THE DEMAND FOR LIFE INSURANCE

by

S.R.DIACON B.Sc.

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ABSTRACT

This thesis is concerned with an examination of consumer purchases of life insurance. The problem is tackled in three main areas: first a theoretical examination of consumption, saving and bequest, based on the Yaari 'life-cycle' model of lifetime consumption. Secondly an analysis is developed to split new premium income into its two components : new savings and payments for protection against the financial effects of premature death. This analysis is then applied to the UK new business ordinary non-group renewable premium in order to derive figures for aggregate expenditure on savings-based and protection-based life insurance for the years 1946 to 1968. Thirdly, econometric techniques are applied to the aggregate data to explain the demand for both new and in force life insurance business: a study of the former is based on a single Demand and Supply model for new business (using the Two Stage Least Squares method); the analysis of contracts in force is undertaken by examining their Surrender Rates.

The major concern of this thesis is to differentiate between the savings and protection elements of life insurance. The results in later chapters show that the demand for these elements is determined by different explanatory factors although some variables (notably permanant income) strongly influence both elements. Notably, Demand(in the form of premium expenditure) is not affedted by market price in either the savings-based or the protection-based case (although the same cannot be

said for supply).

Attention is given to the role of inflationary expectations in the purchase of life insurance. Though the period under study was one of comparatively mild inflation, expectations are found to have a negative effect both on Demand and Supply and Surrender Rates: interesting conclusions can thus be drawn about the structure of long-term inflationary expectations.

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Stephen Diacon

Department of Industrial Economics

University of Nottingham

CHAPTER ONE : AN INTRODUCTION

1.1 The Importance of Life Insurance

Ever since Income Tax was first introduced in the United Kingdom by William Pitt (the Younger), life insurance has been accorded special privileges. Successive governments have encouraged the individual to purchase life insurance for two main reasons: first, to provide protection for heirs and dependents against the financial consequences of the breadwinner's premature death and secondly, to encourage long term saving and the provision for retirement. The purchase of life insurance thus provides the individual with a method of securing the future well-being of himself and his family without undue reliance on the various State welfare schemes.

In the course of the provision of their services, the life insurance companies, of necessity, build up large funds of invested assets which represent the present value of their future liability to policyholders. Over the post-war years, these assets have grown at an annual rate of around 10%, which is faster than the growth of Gross National Product, National Savings and the assets of the London Clearing Banks but not as rapid as the assets of the Building Societies or Unit Trusts (see S. Wynn (12) Ch.1). In 1950, the total assets of the life insurance companies were third in the 'league-table' of

UK Financial Institutions (with National Savings second and the London Clearing Banks first). By 1970, however, the life insurance companies were at the top of the table although they were subsequently deposed by the Building Societies in 1976. Table 1.1.1 shows the size of funds for various UK financial institutions.

Another indicator of the size and importance of the UK life insurance industry is the volume of its new business in comparison with that of its nearest competitors. Table 1.1.2 compares the new business figures of the major institutions with Total Personal Saving for selected years.

It is apparent then that the UK life insurance industry has developed in size and importance over the post-war period. Moreover, to some extent, the figures of Tables 1.1.1 and 1.1.2 under-estimate the worth of life insurance. This is because the payments represented in Table 1.1.2 are the first in a series of contractual payments thus providing considerable stability to the savings so generated. According to R.L. Carter (2) Ch.3, this contractual saving has benefitted the post-1945 UK Economy:

> "Over most of the post-war period since 1945 Britain has enjoyed a very low rate of unemployment and economic opinion has generally been in favour of raising the level of saving relative to Gross National Product in order to provide for higher levels of investment. Thus life insurance can claim to have played a valuable role in the economy."

This view is endorsed by A.R. Threadgold (11) who tentatively concludes that inflows into life insurance and pensions funds have resulted in a net addition to personal sector saving.

Moreover, the Tables give no real idea of the amount of 'protection' provided (against the financial consequences of

Table 1.1.1 Total Assets of Financial Institutions in the UK. (£000m)

	1950	1955	1960	1965	1970	1975
London & Clearing Banks	6.48	7.10	8.26	10.04	11.41	27.37*
National Savings	6.13	6.13	7.29	8.37	8.81	11.62
Life Insurance (UK) (Ordinary and Industrial)	2.38	3.37	5.11	8.24	12.76	20.48
Building Societies	1.26	2.07	3.17	5.53	10.82	24.20
Unit Trusts	-	-	0.19	0.50	1.32	2.56
*1974						

Source: Annual Abstract of Statistics (4) and Financial Statistics (5)

Notes

1) National Savings : Amounts remaining invested on 31 December (except for 1950 and 1955 which are at 31 March)

2) Life Insurance (UK) : Life and annuity funds at the end of the year for companies established in the UK. Book Values except for British government and UK Local Authority securities which are at Nominal Values.

Table 1.1.2 Annual New Business of Financial Institutions in the UK (£m)

	1950	1955	1960	1965	1970	1975
National Savings	-122.5	-141.7	179.6	-235.1	-45.5	223.8
Life Insurance (Ordinary yearly renewal premiums)	22.1	32.9	53.0	87.6	182.3	324.8*
Building Societies	86.4	143.8	111.3	450.0	978.6	2747.7
Unit Trusts (net sales)	-	-	13.5	59.0	97.8	190.1
Total Personal Saving	113.0	521.0	1222.0	2017.0	3079.0	11479.0
*1973						

Source: Annual Abstract of Statistics (4) and Financial Statistics (5)

Notes

1) National Savings : Receipts less Repayments
2) Building Societies : Shares (Subscribed less Withdrawn) plus Deposits (Received less Withdrawn)
3) Life Insurance : After 1968 these figures include annuity, capital redemption and long term personal accident business that were previously excluded. Much overseas business that was previously included has been excluded from 1969 onwards.

premature death) by the purchase of life insurance. The existence of this protection obviously has a significant (but generally unquantifiable) effect on the welfare of beneficiaries with a consequent cushioning of the State - provided social services.

1.2 Why the Demand for Life Insurance?

Following on from the previous section, a study of the demand for life insurance is of interest for three main reasons:

i) to investigate the behaviour of households particularly with respect to the provision of protection and contractual saving;

ii) to determine the total sales and changing structure of the home business for life insurance;

iii) to explain the role of life insurance in the context of the UK Economy.

An investigation of the life insurance purchases of households are of particular interest for a variety of reasons. First of all, various individual economic characteristics such as household income, wealth, liquidity and expectations etc., are known (or are hypothesised) to affect the purchase of life insurance. These factors may have differing effects on the purchase of protection-based and saving-based life insurance. Thus Chapters Two and Three will be concerned with hypothesising the determinants of protection-based life insurance (a definition will be deferred until then) while Chapter Four is concerned with the influences on saving in general and saving-based life insurance in particular. Broadly speaking, the analysis of Chapters Two and Three is based on the life-cycle models of Menahem Yaari (13) while Chapter Four reviews a number of existing studies, many of which are detailed in C. Ferry (3) and J.F. Lee and W.M. Whitaker (6). Later Chapters attempt to examine in detail the actual influence of these individual economic characteristics on the purchase of life insurance in the UK.

Secondly, institutional factors such as social security and State pensions provisions influence the household's purchase of life insurance (eg. see J. Revell (10)). The State can also affect the purchase via its taxation policy and its general succes in controlling the Economy (eg. its influence over inflation, interest rates, unemployment etc.). An examination of these factors is of particular interest and special emphasis is placed upon the role of inflationary expectations. The effects of the social security arrangements unfortunately proved much more difficult to investigate.

Any study of household behaviour is limited, however, by the quality of data available. Ideally cross-section studies providing information on households purchasing life insurance are the most useful (eg. see D.R. Anderson and J.R. Nevin (1)). Unfortunately time series studies (such as those undertaken in Chapters Eight and Nine) are not entirely successful because they cannot observe data pertaining only to those purchasers of life insurance. For example, household income must be approximated by aggregate Personal Disposable Income per capita: this figure, of course, includes the income of those households not purchasing insurance and so is not entirely appropriate. This aggregation problem is common to many time series studies.

A study of the demand for life insurance also reveals interesting results about the role and performance of those life insurers operating in the UK market. In particular the determinants of the level of sales (both demand and supply sides) and also the structure of those sales (eg. the relative importance of saving and protection-based policies). Some view can also be formed about the state of competition in the life insurance market: however, this study specifically excludes any examination of often the most recently competitive products: single premium policies and equity-linked business. Naturally enough, the life insurance industry is also concerned with the reaction of demand to variations in market price and the various economic indicators (income, unemployment, inflation etc.). Because we are now concerned with the potential market for life insurance the aggregation problems are not so severe.

Finally, a study of the demand for life insurance also has repercussions for the management of the Economy. In particular, the behaviour of contractual saving and the provision of protection strongly influences (and is influenced by) business investment and the social security system. The life insurance companies are also large purchasers of Government debt, and these purchases depend on the term and interest structure of new life insurance business.

1.3 Methodology

Any study of the demand for life insurance must cover the two major aspects of new business and (contractual) business in force. Generally, when we speak of demand it is premium income that concerns us since it represents the most accurate measure of the quantity of life insurance purchased by a household. The other possible measures of demand (number of policies, sums insured or change in invested assets) are subject to a variety of disadvantages which preclude their use: the number of policies purchased gives no indication of the size of those policies and consequently throws no light on the amount of protection or saving involved. Sums insured, on the other hand, overestimate the importance of the protection-based policies where the probability of making a claim is relatively small. The change in invested assets depends on a number of factors as well as new business (eg. interest income and claims experience) and in addition cannot be properly broken down into savings and protection elements. Thus premium income has been chosen as the relevant proxy for quantity demanded because, by and large, it adequately reflects the size and importance of the various components of life insurance business: it has also the advantage of measuring actual household expenditure on life insurance.

Although a study of new business premium income presents no real difficulties, an analysis of business in force involves more problems because of the strong trend elements involved: not surprisingly, the pattern of these continuing contractual savings remains particularly stable over time (eg. S. Neumann (8)

Ch.7 p.198, obtains $R^2 = 0.9998$ in a regression of in force premium income on a number of explanatory variables - one of which was the lagged dependent variable). An alternative would be to examine the change in premium income in force (ie. first differences) but it is questionable whether this is really a worthy subject of enquiry since it represents the aggregate of new business (already investigated), claims and voluntary terminations (which are variable at the discretion of the policyholder). This study's examination of the demand for contractual in force business is devoted to an examination of those contractual savings that have been prematurely liquidated by voluntary termination. This approach seems reasonable because terminating his insurance is the only way in which the policyholder can express dissatisfaction with his in force contract (and hence his Demand for that contract). Of the three methods of voluntary termination (surrender, lapse and becoming paid-up) the Surrender Rate has been chosen as the best representative of the demand for in force life insurance for the reasons explained in the course of Chapter Nine.

It has already been intimated that life insurance is purchased for two broad reasons: to provide protection (for dependents) against the financial consequences of premature death and as a method of long term contractual saving. In order to qualify for income tax relief on premiums UK life insurance contracts must include an element of protection. The most popular contracts (Endowment and Whole of Life) typically have both savings and protection elements. Consequently a rigorous analysis of the demand for life insurance is not really possible unless some method is devised to separate out the protection and savings elements of new business premium income. An analysis of the different types of contract is a gross simplification. For similar reasons it is not really possible to use the change in life office assets as a measure of demand because those assets include a small reserve to cover the protection element. Consequently, care has been taken to distinguish between the savings and protection elements and further discussion is taken up especially in Chapters Two, Five and Six.

A large variety of contracts are marketed by the life offices in the UK representing a great diversity of reasons for purchase. The scope of this study has been limited to the traditionally most important element of life office business - ordinary life insurance not including annuities. Single premium and group life insurance have also been excluded because of their peculiar characteristics (single premium business is very volatile and largely dependent on the taxation structure). Membership of group life insurance schemes is generally for reasons related to the nature and conditions of employment (see S. Wynn (12) and B. Reitz (9)). Unit or equity linked contracts have also been excluded where ever possible (again because of their peculiar nature); in any case they are conducted largely on a single premium basis and are a comparatively recent phenomenon (not becoming important until the mid-1960's).(1)

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(1)"A feature of this class of business which is more important in linked life assurance than in life assurance as a whole is the large amount of single premium investment." (Life Offices Association (7), 1969-73).
The prevalence of single premium linked business can be observed from the breakdown of new business reported in (7):

(£m)	1969	1970	1971	1972	1973	1974	1975
New yearly premiums	32	25	28	28	38	40	45
New single premiums	27	53	104	296	346	125	84

The statistical life insurance data for this study was obtained from three main sources:

a) company returns to the Department of Trade (as reported, in aggregate, in the Annual Abstract of Statistics (4));

b) company returns to the Life Offices Association and Associated Scottish Life Offices (as reported, in aggregate, in (7));

c) a survey, undertaken by the author, of fifteen of the largest life offices based in the United Kingdom.

The available data unfortunately limits this study in a number of significant respects, the most important being caused by a major discontinuity in the DOT returns in the year 1968/69 when the 'Insurance Companies (Accounts and Forms) Regulations 1968' substantially altered the reporting requirements. In particular much overseas business that had previously been included was excluded from 1969 onwards. Additionally, the post 1969 figures include capital redemption, long term personal accident business and deferred and immediate annuity business (new business figures) which were all shown separately before. Consequently the multiple regression analysis conducted in the later chapters uses data over the period 1946-1968 inclusive. Of course, there are other problems of data compatibility and these are reported in Chapter Six.

1.4 Chapter Structure

This study can be subdivided into three main parts: first, Chapters Two, Three and Four are concerned with a theoretical treatment of the purchase of protection and savings-based life insurance. In the second part (Chapters Five and Six) consideration is given to the problems of splitting UK new business premium income into its saving and protection elements. Thirdly, Chapters Seven, Eight and Nine constitute a statistical analysis of the data produced and hypotheses formulated in the earlier Chapters. The last Chapter (Ten) forms the Conclusion and pulls together the various results.

Chapter Two is concerned with developing a theoretical background to analyse consumption, lifetime and non-lifetime saving and bequest. The theory is based on the 'lifecycle' model of M. Yaari (13): this theory is extended and developed to handle a variety of situations, including the possibility of surrender of the protection-based life insurance contract.

Chapter Three uses some of the theory developed in Chapter Two to analyse the specific effects of anticipated and unanticipated inflation on the purchase of protection-based life insurance.

Chapter Four summarises some of the existing work on the determinants of saving, saving via life insurance and the demand for life insurance in general. Conclusions are drawn relating to the factors, variables and models that might be applied to explain the purchase of UK life insurance.

Chapter Five discusses the savings and protection content in standard life insurance contracts and develops a formulation to split these two elements of new business premium income. Chapter Six then applies the theory of the preceding Chapter to derive actual series for financial saving via life insurance, and the demand for protection-based and savings-based life insurance in the UK over the period 1946-1968.

Chapter Seven details the models and variables to be used in the multiple regression analysis of the purchase of UK life insurance. Some discussion is devoted to the derivation of an inflationary expectations variable: however, the extent that this variable can be tested is severely restricted by the sample period. Unfortunately, 1946-68 was a period of comparatively mild inflation (with the exception of the years of the Korean War): the most drastic inflation of the early 1970's is, therefore, excluded. Consequently the effects of inflation expectations on the purchase of life insurance are almost certainly less than they would have been had a longer sample period been possible.

Chapter Eight is a summary of the results obtained by running the multiple regression analyses of the new business premium income outlined in Chapter Seven. Two main models are attempted: first using the Financial Saving Ratio (new saving via life insurance as a percentage of Personal Disposable Income) as the dependent variable and secondly a Demand model (incorporating Supply equations and using the technique of Two Stage Least Squares).

Finally, Chapter Nine describes the results of an analysis of the demand for in force business by looking at the Surrender Rate for Ordinary (savings-based) life insurance.

Chapter One : References

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CHAPTER TWO : A THEORETICAL MODEL OF THE PURCHASE OF PROTECTION-BASED LIFE INSURANCE

2.1 Introduction

The objective of this Chapter is to describe a number of theoretical models which throw some light on the consumer's motives for purchasing protection-based life insurance. The underlying assumption behind the models is that the individual arranges his consumption and bequest expenditures so as to maximise the expected utility attainable from consumption over his lifetime and the estate remaining and bequeathed upon his death. The cardinal utility analysis so developed is thus based on the expected utility postulates of J. von Neumann and O. Morgenstern (32).

The models are used to develop hypotheses about an individual's purchase of protection-based life insurance: in particular to investigate how his decision depends on his age and his subjective estimation of his own mortality experience. Later Sections of this Chapter attempt to explain the conditions under which a consumer may wish to Surrender his life insurance arrangements. A discussion of the influence of inflationary expectations - and the resultant effect on the purchase and subsequent surrender of protection-based life insurance - is deferred to Chapter Three but uses the theory developed here.

Chapter Two is divided into four main parts: Section 2.2 introduces and defines the concepts and works through a simplified two-period model as an illustration. Sections 2.3, 2.4 and 2.5 further define and investigate the role of the bequest in a consumer's decision process. Sections 2.6 and 2.9 develop a continuous lifetime consumption model (along the lines first introduced by M. Yaari (33)) which is then used to analyse the structure conditions. Finally Sections 2.10 to 2.12 extend the theory of the previous Sections to analyse the consumer's decision to surrender his life insurance contract.

A key to the notation of each Section is provided in Appendix 2.2.

2.2 Consumption and Lifetime Saving

Wherever possible, notation will be used that corresponds with that agreed in 1898[1] and described in P.F. Hooker and L.H. Longley-Cook (16) (pp. 270-279).

The age of the consumer will be specified by the lower case letter x where x lies in the range $(0, \infty)$, and l_x will be used to denote the number of individuals who expect to attain exact age x (according to any particular mortality table).

We will assume that the consumer expects to live T years (following the work of D. Moffet (22) and (23), S.F. Richard (29), S. Fischer (8) and N.H. Hakansson (14)) where clearly T is a stochastic variable with probability density function $l_{x+T}\,\mu_{x+T} / l_x$ (μ_{x+T} is the force of mortality) as described by M. Yaari (33) and K. Borch (3).

At time t (ie. at age x+t) the consumer's income stream is given by y(t) and his consumption expenditure, c(t) both real-valued, non-negative functions of t on the interval $(0,T)$. The residual $y(t) - c(t)$ then represents the traditional definition of saving, but this can be divided into a further two parts:

$$y(t) - c(t) = s(t) + b(t) \qquad (2.2.1)$$

$$\text{where } c(t) \geq 0 \qquad (2.2.2)$$

b(t) denotes that part of the residual that the consumer does not intend to dissave in his lifetime. That is, b(t) is the deliberate process of laying income aside in order that there may be unrealised savings stock on the death of the consumer. b(t) will be called a 'bequesting flow' and

(1) By the Second International Actuarial Congress, May 1898.

the bequest left to the consumer's heirs, should he die at the time t will be denoted by B(t).

s(t) denotes that part of the residual that the consumer intends to dissave sometime in his lifetime and will be termed 'lifetime saving flow'. s(t) is used to build up 'lifetime savings stock' S(t) and

$$S(t) = e^{\delta t} \int_0^t e^{-\delta \tau} . s(\tau)\, d\tau \qquad (2.2.3)$$

where δ is the force of interest (which for simplicity is assumed constant over the range (0,T)). Differentiating through Equation (2.2.3) by t, we have

$$S'(t) = \delta S(t) + s(t) \qquad (2.2.4)$$

where $S'(t) = dS(t)/dt$. Thus the change in Savings Stock (S') is simply the sum of interest ($\delta S(t)$) and new saving (s(t)). Both s(t) and S(t) are usually allowed to be negative although in order to build a workable model of consumer consumption and saving, it is always necessary to constrain S(t) and s(t) in some way.

However, the definition of 'lifetime' saving implies a 'built-in' constraint to the effect that the consumer saves and dissaves in order to make his discounted value of lifetime savings stocks at death (at time T) equal to zero. This can be represented as[2]

$$_0\int^{\infty} e^{-\delta T} .S(T).\ (\overset{+}{l}_{x+T}\ \overset{+}{\mu}_{x+T})/\overset{+}{l}_x \quad dT = 0$$

(2.2.5)

where $\overset{+}{l}_{x+T}$ denotes the consumer's own subjective estimate of the mortality table. $\overset{+}{l}_{x+T}$ will be represented by the equation

$$\overset{+}{l}_{x+t} = L(t).l_{x+t}$$

(2.2.6)

where $L(0) = 1$ and $L(t)$ is assumed to be a differentiable function of time t.

Unfortunately, we shall see in the analysis of Section 2.6 that Equations (2.2.5) and (2.2.6) above are incompatible in the sense that the standard calculus of variations calculations cannot be resolved when both Equations are utilised.

F. Modigliani and R. Brumberg (21) list four motives for saving:

(i) the desire to add to the estate for the benefit of heirs;

(ii) to 'straighten out' the pattern of income and

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(2) The structure of Equation (2.2.5) and in fact the entire notation in this Chapter implies that all the consumer's decisions are taken at age x. Equation (2.2.5) can be simplified to obtain:

$$_0\int^{\infty} e^{-\delta t}\ (\overset{+}{l}_{x+t}/\ \overset{+}{l}_x).\ s(t)\ dt = 0$$

consumption;

(iii) the precautionary motive - in order to meet possible emergencies;

(iv) because of the presence of uncertainty, it may be necessary to have an equity in certain kinds of assets (such as houses and consumer durables) before an individual can receive services from them.

Motives (ii) and (iii) refer to lifetime saving, however while (ii) is fairly straightforward, it is difficult to reconcile (iii) with Equation (2.2.5) since emergencies may be expected throughout the consumer's lifetime. Friedman (11) has pointed out that the higher the ratio of non-human wealth to permanent income, the less need there is for any additional reserve (ie. saving). Thus the consumer may be in most need of an additional reserve when lifetime savings stocks are zero or negative. Consequently, if we include precautionary saving as part of 'lifetime' saving Equation (2.2.5) must be modified as follows:

$$\int_0^{\infty} e^{-\delta T} . S(T) . (\overset{+}{l}_{x+T} . \overset{+}{\mu}_{x+T}) / \overset{+}{l}_x \, dT = A \qquad (2.2.7)$$

where $A \neq 0$

Equation (2.2.7) can be interpreted to mean that the consumer expects to have non-zero (and most likely positive) savings stocks at his eventual death. These savings stocks will thus contribute towards the consumer's bequest but arise not because of any bequest motive (in fact $A > 0$ in the absence of a bequest motive although an attempt to minimise A would be made) but because of the uncertain situation of the consumer. Note that we have implicitly assumed that the consumer's money income stream y is known in advance; however we shall see in a later Chapter that although money income may be known with certainty, the same may not be said about real income. If the consumer has a non-zero bequest motive, this does not necessarily mean that A will be maximised since any utility attaching to consumption will tend to reduce A. Although A is not a function of x (since there is no a priori reason why the possibility of emergencies should depend on age), A is dependent on the risk aversion of the consumer, so that we might expect the more cautious consumer to have a greater value of A. This facet of A's behaviour may cause difficulties later on, however Modigliani and Brumberg (21), p.428, have indicated that the third and fourth motives for saving appear not to be significant because of the consumer's tendency to accumulate assets at a rapid rate during the early income earning years which are therefore later available as a general reserve against emergency. Consequently precautionary saving can initially be disposed of without much loss of generality either by ignoring it altogether or by analysing it in conjunction with the bequest motive.

Even in the absence of precautionary saving it is not inconceivable that lifetime savings stocks will be non-zero at death. This phenomenon arises because the consumer does not know his exact age at death. In this case S(t) forms part (although an unintended part) of the consumer's bequest.

By including b(t) in the residual y - c we have implicitly indicated that b(t) is one of the components of saving.

Indeed, Modigliani and Brumberg include the bequest motive as the first of their four 'motives for saving' and Moffet (22) p.19, concludes that, in the absence of insurance, "one can say that the existence of a bequest motive creates an incentive to save". However, there is a strong case for arguing that the process of bequesting (ie. allocations out of income) constitutes elements of both consumption and saving: this is particularly true of the purchase of temporary life insurance (whose case will be more fully described in Chapter Five).

The standard type of definition to be found in any basic economics textbook states that consumption is "the combination of goods and services to produce satisfaction for the consumer".[3] This definition may be interpreted in two main ways: first, with the emphasis on the 'combination' of goods and services and second on the fact that this combination produces 'satisfaction'. There can be no doubt that consumption involves both 'combination' of goods and services and also the production of 'satisfaction'. However, if we want to define saving in terms of deferred consumption then it becomes important to determine exactly which element of consumption is deferred: 'combination' or 'satisfaction'. If it is decided that saving occurs when satisfaction is deferred to a later time period, then it is possible to argue that any activity producing immediate satisfaction must be classed as consumption.

Initially it is possible to give a definition of consumption, with emphasis on the 'satisfaction' element, to the effect that consumption results from any activity that produces

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(3) For example see W.W. Cochrane & C.S. Bell (5) p.4.

utility. A. Marshall (20) p.54, envisages much the same type of explanation:

> "Just as man can produce only utilities, so he can consume nothing more. He can produce services and other immaterial products and he can consume them. But as his production of material products is really nothing more than a rearrangement of matter which gives it new utilities so his consumption of them is nothing more than a disarrangement of matter, which diminishes or destroys its utilities."

However, this definition does not differentiate clearly between consumption and (lifetime) saving. Since lifetime saving will eventually be used to produce satisfaction for the consumer, it could be argued that consumption is synonomous with income, ie. there is no saving component at all. Thus we need to say that consumption results in any activity that produces immediate (rather than deferred) satisfaction. This is the type of definition envisaged by I. Fisher (9) p.9:

> "Spending and (saving) differ only in degree, depending on the length of time elapsing between the expenditure and the enjoyment. To spend is to pay money for enjoyments which come very soon. To (save) is to pay money for enjoyments which are deferred to a later time."

The above definition of consumption introduces one further problem in that there is usually some type of time lag between consumption expenditure and the satisfaction that is the counterpart of consumption. Thus the peculiar situation arises that unless consumption occurs simultaneously with expenditure, saving will result because the enjoyments have been deferred to a later time. Consequently, the definition of consumption must be modified to say that consumption results in any activity that produces satisfaction within a reasonable period after acquisition.

One of the consequences of defining consumption as a satisfaction producing activity results in a change in the treatment of certain activities that might have been regarded as saving. Thus, another motive for saving that was not included in Modigliani and Brumberg's list arises if a man wishes to have assets merely for the sake of possessing those assets: that is, if satisfaction is derived from the mere act of saving rather than from the future consumption that the saving will generate.[4] The new definition would then classify this behaviour as consumption rather than saving. However it is obvious that this treatment contradicts the result obtained by defining consumption as a reduction in the net present value of wealth.

Another, much more interesting consequence of the definition of consumption (as a satisfaction producing activity) is that it produces strong justification for including bequesting flow as an element of consumption rather than saving. Although the reasoning is taken up more fully in Section 2.3 it can be argued that if the consumer gains immediate satisfaction from the knowledge that his dependents will not suffer from the financial consequences of his uncertain death, then the payments that provide this protection (the bequesting flow b(t)) will constitute consumption.

At this point it might be constructive to introduce a simple utility maximising model along the lines indicated by M.W. Jones-Lee (19) Ch.4, and especially Moffet (22) and analogous to several of those of H.A.J. Green (13). This

[4]This case if mentioned by J.S. Duesenberry (6) p.34.

model has the advantage that the concepts already introduced can be further elaborated while some interesting results can be obtained.

The ensuing model is a two-period discrete time one where it is assumed that the consumer dies in one or the other of the two time periods. We say that, from a given mortality table, l_k denotes the number alive at the beginning of the k^{th} period, then note that $l_k=0$ for $k \geq 3$. Income, consumption and bequesting flow decisions are assumed to be made at the beginning of each period and are all taken to be positive (ie. $y_k, c_k, b_k \geq 0$; $k = 1,2$) Any bequest to be made by the consumer is undertaken solely through the medium of a two-period temporary life insurance (with office premiums b_1 and b_2) where the bequests (B_1 and B_2) are paid out at the end of the period of death. The balance of income not allocated (ie. lifetime savings flows s_1 and s_2) are assumed to be saved with a banker at rate of interest (per period) i (the same rate as used by the insurer in premium calculations). s_1 and s_2 are used to build up lifetime savings stocks S_1 and S_2 (determined at the beginning of each period): all four components of saving may be negative. The consumer derives satisfaction from both consumption and the bequest provided by the life insurance and this can be denoted by the instantaneous utility functions u(c) and w(B). Finally we assume that in the first instance the utility assigned to the arbitrary consumption and bequest plans is given by[5]:

$$U(c,B) = \sum_{k=1}^{T} (u(c_k) + w(B_k)) \tag{2.2.8}$$

[5]This form for U(c,B) implies that preferences for consumption and bequest are independent. Furthermore intertemporal independence is implicitly assumed.

However since T is a stochastic variable, we need to find the expected utility of the lifetime consumption and bequest plans; this is given by

$$\bar{U}(c,B) = \sum_{1}^{2} \sum_{1}^{T} (u(c_k) + w(B_k)).p(T) \tag{2.2.9}$$

where p(T), the probability density function of T, is given by

$$p(1) = (l_1 - l_2) / l_1$$

$$p(2) = l_2 / l_1 \tag{2.2.10}$$

Equations (2.2.9) and (2.2.10) can then be combined to give

$$\bar{U}(c,B) = \sum_{k=1}^{2} (l_k/l_1). (u(c_k) + w(B_k)) \tag{2.2.11}$$

The objective of the consumer is therefore to maximise $\bar{U}(c,B)$ by varying c and B subject to certain conditions. The restrictions that lifetime savings must be zero at expected time of death can be written

$$p(1).S_1 + v.p(2).S_2 = 0 \tag{2.2.12}$$

where $v = (1 + i)^{-1}$. If the consumer's initial wealth at the start of period one is assumed to be zero then we have

$$S_1 = s_1$$

$$S_2 = s_1(1+i) + s_2 \tag{2.2.13}$$

so that (2.2.12) can be rewritten

$$l_1.s_1 + v.l_2.s_2 = 0 \tag{2.2.14}$$

(incorporating Equations (2.2.10), (2.2.12) and (2.2.13)). Note that we have assumed that the consumer's subjective estimate of mortality is given by the equation $\overset{+}{l}_k = l_k$ (see Equation (2.2.6)): it would not be sensible to do otherwise because of the initial assumption that the consumer dies sometime in the two periods.

Using the principle of equivalence (see Borch (3)) - where the present value of expected premiums is equated to the present value of expected claims - the relationship between b and B is as follows:

$$l_1.b_1 + v.l_2.b_2 = k.(v.(l_1 - l_2).B_1 + v^2.l_2.B_2) \tag{2.2.15}$$

where $k \geq 1$ is the loading on the pure premium to allow for insurer's expenses and profit.

The relationship between y, c, b and s is of course:

$$y_k = c_k + b_k + s_k \quad ; \quad k = 1, 2 \tag{2.2.16}$$

since income (y_k) can either be consumed (c_k), used to provide a bequest (b_k) or 'lifetime' saved (s_k). Now equations (2.2.14), (2.2.15) and (2.2.16) can be combined by substituting for s_1 and s_2 in (2.2.14) ie.

$$l_1.(y_1 - c_1 - b_1) + v.l_2.(y_2 - c_2 - b_2) = 0$$

ie. by rearranging

$$l_1.(y_1 - c_1) + v.l_2.(y_2 - c_2) = (l_1.b_1 + v.l_2.b_2)$$

so that we get

$$l_1.(y_1 - c_1) + v.l_2.(y_2 - c_2) = k.(v.(l_1 - l_2).B_1 + v^2.l_2.B_2) \qquad (2.2.17)$$

by substituting into Equation (2.2.15).

Following the methodology of Moffet (22), we set the partial derivatives of (2.2.11) equal to zero, in order to obtain

$$l_1.u'(c_1) + l_1.w'(B_1)\partial B_1/\partial c_1 + l_2.w'(B_2)\partial B_2/\partial c_1 = 0$$

$$l_1.w'(B_1)\partial B_1/\partial c_2 + l_2.u'(c_2) + l_2.w'(B_2)\partial B_2/\partial c_2 = 0$$

$$l_1.u'(c_1)\partial c_1/\partial B_1 + l_1.w'(B_1) + l_2.u'(c_2)\partial c_2/\partial B_1 = 0$$

$$l_1.u'(c_1)\partial c_1/\partial B_2 + l_2.u'(c_2)\partial c_2/\partial B_2 + l_2.w'(B_2) = 0 \qquad (2.2.18)$$

Noting that the partial derivatives can be found from Equation (2.2.17) we get the final solution

$$\frac{u'(c_1)}{u'(c_2)} = (1 + i) \qquad (2.2.19a)$$

$$\frac{w'(B_1)}{w'(B_2)} = \frac{(l_1 - l_2).(1 + i)}{l_1} \qquad (2.2.19b)$$

Equation (2.2.19a) provides no real surprises in that it corresponds with previously existing results (eg. see Moffet (22)): thus as the rate of interest i increases the consumer will prefer second to first period consumption (ceteris paribus). In order to correctly interpret Equation (2.2.19b) it must be remembered that the mortality rates involved are those used by the insurer and are usually not closely related to the individual characteristics of the consumer. Thus Equation (2.2.19b) indicates that the older of two, otherwise identical, consumers (ie. the one with the lower value of l_2) will prefer a Period Two to a Period One bequest (remember that it has been assumed that death occurs in one or the other of the two periods). Similarly, the younger of the two (ie. the one most likely to die in Period Two) will prefer a Period One bequest (ceteris paribus). There are two elements in the explanation of this phenomenon:

i) a change in the budget constraint: concentration of bequest on those ages with the lowest probability of death will reduce the cost of life insurance protection;

ii) it could be argued that greatest protection is needed (ceteris paribus) for those ages where the probability of death is lowest. This is because death, if it does occur at these ages, will be more unexpected.

To finish off this simplified two-period example, it is of interest to examine the assumptions behind the construction of U(c,B) (Equation 2.2.8). Essentially, (2.2.8) indicates that the consumer derives <u>immediate satisfaction</u>

from the process of bequesting (and therefore protecting dependents from the effects of unexpected death) irrespective of the timing of his death. Most other studies of the process of bequest - based on the Yaari 'Life Cycle' Model (see Yaari (33) and Moffet (23))- assume that the derivation of utility from making a bequest is contingent upon the <u>death of the consumer</u>. In this case the utility of bequest must represent the discounted value of the heir's and dependent's utility of consumption functions <u>given</u> that the consumer dies. Consequently, since utility is only derived at the time of death, the 'Life Cycle' construction of the utility function assigned to consumption and bequest plans is of the form:

$$U(c,B) = \sum_{k=1}^{T} u(c_k) + w(B_T) \tag{2.2.20}$$

The expected value of lifetime consumption and bequest is then

$$\bar{U}(c,B) = \sum_{T=1}^{2} \left(\sum_{k=1}^{T} u(c_k) + w(B_T) \right).p(T) \tag{2.2.21}$$

$$= \sum_{k=1}^{2} (l_k/l_1).u(c_k) + \frac{(l_1-l_2)}{l_1} .w(B_1) + (l_2/l_1).w(B_2) \tag{2.2.22}$$

If we maximise $\bar{U}(c,B)$ by differentiating Equation (2.2.22) while applying the same conditions as before the final solutions so obtained are:

$$\frac{u'(c_1)}{u'(c_2)} = (1 + i) \qquad (2.2.23a)$$

$$\frac{w'(B_1)}{w'(B_2)} = (1 + i) \qquad (2.2.23b)$$

Clearly, these solutions are difficult to explain in the context of the provision of protection for the benefit of heirs and dependents: we would intuitively expect that provision to depend on the age of the consumer. Furthermore, the more uncertain the consumer is about the possibility of death, the greater the amount of protection we would expect him to require; consequently our model (Equations(2.2.19)) predicts that, in the absence of other information, a consumer will require more protection at younger (rather than older) ages.

