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THEORIES OF TARIFFS: TRADE WARS, TRADE AGREEMENTS, AND POLITICAL ECONOMY

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Abstract

This thesis is concerned with the general trade theoretic issue of what explains tariffs. Two possible theories are investigated: (i) the optimum tariff argument, where countries exploit their market power to affect world prices, and (ii) the political economy argument, where well-organised interest groups who have a preference for the tariff protection level can influence their governments through lobbying.

The main contribution of this thesis is the use of the many-country, two-good trade model, which can be found in the customs union literature, to investigate the importance of the (world) market structure on the welfare effects of tariffs. This model, where a good is exported by more than one country, allows us to examine the welfare effects of tariffs which vary with how the goods are divided initially among the countries. The theory of optimum tariffs and retaliation, usually in the two-country, two-good context, suggests that the country whose endowments of goods are relatively large tends to ‘win’ a trade war. Still, the analysis in this chapter shows that there is a greater possibility for a country to win even if the country’s endowments are relatively small if the world market of its exportable moves closer to the monopolistic market, i.e. there are less countries exporting the same good and/or the world endowment of that good is divided more disproportionately among its exporters.

An important feature of the many-country, two-good trade model is that tariffs are strategic complements between countries that have the same trade pattern and are strategic substitutes otherwise. Therefore, two possible trade agreements can be investigated: (i) an agreement between countries whose tariffs are strategic complements, and (ii) an agreement between countries whose tariffs are strategic substitutes. Since these trade agreements imply different sources of gain for a country (gain from an improvement in terms of trade for the former and gain from an increase in volume of trade for the latter), this thesis examines the choice of a country by comparing the welfare implications between the two possibilities. It is found that a country would prefer to have a trade agreement with the country whose endowments of goods are relatively large regardless of the strategic complementarity or substitutability of their tariffs.

Finally, this thesis attempts to endogenise the lobby formation by modelling an individual’s decision to participate in lobbying prior to the stages of interaction between a government and lobbies studied by Grossman and Helpman (1994). It is found that no one lobbies individually in equilibrium if the total population and/or the fixed cost of lobbying are too large. An incentive that leads individuals to form a lobby is the ability of the group to restrain the individuals’ otherwise offsetting lobbying efforts. An interesting result is that, in equilibrium, some individuals might choose to join the lobbies that lobby against their interests to moderate their efforts rather than to join the lobbies that lobby in their favour. This result raises a question whether the standard industry-lobby in the literature might exaggerate the actual lobbying activities.
Contents

1 Introduction

2 Literature Review
   2.1 Terms of trade argument ..................................................... 13
      2.1.1 Optimum tariffs and retaliation ................................... 13
      2.1.2 International trade agreements ................................... 20
      2.1.3 Customs union ......................................................... 26
      2.1.4 Summary ................................................................. 27
   2.2 Political economy of trade policy ....................................... 28
      2.2.1 Lobby formation ......................................................... 32
      2.2.2 Summary ................................................................. 41

3 Optimum Tariffs and Retaliation in the Multilateral Context 43
   3.1 Introduction ................................................................. 44
   3.2 Two-country model .......................................................... 46
      3.2.1 Interior Nash equilibrium ........................................ 49
      3.2.2 An extreme case ....................................................... 56
   3.3 Multi-country model ....................................................... 59
      3.3.1 Nash equilibrium tariffs ........................................... 61
      3.3.2 Welfare analysis ..................................................... 72
      3.3.3 Discussions ............................................................. 77
4 International Trade Agreements: Terms of Trade and Volume of Trade Incentives

4.1 Introduction ........................................ 82
4.2 Two-country model ................................... 85
  4.2.1 Nash bargaining solution .......................... 86
4.3 Multi-country model .................................. 91
  4.3.1 Volume of trade-driven trade agreement ......... 93
  4.3.2 Terms of trade-driven trade agreement .......... 99
  4.3.3 Comparison of results ........................... 104
4.4 Conclusion ............................................ 107

5 An Endogenous Lobby Formation Model ............ 109

5.1 Introduction ......................................... 110
5.2 Basic settings ....................................... 112
5.3 Lobbying individually ............................... 115
  5.3.1 Policy formation ................................ 115
  5.3.2 Individual decision ............................. 120
  5.3.3 Lobbying with fixed cost ....................... 124
5.4 Lobby formation ..................................... 128
  5.4.1 Policy formation ................................ 129
  5.4.2 Lobby contribution .............................. 131
  5.4.3 Individual decision ............................. 134
5.5 Conclusion ............................................ 146

6 Conclusion ............................................... 148

A Chapter 3 ................................................. 150

A.1 Derivation of the Nash equilibrium tariffs in the two-country model .................. 150
A.2 Derivation of the best response tariff functions in the multi-country model. 153
A.3 Simulation results ........................................ 157

B Chapter 4 ........................................ 164
  B.1 Nash bargaining solution in the two-country model ............. 164
  B.2 Efficiency locus between $A$ and $B$ in the multi-country model .... 168
  B.3 Efficiency locus between $A$ and $a$ in the multi-country model .... 170
  B.4 Simulation results ........................................ 173

C Chapter 5 ........................................ 183
  C.1 Proof of (5.39) in the text .................................. 183
  C.2 Proof of (5.46) in the text .................................. 185
  C.3 Proof that $-\theta_2 < \theta_1 < \theta_3 < 0 < -\theta_3 < -\theta_1 < \theta_2$ .... 186
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Syropoulos (2002) Relative size effect.</td>
<td>19</td>
</tr>
<tr>
<td>2-2</td>
<td>Nash bargaining solution</td>
<td>25</td>
</tr>
<tr>
<td>2-3</td>
<td>Mitra (1999) Equilibrium number of lobbies.</td>
<td>35</td>
</tr>
<tr>
<td>3-1</td>
<td>Best response tariff functions and interior Nash equilibrium.</td>
<td>52</td>
</tr>
<tr>
<td>3-2</td>
<td>Degree of specialisation and the outcome of trade war.</td>
<td>56</td>
</tr>
<tr>
<td>3-3</td>
<td>Degree of market domination and Nash equilibrium tariff, given $n_x = n_y = 9$.</td>
<td>69</td>
</tr>
<tr>
<td>3-4</td>
<td>Market concentration and Nash equilibrium tariff, given $\delta_x = \delta_y = 0.5$.</td>
<td>70</td>
</tr>
<tr>
<td>3-5</td>
<td>Degree of market domination and Nash equilibrium world relative price, given $n_x = n_y = 9$.</td>
<td>73</td>
</tr>
<tr>
<td>3-6</td>
<td>Market concentration and Nash equilibrium world relative price, given $\delta_x = \delta_y = 0.5$.</td>
<td>74</td>
</tr>
<tr>
<td>3-7</td>
<td>Degree of market domination and Nash equilibrium utility, given $n_x = n_y = 9$.</td>
<td>75</td>
</tr>
<tr>
<td>3-8</td>
<td>Market concentration and Nash equilibrium utility, given $\delta_x = \delta_y = 0.5$.</td>
<td>76</td>
</tr>
<tr>
<td>5-1</td>
<td>Lobbying individually with fixed cost.</td>
<td>126</td>
</tr>
<tr>
<td>5-2</td>
<td>Equilibrium lobbies, $L^<em>_l$ and $L^</em>_s$.</td>
<td>144</td>
</tr>
</tbody>
</table>
List of Tables

3.1 Complete specialisation by both countries. 58
3.2 Complete specialisation by A. 58

A.1 Degree of specialisation and Nash equilibrium welfare. 158
A.2 Degree of market domination and Nash equilibrium tariff. 159
A.3 Market concentration and Nash equilibrium tariff. 160
A.4 Degree of market domination and Nash equilibrium world relative price. 160
A.5 Market concentration and Nash equilibrium world relative price. 161
A.6 Degree of market domination and Nash equilibrium welfare. 162
A.7 Market concentration and Nash equilibrium welfare. 163

B.1 Equilibrium trade agreement in the two-country model. 173
B.2 Degree of market domination and equilibrium tariff. 174
B.3 Degree of market domination and equilibrium world relative price. 174
B.4 Degree of market domination and equilibrium welfare. 175
B.5 Market concentration and equilibrium tariff. 176
B.6 Market concentration and equilibrium world relative price. 177
B.7 Market concentration and equilibrium welfare. 177
B.8 Degree of market domination and equilibrium tariff. 178
B.9 Degree of market domination and equilibrium world relative price. 178
B.10 Degree of market domination and equilibrium welfare. 179
B.11 Market concentration and equilibrium tariff. 180
B.12 Market concentration and equilibrium world relative price. . . . . . . . . . . . . . . . .180
B.13 Market concentration and equilibrium welfare. . . . . . . . . . . . . . . . . . . . . . . . .181
B.14 Degree of market domination and A’s best strategy. . . . . . . . . . . . . . . . . . . . . . . .181
B.15 Market concentration and A’s best strategy. . . . . . . . . . . . . . . . . . . . . . . . . . . . . .182
Chapter 1

Introduction

This thesis consists of three theoretical works based on the terms of trade and the political economy arguments for tariffs. The theory of optimum tariffs and retaliation is revisited in the multilateral context in which the countries’ tariffs can be strategic complements as well as strategic substitutes. Then the framework is extended to study international trade agreements. Finally, the study is confined to a ‘small’ open country of which the trade policy is formulated politically.

The methodology is based on the general equilibrium analysis. We abstract from modelling the production structure by assuming some fixed endowments of goods to be traded within and between countries. On the consumption side, consumers’ preferences are assumed to be identical across all individuals and all countries. In chapters 3 and 4, they are represented by the Cobb-Douglas utility function. However, in chapter 5, the consumers’ preferences are represented by a quasi-linear utility function so that explicit solutions can be obtained for further investigation on lobby formation. In all chapters, the game theoretical framework is employed to study the interactions between countries and between a government and its citizen.

It is argued that a ‘large’ country can benefit from imposing an import tariff through its effect on world relative price.\(^1\) When a country is large, its contracting import de-

\(^1\)As is typical in the literature, the terms ‘large’ and ‘small’ country are used to discriminate countries
mand due to its import tariff reduces the world relative price of its import which is an improvement in its terms of trade. The term 'optimum tariff' refers to the tariff rate which is not so large that the cost from reduction in volume of trade exceeds terms of trade benefit. However, the terms of trade benefit for one country apparently comes at the cost of its trading partner and it is likely that the trading partner will retaliate also in the form of an optimum tariff. Therefore, the framework in which countries impose the optimum tariffs (non-cooperatively) in retaliation to each other is used to study international trade wars. It has been shown, usually in the two-country, two-good context, that the country whose endowments of goods are relatively large tends to 'win' a trade war. This is due to the positive relationship between country size (measured by the level of its endowments in this thesis) and its price elasticity of import demand and hence its market power- the country with greater market power is able to impose a larger tariff (relative to its trading partner) so that world relative price is shifted in its favour compared to the world relative price under free trade.

However, in chapter 3, we construct a many-country, two-good trade model in which there exists more than one country trading under the same trade pattern so that their tariffs are strategic complements. Therefore, apart from its own tariff, whether a country wins a trade war depends not only on the tariff of its rival (the country with different trade pattern) as is typically examined in the literature but also the tariff of its 'ally' (the country with the same trade pattern). This thesis investigates the importance of the (world) market structure on the welfare effects of tariffs. It is shown that there is a greater possibility for a country to win, even if the country's endowments are relatively small, if the world market of its exportable moves closer to the monopolistic market, i.e.

\[ \text{with respect to their market power to influence world prices in this thesis.} \]

\[2\text{ A country's welfare must rise in the non-cooperative equilibrium relative to the free trade equilibrium for that country to win the trade war.} \]

\[3\text{ Since this thesis also looks at the cooperative tariff setting, it is important to note that the term 'ally' here represents the natural ally that shares common interest over the movement of world relative price, not the ally as a result of an agreement.} \]

\[4\text{ The country whose endowments are relatively small needs not be a small country as long as its endowments are not too small to have the ability to influence the world prices.} \]
there are less countries exporting the same good and/or the world endowment of that
good is divided more disproportionately among its exporters.

Based on the same many-country, two-good trade model, chapter 4 studies the ne­
gotiation of trade agreement. It is argued that countries trade at the level less than
efficient in the trade war equilibrium since their unilateral tariffs are too high. The role
of eliminating this inefficiency is a rationale for trade agreement provided in the liter­
ature. It has been shown that the efficient trade agreement between any two countries
is not unique. In fact, there exists a set of trade agreements which are Pareto efficient.
Recent works on trade agreement, therefore, devote to the study of trade negotiation
to select a unique agreement among those efficient ones. In general, the source of gain
from trade agreement in this line of literature is from an increase in the countries’ trade
volume. However, as recognised in the customs union literature, another source of gain
from the trade agreement is that it enables its member countries to exploit their collective
market power to improve their terms of trade. It is argued that countries that import the
same good can internalise their (positive) tariff externality by setting their tariff jointly
in a customs union.

Within the many-country, two-good trade model, the countries’ tariffs can be strategic
complements as well as strategic substitutes. Therefore, the trade agreement of which
the member countries exploit their collective market power to improve their terms of
trade can be studied as well as the trade agreement of which the member countries agree
to adjust their trade policies to increase their trade volume. Since the two types of trade
agreement imply different sources of gain, chapter 4 investigates the conditions in which
a country might prefer one type of agreement to the other. It is found that a country
would prefer to have a trade agreement with the country whose endowments of goods are
relatively large regardless of the types of agreement.

The results in chapters 3 and 4 are based on the assumption of the benevolent gov­
ernment that maximises the aggregate welfare of its citizen. However, this assumption
subjects to the criticism that it neglects the reality that politics play a significant role
in the trade policy formation. To address this criticism, among others, Grossman and Helpman (1994) view trade policy as the outcome of political lobbying.\(^5\) It is assumed that interest groups can influence the government’s decision by offering campaign contributions contingent on the trade policies implemented by the government. Instead of being benevolent, the government is willing to trade off some reduction in the general welfare in return for the campaign contribution. As a result, groups that can be organised will be able to obtain the trade policies which favour their interests at the expense of the public as a whole. However, while the model can explain the political process in which particular groups can translate their interests into the government policies, the existence of the lobbies is still exogenous.

Chapter 5 extends the influence driven contribution approach of Grossman and Helpman (1994) by endogenising the lobby formation. To restrict their attention to the interaction between the lobbies and the government, the previous works simply assume that individuals or firms will not buy influence individually but through group formation and it is usually assumed that a lobby is formed only within a well-defined group, such as firms within the same industry. By allowing an individual to choose his/her choice freely whether to lobby individually and which lobby to join, this chapter shows explicitly that an individual would not lobby individually when the total population and/or the fixed cost of lobbying are too large. Furthermore, it is shown that a lobby can consist of the individuals whose policy preferences are diverse (for example, relative to free trade, there might be some members who prefer an import subsidy as well as some members who prefer an import tariff within the same lobby). Therefore, if lobbying is the effort to shift one dimensional trade policy from free trade toward one way or the other, not only a lobby’s influence on the government’s decision is moderated by its opposing lobby, its aggressiveness in lobbying (the rate of the tariff/subsidy it is lobbying for) would also be

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\(^5\)There are a few other approaches on the political economy of tariffs. Mayer (1984) views trade policy as the outcome of majority voting over tariff levels. Magee, Brock and Young (1989) view the trade policy as the outcome of electoral competition in which political parties choose their trade policy stances to attract campaign contributions from interest groups which are positively related with their chance of winning the election.
moderated by its own members. The analysis in this chapter also reveals that individuals tend to be better off than lobbying individually when they lobby through groups. This is because, when the individuals lobby individually, their campaign contributions tend to offset each other so that their campaign contributions are merely to prevent the outcome which is even worse given the campaign contributions of the others: a case of the prisoner's dilemma problem. The lobby that emerges in this chapter can be considered as the means through which the individuals can restrain their otherwise offsetting campaign contributions. This finding explains an incentive of the individuals to form a lobby.

In general, this thesis is concerned with the general trade theoretic issue of what explains tariffs. Two possible theories are investigated: the optimum tariff and the political economy arguments. After an introduction in chapter 1 and a literature review in chapter 2, based on the optimum tariff argument, the non-cooperative (trade wars) and the cooperative (trade agreements) international tariff settings are considered in chapter 3 and chapter 4, respectively. Then, chapter 5 considers the tariff setting of a small country based on the political economy argument. A conclusion in chapter 6 closes the thesis.
Chapter 2

Literature Review

Under the assumption of large country, an international trade war is due to the unilateral incentive of each country to impose an import tariff to improve its terms of trade. The trade war is usually modelled as a non-cooperative international tariff setting game. However, it has been shown that the trade war equilibrium is inefficient in the sense that all countries can be made better off if they can coordinate their tariff settings. A trade agreement is viewed as the device to eliminate this inefficiency. Negotiation of a trade agreement is usually modelled as a bargaining game between countries to mutually remove the terms of trade incentive. The trade war and trade agreement based on the terms of trade argument are studied in chapters 3 and 4 of this thesis and their previous theoretical works are reviewed in section 2.1 of this chapter.

Under the terms of trade argument, it is implied that the optimal trade policy for a small country which has no market power is always free trade. This conclusion, however, contradicts to the tariffs empirically observed in most developing countries. The political economy argument for tariffs, the second argument dealt with by this thesis, is capable of dealing with this criticism. Under this argument, a tariff might be the optimal policy for the policy maker even in a small country since the policy maker is assumed to have an objective function which differs from the one who maximises the general welfare under the terms of trade argument. Such an objective function depends on the political and
institutional settings assumed in each particular model. Under the Downsian paradigm of political economy in which the policy maker's prime objective is to stay in power, the policy maker may choose to maximise the welfare of the median voter to please the majority of the population or to trade off some loss in the general welfare with political contribution made by lobbies to increase their chance of winning elections. The study of trade policy in chapter 5 of this thesis is based on this argument and its previous theoretical works are considered in section 2.2 of this chapter.

2.1 Terms of trade argument

This thesis studies the negotiation of trade agreement as a cooperative international tariff setting game. However, we also need a benchmark against which the welfare analysis of the trade agreement can be done, i.e. how the countries' welfare are affected compared to the equilibrium without the agreement. The non-cooperative international tariff setting game is also studied to serve this objective. This section reviews the previous theoretical works on optimum tariffs and retaliation and on international trade agreements in sections 2.1.1 and 2.1.2, respectively. Since the many-country, two-good trade model in this thesis is similar to a trade model used in the customs union literature and since the nature of a trade agreement considered in this thesis is related to the CU agreement in the way that the member countries exploit their collective market power to manipulate the world relative price against the non-member countries, the relevant theoretical literature on CU are reviewed in section 2.1.3. A summary in section 2.1.4 points out the status of the literature and the contributions of this thesis.

2.1.1 Optimum tariffs and retaliation

Conventionally, the term 'optimum tariff' refers to the tariff justified by the terms of trade argument. The terms of trade argument for tariffs is based on the assumption that countries are large enough to influence the world relative prices of their imports
and exports. The contracting import demand of a large country due to its import tariff reduces the world relative price of its import which is an improvement in its terms of trade. Even though the country suffers from a reduction in its volume of trade due to an import tariff (a common argument for free trade), it also benefits from the tariff through an improvement in its terms of trade. The theory of optimum tariffs shows that there exists a tariff rate which is not so large that the cost from reduction in volume of trade exceeds the terms of trade benefit. As the theory of optimum tariffs is based on the assumption that the tariff imposing country possesses the market power to influence world prices, the country's optimisation problem with respect to its tariff choice is analogous to the pricing behaviour of a monopoly firm. Even if the monopoly firm is the price setter in a market, it cannot increase the price without bound to increase the profit since the higher price also means the contracting demand for its product. The extent to which the monopoly firm can increase the price (the firm's market power), therefore, depends on the extent to which the demand for its product reacts to the higher price, i.e. the price elasticity of demand. Similarly, a large country's import tariff has an effect of increasing the world relative price of its export (reducing the world relative price of its import) which is an improvement in its terms of trade, the extent to which the country can impose the tariff depends on the price elasticity of foreign demand for its export. Assume countries A and B trading goods x and y. It is shown in all of the international economics textbooks, such as Krugman and Obstfeld (2003), that the formula for A's optimum tariff is

$$\tau^A = \frac{1}{\epsilon^A - 1}$$

(2.1)

where $\tau^A$ is A's optimum tariff and $\epsilon^A$ is the price elasticity of foreign demand for A's export (B's price elasticity of import demand). The optimum tariff that A can impose is higher when B's demand is less elastic so that there is less reduction in the volume of trade for a marginal increase in A's tariff. A always benefits from imposing $\tau^A$ as long as B has zero tariff.
However, since A's gain from imposing the tariff comes at the cost of B, this apparently invites 'retaliation' from B also in the form of an optimum tariff. The term 'trade war' is used to represent this equilibrium in which the large countries imposing their optimum tariffs in retaliation to each other. It is now uncertain that a large country can win the trade war, i.e. can raise its welfare relative to free trade, but it is certain that at least one country will lose. Note that B's optimum tariff also depends on the price elasticity of foreign demand for its export denoted by $e^B$.

Johnson (1953) represents the countries' trade preferences by a utility function in which their price elasticity of import demand are constant. By simulations, he shows that A will win the trade war if $e^A$ is sufficiently smaller than $e^B$, i.e. A faces sufficiently less elastic foreign demand for its export than B. Since the world relative price depends positively on one country's tariff and negatively on the other's, the world relative price will be shifted from free trade to the advantage of the country whose market power is greater to impose a larger tariff. Analogous to the monopolist, the country that has a greater market power to impose a larger tariff must face less elastic foreign demand for its export. Kennan and Riezman (1988) were the first to relate the country's market power to impose an import tariff to its size. They construct a two-country, two-good trade model in which the Cobb-Douglas preference and a world fixed endowment of each good are assumed (so that they measure the country's size by the level of its endowment). Also with the help of simulations, they show that the country whose endowment is larger, tends to win the trade war. In general, both papers show that both countries will lose when they are not too dissimilar.

More recently, Syropoulos (2002) obtains the same results as in Kennan and Riezman (1988) in a more general framework. Syropoulos clarifies the Kennan and Riezman (1988)

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1The term tariff retaliation is used differently by Baldwin (1990) as having a strategic role of deterring undesirable trade policies of a foreign government. With the foreign government's trade policy determined under lobbying from its private sectors, Baldwin (1990) examines the use of tariff retaliation by a home government to deter the lobbying activities of those foreign lobbies.

2There is no case in which both countries will win the trade war since the tariffs reduce the world welfare as a whole compared to the free trade equilibrium. Therefore, with positive tariffs, for one country's welfare to raise above the free trade equilibrium, the other country's welfare must be reduced.
results in a more general production and consumption structure. Within the standard two-country, two-good setting, he establishes how the country with sufficiently large relative size wins the trade war. An interesting finding is that in the limit where one country is infinitesimally small (which means the other country is infinitely large), the infinitely large country is indifferent between the autarky, free trade and the Nash equilibrium (the trade war) which leaves an open ended question about the incentive of the infinitely large country to intervene in free trade.\(^3\)

Still assume countries A and B trading goods x and y. It is assumed that the consumers have identical and homothetic preferences and the technology used in the production is constant return to scale using a vector \(V^i\) of factors including labour \(L^i\), for \(i = A, B\). The factors are fixed in supply and each country produces both goods. Each citizen owns one unit of labour so that \(L^i\) also represents the number of country i’s population. As the objective of Syropoulos is to reexamine the effect of country size on the trade war outcome, a definition of country size is required. An increase in country i’s size is defined to be the simultaneous increase by the same proportion in the fixed supply of all factors \(V^i\). Since Syropoulos formulates his analysis in per capita term and \(L^i\) is one of the factors \(V^i\), he defines a variable to represent the relative size of country i to be \(\lambda^i = \frac{L^i}{L^j}\) for \(i \neq j = A, B\) and refers to the change in country i’s relative size as a change in \(\lambda^i\) leaving the per capita factor endowment \(v^i = \frac{V^i}{L^i}\) always constant.

For clarity, consider country A. By algebraic manipulations, Syropoulos obtains an expression

\[
e^A du^A = \lambda^B m^B \left[ 1 - \tau^A \left( e^A - 1 \right) \right] (-\tilde{q}^A) + \tau^A m^B \left[ d\lambda^B - (\lambda^B \beta^B) \frac{d\tau^B}{1 + \tau^B} \right] \tag{2.2}
\]

\(e^A\) is A’s aggregate expenditure and \(du^A\) represents the change in A’s per capita utility.

\(^3\)A similar question is raised in this thesis in chapter 3. It is shown that in the limit where both countries monopolise their exports, their optimum tariffs will be trade prohibitive. As the countries always prefer some trade to autarky, the problem whether the countries will intervene in free trade in the limit also arises.
\( m^B \) is the per capita import demand of country \( B \) and \( \tau^A \) is \( A \)'s ad valorem import tariff. \( \epsilon^A = -\frac{\partial m^B}{\partial q^A} \frac{\delta^B}{m^B} > 0 \) is the price elasticity of foreign demand for \( A \)'s export and \( \tilde{q}^A = \frac{d\tilde{q}^A}{q^A} \) is the percentage change in \( A \)'s terms of trade \( \left( q^A = \frac{1}{\tilde{q}^A} \right) \). \( \beta^B = -\frac{\partial m^B}{\partial \tau^B} \frac{1+\tau^B}{m^B} > 0 \) and \( \tau^B \) is \( B \)'s ad valorem import tariff. (2.2) is the well known decomposition of the welfare effect of tariff into the terms of trade and the volume of trade effects. The first term indicates the change in \( A \)'s terms of trade and the second term indicates the change in \( A \)'s volume of trade.

(2.2) allows Syropoulos to fulfill his main objective to identify the channels through which the country relative size affects the outcome of the trade war. In terms of the expression in (2.2), he considers how \( A \)'s per capita utility \( u^A \) is affected by a change in the other country relative size \( \lambda^B \left( = \frac{1}{\lambda^A} \right) \).

Under the trade war with positive trade, i.e. the interior Nash equilibrium, the countries impose their optimum tariffs which satisfy (2.1). Therefore, the first term in (2.2) is eliminated and hence

\[
e^A d\tilde{u}^A = \tau^A m^B \left[ d\lambda^B \left( \lambda^B \beta^B \right) \frac{d\tau^B}{1+\tau^B_N} \right] \quad (2.3)
\]

(2.3) shows that, in the trade war equilibrium, the change in \( B \)'s relative size affects \( A \)'s per capita utility \( u^A_N \), only through its effect on \( A \)'s per capita volume of trade. The effect of the country relative size on the per capita volume of trade consists of a direct effect and a strategic effect. The direct effect is represented by the first term in the square bracket \( d\lambda^B \), an increase in \( \lambda^B \) raises \( u^A_N \) since an increase in \( \lambda^B \) raises \( A \)'s per capita import. As represented by the second term in the square bracket, the strategic effect of an increase in \( \lambda^B \) on \( A \)'s per capita import and hence \( u^A_N \) is through its effect on \( B \)'s optimum tariff \( \tau^B_N \). It is clear that the sign of \( d\tau^B_N \) due to a change in the country relative size cannot be determined in this general setting, and, therefore, the aggregate effect of a change in \( B \)'s relative size on \( A \)'s per capita utility is still ambiguous.

Instead of imposing more assumptions on the consumption and production structure
of the model like the previous works such as Kennan and Riezman (1988). Syropoulos evaluates (2.3) in the limit when \( B \) becomes infinitesimally small \( \lambda^B \to 0 \). As a result, from (2.3), the strategic effect of a change in \( \lambda^B \) on \( u^A_N \) becomes negligible and an increase in \( \lambda^B \) will always benefit \( A \) in the trade war equilibrium. This is because when \( B \) is infinitesimally small, the equilibrium converges to autarky, i.e. there is nearly no trade. When \( \lambda^B \) increases, \( A \) always benefits from an increase in the volume of trade. More formally, \( \lim_{\lambda^B \to 0} \frac{du^A_N}{d\lambda^B} > 0 \). This finding identifies the channel through which a change in the country relative size affects the trade war outcome. However, to determine whether the big country can win the trade war, the free trade equilibrium must be considered for comparison.

Under free trade, \( \tau^i_F = 0 \) for \( i = A, B \), (2.2) reduces to \( e^A du^A_F = \lambda^B m^B_F (-\hat{q}^A) \). Syropoulos shows that, under free trade, \( (-\hat{q}^A) = \frac{\hat{\lambda}^B}{\epsilon^A + \epsilon^B - 1} \) where \( \epsilon^A + \epsilon^B - 1 > 0 \) and \( \hat{\lambda} = \frac{d\lambda^B}{\lambda^B} \). Consequently,

\[
e^A du^A_F = \frac{m^B_F}{\epsilon^A + \epsilon^B - 1} d\lambda^B
\]  

(2.4) states that under free trade, \( A \)'s per capita utility is raised when \( B \)'s relative size increases. This is because when \( B \)'s relative size increases, its import demand increases, and to restore equilibrium, the price of \( A \)'s export must increase which implies an improvement in \( A \)'s terms of trade and its welfare, i.e. from (2.4), \( -\hat{q}^A = \frac{\hat{\lambda}^B}{\epsilon^A + \epsilon^B - 1} > 0 \) for \( \lambda^B > 0 \Rightarrow e^A du^A > 0 \).

In the limit when \( \lambda^B \to 0 \), similar to the trade war equilibrium, the free trade equilibrium also converges to autarky, \( m^B_F \to 0 \) which, by (2.4), implies that \( \lim_{\lambda^B \to 0} \frac{du^A_F}{d\lambda^B} = 0 \). Since \( \lim_{\lambda^B \to 0} \frac{du^A_N}{d\lambda^B} = 0 \), \( \lim_{\lambda^B \to 0} \frac{du^A_F}{d\lambda^B} > 0 \) and both \( u^A_N \) and \( u^A_F \) converge to \( u^A_A \) (\( A \)'s per capita utility under autarky), this means that \( u^A_N > u^A_F \) in the neighbourhood of \( \lambda^B \to 0 \) which implies that \( A \) always wins the trade war when its relative size is very large. This is shown in Figure 2-1.
\( \lambda^A \) is the threshold relative size of A for the relative size above which A can win the trade war. This limit analysis allows Syropoulos to conclude that a country can win a trade war if its relative size is sufficiently large. And as shown in Figure 2-1, \( \lim_{\lambda \rightarrow -0} u^A = \lim_{\lambda \rightarrow -0} u^B = \lim_{\lambda \rightarrow -0} u^N \), i.e. the infinitely large country A is indifferent between autarky, free trade, and the trade war equilibrium, which raises a question of the infinitely large country's incentive to intervene in free trade.

The same threshold value for which country B will win the trade war, \( \lambda^B \), also exists. From the fact that \( \lambda^A = \frac{1}{\lambda^B} \), there exists another threshold value \( \lambda^A = \frac{1}{\lambda^B} \) for A's relative size below which A surely loses. Therefore, country A will lose and B will win for \( \lambda^A \in (0, \lambda^A) \) and country A will win and B will lose for \( \lambda^A \in (\lambda^A, \infty) \). However, within the general structure of the model the trade war outcome when \( \lambda^A \in [\lambda^A, \lambda^A] \) is indeterminate as the functions \( u^A_F \) and \( u^A_N \) in Figure 2-1 do not necessarily intersect only once at \( \lambda^A = \lambda^A \). Syropoulos identifies the sufficient assumptions on the countries' price elasticities of import demand which ensure that the intersection between \( u^A_F \) and \( u^A_N \) in Figure 2-1 at \( \lambda^A = \lambda^A \) is unique. It is shown in Figure 2-1 that if the intersection

Figure 2-1: Syropoulos (2002) Relative size effect.
is unique, both countries lose in the trade war equilibrium for the intermediate values of \( \lambda^A \in [\lambda^A, \lambda^A] \).

In general, in the two-country, two-good model, the theory of optimum tariffs and retaliation argues that the trade war arises from the countries’ unilateral incentive to manipulate their terms of trade using their tariffs. The country that wins the trade war must be able to impose a larger tariff. The relative market power of a country to impose a tariff depends on its own price elasticity of import demand and that of the other country. The country whose price elasticity of import demand is higher will have greater market power to impose the larger tariff. The literature suggests that the country’s price elasticity of import demand is positively related with the country’s size. Therefore, it is argued that the country whose size is larger can win the trade war.

2.1.2 International trade agreements

Due to the countries’ unilateral incentive to impose tariffs and hence their tariffs’ externalities, there exists an inefficiency in the trade war equilibrium. The role of eliminating this inefficiency is a rationale for the trade agreement provided in the literature.\(^1\) The literature views the trade agreement as the product of cooperative international tariff setting. In fact, there exists a set of trade agreements (combinations of the countries’ trade policies) which are Pareto efficient. Each of which implies different welfare effects on the negotiating countries. The Nash bargaining solution models the negotiation of the trade agreement to select a unique agreement among those efficient ones. The static model of trade negotiation, therefore, focuses on how the gain from the cooperation is to be divided among the negotiating countries.\(^5\)

---

\(^1\)In fact, there is another rationale for the trade agreement which is termed by Bagwell and Staiger (2002) as the ‘commitment approach’. The commitment approach views the trade agreement as one way for governments to credibly distance themselves from the domestic political pressure over trade policies. An example of this approach is Mitra (2002).

\(^5\)In light of the fact that there is no supra-national organisation which can punish the countries who fail to comply with the trade agreement, the trade agreement must be self enforcing. The literature addresses this issue in the dynamic repeated game framework. The fact that the equilibrium trade agreement must be sustainable such that no member country has an incentive to defect restricts the set
Role of trade agreements

In the standard two-country, two-good model, Bagwell and Staiger (1999, 2002) show that the inefficiency in the trade war equilibrium arises solely from the countries’ unilateral incentive to manipulate their terms of trade and the trade agreement is the means by which this terms of trade incentive can be removed.

Given countries $A$ and $B$ trading goods $x$ and $y$, $A$ is assumed to export good $x$ and import good $y$. Consumption and production of the two goods take place under perfect competition and are the functions of the domestic relative price denoted by $p_i = \frac{p_j^i}{p_k^i}$ for $i = A, B$, where $p_j^i$ is the domestic price of good $j$ in country $i$. Each country imposes a non-prohibitive ad valorem tariff $\tau^i$ on its imports. Therefore, given $T^i = 1 + \tau^i$, it is implied that $p_A^A = T_A^A p^w \equiv p^A (T_A^A, p^w)$ and $p_B^B = p^w \frac{p_B^2}{p_A^2}$ represents the world relative price. The trade balance and market clearing condition imply that the equilibrium world relative price denoted by $\tilde{p}^w$ can be represented by $\tilde{p}^w (T_A^A, T_B^B)$. Note that $\frac{dp_A^B}{dT_A^A} < 0 < \frac{dp_A^B}{dT_B^B}$ and it is assumed that $\frac{dp_B^w}{dT_A^A} < 0 < \frac{dp_B^w}{dT_B^B}$.

Bagwell and Staiger represent each government’s objective function as $U^i (p^i, \tilde{p}^w)$ to isolate the tariff’s terms of trade effect (the terms of trade effect amounts to the change in $\tilde{p}^w$). Given this general class of the objective function, it is assumed that $U^A_{p_A} = \frac{\partial U^A(p_A, \tilde{p}^w)}{\partial \tilde{p}^w} < 0 < \frac{\partial U^B(p_B, \tilde{p}^w)}{\partial \tilde{p}^w} \equiv U^B_{p_B}$, i.e. given the domestic price, each government will be better off when its country’s terms of trade improves. The non-cooperative first order conditions, given $U^i_{p} \equiv \frac{\partial U^i(p^i, \tilde{p}^w)}{\partial p^i}$, can be represented by

$$
A : \quad U^A_{p_A} + \lambda^A U^A_{\tilde{p}^w} = 0 \\
B : \quad U^B_{p_B} + \lambda^B U^B_{\tilde{p}^w} = 0
$$

of the possible agreements. The sustainable trade agreements may not be fully efficient, however, the resulted equilibrium is more efficient than the non-cooperative one. Examples of works which consider the issue of enforcement are Dixit (1987), Furusawa (1999) and Park (2000).

6 $U^i (p^i, \tilde{p}^w)$ does not necessarily represent the country’s aggregate welfare since Bagwell and Staiger also allow for the government’s political considerations, the second argument for tariffs dealt with by this thesis.
where \( \lambda^A = \frac{\partial w}{\partial T^A} \) and \( \lambda^B = \frac{\partial w}{\partial T^B} \). Bagwell and Staiger define the efficient trade agreement as any tariff combination which satisfies

\[
\left. \frac{dT^B}{dT^A} \right|_{dU^A=0} = \left. \frac{dT^B}{dT^A} \right|_{dU^B=0} \tag{2.6}
\]

\( \left. \frac{dT^B}{dT^A} \right|_{dU^A=0} \) and \( \left. \frac{dT^B}{dT^A} \right|_{dU^B=0} \) are the slopes of A’s and B’s indifference curves in the tariffs space, respectively. Therefore, (2.6) simply defines the locus where the two countries’ indifference curves are tangent.

Bagwell and Staiger show that the non-cooperative equilibrium tariffs implied by (2.5) are not efficient as they cannot satisfy (2.6) simultaneously. This will be possible only if, hypothetically, both countries take \( x_A = 0 = x_B \) in (2.5), i.e. the two governments are not motivated by the terms of trade incentive. Therefore, it is implied that the inefficiency arises solely from the countries’ unilateral incentive to manipulate their terms of trade. Intuitively, the trade war is inefficient since \( A \) and \( B \) can still gain from an increase in their trade volume if they can mutually reduce their unilateral tariffs of which the effects on the world relative price tend to offset each other.

It can be shown without going further into details that the countries act as if they set their tariffs unilaterally taking \( \lambda^A = 0 = \lambda^B \) when they set their tariffs cooperatively.\(^7\) As will be clear shortly, reciprocal free trade is an efficient agreement which satisfies (2.6) and can be achieved only in the negotiation between symmetric countries. It is clear that the reciprocal free trade also satisfies (2.5) if \( \lambda^A = 0 = \lambda^B \).\(^8\) Therefore, it can be argued

\(^7\)Bagwell and Staiger show that the countries act as if they set their tariffs unilaterally taking \( \lambda^A = 0 = \lambda^B \) if they reduce their tariffs under the GATT’s principle of reciprocity. They distinguish the approaches to trade negotiation between the ‘rules-based’ and ‘power-based’ negotiation. Under the rules-based negotiation, as practised under the GATT, the governments agree upon a set of negotiation rules which will be abided by all subsequent negotiations. On the contrary, the agreement under the power-based negotiation purely reflects the bargaining power of the negotiating parties as they are unconstrained by any previous agreed upon rules. They show that the rules-based might be preferable to the power-based negotiation for the relatively weaker countries as it moderates the power asymmetries between countries. The trade negotiation studied in this thesis can be classified as the power-based negotiation.

