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Essays on Aggregate Dynamics:
Externalities, Liquidity and Financial Crises

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Abstract

This dissertation is composed of five chapters. After the introduction, the second chapter pays attention to real aspect of economic fluctuations by analyzing the role of technological progress. The third chapter considers financial aspect of fluctuations. The fourth chapter is studying how the interaction between the corporate and the financial sector results in financial crises. Finally, concluding remarks are provided in the last chapter.

In the second chapter, we consider a mechanism of unstable fluctuations of aggregate investments, which is the main driving factor of business cycles, by means of a coordination game using the global game approach (Carlsson and van Damme (1993), Morris and Shin (2001)). Originally, a coordination game based on strategic complementarities gives rise to multiple equilibria and which equilibrium is realized depends on the expectation of economic agents. However, this multiplicity comes from the underlying assumption that all economic variables in the model are perfect information. A global game is an incomplete information game regarding an underlying economic state and this informational incompleteness produces a unique equilibrium by iterated dominance procedure.

We extend the static global game method to a dynamic version to generate unstable fluctuations of aggregate investments. For this purpose, we will pay attention to the effect of coordination of aggregate investments on future profitability. More precisely, aggregate investments through coordination produce positive externalities...
on the future economic activity since active investments produce tangible and non-tangible assets in the economy. These assets have long term productive effects on the economy. Once this effect of coordination of aggregate investments between periods is taken into account, we can show that firms’ equilibrium strategies of investments become highly volatile over time. Moreover, long persistence of high or low economic activity, which is one of the characteristics of business fluctuations, can be explained by this model as well.

The third chapter examines the effect of firms funding liquidity on macroeconomic dynamics and the role of liquidity markets. While existing studies relating funding liquidity try to address the relationship between market and funding liquidity from the point of view of financial intermediaries, we will consider a different mechanism of creating liquidity from the point of view of non-financial corporate firms. Here, we regard liquidity as firms’ accumulated net worth and analyze the mechanism of transacting their liquidity through liquidity markets. In this regard, Holmström and Tirole (1998a) is along the same line as ours. However, while characteristics of their firms are all identical, we introduce heterogeneity between firms with regard to their productivities and accumulation of their net worth. As a result, we can obtain different implications for macroeconomic dynamics. From our analysis, we show that under existence of externality between probabilities of liquidity shocks 1) the economy without liquidity markets is highly volatile. 2) liquidity markets insulate the economy from liquidity shocks and keep the aggregate outputs and economy-wide liquidity levels high. 3) During an unstable economic environment, the economic activity can sharply drop in the existence of liquidity markets.

The fourth chapter aims at showing risk shifting behaviour of financial intermediaries (banks) in the context of an economic growth model to analyze financial crises. In this economy, there are risk neutral banks that take deposits from households and seek to maximize the benefit of stock holders by investing in safe and risky assets. Al-
though deposit is a simple debt contract and they are protected by limited liabilities, the manager of a bank incurs non-pecuniary bankruptcy cost in case of insolvency.

In the early stage of economic development, the amount of an individual’s assets (deposits) is scarce and the rate of return on corporate investment is very high. Thus, risky assets are not profitable and banks invest all of their available funds in a corporate sector, which leads to further economic growth. However, as a result of economic development, assets (deposits) held by households increase and the rate of return on corporate investment is decreasing, which makes risky assets more profitable. Therefore, banks are willing to hold risky assets.

More precisely, the underlying mechanism of risk shifting behaviour of banks is as follows. In the low capitalized economy in which a rate of return on safe assets is high and households’ assets are scarce, investing in corporate sectors is more profitable than that of risky assets. This is because the option value from investing in risky assets is low compared to expected bankruptcy cost incurred by banks. However, as the economy grows, the rate of return on safe assets is decreasing whereas individual assets are increasing. In this situation, the option values of risky assets are increasing relative to expected bankruptcy costs, which gives banks incentive to invest in risky assets. Holding risky assets leads some of the banks to be insolvent and go bankrupt. Consequently, the number of banks decreases and financial function deteriorates, which results in a credit crunch on firms’ investments. Moreover, under some conditions, this economy shows endogenous periodical movements between high and low capital stocks.
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To the memory of

Tina
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Chapter 1

Introduction

1.1 Motivation

The purpose of this Ph.D dissertation is to improve our understanding of the mechanism behind the cyclical behaviour of economic activity by paying particular attention to the role of corporate investment in developed economies. In other words, the scope of this research mainly covers highly capitalized economies like US, European countries and Japan. Generally speaking, the differences between these countries and other less capitalized economies regarding corporate investments lie in 1) the nature of investments and 2) their way of funding. The nature of corporate investments in less capitalized countries which are the main driving force for their rapid economic growth is characterized as labour intensive skill whereas that of developed economies is rather high technology/knowledge intensive skill. This difference comes primarily from the nature of the main industries leading these economies. In less capitalized countries, manufacturing sectors with low wage workers drive these economies’ development while service sector requiring highly sophisticated knowledge is rapidly expanding in highly capitalized ones. Furthermore, with respect to their funding methods, firms in low capitalized countries rely heavily on bank lending while those
in highly capitalized countries (mainly, large size firms) tend to hold sufficient internal funds and finance their investments. This financial aspect of investments also characterizes the economic cycle in highly capitalized economies through liquidity provision and capital accumulation in an economy.

Needless to say, there exists a great number of works inquiring into economic fluctuations from various perspectives. For instance, research covers specification of shocks on an economy which lead to long run fluctuations, time to build, the role of financial intermediaries in propagation mechanism of cycles and monetary policy of the central bank. This research strongly reflects the main economic issues at each historical event. When the world economy was in the 1930’s recession, the deterioration of banking systems and inappropriate monetary policy were regarded as the main reasons for prolonging the economic turmoil. During the economic stagnation in the 1970’s, the central issue was the effect of monetary policy on the expectation of people, inflation and economic activities. Finally, around 2000 in the boom of IT industries in US, it was said that business cycles disappeared as a result of continuous productivity growth. Although this optimistic claim was not true, it posed a question what the features of business fluctuations, especially in developed countries in which a growth rate of an economy is stable and firms/households hold relatively enough wealth, are. Where and how these countries’ economy is going to be? However, consideration of business cycles so far does not tell anything about this point and few studies contribute to clarify the characteristics of cycles from the point of view of an economic development. Moreover, taking into consideration the difference of growth stage allows us to understand the role of government from a different perspective. There are also various views on an economic policy response to an economic cycle ranging from aggressive intervention by government to “nothing” to do. We will think about these issues by taking into account the growth stage of an economy. Therefore, this study attempts to shed light on the direction and policy stance of developed
countries through investigation of the mechanism of economic cycles and crises.

Although research on economic cycles has a long history and there are various approaches, the basic question that at least we have to address is to explain huge and unstable fluctuations of aggregate economic activities over time. In this regard, it is well known that the key to approaching the economic cycle is to clarify the movement of corporate aggregate investments. This is because movement of aggregate investments is quantitively the main factor of irregular and highly volatile aggregate outputs. For example, Keynes (1936) made this point more clearly.

_The difficulties in the way of maintaining effective demand at a level high enough to provide full employment, which ensues from the association of a conventional and fairly stable long-term rate of interest with a fickle and highly unstable marginal efficiency of capital, should be, by now, obvious to the reader._ (Keynes, 1936, p. 204)

Broadly speaking, in order to explain unstable fluctuations of aggregate investments, existing works have paid attention to both real and financial aspect of an economy. While we share the same approach with them in some respect, there are still differences in another respect. Like the previous research, we pay attention to the role of the financial function in the context of economic fluctuations. In particular, this dissertation studies liquidity management of corporate sectors which characterizes the features of cycle in a developed economy. The difference is that we mainly adopt a coordination game to approach real aspects of fluctuations. Most of the literature on business cycle models takes the stance of a representative agent view. However, the business cycle phenomenon itself has a feature of mass activity of a lot of economic agents (e.g. corporate investment by individual firms). Hence, depending on the strength of coordination between them, the economy shows huge fluctuations. A coordination game is based on this idea and provides simple and strong tools to analyze economic fluctuations. In addition, we will consider the productive effect of
coordination of corporate aggregate investments on economic cycles in order to clarify the interaction between accumulation of knowledge and the cycle which is essential in developed economies.

Furthermore, economic fluctuations often end up with financial crises which cause long run stagnation of an economy in the succeeding periods. Especially, this phenomenon contains two broad characteristics. One of the characteristics of financial crises is that even developed countries which have well-established financial systems suffer crises. The other is that crises occur suddenly with few signals. Before crises, economic conditions are often rather predictable and it is very difficult to anticipate the collapse of financial markets and intermediaries followed by long run stagnation of economic activities. By paying attention to these facts, in this dissertation, a financial crisis is not simply interpreted as an aspect of cyclical behaviour of business cycle. It is deeply rooted into economic growth and may cause a regime change. In order to analyze this regime change, we will consider the distorted behaviour of financial intermediaries and show that financial intermediaries take excessive risk as a result of economic development. As we will show, the mechanism of crises in which financial intermediaries take excessive risk is closely associated with accumulation of capital stocks in an economy. Next, we will briefly discuss the existing theoretical literatures most relevant to this dissertation and focus more on the questions we will try to tackle.

1.2 Real business cycles and financial frictions

In this section, we survey two kinds of models of business fluctuations which are currently in the main stream of economic dynamics modeling. Then, their advantages and disadvantages are critically discussed.
1.2.1 Real business cycle theory

In order to explain the behaviour of corporate investments, existing studies take various methodological stances. However, in modern macroeconomic theories, Real Business Cycle theory (hereafter RBC theory) which is pioneered by Kydland and Prescott (1982) and Long and Plosser (1983), is influential. One of the features of these theories is that business fluctuations are caused by real factors (for example, a productivity shock) and an economy is always in equilibrium. Thus, expansion and recession are thought to be realizations of a particular economic equilibrium. Therefore RBC theories suggest that governmental intervention should be prohibited because the economy is always in an optimal state and any intervention by the government distorts economic efficiency. However, there are three issues for considering the fluctuations of investment through the framework of RBC models.

Firstly, the basic RBC model has a mechanism in which external shocks lead to output fluctuations via intertemporal substitution between leisure and labour supply. However, empirically, the response of labour supply to the shock is known to be trivial. Secondly, as the structure of economic models becomes complicated by containing a lot of economic factors, it becomes difficult to analyze analytically the dynamic characteristics of equilibrium and numerical methods are applied. However, since the economic parameters are generally unstable, it is quite difficult to specify a clear relationship between economic variables by numerical exercises and to assess what the main economic factors to affect fluctuations of investment are. Finally, if adjustment costs are included in the decision-making of investments, fluctuations decrease significantly. In the RBC models, consumption is determined as intertemporal smoothing between present and future. Therefore most of the shocks to the production technology are adjusted through the level of aggregate investment. That is why

\[1\] The basic structures of RBC models have been already established as the neoclassical optimal growth theories and constructed in dynamic general equilibrium models (Koopmans (1965), Cass (1966)).
the behaviour of aggregate investments shows big swings. Instead, if adjustments of investment incur huge costs and it cannot change in response to the change of technological shocks, intertemporal smoothing of investment also works. Hence, the volatility of investment is much less than that without adjustment costs. In fact, some empirical literatures (for instance, Hayashi (1982)\textsuperscript{2}) show that there is a possibility of existence of adjustment costs.

### 1.2.2 Financial frictions model

In order to produce huge volatility in aggregate investments, financial frictions between lenders and borrowers have recently been considered in business cycle models. This is because, in part, current economic fluctuations are closely related to financial markets and intermediaries. Moreover, this strand of research is based on development of information and contract theory which provided theoretical backgrounds for considering financial aspects of business cycle. In these theories, when firms borrow necessary funds from financial markets or intermediaries, they are financially constrained due to informational problems. Under such an environment, collateral and net wealth owned by firms have a significant role in mitigating information problems and financing their projects\textsuperscript{3}. Thus, the change in value of net wealth and collateral can have a huge impact on economic fluctuations through agency costs.

As for net wealth held by firms, Bernanke and Gertler (1990) focuses on the role of net wealth owned by firms and assessed the implication to business cycles. When there is asymmetric information between lenders and borrowers, necessary funds for investment might not be available for borrowers because of adverse selection problems. To avoid these problems, lenders require borrowers to have enough net wealth so that some realized loss is borne by borrowers and they take proper behaviour. That

\textsuperscript{2}Hayashi (1982) empirically presented that Tobin’s Q moves procyclically along the movement of economic activities. If investments adjust quickly, the Q value always becomes one. In other words, adjustment costs may change investments.

\textsuperscript{3}Tirole (2000) analyzes a various issues of moral hazard in corporate finance.
is, in order for borrowers to secure outside funds, they need to mitigate incentive compatibility constraints through provision of their own funds. Thus, available funds for investment could be limited by the amount of firms’ own wealth. In a recession, net wealth in firms decreases due to its huge loss. This causes the reduction of funds available for firms and cumulative adverse effects on aggregate investments and economic activities.

Not only net wealth but also the value of collateral owned by a firm (for instance, land) affects the availability of funds under incompleteness of contracts. This is because there is a possibility that clauses in contracts are not implemented by the borrowers. Borrowers have a bargaining power in that they can renegotiate the contracts \textit{ex post} by refusing to exercise their human-specific managerial skill which cannot be replaced by other managers (Hart and Moore (1994)). Anticipating this possibility, lenders might not lend necessary funds for projects to borrowers. Therefore, borrowers need some collateral to compensate this risk. Kiyotaki and Moore (1997) showed that changing the value of collateral affects the availability of funds for investment and economic fluctuations (which is constrained by the value of assets) through a general equilibrium model. In particular, asset price and economic activity show cyclical movement around the steady state. In this way, a small shock can propagate and induces huge fluctuations of economic activities.

This research shares the features that changes in firms’ balance sheet condition mainly affect the availability of funds. Although this kind of discussion applies to small and medium size firms, a lot of large size firms in developed economies hold significant amount of inner funds and do not necessarily finance by outside funds. In other words, there are two types of firms which have different preferences for their financing methods. However, the existing works still leave open the implications of this different type of raising funds to economic fluctuations.
1.3 Direction of this dissertation

So far, we have seen two typical approaches for analyzing movements of aggregate investment. The RBC model has limitations in that it is not easy to explain the unstable feature of fluctuations. On the other hand, financial friction models can potentially overcome this shortcoming. In this dissertation, firstly, we will analyze a coordination game that delivers unstable movement of aggregate investments and show the implication for developed economies. Then, we will focus on the liquidity management of firms and sketch a financial mechanism of economic fluctuations in developed economies. Finally, a possibility of financial crises and endogenous cycle are investigated in a dynamic model of capital accumulation.

1.3.1 Coordination problems and global games

In order to overcome the shortcomings of the RBC model, we will include strategic interactions between firms deciding on individual investments. While almost all business cycle models are analyzed from the point of view of a representative agent approach, the phenomenon of business cycles is characterized as mass activities of a lot of economic agents (consumers, firms). In particular, it is crucial to understand how firms invest simultaneously. In game theoretical literatures, the phenomenon that many agents take the same action (investment) simultaneously has been analyzed as coordination problems. One of the characteristics of these studies is that they include externalities (informational and payoff externalities) between actions of agents and, as a result, there can be multiple equilibria which are ranked by Pareto criterion. This kind of game in which strategic complementarities are included is analyzed as a supermoduler game (Milgrom and Roberts (1990)). In this setting, decentralized economy may achieve inefficient equilibrium called coordination failure, which is different from traditional neoclassical economics. For this purpose, there are
Various kinds of models to analyze coordination problems, for instance, technological complementarities (Baxter and King (1991)), demand spillovers (Blanchard and Kiyotaki (1987)), a search model (Diamond (1982), Kiyotaki and Wright (1993)), timing of actions (Chamley and Gale (1994), Shleifer (1986))\textsuperscript{4}. The advantages of applying the coordination game for research on business cycle theories are not only that it is easy to explain huge fluctuations of aggregate economic activities. It can also capture the strategic interaction between agents and takes into consideration factors that have not been considered in existing business cycle models (for instance, information flows between agents). As a result, more detailed characteristics about business cycles (for instance, asymmetric span between expansion and recession) can be explained.

However, coordination games do not necessarily have a unique equilibrium and the problem of multiple equilibria makes comparative static analysis intractable. Put differently, they have no predictive powers, for instance, in making economic policies, and cannot decide which equilibrium is actually realized. However, this indeterminacy relies heavily on the assumption that all of the economic parameters are common knowledge among agents. On the other hand, a global game which is introduced by Carlsson and van Damme (1993) is an incomplete information game about underlying economic states and showed that when uncertainty and noisy signals of the economic states are included in coordination games, a unique equilibrium (risk dominant equilibrium) can be selected in the limiting case that the noise of signals becomes vanishingly small\textsuperscript{5}. This special type of coordination games has been applied to various discrete decision making problems such as bank runs (Goldstein and Pauzner (2000)), currency crises (Morris and Shin (1998)) and debt pricing (Morris and Shin (2002)). However, the implications for economic fluctuations are still limited due to technical problems brought about by extending this game from static to dynamic analysis. One of the purposes of chapter 2 is to show the fluctuations of aggregate investments by

\textsuperscript{4}As a comprehensive survey on this topics, see Cooper (1999)
\textsuperscript{5}Morris and Shin (2001) is a comprehensive survey on this topic.
dynamic global game in a simple way. Furthermore, we will consider the meaning of coordination of aggregate investments from the perspective of endogenous growth theory and its implication for economic fluctuations. By doing so, the mechanism of technological improvement which is essential in developed countries is introduced.

1.3.2 Liquidity cycle and crises

Financial friction models based on firms’ net wealth are still essential to consider the fluctuations of aggregate investments, especially, of medium and small sized firms. However, this kind of model is mainly based on consideration of borrowers’ balance sheet conditions which affect financial availability for firms. In contrast, large firms in developed countries hold sufficient amount of internal funds and, often, invest without borrowing from financial markets and intermediaries. However, the implication of this financial structure for economic fluctuations is still unknown. In this dissertation, as the second topic on business fluctuations, we will consider the financial aspect of economic fluctuations by focusing on liquidity availability from the stance of corporate firms. Though there are vast literatures on liquidity, a seminal work on this issue was presented by Holmström and Tirole (1998a). They studied optimal liquidity provision under moral hazard problem of a manager. In their framework, they showed that, due to moral hazard, investors have to guarantee manager’s minimum amounts of payoff in order for her to behave properly and, as a result, optimal policy of liquidity provision is different from that of the first best one. Moreover, aggregate uncertainty causes supply of liquidity in private sector insufficient and government support for liquidity becomes essential. However, their work will not aim to analyze dynamic characteristics of corporate liquidity management.

With regard to macroeconomic implication of liquidity management, there are a few studies in the context of business fluctuations. Kiyotaki and Moore (2008) studied an amplification mechanism under liquidity constraints by firms and showed
the role of money. Their study focused on liquidity services of money when markets are tight. If money is scarce, a liquidity shock to asset markets puts constraints on availability of necessary funds for investments of firms, for instance, through selling holding assets. In turn, a reduction of output induces assets markets to be less liquid because of falling prices. Thus, the function of monetary authorities which convert the proportion of money and other assets through open market operation serves stability of the economy. Here, the concept of liquidity is differently interpreted from ours in that they pay attention to the liquidity aspect of a certain asset (money) while our study focuses on internal funds of firms and liquidity transaction between firms.

1.3.3 Financial crises and cycles

In chapter 4, we will consider the mechanism of financial crises from the point of view of economic growth and show that they are a phenomenon of a regime change. As mentioned before, even developed countries with sophisticated financial systems suffer from crises periodically. In order to investigate this aspect of crises, we take into account the effect of capital accumulation on the incentive of banks to induce risk taking behaviour. Then, we clarified that banks tend to take excessive risk after an economy grows sufficiently. In other words, the economy shifts from the stage of steadily capital accumulation to that of stagnation when the economy is fully developed like high capitalized developed countries. There are a few existing works regarding the interaction between financial function and a regime change. For instance, Azariadis and Smith (1998) studied the impact of economic growth on the asymmetric information problem of firms and, after some stage of economic development, incentive compatibility constraints become binding and credit available are constrained. Mattesini (2005) also pays attention to the relationship between financial function and a regime change. It focuses on monitoring resource of banks and, as a result of economic growth and lowering interest rate, they lose such a resource
and do not monitor borrowers precisely.

The difference of ours from other papers is that we treat the risk shifting incentive of financial intermediaries directly. Financial crises are literally characterized as insolvency of a lot of banks because of taking excessive risks and, accordingly, stagnation of economic growth. However, surprisingly, there are few papers investigating this aspect of crises so far. Thus, our study is the first step to tackle this problem.
Chapter 2

Global games and economic fluctuations

2.1 Introduction

As Keynes (1936) suggested, the most critical factor contributing to the mechanism of business fluctuations is the movement of aggregate corporate investments\(^1\). That is, aggregate output fluctuations follow the movement of aggregate investment. Moreover, it presents not only huge fluctuations but also persistence in which expansion (recession) lasts for a long period.

In order to explain these fluctuations of aggregate investment, economists have provided various theoretical frameworks. A lot of studies on this theme use real business cycle theories (Kydland and Prescott (1982), Long and Plosser (1983)). Although this framework is able to clarify a great variety of aspects on investments, it is not necessarily able to show huge fluctuations of aggregate investments. For instance, in this framework, the cause of fluctuations is mostly attributed to external technology shocks and there are few endogenous mechanisms to produce big swings of output.

\(^1\)Keynes claimed that a typical firm’s marginal efficiency of capital is highly volatile over time.
In addition, it is also difficult to explain long run persistence of aggregate investment and output since the economy has a tendency to converge to a steady state quickly and cannot show long duration.

To address this problem\(^2\), we will adopt a different approach to analyze the mechanism of investment fluctuations and focus on game theoretical methods regarding coordination problems between firms. This approach pays attention to the role of strategic complementarities between firms’ behaviour and makes clear the characteristics of equilibria achieved by the strategic interactions of firms. In a macroeconomic context, the coordination of actions among firms will determine the level of economic activity. In this regard, here, a simple economic cycle model based on a coordination game will be presented using the global game methodology. The global game approach provides us for equilibrium selection method (Carlsson and van Damme (1993), Morris and Shin (2001)). Generally, games with strategic complementarities can produce multiple equilibria depending on the expectations of economic agents. Since these equilibria are all economically rational and self-fulfilling, they cannot be confined to a unique equilibrium\(^3\). However, this indeterminacy relies heavily on the assumption that all of the economic parameters are common knowledge among agents.

