17, terms of trade shock fundamentally explains South Africa structural trade balance surplus, they are strong enough to offset winding government spending shock.

4- Conclusion

This chapter had reported the main results from the applications of the theoretical DSGE model to analysis business cycle fluctuations in a developing Sub-Saharan African economy. The dynamic stochastic general equilibrium model developed and linearized in the second chapter, calibrated and estimated in the third chapter using South Africa macroeconomic data was used to address three important empirical questions in the South African economy.

First, we implemented the sensitivity analysis that assessed the empirical importance of financial, nominal, and real frictions embedded in the model. The sensitivity analysis examined the contribution of each of the frictions to the DSGE model's marginal likelihood, which is a measure of in-sample goodness in Bayesian econometrics and constitutes therefore a summary statistic of the model complexity. In general, financial, nominal, and real frictions embedded in the theoretical model are critical to capture the empirical dynamic of South Africa macroeconomic data.

Second, we conducted the impulse response analysis, described the macroeconomic adjustment to exogenous shocks, and analysed the dynamic properties of the South African economy. The analysis showed that the model implications on the macroeconomic adjustment are essentially in line with the conventional wisdom of the literature. For example, a productivity shock leads to an increase in the output and a fall into the inflation as predicted by the New-Keynesian model. The theoretical DSGE model also emphasises the distinguishing trait of a developing economy when hits by exogenous shocks; financial frictions modified the macroeconomic adjustment and the developing economy is slightly more volatile than developed economy.

Finally, we decomposed the forecast error variances of observable macroeconomic variables simulated at various horizons that characterise the short run, medium run, and long run. The forecast error variances decomposition gives answer to one of the fundamental question of this research: what are the sources of business cycle fluctuations in a developing Sub-Saharan African economy? We argue that the export demand shock, the government spending shock, the term of trade shock, and the productivity shock are the main driving forces of growth rates of the real sector's variables (real GDP, consumption, investment) and the trade balance to GDP ratio as well, the relative importance of each shock varies with the horizon. The productivity shock, the price mark-up shock, and the terms of trade shock drive the aggregate inflation. The financial market shock predominantly explains movements in the domestic interest rate, while the monetary policy shock largely impels the exchange rate fluctuations.

Conclusion

This research has provided insights on the sources of business cycle fluctuations in a developing Sub-Saharan African economy. The thesis answers essentially three macroeconomic questions: what shocks constitute the main driving forces of the South African key macroeconomic variables? Which frictions are empirically essential in explaining the dynamic of the South Africa data? And, how business cycle fluctuations affect the South African economy? In order words, what is the macroeconomic adjustment of the South African economy to various exogenous shocks? These macroeconomic questions as well as their answers are relevant for the South African economy and other Sub Saharan African economies.

Throughout this thesis, we adopted a philosophy that considers the economy as a system with a steady state or a balanced growth path and that embeds real, nominal, and financial frictions. Seven orthogonal structural shocks hit that economic system: a financial market shock that affects both the premium on the assets held by households and the foreign interest rate, a cost-push shock, a productivity shock in the domestically produced goods sector, an export demand shock, a terms of trade shock, a government spending shock, and a monetary policy shock. These shocks move the economic system away from the steady state or the balanced growth path and trigger business cycles fluctuations in macroeconomic variables. Thus, exogenous shocks cause business cycle fluctuations in developing Sub-Saharan African economies, and the dynamic effects of these shocks drive macroeconomic variables from their steady state values to paths marked by peaks and troughs. Real, nominal, and financial frictions

slowdown the macroeconomic adjustment, magnify the economic volatility, and generate the persistence in macroeconomic variables.

To conceptualize that philosophy, in the pure sense of the economy theory, and quantify the effects of exogenous shocks, this research has adopted an analytical framework built upon the effort to understand business cycle fluctuations in industrialized economies. The workhorse of that framework, the dynamic stochastic general equilibrium model, has become the backbone of medium scale models adopted by central banks in the developed world.