\(^8\)Under the terms of trade argument for tariffs, the unilateral optimal trade policy for both countries
that the trade agreement is efficiency enhancing by its ability to remove the countries' terms of trade incentive.

**Efficient trade agreements**

Indeed, the efficient tariff combination between $A$ and $B$ as implied by (2.6) is not unique. Mayer (1981) is the first to show that the efficient tariff combination can be any pair of $T^A$ and $T^B$ which satisfy

$$T^A T^B = 1 \quad (2.7)$$

This is because these combinations of tariffs equalise the domestic relative prices of both countries.\(^9\) The equal domestic relative prices is the condition for both countries to trade at the efficient frontier. If the countries still find their imports cheaper abroad they can still gain from a larger trade volume by reducing their tariffs. Clearly, the reciprocal free trade where $T^A = T^B = 1$ is among those efficient tariff combinations. However, whether the free trade agreement can be attained still depends on the relative market power of the two countries. This is because even if a trade agreement is efficient, both countries must be at least as well off with the trade agreement as in the non-cooperative equilibrium for the trade agreement to be acceptable. If, in the non-cooperative equilibrium, a country already has a utility level above the free trade level, i.e. it can win the trade war, the free trade agreement will not be acceptable to that country.\(^10\) This point

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\(^9\) Recall that the domestic relative prices of both countries are $p^A = T^A p^w$ and $p^B = T^B p^w$. Therefore, $p^A = p^B \Rightarrow T^A T^B = 1$.

\(^{10}\) This conclusion neglects the possibility that the losing country may bribe the winning country in the trade war to move to free trade by offering a lump-sum transfer to compensate the winning country their forgone welfare from imposing tariff. An example of the papers which consider this type of trade agreement is Park (2000). In the repeated game framework, he finds that, relative to the negotiation without direct transfer, the negotiation with direct transfer between a small and a large country can improve the set of sustainable agreements to the advantage of the small country and hence improve its bargaining position. Park (2000) argues that the non-tariff concessions such as stricter protection of
is made by Riezman (1982) who shows that the free trade agreement can be attained only between the symmetric countries. Therefore, from (2.7), the trade agreement between the asymmetric countries must be a subsidy-tariff combination.

**Nash bargaining solution**

As the efficient trade agreement is not unique, additional assumption is needed to select a unique equilibrium trade agreement among those efficient trade agreements. A trade negotiation between two countries is usually modelled as a Nash bargaining game in which the two countries cooperatively maximise the product of their gains over the non-cooperative outcome, i.e.

\[
\max_{U^A, U^B} \left( U^A - U^A_N \right)^\alpha \left( U^B - U^B_N \right)^{1-\alpha} \quad \text{s.t.} \quad U^A \geq U^A_N \quad \text{and} \quad U^B \geq U^B_N
\]

(2.8)

\( U^i \) and \( U^i_N \) are country \( i \)'s cooperative and non-cooperative utility level. \( \alpha \) captures the relative bargaining power of \( A \). The constraints \( U^A \geq U^A_N \) and \( U^B \geq U^B_N \) require that both countries must be at least as well off as in the non-cooperative equilibrium. The curve \( LL' \) in Figure 2-2 represents the efficient frontier of welfare combinations underlain by the efficient tariff combinations which satisfy (2.7).

The unique equilibrium trade agreement is determined by the tangency between the level curve \( (U^A - U^A_N)^\alpha \left( U^B - U^B_N \right)^{1-\alpha} \) and the efficient frontier \( LL' \). Figure 2-2 is drawn assuming that both countries have symmetric market power \( (U^A_N = U^B_N < U^A_P = U^B_P) \). However, it is assumed that \( B \) has greater bargaining power than \( A \), i.e. \( \alpha < \frac{1}{2} \).11

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11 Furusawa (1999) interprets \( \alpha \) as reflecting the government’s time patience. He models a trade negotiation between two governments, which are different only in their time patience, as a Nash bargaining game. The outcome of which will be implemented by the two governments and must be sustained over
Therefore, the distribution of gain from the agreement agreed by the two countries is at point C where B’s gain is higher than A’s. The equilibrium trade agreement is the tariff pair \((T_C^A, T_C^B)\) which implies A’s and B’s utility level at point C.\(^\text{12}\) As mentioned earlier, Riezman (1982) shows that \(F\) will be the tangency between the level curve \((U^A - U_N^A)^\alpha (U^B - U_N^B)^{1-\alpha}\) and the efficient frontier \(LL'\) which implies a reciprocal free trade when both countries are symmetric in terms of their market and bargaining power.\(^\text{13}\)\n
---

\(^{12}\) According to (2.7), \((T_D^A, T_D^B)\) which underlie point C in Figure 2-2 must be a subsidy-tariff combination in which there are an import subsidy by A and an import tariff by B.

\(^{13}\) If an international transfer mechanism is available, both A and B can agree to implement free trade which underlies point \(F\) in Figure 2-2 straightforwardly. However, due to the asymmetry in their bargaining power assumed in Figure 2-2, B must obtain a level of utility at \(U_B^N\) which is higher than the level of utility \(U_B^F\) implied by the reciprocal free trade agreement. Therefore, it is possible that A can bargain the amount of direct transfer required by B to implement the reciprocal free trade. Figure 2-2 shows that the equilibrium agreement between the two countries in addition to the reciprocal trade agreement must be the amount of direct transfer equal to \(U_C^B - U_F^B\) that A must make to support the agreement. It is clear that free trade is still a possible outcome even if one country’s welfare without an agreement is already higher than the level it obtains under the reciprocal free trade.
2.1.3 Customs union

The theory of customs union has concerned the welfare effects of a customs union on its members as well as the rest of the world. Pioneered by Viner (1950), the welfare effects of a customs union is studied along the trade creation-trade diversion approach. However, there is also the terms of trade approach, as called by Riezman (1979a), which usually employs more restrictive models (e.g. the three-country, two good model) but uses general equilibrium analysis. In this approach, such as Riezman (1979a, 1979b), Kennan and Riezman (1990) and Syropoulos (1999), it is recognised that the resulting changes in trade volumes, and possibly in the direction and composition of trade flows due to the intra-union trade liberalisation and the coordination of external tariff policies may improve the terms of trade of its members against the outsiders.

Riezman (1979a, 1979b) examines the condition for two countries to benefit from forming a customs union in the three-country, two-good and the three-country, three good models. Assume that two of the three countries can form a customs union by reducing tariffs on imports from each other but keep their tariffs on import from the third country at the pre-union level. He examines the welfare effects of the decrease in the tariffs. He finds that, in the three-good model, change in the terms of trade between the union members due to the integration is indeterminate while, in the two-good model, one member benefits at the expense of the other. However, he also finds that the union's terms of trade with the third country is necessarily improved. Therefore, it is argued that the pre-union trade between the two members must be small so that the terms of trade benefit against the third country dominates and hence both countries can gain from the customs union.

Kennan and Riezman (1990) construct a three-country, three-good model in which members of the customs union have internal free trade and jointly set a common opti-

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14 In a partial equilibrium analysis, Panagariya (2000) shows that the intra-union trade liberalisation creates the 'revenue-transfer effect' in which the liberalisation causes one country to lose its tariff revenue but the other country gains in the form of the producer surplus relative to the non-cooperative equilibrium.
mum external tariff against the optimum tariff of the third country. They find that the coordination of external tariff policies by the customs union members enables them to internalise the positive terms of trade externalities they generate for each other when they import similar products from the third country.

Syropoulos (1999), within the Kennan and Riezman (1990) framework, argues that there exists a 'trade liberalising' force, generated by the intra-union trade liberalisation, that moderates the collective market power of the union members to impose a common optimum external tariff. However, he finds that the customs union still benefits the members and harms the outsider depending on trade patterns and comparative advantage.

### 2.1.4 Summary

It can be seen that the study of trade negotiation is usually made in the two-country context in which the trade agreement is negotiated between countries that import and export different goods. Therefore, the source of gain from the trade agreement in this line of literature is from an increase in the countries' trade volume. However, as recognised in the customs union literature, another source of gain from the trade agreement is that it enables the member countries to exploit their collective market power to improve their terms of trade. It is argued that the countries that import the same good can internalise their positive tariff externality by setting their tariff jointly in a customs union. Clearly, a country can expect different sources of gain from trade agreement from different negotiating partners.

Chapter 4 examines the choice of a country to negotiate a trade agreement if there are different negotiating partners and hence different sources of gain available to that country. Despite the popularity of the three-country, three-good model in the customs union literature, we base our analysis on a many-country, two-good model which shares

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15Richardson (1994) points out that the popularity of the $3 \times 3$ model stems from its symmetry and the fact that all countries trade with each other - both of which cannot hold in the $3 \times 2$ model. Lloyd (1982) points out that the $3 \times 3$ model introduces several effects which cannot be present in the $3 \times 2$ model.
the same trade patterns with the three-country, two-good model in the customs union literature. The similarity is that there exists more than one country that exports one good.\textsuperscript{16} The advantage of this trade pattern is that it reduces the dimensions of the model which eases the derivations that are complicate even in the two-country, two-good setting. Base on this model, chapter 3 also investigates the importance of the world market structure of a good, which depends on the market share distribution between its exporters and the number of them, in determining a trade war outcome which seems to be missing in the literature.

2.2 Political economy of trade policy

The terms of trade argument for tariffs discussed in the previous section is based on the assumption that the governments maximise the national welfare. However, in the real world where the existence of the governments depends largely on politics, it is reasonable to view the governments’ objectives, particularly with regard to the making of trade policy, as not only to serve the national welfare but also to serve their political concerns. Despite this common belief about the importance of politics in the trade policy formation and the various approaches to explain it, there still exists no coherent theory. This apparently reflects the fact that there are many ways through which individuals can influence their government’s decision on economic policies which in turn affect their individual well-being. Those ways vary across times and countries depending on their specific political and institutional settings. Theoretical works on the political economy of trade policy attempt to model this relationship between the government and its people in determining trade policies through different approaches.

Mayer (1984)’s median voter model views the trade policy as the outcome of majority

\textsuperscript{16}Richardson (1994) chooses the three-country, two-good model to investigate the possibility that the union members can circumvent the commitment to liberalise their trade with the other member by the use of domestic policy since it is critical for his analysis that the union common external tariff has the discriminatory nature and this is not possible if imports of the union member come from only one source.
voting over a tariff level. Given the difference in the individuals’ single-peaked preferences toward the tariff level due to the difference in their factor endowments (equal labour but uneven capital endowments across individuals), over the continuum of individuals ranked in ascending order of their factor endowments hence their tariff level preferences, the political equilibrium trade policy predicted by the model is the one most preferred by the median voter under the majority voting rule. This is because at this tariff level, neither those who prefer smaller nor those who prefer larger tariff rate than the one preferred by the median voter will be the majority. Laussel and Riezman (2001) extend this framework to consider the trade policy formation in the representative democracy. In the two-country two-good setting and the ‘citizen candidate’ model,\textsuperscript{17} they find that the trade policy tends to be biased against trade as, due to the terms of trade effect of tariffs, the citizens in each country tend to delegate the trade policy choice to somebody who is more protectionist than themselves to credibly commit their respective country to a more aggressive trade policy.

Magee et. al. (1989), differently, view the trade policy as the outcome of electoral competition in which political parties choose their trade policy stances to attract campaign contributions from interest groups which are positively related with their chance of winning the election. In the electoral competition model, the lobbies choose their levels of contribution to maximise the sum of their members’ income less contribution expenditure given the policy stances previously announced by the political parties. Anticipating their probabilities of winning the election (which depend on the campaign contributions), the political parties choose their policy stances to maximise such probabilities and the equilibrium trade policy is the outcome of this process.

Hillman (1982) postulates the government’s objective function as the political-support function which positively depends on the industry’s profits and negatively depends on

\textsuperscript{17}As opposed to the standard theory in which the policy maker is not one of the citizens and has no personal preference toward any policy, the citizen candidate model is the model in which a policy maker is elected from the set of citizens who decide to run for election and set the policy according to his/her preference.
the consumer's welfare lose from the trade policy which induces domestic price increase. Similarly, Findlay and Wellisz (1982) simply assumes a functional relationship between lobbying contributions and tariffs, i.e. a tariff function which positively depends on the contribution from pro-tariff lobby and negatively depends on the contribution from anti-tariff lobby. Given the tariff function, the interest groups choose their contribution level to maximise their payoffs. It is found that the trade policy outcome depends on the relative weight the government attaches to the industry profit and the consumer welfare in Hillman (1982) and depends on the marginal rate of substitution between the opposing lobbies' contributions in the tariff function in Findlay and Wellisz (1982). In general, these two works provide a better understanding of the forces that shape the structure of tariffs. However, by abstracting from explaining the political process in which the government interacts with the individuals or interest groups, no fundamental determinant of trade policy is provided by these two approaches.

The interaction between the government and the interest groups is modelled explicitly in Grossman and Helpman (1994). Applying the price-menu auction framework of Bernheim and Whinston (1986), the government is viewed as the auctioneer and the lobbies as the bidders. Within the specific-factor model, it is assumed that the lobby group is formed between the factor owners along the sectoral line and the number of sectors in which a lobby is formed is constant. First, the given set of lobbies non-cooperatively announce their contribution schedules which specify the amount of contribution they will make against every possible trade policy vector which will be implemented by the government. Then, given the contribution schedules announced by the lobbies, the government chooses a trade policy vector to maximise the weighted sum between the contributions from lobbies and the general welfare, implements the policy and collects the contributions. As a result, it is shown that the government is effectively maximising the weighted sum between the interest groups' and the aggregate welfare which is clearly a form of the more general Hillman (1982) political support function.\(^{18}\) The political equilibrium

\(^{18}\)Mitra (1999) claims that the Grossman and Helpman (1994) model provides microfoundations for
The trade policy derived by Grossman and Helpman (1994) is

\[
\frac{\tau_i}{1 + \tau_i} = \frac{I_i - \alpha z_i}{a + \alpha \epsilon_i}
\] (2.9)

for \( i = 1, 2, \ldots, M \), \( M \) is the number of non-numeraire sectors. \( \tau_i \), \( z_i \) and \( \epsilon_i \) are the equilibrium ad valorem tariff, domestic output to imports ratio and the elasticity of import demand in sector \( i \), respectively (\( z_i \) and \( \epsilon_i \) are negative for the export sectors). \( I_i \) equal 1 if sector \( i \) is represented by a lobby and 0 otherwise. \( a \) is the weight the government attaches to the general welfare relative to the contribution from the lobbies in its objective function and \( \alpha \in [0, 1] \) denotes the fraction of the total population whose sectors are organised and represented by a lobby, the set of which is treated as exogenous. It can be seen that sectors that can be organised will be able to obtain the policies which favour their interests (the policies which increase the domestic prices in their sectors), however, at the expense of the public as a whole (higher prices in the protected sectors faced by the consumers). The level of protection for the organised sectors will be higher when the ratio of domestic output to imports in that sector is higher, and lower when the import demand elasticity in that sector, the weight the government attaches to the general welfare and the fraction of population represented by a lobby are higher.

However, while the model can explain the political process in which particular groups can translate their interests into the government policies, the existence of the lobbies is still exogenous. Reuben (2002) points out the need to concentrate on lobby formation as the best way of continuing with research on interest groups. He argues that ‘Not modelling group formation would leave a big gap in our understanding... (and) our models will be limited to explaining short term, static situations in which interest groups neither form nor expire.’
2.2.1 Lobby formation

Mitra (1999) extends the Grossman and Helpman (1994) framework by endogenising the number of lobbies which is kept constant. He analyses the relationship between the number of lobbies and the country income inequality and finds the general conditions of a sector of production in which a lobby group will be organised. He employs the specific-factor model to study the lobbying and trade policy in a small open economy with \( N \) total population and \( M \) sectors. Consumer preferences are identical represented by the additive separable utility function and the technology of production using both sector-specific factors and labour (except production of numeraire good using only labour where the wage rate is normalised to 1) is assumed to be constant return to scale. Each individual owns equal amount of labour and at most one type of \( M \) sector-specific factors.

To simplify his analysis, unlike Grossman and Helpman (1994), all sectors are assumed to be symmetric such that there are equal number of specific factor owners \( n \) and equal amount of sector-specific factor. The only difference between sectors is their fixed cost of forming a lobby group to represent their sectors, \( F_i \), \( \forall i = 1, 2, \ldots, M \).

The lobbies non-cooperatively announce their contribution schedules and then, given the announced contribution schedules, the government chooses a trade policy vector to maximise the weighted sum of contributions and the general welfare, the Grossman and Helpman result identical to (2.9) is obtained.

\[
\frac{\tau_i}{1 + \tau_i} = \frac{I_i - \frac{mn}{N} z_i}{a + \frac{mn}{N} \epsilon_i}
\]

where \( m \in (0, N] \) is the number of sectors which are organised. It is clear that \( \frac{mn}{N} = \alpha \) in (2.9) which represents the fraction of the total population whose sectors are represented by a lobby group. Mitra defines \( \theta = \frac{nM}{N} \) which is the proportion of population that owns some specific factors of production and interprets it as a measure of the degree of income inequality, i.e. a decrease in \( \theta \) means a greater income inequality.
in the economy. Substitute $\theta$ into the result above.

$$\frac{\tau_i}{1 + \tau_i} = \frac{I_i - \theta \frac{m}{M} z_i}{a + \theta \frac{m}{M} \epsilon_i}$$

(2.10)

A decrease in $\theta$ leads to a higher level of protection in all organised sectors which implies a higher benefit from lobbying keeping the number of organised sectors $m$ constant. However, if the number of lobbies is to be endogenised, the higher benefit from lobbying from a decrease in $\theta$ also attracts a greater number of organised sectors $m$ which reduces the level of protection in all organised sectors. Therefore, it is no longer clear that a greater income inequality would lead to more or less protection when the number of lobbies is endogenised. Similarly, the effect of a change in the weight the government attaches to the general welfare, $a$, on the level of protection, predicted to be negative in Grossman and Helpman (1994), is also ambiguous. The effects of a change in $\theta$ and $a$ on the level of protection, therefore, depend on whether the direct effect or the entry effect will dominate. It is from this point which Mitra extends the Grossman and Helpman framework.

Prior to the stages in which a set of lobbies interacts with the government, it is assumed that the owners of each kind of the specific factor decide whether to contribute to the financing of the fixed cost of forming a lobby $F_i$. When there are $m$ lobbies, let $\tilde{\Omega}_o(m)$ and $\tilde{\Omega}_u(m)$ denote the equilibrium gross welfare of an organised group and that of an unorganised group, respectively, and $\tilde{C}(m)$ denotes the equilibrium contribution each organised lobby has to make.19 Let sectors be ranked and indexed in ascending order of their fixed cost $F_i$ such that $F_{\min} \leq F_1 < F_2 \ldots < F_{m-1} < F_m < F_{m+1} \ldots < F_M \leq F_{\max}$. Taking $i - 1$ groups as organised and let the members of another group decide whether to form a lobby or remain unorganised. It is argued that a lobby group will be formed

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19 $\tilde{\Omega}_o(m)$ and $\tilde{\Omega}_u(m)$ are the sum of welfare of the individuals who own the same specific factor. The former represents their aggregate welfare if their sector is represented by a lobby and the latter if not. $\tilde{\Omega}_o(m)$, $\tilde{\Omega}_u(m)$ and $\tilde{C}(m)$ are the same for all sectors since they are assumed to be identical.
in sector $i$ if

$$\tilde{\Omega}_o(i) - \tilde{\Omega}_u(i - 1) - \bar{C}(i) > F_i$$

$\tilde{\Omega}_o(i) - \tilde{\Omega}_u(i - 1) - \bar{C}(i) = NB(i)$ represents the net benefit from forming a lobby for sector $i$ given all sectors indexed by $j < i$ are organised. Therefore, the above expression states that the owners of the specific factor in sector $i$ will form a lobby if the net benefit from forming a lobby to the group as a whole exceeds their fixed organisational costs.

Consider the continuum of sectors normalised to 1. Mitra shows that $\frac{\partial NB(m)}{\partial m} < 0$, i.e. the net benefit from forming a lobby for a sector declines as more sectors are represented by a lobby group (the number of lobbies rises). The net benefit from forming a lobby is diminishing with the number of lobbies because there will be more lobbies working against each other and a smaller unorganised population to exploit. Since the continuum of sectors is ranked in ascending order of their fixed cost of lobbying, the fixed cost can be written as a function of the number of lobbies as $F(m)$ and it is obvious that $\frac{\partial F(m)}{\partial m} > 0$. Consequently, a unique equilibrium number of lobbies can be determined in Figure 2-3.

It is shown that not every sector will always be organised. Given the asymmetry in the fixed cost across sectors and the fact that the net benefit from lobbying is diminishing with the number of lobbies, sectors with high fixed cost will be the ones that are unorganised. Comparative statics with respect to $\theta$ and $a$ reveals that greater income inequality (decrease in $\theta$) results in an increase in the equilibrium number of lobbies (it shifts the function $NB$ in Figure 2-3 outward) and its direct effect on the level of protection dominates the entry effect such that the level of protection is higher. A higher affinity of the government for political contributions (decrease in $a$) also results in an

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20Mitra shows that, for a continuum of identical sectors ranked and indexed in ascending order of their fixed cost, if an infinitesimal mass around sector indexed by $i$ is organised, then masses with a lower fixed cost, i.e. $\forall j < i$, should also be organised.
increase in the equilibrium number of lobbies, however, the level of protection each organised sector obtains in equilibrium will rise or fall depending on the slope of the fixed cost function $F(m)$. Relative to $F(m)$ in Figure 2-3, the direct effect will be more likely to dominate the entry effect, i.e. a decrease in $a$ is more likely to increase the level of protection for organised sectors, when $F(m)$ is steeper so that given the same decrease in $a$, the number of entries is smaller.

Mitra imposes more restrictive assumptions on the model to examine the effects of some other factors in addition to the fixed cost on the lobby formation. In general, he shows that those sectors that have the advantage of having high capital stock, high concentration of the specific-factor owners, more inelastic demand in addition to having less fixed cost examined above, will be organised.

In general, Olson (1971) argues that organised groups are usually small in their number of members since if a group is so big that each member's action is imperceptible, every individual member would decide to free ride on the efforts of the other group members and the collective good is not produced. The literature addresses this issue in the repeated game framework in which the threat of ending cooperation keeps individuals
from free riding.

Pecorino (1998), like Findlay and Wellisz (1982), assumes a general tariff function which positively depends on the total contribution made by the firms in an industry, i.e. 
\[ \tau (S), \quad \tau'(S) > 0, \quad \tau''(S) < 0 \quad \text{and} \quad \tau(0) = 0 \]
where \( S = \sum_{i=1}^{n} s^i \) and \( s^i \) represents the contribution made by firm \( i \) and \( n \) is the number of firms in the industry. Given the tariff function, the firms choose the contribution cooperatively to maximise the industry net profit, i.e. the sum of their profits, which depend positively on the tariff, less the aggregate contribution expenditure. In the cooperative equilibrium, the firms obtain the level of protection denoted by \( \tau_C \). By the assumption that all firms are identical, they pay the cooperative contribution equally, i.e. \( s^i_C = s_C = \frac{S_C}{n} \) for all \( i = 1, \ldots, n \). Given \( \tau_C \) and \( s_C \) and let \( \pi \) denotes each firm profit, the net profit accruing to each firm in this cooperative equilibrium is \( \pi_C - s_C \). In an infinitely repeated game, each firm receives the net profit \( \pi_C - s_C \) in each period as long as no one defected in the previous period. If any one defected in the previous period, the cooperation is terminated and the equilibrium will be reverted to the non-cooperative equilibrium forever. When any firm defects, it chooses to pay an amount denoted by \( s_D < s_C \) (which maximises its individual net profit given that all other firms are paying \( s_C \)) and obtains a higher level of net profit in that period denoted by \( \pi_D - s_D > \pi_C - s_C \).\(^{21}\) In the periods of punishment, each firm makes their contribution non-cooperatively denoted by \( s_N \) and obtains the net benefit lower than the cooperative one denoted by \( \pi_N - s_N < \pi_C - s_C \).\(^{22}\) Therefore, the necessary condition to maintain the cooperation over the infinite periods is

\[
(\pi_D - s_D) + \frac{\delta}{1 - \delta} (\pi_N - s_N) \leq \frac{1}{1 - \delta} (\pi_C - s_C)
\]

\(^{21}\) Since the defecting firm pays a smaller amount of contribution \( s_D < s_C \) and obtains a level of tariff \( \tau_D \) which is not much less than \( \tau_C \) if \( s_C \) is small.

\(^{22}\) Since the level of protection they obtain in the non-cooperative equilibrium is \( \tau_N < \tau_C \). Even if they pay \( s_N < s_C \), their profit loss from the reduction in the level of protection is larger. Compared to the defecting equilibrium, \( \tau_D > \tau_N \) since all firms except the defecting firm still contribute \( s_C > s_N \), i.e. \( (n - 1)s_C + s_D > ns_N \Rightarrow \tau_D > \tau_N \) as \( \tau'(S) > 0 \).
The cooperation can be maintained if the present discounted value of defecting and suffering the punishment is not greater than the present discounted value of cooperating. The difficulty in maintaining the cooperation is measured by the value of the discount factor \( \delta^* \) which satisfies this condition with equality. The cooperation cannot be maintained for the values of discount factor below \( \delta^* \). Therefore, a rise in \( \delta^* \) indicates the greater difficulty in maintaining the cooperation. Solving the above expression with equality, \( \delta^* \) is obtained.

\[
\delta^* = \frac{(\pi_D - s_D) - (\pi_C - s_C)}{(\pi_D - s_D) - (\pi_N - s_N)}
\]

(2.11)

Since \( (\pi_N - s_N) < (\pi_C - s_C) < (\pi_D - s_D) \), \( \delta^* \leq 1 \). Pecorino shows that an increase in the number of firms \( n \) has two effects on the minimum discount factor \( \delta^* \). First, it increases \( \delta^* \) by increasing the desirability for defection, i.e. \( (\pi_D - s_D) \) is increased when the number of firms increases.\(^{23}\) Second, it decreases \( \delta^* \) by increasing the severity of punishment, i.e. \( (\pi_N - s_N) \) is decreased when the number of firms increases.\(^{24}\) The change in the number of firms does not affect the net profit each firm obtains in the cooperative equilibrium \( (\pi_C - s_C) \) since the total contribution made by all firms to maximise the industry net profit hence the level of protection remain unchanged. This is due to the fact that the size of the industry is kept constant and the size of each firm and the contribution each of them makes decreases proportionately as the number of firms rises. Given these two contrasting effects of an increase in \( n \) on \( \delta^* \) and not being able to determine which one dominates, Pecorino concludes that there is no presumption that the free rider

\(^{23}\)When the industry size is kept constant and all firms in the industry are identical, an increase in the number of firms implies a smaller size of each firm. When each firm is smaller, its contribution is smaller and, therefore, a reduction in any one firm’s contribution from \( s_C \) to \( s_D \) reduces less the level of protection, i.e. \( \tau_D \) moves closer to \( \tau_C \) from below when \( n \) increases. Consequently, any one firm can defect by paying less contribution and yet sacrificing less protection when there is larger number of firms.

\(^{24}\)When each firm is smaller as a result of an increase in the number of firms, each of them pays less amount of contribution in the non-cooperative equilibrium which lowers the level of the tariff protection they receive, i.e. \( \tau_N \) decreases as \( n \) increases, leading to their lower profit.
problem is worse when the number of firms is larger.

Magee (2002) follows Pecorino by considering the free rider problem within the repeated game framework and measuring the difficulty in maintaining the firms' cooperation by the minimum discount factor \(\delta^*\). He finds the same expression for \(\delta^*\) as in (2.11). However, by endogenising the Pecorino's tariff function as the outcome of the Nash bargaining game between the government and the industry over the level of contribution against each level of tariff, he finds the condition in which the second effect of an increase in the number of firms \(n\) on \(\delta^*\) (the effect which decreases \(\delta^*\) through its negative effect on \(\pi_N\)) is zero and is able to conclude that, under such condition, an increase in the number of firms always worsens the free rider problem.

Magee employs the specific factor model with only one non-numeraire import-competing sector, producing with constant return to scale technology, and using the specific factor and labour which are fixed in supply. Consumer preferences are identical and represented by the general additive separable utility function. Since there is only one non-numeraire sector, only one lobby operates in this model. In the first stage, the industry initially forms an exploratory group whose purpose is to discover the cost of achieving their policy goals (it is implicit that they can do so without cost). Their representatives meet and bargain over the contribution they will pay the government against each level of tariff. The outcome determines a tariff function which depends on the total contribution made by the industry. In the second stage, given the tariff function, the firms in the industry decide whether to contribute to the lobbying effort by comparing between the present values of their net profit if they defect and are punished by the reversion to the non-cooperative equilibrium and if they cooperate over the infinite periods.

Given that \(n\) firms in the industry are identical and each firm's profit is represented by \(\pi\), let \(U(p)\) and \(\Pi(p) = n\pi(p)\) represent the aggregate welfare and the industry aggregate profit respectively. Both depend on the domestic price \(p\). Given that \(\tau\) is the tariff rate, and \(p^w\) is the world price of the good produced by the industry which is constant, \(p = p^w + \tau\). Due to the fact that all firms are identical, they pay equal amount
of political contribution denoted by $s$ and $S = ns$ represents the total contribution the firms make to the government. The government's objective function is similar to that of Grossman and Helpman (1994), i.e. $S + aU(p)$, where $a$ is the weight the government attaches to the general welfare relative to the industry contribution. Since Magee views lobbying as the Nash bargaining between the government and the industry over the political contribution, the bargaining problem is such that

$$\max_S \{ [S + aU(p^w + \tau)] - aU(p^w) \}^\theta \{ [\Pi(p^w + \tau) - S] - \Pi(p^w) \}^{1-\theta}$$

$\theta$ is the government's bargaining power relative to the industry lobby. As $p^w$ is constant, the government will implement free trade without lobbying, therefore, the threat point if the bargaining breaks down is the equilibrium in which there is free trade, $\tau = 0$, and no political contribution is made, $S = 0$.

Solving the above bargaining problem gives

$$(2.12) \quad S(\tau) = \theta [\Pi(p^w + \tau) - \Pi(p^w)] + (1 - \theta) \ [aU(p^w) - aU(p^w + \tau)]$$

Solving (2.12) for $\tau$, a tariff function $\tau(S)$ is implied. By examining (2.12) in its general form, Magee shows that the tariff function implied by the bargaining between the government and the industry lobby has the properties of the tariff function assumed by Pecorino and all other works which use the tariff formation function approach, that is $\tau'(S) > 0$, $\tau''(S) < 0$, and $\tau(0) = 0$. Therefore, Magee claims that this result provides microfoundations for the tariff formation function approach.

In the second stage of the game where the cooperation is to be maintained over the infinite periods, Magee obtains the minimum discount factor $\delta^*$ represented by (2.11). He points out that if the firms contribute nothing in the non-cooperative equilibrium, i.e. no firm lobbies the government individually, the tariff protection in the non-cooperative
equilibrium will be zero, i.e. \( s_N = 0 \Rightarrow S_N = n s_N = 0 \Rightarrow \tau_N = 0 \) and dividing the industry into smaller firms (an increase in the number of firms) will have no impact on the non-cooperative outcome. Therefore, unlike the Pecorino's result, an increase in the number of firms will only raise the one-period benefit of defecting, i.e. a rise in \( n \) only increases \( (\pi_D - s_D) \) hence \( \delta^* \). Magee finds the condition in which this can occur, i.e. firms do not make contributions in the non-cooperative equilibrium, and hence is able to give a sharper conclusion than Pecorino that, under such condition, an increase in the number of firms will always worsen the free rider problem.

When firms do not cooperate, given the tariff function \( \tau(S) \) implied by (2.12) and the political contributions made by the other firms, each firm chooses an amount of political contribution to maximise its individual net profit, i.e. the optimisation problem for firm \( i \) is

\[
\max_{s^i} \pi^i (\tau(S_{-i} + s^i)) - s^i
\]

where \( S_{-i} \) is the sum of the political contributions made by firms other than \( i \). Since \( \frac{\partial \pi^i}{\partial \tau} > 0 \) and \( \frac{\partial \pi^i}{\partial s^i} > 0 \) for all \( i \), it can be said that the firms' political contributions are strategic complements. However, for firm \( i \) to pay a positive amount of political contribution in the non-cooperative equilibrium, the profit it earns from the first unit of the political contribution must exceed the political contribution it pays at the margin, i.e.

\[
\left. \frac{\partial \pi^i}{\partial \tau} \frac{\partial \tau}{\partial s^i} \right|_{s^i=0} > 1
\]

From (2.12), Magee is able to show that
Therefore, the condition in which firms do not make political contribution in the non-cooperative equilibrium is when \( n \geq 1/\theta \), i.e. when the number of firms is beyond a certain number and that number positively depends on the bargaining power of the industry relative to the government, \((1 - \theta)\). This is because each firm’s benefit from lobbying depends on \( \theta \) and \( n \) such that the lobby surplus (exploited from the general population and divided between the government and the industry lobby) shared by the industry as a whole depends on \( \theta \) and this share is divided equally between all firms in the industry (no matter they lobby cooperatively or non-cooperatively). An increase in either \( \theta \) or \( n \) reduces the lobby benefit accruing to each firm. In the non-cooperative equilibrium, when \( \theta \) and \( n \) are too high such that \( n \geq 1/\theta \) the profit each firm earns from the first unit of the political contribution will be less than the political contribution it pays at the margin and, therefore, discourages them from making a positive amount of contribution. In the symmetric Nash bargaining where \( \theta = \frac{1}{2} \Rightarrow n \geq 2 \), the maximum number of firms in the industry which allows them to lobby non-cooperatively is 1. Therefore, if the industry consists of more than one firm, the firms will not lobby non-cooperatively and an increase in the number of them always makes their cooperation more difficult.

2.2.2 Summary

Among others, the lobbying framework of Grossman and Helpman (1994) is successful in explaining the political process in which particular groups can translate their interests into the government policies. They view the trade policy as the outcome of the political lobbying. It is assumed that interest groups can influence the government’s decision by offering campaign contributions contingent on the trade policies implemented by the government. Instead of being benevolent, the government is willing to trade off some
reduction in the general welfare in return for the campaign contribution. As a result, groups that can be organised will be able to obtain the trade policies which favour their interests at the expense of the public as a whole. However, the existence of the lobbies in the model is still exogenous. In fact, there exist some works such as Pecorino (1998), Mitra (1999) and Magee (2002) which investigate the lobby formation. Still, there remain some basic issues that they tend to ignore and some assumptions which seem to be too restrictive that they can easily lead us to draw incorrect conclusions. First, as pointed out by Reuben (2002), these previous works still do not reflect what really drives collective behaviour. Based on the Olson (1971) argument, Pecorino (1998) and Magee (2002) examine the difficulty in maintaining the collective action as a function of the number of firms in an industry not what makes them act collectively in the first place. Mitra (1999) merely mentions that each firm might be small relative to the fixed cost of lobbying which prevents them from the unilateral action. Second, they seem to restrict a lobby to be formed only within a well-defined group such as among firms within the same industry. Similar to Mitra (1999), chapter 5 of this thesis attempts to endogenise the formation of lobby by extending the lobbying framework of Grossman and Helpman (1994). However, we investigate the individual's decision to join a lobby prior to the stages of interaction between the government and the lobbies based on the Mayer (1984) model. In the Mayer's model, the individuals' single-peaked preferences toward the tariff level are different due to the difference in their factor endowments (endowments of goods in this thesis). Therefore, the individuals are not belong to any industry and hence are not restricted to join any particular lobby. Indeed, they are purely different in their preferences toward the trade policy and their decisions to participate in lobbying are purely strategic. Therefore, the individual's incentive to act collectively can also be examined in this framework.
Chapter 3

Optimum Tariffs and Retaliation in the Multilateral Context

The main contribution of this chapter is the use of the many-country, two-good trade model, which can be found in the customs union literature, to investigate the importance of the (world) market structure on the welfare effects of tariffs. This model, where a good is exported by more than one country, allows us to examine the welfare effects of tariffs which vary with how the goods are divided initially among the countries. The theory of optimum tariffs and retaliation, usually in the two-country, two-good context, suggests that the country whose endowments of goods are relatively large tends to win a trade war. Still, the analysis in this chapter shows that there is a greater possibility for a country to win even if the country’s endowments are relatively small if the world market of its exportable moves closer to the monopolistic market, i.e. there are less countries exporting the same good and/or the world endowment of that good is divided more disproportionately among its exporters.
### 3.1 Introduction

A recent example of trade war is the US-EU steel trade dispute. Under Section 201 of the US trade law which provides for temporary restrictions on imports that have surged in such quantities as to cause or threaten to cause serious injury to a domestic industry, the Bush Administration announced on 5 March 2002 the decision to impose tariffs of up to 30 per cent on its steel imports for 3 years as a remedy to its steel industry which has been reportedly affected by imports rising to more than 20 per cent of its consumption. This invoked the response from the EU on 22 March 2002 which threatened to impose retaliatory tariffs on products such as motorcycles, textiles, and steel encompassing 360 million US dollars worth of US exports to pressure the US to reverse its decision.\(^1\) On the one hand, given the current efforts of the two economic powers to settle the dispute and the present multilateral environment characterised by the existence of the World Trade Organisation (WTO), it may seem far-fetched that a full blown trade war would occur. On the other hand, there has been reportedly no breakthrough on the issue and no sign that the EU is willing to back away from its threat to impose the punitive tariffs. Therefore, studying of the trade war deserves attention.

As both the US and the EU are the two largest trading partners in the world trade, they share 18.4 and 15.4 per cent in the world merchandise exports, and 23.5 and 18.2 per cent in the world merchandise imports, respectively,\(^2\) the implications of their disputes have gone much beyond the question of how they are affected if the trade war occurs. An interesting question is the implications of the disputes for the third countries especially those with less economic power, the developing countries. This chapter re-investigates the theory of optimum tariffs and retaliation in a multi-country setting which is capable of examining this question.

In the standard two-country two-good trade model, Johnson (1953) shows that the

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\(^1\) For more details, see Ahearn (2002) and Ahearn (2003).

country that has more elastic import demand tends to win the trade war. Kennan and Riezman (1988) were the first to relate the country's market power to impose an import tariff to its size. They construct a two-country two-good trade model in which the Cobb-Douglas preference and a world fixed endowment of each good are assumed (so that they measure the country's size by the level of its endowment). With the help of simulations, they show that the country whose endowment is larger, tends to win the trade war. In general, both papers show that both countries will lose when they are not too dissimilar. More recently, Syropoulos (2002) obtains the Kennan and Riezman (1988) results in a more general framework.