In contrast, a global game as introduced by Carlsson and van Damme (1993) is an incomplete information game about underlying economic states. They showed that when uncertainty and noisy signals of the economic states are included in coordination games, a unique equilibrium (risk dominant equilibrium) can be selected in the limiting case that the noise of signals becomes vanishingly small. In addition, this equilibrium survives iterated deletion of strictly dominated strategies according to.

\(^2\)The approach described in the following is not the only way to address this problem. The other way is to include financial frictions coming from informational problems between a lender and a borrower which create financial accelerator effects on aggregate investments. For example, see Bernanke and Gertler (1989). Moreover, there are studies which pay attention to herding of firms. This research considers how firms take actions sequentially by means of informational spillovers and agency problems (Bykhchandani at al. (1992), Scharfstein and Stein (1990)). As a comprehensive guide, see Chamley (2004).

\(^3\)Cooper (1999) overviews a wide range of coordination games.
to Milgrom and Roberts (1990). Since a unique equilibrium can be obtained in this setting, it is easy to analyze various discrete economic problems, for example bank runs (Goldstein and Pauzner (2002)), currency crises (Morris and Shin (1998)), debt pricing (Morris and Shin (2002)) and a regime change (Angeletos et al. (2004)).

There are various applications of this methodology in dynamic settings. Roughly speaking, dynamic global games are classified into two categories. One is backward looking and another is forward looking models. The former considers the effect of the economic activities in the past on current ones. In contrast, the latter thinks that the anticipation of economic state for the future affects the current economic activities. Forward looking models are related to a decision to delay or invest. Firms delay in order to obtain a better opportunity to invest or useful information about underlying economic states. Along this line, Steiner (2005) analyzed equilibrium fluctuations through a simple delay and investment game. The game of his model was reduced to a simple repeated game so that the uniqueness of equilibrium is kept. Giannitsarou and Toxvaerd (2007) characterised equilibria of recursive global games precisely and showed the uniqueness of equilibrium. With regard to backward looking models, the general approach has been applied in sequential actions model of Oyama (2004) who did not introduce incomplete information and analyzed strategic complementarities with preceding and following actions of other firms. Applying the same procedure as Milgrom and Roberts (1990), he showed there is a unique equilibrium which is dominant solvable. However, it is, in general, difficult to create a dynamic global game because a unique equilibrium is not necessarily guaranteed. Angeletos et al (2004) showed that if social learning is included in dynamic global game models, the uniqueness of equilibrium cannot be obtained. Moreover, there are few researches which analyze business fluctuations from the point of view of backward looking models.

In this chapter, we reconsider the role of coordination between firms in the context
of macroeconomics and examine the implication of current coordination for future productivity effects and decision-making of investments. For this purpose, we will capture the effect of coordination between firms as a different way from existing literatures. Almost all studies of coordination games applied to macroeconomic fluctuations regard coordination of aggregate investment as one of the drivers of effective demand. However, in this chapter, borrowing from the endogenous growth theory, we consider the additional role of human capital externalities. For instance, the past coordination between firms generate positive externalities (e.g. through learning by doing) and these efficiency-enhancing externalities make profitability of investments improve in the future. Then, this improvement of profitability by past externalities affects decision-making of investments by firms operating in the present. Intuitively, if a lot of firms invest in the same period, the economy can provide workers with plenty of opportunities for job training and their productivities (human capital) can improve as well. Furthermore, this effect does not vanish within the same period and lasts in the future period. Therefore, from the past to present and future, firms’ profitability and investments can change irregularly over time, which can be interpreted as unstable movements of aggregate investment. Additionally, due to externality between the past and present, long run persistence can also appear in this framework.

In this model, this effect is simply expressed in a way that past aggregate investment is included in a present payoff function. Then, firms make their decision given the influence of past realized aggregate investment and that of current expected aggregate investment. Within this simpler structure, the model is similar to a repeated static game and the uniqueness of equilibrium is guaranteed. As a result, the effect of the past environment can cause fluctuations of equilibrium in the following period. Thus, big cyclical fluctuations over time can be created depending on the path of underlying economic states.

The structure of this chapter is as follows. In section 2, we re-think the role of co-
ordination of investments in macroeconomics. Then, in section 3, the basic structure of the model is described precisely. Section 4 presents the case of no strategic complementarities as a benchmark. After that, equilibrium strategies are found recursively from the past economic activities without and with current externalities in sections 5 and 6, respectively. Based on these equilibria, in section 7, this limiting case of equilibria is examined and followed by its implication for persistence in section 8. Finally, concluding remarks are presented in section 9. Proof of uniqueness of these equilibria is presented in the appendix.

2.2 The role of coordination between firms and firms’ capital in macroeconomics

2.2.1 Forward and backward looking behaviour in macroeconomics

As simply described in the introduction, there are two ways to approach dynamic macroeconomic models. The first is forward looking models and the other is backward looking models. Forward looking models mean that expected future variables affect current economic decisions. A lot of neoclassical macroeconomic theories are basically along this line. For example, firms choose their investments taking into account net present value of payoffs that these investments produce. On the other hand, current activities are mainly determined by past economic situations from the point of view of backward looking models. For example, traditional Keynesian models like IS-LM framework are based on this idea. One of the reasons that past economic results affect the current economic situation is that these theories explicitly or implicitly assume the existence of imperfections of markets which cause financial frictions and liquidity constraints to corporate and household sectors. Thus, firms and households cannot
borrow necessary funds by setting future payoff as collateral and current economic activities can be restricted by past activities.

In this study, the basic structure is based on a backward looking model and current economic conditions are affected by past economic activity. However, here, we do not assume imperfection of markets. The dynamics of the model are driven by past externalities that affect future productivity levels. In the next sub-section, we will explain this point more precisely.

2.2.2 An interpretation of coordination between firms

Conventionally, macroeconomic coordination between firms (e.g. aggregate investments) in business cycle theories considers its effective demand and spillover effects on the economy. But it also creates valuable capital which has long run effects on future productivity. In this case, capital means both tangible capital like building, lands and intangible capital like human capital and accumulation of knowledge. In macroeconomics, these kinds of capital and externality effects are mainly considered in the framework of endogenous growth theory. For instance, active investments by firms provide workers with job training and learning opportunities in practice. These efficiency-improving activities can enhance workers’ productivity (learning by doing). This effect does not vanish within a period and lasts to the future economic productivity. Thus, some of this externality has an influence on profitability in the next period. While endogenous growth theories in macroeconomics try to clarify the mechanism of economic growth by enhancing marginal productivity of capital through externalities like learning by doing and research and development using market structure such as monopolistic competition framework, the model in this paper applies these ideas to the context of economic fluctuations. In developed countries, main industries

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4In this regard, this study is different from the point made by Keynes (1936) in that he mainly investigated the role of forward looking behaviour of firms, for instance, managers’ optimistic and pessimistic decision making.
have been changing from manufacturing to service industries, while main industries of developing countries are manufacturing. The crucial economic factors for service industries are research and development and human capital (especially knowledge attached to labour). Therefore, the productive effects from the past aggregate investments could have an enormous influence on economic fluctuations in developed countries.

2.3 The model

2.3.1 Strategic complementarities within and between periods

In this economy, there is an infinite number of periods \((t = 0, 1 \cdots)\). In each period, there are firms measured \([0, 1]\), which are indexed by \(i\). They have an option to invest each period and make their decision whether to invest or not. The information structure of this economy is incomplete with regard to states of this economy \(\theta\) and these states are not observable by firms. However, firms can obtain a noisy signal \(x^i = \theta + \varepsilon^i\). \(\varepsilon\) is drawn from normal distribution \(N(0, \sigma^2)\) and associated c.d.f. and p.d.f. are \(F\) and \(f\), respectively. It is also independent from \(\theta\). \(\theta\) is assumed to be picked randomly from a distribution which has c.d.f. \(\Phi\) on the real line and associated p.d.f. \(\phi\).

Although firms decide whether to invest or not in each period, the productive effect of aggregate investments of current period is assumed to appear not only in the current period but also in the next period. The intuition of this idea is that aggregate investment generates tangible (e.g. production facilities) and intangible capital (e.g. accumulation of knowledge embedded in labour force) in an economy and they have an effect on the future productivity. Moreover, some fractions of these effects of aggregate investments in the previous period are supposed to be included
in the payoff function of the present. These fractions are expressed as \( \delta \in [0, 1] \). If \( \delta \) equals 1, past coordination perfectly affects current profitability. Conversely, when \( \delta \) equals zero, past coordination has no effects on present economic activity.

Taking into consideration the above discussion, we will define the payoff function of firms. If firms do not invest, their payoffs are zero. If firms invest, payoffs from investments have a characteristic of strategic complementarities and expressed as

\[
\pi(\theta, l, l_{-1}) = \theta + l + \delta l_{-1} - 1
\]  

This function is taken from Morris and Shin (2001). In this expression, \( l \) is the number of firms which invested in the current period and this creates productive capital for firms. \( l_{-1} \) is the number of firms which invested in the previous period. For simplicity, productive effects of capital created in the past are expressed by this term. Therefore, firms face two payoff complementarities; aggregate investment of firms within and between periods.

In other words, aggregate output is linearly composed of three elements in this model. \( \theta \) is interpreted as a productivity parameter which changes stochastically. \( l \) is an effective demand of current aggregate investment which boosts a production level of individual firms, while \( \delta l_{-1} \) is the positive productive effect from the past aggregate investment with a discount \( \delta \). On the other hand, the cost is normalized as one. The strategy of firms is a binary decision making regarding whether to invest or not. If firms decide to invest in the current period, it contributes to the individual and aggregate level of production of the next period.

In this game they follow symmetric monotone strategies, deciding between investing \( I \) or not \( N \) based on thresholds. In considering their expected payoffs, firms have to form their belief about the proportion of other firms which invested in the same period. Following Morris and Shin (2001), the proportion of investing firms is known to be uniformly distributed on \([0, 1]\) (Laplacian beliefs). More precisely, they form
beliefs such that the proportion of firms who obtained signals above the (symmetric) threshold are distributed uniformly over the unit interval. Therefore, the strategy of firms has the following form.

\[
s^i = \begin{cases} 
I & \text{if } x^i \geq \theta^* \\
N & \text{if } x^i < \theta^* 
\end{cases} \tag{2.2}
\]

\(\theta^*\) is a symmetric, unique equilibrium strategy and defined as \(\int_{l} \pi(l, l_{-1}, \theta^*)dl = 0\) after observing \(l_{-1}\). The proof of an existence of a unique equilibrium is discussed in the appendix.

### 2.3.2 Time flows and decision making of firms

Firms make their decisions based on the proportion of firms which invested in the previous period and their belief of proportion of investing in the current period. This is shown in the Figure 2.1. For example, the decision at \(t\) is influenced by aggregate investment at \(t - 1\) and expected aggregate investments at \(t\).
The payoff and information structure of this game is common knowledge among firms, while $\theta_t$ is unknown and $x^i_t$ is the noisy signal of firm $i$.

Next, we solve firms’ decision problems of investment to determine equilibrium strategies. At first, in the next section, equilibrium strategies will be determined in the case of no strategic complementarities, while the role of the past externalities and current coordination (strategic complementarities) of investments are studied in the following sections.

## 2.4 No strategic complementarities case

In this section, we consider the case where there are no strategic complementarities for the purpose of comparing the equilibrium strategies with those in the following sections where externalities are present. Therefore, firms’ payoff function takes the following simple form.

$$\pi_t = \theta_t - 1$$

Because they do not need to form a belief about other firms’ decision to investments, the equilibrium strategy is defined as

$$s^i_t = \begin{cases} 
I & \text{if } \mathbb{E}(\theta_t | x^i_t) \geq 1 \\
N & \text{if otherwise}
\end{cases}$$

When a firm obtains signal $x^i_t$, it infers the value of $\theta_t$ from the distribution $N(x^i_t, \sigma)$. Therefore firms invest if and only if $x^i_t \geq 1$. Hence, when a firm receives a signal above one, it invests. Conversely, if it obtains a signal below one, it does not invest. This strategy is repeated every period and a firm’s decision making regarding investment does not change. Therefore, without strategic complementarities, we cannot explain unstable movement of equilibrium behaviour of firms.
2.5 Economy with past externalities only

Next, we consider the equilibrium strategies of firms with only past externalities. Examining past externalities on their own will allow us to compare with the case in which both past and current externalities exist. Given that only past externalities are included in the payoff function, managers recursively solve equilibrium strategies from the past. Therefore, their payoff function is expressed as follows.

\[ \pi(\theta, l_{-1}) = \theta + \delta l_{-1} - 1 \]

2.5.1 Decision problems of firms with past externalities

**T = 0** In this period, the equilibrium strategy is the same as the no externalities case because there is no past economic activities. Thus, payoff function becomes \( \pi(\theta_0) = \theta_0 - 1 \). Therefore,

\[ s_i^0 = \begin{cases}  I & \text{if } E(\theta_0 | x_i^0) \geq 1 = \theta_0^{S^*} \\ N & \text{if otherwise} \end{cases} \]

\( \theta_t^{S^*} \) is the equilibrium strategy of firms whose payoff function includes only past externalities at \( t \). When a firm obtains signal \( x_t^i \), it infers the value of \( \theta_t \) from normal distribution with mean \( x_t^i \) and standard deviation \( \sigma \). Hence, they decide to invest if their signals are above 1. If not, they do not invest.

**T = 1** From the threshold of \( T = 0 \), the proportion of investing firms at \( T = 0 \) is known at \( T = 1 \). This proportion is the same as the probability that firms obtain their signal above \( \theta_0^{S^*} \). Thus, the proportion of firms investing is \( 1 - F\left(\frac{\theta_0^{S^*} - \theta_0}{\sigma}\right) \). Because \( 1 - F\left(\frac{\theta_0^{S^*} - \theta_0}{\sigma}\right) = F\left(\frac{\theta_0 - \theta_0^{S^*}}{\sigma}\right) \), the number of firms which invested is \( F\left(\frac{\theta_0 - \theta_0^{S^*}}{\sigma}\right) \) at the end of \( T = 0 \).

Therefore, in this period, firms invest if \( E(\theta_1 | x_1^i) \geq 1 - \delta F\left(\frac{\theta_0 - \theta_0^{S^*}}{\sigma}\right) = \theta_1^{S^*} \). Thus,
firms who obtained signals above \( \theta_1^{S*} \) invest, and do not if signals are below this threshold. At this period, the proportion of firms which invested is \( F(\frac{\theta_1-\theta_n^{S*}}{\sigma}) \).

\( T = n \) Similarly, at \( T = n \) firms invest when

\[
E(\theta_n|x_n^i) \geq 1 - \delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) = \theta_n^{S*} \tag{2.3}
\]

### 2.5.2 Movement of equilibrium strategies

To see the movement of equilibrium strategies more clearly, we rewrite (2.3) in the following form.

\[
\Delta \theta_n^{S*} = -\delta \Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) \tag{2.4}
\]

In this expression, \( \Delta \) is the difference between current and past variables. \( \Delta \theta_n^{S*} = \theta_n^{S*} - \theta_{n-1}^{S*} \) and \( \Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) = F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) - F(\frac{\theta_{n-2}-\theta_n^{S*}}{\sigma}) \). Therefore, the movement of \( \Delta \theta_n^{S*} \) is

\[
\Delta \theta_n^{S*} = \begin{cases} 
\text{Positive} & \text{if } \Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) < 0 \\
\text{No change} & \text{if } \Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) = 0 \\
\text{Negative} & \text{if } \Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) > 0
\end{cases}
\]

Moreover,

\[
\Delta F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) = F(\frac{\theta_{n-1}-\theta_n^{S*}}{\sigma}) - F(\frac{\theta_{n-2}-\theta_n^{S*}}{\sigma}) \begin{cases} < \quad \text{or} \quad > \quad \text{or} \quad = \end{cases} 0
\]

\( \Leftrightarrow \)
\[ \begin{align*}
\theta_{n-1} - \theta^S_{n-1} &\begin{cases} < \\ = \\ > \end{cases} \theta_{n-2} - \theta^S_{n-2} \\
\iff \quad \Delta \theta^S_{n-1} &\begin{cases} > \\ = \\ < \end{cases} \Delta \theta_{n-1} \text{ where } \Delta \theta_{n-1} = \theta_{n-1} - \theta_{n-2}
\end{align*} \]

Summing up in Proposition 1, the movement of \( \theta^S_n \) becomes as follows.

**Proposition 1 (Equilibrium fluctuations in the case of past externalities only)**

When the current economy is affected by productive effects of the previous aggregate investments only, firms’ equilibrium strategies at present period fluctuate in the following ways.

\[
\Delta \theta^S_n = \begin{cases} 
\text{Positive} & \text{if } \Delta \theta^S_{n-1} > \Delta \theta_{n-1} \\
\text{No change} & \text{if } \Delta \theta^S_{n-1} = \Delta \theta_{n-1} \\
\text{Negative} & \text{if } \Delta \theta^S_{n-1} < \Delta \theta_{n-1}
\end{cases}
\] (2.5)

Note that an expected value of a signal is the same as a realized economic state \((E(x^i) = \theta)\). In this proposition, the movement of equilibrium strategies depends on the comparison between difference of equilibrium strategies and that of expected values of signals in the previous period. If the former is greater than the latter, the equilibrium strategy is higher than the previous period. Conversely, if the latter is greater, the equilibrium strategy at current period is lower. Thus, this economy
shows fluctuations by changing equilibrium strategies\(^5\). Since the relative strength of these two differences determined the number of firms who invested in the past, this proposition says that an current equilibrium strategy goes up (down) if the number of firms invested in the previous period was less (more) than before.

### 2.6 Economy with both past and current externalities

In this section, we consider the case in which both current and past externalities are included. Thus, the payoff function of firms are presented as (2.1).

#### 2.6.1 Decision problems of firms with past externalities and current coordination of aggregate investments

Similar to the previous section, this dynamic coordination game can be solved recursively from the past. Because of the Laplacian belief mentioned before, thresholds of each period are determined as follows.

\[ T = 0 \] In this period, there is no past time. Therefore, the equilibrium strategy is determined as the following.

\[
\int_{l_0=0}^{1} (\theta_0 + l_0 - 1)dl = 0 \iff \theta_0^{L^*} = \frac{1}{2}
\]

\( \theta_0^{L^*} \) is the equilibrium strategy in the framework with both past and current externalities. Therefore, the equilibrium strategy is \( \frac{1}{2} \) and firms who obtain their private

\(^5\)The following description \( \Delta \theta_n^{S^*} \) is easily translated in terms of \( \Delta l_n \). Provided \( \theta_n^{S^*} \) is determined, \( \Delta l_n = \Delta F(\theta_n - \theta_n^{S^*}) \) since the number of firms invested is \( 1 - F(\theta_n^{S^*} - \theta_n) \). Therefore, in the case of \( \Delta \theta_n > \Delta \theta_n^{S^*} \), \( \Delta l_n \) increases. On the other hand, \( \Delta l_n \) decreases when \( \Delta \theta_n < \Delta \theta_n^{S^*} \) is hold. When \( \Delta \theta_n = \Delta \theta_n^{S^*} \), the current investment does not change.
signals above this threshold invest and those who received below this point do not invest.

T = 1) Since not only current coordination effects but also past aggregate investments have a positive effect on payoff of this period, equilibrium strategy of this period is obtained from the next equation.

\[
\frac{1}{l_1=0} (\theta_1 + l_1 + \delta F(\frac{\theta_0 - \theta_0^*}{\sigma}) - 1)dl = 0
\]

Therefore, the equilibrium strategy at this period becomes

\[
\theta_1^* = \frac{1}{2} - \delta F(\frac{\theta_0 - \theta_0^*}{\sigma})
\]

T = n) As the same procedure mentioned above continues, the equilibrium strategy in period n is

\[
\theta_n^* = \frac{1}{2} - \delta F(\frac{\theta_{n-1} - \theta_n^*}{\sigma})
\]

(2.6)

2.6.2 Movement of equilibrium strategies

To see the movement of equilibrium strategies more clearly, we rewrite (2.6) in the following form.

\[
\Delta \theta_n^* = -\delta \Delta F(\frac{\theta_{n-1} - \theta_n^*}{\sigma})
\]

(2.7)

In this expression, \(\Delta\) is a difference between current and past variables. \(\Delta \theta_n^* = \theta_n^* - \theta_{n-1}^*\) and \(\Delta F(\frac{\theta_{n-1} - \theta_n^*}{\sigma}) = F(\frac{\theta_{n-1} - \theta_n^*}{\sigma}) - F(\frac{\theta_{n-2} - \theta_{n-1}^*}{\sigma})\).

Therefore, the movement of \(\Delta \theta_n^*\) is
\[
\Delta \theta_n^{L^*} = \begin{cases} 
\text{Positive} & \text{if } \Delta F\left(\frac{\theta_{n-1} - \theta_n^{L^*}}{\sigma_{n-1}}\right) < 0 \\
\text{No change} & \text{if } \Delta F\left(\frac{\theta_{n-1} - \theta_n^{L^*}}{\sigma_{n-1}}\right) = 0 \\
\text{Negative} & \text{if } \Delta F\left(\frac{\theta_{n-1} - \theta_n^{L^*}}{\sigma_{n-1}}\right) > 0
\end{cases}
\]

Similar to the case of firms without current coordination, the movement of equilibrium strategies of firms becomes as in Proposition 2.

**Proposition 2 (Equilibrium fluctuations under the existence of both past and current externalities)**

When the current economy is affected by productive effects of the previous aggregate investments and, also, current coordination, firms’ equilibrium strategies at present period fluctuate in the following ways.

\[
\Delta \theta_n^{L^*} = \begin{cases} 
\text{Positive} & \text{if } \Delta \theta_{n-1}^{L^*} > \Delta \theta_{n-1} \\
\text{No change} & \text{if } \Delta \theta_{n-1}^{L^*} = \Delta \theta_{n-1} \\
\text{Negative} & \text{if } \Delta \theta_{n-1}^{L^*} < \Delta \theta_{n-1}
\end{cases}
\] (2.8)

In this proposition, the movement of equilibrium strategies depends on the comparison between difference of equilibrium strategies and that of expected values of signals in the previous period. If the former is greater than the latter, the equilibrium strategy is higher than the previous period. Conversely, if the latter is greater, the equilibrium strategy at current period is lower. Therefore, volatile fluctuations of equilibrium strategies of firms are shown in this framework. Although there is an effect of current aggregate investment in this framework, the interpretation of this proposition is the same as proposition 2.
2.7 The limit case of equilibrium strategies and persistence of economic fluctuations

A global game is characterised such that when incompleteness of information vanishes, a unique equilibrium (risk dominant equilibrium) can be achieved. To show the persistence of cycle more clearly, the limiting case is considered in this section.