The second chapter of this thesis was devoted to the construction of such a model to describe a developing Sub-Saharan African economy. In constructing that model, we have assumed that the current generation of New Keynesian dynamic stochastic general equilibrium models is capable of capturing the time series properties of a developing economy data, as long as those models incorporate some structural characteristics. Two striking structural features, profoundly discernible, and noticeably marked in Sub-Saharan African economies, could reasonably fall inside those desirable structural characteristics. There are: financial frictions that hinder investment financing and amplify the effect of interest rate and exchange rate shocks on the real macroeconomic equilibrium, and the high exchange rate pass-through which reflects exchange rate shocks feeding faster into the domestic price level. While those two features are not specifically restricted to developing Sub-Saharan African economies, as they could feature economies in other regions, the sensitivity analysis conducted in this research had shown that they are necessary to capture the empirical dynamic of the South Africa macroeconomic data. Other distortions crucial for developing economies were excluded to simplify the analytical framework.

The constructed dynamic stochastic general equilibrium model also gave an accent to additional features necessary to capture the empirical persistence in macroeconomic data. An external habit formation in consumption, an internal investment adjustment costs, and a variable capital utilization rate were introduced to capture the sluggish response of consumption, investment, and rental rate to exogenous shocks. Domestically produced goods prices and wages also exhibit nominal rigidities and partial indexation. The represented developing economy is a collection of 28 allocations that dictate the behaviours of households, production firms, capital good producing firms, entrepreneurs, importing firms and the government.

The third chapter had laid down the ground for an empirical application of the log-linearized version of the DSGE model developed in the second chapter. The log-linearization has indicated the choice of a perturbation method to apply empirically the DSGE model of this thesis. Two methods were used to fit the DSGE model to the South African economy. First, parameters that are functions of steady state ratios and that are weakly identified by estimating the model with observed variables expressed as deviations from the steady state were calibrated. The calibration yielded a set of parameter values consistent with the South African economy's long-run properties and steady state ratios. Second, the remaining structural parameters were inferred by estimating the linearized DSGE model using the South Africa macroeconomic data transformed into mean-zero covariance stationary stochastic processes. Overall, the DSGE model fit the South Africa data quite well, exogenous shocks and structural parameters are estimated to be significantly different from zero at a step size of 5%, but the portfolio adjustment cost parameter and the coefficient to output in the monetary policy rule. The developing economy is confronting exogenous shocks of larger magnitude than values generally estimated in advanced economies.

The fourth chapter has used the ingredients provided by the first three chapters and discussed three important applications of the estimated DSGE model. First, a sensitivity analysis assessed the empirical importance of financial, nominal, and real frictions embedded in the model. In general, based on their contributions to the marginal likelihood, frictions are critical to capture the empirical dynamic of the South Africa macroeconomic data. Second, the macroeconomic adjustment to exogenous shocks was described as well as the dynamic properties of the South African economy. The analysis showed that the model implications on the macroeconomic adjustment are essentially in line with the conventional wisdom of the literature. The theoretical DSGE model also emphasises the distinguishing trait of a developing economy when hits by exogenous shocks; financial frictions modified the macroeconomic adjustment and the developing economy is slightly more volatile than developed economies. Finally, the forecast variance decomposition was conducted to unveil the short-term driving forces of observable macroeconomic variables. We argue that the export demand shock, the government spending shock, the terms of trade shock, and the productivity shock are the main driving forces of the growth rates of the real sector's variables (real GDP, consumption, investment) as well as the trade balance to GDP ratio; the relative importance of each shock varies with the horizon. The productivity shock, the price mark-up shock, and the terms of trade shock drive the aggregate inflation. The financial market shock predominantly explains movements in the domestic interest rate, while the monetary policy shock largely impels the exchange rate fluctuations.

The model's difficulties in replicating the joint behaviour of the current exchange rate and future domestic inflation, or that of the current interest rate and future domestic inflation, suggest rooms for improvements in some of its aspects, notably the external sector and policies.