In general, the theory of optimum tariffs and retaliation argues that the trade war arises from the countries' unilateral incentive to manipulate their terms of trade. The country that wins the trade war must be able to impose a larger tariff. The relative market power of a country to impose a tariff depends on its own price elasticity of import demand and that of the other country. The country whose price elasticity of import demand is higher will have greater market power to impose the larger tariff. The literature suggests that the country's price elasticity of import demand is positively related with the country's size. Therefore, it is argued that the country whose size is larger can win the trade war. The analysis in this chapter is based on the Kennan and Riezman (1988) model but there are more than two countries sharing and exchanging the world fixed endowments of two goods. In contrast to the result obtained in the standard two-country two-good trade model, this chapter shows that, in the multi-country context, there is a greater possibility for a country to win a trade war even if the country's endowments are relatively small if the world market of its exportable moves closer to the monopolistic market, i.e. there are less countries exporting the same good and/or the world endowment of that good is divided more disproportionately among its exporters.

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3 A country's welfare must rise in the non-cooperative equilibrium relative to the free trade equilibrium for that country to win the trade war.

4 This many-country, two-good trade model is similar to the three-country, two-good model that has been used in the customs union literature, for example, Riezman (1979a) and Richardson (1994).
To give a sense of how the introduction of more countries affects the results, in section 3.2, a two-country two-good model is constructed in which each country sets its import tariff non-cooperatively and simultaneously to maximise its national welfare. We re-derive those results in the literature. However, in an extreme case where each country monopolises its exportable, we argue that the optimum tariffs (if they are to impose against each other) hence their welfare loss will be too high such that free trade can be the dominating strategy for both countries.

More countries are added to participate in the tariff game in section 3.3. With more countries but still two goods, there exists more than one country exporting the same good. This setting provides the possibility that the countries’ tariffs are strategic complements and hence the possibility that a relatively small-endowed country wins the trade war.\footnote{Some countries might even be able to benefit from other countries’ tariffs without having to impose positive tariff and sacrificing their own trade volume. Syropoulos (2002) proposes that identifying the exact circumstances under which this possibility might arise is important and theoretically challenging. This point will also be addressed in this section.} Conclusion is made in section 3.4.

### 3.2 Two-country model

The model is a pure exchange model between two countries, $A$ and $B$, trading two goods $x$ and $y$. Consumers in both countries have identical and homothetic preferences represented by the Cobb-Douglas utility function,

$$U^i = \left( c^i_x \right)^{\frac{1}{2}} \left( c^i_y \right)^{\frac{1}{2}}$$

where $c^i_x$ and $c^i_y$ are country $i$’s consumption of $x$ and $y$, $i \in \{A, B\}$.

Each country is initially endowed with both goods proportionately to the world endowment. Assume that the world endowment of $x$ is $X$ and $y$ is $Y$. Let $X^i$ and $Y^i$ are the country $i$’s endowment of $x$ and $y$. The endowment structure is
$$X^A = \beta_x X \quad Y^A = (1 - \beta_y) Y$$

$$X^B = (1 - \beta_x) X \quad Y^B = \beta_y Y \quad \text{(3.1)}$$

$0 \leq \beta_x, \beta_y \leq 1$. Assume that $A$ has the comparative advantage in $x$ and $B$ has the comparative advantage in $y$. Given the identical homothetic preferences, it is implied that $\beta_x + \beta_y > 1$ must be satisfied for trade to be positive.\textsuperscript{6} Since $\beta_j$ reflects the comparative advantage of the country exporting good $j$, $\beta_x$ and $\beta_y$ will be called the 'degree of specialisation' of $A$ and $B$, respectively.\textsuperscript{7} The greater the $\beta_j$ is, the greater is the trade interdependence (the trade volume) between the two countries.

Let $p_j^i$ be the domestic price of good $j$ in country $i$. Consumers' optimisation implies

$$\frac{c_j^i}{c_j} = \frac{p_j^i}{p_j^A} \quad \text{(3.2)}$$

Let $q_j$ be the world price of good $j$, choose the world price of $x$ as numeraire hence $q_x = 1$ and let $q_y = q$. Assume that both countries impose ad-valorem tariff rate $\tau^i \in (-1, \infty)$ on their imports.\textsuperscript{8} Therefore, (3.2) becomes

\textsuperscript{6}Consider $A$'s import, $M^A = c_x^A - Y^A$. As can be verified by (3.5) and (3.7) below, under free trade, $c_x^A = \frac{1}{2} \left( \frac{X^A + Y^A}{q} \right)$ and $q = \frac{X}{Y}$. From (3.1), $M^A = \frac{1}{2} \left[ \beta_x + (1 - \beta_y) \right] Y - (1 - \beta_y) Y$. Therefore for $M^A > 0$, $\beta_x + \beta_y > 1$ must be satisfied.

\textsuperscript{7}Note that $\beta_x$ and $\beta_y$ also reflect the countries' sizes. However, as a country's size depends on both $\beta_x$ and $\beta_y$, this thesis chooses to discuss $\beta_j$ in terms of the degree of specialisation so that the asymmetry between the two countries can be discussed simply in terms of the difference between $\beta_x$ and $\beta_y$.

\textsuperscript{8}The lower bound -1 is required for the domestic price of the importable goods to be positive. $\tau^i < 0$ indicates an import subsidy.
The equilibrium consumption in terms of the world relative price, $q = \frac{q_x}{q_z}$, and both countries' tariffs, $\tau^A$ and $\tau^B$, can be solved for using the budget constraints of both countries which require that their consumption expenditure must equal to their income. Formally,

$$c_x^A = \frac{1}{q (1 + \tau^A)}$$

$$c_y^A = \frac{(1 + \tau^A)}{q}$$

(3.3)

$$c_x^B = \frac{(1 + \tau^B)}{q}$$

$$c_y^B = \frac{(1 + \tau^B)}{q}$$

(3.4)

when evaluated at the world prices.\(^9\)

Utilising (3.3), (3.4) implies

$$c_x^A = s^A (X^A + qY^A)$$

$$c_y^A = (1 - s^A) \frac{(X^A + qY^A)}{q}$$

$$c_x^B = s^B (X^B + qY^B)$$

$$c_y^B = (1 - s^B) \frac{(X^B + qY^B)}{q}$$

(3.5)

where $s^A = \frac{T^A}{1 + T^A}$, $s^B = \frac{1}{1 + T^B}$, and $T^i = 1 + \tau^i$. From (3.5), $s^i$ can be interpreted as the share of income country $i$ spends on consumption of $x$.

\(^9\)This expression is equivalent to the budget constraint evaluated at domestic prices. When evaluating at domestic prices, $A$'s budget constraint is $c_x^A + q (1 + \tau^A) c_y^A = X^A + q (1 + \tau^A) Y^A + q \tau^A M^A$. Since $c_y^A = Y^A + M^A$, $c_x^A + qc_y^A + q \tau^A (Y^A + M^A) = X^A + q (1 + \tau^A) Y^A + q \tau^A M^A$ which is equivalent to $c_x^A + qc_y^A = X^A + q Y^A$.

48
By substituting (3.5) into the utility function, country $i$'s utility in terms of its tariff, the world relative price, and its endowments can be obtained.

$$U^i = \frac{\sqrt{T^i}}{(1 + T^i)} \frac{(X^i + qY^i)}{\sqrt{q}}$$  \hspace{1cm} (3.6)

Since both countries are large such that their demands can influence the world relative price, therefore, $q$ is a function of both countries' tariffs. The equilibrium $q$ which clears all markets must satisfy the market clearing condition $X^A + X^B = c_A^y + c_B^y$. Substitute (3.5) into the market clearing condition and solve for $q$.

$$q = \frac{[(1 - s^A) X^A + (1 - s^B) X^B]}{(s^A Y^A + s^B Y^B)}$$  \hspace{1cm} (3.7)

Since the world relative price $q$ represents the terms of trade of the country exporting $y$, an increase in $q$ means an improvement in $B$'s terms of trade while a deterioration in those of $A$.

Assuming that $A$ and $B$ choose their tariffs non-cooperatively and simultaneously, country $i$'s best response tariff to the other's tariff can be obtained by the maximisation of (3.6) with respect to $T^i$ taking the other's tariff as given. The interior Nash equilibrium of the non-cooperative tariff game is derived in the next section.

### 3.2.1 Interior Nash equilibrium

From (3.6) and (3.7), the first order condition for country $i$'s optimum tariff is

$$\frac{dU^i}{dT^i} = \frac{\partial U^i}{\partial T^i} + \frac{\partial U^i}{\partial q} \frac{\partial q}{\partial T^i} = 0$$  \hspace{1cm} (3.8)

From (3.6) and (3.7),
\[
\frac{\partial U^i}{\partial T^i} = \frac{(1 - T^i)}{2\sqrt{T^i}(1 + T^i)^2} \frac{(X^i + qY^i)}{\sqrt{q}}
\]
\[
\frac{\partial U^i}{\partial q} = \frac{(qY^i - X^i)}{2\sqrt{qq} \frac{\sqrt{T^i}}{(1 + T^i)}}, \quad \text{(3.9)}
\]

and

\[
\frac{\partial q}{\partial T^A} = -\frac{1}{(1 + T^A)^2} \frac{[(1 - s^A) X^A + (1 - s^B) X^B] Y^A}{(s^AY^A + s^BY^B)^2}
\]
\[
= -\frac{1}{(1 + T^A)^2} \frac{(X^A + qY^A)}{(s^AY^A + s^BY^B)}
\]
\[
\frac{\partial q}{\partial T^B} = \frac{1}{(1 + T^B)^2} \frac{[(1 - s^A) X^A + (1 - s^B) X^B] Y^B}{(s^AY^A + s^BY^B)^2}
\]
\[
= \frac{1}{(1 + T^B)^2} \frac{(X^B + qY^B)}{(s^AY^A + s^BY^B)} \quad \text{(3.10)}
\]

It can be shown that \(\frac{\partial U^i}{\partial T^i}\) represents the negative direct effect of an import tariff on welfare as it reduces the volume of trade\(^{10}\) and \(\frac{\partial U^i}{\partial q} \frac{\partial q}{\partial T^i}\) is the positive indirect effect of an import tariff through its effect on the terms of trade.\(^{11}\) Therefore, the country \(i\)'s optimum tariff is the tariff that equates the marginal gain from improvement in the terms of trade and the marginal loss from reduction in the volume of trade, given the

\(^{10}\)An increase in the import tariff reduces the domestic consumption of importable good while increases the domestic consumption of exportable good, therefore, there are less imports and exports by the country. From (3.9), \(\frac{\partial U^i}{\partial T^i} = \frac{(1 - T^i)}{2\sqrt{T^i}(1 + T^i)^2} \frac{(X^i + qY^i)}{\sqrt{q}} < 0\) for \(T^i > 1\).

\(^{11}\)It can be shown that \(\frac{\partial U^i}{\partial q} \frac{\partial q}{\partial T^i}\) is positive for positive volume of trade. Consider country \(A\), \(\frac{\partial U^A}{\partial q} = \frac{(X^A - X^Y)}{2\sqrt{q^A} \frac{\sqrt{T^A}}{(1 + T^A)}} < 0\) for \(q < \frac{X^A}{T^A}\). It can be shown that \(q < \frac{X^A}{T^A}\) must hold for \(A\) to import \(y\). For \(A\) to trade, the domestic relative price of its importable must be cheaper under trade, i.e. \(qT^A < \frac{X^A}{T^A} \Rightarrow q < \frac{X^A}{T^A}\). Therefore, \(q < \frac{X^A}{T^A}\) for non-negative import tariff \(T^A \geq 1\). In addition, it is clear that \(\frac{\partial q}{\partial q} = \frac{-(1 + T^A)^2}{(s^AY^A + s^BY^B)} < 0\).
other country’s tariff.

Substituting (3.9) and (3.10) back into (3.8) and solve for $T^A$ and $T^B$, the two countries’ best response tariff functions can be obtained.¹²

\[
\tilde{T}^A(T^B) = \sqrt{\frac{\beta_y}{(1 - \beta_x) \left[(1 - \beta_y)T^B + 1\right]}}
\] (3.11)

\[
\tilde{T}^B(T^A) = \sqrt{\frac{\beta_x}{(1 - \beta_y) \left[(1 - \beta_x)T^A + 1\right]}}
\] (3.12)

The two best response tariff functions are represented in Figure 3-1. The negative slope of the best response tariff functions shows that both countries’ tariffs are strategic substitutes.¹³ An increase in $A$’s tariff undermines the terms of trade of $B$ hence reducing the marginal gain of $B$’s tariff and vice versa. $T_0^A$ and $T_0^B$ represent the optimum tariffs of $A$ and $B$ when the other country’s tariff is zero.¹⁴ This is the most preferred point by each country as they can obtain the highest utility from an import tariff, however, at the cost of the other. Each country’s utility decreases along its best response tariff function as the other country’s tariff rises and reaches the lowest when the other country’s tariff equal to the trade prohibitive rate, $T_{pr}$, under which no trade occurs. As shown in Figure 3-1, the best response tariff of a country is zero when the other’s tariff is trade

¹²The derivations of the countries’ best response tariff functions and the Nash equilibrium tariffs are provided in Appendix A.1.

¹³From (3.11) and (3.12), $\frac{\partial T^A}{\partial T^B} = -\frac{1}{2} \frac{1}{\sqrt{T^B}} \frac{\beta_y}{(1 - \beta_x)} \frac{[(1 - \beta_y)T^B + 1] - \beta_x (\frac{\beta_y}{T^B} + 1)(1 - \beta_x)}{[(1 - \beta_y)T^B + 1]^2} < 0$ and $\frac{\partial T^B}{\partial T^A} = -\frac{1}{2} \frac{1}{\sqrt{T^A}} \frac{\beta_x}{(1 - \beta_y)} \frac{[(1 - \beta_x)T^A + 1] - \beta_y (\frac{\beta_x}{T^A} + 1)(1 - \beta_x)}{[(1 - \beta_x)T^A + 1]^2} < 0$.

¹⁴Consider country $A$, setting $T^B = 1$ in (3.11) yields $T_0^A = \frac{\beta_y}{(1 - \beta_x) (1 - \beta_y + 1)}$. Similarly, $T_0^B = \sqrt{\frac{\beta_x}{(1 - \beta_x) (1 - \beta_x + 1)}}$. 
Figure 3-1: Best response tariff functions and interior Nash equilibrium.

prohibitive. There is no trade for any tariff pairs in the grey area on the top right corner of the diagram in Figure 3-1.

The interior Nash equilibrium tariffs, \((T^A_N, T^B_N)\), can be solved for by substituting (3.12) into (3.11). Solving for \(T^A\), A’s interior Nash equilibrium tariff is

\[
T^A_N = \sqrt{\frac{\beta_y}{(1 - \beta_x)}}
\] (3.13)

\(^{15}\)The prohibitive tariff rate means one country’s minimum tariff rate which eliminates trade given the other country’s tariff equal to zero. There will be positive volume of trade if \(qT^A < \frac{X^A}{Y_A} \) or \(\frac{T^B}{Y_B} < \frac{X^B}{X_B}\), i.e. the domestic relative price of one country’s importable under positive trade must be less than the domestic relative price of that country’s importable under autarky. The two inequalities imply

\[
T^B \frac{X^B}{Y_B} < q < \frac{X^A}{Y_A} \quad \Rightarrow \quad T^A T^B \frac{X^B}{Y_B} \frac{X^A}{X_A} < 1 \quad \Rightarrow \quad T^A T^B < \frac{X^A}{Y_A} \frac{Y_B}{X_B} \Rightarrow T^A T^B < \frac{\beta_x \beta_y}{(1 - \beta_x)(1 - \beta_y)}
\]

for trade to be positive. Therefore, by the definition of the prohibitive tariff rate, \(T_{pr} = \frac{(1 - \beta_x)(1 - \beta_y)}{\beta_x \beta_y}\), which is equal for both countries. Consider country A, substituting \(T_{pr}\) for \(T^B\) in (3.11), the best response tariff of A is zero (indeed, any choices of A’s tariff given \(T^B = T_{pr}\) are indifferent as there is no trade).
Substituting (3.13) back into $B$’s best response tariff function (3.12) gives the interior Nash equilibrium tariff of $B$.

$$T_N^B = \sqrt{\frac{\beta_x}{(1 - \beta_y)}} \quad (3.14)$$

The equilibrium tariff pair $(T_N^A, T_N^B)$ is similar to the Kennan and Riezman (1988) result and is represented by point $N$ in Figure 3-1. Substituting (3.13) and (3.14) back into (3.7) yields

$$q_N = \frac{X}{Y} \frac{T_N^B}{T_N^A}$$

It can be verified that

$$T_N^A = \sqrt{\frac{\beta_y}{(1 - \beta_x)}} = \frac{1}{1 - \frac{1}{\epsilon_x}}$$

$$T_N^B = \sqrt{\frac{\beta_x}{(1 - \beta_y)}} = \frac{1}{1 - \frac{1}{\epsilon_y}}$$

where $\epsilon^i$ is the price elasticity of foreign demand for country $i$’s export.\(^\text{16}\)

---

\(^{16}\)Consider $\epsilon^A$, $\epsilon^A = \frac{\partial (c^B - X^B)}{\partial q} \frac{q}{(c^A - X^A)}$. From (3.5), $\epsilon^A = \frac{q^B Y^B \beta^B}{q^B Y^B \beta^B - (1 - \beta^B)X^B}$. Substitute $q_N = \frac{X}{Y} \frac{T_N^B}{T_N^A}$ for $q$, and $T_N^B$ for $T^B$ in $s^B$, $\epsilon^A = \frac{\frac{T_N^B}{T_N^A} \beta^B Y^B}{\frac{T_N^B}{T_N^A} \beta^B Y^B - \frac{1}{(1 + \beta_Y)}(1 - \beta_B)X^B} = \frac{\sqrt{\beta_y}}{\sqrt{\beta_y - \sqrt{(1 - \beta_x)}}}$. 

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53
\[
\epsilon^A = \frac{\sqrt{\beta_y}}{\sqrt{\beta_y} - \sqrt{(1 - \beta_z)}} \\
\epsilon^B = \frac{\sqrt{\beta_z}}{\sqrt{\beta_z} - \sqrt{(1 - \beta_y)}}
\]

It can be seen that the world relative price is shifted to the advantage of the country of which the Nash equilibrium tariff is larger. A country’s Nash equilibrium tariff will be larger than the other’s when the price elasticity of demand for its exportable is smaller. The price elasticity of demand for a country’s exportable will be smaller than the other’s when its degree of specialisation is larger, i.e. \(\beta_x > \beta_y \Rightarrow \epsilon^A < \epsilon^B \Rightarrow T^A_N > T^B_N \Rightarrow q_N < q_F = \frac{x}{\bar{y}}\). This is analogous to the monopolist setting his/her price, the country that faces less elastic demand for its export will have greater market power to impose a tariff. The greater the degree to which it specialises in its export, the greater is the other country’s dependence on import and hence its less elastic import demand.

Define \(U^i_N\) as the Nash equilibrium utility level of country \(i\) and \(U^i_F\) as the utility level of country \(i\) under free trade. Substituting \(T^A_N, T^B_N\) for the tariffs and \(q_N\) for the world relative price \(q\) in (3.6), the Nash equilibrium utility level are

\[
U^A_N = \frac{T^A_N}{(1 + T^A_N)} \frac{1}{\sqrt{T^B_N}} \left[ \beta_x + \frac{T^B_N}{T^A_N} (1 - \beta_y) \right] \sqrt{XY} \\
U^B_N = \frac{1}{(1 + T^B_N)} \sqrt{T^A_N} \left[ (1 - \beta_x) + \frac{T^B_N}{T^A_N} \beta_y \right] \sqrt{XY} \quad (3.15)
\]

The countries’ utility levels under free trade can be derived by substituting 1 for the tariffs in the above expressions. Therefore,
\[ U_F^A = \frac{1}{2} \left[ \beta_z + (1 - \beta_y) \right] \sqrt{XY} \]

\[ U_F^B = \frac{1}{2} \left[ (1 - \beta_z) + \beta_y \right] \sqrt{XY} \]  

(3.16)

For country \( i \) to win the trade war, \( \frac{U_A^i}{U_F^i} \) must exceed 1. From (3.15) and (3.16),

\[ \frac{U_A^i}{U_F^i} = \frac{2 \left[ T_N^A \beta_z + T_N^B \left( 1 - \beta_y \right) \right]}{(1 + T_N^A) \sqrt{T_N^B \left[ \beta_z + (1 - \beta_y) \right]}} \]

\[ \frac{U_B^i}{U_F^i} = \frac{2 \left[ T_N^A (1 - \beta_z) + T_N^B \beta_y \right]}{\sqrt{T_N^A \left( 1 + T_N^B \right) \left[ (1 - \beta_z) + \beta_y \right]}} \]  

(3.17)

Table A.1 in the Appendix A.3 shows the values of \( \frac{U_A^i}{U_F^i} \) and \( \frac{U_B^i}{U_F^i} \) in (3.17) corresponding to the values of \( \beta_z, \beta_y \in (0, 1) \) which induce positive trade \( (\beta_z + \beta_y > 1) \). Figure 3-2 summarises the simulation results.

Since no trade occurs for low values of \( \beta_z \) and \( \beta_y \), only the values of \( \beta_z, \beta_y \in \left( \frac{1}{2}, 1 \right) \) are shown in Figure 3-2 for ease of representation. The curves \( AA' \) and \( BB' \) are drawn to represent the combinations of \( \beta_z \) and \( \beta_y \) which make \( \frac{U_A^i}{U_F^i} = 1 \) and \( \frac{U_B^i}{U_F^i} = 1 \) respectively, i.e. they are the locus of \( \beta_z \) and \( \beta_y \) which make the countries indifferent between the Nash equilibrium and free trade. Figure 3-2 replicates the Kennan and Riezman (1988) results that \( A \) wins the trade war \( (\frac{U_A^i}{U_F^i} > 1) \) when the locus of \( \beta_z \) and \( \beta_y \) is within the area to the lower-right of the curve \( AA' \) and \( B \) wins the trade war \( (\frac{U_B^i}{U_F^i} > 1) \) when the locus of \( \beta_z \) and \( \beta_y \) is within the area to the upper-left of the curve \( BB' \); otherwise, both are worse off in the Nash equilibrium as compared to free trade.

It can be seen that for one country to win the trade war, its degree of specialisation must be sufficiently larger than the other’s. This is because for a country’s welfare to be
improved from the free trade level, an improvement in its terms of trade must be large enough to the extent which is more than just compensating the country's welfare loss from reduction in its trade volume. This requires the country's tariff to be sufficiently larger than the other country's tariff. As found earlier, a country's market power to impose an import tariff is positively related to its degree of specialisation. Although the discussions with regard to $\beta_j$ are made in terms of the degree of specialisation instead of the country size, the results in this section can also be stated in terms of the country size that the country which is sufficiently larger than the other will win the trade war.

3.2.2 An extreme case

The results discussed above replicate the results in the literature. However, an interesting case arises when the limit of the degree of specialisation is considered. From (3.13) and (3.14), a country's optimum tariff will be trade prohibitive when the country's degree of specialisation approaches one since in which case the country's market power reaches
its maximum, i.e. the price elasticity of foreign demand for its export is inelastic.\footnote{Consider $A$. $\beta_x \to 1 \Rightarrow \epsilon^A = \frac{\sqrt{\beta_x}}{\sqrt{\beta_x - (1-\beta_x)}} \to 1$. From footnote 15, the prohibitive tariff rate is $T_{pr} = \frac{\beta_x \beta_y}{(1-\beta_x)(1-\beta_y)} \to \infty$ when either $\beta_x$ or $\beta_y$ approaches 1.}

As a prohibitive tariff by one country eliminates all trade regardless of the other country's tariff while the countries always prefer some trade to autarky, whether the prohibitive tariff can be the equilibrium when a country monopolises its exportable is still ambiguous.

Under autarky, the countries' consumption of each good is equal to their endowment, i.e. $c^i_x = X^i$ and $c^i_y = Y^i$. Therefore, from the utility function $U^i = (c^i_x)^{\frac{1}{2}} (c^i_y)^{\frac{1}{2}}$,

\[
U^A_A = \sqrt{\beta_x (1-\beta_y)} \sqrt{XY}
\]

\[
U^B_A = \sqrt{(1-\beta_x) \beta_y} \sqrt{XY}
\]  

(3.18)

It can be verified that free trade is always preferred to autarky by both countries by showing that $\frac{U^A_A}{U^B_A} < 1$.\footnote{From (3.16) and (3.18), $\frac{U^A_A}{U^B_A} = \frac{2 \beta_x (1-\beta_y)}{\beta_x (1-\beta_y) + (1-\beta_x)} < 1 \Rightarrow 2 \sqrt{\beta_x (1-\beta_y)} - \beta_x - (1-\beta_y) < 0 \Rightarrow - \left[ \sqrt{\beta_x} - \sqrt{1-\beta_y} \right]^2 < 0$ which is true. It can be shown that $\frac{U^A_A}{U^B_A} < 1$ likewise.}

Following Riezman (1982), assume that each country simultaneously chooses a strategy between to impose and not to impose tariff; and if chooses to impose, the tariff must be the optimum tariff.\footnote{Riezman (1982) argues that allowing only two strategies is not restrictive as it seems. This is because the strategy of imposing optimum tariff consists of many possible tariff rates given the other country's tariff. Indeed, the above results where both countries unilaterally impose their optimum tariffs when $\beta_x$ and $\beta_y$ are smaller than one still hold with this assumption.}

When both countries monopolise their exportables, i.e. $\beta_x = \beta_y = 1$, the payoff matrix can be represented by Table 3.1.

$F$ and $T$ represent the no tariff and optimum tariff strategy, respectively. When a country monopolises its exportable, its optimum tariff is the tariff rate that eliminates
Table 3.1: Complete specialisation by both countries.

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<td>$T$</td>
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<td>$U^A_F$, $U^B_F$</td>
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<td>$U^A_T$, $U^B_T$</td>
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Table 3.2: Complete specialisation by A.

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<tr>
<td>$U^A_F$, $U^B_F$</td>
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all trade regardless of the other country's tariff. Therefore, the payoffs to both countries if either one of them chooses strategy $T$ will be equal to the autarky level, i.e. the pair of payoffs in which case is $(U^A_A, U^B_B)$.

Since $U^A_A > U^A_F$ and $U^B_B > U^B_T$, it is clear from Table 3.1 that the dominating strategy for both countries is to implement free trade. Each country chooses to implement free trade unilaterally in this case since, given any strategy chosen by the other country, they are always worse off than under free trade with the prohibitive tariff. The intuition behind this result is that even though when the countries gain their maximum market power when they monopolise their exportables, their trade interdependence is also at its maximum, i.e. each country's demand for one of the two goods has to be met only by imports from the other country. Therefore, they cannot afford to engage in a trade war which eliminates all trade.\(^{20}\)

Suppose that $\beta_y < \beta_x = 1$, i.e. only $A$ monopolises its exportable. In which case, $A$'s optimum tariff is still the prohibitive rate but $B$'s optimum tariff will be less than trade prohibitive. Therefore, the payoff matrix can be represented by Table 3.2.

\(^{20}\)This 'balance of terror' would prompt the continuous efforts between the US and the EU to settle their trade disputes. For example, with respect to the steel trade dispute, the latest US decision to exempt 50 per cent of 2.3 billion US dollars of steel imported from the EU from its safeguard tariffs and the EU suspension of its retaliatory threat until a formal ruling on the dispute will be made by the WTO.
If the strategies chosen by $A$ and $B$ are $F$ and $T$, respectively, the situation is analogous to the situation when $B$ imposes its optimum tariff without retaliation. This clearly implies $U^A_N < U^F_N$ and $U^B_N > U^F_P$. Note that even though $A$ is worse off than under the reciprocal free trade, it still prefers this situation to autarky ($U^A_N > U^A_1$) since there is still positive trade. Consequently, from Table 3.2, the unilateral free trade is still the dominating strategy for $A$ and strategy $T$ will be the weakly dominating strategy for $B$.$^{21}$ However, it seems to be counter-intuitive if the country with greater market power $A$ chooses not to engage in the trade war and be exploited by $B$'s optimum tariff. The result will be different if there is a limit which is less than the prohibitive rate that a country can impose its tariff. Suppose that there is some minimum level of consumption of importable that $A$ must maintain and that minimum level exceeds $A$'s endowment of the importable. Consequently, there is a maximum level of tariff $A$ can impose which is less than the prohibitive rate. Given $\beta_y < \beta_x = 1$, $A$ will impose this maximum tariff if it chooses strategy $T$. Since this maximum tariff does not eliminate all trade, it is the dominating strategy for $A$ and the equilibrium is similar to the case when $\beta_y, \beta_x < 1$ where both countries impose tariffs. Whether $A$ or $B$ will win or both will lose the trade war depends on $A$'s required minimum level of importable consumption hence the maximum tariff it can impose. The larger the maximum tariff $A$ can impose, the larger the possibility that it can win the trade war.

### 3.3 Multi-country model

The previous section has considered the tariff setting in the world of two countries. In this section, the model is extended to incorporate more countries to investigate the importance of the (world) market structure on the welfare effects of tariffs.

The model is still a pure exchange model of two goods, $x$ and $y$ with the world en-

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$^{21}$This result coincides with Syropoulos (2002) in which in the limit when one country is arbitrarily large relative to the other country, the country's non-cooperative payoff equals the autarky level. Therefore, Syropoulos questions the incentive of a large country to depart from free trade in such case.
endowments of \( X \) and \( Y \). However, there are a number of \( n_x \geq 1 \) countries sharing the endowments which were available to \( A \) in the previous section, namely \( a^1, a^2, \ldots, a^{n_x} \). Similarly, there are a number of \( n_y \geq 1 \) countries sharing the endowments which were available to \( B \), namely \( b^1, b^2, \ldots, b^{n_y} \). In summary, there are \( 2 + n_x + n_y \) countries in the country set \( \{A, B, a^1, a^2, \ldots, a^{n_x}, b^1, b^2, \ldots, b^{n_y}\} \). Therefore, there are two groups of countries: countries \( \{A, a^1, a^2, \ldots, a^{n_x}\} \) which export good \( x \) and import good \( y \) and countries \( \{B, b^1, b^2, \ldots, b^{n_y}\} \) which export good \( y \) and import good \( x \). The tariffs are strategic substitutes between countries with different trade patterns and are strategic complements between countries with similar trade patterns. It is shown in this section that the outcome of the trade war is determined by the world market structure of both goods, i.e. the numbers of their exporters and how the world endowments are divided initially.

Identical and homothetic preferences represented by the Cobb-Douglas utility function is still assumed across all countries. Therefore, (3.5) hence (3.6) still hold for all countries, i.e.

\[
\begin{align*}
c_x^i &= s^i \left( X^i + qY^i \right) \\
c_y^i &= (1 - s^i) \frac{\left( X^i + qY^i \right)}{q}
\end{align*}
\]

where \( s^i = \frac{T^i}{(1 + T^i)} \) for all \( x \)-exporting countries, \( i \in \{A, a^1, a^2, \ldots, a^{n_x}\} \), and \( s^i = \frac{1}{(1 + T^i)} \) for all \( y \)-exporting countries, \( i \in \{B, b^1, b^2, \ldots, b^{n_y}\} \), and

\[
U^i = \frac{\sqrt{T^i}}{(1 + T^i)} \frac{\left( X^i + qY^i \right)}{\sqrt{q}}
\]

for all \( i \in \{A, B, a^1, a^2, \ldots, a^{n_x}, b^1, b^2, \ldots, b^{n_y}\} \) and \( \Sigma X^i = X \) and \( \Sigma Y^i = Y \).

As in the previous section, the world relative price \( q \) can be derived from the market clearing condition which is now
\[ X^A + X^B + \Sigma_{i=1}^{n_x} X^{a_i} + \Sigma_{i=1}^{n_y} X^{b_i} = c_x^A + c_x^B + \Sigma_{i=1}^{n_x} c_x^{a_i} + \Sigma_{i=1}^{n_y} c_x^{b_i} \]

Substitute the above expression for \( c_x^i \) and solve for \( q \), (3.7) now becomes

\[ q = \frac{[(1 - s^A) X^A + (1 - s^B) X^B + \Sigma_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} + \Sigma_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i}]}{(s^A Y^A + s^B Y^B + \Sigma_{i=1}^{n_x} s^{a_i} Y^{a_i} + \Sigma_{i=1}^{n_y} s^{b_i} Y^{b_i})} \quad (3.19) \]

Assume that all countries choose their tariffs simultaneously and non-cooperatively, how the countries set their tariffs in this multi-country setting is discussed in the next section.

### 3.3.1 Nash equilibrium tariffs

The first order condition for each country optimum tariff is still the same as in (3.8)

\[ \frac{dU^i}{dT^i} = \frac{\partial U^i}{\partial T^i} + \frac{\partial U^i}{\partial q} \frac{\partial q}{\partial T^i} = 0 \]

and \( \frac{\partial U^i}{\partial T^i} \) and \( \frac{\partial U^i}{\partial q} \) are still the same as in (3.9).

From (3.19),

\[ \frac{\partial q}{\partial T^i} = -\frac{1}{(1 + T^i)^2} \frac{(X^i + qY^i)}{(s^A Y^A + s^B Y^B + \Sigma_{i=1}^{n_x} s^{a_i} Y^{a_i} + \Sigma_{i=1}^{n_y} s^{b_i} Y^{b_i})} \quad (3.20) \]

for all \( x \)-exporting countries, and

\[ \frac{\partial q}{\partial T^i} = \frac{1}{(1 + T^i)^2} \frac{(X^i + qY^i)}{(s^A Y^A + s^B Y^B + \Sigma_{i=1}^{n_x} s^{a_i} Y^{a_i} + \Sigma_{i=1}^{n_y} s^{b_i} Y^{b_i})} \quad (3.21) \]
for all $y$-exporting countries.

Substitute (3.9), (3.20) and (3.21) into the first order condition and solve for $T_i$ accordingly, the best response tariff functions of all countries can be obtained. The derivation shown in Appendix A.2 gives the best response tariff functions below.

$$
\widetilde{T}^A(T^n, \{T^n_i, \{T^n\}_i\}) = \sqrt{\frac{X^n_A}{(1 - s^B)X^n_B + \sum_{i=1}^{n_x} (1 - s^{a^i})X^{a^i} + \sum_{i=1}^{n_y} (1 - s^{b^i})X^{b^i}}} + 1
$$

$$
\widetilde{T}^B(T^n, \{T^n_i, \{T^n\}_i\}) = \sqrt{\frac{Y^n_B}{s^n_AY^n_A + \sum_{i=1}^{n_x} s^{a^i}Y^{a^i} + \sum_{i=1}^{n_y} s^{b^i}Y^{b^i}}} + 1
$$
\[ \tilde{\bar{\gamma}}^i(T^A, T^B, \{T^{a^i}\}_j\neq i, \{T^{b^i}\}) = \sqrt{\left(\begin{array}{c}
\begin{array}{c}
(1 - s^A)X^A \\
(1 - s^B)X^B \\
+ \sum_{j \neq i}(1 - s^{a^j})X^{a^j} \\
+ \sum_{i=1}^{n^x}(1 - s^{b^i})X^{b^i}
\end{array}
\end{array}\right) + 1} \]

\[ \tilde{\bar{\gamma}}^b(T^A, T^B, \{T^{a^i}\}_j\neq i, \{T^{b^i}\}_j\neq i) = \sqrt{\left(\begin{array}{c}
\begin{array}{c}
S^A Y^A \\
S^B Y^B \\
+ \sum_{j \neq i} S^{a^j} Y^{a^j} \\
+ \sum_{i=1}^{n^x} S^{b^i} Y^{b^i}
\end{array}
\end{array}\right) + 1} \]

(3.22)

where \{T^{a^i}\} and \{T^{b^i}\} represents the set of all tariffs of countries \(a^i\) and \(b^i\); and \(\{T^{a^j}\}_j\neq i\) and \(\{T^{b^j}\}_j\neq i\) are \(\{T^{a^i}\} - T^{a^i}\) and \(\{T^{b^i}\} - T^{b^i}\), respectively. It can be shown that the tariffs of the countries with similar trade patterns are strategic complements while the tariffs of the countries with different trade patterns are strategic substitutes.\(^{22}\)

\(^{22}\)Consider the best response tariff function of \(A\) in (3.22), for example, \(\frac{\partial T^A}{\partial T^b} =\)
This is because, from (3.20) and (3.21), the tariffs of the countries exporting the same goods affect the world relative price in the same way but there are different signs for the tariffs of the countries exporting different goods.

To simplify the analysis, the endowment structure is assumed to be

\[
\begin{align*}
X^A &= \delta_x \beta_x X \\
Y^A &= \delta_x (1 - \beta_y) Y \\
X^B &= \delta_y (1 - \beta_x) X \\
Y^B &= \delta_y \beta_y Y \\
X^{a_i} &= \frac{(1 - \delta_x)}{n_x} \beta_x X \\
Y^{a_i} &= \frac{(1 - \delta_x)}{n_x} (1 - \beta_y) Y \\
X^{b_i} &= \frac{(1 - \delta_y)}{n_y} (1 - \beta_x) X \\
Y^{b_i} &= \frac{(1 - \delta_y)}{n_y} \beta_y Y \\
\forall i &= 1, 2, \ldots, n_x, n_y
\end{align*}
\]

(3.23)

In addition to the degree of specialisation $\beta_x$ and $\beta_y$ which indicate the allocation of endowments hence the volume of trade between the countries with different trade patterns, new parameters, which indicate the allocation of endowments between countries with similar trade patterns (the world market structure of each good), are introduced into the model. $0 \leq \delta_x \leq 1$ is the proportion of all endowments available to the countries exporting good $x$ shared by country $A$ while the rest $(1 - \delta_x)$ is shared equally among countries $a^1, a^2, \ldots, a^{n_x}$. Likewise, $0 \leq \delta_y \leq 1$ is the proportion of all endowments available to the countries exporting good $y$ shared by country $B$ while the rest $(1 - \delta_y)$ is shared equally among countries $b^1, b^2, \ldots, b^{n_y}$. In this way, countries $a^1, a^2, \ldots, a^{n_x}$ are identical as are countries $b^1, b^2, \ldots, b^{n_y}$, and their superscripts will be omitted henceforth. When $\delta_x$ and $\delta_y$ are large enough $A$ and $B$ will share larger endowments hence larger market

\[
\begin{align*}
\frac{1}{2 \Gamma A^A} \frac{1}{1 + \Gamma} \frac{X^A X^B + (X^A)^2 Y^B}{\frac{X^A}{Y^A} + 1} &< 0, \quad \frac{\partial T^A}{\partial X^A} = \frac{1}{2 \Gamma A^A} \frac{1}{1 + \Gamma} \frac{X^A X^B + (X^A)^2 Y^B}{\frac{X^A}{Y^A} + 1} > 0, \quad \frac{\partial T^A}{\partial X^A} = -\frac{1}{2 \Gamma B^A} \frac{1}{1 + \Gamma} \frac{X^A X^B + (X^B)^2 Y^B}{\frac{X^B}{Y^B} + 1} < 0, \quad \text{where} \quad \Gamma \equiv \left[ (1 - s^B) X^B + \sum_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \right] \quad \text{and} \quad \Psi \equiv \left( s^B Y^B + \sum_{i=1}^{n_x} s^{a_i} Y^{a_i} + \sum_{i=1}^{n_y} s^{b_i} Y^{b_i} \right).
\end{align*}
\]
shares in their exportable markets than their counterparts which export the same goods. Therefore, we call country A and B as the ‘dominating exporters’ of their exportable markets and countries a and b as the ‘small-size exporters’ since they own less endowments hence less market shares than their dominating counterparts. Consequently, $\delta_x$ and $\delta_y$ will be called ‘the degree of market domination’, an increase in $\delta_x$ ($\delta_y$) will be interpreted as the world market of good $x$ ($y$) is more dominated by country $A$ ($B$) so that it moves toward a monopolistic market.