It will be seen in the next Section that the conventional 'Life Cycle' Model (Equation (2.2.23b)) is more useful in explaining those benefits that do not provide for the protection of dependents ie. non-lifetime saving. In this context the statement that the bequest is invariant to age is easier to understand.

2.3 The Bequest Motive

Convention demands that b(t) (the bequesting flow) and B(t) (the bequest at time t) are always positive for several reasons:

i) we normally assume that the consumer will prefer to leave his dependents with an estate rather than with debts although the legal/taxation system determines the extent of the transference of wealth. Thus to quote Alfred Marshall (20) p.228:

> "were it not for family affections, many who now work hard and save carefully would not exert themselves to do more than secure a comfortable annuity for their lives";

ii) the structure of a rational capital market should ensure that loans are not available to individuals who are highly likely to die before the repayment;

iii) we do not normally allow individuals to sell insurance on their own lives (this is a feature of several of the works on the lifetime allocation of consumption eg. Fischer (8)).

However, it will not normally be necessary to impose an ex ante restriction on b(t) and B(t) since the pull of any bequest motive should guarantee that b(t) and B(t) turn out to be positive for all values of t. In the absence of a bequest motive it may indeed prove necessary to introduce some form of constraint; for example, Yaari (33) stipulates S(T) = 0, where T is the age at death, while Moffet (22) stipulates the local constraint that the consumer must always be solvent (ie. $S(t) \geq 0$ for all t).

We now return to the argument in the middle paragraphs of Section 2.2 to show that the process of bequesting constitutes some element of consumption as well as saving. If consumption is defined as a satisfaction producing event then the bequesting allocation out of income would count entirely as saving if the consumer obtained no immediate satisfaction from laying aside b(t). (Note that this would also be true if the 'combination' definition of consumption was used). Satisfaction would then be deferred and would accrue only after death with the ultimate consumption of the bequest by heirs and dependents. This is the type of situation envisaged by most of the authors who have discussed the lifetime allocation of consumption eg. Yaari, Moffet, Borch, Fischer and Hakansson. However, it can be argued that the consumer gains immediate satisfaction from the knowledge that his dependents will not suffer from the financial consequences of his premature death. Thus bequesting flow b(t) can be regarded as payment for protection (against the uncertainty of death) which realises immediate satisfaction and is therefore classed as consumption. But, this is obviously not the end of the argument since, although we have indicated that bequesting was not entirely saving, it must also be noted that neither is it entirely consumption. Consequently, it is necessary to examine the process of bequesting in greater detail.

Alfred Marshall (20) p.190, details two main reasons why the consumers may have a bequest motive: first, family affections lead to a desire to protect dependents against the consequences of premature death. Secondly, to leave the dependents 'better off' financially than they were in his lifetime:

> "A man can have no stronger stimulus to energy and enterprise than the hope of rising in life, and leaving his family to start from a higher round of the social ladder than that on which he began".

To this list, we may add a third minor reason to cover gifts to charities and the like, however the problems caused by altruistic behaviour will be ignored in this context.

Thus the first of Marshall's reasons for bequest is to provide protection for heirs and dependents and the second reason is basically to provide a bequest that is in excess of an amount needed to provide adequate protection.

Intuitively, we might expect that 'adequate protection' would result if the dependents were able to continue at more or less the same standard of living after the death of the consumer as they had attained before. Another similar but more rigorous definition would equate a bequest providing 'adequate protection' with the expected discounted cash value of the consumer's future lifetime earnings less that part which would be consumed by the consumer himself (ie. the consumer's human capital). Both these definitions aim at a situation analagous to the Principle of Indemnity (although we note that the legal Principle of Indemnity has not applied to insurances on human life since 1854[6]). Consequently, a bequest that provides protection of dependents against premature death will be defined as having an upper bound determined according to the indemnity principle.

Bequesting flow to provide a bequest in excess of the

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(6) Dalby v. India and London Life (1854) 15CB 365

upper bound cannot be used to provide protection for dependents: it must therefore be classified as saving rather than consumption. This, of course, is not lifetime saving, but consumption by heirs and dependents that is deferred until the individual's death - this will be termed 'non-lifetime' saving.

To sum up, bequesting flow b(t) can be divided into two main classes: to provide protection against premature death and to provide more than adequate protection. The former can be classed as consumption and the latter as saving since consumption is deliberately deferred until a later date. Obviously, the second class cannot exist until the first reason for bequesting has already been fully satisfied.

Should the consumer die at age x+t, then in the <u>second</u> case above, the bequest that he makes to heirs and dependents ie. B(t) - B*(t) where B* is the upper bound on protection can just be classified as realised savings stocks. Similarly, there seems to be no a priori reason why B*(t) should not be classed as realised savings stocks except that it has already been argued that B*(t) is connected with consumption and not saving. However, this is not a problem since bequesting for protection purposes is essentially just a method of paying in advance for <u>known</u> future consumption. Thus the realisation of B*(t) is akin to the consumption of say, a consumer durable and is therefore more closely connected with consumption than with saving.

The classification of bequesting flow into consumption and saving has interesting implications when applied to the purchase of temporary life insurance. Theoretically, since the Principle of Indemnity does not apply to life insurance, the budget constraint is the only restriction on the amount of temporary life insurance that can be purchased by an individual. Thus, subject to his budget constraint, it is possible for the consumer to purchase life insurance in excess of the upper bound mentioned above. This would imply that part of the temporary life insurance premium would be classed as saving and this is contrary to all current opinions on the matter.(7)

It must be pointed out, however, that although theoretically the consumer can purchase unlimited amounts of temporary life insurance, it is possible that the insurer might suspect moral hazard (ie. that the individual influences the possibility of loss) unless the consumer can demonstrate a valid reason for the large sum insured.(8) Thus the insurer may effectively impose an untenable budget constraint on the consumer by increasing the calculation mortality rates (see Section 2.5).

'Adequate protection' has already been defined in terms

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(7) For example, see: J.S. Duesenberry (7) and S. Neumann (26). This type of saving would, of course, be classed as 'non-lifetime' saving.

(8) For example, to quote from Peter Edwards "Life Assurance Financial Underwriting" Planned Savings November 1978, Wootten Publications Ltd:

> "In underwriting any proposal, we not only have to look at the medical aspects but we have to satisfy ourselves that the sum assured can be justified, whether there is financial need and more importantly perhaps, we must know the reasons for effecting the policy."
>
> "To help the underwriter, we therefore have to apply a rule of thumb in deciding what is reasonable by way of sum assured. If any individual wishes to pay more than 20% of total income in the provision of life assurance then great care should be taken before acceptance."

of the amount of the bequest at any one time, but there is also a 'time-scale' problem. This arises because it is unlikely that protection for dependents will be needed indefinitely. After the consumer has reached a certain age (for instance, if he has outlived his spouse and the children have left home) then protection for dependents may no longer be required. This does not mean however, that the consumer has no bequest motive. The time-scale consideration has an interesting application to whole of life insurance: this arises since we can reasonably assume that protection for dependents will not be needed indefinitely. Thus obviously, whole of life insurance premiums cannot be classified as solely consumption but also include an element of 'non-lifetime' saving. This problem will be further discussed in Chapter Five.

2.4 The Utility of Bequest

It is the consequence of Section 2.3 that the instantaneous utility of bequest function associated with the amount of bequest at any time is a hybrid composed of two parts. The first is associated with the immediate satisfaction obtained from the knowledge that dependents are protected against the effects of premature death. This utility function has the form:

$$\text{Utility at age } (x+t) = \begin{cases} w_1(B(t)), & B(t) \leq B^*(t) \\ 0, & B(t) > B^*(t) \end{cases} \qquad (2.4.1)$$

where $B^*(t)$ is the upper bound on protection defined by the indemnity principle at age $x + t$ and $w_1'(B(t)) > 0$ where $w_1'(B) = \frac{d}{dB} w_1(B)$.

The second part of the utility of bequest function is associated with the deferred utility of heirs and dependents resulting from the consumption of bequests in excess of B^* ie. 'non-lifetime' saving. This utility has the form:

$$\begin{array}{l}\text{Deferred utility} \\ \text{at age } (x+t)\end{array} = \begin{cases} 0, & B(t) \leq B^*(t) \\ w_2(B(t)-B^*(t)), & B(t) \leq B^*(t) \end{cases} \qquad (2.4.2)$$

The use of utility function $w_2(B)$ is slightly misleading in that it seems to imply that the consumer gains immediate satisfaction from the non-protecting consumption of heirs and dependents (the situation mentioned before and noted by Duesenberry (7)). However, (because of our definition of consumption) w_2 (B(t) - B*(t)) is really only a simplification representing the discounted value of the heirs' and dependents' utility of consumption functions <u>given</u> that the consumer dies at age x + t. ie.

$$w_2(B(t) - B^*(t)) = {}_t\int^{\infty} \alpha_h(\tau).u_h(c_h(\tau))\, d\tau \qquad (2.4.3)$$

where the consumption stream of heirs and dependents $c_h(\tau)$ is generated by the bequest B(t) - B*(t). ie.

$$B(t) - B^*(t) = {}_0\int^{\infty} e^{-\delta\tau}.\, c_h(\tau)\, d\tau \qquad (2.4.4)$$

and $\alpha_h(\tau)$ is the discounting factor (ie. measure of the impatience of heirs and dependents).(9)

$w_1(B)$ and $w_2(B)$ may be combined to produce the hybrid utility of bequest function w(B) where:

$$w(B) = \begin{cases} w_1(B) & , \; B \leq B^* \\ w_1(B^*) + w_2(B) & , \; B > B^* \end{cases} \qquad (2.4.5)$$

The hybrid instantaneous utility function of Equation (2.4.5) is illustrated below in Figure 2.4.1 for the case of a risk-averse consumer (so that the function is concave to the origin):

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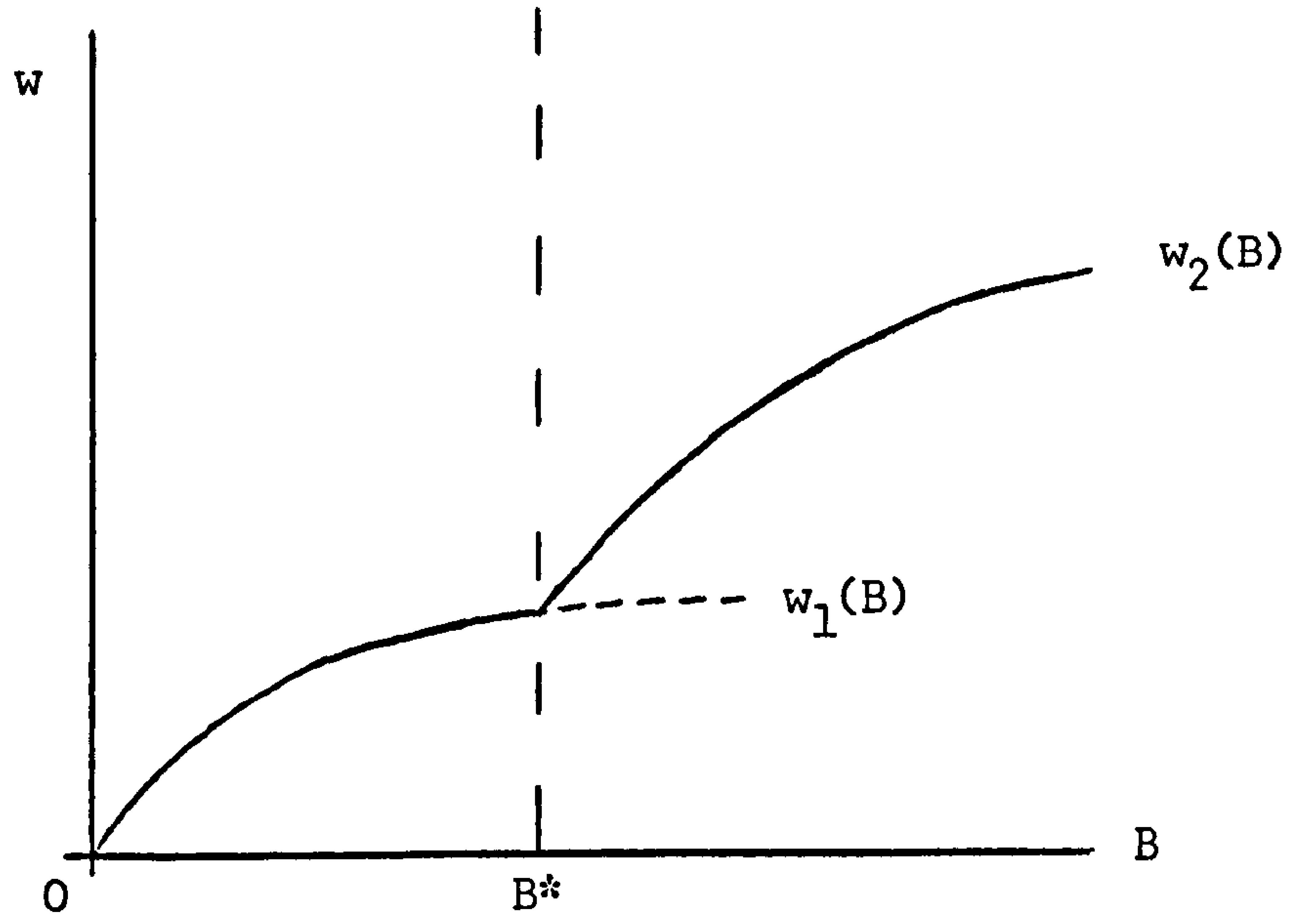
(9) w_2 is just a 'utility related' form of the utility function for an altruistic consumer (see, for example, D. Collard, "Altruism and Economy" p.7)

Figure 2.4.1 The hybrid instantaneous utility of bequest function.



The main problem with the use of the hybrid instantaneous utility function $w(B)$ arises when considering an expression for the expected utility derived from Consumption and Bequest plans $\bar{U}(c,B)$ (this is highlighted in the difference between Equations (2.2.8) and (2.2.20)). Thus w_1 is associated with immediate satisfaction while w_2 is dependent upon the death of the consumer as in the Yaari 'Life Cycle' Models (ie. Equation (2.4.3) is only well-defined if the consumer dies at time t). At this point, it is appropriate to note also that the instantaneous utility of bequest function is also a function of the age of the consumer (this phenomenon has been referred to as the 'time-scale' problem in Section 2.3). Thus, after some sufficiently large time (say after t^*) the consumer does not require a bequest for protection purposes since, for example, all his former heirs and dependents are no longer dependent on his income. Consequently we require that

$$w_1(B(t)) = 0 \quad \text{for all } t \geq t^*$$

To conclude this section, the discrete time model of Section 2.2 can be generalised to show the effects of the hybrid instantaneous utility of bequest function (of Equation (2.4.5)). We assume again that bequests are made solely via life insurance. The utility derived from the instantaneous consumption and bequest plans $u(c)$, $w_1(B)$ and $w_2(B)$ can then be denoted by

$$U(c,B) = \sum_{k=1}^{T} \left(u(c_k) + w_1(B_k) \right) + w_2(B_T - B_T^*) \qquad (2.4.6)$$

The expected utility from consumption and bequest plans is then

$$\bar{U}(c,B) = \sum_{k=1}^{2} (l_k/l_1).\left(u(c_k) + w_1(B_k) \right) + \left(\frac{l_1 - l_2}{l_1} \right).w_2(B_1-B_1^*) + (l_2/l_1)w_2(B_2-B_2^*) \qquad (2.4.7)$$

Before we can proceed, it is necessary to determine expressions for B_1^* and B_2^* since it is unrealistic to assume that they are both constant. In Section 2.3 it was suggested that B^* might be given by the expected discounted cash value of the consumer's future lifetime earnings less that part which would be consumed by the consumer himself.[10]

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[10] "The most important piece of information the underwriter needs relates to the estimated net worth of the life assured." Peter Edwards, Planned Savings, November 1978.

But in this discrete-time model, the consumer was assumed to die in one or the other of the two periods; this, in conjunction with the assumption that bequests are made at the end of the year of death yields:

$$B_1^* = \frac{l_1 - l_2}{l_1} \cdot (y_2 - c_2)$$

$$B_2^* = 0 \qquad (2.4.8)$$

Combining (2.4.8) with Equation (2.4.7) and using the method of relative maximum values (See Allen (1) Ch. 14) we note that the function $f(x,y)$ is maximised subject to the side relation $\emptyset(x,y) = 0$ when

$$\frac{\partial f}{\partial x} \Big/ \frac{\partial \emptyset}{\partial x} = \frac{\partial f}{\partial y} \Big/ \frac{\partial \emptyset}{\partial y} \quad \text{etc.,}$$

From Equation (2.2.17), we can write the side relation as

$$\emptyset = k \cdot (v(l_1 - l_2)B_1 + v^2 l_2 B_2) - l_1(y_1 - c_1) - v l_2(y_2 - c_2) \qquad (2.4.9)$$

Thus $\overline{U}(c,B)$ is maximised when

$$\frac{u'(c_1)}{l_1} = \frac{\frac{l_2}{l_1} u'(c_2) + \left(\frac{l_1 - l_2}{l_1}\right)^2 w_2'(B_1 - B_1^*)}{v l_2}$$

$$= \frac{w_1'(B_1) + \left(\frac{l_1 - l_2}{l_1}\right) w_2'(B_1 - B_1^*)}{k.v.(l_1 - l_2)}$$

$$= \frac{\frac{l_2}{l_1} w_1'(B_2) + \frac{l_2}{l_1} w_2'(B_2)}{k.v^2 l_2} \qquad (2.4.10)$$

Now Equation (2.4.10) is principally of interest when $B_1 > B_1^*$ and $B_2 > B_2^*$ (otherwise $w_2(B_1 - B_1^*) = w_2(B_2) = 0$) so that, as $w_1(B_1)$ is constant, we have $w_1'(B_1) = 0$. Noting that $w_1(B_2) = 0$ (because $B_2^* = 0$) we can simplify Equations (2.4.10) to obtain:

$$u'(c_1) = \frac{1}{v}u'(c_2) + \frac{1}{v}\frac{(l_1-l_2)^2}{l_1 l_2} w_2'(B_1-B_1^*)$$

$$= \frac{1}{kv} w_2' (B_1-B_1^*) \ .$$

$$= \frac{1}{kv^2} w_2'(B_2) \qquad (2.4.11)$$

Equations (2.4.11) can be then solved to obtain:

$$\frac{u'(c_1)}{u'(c_2)} = \frac{(1 + i)}{1-k.\frac{(l_1-l_2)^2}{l_1 \cdot l_2}} \qquad (2.4.12a)$$

$$\frac{w_2'(B_1-B_1^*)}{w_2'(B_2)} = (1 + i) \qquad (2.4.12b)$$

The solution of Equations (2.4.11) is especially interesting because, in certain circumstances, $\frac{u'(c_1)}{u'(c_2)}$ may be negative. In particular, if $k \geq 1$ is large or if l_2 is small relative to l_1 (ie. that the consumer has a low probability of surviving the first year) then $\frac{u'(c_1)}{u'(c_2)}$ could be negative.

It is difficult to visualise why this may be the case but it may be that, in order to make a bequest of the size denoted by Equations (2.4.8), the consumer must cut back on consumption to an extraordinary extent. Alternatively, Equation (2.4.12a) may be interpreted by assuming that the marginal rate of substitution $\frac{u'(c_1)}{u'(c_2)}$ should always be positive. Then, a side condition for bequests of the size $B_1 > B_1^*$ and $B_2 > B_2^*$ is that

$$\frac{k.(l_1 - l_2)^2}{l_1 \cdot l_2} < 1 \qquad (2.4.13)$$

Thus Equation (2.4.13) indicates that the consumer must have a sufficiently low probability of dying in Year One (ie. that l_2 is sufficiently large in relation to l_1) before a bequest of the size indicated by Equations (2.4.8) becomes feasible. Thus we can conclude that the elder of the two otherwise identical consumers is less inclined to make a saving-type bequest (ceteris paribus).

Table 2.4.1 gives the maximum age at which savings-type bequests will be made for various values of the loading factor k: that age obviously falls as k increases.

Table 2.4.1 Maximum Age at which Savings-type Bequests are Made.

k	Maximum Age
1	51
2	44
3	40
4	37
5	35

(Mortality rates from Institute of Actuaries (17))

The calculations for Table 2.4.1 assume that a policy-holder at inception age x, is certain to die before he attains age 109: this maximum lifetime is then divided into two equal periods of length $\frac{1}{2}(x + 109)$ which correspond to those of the preceding model. The above Table shows that, in practice, Equation (2.4.13) can impose a major constraint on the consumer's type of bequest. For example, if $k \geq 3$ then a policyholder exceeding age 40 cannot afford a bequest in excess of B^*_{75}; any bequest he then makes will provide a sum less than that needed to compensate his family for the financial loss resulting from his death.

In Section 2.6, more formal models of the lifetime allocation of income are constructed on a continuous-time basis. A number of simplifying assumptions will be made, the chief of which is that bequests will be made for protection purposes only (so that there is no element of non-lifetime saving): under these circumstances, the utility of bequest function $w_2(B)$ is identically equal to zero for all values of B. This assumption represents a significant departure from the usual statements of Yaari's 'Life Cycle' model (Yaari (33) and Moffet (23)) which instead assumes that $w_1(B) = 0$. The reasons for this departure have been developed in the preceding Sections.

2.5 The Vehicle of Bequest

For the purposes of this lifetime consumption model, it is convenient to initially assume that all bequests are made through the medium of an insurer. This assumption is obviously slightly unrealistic since most consumers (with a non-zero bequest motive) plan to leave personal assets such as their house, jewellery, bank accounts etc., in addition to any sums insured. Obviously, the vehicle for bequest varies according to the motives for bequest. For example, we would expect the consumer to be more risk averse (in relation to the amount of the bequest) when bequesting for protection purposes than for other motives. Thus if the consumer is bequesting for protection purposes, we would expect him to choose the least risky vehicle for bequest ie. the one that best guarantees the amount of his desired bequest (earning presumably a lower expected return). Consequently, temporary life insurance is the ideal vehicle for bequest from the protection point of view since:

a) it is 'risk-free' from the point of view that the planned bequest will always equal the actual bequest at any age (assuming no risk of insolvency for the insurer);

b) in comparison with, say, the banker the insurer is much cheaper when providing a guaranteed level of bequest (see Table 2.5.1);

c) the problem of 'time-scale' is automatically solved in the insurance case.

If, however, the consumer wishes to provide a bequest for non-protection purposes, then temporary life insurance may not necessarily be the best vehicle for that bequest since first, a better rate of return may be obtained from an institution that does not guarantee the amount of the bequest and second, the consumer may not want to take the risk that any money 'tied-up' with the insurer would be lost should he survive (this will be called the 'survival risk').

A similar type of situation may arise in connection with bequesting for protection purposes if the consumer is uncertain of the time-scale involved (for protection purposes) (see Section 2.3): there would then be the risk that money tied up in an insurance contract would be 'wasted' once the consumer reaches a certain age ($x + t^*$ in Section 2.4).

In order to analyse the effects of 'survival risk' on the behaviour of the consumer, a single one-period model has been developed to give an indication of the conditions under which the insurer will be preferred to the banker using the assumption that the consumer maximises his expected utility of bequest. It is assumed that the consumer is risk-neutral in relation to the amount of the bequest. Budget constraints and consumption possibilities will also be ignored. The resulting analysis is based on that of Professors M. Friedman and L.J. Savage (12) and is similar, although not identical, to that of P. Fortune (10).

We assume that the consumer at age x, has an uncertain lifetime and this is characterised by the probability $q_x (< 1)$ that he will die before attaining age $x + 1$. It will also be assumed that the consumer prefers to leave a large rather than a small bequest - therefore the utility of bequest function $w(B)$ has positive first difference.

w(B) represents the utility associated with the certain bequest B.

We now go on to give the consumer a choice of two alternatives, both providing certain bequests at the end of the year: a one-year temporary life insurance or a bank deposit. These will be termed Alternatives I and II respectively. Alternative I incorporates an element of 'survival-risk' in that the policyholder is not certain to die at age x - if he survives he will obtain no monetary return on his initial outlay. Alternative II is riskless in that the amount B_2 is not contingent on the death of the consumer.

Thus, if the consumer purchases the life insurance, his estate will either increase by the sum insured B_1 with probability q_x or remain unaltered with probability $1-q_x$. For this alternative the policyholder has to pay a premium equal to the actuarial present value of B_1 plus a loading to cover the insurer's expenses. This payment is then given by:

$$P = k_1.q_x.v.B_1 \tag{2.5.1}$$

where the loading $k_1 \geq 1$ and $v = (1+i)^{-1}$ where i is the rate of interest earned by both the life office and the banker.

If the consumer uses the second riskless Alternative and makes the same payment P, he will obtain an amount B_2 at the end of the year whether he dies or not. B_2 is given by:

$$B_2 = k_2.P.(1+i) \tag{2.5.2}$$

This equation assumes that the expenses loading charged by the banker is represented by $k_2 \leq 1$

Note that for reasonably young ages, B_1 is greatly in excess of B_2; alternatively, if B_1 and B_2 were constructed to be equal then the premium for the insurer would be much smaller than the payment to the banker. These points are best illustrated in the following Table 2.5.1 where k_1 and k_2 are assumed (for simplicity) to be equal to unity (see Equation (2.5.4)).

Table 2.5.1 A Comparison of Bequests provided by the Insurer (B_1) and the Bank (B_2) with equivalent outlay (P). (11)

Age	B_1(£)	B_2(£)
25	1000	0.7
50	1000	4.8
100	1000	412.3

In order to proceed along the lines suggested by Friedman and Savage, it is convenient at this point, to return to some notation described early in Section 2.2, in connection with the consumer's own subjective estimate of his mortality rate (which will be termed $\overset{+}{q}_x$). It is an instance well-known to actuaries that persons who have a shorter expectation of life (ie. higher mortality rates)

(11) The Mortality Rates are taken from the A1967/70 Tables for Assured Lives (17) (ultimate).

than average (independent of their medical condition) are aware of the fact from some sixth sense and, therefore, tend to prefer policies such as temporary life insurances or family income benefits where the sum insured is high for a given rate of premium.[12] Fischer (8) refers to such phenomenon as 'intimations of mortality' and R.D.C. Brackenridge (4) p.13, notes that:

> "it is not altogether surprising that an applicant (for life insurance) who has some reason to believe that he might be suffering from a mortal condition would wish to insure his life for as large a sum as possible at as little cost to himself provided that any defect could go undetected at medical examination".

The expected utility of the first alternative is given by: $\bar{w}(I) = \overset{+}{q}_x . w(B_1)$ and that of the second by: $\bar{w}(II) = w(B_2)$. If we let $\bar{B}_1$ be the actuarial expected value of the benefit in the first alternative then:

$$\bar{B}_1 = \overset{+}{q}_x . B_1 \qquad (2.5.3)$$

The relationship between B_1 and B_2 is obtained by solving Equations (2.5.1) and (2.5.2) to give:

$$B_2 = k_1 . k_2 . q_x . B_1 \qquad (2.5.4)$$

Thus from Equation (2.5.3)

$$\bar{B}_1 = \frac{\overset{+}{q}_x . B_2}{q_x . k_1 k_2} \qquad (2.5.5)$$

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(12) See for example, the Chartered Insurance Institute Tuition Service Manual No.61 "The Mathematical Basis for Life Insurance" Lesson VI, para. 12.

and so if $\frac{\overset{+}{q}_x}{q_x.k_1k_2} = 1$, the insurance is 'fair'. Otherwise, the insurance is unfair and $\bar{B}_1 > B_2$ if and only if $(\overset{+}{q}_x / q_x) > k_1.k_2$

Thus an immediate result is obtained to the effect that it is possible for a temporary life insurance to be actuarially fair even though it carries a positive loading. This is because of the offsetting effect of $\overset{+}{q}_x$.

Let B_1^* be the certain benefit equivalent to Alternative I so that:

$$w(B_1^*) = \bar{w}(I) = \overset{+}{q}_x.w(B_1) \tag{2.5.6}$$

Then if $B_1^* < \bar{B}_1$ the consumer is a 'survival' certainty preferrer (so that he prefers the banker) and is willing to pay up to $\bar{B}_1 - B_1^*$ to obtain certainty in preference to the risky situation. Similarly, if $B_1^* > \bar{B}_1$ the consumer is a survival risk preferrer and is willing to pay up to $B_1^* - \bar{B}_1$ for his life insurance contract.

Suppose initially that $B_1^* < \bar{B}_1$ (ie. the consumer is a certainty preferrer), then the insurer will only be used if the price to be paid for 'survival' certainty is too high ie. if $B_2 < B_1^*$ As has been seen already, since $\overset{+}{q}_x$ is the only variable, this situation is possible if and only if $\overset{+}{q}_x$ exceeds $k_1k_2q_x$ by some sufficiently large amount. Let $\overset{++}{q}_x$ be the value of $\overset{+}{q}_x$ that equates B_2 with B_1^*:

$$\text{ie.} \quad w(B_2) = \overset{++}{q}_x.w(B_1) \tag{2.5.7}$$

Then any value of $\overset{+}{q}_x > \overset{++}{q}_x$ will be sufficient to persuade the survival risk-averse consumer to purchase temporary life insurance.

Note that $\overset{++}{q}_x$ depends only on w, k_1, k_2, q_x and B_1 and of these, only w is at the discretion of the policy-holder. Also, since w is assumed to have positive first difference, any combination of q_x, k_1 and k_2 that produces $k_1 . k_2 . q_x \geq 1$ will imply that the survival risk averse consumer will always choose the banker, irrespective of his subjective mortality rate $\overset{+}{q}_x$. This situation would be very unusual however, because of the relatively small values of q_x except at high ages. Finally, we note that the larger the value of $\overset{+}{q}_x$, the more likely it becomes that the consumer will purchase insurance in preference to the banker.

If we suppose that there is now a situation of risk preference (ie. $B_1^* > \bar{B}_1$) then the consumer will only use the banker if the price he has to pay for risk is too high ie. if $B_2 > B_1^*$ This situation is only possible if

$\overset{+}{q}_x < \overset{++}{q}_x < k_1 . k_2 . q_x$

Choosing as an example, the particular utility of bequest function $w(B) = c.B^\beta$ where c is a constant and $0 < \beta < 1$ [13] then $\overset{+}{q}_x$ can be obtained from Equations (2.5.4) and (2.5.7) to give $\overset{++}{q}_x = (k.q_x)^\beta$

Results for selected values of β are set out in Table 2.5.2 below with mortality rates from the latest mortality table published by the Institute of Actuaries (17).

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[13] This type of utility function is also used for illustratory purposes by Fischer (8) and Moffet (22).

Table 2.5.2 A comparison of $\overset{++}{q}_x/q_x$ using the utility of bequest function $w(B) = cB^{\beta}$.

($k_1 = 1.5$, $k_2 = 1$)

	β	q_x	$\overset{++}{q}_x$	$\overset{++}{q}_x/q_x$
Age 20	0.25	0.00089	0.19112	214.9
	0.50	0.00089	0.03653	41.1
	0.75	0.00089	0.00698	7.8
Age 50	0.25	0.00479	0.29113	60.8
	0.50	0.00479	0.08454	17.7
	0.75	0.00479	0.02467	5.2
Age 100	0.25	0.41229	0.88680	2.2
	0.50	0.41229	0.78640	1.9
	0.75	0.41229	0.69738	1.7

The results in the above Table show that with the particular utility function, the ratio $\overset{++}{q}_x/q_x$ is very sensitive to changes in β : $\overset{++}{q}_x/q_x$ was found to be insensitive to changes in k_1k_2 within the range (1.5,3.0). As expected, high values of β which are associated with decreased survival risk aversion produce smaller values of $\overset{++}{q}_x/q_x$.

The theory developed for the one period model can quite easily be extended to an n-period analysis. In the n-period analysis Equations (2.5.1) and (2.5.2) become:

$$P = k_1.B_1.A'_{x:\overline{n}|}$$

and $$B_2 = k_2.P.(1+i)^m$$

where $A^{1}_{x:\overline{n|}}$ is the single pure premium on an n-period temporary life insurance and m denotes the number of years before the death of the consumer (unknown).

The n-period analysis shows that the survival risk-averse consumer is even more willing to choose the insurer than he was in the one period analysis. This situation is particularly noticeable when n is large in relation to m.

Finally, it must be pointed out that if the sum insured was sufficiently large for the insurer to suspect moral hazard (see Section 2.3) then it would be reasonable to suppose that the insurer would calculate the premium P using a rate of mortality closely corresponding to $\overset{+}{q}_x$. Thus Equation (2.5.1) would become:

$$P = k_1 \overset{+}{q}_x . v . B_1$$

and Equation (2.5.4) $B_2 = k_1 . k_2 . \overset{+}{q}_x . B_1$

Thus $\bar{B}_1$ becomes $\bar{B}_1 = {}^{B_2}/k_1 k_2$ and it is immediately obvious that if $k_1 . k_2 \geq 1$ the risk-averse consumer will never purchase insurance (since B_1 and B_2 cannot be varied).

2.6 An Anticipated Model of Lifetime Consumption, Saving and Bequest.

Borch (3), Yaari (33), (34), (35) and Moffet (23) have dealt quite comprehensively with the problem of consumer income allocation over time by using the continuous time model (pioneered by R.H. Strotz (30)) and applying the calculus of variations that was first introduced by F.P. Ramsey (27). Other authors such as Moffet (22), Hakansson (14), Fischer (8) and Richard (29) have chosen to use a discrete time model sometimes allied with dynamic programming techniques.[(14)]

In this section, a continuous-time model will be developed on the assumption that the constants y(t) can be anticipated with certainty at age x (ie. time zero) and the optimal level of variables c(t), B(t) remain fixed once they have been determined. I have called this an anticipated model of lifetime consumption.

In order to develop the anticipated model a number of (conventional) assumptions have to be made. Most of these assumptions are very much in line with those made by other writers on the subject although significant differences have been noted below.

First of all, it is assumed that the consumer's objective is to maximise expected utility from consumption and bequest.

Second, the force of interest δ is assumed known and constant throughout the consumer's lifetime. We also assume that the individual can lend and borrow at the same rate δ.

[(14)] See, for example, R. Bellman (2) and J. Mossin (24).

Third, the individual's potential future income stream is exogenously determined and is known with certainty at age x (ie. at time zero).

Fourth, it is assumed for simplicity that there are no transactions costs involved in the usage of either the banker or insurer (ie. that the k_1 and k_2 of the preceding Section both equal unity). The relaxation of this assumption would simply introduce these constants into the arithmetic in a very simple way and no benefit is gained from this complication.