Appendix 7: The Dynare model file

```
var dYobs dCobs dIobs dTBYobs PIobs iobs dSobs Cn Cm P PI C D H Y Rk W We MC
Pn PIn K Q I u De wbar Ce NW Re i X Pm PIm MCm Ym S E_B E_P E_Y E_G istar
Pstar Cstar BMA IMA TMA;

varexo ETA_B ETA_P ETA_Y ETA_X ETA_T ETA_G ETA_R;

parameters alpha beta delta gamma theta omega nu epsilon_w phi eta hab phi_c
psi_D l_n xi_n l_m xi_m l_w xi_w chi zeta_u zeta_e zeta_l zeta_c zeta_p tau
rho_r psi_pi psi_y psi_ex rho_b rho_istar rho_p rho_y rho_x rho_t rho_g
mu_istar mu_p mu_t theta1 C_Yss Ce_Yss I_Yss X_Yss Ym_Yss A_Yss D_Yss De_Yss
Ce_NWss
We_NWss Pn_Pss Pm_Pss Pstar_Pss Pm_Pstar awbar bwbar cwbar D_ss Rk_ss Re_ss
istar_ss S_ss i_ss pi_ss ggtrend gtrend betabar deltabar;

//Calibrated parameters

alpha=0.368;
beta=0.989;
delta=0.011;
theta=0.492;
gamma=0.749;
omega=0.955;
nu=0.985;
epsilon_w=1.01;

//Initial values of estimated parameters

phi=1.5014;
eta=3.0619;
hab=0.2847;
phi_c=1.5030;
psi_D=.0007;
l_n=0.3508;
xi_n=0.5844;
l_m=0.4562;
xi_m=0.5966;
l_w=0.4294;
xi_w=0.8280;
chi=1.8826;
zeta_c=0.4374;
zeta_e=0.4993;
zeta_l=0.6915;
zeta_p=3.7493;
zeta_u=0.1019;
tau=0.4965;
rho_r=0.8407;
psi_pi=1.1434;
psi_y=0.0002;
psi_ex=1.1389;
rho_b=0.7655;
rho_istar=0.6785;
```

```
rho_g=0.9034;
rho_p=0.7965;
rho_y=0.9998;
rho_x=0.8877;
rho_t=0.9800;
mu_istar=0.1206;
mu_p=0.6111;
mu_t=0.9297;
theta1=0.4957;

//Steady state ratios and steady state values

C_Yss=0.517;       //steady state ratio household consumption to output
Ce_Yss=0.051;      //steady state ratio entrepreneur's consumption to output
I_Yss=0.158;       //steady state ratio investment to output
X_Yss=0.241;       //steady state ratio exports to output
Ym_Yss=0.221;      //steady state ratio imports to output
A_Yss=0.910;       //steady state ratio domestic absorption to output
D_Yss=0.115;       //steady state ratio household debt denominated in foreign
currency to output
De_Yss=0.121;      //steady state entrepreneurs debt denominated in foreign
currency to output
Ce_NWss=0.015;     //steady state ratio entrepreneurs consumption to net
worth
We_NWss=0.007;     //steady state ratio entrepreneurs wage to net worth
Pn_Pss=1.384;      //steady state ratio domestically produced goods price to
CPI
Pm_Pss=1.067;      //steady state ratio import goods price to CPI
Pstar_Pss=1.545;   //steady state ratio foreign country CPI to home country
CPI