The number of small-size exporters $n_x$ and $n_y$ can be interpreted as representing the ‘market concentration’ of good $x$ and $y$, respectively. An increase in $n_x$ ($n_y$) means more exporters of good $x$ ($y$) in the world market, i.e. the market is less concentrated, which means the world market of good $x$ ($y$) moves toward more competition. From (3.20), (3.21), and (3.23), it can readily be established that when $n_x$ and $n_y$ are very large, the small-size exporters are so small that they cannot influence the world prices and free trade is their optimal policy. There will be only two dominating exporters, A and B, whose market power to manipulate the world prices still remains.

Substituting (3.23) into (3.22) yields

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23 The term ‘small-size’ is used to avoid confusion with the term ‘small’ country which means the country that has no market power to influence the world prices.

24 As $n_x, n_y \rightarrow \infty$, $X^a$, $Y^a$, $X^b$, and $Y^b$ approach zero hence $\frac{\partial q}{\partial \tau^a}$ and $\frac{\partial q}{\partial \tau^b}$ approach zero. From (3.22), the best response tariff for all countries $a^i$ and $b^i$ are always free trade. Another possibility for countries $a^i$ and $b^i$ to be ‘small’ is $\delta_x, \delta_y \rightarrow 1$. 

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65
\[
\widetilde{T}^A(T^B, T^a, T^b) = \sqrt{\frac{\delta_x \beta_x + (1 - s^B)\delta_y (1 - \beta_x) + (1 - s^a) (1 - \delta_x) \beta_x + (1 - s^b)(1 - \delta_y)(1 - \beta_x)}{\delta_x(1 - \beta_y) + s^B \delta_y \beta_y + s^a (1 - \delta_x)(1 - \beta_y) + s^b (1 - \delta_y) \beta_y}} + 1
\]

\[
\widetilde{T}^B(T^A, T^a, T^b) = \sqrt{\frac{\delta_y \beta_y + s^A \delta_x (1 - \beta_y) + s^a (1 - \delta_x)(1 - \beta_y) + s^b(1 - \delta_y) \beta_y}{\delta_y(1 - \beta_x) + (1 - s^A)\delta_x \beta_x + (1 - s^a) (1 - \delta_x) \beta_x + (1 - s^b)(1 - \delta_y)(1 - \beta_x)}} + 1
\]
Using the best response tariff functions in (3.24)\(^{25}\), a simulation analysis is conducted to investigate the effects of the model parameters on the non-cooperative equilibrium.

Since the focus of our analysis is on how the world market structure of both goods

\(^{25}\)Note that the best response tariff functions for the small-size countries in (3.24) are not final since the terms \(s^a\) and \(s^b\) still exist on the right hand sides to keep the expressions simple.
affects the Nash equilibrium while $\beta_x$ and $\beta_y$ indicate only the volume of trade between countries, they are assumed to be equal and constant throughout the analysis to reduce the volume of complexity. The simulation results are obtained assuming $\beta_x = \beta_y = 0.7$.\textsuperscript{26}

The effects of the degree of market domination ($\delta_x$ and $\delta_y$) and market concentration ($n_x$ and $n_y$) on the Nash equilibrium are considered one at a time by keeping the other constant and symmetric between the $x$-exporting and the $y$-exporting countries. The simulation results are shown in Appendix A.3. The results with regard to the Nash equilibrium tariffs are discussed in detail below.

Degree of market domination

The effects of $\delta_x$ and $\delta_y$ on the Nash equilibrium tariffs are shown in Table A.2 and can be represented by Figure 3-3.

The diagrams in Figure 3-3 are obtained arbitrarily assuming that $n_x = n_y = 9$. The relationship between the Nash equilibrium tariffs and the degree of market domination in the market of one good is drawn keeping the degree of market domination in the market of the other good fixed at 0.5. Since the degree of specialisation and the number of small-size exporters between the $x$-exporting and the $y$-exporting countries are kept equal, i.e. $\beta_x = \beta_y$ and $n_x = n_y$, the two diagrams in Figure 3-3 mirror each other. Therefore, discussion of only the change in $\delta_x$ is sufficient. When $\delta_x$ is equal to 0, there is no country $A$ hence its tariff is zero. As $\delta_x$ increases from zero, $T^A_N$ increases and $T^B_N$ decreases. $T^B_N$ and $T^H_N$ increase with $\delta_x$ over the interval in which $T^A_N < T^x_N$.\textsuperscript{27}

\textsuperscript{26}Note that by introducing more countries exporting the same goods, the limit case in which a country’s optimum tariff is trade prohibitive will never occur since no single country can monopolise its export. This can be seen from (3.24), for example, assuming that $\beta_x = 1$, the best response tariff of country $A$ will be $\sqrt{\frac{1}{(1-\delta_x)(1-\delta_y)+1}}$ which is always finite, given that $\delta_x < 1$, i.e. there exists other countries apart from $A$ who are exporting $x$.

\textsuperscript{27}i.e. until $\delta_x = 0.1$, the point where all countries exporting good $x$ including $A$ are identical. For $\delta_x \leq 0.1$, $A$ has smaller endowment than each of country $a$ hence the world market of good $x$ is not dominated by $A$. 68
Figure 3-3: Degree of market domination and Nash equilibrium tariff, given \( n_x = n_y = 9 \).

decrease thereafter. \( T_N^a = 1 \) when \( \delta_x \) equals to 1 (there are no countries \( a \)).\(^{28}\) \( T_N^A \geq T_N^B \) and \( T_N^a \leq T_N^b \) when \( \delta_x \geq \delta_y \).

An increase in the degree of market domination in one market directly increases the tariff of the dominating exporter in that market since it obtains a greater market power due to its larger market share. On the contrary, this means the smaller market share hence the smaller tariffs of the small-size exporters in the same market.\(^{29}\) Due to the strategic substitutability between tariffs of the countries that have different trade patterns; provided that the market is dominated by the dominating exporter, the strategic effect of the increase in the dominating exporter's tariff dominates the strategic effect of the decrease in the small-size exporters' tariffs so that the tariffs of those exporters in the other market are reduced.

\(^{28}\)Note that when both \( \delta_x \) and \( \delta_y \) equal to 1, the model reduces to the 2-country model as in the previous chapter.

\(^{29}\)Due to the strategic complementarity of their tariffs, the larger tariff of the dominating exporter at the same time strategically increases the tariffs of its small-size counterparts. However, the simulation results show that, with respect to an increase in \( \delta_j \), the direct effect dominates the strategic effect.
Figure 3-4: Market concentration and Nash equilibrium tariff, given $\delta_x = \delta_y = 0.5$.

Market concentration

The effects of $n_x$ and $n_y$ on the Nash equilibrium tariffs are shown in Table A.3 and can be represented by Figure 3-4.

The diagrams in Figure 3-4 are obtained arbitrarily assuming that $\delta_x = \delta_y = 0.5$. The relationship between the Nash equilibrium tariffs and the number of small-size exporters of one good is drawn keeping the number of small-size exporters of the other good fixed at 10. Since the degree of specialisation and the degree of market domination between the $x$-exporting countries and the $y$-exporting countries are kept equal, i.e. $\beta_x = \beta_y$ and $\delta_x = \delta_y$, the two diagrams in Figure 3-4 mirror each other. Therefore, only the change in $n_x$ will be discussed. From Figure 3-4, when $n_x$ is equal to 1, $T_{N}^x = T_{N}^a$ since $\delta_x = 0.5$ is assumed so that there are two identical $x$-exporting countries. It can be seen that $T_{N}^A$ and $T_{N}^a$ decrease while $T_{N}^b$ and $T_{N}^b$ increase with $n_x$. When $n_x$ is very large, $T_{N}^a$ approaches 1 as the small-size $x$-exporters become small. $T_{N}^A \leq T_{N}^B$ and $T_{N}^a \leq T_{N}^b$ when $n_x \leq n_y$.

Given a constant market size, an increase in the number of small-size exporters in one market clearly reduces the market share of each of them hence their smaller tariffs. An increase in $n_x$ leaves the market shares of the other countries unchanged. This implies
that only $T_N^a$ are directly affected by an increase in $n_x$ which is reflected by the sharp decrease of $T_N^a$ in the left diagram of Figure 3-4 while the others respond to the change in $n_x$ only gradually. Considering the best response tariff functions of the other countries apart from country $a$ reveals that the effects of the increase in $n_x$ on those countries' tariffs are only through its effect on $T_N^a$.\textsuperscript{30} Since $T_N^a$ and $T_N^A$ are strategic complements and $T_N^a$ and $T_N^B$, $T_N^b$ are strategic substitutes, the decrease in $T_N^a$ strategically decreases the equilibrium tariff of $A$ while increases the equilibrium tariff of $B$ and $b$ as shown in the left diagram.

Proposition 1 summarises the above results which are necessary for the main argument of this chapter.

**Proposition 1** (i) Given that $n_x = n_y$, $T_N^A \leq T_N^B$ and $T_N^A \geq T_N^B$ when $\delta_x \geq \delta_y$.

(ii) Given that $\delta_x = \delta_y$, $T_N^A \leq T_N^B$ and $T_N^A \geq T_N^B$ when $n_x \leq n_y$.

Proposition 1 (i) states that given equal numbers of exporters in both markets, in the more dominated market, the dominating exporter imposes a larger tariff and the small-size exporters impose smaller tariffs relative to the tariffs of the exporters of the same types in the other market. This is because the degree of market domination indicates the distribution of endowments among the exporters in a market. As compared to the exporters in the other market, a larger degree of market domination indicates the larger endowments for the dominating exporter but the smaller endowments for each of the small-size exporters which imply their market power to impose tariffs.

Proposition 1 (ii) states that given equal degree of market domination in both markets, the exporters in the more concentrated market impose larger tariffs. This is because, as compared to the exporters in the other market, the smaller number of small-size exporters in a market indicates the larger endowments available to each of them. This implies the greater market power of each of them to impose larger tariffs. As discussed above, their larger tariffs strategically increase their dominating co-exporter's tariff.

\textsuperscript{30}It can be seen in (3.24) that there is no $n_z$ in $A$’s, $B$’s and $b$’s best response tariff functions.
In general, Proposition 1 states that the exporters (at least the dominating exporter) in the less competitive market (larger degree of market domination and smaller number of exporters) tends to have the greater market power to impose larger tariffs.

### 3.3.2 Welfare analysis

For a country to win a trade war, the necessary condition is that the world relative price must be shifted to its advantage. Therefore, to examine how the market structure affects the countries’ payoffs in the trade war, its effects on the equilibrium world relative price should be discussed.

**World relative price**

Recall that $q_N$ represents the Nash equilibrium world relative price and $q_F = \frac{\bar{X}}{\bar{Y}}$ represents the world relative price under free trade. Substitute (3.23) into (3.19),

\[
\frac{q_N}{q_F} = \frac{(1 - s^A) \delta_x \beta_x + (1 - s^B) \delta_y (1 - \beta_x) + (1 - s^C)(1 - \delta_x) \beta_x + (1 - s^D)(1 - \delta_y)(1 - \beta_x)}{s^A \delta_x (1 - \beta_y) + s^B \delta_y \beta_y + s^C (1 - \delta_x)(1 - \beta_y) + s^D (1 - \delta_y) \beta_y}
\]

\[\frac{q_N}{q_F} < 1 \quad \left( \frac{q_N}{q_F} > 1 \right) \] indicates an improvement in the terms of trade of the $x$-exporting ($y$-exporting) countries. The effects of a change in the degree of market domination and the number of small-size exporters can be examined by substituting the tariffs in Tables A.2 and A.3 into (3.25) correspondingly. The results obtained in Tables A.4 and A.5 in
It can be observed in Figure 3-5 that the Nash equilibrium world relative price increases with $\delta_x$ to a certain value and decreases thereafter while it decreases with $\delta_y$ to a certain value and increases thereafter. In addition, $\frac{\delta x}{q_F} > 1$ when $\delta_x < \delta_y$ and $\frac{\delta x}{q_F} < 1$ when $\delta_x > \delta_y$. Similarly, Figure 3-6 shows that the Nash equilibrium world relative price is increasing with $n_x$ while decreasing with $n_y$, and $\frac{\delta x}{q_F} > 1$ when $n_x > n_y$ and $\frac{\delta x}{q_F} < 1$ when $n_x < n_y$. These results imply that the Nash equilibrium world relative price will be in favour of the countries whose exportable market is less competitive as compared to the world market of the other good (larger degree of market domination and smaller number of exporters). This is because, by Proposition 1, the exporters (at least the dominating exporter) in the less competitive market tends to have the greater market power to impose larger tariffs.\textsuperscript{31}

\textsuperscript{31}However, it can be seen in Figure 3-5 that a larger degree of market domination in a market tends to work against the interests of the exporters in that market when the degree of market domination is small. This is because over the interval of small values of $\delta_j$, the market is not dominated by its dominating exporter. Therefore, the strategic effect of the reduction in the small-size exporters’ tariffs that raises
Figure 3-6: Market concentration and Nash equilibrium world relative price, given $\delta_x = \delta_y = 0.5$.

However, for a country to win in equilibrium, the welfare gain from an improvement in the country’s terms of trade must be large enough to overcome the welfare loss from reduction in its trade volume. As discussed above, there is a monotonic relationship between the countries’ terms of trade and the degree of incompetitiveness in their exportable markets. Therefore, it is readily implied that the degree of market domination and market concentration in the market of one good must be ‘sufficiently large’ relative to the degree of market domination and market concentration in the other market to make the dominating and/or the small-size exporters of that good win the trade war. In the symmetric case, Figure 3-5 and Figure 3-6 suggest that the Nash equilibrium world relative price is equal to the free trade level when $\delta_x = \delta_y$ and $n_x = n_y$. In which case, all countries are bound to lose in equilibrium since neither of them are successful in shifting the world relative price to their advantage despite the welfare cost of tariffs that they impose. These points can be established more concretely when the countries’ equilibrium tariffs of the exporters in the other market dominates so that the equilibrium world relative price is shifted from their advantage.
Figure 3-7: Degree of market domination and Nash equilibrium utility, given $n_x = n_y = 9$. Welfare is considered.

Equilibrium welfare

The payoff outcome can be examined by considering the ratio of the Nash equilibrium to free trade utility. From (3.6),

$$\frac{U_N^i}{U_F^i} = \frac{\sqrt{T_N^i} (X^i + q_N Y^i)}{(1 + T_N^i) \sqrt{q_N}} \frac{1}{2 \left( X^i + q_N Y^i \right)}$$

Substituting results in Tables A.2 and A.3 for $T_N^i$ and results in Tables A.4 and A.5 for $q_N$ correspondingly to the values of $\delta_x$, $\delta_y$, $n_x$, and $n_y$ yields the values of the Nash equilibrium to free trade utility ratio in Tables A.6 and A.7 in Appendix A.3. Country $i$ wins the trade war if $\frac{U_N^i}{U_F^i} > 1$.

The results in A.6 and A.7 can be represented by Figure 3-7 and Figure 3-8. In Figure 3-7 and Figure 3-8, the countries’ Nash utilities increase with the degrees of market...
Proposition 2 An exporter of one good can win the trade war if the world market of that good is 'sufficiently' dominated and/or 'sufficiently' concentrated relative to the world market of the other good.
trade war, even if the country's endowments are relatively small, if the world market of its exportable moves closer to the monopolistic market, i.e. there are less countries exporting the same good and/or the world endowment of that good is divided more disproportionately among its exporters. This is understandable when we consider a monopolistic market in which the monopolist sets the price of its product to maximise profit of the whole industry. The price set by the monopolist gives the highest profit that the firm can obtain. However, when there is more than one firm competing in the market, those firms set their prices unilaterally to maximise their own profits instead of the industry profit as a whole. Therefore, they end up with a price hence their profits lower than what they would obtain otherwise. If the world market of one good is a monopolistic market which has only one exporter, that exporter setting a tariff is analogous to the monopolist setting the price. When there is more than one exporter, they unilaterally set their tariffs which are smaller than what are optimal for them as a whole. The greater the extent to which a market differs from the monopolistic market, the greater is the difference between its exporters' unilateral tariffs and the collective optimal levels. Therefore, when the world market of one good moves closer to the monopolistic market, the exporters in that market are able to impose larger unilateral tariffs and hence a greater possibility for them to win the trade war.

3.3.3 Discussions

Furthermore, there are a few points that are worth discussing.

More competition, less restrictive trade From the discussion of Proposition 2 above, the ability of a country to impose a unilateral tariff seems to be positively related with the degree of incompetitiveness in the world market of its exportable. Therefore, it is logical to argue that the world trade will be less restrictive in the non-cooperative

33 Each exporter sets a smaller tariff in equilibrium when there are more of them since (i) each of them is relatively smaller, and (ii) due to the strategic complementarity of their tariffs, their co-exporters' tariffs are smaller.
equilibrium if there are more competitions in the world markets (smaller degrees of market domination and larger numbers of exporters). This point can be illustrated clearly by considering the Nash equilibrium tariffs in the cells along the diagonal of Table A.3 where \( n_x = n_y \). The Nash equilibrium tariffs of all countries decrease as \( n_x = n_y \) increase. The standard two-country two-good model is a special case of the many-country two-good model in this section. It consists of two (world) markets and each of which has only one exporter so that we can expect a more restrictive trade in equilibrium as compared to the model consisting of more countries.

The advantage of being small It can be seen from Figures 3-7 and 3-8 that the small-size exporters have greater tendency to win the trade war than their dominating counterparts as they require less degree of market domination and market concentration to win the trade war. For example, from the left diagrams in Figure 3-7 and 3-8, the minimum degree of market domination and the minimum degree of market concentration required by the small-size \( x \)-exporters to win the trade war is less than that required by the dominating exporter, i.e. \( \delta_x^A < \delta_y^A \) and \( \Pi_x^A > \Pi_y^A \). This is because the small-size exporters can obtain the same level of the equilibrium world relative price as their dominating counterparts while they impose smaller tariffs hence smaller loss from the reduction in their trade volume.

Figure 3-7 shows that even when the small-size countries are 'small' as most of their exportable markets are dominated by their dominating counterparts, i.e. \( \delta_x \) or \( \delta_y \) approaches 1, they still be able to benefit from the global trade war. This result is in contrast with the previous literature which shows that the small countries always lose the trade war. This is because, in this model, even a 'small' small-size country set zero tariff, it can still free ride its dominating counterpart's effort to manipulate the world

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34 Since a change in the degree of market domination and market concentration increases the tariffs of some countries while decreases the tariffs of other countries in equilibrium, the degree of trade restrictiveness could be measured by the average of the countries' Nash equilibrium tariffs.

35 For example, Mayer (1981), Kennan and Reizman (1988), etc.

36 When \( n_x, n_y \to \infty \) and/or \( \delta_x, \delta_y \to 1 \), \( X^a, Y^a, X^b \), and \( Y^b \) approach zero. From (3.22), the best
3.4 Conclusion

This chapter re-investigated the theory of optimum tariff and retaliation in the multi-country setting. We started from constructing a two-country two-good trade model as the benchmark in section 3.2. In the two-country setting, which country could win the trade war depended on which one was able to impose a tariff that was sufficiently larger than the other's to shift the world relative price to its advantage. Similar to the pricing behaviour of the monopoly firm in a market, the less elastic foreign demand faced by a country, the larger the market power of that country to impose a larger tariff. As the two countries shared the same pool of endowment, when the exporting country was endowed with more of its exportable, the importing country was endowed with less of the same good. This implied a greater degree the importing country had to rely on imports from the exporting country hence its less elastic demand. Consequently, it was established that the greater the degree of specialisation of a country, the greater the level of tariff that country could impose and the greater the possibility that country could win a trade war. These results replicated the results in the previous literature. However, it was shown in section 3.2.2 that, in the limit where both countries completely relied on trade (both of them monopolised their exportables), the welfare loss from engaging in a trade war was too high such that they unilaterally implemented free trade.

In the multi-country two-good model in section 3.3, the trade war outcome did not depend on the tariffs of only two countries which were strategic substitutes. There was also a possibility that the countries' tariffs could be strategic complements since there was more than one country trading under the same trade pattern. With the help of simulation response tariffs for all countries $a$ and $b$ are always free trade.

The possibility that the 'small' countries would be able to free-ride on the tariffs of the 'large' countries with similar trade pattern as there may be positive terms of trade externalities between them is mentioned by Syropoulos (2002).
analysis, it was revealed that all countries’ Nash equilibrium tariffs are positively related with their market shares which depended on how the world endowments are divided among the countries, the degree of market domination and market concentration in their respective markets in particular. The welfare analysis implied that any one country could win the trade war if the world market of its exportable was ‘sufficiently’ dominated and/or ‘sufficiently’ concentrated relative to the world market of the other good. This was due to the fact that the more exporters in the world market of one good, the smaller the tariffs those exporters would impose unilaterally relative to what are optimal for them as a whole. Since the outcome of the trade war depended on the relative size of the opposing tariffs to shift the world relative price from free trade, it was those exporters in the sufficiently dominated and sufficiently concentrated market which imposed sufficiently larger tariffs than the tariffs of the exporters in the other market that won the trade war. As opposed to the results in the literature, these finding lead us to conclude that, due to the positive tariff externalities between countries that have the same trade pattern, a country would be able to win a trade war when the world market of its exportable is relatively less competitive even if that country is small.

The multi-country two-good trade model in this chapter can be extended to study the cooperative tariff setting. Analogous to the Cournot duopoly game where the non-cooperative price is lower than what the firms can achieve if they can determine their production cooperatively, there is a possibility that the non-cooperative tariffs between countries that have the same trade pattern are too low that they exploit less than efficient terms of trade. Therefore, the trade agreements as the collusions between countries to further improve their terms of trade should be worth investigating. This type of trade agreement is studied in the next chapter.
Chapter 4

International Trade Agreements:
Terms of Trade and Volume of Trade Incentives

This chapter extends the many-country two-good trade model in chapter 3 to investigate the negotiation of trade agreement among a subset of the countries. An important feature of the many-country two-good trade model is that tariffs are strategic complements between countries that have the same trade pattern and are strategic substitutes otherwise. Therefore, two possible trade agreements can be investigated: (i) an agreement between countries whose tariffs are strategic complements, and (ii) an agreement between countries whose tariffs are strategic substitutes. Since these trade agreements imply different sources of gain for the negotiating countries (gain from an improvement in terms of trade for the former and gain from an increase in trade volume for the latter), this chapter examines the choice of a country by comparing the welfare implications of the two possibilities. It is found that a country would prefer to have a trade agreement with the country whose endowments of goods are relatively large regardless of the strategic complementarity or substitutability of their tariffs.
4.1 Introduction

It has been accepted, at least in neoclassical economics, that trade liberalisation can raise countries' welfare and the world welfare at large. The trade liberalisation has been conducted on the multilateral basis in which all countries participate in the negotiation, drafting, and adoption of the common trade rules. On the one hand, given diverse economic and political interests among countries, multilateral trade negotiation under the GATT had proved itself to be a slow process. It had gone through eight rounds of negotiation from 1947 to 1994 before the establishment of the WTO in 1995. On the other hand, the process had delivered significant results in liberalising world trade. Negotiated tariffs for industrial goods had been reduced close to zero, new sectors such as services and even the trouble field of agriculture were brought into the negotiations, etc.

Given the increasing complexity of agendas incorporated into the negotiations such as environmental and intellectual property issues and more participants with even more diverse interests such as those transitional economies in Eastern Europe and the joining of China, the multilateral trade negotiation has now faced with new challenges. Some such as Steinberg (1997) and Rode (2003) advocate the cooperation between the US and the EU to deal with the challenges by pressuring all other countries to be in line with the multilateral trade liberalisation as they succeeded in closing the Uruguay Round.¹ Proposals have been made to foster the two economic powers closer tie such as a call for the negotiation of a Transatlantic Free Trade Agreement (TAFTA) and even deeper cooperation, "the New Transatlantic Marketplace (NTM) that would progressively eliminate tariff and non-tariff barriers to trade, and would presumably address deeper trade-related policies such as standards, subsidies, intellectual property protection, investment measures, services, and competition policy" (Steinberg, 1997). However, neither of them has

¹"By withdrawing from the GATT 1947 and joining the WTO, the transatlantic powers forced the rest of the world to join the WTO or lose their Most-Favoured-Nation access to Europe and the United States, which no GATT Contracting Party could afford." (Steinberg, 1997).
A number of barriers to transatlantic trade policy cooperation have been suggested in the literature. Among others, Steinberg (1997) argues that the US or the EU itself might have other attractive alternatives to the cooperation with each other or even to the multilateral trade liberalisation. Many trade arrangements with the third countries have been made by both the US and the EU to secure their market access to those countries or even competitively to put their own producers on a better footing in those markets than the producers of the other. He argues further that the US and the EU appear in some respects to be rivalling each other to establish trade arrangements with third countries or regions. The excellent examples are the US’s economic cooperation with Asia through the Asia Pacific Economic Cooperation (APEC) which was perceived in Europe as an American alternative to Atlanticism (Rode, 2003) and the EU’s counter arrangement with East Asian economies including China through the Asia Europe Meeting (ASEM) (Brennan, 2001). It is this point of argument which motivates this chapter to study why one country would prefer to coordinate its trade policy with one group of countries rather than others.

It is argued that countries trade at the level less than efficient in the trade war equilibrium since their unilateral tariffs are too high. The role of eliminating this inefficiency is a rationale for the trade agreement provided in the literature. Bagwell and Staiger (1999, 2002) show that the inefficiency in the trade war equilibrium arises solely from the countries’ unilateral incentive to manipulate their terms of trade and the trade agreement is the means by which this terms of trade incentive can be removed. It has been shown that the efficient trade agreement between any two countries is not unique. In fact, Mayer (1981) show that there exists a set of trade agreements which are Pareto efficient. The recent works on trade agreement, therefore, devotes to the study of trade negotiation to select a unique agreement among those efficient ones. In general, the gain

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2 The static model of trade negotiation focuses on how the gain from the cooperation is to be divided among the negotiating countries using the Nash bargaining solution. In addition, in light of the fact that the international trade agreement must be self-enforcing, the negotiation of trade agreement is
from trade agreement in this line of literature comes from an increase in the countries' trade volume. However, as recognised in the customs union literature, such as Kennan and Riezman (1990) and Syropoulos (1999), another source of gain from trade agreement can be the collective market power of the member-countries to exploit their terms of trade. It is argued that the countries that import the same good can internalise their (positive) tariff externality by setting their tariff jointly in a customs union.

This chapter is based on the multi-country two-good trade model in chapter 3 in which the countries’ tariffs can be strategic complements as well as strategic substitutes. Therefore, the trade agreement of which the negotiating countries collectively exploit their terms of trade can be studied as well as the trade agreement of which the negotiating countries agree to adjust their trade policies to increase their trade volume. We shall term the former as ‘the terms of trade-driven trade agreement’ and the latter as ‘the volume of trade-driven trade agreement’ henceforth. Since the two types of trade agreement imply different sources of gain and, especially, different negotiating parties, this chapter investigates the conditions in which a country might prefer one type of agreement to the other.

The analysis is made by considering the two types of trade agreement separately. Then their welfare implications are compared from a perspective of a country. Section 4.2 studies the trade agreement within a two-country two-good trade model to re-derive the results in the literature against which the results in the following sections can be compared. Section 4.3 develops the multi-country model in which the volume of trade-driven and the terms of trade-driven trade agreement are studied in sections 4.3.1 and 4.3.2, respectively. Section 4.3.3 compares the welfare gains over the non-cooperative equilibrium (studied in chapter 3) between the two possibilities. Conclusion is made in section 4.4.

studied in the dynamic repeated game framework. The fact that the equilibrium trade agreement must be sustainable such that no member country has an incentive to defect restricts the set of the possible agreements. The sustainable trade agreements may not be fully efficient, however, the resulted equilibrium is more efficient than the non-cooperative one. Examples of works which consider the issue of enforcement are Dixit (1987), Furusawa (1999) and Park (2000).
4.2 Two-country model

As in chapter 3, there are country A and B trading good x and y under the world relative price \( q \) in which the world price of x is chosen as numeraire. Consumers’ preferences are identical and are represented by the Cobb-Douglas utility function, \( U^i = (c_x^i)^{\frac{1}{\alpha}} (c_y^i)^{\frac{1}{\beta}} \) for \( i \in \{A, B\} \).

Both countries are initially endowed with both goods as follows.

\[
X^A = \beta_x X \quad \quad \quad \quad Y^A = (1 - \beta_y) Y
\]

\[
X^B = (1 - \beta_x) X \quad \quad \quad \quad Y^B = \beta_y Y \quad \quad \quad \quad (4.1)
\]

where \( 0 \leq \beta_x, \beta_y \leq 1 \). Recall that \( \beta_x \) and \( \beta_y \) are the ‘degree of specialisation’ in good x and y respectively, and it is assumed that A is the exporter of x and B is the exporter of y. \( \beta_x + \beta_y > 1 \) must be satisfied for trade to be positive.

Both countries impose an ad-valorem tariff on their imports, \( \tau^A, \tau^B \in (-1, \infty) \) for domestic prices to be positive. As was derived in chapter 3, country i’s utility in terms of its endowments, tariff and the world relative price, and the world relative price as a function of the countries’ tariffs and endowments are

\[
U^i = \frac{\sqrt{T^i}}{(1 + T^i)} \frac{(X^i + qY^i)}{\sqrt{q}} \quad \quad \quad \quad (4.2)
\]

\[
q = \frac{[\{(1 - s^A) X^A + (1 - s^B) X^B\}]}{\left(s^A Y^A + s^B Y^B\right)} \quad \quad \quad \quad (4.3)
\]

where \( T^i = 1 + \tau^i \), \( s^A = \frac{T^A}{1 + T^A} \), and \( s^B = \frac{1}{1 + T^B} \).
4.2.1 Nash bargaining solution

The symmetric Nash bargaining solution is employed to study the policy coordination between $A$ and $B$ using the Nash equilibrium as the threat point if the agreement cannot be reached. Even though the symmetric concept abstracts from any differences in the 'bargaining ability' between the two players\(^3\); in the context of this model, the asymmetry between them (difference in the degrees of specialisation in the countries' exportables) is readily captured by the fact that the two countries face different threat points as they obtain different payoffs in the non-cooperative game.\(^4\)

Suppose that $A$ and $B$ have a chance to negotiate their trade policies, the agreement reached by the two countries is the tariff pair $\left( T^A_C, T^B_C \right)$ which maximises the product of the countries’ gains in utility over the non-cooperative outcome. This is given by

$$
\left( T^A_C, T^B_C \right) = \arg \max_{\left( T^A, T^B \right)} \left( U^A - U^A_N \right) \left( U^B - U^B_N \right)
$$

where $U^A_N$ and $U^B_N$ are the utility level each country receives in the non-cooperative equilibrium.

It can be seen from (4.2) and (4.3) that a country's tariff both directly and indirectly (through the world relative price) affects the country's own utility but only indirectly affects its trading partner through the world relative price. Formally,

\(^3\)The bargaining ability of a country depends on many factors, such as the negotiation skills of the negotiators, political relations between countries, domestic political support and opposition, etc. Furusawa (1999) uses the governments' future discount rates to represent their difference in bargaining power to examine the issue of enforcement in a repeated game.

\(^4\)Alternatively, one can use the model of alternating offers in which there is infinitesimal lag between offers. In fact, the Nash bargaining solution is the solution to the limit case of the model of alternating offers as the lag between offers approaches zero. See Furusawa (1999) for example.
\[ \begin{align*}
dU^A &= \left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) dT^A + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^B} dT^B \\
dU^B &= \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^A} dT^A + \left( \frac{\partial U^B}{\partial T^B} + \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^B} \right) dT^B \\
\end{align*} \] (4.4)

The first order conditions for the above optimisation problem are, therefore,

\[ \left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) + \lambda \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^A} = 0 \] (4.5)

\[ \left( \frac{\partial U^B}{\partial T^B} + \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^B} \right) + \frac{1}{\lambda} \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^B} = 0 \] (4.6)

where \( \lambda = \frac{U^A - U^B}{U^A_U - U^B_U} \).

Rearrange (4.5) and (4.6) and divide (4.5) by (4.6), the following expression can be obtained.

\[ \frac{\frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A}}{\frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^A}} = \frac{\frac{\partial U^A}{\partial T^B} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^B}}{\frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^B}} \] (4.7)

(4.7) defines the efficiency condition in which (4.5) and (4.6) are satisfied simultaneously. By (4.7), it is straightforward to show that the Nash equilibrium tariffs are inefficient. Recall that the Nash equilibrium tariffs must satisfy \( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} = 0 \) and \( \frac{\partial U^B}{\partial T^B} + \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^B} = 0 \) simultaneously. Since (4.7) will be violated to satisfy these two conditions, as shown by Bagwell and Staiger (1999), the Nash equilibrium tariffs are inefficient.

The Nash equilibrium tariffs are inefficient since each country does not internalise the negative terms of trade externalities of its tariff when it chooses the tariff unilaterally. Therefore, both countries end up with higher tariffs and hence lower volume of trade relative to the efficient level.
The trade agreement to eliminate this inefficiency can be determined by solving (4.7), (4.5), and (4.6) simultaneously. The derivation is provided in Appendix B.1 and only the key expressions are presented below.

First, solve (4.7) for \( T^B \), the efficient combinations of trade policies can be obtained.

\[ T_C^A T_C^B = 1 \]  \hspace{1cm} (4.8)

Since \( T_C^A T_C^B > 1 \) when both \( T_C^A \) and \( T_C^B \) are greater than 1, the trade agreements which entail positive import tariffs by both countries will never be adopted in equilibrium since they are inefficient and both countries can still be made better off by altering the negotiating tariff pairs. The efficient trade agreement must either entail free trade by both countries or there must be an import tariff by one country and an import subsidy by the other.\(^5\)

It can be seen that the efficient trade agreement is not unique as it can be any pair of tariffs that satisfy (4.8). To determine a unique equilibrium trade agreement \((T_C^A, T_C^B)\), substituting (4.8) into (4.3), the cooperative world relative price can be represented by

\[ q_C = \frac{1}{T_C^A} \frac{(X^A + X^B)}{(Y^A + Y^B)} = \frac{1}{T_C^A} \frac{X}{Y} \]  \hspace{1cm} (4.9)

Substituting (4.8) and (4.9) back into (4.5), it is found that

\[ \lambda = \frac{(U_C^A - U_A)}{(U_C^B - U_B)} = 1 \]  \hspace{1cm} (4.10)

\(^5\)This result is similar to Mayer (1981) since the governments' objectives in this model are simply to maximise national welfare. However, in Bagwell and Staiger (1999), it is possible that the efficient trade agreement might entail positive import tariffs by both countries if the governments of both countries have political motive toward having positive tariffs, i.e. the governments might want to protect their import-competing industries etc.
(4.10) suggests that the trade agreement reached by the two countries must yield equal amount of gain to both of them regardless of their different threat points. This is because, by employing the symmetric Nash bargaining solution, it is assumed that both countries have the same bargaining ability.\(^6\)

Substitute (4.8) and (4.9) into (4.2),

\[
U_A^C = \frac{[\beta_x T_C^A + (1 - \beta_y)]}{(1 + T_C^A)} \sqrt{XY}
\]

\[
U_B^C = \frac{[(1 - \beta_x) T_C^A + \beta_y]}{(1 + T_C^A)} \sqrt{XY}
\]  

(4.11)

Substitute (4.11) back into (4.10), the expression for \(T_C^A\) can be solved for. Then substituting \(T_C^A\) back into (4.8) gives the expression for \(T_C^B\). Therefore, the trade agreement \((T_C^A, T_C^B)\) reached by the two countries are

\[
T_C^A = \frac{(u_A^A - u_B^B)}{\sqrt{XY}} + \left[\beta_y - (1 - \beta_y)\right]
\]

\[
T_C^B = \frac{[\beta_x - (1 - \beta_x)]}{\sqrt{XY}} - \frac{(u_A^A - u_B^B)}{\sqrt{XY}} + \left[\beta_y - (1 - \beta_y)\right]
\]  

(4.12)

From chapter 3, the non-cooperative tariffs and world relative price are \(T_N^A = \sqrt{\frac{\beta_x}{(1 - \beta_x)}}\), \(T_N^B = \sqrt{\frac{\beta_x}{(1 - \beta_x)}}\), and \(q_N = \frac{T_B^A X}{T_A^A Y}\). Substitute these facts into (4.2),

\[^6\text{Another point with respect to (4.10) is worth mentioning when (4.5) and (4.6) are reconsidered. As } \lambda = 1, \text{ the use of the symmetric Nash bargaining solution to study the trade policy coordination between countries is similar to the approach of maximising the linear sum of the two countries' utilities. However, the advantage of the former is that it gives a unique solution. Derivation along the latter approach merely suggests that the trade agreement can be any pair of tariffs that satisfy (4.8) while the former imposes another condition for uniqueness of the solution which is \((T_C^A, T_C^B)\) must also maximise the product of the countries' gains over the non-cooperative outcome \((U_A^A - U_B^A)(U_B^B - U_B^B))\).\]
From (4.12) and (4.13), it is clear that when the two countries are symmetric, \( \beta_x = \beta_y \)
(which implies \( T_N^A = T_N^B \) and \( U_N^A = U_N^B \)), the agreement reached between them will be
free trade, i.e. \( T_G^A = T_G^B = 1 \).