We will need to use the following relationships (which were described in Section 2.2):

$$y(t) = c(t) + s(t) + b(t) \tag{2.2.1}$$

$$S(t) = e^{\delta t} . {}_0\int^t e^{-\delta\tau} .s(\tau)\, d\tau \tag{2.2.3}$$

$$S'(t) = \delta.S(t) + s(t) \tag{2.2.4}$$

$$S(0) = 0 \tag{2.6.1}$$

and since bequests (for protection purposes) are provided through the medium of an insurer, we have:

$${}_0\int^\infty e^{-\delta t}.l_{x+t}.b(t)\, dt = {}_0\int^\infty e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\, dt \tag{2.6.2}$$

(by the principle of equivalence, eg. see Borch (3)). Equation (2.6.2) can be rewritten by incorporating Equations

(2.2.1) and (2.2.4) to obtain

$$_0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\ dt \ = \ _0\int^{\infty} e^{-\delta t}.l_{x+t}.(\ y(t) - c(t) + \delta.S(t) - S'(t)\ )\ dt$$

and integrating $e^{-\delta t}.l_{x+t}.S'(t)$ by parts we obtain

$$_0\int^{\infty} e^{-\delta t}.l_{x+t}.S'(t)\ dt \ = \ -l_x.S(0) \ + \ _0\int^{\infty} S(t).(\ e^{-\delta t}.l_{x+t}\mu_{x+t} + \delta.l_{x+t}.e^{-\delta t}\ )\ dt$$

ie.

$$_0\int^{\infty} e^{-\delta t}.l_{x+t}.S'(t)\ dt \ = \ _0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.S(t)\ dt \ + \ \delta.{_0\int^{\infty}} S(t).l_{x+t}.e^{-\delta t}\ dt$$

since $S(0) = 0$.

Thus we obtain

$$_0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\ dt \ = \ _0\int^{\infty} e^{-\delta t}.l_{x+t}.(\ y(t) - c(t)\ )\ dt \ - \ _0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.S(t)\ dt$$

ie.

$$ {}_0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.(\ S(t)\ +\ B(t)\ )\ dt $$

$$ = {}_0\int^{\infty} e^{-\delta t}.l_{x+t}.(\ b(t)\ +\ S'(t)\ -\ \delta.S(t)\ )\ dt $$

(2.6.3)

If we follow the method of Borch (3), we find that the prospective reserve for this insurance is given by

$$ V(t) = e^{\delta t}.\ {}_t\int^{\infty} e^{-\delta\tau}.\frac{l_{x+\tau}}{l_{x+t}}.(\ \mu_{x+\tau}B(\tau)\ +\ \mu_{x+\tau}S(\tau)\ -\ b(\tau) $$

$$ -\ S'(\tau)\ +\ \delta.S(\tau)\ )\ d\tau $$

(2.6.4)

where $V(0) = 0$.

From which we obtain:

$$ V'(t) = (\delta + \mu_{x+t}).V(t) + b(t) + S'(t) - (\delta + \mu_{x+t}).S(t) $$

$$ -\ \mu_{x+t}B(t) $$

(2.6.5)

The next three Sections will be concerned with two types of Anticipated model: in the first type, we make the simplifying assumption that the consumer deposits his lifetime savings with a banker (say) and makes his bequest (for protection purposes only) through the sole medium of a life insurance contract. Consequently, the premium payments are synonymous with the bequesting flows $b(t)$ and the saving flows $s(t)$ represent lifetime saving only (as defined in Section 2.2). The second type of Anticipated model drops the constraint that $s(t)$ represents only lifetime saving (so that Equation (2.2.5) will no longer hold).

Thus the first Anticipated model type of Sections 2.7 and 2.8 effectively assumes that the consumer intends to have dissaved his entire savings stocks by the date of his death. Thus the consumer does not intend that any of his savings S(t) should remain after the time of his eventual death. In practice, this assumption seems open to criticism since savings stocks tied-up in consumer durables and particularly owner-occupied housing often remain unliquidated at the time of death. There are, however, a number of factors which can explain this phenomenon without invalidating the initial assumption. First of all, many elderly people do actually 'run-down' their savings stocks both by spending liquid assets and by moving into smaller houses. Secondly, we have already noted that savings stocks are sometimes retained for precautionary motives since the consumer does not know the date of his eventual death.[15] Thirdly, we may in fact be able to define stocks of property and consumer durables as 'fully depreciated' consumption - not savings stocks at all.

The first Anticipated model then proceeds by assuming that the consumer's objective is to maximise his expected lifetime utility from consumption and bequest. Thus the consumer must vary his consumption pattern c(t) and his bequests B(t) so as to maximise the function:

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(15) It has been demonstrated by Borch (3) and Moffet (22) that the consumer can avoid the need for precautionary saving by paying over his entire income to the insurer. This optimal solution then requires the insurer to provide a flow of money for consumption purposes and a bequest at each age: the consumer then has no need for further saving.

$$\bar{U}(c,B) = {}_0\int^{\infty} \left({}_0\int^{T} (\alpha(t)u(c(t)) + \beta(t)w(B(t)))dt \right) . \frac{\ell_{x+T}\mu_{x+T}}{\ell_x} dT$$

(2.6.6)

which can be simplified to produce

$$\bar{U}(c,B) = {}_0\int^{\infty} \frac{\ell_{x+t}}{\ell_x} . (\alpha(t)u(c(t)) + \beta(t)w(B(t))) dt$$

(2.6.7)

α and β may be defined as the consumer's subjective discount function for utility of consumption and bequest respectively. Thus α and β represent the consumer's 'impatience', that is his preference for early rather than later utility.(16) α is normally a decreasing function of time t, while Yaari (33) argues that β should be 'humped'.

No constraints need to be imposed on Equation (2.6.6) except that c be positive, bounded and measureable.(17) Note, however, that (2.6.6) implicitly assumes that consumers' preferences are independent over time.

If the standard Yaari 'Life Cycle' model had been used then, of course, Equations (2.6.6) and (2.6.7) would take the alternative form given by the following:

$$\bar{U}_Y(c,B) = {}_0\int^{\infty} \left({}_0\int^{T} \alpha(t)u(c(t)) dt + \beta(T)w(B(T)) \right) . \frac{\ell_{x+T}\mu_{x+T}}{\ell_x} dT$$

(2.6.6a)

which can be simplified to produce

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(16) Irving Fisher's concept of impatience is further explained in (9) Ch.IV.

(17) For a further description, see Yaari (34)

$$\bar{U}_Y(c,B) = {}_0\int^{\infty} \frac{\overset{+}{l}_{x+t}}{\overset{+}{l}_x}.(\ \alpha(t)u(c(t)) + \mu_{x+t}\beta(t)w(B(t))\ )\ dt \qquad (2.6.7a)$$

Equation (2.6.7) is then maximised subject to the constraint given, in Equations (2.2.1) - (2.2.4). Additionally, we impose the lifetime saving constraint given by Equation (2.2.5) .

ie.

$$\frac{1}{\overset{+}{l}_x}.{}_0\int^{\infty} e^{-\delta T}.S(T).\overset{+}{l}_{x+T}.\overset{+}{\mu}_{x+T}\ dT \ = \ 0 \qquad (2.2.5)$$

However if we substitute for $\overset{+}{l}_{x+T}$ and $\overset{+}{\mu}_{x+T}$ (using Equation (2.2.6) - so that $\overset{+}{l}_{x+T} = L(T).\overset{+}{l}_{x+T}$ and $\overset{+}{\mu}_{x+T} = \mu_{x+T} - L'(T)/L(T)$) we obtain:

$${}_0\int^{\infty} e^{-\delta T}.S(T).L(T).l_{x+T}.(\ \mu_{x+T} - L'(T)/L(T)\ )\ dT = 0 \qquad (2.6.8)$$

since $\overset{+}{l}_x$ is independent of T.

Equation (2.6.8) can be rewritten as

$${}_0\int^{\infty} e^{-\delta t}.S(t).L(t).l_{x+t}\mu_{x+t}\ dt = {}_0\int^{\infty} e^{-\delta t}.S(t).L'(t).l_{x+t}\ dt \qquad (2.6.9)$$

and a comparison with Equation (2.6.3) shows that the constraint given in Equation (2.2.5) can only be incorporated if L(t) = 1 for all t (ie. if $\overset{+}{l}_{x+t} = l_{x+t}$ for all t). Consequently, since in the first Anticipated model, we are especially concerned with lifetime saving, the generalised constraint (Equation (2.2.5)) must be dropped in favour of the simpler version:

$$\int_0^\infty e^{-\delta t}.S(t).l_{x+t}\mu_{x+t}\, dt = 0$$

(2.6.10)

The second type of Anticipated model drops the assumption that all saving is of the 'lifetime' variety. Consequently, we do not assume that the expected present value of future savings stocks is zero at time $t = 0$. Additionally the form of the consumer's expected lifetime utility function must be altered to allow for the fact that the consumer now gains satisfaction from unrealised savings stocks at the time of death. We continue to assume, however, that all bequests are for the purpose of protecting dependents against the financial consequences of premature death. The expected utility function is then of the following form:

$$\bar{U}(c,B) = \int_0^\infty \left(\int_0^T (\alpha(t)u(c(t)) + \beta(t)w(B(t)+S(t)))\, dt \right).\frac{\overset{+}{l}_{x+T}\overset{+}{\mu}_{x+T}}{\overset{+}{l}_x}\, dT$$

(2.6.11)

which can be simplified to produce

$$\bar{U}(c,B) = \int_0^\infty \frac{\overset{+}{l}_{x+t}}{\overset{+}{l}_x}.(\alpha(t)u(c(t)) + \beta(t)w(B(t)+S(t)))\, dt$$

(2.6.12)

By substituting into Equations (2.6.7) and (2.6.12) for $c(t)$ and $B(t)$ in terms of $b(t)$, $S(t)$, $S'(t)$, $V(t)$ and $V'(t)$ we can obtain an expression for $\bar{U}(c,B)$ of the following form:

$$\overline{U}(c,B) = \int_{t_2}^{t_1} f(t,S,V,S',V',b)\,dt$$

so that the problem can be solved using the Calculus of Variations. The solution to the problem must therefore satisfy the Euler Equations[18]:

$$\partial f/\partial S = d/dt\,(\,\partial f/\partial S'\,) \tag{2.6.13a}$$

$$\partial f/\partial V = d/dt\,(\,\partial f/\partial V'\,) \tag{2.6.13b}$$

and

$$\frac{\alpha(t)u'(c(t))}{\beta(t)w'(B(t))} = \frac{1}{\mu_{x+t}} \tag{2.6.13c}$$

Equation (2.6.13c) indicates that the marginal utility of consumption ($\alpha(t)u'(c(t))$) is equal to the marginal utility of bequest ($\beta(t)w'(B(t))$) multiplied by a factor of $1/\mu_{x+t}$ (which is >1). Furthermore, since μ_{x+t} is generally an increasing function of x (for example see (17)), Equation (2.6.13c) indicates a preference for consumption as inception age x or time t increases.

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(18) See R.G.D. Allen (1) Ch.XX
Note that $S'(t) = \frac{dS(t)}{dt}$ and $V'(t) = \frac{dV(t)}{dt}$

2.7 The Anticipated Model : Lifetime Saving Only

The simplest case for the solution maximising Equation (2.6.7) is obtained by making the following assumptions:

i) that $\tilde{l}_{x+t} = l_{x+t}$ for all t (see Equation (2.2.6)).

ii) Equation (2.6.10)

ie. $${}_0\int^{\infty} e^{-\delta t}.S(t).l_{x+t}\mu_{x+t}\,dt = 0$$

so that the expected discounted value of life-time savings is zero at death.

The above assumptions have a simplifying effect on Equation (2.6.3) to produce

$${}_0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\,dt = {}_0\int^{\infty} e^{-\delta t}.l_{x+t}.(\,b(t) + S'(t) - \delta.S(t)\,)\,dt$$

The prospective reserve for this insurance then gives

$$V'(t) = \{\delta + \mu_{x+t}\}\,.\,V(t) + b(t) + S'(t) - \delta.S(t) - \mu_{x+t}\,B(t) \qquad (2.7.1)$$

where $V(0) = 0$ (note that the terms V and V' are different from those used in Equation (2.6.5)).
Thus we can use Equations (2.2.1), (2.2.4) and (2.7.1) to rewrite Equation (2.6.7) as

$$\bar{U}(c,B) = {}_0\int^\infty \alpha(t).\frac{l_{x+t}}{l_x}.u(y(t)+\delta.S(t)-b(t)-S'(t))\,dt \; +$$

$${}_0\int^\infty \beta(t).\frac{l_{x+t}}{l_x}.w(1/\mu_{x+t}.((\delta+\mu_{x+t}).V(t)-V'(t)+b(t)+S'(t)-\delta.S(t)))\,dt$$

$$(2.7.2)$$

Equation (2.7.2) is maximised if the following Euler Equations are satisfied

$$\frac{l_{x+t}}{l_x}.(\alpha(t).u'(c(t)).\delta - \beta(t).w'(B(t)).\delta/\mu_{x+t}) =$$

$$\frac{d}{dt}\left(\frac{l_{x+t}}{l_x}.(-\alpha(t).u'(c(t)) + \beta(t).w'(B(t)).1/\mu_{x+t})\right) \qquad (2.7.3)$$

and

$$\frac{l_{x+t}}{l_x}.\ \beta(t).w'(B(t)).(1+\delta/\mu_{x+t}) = \frac{d}{dt}\left(\frac{-l_{x+t}}{l_x}.\beta(t).w'(B(t)).1/\mu_{x+t}\right)$$

$$(2.7.4)$$

Equations (2.7.3), (2.7.4) and (2.6.13c) produce an optimal solution satisfying:

$$c'(t) = \frac{u'(c(t))}{u''(c(t))}.\left(\frac{-\alpha'(t)}{\alpha(t)} - \delta\right) \qquad (2.7.5)$$

and

$$B'(t) = \frac{w'(B(t))}{w''(B(t))}.\left(\frac{\mu'_{x+t}}{\mu_{x+t}} - \frac{\beta'(t)}{\beta(t)} - \delta\right) \qquad (2.7.6)$$

There are several points of interest in Equations (2.7.5) and (2.7.6):

1) Equations (2.7.5) and (2.7.6) are independent in the sense that the former does not involve $B(t)$ and the latter does not involve $c(t)$. Thus the consumer can locally separate his consumption decisions from his bequest decisions;

2) There are an infinite number of ways in which the insurance premiums $b(t)$ may be arranged (because $s(t)$ and $b(t)$ are co-determinant). However we would normally expect institutional factors to constrain the $b(t)$ in some way (eg. $b(t) = b$ (a constant) for all t : in this case, a unique value for b can be obtained);

3) The inclusion of $\frac{\mu'_{x+t}}{\mu_{x+t}}$ in (2.7.6) prevents Equations (2.7.5) and (2.7.6) from being symmetrical in the sense described by Yaari (33). Thus (2.7.5) describes a different behaviour to (2.7.6);

The inclusion of the term $\frac{\mu'_{x+t}}{\mu_{x+t}}$ in (2.7.6) has several interesting repercussions[19]:

4) Since μ'/μ is positive for the majority of adult ages (all ages over 30 in (17)) it has the effect of slowing down the rate of increase (quickening

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[19] Note that the presence of this term is the only difference between the present results and those that would have been obtained using the Yaari 'Life Cycle' Equation (2.6.7a)

the rate of decrease) of the bequest plan;

5) However, for certain age ranges, μ'/μ may be negative (for example 0-12 and 17-29 in (17)). Consequently, if the inception age is say, in the 17-29 age range then, ceteris paribus, the bequest plan will initially increase before 'flattening out' later on. (Since w' is commonly assumed to be positive while w" is < 0);

6) For (17) there is some evidence that μ'/μ passes through a point of inflexion in 'middle age'. Consequently, after this point, the rate of decrease in the bequest plan (after accelerating initially) begins to decelerate (ceteris paribus).

The latter three points above indicate that the bequest plan will broadly tend to be 'humped' shaped: initially $\delta > (\mu'/\mu - \beta'/\beta)$ but as t increases, $(\mu'/\mu - \beta'/\beta)$ increases in size. (Note that β'/β will be positive for small t but getting smaller as t increases). However, the greater the inception age x, the more likely a downward sloping bequest plan becomes. In practical terms, this result makes more sense than that obtained by using the standard 'life cycle' model (where the bequest plan does not contain the term μ'/μ and is therefore independent of the inception age).[20] This is because increased

[20] We note that the standard life-cycle theory of consumption has been criticised (by L.C. Thurow (31), K. Nagatani (25), J. Heckman (15) and I Irvine (18)) because empirical studies indicate that household consumption over the lifetime is not a steadily increasing or decreasing function (as in Equation (2.7.5) but is 'humped' in the middle years.

protection is provided at younger ages where that protection is most needed (when uncertainty over the timing of death is greatest and when premature death would do most harm to dependents, eg. see L.C. Thurow (31) p.328:

> "There is no a priori method to determine when risk is highest. Risks may be higher when old since the probability of death and sickness is higher, but they might equally well be higher when young since fewer assets have been accumulated and there are more dependents who would suffer for a longer period of time.").

2.8 The Anticipated Model : Lifetime and Precautionary Saving.

In this model we keep the assumption that $\bar{l}_{x+t} = l_{x+t}$ but introduce a new second assumption to deal with the problem of precautionary saving mentioned in Section 2.2, ie.

i) $\bar{l}_{x+t} = l_{x+t}$ for all t

ii) Equation (2.2.7) ie. $\int_0^\infty e^{-\delta t}.l_{x+t}\mu_{x+t}.S(t)\,dt = A$

iii) A is totally independent of time t.

Again the assumptions have a simplifying effect on Equation (2.6.3) to produce:

$$\int_0^\infty e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\,dt + A = \int_0^\infty e^{-\delta t}.l_{x+t}.(b(t)+S'(t)-\delta.S(t))\,dt$$

The prospective reserve for this insurance then gives:

$$V'(t) = (\delta+\mu_{x+t}).V(t) + b(t) + S'(t) - \delta.S(t) - \mu_{x+t}.B(t) - A.(\delta+\mu_{x+t}-1) \qquad (2.8.1)$$

where $V(0) = -A$.

Following the method of Section 2.7, Equation (2.8.1) can be used to derive an expression for B(t) which can then be used to rewrite Equation (2.6.7). Maximisation of $\bar{U}(c,B)$ then produces the same Euler Equations as before (ie. (2.7.3) and (2.7.4)) because the coefficients of S(t),

$S'(t)$, $V(t)$ and $V'(t)$ are identical for Equations (2.7.1) and (2.8.1).

Since Equations (2.7.5) and (2.7.6) are independent, this implies that the optimal consumption pattern in this model will be identical to that of Section 2.7. It is immediately obvious (by comparing (2.7.1) and (2.8.1)) that the consumer makes a smaller bequest for each age in this model although the pattern of bequests (as dictated by Equations (2.7.6)) will be identical.

There are other points of interest in Equation (2.8.1):

1) the amount by which the bequest in Section 2.7 exceeds that in this Section is not constant but depends on the age of the consumer and the force of interest δ ;

2) the greater the value of A, the smaller the bequest;

In one sense, the inclusion of Precautionary saving in this Chapter is unnecessary since the consumer's future income stream is assumed to be known with certainty (ie. the consumer behaves as if future income were known with certainty) although, of course, the time of death is unknown. However it has been argued that uncertainty about future price levels produces precautionary saving on the part of the consumer: consequently the results of this Section will be needed in later Chapters.

The results of this Section are also important in another respect. The first Anticipated model of this Chapter makes the assumption that bequests planned by the consumer can only be provided by life insurance. Consequently 'lifetime savings' are defined as those savings destined for future spending. Thus, there seems to be no room in the model for the purchase of consumer durables which would be expected to survive the consumer (and would not be sold to finance future consumption). This Section suggests two ways of circumventing the problem:

a) to treat the purchase of consumer durables as consumption. This contradicts the Friedman (11) and Modigliani (21) definition of consumption which includes only the real consumption of goods and services rather than monetary expenditures; durables are consumption expenditures only to the extent that they are depreciated in a particular period, not the amount spent for their acquisition. However, this treatment is in line with the description of PIA saving in Chapter Five (which is described as more akin to consumption than financial saving). Thus it could be argued that the consumer has no positive intention of saving when he purchases a durable: he is constrained to make the purchase in order to enjoy the services that the durable will yield; or

b) if the purchase of durables must be considered as saving then these savings stocks will remain after the consumer's death (and be represented by the quantity A). However, these durables do <u>not</u> yield the consumer any utility of bequest and can, therefore, be treated in the same way as precautionary saving. Thus, the interesting conclusion to be drawn is that the greater the accumulation of consumer durables (ie. the larger the value of A) the smaller the level of any bequest: this result ties in exactly with Fortune's (10) Theorem 4.

2.9 The Anticipated Model : The Generalised Case.

This Section illustrates the completely generalised model with no restrictions on the behaviour of ℓ_{x+t} or on lifetime savings.

The Equation (2.6.4) is used without any modifications to obtain Equation (2.6.5) and hence an expression for B(t). Then B(t) and c(t) can be substituted in Equation (2.6.12). Maximisation of (2.6.12) produces the pair of simultaneous Euler Equations:

$$\ell_{x+t}.(\alpha(t).\delta.u'(c(t)) - \beta(t).w'(B(t)+S(t)).\delta/\mu_{x+t}) =$$

$$\frac{d}{dt}(\ell_{x+t}.(-\alpha(t).u'(c(t)) + \beta(t).w'(B(t)+S(t)).1/\mu_{x+t})) \tag{2.9.1}$$

and

$$\ell_{x+t}.\beta(t).w'(B(t)+S(t)).(1+\delta/\mu_{x+t}) = \frac{d}{dt}(-\ell_{x+t}.\beta(t).w'(B(t)+S(t)).1/\mu_{x+t}) \tag{2.9.2}$$

Solving Equations (2.9.1) and (2.9.2) produces an optimal solution of the form[21]:

$$S'(t) + B'(t) = \frac{w'(B(t)+S(t))}{w''(B(t)+S(t))}.\left(\frac{\mu'_{x+t}}{\mu_{x+t}} - \frac{\beta'(t)}{\beta(t)} - \delta - \frac{L'(t)}{L(t)}\right) \tag{2.9.3}$$

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[21] See Appendix 2.1 for a proof of this result.

and

$$c'(t) = \frac{u'(c(t))}{u''(c(t))} \cdot \left(-\delta - \frac{\alpha'(t)}{\alpha(t)} - \frac{L'(t)}{L(t)} \right) \tag{2.9.4}$$

It is interesting to examine the effects of the subjective rate of mortality by examining Equation (2.9.3): if L'/L is positive this will accentuate the 'humped' shape of the bequest plan (ie. defer the decline of the plan to a later age/time) while if L'/L is negative, the bequest plan will start to decline at an earlier time. Now we can associate a negative figure for L'/L with the consumer who is pessimistic about the course of his future mortality: he will therefore concentrate his consumption and bequests towards the younger ages.

Several authors have noted that institutional constraints generally prevent an individual from dying with negative net worth (eg. see Yaari (33) and Moffet (23)). Consequently, since our lifetime savings constraint is no longer applicable, the constraint that $(S(t) + B(t)) \geq 0$ should be imposed. Moffet has shown that this constraint implies the existence of a local constraint on borrowing and the same argument applies in this case.

Before we progress to the Unanticipated Models, it is constructive to point out that the income stream $y(t)$ was assumed to be known in advance for all time at $t = 0$. While this assumption is necessary in this theory to avoid undue complications, it does have some connection with economic theory as well. In particular the assumption that $y(t)$ is known in advance is a good approximation to the Permanent Income Hypothesis of Professor Milton

Friedman (11). Essentially the theory of Sections 2.2 to 2.9 treats all income as permanent income (with no transitory income component). Consequently, all consumption expenditures must also be regarded as permanent.

A central hypothesis of the Friedman theory is that that proportion of permanent income saved by the consumer unit is independent of its income in a particular period. This fits in very well with the results developed so far.

2.10 An Unanticipated Model of Consumption, Lifetime Saving and Bequest.

For the purposes of the anticipated model, it was assumed that $y(t)$ could be predicted with certainty at time zero and that the original level of variables c and B would remain fixed once determined. In the unanticipated model it is assumed that the consumer experiences a once-and-for-all change of circumstances at time τ that could not have been foreseen at time zero. For simplifying purposes, circumstances are assumed to alter once and once only. The consumer therefore has to reconsider his consumption and bequest decisions at time τ where the same assumptions as before (Section 2.6) apply. The only complication involves the surrender value of any life insurance that the consumer contracted at time zero. Again for simplification only changes of circumstances directly involving the possible surrender of the life insurance contract are considered. Thus at time τ , Equations (2.2.1) and (2.2.4), (2.6.2) and (2.6.3) must be rewritten:

$$y_\tau(t) = c_\tau(t) + s_\tau(t) + b_\tau(t) \qquad (2.10.1)$$

$$S'_\tau(t) = \delta . S_\tau(t) + s_\tau(t) \qquad (2.10.2)$$

$$\int_\tau^\infty e^{-\delta t} . l_{x+t} . b_\tau(t)\, dt = \int_\tau^\infty e^{-\delta t} . l_{x+t} \mu_{x+t} . B_\tau(t)\, dt \qquad (2.10.3)$$

for $t \geq \tau$

The reasons why the consumer may wish to surrender his existing life insurance are discussed in more detail in Chapter Nine; however, they can be summarised under three main headings:

1) The policyholder has no further use for his life insurance.

2) A cash lump sum is required for consumption.

3) To avoid the expenditure on premiums,

The three reasons can be summarised, using utility function notation as follows:

Case One : $w(B(t)) = 0$ for all $t \geq \tau$

Case Two : $S_\tau(\tau) = S(\tau) + SV$ where SV is the surrender value.

Case Three : $y_\tau(t) \leq y(t)$ for all $t \geq \tau$

It is immediately obvious that unless the Surrender Value is very large and negative, Case One above implies that the consumer will immediately cease to make any further bequesting payments, ie.

$$b_\tau(t) = 0 \text{ for all } t \geq \tau$$

However, before the two remaining cases are discussed, some further evaluation of the Surrender Value SV is necessary.

The Surrender Value at time τ is a direct function of the retrospective reserve of the insurance contract after τ years.

This is given by:

$$V(\tau) = {}_0\int^{\tau} e^{-\delta t}.(l_{x+t}.b(t) - l_{x+t}\mu_{x+t}.B(t))\, dt \tag{2.10.4}$$

so that the Surrender Value is given by

$$SV = a.V(\tau) + g$$

where

$$0 < a \leq 1 \quad \text{and} \quad g \leq 0 \tag{2.10.5}$$

If $a = 1$ and $g = 0$ in Equation (2.10.5), the position of the consumer at time τ could remain unchanged should he surrender. This results from the fact that if the Surrender Value was 'ploughed back' into bequesting the consumer could still continue with his original bequesting plan, ie.

$$V(\tau) + {}_{\tau}\int^{\infty} e^{-\delta t}.l_{x+t}.b(t)\, dt = {}_{\tau}\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\, dt$$

implies

$${}_0\int^{\infty} e^{-\delta t}.l_{x+t}.b(t)\, dt = {}_0\int^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\, dt$$

ie. the original bequesting plan is attainable. But if $a \neq 1$ and $g \neq 0$ then the old bequesting plan is not available and furthermore it is more than probable that the level of bequests originally planned (ie. $B(t)$ for $t \geq \tau$) cannot be maintained by the original level of planned bequesting flows (ie. $b(t)$; $t \geq \tau$)

ie.

$$\int_{\tau}^{\infty} e^{-\delta t}.l_{x+t}.b(t)\ dt \leq \int_{\tau}^{\infty} e^{-\delta t}.l_{x+t}\mu_{x+t}.B(t)\ dt$$

Thus, because the consumer had originally entered into a contractual arrangement with the insurer, he is to some extent tied to his original consumption and bequest plans (R.H. Strotz (30) has referred to similar actions as 'strategies of precommitment').

2.11 The Unanticipated Model : Case Two

In Case Two, the consumer's utility of consumption is either up-valued at time τ so that $u_\tau(c(t)) > u(c(t))$ or the consumer's impatience function $\alpha(t)$ changes (for $t \geq \tau$). Thus the consumer must readjust his optimal consumption and bequest plans. Furthermore, the Surrender Value paid on the old insurance arrangements acts as an additional incentive for surrender.

The Case Two Equations can be stated as:

$$y(t) = c_\tau(t) + s_\tau(t) + b_\tau(t) \qquad (2.11.1)$$

$$S'_\tau(t) = \delta . S_\tau(t) + s_\tau(t) \qquad (2.11.2)$$

$$S_\tau(\tau) = S(\tau) + SV \qquad (2.11.3)$$

$$\int_\tau^\infty e^{-\delta t} . \frac{l_{x+t}\mu_{x+t}}{l_{x+\tau}} . S_\tau(t)\, dt = 0 \qquad (2.11.4)$$

and

$$\int_\tau^\infty e^{-\delta t} . l_{x+t} . b_\tau(t)\, dt = \int_\tau^\infty e^{-\delta t} . l_{x+t}\mu_{x+t} B_\tau(t)\, dt \qquad (2.11.5)$$

for $t \geq \tau$.

In the case of a change in u , we seek to maximise the expression

$$\bar{U}_\tau(c,B) = {}_\tau\int^\infty \alpha(t).\frac{l_{x+t}}{l_{x+\tau}}.u_\tau(c_\tau(t))\, dt + {}_\tau\int^\infty \beta(t).\frac{l_{x+t}}{l_{x+\tau}}.w(B_\tau(t))\, dt \tag{2.11.6}$$

for $t \geq \tau$.[22]

Then we can proceed in the same manner as that used to derive Equation (2.6.3) to obtain (from Equation (2.11.5)):

$$ {}_\tau\int^\infty e^{-\delta t}.l_{x+t}.(b_\tau(t)+S'_\tau(t)-\delta.S_\tau(t))\, dt + e^{-\delta\tau}.l_{x+\tau}.S_\tau(\tau)$$

$$= {}_\tau\int^\infty e^{-\delta t}.l_{x+t}\mu_{x+t}.(S_\tau(t)+B_\tau(t))\, dt$$

Incorporating Equation (2.11.4), we get

$$ {}_\tau\int^\infty e^{-\delta t}.l_{x+t}\mu_{x+t}.B_\tau(t)\, dt = {}_\tau\int^\infty e^{-\delta t}.l_{x+t}.(b_\tau(t)+S'_\tau(t)-\delta.S_\tau(\tau))\, dt + e^{-\delta\tau}.l_{x+\tau}.S_\tau(\tau) \tag{2.11.7}$$

The prospective reserve for this insurance is given by:

$$V_\tau(t) = e^{\delta t} . {}_t\int^\infty e^{-\delta z}.\frac{l_{x+z}}{l_{x+t}}.(\mu_{x+z}.B_\tau(z) - b_\tau(z) - S'_\tau(z) + \delta.S_\tau(z))\, dz - e^{-\delta\tau}.l_{x+\tau}.S_\tau(\tau) \tag{2.11.8}$$

for $t \geq \tau$.

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(22) It should be noted that this behaviour is basically different from the myopia propounded by Strotz (30). In particular, Strotz makes α (and by implication β) a function of $(t-\tau)$: no benefit is to be derived from an extension in this case.

and from Equation (2.11.8), we obtain by differentiation

$$V'_\tau(t) = (\delta+\mu_{x+t}).V_\tau(t) + b_\tau(t) + S'_\tau(t) - \delta.S_\tau(t)$$

$$- \mu_{x+t}.B_\tau(t) + (\delta+\mu_{x+t}+1).e^{-\delta\tau}.l_{x+t}.S_\tau(\tau) \qquad (2.11.9)$$

Now Equations (2.11.9) and (2.7.1) are identical so far as the coefficients of V, V', S, S' and b are concerned and so the calculus of variations (which maximises $\bar{U}_\tau(c,B)$ will produce Euler Equations analogous to Equations (2.7.5) and (2.7.6) (for $t \geq \tau$)
ie.

$$c'_\tau(t) = \frac{u'_\tau(c_\tau(t))}{u''_\tau(c_\tau(t))} . (-\alpha'(t)/\alpha(t) - \delta) \qquad (2.11.10)$$

and

$$B'_\tau(t) = \frac{w'(B_\tau(t))}{w''(B_\tau(t))} . (\mu'_{x+t}/\mu_{x+t} - \beta'(t)/\beta(t) - \delta) \qquad (2.11.11)$$

Finally, in order to decide whether to surrender his existing insurance arrangements the consumer must compare the maximised value of $\bar{U}_\tau(c,B)$ so obtained from Equation (2.11.6) with the value of expected lifetime utility at time τ had the existing insurance arrangements been continued but with the new instantaneous utility functions u_τ (to be termed $U^*_\tau(c,B)$).

Thus the consumer will surrender if $\bar{U}_\tau > U^*_\tau$ where

$$U^*_\tau(c,B) = {}_\tau\int^\infty \alpha(t).\frac{l_{x+t}}{l_{x+\tau}}.u_\tau(c^*(t))\,dt + {}_\tau\int^\infty \beta(t).\frac{l_{x+t}}{l_{x+\tau}}.w(B(t))\,dt$$

$B(t)$ is the optimal bequest plan, determined at time zero (age x) by solving Equation (2.7.6) and $c^*(t)$ is the balance available for consumption (in the new circumstances) after meeting the insurance premiums for the existing bequest plan allocated in an optimal way.

Although it is not possible to proceed any further with the comparison of $\bar{U}_\tau$ and U^*_τ, the following points can be made:

1) The surrender does not alter the basic pattern of consumption and bequest;

2) As expected, the larger the Surrender Value, the greater the value of $\bar{U}_\tau$ (substitute $B_\tau(t)$ from Equation (2.11.9) into (2.11.3) and then into (2.11.6)) and hence the greater the probability of Surrender;

3) Any deterioration in the health of the consumer between ages x and $x+\tau$ will make the new insurance arrangements more expensive (ie. make the b's larger in relation to the B's);

4) In any case, unless the $b(t)$, $t \geq 0$ are monotonically increasing, Equation (2.11.5) will produce smaller levels of bequest per unit premium than Equation (2.6.3) for time $t \geq \tau$

(because of the effects of mortality);

5) Because the maintenance of the original bequest also involves $b(t)$, $t \geq \tau$ remaining unchanged this severely limits the extent to which income can be redistributed towards consumption. Consequently, surrender is quite likely to occur unless only a redistribution of consumption is required: in this case u would remain unchanged but $\alpha(t)$ would alter.

2.12 The Unanticipated Model : Case Three

In Case Three the only change of circumstances at time τ is a downward revaluation of the future income stream $y_\tau(t) \leq y(t)$ for all $t \geq \tau$. Neither the utility of consumption nor bequest functions are altered.

Thus the Case Three Equations can be formulated as

$$y_\tau(t) = c_\tau(t) + s_\tau(t) + b_\tau(t) \qquad (2.12.1)$$

$$S'_\tau(t) = \delta . S_\tau(t) + s_\tau(t) \qquad (2.12.2)$$

$$S_\tau(\tau) = S(\tau) + SV \qquad (2.12.3)$$

$$\int_\tau^\infty e^{-\delta t} . l_{x+t} \mu_{x+t} . S_\tau(t)\, dt = 0 \qquad (2.12.4)$$

$$\int_\tau^\infty e^{-\delta t} l_{x+t} . b_\tau(t)\, dt = \int_\tau^\infty e^{-\delta t} . l_{x+t} \mu_{x+t} . B_\tau(t)\, dt \qquad (2.12.5)$$

and

$$\bar{U}_\tau(c,B) = \int_\tau^\infty \alpha(t) . \frac{l_{x+t}}{l_{x+\tau}} . u(c_\tau(t))\, dt + \int_\tau^\infty \beta(t) . \frac{l_{x+t}}{l_{x+\tau}} . w(B_\tau(t))\, dt$$

The solution maximising $\bar{U}_\tau$ yields the same Euler Equations as before (Equations (2.11.10) and (2.11.11)) but before comparing $\bar{U}_\tau$ and U^*_τ, some further assumptions must be made about the existing insurance contract.