2.7.1 The limit case of equilibrium strategies

When the noise of signals becomes vanishingly small, in other words, \( \sigma \to \infty \), the strategy (2.2) becomes

\[
s^i = \begin{cases} 
  I & \text{if } \theta \geq \theta^* \\
  N & \text{if } \theta < \theta^* 
\end{cases}
\]

Under this strategy, there are three cases in (2.3) and (2.6), which are summarized as corollaries 1 and 2.

Corollary 1 (The limiting case of equilibrium strategies where the effect of current aggregate investment is taken into account)

In the limiting case of vanishing incompleteness of signals, the fluctuations of equilibrium strategies of firms taken into consideration the effect of current aggregate investment can be characterized as follows.

1) if \( \theta_{n-1} > \theta^{L*}_{n-1} \) and \( \sigma \to 0 \), \( \theta^{L*}_n \to \frac{1}{2} - \delta \)

2) if \( \theta_{n-1} < \theta^{L*}_{n-1} \) and \( \sigma \to 0 \), \( \theta^{L*}_n \to \frac{1}{2} \)

3) if \( \theta_{n-1} = \theta^{L*}_{n-1} \) and \( \sigma \to 0 \), \( \theta^{L*}_n \to \frac{1}{2} - \delta F(0) \). Since the distribution \( F \) is standard normal, \( \theta^{L*}_n \to \frac{1}{2}(1 - \delta) \).

Proof. In this corollary, if \( \sigma \to 0 \), \( F\left(\frac{\theta_{n-1} - \theta^{L*}_{n-1}}{\sigma}\right) \) of (2.6) becomes 1 and 0 depending on the relative strength of the state of economy in the previous period and the past
equilibrium strategy. If \( n_1 > \theta^{L*}_{n-1} \), \( \frac{n_1 - \theta^{L*}_{n-1}}{\sigma} \) is +\( \infty \) when \( \sigma \to 0 \) and \( F(\frac{n_1 - \theta^{L*}_{n-1}}{\sigma}) = 1 \). Thus, from (2.6) \( \theta^{L*}_n \) converges to \( \frac{1}{2} - \delta \). Conversely, if \( n_1 < \theta^{L*}_{n-1} \), \( \frac{n_1 - \theta^{L*}_{n-1}}{\sigma} \) becomes \( -\infty \) and \( F(\frac{n_1 - \theta^{L*}_{n-1}}{\sigma}) = 0 \). Then, \( \theta^{L*}_n \) converges to \( \frac{1}{2} \). Finally, when \( n_1 = \theta^{L*}_{n-1} \), \( F(\frac{n_1 - \theta^{L*}_{n-1}}{\sigma}) = F(0) = \frac{1}{2} \). Therefore, \( \theta^{L*}_n \) converges to \( \frac{1}{2}(1 - \delta) \).

In this case, according to the previous state, threshold strategies change between \( \frac{1}{2} \) and \( \frac{1}{2} - \delta \). If the state of the previous period is good and the signal is higher than the threshold, the threshold of the next period reduces to \( \frac{1}{2} - \delta \). However, once the signal decreases below this threshold sometime in the following period, the threshold after that period increases to \( \frac{1}{2} \). In the case of \( n_1 = \theta^{L*}_{n-1} \), the threshold changes to \( \frac{1}{2}(1 - \delta) \) because the distribution \( F \) is standard normal.

Intuitively, when the state of the previous period was high and above the threshold of that period, almost all firms invested because they obtained extremely accurate signals (\( \sigma \to 0 \)). In this situation, the effects of past aggregate investments on the present economic activities through positive externality from the past are maximized. Therefore, the threshold of present period reaches the lowest point \( \frac{1}{2}(1 - \delta) \). In contrast, when the past state of economy was low and below the threshold of that period, the threshold of the current period becomes highest \( \frac{1}{2} \) since almost all firms did not invest previously and the effects of the externality of aggregate investments from the past is weak.

The same analysis so far can be applied to the case of firms without current coordination and the result is summarized as follows. The proof is the same as the case of firms with current coordination.

**Corollary 2 (The limiting case of equilibrium strategies where the effect of current aggregate investment is not taken into account)**

In the limiting case of vanishingly incompleteness of signals, the fluctuations of
equilibrium strategies of firms who do not take into account the effect of current aggregate investment can be characterized as follows.

1) if $\theta_{n-1} > \theta_{n-1}^*$ and $\sigma \to 0$, $\theta_n^* \to 1 - \delta$.

2) if $\theta_{n-1} < \theta_{n-1}^*$ and $\sigma \to 0$, $\theta_n^* \to 1$.

3) if $\theta_{n-1} = \theta_{n-1}^*$ and $\sigma \to 0$, $\theta_n^* \to 1 - \delta F(0)$. Since the distribution $F$ is standard normal, $\theta_n^* \to 1 - \frac{1}{2} \delta$.

2.7.2 Persistence of cycle

One of the characteristics of this model is that not only the turning point of cycles, but also the persistence of cycles can be explained depending on the movement of underlying economic states. Here, persistence of cycle can be defined in terms of the probability of a certain economic state (expansion or recession) lasting given the same state in the previous period. If this probability is increasing over time, we say that there is persistence in the economic cycle. Since underlying states move randomly, equilibrium strategies and economic situations change, for instance, as in the Figure 2.2. In this Figure, beginning with $\frac{1}{2} - \delta$ as reflected expansion in the previous period, the economic situation at $T + 1$ becomes $\frac{1}{2} - \delta$ if period $T$ is in boom and $\frac{1}{2}$ if $T$ is in recession. The cumulative probability that the economy is in expansion is $1 - \Phi(\frac{1}{2} - \delta)$, where $\Phi$ is c.d.f. of $\theta$, and that of recession at $T$ is $\Phi(\frac{1}{2} - \delta)$. Further, the economic situation at $t = T + 1$ can be also expressed in a similar way. When $T$ is in expansion and the equilibrium strategy is $\frac{1}{2} - \delta$, the same economic situation (expansion) continues with probability $1 - \Phi(\frac{1}{2} - \delta)$. In contrast, after recession at $T$, the probability of expansion at $T + 1$ is $1 - \Phi(\frac{1}{2})$. Therefore, the probability of expansion in the current period when the previous economic situation is in boom is greater than that when the past economy is in recession. As a result, the economic cycle can persist over time in this model. The same consideration can be applied when the current and past economy are in recession.
Intuitively, when a state is above a threshold, expansions tend to last after this period because the next threshold decreases. In contrast, if the signal drops under the equilibrium point, it is not easy to coordinate to invest after this period since the threshold goes up. In this situation, recessions can last for a number of periods. Thus, booms and recessions continue for a while until the actual states go up or down beyond the relevant thresholds.

Next, we will examine the effect of past externality on persistence. When the extent of externality $\delta$ converges to zero, the equilibrium strategy converges to $\frac{1}{2}$. On the other hand, the equilibrium strategy becomes $-\frac{1}{2}$ when $\delta$ is one. This argument for persistence shows that persistence of expansion becomes longer when the effect of past externality is stronger ($\delta$ is greater) than the case of lower $\delta$. This is because the economy is easy to drop into recession in the case of no externality, while expansion may continue to last for a long time when the externality is stronger due to the reduction of the equilibrium strategy.
2.8 The implications for persistence with and without current coordination

Until now, we divided the analysis between games with and without current coordination. Finally, we will examine the implication of this difference for the persistence of a cycle. As shown in the previous sections, there is persistence in the cycle in both cases. However, the probability that the economy falls into recession and recovers from this situation is different between these two cases. For instance, when the economy is in boom at some point of time, the economy continues to expand with probability \(1 - \Phi(\frac{1}{2} - \delta)\) in the case of current coordination in the next period, while \(1 - \Phi(1 - \delta)\) in the case without current coordination. Also, after the economy drops into recession, expansion appears with probability \(1 - \Phi(\frac{1}{2})\) for firms with current coordination. Conversely, firms which have a payoff without current coordination can recover from recession at the probability \(1 - \Phi(1)\). In other words, once economy drops into recession, this situation tends to last in the following periods under the case without current coordination. Though the expansion has persistence, this effect is limited compared to the case with current coordination. Therefore, firms under no current coordination tend to be stuck in stagnation and consequently are prevented from accumulation of capital through coordination of aggregate investments.

2.9 Conclusion

In this chapter, in order to analyze unstable fluctuations of aggregate corporate investments, firms’ decision making of corporate investments was examined by means of a dynamic global game. For this purpose, the externality of the past aggregate investments rationalized on endogenous growth theory (learning by doing) is supposed to have an impact on the current economic activities. Due to the simplicity of this
setting, these coordination games have a unique equilibrium at each stage and unstable behaviour of equilibrium strategies can be produced over time. On the other hand, without these externalities from the past, huge fluctuations cannot emerge. In addition, by including past externalities, we can show persistence in which a certain economic situation can persist in the subsequent periods.

As for the implication for business cycles, these movements of equilibrium strategies can explain two characteristics of business fluctuations. Firstly, turning points of expansions and recessions can be determined depending on the path of underlying economic states. In other words, firms coordinate or fail to coordinate on investments relying on the equilibrium strategies of each period. Consequently, huge fluctuations of investments can appear over time. Secondly, any sufficiently long run duration of cycles can be explained in this model. In particular, compared to the case without a past externality, the existence of it can prolong periodicity of cycles. Once firms coordinate (do not coordinate) on investments and the economy goes into expansion (recession), the economic environment of the future tends to be favourable (unfavourable) because of externalities from past aggregate investments.

In addition, a few comparative statics say that the greater the strength of past externalities is, the longer the persistence of expansion is. Furthermore, existence of current coordination makes persistence of expansion more prolonged than without current coordination.
2.10 Appendix

Proof of the uniqueness of equilibrium

Generically, a dynamic global game does not necessarily have a unique equilibrium. However, this game is highly simplified to keep each stage isolated except for the influence of past investment. This simplification can make each stage equivalent to a static global game. One caveat is about the linearity of function (2.1). Linearity is a sufficient condition for uniqueness and, although the function (2.1) is composed of a linear relation between $\theta$, $l$ and $\delta_{l-1}$, there might be several other ways to set the functional form with satisfying the uniqueness of the equilibrium.

The uniqueness of equilibrium in this game can be proved by the following. At first, the difference of payoff between investment and not are defined as $V(\theta, l, l_{-1}) = \pi(I, \theta, l, l_{-1}) - \pi(N, \theta, l, l_{-1})$. Then,

$$V(\theta, l, l_{-1}) = \pi(\theta, l, l_{-1}) = \theta + l + l_{-1} - 1$$

This function is continuous and increasing in $\theta$ and $l$. Moreover, there is a unique $\theta^*$ satisfying $\int_{l=0}^{1} \pi(l, l_{-1}, \theta^*)dl = 0$. Limit dominance (in the extreme value of $\theta$, investment or not investment dominates in all values of $l$) is guaranteed because $\pi$ is increasing in $\theta$ and $l$ is bounded in $[0, 1]$. Finally, expectations of signals are well defined since the mean of $\varepsilon$ is zero. In our model, the structure of payoff function is the same as that of Morris and Shin (2001) except past externality $\delta_{l-1}$ and each firm makes a decision given the effect of this externality in every period. Therefore, the same procedure of proof in Morris and Shin (2001) can apply to our case. From Lemma 2.3 in Morris and Shin (2001), there is a (symmetric) unique switching strategy equilibrium $\theta^*$ in which firms invest when $x^i \geq \theta^*$ and not when $x^i < \theta^*$ and it survives iterated elimination of dominated strategies in this game.
Chapter 3

Liquidity, firms’ productivity and economic dynamics

3.1 Introduction

One of the important aspects of current business fluctuations is that the financial markets have had significant influence on economic fluctuations. In order to understand the mechanism linking real and financial markets, there have appeared a lot of studies in economics, especially focused on principal-agent problems and contractual incompleteness (e.g. Bernanke and Gertler (1989), Kiyotaki and Moore (1997))\(^1\). In these works, financial frictions coming from informational problems between lenders and borrowers put restrictions on the availability of investment funds for borrowers. As a result, net worth and collateral held by firms have an essential role to mitigate financial frictions and resulting credit contractions. At the same time, how firms deal with liquidity shocks in the middle of production has become a growing issue through the experiences of financial crises in the last decades. Liquidity management is an essential part of daily decision making of managers and if firms fail to meet liquidity

\(^1\)See Freixas and Rochet (1997) for a broad survey.
needs in the middle of the production, they might go bankrupt even if they have positive net present value projects. Therefore, smooth functioning of liquidity markets has a crucial role for stability of economic activities. The purpose of this chapter is to examine the role of a liquidity management in a corporate sector and liquidity markets from the point of view of macroeconomic dynamics.

Here, we need to discuss the concept of liquidity. The meaning of liquidity depends on the context in which it is used in economics. Roughly speaking, there are three patterns regarding how to use this term\(^2\). Firstly, it means *currency and money*. This is because money is the most liquid asset in that its value is stable relative to other financial assets when it is transacted as a medium of exchange. In contrast, other financial assets are transacted more or less with costs in a form of elapsing time to realize its full value (e.g. selling real estate) and volatility of their values. Secondly, liquidity is often referred in a context of *market liquidity*. If there are a lot of agents to buy and sell for a financial asset and the volume of transaction in a market is huge, this market is deemed to be liquid. In such a market, an appropriate price of a financial asset can be easily found and induces investors to act properly. On the other hand, when there are few participants in a market and the price on this market is enormously volatile, the market is illiquid and participants might give up transacting their assets. Most OTC (over-the-counter) markets like securitization markets are vulnerable to this kind of illiquidity. Finally, though it is closely related to market liquidity, *funding liquidity* refers to temporary funds need for precautionary reason. In this case, in order to obtain liquidity, firms try to negotiate with lenders and/or hoard liquid assets (e.g. government bonds) in advance against the liquidity shock.

Economic studies regarding liquidity so far are mainly based on the last two aspects mentioned above; market and funding liquidity\(^3\). With regard to market li-

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\(^2\)This classification is according to Nikolaou (2009).

\(^3\)Needless to say, the vast amounts of analysis relating the first point of liquidity as currency exist, for instance liquidity preference theory (Keynes (1936)). However, we will not mention it further since new research lies mainly in market and funding liquidity in reflection to the current financial
M. Morris and Shin (2003) studied fire sales in an asset market by the method of global games. In their study, investors with short term views, faced by a loss limit, rush to sell their asset holdings when the price of an asset decreases close to the limit, which causes illiquidity in a market. Empirical evidence regarding this kind of liquidity also has been shown in various markets (e.g. corporate bond market (Dittmar et. al (2002))). With respect to the accessibility to liquidity, Holmström and Tirole (1998a) studied optimal liquidity provision under moral hazard of a manager. In their framework, they showed that, due to the moral hazard of a manager, investors have to guarantee minimum amounts of payoff for him in order to make him behave appropriately and, as a result, optimal policy of liquidity provision is different from the first best one. Moreover, aggregate uncertainty causes a shortage of supply of liquidity in the private sector and government support for liquidity in the form of government bonds and monetary injection becomes essential. As mentioned before, market liquidity and accessibility are closely related with each other. For example, when firms need temporal funds, firms have to raise money by several ways. One of the options is to ask outside investors to lend the necessary money. The other is to sell their holdings of liquid assets in markets. In such a situation, market liquidity and accessibility is related to firms’ liquidity management since market condition also affects the availability of funds for firms.

With regard to macroeconomic implications of liquidity management, there are a few studies in the context of business fluctuations. Kiyotaki and Moore (2008) studied an amplification mechanism and the role of money when firms face liquidity constraints. More precisely, although there are several kinds of assets in an economy (e.g. equity) other than money and a rate of return of these assets is higher than crises.

As other studies, Geanakoplos (2001) also analyzed the market crash induced by liquidity shortage in a general equilibrium model.

Brunnermeier and Pedersen (2008) theoretically studied the relationship between these two concepts in a security market. As an empirical study, Nathaniel et al. (2008) examined these two relationships by looking at events during the subprime crises of 2007.
that of money, money offers sufficient liquidity services when markets are in turmoil. Firms, therefore, take into consideration future liquidity constraints and try to hold enough money. If money is scarce, a liquidity shock to asset markets puts constraints on the availability of necessary funds for investment. In turn, the reduction of output induces asset markets to be less liquid because the markets’ size shrinks. Thus, the function of monetary authorities which converts the proportion of money and other assets through open market operations serves the stability of an economy.

The difference between existing studies and ours is that we focus on liquidity management of corporate sectors and contribute to make clear how liquidity funds are produced in an economy and can be transacted between firms. For this question, the previous literature mainly paid attention to market liquidity and feedback effects of market and funding liquidity in a financial sector (Adrian and Shin (2007), Brunnermeier and Pedersen (2008)) in reference to the current financial crises. Financial intermediaries facing funding liquidity risk sell their securities holdings, which leads to a crash in these markets. Therefore, market liquidity evaporates due to fire sales by financial intermediaries. In turn, lower prices of these assets deteriorate the balance sheet of these intermediaries and induce further funding problems. Moreover, Adrian and Shin (2007) states that during booms financial intermediaries tend to increase their leverage, while during downturns they are inclined to reduce leverage. Hence, liquidity availability in financial markets and financial cycles are exacerbated by the behaviour of financial intermediaries.

In this study, we will consider a different mechanism for creating liquidity. Here, we regard liquidity as firms’ net worth and analyze the relationship between lifecycle productivity change in a corporate sector and accumulation of net worth. In this regard, we use two empirical findings of characteristics of new and old firms;

Allen and Carletti (2008) also stress the close relationship between banking activities and financial markets in current financial crises, claiming that a liquidity shortage due to incomplete financial markets leads to liquidity crises and possible contagion, which damages the real economy.
productivity difference and frequency of bankruptcy. The first is Cooley and Quadrini (2001) where they found that new firms are more likely to go bankrupt than old ones. The other is Caballero and Hammour (1994) who showed that new firms’ productivity is higher than the productivity of older firms. These two facts are easy to understand if we take into account firms’ net worth. When firms are young and start business, they try to keep their productivity to be high because, otherwise, they might go bankrupt in the future and lose the future benefit obtained from continuation of their business. However, in general, their net worth is limited and if a liquidity shock hits the economy, they are likely to go bankrupt unless other firms provide liquidity support. On the other hand, old firms hold enough net worth since they survived the preceding economy and accumulated sufficient internal funds. However, they are inclined to adopt low productivity projects because their continuation value is low and their managerial discipline tends to be destroyed. As a result, a manager attempts to pursue his private goal which does not contribute to the firm’s profitability. Summing up, old and mature firms have sufficient liquidity, while new firms hold insufficient liquidity. In contrast, new firms engage in high productive projects whereas old ones tend to have low productive projects.

Moreover, provided that there are two types of firms with regard to amounts of liquidity holdings, there is the possibility that liquidity markets emerge where trade takes place between these types of firms. Put differently, firms with sufficient liquidity holdings supply excess liquidity for ones with insufficient liquidity. By doing so, firms with low liquidity holdings can be insulated from stochastic liquidity shocks over time. From this point of view, we will compare the effect of liquidity shocks on the dynamics of an economy both with and without liquidity markets. However, these dynamics rely heavily on characteristics of liquidity shocks; how do these probabilities change? On this point, we assume that previous economic states have an externality on the current probability of a shock. That is, if the economy was previously booming, then
the current economy is unlikely to be hit by a shock. In contrast, if the economy was previously in recession, then the current economy is likely to suffer a shock. By interaction between liquidity markets and past externality, we can observe features of dynamics of aggregate output. For instance, liquidity markets have a role of keeping aggregate economic activities at the highest level and stabilize through liquidity transactions between firms. However, introducing liquidity markets may also create a sharp reduction of aggregate output when the economy experiences continuous downside pressure. On the other hand, without liquidity markets, the economy significantly fluctuates because of liquidity shocks. Regarding liquidity management of a non-financial corporate sector, Holmström and Tirole (1998a) is close to our model. However, there are two differences between their and our model. The first is that they basically do not analyze the macroeconomic implications of liquidity management, for instance, fluctuations and persistence, while we show features of them. The other is that firms in their model are all identical and, if liquidity shocks are at an aggregate level which hit all firms at the same time, liquidity markets between firms cannot emerge. On the other hand, our model introduces a productivity difference between firms and an opportunity to transact liquidity through markets even if the shocks are at an aggregate level. Moreover, we evaluate the role of this liquidity market in the context of macroeconomic dynamics.

The structure of this chapter is as follows. In section 2, we introduce the basic model of liquidity to feature the two empirical findings; productivity difference and frequency of bankruptcy between firms. Then, we will analyze macroeconomic dynamics in the case without liquidity markets in section 3, while section 4 is dedicated to the case with liquidity markets. The role of liquidity markets and a past externality will be evaluated in section 5. Finally, concluding remarks will be made in section 6.
3.2 The model

3.2.1 Description

There is a continuum of firms of measure one and they last two periods. The discount factor is normalized to zero. At the beginning of each period they invest their endowment of one unit. There are two types of projects; high productivity projects ($H$ projects, $\theta_H$) and low productivity ones ($L$ projects, $\theta_L$) and firms choose one of them at the beginning of each period ($\theta_H > \theta_L$). Moreover, suppose that $L$ projects bring about non-pecuniary, private benefit $B$ to firms. This benefit is interpreted as perks for managers when they choose low productive projects which bring about positive utility only for them. For instance, a manager tries unnecessarily to expand its business scale by mergers and acquisitions in order to enhance his/her reputation. That is, launching a low productive project generates private benefit for managers in the expense of firms’ value\textsuperscript{7}.

After the initial investment, firms may be hit by a liquidity shock and need additional amount of funds. This amount is assumed to be $\rho$ and it happens with probability $\phi$\textsuperscript{8}. Here, the probability of a shock $\phi$ is assumed to be dependent on an aggregate state in the previous period; $\phi = \Phi(Y_{-1})$. $Y_{-1}$ means an aggregate output in the previous period and $\Phi(\cdot)$ is assumed to be a decreasing function with regard to $Y_{-1}$. In other words, when a past aggregate output is high, the probability of occurrence of a shock declines. On the other hand, if a past aggregate state is low, the probability of a shock becomes high. Intuitively, firms are likely to suffer liquidity events under a bad economic environment because of uncertainty about their daily business. For instance, when the economy is in recession in the past, available funds

\textsuperscript{7}This condition is similar to free cash flow hypothesis a la Jensen (1986). In his work, he suggested that when a firm holds excess liquidity, the manager tends to waste this money for his own purpose, for instance empire building.