However, in the situation in which one country has greater market power than the
other\(^7\), their threat points differ, \( U_N^A \neq U_N^B \), and free trade between them is definitely not
the outcome.\(^8\) From (4.8), there must be an import tariff by one country and an import
subsidy by the other. In the mean time, from (4.10), the tariff-subsidy combination must
yield equal amount of gain over the non-cooperative outcome to both of them as they
have equal bargaining ability.

Numerical calculations of (4.12) and (4.13) for some hypothetical values of \( \beta_x \) and
\( \beta_y \) (shown in Table B.1 in Appendix B.4) suggest that the country with greater degree
of specialisation in its exportable imposes an import tariff while the other imposes an
import subsidy. Intuitively, instead of both countries non-cooperatively setting positive
import tariffs, the country with less market power agrees to subsidise its imports in
exchange for less aggressive import tariff by the other. In doing so, the country with
less market power can obtain better terms of trade and both countries can gain from
an increase in their trade volume, relative to the non-cooperative equilibrium.\(^9\) These

\(^7\)The countries' market power is defined by their price elasticities of foreign demand for their exports
as in chapter 3. The country with greater degree of specialisation in its exportable \( (\beta_j) \) faces less elastic
foreign demand for its export hence it has greater market power than the other.

\(^8\)Riezman (1982) shows that when the gains from moving to free trade are unevenly distributed, the
attainment of free trade is more unlikely.

\(^9\)The two countries also have losses associating with the agreement relative to the non-cooperative
equilibrium. They are losses in the tariff revenue and the cost of subsidisation for the country with less
market power and the deterioration in terms of trade for the other. However, since the agreement is
efficient, those gains always dominate the losses from implementing the tariff-subsidy combination.
results are similar to Mayer (1981) who shows that a trade agreement between a ‘small’ and a ‘large’ country must entail an import subsidy by the former and an import tariff by the latter. However, due to the more structural assumptions that we impose and the use of the Nash bargaining solution, we can derive a continuous set of equilibrium agreements corresponding to different configurations of the negotiating countries. The greater is the market power asymmetry, the larger the subsidy and the tariff agreed by the two countries.\(^\text{10}\)

4.3 Multi-country model

In this section, the model is still a pure exchange model of two goods, \(x\) and \(y\) with the world endowments of \(X\) and \(Y\), however, they are shared by more countries. There are countries \(\{A, a^1, a^2, \ldots, a^n\}\) exporting good \(x\) and countries \(\{B, b^1, b^2, \ldots, b^n\}\) exporting good \(y\). Recall that \(A\) and \(B\) are the dominating exporters, and \(\{a^1, a^2, \ldots, a^n\}\) and \(\{b^1, b^2, \ldots, b^n\}\) are the small-size exporters. Assuming that the small-size exporters of the same good are identical, the distribution of the world endowment is

\[
\begin{align*}
X^A &= \delta_x \beta_x X \\
Y^A &= \delta_x (1 - \beta_y) Y \\
X^B &= \delta_y (1 - \beta_x) X \\
Y^B &= \delta_y \beta_y Y \\
X^a &= \frac{(1 - \delta_x)}{n_x} \beta_x X \\
Y^a &= \frac{(1 - \delta_x)}{n_x} (1 - \beta_y) Y \\
X^b &= \frac{(1 - \delta_y)}{n_y} (1 - \beta_x) X \\
Y^b &= \frac{(1 - \delta_y)}{n_y} \beta_y Y
\end{align*}
\]

(4.14)

Recall that \(\delta_x\) and \(\delta_y\) and \(n_x\) and \(n_y\) are the ‘degree of market domination’ and the ‘degree of market concentration’ in the world market of good \(x\) and \(y\), respectively.

\(^\text{10}\) However, as implied by (4.10), both countries gain equally relative to the non-cooperative equilibrium. The use of the asymmetric Nash bargaining solution in which both countries have asymmetric bargaining ability may alter the distribution of gain to both countries. However, the qualitative results of this section will not change.
Identical and homothetic preferences represented by the Cobb-Douglas utility function is still assumed across all countries. Therefore, (4.2) still holds for all countries. It is restated in (4.15).

\[
U^i = \frac{\sqrt{T^i}}{(1 + T^i)} \frac{(X^i + qY^i)}{\sqrt{q}}
\]  

(4.15)

for all \( i \in \{A, a, B, b\} \) and \( T^i = 1 + \tau^i \).

With more countries, the world relative price (4.3) becomes

\[
q = \frac{[(1 - s^A) X^A + (1 - s^B) X^B + n_x(1 - s^a)X^a + n_y(1 - s^b)X^b]}{(s^A Y^A + s^B Y^B + n_x s^a Y^a + n_y s^b Y^b)}
\]  

(4.16)

where \( s^i = \frac{T^i}{(1 + T^i)} \) for all \( x \)-exporting countries, \( i \in \{A, a\} \), and \( s^i = \frac{1}{(1 + T^i)} \) for all \( y \)-exporting countries, \( i \in \{B, b\} \).

Consider from the perspective of country \( A \).\(^{11}\) In the previous section, the only trade agreement available to \( A \) is an agreement with \( B \) of which the source of gain is an increase in their trade volume. Another alternative available to \( A \) in this section is an agreement with its small-size co-exporters \( a \) of which the source of gain is their collective market power to manipulate their terms of trade against the \( y \)-exporting countries. Since these two types of agreement imply different source of gain and different source of negotiating partner to country \( A \), this section examines the conditions in which country \( A \) prefers one type of agreement to the other.

Therefore, the trade negotiations to be considered are the bargaining games between (i) \( A \) and \( B \) (the volume of trade-driven trade agreement) and (ii) \( A \) and \( a \) (the terms

\(^{11}\)Recall the discussion on the barriers to the US-EU cooperation as the motivation of this chapter. It is argued that they might have more preferable alternatives to the cooperation with each other as can be reflected by their recent emphasis on the trade arrangements in many regions rather than the cooperation with each other. Therefore, to examine a country’s choice of negotiating partners, it is reasonable to consider from the perspective of a dominating country if the two dominating countries in this model are to be interpreted as representing the US and the EU.
of trade-driven trade agreement). It is assumed that the two bargaining games are independent of each other, i.e. the bargaining between country A and any one country does not take into account the possibility that country A can also bargain with the other. Therefore, the threat points in all cases are the countries' non-cooperative utilities. Since the purpose of this section is to compare the negotiating choices of country A, it is assumed that only country A is active in seeking a trade agreement and it can only be either (i) or (ii) at a time. Therefore, when country A cooperatively setting its tariff with any one country, the other countries' tariffs will be set non-cooperatively. The two cases are considered separately and their comparisons are made at the end of this section.

4.3.1 Volume of trade-driven trade agreement

Suppose that there is a chance that A and B can coordinate their trade policies while, at the same time, the small-size exporters set their trade policies unilaterally. With symmetric Nash bargaining, the bargaining solution between A and B in the presence of small-size exporters is still the same as in section 4.2. Denote the cooperative tariffs as $T^A_{(A+B)}$ and $T^B_{(A+B)}$, the bargaining solution is

$$\left( T^A_{(A+B)}, T^B_{(A+B)} \right) = \arg \max_{(T^A, T^B)} (U^A - U^A_N) (U^B - U^B_N)$$

Therefore, the first order conditions are the same as in (4.5) and (4.6), hence the efficient condition (4.7) which is implied by (4.5) and (4.6) still holds. Therefore, A's and B's Nash equilibrium tariffs are still inefficient in this multilateral context. Solving (4.7) for $T^B$ given that $q$ is represented by (4.16), the efficient trade policy combinations between the two countries are defined by

\[^{12}\text{If the bargaining game between A and any one country takes into account that A can negotiate another trade agreement with the other, the bargaining position of A in that bargaining game is improved as it alters the threat point to the advantage of A. However, as we compare only the welfares of A across the two bargaining games and the bargaining position of A is improved symmetrically between them, removing the assumption that the two bargaining games are unrelated would not change the results.}\]
\[
T^B = \frac{\sqrt{\frac{T^A (X^A Y^B + X^B Y^A)}{-(T^A)^2 (X^A Y^B + X^B Y^A)^2}} - 4 \left[ \left( (X^A + W) (Y^A + Z) \right) \right] - \left[ \left( (X^A + W) (Y^B + Z) \right) \right]}{\left[ \left( X^A + X^B + W \right) \right] - \left[ \left( X^A + Y^A + Y^B + Z \right) \right]}
\]

(4.17)

where \( W \equiv [n_x (1 - s^a) X^a + n_y (1 - s^b) X^b] \) and \( Z \equiv (n_x s^a Y^a + n_y s^b Y^b) \).\(^{13}\) It can be seen that the efficiency locus between the two dominating exporters also depends on the small-size exporters’ trade policies through their effects on the world relative price \( q \). Algebraically, substituting (4.17) for \( T^B \) in (4.5) and solving for \( T^A \), then substituting the result back into (4.17), the equilibrium agreement between the two countries in terms of the small-size exporters’ tariffs can be obtained. In other words, the solutions \( T^A (T^a, T^b) \) and \( T^B (T^a, T^b) \) are their cooperative best response tariff functions to the small-size exporters’ tariffs. Since the small-size exporters’ tariffs are chosen non-cooperatively, their best response tariff functions are still the same as in chapter 3. The equilibrium tariffs of all countries can be obtained by solving all of these best response tariff functions simultaneously. Unfortunately, due to the complexity of the multi-country settings, the equilibrium tariffs can be solved only numerically by assuming some specific values of the model parameters. Therefore, the analysis of this section is based on the simulation results for some hypothetical values of those parameters.

According to the derivation above, in addition to (4.17), the expressions for (4.5) and the small-size exporters’ best response tariff functions are needed for the simulation to

\(^{13}\)The derivation of (4.17) is shown in Appendix B.2. It can be verified that when there are only country \( A \) and \( B \), i.e. when the degrees of market domination \( \delta_x = \delta_y = 1 \) which makes \( W = 0 \) and \( Z = 0 \), (4.17) is reduced to \( T^B = 1/T^A \) as in (4.8).
be conducted.

From (4.2) and (4.16), (4.5) is

$$\left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) + \lambda \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^A} = 0$$

$$\begin{bmatrix}
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)^2}} \left( \frac{X^A+qY^A}{\sqrt{q}} \right) - \frac{(qY^A-X^A)}{2\sqrt{q}} \left( 1+T^A \right) \left( 1+T^A \right)^2 \left( s_A^{Y^A} + s_B^{Y^B} + Z \right) \\
-\lambda \frac{(Y^B-X^B)}{2\sqrt{q}} \left( 1+T^B \right) \left( 1+T^A \right)^2 \left( s_A^{Y^A} + s_B^{Y^B} + Z \right)
\end{bmatrix} = 0$$

The expression can be simplified to

$$\lambda = \frac{(1-T^A)(1+T^A)}{(qY^B-X^B)\sqrt{T^B}} \left[ \frac{(1-s^A)X^A + (1-s^B)X^B + W}{\sqrt{T^A(1+T^A)} - (qY^A-X^A)T^A} \right]$$

Recall that $\lambda = \frac{(U^A-U^B)}{(U^B-U^R)}$, therefore,

$$\begin{bmatrix}
\frac{\sqrt{T^A}}{(1+T^A)} \left( \frac{X^A+qY^A}{\sqrt{q}} \right) - U^A_N \\
\frac{\sqrt{T^B}}{(1+T^B)} \left( \frac{X^B+qY^B}{\sqrt{q}} \right) - U^B_N
\end{bmatrix} = \frac{(1+T^B)}{(qY^B-X^B)\sqrt{T^B}} \left[ \frac{(1-s^A)X^A + (1-s^B)X^B + W}{\sqrt{T^A(1+T^A)} - (qY^A-X^A)T^A} \right]$$

(4.18)
(4.18) defines A’s cooperative best response tariff function to the small-size countries’ tariffs in which \( q \) is represented by (4.16). \( U^A_N \) and \( U^B_N \) are country A’s and B’s non-cooperative utility levels. They are available in Tables A.6 and A.7 in Appendix A.3.

Since the small-size exporters set their trade policies non-cooperatively, the small-size exporters’ best response tariff functions remain the same as in (3.24) in chapter 3. They are implicitly defined by\(^{14}\)

\[
T^a = \sqrt{\frac{X^a}{[(1-s^a)X^A+(1-s^B)X^B+(n_x-1)(1-s^a)X^a+n_y(1-s^B)X^B] + 1}}
\]

\[
T^b = \sqrt{\frac{Y^b}{[(1-s^A)Y^A+(1-s^B)Y^B+(n_x-1)s^a Y^a+n_y(1-s^B)Y^B] + 1}}
\]

The simulation is done by solving (4.17), (4.18), and (4.19) simultaneously for \( T^A \), \( T^B \), \( T^a \), and \( T^b \) for some hypothetical values of \( \beta_x \) and \( \beta_y \), \( \delta_x \) and \( \delta_y \), and \( n_x \) and \( n_y \). \( U^A_N \) and \( U^B_N \) in (4.18) are substituted correspondingly by the results in Tables A.6 and A.7 in Appendix A.3. Assume that \( \beta_x = \beta_y = 0.7 \) throughout the rest of the analysis for the convenience of comparison with the non-cooperative results in chapter 3 (\( \beta_x = \beta_y = 0.7 \) is assumed there). The effects of the degrees of market domination (\( \delta_x \) and \( \delta_y \)) and market concentration (\( n_x \) and \( n_y \)) are considered separately by assuming the other being equal and constant. The results, to be represented by \( T^A_{(A+B)} \), \( T^B_{(A+B)} \), \( T^a_{(A+B)} \), and \( T^b_{(A+B)} \), are substituted back into (4.16) for the equilibrium world relative price \( q_{(A+B)} \) and into (4.2) for the equilibrium utilities \( U^i_{(A+B)} \). As mentioned earlier, to evaluate the effects of the trade agreement, these results will be analysed relative to the non-cooperative results in chapter 3. The results are shown in Tables B.2 to B.7 in Appendix B.4.

As in chapter 3, the effects of the degree of market domination are investigated

\(^{14}\)Note that \( s^a \) (\( T^a \)) and \( s^b \) (\( T^b \)) are left on the right hand sides of the expressions for the convenience of presentation. If there are only one \( a \) and one \( b \), the terms \( s^a \) and \( s^b \) on the right hand sides are eliminated and the expressions in (4.19) are their best response tariff functions. See their derivation in Appendix A.2.

96
by assuming $n_x = n_y = 9$ while the effects of the degree of market concentration are investigated by assuming $\delta_x = \delta_y = 0.5$. The results are shown in Tables B.2 and B.5 for the equilibrium tariffs, Tables B.3 and B.6 for the equilibrium world relative price, and Tables B.4 and B.7 for the equilibrium utilities. The non-cooperative results are also presented for comparison. In Tables B.3 and B.6, the world relative prices are normalised by the free trade world relative price $q_F = \frac{x}{y}$ to eliminate the term $\frac{x}{y}$, therefore, the values shown indicate the percentage values relative to the free trade level. In Tables B.4 and B.7, the equilibrium utilities are normalised by the corresponding non-cooperative utilities so that the values which exceed 1 indicate the welfare improvement from the non-cooperative equilibrium and the other way round for the values less than 1.

Similar to the result in the two-country model, it can be observed from Tables B.2 and B.5 that free trade agreement between $A$ and $B$ is achieved only in the symmetric cases where $\delta_x = \delta_y$ and $n_x = n_y$. Otherwise, the trade agreement must entail a lower import tariff (relative to the Nash equilibrium tariff) by the country that has greater market power (the country whose exportable market is more dominated and more concentrated) in exchange for an import subsidy by the other.

From Tables B.4 and B.7, both $A$ and $B$ gain from the trade agreement over the non-cooperative outcomes in all cases since the non-cooperative tariffs are inefficient between them. However, despite their symmetric bargaining ability (reflected by the use of symmetric Nash bargaining solution), $A$ and $B$ do not always gain an equal amount in equilibrium as in the two-country model. This is because the efficient tariff combinations between them still depends on the small-size exporters’ endowments and their trade policies (the terms $W$ and $Z$ in (4.17)) hence their utilities are not always perfectly substitutable in their bargaining (the efficient frontier in the utility space is not linear with the slope $-1$, therefore, the level curve $(U^A - U^A_N) (U^B - U^B_N)$ need not always be tangent with the efficient frontier at its slope $-\frac{(u^A-u^A_N)}{(u^A-u^A_N)} = -1$, i.e. when both countries gain an equal amount).

Consider the effects of the trade agreement on the small-size countries. Tables B.4 and
B.7 suggest that all small-size exporters will gain only in the symmetric cases. Otherwise, only the small-size exporters in the less competitive market will lose in the equilibrium. This is due to the strategic effects of the dominating countries' tariffs on the small-size exporters'. In the symmetric cases, free trade between the dominating countries under the agreement allows all the small-size countries to impose lower tariffs and obtain a higher utility level at the world relative price equal to the free trade level (see Tables B.2, B.3, B.5, and B.6). In the asymmetric cases, it can be observed that the world relative price will be shifted from the interests of the countries whose exportable market is less competitive \((q_{A+B}) > q_N\) when \(\delta_x > \delta_y\) and \(n_x < n_y\); and \(q_{(A+B)} < q_N\) when \(\delta_x < \delta_y\) and \(n_x > n_y\) in Tables B.3 and B.6) due to the trade agreement. Even if Tables B.2 and B.5 suggest that the losing small-size countries impose smaller tariffs than in the non-cooperative equilibrium\(^{15}\), the gain from their increase in trade volume cannot compensate such loss in their terms of trade relative to the non-cooperative outcomes.

In general, the \(A + B\) trade agreement is aimed at increasing the trade volume by shifting the world relative price from the non-cooperative toward the free trade level. Therefore, the dominating country for which the world relative price is shifted from free trade in its favour in the non-cooperative equilibrium (the country whose market power is larger) has to lower its tariff to bring back the world relative price toward free trade. This change in the world relative price, therefore, affects the small-size exporters. The small-size exporters whose exportable market is less competitive are clearly worse off since the trade agreement undermines the terms of trade they once enjoyed in the non-cooperative equilibrium \((q_F > q_{A+B}) > q_N\) when \(\delta_x > \delta_y\) and \(n_x < n_y\); and \(q_F < q_{(A+B)} < q_N\) when \(\delta_x < \delta_y\) and \(n_x > n_y\) in Tables B.3 and B.6). These results are summarised in Proposition 3.

**Proposition 3** Relative to the non-cooperative equilibrium, a trade agreement between the dominating countries (i) makes all countries better off when both markets are sym-

\(^{15}\)The small-size \(x\)-exporters will impose lower (higher) tariff and the small-size \(y\)-exporters will impose higher (lower) tariff than under the non-cooperative equilibrium when \(A\) has greater (less) market power than \(B\) as the strategic effect of \(A\)'s tariff will dominate (be dominated by) \(B\)'s tariff.
metric (ii) makes all countries except the small-size countries whose exportable market is less competitive better off (makes the small-size countries whose exportable market is less competitive worse off).

4.3.2 Terms of trade-driven trade agreement

Now suppose that $A$ can negotiate a number of $n_x$ bilateral trade agreements with each of country $a$, while leaving the $y$-exporting countries setting their trade policies unilaterally. Since countries $A$ and $a$ have the same motive to lower the world relative price $q$ through their import tariffs, it can be expected that the collusion between them results in their higher tariffs relative to the non-cooperative tariffs to shift the world relative price to their advantage collectively. However, such gain definitely comes at the cost of the $y$-exporting countries. Since the purpose of this chapter is to compare the expected gains for a dominating country between its negotiating partner alternatives, we abstract from the possibility that the remaining countries can also collaborate their trade policies in retaliation.

Denote the cooperative tariffs as $T^A_{(A+a)}$ and $T^a_{(A+a)}$. Since countries $a$ are identical, the generalised bargaining solution between country $A$ and $n_x$ of countries $a$ is

$$(T^A_{(A+a)}, T^a_{(A+a)}) = \arg \max_{(T^A, T^a)} \left( U^A - U^A_N \right) n_x \left( U^a - U^a_N \right)$$

---

16 By a rough experiment not presented in this work, when country $A$ can negotiate with a subset of countries $a$, those countries $a$ who remain outside of the trade agreement and set their tariffs unilaterally obtain higher gain than those setting their trade policies under the trade agreement. This is because those remain outside of the agreement still enjoy an improvement in the terms of trade from the collusive effort of those in the trade agreement at a lower tariff rate they impose unilaterally. It is found that the gain for those in the trade agreement including country $A$ increases when more countries $a$ are joining. Since all countries $a$ are identical, there is no justification that a subset of them will act differently by staying out of the agreement and free ride the collusive effort. Therefore, all countries $a$ will be negotiating with country $A$ in this section.

17 If retaliatory trade agreement between $y$-exporters is allowed, the world consists of two export cartels. Each export cartel imposes tariff against each other, therefore, the tariff setting game is analogous to the tariff war between two countries in chapter 3. See Krugman (1991) and Bond and Syropoulos (1996) which consider the world consisting of trading blocs imposing optimum tariffs against each other.
From (4.15) and (4.16), the first order conditions for the above solution are

\[
\left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) + \mu \frac{\partial U^a}{\partial q} \frac{\partial q}{\partial T^A} = 0 \tag{4.20}
\]

and

\[
\left( \frac{\partial U^a}{\partial T^a} + \frac{\partial U^a}{\partial q} \frac{\partial q}{\partial T^a} \right) + \frac{1}{\mu} \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^a} = 0 \tag{4.21}
\]

where \( \mu = \frac{(U^A - U^\alpha)}{(U^a - U^\alpha)} \).

It can be seen that the number of small-size countries \( n_x \) can be eliminated from the first order conditions.\(^{18}\)

(4.20) and (4.21) imply the efficiency condition between all \( x \)-exporting countries which is

\[
- \left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) = - \frac{\partial U^a}{\partial q} \frac{\partial q}{\partial T^a} \left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^a}{\partial q} \frac{\partial q}{\partial T^a} \right) \tag{4.22}
\]

Consider the efficiency between all \( x \)-exporting countries. It can be seen from (4.22) that the \( x \)-exporting countries’ tariffs are inefficient and they can gain from cooperation. However, the countries impose the Nash equilibrium tariffs lower than the efficient level in this case.\(^{19}\)

\(^{18}\)If \( A \) negotiate a plurilateral trade agreement with all of countries \( a \) at once, the bargaining solution will be \( (T^A_{(A+\alpha)}, T^\alpha_{(A+\alpha)}) = \arg \max_{(T^A,T^\alpha)} (U^A - U^\alpha)(U^a - U^\alpha)^{n_x} \) where \( n_x \) now indicates the collective bargaining power of the small-size countries. Therefore, it is presumable that \( A \) would be better off with the \( n_x \) of bilateral agreements than with the plurilateral agreement. So we consider the former in this section.

\(^{19}\)It can be shown that when countries \( a \) are small, the \( x \)-exporting countries’ Nash equilibrium tariffs are already efficient so that they cannot gain from a trade agreement. The RHS in (4.22) converges to zero if countries \( a \) are small, i.e. \( \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^a} = 0 \). Therefore, (4.22) is satisfied even if \( A \) sets its
Solving (4.22) for $T^A$ the efficiency locus between the dominating and the small-size exporters of good $x$, given the tariffs of $y$-exporting countries, can be obtained. The derivation shown in Appendix B.3 gives

$$
T^A = \frac{-T^a (X^A Y^a + X^a Y^A)}{(T^a)^2 (X^A Y^a + X^a Y^A)^2} + \sqrt{-4 \left[ (T^a)^2 E (Y^A + Y^a + F) \right] \left[ F (X^A + X^a + E) \right] \left[ - (T^a)^2 (X^A + E) (Y^a + F) \right]} \left[ - (X^a + E) (Y^A + F) \right]}
$$

(4.23)

where $E \equiv [(1 - s^B) X^B + (n_x - 1) (1 - s^a) X^a + ny (1 - s^b) X^b]$ and $F \equiv [s^B Y^B + (n_x - 1) s^a Y^a + ny s^b Y^b]$.

(4.23) and (4.21) must be solved simultaneously with country $B$’s and $b$’s non-cooperative best response tariff functions to determine the trade agreement between countries $A$ and $a$ and the general equilibrium as a whole. Simulations over the same hypothetical values of the model parameters as in the previous section are conducted to analyse the negotiation between countries $A$ and $a$.

In addition to (4.23), the expressions for (4.21) and the $y$-exporting countries’ best response tariff functions are needed for the simulation to be conducted. From (4.15) and (4.16), (4.21) is

$$
tariff\ unilaterally \left( \frac{\partial U^A}{\partial x} + \frac{\partial U^A}{\partial y} \frac{\partial y}{\partial x} = 0 \right). \text{ This finding also applies to the efficiency between the Nash equilibrium tariff of } A \text{ and the Nash equilibrium tariff of ‘small’ country } b (\text{zero}) \text{ that have different trade patterns. Therefore, it contradicts the Mayer (1981) result that a small country can still gain from a trade agreement with a large country.}
$$

101
\[
J = \frac{(1-T^a)^2}{2\sqrt{T^a(1+T^a)}^2} \frac{(X^a+qY^a)}{\sqrt{q}}
- \frac{1}{2\sqrt{q}} \frac{(qY^a-X^a) \sqrt{T^a}}{(1+T^a)^2} \frac{1}{(1+T^a)^2} \frac{(X^a+qY^a)}{(sA^A+sB^B+s^a+sx^a+s^b+y^b)}
- \frac{1}{\mu} \frac{1}{2\sqrt{q}} \frac{(qY^a-X^a) \sqrt{T^a}}{(1+T^a)^2} \frac{1}{(1+T^a)^2} \frac{(X^a+qY^a)}{(sA^A+sB^B+s^a+sx^a+s^b+y^b)}
\]

= 0

Simplify the expression.

\[
\mu = \frac{(qY^A - X^A) \sqrt{T^A}}{(1 + T^A)} \frac{\sqrt{T^a} (1 + T^a)}{(1 - T^a) (1 + T^a)} \left( \frac{(1 - s^A) X^A + (1 - s^B) X^B + n_x (1 - s^a) X^a + n_y (1 - s^b) X^b}{(1 - T^a) (1 + T^a)} \right) - (q Y^a - X^a) T^a
\]

Substitute for \( \mu = \frac{U^A - U^a}{U^a - U^N} \).

\[
\frac{\sqrt{T^A} (X^A+qY^A)}{(1+T^A)^2} \frac{\sqrt{q}}{-U^A_N} = \frac{(qY^A - X^A) \sqrt{T^A}}{(1 + T^A)} \frac{\sqrt{T^a} (1 + T^a)}{(1 - T^a) (1 + T^a)} \left( \frac{(1 - s^A) X^A + (1 - s^B) X^B + n_x (1 - s^a) X^a + n_y (1 - s^b) X^b}{(1 - T^a) (1 + T^a)} \right) - (q Y^a - X^a) T^a
\]

(4.24)

(4.24) defines the small-size country \( a \)'s cooperative best response tariff function to \( y \)-exporting country tariffs, given \( q \) represented by (4.16). \( U^a_N \) and \( U^a_N \) are country \( A \)'s and \( a \)'s non-cooperative utility levels. They are available in Tables A.6 and A.7 in Appendix A.3.

From (3.24) in chapter 3, the \( y \)-exporting countries' unilateral best response tariff
functions are

\[
T^B = \sqrt{\frac{Y^b}{(s^A Y^A + n_x s^{\alpha} Y^{\alpha} + n_y \delta^\alpha Y^B)} + 1}
\]

\[
T^b = \sqrt{\frac{Y^b}{[s^A Y^A + s^B Y^B + n_x s^{\alpha} Y^{\alpha} + (n_y - 1) \delta^\alpha Y^B] + 1}}
\]

The simulation is done by solving (4.23), (4.24), and (4.25) simultaneously for \(T^A\), \(T^B\), \(T^a\), and \(T^b\) over some hypothetical values of \(\beta_x\) and \(\beta_y\), \(\delta_x\) and \(\delta_y\), and \(n_x\) and \(n_y\). \(U_N^A\) and \(U_N^a\) in (4.24) are substituted correspondingly by the results in Tables A.6 and A.7 in Appendix A.3. Similar to the previous section, the degrees of specialisation (\(\beta_x\) and \(\beta_y\)) are chosen to be \(\beta_x = \beta_y = 0.7\) throughout the analysis. The effects of the degrees of market domination (\(\delta_x\) and \(\delta_y\)) and market concentration (\(n_x\) and \(n_y\)) are considered separately by assuming the other being equal and constant. The results, to be represented by \(T^A_{(A+a)}\), \(T^B_{(A+a)}\), \(T^a_{(A+a)}\), and \(T^b_{(A+a)}\), are substituted back into (4.16) for the equilibrium world relative price \(q_{(A+a)}\) and into (4.2) for the equilibrium utilities \(U^i_{(A+a)}\). The results are shown in Tables B.8 to B.13 in Appendix B.4.

In all cases except when countries a are 'small' (when \(n_x \to \infty\) in Tables B.11 to B.13), the trade agreement between the \(x\)-exporters pushes up their tariffs which strategically reduces \(y\)-exporters’ tariffs. The result is that the world relative price \(q\) is shifted from the non-cooperative level to the advantage of the \(x\)-exporters (Tables B.9 and B.12). Therefore, the \(x\)-exporters are better off, however, at the cost of the \(y\)-exporters relative to the non-cooperative equilibrium (Table B.10 and B.13). This is because when the \(x\)-exporters internalise the effects of their trade policies, they choose their tariff rates not only to maximise their own welfare but the weighted sum of all the \(x\)-exporting countries’ welfare as a whole. This requires a tariff rate higher than the unilateral one. Facing the higher tariff from the \(x\)-exporting countries to which the \(y\)-exporting countries’ tariffs are
strategic substitutes, relative to their non-cooperative equilibrium tariffs (without the x-exporters' trade agreement), the y-exporting countries have to lower their tariffs along their unilateral best response functions. Consequently, the world relative price is shifted to the advantage of the x-exporting countries and to the disadvantage of the y-exporting countries.

Given the symmetric settings between x-exporters and y-exporters, similar results will be obtained when tariff coordination between y-exporting countries is considered. Therefore, the results can be generalised in Proposition 4.

Proposition 4 Relative to the non-cooperative equilibrium, a trade agreement between countries that export the same good makes all countries exporting that good better off but countries that export the other good worse off.

Proposition 4 is straightforward. As recognised in the customs union literature, the coordination of external tariff policies by CU members enables them to internalise the positive terms of trade externalities they generate for each other when they import similar products from the rest of the world. Syropoulos (1999) argues that since the CU members also liberalise their internal trade, there must be a 'trade liberalising' force that tempers this market power. Therefore, the effect of the CU's common external tariff on the rest of the world is still ambiguous. However, we obtain a clear cut result that the trade policy coordination between countries that import the same good always harms the rest of the world, as stated in Proposition 4, since, in our model, the negotiating countries import and export the same goods so that there is virtually no trade between them to be liberalised, consequently, the trade policy coordination generates only the 'externality internalising' force that enhances their collective market power in world trade.

4.3.3 Comparison of results

This section makes the main argument of this chapter: why would a country prefer to negotiate a trade agreement with one country rather than others? We investigate the
conditions in which the terms of trade-driven agreement will be preferred to the volume of trade-driven agreement by A by comparing the country A's payoffs between the two agreements.

The payoffs to country A from the two types of trade agreement are presented in Tables B.4, B.7, B.10, and B.13. All of them are normalised by the non-cooperative utility and are readily comparable. In general, the criterion to determine which of the two agreements is preferred by country A in a specific circumstance is which of the two agreements gives higher utility to country A. Given the normalisation of the results, the criterion will be which of the two agreements makes country A better off to the larger extent relative to the non-cooperative outcome, i.e. country A will prefer a trade agreement with country a rather than the one with country B when \( \frac{U_A^{(A+a)}}{U_A^{(N)}} > \frac{U_A^{(A+B)}}{U_A^{(N)}} \) and vice versa. Comparing the results between Tables B.4 and B.10 gives the results in Table B.14; and between Tables B.7 and B.13 gives the results in Table B.15. The letter in each cell indicates the country whom country A prefers its trade agreement with.

Table B.14 suggests that, given equal degree of market concentration in both markets, the \( A+a \) trade agreement will be preferred to the \( A+B \) trade agreement when country A dominates not too much of the world \( x \)-market, i.e. when \( \delta_x \) is sufficiently low for which the threshold value decreases with \( \delta_y \). Table B.15 suggests that, given equal degree of market domination in both markets, the \( A+a \) trade agreement will be preferred to the \( A+B \) trade agreement when the world market of good \( x \) is highly concentrated, i.e. \( n_x \) is sufficiently low. Given the degree of market domination \( \delta_x = \delta_y = 0.5 \) in B.15, the suggested maximum number of small-size \( x \)-exporter is \( n_x = 1 \) which implies that country A and a are identical, i.e. they share the world endowments equally (\( \delta_x = (1 - \delta_x) = 0.5 \)). In addition, it should be noted that the results are invariant with the degree of market concentration in the world market of good \( y \) (\( n_y \)). These results are summarised in Proposition 5.

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20In fact, they are readily comparable even without the normalisation since the comparison across the two cases is made between the same country and the same values of model parameters. However, if we have to compare the unnormalised ones, another table showing them is needed which is unnecessary.
Proposition 5 Given the degree of market domination in the \( y \)-market \( \delta_y \), \( \tilde{\delta}_x \) and \( \tilde{n}_x \) are the degrees of market domination and market concentration in the \( x \)-market which make country \( A \) indifferent between the \( A+a \) and the \( A+B \) trade agreement. The former will be preferred to the latter by country \( A \) when \( \delta_x \leq \tilde{\delta}_x \) and \( n_x \leq \tilde{n}_x \).

To help explain Proposition 5, it should be noted from sections 4.3.1 and 4.3.2 that there are different sources of gain for country \( A \) between the \( A+a \) and the \( A+B \) trade agreement. Relative to non-cooperative equilibrium, under the former, country \( A \) sacrifices its trade volume for a better terms of trade by an increase in its tariff while, under the latter, country \( A \) sacrifices its terms of trade for a larger trade volume by a decrease in its tariff. Therefore, in any circumstances, for country \( A \) to prefer the \( A+a \) to the \( A+B \) trade agreement, the net gain in its terms of trade from the former must outweigh the net gain in its trade volume from the latter. This means that, in addition to \( A \)'s tariff, the tariffs imposed by the small-size countries \( a \) under the \( A+a \) agreement must also be large enough. Each small-size country \( a \) will be able to impose the large enough tariff when it possesses a large enough endowments (which is positively related with their market power); and for the small-size country \( a \) to possess such a large enough endowments, the \( x \)-market must be sufficiently low dominated by country \( A \) and sufficiently high concentrated.\(^{21}\) In general, the terms of trade-driven trade agreement between the exporters of one good will be more likely when the structure within the world market of that good allows the small-size exporters to have sufficient market power to attract its dominating exporter. Otherwise, the volume of trade-driven agreement with the other dominating country will be preferable. It can be concluded that the choice of a country to negotiate a trade agreement does not depend on the strategic complementarity or substitutability of its negotiating partners but their levels of endowments that generate sufficiently large gain (no matter it is the gain in terms of trade or in volume of trade)

\(^{21}\)Therefore, \( \tilde{n}_x \), the maximum number of the small-size \( x \)-exporters to make \( A+a \) trade agreement preferable, could be greater than 1 if the degree of market domination by the dominating countries is lower than that assumed in Table B.15.
relative to the non-cooperative equilibrium.\footnote{Therefore, the results are invariant with \( n_y \) since a change in \( n_y \) affects neither the endowments of \( B \) nor \( a \).}

### 4.4 Conclusion

This chapter extended the multi-country two-good trade model in chapter 3 to consider the possibility that a subset of the countries could coordinate their trade policies. A trade agreement within the two-country model was first examined so that the results in the multi-country one can be compared. It was shown that the Nash equilibrium tariffs are inefficient and there was a rationale for a trade agreement. Under symmetric Nash bargaining, the two countries agreed to implement free trade when they were symmetric, otherwise, they adopted a tariff-subsidy combination scheme in which the country with greater market power reduced its tariff in exchange for an import subsidy by the other country.

In the multi-country model, there were more alternatives available to a country to negotiate its trade policy. One was the volume of trade-driven trade agreement negotiated between countries with different trade patterns and the other was the terms of trade-driven trade agreement negotiated between countries with similar trade pattern. The volume of trade-driven agreement between the two dominating countries made all countries better off relative to the non-cooperative equilibrium when the market structures in \( x \) and \( y \) markets were symmetric. Otherwise, the small-size countries who had enjoyed the non-cooperative terms of trade would be worse off. The terms of trade-driven agreement between the dominating and the small-size exporters of one good, by virtue of their collective market power, raised their tariffs to improve their terms of trade and made themselves better off at the cost of the exporters of the other good.

Since the two types of trade agreement implied different sources of gain and different negotiating partners, this chapter examined the conditions in which one was preferred to the other by a country. By comparing the welfare implications between the two
types of agreement from the perspective of a country, it was found that the choice of a country to negotiate a trade agreement did not depend on the strategic complementarity or substitutability of its negotiating partners' tariffs but their levels of endowments that generated sufficiently large gain (no matter it was the gain in terms of trade or in volume of trade) relative to the non-cooperative equilibrium.

Since the study of tariffs in this chapter as well as that in chapter 3 is based solely on the terms of trade incentive of a large country to impose a positive import tariff, more dimensions can be added to the model if the political economy aspect of tariff is considered. The country's preference between the two types of trade agreement suggested in this chapter would be changed if private sectors with diverse interests in the trade volume and the terms of trade are allowed to influence the governments' decision. Relative to the governments' trade policies in absence of the political motive, the governments have to reformulate them to strike a new balance between the terms of trade and the political motive. The political economy aspect is, therefore, an interesting candidate for an extension of the model.

In the next chapter the political economy of trade policy is considered. However, we confine to the trade policy formation in a small country in which the terms of trade argument is dropped to examine the issue of domestic lobby formation.
Chapter 5

An Endogenous Lobby Formation Model

This chapter presents a political economy model of tariff in a small open economy. It is assumed that the government’s trade policy is a result of lobbying activities and hence the influence driven contribution approach of Grossman and Helpman (1994) is employed. While previous works on lobbying and international trade policy tend to take the existence of a lobby as given, this chapter goes further by attempting to endogenise the lobby formation. It is shown explicitly that individuals would not lobby individually when the total population and/or the fixed cost of lobbying are too large. An incentive which leads the individuals to form a lobby is the ability of the group to restrain their otherwise offsetting lobbying efforts. An interesting result is that, in equilibrium, some individuals might choose to join the lobbies that lobby against their interests to moderate their efforts rather than to join the lobbies that lobby in their favour. This result raises a question whether the standard industry-lobby in the literature might exaggerate the actual lobbying activities.
5.1 Introduction

Grossman and Helpman (2001) note that “political action committees (PACs), which are organisations that have registered with the US government under the reporting requirements of the Federal Election Campaign Act of 1974 so as to be able to contribute legally to political candidates, numbered 3,835 at the end of 1999. This represents a dramatic increase from the 608 PACs that had registered by the end of 1974...The overall impression that these figures give is that the number of SIGs (special interest groups) active in national politics in the United States is by no means small, and probably continues to grow.”