Suppose that the existing arrangements (undertaken at time zero) were those described in Section 2.7 but with the additional constraint that the insurance premiums $b(t)$ are constant throughout the consumer's lifetime (ie. $b(t) = b$ for all t).

The original optimal consumption and bequest plans were obtained by integrating Equations (2.7.5) and (2.7.6) ie.

$$c(t) = {}_0\int^t \frac{u'(c(z))}{u''(c(z))} . \left(-\alpha'(z)/\alpha(z) - \delta \right) dz \tag{2.12.6}$$

$$B(t) = {}_0\int^t \frac{w'(B(z))}{w''(B(z))} . \left(\mu'_{x+z}/\mu_{x+z} - \beta'(z)/\beta(z) - \delta \right) dz \tag{2.12.7}$$

Now, from Footnote (2), the original lifetime savings constraint can be written as:

$${}_0\int^\infty e^{-\delta t}.l_{x+t}.s(t)\, dt = 0$$

and by substitution, we get

$${}_0\int^\infty e^{-\delta t}.l_{x+t}.(y(t)-c(t))\, dt = b.\ {}_0\int^\infty e^{-\delta t}.l_{x+t}\, dt \tag{2.12.8}$$

Since $y(t)$ and $c(t)$ are known for all t we can therefore derive the unique value for b.

The maximised value for $\bar{U}_T$ must then be compared with that of U^*_T where

$$U^*_\tau = \int_\tau^\infty \alpha(t) . \frac{l_{x+t}}{l_{x+\tau}} . u(c^*_\tau(t))\, dt + \int_\tau^\infty \beta(t) . \frac{l_{x+t}}{l_{x+\tau}} . w(B(t))\, dt$$

(2.12.9)

$B(t)$ is obtained from Equation (2.12.7) and $c^*_\tau(t) = y_\tau(t) - s_\tau(t) - b$ (where b is determined by Equation (2.12.8)) allocated in some optimal way but constrained by Equation (2.12.4) and $c^*_\tau(t) \geq 0$ for all t.[23]

The obvious conclusions to be drawn from this analysis are that, first, if $b \geq y_\tau(t)$ for all $t \geq \tau$ then surrender of the existing insurance arrangements is the only alternative. Secondly, the larger the existing premium (b) the more likely surrender becomes in this case (although a large value of b will be associated with a large value of SV). Thirdly, as in the previous case, the decision to surrender obviously exhibits some 'stickiness' since $\bar{U}_\tau$ must exceed U^*_τ before such a decision would be taken.

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[23] Maximisation of U^*_τ by varying $c^*_\tau(t)$ is just a simple business because the second term in Equation (2.12.9) remains fixed.

2.13 Conclusion

This Chapter has attempted to examine consumers' motives for the purchase of protection-based life insurance. We have seen that the consumer's disposable income can be divided into three elements - consumption, lifetime saving and bequesting flow. Bequesting flow contains both consumption and savings elements but for the purposes of this Chapter - where most bequests are made via the purchase of protection-based life insurance - bequesting flow and therefore the life insurance premium is taken to be analogous to consumption.

Section 2.5 examines the consumer's choice for the vehicle of bequest. In particular, he is assumed to be able to choose either a life insurance policy or the services of a banker. His actual choice will depend on a number of factors including the 'survival risk' (which reduces as the policy term increases) and his subjective estimate of mortality. The longer the consumer's lifetime horizon, the more likely he is to use life insurance as the preferred bequesting method.

Sections 2.6 to 2.12 (with the exception of 2.9) then assume that all bequests are made via the purchase of life insurance. Furthermore these bequests are for the purpose of protecting dependents against the financial risks of premature death (Sections 2.4 and 2.9 briefly examine the situation if this is not the case).

The shape of the bequest plan - as discussed in Sections 2.6 to 2.9 - is affected by a number of factors including preferences, the force of interest, age, impatience and the passing of time. In general, we might expect the bequest plan to be 'humped' in shape although the greater the inception age x, the more likely a downward sloping bequest plan becomes.

The purchase of consumer durables - which might be expected to survive the consumer and therefore contribute to his bequest - was analysed in Section 2.8. The analysis predicted that the greater the accumulation of consumer durables, the smaller the level of any bequest (although the general shape of the bequest plan remained unaltered).

Section 2.9 examines the effect of the consumer's subjective estimate of mortality and reveals that a consumer who is pessimistic about the course of his future mortality will concentrate his bequests towards the younger ages.

The remaining Sections 2.10 to 2.12 attempted to analyse what the consumer would do if, at some time τ, a once-and-for-all change of circumstances occurred which could not have been foreseen at time zero. Although the result in part depends on the nature of the change in circumstances, the analysis reveals that there is always a certain 'stickiness' - in that a change of circumstances does not automatically imply that the consumer will surrender his life insurance arrangements. The larger the Surrender Value paid, the more likely is the consumer to Surrender.

Appendix 2.1 : A Derivation of Equations (2.10.3) and (2.10.4).

From Equation (2.6.5) we obtain

$$\mu_{x+t}.B(t) = (\delta+\mu_{x+t}).V(t) - V'(t) + b(t) + S'(t) - (\delta+\mu_{x+t}).S(t)$$

Consequently from Equations (2.6.7) we have the Euler Equations (2.9.1) and (2.9.2).

Additionally we note that if $\overset{+}{l}_{x+t} = L(t).l_{x+t}$ then

$$\overset{+}{\mu}_{x+t} = \mu_{x+t} - L'(t)/L(t) \tag{A2.1.1}$$

Then using the notation of Equation (2.6.13) we note that

$$\partial f/\partial S = \frac{d}{dt}(\partial f/\partial S') = \frac{d}{dt}\,\overset{+}{l}_{x+t}.\alpha(t).u'(c(t)).(-1) \quad +$$

$$\frac{d}{dt}\,\overset{+}{l}_{x+t}.\beta(t).w'(B(t)+S(t)).1/\mu_{x+t}$$

But

$$\frac{d}{dt}\,\overset{+}{l}_{x+t}.\beta(t).w'(B(t)+S(t)).1/\mu_{x+t} = -\frac{d}{dt}(\partial f/\partial V') = -\partial f/\partial V$$

So

$$\partial f/\partial S + \partial f/\partial V = \frac{d}{dt}\left(-\overset{+}{l}_{x+t}.\alpha(t).u'(c(t)) \right) \tag{A2.1.2}$$

However

$$\partial f/\partial S + \partial f/\partial V = \overset{+}{l}_{x+t}.\alpha(t).\delta.u'(c(t)) + \overset{+}{l}_{x+t}.\beta(t).w'(B(t)+S(t)) \tag{A2.1.3}$$

Equating Equations (A2.1.2) and (A2.1.3), we have

$$\overset{\dagger}{l}_{x+t}.\alpha(t).\delta.u'(c(t)) + \overset{\dagger}{l}_{x+t}.\beta(t).w'(B(t)+S(t)) =$$

$$- \overset{\dagger}{l}_{x+t}.u'(c(t)).(\alpha'(t) - \alpha(t).\overset{\dagger}{\mu}_{x+t}) - \overset{\dagger}{l}_{x+t}.\alpha(t).u''(c(t)).c'(t)$$

Substituting in Equation (A2.1.1) and simplifying we get

$$\beta(t).w'(B(t)+S(t)) = u'(c(t)).(-\delta.\alpha(t) - \alpha'(t) + \mu_{x+t}.\alpha(t) -$$

$$\alpha(t).L'(t)/L(t)) - \alpha(t).u''(c(t)).c'(t)$$

and dividing through by $\alpha(t)$

$$\beta(t).w'(B(t)+S(t))/\alpha(t) = u'(c(t)).(\mu_{x+t} - \delta - \alpha'(t)/\alpha(t) - L'(t)/L(t))$$

$$- u''(c(t)).c'(t)$$

Then noting from Equation (2.6.13c) that

$$\beta(t).w'(B(t)+S(t)) = \mu_{x+t}.\alpha(t).u'(c(t))$$

we get

$$c'(t) = \frac{u'(c(t))}{u''(c(t))} . (- \delta - \alpha'(t)/\alpha(t) - L'(t)/L(t))$$

Appendix 2.2 A summary of Notation

Section 2.2

x	The Inception Age of the Consumer (at time $t = 0$).
l_{x+t}	The number of people alive at exact age $(x+t)$ according to a particular mortality table.
T	The time at death (a stochastic variable).
μ_{x+t}	The 'force' of mortality.
$y(t)$	Disposable Income at time t.
$c(t)$	Consumption at time t.
$s(t)$	Lifetime Saving at time t.
$b(t)$	Bequesting Flow at time t: later assumed to be equivalent to the life insurance premium paid.
$S(t)$	Accumulated savings stocks at time t.
$S'(t)$	The change in $S(t)$.
δ	The continuously compounded 'force of interest'.
$\hat{l}_{x+t} = L(t).l_{x+t}$	The consumer's subjective version of
B	The amount of the bequest.
$u(c)$	The instantaneous utility of consumption.
$w(B)$	The instantaneous utility of bequest.
$\overline{U}(c,B)$	The expected utility of the life-time consumption and bequest.

Section 2.3

$B^*(t)$	The upper bound on bequests for protection purposes.

Section 2.4

$w_1(B)$	Utility function for bequests for protection purposes.
$w_2(B)$	Utility function for bequests for savings purposes.

Section 2.5

q_x	Probability of death between ages x and x+1.
$w(B)$	Utility of bequest function.
B_1	Sum insured (Bequest via Life Insurance - one year temporary).
B_2	Amount of bequest via bank deposit for equivalent outlay to life insurance premium.
P	Life Office premium on one year temporary policy.
$k_1 \geq 1$	Loading for Life Office expenses.
$k_2 \leq 1$	Loading for Banker's expenses.
$\overset{+}{q}_x$	Consumer's subjective version of q_x.
$\bar{w}(I)$	Expected utility in 'Life Insurance' Alternative.
$\bar{w}(II)$	Expected utility in 'Banker' Alternative.
$\bar{B}_1$	Expected value of sum insured (B_1).
B_1^*	Certain benefit equivalent of 'Life Insurance' Alternative, ie. $w(B_1^*) = \bar{w}(I)$.
$\overset{++}{q}_x$	That value of $\overset{+}{q}_x$ that equates B_2 with B_1^* ie. $w(B_2) = \overset{++}{q}_x w(B_1)$
$v = \frac{1}{1+i}$	The discount factor.

Section 2.6

Largely Section 2.2 with the addition of:

$\alpha(t)$	Consumer's subjective discount function ('impatience') for the utility of Consumption.
$\beta(t)$	Consumer's subjective discount function for the utility of Bequest.

$B(t)$	The amount of the Bequest at time t (for protection purposes only solely through the purchase of Life Insurance).
$V(t)$	The prospective reserve for the Life Insurance at time t.
$V'(t)$	The change in $V(t)$ $(= \frac{d}{dt} V(t))$

Section 2.10

	The subscript τ ('tau')represents the value of the article after the once-and-for-all change at time τ Thus eg.
$y_\tau(t)$	Disposable income at time t for $t \geq \tau$

Section 2.11

$\overline{U}_\tau(c,B)$	The expected utility of lifetime consumption and bequest after the once-and-for-all change in circumstances at time τ
$U^*_\tau(c,B)$	The expected lifetime utility at time τ, given the new arrangements at that time, but still under the 'old' (ie. at time zero) pattern of bequest with consumption arranged in a new optimal fashion.

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CHAPTER THREE : INFLATION AND PROTECTION-BASED LIFE INSURANCE

3.1 Introduction

Before the effects of inflation on the demand for life insurance can be quantitatively examined, it is of interest to investigate the theoretical effects of inflation on the purchase of life insurance bought for protection purposes. Consequently, this Chapter will use the theory developed in Chapter Two to analyse the effects of inflation expectations on the consumption element of life insurance. (The effect of inflation on saving will be surveyed in Chapter Four). Chapter Three is not concerned with the formulation of a consumer's price or inflation expectations (these are examined in greater detail in Chapter Seven): we begin with the assumption that the rational purchaser of life insurance will make some kind of estimate of future price levels (even if the eventual estimate turns out to be 'no change').

Section 3.2. describes the difference between 'anticipated' and 'unanticipated' inflation and develops some of the notation that will be used in later Sections. The concept of 'money illusion' is introduced although it will be explained in greater detail in Chapter Four.

Section 3.3. attempts to clarify the notation and concepts developed in Chapter Two in order that the analyses of the later Sections may proceed. This clarification is necessary because the theory of Chapter Two implicitly assumes that first, there is no inflation and second, that the consumer has perfect foresight. Thus the real value of future income streams (y(t)) and future real rates of interest are known with certainty. These rigid assumptions are relaxed a little and an artificial construction is introduced to circumvent the problems caused by unanticipated inflation.

Section 3.4. develops a simple two-period model along the lines first suggested in Section 2.2., and Section 3.5. then uses the corresponding continuous-time model to analyse how lifetime consumption and protection-based bequests are influenced by the existence of inflation expectations.

Finally, Section 3.6. uses the Unanticipated Continuous-Time Model to throw more light upon the unanticipated nature of inflationary expectations.

The notation of this Chapter is based on that developed in Chapter Two and thus Appendix 2.2. serves as a suitable 'key'. The only major addition is the symbol $\dot{P}$ which denotes the 'certain equivalent' expected rate of price changes.

3.2 Anticipated and Unanticipated Inflation

Originally, unanticipated inflation was defined to occur if price levels were expected to remain constant whereas, in fact, they rose. Thus R.A. Kessel and A.A. Alchain (9) comment that:

> "Anticipated inflation is characterised by market phenomena implied by the postulate that prices are expected to rise. Unanticipated inflation is characterised by market phenomena implied by the alternative postulate that the contemporaneous level of prices is expected to persist."

More modern definitions emphasise that unanticipated inflation is purely the difference between the actual and the expected price increases. Thus, to quote from J.A. Trevithick and C. Mulvey (19) p.110:

> "The current rate of inflation ($\dot{P}$) can be divided into two parts: that part which is fully anticipated $\dot{P}^e$ and that part which is unanticipated $\dot{P}-\dot{P}^e$".

Research into the methods that consumers and savers use to make their price expectations has taken two distinctive paths: the first (see, for example, P. Cagan (2) and R.M. Solow (17)) says that inflation expectations adjust by a constant fraction of the difference between the most recently recorded actual inflation rate and the previously formed expectation. (This is known as the 'error learning' or 'adaptive expectations' hypothesis). A slightly more sophisticated second order error learning mechanism has been postulated by D. Rose (15): this specifies that the consumer takes account of both the recent rate of inflation and its rate of change.

The second main area of research into expectations has led some investigators to attempt to estimate inflation expectations directly. In particular J.A. Carlson and J.M. Parkin (3) have constructed a series for the UK based on observations of the percentage of the population who believe inflation will fall as compared to the percentage who believe it will rise.

Criticisms of the use of arbitrary weighting schemes to generate expectations from past values of actual inflation have been summarised by D.E.W. Laidler and J.M. Parkin (10), Trevithick and Mulvey (1), Carlson and Parkin (3) and J.S. Flemming (6). In particular there is no guarantee that expectations are formed in the way that the error-learning process suggests; and secondly, it is not at all clear whether the error learning process should be applied to the price level P, the rate of inflation $\dot{P}$ or even the change in the rate $\ddot{P}$. The advantage of the error learning mechanism is, of course, that it is very convenient from the point of view of econometric estimation. If the mechanism must be disregarded then, in the absence of any direct accurate information on price expectations (such as those derived by Carlson and Parkin) it becomes almost impossible to include a price expectations variable in any statistical analysis. Fortunately, the results generated by Carlson and Parkin (and similar studies (see Trevithick and Mulvey p.115)) have served to restore confidence in the error learning models because of the similarity in the results produced:

> "To summarise, these results suggest that when inflation is rapid, expectations approximate a second-order error-learning process, whilst when inflation is mild, expectations approximate an autoregressive scheme. In the sample

> period, the only additional variable to have had any significant impact on the expected inflation rate is the devaluation of the exchange rate." Carlson and Parkin (3) p.135

Before we consider unanticipated inflation in greater detail, it is beneficial to return to a question that was formulated by Laidler and Parkin (10) p.770 :

> "what precisely is the expected rate of inflation, is this a unique variable or may several measures of it coexist?".

Laidler and Parkin note that expectations can vary over at least three important dimensions and these are particularly critical when we come on to consider the expectations of purchasers of life insurance. First,

> "different individuals and other agents will, in general, form different expectations of the same variable over the same future time horizon".

In particular, we must ask if the expectations of life insurance purchasers are different from those of other consumers. Second,

> "Different price indices will be relevant to different decisions and to different individuals".

Third,

> "the time horizon over which an expectation is formed will depend on the problem for whose solution it is necessary to form the expectation, and the same person may easily have very different expectations about the course of the prices over, for example, the next year and the next decade".

This problem is especially relevant for life insurance where long term contracts are involved. The only way that these problems may be answered in the life insurance case is by an examination of market behaviour and by analysis of the replies to the few life insurance market surveys that have been undertaken (and made public) (such as those by E.V. Morgan (12), 'Which' (4) and Southern Television (18)).

It has already been stated that the actual level of the variable of concern (whether it be the price level P, the rate of change of these prices $\dot{P}$ or even some higher order of price level) can be split into two component parts, the anticipated and the unanticipated. From the point of view of the consumer (or saver) this can be interpreted as meaning that the anticipated rate of inflation (say) is anticipated with complete certainty and any uncertainty surrounding future inflation rates manifests itself in the unanticipated part (which may be either positive, negative or zero). Furthermore, the anticipated rate of future inflation is known to the consumer to be the best possible estimate that can be made at that moment of estimation.

Thus we are able to say that, as far as future inflation rates are concerned, the consumer, if he reacts at all, responds to his anticipated rate of inflation ($\dot{P}^e$). However, the degree to which he responds to any expected inflation rate $\dot{P}^e$ will depend on the extent of his money illusion (see A. Deaton (5) and F.T. Juster and P. Wachtel (8)). Thus, if we assumed that consumption C_t depended on both Money Income at time t (Y_t) and the expected rate of change of prices during year t ($\dot{P}^e_t$) then we might formulate the simplified model:

$$C_t = C\left[\frac{Y_t}{P_{t-1}.(1 + \emptyset.\dot{P}^e_t)}\right] \qquad (3.2.1)$$

where $0 \leq \emptyset \leq 1$. Then if $\emptyset = 1$ (ie. no money illusion) consumption in year t depends on anticipated real income in year t (ie. $\frac{Y_t}{P_{t-1}(1 + \dot{P}^e_t)}$). If, on the other hand there is complete money illusion (ie. $\emptyset = 0$) then expected price increases are completely disregarded for consumption purposes. Various empirical studies to determine the general level of money illusion in the UK economy have produced differing results: Solow's calculations (17) indicate substantial money illusion, while others (such as P.G. Saunders and A.R. Nobay (16), J.M. Parkin, M.T. Sumner and R. Ward (14)) imply a value of $\emptyset$ much nearer to unity.

However, just because the consumer reacts according to the level of anticipated inflation does not mean that he regards that anticipated level as the correct rate of future inflation. We assume that the consumer is conscious of the existence of possible unanticipated inflation or deflation (although not, of course, its size) and that consequently there is some degree of uncertainty as to the actual level of future prices. The size of this variation about the mean predicted value will therefore affect the reactions of the consumer, (especially as the variation itself will be difficult to estimate); Flemming argues that the size of this variation about the mean predicted value will increase as the expected rate of inflation increases ((6) p.105).

Theoretically, the existence of unanticipated inflation can be illustrated by allocating a probability distribution to the rate of future inflation $\dot{P}$. This probability distribution relates to the state of the individual's belief about the size of (and not to the actual, correct level of $\dot{P}$ - thus it is immaterial whether estimates of $\dot{P}$ are unbiased or not). Thus the rate of anticipated inflation is given by:

$$E(\dot{P}) \equiv \dot{P}^e = \int_{-\infty}^{\infty} \dot{P}.\, f(\dot{P})\, d\dot{P} \tag{3.2.2}$$

where $f(\dot{P})$ is the subjectively formed probability density function of the value of future inflation. Similarly, the variation caused by unanticipated inflation is shown up by the standard deviation $\sigma(\dot{P})$ where

$$\sigma^2(\dot{P}) = \int_{-\infty}^{\infty} (\dot{P}-\dot{P}^e)^2 .\, f(\dot{P})\, d\dot{P} \tag{3.2.3}$$

Any uncertainty about the correctness of the estimate $\dot{P}^e$ is therefore reflected in a wider confidence interval surrounding $\dot{P}^e$ (which, of course, is reflected in turn by the value of $\sigma(\dot{P})$). Furthermore, the existence of unanticipated inflation is not the only factor affecting σ : for example, it is most likely that the further the consumer is required to predict into the future, the greater is the uncertainty about the accuracy of the predictions (ie. that σ is an increasing function of the time period involved).

With the introduction of Equations (3.2.2) and (3.2.3) it should be possible to indicate how the consumer (or saver) is influenced by both anticipated and unanticipated inflation. In the above context unanticipated inflation will have an important effect if the individual is particularly susceptible to the variations produced by that inflation.

The effects of price level uncertainty have been discussed by Flemming (6) who mentions particularly the influence on life insurance and pensions contracts. He points out that because of their uncertainty about future price levels, individuals may save more than is strictly necessary in the earlier part of their lifetime so that sufficient savings stocks be available to tap later on. Thus Flemming effectively argues that unanticipated inflation affects directly the consumer's precautionary motive for saving and that therefore, unanticipated inflation will alter the behaviour discussed in Section 2.1. Consequently, it becomes of interest to examine the theory of Chapter Two in greater detail in the light of the consumer's inflationary expectations.

3.3 Some Clarifications

At this stage it is necessary to clarify the exact meaning of some of the variables used in Chapter Two.

From Chapter Two we have the following Equations:

$$y(t) = c(t) + s(t) + b(t) \tag{2.2.1}$$

$$S(t) = e^{\delta t} \cdot {}_0\int^t e^{-\delta t} . s(\tau)\, d\tau \tag{2.2.3}$$

and

$$ {}_0\int^\infty e^{-\delta t} l_{x+t} b(t) dt = {}_0\int^\infty e^{-\delta t} l_{x+t}\, \mu_{x+t} B(t) dt \tag{2.6.3}$$

These were used, in conjunction with a particular expected utility function, to produce expressions describing the optimal consumption and bequest plans $(c^*(t), B^*(t))$ under differing circumstances. The whole analysis was worked on the assumption that:

a) money income flow $y(t)$ was known in advance with certainty (although not necessarily with accuracy);

b) decisions relating to future consumption and bequest were made at time zero (ie. age x) and once determined, $c(t)$ and $B(t)$ remained unchanged;

c) the ruling market rate of interest was constant and known throughout the consumer's lifetime and was denoted by force of interest δ.

The only modification made to make some allowance for reality was the inclusion of the sections entitled 'The Unanticipated Case'. Here it was assumed that at time τ , the consumer perceived that his initial assumptions had been wrong in some way. Consequently he altered the content (although not the structure) of his basic assumptions and then proceeded as before: the only difference being that he had to take into account the existence of a contractual agreement between himself and his insurer. Once the existence of inflation is allowed for, it becomes necessary to re-examine the assumptions of Chapter Two.

First of all, the assumption that money income is known in advance and with certainty cannot and must not be altered: once we allow the consumer to be uncertain of his future income, the whole tenor of the analysis changes. Consequently, in some way we must still assume that the consumer knows his money income in advance (and with certainty). Initially, this would seem to rule out the existence of unanticipated inflation; however, it is possible to circumvent this difficulty by making an artificial construction. To this end, we define that rate of inflation which is known in advance (at time zero) and expected <u>with certainty</u> as $\dot{p}$, where

$$\dot{p} = \dot{P}^e + g(\sigma^2(\dot{P})) \tag{3.3.1}$$

$\dot{P}^e$ and σ^2 are obtained from Equations (3.2.2) and (3.2.3): for the sake of simplicity we assume that the future expected rate of inflation $\dot{P}^e$ is constant and does not depend on time.[1]

[1] This is no worse than assuming that the force of interest is constant. As M. Yaari (20) shows, the relaxation of this assumption can be easily accommodated.

$\dot{P}$ can be interpreted as the 'certain inflation equivalent' of expected inflation $\dot{P}^e$; consequently the function g (which is a function of the level of possible unanticipated inflation) indicates the consumer's viewpoint on the uncertainty of future inflation. $\dot{P}$ therefore incorporates both the level of anticipated inflation $\dot{P}^e$ and also unanticipated inflation. If the consumer is risk averse or just pessimistic about the course of future inflation then the existence of uncertainty penalises the consumer so that g is positive (ie. the individual is prepared to trade a higher certain level of inflation $\dot{P}$ against the possibility of unanticipated inflation or deflation: this view is supported by Juster and Wachtel (8) p.87). The greater the level of uncertainty, the higher the corresponding certain inflation equivalent, ie. $g' > 0$. However, if the consumer is optimistic about unanticipated inflation then g may well be negative: again we would expect $g' > 0$.

Equation (3.3.1) can easily be adapted to allow for the possible existence of money illusion:

$$\dot{P} = \emptyset\,(\dot{P}^e + g\,(\sigma^2(\dot{P})\,)\,) \tag{3.3.2}$$

where $0 \leq \emptyset \leq 1$. If $\emptyset = 1$ (ie. no money illusion) then expected prices are fully taken into account in consumption and bequest decisions whereas if $\emptyset = 0$ (ie. complete money illusion) then no account is taken of future inflation.

It is now possible to rework some of the theory of Chapter Two by assuming that the consumer 'weighs' all his decisions in real (with allowance made for partial money illusion) rather than in money terms. Values are converted from money into real terms by utilising Equation (3.3.2).

Again we assume that all decisions relating to future consumption and bequest are made at time zero; however, when we come to examine the 'Unanticipated Case' at time τ , we have to include the possibility that the consumer could have been wrong in his initial assumptions about the rate of inflation.

Finally, when it comes to considering Equation (2.6.3) we assume that the form of this expression is determined by the insurer and not by the consumer. In particular, since life insurance is arranged in money terms with no allowance made for the progress of inflation, we will assume that Equation (2.6.3) remains unaltered, being part-and-parcel of the contractual agreement between insurer and consumer.

3.4 A Simple Two-Period Example

Essentially, the ensuing example follows the same methodology as that of Section 2.2 with appropriate allowances made for the existence of inflation.

The consumer is assumed to maximise the expected utility of the real-valued consumption and bequest plans c^R and B^R. ie. the consumer wishes to maximise

$$\bar{U}(c^R, B^R) = \sum_{k=1}^{2} (l_k/l_1) \cdot (u(c_k^R) + w(B_k^R)) \tag{3.4.1}$$

where

$$c_1^R = c_1 \quad ; \quad B_1^R = B_1 / (1+ \;)$$
$$c_2^R = c_2 / (1+\dot{P}) \quad ; \quad B_2^R = B_2 / (1+\dot{P})^2$$

and $\dot{P}$ is obtained from Equation (3.3.2). (Note that income and consumption flows are assumed to accrue at the beginning of the period while bequests are paid at the end).

Following the methodology of D. Moffet (11), the partial derivatives of Equation (3.4.1) are then set equal to zero. Values for $\frac{\partial B_i}{\partial c_j}$, $i,j = 1,2$ are again obtained from Equation (2.2.17) ie.

$$l_1(y_1-c_1) + v.l_2(y_2-c_2) = K.(v.(l_1-l_2).B_1 + v^2 l_2 B_2) \tag{2.2.17}$$

The four resultant equations are therefore:

$$l_1.u'(c_1^R) + l_1.w'(B_1^R).\partial B_1^R/\partial c_1^R + l_2.w'(B_2^R).\partial B_1^R/\partial c_1^R = 0$$

$$l_1.w'(B_1^R).\partial B_1^R/\partial c_2^R + l_2.u'(c_2^R) + l_2.w'(B_2^R).\partial B_2^R/\partial c_2^R = 0$$

$$l_1.u'(c_1^R).\partial c_1^R/\partial B_1^R + l_1.w'(B_1^R) + l_2.u'(c_2^R).\partial c_2^R/\partial B_1^R = 0$$

$$l_1.u'(c_1^R).\partial c_1^R/\partial B_2^R + l_2.u'(c_2^R).\partial c_2^R/\partial B_2^R + l_2.w'(B_2^R) = 0$$

(3.4.2)

Noting that

$$\frac{B_1^R}{c_1^R} = \frac{1}{(1+\dot{P})} \cdot \frac{\partial B_1}{\partial c_1}$$

$$\frac{B_1^R}{c_2^R} = \frac{\partial B_1}{\partial c_2}$$

$$\frac{B_2^R}{c_1^R} = \frac{1}{(1+\dot{P})^2} \cdot \frac{\partial B_2}{\partial c_1}$$

and $$\frac{B_2^R}{c_2} = \frac{1}{(1+\dot{P})} \cdot \frac{\partial B_2}{\partial c_2}$$

we get the final solution

$$\frac{u'(c_1^R)}{u'(c_2^R)} = \frac{(1+i)}{(1+\dot{P})} \qquad (3.4.3a)$$

$$\frac{w'(B_1^R)}{w'(B_2^R)} = \frac{(1+i)}{(1+\dot{P})} \cdot \frac{l_1-l_2}{l_1} \qquad (3.4.3b)$$

$$\frac{u'(c_1^R)}{w'(B_1^R)} = \frac{(1+i)}{(1+\dot{P})} \cdot \frac{2.\,l_1}{K.(l_1-l_2)} \qquad (3.4.3c)$$

Equations (3.4.3) indicate that as the level of the certain inflation equivalent $\dot{P}$. increases, the Marginal Rate of Substitution decreases. So Equation (3.4.3a) shows that if $\dot{P}$ rises, then real Period One Consumption will rise at the expense of that for Period Two. ie. that the consumer will bring forward his consumption. Equation (3.4.3b) gives a similar result for the level of bequests in the two Periods.

Equation (3.4.3c) indicates that if the certain inflation equivalent $\dot{P}$ rises then the real Period One bequests will fall in favour of consumption. However, if the rise in the rate of interest is greater than that for $\dot{P}$, then Period One bequests will increase.

Additionally, it can be seen from Equation (3.4.3b) that the effect of inflation (both anticipated and unanticipated) is to increase the real size of B_1 relative to B_2. This can be interpreted to mean that, ceteris paribus, the bequest level at younger ages will be increased relative to that for older ages.

If the assumption of a constant certain inflation equivalent is revised to allow for the possibility of different rates $\dot{P}_1$ and $\dot{P}_2$ respectively (for Periods One and Two) then Equations (3.4.3) can be reworked to obtain:

$$\frac{u'(c_1^R)}{u'(c_2^R)} = \frac{(1+i)}{(1+\dot{P}_1)} \qquad (3.4.4a)$$

$$\frac{w'(B_1^R)}{w'(B_2^R)} = \frac{(1+i)}{(1+\dot{P}_2)} \cdot \frac{\ell_1 - \ell_2}{\ell_1} \qquad (3.4.4b)$$

and

$$\frac{u'(c_1^R)}{w'(B_1^R)} = \frac{(1+i)}{(1+\dot{P}_1)} \cdot \frac{\ell_1}{K.(\ell_1 - \ell_2)} \qquad (3.4.4c)$$

where $$B_2^R = \frac{B_2}{(1+\dot{P}_1) \cdot (1+\dot{P}_2)}$$

This complication allows us to accommodate the fact that the uncertainty surrounding future levels of inflation may increase as the time scale involved increases. If this is the case, then a risk-averse consumer (who is pessimistic about the effects of inflation) will have $\dot{P}_2 > \dot{P}_1$, even though $P_1^e = P_2^e$. Consequently we can infer from Equation (3.4.4b) that, even if the expected (anticipated) rate of inflation is the same for the two periods, the real bequest plan will decline at a greater rate than real consumption. We also note that the differences between bequest and consumption are further highlighted in a comparison of Equations (3.4.3) and (3.4.4). Because $\dot{P}$ was assumed constant (so that it 'averaged out' the effects of $\dot{P}_1$ and $\dot{P}_2$), it must be that $\dot{P}_1 < \bar{P}$ while $\dot{P}_2 > P$.

If we make the usual assumptions that first, the probability of dying increases as age increases and second, that the rate of interest i is reasonably small, then the right hand side of Equation (2.2.17) will reduce in size as the bequests are brought forward (ie. B_1 gets larger relative to B_2). This may offset, to some extent, the degree of initial borrowing necessary to finance the increased consumption in Period One (from Equation (3.4.4a)). It may also help to explain why inflation may lead to increased (rather than decreased) saving (for example, see Morgan Grenfell (13) and Bank of England (1)).

3.5 The Anticipated Continuous-Time Model

This Section follows the methodology of Section 2.7 to show how the existence of anticipated and unanticipated inflation affects the behaviour of a utility-maximising consumer (lifetime saving only).

The consumer is therefore assumed to maximise the expected utility of the real-valued consumption and bequest plans c^R and B^R, ie. he wishes to maximise:

$$\bar{U}(c^R,B^R) = {}_0\!\int^{\infty} \frac{l_{x+t}}{l_x}(\alpha(t)u(c^R(t)) + \beta(t)w(B^R(t)))dt \qquad (3.5.1)$$

where

$$c^R(t) = c(t).e^{-\dot{P}t}$$

$$B^R(t) = B(t).e^{-\dot{P}t}$$

ie. $c(t)$ and $B(t)$ refer to the money values of consumption and bequest and $\dot{P}$ is obtained from Equation (3.3.2). (Note that $\dot{P}$ now represents the 'instantaneous' certain inflation equivalent).