\textsuperscript{8}This liquidity shock can be interpreted in various ways. For instance, it means cost overrun or uncertainty about future inflow of money from other transacted parties when they suffer from managerial problems.
for running firms on a daily basis may decrease due to a sudden stop of inflow of payments from other transacted parties which have managerial problems. On the other hand, a good economic state is unlikely to bring about such problems. $\phi_1$ and $\phi_2$ are defined as probabilities of a shock in the period 1 and 2 because these values are different in general.

Firms that pledge to their interim liquidity needs can realize their final output. Otherwise, firms cannot meet the necessary funding and cannot continue their projects. In this case, they are liquidated and their liquidation values are zero. While firms aim at maximizing wealth of the second period and private benefits of both periods (in the case of investment in low productive projects), the amount of wealth created in the past may be spent as necessary costs for next period’s projects. Given that there is a single good in this economy, the wealth at the beginning of period 1 ($W_0$) is one. The levels of firms’ utility obtained at the end of period 1 and 2 are denoted as $W_1$ and $W_2$, respectively, and they are defined as $W_i = W^*_i + B$, $i = 1, 2$. Here, $W^*_i$ is the amount of liquidity available to firms. As we will show later, the amount of $W^*$ affects the possibility of a survival of firms in the case of a liquidity shock, which determines the utility of them $W$. The timeline of this economy is depicted as Figure 3.1.

Next, we make three assumptions regarding a change in wealth by selection of project types, a manager’s private benefit and selection of projects’ type by firms.

**Assumption 1:** $\theta_H > 1 + \rho > \theta_L > 1$

Under this assumption, wealth (=liquidity availability) generated from a high productive project can cover all of the costs for investment (initial investments and liquidity needs) while that from a low productive can only cover the initial investments cost. Therefore, $L$ projects are liquidated if a liquidity shock happens.

**Assumption 2:** $\theta_H < \theta_L + B$

This assumption says that, from the point of view of a manager, low productive
Liquidity shock (ρ) occurs with probability 1φ. Firms choose H or L projects. Final output realized.

\[ W_0 = 1 \quad W_1^* \rightarrow W_1^* \quad W_2 \]

Figure 3.1: Timing of investments and creation of liquidity

projects are preferred to high productive ones.

**Assumption 3:** \( \theta_H - \Phi_2 \rho > (1 - \Phi_2)(\theta_L + B) \)

Probability \( \Phi_2 \) takes several values depending on the past aggregate outcomes \( \Phi_2 = \Phi(Y_{-1}) \). This assumption guarantees that, regardless of the value of the probability, \( H \) projects are more profitable for firms in the first period even if private benefit is taken into consideration. In other words, the continuation value for firms is high enough to take on high productive projects.

### 3.2.2 Decision-making at the beginning of period 2

Next, we will consider the dynamic relationship between the movement of firms’ wealth and the choice of project’s type by firms. Since firms last two periods of time and cannot launch investments after period 2, we will solve the model by backward induction. At the end of period 2 firms die and have no future projects. Therefore, they will choose \( L \) projects from Assumption 2. Depending on wealth owned by firms at the beginning of period 2, the amount of wealth of firms at period 2 becomes as follows. Here, \( W_1^* \) is defined as \( W_1^* = W_1 - B \) if firms choose \( L \) project at period 1.
since $B$ is not pecuniary.

If $W_1^* < 1$, then

$$W_2 = W_1$$

If $W_1^* > 1 + \rho$, then

$$\begin{cases} 
W_2 = W_1 - 1 - \rho + \theta_L + B & \text{when a liquidity shock occurs.} \\
W_2 = W_1 - 1 + \theta_L + B & \text{when a liquidity shock does not occur.}
\end{cases}$$

Thus, expected $W_2$ conditional on $W_1^* > 1 + \rho$ becomes

$$E(W_2 | W_1^* > 1 + \rho) = W_1 - 1 + \theta_L + B - \phi_2 \rho$$ \hspace{1cm} (3.1)

If $1 < W_1^* < 1 + \rho$, then

$$\begin{cases} 
W_2 = W_1 - 1 & \text{when a liquidity shock occurs.} \\
W_2 = W_1 - 1 + \theta_L + B & \text{when a liquidity shock does not occur.}
\end{cases}$$

Thus, expected $W_2$ conditional on $1 < W_1^* < 1 + \rho$ becomes

$$E(W_2 | 1 < W_1^* < 1 + \rho) = W_1 - 1 + (1 - \phi_2)(\theta_L + B)$$ \hspace{1cm} (3.2)

### 3.2.3 Decision-making at the beginning of period 1

Taking into account the above discussion and $W_0 = 1$, decision making of firms with regard to a project’s type at the beginning of period 1 becomes as follows.

If firms choose $H$ projects, then
\[
\begin{aligned}
W_1 &= 0 \quad \text{when a liquidity shock occurs.} \\
W_1 &= \theta_H \quad \text{when a liquidity shock does not occur.}
\end{aligned}
\]

If firms choose \( L \) projects, then
\[
\begin{aligned}
W_1 &= 0 \quad \text{when a liquidity shock occurs.} \\
W_1 &= \theta_L + B \quad \text{when a liquidity shock does not occur.}
\end{aligned}
\]

Given Assumption 1, (3.1) and (3.2), expected values of \( W_2 \) conditional on the choice of each project are determined as follows.

\[
E(W_2 \mid \text{firms choose } H) = (1 - \phi_1)(\theta_H - 1 + \theta_L + B - \phi_2 \rho) \quad (3.3)
\]

\[
E(W_2 \mid \text{firms choose } L) = (1 - \phi_1)(\theta_L + B - 1 + (1 - \phi_2)(\theta_L + B)) \quad (3.4)
\]

If (3.3) > (3.4), then firms will choose \( H \) project rather than \( L \) one and this condition is guaranteed by Assumption 3.

Summing up, firms choose an \( H \) project at the beginning of period 1 and an \( L \) project at the beginning of period 2. Therefore, young firms have a tendency to be high productive while old ones are likely to be low productive, which matches an empirical result (Cooley and Quadrini (2001)). Moreover, without liquidity support, young firms in period 1 cannot survive when they are hit by a shock because they have only one unit of money and cannot meet their liquidity need. On the other hand, old firms can survive with certainty because they chose \( H \) projects and held sufficient wealth when they were young. Hence, the existence of liquidity markets has a crucial role for the implications of aggregate dynamics. Next, we will consider this point more precisely.
3.3 Dynamics without liquidity markets

In order to consider the dynamics of this simple model, we will construct an overlapping generations model in which new firms are born at the second period of old firms (See Figure 3.2). The crucial point of the dynamics is that if young firms are hit by a liquidity shock, they cannot survive to the end of period 1 or start to invest at the period 2 because of a shortage of liquidity. Therefore, the effect of a liquidity shock continues to the next period.

3.3.1 The case of no past externality

This model assumes that the previous aggregate output affects the current probability of a liquidity shock; $\phi = \Phi(Y_{-1})$. However, in order to evaluate the role of past externality clearly, at first, we will consider the case of no externality. That is, if the current probability is not sensitive to the previous economy, how does this economy move, in other words, what is the implication of the past externality for
Figure 3.3: The stochastic process without liquidity markets

macroeconomic dynamics? In order to address this question, we assume that liquidity shocks are completely independent at every period and there is no influence from the past. We denote this independent shock as $\phi$. Moreover, as a result of realization of liquidity shocks (or no shocks), there are four possible scenarios of economic states in the case of no liquidity markets; $HL$, $H$, $L$ and $N$. These respectively mean that the economy is under the highest aggregate outputs (both old and young firms invest), only $H$ projects (only young firms invest), only $L$ projects (only old firms invest) and no investments. More precisely, regarding each state as a stochastic process, we can formalize this as Markov process as follows.

$$
Pr(Y_{t+1} | Y_t, Y_{t-1}, \ldots) = Pr(Y_{t+1} | Y_t)
$$

$Y_t$ is aggregate output at period $t$. The stochastic process of this economy is shown in Figure 3.3. Because there is no past externality, $\phi_{HL} = \phi_H = \phi_L = \phi_N = \phi$. 
In this setting, the Markov process follows the one step transition matrix.

\[
P_{I,N} = \begin{pmatrix}
H & L & N & HL \\
0 & \phi & 0 & 1-\phi \\
1-\phi & 0 & \phi & 0 \\
1-\phi & 0 & \phi & 0 \\
0 & \phi & 0 & 1-\phi \\
\end{pmatrix}
\]

(3.5)

\(P_{I,N}\) is matrix in which the transition probabilities are not affected by the past economy. When two steps are taken from this transition matrix, we can obtain the following.

\[
P_{I,N}^2 = \begin{pmatrix}
H & L & N & HL \\
\phi(1-\phi) & \phi(1-\phi) & \phi^2 & (1-\phi)^2 \\
\phi(1-\phi) & \phi(1-\phi) & \phi^2 & (1-\phi)^2 \\
\phi(1-\phi) & \phi(1-\phi) & \phi^2 & (1-\phi)^2 \\
\phi(1-\phi) & \phi(1-\phi) & \phi^2 & (1-\phi)^2 \\
\end{pmatrix}
\]

Thus, taking two steps of matrix (3.5) leads the economy to limiting matrix where all columns are the same. That is, this economy without past externality and liquidity markets does not exhibit any persistence.

### 3.3.2 The persistence of cycles

Next, we will examine the case with a past externality on liquidity shocks. That is, past economic states affect the probability of a liquidity shock in the next period as follows.

\[\phi = \Phi(Y_{-1}) \text{ where } Y_{-1} \in \{Y_{HL}^{-1}, Y_H^{-1}, Y_L^{-1}, Y_N^{-1}\}\]

Here, \(Y_{i}^{-1} i = HL, H, L, N\) is aggregate output of the previous period when the state was \(i\). In other words, since firms are measured as mass one, \(Y_{HL}^{-1} = \theta_H + \theta_L\),
\( Y_H^{-1} = \theta_H, Y_L^{-1} = \theta_L \) and \( Y_N^{-1} = 0 \). In addition, we will define the probabilities based on each case of past economic states.

\[
\begin{align*}
\phi_{HL} &= \Phi(Y_{HL}^{-1}) \text{ when the previous state is } HL. \\
\phi_H &= \Phi(Y_H^{-1}) \text{ when the previous state is } H. \\
\phi_L &= \Phi(Y_L^{-1}) \text{ when the previous state is } L. \\
\phi_N &= \Phi(Y_N^{-1}) \text{ when the previous state is } N.
\end{align*}
\]

\( \phi_{HL} < \phi_H < \phi_L < \phi_N \) since \( \Phi(\cdot) \) is decreasing function. Thus, the interaction between accumulation of firms’ wealth through two periods and probability changes via past economic states produces fluctuations of aggregate output over time.

One step transition matrices from one state to another corresponding to this stochastic process are expressed in the following. \( P_N \) means the transition matrix under no liquidity markets.

\[
P_N = \begin{pmatrix}
H & L & N & HL \\
0 & \phi_H & 0 & 1 - \phi_H \\
1 - \phi_L & 0 & \phi_L & 0 \\
1 - \phi_N & 0 & \phi_N & 0 \\
0 & \phi_{HL} & 0 & 1 - \phi_{HL}
\end{pmatrix}
\]

(3.6)

From this matrix, we can see that, depending on a starting state of economy, the probability of reaching other states is different. For instance, if we set \( \phi_{HL} = 0.1, \phi_H = 0.4, \phi_L = 0.6, \phi_N = 0.9 \), graph 1.a-d (attached in the end of this chapter) shows changes in probabilities of reaching each state from a given state as time passes by.

Therefore, there is a possibility that a different initial state of the economy goes along a different path and reaches different states if we restrict the stochastic process in finite periods. More precisely, according to the above example, all starting states are likely to reach \( HL \) or \( N \) with high probabilities. Moreover, if the economy starts from \( HL \) or \( H \), succeeding states tend to become \( HL \) whereas, in the case of \( L \) or no
investments as a starting point, no investments are likely to appear. In this sense, the
dynamics of this economy has persistence. However, the difference in probabilities
between $HL$ and $N$ becomes smaller as times passes by since the probability of
reaching $HL$ decreases while that of $N$ increases.

Finally, we will look at the feature of the limiting case of matrix (3.6). Matrix
(3.6) is classified as regular matrix in which all entries are positive values after several
steps. For instance, all entries of matrix (3.6) become positive after four steps. It is
known that in this class of matrix there is a unique limiting matrix in which all rows
are the same vectors as steps go to infinity (Grinstead and Snell (1997), Theorem
11.7 pp.434). In other words, all columns are the same values in the limit regardless
of the starting state. When such a vector is denoted as $(w_H, w_L, w_N, w_{HL})$, the
limiting matrix of (3.6) is calculated as follows (as for the procedure of calculation,
see appendix). $\overline{P}_N$ means the limiting matrix of (3.6).

\[
\overline{P}_N = \begin{pmatrix}
H & L & N & HL \\
 w_H & w_L & w_N & w_{HL} \\
 w_H & w_L & w_N & w_{HL} \\
 w_H & w_L & w_N & w_{HL} \\
 w_H & w_L & w_N & w_{HL} \\
\end{pmatrix}
\]

(3.7)

\[
w_H = w_L = \frac{\phi_{HL}(1 - \phi_N)}{A}, w_N = \frac{\phi_L\phi_{HL}}{A}, w_{HL} = \frac{(1 - \phi_H)(1 - \phi_N)}{A}
\]

Here,

\[
A = 2\phi_{HL}(1 - \phi_N) + \phi_L\phi_{HL} + (1 - \phi_N)(1 - \phi_H)
\]

From this limiting matrix, the signs of movement of $(w_H, w_L, w_N, w_{HL})$ can be
obtained in changing the probabilities (Table 3.1).

In the limit of the transition, as any probabilities of a liquidity shock increase,
Table 3.1: Movement of limiting matrix in changing of probabilities; no liquidity markets

<table>
<thead>
<tr>
<th></th>
<th>$\Delta w_H$</th>
<th>$\Delta w_L$</th>
<th>$\Delta w_N$</th>
<th>$\Delta w_{HL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \phi_{HL}$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \phi_H$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \phi_L$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \phi_N$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.2: Consecutive and intermittent cases; no liquidity markets

<table>
<thead>
<tr>
<th>Time</th>
<th>$T - 2$</th>
<th>$T - 1$</th>
<th>$T$</th>
<th>$T + 1$</th>
<th>$T + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three consecutive shocks</td>
<td>$L$</td>
<td>$N$</td>
<td>$N$</td>
<td>$H$</td>
<td>$HL$</td>
</tr>
<tr>
<td>Every two period shock</td>
<td>$L$</td>
<td>$H$</td>
<td>$L$</td>
<td>$H$</td>
<td>$L$</td>
</tr>
</tbody>
</table>

the frequency of reaching the highest state ($HL$) decreases whereas the possibility of arriving at the lowest state ($N$) increases. With regard to states $H$ and $L$, while an increase in the probabilities of a liquidity shock from $HL$ and $H$ causes high incidence of state $H$ and $L$, an increase from $L$ and $N$ induces a low frequency of $H$ and $L$.

### 3.3.3 Fluctuations

So far, we have examined the duration of a cycle as a Markov process. In order to observe characteristics of fluctuations in the case of no liquidity market more closely, we will try a thought experiment. In particular, we are interested in the characteristics of the downturn and recovery of the economy. For this purpose, we will consider two simple cases of shocks; 1) three period consecutive shocks, and 2) shocks every two periods. This situation is depicted in Table 3.2 and Figure 3.4. In this table, there were no shocks before $T - 2$ and shocks begin from $T - 2$ in each case. Under the case of three consecutive shocks, young firms at $T - 2$ are eliminated by the shocks because of no liquidity funds and cannot continue to the next period. Therefore, aggregate output cannot be realized until shocks stop after old firms invest at $T - 2$. On the other hand, high and low projects generate cycles when shocks occur every two period.
Figure 3.4: **Consecutive and intermittent cases; no liquidity markets.** In this figure high (low) productive projects are assumed to produce 2 (1) units of outputs.

From the above consideration, we can see the following features of economic dynamics with no liquidity markets.

**Proposition 3 (The economic dynamics in the case of no liquidity markets)**

*Provided that there were no liquidity shocks before,*

1) *When a shock happens in only one period, aggregate output drops to the level corresponding to low productive projects.*

2) *When shocks continue for more than two periods, there is no output in the economy.*

3) *As soon as liquidity shocks disappear, high productive firms emerge and the economy goes up abruptly.*

4) *When shocks occur every two periods, cycles between high and low outputs are produced.*

As we can see, after the shocks, the economy stagnates with low productive and no investments up to two periods. In contrast, intermittent shocks give birth to cycles
between $H$ and $L$ projects. Thus, liquidity shocks bring about huge instability in the economy\textsuperscript{9}.

### 3.4 Dynamics with liquidity markets

In the model of the last section, depending on shocks, the economy can significantly fluctuate. One of the reasons for this situation is that firms with sufficient liquidity cannot lend excess liquidity to ones with insufficient liquidity because there are no markets and financial intermediaries to accomplish such a financial transaction. In this sense, liquidity funds owned by old firms are “wasted” in the framework of no liquidity markets. If financial markets or intermediaries exist in this economy, there is a possibility that such a situation is mitigated. Figure 3.5 shows one example of this kind of financial transaction in an overlapping generations model. In period 2, old generations with enough liquidity at the beginning of time lend to new entrants with insufficient one, while at the end of time the borrowing firm repays the lender. Indeed, Campello et al (2009) analyzed a broad survey data and found that old firms are not sensitive to external finance and highly dependent on internal funds, while young firms are sensitive to external financing. Moreover, Lins et al (2008) empirically found that non-operational cash in a firm is mainly used for liquidity insurance, while a line of credit is for a corporate growth opportunity.

This financial transaction can be interpreted in several ways. For instance, firms with enough liquidity deposit their money in banks and banks set credit lines for firms with insufficient liquidity. Recent empirical research by Ivashina et al. (2009) has claimed that the availability of deposit is crucial to new lending of financial

\textsuperscript{9}In this model, if there are two consecutive aggregate shocks, no firms invest and this economy disappears because firms in each type are assumed to be identical. One of the ways to overcome this problem is to introduce idiosyncratic survival disturbance on firms. That is, we assume there are idiosyncratic lucky incidences preventing firms from failure. If a scale of this disturbance does not dominate aggregate shocks, some of the firms still could survive and the economy would not die out. However, a formal analysis of macroeconomic dynamics with this survival disturbance becomes complicated.
intermediaries, especially during financial distress since inter bank markets dry up. In another way, firms can issue commercial papers in markets for lending and borrowing excess liquidity. If this financial contract can succeed over time, firms do not easily go bankrupt due to the liquidity shocks and the economy becomes stable. However, this point depends on the characteristics of a liquidity shock in that the shock is affected by past economic outcomes or not, it occurs intermittently or consecutively\textsuperscript{10}. Here, we will consider these cases with liquidity markets. Since we would like to focus on the role of liquidity markets, the financial problem of initial investment will be omitted and an outlay of the initial investment (one unit) of new firms is assumed to be given at the beginning of period 1.

To begin with, we will pay attention to the timing of liquidity finance. In considering liquidity funding, timing of finance is also crucial from the point of view of informational problems. In the context of our model, there are two possibilities of timing to obtain liquidity along a time line; before and after a shock. If realized output as a result of investment can be observed by both lenders and borrowers but cannot be verifiable to a third party (e.g. court), the parties cannot make a liquidity contract based on it. Thus, when a liquidity transaction takes place after a shock, relative bargaining power between them determines the share of realized proceeds from the transaction. However, depending on this share reflecting their bargaining power, lenders are not willing to lend to borrowers since their share can be small. For simplicity, we assume that a liquidity shock is observable but non-verifiable and, through bargaining between a lender (an old firm) and a borrower (a new firm), a

\textsuperscript{10}In this regard, Holmström and Tirole (1998a) suggested that if a liquidity shock is idiosyncratic, there is a possibility that, through financial intermediaries, firms who were not hit by the shock and hold excess liquidity lend to ones suffered a liquidity shock and holding insufficient liquidity. On the other hand, if the shock is an aggregate shock, all firms are hit by a liquidity shock and cannot survive without outside liquidity support. However, there is a difference between their model and ours regarding the mechanism of creating liquidity. While Holmström and Tirole model considers identical firms and they all have a same amount of wealth, liquidity is produced by matured firms who have survived in the previous period and hold enough internal funds to prevent a liquidity shortage in our model. Therefore, an aggregate shock does not necessarily cause liquidity problems for firms in our model.
lender can recoup the amount $\alpha$ of realized output $\theta_H$. Thus, $\alpha$ is a parameter of bargaining power, coming from, for instance, their outside options or managers’ special skill for running a firm (Hart and Moore (1994)). In addition, if liquidity shocks do not hit the economy with probability $(1 - \phi)$, $\rho$ is returned to lenders.

Here, the following assumption is imposed to exclude the possibility of funding after realization of a shock.

**Assumption 4:** $\alpha_\theta H < \rho, (1 - \phi)\rho + \alpha_\theta H > \rho$

The first part of this assumption says that it is impossible for new firms to raise funds after the realization of a shock since lenders cannot be fully repaid due to the incomplete contract. On the other hand, in the second part, new firms are able to obtain necessary liquidity before a shock. In other words, the probability of a shock $\phi$ is small enough to satisfy these relations\(^{11}\).

Next, we will add a liquidity market to this model. Through this market, old firms

---

\(^{11}\)Holmström and Tirole (1998a) studied the effect of a moral hazard problem on liquidity funding and showed that, after a shock, financial constraint becomes harsh compared to that before a shock. The reason is that the need of liquidity is evaluated stochastically, which is the same as ours.
who have sufficient liquidity can provide young firms who hold insufficient liquidity with excess funds. For this purpose, we impose an additional assumption on this model.

**Assumption 5:**

\[
\begin{align*}
1 + 2\rho &\leq W_1^* & \text{when liquidity shocks do not occur.} \\
1 + 2\rho &> W_1^* > 1 + \rho & \text{when liquidity shocks occur.}
\end{align*}
\]

This assumption says that if a liquidity shock does not happen, old firms in the next period can hold enough liquidity for both young firms and their own. However, if the shock occurs, it deteriorates young firms’ balance sheets and their wealth in the beginning of the second period (when they become old firms) decreases to the level that it can afford to finance only their own liquidity needs even if they can survive due to financing from old firms\(^{12}\). In reality, it is well known from recent financial crises that deterioration of balance sheets of lenders causes difficulties in liquidity availability for firms and financial intermediaries (Nathaniel et al. (2008)). This is because lenders are concerned about future availability of liquidity for their own purposes and are not willing to supply their excess funds.