Pm_Pstar=0.691;    //steady state ratio import good price to foreign country
price
awbar=0.242;       //steady state entrepreneur share
bwbar=0.751;       //steady state foreign lender share
cwbar=0.007;       //steady state share of monitoring cost
D_ss=0.027;        //steady state household debt denominated in foreign
currency (in trillions of US$)
Rk_ss=0.023;       //steady state rental rate of capital
Re_ss=1.034;       //steady state entrepreneur gross return
istar_ss=0.021;    //steady state foreign interest rate
S_ss=1;            //steady state exchange rate
i_ss=0.0336;       //steady state central bank interest rate
pi_ss=8.712;       //steady state annual inflation rate
ggtrend=1.00784;   //quarterly gross long run growth rate
gtrend= (ggtrend-1)*100;
betabar= beta*ggtrend^(-phi);
deltabar=(1-(1-delta)/ggtrend);

model(linear);
          Cn = -theta*(Pn-P)+C;
          Cm = -theta*(Pm-P)+C;
          P = gamma*(Pn_Pss^(1-theta))*Pn+(1-gamma)*(Pm_Pss^(1-theta))*Pm;
          PI = P-P(-1);
          C = (1/(1+hab/ggtrend))*(C(1)+(hab/ggtrend)*C(-1))-((1-
hab/ggtrend)/((1+hab/ggtrend)*phi))*((i_ss/(1+i_ss))*i-PI(1)+E_B);
```

```
          psi_D*(D_ss/S_ss)*D = (istar_ss/(1+istar_ss))*istar-
(i_ss/(1+i_ss))*i+PI(1)+S(1)-S-E_B;
          W =  (1/(1+betabar*ggtrend))*(betabar*ggtrend*W(+1)+W(-1))
               +(betabar*ggtrend/(1+betabar*ggtrend))*(PI(+1))
               -((1+betabar*ggtrend*l_w)/(1+betabar*ggtrend))*(PI)
               +(l_w/(1+betabar*ggtrend))*(PI(-1))
               -(1/(1+betabar*ggtrend))*(((1-betabar*ggtrend*xi_w)*(1-
xi_w))/(((1+(((1+epsilon_w)*eta)/(epsilon_w))))*xi_w))*(W-eta*H-(phi/(1-
hab/ggtrend))*(C-(hab/ggtrend)*C(-1)));
          Y = gamma*(A_Yss*Pn_Pss^(-theta))*(-theta*(Pn-
P)+(1/A_Yss)*(C_Yss*C+Ce_Yss*Ce+I_Yss*I+E_G))+X_Yss*X;
          Y = phi_c*(alpha*(u+K(-1))+(1-alpha)*omega*H+E_Y);
          We = W+H;
          H = Rk-W+u+K(-1);
          MC = alpha*Rk+omega*(1-alpha)*W+(1-alpha)*(1-omega)*We-E_Y;
          PIn = (1/(1+betabar*ggtrend*l_n))*
                (betabar*ggtrend*PIn(1)+l_n*PIn(-1)
                +((1-xi_n)*(1-betabar*ggtrend*xi_n)/xi_n)*(MC))+E_P;
          Pn = Pn(-1)+PIn;
          Q =((1-delta)/(Rk_ss+1-delta))*Q(1)+(Rk_ss/(Rk_ss+1-delta))*Rk(1)-
((i_ss/(1+i_ss))*i-PI(1)+E_B);
          I = (1/(1+betabar*ggtrend))*(I(-
1)+betabar*ggtrend*I(1)+(1/((ggtrend^2)*chi))*Q);