The prospective reserve for the insurance is given by Equation (2.7.1), ie.

$$V'(t) = (\delta+\mu_{x+t})V(t) + b(t) + S'(t) - \delta S(t) - \mu_{x+t}B(t) \qquad (2.7.1)$$

and this is substituted back into Equation (3.5.1) to obtain

$$\tilde{U}(c^R,B^R) = {}_0\!\int^\infty \alpha(t)\,\frac{l_{x+t}}{l_x}\,u(e^{-\dot{\rho}t}(y(t) + \delta S(t) - b(t)$$

$$-S'(t)))dt + {}_0\!\int^\infty \beta(t)\,\frac{l_{x+t}}{l_x}\,w(e^{-\dot{\rho}t}(\frac{1}{\mu_{x+t}}((\delta + \mu_{x+t})V(t)$$

$$-V'(t) + b(t) + S'(t) - \delta S(t)))dt$$

(3.5.2)

Equation (3.5.2) is maximised if the following Euler equations are satisfied:

$$\frac{\alpha(t)\,.\,u'(c(t))}{\beta(t)\,.\,w'(B(t))} = \frac{1}{\mu_{x+t}}$$

(2.6.7c)

with

$$\frac{l_{x+t}}{l_x}\,.\,e^{-\dot{\rho}t}.\,(\alpha(t).u'(c^R(t)).\delta - \beta(t).w'(B^R(t)).\delta/\mu_{x+t})$$

$$= d/dt\,(\frac{l_{x+t}}{l_x}.e^{-\dot{\rho}t}.(\alpha(t).u'(c^R(t)).(-1) + \beta(t).w'(B^R(t)).\frac{1}{\mu_{x+t}}))$$

(3.5.3)

and

$$\frac{l_{x+t}}{l_x}.e^{-\dot{\rho}t}.\,(\beta(t).w'(B^R(t)).(1+\delta/\mu_{x+t})) =$$

$$d/dt\,(\frac{l_{x+t}}{l_x}.e^{-\dot{\rho}t}.(\beta(t).w'(B^R(t)).(-1/\mu_{x+t})))$$

(3.5.4)

Equations (2.6.7c), (3.5.3) and (3.5.4) produce an optimal solution satisfying

$$c^{R'}(t) = \frac{u'(c^R(t))}{u''(c^R(t))} \cdot \left(\frac{-\alpha'(t)}{\alpha(t)} - (\delta - \dot{P})\right) \tag{3.5.5}$$

$$B^{R'}(t) = \frac{w'(B^R(t))}{w''(B^R(t))} \cdot \left(\frac{\mu'_{x+t}}{\mu_{x+t}} - \frac{\beta'(t)}{\beta(t)} - (\delta - \dot{P})\right) \tag{3.5.6}$$

Thus we observe that, in general, an increase in the level of the certain inflation equivalent $\dot{P}$ has the effect of decreasing[2] the rate of increase in real consumption ($c^R(t)$) and real bequest ($B^R(t)$). However, <u>total</u> real consumption will not decrease over time unless the rate of impatience ($\frac{-\alpha'}{\alpha}$) is greater than the real rate of interest ($\delta - \dot{P}$). Similarly the total real bequest level will not decrease over time unless ($\frac{\mu'_{x+t}}{\mu_{x+t}} - \frac{\beta'(t)}{\beta(t)}$) exceeds ($\delta - \dot{P}$). Note that since ($\frac{-\alpha'}{\alpha}$) is commonly assumed to be positive (for example see Yaari (20)), total real consumption will certainly decrease over time if $\dot{P} > \delta$. However, because the quantity $\frac{\mu'_{x+t}}{\mu_{x+t}}$ is positive for most adult ages (in fact, for all $x+t \geq 30$; see A1967/70 Tables of Assured Lives (7)), Equation (3.5.6) shows that for a given level of $\dot{P}$, the total level of real bequests are less resistant to the effects of inflation than the corresponding levels of real consumption. However, notice that, under the circumstances mentioned above, inflation will not <u>decrease</u> the level of bequests (this point has

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[2] Since u' and w' are normally assumed positive while u'' and w'' are negative.

also been made by Flemming (6) p.106).

A comparison of Equations (3.5.6) and (2.7.6) is interesting in that the certain inflation equivalent $\dot{P}$ reduces the 'humped' shape of the real bequest plan, ie. the real bequest plan will start to decline at an earlier time the higher the rate of $\dot{P}$. Consequently, we are able to deduce that inflation will certainly reduce the real level of bequests made at older rather than younger ages with the 'substitution' effect of inflation. The actual situation may be complicated by the existence of an 'income' effect because a reduction in the level of money bequest made at older ages effectively reduces the overall 'price' of the bequest plan which can be illustrated by reference to the single premium. It follows that it is not possible to quantify the effects of inflation on the amount of money bequest at time t until this 'income' effect has been further examined.

Later Chapters will be concerned with the effects of inflation on the consumer's initial purchase of life insurance (ie. at time zero/age x) and we will be particularly interested in the effects on new premium income so generated. Since the concept of 'new premium income' cannot be applied to this model, attention must be concentrated on the single premium which, in terms of the notation of the preceding Sections, can be written as

$$A_x = \frac{1}{l_x} \cdot {}_0\int^{\infty} e^{-\delta t} . l_{x+t}\mu_{x+t} \, B(t) \, dt$$

(Note that in practice, most conventional life insurance contracts have premiums that are paid annually - or even more frequently. Thus it is not usually possible to observe the single premium).

It turns out that the quantity

$$A_x = \frac{1}{l_x} \cdot {}_0\int^{\infty} e^{-\delta t} . l_{x+t}\mu_{x+t} . B(t)\, dt$$

is of interest for at least two reasons:

a) in order to examine the 'income' effects of inflation, and

b) to give an indication of the effects of inflation on the demand for new life insurance.

From Equation (3.5.1) we note that $B^R(t) = B(t)\, e^{-\dot{P}t}$ where $B(t)$ denotes the money value of the bequest made at time t. Equation (3.5.6) gives an indication of the behaviour of the real value of bequests and in order to ascertain that of $B(t)$, we must look to

$$B(t) = e^{\dot{P}t} \, . \, B^R(t) \tag{3.5.7}$$

Equation (3.5.7) can be substituted into the Single Premium formula to give

$$A_x = \frac{1}{l_x} \cdot {}_0\int^{\infty} e^{-(\delta-\dot{P})t} . l_{x+t}\mu_{x+t} . B^R(t)\, dt \tag{3.5.8}$$

The final stage of the analysis would then be to form the expression $\delta A_x/\delta\dot{P}$ which would enable an examination of the responsiveness of A_x to changes in $\dot{P}$. Unfortunately, Equation (3.5.8) is very difficult to compute and impossible for arbitrary utility of consumption and bequest functions.

Consequently, only a very few comments can be made on the general behaviour of the Equations:

$$A_x = \frac{1}{l_x} \cdot {}_0\int^{\infty} e^{-(\delta-\dot{P})t} \cdot l_{x+t}\mu_{x+t} \cdot B^R(t)\, dt \tag{3.5.8}$$

$$B^{R'}(t) = \frac{w'(B^R(t))}{w''(B^R(t))} \cdot \left(\frac{\mu'_{x+t}}{\mu_{x+t}} - \frac{\beta'(t)}{\beta(t)} - (\delta - \dot{P}) \right) \tag{3.5.6}$$

i) if we assume that $\frac{\mu'_{x+t}}{\mu_{x+t}}$ is consistently positive (ie. $x \geq 30$ in (7)) while $\dot{P}$ is greater than δ then the greater the value of $\dot{P}$, the more rapid the decline of $B^R(t)$. Furthermore, $B^R(t)$ is declining for all time $t \geq 0$. However, from Equation (3.5.8) the decline in the value of B^R as t progresses is offset by the factor $e^{-(\delta-\dot{P})t} \cdot l_{x+t}\mu_{x+t}$ which is positive and increasing. (Note that since $e^{-(\delta-\dot{P})t}$ increases more rapidly than any power of t[(3)] we might expect this factor to outweigh the smaller values of $B^R(t)$ for large t). Thus we come to the first tentative conclusion that if $\dot{P} > \delta$ and $x \geq 30$ then $\partial A_x/\partial\dot{P} > 0$ ie. an increase in $\dot{P}$ will increase A_x:

ii) alternatively, if $\dot{P} < \delta$ (where δ is the 'long-run' force of interest and $\dot{P}$ is the 'long-run' certain inflation equivalent) while $x \geq 30$, then the opposite result might be expected if $\dot{P}$

(3) For proof, see for example, 'Real Analysis' J.A.Anderson, Logos Press Ltd., London 1969. Ch.9

is small enough, ie. an increase in $\dot{P}$ will have little (or a negative) effect on A_x:

iii) if $\dot{P} < \delta$ and, furthermore $x < 30$ (note, the inception age 30 corresponds to that age after which μ_{x+t} ceases to fall in (7)) then $B^R(t)$ will almost certainly be 'humped' shaped. This means that less weight is contributed by the older bequest levels (since $l_{x+t}\,\mu_{x+t}$ is generally increasing). In this case therefore, we might expect an increase in $\dot{P}$ to have a negative effect on A_x (since the decline in the level of $B^R(t)$ at high ages more than offsets the increased weight of these bequests determined by the factor $e^{-(\delta-\dot{P})t}$):

iv) in all other cases, the effect of $\dot{P}$ on A_x is uncertain:

v) all inferences about the behaviour of A_x have been made on the basis of the slope rather than the levels of the functions involved (eg. $B^{R'}(t)$ rather than $B^R(t)$ directly); this is because it is not possible to derive $B^R(t)$ without solving the full set of simultaneous equations involving consumption, lifetime saving and bequest.

3.6 The Unanticipated Continuous-Time Model

This Section follows the methodology of Sections 2.10, 2.11 and 2.12 to show how the consumer will react to an unforeseen, once-and-for-all change of circumstances at time τ that could not have been anticipated at time zero.

We now assume that at time τ the consumer changes his level of the certain inflation equivalent from $\dot{P}$ to $\dot{P}_\tau$ where, from Equation (3.3.2)

$$\dot{P}_\tau = \emptyset.(\dot{P}^e_\tau + g_\tau(\sigma^2(\dot{P}))) \tag{3.6.1}$$

and $0 \leq \emptyset \leq 1$.

There are several pertinent features to note about $\dot{P}_\tau$:

i) we assume that the degree of money illusion $\emptyset$ remains unaltered;

ii) $\dot{P}^e_\tau$ may be greater or smaller than the original $\dot{P}^e$ at time zero;

iii) we would expect the degree of uncertainty g_τ to be smaller than the original at time zero. This is because the consumer will have a smaller time horizon over which to anticipate the course of inflation - thus his level of uncertainty should decrease as τ increases.

It also seems reasonable to assume that, if the consumer's inflation expectations have changed, then so too have his expectations of future money income and force of interest. We therefore denote the subsequent money income by $y_\tau(t)$ $(t > \tau)$ and force of interest by δ_τ .

We now assume that the consumer wishes to maximise the expected utility of the real valued consumption and bequest plans c_τ^R and B_τ^R , ie. to maximise:

$$\bar{U}_\tau(c_\tau^R, B_\tau^R) = {}_\tau\int^\infty \frac{l_{x+t}}{l_{x+\tau}} . (\alpha(t).u(c_\tau^R(t)) + \beta(t).w(B_\tau^R(t))) \, dt \tag{3.6.2}$$

for $t \geq \tau$

where $$c_\tau^R(t) = c_\tau(t).e^{-\dot{P}_\tau t}$$

$$B_\tau^R(t) = B_\tau(t).e^{-\dot{P}_\tau t}$$

and $\dot{P}_\tau$ is obtained from Equation (3.6.1).
The prospective reserve for any life insurance (for protection purposes) is given by Equation (2.11.9):

$$V_\tau'(t) = (\delta_\tau + \mu_{x+t}).V_\tau(t) + b_\tau(t) + S_\tau'(t) - \delta_\tau S_\tau(t)$$
$$- \mu_{x+t} B_\tau(t) + (\delta_\tau + \mu_{x+t} + 1).e^{-\delta_\tau \tau}.l_{x+\tau}.S_\tau(\tau) \tag{3.6.3}$$

where $$S_\tau(\tau) = S(\tau) + SV \tag{2.11.3}$$

Now Equations (2.11.9) and (2.7.1) are identical as far as the coefficients of V, V', S, S' and b are concerned and so the calculus of variations (which maximises $\bar{U}_\tau(c_\tau^R, B_\tau^R)$) will produce Euler equations analogous to Equations (3.5.5) and (3.5.6) for $t \geq \tau$

$$c_\tau^{R'}(t) = \frac{u'(c_\tau^R(t))}{u''(c_\tau^R(t))} . (-\alpha'(t)/\alpha(t) - (\delta_\tau - \dot{P}_\tau)) \tag{3.6.4}$$

and

$$B_\tau^{R\prime}(t) = \frac{w'(B_\tau^R(t))}{w''(B_\tau^R(t))} \cdot \left(\mu'_{x+t}/\mu_{x+t} - \beta'(t)/\beta(t) - (\delta_\tau - \dot{P}_\tau) \right) \qquad (3.6.5)$$

In order to decide whether to change his existing insurance arrangements, the consumer must compare the maximised value of $\bar{U}_\tau$ so obtained from Equation (3.6.2) with the value of expected lifetime utility at time τ had the original insurance arrangements been continued (to be termed $U_\tau^*(c^R, B^R)$).

Thus the consumer will surrender his old insurance arrangements and change his consumption pattern if $\bar{U}_\tau > U_\tau^*$ where

$$U_\tau^*(c_\tau^R, B_\tau^R) = {}_\tau\int^\infty \frac{l_{x+t}}{l_{x+\tau}} \cdot \left(\alpha(t).u(e^{-\dot{P}_\tau t}.c^*(t)) + \beta(t).w(e^{-\dot{P}_\tau t}.B(t)) \right) dt \qquad (3.6.6)$$

$B(t)$ is the optimal money-valued bequest plan determined at time zero by solving Equation (3.5.6) and $c^*(t)$ is the new consumption plan allocated in an optimal way while still meeting existing insurance premiums.

Although it is not possible to proceed any further with the comparison of $\bar{U}_\tau$ and U_τ^*, the following points can be made:

i) a change in the certain inflation equivalent at time τ does not alter the basic pattern of real consumption and bequest;

ii) obviously, if the effects of $\frac{u'}{u''}$ and $\frac{w'}{w''}$

are ignored then a <u>decrease</u> in the real force of interest ($\delta_\tau - \dot{P}_\tau$) will reduce the rate of increase in real consumption and bequest over time (or alternatively heighten the rate of decrease);

iii) if the consumer associates a rise in inflation expectations with an increase in the force of interest then, provided the same view is shared by the insurer, a given pattern of premium payments will provide <u>higher</u> real bequests than before. Thus, ceteris paribus, a rise in inflation expectations provides higher future bequest levels on surrender. Conversely, a fall in inflation expectations is not so likely to cause the surrender of the existing insurance arrangements;

iv) similarly, if the consumer associates a rise in inflation expectations with an increase in future money incomes (ie. $y_\tau(t) > y(t)$ for $t > \tau$) then he will be able to increase his life insurance premiums.

v) Equations (3.6.3) and (2.11.3) indicate that, as before, the smaller the Surrender Value (SV) at time τ , the smaller the future potential bequest ($B_\tau(t)$). It should be noted however that, in practice, the Surrender Value paid by insurers probably decreases in money terms as inflation increases because Surrender Values are supposedly funded by the sale of securities (which may decrease in value);

vi) although it is not possible to draw any conclusions about the size of the real consumption and bequesting flows of Equation (3.6.2), it does seem as if an increase in inflation expectations has the effect of concentrating consumption and bequest towards the younger ages. Thus, ceteris paribus, the more 'impatient' the consumer (ie. the more downward sloping are the functions $\alpha(t)$ and $\beta(t)$) the more probable it becomes that $\bar{U}_\tau$ will exceed U^*_τ.

In conclusion, we may say that the surrender of existing (protection-based) life insurance may take place following a change in inflation expectations although surrender is more likely after an upward revision in expectations. However, the surrender becomes less probable with a decrease in the size of the Surrender Value paid and also if consumers are not highly 'impatient' for early consumption and/or bequest.

We also note that any changes in the certain inflation equivalent are more probably caused by a revision of inflation expectations since an increase in the uncertainty caused by unanticipated inflation is offset to some extent by the smaller time period involved in estimating future inflation.

3.7 Conclusion

After a brief introduction on the meaning and determinants of anticipated and unanticipated inflation, an expected utility maximising model was used to determine the various effects of inflation on the purchase of life insurance for the purpose of providing protection for heirs and dependents (this is implicit in the structure of Equations (3.4.1) and (3.5.1)).

In order to accommodate the requirements of the models, a 'certain inflation equivalent' $\dot{P}$ was used. In the construction of $\dot{P}$ it was found possible to incorporate elements of anticipated inflation ($\dot{P}^e$), unanticipated inflation and money illusion.

The certain inflation equivalent was then used to determine the effects of its three components on the purchase of protection-based life insurance. In brief, the results showed that the consumer who is conscious of the effects of inflation on the real value of his consumption and bequest would:

i) bring forward in time his real consumption and bequest expenditure;

ii) actually decrease his real consumption and bequest expenditure in certain instances: notably if $\dot{P}$ is greater than the force of interest δ ;

iii) (under the circumstances described in ii) above) decrease in the real value of his bequest over time at probably a greater rate than that for real consumption;

iv) consider the termination of the existing life insurance contract at time τ should he find that his inflation expectations have changed. Surrender is more probable if $\dot{P}_{\tau} > \dot{P}$ although any decrease in the Surrender Value paid will reduce the tendency to surrender.

Chapter Three : References

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CHAPTER FOUR : INFLATION, FINANCIAL SAVING AND THE DEMAND FOR LIFE INSURANCE

4.1. Introduction

This Chapter represents a theoretical stepping-stone between Chapters Two and Three - which concentrate on the purchase of protection-based life insurance - and Chapters Eight and Nine which will attempt some empirical tests. Basically there are two elements of financial saving through life insurance: lifetime saving and 'non-lifetime' saving. Lifetime saving via life insurance will not be discussed until Chapter Five - and it is mainly concerned with saving through the medium of endowment policies. 'Non-lifetime' saving (ie. bequests for saving purposes) has already been mentioned briefly in Chapter Two but will be analysed in more detail again in Chapter Five - it is mainly concerned with saving through the medium of whole of life policies.

The main objectives of this Chapter are twofold: first, to survey the major theoretical models of demand consumption and saving in order to assess: (a) their applicability for use in later Chapters and (b) to see whether they yield any a priori indications on the behaviour of lifetime financial savings via life insurance (which has not previously been discussed); and second, to make allowances both in the models of (a) above and also of those in Chapters Two and Three, for the institutional constraints affecting life insurance. These constraints are centred on the view of life insurance as a method of long term saving.

4.2 Models of Consumption and Saving

Models of aggregate consumption (and saving) and consumer demand functions have been recently summarised by R. Ferber (15) and J. Brown and A. Deaton (7) respectively. These two syntheses describe the major theories and developments in the field of consumer economics and provide the basic framework for this Section. The objective of this Section is therefore to briefly summarise the major theories of consumption (and saving) so that a later attempt can be made to assess their relevance to the study of life insurance.

a) The Keynesian Theory

The original concept of a consumption function relating current consumption to current disposable income was described by J.M. Keynes in his 'General Theory' (30) and was later to be known as the absolute income hypothesis. Keynes' propositions were basically that first, there is a break-even level of income at which the average propensity to consume equals unity. Below this level APC is greater than unity and above it, APC is less than unity. Second, the marginal propensity to consume is greater than zero but less than unity for all levels of income. A corollary of these two hypotheses is that the marginal propensity to consume is always less than the average propensity to consume.

Many statistical studies have confirmed the existence of some type of relationship between current income and current consumption. However, for several reasons, doubt has been cast on the usefulness of the absolute income hypothesis:

i) a large number of other variables were also found to affect consumption - this could make the consumption function unstable;

ii) the theory proved unable to predict post-1945 aggregate consumption behaviour; and

iii) the proven existence of two distinct forms of consumption function: for the short and long-runs (for example see R.G. Lipsey (36)). Evidence from U.S. aggregate consumption data indicated that the long-run APC had been virtually constant (since 1870) while the short-run APC had been declining as incomes rose.

It would be fair to say that the above criticisms of the absolute income hypothesis were derived from analysis of aggregate consumption (and therefore savings) data. However, all three criticisms also apply to the consumption of (demand for) individual goods and services. In particular we would expect the prospective purchaser of life insurance to take account of some longer term concept of income since life insurance is essentially a long-term contractual commitment. The only conceivable situation where the purchase of life insurance might be directly linked to current income is in the case of life insurance allied to the purchase of a house: in this case, the amount of the mortgage required is frequently determined by current income (and so therefore, will be the life insurance).

b) Relative Income Hypothesis

The foundation of the relative income hypothesis is that the propensity to consume depends on relative income, that is, income relative to some prior standard (in the case of time-series) or relative to income of a reference group (in the case of cross section data). Ferber (15) reports that this hypothesis was first suggested by D. Brady and R. Friedman (5) who suggested the following relationship:

$$\frac{s_t}{y_t} = a + b \cdot \frac{y_t}{\bar{y}} \qquad (4.2.1)$$

where s_t and y_t represent current individual saving and income and $\bar{y}$ represents average income.

The relative income hypothesis was further supported by J.S. Duesenberry (13) who argued that the preferences of individual consumers are not independent (as they formally were assumed to be). Duesenberry concludes that in periods of steadily rising income, the aggregate savings ratio tends to be independent of current income. Furthermore, this savings ratio (s/y) is principally dependent on the ratio of current income to the previous peak income (y_o)

$$\left(\text{ie. } \frac{s_t}{y_t} = a + b \cdot \frac{y_t}{y_o}\right) \qquad (4.2.2)$$

Although the relative income hypothesis might well apply to the purchase of certain goods and services (and especially consumer durables), it does not intuitively relate to the purchase of life insurance. It is difficult to imagine that a consumer's purchase of life insurance (whether for protection or financial saving) could be determined by the size of his income relative to that of his contemporaries. The only way in which this could conceivably occur is via the salesman of that life insurance who might provide information on amounts purchased by the consumer's contemporaries.

c) Permanent Income and Life Cycle Hypotheses

The basic idea underlying these theories is that the consumer plans his consumption not on the basis of the income received during the current period but rather on the basis of his long-run or lifetime income expectation. In the Life Cycle Hypothesis of Franco Modigliani (39 a-d), this income is that which the household expects to earn over its lifetime, ie. the maximum amount the household could spend on consumption each year without accumulating debts that are passed on to future generations. In the Milton Friedman theory (19), permanent income is the amount the household could consume forever without increasing or decreasing its present stock of wealth.

The central hypothesis of both theories is that the household's actual consumption is related to its permanent rather than its current income. Thus a change in a household's current income will affect its actual consumption only so far as it affects its permanent income. Any change in current income that is thought to be temporary will have little or no effect on permanent income, and hence on consumption.

Note that in both the approaches of Friedman and Modigliani, consumption is defined to include the real consumption of goods and services rather than monetary expenditures. Consumer durables are expenditures only to the extent that they are depreciated in a particular period, not the amount spent for their acquisition: the balance therefore constitutes saving.

The theory of Chapter Two provides some justification for applying the Modigliani and Friedman theories to the study of life insurance. In Chapter Two, it was assumed that income was known perfectly in advance so that there was effectively no transitory component. Furthermore, the construction of the model of Chapter Two fits in neatly with the Modigliani definition of permanent income.

Intuitively as well, the theories should repay application to life insurance because of the long-term contractual nature of the contracts (especially renewable premium ones). In undertaking an annual premium life insurance, the consumer must pay regard to future income levels as well as current income.

Additionally, the element of lifetime saving in life insurance dovetails nicely with Modigliani's concept of the 'life cycle' of consumption, ie. saving at young ages to be followed by dissaving at older ages (eg. after retirement).

d) Stock Adjustment Theories

As Ferber describes, originally, the many stock adjustment models (which assume that current decisions are influenced by past behaviour) seemed to be split into two camps: one approach was to view stocks in the form of tastes and habits which conditioned human behaviour over a long period of years and which accounted to a large extent for the positive autocorrelation in consumer purchases of many non-durables and similar goods and services. This approach lends itself to the purchase of life insurance in at least two respects:

i) if we wish to analyse the amount of premium income paid by consumers, then the existing stock of life insurance will obviously be very important because of the long term contractual (habit-forming) nature of the premium payments.

ii) of course, i) above will not necessarily apply to new premium income. However the theory would be applied to the purchase of new life insurance if it could be demonstrated that it was common for life insurance policyholders to hold more than one policy (as is certainly the case for industrial life insurance).

The second approach sought to explain why consumer purchases of many durables followed an opposite pattern to that described above. This approach viewed stocks as a physical accumulation of goods desired by consumers which led to additional purchases as desired stocks deviated from actual stocks, and thus explained any negative autocorrelation between purchases. This second approach could also be applied to the purchase of life insurance since it could be argued that the existence of a life insurance policy in force, will discourage further purchases.

The two approaches have been combined in one single theory by H.S. Houthakker and L.D. Taylor (26), who proposed, in its simplest form the following relationship:

$$q_t = \alpha + \beta . s_t + \gamma . y_t \tag{4.2.3}$$

where q_t denotes purchases (demand) at time t, s_t is the stock and y_t is income. If β is positive, this denotes that the purchases are habit forming (first approach); if β is negative then the stock adjustment effect is predominant (second approach).

Houthakker and Taylor then proceed to eliminate s_t (which, in the majority of cases is not observable) by relating it to purchases and depreciation. However, for life insurance purchases, s_t can be estimated directly by observing the relevant In Force figures (either sums insured or numbers of policies).

In a later chapter, Houthakker and Taylor discuss the possibility of adapting Equation (4.2.3) to include Friedman's Permanent Income Hypothesis (this combination is also attempted by K. Hilton and D.H. Crossfield (23)) and they suggest the following:

$$q(t) = \alpha + \beta s(t) + \gamma_p y_p(t) + \gamma_T y_T(t) \tag{4.2.4}$$

where y_p is permanent income and y_T is transitory income. However, since the permanent income is not directly observable, Houthakker and Taylor include an additional assumption (which is much in line with Friedman's own ideas) that the change in permanent income is proportional to the change in current income, ie.

$$\dot{y}_p(t) = k\dot{y}(t) \tag{4.2.5}$$

Equation (4.2.4) can then be rewritten as

$$\dot{q}(t) = \beta\,(q(t) - \delta\, s(t)\,) + (\gamma_p k + \gamma_T(1-k)\,)\,\dot{y}(t) \tag{4.2.6}$$

where δ represents the constant rate of depreciation.

Naturally, when examining the new purchases of life insurance, we would wish to test both the Houthakker and Taylor and Milton Friedman models. Thus Equation (4.2.6) (in its discrete time form) represents a useful method of approach.

It should be pointed out that in all four models so far under discussion, whenever aggregate time series expenditure or income data has been used, it has always been deflated by some relevant price index (for example see Houthakker and Taylor (26) p.54, Duesenberry (13)p.90 and Friedman (19) Ch.V). This has been done on the plausible assumption that consumers are influenced by variables in real rather than monetary terms. However, evidence is accumulating which indicates that consumers are in fact subject to money illusion. This view (which will be the subject of the next Section) has been supported by various authors such as W.H. Branson and A.K. Klevorick (6), J.T. Juster and P.Wachtel (28), D.H. Howard (27) and A. Deaton (12a and 12b).

Finally it should be noted that all four models refer to saving in the aggregate sense rather than to the 'product' of any individual savings institution in particular. It is not our intention to delve deeply into the theory of portfolio selection; however M. Hamburger (20) and R.S. Headen and J.F. Lee (22) provide a good example of the concepts involved while J. Revell (47) describes the effects of inflation on individual assets. The basic difference between the models of Hamburger and Friedman, Houthakker etc., is the inclusion of a 'yield' variable for the individual assets: this proves difficult to calculate for life insurance anyway (and its omission from the Hamburger model leaves the basic models of this Section remaining).

4.3 Models of Inflation and Saving

This Section is concerned with two further developments of the consumption/saving models of Section 4.2. In the first place, models will be examined that specifically include inflation as an independent variable and secondly we provide a brief description of those, comparatively recent models that attempt to explain the relationship between rises in the savings ratio and increases in the rate of inflation (these have been summarised in Bank of England (3)). Again the basic objective is to ascertain whether these models can throw any light on the purchase of life insurance. Note that several specific models of life insurance and inflation have been formulated by other authors (notably S. Neumann (44) and P. Fortune (18)), but we shall defer an examination of these until Section 4.5.

Money illusion has been defined by Don Patinkin (46) p.22, to occur if an equiproportionate change in all accounting prices - including that of paper money - produces some change in the amounts of commodities demanded by the consumer. Thus the economic behaviour of an individual free of money illusion depends solely on money prices, it does not depend solely on relative prices. An individual who is suffering from money illusion also reacts to changes which affect only the absolute level of accounting prices: his economic behaviour does not depend solely on the money prices[1].

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[1] In Patinkin's notation (p.16), the accounting prices of an economy of n goods are represented by $P_1, \ldots, P_n$ where the nth good is paper money. The respective money prices are therefore $P_1/P_n, P_2/P_n, \ldots, 1$. The relative prices of the (n-1) commodities in terms of the first one are $1, P_2/P_1, \ldots, P_{n-1}/P_1$

Branson and Klevorick (6) represent money illusion in a more specific form. They argue that, for a consumption function of the form

$$C_t = b_0(y_t)^{b_1} \cdot (w_t)^{b_2} \cdot (P_t)^{b_3} \tag{4.3.1}$$

money illusion exists (in the Patinkin sense) if $b_3 > 0$ while money illusion is absent if $b_3 = 0$ (where C_t represents real consumption, y_t real net labour income and w_t real consumer net worth, all on a per capita basis). Branson and Klevorick go on to test a distributed lag model with the basic equation:

$$\ln C_t = \beta_0 + \sum_{i=0}^{I} \gamma_i \cdot \ln y_{t-i} + \sum_{j=0}^{J} \delta_j \cdot \ln w_{t-j} + \sum_{k=0}^{K} \eta_k \cdot \ln P_{t-k} \tag{4.3.2}$$

where the lags for income, wealth and prices are I, J and K quarters respectively. They come to the conclusion that, since $\sum_{k=0}^{K} \eta_k > 0$, consumers exhibit money illusion via a distributed lag adjustment to the price level.

In order to explain this phenomenon, Branson and Klevorick argue that where prices, money income and money wealth all increase proportionately, consumers notice the income and wealth increases more than they do the price level rise and therefore increase their real consumption.

Deaton (12) and Juster and Wachtel (28) all agree with Branson and Klevorick on the existence of money illusion but disagree on the direction of its effects. In particular, Deaton objects vehemently to their conclusions since his own results confirm that $\sum_{k=0}^{K} \eta_k$ is negative, ie. that the presence of money illusion has a positive effect on saving rather than on consumption.

Deaton (12) disputes the traditional economic theory which assumes money illusion to occur when individuals are more conscious of changes in nominal incomes than of changes in prices (so that identical changes in incomes and prices lead to higher consumption and therefore lower saving). Deaton argues that, because goods are purchased sequentially, the consumer will have accurate information only on the goods actually bought. Additionally, because generally price indices are always out of date, at least in the first instance, individual consumers have no possible means of distinguishing relative price changes from absolute price changes. Consequently, in times of generally rising prices there is a mass illusion that all goods are relatively more expensive so that, as each consumer attempts to adjust his purchase, consumption falls and saving rises. If inflation continues to accelerate while expectations lag behind reality then consumers never fully adjust and the saving ratio remains abnormally high.

The Deaton, and Juster and Wachtel models are similar in that they both rely on the effects and interactions of anticipated (expected) and unanticipated inflation. Deaton argues that the consumer confuses unanticipated inflation with a rise in absolute (rather than relative) prices : this induces a rise in the saving ratio. Similarly, unanticipated rises in permanent income are confused with transitory income which again induces saving (and this view is partially supported by the results of Bank of England (3)).

On the other hand, Juster and Wachtel concentrate on the uncertainty generated by unanticipated rises in income and prices. They argue that high rates of inflation have been historically associated with variable rates of inflation, so that if money income is not expected to match this variation, real income will be subject to greater uncertainty in times of high inflation. Furthermore, a wider dispersion of possible real incomes may not have symmetrical effects on behaviour, in that the prospect of declining real income may carry more weight on consumer decisions than the (equally likely) prospect of rising real income. Thus consumers would attempt to curtail spending in an attempt to guard against declining real income.

In their model, Juster and Wachtel include the variables CPI and CPI* where CPI denotes the actual rate of change of consumer prices and CPI* refers to the expected rate of change of consumer prices derived from survey data produced by the Survey Research Center at the University of Michigan. Thus, if these variables are included in the model in the form

$$a_1 \, CPI + a_2 \, CPI^*$$

where a_1 and a_2 are regression coefficients, then $(a_1 + a_2)$ is interpreted as the effect of anticipated inflation and a_1 is interpreted as the effect of unexpected inflation. From their results, Juster and Wachtel tentatively conclude that unanticipated inflation has a negative effect on consumption (which supports their hypothesis). Fully anticipated inflation, however, raises consumption expenditures on non-durables (and services) at the expense of both saving and expenditure on durable goods.

The implications to be drawn from these models, when investigating the purchase of life insurance, are fairly clear:

a) there is definite justification for the inclusion of both anticipated and unanticipated inflation as explanatory variables (this, of course, is one of the main conclusions of Chapter Two). However, it is not certain whether the level of prices or the rate of change of prices should be used. Furthermore, the effect of inflation on saving is uncertain with Deaton arguing that the effect is positive;

b) again, both transitory and permanent income variables prove useful (Deaton, and Juster and Wachtel used both). Furthermore, these studies give some indication on the method of calculating permanent income, should it prove necessary (we have already noted one alternative method in Equation (4.2.5));

c) although the fact has not been mentioned explicitly, it has been found that certain advantage is to be gained by splitting up (or analysing separately) the different sources of income. Thus Hilton and Crossfield (23) comment that the distinction between human and non-human income is a useful one. M.J.C. Surrey (51) splits personal income into three components (wages and salaries, current grants from public authorities, and all other personal income);

d) Juster and Wachtel (28) also find some support for introducing both the level of unemployment and the change in unemployment as explanations of the saving ratio. The reasoning is that when unemployment is at high levels, consumers dissave to maintain consumption. Additionally, when unemployment is rising, fear of unemployment is probably rising too: this leads to a building up of reserves on the part of people who continue to hold jobs (ie. increased saving).

Finally, we close this Section with an examination of the various (new) models designed specifically to justify the high levels of the saving ratio in the mid-1970's. A survey of these various models was undertaken by the Bank of England (3). In its introduction the paper states that weak evidence was found to support the Deaton model; no evidence at all could support the model of Juster and Wachtel but strong evidence was found to support a theory proposed in the Morgan Grenfell Economic Review (42). This was the view that the value of liquid assets held by the personal sector, when adjusted for the effects of inflation, has had a significantly negative effect on the personal saving ratio. (Note that the idea of liquid assets as an explanation of consumption goes back further than 1975, eg. Hilton and Crossfield 1969 (23) and D.B. Suits 1963 (50)).

Holdings of liquid assets can affect savings patterns either because of their absolute size (a 'wealth effect') or because of their relative size (a 'portfolio balance effect'). Thus individuals may regard their holdings of liquid assets as the most important part of their available resources when planning expenditure. Alternatively, consumers may have a preferred amount of liquid balances to satisfy both precautionary and transaction motives, and any reduction below this amount may cause expenditure to be restrained (and saving increased).

In his study, Ferber (15) concludes that the weight of evidence favoured the inclusion of liquid assets in the consumption function. However, liquid assets could only directly affect the purchase of life insurance through the 'portfolio balance effect' described above since expenditure on new life insurance takes place over a long period of time. Thus we expect the effect of liquid assets, if any, to be negative in that a rise in liquid assets will reduce new purchases of life insurance.