With regard to financial availability, the Assumption 4 says that firms with high productivity can raise necessary funds through liquidity markets before a shock. Since the structure of this model is fully recognized by all agents, there are no asymmetric information problems (adverse selection and moral hazard problems). As mentioned before, while there is a verifiability problem regarding liquidity shocks to the third party, old firms know the type of firms to whom they try to lend.

Although the same consideration regarding Markov process in the case of no liq-

\(^{12}\)Strictly speaking, this point is also affected by repayment for investors. In other words, if repayment is huge, the amount in hand of the borrowers is little and may be less than \(1 + \rho\) when a liquidity shock occurs. In this model, this possibility is assumed to be excluded by borrowers holding a majority of bargaining power, which was mentioned in the context of non-verifiability of a realized output.
Liquidity markets can be applied to that with liquidity markets, one caveat is that there are three scenarios in the situation of $HL$: 1) A shock hits the economy in the previous period and $L$ firms’ balance sheets at current period are heavily deteriorated so that they cannot lend money for $H$ firms if a shock again hits in the current period. 2) No shock happened in the previous period. Thus, $L$ firms’ balance sheets are not hurt. However, the current economy suffers a shock and balance sheets of $H$ firms are damaged. 3) There was no shock in the previous period and balance sheets of $L$ firms are not hurt in the current period. Moreover, there is no shock in the current period as well and balance sheets of $H$ firms are sound. These three scenarios regarding $HL$ have a different effect on the economic consequences in the next period, which determines the aggregate investments and outputs in the next period.

The movement of this economy is shown in Figure 3.6. In this figure, the notation is as follows; in the scenario of 1) we denote “$HL_w$”. The subscript “$w$” means weak balance sheets of $L$ firms. Likewise, “$HL_{s,s}$” and “$HL_{s,n}$” indicate the scenario 2) and 3), respectively. “$s,s$” means strong balance sheets of $L$ firms with a current shock while “$s,n$” means strong balance sheets with no current shocks. However, the probability of shocks from these three states is the same, $\phi_{HL} = \Phi(Y_{HL}^{-1})$.

### 3.4.1 The case of no past externality

At first, as in the case without liquidity markets, we will look at the case of liquidity markets, but without a past externality. The transition matrix of this scenario $P_{I,L}$ is as follows.
As we take three steps from this transition matrix, the next limiting matrix can be obtained and each column converges to the same value.

\[ P_{I,L}^3 = \]

\begin{bmatrix}
H & 0 & 0 & 0 & 0 & \phi & 1 - \phi \\
L & 1 - \phi & 0 & \phi & 0 & 0 & 0 \\
N & 1 - \phi & 0 & \phi & 0 & 0 & 0 \\
HL_w & 0 & 0 & 0 & 0 & \phi & 1 - \phi \\
HL_{s,s} & 0 & \phi & 0 & 1 - \phi & 0 & 0 \\
HL_{s,n} & 0 & 0 & 0 & 0 & \phi & 1 - \phi \\
\end{bmatrix}

Figure 3.6: Stochastic process with liquidity markets
Therefore, similar to the case of no liquidity markets, if the economy with liquidity markets is not exposed to a past externality, there is not enough persistence as we can see in actual business fluctuations.

### 3.4.2 Persistence of a cycle

Next, we will consider the case of existence of a past externality. Similar to the case of no liquidity markets, the one-step transition matrix of the economy with liquidity markets can be shown as follows. $P_L$ means transition matrix under liquidity markets.

\[
\begin{array}{ccccccc}
H & L & N & HL_w & HL_{s,s} & HL_{s,n} \\
\phi^2(1 - \phi) & \phi^2(1 - \phi) & \phi^3 & \phi(1 - \phi) + \phi(1 - \phi)^2 & \phi(1 - \phi)^2 + (1 - \phi)^3 \\
\phi^2(1 - \phi) & \phi^2(1 - \phi) & \phi^3 & \phi(1 - \phi) + \phi(1 - \phi)^2 & \phi(1 - \phi)^2 + (1 - \phi)^3 \\
\phi^2(1 - \phi) & \phi^2(1 - \phi) & \phi^3 & \phi(1 - \phi) + \phi(1 - \phi)^2 & \phi(1 - \phi)^2 + (1 - \phi)^3 \\
\phi^2(1 - \phi) & \phi^2(1 - \phi) & \phi^3 & \phi(1 - \phi) + \phi(1 - \phi)^2 & \phi(1 - \phi)^2 + (1 - \phi)^3 \\
\phi^2(1 - \phi) & \phi^2(1 - \phi) & \phi^3 & \phi(1 - \phi) + \phi(1 - \phi)^2 & \phi(1 - \phi)^2 + (1 - \phi)^3 \\
\end{array}
\]

This matrix also shows that each state goes along its own path in the next period. Again, if we set $\phi_{HL} = 0.1$, $\phi_H = 0.4$, $\phi_L = 0.6$, $\phi_N = 0.9$, graphs 2.a-f shows changes
in probabilities of reaching each state from a given state as time passes by\textsuperscript{13}.

Thus, each state has different paths and reaches different states in successive periods. In the framework of existence of liquidity markets, the economy is likely to reach $HL$ or no investment. Therefore, dynamics with liquidity markets also include persistence effects. However, compared to the no liquidity markets’ case, the existence of liquidity markets has tendency to keep the economy at the highest level ($HL$). In particular, even $L$ and $N$ as starting states gradually approach to state $HL$ with high probability. Graph 2.b, c shows that the probability of reaching $N$ can be overwhelmed by that of $HL$ after several steps. Hence, liquidity provisions through liquidity markets play a significant role for keeping the economy the highest state.

Finally, we will examine the feature of limiting matrix of (3.9) as before. The limiting matrix of (3.9) is calculated as follows (see also appendix). $P_L$ is the limiting matrix of (3.9).

\[
\begin{pmatrix}
H & L & N & HL_w & HL_{s,s} & HL_{s,n} \\
H & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}} \\
L & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}} \\
N & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}} \\
HL_w & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}} \\
HL_{s,s} & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}} \\
HL_{s,n} & w_H & w_L & w_N & w_{HL_w} & w_{HL_{s,s}} & w_{HL_{s,n}}
\end{pmatrix}
\]

From Assumption 4, the probabilities have restriction on their values. However, the qualitative results do not change in the current discussion.

\[
w_H = w_L = \frac{\phi^2_{HL}(1 - \phi_N)}{A'}, w_N = \frac{\phi_L \phi^2_{HL}}{A'}, w_{HL_w} = \frac{\phi_{HL}(1 - \phi_N)(1 - \phi_{HL})}{A'}
\]

\textsuperscript{13}From Assumption 4, the probabilities have restriction on their values. However, the qualitative results do not change in the current discussion.
Table 3.3: Movement of limiting matrix in changing of probabilities (liquidity markets)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta w_H$</th>
<th>$\Delta w_L$</th>
<th>$\Delta w_N$</th>
<th>$\Delta w_{HL_w}$</th>
<th>$\Delta w_{HL_{s,s}}$</th>
<th>$\Delta w_{HL_{s,n}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \phi_{HL}$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\Delta \phi_H$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \phi_L$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta \phi_N$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$$w_{HL_{s,s}} = \frac{\phi_{HL}(1 - \phi_N)}{A'}, w_{HL_{s,n}} = \frac{(1 - \phi_N)(1 - \phi_{HL}(1 - \phi_{HL}) - \phi_H\phi_{HL})}{A'}$$

Here,

$$A' = (1 - \phi_N)(2\phi_{HL}^2 + (1 - \phi_H)\phi_{HL} + 1) + \phi_L\phi_{HL}^2$$

As before, the signs of movement of $(w_H, w_L, w_N, w_{HL_w}, w_{HL_{s,s}}, w_{HL_{s,n}})$ are presented in Table 3.3 when probabilities of a shock change.

In this limiting case, a movement of the probability at $HL$ in the case with liquidity markets is more complicated than that with no liquidity markets. Especially, when the probability of a liquidity shock from $HL$ changes, the signs of probabilities $HL_w$, $HL_{s,s}$ and $HL_{s,n}$ cannot be determined. However, the overall effect on $HL$, in other words, probability of $HL_w + HL_{s,s} + HL_{s,n}$ becomes negative ($\frac{\partial(w_{HL_w} + w_{HL_{s,s}} + w_{HL_{s,n}})}{\partial \phi_{HL}} < 0$). Moreover, a change in the probability of $H$ has an influence on $HL_w$, $HL_{s,s}$ positively and $HL_{s,n}$ negatively. On the other hand, the effect on $H$, $L$ and $N$ is the same as the case with no liquidity shocks. In particular, the probability of no investments always increases.
3.4.3 Fluctuations

As in the case of no liquidity markets, for the purpose of looking at the feature of dynamics, we will consider the two simple cases: 1) three period consecutive shocks, and 2) shocks every two periods. As usual, there are no shocks before period $T - 2$ and shocks start from that period (See Table 3.4 and Figure 3.7). In the case of three consecutive shocks, both new and old firms can survive in the first shocks because liquidity provision by old firms helps new firms to meet liquidity needs. However, in the next period, while old firms can deal with their own liquidity problem, they have no capacity for lending excess liquidity to young firms because their balance sheet has deteriorated by the previous shock. In other words, a liquidity market evaporates due to no liquidity supply by investors. Therefore, if shocks are for three consecutive periods, after both old and young firms can invest, only old firms can invest followed by no investments afterward ($HL \rightarrow L \rightarrow no investment$) until shocks stop. On the other hand, if shocks occur every two periods, the economy becomes stable due to liquidity provision through the market ($HL \rightarrow HL \rightarrow HL$).

The following characteristics of dynamics can be seen from these considerations.

**Proposition 4 (The economic dynamics in the existence of liquidity markets)**

Provided that there are no liquidity shocks before,

1) When a shock happens in only one period, the economy maintains the highest level of aggregate output from both high and low productive projects.

2) When there are two consecutive shocks, only low productive projects succeed. On the other hand, when three consecutive shocks occur, there is no aggregate investment
Figure 3.7: **Consecutive and intermittent cases; liquidity markets.** In this figure high (low) productive projects are assumed to produce 2 (1) units of outputs or output.

3) **When shocks occur every two periods, liquidity markets insulate firms from the shocks and high outputs continue.**

4) **After three consecutive shocks end, high productive firms emerge and the economy goes up abruptly.**

The above consideration implies that consecutive shocks lead to a sharp decline of aggregate output followed by an abrupt increase in the aggregate economy. In contrast, the economy becomes completely stable unless the shocks are consecutive. Interpreting liquidity shocks as downside pressure gives us the implication that the unstable time in which an economy is under consecutive downturn pressure is likely to cause liquidity squeeze in liquidity markets and a substantial drop in the level of aggregate output. Then, the economy recovers sharply by high productive firms.
3.5 The role of past externality and liquidity markets in business fluctuations

Finally, so far, we have examined the effect of past externality on the probability of shocks and its implication for persistence of aggregate investments in the case both with and without liquidity markets. As a result, the effect of past aggregate output on the current probability of a liquidity shock is crucial to create persistence of macroeconomic dynamics in this model. More precisely, when the aggregate economy is favourable in the current period, the next period economy is unlikely to be hit by a shock and has a tendency to achieve a high aggregate output. On the other hand, when firms are in recession, realization of a high aggregate output is likely to be disrupted due to a high incidence of a liquidity shock.

Summing up, in the case with liquidity markets, the highest state can be easily achieved and last through interaction between past externalities and liquidity markets. Since high states lower the probability of shocks, once the economy goes into higher states, firms are rarely hit by liquidity shocks. Additionally, even if shocks hit the economy, firms are protected by liquidity markets and seldom go bankrupt. In turn, keeping higher states causes low probability of shocks. Therefore, this feedback effect of past externalities and liquidity markets has a significant role to keep the economy and economy-wide liquidity level the highest.

3.6 Conclusion

In this chapter, we attempted to analyze the role of corporate liquidity in macroeconomic dynamics and showed several macroeconomic characteristics. Starting from two empirical facts regarding firms’ productivity differences and frequency of bankruptcy, the concept of “liquidity” is captured as firms’ internal wealth produced through their
investments and "liquidity markets" as its availability for other firms which are hit by liquidity shocks. Then, the mechanism of its creation in a corporate sector was clarified by introducing a difference between managerial decisions in old and new firms. As a result of preferences for high or low productive projects, in other words, continuation value or private benefit, new firms choose high productive projects to obtain enough liquidity for surviving in the next period, while old ones select low productive projects to receive private benefit because they do not have to care about their future. Moreover, new firms are likely to go bankrupt due to scarce internal funds whereas old ones can survive because of sufficient liquidity funds in suffering the shock.

Based on the above considerations, next, we examined the macroeconomic consequence of liquidity markets. In order to consider the dynamics, we assumed that the probability of a liquidity shock relies on the past economic state. In other words, an externality from the past exists in this economy. If old firms hold excess liquidity, there is possibility that they lend to new firms facing liquidity needs. As a consequence, we can find the following features of macroeconomic dynamics.

1) The economy with both liquidity and non-liquidity markets show persistence.
2) Under liquidity markets, the economy on average achieves higher level of activity.
3) Without liquidity markets, an economy shows high volatility.
4) Consecutive liquidity shocks lead to sharp drop in an aggregate output in the case of liquidity markets.

Therefore, the interaction between liquidity markets and past externalities on the probability of shocks keeps the economy and economy-wide liquidity levels high.

Though we have so far considered the role of liquidity markets in the context of macroeconomic dynamics, we have not paid much attention to practical and policy implications regarding how to achieve this transaction. This point is closely related
to the question whether a private sector can create their own liquidity themselves. As explained in footnote 10, Holmström and Tirole (1998a) suggested that financial intermediaries play a crucial role to insure the liquidity risk between firms when liquidity shocks are idiosyncratic. This role is analogous to consumer liquidity insurance analyzed by Diamond and Dybvig (1983) and Diamond and Rajan (2003) where demand deposits supplied by banks insure investors against their liquidity demands, while firms can complete their projects without inefficient liquidation in the middle of production. Therefore, financial intermediaries and their insurance function can improve economic efficiency. Hence, a private sector can create its own liquidity and insulate firms from liquidity shocks in the case of idiosyncratic shocks. However, this sort of insurance fails when shocks are aggregate level. This is because all firms need liquidity support at the same time. In this situation, outside liquidity support like liquid assets (e.g. government bonds) and monetary injection by the central bank play a crucial role as means of liquidity holding.\footnote{This point also calls for other interesting implications of asset pricing models. If, for example, government bonds are held for the purpose of future liquidity needs, the pricing of these assets is different from that of normal asset pricing models. From this point, Holmstrom and Tirole (1998b) extended the CAPM (capital asset pricing model) to include a liquidity factor and explained the liquidity premium attached on liquid assets.}

In contrast to these models, liquidity markets in our model can work even if the shocks are aggregate since there is a difference of amounts of liquidity holdings between old and new firms due to different choice of their projects. Thus, even in aggregate shocks, the government and central bank’s support is not necessarily essential. Rather, if they provide liquidity in such a situation, there is a possibility that a moral hazard problem among private sectors arises due to excess liquidity. However, in the time when downside pressure is severe, their role is crucial to prevent evaporation in liquidity markets. In such a situation, liquidity shocks tend to damage balance sheets of lenders and they are reluctant to supply liquidity.

Finally, this model can also shed light on the controversial arguments regarding
the welfare effect of business cycles. Since Schumpeter (1934), it is often said that business cycle has a welfare improving effect on an economy in that it can wipe out low productive firms and make only high productive ones remain in the economy. However, our model suggested that such an effect cannot be expected since high productive firms do not have enough liquidity and, without outside liquidity support, they go bankrupt during the periods of downside pressures. In contrast, low productive firms still survive due to holding sufficient amount of liquidity. In this sense, there is a possibility that government fiscal and monetary policy to tackle against recession can be welfare-improving.
3.7 Appendix

After an infinite number of iterations, the regular matrix converges to a unique limiting matrix in which all row vectors are the same in every starting state. Thus, this limiting row vector can be expressed as \((w_H, w_L, w_N, w_{HL})\) in the case of no liquidity markets and the following relation is satisfied.

\[
(w_H, w_L, w_N, w_{HL}) \begin{pmatrix}
0 & \phi_H & 0 & 1 - \phi_H \\
1 - \phi_L & 0 & \phi_L & 0  \\
1 - \phi_N & 0 & \phi_N & 0  \\
0 & \phi_{HL} & 0 & 1 - \phi_{HL} \\
\end{pmatrix} = (w_H, w_L, w_N, w_{HL})
\]

Therefore,

\[
w_H + w_L + w_N + w_{HL} = 1
\]

\[
w_L(1 - \phi_L) + w_N(1 - \phi_N) = w_H
\]

\[
w_H\phi_H + w_{HL}\phi_{HL} = w_L
\]

\[
w_L\phi_L + w_N\phi_N = w_N
\]

\[
w_H(1 - \phi_H) + w_{HL}(1 - \phi_{HL}) = w_{HL}
\]

Given the probabilities of each state, solving these equations gives us the value of \((w_H, w_L, w_N, w_{HL})\).
\[ w_H = w_L = \frac{\phi_{HL}(1 - \phi_N)}{A}, w_N = \frac{\phi_L \phi_{HL}}{A}, w_{HL} = \frac{(1 - \phi_H)(1 - \phi_N)}{A} \]

Here,

\[ A = 2\phi_{HL}(1 - \phi_N) + \phi_L \phi_{HL} + (1 - \phi_N)(1 - \phi_H) \]

Similarly, in the case of liquidity markets, the row vector \((w_H, w_L, w_N, w_{HLw}, w_{HLs,s}, w_{HLs,n})\) can be solved by following relations.

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & \phi_H & 1 - \phi_H \\
1 - \phi_L & 0 & \phi_L & 0 & 0 & 0 \\
1 - \phi_N & 0 & \phi_N & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_{HL} & 1 - \phi_{HL} \\
0 & \phi_{HL} & 0 & 1 - \phi_{HL} & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_{HL} & 1 - \phi_{HL}
\end{pmatrix}
= (w_H, w_L, w_N, w_{HLw}, w_{HLs,s}, w_{HLs,n})
\]

Therefore,

\[ w_H = w_L = \frac{\phi_{HL}^2(1 - \phi_N)}{A'}, w_N = \frac{\phi_L \phi_{HL}^2}{A'}, w_{HLw} = \frac{\phi_{HL}(1 - \phi_N)(1 - \phi_H)}{A'} \]

\[
w_{HLs,s} = \frac{\phi_{HL}(1 - \phi_N)}{A'}, w_{HLs,n} = \frac{(1 - \phi_N)(1 - \phi_{HL}(1 - \phi_H) - \phi_H \phi_{HL})}{A'}
\]

Here,
\[ A' = (1 - \phi_N) (2\phi_{HL}^2 + (1 - \phi_H)\phi_{HL} + 1) + \phi_L \phi_{HL}^2 \]
Graph 1-a: the case of starting state $H$

Graph 1-b: the case of starting state $L$

Graph 1-c: the case of starting state $N$

Graph 1-d: the case of starting state $HL$
Graph 2-a: the case of starting state $H$

Graph 2-b: the case of starting state $L$

Graph 2-c: the case of starting state $N$

Graph 2-d: the case of starting state $HL_w$

Graph 2-e: the case of starting state $HL_{s,s}$

Graph 2-f: the case of starting state $HL_{s,n}$
Chapter 4

Growth Driven Financial Crises and Cycles

4.1 Introduction

Financial crises are characterised by runs on financial intermediaries, deterioration of financial positions in the corporate sector and serious real macroeconomic effects on an aggregate economy. Besides these general characteristics, many crises also share other special features. Firstly, they happen periodically. During twentieth century, the most prominent crises were Great Depression followed by the stock market crash in New York Stock Exchange in 1929. As the recent events, in 1990’s US, Japan and Northern European countries (Norway, Sweden and Finland) suffered crises at the same time. In most recent cases, the subprime shock hit the world economy and a lot of countries are still struggling with it. Secondly, it breaks out suddenly after good economic performance. US economy, after the First World War, flourished most among any other countries. Japanese economy in 1980’s was in good condition even though it experienced the asset bubbles. The early 2000’s had been good and relatively stable in the world economy after Asian crises finished. Finally, not only
developing countries but also developed countries with established financial systems are hit by financial crises. Before the subprime crises, the US financial system based on highly sophisticated capital markets was appealing to other countries. The Japanese banking system was said to be one of the driving factors of its development until 1970’s.

From the point of view of these features, we can somewhat anticipate that financial crises can be regarded as a “regime change” phenomenon where there is a sudden change in the prevailing economic environment. That is, after an expansionary phase, the economy is suddenly dropped into a long stagnation with financial turmoil. Intuitively, we can explain such a phenomenon as a consequence of economic development. During a rapid economic growth, in contrast to the preferable economic environment, various negative factors gradually accumulate overtime behind the scene. However, as an economy develops sufficiently, those factors suddenly appear by a shock such as a crash of bubbles, which is followed by long economic stagnation. In this chapter, we aim at analyzing the mechanism of financial crises in the context of risk shifting behaviour of financial intermediaries through paying much attention to 1) accumulation of household’s wealth and 2) decrease in the rate of return on capital stocks as those negative factors. Before moving to the precise model, we will make a brief review of existing literatures on financial crises.

Roughly speaking, the past literature on financial crises is divided into two views\(^1\). The first is that financial crises are the phenomenon of panic or sunspot equilibrium. Although there are classical works on it\(^3\), the seminal work based on this view is Bryant (1980), Diamond and Dybvig (1983). They analyzed bank runs induced by the deposit contract which indicates that depositors can withdraw their deposit on demand. However, because banks borrow highly liquid deposit and lend to highly

\(^1\)These two views come from Allen and Gale (2009).
\(^2\)There are also vast amount of literatures on currency and banking crises. However, they are omitted here.
\(^3\)For instance, Kindleberger (1978)
illiquid assets (corporate lending), they might not meet depositors' demand for withdrawal. As a result, they withdraw their deposit from banks when they expect others to do so, while do not if they expect others not to do. Therefore, there are multiple equilibria and which equilibrium is realized depends on the expectation of depositors and cannot be predicted *ex ante*.