One of the theories that deal with the lobbying activities is the influence driven contribution model of Grossman and Helpman (1994) in which interest groups can influence the government’s policies by offering contributions contingent on the trade policies implemented by the government. It is assumed that the government is willing to trade off some reduction in the general welfare in return for a the contribution. As a result, groups that can be organised are able to obtain the trade policies in favour of their interests at the expense of the public as a whole. While the model can explain the political process in which particular groups can translate their interests into the government policies, the existence of the lobbies is still exogenous. This chapter, therefore, attempts to endogenise the lobby formation.

In fact, there have been a few papers investigating the lobby formation. Mitra (1999) shows that the benefit from lobbying to each industry lobby is decreasing with the number of them and hence not all industries can be organised in the long run. By assuming that there exist fixed costs of lobbying which are heterogeneous across industries, the industries which can be organised are those whose fixed costs of lobbying not exceeding the lobbying benefit to their industries.

In general, Olson (1971) points out the free rider problem in group formation. He argues that since the good provided by a collective action tends to be a public good, it is in an individual’s interest not to contribute to the cost of providing the good which still
be provided by the contributions of the others. Therefore, groups consisting of so large
number of members that their members' actions are imperceptible may find it difficult to
act collectively. Groups that can be organised tend to be the groups with small number
of members.

Following Olson (1971), Pecorino (1998) analyses the ability of firms in an industry to
overcome the free rider problem in a repeated tariff lobbying game where their cooperation
in lobbying is maintained through the use of trigger strategy. The difficulty of maintaining
cooperation is measured by the critical value of the discount parameter below which the
cooperation cannot be maintained. He finds that the critical level does not rise to the
cooperation prohibitive level (one) as the number of firms approaches infinity so he argues
that there is no presumption that the cooperation must break down as the number of
firms increases without bound. Magee (2002), also in a repeated game framework, finds
a more clear cut result that an increase in the number of firms necessarily worsens the
free-rider problem.

However, it should be noted that these works implicitly assume that the firms lobby
the government only through forming a lobby and each lobby will be organised only
among the firms within the same industry. Based on the model of a small open economy
consisting of individuals whose endowments of goods hence trade policy preferences are
different, this chapter examines the individuals' decision to influence the government in
a more general framework in which they are allowed to lobby individually and join any
lobby. Section 5.2 provides the basic setting of the model. Section 5.3 formally shows
that individual would not lobby individually when the total population and/or the fixed
cost of lobbying are too large. Section 5.4 considers the lobby formation and Section 5.5
concludes.
5.2 Basic settings

Consider a small open country consisting of $N$ individuals endowed with and consuming two goods, $x$ and $y$. Preferences toward consumption of the two goods are identical across all individuals and represented by a quasi-linear utility function.

\[ u^i = \alpha c_y^i - \frac{1}{2} (c_y^i)^2 + c_x^i \]  

(5.1)

where $u^i$, $c_x^i$ and $c_y^i$ are individual $i$'s utility level, consumption of good $x$ and good $y$ respectively; and $\alpha > 0$ is a constant.\(^1\)

Lagrangean optimisation implies equal consumption of good $y$ for all individuals, presuming each individual's income is high enough,

\[ c_y^i = \alpha - \frac{p_y}{p_x} \]  

(5.2)

$p_x$ and $p_y$ are goods $x$ and $y$ domestic prices. Assume that the economy is a net importer of good $y$ whose world price is $q$ and the government imposes an ad valorem tariff or subsidy rate $\tau$ on the imports. Choose $p_x = 1$ as numeraire, (5.2) becomes

\[ c_y^i = \alpha - q (1 + \tau) = \alpha - qT \]  

(5.3)

where $T = 1 + \tau$. It can be seen that, given the utility function assumed, consumption of good $y$ is affected by the import tariff/subsidy uniformly across all individuals.

The rest of the income is spent on consumption of good $x$, therefore, individual $i$'s expenditure on good $x$ is his/her income, $m^i$, less expenditure on the consumption of

\(^1\)This quasi-linear utility function is adopted from Laussel and Riezman (2001). Its advantage is that it generates a linear expenditure function which eases subsequent derivations.
good $y$ represented by (5.3).

$$c^i_x = m^i - qT(\alpha - qT) \quad (5.4)$$

Individual $i$’s income depends on his/her endowment of both goods, $x^i$ and $y^i$, and the tariff revenue transfer from the government, $r^i$.

$$m^i = x^i + qTy^i + r^i$$

Assume that the economy aggregate endowment of good $x$ and $y$ are $X$ and $Y$, each individual shares equal amount of good $x$ but uneven amount of good $y$ such that

$$x^i = \frac{X}{N}$$
$$y^i \geq 0; \sum_{i=1}^{N} y^i = Y$$

Assume that the government tariff revenue is redistributed equally among all individuals in the economy.\(^2\) Therefore,

$$r^i = \frac{q(T-1)(NC_y^i - Y)}{N} = \frac{q(T-1)(N(\alpha - qT) - Y)}{N}$$

From (5.3), individuals consume good $y$ equally, the aggregate consumption of good $y$ is, therefore, individual consumption of good $y$ times the number of population, $NC_y^i$.

Substituting these facts into (5.4), individual $i$’s consumption of good $x$ is

\(^2\)As opposed to Mayer (1984) who assumes that the tariff revenue is redistributed according to individuals’ factor income, this assumption better reflects the fact that the government’s revenues are usually spent on providing public goods which any individual can consume.
\[ c_i = \left( \frac{(X + qY)}{N} - \alpha q \right) + q (\theta_i + q) T \quad (5.5) \]

where \( \theta_i = y_i - \frac{X}{N} \). \( \theta_i \geq 0 \) means individual \( i \)'s endowment of good \( y \) is larger, equal and lower than average and, therefore, \( \sum_{i=1}^{N} \theta_i = 0 \). Substituting (5.3) and (5.5) back into (5.1), individual \( i \)'s utility is

\[ u^i = \left( \frac{(X + qY)}{N} - \alpha q + \frac{\alpha^2}{2} \right) - \frac{q^2}{2} T^2 + q (\theta_i + q) T = u^i (\theta_i, T) \quad (5.6) \]

It can be seen that individual utility is a function of tariff and individual ownership of good \( y \). By (5.6), it can be shown that individuals have different preferences toward the government's tariff policy. Differentiate (5.6) with respect to \( T \).

\[ \frac{\partial u^i}{\partial T} = q \left[ \theta_i^i + q (1 - T) \right] \quad (5.7) \]

Setting \( \frac{\partial u^i}{\partial T} = 0 \), individual \( i \)'s optimal tariff rate is implied,

\[ T^i = 1 + \frac{\theta_i^i}{q} \quad (5.8) \]

It can be seen that the individual optimal tariff rate depends on individual ownership of good \( y \). \( T^i \geq 1 \) when \( \theta_i^i \geq 0 \), i.e. an individual prefers an import tariff, free trade, or an import subsidy when his/her endowment of good \( y \) is larger, equal or less than average. The further the deviation from the average, the higher is his/her optimal tariff/subsidy rate.
5.3 Lobbying individually

This section studies tariff policy formation within the framework of Grossman and Helpman (1994) and individuals' decision to lobby when they set their contribution schedules and offer to the government individually. The main objective of this section is to examine why the individuals would prefer to lobby the government through groups rather than to lobby on their individual efforts. The main argument is that the individuals consider themselves too small relative to the lobbying costs to make the individual efforts. This point is often argued by the previous works, such as Mitra (1999), and it is to be shown explicitly in this section.

In the first stage of the game, each individual decides whether to lobby or not to lobby and announces a contribution schedule to the government, given the contribution schedules of the others, if he/she decides to lobby. It is assumed for the time being that the individual can announce a contribution schedule without any fixed cost. The contribution schedule specifies how much contribution the individual will be making against every possible tariff rate chosen by the government. In the second stage, given the contribution schedules, the government chooses and implements a tariff rate and collects the contributions.

5.3.1 Policy formation

Let $u^i (\theta^i, T)$ represents individual $i$'s utility. If $N$ is the set of all population, define $L \subseteq N$ as a set of all individuals who decide to lobby. Every $i \in L$ chooses a contribution schedule (function), $s^i (T)$, which maximises his/her net utility, i.e.

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3Before an individual can convey his/her desire to the government, some costs might be incurred and those costs are viewed as sunk investment by many authors, inter alia, Mitra (1999). Mitra (1999) argues that the costs may consist of the costs of establishing links with politicians, communication, etc. In case of a lobby group which is to be considered in the next section, the fixed costs may be the costs of forming an organisation, hiring professional lobbyists, building a communications network among members, designing a scheme of punishments for defaulting members, etc.
Given the contribution schedules, in the second stage, the government’s objective is to maximise the weighted sum of the aggregate contribution and the general welfare, i.e.

$$\max_{\theta^i, T} u^i(\theta^i, T) - s^i(T)$$

where $u^i(\theta^i, T)$ is the aggregate utility and $a \geq 0$ represents the government’s concern over its political survival which depends on the general welfare at large.

Let $T_p$ be the government’s political optimal tariff defined by

$$T_p = \arg \max_{T} \sum_{i \in L} s^i(T) + aU(T)$$

and $T_p^{-i}$ be the government’s optimal policy were individual $i$ not participating in lobby, defined by $T_p^{-i} = \arg \max_{T} \sum_{j \in L^{-i}} s^j(T) + aU(T)$, where $L^{-i} \equiv L - \{i\}$. Without imposing any additional assumption on the contribution function, a contribution function chosen by every individual can take many forms as long as two conditions below are satisfied. The first is

$$s^i(T_p) + \sum_{j \in L^{-i}} s^j(T_p) + aU(T_p) \geq \sum_{j \in L^{-i}} s^j(T_p^{-i}) + aU(T_p^{-i})$$

for all $i \in L$. This condition states that to induce the government to implement $T_p$, the contribution made by every individual $i \in L$ must make the government at least as
well off as implementing its optimal policy were he/she not participating in lobby. Since the individuals have no desire to pay the government more than necessary to induce it to implement \( T_p \), the condition is, therefore, met with equality. Rearranging gives

\[
s^i (T_p) = \left[ \sum_{j \in L - i} s^j (T_p^{-i}) + aU (T_p^{-i}) \right] - \left[ \sum_{j \in L - i} s^j (T_p) + aU (T_p) \right]
\]  \hspace{1cm} (5.10)

By the definition of \( T_p^{-i} \), \( s^i (T_p) \) is, therefore, non negative.

The second condition is that, in equilibrium, the contribution made by every individual must not exceed his/her gain from participation in lobby, i.e.

\[
s^i (T_p) \leq u^i (\theta^i, T_p) - u^i (\theta^i, T_p^{-i})
\]  \hspace{1cm} (5.11)

for all \( i \in L \), otherwise there would be some individuals in the set \( L \) who find it beneficial not to lobby. Substitute (5.10) into (5.11),

\[
\left[ \sum_{j \in L - i} s^j (T_p^{-i}) + aU (T_p^{-i}) \right] - \left[ \sum_{j \in L - i} s^j (T_p) + aU (T_p) \right] 
\leq u^i (\theta^i, T_p) - u^i (\theta^i, T_p^{-i})
\]

\[
[u^i (\theta^i, T_p) - s^i (T_p)] + \left[ s^i (T_p) + \sum_{j \in L - i} s^j (T_p) + aU (T_p) \right] 
\geq [u^i (\theta^i, T_p^{-i}) - s^i (T_p^{-i})] + \left[ s^i (T_p^{-i}) + \sum_{j \in L - i} s^j (T_p^{-i}) + aU (T_p^{-i}) \right]
\]
which implies that for every $i \in L$, the political optimal tariff $T_p$ must also maximise the joint welfare of that individual and the government, given the contribution schedules offered by $j \in L - i$, i.e.

$$T_p = \arg \max \ [u^i (\theta^i, T) - s^i (T)] + \left[ \sum_{i \in L} s^i (T) + aU (T) \right]$$

(5.12)

for all $i \in L$.\(^4\)

Assume further that the contribution function offered by every individual is differentiable at least around $T_p$. (5.9) and (5.12) imply that there are two first order conditions that must be satisfied simultaneously by $T_p$.

$$\sum_{i \in L} \frac{\partial s^i}{\partial T} + a \frac{\partial U}{\partial T} = 0$$

(5.13)

$$\left[ \frac{\partial u^i}{\partial T} - \frac{\partial s^i}{\partial T} \right] + \left[ \sum_{i \in L} \frac{\partial s^i}{\partial T} + a \frac{\partial U}{\partial T} \right] = 0$$

(5.14)

Therefore, it is implied that

$$\frac{\partial u^i}{\partial T} = \frac{\partial s^i}{\partial T}$$

which means that every $i \in L$ will set his/her contribution schedule such that the marginal change in the contribution for a small change in the government tariff matches the effect of tariff change on his/her utility, at least around $T_p$. Substituting this fact back into the government’s first order condition (5.13), it is clear that within the lobbying

\(^4\)This is Proposition 1 (c) of Grossman and Helpman (1994).
framework of Grossman and Helpman (1994), the political optimal tariff is a tariff which maximises the weighted sum of the utilities of every \(i \in L\) and the aggregate utility.

\[
\sum_{i \in L} \frac{\partial u^i}{\partial T} + a \frac{\partial U}{\partial T} = 0 \tag{5.15}
\]

Given set \(L\), the political optimal trade policy \(T_p\) can be derived by solving (5.15) for \(T\). From (5.7), the first order condition is

\[
\sum_{i \in L} q \left[ \theta^i + q (1 - T) \right] + aNq^2 (1 - T) = 0
\]

Eliminate the common \(q\) and expand.

\[
\sum_{i \in L} \theta^i + N_Lq (1 - T) + aNq (1 - T) = 0
\]

where \(N_L\) is the number of individuals in \(L\). Solving for \(T\).

\[
T_p = 1 + \frac{\sum_{i \in L} \theta^i}{q (N_L + aN)} = 1 + \frac{N_L}{q (N_L + aN)} \bar{\theta}_L \tag{5.16}
\]

where \(\bar{\theta}_L = \frac{\sum_{i \in L} \theta^i}{N_L}\). It can be seen that the political optimal tariff policy will be an import tariff, free trade or an import subsidy, i.e. \(T_p \gneq 1\), when the average lobby ownership is larger, equal or less than the population average, i.e. \(\bar{\theta}_L \gneq 0\). Would the trade policy be tariff/subsidy, the extent to which it deviates from free trade positively relates to the deviation of \(\bar{\theta}_L\) from the mean (zero)\(^5\) and the weight the government

\[^5\frac{\partial T_p}{\partial \bar{\theta}_L} = \frac{1}{q (N_L + aN)} N_L > 0\]. The political optimal tariff is shifted away from free trade in favour of the individual whose ownership is \(\bar{\theta}_L\) since, from (5.15), his/her utility is effectively given an extra weight in

119
attaches to all individual $i \in L$ as a whole relative to the total population $\frac{N_L}{(N_L+aN)}$. Both $\overline{\theta}_L$ and $N_L$ are determined in the first stage of the game when every individual decides whether to lobby or not.

### 5.3.2 Individual decision

To determine the set $L$, individual decisions in the first stage of the game must be considered. It will now be shown that, without fixed cost of lobbying, all individuals will decide to lobby in equilibrium. The analysis is done by showing that given any set $L = L^o$ (can be empty), individuals not in $L^o$ always find it beneficial to be included into the set; and once everyone is in the set, i.e. $L = N$, no one finds it beneficial to leave.

From (5.15), given any set $L = L^o$, the political optimal tariff for the government $T^o$ is defined by

$$T^o = \arg \max \sum_{j \in L^o} u^j (\theta^j, T) + aU(T)$$

Similarly, if an individual $i \notin L^o$ were to lobby, the political optimal tariff for the government $T^{+i}$ is defined by

$$T^{+i} = \arg \max u^i (\theta^i, T) + \sum_{j \in L^o} u^j (\theta^j, T) + aU(T)$$

the government's objective function in return for the contributions made by all individual $i \in L$. More specifically, the utility of the individual with ownership $\overline{\theta}_L$ is given the weight of $(N_L + a)$ while the others' utilities are given a common weight of $a$.

The political optimal tariff/subsidy rate will be greater when $N_L$ is larger and/or the weight the government put on the aggregate welfare, $a$, is smaller. Intuitively, the term $\frac{N_L}{(N_L+aN)}$ in (5.16) can be thought of as the relative weight the government put on the individuals who lobby as a whole against the general welfare, an increase in $\frac{N_L}{(N_L+aN)}$, either $N_L$ increases and/or $a$ decreases, will make the political optimal tariff deviate more from free trade.
From (5.11), an individual $i \notin L^o$ will participate in lobbying if

$$\left[u^i (\theta^i, T_p^{+i}) - s^i (T_p^{+i})\right] - u^i (\theta^i, T_p^o) \geq 0$$

(5.19)

From (5.10), the contribution $s^i (T_p^{+i})$ made by the individual $i$ must be just large enough to induce the government to switch from implementing $T_p^o$ to $T_p^{+i}$.

$$s^i (T_p^{+i}) = \left[\sum_{j \in L^o} s^j (T_p^o) + aU (T_p^o)\right] - \left[\sum_{j \in L^o} s^j (T_p^{+i}) + aU (T_p^{+i})\right]$$

(5.20)

Since the contribution function can take many forms as long as (5.19) and (5.20) are satisfied, an additional assumption is needed to be able to select a contribution function among those forms. That assumption is the 'truthfulness' of the contribution function. A contribution function which is truthful is the contribution function which reflects the true willingness of the individual to pay for every possible tariff rate chosen by the government. Since the lobbying game is viewed as a menu-auction problem, an individual’s true willingness to pay is reflected by the maximum amount he/she wishes to pay for a given tariff rate chosen by the government. Such maximum amount that an individual will wish to pay is the amount which leave him/her some base level of net utility. For clarity, denote individual $i$’s base level of net utility by $b^i$, the maximum amount of contribution that individual $i$ wants to pay for a given tariff rate is $u^i (\theta^i, T) - b^i$. The truthful contribution function, therefore, takes the form $s^i (T) = u^i (\theta^i, T) - b^i$ for every possible tariff rate chosen by the government.

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7It would be clearer to think of the base level of net utility, $b^i$, as the minimum amount of money a bidder wants to have left in his/her pocket after the auction. In this work, $b^i$ is chosen by each individual to be as high as possible such that his/her contribution made to the government is not too small to be able to influence the government’s decision. See Grossman and Helpman (1994) for details.

8If any $T$ makes $u^i (\theta^i, T) \leq b^i$, the maximum amount individual $i$ wishes to pay for that tariff is
Following Grossman and Helpman (1994), assume that every individual plays the truthful strategy by always announcing the truthful contribution schedule to the government. Since every individual plays a truthful strategy, (5.20) becomes

\[ s^i (T_p^{+i}) = \left[ \sum_{j \in L^o} (w^j (\theta^j, T_p^o) - b^j) + aU (T_p^o) \right] - \left[ \sum_{j \in L^o} (w^j (\theta^j, T_p^{+i}) - b^j) + aU (T_p^{+i}) \right] \]

(5.21)

Substitute this fact back into (5.19) and rearrange.

\[ u^i (\theta^i, T_p^{+i}) + \sum_{j \in L^o} u^j (\theta^j, T_p^{+i}) + aU (T_p^{+i}) \]

\[ \geq u^i (\theta^i, T_p^o) + \sum_{j \in L^o} u^j (\theta^j, T_p^o) + aU (T_p^o) \]

which is always true by the definition of \( T_p^{+i} \) in (5.18) regardless of the individual ownership \( \theta^i \). Even if individual \( i \)'s participation in the given set \( L^o \) does not change the political optimal tariff, i.e. \( T_p^{+i} = T_p^o \), he/she can always pay zero contribution according to his/her contribution schedule which had been announced prior to the government’s decision making. Therefore, without fixed cost of lobbying, every individual \( i \notin L^o \) will always find it beneficial to participate in lobby.

---

\[ q \text{From (5.21), when } T_p^{+i} = T_p^o, \ s^i (T_p^{+i}) = \left[ \sum_{j \in L^o} w^j (\theta^j, T_p^o) + aU (T_p^o) \right] - \left[ \sum_{j \in L^o} w^j (\theta^j, T_p^{+i}) + aU (T_p^{+i}) \right] = 0. \]
Once every individual is in set $L$, i.e. $L = N$, the political optimal tariff $T_p^*$ is defined by

$$T_p^* = \arg \max u^i (\theta^i, T) + \sum_{j \in N_{-i}} u^j (\theta^j, T) + aU (T)$$  \hspace{1cm} (5.22)$$

where $N_{-i} = N - \{i\}$. If an individual $i$ leaves set $L$, i.e. decides not to lobby, the political optimal tariff $T_{p}^{-i}$ is defined by

$$T_{p}^{-i} = \arg \max \sum_{j \in N_{-i}} u^j (\theta^j, T) + aU (T)$$  \hspace{1cm} (5.23)$$

The individual $i$ will find it beneficial not to lobby if

$$u^i (\theta^i, T_{p}^{-i}) - [u^i (\theta^i, T_p^*) - s^i (T_p^*)] \geq 0$$  \hspace{1cm} (5.24)$$

The truthful contribution schedule made by individual $i$ to support $T_p^*$ is

$$s^i (T_p^*) = \left[ \sum_{j \in N_{-i}} u^j (\theta^j, T_{p}^{-i}) + aU (T_{p}^{-i}) \right] - \left[ \sum_{j \in N_{-i}} u^j (\theta^j, T_p^*) + aU (T_p^*) \right]$$  \hspace{1cm} (5.25)$$

Substitute (5.25) back into (5.24) and rearrange.

$$u^i (\theta^i, T_p^*) + \sum_{j \in N_{-i}} u^j (\theta^j, T_p^*) + aU (T_p^*)$$  \hspace{1cm} (5.26)$$

$$\leq u^i (\theta^i, T_{p}^{-i}) + \sum_{j \in N_{-i}} u^j (\theta^j, T_{p}^{-i}) + aU (T_{p}^{-i})$$
which is a contradiction by the definition of $T_p^*$ in (5.22). Therefore, no one finds it beneficial to leave the set $L = N$ and it can be concluded that everyone will decide to lobby individually in equilibrium. This is due to the truthful strategy played by every individual. Whatever the political tariff outcome, every individual pays only the amount that they are willing to pay to support that outcome. As illustrated in footnote 10, an individual pays zero contribution if his/her participation in lobbying does not alter the political tariff. Therefore, the net benefit from lobbying for every individual is always non negative.

From (5.16), $T_p^* = 1 + \sum_{i \in L} \theta_i^* = 1 + \sum_{i \in N} \theta_i^* = 1$, once $L = N$. Consequently, it is obvious that individuals are worse off than with no lobbying since they have to pay contributions to the government for the free trade policy which would always be chosen by the government even without the contributions. This prisoner's dilemma occurs since every individual has a unilateral incentive to lobby. Therefore, their lobbying efforts neutralise each other in equilibrium and their political contributions are only to prevent the trade policy from being shifted away from their interests.

Recall that these conclusions are subject to the assumption that there is no fixed cost of lobbying. As discussed earlier, there may exist some fixed costs of lobbying in reality which may consist of the costs of establishing links with politicians, communication, etc. Therefore, this assumption is relaxed in the next section.

### 5.3.3 Lobbying with fixed cost

Suppose that all individuals face a homogenous fixed cost of lobbying $F \geq 0$. The condition for an individual $i$ not to lobby in equilibrium in (5.26) becomes

\[ \text{By the definition of } T_p^* \text{ in (5.22), it is implied that } \sum_{j \in N_i} \frac{\partial u^i}{\partial T} + a \frac{\partial u^i}{\partial T} = -\frac{\partial u^i}{\partial T} \text{ at } T = T_p^*. \text{ Therefore, (5.25) is equivalent to } \left. -\frac{\partial u^i}{\partial T} \right|_{T=T_p^*} (T_p^* - T_p^*) \text{ which is, from (5.6), } -q (\theta^i + q (1 - T_p^*)) (T_p^* - T_p^*). \]

Since $T_p^* = 1$, it is clear that the larger the deviation of individual $i$'s ownership from the mean (zero), the larger contribution he/she has to make while individuals whose ownership is equal to the mean makes zero contribution in equilibrium.
The expression on the LHS is non negative by the definition of $T_p^*$ in (5.22). (5.27) simply states that an individual $i$ will not lobby in equilibrium if the net benefit (net of political contribution but gross of the fixed cost) from lobbying he/she can expect unilaterally is less than the fixed cost.

Using linear approximation, the net benefit from lobbying on the LHS in (5.27) becomes

$$\left\{ \left[ u_i \left( \theta^i, T_p^* \right) + \sum_{j \in N - i} u_j \left( \theta^j, T_p^* \right) + a U \left( T_p^* \right) \right] - \left[ u_i \left( \theta^i, T_p^{-i} \right) + \sum_{j \in N - i} u_j \left( \theta^j, T_p^{-i} \right) + a U \left( T_p^{-i} \right) \right] \right\} \leq F$$

Using linear approximation, the net benefit from lobbying on the LHS in (5.27) becomes

$$\left\{ \left( \frac{\partial u_i}{\partial T} + \sum_{j \in N - i} \frac{\partial u_j}{\partial T} + \frac{\partial U}{\partial T} \right) \right\} \bigg|_{T = T_p^{-i}} \left( T_p^* - T_p^{-i} \right) \leq F$$

Since $\frac{\partial u_i}{\partial T} + \sum_{j \in N - i} \frac{\partial u_j}{\partial T} = \sum_{i \in N} \frac{\partial u_i}{\partial T} = \frac{\partial U}{\partial T}$ and $T_p^* = 1$,

$$(1 + a) \left. \frac{\partial U}{\partial T} \right|_{T = T_p^{-i}} (1 - T_p^{-i}) \leq F$$

From (5.7),

$$(1 + a) N q^2 (1 - T_p^{-i})^2 \leq F$$

From (5.16), $T_p^{-i}$, defined by (5.23), is $T_p^{-i} = 1 + \frac{\sum_{j \in N} \theta^j - \theta^i}{q(1+a)N-1} = 1 - \frac{\theta^i}{q(1+a)N-1}$.

$$NB \left( \theta^i \right) \equiv \frac{(1 + a) N \left( \theta^i \right)^2}{[(1 + a) N - 1]^2} \leq F$$

(5.28)
It is clear that $NB(\theta^i)$ is strictly convex in $\theta^i$ as
$$\frac{\partial^2 NB(\theta^i)}{\partial \theta^i}\left(\frac{(1+a)N}{[1+(1+a)N-1]^2}\right) > 0$$
and
$$\frac{\partial NB(\theta^i)}{\partial \theta^i} = 2(1+a)N^i \frac{[1+aN]}{[1+(1+a)N-1]^2} > 0.$$ Therefore, $NB(\theta^i)$ can be represented in Figure 5-1.

$\theta_{\min}$ and $\theta_{\max}$ are the minimum and maximum ownerships in the economy, respectively. It can be seen that the closer an individual’s policy preference is to free trade, the smaller is the net benefit from lobbying that individual can expect unilaterally. Therefore, given a positive fixed cost, there will be some individuals who do not find it beneficial to lobby individually. They are those whose ownerships are within the interval $(\theta_1, \theta_2)$ in Figure 5-1. The equilibrium trade policy is now not necessarily free trade as in the previous section because not every individual participates in lobbying. From (5.16), the political equilibrium trade policy depends on the average ownership and on the welfare of individuals participating in lobbying which depends on the distribution of ownership at the two tails of the distribution. The political equilibrium trade policy will be an import subsidy, free trade, or an import tariff when the ownerships on the left

11 This is because when all individuals are lobbying, the optimal political tariff is free trade and hence the greater the deviation of an individual’s ownership is from the population average, the greater is the net welfare loss of that individual were he/she not to lobby.
The purpose of this section is to identify the cases where no individual can afford to lobby individually. From Figure 5-1, no one will lobby if the fixed cost is too large. In addition, from (5.28),

\[
\frac{\partial^2 NB(\theta^i)}{\partial \theta^i \partial N} = -\frac{2(1+a)((1+a)N+1)}{[(1+a)N-1]^2} \theta^i
\]

which is greater (less) than zero if \( \theta^i < 0 \) (\( \theta^i > 0 \)). This means that the slope of \( NB(\theta^i) \) in Figure 5-1 is increasing (decreasing) with \( N \) for \( \theta^i < 0 \) (\( \theta^i > 0 \)) which implies that \( NB(\theta^i) \) will be flatter if the total population \( N \) is larger and no one will lobby individually if the total population is sufficiently large. This is depicted by the function \( NB'(\theta^i) \) in Figure 5-1. The number of total population \( N' \) which underlying \( NB'(\theta^i) \) is larger than the number of total population \( N \) which underlying \( NB(\theta^i) \). This result is understandable when \( T_{p-i} = 1 - \frac{\theta^i}{q[(1+a)N-1]} \) is considered. It can be seen that \( T_{p-i} \) will be closer to \( T_p^* = 1 \) as \( N \) becoming larger. This is because as an individual becomes smaller relative to the larger total population, his/her political contribution influence on the government's trade policy becomes smaller. From (5.27), if \( N \) is arbitrarily large, \( T_{p-i} \) converges to \( T_p^* \) and the net benefit any individual can expect from lobbying converges to zero and hence no individual will find it beneficial to lobby.\(^{13} \) These results can be summarised in Proposition 6.\(^{14} \)

**Proposition 6** No one lobbies individually in equilibrium if the total population and/or the fixed cost of lobbying are too large.

Proposition 6 provides support for Mitra (1999)'s claim that individuals do not lobby individually since they consider themselves too small to communicate their offers or

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\(^{12}\)This result coincides with the majority voting model of Mayer (1984) in which the political equilibrium trade policy are those preferred by the majority.

\(^{13}\)In terms of Figure 5-1, \( NB(\theta^i) \) will lie thoroughly on the horizontal axis in which case.

\(^{14}\)Needless to say, the effect of the government concern on the aggregate welfare, \( a \), on the individual decision to lobby is similar to the effect of \( N \). It is obvious that as the government has greater concern on the aggregate welfare relative to the political contribution, an individual’s influence on the government and hence the net benefit he/she can expect from lobbying will be smaller. This can be illustrated by

\[
\frac{\theta^i NB(\theta^i)}{\partial \theta^i \partial a} = -\frac{2N((1+a)N+1)}{[(1+a)(N-1)]} \theta^i
\]

which is greater (less) than zero if \( \theta^i < 0 \) (\( \theta^i > 0 \)), similar to the effect of \( N \). However, Mitra (1999) finds that the lobbying benefit could increase with \( a \) since an increase in \( a \) reduces the number of organised industries.
persuade the government to formulate economic policy one way or the other since the transactions costs for these to be done at the level of the individual may be very high. This section can be considered as providing a formal proof of the claim. We proceed to the case where the individuals influence the government through forming lobbies in the next section.

5.4 Lobby formation

As opposed to the previous works which usually assume that a lobby is formed only within a well-defined group, such as identical firms within the same industry, this section concerns the lobby formation in which individuals are free to join any lobby. Before we proceed, it should be noted that the results of this section show that an individual might choose to join the lobby that lobbies against his/her interest to moderate its effort rather than the lobby that lobbies in his/her favour. Therefore, in equilibrium, a lobby group might consist of members whose policy preferences are diverse, e.g. an individual who prefers an import tariff might be a member of the same lobby as the individual who prefers an import subsidy. Given the complete information on the individuals' policy preferences, one may wonder why the lobby does not reject the entrance of some members knowing that they want to mitigate its objective. Note that the main assumption in the lobbying framework is that a lobby can make the government be aware of its policy preference by the use of money. Given that the lobby normally has to face with fierce competition in the political arena but has to rely on the donations from its members,\textsuperscript{15} it is reasonable that the lobby might be willing to trade off its policy stance for a higher donation like the government which values contributions while concerning the general welfare. This two-level politics is able to generate such an outcome that seems to support what is often

\textsuperscript{15}One might even argue that lobbies certainly have interests aside from influencing policy, such as profits for the lobby entrepreneurs (Salisbury, 1990).
cited as “politics makes strange bedfellows”.\textsuperscript{16} Therefore, the passive role of the lobbies is assumed in this section.

The political game consists of three stages. In the first stage, individuals decide whether to form and join the lobbies. In the second stage, each organised lobby non co-operatively chooses a contribution function which maximises the aggregate utility of its members and announces to the government. In the last stage, given the contribution schedules announced by the lobbies, the government chooses and implements a tariff rate to maximise the weighted sum of the contribution and the general welfare, and then collects the contribution from the lobbies accordingly. Each lobby’s contribution will be shared equally among its members. To keep the complexities of the model at the minimum, it is assumed in this section that there is no fixed cost of lobbying.

5.4.1 Policy formation

If lobbying is the effort to shift the one dimensional trade policy away from free trade, it is presumable that there are two opposing forces pulling the trade policy toward an import tariff or an import subsidy. This section concerns the equilibrium in which there exist two opposing lobbies, one lobbying for an import tariff and the other lobbying for an import subsidy, represented by $L_t$ and $L_s$, respectively. Given the other lobby’s contribution schedule, each lobby maximises the net aggregate welfare of its members.

$$\max_{S_k(T)} \sum_{j \in L_k} u^j (\theta^j, T) - S_k(T)$$

(5.29)

where $S_k(T)$ is lobby $L_k$’s contribution schedule, $L_k \in \{L_t, L_s\}$.

\textsuperscript{16}Being aware of the fact that interest groups often enter into coalitions so as to reduce the cost of issue advocacy, Almeida (2003) argues that the opposing interest groups might have a common short-term interest to work together. An example in the US is the Cigarette Tax for Health Care coalition which campaigns for a raise in the cigarette tax. It has brought together three public-relations firms who often find themselves dueling against one another: Chernoff Newman Silver Gregory, McAlister Communications and Richard Quinn & Associates.
If the lobbies announce truthful contribution schedules, the government is effectively maximising the weighted sum of the lobbies' aggregate utilities and the economy aggregate utility.

\[
\max_T \sum_{j \in I_t} u^j (\theta^j, T) + \sum_{j \in I_s} u^j (\theta^j, T) + aU (T) \tag{5.30}
\]

This is because the objective of the government is to choose \( T_p \) such that 
\[
S_t (T_p) + S_s (T_p) + aU (T_p) \geq S_t (T) + S_s (T) + aU (T). 
\]

The truthful contribution made by lobby \( L_k \) for any \( T \) chosen by the government must leave the lobby a base level of aggregate utility, \( B_k \). Therefore, lobby \( k \)'s truthful contribution function must satisfy \( \sum_{j \in L_k} u^j (\theta^j, T) - S_k (T) = B_k \Rightarrow S_k (T) = \sum_{j \in L_k} u^j (\theta^j, T) - B_k \) for all possible \( T \) chosen by the government. Therefore, the government optimisation condition is

\[
\begin{bmatrix}
(S_i \in L_t) u^i (\theta^i, T_p) - B_t \\
+ (S_i \in L_s) u^i (\theta^i, T_p) - B_s \\
+ aU (T_p)
\end{bmatrix} \geq \begin{bmatrix}
(S_i \in L_t) u^i (\theta^i, T) - B_t \\
+ (S_i \in L_s) u^i (\theta^i, T) - B_s \\
+ aU (T)
\end{bmatrix}
\]

which implies that

\[
\begin{bmatrix}
\sum_{j \in L_t} u^j (\theta^j, T_p) \\
+ \sum_{j \in L_s} u^j (\theta^j, T_p) \\
+ aU (T_p)
\end{bmatrix} \geq \begin{bmatrix}
\sum_{j \in L_t} u^j (\theta^j, T) \\
+ \sum_{j \in L_s} u^j (\theta^j, T) \\
+ aU (T)
\end{bmatrix}
\]

From \( (5.30) \), the first order condition for the government's optimisation problem is

\[
\sum_{j \in L_t} \frac{\partial u^j}{\partial T} + \sum_{j \in L_s} \frac{\partial u^j}{\partial T} + a \frac{\partial U}{\partial T} = 0 \tag{5.31}
\]

From \( (5.7) \), \( (5.31) \) is
Solving for $T$, the political optimal tariff can be represented by

$$T_p = 1 + \frac{\sum_{j \in L_t} \theta^j + \sum_{j \in L_s} \theta^j}{q (N_t + N_t + aN)} = 1 + \frac{N_t \theta_t + N_s \theta_s}{q (N_t + N_s + aN)}$$

(5.32)

$$\theta_k = \frac{\sum_{j \in L_k} \theta^j}{N_k}$$

where $N_k$ is the number of lobby $L_k$'s members. Similar to the previous section, the government's political optimal tariff is the policy compromise between its policy preference and those of the lobbies. It will be an import tariff, free trade, or an import subsidy, $T_p \overset{\geq}{=} 1$, when the average ownership of the lobbies is larger, equal or less than the population average, $N_t \theta_t + N_s \theta_s \overset{\geq}{=} 0$. It can be seen that when the lobbies' objectives are to maximise the aggregate welfare of their members, individuals are indifferent between the two lobbies in terms of the gross benefit from lobbying since they always gain extra weight in the government's objective function regardless of which lobby they join. As is to be shown in the next section, the only difference between joining the two lobbies is the lobbies' contributions that the individuals will have to share.