Similarly, Howard (27) concludes that holdings of liquid assets have a significantly negative effect on the savings ratio and that this relationship is particularly important in the United Kingdom. A number of reasons are advanced for this behaviour relying on the role of liquid assets as transactions balances and as a 'buffer-stock' against unexpected short-falls in income. Howard has also something to say about the effects of inflation:

> "in two countries - Japan and the United States - unexpected inflation is positively related to personal saving, and in Canada, the United Kingdom and the United States expected inflation has a similar effect. In addition, in Japan, inflation expectations may discourage personal saving, but the evidence is mixed. (In the UK equation the effects of a nominal long rate of interest may be capturing the effects of long run inflation expectations. If so, the positive sign of δ_1 (the coefficient of the relevant interest rate) reported for the UK also constitutes

evidence of a positive relation between such expectations and personal saving)." (27) p.554

4.4 Special Characteristics of Saving Through Life Insurance

Virtually all of the models described in Sections 4.2 and 4.3 are concerned with a description of aggregate saving or the savings ratio (Saving divided by Personal Disposable Income) rather than saving via any particular savings institution. The main purpose of Sections 4.2 and 4.3 was to examine these models so that their possible application to a study of life insurance could be assessed. However, there are several unique features of both life insurance saving and the saver/policyholder that deserve some comment; these must also be borne in mind when it comes to drawing up a model of the purchase of life insurance.

In the first place, the vast majority of saving via life insurance is long-term saving. The average term on endowment contracts is around 20-25 years (an average of 23½ years was reported by Southern Television Ltd., (49) p.126 in Dec. 1975) and tax relief on premiums is only available for contract terms exceeding ten years (see R.L. Carter (9) p. 2.1.4 -09). Furthermore the construction of the reversionary bonus system encourages savings via longer term policies. Consequently, the prospective purchaser of new life insurance is encouraged to take a long term view of future real income and prices. Obviously, for whole of life policies, the consumer must make a decision on real income throughout his lifetime: this lends great weight to the 'life-cycle' hypothesis as an explanation of the purchases of life insurance.

Secondly, the majority of life insurance (and certainly the types of interest in Chapter Six) is on a contractual basis. This means that:

i) premiums must be paid regularly, and

ii) savings cannot be readily liquidated without penalty.

The objective of contractual saving is to make that saving habit forming: this is encouraged by the granting of tax relief on these types of policies. Furthermore, the contractual element of life insurance is further emphasised because a large proportion of premiums are paid by Bankers Order so that the policyholder does not even have to remember to pay them.

It has been argued that contractual saving (and contractual payments) exhibit slower and smaller fluctuations than savings (and payments) that are discretionary (for example see G. Katona (29) p.15, and P.L. Cheng (10)). Similarly, contractual saving is less responsive to factors like inflation (for example see Midland Bank (38) p.12 and Barclays Bank (4) p.4).

It has been demonstrated in Chapters Two and Three that the penalty for breaking the life insurance contractual agreement can play a considerable role in the decision on whether to do so or not. Not surprisingly, it turns out that the smaller the penalty, the more 'liquid' life insurance savings become (it must be pointed out that, in practice, the penalty is fairly substantial). The factors affecting the surrender of life insurance contracts will be explained in greater detail in Chapter Nine.

However, it must be noted that although life insurance is basically long term, contractual saving, the decision to purchase a new life insurance is not a contractual one but discretionary:

> "The decision to purchase a new life insurance policy is thus a genuine one; however, once the first or second annual premiums have been paid, the periodic decision involved in paying renewal premiums is of a routine nature. Renewal premium is a contractual outlay which does not necessarily depend on developments during the period of the payment." Neumann (44) p.112

Consequently, the effects of inflation on the purchase of life insurance must be examined in two areas:

a) the effects on discretionary saving and consumption,

b) the effects on contractual saving and consumption payments.

The effects of inflation on the discretionary part of life insurance form the major part of this study and Chapters Two - Eight are devoted to it. Chapter Nine examines the effects of inflation on contractual saving and consumption combined by looking at Surrenders of life insurance.

The third feature of life insurance in the United Kingdom has already been mentioned: that 'qualifying policies'[(2)] receive income tax relief, therefore effectively reducing the 'price' of life insurance. This tax relief has been at half the basic rate of income tax for many years and therefore can be assumed to be dependent entirely on the level of income tax (for the consumer who pays such tax). Thus the basic rate of income tax should

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[(2)] See Carter (9) p.2.1.4 - 08

be applied as another variable to explain the purchase of life insurance.

As the fourth point, we reiterate the question posed by Neumann (44) p.92, namely "Is life insurance really a Saving Medium?". While this question might have some relevance in the U.S.A. (where the majority of life insurance is protection-based) it seems intuitively obvious that, in the UK., a vast amount of saving is undertaken via life insurance. However, some doubt has been sown by Money Which? (40) who undertook a survey of over 500 Consumer Association members. This survey investigated the main reasons for buying the (latest) insurance policy and found that, for those respondents who wanted protection only, the vast majority (66%) purchased savings-type life insurance. The results are set out in Table 4.4.1 below:

Table 4.4.1 : Reasons Consumer Association members gave for buying life insurance and what they bought.

Main reason for buying	Number of members giving this reason	Proportion of members giving each reason who bought		
		Protection based %	Saving-type %	Other %
To protect dependents against member's premature death	221	31	66	3
To provide lump sum later on	141	4	92	4
To pay off mortgage	122	24	75	1
Other (eg. to get tax relief)	30	7	87	6

Source: Money Which? Dec.'75

The results of Table 4.4.1 would seem to indicate that some policyholders have purchased 'savings-type' life insurance ('Money Which' define this as endowment plus whole-life) when they should have purchased 'protection-type' ('Money Which' - temporary life insurance). However, the theory of Chapter Five clearly indicates that both endowment and whole-of-life insurance have a protection element so that the results of Table 4.4.1 can, to some extent, be understood.

The next point of interest is the very close connection between the purchase of life insurance and the purchase of a house or property. A large proportion of protection-type life insurance is associated with mortgage protection while saving-type life insurance is often used as a method of repaying the loan to a building society. Additionally, life insurance is not a homogeneous product: there are considerable variations in the type of product, the quality of service and the performance of the various ordinary (as opposed to industrial) life insurance companies (this aspect is monitored in publications like 'Planned Savings' (Wootten Publications Ltd.) and in an annual review in 'The Economist').

The last point is concerned with the state of knowledge of the average purchaser of life insurance: it would be fair to say that the average consumer does not have an intimate understanding of life insurance (eg. see Southern Television Ltd. (49) p.75) and, in particular, cannot accurately assess the price or rate of return offered. However, a considerable proportion of ordinary life insurance is purchased by better educated consumers from the higher social groupings (see E.V. Morgan (41) Section II).

Furthermore, a significant amount of ordinary life insurance purchases are undertaken under the aegis of some kind of insurance adviser. In their survey, Southern Television Ltd. (49) p.53, reported that 44% of insurance purchasers (ordinary and industrial life insurance combined) obtained their information from an 'Insurance Representative' (this figure must be treated with caution, however, because a larger proportion of Industrial Branch business is conducted via an 'Insurance Representative').

4.5 Studies of Life Insurance and other forms of Contractual Saving

The objective of this final section of Chapter Four is to review those studies that investigate contractual saving specifically: again with the intention of making an assessment of their relevance to this particular study of life insurance. These studies fall into two main groups: non-insurance contractual saving and saving through life insurance.

Alan R. Roe (48) defines contractual saving as saving flow "which is predetermined in the sense of being dependent on a decision of a previous period". In practice, only two financial transactions are normally defined as contractual saving; these are the payment of life insurance premiums (including contributions to pension schemes) and the repayment of capital sums in respect of house-purchase loans. One of the outstanding features of these contractual savings is their stability when examined over a period of time.

But, this conventional definition of contractual saving is inaccurate. First, to take up Neumann's point in Section 4.4, life insurance premiums paid in respect of policies taken out in the current period are rather different from premiums paid in respect of policies taken out in some past period. Roe comments that,

> "The former will be contractual in respect of subsequent periods, but are scarcely contractual in respect of the current periods since they are not predetermined as the result of a savings decision of a previous period."

In Chapter Nine, these initial premiums will be further examined (in the context of surrendering policyholders) and will be redefined as 'Not Yet Contractual' payments.

The second deficiency of the conventional definition of contractual saving is that it excludes a number of smaller but significant items like the accumulation of shares and deposits in building societies, unit trusts and National Savings Treasury Securities (eg. SAYE) over a number of years at some pre-agreed rate of accumulation. In his article, Roe proceeds to examine building society shares which involve a commitment to regular saving (these shares generally amount to around 3% of total building society shares outstanding).

Unfortunately, Roe's conclusions are not encouraging, or indeed very helpful. It would appear that over the period of his study (1958-67) building society shares involving a contractual commitment exhibited greater instability than those without any contractual commitment. Roe advances a number of explanations of this absurd phenomenon and concludes that "the possibilities for making use of the contractual nature of the saving inflow for forecasting may be non-existent".

Although his study of the consumption of non-durable goods is not explicitly concerned with contractual payments, Cheng (10) hypothesises that these payments (contractual savings plus durable goods financing) have surreptitiously affected consumption expenditures for non-durable goods and services. Allowance is made for these contractual payments by deducting them from current disposable income to give the actual income available for expenditure on non-durables. Cheng concludes that,

> "The increased contractual commitments of consumers' disposable income has increased the income elasticity of demand for non-durables, and the demand for non-durables has become

more cyclical than in the past."

The second (and major) group of works studying contractual savings/payments comprises those empirical studies of the purchase of life insurance, the majority of which have been summarised by C. Ferry (16) and J. Finley Lee and W.M. Whitaker (33). The remainder of this Chapter will be devoted to an illumination of the major features of these studies (which relate, without exception, to the U.S.A. life insurance industry). Note however, that all these models are open to the criticisms of A. Deaton (11) Ch.2, especially the problems involved in aggregation over consumers. Essentially, there is no guarantee, even if every single consumer in the economy behaved according to the predictions of some utility theory, that their aggregate behaviour will likewise conform. Deaton also points out that certain aggregative models of demand do not conform to the basic propositions of utility theory (which he describes as homogeneity, symmetry, negativity and budget constraint).

The following models of the purchase of life insurance fall into two clear-cut groups: those concerned with the 'demand' for life insurance and those attempting to explain the importance of saving via the medium of life insurance. All the models are, however, subject to the same criticism: that, by failing to distinguish clearly between the saving and protection (consumption) elements of life insurance, they oversimplify the analysis. This is especially true of the second group of models where a failure to strip out the expenses loadings from office premiums can result in a gross overestimation of the importance of saving flow in the so-called 'savings-based' policies (this point will become more obvious in Chapter Six).

An additional major criticism of the first group of models is that they all neglect any possible identification problems caused by the exclusion of the Supply equation from the model. Thus it is conceivable that, instead of measuring a Demand relationship, they are measuring Supply instead (this point will be taken up more fully in Chapter Seven).

We note that only the models of the second group are properly aimed at an analysis of Contractual Saving since, by their very nature, the Demand models are concerned with new purchases of 'Not Yet Contractual' Life Insurance.

One of the first studies investigating the effects of inflation on life insurance seems to have been undertaken by D.B. Houston (25) in 1960. Houston's objective was to determine the existence and extent of any empirical relationship between changes in the price level and the pattern of savings through life insurance (which were defined as the net increase in life insurance company reserves in any given year). In particular, Houston related the proportion of premiums devoted to saving to the Consumer Price Index and determined the following relationship for 1946-58:

$$\frac{\text{Change in reserve x 100}}{\text{Premiums}} = 92.5 - (0.5277)\ \text{CPI} \tag{4.5.1}$$

He therefore concludes that since 1946, there has been a steady decrease in the relative importance of saving through life insurance. Houston's results are difficult to interpret however, because of a number of criticisms, the major one from our point of view being that he does not differentiate between renewal premium income and new premium income (and so confuses contractual and non-contractual elements). His results are also open to doubt because of the extreme simplicity of his model.

A.E. Hofflander and R.M. Duvall (24) consider that inflation would have two separate effects on the demand for life insurance: first, an 'income effect' which causes the total purchases of insurance to decrease because real income has decreased. Secondly, there might be a 'substitution effect' where term insurance (protection-based) may increasingly be substituted for part of the savings-based life insurance. However, as Neumann (44) p.125 points out, the Hofflander/Duvall indifference analysis can have a number of interpretations entirely unconnected with inflation. In particular, the Hofflander/Duvall analysis depends on at least two critical factors:

i) that higher anticipated rates of future inflation increase the price of real protection but not that of Other Goods and Services;

ii) that the income consumption curve (between sales of protection and savings-based insurance) has positive first and second derivatives.

Hofflander and Duvall then proceed to derive a model to explain the 'total issuance of savings-based life insurance' (Y_t)

$$\text{ie.} \quad Y_t = A_0 + A_1X^a_{1t+s} + \sum_{i=2}^{k} A_iX_{it} + U_t \tag{4.5.2}$$

where the A's are unknown coefficients, X^a_{1t+s} is the anticipated price level at time t+s and the other X_i include population, births and per capita income. Two different models of expected price level were used:

i) where the price level at time t+s consists of a trend term plus a function of the current difference between the actual price level and the trend ie.

$$X^a_{1t+s} = b_0 + b_1(t+s) + b_2(X_{1t}-b_0-b_1t)$$

where X_{1t} denotes the Consumer Price Index.

ii) where the price level at time t was given by a weighted average of past levels[(3)] ie.

$$X^a_{1t} = bX_{1t} + b(1-b)X_{1t-1} + b(1-b)^2X_{1t-2} + \ldots$$

In both models, the coefficient of CPI was negative (however the results of the second model must be treated with caution as the model was misspecified by the omission of other lagged explanatory variables).

Both models were then repeated for sales of protection-based life insurance and similar results were obtained. They therefore conclude that the 'income effect' has been stronger than any 'substitution effect'.

The model of J. Hammond, D. Houston and E. Melander (21) differs from the others because it is based on cross-section rather than time-series data. An attempt is made

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(3) This type of model was first introduced by L.M. Koyck (32) in 1954.

to examine the relationships between life insurance premium expenditures and various economic and demographic characteristics of households. This is achieved by postulating a model of the form:

$$P = a + \sum_{i=1}^{k} b_i X_i$$

where P refers to the amount of money expended by households for life insurance premiums, and the various explanatory variables include income, net worth, education, age, marital status and occupation.

The Hammond, Houston and Melander results are, however, unsuitable for comparison with the present ones because

i) a cross-section study measures long-run elasticity coefficients while a time-series study examines the short-run;

ii) new premium income and renewal premium income are aggregated;

iii) premium income on all types of life insurance (both saving and protection-based) are aggregated; and

iv) the model produces poor results in any case.

The work by Neumann (44) and (45) seems to be the most rigorous examination to date of the purchase of life insurance. The hypothesis tested is that there was no significant effect of the post-war American inflation on saving through life insurance: time series data was used from 1946 to 1964.

Neumann introduces a price expectations variable into his analysis which incorporates the basis of the adaptive expectations approach of M. Nerlove (43). A Koyck model is then utilised to produce:

$$L_T = a_0 + a_1 P_T + a_2 L_{T-1} + \sum_{i=1}^{n} b_i X_{iT} + U_T \tag{4.5.3}$$

where L_T is 'some measure of saving through life insurance', P_T is the price level (Consumer Price Index) at time T and the X_{iT} are additional explanatory factors such as births, disposable income and marriages. At first sight, it would seem that Neumann's model is subject to the same specification error as the model of Hofflander and Duvall: to quote Fortune (17),

> "(Neumann) uses the 'adaptive expectations' model of the expected price level in an appropriate manner which gives rise to a specification error in the form of excluding the lagged values of all non-price level explanatory variables in his demand function."

However, Neumann comments in a later footnote that these lagged explanatory variables had in fact been included but their coefficients were found not to be significant. Moreover, the omission of the lagged explanatory variables did not change the coefficients of Equation (4.5.3).

Neumann undertook several different investigations using the model of Equation (4.5.3) with various measures of the dependent variable L_T. The model was also tested in a first difference form. The objective of these different approaches was to test whether inflation not only caused a contraction of the total purchases of life insurance but also whether there was a shift from savings-based to protection-based life insurance. In general, Neumann's models confirmed his original hypothesis that inflation (over the period of the study) had no discernable effect on saving through life insurance.

In his critique of Neumann's study, Peter Fortune (17) hypothesises a model that makes allowance for the effects of both higher price levels and also higher expected rates of inflation. Thus, denoting the implicit nominal yield on 'saving through life insurance' as ρ, Fortune suggests a model of the following kind:

$$L = a_0 + a_1 P + a_2 RINF^* + a_3(\rho - R) + a_4(R - RINF^*) + a_5 Y^* + a_6 W_{-1} \qquad (4.5.4)$$

where R denotes the market rate of interest on financial assets;

RINF* - the expected rate of inflation;

Y* - 'permanent' real disposable labour income of households, ie. the amount of income lost if the individual should die;

W_{-1} - the initial real financial assets of households other than wealth in insurance;

and L - life insurance policy reserves per dollar of insurance.

Fortune then assumes that the expected rate of future inflation is formed through the adaptive expectations mechanism, and then tests his model over the period 1953I to 1967IV. He comes to the following conclusions:

a) an increase in Y* reduces the ratio L by a small amount;

b) the coefficient of W_{-1} is not significant;

c) L is positively related to the level of expected future prices; and

d) the impact of RINF* on L through its effects on expected future price levels is also positive.

The basic objective of the G. Mantis and R. Farmer (37) paper is to facilitate quick and easy predictions of the demand for life insurance. The basic features of their model are similar to those of preceding models except in the one respect: instead of using marriages and births as explanatory variables, the 8-year period lag of marriages and 2-year lag of births were used (the lags being determined by maximising simple correlation with the dependent variable). Other independent variables were:

Price of life insurance relative to other consumer prices (this figure was computed by comparing the CPI to a composed price index for life insurance);
Personal income;
Population; and
Employment.

The dependent variable was described as 'Sales of Life Insurance' and it is supposed that this refers to sums insured in force each year. Consequently, the conclusions of this model cannot really be of value to this present study. In addition, the model must be regarded of dubious value because of the negative coefficients of the Births, Marriages and Employment variables as well as a positive coefficient of Relative Prices (which were not explained by Mantis and Farmer).

The first model of J. Lee and W. Whitaker (34) is especially interesting because of the inclusion of an explanatory variable that no previous study had considered:

> "Life insurance in force was incorporated in order that the effect of the existing stock of life insurance on additional purchases might be measured - in other words, to test for significant market saturation."

This inclusion of the 'stock' of life insurance fits exactly with the stock adjustment model of Houthakker and Taylor (26). It is interesting to note that this variable turned out to be insignificant for all types of life insurance except industrial life (where multiple-policy purchases are fairly common - at least in the UK).

Lee and Whitaker then proceed to build a simultaneous equation model in order to explain sales of ordinary, group and industrial life insurances. The parameters were then estimated by the method of three-stage least squares. The main conclusions were that there was not a great deal of inter-industry competitiveness(between the different types of life insurance). Income was found to have a positive effect on purchases while the CPI had a negative effect.

The second model of Lee and Whitaker (35) attempts to measure the income elasticity of new life insurance sums insured by fitting a time-series model of the form:

$$L = P^{\alpha} . Y^{\beta} . C^{-\gamma}$$

where

L - Life insurance purchases

P - Population

Y - Disposable personal income, and

C - Consumer Price Index.

The results confirm those of their previous study and the method throws no new light on methodology.

In his study of optimal life insurance, P. Fortune (18) develops a theoretical framework based on the expected utility hypothesis. This theoretical model implies that the optimal amount of net life insurance for protection purchases (defined as life insurance in force less policy reserves) depends on three key variables: the amount of real per capita wage and salary income, the amount of real per capita non-human wealth (W) and the real rate of discount (R). Fortune's model predicts (as do the results of Section 2.8) that the coefficient of W be negative.

The actual model tested by Fortune is linear/logarithmic using quarterly time-series data from 1964 I to 1971 IV

$$\log(NINS/Np) = A_0 + A_1 \log ICS + A_2 \log(W/Np) + A_3 \log(WAGE/Np) + A_4 \log(R-XINF) \tag{4.5.5}$$

where NINS/Np denotes the amount of real per capita net insurance;

ICS - an index of Consumer Sentiment;

W/Np - real per capita net worth;

WAGE/Np - real per capita wage income;

R-XINF - the yield on corporate bonds less the expected rate of inflation (derived from the study of M. Feldstein and O. Eckstein (14)).

Fortune used the Almon polynomial distribution lag technique (see S. Almon (1)) to capture lags in the response to changes in W, WAGE and R. Furthermore, inflation has a significantly negative effect on demand via its influence on the Index of Consumer Sentiment.

The model of Headen and Lee (22), like that of Hamburger (20) concentrates on the purchase of life insurance as part of the household's portfolio behaviour: "life insurance demand may be determined, at least partially, by household financial asset decisions". Headen and Lee (22) then proceed to use a cost model of the process of portfolio adjustment where the household is assumed to purchase a combination of four assets (Y_k): corporate stocks and bonds, money, time deposits and ordinary life insurance (measured by new sums insured). The 'target' level of any given asset is assumed to be principally dependent on variables influencing the household's portfolio behaviour (Z_i): net savings, consumer sentiment, market yields and a stock market price index. These hypotheses are combined into a model of the form:

$$Y_{kt} = \sum_{i=1}^{4} b_i Z_{it} + \sum_{j=1}^{4} a_j Y_{jt-1} + U_t \qquad k = 1, .., 4$$

where Y_{kt} denotes the consumers' holdings of asset k at time t and Z_i refer to the dependent variables described above.

The final model described in this Section is one that attempts to analyse the determinants of the purchase of life insurance by young married couples (D. Anderson and J. Nevin (2)). Information was gathered from a survey of 230 couples between 1968 and 1971. The principle results of this study suggest that middle income households purchase less life insurance than either lower or higher income households. Additionally, 'expected income' explained more variation in the amount of insurance purchased than did current income: this emphasises Friedman's view of the importance of permanent income.

4.6 Saving and Protection

There are a number of interesting conclusions to be drawn from the preceding Sections and here, we concentrate on those relating to the distinction between saving and protection in life insurance.

None of the models examined in Section 4.5 effectively distinguish between protection-based and savings-based life insurance. Thus some studies aggregated life insurance purchases as if life insurance contracts were homogeneous (eg. Houston; Hammond, Houston and Melander; Mantis and Farmer). Even the more specific studies (such as Hofflander and Duvall; Fortune; Neumann) overlooked the fact that savings-based life insurance commonly incorporates an element of protection.

It has already been pointed out near the beginning of Section 4.5 that the models of the purchase of life insurance fall into two fairly clear-cut groups: those concerned with the demand for life insurance and those explaining saving via the medium of life insurance. Both of these models need to clearly establish the difference between protection-based and savings-based life insurance because these two aspects behave in different ways and are affected by different factors.

The distinction between protection-based and savings-based life insurance is of special interest since Hofflander and Duvall (24) (and later Neumann (44)) have hypothesised that they are co-determinant. A similar argument (expressed in a slightly different form) is taken up by Lee and Whitaker (34) where the sales of one type of life insurance product (say ordinary) were possibly determined by the sales of other product types (eg. industrial and group).

It seems possible that protection-based and savings-based life insurance could indeed be substitutes as Hofflander and Duvall hypothesise. For example, it is immediately obvious from the definition of non-lifetime saving via life insurance (Section 2.3) that a decrease in either income or wealth would cause the consumer to shift his bequesting emphasis from savings-type to protection-type (because of the 'indemnity' definition of protection). Hofflander and Duvall suggest a similar response as an effect of inflation.

On the other hand, a complementary relationship between saving and protection also seems possible given that savings and protection elements are often incorporated in the same contract.

4.7 Conclusion

There are a number of important additional conclusions to be drawn from this review of the various models of consumption and saving. These conclusions relate principally to the factors, variables and models that might be applied to explain the purchase of UK life insurance.

a) Contractual versus Not Yet Contractual

Although it does not seem to be a universal approach, most studies separated the data on new contracts and existing contracts. A study of the existing contracts would therefore capture the influence of contractual saving, whereas an examination of new contracts of life insurance will have both elements of discretionary and contractual saving.

b) Measures of Saving

There seems to be controversy over how saving through life insurance should be defined (eg. see J. Kindahl (31)). Additionally there seems to be considerable disagreement on how the more basic demand for life insurance should be measured. If we are concerned with new life insurance purchases then the possible measures include:

- new premium income (eg. Neumann)
- new sums insured
- numbers of new policies
- increases in policy reserves (eg. Fortune)

Our basic concern is over consumers' expenditure on life insurance and therefore the first or the last items in the above list are the most relevant. However, both items are subject to measurement difficulties: these will be discussed in greater detail in Chapters Five and Six where an attempt will be made to modify new premium income so as to produce a more relevant measure of consumption and saving through life insurance.

c) Income

Virtually all the models of Section 4.5 include disposable income as a determinant of the purchase of life insurance. Cheng (10) implies that this should be disposable income net of contractual commitments. Other authors (eg. Anderson and Nevin (2) and Fortune (18)) note that this should be Friedman's Permanent Income. Surrey (51) notes that the source of the income may be important.

d) Variables specifically affecting Saving

Some of the models have variables that specifically aim to explain saving through life insurance rather than just life insurance purchases in general (which also include consumption). These additional variables are derived from either simple Keynesian theory (and so include income, wealth and the rate of interest) or from portfolio behaviour models (relative yields, stock market index, consumer confidence).

e) Variables specifically affecting Protection

Similarly, some models have variables that seek, principally to explain the purchase of protection-based life insurance. These include unemployment, births, marriages, number of children, age of consumer and wealth.

f) Inflation

The majority of time series models allow partially for inflation by discounting monetary values by some price index and by calculating market yields in real terms. This procedure makes the implicit assumption of zero money illusion in the Patinkin sense. However some studies allow inflation to directly affect purchases of life insurance (ie. assuming some degree of money illusion); however there seems to be disagreement on whether the rate of inflation (eg. Fortune (18)) or the price level should feature as the explanatory variable.

g) Expectations

Of course those models using permanent income as an explanatory variable automatically allow for income expectations.

Three other authors also include expectations of future prices (Hofflander and Duvall; Neumann; Fortune) however only Hofflander and Duvall, and Fortune attempt to carry these expectations into the future: Fortune attempts this by using the expected rate of inflation (rather than the current expected price level as used by Neumann).

Price expectations are mainly formulated by the 'adaptive expectations' method although survey-based expectations (similar to those generated by J.A. Carlson and M. Parkin (8)) were used by Fortune (18).

h) Results

One of the major features of the results from the preceding models (viewed as a whole) is their variability:

although most of the studies were conducted over the same time period on U.S.A. data they occasionally report widely differing results. In addition, many models indicate that sales of life insurance are very sensitive to changes in the explanatory variables.

Another feature of the models of Section 4.5 is their obvious statistical susceptibility: most models indicate considerable multicollinearity between explanatory variables as well as a significant degree of autocorrelation of residuals (in the time-series cases). Alternatively, the models have very low R^2 figures (eg. Anderson and Nevin) or have poor predictive ability.

4.8 Implications

It has already been stated that this study is concerned with an analysis of consumers' expenditure on life insurance. Later Chapters will attempt to build a demand model to explain these purchases of life insurance. As a result of this emphasis of the later Chapters, our attention will be focussed on the determinants of new life insurance acquisitions. Consequently, the models of the preceding Section, using new premium income as the dependent variable, provide the best examples of existing work on similar lines.

An analysis of the existing models has yielded much information on possible relationships and explanatory variables. This will be of great use when we come to derive our own demand model to explain the purchase of new life insurance.

The models of Section Five can be criticised as no one of them gives proper consideration to the two main reasons for the purchase of life insurance: financial saving and protection of dependents. It is important to separate these two elements since they will be affected by differing sets of factors and some individual factors may well have a positive effect on one element but a negative effect on the other. Chapters Seven and Eight are concerned with formulating a demand model of life insurance which distinguishes between the two elements: saving and protection. Additionally we will also be interested in examining the characteristics of saving via the medium of life insurance. However, before that can be accomplished, the saving and consumption elements of new office premium income must be separated and that task is undertaken in Chapters Five and Six.

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CHAPTER FIVE : CONSUMPTION AND FINANCIAL SAVING IN STANDARD LIFE INSURANCE CONTRACTS

5.1 Introduction

In Chapter Two the generalised distinction between consumption and lifetime saving was discussed and it was demonstrated that bequesting flow can be classified both as consumption and as 'non-lifetime' saving, depending on the nature of the bequest. Life insurance was included in the analysis as solely a vehicle for bequest and it was assumed that any lifetime saving was via a bank deposit.

In this Chapter, we will be mainly concerned with two extensions first, in order to examine contracts of life insurance that include elements of lifetime saving (eg. endowment and pure endowment contracts) and second, to further examine the financial savings content of all the standard life insurance contracts (temporary, whole of life and endowment).

Throughout this Chapter we will assume that the subject of the life insurance and the policyholder are one and the same person. Similarly we will not be concerned with joint-life, group-life or 'second-hand' life insurance contracts. Whenever specific mention is made of the life insurance industry the emphasis is laid on ordinary life insurance rather then industrial life.[1]

Particular attention will be paid to the difference between financial saving flows and financial savings stocks: thus when talking about saving flow we are interested in that proportion of the life insurance premium that constitutes financial saving flow. When referring to savings stocks, we mean the capitalised built-up value of saving inflow held by the insurer in trust for the policyholder.

The objectives of Chapter Five are two-fold: first, to develop a formularisation of the financial savings/consumption distinction for life insurance premiums and funds. Second, to develop the saving/consumption distinction so that it may be applied to collective life insurance data. Chapter Six will be devoted to a description of the available life insurance data along with its actual breakdown into savings and consumption elements using the results of this Chapter.

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[1] The terms joint-life, group-life, ordinary and industrial are further explained in R.L. Carter (ed) (3). The second-hand market for life insurance is described by J. Gaselee (5).

5.2 PIA Saving

At this point it seems appropriate to introduce a new definition of the type of saving that can result when there is a time lag between purchase and consumption. We shall say that Payment in Advance (PIA) Saving occurs when payment (ie. expenditure on specific goods and services) is in advance of anticipated future consumption. Thus saving arises not from any positive decision (on the part of the individual) to set money aside himself but from the method of payment for future consumption (which involves acquiring proprietorial rights). The other main form of saving, derived from a conscious voluntary decision to set money aside, out of income, to cover some form of future expenditure will be called 'free' saving.

Payment in Advance Saving has several peculiar characteristics: first, while PIA saving can arise from a voluntary and intentional decision on the part of the individual to purchase some time ahead of consumption (as for example, in the building up of stocks/stores of commodities), it can also arise unintentionally. Thus unintentional PIA saving often results from an institutionalised pattern of payments (in advance of consumption) that are beyond the control of the individual.

Secondly, the main difference between PIA saving and all other types of saving is that PIA saving occurs through the process of making a purchase: the resultant stock of 'savings' are thus tied up in goods and services and not in terms of money and other fairly liquid financial claims (the money being transferred to the vendor of the goods and services).

Thirdly, the whole of PIA saving is unintentional to the extent that the primary objective of the individual lies in consumption and not in saving as a means of allocating the flow of Disposable Income. Thus PIA saving is not trading in commodities: goods and services are bought to hold and ultimately to consume.

Consequently, although theoretically defined as 'saving', PIA saving will not be viewed as such by the individual but as expenditure (as if for current consumption). Another aspect of PIA saving is that it will always be followed by an equal and opposite amount of dis-saving if we take a time span long enough for the completion of consumption.

Finally, if we carry the argument of payment in advance of consumption to its logical extreme, then it would seem that most transactions would involve PIA saving because very few goods or services are 'currently consumed' to the extent that consumption starts instantaneously after purchase. In Keynesian macroeconomics the problem does not arise because emphasis is placed on consumption expenditure rather than on consumption. However, an acceptable alternative is to replace 'instantaneously after purchase' with something like 'within a reasonable period of purchase'. The problem then boils down to defining this reasonable period after purchase: obviously the length of this period would depend on the commodity in question. Consequently, any expenditure on a good or service, not consumed within a 'reasonable period' would constitute PIA saving. So, we may split income flows in the following way:

Income flow	=	Consumption expenditures (within a reasonable period)	+	Consumption expenditures (not within a reasonable period) (PIA saving)	+	Free saving

The second main element in the flow of personal saving has already been introduced - Free saving; this arises from a decision to set money aside out of the flow of income in order to build up a stock of savings - held in the form of financial claims.(2)

Although there are many different ways of subdividing free saving (for example, according to the liquidity of the resultant stock of savings), it is convenient (when later thinking about applications to insurance) to classify by uses of the savings stock. Thus, money from the flow of income may be withdrawn completely from the economy by the saver and held as coin and notes. Alternatively, the saver may intend to hold the equivalent of notes and coin (where the interest yield to the saver is zero) but in fact, the money is returned to the circular flow (eg. through the use of a current account in a joint-stock bank). Thirdly, the non-consumption expenditure income

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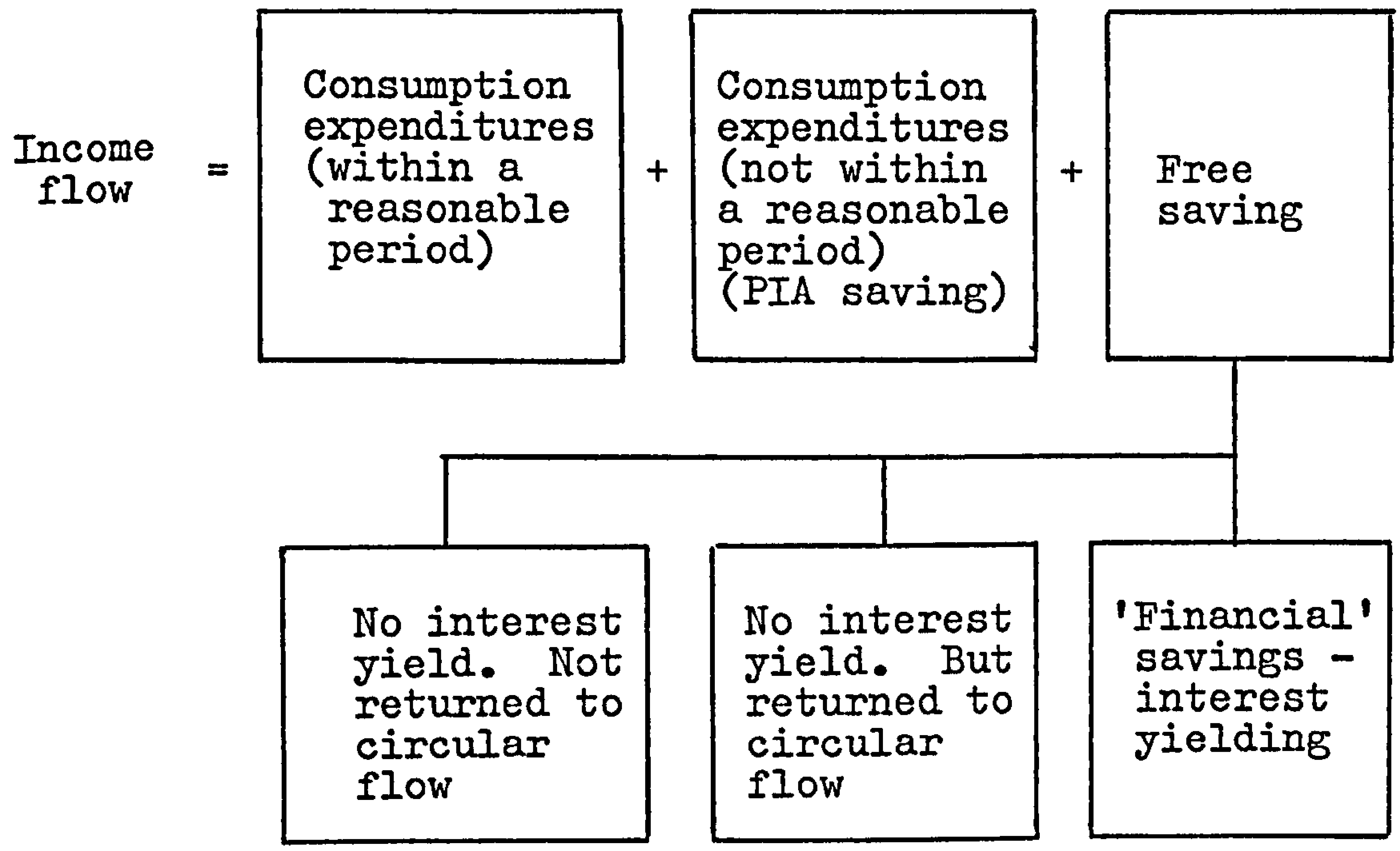
(2) A financial claim can be defined as 'a claim to the payment of a future sum of money and/or a periodic payment of money'. J. Revell (25) Ch.2.

may be returned directly to the economy, by the direct or indirect purchase of non-money financial claims; in this case the saver can expect to receive a positive money interest return on his savings. In future, this last element will be referred to as 'Financial' saving. We can sum up this Section in the following form:



Finally, we note from Chapter Two that Financial saving can be divided into two further forms: 'lifetime' and 'non-lifetime'.