          u =  (1/(zeta_u/(1-zeta_u)))* Rk;
          K = (1-deltabar)*K(-1)+deltabar*I;
          De+(istar_ss/(1+istar_ss))*istar(1)-Re(1)+S(1) =
Q+K+(zeta_l/bwbar)*wbar;
          Re(1)-zeta_p*wbar= (istar_ss/(1+istar_ss))*istar(1)+S(1)-S;
          Ce = Re+Q(-1)+K(-1)+(zeta_e/awbar)*wbar(-1);
          NW = (nu/(1-nu))*Ce_NWss*Ce+We_NWss*We;
          Q(-1)+Re = 2*(Rk_ss/Re_ss)*(Rk+u)+((1-delta)/Re_ss)*Q;
          (i_ss/(1+i_ss))*i =  rho_r*(i_ss/(1+i_ss))*i(-1)
                               +(1-rho_r)*(psi_pi*PI+psi_y*Y+psi_ex*(S-S(-
1)))+ETA_R;
          X = -theta1*(Pn-S-Pstar+tau*(istar_ss/(1+tau*istar_ss))*istar(-
1))+Cstar;
          PIm = (1/(1+betabar*ggtrend*l_m))*(betabar*ggtrend*PIm(1)+l_m*PIm(-
1)+((1-xi_m)*(1-betabar*ggtrend*xi_m)/xi_m)*MCm);
          MCm = S+Pstar-Pm;
          Pm = Pm(-1)+PIm;
          C_Yss*C+Ce_Yss*Ce+I_Yss*I+E_G+D_Yss*(S+D)

+(1+istar_ss)*De_Yss*(1/ggtrend)*((istar_ss/(1+istar_ss))*istar+S+De(-
1))+cwbar*Re_ss*(1/ggtrend)*(1/deltabar)*I_Yss*((zeta_c/cwbar)*wbar(-
1)+Re+Q(-1)+K(-1))
          =Y+S_ss*Pstar_Pss*Ym_Yss*((Pm_Pstar*(1/S_ss)*Pm-S-Pstar)-
P+Ym)+Re_ss*(1/deltabar)*I_Yss*u+(1+istar_ss)*(1/ggtrend)*D_Yss*((istar_ss/(1
+istar_ss))*istar+S+D(-1))
                          +De_Yss*(S+De);
          Ym = -theta*(Pm-P)+(1/A_Yss)*(C_Yss*C+Ce_Yss*Ce+I_Yss*I+E_G);

          //exogenous processes

          E_B = rho_b*E_B(-1)+BMA;
          (istar_ss/(1+istar_ss))*istar =
rho_istar*(istar_ss/(1+istar_ss))*istar(-1)+BMA-mu_istar*BMA(-1);
```

```
            BMA = ETA_B;
            E_P = rho_p*E_P(-1)+IMA-mu_p*IMA(-1);
            IMA = ETA_P;
            E_Y = rho_y*E_Y(-1)+ETA_Y;
            Cstar = rho_x*Cstar(-1)+ETA_X;
            Pn-Pstar-S = rho_t*(Pn(-1)-Pstar(-1)-S(-1))+TMA+mu_t*TMA(-1);
            TMA=-ETA_T;
            E_G = rho_g*E_G(-1)+ETA_G;

// measurment equations

dYobs = gtrend+Y-Y(-1);
dCobs = gtrend+C-C(-1);
dIobs = gtrend+I-I(-1);
dTBYobs = 100*(X_Yss-Ym_Yss)+X_Yss*(X-Y)-Ym_Yss*(Ym-Y);
PIobs = pi_ss+4*PI;
iobs = 400*i_ss+4*i;
dSobs = S-S(-1);

end;

shocks;
var ETA_B; stderr 0.0391;
var ETA_P; stderr 0.7636;
var ETA_Y; stderr 0.8482;
var ETA_X; stderr 4.9305;
var ETA_T; stderr 1.9104;
var ETA_G; stderr 1.9063;
var ETA_R; stderr 0.1591;
end;

steady;
check;

estimated_params;
// PARAM NAME, INITVAL, LB, UB, PRIOR_SHAPE, PRIOR_P1, PRIOR_P2, PRIOR_P3,
PRIOR_P4, JSCALE