5.4.2 Lobby contribution

By implementing $T_p$, the government collects contributions from the lobbies according to their pre-announced contribution schedules. Consider lobby $L_t$. The government's political optimal tariff were lobby $L_t$ not lobbying is $T_p^{-t}$, defined by

$$T_p^{-t} = \arg \max_{j \in L_s} \sum_{j \in L_s} u^j (\theta^j, T) + aU(T_p)$$

From (5.32),
Denote the contribution that lobby \( L_t \) must make to support \( T_p \) by \( S_t(T_p) \). Since lobby \( L_t \) has no incentive to pay more than the amount that just compensates the government’s welfare were it not lobbying, from (5.30),

\[
S_t(T_p) = \left[ \sum_{j \in L_s} u_j^i (\theta^j, T_p^{-t}) \right] - \left[ \sum_{j \in L_s} u_j^i (\theta^j, T_p) \right]
\]

Using linear approximation,

\[
S_t(T_p) = \left( \sum_{j \in L_s} \frac{\partial u^i_j}{\partial T} + a \frac{\partial U}{\partial T} \right) \bigg|_{T=T_p} (T_p^{-t} - T_p)
\]

From (5.7),

\[
S_t(T_p) = \left\{ \sum_{j \in L_s} q [\theta^j + q (1 - T_p)] + aNq^2 (1 - T_p) \right\} (T_p^{-t} - T_p)
\]

\[
S_t(T_p) = q (N_s + aN) \left[ \sum_{j \in L_s} \theta^j \frac{q}{(N_s + aN)} + q (1 - T_p) \right] (T_p^{-t} - T_p)
\]

Substitute (5.32) for \( T_p \) and (5.33) for \( T_p^{-t} \). Recall that \( \theta_s = \frac{\sum_{j \in L_s} \theta^j}{N_s} \).
where $B_t = (N_t + N_s)\theta_s / (N_t + N_s + aN)$ is the ownership of the individual whose most preferred policy is $T_p^t$. Lobby $L_a$'s contribution to support $T_p$ can be derived likewise. Therefore, the contribution each member of each lobby has to pay for $T_p$ is

$$S_t(T_p) = (N_s + aN) \frac{(N_t)^2 \left[ \frac{N_t \theta_s}{(N_s + aN)} - \theta_t \right]^2}{(N_t + N_s + aN)^2}$$

Similarly, $B_s = (N_s + aN)\theta_s / (N_t + N_s + aN)$ is the ownership of the individual whose most preferred policy is the political optimal tariff were lobby $L_s$ not participating in lobbying. It is clear that the contribution each lobby has to pay for $T_p$ is larger when there is larger deviation between its policy preference and the policy preferences of the government and its opponent. This is intuitive since the greater the difference between the policy preference of a lobby and the policy preferences of the government and its opponent, the greater is the effort of the lobby is required to shift the outcome in its favour.

$$\frac{S_t(T_p)}{N_t} = \frac{N_t (N_s + aN)}{(N_t + N_s + aN)^2} (\theta_t - \theta_s)^2$$
$$\frac{S_s(T_p)}{N_s} = \frac{N_s (N_t + aN)}{(N_t + N_s + aN)^2} (\theta_t - \theta_s)^2$$

Similarly, $\theta_t = N_t \theta_t / (N_t + aN)$ is the ownership of the individual whose most preferred policy is the political optimal tariff were lobby $L_t$ not participating in lobbying. It is clear that the contribution each lobby has to pay for $T_p$ is larger when there is larger deviation between its policy preference and the policy preferences of the government and its opponent. This is intuitive since the greater the difference between the policy preference of a lobby and the policy preferences of the government and its opponent, the greater is the effort of the lobby is required to shift the outcome in its favour.

It can be seen that the political optimal tariff in (5.32) and the contribution each lobby member has to pay in (5.34) depend on the lobbies' identities (the average ownership among their members) and sizes (the numbers of their members). They are pre-
determined by the individuals’ decision to join the lobbies in the first stage of the game.

5.4.3 Individual decision

In the first stage of the game, the individuals choose between staying out of the lobbies and joining the tariff or subsidy lobby. Among the three choices, given the choices of the other individuals, every individual chooses the strategy that gives him/her the highest net utility in equilibrium. The objective of this section is to identify the possible forms of coalitions when the individuals choose their lobbies endogenously.

Denote \( L_t^* \) and \( L_s^* \) as the equilibrium sets of the tariff and subsidy lobbies, respectively, by which the political equilibrium tariff \( T_p^* \) is implied.\(^{17}\) From (5.32),

\[
T_p^* = 1 + \frac{N_t^* \theta_t^* + N_s^* \theta_s^*}{q (N_t^* + N_s^* + aN)}
\]  

(5.35)

The equilibrium in the first stage of the game must be such that no one finds it beneficial to change his/her course of action. This means that the two following conditions must be satisfied in equilibrium.

**Condition 1** At \( T_p^* \), every individual \( i \in L_t^* \) (\( i \in L_s^* \)) must not find it beneficial to leave \( L_t^* \) (\( L_s^* \)) and stay out of the lobbies.

**Condition 2** At \( T_p^* \), every individual \( i \in L_t^* \) (\( i \in L_s^* \)) must not find it beneficial to switch to \( L_s^* \) (\( L_t^* \)).

Condition 1 is the necessary condition for an individual \( i \) to be a member of a lobby in equilibrium. It ensures that each lobby consists of the members who prefer participating in rather than staying out of the lobbies. Condition 2 is the condition in which that

\(^{17}\)From (5.30), \( T_p^* \) is defined by 

\[
T_p^* = \arg \max_{T} \sum_{j \in L_t^*} u_j (\theta^i, T) + \sum_{j \in L_s^*} u_j (\theta^i, T) + aU (T).
\]

The superscript (*) will be used to indicate the equilibrium values of all the variables henceforth.
individual \(i\) prefers staying in that lobby rather than the other. Therefore, to identify the equilibrium sets of the tariff and the subsidy lobbies, the set of individuals who do not violate these conditions must be identified. These conditions are considered in turn.

**Condition 1**

Consider \(i \in L_i^*\). At \(T_p^*\), every individual \(i \in L_i^*\) must not find it beneficial to leave \(L_i^*\) and stay out of the lobbies. This condition can be represented by

\[
\begin{align*}
  u^i (\theta^i, T_p^{i-1}) - \frac{S_i^* (T_p^*)}{N_i^*} &\leq 0 \\
\end{align*}
\]  

(5.36)

for all \(i \in L_i^*\).

\(T_p^{i-1}\) is the political optimal tariff were \(i \in L_i^*\) not to participate in lobbying. From (5.30), it is defined by

\[
T_p^{i-1} = \frac{(N_i^* \theta^i - \theta^i) + N_s^* \theta^*}{q (\Delta_i^* + N_s^* + a N)}
\]

(5.37)

Therefore, \(u^i (\theta^i, T_p^{i-1})\) is the net utility individual \(i \in L_i^*\) would obtain were he/she to leave the lobby while \(u^i (\theta^i, T_p^*)\) is the net utility of the individual \(i \in L_i^*\) in the equilibrium. Therefore, (5.36) states that, in equilibrium, all individual \(i \in L_i^*\) must have non positive gain to leave \(L_i^*\) and stay out of lobbying.

Rearrange, use linear approximation, and substitute (5.34) for \(\frac{S_i^* (T_p^*)}{N_i^*}\) in (5.36), (5.36) becomes

\[18\] Another condition that, at \(T_p^*\), every individual \(i \notin \{L_i^*, L_s^*\}\) must not find it beneficial to join either \(L_i^*\) or \(L_s^*\), is not necessary since it is already implied by the satisfaction of Condition 1 and Condition 2.
\[
\frac{\partial u^i}{\partial T^i} \bigg|_{T=T_p^*} (T_{p^*}^t - T_{p^*}^*) + \frac{N^*_i (N^*_s + aN)}{(N^*_i + N^*_s + aN)^2} (\theta_{-t}^* - \theta_t^*)^2 \leq 0
\]

\[\theta_{-t}^* = \frac{N^*_s \theta_t^*}{(N^*_s + aN)}\] is the ownership of the individual whose most preferred policy is the political optimal tariff were \(L_t^*\) not lobbying at \(T_p^*\), i.e. \(T_{p^*}^t\). From (5.7),

\[q \left[ \theta^i + q \left( 1 - T_{p^*}^* \right) \right] (T_{p^*}^t - T_{p^*}^*) + \frac{N^*_i (N^*_s + aN)}{(N^*_i + N^*_s + aN)^2} (\theta_{-t}^* - \theta_t^*)^2 \leq 0\]

Substitute (5.35) for \(T_{p^*}^*\) and (5.37) for \(T_{p^*}^{t-i}\).

\[- \frac{(\theta^i - \tilde{\theta})^2}{\left[ (N^*_i - 1) + N^*_s + aN \right]} + \frac{N^*_i (N^*_s + aN)}{(N^*_i + N^*_s + aN)^2} (\theta_{-t}^* - \theta_t^*)^2 \leq 0\] (5.38)

\[\tilde{\theta} = \frac{N^*_s \theta_t^* + N^*_i \theta_t^*}{(N^*_i + N^*_s + aN)}\] is the ownership of the individual whose most preferred policy is \(T_{p^*}^*\).

Given that \(\theta_s^* < 0 < \theta_t^*\),\(^{19}\) it is shown in Appendix C.1 that (5.38) will be violated if

\[\left[ \theta^* + \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) \right] < \theta^i < \left[ \theta^* - \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) \right] \] (5.39)

\[\mu_t = \frac{N^*_i (N^*_s + aN) [(N^*_i - 1) + N^*_s + aN]}{(N^*_i + N^*_s + aN)^2} > 0\] (5.40)

Since \(\theta_{-t}^* = \frac{N^*_s \theta_t^*}{(N^*_s + aN)}\) is the weighted average between \(\theta_s^*\) and 0, \((\theta_{-t}^* - \theta_t^*) < 0\) by the assumption that \(\theta_s^* < 0 < \theta_t^*\). Therefore, \[\left[ \theta^* + \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) \right] < \theta^* < \left[ \theta^* - \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) \right].\]

\(^{19}\)Since it is assumed that \(L_t^*\) is lobbying for an import tariff and \(L_s^*\) is lobbying for an import subsidy. \(\theta_s^* < 0 < \theta_t^*\) is the natural assumption.
(5.39) is the set of individuals who violate Condition 1 and hence they cannot be a member of the tariff lobby in equilibrium, i.e. they cannot be in $L^*_t$. It can be seen that the individual who could be a member of the tariff lobby in equilibrium must have the ownership sufficiently far from $\theta^*$, the ownership of the individual whose most preferred policy is $T^*_p$. This is understandable when (5.38) is considered. The first term in (5.38) is the welfare loss of an individual $i \in L^*_t$ from his/her utility not being given an extra weight in the government's objective function while the second term is the welfare gain in terms of the political contribution he/she can forgo were he/she to leave $L^*_t$. It can be seen that the welfare gain is constant with respect to $\theta^i$. Therefore, when comparing across individuals, the individuals whose ownerships are closer to $\theta^*$ will face smaller welfare loss from leaving $L^*_t$. If the individual whose ownership is $\left[\theta^* + \sqrt{\mu_t} (\theta^* \tau - \theta^*_t)\right] < \theta^i < \left[\theta^* - \sqrt{\mu_t} (\theta^* \tau - \theta^*_t)\right]$ was in $L^*_t$, he/she would face sufficiently small welfare loss to have a positive net benefit to leave $L^*_t$. Hence being in $L^*_t$ is apparently not his/her optimal strategy.

The set of individuals who violate Condition 1 were they in $L^*_t$ can be obtained likewise. They are the individuals whose ownerships are

$$\left[\theta^* - \sqrt{\mu_s} (\theta^* \tau - \theta^*_s)\right] < \theta^i < \left[\theta^* + \sqrt{\mu_s} (\theta^* \tau - \theta^*_s)\right]$$

(5.41)

$$\mu_s = \frac{N^*_s (N^*_s + aN) [N^*_t + (N^*_s - 1) + aN]}{(N^*_t + N^*_s + aN)^2} > 0$$

(5.42)

---

20 As Condition 2 must also be satisfied, satisfaction of Condition 1 is necessary but not sufficient for an individual to be a member of the tariff lobby in equilibrium.

21 From (5.38), it is clear that the individual whose ownership is $\tilde{\theta}^*$ have the maximum gain from leaving $L^*_t$ (were he/she in $L^*_t$) comparing to the other individuals, since he/she faces zero welfare loss from doing so.
where \[ \hat{\theta} - \sqrt{\mu_s} (\theta^{*}_{-s} - \theta^{*}_s) \] \[ < \hat{\theta} < \sqrt{\mu_s} (\theta^{*}_{-s} - \theta^{*}_s) \]. 22

**Condition 2**

Consider individual \( i \in L^*_i \). At \( T^*_p \), every individual \( i \in L^*_i \) must not find it beneficial to switch to \( L^*_s \). This condition can be represented by

\[
\left[ u^i (\theta^i, T^*_p) - \frac{S_s (T^*_p) - S^*_i (T^*_p)}{N^*_s + 1} \right] - \left[ u^i (\theta^i, T^*_p) - \frac{S^*_i (T^*_p)}{N^*_i} \right] \leq 0
\]

for all \( i \in L^*_i \).

\( T^*_p \) is the political optimal tariff were individual \( i \in L^*_i \) to switch to \( L^*_s \). From (5.30), it is defined by

\[
T^*_p = \arg \max \left[ \sum_{j \in L^*_i} u^j (\theta^j, T) - u^i (\theta^i, T) \right] + \left[ \sum_{j \in L^*_i} u^j (\theta^j, T) + u^i (\theta^i, T) \right] + aU (T) \]

which is clearly identical to \( T^*_p \). This is because, when the lobbies maximise the aggregate utilities of their members, the individuals’ utilities are always given an extra weight in the government’s objective function regardless of which lobby they join. Therefore, switching lobby does not alter the gross utility of an individual. At \( T^*_p \), an individual \( i \in L^*_i \) can gain from switching to \( L^*_s \) only if he/she can expect to pay a lower contribution. The above condition reduces to

\[
\frac{S^*_i (T^*_p)}{N^*_i} - \frac{S_s (T^*_p - i, s + i)}{N^*_s + 1} \leq 0
\]

for all \( i \in L^*_i \). This means that all individual \( i \in L^*_i \) must not be able to expect to pay a lower contribution by switching to \( L^*_s \). From (5.34), the above condition is

\[
22 \theta^{*}_{-s} = \frac{N^*_s \theta^*_s}{N^*_s + aN} \] is the weighted average between \( \theta^*_s \) and 0, therefore, \( \theta^{*}_{-s} - \theta^*_s > 0 \) by the assumption that \( \theta^*_s < 0 < \theta^*_s \).
Rearrange the second term.

\[
\frac{N_t^* (N_s^* + aN)}{(N_t^* + N_s^* + aN)^2} (\theta_{t}^* - \theta_{t}^*)^2 - \frac{1}{(N_s^* + 1) [(N_t^* - 1) + aN]} (\hat{\theta}_t - \theta^*)^2 \leq 0
\]  

(5.44)

\[
\hat{\theta}_t = \bar{\theta}^* + \frac{N_t^* (N_t^* + aN)}{(N_t^* + N_s^* + aN)} (\theta_{*-t}^* - \theta_{-s}^*)
\]  

(5.45)

\(\hat{\theta}_t\) is the ownership of the individual who, if switches from \(L_t^*\) to \(L_s^*\) at \(T^*_p\), will alter the average ownership of \(L_s^*\) to be equal to the new \(\theta_{-s}^*\) which, from (5.34), means that, ceteris paribus, the new contribution made by the new subsidy lobby \(L_s^* + \{i\}\) equals to zero.\(^{23}\) Note that \(\hat{\theta}_t > \bar{\theta}^*\) since \((\theta_{*-s}^* - \theta_{s}^*) > 0\).

It is shown in Appendix C.2 that (5.44) will be violated if

\[
\left[ \hat{\theta}_t + \sqrt{\lambda_t} (\theta_{*-t}^* - \theta_{t}^*) \right] < \theta^* < \left[ \hat{\theta}_t - \sqrt{\lambda_t} (\theta_{*-t}^* - \theta_{t}^*) \right]
\]  

(5.46)

\[
\lambda_t = \frac{N_t^* (N_t^* + aN) (N_s^* + 1) [(N_t^* - 1) + aN]}{(N_t^* + N_s^* + aN)^2}
\]  

(5.47)

\(^{23}\)See the second term in (5.43), if an individual \(i \in L_t^*\) switches to \(L_s^*\) at \(T^*_p\), the new average ownership of \(L_s^*\) is \(\frac{N_s^* \theta_{s}^* + \theta^*}{(N_s^* + 1)}\) and the new \(\theta_{-s}^*\) is \(\frac{(N_t^* - 1) + aN}{(N_t^* - 1) + aN}\). If \(\theta^* = \hat{\theta}_t\), these two are equal which means zero contribution paid by the new subsidy lobby \(L_s^* + \{i\}\).
Since \((\theta^*_{-t} - \theta^*_t) < 0, [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^*_{-t} - \theta^*_t)] < [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^*_{-t} - \theta^*_t)]\).

From (5.44), it can be seen that the first term, which represents the contribution cost paid by \(i \in L^*_t\) at \(T^*_p\), is constant with respect to \(\theta^i\). Therefore, the gains from switching from \(L^*_t\) to \(L^*_s\) are different across individuals with respect to the extent to which the individuals can affect the average ownership of \(L^*_s\), i.e. \(\theta^*_s\), and hence its political contribution. As discussed above, the individual whose ownership is \(\hat{\theta}_t\) can expect to pay zero contribution were he/she in \(L^*_t\) and switched to \(L^*_s\) at \(T^*_p\). Therefore, the individuals whose ownerships are close to \(\hat{\theta}_t\) will be more likely to violate (5.44) as compared to the others since they can expect to pay a lower contribution\(^{24}\) were they in \(L^*_t\) and switched to \(L^*_s\). This is stated by (5.46) in which the boundaries of the set of ownerships which violate (5.44) deviate symmetrically around \(\hat{\theta}_t\), i.e. \([\hat{\theta}_t + \sqrt{\lambda_t} (\theta^*_{-t} - \theta^*_t)] < \theta^i < [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^*_{-t} - \theta^*_t)]\).

The individuals whose ownerships are \([\hat{\theta}_s + \sqrt{\lambda_s} (\theta^*_{-s} - \theta^*_s)] < \theta^i < [\hat{\theta}_s - \sqrt{\lambda_s} (\theta^*_{-s} - \theta^*_s)]\), therefore, cannot be in \(L^*_t\) since they would have an incentive to switch to \(L^*_s\) were they in \(L^*_t\) at \(T^*_p\).

The set of individuals who violate Condition 2 were they in \(L^*_s\) can be obtained likewise. They are the individuals whose ownerships are

\[
[\hat{\theta}_s - \sqrt{\lambda_s} (\theta^*_{-s} - \theta^*_s)] < \theta^i < [\hat{\theta}_s + \sqrt{\lambda_s} (\theta^*_{-s} - \theta^*_s)]
\]

(5.48)

\[
\lambda_s = \frac{N^*_s (N^*_s + aN) (N^*_s + 1) [(N^*_s - 1) + aN]}{(N^*_t + N^*_s + aN)^2}
\]

(5.49)

\(^{24}\)The closer an individual \(i\)'s ownership is to \(\hat{\theta}_t\) (either \(\theta^i\) is larger or smaller than \(\hat{\theta}_t\)), the smaller is the contribution the individual \(i\) can expect from switching from \(L^*_t\) to \(L^*_s\).
\[
\tilde{\theta}_s = \theta^* + \frac{N_t^* (N_t^* + aN)}{(N_t^* + N_s^* + aN)} (\theta_{-t}^* - \theta_t^*)
\] (5.50)

Since \((\theta_{-s}^* - \theta_s^*) > 0\), \([\tilde{\theta}_s - \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)] < \tilde{\theta}_s < [\tilde{\theta}_s + \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)].\)

(5.39) and (5.46) are the sets of individuals who cannot be the members of the tariff lobby in equilibrium, however, it is not necessary that all the rest of the individuals will be the tariff lobby's members. Likewise, all the rest of the individuals from those in (5.41) and (5.48) need not be the subsidy lobby's members. Therefore, to determine how the individuals group themselves in equilibrium (5.39), (5.41), (5.46), and (5.48) have to be compared. They are summarised in Summary 1 as the point of reference.

**Summary 1**

(i) The equilibrium tariff lobby \(L_t^*\) does not consist of the individuals whose ownerships are

Condition 1: \([\tilde{\theta}_t^* + \sqrt{\lambda_t} (\theta_{-t}^* - \theta_t^*)] < \theta_t^* < [\tilde{\theta}_t^* - \sqrt{\lambda_t} (\theta_{-t}^* - \theta_t^*)]\)

Condition 2: \([\tilde{\theta}_t^* + \sqrt{\lambda_t} (\theta_{-t}^* - \theta_t^*)] < \theta_t^* < [\tilde{\theta}_t^* - \sqrt{\lambda_t} (\theta_{-t}^* - \theta_t^*)]\)

(ii) The equilibrium subsidy lobby \(L_s^*\) does not consist of the individuals whose ownerships are

Condition 1: \([\tilde{\theta}_s^* + \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)] < \theta_s^* < [\tilde{\theta}_s^* - \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)]\)

Condition 2: \([\tilde{\theta}_s^* + \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)] < \theta_s^* < [\tilde{\theta}_s^* - \sqrt{\lambda_s} (\theta_{-s}^* - \theta_s^*)]\)

Since the critical values in Summary 1 still depend on the numbers of the lobbies' members which depend on the distribution of ownership in the economy, their comparison is not possible without knowing the distribution function.

Suppose that the distribution of ownership is symmetric around the population mean \(\bar{\theta} = 0\). Therefore, it is presumable that the equilibrium will be symmetric such that
the two lobbies' efforts neutralise each other. Specifically, with symmetric distribution. \( N_t^* = N_s^* \) and the political equilibrium tariff is free trade, \( T_p^* = 1 \). From (5.35), it is implied that \( \overline{\theta} = 0 \) and hence \( \theta_s^* = -\theta_t^* \).

Therefore, the results in Summary 1 can be simplified and be compared in terms of the variables associated with only one lobby that chosen to be the tariff lobby. From (5.40), (5.42), (5.47) and (5.49), the symmetric equilibrium implies that \( \mu_s = \mu_t \) and \( \lambda_s = \lambda_t \). The symmetric equilibrium also implies that \( \theta_{s-t}^* = -\theta_{s-t}^* \) and hence, from (5.45) and (5.50), \( \widehat{\theta}_s = -\widehat{\theta}_t \). Therefore, the results in Summary 1 can be simplified to

\[
(i) \quad i \notin L^*_t
\]

Condition 1 : \( \sqrt{\mu_t} (\theta_{s-t}^* - \theta_t^*) < \theta_i < -\sqrt{\mu_t} (\theta_{s-t}^* - \theta_t^*) \)
Condition 2 : \( \left[ \widehat{\theta}_t + \sqrt{\lambda_t} (\theta_{s-t}^* - \theta_t^*) \right] < \theta_i < \left[ \widehat{\theta}_t - \sqrt{\lambda_t} (\theta_{s-t}^* - \theta_t^*) \right] \)

\[
(ii) \quad i \notin L^*_s
\]

Condition 1 : \( \sqrt{\mu_t} (\theta_{s-t}^* - \theta_t^*) < \theta_i < -\sqrt{\mu_t} (\theta_{s-t}^* - \theta_t^*) \)
Condition 2 : \( \left[ \widehat{\theta}_s - \sqrt{\lambda_t} (\theta_{s-t}^* - \theta_t^*) \right] < \theta_i < \left[ \widehat{\theta}_s + \sqrt{\lambda_t} (\theta_{s-t}^* - \theta_t^*) \right] \)

(5.51)

It can be seen that, under the symmetric distribution, only three critical values need to be considered.
\[
\begin{align*}
\theta_1 & \equiv \sqrt{\mu_x} (\theta_{t-1}^* - \theta_t^*) \\
\theta_2 & \equiv \left[ \hat{\theta}_t - \sqrt{\lambda_x} (\theta_{t-1}^* - \theta_t^*) \right] \\
\theta_3 & \equiv \left[ \hat{\theta}_t + \sqrt{\lambda_x} (\theta_{t-1}^* - \theta_t^*) \right] \\
\end{align*}
\] (5.52)

It is shown in Appendix C.3 that they are in the following order.

\[ -\theta_2 < \theta_1 < \theta_3 < 0 < -\theta_3 < -\theta_1 < \theta_2 \]

Figure 5-2 (a), in which \([\theta_{\text{min}}, \theta_{\text{max}}]\) is the set of all possible ownerships, summarises these relationships and the conditions stated in (5.51).

It can be seen that the set of those who violate Condition 1 and 2 for \(i \in L^*_t\) are those whose ownership \(\theta^i \in (\theta_1, \theta_2)\). Therefore, the possible tariff lobby's members in equilibrium should be those individuals whose ownerships are \(\theta^i \in [\theta_{\text{min}}, \theta_1]\) and \(\theta^i \in [\theta_2, \theta_{\text{max}}]\). Similarly, the possible equilibrium subsidy lobby should consist of those individuals whose ownerships are \(\theta^i \in [\theta_{\text{min}}, -\theta_2]\) and \(\theta^i \in [-\theta_1, \theta_{\text{max}}]\). In fact, there are two possible forms of symmetric coalitions suggested by Figure 5-2 (a). One is represented by Figure 5-2 (b) and the other by Figure 5-2 (c). It can be seen that both forms of coalitions do not violate Condition 1 and Condition 2 for \(i \in L^*_t\) and \(i \in L^*_t\). However, the form of coalitions in Figure 5-2 (b) implies that the lobbies' average ownerships are \(\theta^*_t < 0 < \theta^*_i\) in equilibrium which is in contradiction with the assumption that \(\theta^*_t < 0 < \theta^*_i\). Therefore, it can be concluded that the possible form of coalitions in equilibrium when individuals choose their lobbies endogenously is as suggested by Figure 5-2 (c).

It can be seen that those who stay out of the lobbies in equilibrium are those whose policy preferences close to the political equilibrium tariff \(T^*_p\). This is straightforward since the equilibrium trade policy is close to their preferences so that they would obtain too small change in the tariff and hence too small improvement in their welfare to
Figure 5-2: Equilibrium lobbies, $L_t^*$ and $L_s^*$. 
cover the lobbies' contribution costs they had to share were they to join the lobbies for their utilities to be given an extra weight in the government's decision. Note that this is different from the case if they lobby individually. As discussed earlier, when the individuals lobby individually, they offer the government their truthful contribution schedules so that they pay the amounts of contribution according to their true willingness to pay given the tariff outcome. They can even pay zero contribution if the tariff turns out to be their most preferred tariff so that every individual always obtains non negative net benefit from lobbying in absence of the fixed cost. In contrast, when the individuals are restricted to lobby only through the lobbies in this section, they have to pay the contribution according to the true willingness to pay of the individuals whose ownerships are the average ownerships among the members of the lobbies they join. Therefore, the amount of contribution per head can be too high for the individuals whose policy preferences are close to the equilibrium tariff to be able to join.

In addition, it can be seen that each lobby consists of the members whose policy preferences are diverse. When group formation is not restricted to be only among a well-defined group as in all previous works, an individual may choose to join the lobby that lobbying against his/her interest to moderate the lobby's effort instead of joining the lobby that lobbying in his/her interest. Consider an individual whose ownership is $-\theta_2 < \theta^i < \theta_1$ in Figure 5-2 (c). If he/she left $L_i$ and stayed out of the lobbies, the political equilibrium tariff would move away from his/her interest. Even if the individual might gain from the contribution he/she could forgo, the above derivation ensures that the loss from the policy change dominates. Alternatively, if the same individual switched from $L_i$ to $L_l$, even if the switching did not alter the political equilibrium tariff (see (5.32)), he/she would have to pay a higher contribution with the subsidy lobby. Therefore,

\begin{itemize}
  \item Note also that, from Figure 5-2 (c), the individual whose ownership is the average among the members of a lobby need not be one of the members as the set of the lobby’s members is not continuous.
  \item The switching has two effects on the contribution of the subsidy lobby that the individual would expect to pay were he/she to switch to. It lowers the contribution of $L_l$ since the individual makes the average ownership of $L_l$ closer to zero. However, it also raises the contribution of $L_l$ since the leaving of the individual makes the average ownership of $L_l$ farther from zero. The above derivation ensures that
\end{itemize}
being a member of the tariff lobby in equilibrium is his/her optimal strategy. It can be concluded that the political equilibrium trade policy tends not to be extreme not only due to the opposing forces between the lobbies but also due to the fact that each lobby's effort is moderated by its own members. This result raises a question whether the standard industry lobby approach might exaggerate the lobbying activities.

Comparing the welfare implications between this two-lobby equilibrium and the individual-lobby equilibrium in section 5.3, a point is worth discussing. It was shown in section 5.3 that all individuals lobby individually in equilibrium in the absence of the fixed cost to support free trade. Since there is also free trade in the symmetric two-lobby equilibrium, the welfare comparison can be done in terms of the contributions that the individuals have to pay. It is obvious that those whose $\theta^i$ close to zero are better off in the two-lobby equilibrium since they do not have to pay any contribution. For those at the two extremes, it is apparent that they are also better off in the two-lobby equilibrium since they can pay smaller contributions. This is because, when the individuals lobby through groups, they do not have to pay according to their own deviations from the mean but according to the average deviations of their respective lobbies which are smaller. Therefore, the lobbies that emerge in this model can be considered as the means through which the individuals can restrain their otherwise offsetting contributions thus providing an incentive for the individuals to lobby collectively.

5.5 Conclusion

This chapter examined the individuals' decision to participate in lobbying prior to the stages of interaction between the government and the lobbies studied by Grossman and Helpman (1994). It was found that no one lobbied individually if the total population and/or the fixed costs were too large since the individuals would find themselves too small to influence the government's decision. When the lobby formation was concerned. It was

the latter effect dominates.
found that when the individuals were not restricted to join a particular lobby as in the previous works, some individuals might want to join the lobbies which lobbied against their interests to moderate their efforts rather than joining the ones which lobbied in their favour. Welfare comparison between the two equilibriums suggested that a lobby could be the means through which the individuals could restrain their otherwise offsetting contributions thus providing them an incentive to lobby collectively.

It can be seen that the model in this chapter shares the same basic settings as the models in the previous chapters on trade disputes and trade agreements. As pointed out by Putnam (1988) and is generally recognised in the literature, there are the entanglements between domestic and international politics. Therefore, an interesting extension of this model is to combine it with the framework in the previous chapters to study the effect of domestic politics on the international front. This will constitute a more comprehensive framework to analyse the international tariff policy whether it is in conflicts or coordinations. More generally, the framework developed in this chapter can also be applied to the determination of the government’s policies other than tariff.
Chapter 6

Conclusion

This thesis is a collection of theoretical works on trade wars, trade agreements, and the political economy of trade policies. A multi-country, two-good model that shares the same trade patterns with a trade model in the customs union literature was employed in chapters 3 and 4. We examined the importance of the world market structure on the outcome of a trade war and the choice of a country to negotiate a trade agreement when the alternatives which imply different sources of gain were available. Chapter 5 concerned the lobby formation which was exogenous in the previous literature.

In chapter 3, it was shown that there was a greater possibility for a country to win even if the country’s endowments were relatively small if the world market of its exportable moved closer to the monopolistic market, i.e. there were less countries exporting the same good and/or the world endowment of that good was divided more disproportionately among its exporters, while the previous literature suggested that only the country whose endowments of goods were relatively large could win a trade war.

In chapter 4, as there were two types of trade agreement which implied different sources of gain and different negotiating partners, the conditions in which one was preferred to the other by a country were examined. By comparing the welfare implications between the two types of agreement from the perspective of a country, it was found that the choice of a country to negotiate a trade agreement did not depend on the strategic
complementarity or substitutability of its negotiating partners' tariffs but their levels of endowments that generated sufficiently large gain (no matter it was the gain in terms of trade or in volume of trade) relative to the non-cooperative equilibrium.

In the last chapter, it was found that no one lobbied individually if the total population and/or the fixed costs were too large since the individuals would find themselves too small to influence the government's decision. When the lobby formation was concerned. It was found that when the individuals were not restricted to join a particular lobby as in the previous works, some individuals might want to join the lobbies which lobbied against their interests to moderate their efforts rather than joining the ones which lobbied in their favour. Welfare comparison between the two equilibriums suggested that a lobby could be the means through which the individuals could restrain their otherwise offsetting contributions thus providing them an incentive to lobby collectively.

As the last chapter is closely linked with the framework in the first two chapters, a possible and interesting extension of this thesis is to combine the two frameworks together to obtain a more comprehensive framework for the trade policy analysis.
Appendix A

Chapter 3

A.1 Derivation of the Nash equilibrium tariffs in the two-country model.

Substituting (3.9) and (3.10) back into (3.8) and solve for $T^A$ and $T^B$, the two countries’ best response tariff functions can be obtained. Consider $A$.

\[
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)^2}} \left( X^A (q^{Y^A}) \right) - \frac{(q^{Y^A} - X^A)}{2\sqrt{q^{Y^A}}} \left( \frac{T^A}{1+T^A} \right) \left( \frac{Y^A}{T^A} \right) = 0
\]

Take out the common terms and substitute (3.7) for $q$ in the denominator of the second term.

\[
\frac{(1-T^A)}{\sqrt{T^A}} - (q^A - X^A) \left( \frac{(s^A) Y^A + (1-s^A) Y^B}{\left[(1-s^A) Y^A + (1-s^B) Y^B\right]} \frac{T^B}{1+T^B} \right) = 0
\]

\[
(1 - T^A)(1 - T^B)((1 - s^A) X^A + (1 - s^B) X^B) - (q^B - X^B)T^B = 0
\]

Use the fact that $(1 - s^A) = \frac{T^A}{1 + T^A}$, and rearrange.

\[
\frac{X^A}{1+T^A} + \frac{(1-T^B)(1-T^A)T^B}{T^A Y^A} = q
\]

Use the fact that $s^A = \frac{T^A}{1 + T^A}$, and substitute (3.7) for $q$.

\[
\frac{X^A}{1+T^A} + \frac{(1-T^B)(1-T^A)T^B}{s^A Y^A} = \frac{(1-s^B) X^B}{s^B Y^A + (1-s^B) Y^B}
\]

(150)
Rearrange.

\[
\begin{bmatrix}
  s^A X^A \\
  + T^A (1 - T^A) (1 - s^B) X^B \\
  \end{bmatrix}
\begin{bmatrix}
  s^A Y^A + s^B Y^B \\
  \end{bmatrix}
= T^A s^A Y^A
\begin{bmatrix}
  (1 - s^A) X^A \\
  +(1 - s^B) X^B \\
  \end{bmatrix}
\]

Using the fact that \( s^A = \frac{T^A}{(1 + T^A)} \) and \( (1 - s^A) = \frac{1}{(1 + T^A)} \), the first term on the left and the first term on the right are equal thus cancel each other out. Rearrange.

\[
\begin{bmatrix}
  s^A X^A Y^A \\
  + T^A (1 - T^A) s^A Y^A (1 - s^B) X^B \\
  + s^A X^A s^B Y^B \\
  + T^A (1 - T^A) s^B (1 - s^B) X^B Y^B \\
  \end{bmatrix}
= \begin{bmatrix}
  T^A s^A (1 - s^A) X^A Y^A \\
  + T^A s^A Y^A (1 - s^B) X^B \\
  \end{bmatrix}
\]

Multiply through by \( \frac{1}{s^A} \).

\[
X^A s^B Y^B + \frac{1}{s^A} T^A (1 - T^A) s^B (1 - s^B) X^B Y^B = (T^A)^2 Y^A (1 - s^B) X^B
\]

\[
X^A s^B Y^B + [1 - (T^A)^2] s^B (1 - s^B) X^B Y^B = (T^A)^2 Y^A (1 - s^B) X^B
\]

Solve for \( T^A \), A's best response tariff function can be obtained.

\[
\widetilde{T}^A(T^B) = \sqrt{\frac{s^B Y^B}{(1 - s^B) X^B} \frac{[X^A + (1 - s^B) X^B]}{Y^A + s^B Y^B}}
\]

Substituting for \( s^B \) and (3.1) for \( X^i \) and \( Y^i \) and rearranging yields

\[
\widetilde{T}^A(T^B) = \sqrt{\frac{\beta_y}{(1 - \beta_x) [(1 - \beta_y) T^B + 1]}}
\]

Working similarly, B's best response tariff function can also be obtained.

151
\[ \tilde{T}^B(T^A) = \sqrt{\frac{\beta_x}{1 - \beta_y} \frac{\beta_x}{(\frac{x}{T^A} + 1)} + \frac{1}{(1 - \beta_x)T^A + 1}} \]  

(3.12)

The interior Nash equilibrium tariffs, \((T^A_N, T^B_N)\), can be solved for by substituting \(\tilde{T}^B(T^A)\) into \(\tilde{T}^A(T^B)\).

\[ (T^A)^2 = \beta_y \left( 1 - \beta_x \right) \frac{\beta_x}{(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \]

\[ (T^A)^2 = \frac{\beta_y}{(1 - \beta_x)} \sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \]

\[ (T^A)^2(1 - \beta_x) \left\{ \frac{\sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)}}{\frac{\beta_x}{T^A} + 1} \right\} = 0 \]

\[ \beta_y \left\{ \sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \right\} \]

\[ + \frac{(T^A)^2(1 - \beta_x)}{(1 - \beta_x)(\frac{\beta_x}{T^A} + 1)} \sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \]

\[ - \beta_y \left[ (1 - \beta_x)T^A + 1 \right] \sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \]

\[ - \beta_y \left[ (1 - \beta_x)T^A + 1 \right] \sqrt{\beta_x(1 - \beta_y)\left( \frac{\beta_x}{T^A} + 1 \right) + \left( \frac{\beta_x}{T^A} + 1 \right)} \]

\[ = 0 \]

\[ \left[ \sqrt{\beta_x(1 - \beta_y)\left( (T^A)^2(1 - \beta_x)(\frac{\beta_x}{T^A} + 1) - \beta_y \left( 1 - \beta_x \right) \left( T^A \right)^2 + 1 \right) \right] = 0 \]

The values of \( T^A \) which make the second parenthesis equal to zero are all negative.

Since \( T^A \) has to be positive for the domestic price of \( A \)'s importable to be positive, the \( A \)'s interior Nash equilibrium tariff is

152
which makes the first parenthesis equal to zero. Substituting $T^A_N$ back into $B$’s best response tariff function $\tilde{T}^B(T^A)$ gives the interior Nash equilibrium tariff of $B$.

$$T^B_N = \sqrt{\frac{\beta_x}{(1 - \beta_y)}}$$ (3.14)

### A.2 Derivation of the best response tariff functions in the multi-country model.

Consider country $A$. Substitute (3.9), (3.20) and (3.21) into the first order condition.

$$\left[ -\frac{(1-T^A)}{2\sqrt{q}} \frac{(X^A+q{Y^A})}{\sqrt{q}} \frac{1}{(1+T^A)^2} \frac{(X^A+q{Y^A})}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \right] = 0$$

Take out the common terms and substitute (3.19) for $q$ in the denominator of the second term.

\[
\begin{aligned}
\frac{(1-T^A)}{\sqrt{T^A}} - \left( qY^A - X^A \right) &= 0 \\
\left( \frac{1 - s^A}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \frac{1}{(1+T^A)^2} \right) &= 0 \\
\end{aligned}
\]
Use the fact that \((1 - s^A) = \frac{1}{(1 + T^A)}\), and rearrange.

\[
\frac{X^A}{T^A Y^A} + \frac{X^B}{T^A Y^A} + \sum_{i=1}^{n_s} (1 - s^a_i) X^a_i + \sum_{i=1}^{n_y} (1 - s^b_i) X^b_i = q
\]

Use the fact that \(s^A = \frac{T^A}{(1 + T^A)}\), and substitute (3.19) for \(q\).