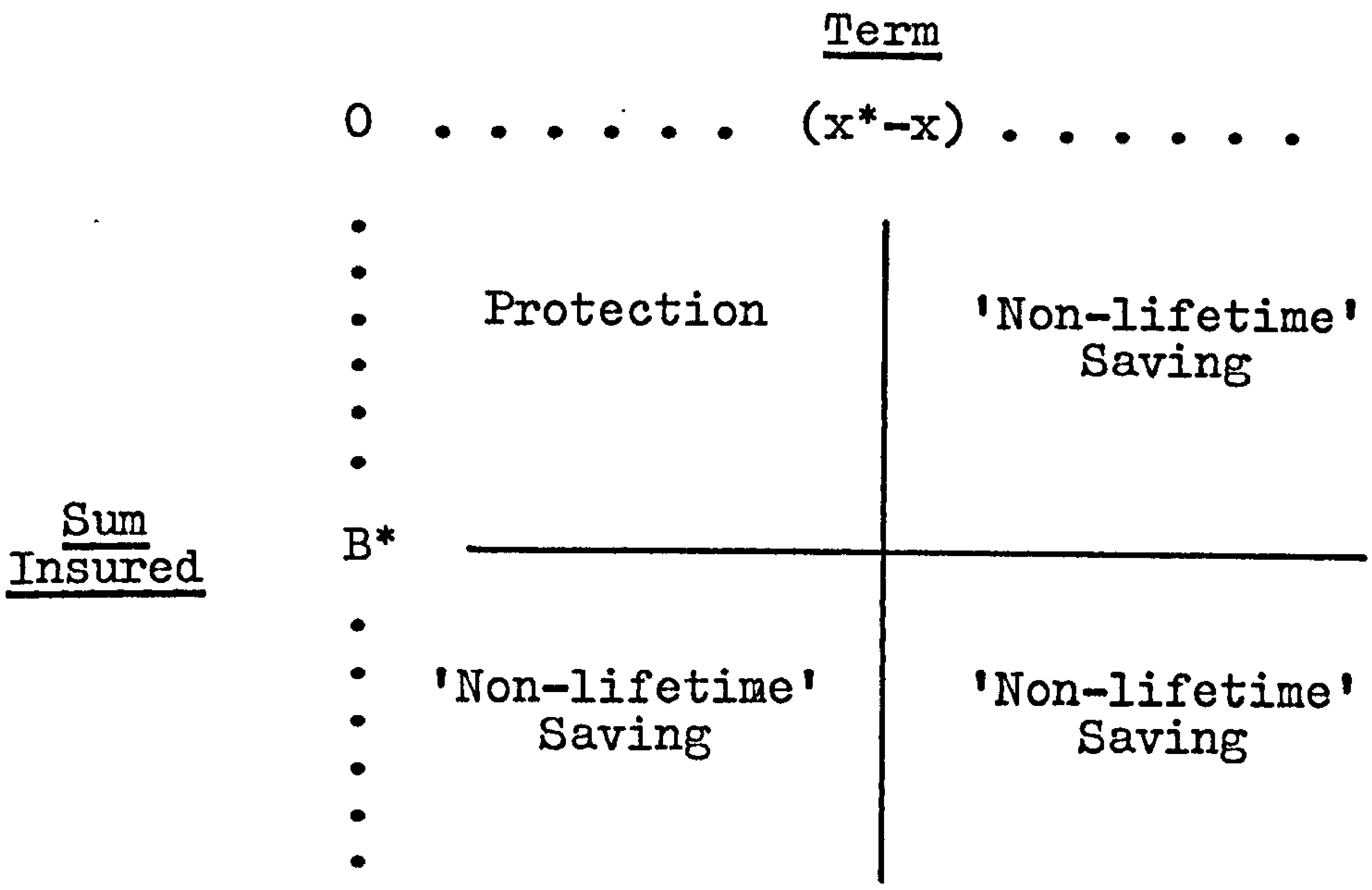
5.3 Temporary Life Insurance

Under a temporary life insurance contract, the payment of benefits to the policyholder occurs only if the policyholder dies within a prespecified time period (called the 'term' of the policy and assumed in general to be n years). Thus temporary life insurance is ideally suitable for the provision of protection-type bequests since the time-scale problem of Section 2.3 can be solved by equating n with (x^*-x) where x is the inception age of the policyholder and x^* is the age after which no protection for dependents is required.

Unlike the situation portrayed in Chapter Two however, temporary life insurance is normally arranged on a level premium basis so that all the premium instalments are identical. We will also assume that all temporary life insurances have a fixed sum insured. This assumption is particularly necessary when examining endowment life insurance although, in practice, it is quite common to find sums insured that decline throughout the term of the temporary life policy.

As far as temporary life insurance is concerned the following table summarises the protection/savings distinction:

Figure 5.3.1. The Structure of Temporary Life Insurance



where B* is the upper bound on the sum insured for protection purposes.

However, it will be assumed for the future that temporary life insurance is not purchased for a term exceeding (x*-x) (so dispensing with the North-east and South-east quadrants). This assumption is not such an unreasonable one because:

- it seems unlikely that the need for a saving-type bequest will cease after age x + n;
- it does not seem sensible that the policyholder pays money for a saving-type bequest that may not be made (if he lives beyond age x+n) (this is the 'survival risk' of Section 2.5) ;
- there are other less risky mediums (from the

point of view of the preceding comment) for saving-type bequests (again, see Section 2.5).

Additionally, it will be assumed again that temporary life insurance is not purchased for a sum insured in excess of B*. This assumption, while not necessarily having any strong theoretical justification, is a useful simplifying one:

- even if the bequest level (ie. sum insured) were allowed to vary with age, the need to deal with a level premium makes further analysis very difficult if the type of bequest varies from age to age. One approach however, would be to allocate a part of each premium (independent of the size of premium) to saving corresponding to the relationship between the bequest (B) and B* ;
- the assumption of a constant sum insured makes the situation very difficult if it is not assumed that the sum insured is less than B*(x) for all relevant ages x;
- again it does not seem sensible that the policyholder pays money for a saving-type bequest that may not be made ;
- again there are less risky alternatives available.

Essentially, the above points concerning the validity of temporary life insurance for saving-type bequests relate back to Section 2.5. These points can be taken to mean that in the Friedman and Savage analysis of that Section the policyholder is more risk-averse in relation to bequests for savings purposes (ie. risk-averse in the sense that the policyholder dislikes the possibility that the intended provision for bequest will be 'wasted' if he survives). Thus it is more probable that a bequest for non-lifetime savings purposes will be made via a bank deposit rather than by using temporary life insurance.

The effect of the above assumptions is to preclude the purchase of temporary life insurance for any other reasons than for the provision of protection for heirs and dependents.

Additionally we note from Section 5.2 that as the payment of benefits under temporary life insurance is conditional on the occurrence of death within the term of the policy, that policy cannot be considered as a financial claim:

> "For a contractual right to receive a future payment, either of a lump sum or of periodic interest, to rank as a financial claim the promise to pay must be unconditional."
> (Revell (15) p. 31)

Thus the purchase of a temporary life insurance policy cannot be termed 'Free' saving and this reinforces our intuitive allocation of temporary insurance to the provision of protection.

In summary we can conclude that any temporary life insurance premiums paid by the policyholder are for protection purposes and therefore count as consumption. Thus since temporary insurance is the basic building block of most other life insurances, it is only necessary to separate out the temporary insurance element in order to ascertain that part of a particular life insurance contract that provides protection: the remainder therefore counts as saving.(3)

However, there is one further issue that arises especially if the term of the temporary contract exceeds one year. The above analysis and that of Chapter Two have allocated the entire temporary life premium to consumption irrespective of the term of the contract. If we assume a level premium system for simplicity (although the same arguments apply for non-level premiums) then, in most years, the actual risk premium (to cover the risk of death in that year) will differ from the overall level premium: in the early years of the contract, the level premium (which is a weighted 'average' of the risk premiums) will exceed the premium for a one-year temporary policy while in the later policy years, the reverse applies. Thus (in the early years) only a part of the level premium actually covers the risk of death in that year and it has been argued (eg. by Neumann (14), H. Levy and J. Kahane (10) and Kindahl (9)) that the balance must therefore represent saving. However this is seen to be unhelpful for two main reasons:

- because payment to cover the risk of death is not necessarily identical to payment for protection in any one year;
- the balance between the level premium and the rising annual premium can be regarded as PIA

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(3)The same basic approach is used by other authors such as S. Neumann (14) and J.K. Kindahl (9).

saving and is therefore more akin to consumption than financial saving.

5.4 PIA Saving and Temporary Life Insurance

Although it has been demonstrated that, in practice, Free Saving is unlikely in a temporary life insurance contract, Payment in Advance Saving does occur:

> "But the term premium includes an element of saving as well. Since the probability of death increases with age over a larger range of ages, the annual premium ought to rise in accordance. Instead, insurers divide the premium equally over the whole period to charge a level premium. As a result, the policy-holder pays in excess of the true premium in the first years of the policy and rather less in the subsequent years."(4)

We can represent the above effect for a level premium temporary life insurance pictorially in Figure 5.4.1. (5)

We assume that the premiums in Figure 5.4.1 are actuarially fair and also that the rates of interest and mortality used in the premium calculation are equivalent to those actually experienced by the life office. Note that if the imputed rate of interest were zero then the two diagonally shaded areas would be equal in size.

The picture for any generalised temporary life insurance with non-constant annual premium is more complex, but has the basic similarity that the shaded area above the rising one-year premium line is equal to the area below at zero rate of interest (see Figure 5.4.2). However, there is little to be gained from this complication and it will hence be assumed that all premiums are paid on the level annual premium basis.

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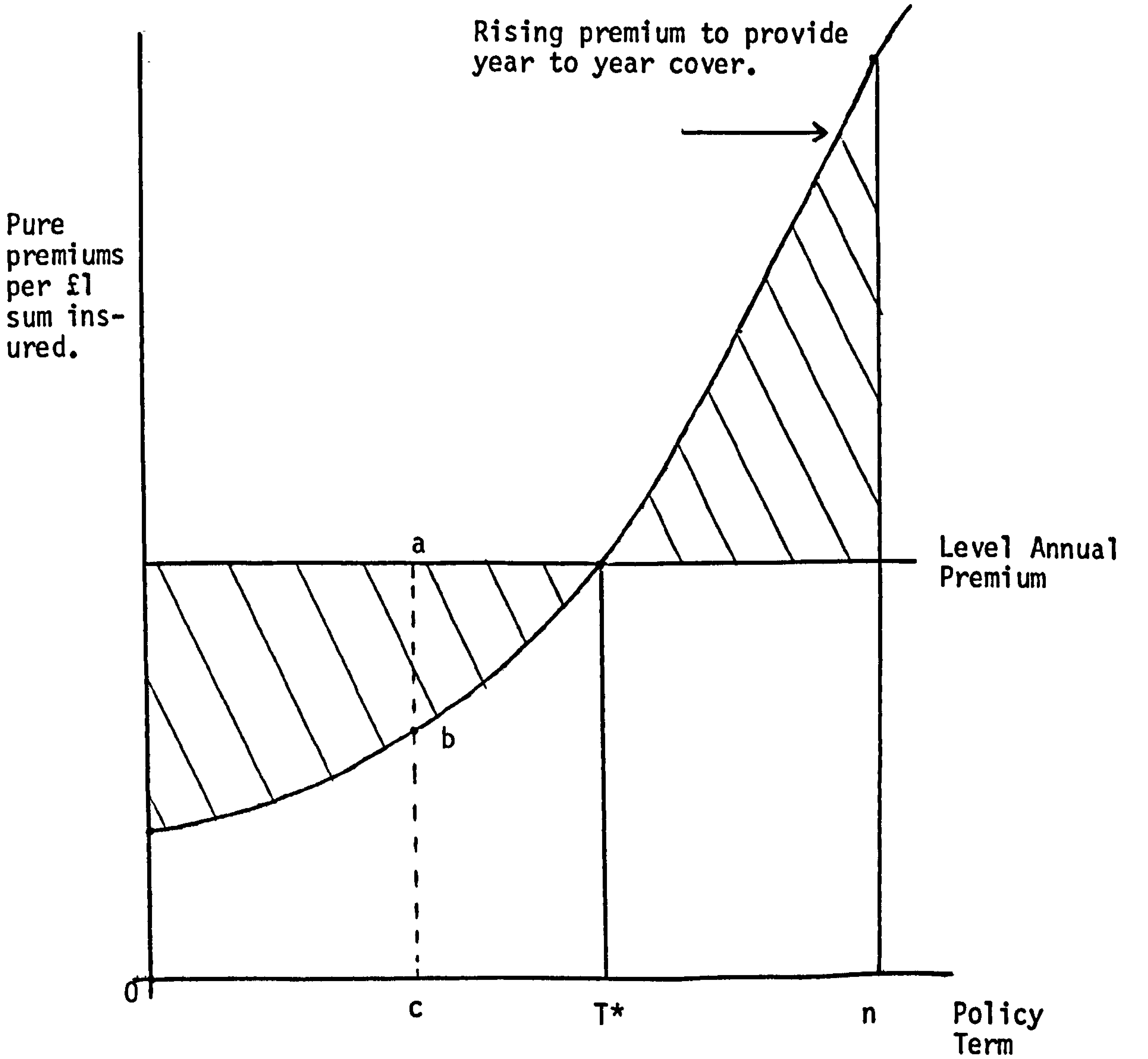
(4) Levy and Kahane (9)

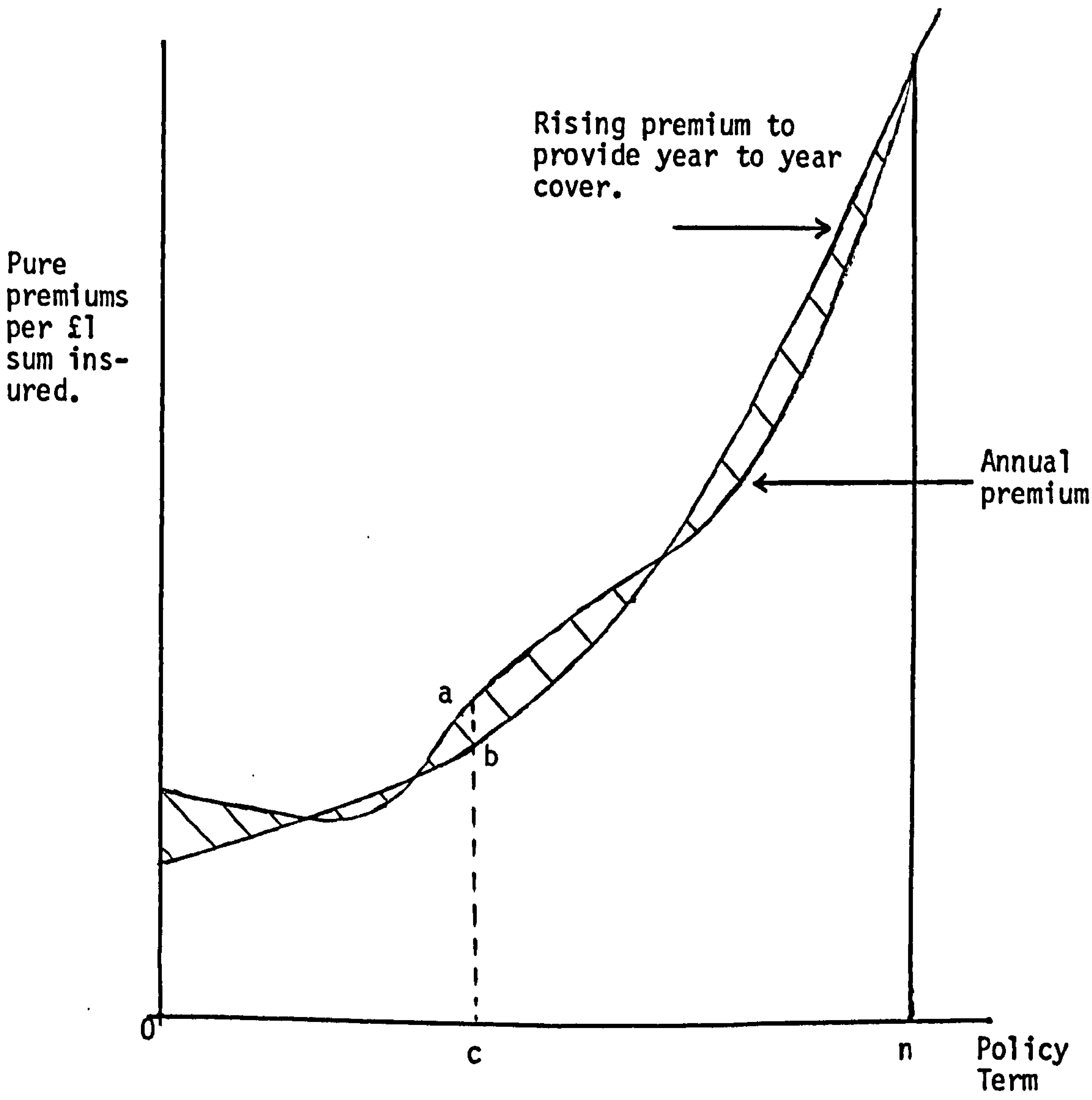
(5) See G. Clayton and W. Osborn (4) p.46

Figure 5.4.1 Temporary Life Insurance Level Premium



Source : Clayton and Osborn (4) p46

Figure 5.4.2 Temporary Life Insurance Non-Level Premiums



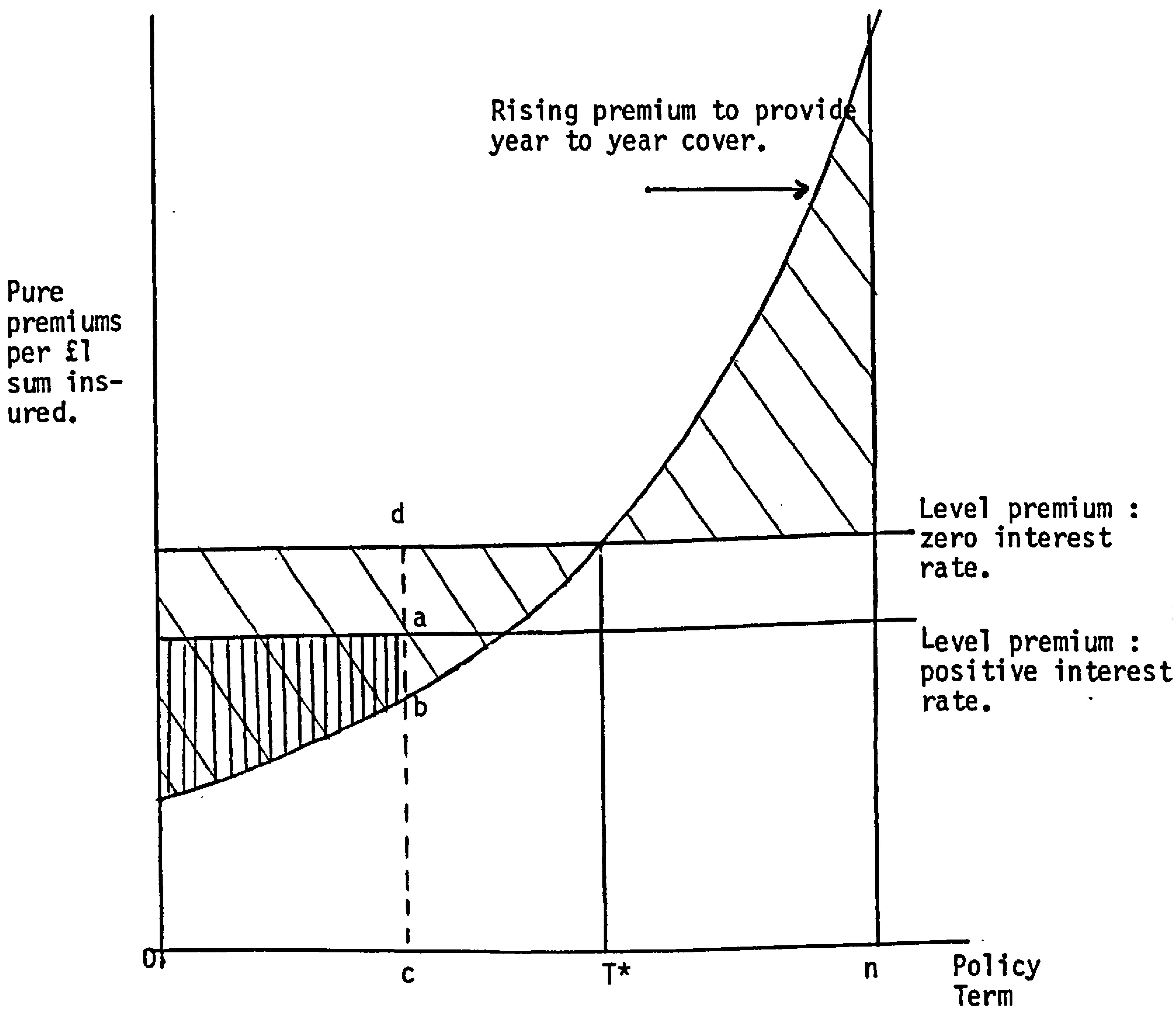
Now, using our definition of PIA saving it is obvious that in Figures 5.4.1 and 5.4.2, PIA saving constitutes that part of consumption expenditure (ac) that is not consumed within the (undefined) reasonable time period (ie. ab). Similarly when the rising one-year premium exceeds the level premium, PIA dissaving occurs.

The position becomes a little more complicated when we come on to consider the PIA savings stock at time oc. If we assume that the imputed rate of interest is equal to zero, then the PIA savings stock at time oc is given by the shaded area to the left of the vertical line ac. PIA saving continues till time oT*, where the stock of PIA Savings held in trust by the life office for the policyholder is at a maximum. After time oT*, PIA dissaving commences until, at the end of the policy term, the stock of PIA avings is exhausted.

Complications arise when the restrictive assumption of zero rates of interest is relaxed. The resultant situation is pictured in Figure 5.4.3: as soon as a positive rate of interest is introduced into the level pure premium calculation the resultant PIA savings stock cannot be determined diagrammatically. This situation arises because the PIA savings stock collected from the policyholder (given by the vertically shaded area to the left of the line ab, and to be called 'Policyholders PIA Savings Stocks') is then supplemented with interest earnings to give the total PIA savings stock held by the life office (to be called 'Office PIA Savings Stocks'). This situation is examined in Figure 5.4.4.(6)

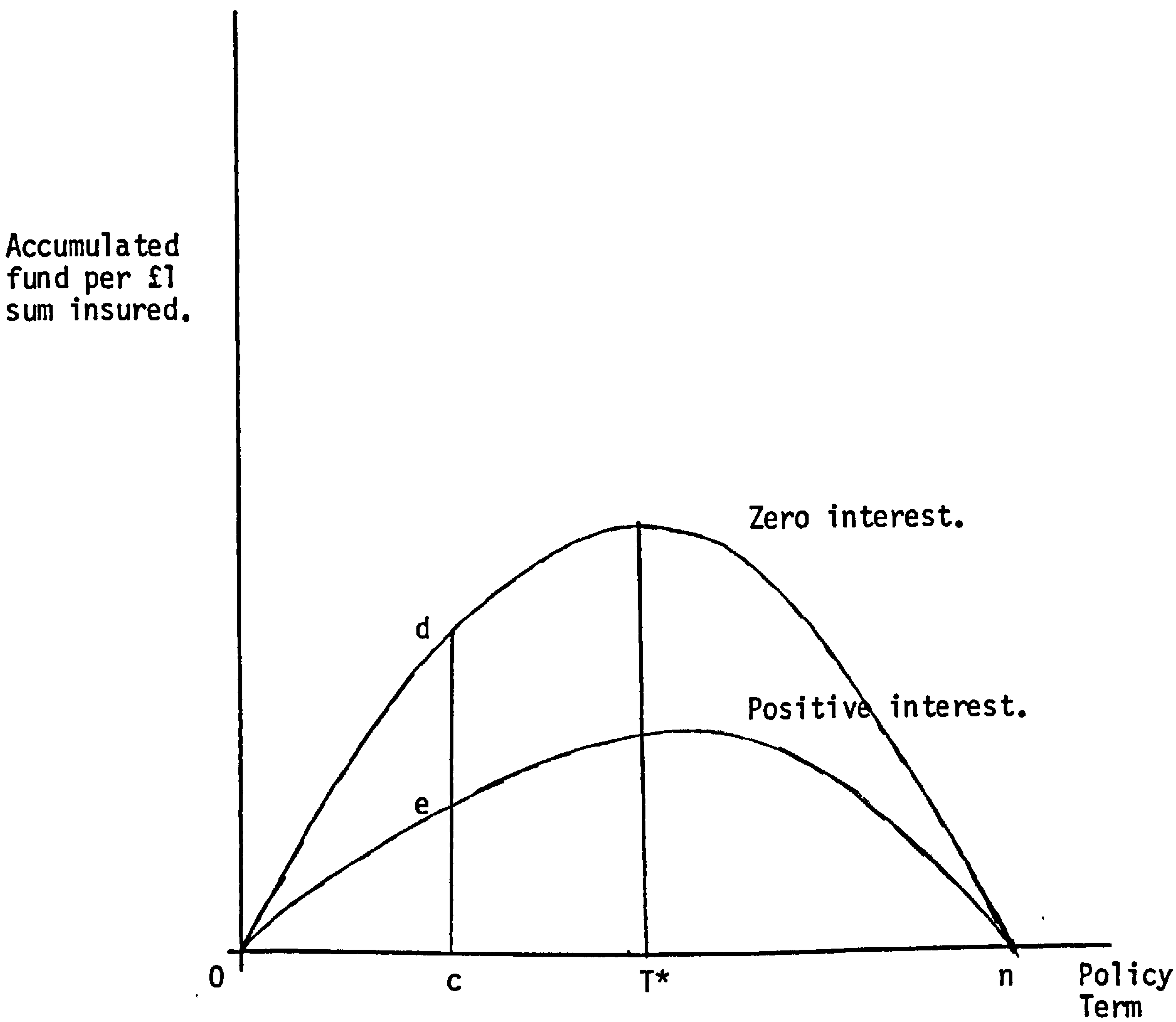
(6) See Clayton and Osborn (4) p.48

Figure 5.4.3 Temporary Life Insurance Premiums (Non-Zero Rates of Interest)



Note : For simplicity, the rising premium schedule is shown unaltered by the difference in interest rates.

Figure 5.4.4 PIA Savings Stocks (Temporary Insurance)



Source : Clayton and Osborn (4) p48

From Figure 5.4.4 the accumulated stock of PIA savings at time oc is given by cd in the case of accumulation without interest (corresponding to the diagonally shaded area to the left of the vertical line cd in the Figure 5.4.3) and ce in the case of accumulation with a positive rate of interest.

It is immediately obvious that the PIA savings stock at time oc is equivalent to the reserve value of the temporary life insurance at time oc. Since PIA savings stocks arise out of an accumulation of past PIA premium saving flow, the retrospective reserve valuation method should most properly be used to calculate them.

Let $P'_{x:\overline{n|}}$ be the level annual premium for a temporary life insurance of unit sum insured, policy term n years, at inception age x. Then using the standard mortality table notion, we can say that at inception, there were l_x policyholders, d_x of whom will die before age x+1.

Let $v = (1+i)^{-1}$ where i is the imputed calculation rate of interest and suppose that time oc is t years after inception.

The rising premiums to provide year to year cover, say k years after inception, is then given $(v.d_{x+k}/l_{x+k})$, $0 \leq k \leq n$ where premiums are assumed paid at the beginning of each year and claims at the end.

Then the total PIA saving flow from the l_{x+k} policyholders alive at age (x+k) is given by:

$$l_{x+k} . \left(P'_{x:\overline{n|}} - \frac{v.d_{x+k}}{l_{x+k}}\right)$$ per unit sum insured.

Compounding this flow for t years gives the PIA savings stocks for each policyholder alive t years after inception:

$$\frac{1}{l_{x+t}} \cdot \sum_{k=0}^{t-1} l_{x+k} \cdot (P'_{x:\overline{n}|} - v.d_{x+k}/l_{x+k}).(1+i)^{t-k} \tag{5.4.1}$$

Multiplying both numerator and denominator of Equation (5.4.1) by v^{x+t} gives

$$\frac{1}{v^{x+t}l_{x+t}} \cdot \sum_{k=0}^{t-1} (v^{x+k}.l_{x+k}.P'_{x:\overline{n}|} - v^{x+k+1}.d_{x+k}) \tag{5.4.2}$$

Equation (5.4.2) is identical to the reserve value of a temporary life insurance policy (denoted by ${}_tV'_{x:\overline{n}|}$) calculated retrospectively. This can be rewritten using standard commutation function notation to obtain(7):

$${}_tV'_{x:\overline{n}|} = \frac{1}{D_{x+t}} \left(P'_{x:\overline{n}|}.(N_x-N_{x+t}) - (M_x-M_{x+t})\right) \tag{5.4.3}$$

To make the above analysis slightly more realistic, we must allow for the fact that the rates of interest and mortality normally used by the life office to calculate the pure premiums are fairly conservative. Thus in real life, the 'calculation' rate of interest is commonly lower than the 'true' rate earned on invested life office funds; similarly the 'calculation' rates of mortality are commonly

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(7)See P.F. Hooker and L.H. Longley-Cook (6) p.110

more conservative than the 'true' rates. Until recently it has been the practice to regard mortality as generally improving over time for all ages, however the most recent mortality tables issued by the Institute of Actuaries (8) demonstrate that for some ages (notably males between 17 and 22) this is no longer the case.

If the 'calculation' rates of interest and mortality (denoted by the presuperscript C) were conservatively chosen then the level calculation pure premium will be higher than necessary in order to cover future liabilities. Then from Equation (5.4.1), the PIA savings stocks at time t become:

$$\frac{1}{{}^{T}l_{x+t}} \sum_{k=0}^{t-1} {}^{T}l_{x+t} \cdot ({}^{C}P'_{x:\overline{n}|} - {}^{T}v \cdot {}^{T}d_{x+k} / {}^{T}l_{x+k}) \cdot (1+{}^{T}i)^{t-k} \tag{5.4.4}$$

where, of course, the rising premium is calculated using the true rates of interest and mortality (denoted by presuperscript T).

Alternatively, Equation (5.4.4) can be expressed as:

$${}^{T}_{t}V'_{x:\overline{n}|} = \frac{1}{{}^{T}D_{x+t}} \cdot ({}^{C}P'_{x:\overline{n}|} \cdot ({}^{T}N_{x} - {}^{T}N_{x+t}) - ({}^{T}M_{x} - {}^{T}M_{x+t})) \tag{5.4.5}$$

which can be rewritten as

$${}^{T}_{t}V'_{x:\overline{n}|} = ({}^{C}P'_{x:\overline{n}|} - {}^{T}P'_{x:\overline{t}|}) \cdot {}^{T}\ddot{a}_{x:\overline{t}|}) \tag{5.4.6}$$

where $$ {}^{T}\ddot{a}_{x:\overline{t}|} = \frac{{}^{T}N_{x} - {}^{T}N_{x+t}}{{}^{T}D_{x+t}} $$

It is obvious that, at the end of the policy term (ie. when t = n), PIA savings stocks are not equal to zero when true and calculation rates of interest and mortality are unequal. In particular, if the calculation rates are conservatively chosen then obviously ${}^{T}_{n}V'_{x:\overline{n}|} > 0$ (since ${}^{C}P'_{x:\overline{n}|} > {}^{T}P'_{x:\overline{n}|}$)

The above analysis shows up an important point: that by not using true rates of interest and mortality to calculate pure premiums, the life office is deliberately overcharging the policyholder on the pure premium basis. However it is also important to examine the reasons for this overcharging. In practice, it seems likely that life offices deliberately overestimate pure premiums <u>in order to provide for</u> surplus, expenses, shareholders' profit and a small safety margin to cover unexpected fluctuations. This does not appear to be a logical procedure since surplus, expenses and shareholders' profit can be covered by the loadings in the Office Premium.(8) It will be subsequently assumed that in calculating level <u>pure premiums</u>, life offices choose calculation rates of interest and mortality that are equal to the best available forecast of true (long term) rates of interest and mortality at the time of premium calculation. Effectively, this is defining the pure premium as that premium just

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(8) It is not current actuarial policy to specifically include a loading to cover fluctuations. This is done by making conservative assumptions on interest and mortality. I am indebted to Professor R.E. Beard for this point.

sufficient to cover future liabilities to policyholders using the current expected value of long term future interest and mortality rates. Any allowance or safety margin for deviation between expected and actual rates is therefore assumed to be included in the Office Premium and not the Pure Premium. Consequently, since ${}^{T}_{t}V'_{x:\overline{n}|}$ is the policy reserve based on the pure premium (ie. with no loading to cover expenses, profit or a safety margin), any excess PIA savings stocks at the end of the policy term are purely fortuitous and cannot in any way be regarded as belonging to the policyholder. We will therefore assume that any excess PIA savings stocks are not returned to the policyholder in any way but contribute to what is sometimes known as the life office's 'Estate'. This assumption seems reasonable since, because calculation rates are equivalent to the expected future rates of interest and mortality, the life office is subject to deviation in both directions (so that there is also the possibility of negative PIA savings stocks). As we would not expect the life office to recover any negative PIA savings stocks from the policyholder it seems only fair that any excess should not go to the policyholder either.

One consequence of the above assumption is that pure premiums must be regarded as time-specific in the sense that the expected long term future rates of interest and mortality change with time. Thus when dealing with time-specific problems, it is not possible to continue with the current industry-wide practice of using out-of-date mortality tables and conservative interest assumptions since the calculation pure premiums so generated are not truly pure but include an element of loading. In Chapter Six, when time-specific calculation pure premiums are needed, these will be generated with the use of reasonable assumptions about long term future interest and mortality rates made at the time of calculation.