// PRIOR_SHAPE: BETA_PDF, GAMMA_PDF, NORMAL_PDF, INV_GAMMA_PDF
stderr ETA_B,0.0391,0.01,4,INV_GAMMA_PDF,0.1,2;
stderr ETA_P,0.7636,0.01,4,INV_GAMMA_PDF,0.65,2;
stderr ETA_Y,0.8482,0.01,4,INV_GAMMA_PDF,0.9,2;
stderr ETA_X,4.9305,0.01,8,INV_GAMMA_PDF,3.5,2;
stderr ETA_T,1.9104,0.01,4,INV_GAMMA_PDF,1.35,2;
stderr ETA_G,1.9063,0.01,4,INV_GAMMA_PDF,1.25,2;
stderr ETA_R,1.3268,0.01,4,INV_GAMMA_PDF,0.15,2;
rho_b,0.7655,.1,.9999,BETA_PDF,0.85,0.10;
rho_istar,0.6785,.1,.9999,BETA_PDF,0.85,0.10;
rho_p,0.7965,.1,.9999,BETA_PDF,0.85,0.10;
rho_y,0.9998,.1,.9999,BETA_PDF,0.85,0.10;
rho_x,0.8877,.1,.9999,BETA_PDF,0.85,0.10;
rho_t,0.9800,.1,.9999,BETA_PDF,0.85,0.10;
rho_g,0.9034,.1,.9999,BETA_PDF,0.85,0.10;
mu_istar,0.1206,0.01,.9999,BETA_PDF,0.5,0.2;
mu_p,0.6111,0.01,.9999,BETA_PDF,0.7,0.2;
mu_t,0.9297,0.01,.9999,BETA_PDF,0.7,0.2;
phi,2.3014,0.25,4,NORMAL_PDF,2.1,0.375;
```

```
eta,3.0619,0.5,5,NORMAL_PDF,2.5,0.75;
hab,0.2847,0.1,0.95,BETA_PDF,0.57,0.1;
phi_c,1.5030,1.0,3,NORMAL_PDF,1.50,0.125;
psi_D,0.0007,0.00001,2,NORMAL_PDF,0.001,0.05;
l_n,0.3508,0.1,0.99,BETA_PDF,0.40,0.15;
xi_n,0.5844,0.1,0.99,BETA_PDF,0.57,0.05;
l_m,0.4562,0.1,0.99,BETA_PDF,0.40,0.15;
xi_m,0.5966,0.1,0.99,BETA_PDF,0.57,0.05;
l_w,0.4294,0.1,0.99,BETA_PDF,0.50,0.15;
xi_w,0.8280,0.1,0.99,BETA_PDF,0.75,0.05;
chi,1.8826,0.1,5,NORMAL_PDF,1.48,1.5;
zeta_c,0.4374,0.01,0.99,BETA_PDF,0.45,0.15;
zeta_e,0.4993,0.01,0.99,BETA_PDF,0.50,0.15;
zeta_l,0.6915,0.01,0.99,BETA_PDF,0.65,0.15;
zeta_p,3.7493,0.001,7.5,NORMAL_PDF,3.75,0.5;
zeta_u,0.1019,0.01,0.99,BETA_PDF,0.5,0.15;
tau,0.4965,0.01,0.9999,BETA_PDF,0.495,0.15;
rho_r,0.8407,0.3,0.9999,BETA_PDF,0.75,0.15;
psi_pi,1.1434,1.0,3,NORMAL_PDF,1.5,0.25;
psi_y,0.0002,0.000001,0.25,NORMAL_PDF,0.125,0.05;
psi_ex,1.1389,0.01,2.0,NORMAL_PDF,1.112,0.05;
gtrend,0.784,0.10,1.10,GAMMA_PDF,0.784,0.011;
pi_ss,8.709,5.01,10.0,GAMMA_PDF,8.712,0.056;
theta1,0.430,0.001,10.0,GAMMA_PDF,0.430,0.1;
end;

varobs dYobs dCobs dIobs dTBYobs PIobs iobs dSobs;

estimation(optim=('MaxIter',1000000000,'MaxFunEvals',500000000),datafile=SAda
ta,first_obs=80,mode_check,presample=15,nobs=100,lik_init=1,mode_compute=5,mh
_nblocks=2,mh_jscale=0.2,mh_replic=2000000,bayesian_irf)dYobs dCobs dIobs
dTBYobs PIobs iobs dSobs;

stoch_simul(ar=10,irf=20,conditional_variance_decomposition = [1 2 4 8 16 32
64 100])dYobs dCobs dIobs PIobs iobs dSobs dTBYobs;
```

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