The other view of financial crises is that they are caused by poor growth prospects coming from a recession. Especially, this stance is closely related to the condition of asset markets, e.g. bubbles and burst. When asset bubbles grow and collapse, financial intermediaries involved in these markets face insolvency and the severe macro-economic consequence is followed. Allen and Gale (2000) analyzed asset bubbles and banking crises on this idea. They showed that bank risk shifting behaviour due to deposit contract and its limited liability constraint (asset substitution problem (Jensen and Meckling (1976))) causes asset bubbles and burst, which leads to the total collapse of banking sector.

As for the relationship between financial intermediaries and asset markets, the recent financial crises have characteristics of liquidity crises in financial markets. This is because business of financial intermediaries has been closely associated with trading in financial markets in recent years. It means that traditional corporate lending business is getting smaller parts on their balance sheets and trading in markets is gaining higher weights. Then, the sudden liquidity needs by financial intermediaries can cause liquidity shortage in a whole market and, as a result, this market becomes evaporated or highly volatile. From the point of this view, Allen and Gale (1994), Shin (2010), precisely examined the fragility of securities markets. Allen and Gale (1994) claimed the role of cash-in-the-market pricing in which asset prices are not determined by the net present value but the amount of liquid assets (for instance, cash) available in a market when liquidity is scarce. Shin (2010) made clear the domino effects leading a small liquidity shock to the overall liquidity evaporation in
a market.

Generally speaking, the recent models of financial crises capture the fact that it is closely associated with capital markets and try to make clear the mechanism of asset bubbles and liquidity problem of financial intermediaries. However, these theories do not explain why asset bubbles occur or, more precisely, what the underlying economic environments of the onset of bubbles are. That is, the existing models on financial crises have searched for the characteristics of their mechanism leading to the devastating effects on markets and a real economy. However, they do not necessarily show the underlying economic conditions causing such phenomenon which they have analyzed. As mentioned before, bubbles and the excessive risk taking behaviour of economic agents are preceded by a good economic performance under well-functioning of financial systems. Therefore, we need to clarify the “missing” piece between a good economic performance and the occurrence of crises. Moreover, crises break out even in developed countries which have relatively well-established financial systems. It implies that financial crises are not only the result of (mis-) behaviour of investors and depositors under poorly organized financial systems. But they might also be an inevitable economic phenomenon as a result of economic development. Thus, in the argument that follows, we will pursue the cause of financial crises from the perspective of economic growth and its consequence and try to fill in this missing piece mentioned above.

As to the relevant literatures, our study is highly motivated by three works. The first is Tirole (1985) which analyzed the economic condition of originating asset bubbles in the framework of overlapping generations model. In his study, assets bubbles are created in the phase of over-accumulation of capital stocks (dynamic inefficiency) and have a role to resolve this problem. That is, from the point of view of investors, investment for bubble assets is more profitable than that for real investments which generate a low rate of return and investing in bubble assets becomes socially effi-
cient due to correcting dynamic inefficiency. As a result of increase in investment for bubbles, capital stocks decrease and a rate of return on real investment becomes higher\(^4\).

The second and third work, more directly related to our study, are a regime change studied by Azariadis and Smith (1998) and Mattesini (2005) which are also based on the OLG model\(^5\). Azariadis and Smith (1998) pays attention to the effect of the accumulation of capital stock on incentive of depositors under asymmetric information regarding firms’ types. As a result of lowering the rate of return on capital stock, incentive compatibility constraint to reveal the types of firms becomes binding. As a consequence, depositors prefer not to deposit their money and aggregate investment discontinuously decreases. Moreover, under the several assumptions, this economy shows cyclical behaviour between high and low output. Therefore, this study also focuses on the macroeconomic effect of accumulation of capital stock by introducing informational asymmetry.

Mattesini (2005) is another paper that considers a regime change. He included moral hazard by firms and costly monitoring activities by banks into OLG framework. In his model firms always engage in low productive projects unless banks monitor them. When capital stock level is low and a rate of return on corporate investment is high, firms are monitored by banks and high productive projects are realized. On the other hand, when the economy is developed and the rate of return becomes low, banks cannot earn profits for monitoring activities and firms adapt to low productive projects. At this point, the output and capital stock decrease abruptly and this economy shows cycles between high and low aggregate output.

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\(^4\)Abel et al (1989) is a comprehensive study on this topic about US economy. However, they rejects the possibility of dynamic inefficiency. Farhi and Tirole (2010) also shows the occurrence of bubbles and their effects on corporate investments.

\(^5\)OLG model is frequently used for the analysis of financial intermediaries in the context of economic growth. These researches are mostly based on costly state verification (CSV) approach (Williamson (1986, 1987)). For instance, cycle in monetary economy (Boyd and Smith (1998)), poverty trap in developing countries (Boyd and Smith (1997)) and rate of return difference between countries (Boyd and Smith (1992)).
Finally, our analysis is partly in similar spirit with Matsuyama (2004) where financial constraints give birth to the cyclical movement of aggregate investment between “Good” and “Bad” projects. Here, Good projects have characteristics of low profitability and high demand spillover bringing about high net worth for firms. On the other hand, Bad projects are high productive though they exhibit low spillover for the economy. At first, firms engage in Good projects since they are financially constrained and cannot launch Bad projects with a high return. However, as they do business in Good projects, they can accumulate net worth because these projects generate high spillover in an economy. After several periods, firms who obtain enough net worth and are financially less constrained start to invest in Bad projects with a high return. Although Bad projects are high productive, they produce low spillover in an economy and, as a result, firms are gradually financially constrained. Therefore, this economy shows oscillation between Good and Bad projects.

The previous studies mentioned so far have mainly taken into consideration firms’ financial positions or the monitoring activities of banks. However, actual financial crises are characterized as bankruptcy of a lot of banks at the same time due to taking excessive risk (for instance, investment in bubbles). Different from papers above, our study is based on risk shifting behaviour by financial intermediaries (banks). One of the characteristics of financial crises is that financial intermediaries take excessive risk before crises brake out. In order to model this aspect, two major elements are included in our model; household assets which increase as a result of economic growth and the effect of a rate of return on corporate investments on the option value of risk shifting by financial intermediaries. Our model is mainly based on Allen and Gale (2000) which analyzes asset bubbles through risk shifting behaviour of banks\(^6\) and considers risk neutral banks that take deposits from households and seek to maximize the benefit of stock holders by investing in safe and risky assets. Although deposit is

\(^6\) Acharya (2000) also extended their framework to analyze systemic risk in a banking sector.
a simple debt contract and they are protected by limited liability, the manager of a bank incurs non-pecuniary bankruptcy cost in the case of insolvency. When economic development is in the early stage, the amount of individual’s assets (deposits) is scarce and the rate of return on corporate investment is higher than that of risky assets. In this circumstance, investing in a corporate sector is more profitable than that of risky assets. This is because the option value from investing in risky assets is low compared to the expected bankruptcy cost incurred by banks. Thus, banks invest all of their available funds in a corporate sector, which leads to further economic growth. However, as a result of economic development, assets (deposits) held by households increase and the rate of return on corporate investment is decreasing, which makes risky assets more profitable. This is because the option values of risk shifting are increasing relative to expected bankruptcy costs and gives banks incentive to invest in risky assets. Therefore, banks are willing to hold risky assets.

Holding risky assets leads some of the banks to be insolvent and go bankrupt. Consequently, the number of banks decreases and financial function is deteriorated, which results in credit crunch on firms’ investments. Moreover, there is a possibility that this economy shows periodical movement between high and low capital stock.

The structure of this chapter is as follows. In section 2, the basic setup of the model is explained. Then, section 3 is followed by the equilibrium conditions of this economy and the trajectory of this model is described. In section 4, the analysis is dedicated to the accumulation of capital stock in the low capitalized economy. After that, in section 5, we show the possibility of financial crises and endogenous economic cycles in the high capitalized economy. Finally, concluding remarks are made in

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7 Needles to say, this is not the sole way to approach risk shifting behaviour of financial intermediaries. It is often claimed that deregulation since 1980’s ensued fiercely competition in banking sector also gives banks an incentive to take excessive risk (Dewatripont et al (2010)).

8 This model is not supposed to be hit by any shocks. When the initial wealth is given, the economy depicts cyclical behaviour endogenously. In this sense, this model is a kind of endogenous cycle like Benhabib and Nishimura (1985). As to a comprehensive guide on endogenous cycle, see Azariadis (1993).
4.2 The basic model

Here, we will think about an infinitely lasting two-period overlapping generations model. There are risk neutral agents with unit mass and they are divided into two types of agents; lenders (depositors, households) and borrowers (firms). The proportion of each type is fixed; $\alpha$ and $1 - \alpha$, respectively. Both of them do not consume in the first period and seek to maximize the consumption in the second period. In addition, there are financial intermediaries (banks) with unit mass who seek to maximize the second period profit as well. Banks are everlasting entities unless they go bankrupt. However, bank managers stay at their position just in two periods and, after the second period, they retire and new managers take on their position. The new manager collects new deposits and starts business, again. In this regard, managers are assumed to maximize their own utility and not to take into account the utility of their successors. Therefore, structurally, OLG model can still be applied to this framework. The point of this setting is that bankruptcies of banks have an adverse effect on the future economy, while the simplicity of OLG framework is preserved.

There are several kinds of markets in this economy; capital goods markets, labour markets, loan markets for safe and risky assets and deposit markets.

While each lender has a unit labour force, they do not have a skill for running productive investment. Thus, they supply their labour force to firms which are run by the previous generation inelastically and, in return, obtain wage $w$ in the middle of the first period. Furthermore, since they do not also have technique for allocating their wage in financial assets properly, their wage in the first period can be deposited in banks and this generates deposit rate $r^D$ in the second period. As a timing of deposit, they deposit their money after realizing whether banks become insolvent or
not as a result of lending activities in the previous period. Thus, banks do not go bankrupt just after new deposits come in. Instead of deposit, households can also hoard their wages in hand. However, in this case, their wages are depreciated with some depreciation rate (for instance, $\varepsilon$) and workers prefer deposit to hoarding. As we will explain later, when the economy is hit by crises, some depositors cannot find banks to deposit their wages and they have no option besides hoarding.

Borrowers have a skill for launching investment and they can produce goods. However, they do not have resources (labour force and funds) to accomplish the production activities. In the first period, they borrow a necessary amount of funds from banks and invest them in capital stock. In the second period, after employing labour force, they conduct non-stochastic, constant returns to scale projects; $Y = F(K^S, L)$. $F$ is a increasing, concave production function and $K^S$, $L$ is capital stock and an amount of labour force, respectively. $K^S_{t+1}$ is capital stock formed in the period $t$ and used in production in the period $t + 1$. If $k^S$ is denoted as capital-labour ratio ($k^S \equiv \frac{K^S}{L}$), then the production function becomes $f(k^S)$. Here, $f(k^S)$ has usual neo-classical assumption $f(0) = 0$, $f'(k^S) > 0$, $f''(k^S) < 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. A rate of return on this project is denoted as $r^S$. Since firms have no wealth at the beginning of the period, they must borrow necessary funds from banks to start projects which are of variable sizes ($k^S$). Because the projects are assumed to completely depreciate within a period, $k^S$ also represents capital stock in this economy at each period. All lending contracts are a form of simple debt contract.

On the other hand, the supply of a risky project is assumed to be determined by investments of banks and it has characteristics that it generates a stochastic rate of return $r^R$ with a density function $h(r^R)$ and the corresponding cumulative function is $H(r^R)$. In addition, the expected value of the return is $\bar{r}_R$ and its support is $[0, r^R_{MAX}]$. The crucial feature of this project is that it has not only stochastic features. But also, it does not generate any productive capital stock and contribute to capital formation.
of this economy. Moreover, the return on these assets are perfectly correlated and short sale is not allowed in this economy \((k^i \geq 0, i = S, R)\). An example of this kind of asset is trading activities of volatile financial assets (e.g. mortgage related financial assets and other bubbly assets). Moreover, households are assumed not to have access to risky assets.

Although banks are protected by limited liability, bank managers are assumed to bear non-pecuniary bankruptcy cost \(Z\) when banks become insolvent. This cost is interpreted as loss of reputation of a manager due to failure of his job\(^9\).

Finally, there is a continuum of symmetric banks in both a loan and a deposit market and they behave as a price taker. However, because the rate of return on risky assets are perfectly correlated, those who invested in risky assets go bankrupt in the case of realization of the low rate of return on risky assets. This state is regarded as financial crises\(^{10}\). The time flow of this economy is described in Figure 4.1.

### 4.3 Equilibrium conditions

In this section, we will describe equilibrium conditions of all markets of this economy.

#### 4.3.1 Capital goods market

The capital goods market is competitive and firms behave as a price taker. Therefore,

\(^9\)Z can be interpreted as future (expected) charter value for a manager when banks survive. In this regard, Brito and Kose (2001) examined the effect of growth opportunity held by banks on their risk taking behaviour. They showed that if decision making is concerned with just one period, they take excessive risk, which is usually claimed by the existing literatures on risk shifting phenomenon. However, when continuation value (growth opportunity) is taken into consideration, banks try to avoid taking too much risk which causes high frequent bankruptcy. Our model builds the mechanism of the change in this charter value and analyzes the effect on risk taking incentive of banks over time.

\(^{10}\)The reason why banks attempt to invest in financial assets whose returns are correlated is still open to question. Acharya (2000) and Acharya and Yorulmazer (2006) considered this point and they showed that there is possibility that banks prefer correlated assets to those whose returns are not correlated under limited liability of deposit contract (they called this phenomenon “systemic risk shifting”).

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Figure 4.1: The flow of the economy
\[ f'(k^S) = r^S \]  \hspace{1cm} (4.1)

4.3.2 Labour market

Each lender who has a unit of labour force supplies it inelastically and the wage is determined by a usual efficiency condition.

\[ w = f(k^S) - k^S f'(k^S) \equiv W(k^S) \]  \hspace{1cm} (4.2)

\[ W'(k^S) > 0 \] is satisfied from the concavity of the production function. In addition, we assume the concavity of optimal wage function (4.2); \[ W''(k^S) < 0 \].

4.3.3 Loan market

The equilibrium condition of the market for loan to safe and risky assets is expressed as follows.

\[ \alpha D = \alpha W = (1 - \alpha)k^S + k^R \]  \hspace{1cm} (4.3)

Again, \( \alpha \) and \( 1 - \alpha \) is a fraction of depositors and borrowers in this economy, respectively.

4.3.4 Deposit market

Depositors do not have a skill for investment and, therefore, deposit their wages \( (W(k^S)) \) in banks, earning the deposit rate \( r^D \) in the second period unless banks go bankrupt. \( D_t \) is denoted as the amount of deposits. We assume that the deposit rate is determined in a way that the demand for deposit is equated to the supply of them.
Furthermore, \( r^D \) cannot be conditioned by the amount of investment by banks and its risk. Moreover, banks are a price taker as to the deposit rate since there are a lot of banks in this economy. In the later sections, we will show that some of the banks become insolvent and go bankrupt with some probability.

### 4.3.5 The relationship between the interest rate on safe assets and deposit

At first, the critical value of the rate of return on risky assets \( r_t^R \) below which banks are insolvent is expressed as follows.

\[
\begin{align*}
  r_t^R k_t^R + r_t^S (1 - \alpha) k_t^S &= r_t^D \{ k_t^R + (1 - \alpha) k_t^S \} \\
  \iff \\
  r_t^R &= r_t^D + (1 - \alpha) (r_t^D - r_t^S) \frac{k_t^S}{k_t^R} = \tilde{r}_t^R \tag{4.4}
\end{align*}
\]

Therefore, a profit of banks can be expressed as follows.

\[
\Pi(r_t^R, r_t^S, r_t^D, k_t^R, k_t^S) = \begin{array}{c}
\int_{\tilde{r}_t^R}^{r_t^R} \\
\int_{\tilde{r}_t^R}^{r_t^R} \\
\int_{\tilde{r}_t^R}^{r_t^R}
\end{array} \{ r_t^R k_t^R + r_t^S (1 - \alpha) k_t^S - r_t^D \{ k_t^R + (1 - \alpha) k_t^S \} \} h(r_t^R) dr_R \\
- \int_{\tilde{r}_t^R}^{r_t^R} Z h(r_t^R) dr_R
\]

Rearranging this equation leads to
\[ \Pi(r_t^R, r_t^S, r_t^D, k_t^R, k_t^S) = \int_{r_t^R}^{r_t^R_{\text{MAX}}} \{ (r_t^R - r_t^D)k_t^R + (1 - \alpha)(r_t^S - r_t^D)k_t^S \} h(r_t^R)dr_t^R \]

\[ - \int_{0}^{r_t^R} Z h(r_t^R)dr_t^R \]  

(4.5)

However, in equilibrium \( r_t^S = r_t^D \), is satisfied\(^{11}\). If \( r_t^S < r_t^D \), \( r_t^R \) increases in \( k_t^S \) and, as a result, \( \Pi(r_t^R, r_t^S, r_t^D, k_t^R, k_t^S) \) are decreasing in \( k_t^S \) from the function (4.5). That is, any investment in safe assets leads to loss. Thus, \( k_t^S = 0 \). However, from the assumption of \( f(k^S) \), \( f'(0) = \infty \) and it contradicts with \( r_t^S < r_t^D \). Conversely, if \( r_t^S > r_t^D \), \( r_t^R \) decreases and \( \Pi(r_t^R, r_t^S, r_t^D, k_t^R, k_t^S) \) can increase to infinity by demanding infinite amount of \( D \). That is, if banks demand an infinite amount of deposit and put it into \( k_t^S \), they can earn an infinite profit because safe assets generate constant returns \( r_t^S - r_t^D \) (\( > 0 \)). In this situation, given the fixed supply of deposit at each period, we cannot find out the market clearing deposit rate at which the demand for deposit is equated to the supply of them. This is inconsistent with a market clearing. Therefore, in equilibrium, \( r_t^S = r_t^D \) should be satisfied.

Taking into consideration the above discussion, the profit function (4.5) is boiled down as follows. We set \( r_t^S = r_t^D = r_t^R = r_t \).

\[ \Pi(r_t^R, r_t, k_t^R) = \int_{r_t}^{r_t^R_{\text{MAX}}} (r_t^R - r_t)k_t^R h(r_t^R)dr_t^R - \int_{0}^{r_t} Z h(r_t^R)dr_t^R \]  

(4.6)

Banks seek to determine the amount of \( k_t^S \) and \( k_t^R \) in order to maximize their profit. From function (4.6), we can observe that an individual bank will invest all of their deposits in either safe or risky assets since this function is increasing with risky

\(^{11}\)This discussion about the equalization of the rate of return on between safe and deposit is mainly based on Allen and Gale (2000).
investment $k_t^R$. Recall that each individual bank is a price taker and they regard $r_t$ as a fixed value. Additionally, this asset allocation problem depends on the stage of the economic development and the rate of return on safe assets.

### 4.3.6 Risk shifting incentive of banks

The analysis of the equilibrium dynamics of this economy is divided into two regions with regard to the accumulation of capital stock; in the case of high and low level of capital stock. This is because, as we will show later, the trajectory of this economy is different between those two regions. Furthermore, these two cases are corresponded to the early and later stage of the economic development, respectively. To begin with, we will define the deviation value of each bank to risky assets. It is meant to be the option value for each bank to take risk when the others invest in safe assets. From the function (4.6) and the equilibrium condition on capital market (4.1), the deviation function of an individual bank when the others invest in safe assets is expressed as the following.

$$
\Pi(W(k_{t-1}^S), k_t^S) = \alpha W(k_{t-1}^S) \int_{f'(k_t^S)}^{r_{\text{MAX}}^R} (r_t^R - f'(k_t^S))h(r_t^R)dr^R - \int_{0}^{f'(k_t^S)} Z h(r_t^R)dr^R \tag{4.7}
$$

Note, again, that the strategy of an each individual bank is whether to invest all deposit ($\alpha W(k_{t-1}^S)$) into risky or safe assets. Thus, this function generates the value of investment in risky assets after the adjustment of the rate of return on capital stock.

For the purpose of the analysis of the trajectory of this economy, we will define the threshold capital stock and the amount of deposit at which the function (4.7) becomes zero. This threshold level of capital stock $k_S$ and $W(k_S^S)$ at which the profitability between the safe and risky assets is equivalent is defined as the following.
\[
\Pi(W(\hat{k}^S), \overline{k}^S) = \alpha W(\hat{k}^S) \int_{f'(\overline{k}^S)}^{r^R_{t+1} \text{MAX}} (r^R_t - f'(\overline{k}^S))h(r^R_t)dr^R - \int_0^{f'(\overline{k}^S)} Z h(r^R_t)dr^R = 0 \quad (4.8)
\]

In words, this threshold is interpreted that banks do not have an incentive to deviate to risky investment until the economy reaches this point. The profit from safe assets is zero since \(r^S_t = r^D_t\) is satisfied in equilibrium. As explained precisely later, \(\hat{k}^S\) is the capital stock a period before \(\overline{k}^S\) through capital accumulation \((k^S_{t+1} = AW(k^S_t))\). That is, \(\hat{k}^S\) is defined as \(\overline{k}^S = AW(\hat{k}^S)\) and \(\hat{k}^S\) is the amount of capital stock just before reaching threshold \(\overline{k}^S\) in the next period; \(\hat{k}^S = W^{-1}(\frac{Z}{A})\). \(W^{-1}(\cdot)\) is an inverse function of the wage function \(W(\cdot)\) whose existence is guaranteed by the assumption that the wage function is monotonic.

The function (4.7) has the key role to determine the division between low and high capitalized economy. For instance, this function takes the shape in the Figure 4.2.

In this figure, banks invest in safe assets on the left side of the capital stock level \(\overline{k}^S\), while they have an incentive to invest in risky assets on the right side of it. As explained in Appendix, since the function (4.7) is monotonically increasing with regard to \(k^S_t, \overline{k}^S\) at which \(\Pi = 0\) is uniquely determined.

Next, we will turn to analyze the trajectory of this economy. At first, we assume the following inequality about \(Z\).

**Assumption 1:**

At the initial date, \(Z\) satisfies the following condition at \(k^R_t\). \(W_0\) is the initial value of assets held by households and deposited in banks.
Assumption 1 says that $Z$ is large enough at the initial date and the deviation value of risky investment is very low.

Because firms who engage in safe assets can contribute to capital formation in this economy, the equilibrium dynamics of this economy is characterized by equations (4.1), (4.2) and (4.7).