Rearrange.

\[
\begin{align*}
(1 - T^A)(1 + T^A) & \begin{bmatrix}
(1 - s^A) X^A \\
+ (1 - s^B) X^B \\
+ \sum_{i=1}^{n_s} (1 - s^a_i) X^a_i \\
+ \sum_{i=1}^{n_y} (1 - s^b_i) X^b_i
\end{bmatrix} - (q Y^A - X^A) T^A = 0
\end{align*}
\]
Using the fact that $s^A = \frac{T^A}{(1+T^A)}$ and $(1 - s^A) = \frac{1}{(1+T^A)}$, the first term on the left and the first term on the right are equal thus cancel each other out. Rearrange.

$$
= T^A s^A Y^A \left[ \begin{array}{c} (1 - s^A) X^A \\ + (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
+ T^A (1 - T^A) s^A Y^A \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
+ s^A X^A \left( \begin{array}{c} s^B Y^B \\ + \sum_{i=1}^{n_z} s^{a_i} Y^{a_i} \\ + \sum_{i=1}^{n_y} s^{b_i} Y^{b_i} \end{array} \right) 
$$

$$
+ T^A (1 - T^A) \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
T^A s^A (1 - s^A) X A Y A 
$$

Using the fact that $s^A = \frac{T^A}{(1+T^A)}$ and $(1 - s^A) = \frac{1}{(1+T^A)}$, the first term on the left and the first term on the right are equal thus cancel each other out. Rearrange.

$$
= T^A s^A Y^A \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
+ T^A s^A Y^A \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
= T^A s^A Y^A \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
+ T^A (1 - T^A) \left[ \begin{array}{c} (1 - s^B) X^B \\ + \sum_{i=1}^{n_z} (1 - s^{a_i}) X^{a_i} \\ + \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \end{array} \right] 
$$

$$
+ s^A X^A \left( \begin{array}{c} s^B Y^B \\ + \sum_{i=1}^{n_z} s^{a_i} Y^{a_i} \\ + \sum_{i=1}^{n_y} s^{b_i} Y^{b_i} \end{array} \right) 
$$

155
\[
\begin{align*}
T^A s^A Y^A & \begin{bmatrix}
(1 - s^B) X^B \\
+ \Sigma_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ \Sigma_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i}
\end{bmatrix} \\
- T^A (1 - T^A) s^A Y^A & \begin{bmatrix}
(1 - s^B) X^B \\
+ \Sigma_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ \Sigma_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i}
\end{bmatrix}
\end{align*}
\]

Rearrange.

\[
\begin{align*}
&T^A (1 - T^A) s^A Y^A + \Sigma_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ &\Sigma_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i}
\end{align*}
\]

Multiply through by \( \frac{1}{s^A} \).

\[
\begin{align*}
&T^A (1 - T^A) + \Sigma_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ &\Sigma_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i}
\end{align*}
\]

156
\[
\begin{aligned}
\left\{ \begin{array}{c}
X^A \\
(1 + T^A)(1 - T^A) \\
- (T^A)^2 Y^A \\
\end{array} \right. 
\begin{bmatrix}
\frac{s^B Y^B}{1 - s^B} X^B \\
+ \sum_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \\
\end{bmatrix} 
\begin{bmatrix}
\frac{s^B Y^B}{1 - s^B} X^B \\
+ \sum_{i=1}^{n_x} (1 - s^{a_i}) X^{a_i} \\
+ \sum_{i=1}^{n_y} (1 - s^{b_i}) X^{b_i} \\
\end{bmatrix} = 0
\end{aligned}
\]

Solving for \(T^A\), country A’s best response tariff function can be obtained.

\[
\overline{T^A}(T^B, \{T^{a_i}\}, \{T^{b_i}\}) = \frac{X^A}{Y^A} + 1
\]

The other countries’ best response tariff functions in (3.22) can be obtained likewise.

### A.3 Simulation results
The numbers in each cell are \( U_n^f / U_n^e \).

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Table A.1: Degree of specialisation and Nash equilibrium welfare.
The numbers in each cell are \((T^A_N, T^B_N, T^a_N, T^b_N)\), given \(\beta_x = \beta_y = 0.7\) and \(n_x = n_y = 9\).

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Table A.2: Degree of market domination and Nash equilibrium tariff
The numbers in each cell are \((T^A_N, T^B_N, T^C_N, T^D_N)\), given \(\beta_x = \beta_y = 0.7\) and \(\delta_x = \delta_y = 0.5\).

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Table A.3: Market concentration and Nash equilibrium tariff.

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Table A.4: Degree of market domination and Nash equilibrium world relative price.
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Table A.5: Market concentration and Nash equilibrium world relative price.
The numbers in each cell are $(\frac{U^a}{U^b}, \frac{U^b}{U^c}, \frac{U^c}{U^d}, \frac{U^d}{U^e})$, given $\beta_x = \beta_y = 0.7$ and $n_x = n_y = 9$.

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Table A.6: Degree of market domination and Nash equilibrium welfare.
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Table A.7: Market concentration and Nash equilibrium welfare.
Appendix B

Chapter 4

B.1 Nash bargaining solution in the two-country model

The optimisation problem is

\[ \max_{T^A, T^B} \left( (U^A - U^A_{\pi}) (U^B - U^B_{\pi}) \right) \]

Therefore, the first order conditions are

\[ \left( \frac{\partial U^A}{\partial T^A} + \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^A} \right) + \lambda \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^A} = 0 \] (1.5)

\[ \left( \frac{\partial U^B}{\partial T^B} + \frac{\partial U^B}{\partial q} \frac{\partial q}{\partial T^B} \right) + \frac{1}{\lambda} \frac{\partial U^A}{\partial q} \frac{\partial q}{\partial T^B} = 0 \] (1.6)

where \( \lambda = \frac{(U^A - U^A_{\pi})}{(U^B - U^B_{\pi})} \).

The first order conditions imply
Rearranging gives

\[
\frac{\partial U^A}{\partial T^B} \frac{\partial U^B}{\partial T^A} + \frac{\partial U^A}{\partial T^A} \frac{\partial U^B}{\partial T^B} + \frac{\partial U^B}{\partial T^B} \frac{\partial U^A}{\partial T^A} = 0
\]

From (4.2) and (4.3), the above expression is

\[
\begin{align*}
&\frac{(1-T^A)}{\sqrt{Q}} \frac{(X^A+qY^A)}{(qY^B-X^B)} \frac{(1-T^B)}{\sqrt{Q}} \frac{(X^B+qY^B)}{(qY^A-X^A)} \\
&+ \frac{1}{\sqrt{Q}} \frac{(1-T^A)}{\sqrt{Q}} \frac{(X^A+qY^A)}{(qY^A-X^A)} \frac{(1-T^B)}{\sqrt{Q}} \frac{(X^B+qY^B)}{(qY^B-X^B)} \frac{2\sqrt{Q}}{2\sqrt{Q}} \frac{(1-T^B)}{\sqrt{Q}} \frac{(1-T^B)}{\sqrt{Q}} \frac{(1-T^A)}{\sqrt{Q}} \frac{(1-T^A)}{\sqrt{Q}} = 0
\end{align*}
\]

Take out the common terms and substitute for \( q \) remaining in the denominators of the second and third term.

\[
\begin{align*}
&\left\{ \frac{(1-T^A)(1-T^B)}{\sqrt{Q}} \frac{(X^A+qY^A)}{(qY^B-X^B)} + \frac{1}{\sqrt{Q}} \frac{(1-T^A)(X^A+qY^A)}{(qY^A-X^A)} \frac{(1-T^B)(X^B+qY^B)}{(qY^B-X^B)} \right\} = 0
\end{align*}
\]

\[
\begin{align*}
&\left\{ (1-T^A)(1+T^A)(1-T^B)(1+T^B) \left[ (1-s^A)X^A + (1-s^B)X^B \right] \right\} + T^B (1-T^A)(1+T^A)(qY^B-X^B) \\
&- T^A (1-T^B)(1+T^B)(qY^A-X^A) \\
&\left\{ (1-T^B)(1+T^B)X^A - (1-T^A)(1+T^A)(T^B)^2X^B \right\} = q \left\{ T^A (1-T^B)(1+T^B)Y^A - T^B (1-T^A)(1+T^A)Y^B \right\}
\end{align*}
\]

Substitute for \( q \) and rearrange.

\[
\begin{align*}
&\begin{bmatrix}
(1-T^B)(1+T^B)X^A \\
- (1-T^A)(1+T^A)(T^B)^2X^B
\end{bmatrix}
= \begin{bmatrix}
T^A (1+T^B)Y^A \\
+ (1+T^A)Y^B
\end{bmatrix}
\end{align*}
\]
\[
\begin{bmatrix}
(1 + T^B) X^A \\
+ T^B (1 + T^A) X^B \\
(1 + T^A) (1 + T^B) X^A Y^B (1 - T^A T^B) \\
- (1 + T^A) (1 + T^B) T^A T^B X^B Y^A (1 - T^A T^B)
\end{bmatrix}
\begin{bmatrix}
T^A (1 - T^B) (1 + T^B) Y^A \\
- T^B (1 - T^A) (1 + T^A) Y^B \\
(1 - T^A T^B) X^A Y^B (1 - T^A T^B) \\
(1 - T^A T^B) X^B Y^A (1 - T^A T^B)
\end{bmatrix}
= 0
\]

Eliminate the common term \((1 + T^A) (1 + T^B)\) and factor out.

\[(1 - T^A T^B) (X^A Y^B - T^A T^B X^B Y^A) = 0\]

Therefore, there are two conditions which solve the above expression, \(T^B = 1/T^A\) and \(T^A T^B = \frac{X^A Y^B}{X^B Y^A} = \frac{\beta_x \beta_y}{(1-\beta_x)(1-\beta_y)}\). It was shown in chapter 3 that the latter condition eliminates all trade between the two countries. The former is, therefore, the solution.

\[T_C^A T_C^B = 1\]  \(\text{(4.8)}\)

Substitute (4.8) into (4.3), the equilibrium world relative price is

\[q_C = \frac{\frac{1}{(1+T_C^A)} X^A + \frac{T_B}{(1+T_C^B)} X^B}{\frac{T_A^C}{(1+T_C^A)} Y^A + \frac{1}{(1+T_C^B)} Y^B}
= \frac{\frac{1}{(1+T_C^A)} X^A + \frac{1}{(1+T_C^A)} X^B}{\frac{T_A^C}{(1+T_C^A)} Y^A + \frac{T_B^C}{(1+T_C^A)} Y^B}
= \frac{1}{T_C^A} \frac{X^A + X^B}{Y^A + Y^B}
= \frac{1}{T_C^A} \frac{X}{Y}\]  \(\text{(4.9)}\)

From (4.2) and (4.3), (4.5) is

166
Simplify and rearrange.

\[
\begin{bmatrix}
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)^2}} \frac{(X^A + qY^A)}{\sqrt{q}} \\
- \frac{(q^2Y^A - X^A)}{2\sqrt{q}} \frac{1}{(1+T^A)(1+T^A)^2} \frac{(X^A + qY^A)}{2\sqrt{q}} \\
- \frac{(q^2Y^B - X^B)}{2\sqrt{q}} \frac{1}{(1+T^B)(1+T^A)^2} \frac{(X^A + qY^A)}{2\sqrt{q}} \\
\end{bmatrix} = 0
\]

Substitute (4.8) for \(TB\) and (4.9) for \(q\).

\[
\lambda = \frac{(1-T^A)}{(q^2Y^A - X^A)\sqrt{T^A}} \left\{ \frac{(1 - T^A)}{(1 + T^A)} \left[ \left(1 - s^A\right)X^A + \left(1 - s^B\right)X^B \right] \right\} - \frac{(qY^A - X^A)}{\sqrt{T^A(1+T^A)}} T^A
\]

Therefore,

\[
\frac{(U^A_C - U^A_N)}{(U^B_C - U^B_N)} = \lambda = 1
\]

From (4.11) in the text, the above expression is

\[
U^A_C - U^B_C = U^A_N - U^B_N
\]
Solve for $T^A_C$, then by (4.8), $T^B_C$ can be obtained.

\[
T^A_C = \frac{(U^A_C - U^B_C)}{\sqrt{XY}} - \frac{[(1 - \beta_z) T^A_C + \beta_z]}{\sqrt{XY}} = (U^A_C - U^B_C)
\]

\[
T^B_C = \frac{[\beta_z - (1 - \beta_z)] - \frac{(U^A_C - U^B_C)}{\sqrt{XY}}}{\sqrt{XY}} + [\beta_z - (1 - \beta_z)]
\]

(4.12)

B.2 Efficiency locus between $A$ and $B$ in the multi-country model

Since the optimisation problem is still the same as in the two-country model, solving for the efficiency locus between $A$ and $B$ in the multi-country model can be started from (4.7). Given the world relative price $q$ represented by (4.16), (4.7) becomes

\[
\begin{bmatrix}
\frac{(1-T^A)}{2\sqrt{T^A(1+TA)^2}} \frac{(X^A+qY^A)}{\sqrt{q}} & \frac{(1-T^B)}{2\sqrt{T^B(1+TB)^2}} \frac{(X^B+qY^B)}{\sqrt{q}} \\
+ & \frac{(1-T^A)}{2\sqrt{T^A(1+TA)^2}} \frac{(Y^B-X^B)}{\sqrt{q}} & \frac{(1-T^B)}{2\sqrt{T^B(1+TB)^2}} \frac{(Y^A-X^A)}{\sqrt{q}} \\
- & \frac{(1-T^A)}{2\sqrt{T^A(1+TA)^2}} \frac{(Y^B-X^B)}{\sqrt{q}} & \frac{(1-T^B)}{2\sqrt{T^B(1+TB)^2}} \frac{(Y^A-X^A)}{\sqrt{q}}
\end{bmatrix} = 0
\]

where $Z \equiv (n_x^a s^a Y^a + n_y^b s^b Y^b)$.

Take out the common terms and substitute for $q$ remaining in the denominators of the second and third term.

\[
\left\{ \begin{array}{l}
\frac{(1-T^A)(1-T^B)}{\sqrt{T^A(1+TA)^2}} \frac{(X^A+qY^A)}{\sqrt{q}} \\
+ & \frac{(1-T^A)}{\sqrt{T^A(1+TA)^2}} \frac{(Y^B-X^B)}{\sqrt{q}} \\
- & \frac{(1-T^A)}{\sqrt{T^A(1+TA)^2}} \frac{(Y^B-X^B)}{\sqrt{q}} \\
\end{array} \right\} = 0
\]

168
where \( W \equiv [n_x (1 - s^a) X^a + n_y (1 - s^b) X^b] \).

\[
\begin{bmatrix}
(1 - T^A) (1 + T^A) (1 - T^B) (1 + T^B) & (1 - s^A) X^A + (1 - s^B) X^B + W \\
+ T^B (1 - T^A) (1 + T^A) (qY^B - X^B) \\
- T^A (1 - T^B) (1 + T^B) (qY^A - X^A)
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
(1 - T^B) (1 + T^B) X^A \\
- (1 - T^A) (1 + T^A) (T^B)^2 X^B \\
+ (1 - T^A) (1 + T^A) (1 - T^B) (1 + T^B) W
\end{bmatrix} = q \begin{bmatrix}
T^A (1 - T^B) (1 + T^B) Y^A \\
- T^B (1 - T^A) (1 + T^A) Y^B
\end{bmatrix}
\]

Substitute (4.16) for \( q \) and rearrange.

\[
\begin{bmatrix}
(1 - T^B) (1 + T^B) X^A \\
- (1 - T^A) (1 + T^A) (T^B)^2 X^B \\
+ (1 - T^A) (1 + T^A) (1 - T^B) (1 + T^B) W
\end{bmatrix} = \begin{bmatrix}
(1 + T^A) (1 + T^B) Y^A \\
+ (1 + T^A) Y^B \\
+ (1 + T^A) (1 + T^B) Z
\end{bmatrix}
\]

Eliminate the common term \((1 + T^A) (1 + T^B)\), expand and rearrange.

\[
\begin{bmatrix}
(T^A)^2 (T^B)^2 (X^B + W) (Y^A + Z) \\
- (T^B)^2 Z (X^A + X^B + W) \\
- T^A T^B (X^A Y^B + X^B Y^A) \\
+ (X^A + W) (Y^B + Z) \\
- (T^A)^2 W (Y^A + Y^B + Z)
\end{bmatrix} = 0
\]
Solve for $T^B$. The solution defines the efficient combination of country $A$ and $B$ trade policies given the small-size countries' tariffs (contained in $W$ and $Z$). However, there exist two solutions to the above expression. Recall that the efficiency locus is defined by $T^B = 1/T^A$ in the two-country model. Therefore, to determine which solution defines the efficiency locus in this multi-country model, that solution must reduce to $T^B = 1/T^A$ if there are only country $A$ and $B$, i.e. $\delta_x = \delta_y = 1 \Rightarrow W = 0$ and $Z = 0$. The efficiency locus is defined by

$$
T^B = \frac{T^A (X^A Y^B + X^B Y^A)}{-\sqrt{-4 \left( \frac{(T^A)^2 (X^A Y^B + X^B Y^A)^2}{(T^A)^2 (X^B + W) (Y^A + Z)} \right) \left[ \frac{X^A + W}{Y^A + Y^B + Z} \right] - \frac{Z (X^A + X^B + W)}{2 \left( \frac{(T^A)^2 (X^B + W) (Y^A + Z)}{Z (X^A + X^B + W)} \right)}}
$$

(4.17)

### B.3 Efficiency locus between $A$ and $a$ in the multi-country model

From (4.15) and (4.16), the efficiency condition (4.22) in the text can be simplified to

$$
\frac{\partial U^A}{\partial T^A} \frac{\partial U^a}{\partial T^a} + \frac{\partial U^A}{\partial T^a} \frac{\partial q}{\partial T^a} + \frac{\partial U^a}{\partial T^a} \frac{\partial q}{\partial T^A} = 0
$$

$$
\begin{bmatrix}
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} & \frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} \\
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} & \frac{(1-T^A)}{(1+T^A)^2} & \frac{(X^A + q Y^A)}{\sqrt{q}} \\
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} & \frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} \\
\frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}} & \frac{(1-T^A)}{2\sqrt{T^A(1+T^A)}} & \frac{(X^A + q Y^A)}{\sqrt{q}}
\end{bmatrix} = 0
$$
Take out the common terms and substitute (4.16) for \( q \) remaining in the denominators of the second and third term.

\[
\left\{ \frac{(1-T^A)(1-T^a)}{\sqrt{T^A\sqrt{T^a}}} \right. \\
\frac{-T^a (1 - T^A) (1 + T^A) (1 - T^a) (1 + T^a)}{\sqrt{T^A(1-T^a)}(qY^a - X^a)} \\
\left. \quad + (1 - T^a) (1 - T^a) (1 - T^a) (1 + T^a) E \right\} = 0
\]

\[
\left\{ \frac{1 - s^A}{\sqrt{T^A\sqrt{T^a}}}X^A \right. \\
\frac{(1 - s^B) X^B}{\sqrt{T^A(1-T^a)}(qY^a - X^a)} \\
\left. \quad + n_x (1 - s^a) X^a + n_y (1 - s^b) X^b \right\} = 0
\]

where \( E \equiv [(1 - s^B) X^B + (n_x - 1) (1 - s^a) X^a + n_y (1 - s^b) X^b]. \) Substitute (4.16) for \( q \) and rearrange.

\[
\left[ \begin{array}{c}
(1 - T^a) (1 + T^a) X^A \\
+ (1 - T^A) (1 + T^A) X^a \\
+ (1 - T^A) (1 + T^A) (1 - T^a) (1 + T^a) E \\
\end{array} \right] = q \left[ \begin{array}{c}
T^A (1 - T^a) (1 + T^a) Y^A \\
+ T^a (1 - T^A) (1 + T^a) Y^a \\
+ (1 + T^A) (1 + T^A) E \\
\end{array} \right]
\]

where \( F \equiv [s^B Y^B + (n_x - 1) s^a Y^a + n_y s^b Y^b]. \)


\[
\begin{bmatrix}
T^a (1 + T^A) (1 + T^a) X^A Y^a (T^A - T^a) \\
-T^a (1 + T^A) (1 + T^a) X^a Y^A (T^A - T^a) \\
-(T^a)^2 (1 + T^A) (1 - T^a) (1 + T^a)^2 Y^A E \\
-(T^a)^2 (1 - T^A) (1 + T^A)^2 (1 + T^a) Y^a E \\
+ (1 + T^A) (1 - T^a) (1 + T^a)^2 X^A F \\
+ (1 - T^A) (1 + T^A)^2 (1 + T^a) X^a F \\
+ (1 - T^A) (1 + T^A)^2 (1 - T^a) (1 + T^a)^2 E F
\end{bmatrix} = 0
\]

Eliminate the common term \((1 + T^A) (1 + T^a)\), expand and rearrange.

\[
\begin{bmatrix}
(T^A)^2 (T^a)^2 E (Y^A + Y^a + F) \\
-(T^A)^2 (X^a + E) (Y^A + F) \\
-(T^a)^2 (X^A + E) (Y^a + F) \\
+T^A T^a (X^A Y^a + X^a Y^A) \\
+F (X^A + X^a + E)
\end{bmatrix} = 0
\]

Solve for \(T^A\). The solution defines the efficient combination of country A's and a's trade policies given the \(y\)-exporting countries' tariffs. However, there exist two solutions to the above function. From the discussion in footnote 19 in the text, the solution which defines the efficient combination of \(T^A\) and \(T^a\) should reduce to the non-cooperative best response tariff function of \(A\) (see (3.22) in chapter 3) when \(a\) is small. It is found that the solution which defines the efficient combination of \(T^A\) and \(T^a\) is

172
The numbers in each cell are \((T^A_C, T^B_C)\).

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Table B.1: Equilibrium trade agreement in the two-country model.

\[
T^A = \frac{-T^a (X^A Y^a + X^a Y^A)}{ \sqrt{-4 \left( (T^a)^2 E (Y^A + Y^a + F) \right) \left[ - \left( (T^a)^2 (X^A + E) (Y^a + F) \right) \right] - \left( (T^a)^2 (X^A Y^a + X^a Y^A) \right)^2}} + \frac{(T^a)^2 (X^A Y^a + X^a Y^A) \left[ F (X^A + X^a + E) \right]}{2 \left[ - (X^a + E) (Y^A + F) \right]}
\]

(4.23)

**B.4 Simulation results**
The numbers in each cell are \( T^A_{(A+B)} \), \( T^B_{(A+B)} \), \( T^a_{(A-B)} \) on the left and \( T^A_N, T^B_N, T^n_N \) on the right, given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.2: Degree of market domination and equilibrium tariff.

The numbers shown in each cell are \( \frac{g(A+B)}{g_F}, \frac{g_N}{g_F} \) given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.3: Degree of market domination and equilibrium world relative price.
The numbers shown in each cell are \( \frac{U^A_{(A+B)}}{U^A_N}, \frac{U^B_{(A+B)}}{U^B_N}, \frac{U^A_{(A+B)}}{U^A_N}, \frac{U^B_{(A+B)}}{U^B_N} \).

given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.4: Degree of market domination and equilibrium welfare.
The numbers shown in each cell are $\left( T^A_{(A+B)}, T^B_{(A+B)}, T^a_{(A-B)}, T^b_{(A-B)} \right)$ on the left and $\left( T^N_A, T^B_N, T^a_N, T^b_N \right)$ on the right, given $\beta_x = \beta_y = 0.7$ and $\delta_x = \delta_y = 0.5$.

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Table B.5: Market concentration and equilibrium tariff.
The numbers in each cell are \((\frac{U^A_{(A+B)}}{q^A}, \frac{U^B_{(A+B)}}{q^B})\).

Given \(\beta_z = \beta_y = 0.7\) and \(\delta_x = \delta_y = 0.5\).

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Table B.6: Market concentration and equilibrium world relative price.

The numbers in each cell are \((U^A_{(A+B)}/U^A_N, U^B_{(A+B)}/U^B_N, U^a_{(A+B)}/U^a_N, U^b_{(A+B)}/U^b_N)\).

Given \(\beta_x = \beta_y = 0.7\) and \(\delta_x = \delta_y = 0.5\).

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Table B.7: Market concentration and equilibrium welfare.
The numbers in each cell are \( \left( T^A_{(A+a)}, T^B_{(A+a)}, T^\alpha_{(A-a)}, T^\phi_{(A+a)} \right) \) on the left and \( \left( T^A_N, T^B_N, T^\alpha_N, T^\phi_N \right) \) on the right, given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.8: Degree of market domination and equilibrium tariff.

The numbers in each cell are \( \left( q_{(A+a)}, q_N \right) \), given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.9: Degree of market domination and equilibrium world relative price.
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given \( \beta_x = \beta_y = 0.7 \) and \( n_x = n_y = 9 \).

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Table B.10: Degree of market domination and equilibrium welfare.
The numbers in each cell are \( \left( T_A^{(A+a)\cdot T_B^{(A+a)\cdot T_N^{(A+a)}}} \right) \), on the left and \( \left( T_A^{N} T_B^{N} T_N^{N} \right) \) on the right.

given \( \beta_x = \beta_y = 0.7 \) and \( \delta_x = \delta_y = 0.5 \).

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Table B.11: Market concentration and equilibrium tariff.

The numbers in each cell are \( \left( \frac{q(A+a)}{qF} \frac{qN}{qF} \right) \),

given \( \beta_x = \beta_y = 0.7 \) and \( \delta_x = \delta_y = 0.5 \).

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Table B.12: Market concentration and equilibrium world relative price.
The numbers in each cell are \( \left( \frac{U^A_{(A,x)}}{U^N_A} \right) \), \( \left( \frac{U^B_{(A,y)}}{U^N_B} \right) \), \( \left( \frac{U^A_{(A,z)}}{U^N_A} \right) \), \( \left( \frac{U^B_{(A,y)}}{U^N_B} \right) \),\( \left( \frac{U^A_{(A,x)}}{U^N_A} \right) \), given \( \beta_x = \beta_y = 0.7 \) and \( \delta_x = \delta_y = 0.5 \).

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<td>.99052</td>
<td>.99039</td>
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<td>.99251</td>
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<td>.99211</td>
<td>.99218</td>
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<tr>
<td>( \rightarrow \infty )</td>
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<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
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Table B.13: Market concentration and equilibrium welfare.

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<th>( \delta_x ) ( \delta_y )</th>
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<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
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<td>a</td>
<td>a</td>
<td>a</td>
<td>B</td>
</tr>
<tr>
<td>0.3</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>B</td>
<td>B</td>
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<tr>
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<td>B</td>
<td>B</td>
<td>B</td>
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<td>B</td>
<td>B</td>
<td>B</td>
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<td>B</td>
<td>B</td>
<td>B</td>
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Table B.14: Degree of market domination and A's best strategy.
Given $\beta_x = \beta_y = 0.7$ and $\delta_x = \delta_y = 0.5$.

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<th>1</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>$\rightarrow \infty$</th>
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<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
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<tr>
<td>5</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
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<tr>
<td>7</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>$\rightarrow \infty$</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

Table B.15: Market concentration and A’s best strategy.
Appendix C

Chapter 5

C.1 Proof of (5.39) in the text

\[
- \frac{(\theta^i - \bar{\theta})^2}{[(N_t^* - 1) + N_s^* + aN]} + \frac{N_t^* (N_s^* + aN)}{(N_t^* + N_s^* + aN)^2} (\theta_t^* - \theta_t^*)^2 \leq 0 \quad (5.38)
\]

\[
\frac{1}{[(N_t^* - 1) + N_s^* + aN]} \left[ \mu_t (\theta_{-t}^* - \theta_t^*)^2 - (\bar{\theta}^* - \bar{\theta})^2 \right] \leq 0
\]

\[
\mu_t = \frac{N_t^* (N_s^* + aN)(N_t^* - 1) + N_s^* + aN}{(N_t^* + N_s^* + aN)^2}.
\]

Therefore, the above condition will be satisfied if

\[
\left[ \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) - (\bar{\theta}^* - \bar{\theta}) \right] \left[ \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) - (\bar{\theta}^* - \bar{\theta}) \right] \leq 0
\]

which means either

\[
\left[ \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) - (\bar{\theta}^* - \bar{\theta}) \right] \leq 0 \quad \text{and} \quad \left[ \sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) \cdot (\bar{\theta}^* - \bar{\theta}) \right] \geq 0
\]
or

\[ \left[ \sqrt{\mu_t} (\theta^*_t - \theta^*_i) - (\theta^i - \bar{\theta}) \right] \geq 0 \text{ and } \left[ \sqrt{\mu_t} (\theta^*_t - \theta^*_i) + (\theta^i - \bar{\theta}) \right] \leq 0 \]

which implies

\[ \theta^i \geq \left[ \bar{\theta} + \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \text{ and } \theta^i \geq \left[ \bar{\theta} - \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \]

or

\[ \theta^i \leq \left[ \bar{\theta} + \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \text{ and } \theta^i \leq \left[ \bar{\theta} - \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \]

Since \( \theta^*_t = \frac{N_t^i\theta^*_t}{N_t^i+aN} \) is the weighted average between \( \theta^*_i \) and \( 0 \), \( (\theta^*_t - \theta^*_i) < 0 \) by the assumption that \( \theta^*_i < 0 < \theta^*_t \), which implies

\[ \left[ \bar{\theta} + \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] < \theta^* < \left[ \bar{\theta} - \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \]

Therefore, the condition for (5.38) to be satisfied is

\[ \theta^i \leq \left[ \bar{\theta} + \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \text{ or } \theta^i \geq \left[ \bar{\theta} - \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \]

which means the ownership \( \theta^i \) that violates (5.38) in the text is

\[ \left[ \bar{\theta} + \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] < \theta^i < \left[ \bar{\theta} - \sqrt{\mu_t} (\theta^*_t - \theta^*_i) \right] \]  \hspace{1cm} (5.39)
C.2 Proof of (5.46) in the text

\[
\frac{N_t^* (N_s^* + aN)}{(N_t^* + N_s^* + aN)^2} (\theta_{-t} - \theta_t^*)^2 - \frac{1}{(N_t^* + 1) [(N_t^* - 1) + aN]} (\hat{\theta}_t - \theta^*)^2 \leq 0 \tag{5.44}
\]

\[
\frac{1}{(N_s^* + 1) [(N_s^* - 1) + aN]} \left[ \lambda_t (\theta_{-t} - \theta_t^*)^2 - (\hat{\theta}_t - \theta^*)^2 \right] \leq 0
\]

\[
\lambda_t = \frac{N_t^* (N_s^* + aN)(N_t^* + 1)(N_t^* - 1) + aN}{(N_t^* + N_s^* + aN)^2}
\]

Therefore, the above condition will be satisfied if

\[
\left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) - (\hat{\theta}_t - \theta^*) \right] \left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) + (\hat{\theta}_t - \theta^*) \right] \leq 0
\]

which means either

\[
\left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) - (\hat{\theta}_t - \theta^*) \right] \leq 0 \text{ and } \left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) + (\hat{\theta}_t - \theta^*) \right] \geq 0
\]

or

\[
\left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) - (\hat{\theta}_t - \theta^*) \right] \geq 0 \text{ and } \left[ \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) + (\hat{\theta}_t - \theta^*) \right] \leq 0
\]

which implies

\[
\theta^i \leq \left[ \hat{\theta}_t - \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) \right] \text{ and } \theta^i \leq \left[ \hat{\theta}_t + \sqrt{\lambda_t} (\theta_{-t} - \theta_t^*) \right]
\]

or

\[
\theta_1^i \leq \left[ \hat{\theta}_1 - \sqrt{\lambda_1} (\theta_{1-} - \theta_1^*) \right] \text{ and } \theta_2^i \leq \left[ \hat{\theta}_2 + \sqrt{\lambda_2} (\theta_{2-} - \theta_2^*) \right]
\]
\[ \theta^i \geq [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \text{ and } \theta^i \geq [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \]

Since \((\theta^* - \theta^i_t) < 0\), it is implied that

\[ [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^* - \theta^i_t)] < \theta^i < [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \]

Therefore, the condition for (5.44) to be satisfied is

\[ \theta^i \leq [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \text{ or } \theta^i \geq [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \]

which means the ownership \(\theta^i\) that violates (5.44) is

\[ [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^* - \theta^i_t)] < \theta^i < [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \]  \hspace{1cm} (5.46)

### C.3 Proof that \(-\theta_2 < \theta_1 < \theta_3 < 0 < -\theta_3 < -\theta_1 < \theta_2\)

\[
\begin{align*}
\theta_1 & \equiv \sqrt{\mu_t} (\theta^* - \theta^i_t) \\
\theta_2 & \equiv [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^* - \theta^i_t)] \\
\theta_3 & \equiv [\hat{\theta}_t + \sqrt{\lambda_t} (\theta^* - \theta^i_t)]
\end{align*}
\]  \hspace{1cm} (5.52)

(i) Since \((\theta^* - \theta^i_t) < 0\), it is clear that \(\theta_1 < 0\).

(ii) In the symmetric equilibrium, (5.45) becomes
\[
\hat{\theta}_t = -\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} (\theta^*_t - \theta^*_i) > 0
\]

Therefore,

\[
\theta_2 = \left[\hat{\theta}_t - \sqrt{\lambda_t} (\theta^*_t - \theta^*_i)\right] = - (\theta^*_t - \theta^*_i) \left[\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} + \sqrt{\lambda_t}\right] > 0
\]

since \((\theta^*_t - \theta^*_i) < 0\), and,

\[
\theta_3 = - (\theta^*_t - \theta^*_i) \left[\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} - \sqrt{\lambda_t}\right] < 0
\]

since \(\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} - \sqrt{\lambda_t} < 0\).

Proof that \(\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} - \sqrt{\lambda_t} < 0\). From (5.47), \(\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} - \sqrt{\lambda_t} < 0\) is

\[
\frac{N_i^* (N_i^* + aN)}{(2N_i^* + aN)} - \sqrt{N_i^* (N_i^* + aN)(N_i^* + 1)[(N_i^* - 1) + aN]} < 0
\]

\[
\sqrt{N_i^* (N_i^* + aN)} - \sqrt{(N_i^* + 1)(N_i^* - 1) + aN} < 0
\]

\[
N_i^* (N_i^* + aN) - (N_i^* + 1)[(N_i^* - 1) + aN] < 0
\]

187
\[ 1 - aN < 0 \]

which is true for \( a > 0 \).

(iii) From (i) - (ii), it is sufficient to show that \(-\theta_3 < -\theta_1 < \theta_2\) to complete the proof.

First,

\[ \theta_2 - (-\theta_1) > 0 \]

From (5.52),

\[ [\hat{\theta}_t - \sqrt{\lambda_t} (\theta^*_t - \theta^*_t)] + \sqrt{\mu_t} (\theta^*_t - \theta^*_t) > 0 \]

\[ \hat{\theta}_t - (\theta^*_t - \theta^*_t) (\sqrt{\lambda_t} - \sqrt{\mu_t}) > 0 \]

It has been shown that \( \hat{\theta}_t > 0 \) and \( (\theta^*_t - \theta^*_t) < 0 \), therefore, the above expression is true if \( (\sqrt{\lambda_t} - \sqrt{\mu_t}) > 0 \).

From (5.40) and (5.47), in the symmetric equilibrium, \( (\sqrt{\lambda_t} - \sqrt{\mu_t}) > 0 \) is

\[ \frac{\sqrt{N_t^* (N_t^* + aN)} (N_t^* + 1) [(N_t^* - 1) + aN]}{(2N_t^* + aN)} - \frac{\sqrt{N_t^* (N_t^* + aN)} (2N_t^* - 1 + aN)}{(2N_t^* + aN)} > 0 \]
\[(N_t^* + 1) [(N_t^* - 1) + aN] - (2N_t^* - 1 + aN) > 0\]

\[N_t^* (N_t^* + aN - 2) > 0\]

which is true.

Second,

\[-\theta_1 - (-\theta_3) > 0\]

From (5.52) and the expression for \(\theta_3\) in (ii) above,

\[-\sqrt{\mu_t} (\theta_{-t}^* - \theta_t^*) - (\theta_{-t}^* - \theta_t^*) \left[ \frac{N_t^* (N_t^* + aN)}{(2N_t^* + aN)} - \sqrt{\lambda_t} \right] > 0\]

\[-(\theta_{-t}^* - \theta_t^*) \left[ \sqrt{\mu_t} + \frac{N_t^* (N_t^* + aN)}{(2N_t^* + aN)} - \sqrt{\lambda_t} \right] > 0\]

Since \((\theta_{-t}^* - \theta_t^*) < 0\), it is sufficient to show that

\[\left[ \sqrt{\mu_t} + \frac{N_t^* (N_t^* + aN)}{(2N_t^* + aN)} - \sqrt{\lambda_t} \right] > 0\]

From (5.40) and (5.47), in the symmetric equilibrium,
\[
\left[ \frac{\sqrt{N_t^* (N_t^* + aN)(2N_t^* - 1 + aN)}}{(2N_t^* + aN)} + \frac{N_t^* (N_t^* + aN)}{(2N_t^* + aN)} \right]
- \sqrt{\frac{N_t^* (N_t^* + aN)(N_t^* + 1)}{(2N_t^* + aN)}} \right] > 0
\]

\[
\sqrt{(2N_t^* - 1 + aN) + \sqrt{N_t^* (N_t^* + aN)} - \sqrt{(N_t^* + 1)[(N_t^* - 1) + aN]} > 0}
\]

\[
\left[ \frac{(2N_t^* - 1 + aN)}{+2\sqrt{N_t^* (N_t^* + aN)(2N_t^* - 1 + aN) + N_t^* (N_t^* + aN)}} \right] > (N_t^* + 1)[(N_t^* - 1) + aN]
\]

It is sufficient to show that

\[
(2N_t^* - 1 + aN) + N_t^* (N_t^* + aN) - (N_t^* + 1)[(N_t^* - 1) + aN] > 0
\]

for the above expression to be true.

\[
2N_t^* - 1 + aN + (N_t^*)^2 + N_t^* aN - (N_t^*)^2 - N_t^* aN - aN + 1 > 0
\]

\[
2N_t^* > 0
\]

which is true.
Bibliography


193