In the next section, we will analyze the trajectory of this economy step by step. Then, the low and high capitalized economy is defined as before and after $\tilde{k}^S$. That is, the low capitalized economy is defined as $k_t^S \leq \tilde{k}^S$ and high one is $k_t^S > \tilde{k}^S$. Before that, we will set an example to make clear the trajectory of this economy in the later sections.
4.3.7 An example of this economy

Here, we assume the following Cobb-Douglas, constant returns to scale production function.

\[ Y = \theta[K^S]^\beta L^{1-\beta} \quad (4.9) \]

\( \theta \) is represented as a technological parameter. Moreover, the probability distribution of the rate of return on risky assets is supposed to be uniform over support \([0, R]\). From the production function, \( y = f(k^S) = \theta[k^S]^\beta \) and this function has the increasing and concave property.

\[ f'(k^S) = \theta \beta [k^S]^{\beta-1} > 0 \]

\[ f''(k^S) = \theta \beta (\beta - 1) [k^S]^{\beta-2} < 0 \]

Equilibrium conditions

The relevant equilibrium conditions become as follows.

Capital goods market

\[ r^S = f'(k^S) = \theta \beta [k^S]^{\beta-1} \quad (4.10) \]

Labour market

\[ w = W(k^S) = \theta[k^S]^\beta - [k^S] \theta \beta [k^S]^{\beta-1} = \theta (1 - \beta)[k^S]^\beta \quad (4.11) \]

The wage function also satisfies the increasing and concave property.

\[ w' = \theta (1 - \beta) \beta [k^S]^{\beta-1} > 0 \]
\( w'' = \theta(1 - \beta)\beta(\beta - 1)[k^S]^{\beta-2} < 0 \)

**Loan market**

\( \alpha w = (1 - \alpha)k^S + k^R \) \hspace{1cm} (4.12)

**Bank’s profit**

Since the rate of return on risky assets is a uniform distribution, the banks’ profit is described as the following.

\[
\Pi = \int_{r_l}^R (r^R - r_t) \frac{k^R}{R} \, dr^R - \int_0^{r_t} \frac{Z}{R} \, dR
\]

(4.13)

The first term is calculated as \( \frac{k^R}{R} (\frac{1}{2} R^2 - r_t R + \frac{1}{2} r_t^2) = \frac{k^R}{2R} (R - r_t)^2 \), while the second term is \( \frac{Z}{R} r_t \). Summing up,

\[
\Pi = \frac{k^R}{2R} (R - r_t)^2 - \frac{Z}{R} r_t
\]

After substituting \( r^S_t = r_t = f'(k^S_t) \), we can obtain the function of an individual bank’s deviation value.

\[
\Pi(W(k^S_{t-1}), k^S_t) = \frac{1}{R} \left\{ \frac{\alpha W(k^S_t)}{2} (R - \theta \beta [k^S_t]^{\beta-1})^2 - Z \theta \beta [k^S_t]^{\beta-1} \right\}
\]

(4.14)

**4.4 A low capitalized economy \((k^S_t \leq \tilde{k}^S)\)**

Firstly, we will investigate the case of low level of capital stock. This case is characterized as that the accumulation of capital stock is low and an individual wealth deposited in banks is scarce.
4.4.1 Movement of a low capitalized economy

When an initial value of individual wealth $W_0$ is given and its value is sufficiently low, the rate of return on safe assets $r_1(=r_1^S = r_1^D)$ is very high since $r_1^S = f'(k_1^S)$. This situation affects the decision making of banks for investment of risky assets. That is, in this case, it is possible that the equation (4.7) becomes negative if banks invest in risky assets. More precisely, consider the situation that all banks except one invest in safe assets and one bank are thinking about which assets it should invest in. Even if this bank would invest the whole amount of deposit available $W_0$ in risky assets $k_1^R$, the deviation value of risky assets would become negative because the rate of return on safe assets $r_1^S(= f'(k_1^S) = f'(AW_0))$ under the investment in safe assets by the others is high enough. This is because the option value of investing in risky assets coming from limited liability of a deposit contract is low in comparison with the expected non-pecuniary bankruptcy costs and they have no incentive to invest in risky assets. This negative profitability of risky assets under the low level of individual assets is guaranteed by Assumption 1.

As a consequence, banks prefer to invest all their deposits from households in safe assets and aggregate capital stock at the first period is as follows.

$$k_1^S = AW_0$$

The same argument that the deposits in banks are all invested in safe assets applies to the further periods until $k_t^S$ reaches $\tilde{k}_t^S$ and, generally, the movement of this economy from the equilibrium condition on the loan market (4.3) becomes the same as that of capital stock of the usual neoclassical growth model.

$$k_{t+1}^S = AW(k_t^S) \quad (4.15)$$

Here, $A \equiv \frac{\alpha}{1-\alpha}$. Furthermore, a steady state $k^{S*}$ of this economy is defined as
Again, $W(k_t^S)$ is an increasing and concave function. When we assume $W'(0) > 1$, the trajectory of this economy is, for example, depicted in Figure 4.3 and the flow of capital accumulation is briefly shown in Figure 4.4. The assumption $W'(0) > 1$ guarantees the uniqueness of the steady state, if any.

There are two cases to be considered in this region; 1) $k^{S*} \leq \overline{k}^S$ and 2) $k^{S*} > \overline{k}^S$. In the first case, the economy converges to the unique steady state $k^{S*}$ and there is no cyclical behaviour in this economy. Figure 4.3 shows this case. The second case is that the economy changes its pass at $\overline{k}^S$ and does not converge to the steady state. Instead, it shifts from the region $k_t^S \leq \overline{k}^S$ to $k_t^S > \overline{k}^S$ (not shown in the figure) and the analysis made in the next section is applied. Since banks invest only in safe assets and, in equilibrium, $r_t^S = r_t^D$, they earn zero profit from this investment. Finally, banks keep solvent and there are no cycles and financial crises in this economy.
Figure 4.4: Capital accumulation in the case of a low capitalized economy ($k_t^S \leq \tilde{k}^S$)

**Proposition 5 (The trajectory of capital stock in a low capitalized economy)**

In the case of a low capitalized economy $k_t^S \leq \tilde{k}^S$, the trajectory of capital stock in an economy is same as the neoclassical economic growth model and, when $k^{S*} \leq \tilde{k}^S$, this economy converges to the unique steady state.

### 4.4.2 An example of a low capitalized economy

Here, we will show a numerical example of this economy by making use of the example presented in the last section. In order to calculate the function (4.14), we set the value of the parameters as follows. $\alpha = 0.7$, $\beta = 0.5$, $\theta = 1.0$, $\bar{R} = 8.0$, $Z = 40$. The initial capital stock is supposed to be 0.05. The question is whether banks invest in risky assets before reaching the steady state once the amount of deposit $W_{t-1}$ was determined. Note that the term $\bar{R} - r_t$ must be positive because the upper bound of the rate of return on risky assets is defined as $\bar{R}$.

Under these parameter values, the capital accumulation takes the shape in the Figure 4.5 if banks invest all of their deposits in safe assets. This figure is calculated
Figure 4.5: The accumulation of capital stock when $\theta = 1.0$

by means of the function (4.15) specifying a pass of the capital accumulation: $k_{t+1}^S = AW(k_t^S) = A\theta(1 - \beta)[k_t^S]^\beta$.

From this figure, the steady state is around 1.3611 and the total amount of deposits available ($\alpha D$) corresponding to this steady state is 0.4083. Next, we will investigate the possibility of investment in risky assets by banks in response to the capital accumulation. Given the deviation value (4.14), we will check the possibility of $\Pi > 0$ when capital stock $k_t^S$ (and deposit) increases as a result of economic growth. The result is in Figure 4.6. This figure is calculated from the function (4.14) by varying the value of deposit ($\alpha W_{t-1}$) and capital stock ($k_t^S$) as the economy grows.

At any values of deviation from safe to risky investment, the profit from risky investment does not become above zero. That is, deviating to investment in risky
assets is not profitable for banks and they invest all of their deposits in safe assets. That is, this economy converges to the steady state without financial crises.

4.5 **A high capitalized economy** \((k^S_t > \bar{k}^S)\)

Next, we will move on to the case of an economy with high level of capital stock. The feature of this stage is that, as a result of economic growth, an individual household holds a great amount of wealth and the rate of return on safe assets \(r_t (= r^S_t = r^D_t)\) is very low. In this case, the value of the equation (4.7) increases and there is a possibility that it becomes positive. Because banks earn at most zero profit from safe assets, they begin to invest their deposits in risky assets. To begin with, we will describe the flow of the economy after crises.
4.5.1 After the failure of banks

As we mentioned before, since the return on risky assets are perfectly correlated, banks who invested in risky assets go bankrupt with probability $\delta_t$ (we call this phenomenon “financial crises”). Here, $\delta_{t+1} = \int_0^{r_{t+1}} h(r_{t+1}) dr$. Usually, it takes a while to restore financial function after financial crises. For instance, failed banks should be recapitalized by the government to start their business again. Or, new banks need to acquire the failed banks to resume the management after cutting their loss. Generally, this process might take a time to negotiate the condition of recapitalization and acquisition among the relevant parties (financial intermediaries and the government) in practice. We will simplify this process in such a way that new banks are born, so that on a move the measure of banks is equal to one. Thus, the reduction of capital stock by the crises is limited to one period.

After the crises, there are a few banks in this economy. Here, we will describe the negative effect of crises such that depositors who live in the area where banks became insolvent have no opportunity to deposit their wages. In other words, they have an only option to hoard their wages in hand. Hence, deposits available for the formation of capital stock in this economy reduce as a result of inaccessibility to banks by depositors.

4.5.2 Movement of a high capitalized economy

Suppose the capital stock of the previous period is $k_t^S > \tilde{k}_S$. The wage of households is, then, $W(k_t^S)$ and deposited in banks. The total amount of deposit ($\alpha W(k_t^S)$) is entirely allocated into either safe or risky assets. However, in this stage, banks have an incentive to make risky investment because the deviation value of risky investment becomes positive. From the definition of $\tilde{k}_S$, $k_{t+1}^S = AW(k_t^S) > AW(\tilde{k}_S) \equiv \overline{k}_S$ and the zero deviation value condition between safe and risky assets expressed in (4.8) is
violated. That is, once the function (4.7) is positive, banks would try to invest all deposit in risky assets since their profit monotonically increases in $k_{t+1}^R$. Then, the safe interest rate would soar up infinitely since the amount of capital stock becomes zero, which induces (4.7) to negative again. If banks are rational, some of them who anticipate this situation would be willing to keep investing in safe assets and, as a consequence, the safe interest rate would be pushed down. Since they have incentives to deviate from investing in safe assets as long as (4.7) is positive, this adjustment process lasts till some of the banks (for instance, the proportion $\mu_{t+1}$) invest in safe assets $k_{t+1}^S$ and the others invest in risky assets to restore the zero deviation value condition between safe and risky assets. Thus, in equilibrium, aggregate investment in safe assets $\alpha W(k_t^S)$ is at the level $\mu_{t+1} \alpha W(k_t^S)$, while $(1-\mu_{t+1}) \alpha W(k_t^S)$ is invested in risky assets $k_{t+1}^R$ so that $\Pi = 0$ is satisfied. Therefore, in the case of $k_t^S > \bar{k}_S$, the capital stock of this economy becomes the following.

$$k_{t+1}^S = AW(k_t^S) - Bk_{t+1}^R$$

(4.17)

This function comes from the equilibrium condition on loan market (4.3). Here, we set $A \equiv \frac{\alpha}{1-\alpha}$, $B \equiv \frac{1}{1-\alpha}$. Next, we will investigate the movement of capital stock in the region $k_t^S > \bar{k}_S$. Consider the deposit and corresponding capital stock satisfying the equation (4.8). Once the economy passes the threshold capital stock $\bar{k}_S$, the amount of deposit increases and is more than $W(\bar{k}_S)$ (denote $W_t^f(> W(\bar{k}_S))$). To ensure the zero deviation value condition, $\bar{k}_S > k_{t+1}^{St}$ should be satisfied. $k_{t+1}^{St}$ is the capital stock at which banks zero deviation value condition is met at the amount of deposit $W_t^f$. Therefore, the level of capital stock is lower than $\bar{k}_S$ and the pass of capital stock becomes a decreasing function. The details are presented in Appendix.

That is, since the individual assets (deposits) increase enough as a result of economic growth, bank’s deviation value also increases. To offset this effect and to achieve the equilibrium condition $\Pi = 0$, the capital stock must decrease to go up
Figure 4.7: The trajectory of a high capitalized economy ($k_t^S > \tilde{k}^S$)

the rate of return on capital stock sufficiently. This situation is, for instance, de-
picted in Figure 4.7 and the flow of capital accumulation is shown in Figure 4.8. In
consequence, the typical trajectory of this economy is shown in Figure 4.9.

In words, an economy which grew steadily from the initial wealth of households
$W_0$ reaches above $\tilde{k}^S$ and the capital stock declines because banks start to invest in
risky assets. After the reduction of capital stock, the rate of return on them restores.
If the effect of financial crises is ignored at the moment, from Figure 4.9, it is possible
to emerge oscillations and the new steady state $\tilde{k}^S$ (denoted as $S$ in Figure 4.9) which
is different from the neoclassical one $k^{S*}$. In Figure 4.9, this capital stock is depicted
at the point where the 45 degree line crosses the curve of the capital accumulation.
Under the amount of deposit ($\alpha W(\tilde{k}^S)$) corresponding to $\tilde{k}^S$, the capital stock in the
next period also becomes $\tilde{k}^S$ to ensure $\Pi(W(\tilde{k}^S), \tilde{k}^S) = 0$. Therefore, this economy
does not converge to the neoclassical steady state $k^{S*}$. Moreover, this new steady
Figure 4.8: Capital accumulation in the case of a high capitalized economy \((k^s_t > \tilde{k}^s)\)

Figure 4.9: An example of the trajectory of a high capitalized economy \((k^s_t > \tilde{k}^s)\)
state is lower than the neoclassical one \((k^{S*} > \hat{k}^S)\).

**Proposition 6 (The trajectory of capital stock in a high capitalized economy)**

In a high capitalized economy \(k^S_t > \hat{k}^S\), the pass of capital stock changes at \(\hat{k}^S\) and drops below \(\hat{k}^S\) due to the risk shifting behaviour of banks. As a result, this economy does not converge to the neoclassical steady state \(k^{S*}\) and the new steady state \(\hat{k}^S\) which is lower than the neoclassical one appears.

However, the economy does not necessarily converge to the new steady state. Instead, there is a possibility that this economy shows endogenous cycles between low and high capital stock. This depends highly on the level of the initial capital stock and the slope of the curve of the capital accumulation. As we showed in the appendix, the capital accumulation curve becomes downward sloping and, more precisely, a convex function; \(\frac{\partial k^S_t}{\partial W} < 0\) and \(\frac{\partial^2 k^S_t}{\partial W^2} > 0\). Moreover, depending on the slope of \(\frac{\partial k^S_t}{\partial W}\), cycles become as follows in the neighbourhood of the new steady state.

1) If \(\frac{\partial k^S_t}{\partial W} \in (-1, 0)\) at \(\hat{k}^S\), the trajectory is a damped oscillation and the pass gradually converges to \(\hat{k}^S\) with a cyclical behaviour.

2) If \(\frac{\partial k^S_t}{\partial W} < -1\) at \(\hat{k}^S\), the trajectory is an explosive oscillation and the capital accumulation pass diverges from \(\hat{k}^S\).

However, in the case of 2), the pass does not diverge infinitely because this economy will get into the stage of a low capitalized economy \((k^S_t \leq \hat{k}^S)\) and \(\frac{\partial k^S_t}{\partial W}\) becomes positive. Therefore, taking into account the effect of divergence from \(\hat{k}^S\) and its limit, there is a possibility to generate cycles like Figure 4.10\textsuperscript{12}. Summing up, there are two trajectories around the new steady state \(\hat{k}^S\) in this economy; a pass converging to \(\hat{k}^S\) in the neighbourhood of \(\hat{k}^S\) and cycles between high and low capital stocks.

Moreover, once we take into consideration the effect of financial crises, this economy is prevented from convergence to \(\hat{k}^S\) and cycles become more complicated by

\textsuperscript{12}In this figure, I draw a cycle with two periods. However, it can be multi-period cycles.
its capital destruction effect. Because banks hold risky assets within the region of \( k_t^S > \bar{k}^S \), financial crises occur in the next period with probability \( \delta_{t+1} \). Then, some households (the proportion \( 1 - \mu_{t+1} \)) cannot have access to banks and the capital stock declines to the level of the following.

\[
k_{t+2}^S = \mu_{t+1} AW(k_{t+1}^S) - Bk_{t+2}^R
\]  

(4.18)

In words, the capital stock in the next period is determined as the fraction of wage (deposit) which reflects accessibility to banks by some depositors in the case of a crisis minus the amount of risky investment. Again, individual households cannot invest in risky assets and those who cannot deposit in banks continue to hold their wages in hand. For example, this situation is shown at dot \( a \) in Figure 4.11. Even if the economy are converging to the steady state \( \bar{k}^S \), it is disrupted from reaching this steady state due to financial crises.

Summing up, given the initial wealth of households, the economy shows periodical cycle endogenously. That is, at first, the economy starts at the low level of capital stock with the high safe interest rate and economic growth gradually results in high capital stock with the low safe interest rate. After passing through critical level \( \bar{k}^S \), financial intermediaries begin to take excessive risk.

The reason why the economy is hit by crises in a high capitalized economy and, as a consequence, the pass of accumulation of capital stock is different from that of standard neoclassical one comes from the existence of deposit contract with limited liability and high accumulation of households assets. Since the downside loss due to bankruptcy is not burden by banks and only the upper side of profits can be obtained, they have a put option on their insolvency. Therefore, they have tendency to invest in risky assets too much. Yet, if this option value is smaller than the expected bankruptcy costs, they attempt to refrain from taking excessive risk. A low capitalized economy is characterized as this situation. That is, when capital stock is
Figure 4.10: An example of a cycle in a high capitalized economy
Figure 4.11: The effect of financial crisis on the pass of capital accumulation
low, the amount of individual assets of an economy is also low and the rate of return on capital stock is quite high. Thus, the option value from insolvency is low compared to an expected value of bankruptcy costs. In this environment, banks are willing not to take excessive risk. However, the economy grows and an amount of capital stock increases, the rate of return on them declines and the amount of households assets increases enormously. At some point, the option value of bankruptcy becomes greater than the expected bankruptcy costs. This is the origin of risk taking behaviour of banks and financial crises in this model.

Finally, we summarize the dynamic movement of capital stock of this economy as follows.

\[ k_{i+1}^S = \varphi(k_i^S) = \begin{cases} 
AW(k_i^S) & \text{if } k_i^S \leq \tilde{k}^S \\
AW(k_i^S) - Bk_{i+1}^R & \text{if } \tilde{k}^S < k_i^S \\
\mu_iAW(k_i^S) - Bk_{i+1}^R & \text{if } \text{a financial crisis happens with probability } \delta_i.
\end{cases} \] (4.19)

### 4.5.3 An example of a high capitalized economy

Using the same example we used so far, we will study the effect of increase in technological progress \( \theta \) on the trajectory of this economy. For this purpose, we will change the value of \( \theta \) from 1.0 to 3.0. The capital accumulation is shown in Figure 4.12. In a similar way as before, we can know the steady state is around 12.2499.

However, in this case, this economy does not reach the neoclassical steady state. Before converging to it, banks start to take risk. This situation is shown in the Figure 4.13. Again, this figure is based on the function (4.14). The reason why an improvement of productivity causes emergence of risk taking behaviour is that it also induces an acceleration of an increase in the amount of individual wealth whereas it enhances productivity of the capital stock. As a consequence, the first effect offsets
Figure 4.12: The accumulation of capital stock when $\theta = 3.0$
the second one and the deviation value of banks is inclined to be positive.

From this figure, we can know that the deviation value becomes indifferent between safe and risky assets when the other banks invest in capital stock at the level around 3.8268. The deposit available for each bank to achieve this indifference is around 1.1480.

4.6 Conclusion

In this chapter, we attempted to make clear the mechanism of financial crises as a consequence of economic development. Summing up the underlying mechanism of risk shifting behaviour of banks, in a low capitalized economy in which an amount of household assets is scarce and a rate of return on safe assets is high, investing in a corporate sector is more profitable than that in risky assets. This is because the
return from risky assets becomes negative after taking into consideration the expected bankruptcy cost. However, as the economy grows, the amount of household assets increases substantially and the rate of return on safe assets is decreasing. Correspondingly, the option value of risky assets is increasing and the expected bankruptcy costs become smaller, which induces banks to hold risky assets. Put differently, banks behave recklessly due to the reduction of the probability of bankruptcy. Hence, the main reasons to cause financial crises are both the huge amount of individual assets and low productivity in capital stock as a result of economic development, which brings about incentive to take excessive risk for banks. That is, after an economy sufficiently grows, there appears an “imbalance” regarding the relationship between real and financial sectors. In financial sectors, the household assets are accumulated and flow into bank deposits to invest in productive capital stock. However, in real sectors, the rate of return on capital stock decreases and it is not profitable from the point of view of banks. In order to resolve this mismatch, the rate of return increase and the amount of deposit decreases through the reduction of capital stock in the economy. Risk shifting behaviour by banks and resulting financial crises are interpreted as the reaction to adjust this imbalance.

The focus of our model is different from the previous research. Generally, the macroeconomic impacts of financial fragility are focused on financial constraints based on an information gap between lenders and borrowers. There, the main problem is the limitation of funds available for corporate firms due to lack of information on them and, as a consequence, aggregate investments are negatively affected. In other words, the central problem lies in "demand (borrowers) side" of the financial interaction in the previous works. On the other hand, our model pays less attention to this point. Rather, the main issue in our model is that the capital stock in an economy are hugely accumulated and households hold a great amount of assets which are brought into banks as a deposit. In other words, the problem lies in "supply (lenders) side" and
it can be said that too much accessibility of funds have also adverse effects on an economy by distorting an incentive of financial intermediaries.

Yet, we can point out several topics remained to be addressed in the future. At first, although we showed the possibility of cycles in an economy, the rigorous analysis on stability was not yet to be done. The convergence to the steady state $k^S$ and existence of multi-period oscillations are still not clear in the analysis so far. Similarly, the comprehensive analysis of the influence of failure of banks on the cycles should be done. Moreover, this model is based on a domestic economy. An extension of this model to the international framework with capital flow between countries seems to be an interesting topic.

Finally, a short remark on solvency policy by the government will be made. Needless to say, financial crises have various aspects and there are a lot of other factors to drive banks’ risk shifting behaviour. For instance, the existence of deposit insurance also misleads banks to gamble for resurrection on the verge of bankruptcy at the cost of taxpayers. Therefore, it is not appropriate to conclude that the government policy to prevent crises is not effective. In order to prevent such behaviour, banks should be required to hold enough capital cushions. Nonetheless, the core of the problem lies in the low productivity of capital stock and highly accumulated households assets. That is, it can be said that financial crises appear in the circumstance where capital stock under the existing technology becomes obsolete. Therefore, even if solvency policy is well articulated and succeeds to prevent financial crises temporally, the underlying issue to cause crises is still remained. Thus, the measure for solving (mitigating) financial crises is to take a policy to boost productivity for capital stocks in an economy by, for instance, enhancing technological progress. Although there are several ways to improve productivity of an economy, one of the promising ones is to promote to change an industrial structure in an economy from old and low productivity sectors to new and high productivity ones. For instance, in developed
countries in which capital stocks in manufacturing sectors are fully accumulated, the competitiveness of these sectors is under pressure from the emerging countries with high cost advantage. Hence, instead of manufacturing sectors, the main industries in these developed countries should shift to service sectors (high technology and knowledge intensive industries) with the comparative advantage. These industries enhance productivity of these countries and, at least, can postpone the occurrence of financial crises though they might not be extinguished\textsuperscript{13}. 

\textsuperscript{13}One caveat is needed. As we have shown in the examples, an improvement of technological parameter $\theta$ brings about the possibility of crises. This is due to the further increase in the amount of deposits. In order to prevent the risk shifting behaviour of banks, the economy must continue to accomplish a drastic technological progress which offset the effect of the accumulation of deposits at each date.
4.7 Appendix

The uniqueness of $k^S$ and $\tilde{k}^S$

Here, we will show that the deviation value of banks (equation (4.7)) is monotonically increasing in $k^S_t$. At $t - 1$, banks hold the amount of deposit available for safe assets $\alpha W(k^S_{t-1}) = (1 - \alpha)k^S_t$. The deviation value is meant to be the one if an individual bank invests this amount of deposit in risky assets, while the others invest in safe assets. Hence,

$$\Pi(k^S_t) = (1 - \alpha)k^S_t \int_{f'(k^S_t)}^{r^R_{MAX}} (r^R_t - f'(k^S_t))h(r^R_t)dr^R - \int_0^{f'(k^S_t)} Z h(r^R_t)dr^R$$

Taking derivative of this function induces to the following.

$$\frac{\partial \Pi}{\partial k^S_t} = (1 - \alpha) \int_{f'(k^S_t)}^{r^R_{MAX}} (r^R_t - f'(k^S_t))h(r^R_t)dr^R - (1 - \alpha)k^S_t \{H(R) - H(r_t)\} f''(k^S_t) - Z h(r_t) f''(k^S_t)$$

From the assumption of the concavity of the production function, we can obtain

$$\frac{\partial \Pi}{\partial k^S_t} > 0$$

Therefore, $k^S$ is uniquely determined to satisfy $\Pi(W(\tilde{k}^S), k^S) = 0$. In addition, from the monotonicity of $W(\cdot)$, $\tilde{k}^S$ is also determined uniquely.

The capital accumulation in a high capitalized economy

In order to know the capital accumulation in the region $k^S_t > \tilde{k}^S$, we will consider the zero deviation value condition (equation (4.8)). After totally differentiating, we
can obtain the following. Here, in order to avoid the notational burden, we describe $W(k_{t-1}^S) = W$.

\[
\frac{\partial k_t^S}{\partial W} = -\frac{\alpha \int_{r_t^R}^{r_{R}^{MAX}} r_t^R h(r_t^R) dr_t^R}{f'(k_t^S)} \left\{ \alpha W \{ H(R) - H(f'(k_t^S)) \} + Z h(f'(k_t^S)) \right\} f''(k_t^S) < 0
\]

That is, the capital stock is a decreasing function of the amount of deposit. Furthermore, taking the second derivative yields the following.

\[
\frac{\partial^2 k_t^S}{\partial W^2} = -\frac{\alpha r_{R}^{MAX} \int_{r_t^R}^{r_{R}^{MAX}} r_t^R h(r_t^R) dr_t^R}{f'(k_t^S)} \left\{ \alpha W \{ H(R) - H(f'(k_t^S)) \} + Z h(f'(k_t^S)) \right\}^2 \cdot \alpha \{ H(R) - H(f'(k_t^S)) \} \cdot (f''(k_t^S))^3 > 0
\]

In other words, the degree of decreasing in capital stock becomes smaller.
Chapter 5

Conclusion

In this dissertation, we have considered economic fluctuations and financial crises from the perspective of developed countries. This conclusion is dedicated to a brief summary of each chapter and some policy implications that are derived from our analysis. First, the basic results of each chapter are simply summarized. Then, possible policy implications of these studies are pointed out. Especially, we will primarily consider the role of economic policies, keeping in mind the question that “should the government stabilize an economy?” or, put differently, “do business fluctuations have harmful effect on economic efficiency?” Next, possible ex ante and ex post economic policies during financial crises will be considered from the point of view of our understanding of the crises. Because we have considered the mechanism and characteristics of financial crises from the perspective of economic growth, we will base the policy suggestions on the viewpoint of growth strategies.
5.1 Summary of each chapter

5.1.1 Global game and economic fluctuations

In the second chapter, we tried to show huge fluctuations and long run persistence of corporate aggregate investments by means of a special type of coordination game called a global game. Although, generally speaking, coordination games give rise to multiple equilibria, this method which includes strategic ambiguity regarding underlying economic states guarantees a unique equilibrium by iterated dominance in the static coordination game. Moreover, to apply a dynamic structure, we took into consideration a past externality by which past aggregate investments have positive effects on current productivity of investments. As a consequence, huge volatility and persistence of aggregate economic activities can be shown in this simple framework. In recent years, most developed countries have been urged to shift their main industry from a manufacturing to service sector which requires human capital with sophisticated knowledge and skills as a resource. There is a possibility that the business fluctuation model in this chapter sheds light on some features of business cycles of these countries.

5.1.2 Liquidity, firms’ productivity and economic dynamics

In the third chapter, we considered the situation that some firms in an economy hold sufficient internal funds, while the other firms have less amount of liquidity. Then, we examined the implication of this financial structure for aggregate investment and its dynamic characteristics. This study is based on two empirical findings: 1) young firms are high productive, while old ones are low, 2) young firms are vulnerable to bankruptcy, while old ones are not. We showed the mechanism of producing corporate liquidity in an economy by introducing a productivity difference between young and old firms and continuation value for them. As a consequence, young firms whose
continuation value is high prefer to adopt high productivity projects which bring about sufficient amount of liquidity for surviving in the future. However, they have tendency to go bankrupt due to insufficient liquidity. On the other hand, old firms whose continuation value is low prefer to have low productive projects for obtaining private benefits. However, since they have enough liquidity carried from young age, they are not vulnerable to bankruptcy. Furthermore, from the above consideration, we set liquidity markets between young firms with insufficient liquidity and old ones with sufficient liquidity. In addition, the probability of liquidity shocks is supposed to have a characteristic of externality from the past. Then, we showed that 1) liquidity markets insulate firms from liquidity shocks and the economy becomes stable, however, 2) in times of instability of the economy showing consecutive liquidity shocks, aggregate economic activity drops sharply even under the existence of liquidity markets. In most developed countries, large firms hold sufficient internal funds whereas small and medium sized firms have less. Our study might illuminate the characteristics of business fluctuations under such a structure.

5.1.3 Growth driven financial crises and cycles

In the forth chapter, we made clear the mechanism of financial crises from the point of view of economic growth. Even developed countries with sophisticated financial systems often suffer crises, especially after good economic performance. Thus, we regard crises as a trigger of a regime change from a current well-working economic system to sudden malfunctioning. In the early stage of economic development, the amount of an individual’s assets is scarce and profitability of risky assets is low. Thus, banks invest only in the corporate sector, which accelerates economic growth. However, as a result of economic development, assets held by households increase and the rate of return on corporate investment is decreasing, which makes risky assets more profitable. Therefore, banks are willing to invest in risky assets. In consequence,
economic growth drops and financial crises occur. The underlying elements which induce crises are the movement of the safe interest rate, the volume of households assets and, accordingly, the change in the option value of taking risk. With low capital stocks, since the safe interest rate is high and the amount of individual wealth is low, the option value for banks is quite low. Therefore, they refrain from taking excessive risk. However, as a result of economic development, the safe interest rate decreases enough and an individual assets are hugely accumulated. Hence, the option value becomes high, which gives a risk shifting incentive to banks, and crises follow. The low safe interest rate and highly accumulation of households’ assets are common features of almost all developed economies. This implies that these countries are always under threat of crises.

5.2 Economic cycles and government regulations

5.2.1 Should business cycles be stabilized?

In the elementary macroeconomics which is strongly affected by Keynesian economics, business fluctuations are taught to be flattened by fiscal and monetary policy. However, this is not necessarily true because the real cost of business cycles is still not well known. For instance, real business cycle theorists claim that the economy is always in equilibrium, and attempts to diminish cycles by the policies of government just causes misallocation of economic resources. Therefore, the opponents of RBC theories are against interventions by government into private economic activities. Moreover, Lucas (2003) regards the welfare cost of business cycles as the volatility of risk neutral individual’s consumption and measured this cost through an RBC framework. As a consequence, he showed that the cost of business fluctuations is not necessarily huge. Besides RBC theory, Schumpeter (1934) also claimed the beneficial effects of business cycles on economic efficiency as “creative destruction”. In times of expansion, almost
all firms can obtain benefit from preferable macroeconomic performance and their managerial disciplines are inclined to be loose. In contrast, during a recession, this kind of inefficient firms is likely to go bankrupt in the tough economic environment. In other words, recession has a role of wiping out inefficient firms and it improves economic efficiency. Aghion and Howitt (1997) listed some of the merits of business cycles, especially recession. 1) A recession cleans out inefficient firms in an economy as mentioned above. 2) During a recession, factor prices decrease and productivity of investments increases. 3) The ratio of capital to debt decreases and this high leverage disciplines a firm’s management.

Conversely, there are also a lot of papers which claim that the cost of business cycles, especially recession, is significant. The seminal paper which pointed out this regard is Fisher (1933). From the incidence of the Great Depression in 1933, he showed the cumulative process of changing price, the real interest rate and the debt burden of firms. During a recession, firms’ debt increases and new investments are hampered by squeezing financial availability\textsuperscript{1}. Moreover, Cabarello and Hammour (1994) theoretically examined the mechanism of a “cleansing effect” of recession where old and inefficient firms are forced to leave markets and new entries are prompted. As a result, they showed that fluctuations of aggregate demand are closely associated with new entries and recession in which aggregate demand declines do not necessarily lead to exit by old firms. As for job creation and destruction, Cabarello and Hammour (1996) examined the reallocation effects of recession through job creation and destruction. They modelled a general equilibrium framework including a search cost and, then, studied job creation and destruction in a phase of business cycle. If the economy has no contractual frictions, job creation and destruction is smoothly accommodated at the bottom of economic activities. However, once contracting incompleteness matters, this synchronization breaks up and job creation and destruction cannot synchronize in

\textsuperscript{1}Nowadays, this point is elaborated from the point of view of financial constraints framework such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997).
a recession. Put differently, the anticipated role of reallocation of economic resources in recession cannot necessarily be achieved. In what follows, we will think about a policy stance on this problem from the perspective of our analysis.

### 5.2.2 Some policy implications of our analysis

Schumpeter argued that recession brings about job destruction in low productivity sector, while job creation takes place in high productivity sector. However, according to Cabarello and Hammour (1996), this mechanism does not work smoothly with incompleteness of financial and labour contracts. Instead of the acceleration of entry of efficient firms with high productive skills, recession prevents new entry and tends to preserve old and inefficient firms in a market.

Our business fluctuations model analyzed in the second chapter illuminated one of the efficiency aspects of business cycles. It is shown that a business cycle is closely associated with coordination of individual corporate investments and its productivity enhancing effects on economic growth. Put differently, coordination on aggregate investments leads to economic efficiency by providing workers with opportunities for training, which improves productivity of production and economic growth. Therefore, government intervention in a recession to boost economic activities generally improves economic efficiency by supporting learning-by-doing in private sectors, which is also an essential factor of endogenous economic growth. Our model, on the other hand, implies that expansion should not be interrupted by governmental interventions. However, since this model does not consider the movement of other crucial economic variables like inflation, it is not necessarily appropriate for us to conclude that expansion should be left without tightening of fiscal and monetary policy.

The model in the second chapter has a special structure based on coordination games and it is difficult to derive more detailed policy implications. The study in the third chapter which investigated the interaction between corporate liquidity and
aggregate investments can illuminate firms’ behaviour in the face of business fluctuations. In this study, we showed that business fluctuations do not necessarily get rid of low profitability firms and improve economic efficiency. That is, with the incidence of liquidity shocks, high productivity firms with little liquidity have tendency to go bankrupt, while low productive ones with huge amount of liquidity can survive. In other words, in contrast to Schumpeter’s discussion, business cycles do not have a “cleansing effects” on the economy and it is possible that economic policies have an important role in stabilizing the economy. In order to smooth volatility of aggregate investment, the central bank might inject money into the economy when young firms have difficulty to access funds after a liquidity shock. However, this support from the central authority might also have negative effects on economic efficiency in the form of moral hazard. The reason that young firms choose high productive projects is that they can hold sufficient liquidity and be protected from liquidity shocks when they become old. This is because their continuation value is higher than short term private benefit. If young firms anticipate that they are rescued by the central bank when they become old, they have no incentive to adopt high productivity projects when they are young and all firms in an economy have low productive projects. In other words, the central bank intervention induces firms to select low productive projects and low productive ones are not only preserved but also created in an economy. As a result, overall liquidity in this economy becomes scarce. This policy dilemma comes from an information asymmetry between firms and the central bank with regard to firms’ type. If the government could clearly distinguish young firms from old ones and commit not to supply funds for old firms, it could inject necessary liquidity only for young ones and avoid moral hazard. However, in practice, it is not clear if the central bank can make such a policy because, firstly, they may not be able to distinguish firms clearly between high and low productivity by their age. Moreover, they may not have a policy tool to accomplish such a purpose even if they can tell the type
of firms. This is because the central bank conducts open market operations through markets and not through individual firms.

5.2.3 Financial crises and regulations

Generally, it is said that financial crises have to be avoided because they put huge costs on an economy. However, it is not necessarily clear what the costs of financial crises are. It is often said that the fiscal burden on taxpayers due to failure of banks is a cost of financial crises. However, this is not the cost of the crises but a redistribution of wealth among banks, depositors and taxpayers. With regard to cost of financial crises, Allen and Gale (2003) regarded the welfare cost of financial crises as the inability to set complete contracts in which all relevant payoff streams depend on any contingent states. A typical contract of a bank is a deposit style contract and it is, apparently, an incomplete contract. However, this welfare cost cannot be avoided since provision of complete contract is prohibitively costly and even a central planner who pays attention to social welfare cannot solve this problem. Therefore, from their point of view, financial crises are not necessarily regarded as a market failure.

The suggestion that financial crises are not necessarily inefficient is partly shared with ours in the fourth chapter. This chapter showed that there is a possibility that financial crises occur when real economic activity is fully matured in terms of accumulation of capital stocks. In other words, since existing technologies in an economy become obsolete, adjustment to recover productivity of capital through capital destruction is inevitable. From a different point of view, financial crises are a “signal”, pointing out the mature of technology of this economy. Therefore, current proposed policy interventions are not necessarily effective to get rid of the deep-rooted cause underlying financial crises: high accumulation of capital stocks and a low rate of

\footnote{They called this Pareto efficiency under deposit contract as constrained efficient.}
return. Rather, inappropriate policy interventions can end up making the situation worse and lead to long run stagnation of the economy. For instance, after financial crises in 1990’s, the Japanese government has adopted Keynesian policy for nearly ten years by aggressive fiscal and monetary policy. However, the effect of these policies on economic activities was limited and the Japanese economy did not recover from stagnation. Why were these active government policies inactive? Though there is a possibility that the transmission mechanism of fiscal and monetary policy had changed in the Japanese economy, the main reason seems to be misunderstanding of the characteristics of financial crises and stagnation. The government thought that stagnation is due to a temporary drop of aggregate economic activities via the adverse influence of bubble crash and can be overcome by fine tuning government policy. However, as we showed, the core of the problem is deep-rooted when we take account of the development stage of the Japanese economy. That is, the financial crises and resultant stagnation are brought about by low productivity of capital stocks as a result of accumulation. Without resolving the problem of low productivity of capital stocks, the cause of financial crises cannot be removed. However, government investments which are often low productive exacerbates the problem. To make matters worse, the government saved failing banks and, indirectly, rescued low productive firms (sometimes called “Zombie firms”). This myopic policy also worsened the problem of low productivity in the Japanese economy.

If so, are there no roles of government to prevent crises? Generally speaking, there are two types of measures against crises; \textit{ex ante} and \textit{ex post}. \textit{Ex ante} regulation is to prevent crises.

\footnote{Doing “nothing” and adjustment of capital stocks naturally to restore high productivity might be one of the policies. Actually, our model described such a situation. However, it seems difficult to accept this policy in a political sense.}

\footnote{The reason why the banking industry should be regulated strictly is considered in Tirole (1994). He regards banking regulation as corporate governance by the government in that small depositors cannot exercise their power to discipline a manager of a bank and the government must do so on their behalf (Representation Hypothesis).}
in advance. For instance, a capital adequacy requirement is a typical measure of this kind of policy for restricting excessive risk taking by banks. In what follows, as *ex ante* regulation, we will keep in mind capital regulation. Moreover, restriction on competition between banks which took place until the beginning of 1980’s in most countries, for instance interest rate regulation, is also considered as this kind of policy. An *ex post* measure is to circumvent the negative effect of crises once they occur. For instance, central bank intervention is included in this kind of measure. Others are deposit insurance in order to secure individual deposits and a smooth procedure to deal with insolvent banks. Here, we will consider the policy implications of our study for *ex ante* and *ex post* policy.

One of the crucial implications of this study is that *ex ante* financial regulations are not always effective for crises\(^5\). This is because financial crises in our study are regarded as an inevitable phenomenon in the sense that an economy in which capital stocks become obsolete as a result of economic development is required to recover its productivity through crises resulting in reduction of capital stocks. Therefore, crises are a rather natural phenomenon to restore profitability of an economy and there is the possibility that *ex ante* regulations might hamper this process. In other words, even if risk shifting behaviour of banks can be avoided by financial regulations, an underlying cause of the problem still remains. Moreover, introduction of regulation itself might bring about additional distortions on the incentives of financial intermediaries. As to capital regulation, there are a lot of theoretical and empirical studies on the effect of regulation on banks’ risk taking behaviour\(^6\). However, the results are mixed and we cannot obtain decisive conclusions so far. Finally, current capital regulation is not perfectly structured and there is always space for excessive risk taking by financial intermediaries. This is because financial technologies and their risk

\(^5\)Needless to say, the capital structure of banks in the model of the fourth chapter is based on a deposit (debt contract) and we cannot precisely study the effect of capital regulation.

\(^6\)See Santos (2000) as a survey
structure are quite complex and, in fact, the financial regulators cannot catch up with financial risk management. Under these circumstances, it is very difficult to prevent excessive risk taking by financial intermediaries. Yet, there are two main reasons to rationalize the introduction of *ex ante* financial regulations. Firstly, taking account of the destructive feature of crises, trying to prevent crises in advance is meaningful. Secondly, introduction of *ex post* intervention mentioned later might cause banks to take moral hazard behaviour, for instance, risk shifting behaviour induced by deposit insurance. In order to make the *ex post* intervention work well, *ex ante* regulation must be put in place in the form of capital requirements by the financial authority.

On the other hand, *ex post* regulations can restore financial function quickly and start to rebuild capital stocks smoothly. Liquidity injection into financial markets by the central bank is effective to some extent so as not to spread the influence of crises through the financial system and the economy as a whole. As mentioned before, given the fact that financial institutions are closely associated with financial markets, central bank intervention through injecting liquidity into markets has a crucial role to keep the markets viable. Moreover, restoring an impaired financial system quickly is essential to start new capital accumulation of an economy. In this regard, there are two countervailing historical cases. When North European countries suffered severe financial crises at the beginning of 1990’s, they took drastic measures to restore financial function, for instance, by consolidation of failed banks. As a result, they could return to pre-crisis economic condition quickly. By contrast, though the Japanese economy also experienced crises in the same period, they did not any decisive policies against it. Consequently, the recovery of the economy became sluggish and "the lost decade" began.

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7 Reflecting this fact, the architecture of capital adequacy requirements seems to have a direction that banks are given incentives to each calculate and hold a proper amount of capital instead of imposing a government-made formula on banks in calculating the necessary amount of capital (an incentive compatible approach). In Basel 2, banks were allowed to use their own risk management model to calculate market risks. Moreover, in Basel 3, they can make use of the information of rating agencies.
5.2.4 Policy over the cycle

In our study, we regarded financial crises as a trigger of a regime change and have claimed that crises tend to occur when a high productive economy shifts to a low one. Although *ex ante* financial regulation like a capital adequacy requirement has a crucial role to prevent devastating crises, it seems not necessarily to be the right direction to deal with crises. Combined with well-structured financial regulation, various measures to improve productivity of an economy should be taken. For instance, giving an incentive to adopt new technology to a firm through competition policy or change the industrial structure of an economy are essential ways to improve productivity. That is, old and low productive sectors shrink, while new and high productive ones should be prompted to grow. Conversely, the worst policy is to make low productivity firms and sectors survive in an economy and keep high productive ones from entering markets.

In this respect, economic fluctuations, especially recessions, do not contribute to improve productivity of an economy through wiping out inefficient sectors as we mentioned before. Rather, it can result in a reduction of productivity by allowing low productive firms to survive. Furthermore, from the study of the third chapter, an economic activity fluctuates significantly because a liquidity shock mainly has an adverse effect on high productivity firms with low amount of liquidity. Thus, as long as there are capacities for fiscal and monetary policies in terms of sustainability of fiscal and monetary balances, the government of developed countries can take measures to decrease volatility of economic fluctuations in addition to attempting to improve

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8 In this regard, as we mentioned before, the Japanese government took aggressive fiscal and monetary policy to deal with recession after the bubble crash. However, for the purpose of treating with the problem underlying the crises, measures inducing change in industrial structure and technology should have been taken.

9 There are also difficulties in this respect. As we pointed out in footnote 13 in the fourth chapter, a technological progress leads to an increase in the volume of deposit and the possibility of risk shifting by banks increases as well. To avoid this situation, the economy must improve its productivity in every period. However, it seems very hard to improve a technology continuously because it cannot be easily and precisely controlled by a policy.
productivity of the economy as a whole. “Liquidationism” in which business cycles get rid of all bad things in an economy is not necessarily true.
Bibliography