Throughout this Section on temporary life insurance, PIA savings generated by pure premiums only have been considered: ie. no allowance has been made for the difference between pure premiums (as defined) and office premium (the amount actually paid by the policyholder). In fact the loadings for expenses, profit and safety margin do not alter the amount of PIA saving/s inherent in a temporary life insurance policy. This is because the payment of the loading by the policyholder is widely regarded as consumption within a 'reasonable' time period:

> "The estimates of consumers' expenditure each year on all forms of life assurance are taken to be the sum of the expenses of management and shareholder's surplus appropriate to life business ..."
> (R. Maurice (12))

and

> "In so far as personal saving consists of individuals' abstentions from consumption out of income it is clear that contributions paid to life insurance companies and pension funds are part of current savings. But it would be wrong to assume that the total saving arising in this way can be calculated by simply adding all insurance premiums and superannuation payments in a given period. The portion of such premiums or contributions which are used to pay the costs of administration must be allocated to consumption."
> (Clayton and Osborn (4) p.26)

5.5 Endowment Life Insurance : Saving Flow Theory

Under an endowment contract the benefits are paid either on survival of the life insured to the end of the policy term or upon earlier death. Although there is no a priori reason why the death benefit and the survival benefit should be equal, they will be treated as such for the rest of this analysis: in any case most endowment business in the UK is undertaken in this fashion.[9]

Thus the n-period non-participating endowment is just a combination of an n-period temporary life insurance and a fixed lump sum paid out if the policyholder survives to the end of the policy term. This fixed sum represents lifetime saving stocks while the premium contribution towards it is simply lifetime saving flow. The non-participating endowment life insurance is thus split into two parts: protection against premature death (ie. consumption) and lifetime financial saving.

If we assume that the lump sum survival benefit is fixed[10] then it automatically follows that the protection part of the level pure premium is exactly equal to the corresponding temporary life insurance pure level premium. Any PIA savings stocks will then be associated

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(9) In fact, endowment life insurance is commonly defined to have equal death and survival benefits. For example, from P.D. Bacon and L.J. New (2):

> "The sum assured is payable at the expiration of a certain period or at earlier death."

(10) This assumption is certainly in accord with the normal method of calculating the endowment premium, eg. see Hooker and Longley-Cook (6).

only with the corresponding temporary insurance and can be treated in exactly the same way as before. Thus the lifetime saving part of the level endowment pure premium is just given by $\frac{S.A_{x:\overset{1}{\overline{n|}}}}{\ddot{a}_{x:\overline{n|}}}$ where $A_{x:\overset{1}{\overline{n|}}}$ is the symbol for the value of (or the single premium for) a 'pure endowment' without return on death within the policy term and S is the sum insured.

However, a non-participating endowment life insurance can also be regarded as a combination of a decreasing temporary life insurance and an increasing lump sum. This viewpoint is taken by Gilling-Smith (3) Section 2.1.2, Walter Williams (17) and M.A. Linton (11):

> "because of internal savings resulting from combining the protection and investment elements in one contract, the charge for the (temporary) insurance element is lower than the rates on a separate policy."

This view will be ignored in preference to the more common constant lump sum version for several reasons:

- the saving/protection combination of each level premium would then not be constant over the policy term;
- as explained in Note (9) above, the method of calculating an endowment premium assumes a constant lump sum survival benefit;
- in the matter of savings stocks, the two methods will produce the same results;
- the former version is much simpler when it comes to dealing with actual endowment insurance premium statistics.

Thus we can say that the corresponding temporary life insurance pure premium $P'_{x:\overline{n|}}$ constitutes the consumption expenditure element emanative in the level non-participating endowment life insurance pure premium $P_{x:\overline{n|}}$. The proportion of non-participating endowment pure premium contributing towards financial lifetime saving flow is then given by:

$$f = 1 - P'_{x:\overline{n|}} / P_{x:\overline{n|}} \tag{5.5.1}$$

and using the conventional commutation table notation, this can be rewritten as:

$$\begin{aligned} f &= 1 - \frac{M_x - M_{x+n}}{M_x - M_{x+n} + D_{x+n}} \\ &= \frac{D_{x+n}}{M_x - M_{x+n} + D_{x+n}} \end{aligned} \tag{5.5.2}$$

where x is the age at inception and n is the policy term.

One further complication in dealing with financial saving flow arises when we consider the loadings that are appended to the level non-participating pure premium in order to obtain the Office Premium.

The level premium per unit sum insured paid by the policyholder to the life office can be split up in the following manner:

Office Premium = Non-participating Pure Premium + Bonus Loading + Expenses Loading. (5.5.3)

This can be further subdivided by splitting the non-participating pure premium into its component parts:

Pure Premium = Consumption Expenditure

+ Financial Lifetime Saving

where 'Consumption Expenditure' is provided by the temporary life insurance element ($P'_{x:\overline{n}|}$) and the financial lifetime saving denotes the return of the capital (lump sum) element.

It has already been explained that the expenses loading of office premium counts as part of consumption expenditure. It is not clear, however, in the light of Chapter Two, whether the bonus loading on premium should be treated entirely as financial saving. Certainly if the basic sum insured is more than adequate to provide protection for dependents (by the indemnity principle of Section 2.3) then the bonus loading can be regarded as financial saving. However there are certain forms of endowment where the sums insured provide an inadequate level of protection with reliance on bonuses to make up the difference(11). In these cases, it would seem that part at least of the bonus loading must be counted as consumption expenditure. However, in spite of this obvious counter example, it will be assumed that the bonus loading counts entirely as financial saving flow:

- if it is known that a certain proportion of the loading is classed as consumption then the

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(11)These policies are variously termed 'low-cost', 'build-up' and 'bonus-reinforced'.

ensuing theory can easily be altered to allow for this;

- since bonuses cannot be guaranteed to the policyholder as of right (ie. there is some default risk involved) it would seem logical that the consumer with a risk-averse utility of bequest function would prefer not to include bonuses as part of his bequest for protection purposes. This is because it is likely that the consumer would be more (default) risk-averse in relation to bequest than lifetime saving;
- the rationale for the purchase of a participating life insurance is usually couched in savings terms. For example, from 'The Book of Money' (16):

  'Regular payments make life assurance easy saving, with-profit payments make it profitable saving';
- although bonus reinforced policies are now quite popular their introduction was quite recent and therefore will not affect the main bulk of premium statistics of Chapter Six.

Consequently the office premium can be divided up as follows:

Office Premium = Consumption Expenditure
+ Financial Saving flow

where

Consumption expenditures = Expenses loading $+ P'_{x:\overline{n}|}$

and

Financial saving flow = $(P_{x:\overline{n}|} - P'_{x:\overline{n}|})$ + Bonus loading

It is therefore possible to determine that the proportion of financial saving in a level endowment life insurance office premium is given by:

$$\phi_{\alpha,\beta} = \frac{(P_{x:\overline{n}|} - P'_{x:\overline{n}|}) + BL}{P_{x:\overline{n}|} + BL + EL} \qquad (5.5.4)$$

where BL is the Bonus Loading and EL the Expenses Loading.

Equation (3.5.4) may be rewritten as

$$\phi_{\alpha,\beta} = \frac{f + \alpha}{1 + \alpha + \beta} \qquad (5.5.5)$$

where f is defined by Equation (5.5.1), $\alpha = \frac{BL}{P_{x:\overline{n}|}}$ and $\beta = \frac{EL}{P_{x:\overline{n}|}}$.

In the next Section, the behaviour of the financial saving ratios f and ϕ will be examined in greater detail; however it is possible to make some preliminary comments on Equations (5.5.1) and (5.5.4):

- the financial saving ratios are obviously affected by the determinants of the pure premium viz. mortality, interest, inception age and policy term. The next Section briefly investigates how these factors influence the ratios;

- in addition, the ratio $\emptyset$ is also affected by the Bonus and Expenses Loadings. Obviously the greater the Bonus Loading the larger the value of $\emptyset$ and the larger the Expenses Loading the smaller the value of $\emptyset$.

Finally, we note that the financial saving flow (out of endowment office premiums) is not the same as expenditures on saving-based life insurance. This is because expenditures on saving-based endowment life insurance must include some proportion of the endowment expenses loading (similarly expenditure on protection-based endowment life insurance is not synonymous with consumption expenditures). This distinction is necessary when it comes to an examination of the reasons for purchasing life insurance (Chapters Seven and Eight) when, in order to build a demand model, the expenditures on savings-based and protection-based life insurance must be differentiated (each with its share of the expenses loading).

It seems reasonable to allocate the expenses loading in proportion to the size of $P'_{x:\overline{n}|}$ relative to the Office Premium, ie.:

Expenditure on protection-based endowment life insurance $= P'_{x:\overline{n}|} + \frac{P'_{x:\overline{n}|}}{OP} \cdot EL$ (5.5.6)

and

Expenditure on savings-based endowment life insurance $= P_{x:\overline{n}|} - P'_{x:\overline{n}|} + BL + \frac{(OP - P'_{x:\overline{n}|})}{OP} \cdot EL$ (5.5.7)

5.6 Endowment Life Insurance : Saving Flow Behaviour

Figure 5.6.1 shows how the financial saving ratio f varies with inception age x for different rates of interest and policy terms. The initial simplifying assumption of zero Bonus and Expenses Loadings will be dropped in due course.

Figure 5.6.1, like all other diagrams in this Section, is based on the Tables of Mortality still most commonly used for premium calculations (7). The mortality experience expressed in these Tables dates from the period 1949-1952 and they are therefore very out-of-date. The new Mortality Tables published by the Institute of Actuaries (8) have recently begun to supercede the A1949/52 Tables (7).

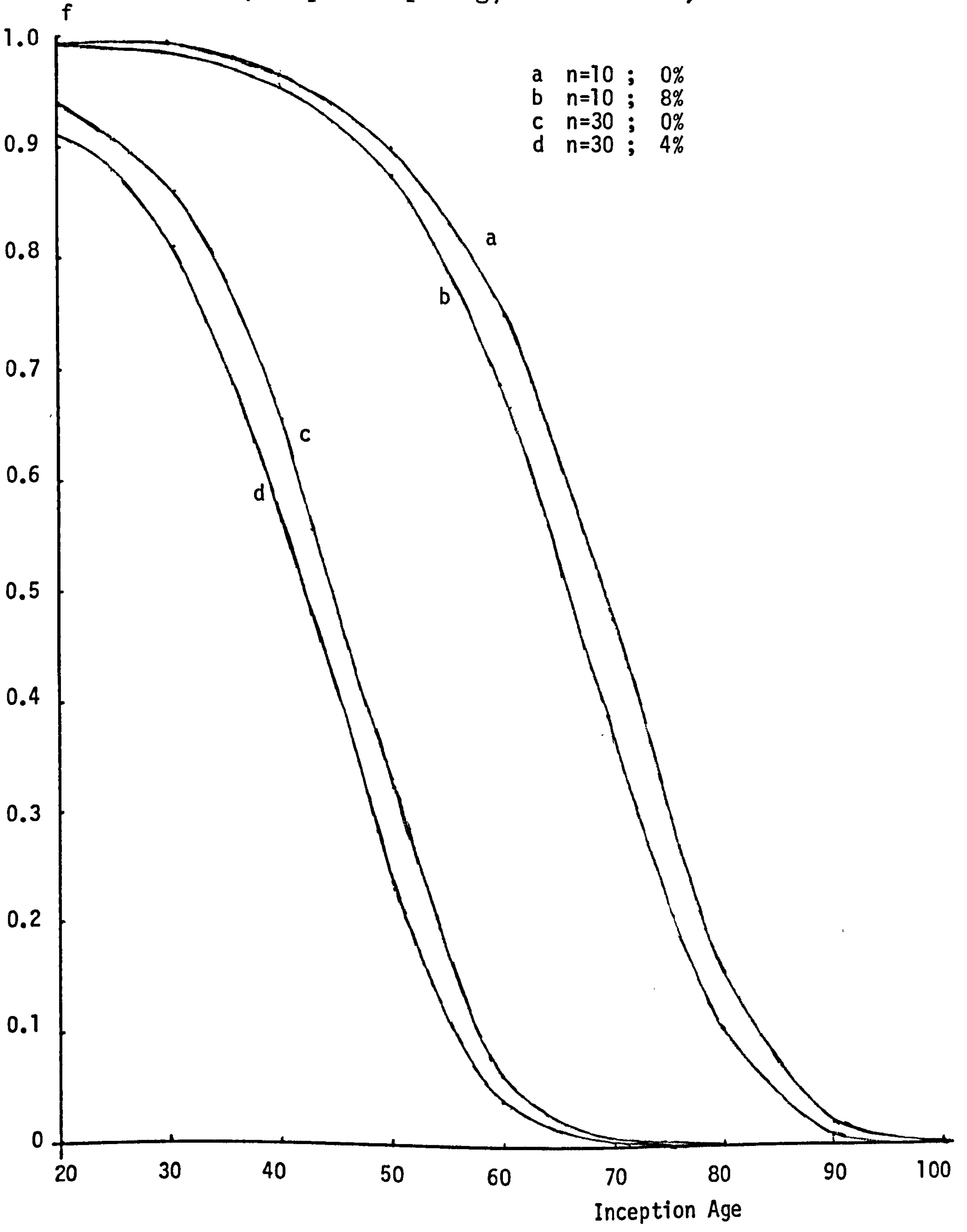
The Shape of the Financial Saving Ratio

The ratio does not peak, as in the whole of life case, but declines continuously as inception age increases. However, the shape of the ratio is still of some interest because of the obvious existence of a point of inflexion[12] where the curve changes from being concave to the origin to being convex to the origin.

Thus for a policyholder who wishes to maximise his financial saving ratio based on pure premiums, it would seem that the earlier the inception age the better; however it can be seen from Figure 5.6.1 that for a Policy Term of 10 years the curve is so shallow at early inception ages that the ratio changes very little in the 10 to

[12] See R.G.D. Allen (1) Ch. 8.

Figure 5.6.1 Endowment Saving Flow by Inception Age
(Non-participating, Pure Premium)



Source: Equation (5 5 2)

12 years following the highest point on the curve. A reference to Figure 5.6.3 shows that the same reasoning applies for the financial saving ratio based on office premiums.

As inception age increases, the ratio decreases at an increasing rate which reaches a maximum at the point of inflexion. Thus a change in inception age has the greatest effect on the ratio at or around the point of inflexion and the least effect at very low and very high inception ages.

The Response to the Rate of Interest

It is obvious from Figure 5.6.1 that, for any inception age and any maturity term, the proportion of pure premium contributing to the Financial Saving Ratio decreases as the rate of interest assumed in premium calculations increases. Appendix 5.1 shows that the response of the financial saving ratio to changes in the rate of interest is always negative, irrespective of inception age and maturity term. It can also be seen from Figure 5.6.1 that a change in the rate of interest has a greater effect for the longer maturity term.

It is worth examining the effect of the rate of interest in greater detail. Recalling Equation (5.5.2) we have,

$$f = \frac{D_{x+n}}{M_x - M_{x+n} + D_{x+n}} \tag{5.5.2}$$

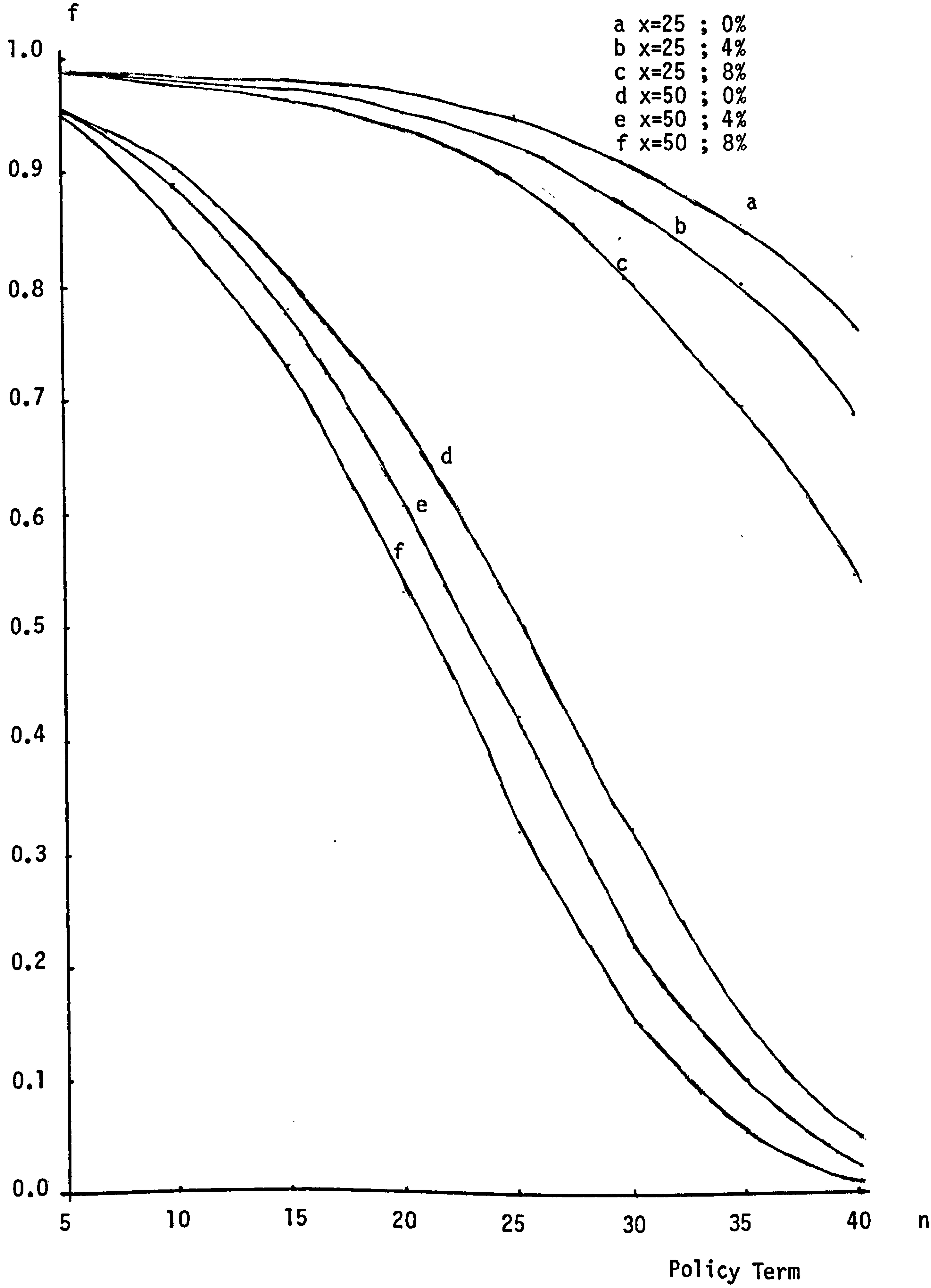
which can be rewritten in its more basic form

$$f = \frac{v^n . l_{x+n}}{v.d_x + v^2.d_{x+1} + \ldots + v^n.d_{x+n-1} + v^n.l_{x+n}} \tag{5.6.1}$$

Now unless the policyholder is very old at inception, the component $(v^n.l_{x+n})$ will dominate in the endowment pure premium $P_{x:\overline{n|}}$ because of the size of l_{x+n} in comparison with the d_x's. Similarly any change in the rate of interest will have a greater effect the larger is the exponent of v (ie. the greatest effect will be on $v^n.l_{x+n}$ again): this explains why a change in the calculation rate of interest produces a greater effect when the policy term increases.

In order to examine the effect on f, three different bands of inception age must be investigated. First, at young inception ages the component $(v.d_x + v^2.d_{x+1} + \dots + v^n.d_{x+n-1})$ will be comparatively small because of the low incidence of death at younger ages. Thus $v^n.l_{x+n}$ and $(v.d_x + \dots + v^n.d_{x+n-1} + v^n.l_{x+n})$ will be similar in size and so a change in the rate of interest will have small overall effect on f. Second, at middle inception ages, the quantity $(v.d_x + \dots + v^n d_{x+n-1})$ cannot be disregarded but is still small in comparison with $v^n.l_{x+n}$. In this case, an increase in the calculation rate of interest (which reduces $v^n.l_{x+n}$) will have a greater effect on the numerator of Equation (5.6.1) than on the denominator. Again the overall effect is a decrease in the size of f. Third, at high inception ages, the quantity $(v.d_x + \dots + v^n.d_{x+n-1})$ will be large in relation to $v^n.l_{x+n}$ and thus the ratio f will be small. Because $v^n.l_{x+n}$ is small, the numerator again dominates the size of f, so that again, a decrease in $v^n l_{x+n}$ decreases f.

Figure 5.6.2 Endowment Saving Flow by Policy Term
(Non-participating, Pure Premium)



Source : Equation (5 5 2)

The Response to the Maturity Term

The trace representing the response of the Financial Saving Ratio to changes in endowment maturity term is shown in Figure 5.6.2 for selected inception ages. It can be seen that as the maturity term increases, the financial saving ratio decreases. Again there is some evidence of a point of inflexion, which is not surprising since Figures 5.6.1 and 5.6.2 are similar in their construction.

A Note on Loadings

If the initial assumption of zero expense and bonus loadings is removed, the financial saving ratio based on office premiums can be expressed by

$$\phi_{\alpha,\beta} = \frac{f + \alpha}{1 + \alpha + \beta} \qquad (5.5.5)$$

where f is the financial saving ratio based on pure premiums,

$$\alpha = \frac{\text{Bonus Loading}}{P_{x:\overline{n}|}} \quad \text{and} \quad \beta = \frac{\text{Expenses Loading}}{P_{x:\overline{n}|}}$$

In the case of a simple reversionary bonus of b per unit sum insured per annum

$$\alpha = \frac{b\cdot(IA)_{x:\overline{n}|}}{P_{x:\overline{n}|}\cdot\ddot{a}_{x:\overline{n}|}}$$

$$= \frac{b\cdot(R_x - R_{x+n} - nM_{x+n} + nD_{x+n})}{D_x\cdot P_{x:\overline{n}|}\cdot\ddot{a}_{x:\overline{n}|}}$$

$$= \frac{b\cdot(R_x - R_{x+n} - nM_{x+n} + nD_{x+n})}{(M_x - M_{x+n} + D_{x+n})}$$

For a compound reversionary bonus

$$\alpha = \frac{A^{(r)}_{x:\overline{n}|} - A_{x:\overline{n}|}}{P_{x:\overline{n}|} \cdot \ddot{a}_{x:\overline{n}|}}$$

$$= \frac{A^{(r)}_{x:\overline{n}|} \cdot D_x}{(M_x - M_{x+n} + D_{x+n})} - 1$$

where $A^{(r)}_{x:\overline{n}|}$ denotes the single premium of an endowment policy, term n, calculated at rate of interest r. If the compound rate of bonus is b, then $r = \frac{i-b}{1+b}$ where i is the rate of interest used in the premium calculations.

The expenses element β is given by:

$$\beta = \frac{1}{(1-k)P_{x:\overline{n}|}} \left(k \cdot P_{x:\overline{n}|} + \frac{I}{\ddot{a}_{x:\overline{n}|}} + c\right)$$

where k allows for those expenses which vary in direct proportion to premium (ie. renewal expenses such as renewal commission), I represents the excess of initial expenses over the renewal expenses (per unit sum insured) and c is a fixed loading which allows for those expenses which vary in direct proportion to the sum insured (eg. the work of the claims and investment departments and of higher management).

The trace illustrating the behaviour of ratios $\phi_{\alpha,0} = \frac{f+\alpha}{1+\alpha}$ and $\phi_{\alpha,\beta} = \frac{f+\alpha}{1+\alpha+\beta}$, for a 10 year endowment, as inception age changes are pictured in Figure 5.6.3 using a rate of interest of 8%. The rate of bonus was chosen to be £4% of sum insured for both simple and compound bonuses. For the expenses loadings, the following figures were chosen:

$$k = 0.05$$

$$I = 0.03$$

$$\text{and } c = 0.002$$

It can be seen from Figure 5.6.3 that the addition of the various loadings has some interesting effects on the position of the financial saving ratio.

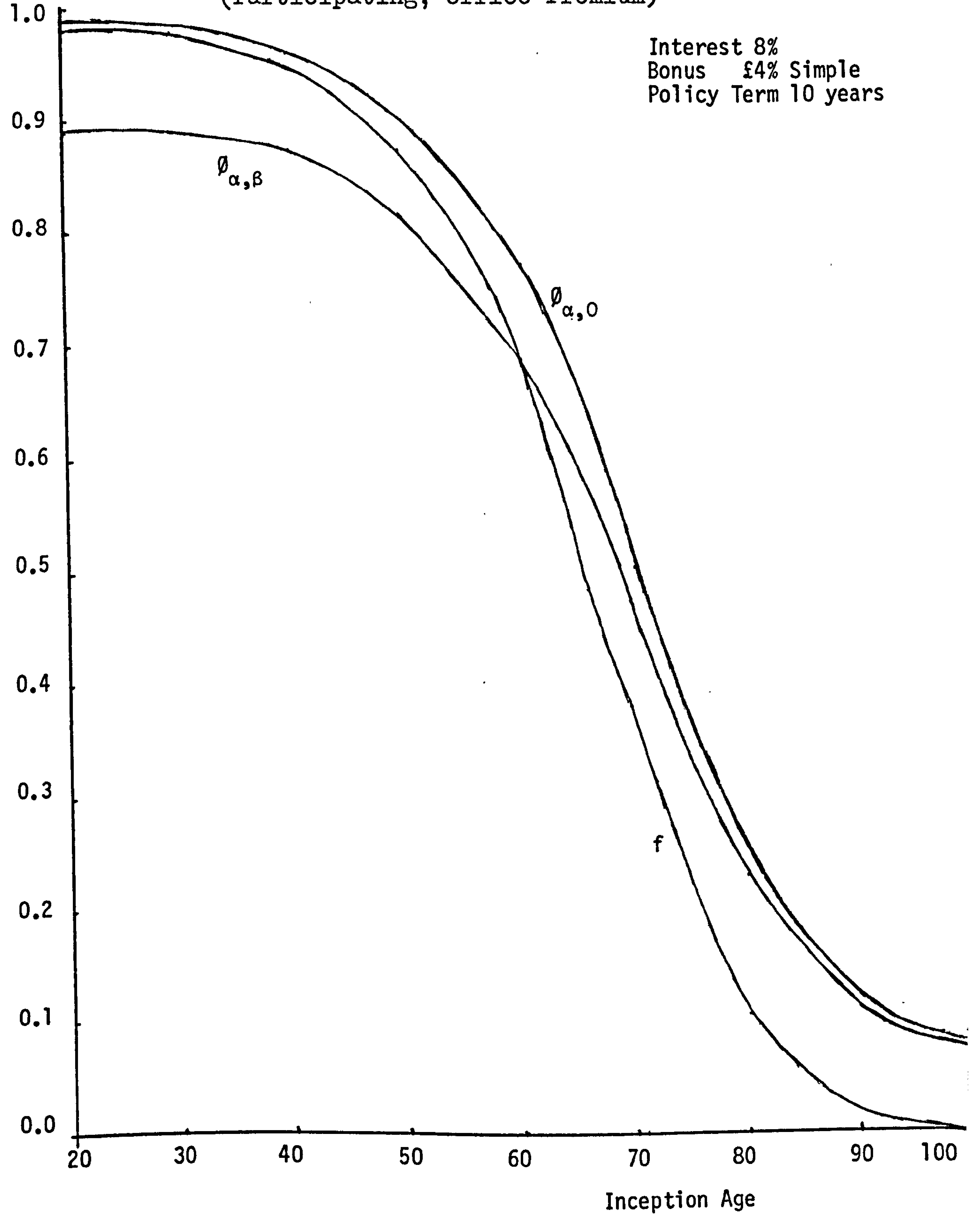
The addition of the bonus loading, to form the ratio $\phi_{\alpha,0}$ seems to pivot the ratio f in an anticlockwise direction about its <u>highest</u> point, as well as moving it upwards. The addition of the expenses loading, to form the ratio $\phi_{\alpha,\beta}$, seems to pivot the ratio $\phi_{\alpha,0}$ in an anticlockwise direction about its <u>lowest</u> point, as well as moving it downwards.

This 'swivelling' of the ratio $\phi_{\alpha,\beta}$ in comparison with f has some interesting implications for the policyholder who wishes to maximise his financial saving ratio. The addition of the <u>bonus</u> loading only makes a real difference at high inception ages, while the addition of the <u>expenses</u> loading has its greatest effect at low inception ages. Thus the younger policyholder should choose a life office with low expenses loadings and ignore the size of the bonus loadings. Similarly, the older policyholder should choose a life office with high bonus loadings and not pay much attention to the size of the expenses loadings.

Simple and compound bonuses have not been separated: the trace for the compound bonus lies marginally above that of the simple bonus for the same rate of bonus. Again, mortality experience is based on (7).

Finally, an investigation of the ratio $\frac{f}{1+\beta}$ indicates

Figure 5.6.3 Endowment Saving Flow by Inception Age
(Participating, Office Premium)



Source : Equation (5 5 4)

i=0.08 ; b=0.04 ; k=0.05 ; I=0.03 ; c=0.002 ; n=10 Simple Bonus that all three elements of the expenses loading, ie. k, I and c, have the effect of pivotting the ratio f in an anticlockwise direction about its lowest point. Thus whatever the construction of the expenses loading (ie. whatever the relative size of k, I and c) an expenses loading will always adversely affect young rather than older policyholders: this can be important when dealing with life offices with different expenses ratios.

5.7 Whole of Life Insurance : Saving Flow Theory

Under a whole of life insurance contract, the payment of benefits to the policyholder's heirs and dependents occurs on the death of the policyholder whenever that may occur. Thus unlike temporary and endowment life insurance, whole of life does not have a fixed policy term so that the contract of insurance continues until terminated by the death of the policyholder and the payment of benefits to his heirs.

When the timescale problem of Section 2.3 was discussed, we commented that it was unlikely that protection (of dependents) would be required for the entire foreseeable lifetime of the policyholder so that a whole of life insurance includes an element of financial non-lifetime saving as well as protection. This saving, of course, cannot be lifetime financial saving but is saving for the benefit of heirs.

The timescale problem of Section 2.3 poses an interesting question in relation to whole of life insurance, namely: at what age (x^*) does the policyholder cease to require protection for dependents? It may be that protection is required indefinitely, so that x^* effectively tends to infinity. On the other hand it may be that x^* is very close to inception age (x) in which case, the whole of life policy would be closely connected with non-lifetime saving. We note that there is no question of any bequest being 'wasted' (ie. no 'survival risk' in the sense that it might not be made). Additionally the policyholder is not troubled by any choice of policy term so that it does not matter if the age x^* is not known with certainty. Because there is no question of any bequest being 'wasted' there is then no a priori reason why the sum insured should be limited to B^* (as was argued in Section 5.3). Thus the saving/protection split in a non-participating whole of life insurance with sum insured B can be described by the following equations:

Whole of Life Pure Premium = Consumption expenditure
+ financial saving (non-lifetime)

$$\text{ie.} \quad B.P_x = (B^*.P'_{x:\overline{x^*-x}|}) + (B.P_x - B^*.P'_{x:\overline{x^*-x}|}) \qquad (5.7.1)$$

where B^* ($\leq B$) is the upper bound for protection purposes on the sum insured,

Equation (5.7.1) is not very informative as it stands because both x^* and B^* are at the discretion of the policyholder: the equation is therefore for descriptive rather than analytic use. The basic problem with whole of life insurance is that the primary determinants of the saving/protection split are solely at the discretion of the policyholder rather than being mechanically predetermined (as in the temporary and endowment life insurance cases). Thus a solution to the problem involves not only a mechanical method of separating the financial non-lifetime saving and protection elements but also some assumptions about the behaviour of whole of life policyholders in general.

Intuitively, one would expect that most policyholders were underinsured to the extent that $B \leq B^*$. This observation is born out by the findings of a survey organised by Money Which? in December 1975 (13), where half of a survey of 560 members were judged to be underinsured. Therefore, as a first simplification to Equation (5.7.1), B and B^* will be assumed to be equal for the typical policyholder, so that there is no upper bound above which bequests are made for savings purposes.

The other problem with Equation (5.7.1) concerns the assumptions about the behaviour of x^* which can theoretically vary between inception age x and infinity. In order to make practical use of Equation (5.7.1) some value for x^* must be assumed so that the protection element of $B.P_x$ can be removed by using the existing theory developed for endowment life insurance.

Given that an assumption must be made about the value of x^*, it seems reasonable to put x^* equal to $x+e_x$ where e_x is the expectation of life of the typical policyholder at inception age x. Values of e_x for the average policyholder can, of course, be obtained from standard Mortality Tables. Choosing $x^* = x + e_x$ effectively makes at least two implicit assumptions. First, that although the individual's lifetime is uncertain, protection for dependents is required over the policyholder's expected lifetime. Second, in view of the assumption that $B = B^*$, any saving for the purpose of bequest is achieved only because the consumer has the chance of dying at a later age than x^*.

The assumption that $x^* = x + e_x$ is not such an unreasonable one for the typical policyholder since if x^* was known with certainty (and was less than $x + e_x$) then the consumer would probably be better off with the purchase of an $(x^* - x)$ year temporary life insurance in conjunction with the use of a bank deposit. This argument is a bit more tenuous if the combination temporary insurance plus bank deposit is compared with a participating whole of life insurance rather than a non-participating policy. However, if we assume that the consumer values the flexibility and liquidity of his savings stocks then the temporary insurance/bank combination might be preferred to any whole of life policy for reasonably small x^*. Thus the value of a whole of life policy is that it provides protection for dependents, over the lifetime of the policyholder, even though that lifetime is uncertain in its duration.

Consequently, it can be argued that, like an endowment policy, whole life policies can be split into three parts: a temporary life insurance element, a lump sum element and a bonus loading on policies participating in bonuses. However, the main differences are that the term of the temporary insurance element is not fixed but varies according to the lifespan of the policyholder and that the lump sum now represents non-lifetime rather than lifetime saving.

Suppose we have a non-participating whole of life policy - sum insured equal to unity - with level pure premium P_x. Now suppose that we have a non-profit endowment policy for the same unit sum insured with level premium $P_{y:\overline{m}|}$. If we assume that the same interest and mortality rates are used in both calculations, then if y denotes the inception age and m the policy term of the endowment, we require values of y and m such that $P_x = P_{y:\overline{m}|}$. Once appropriate values of y and m have been chosen, the analysis can proceed as in the previous Section using $P_{y:\overline{m}|}$ as a surrogate for P_x. The analysis proceeds in this manner, rather than in the way indicated by Equation (5.7.1), in order to correctly ascertain the value of the temporary insurance premium (which, of course, will be given by $P'_{y:\overline{m}|}$). The obvious solution for y and m would be to put $y = x$ and $m = e_x$ so as to obtain $P'_{y:\overline{m}|} = P'_{x:\overline{e_x}|}$ as in Equation (5.7.1). However, in this instance, $P_{x:\overline{e_x}|} \neq P_x$ so that the temporary insurance element of P_x has been incorrectly allocated.

Once both y and m become 'variable' there are very many combinations that produce equality between the whole life and endowment surrogate premiums. Hence it is necessary to choose y and m in a systematic way.

Now $P_x = P_{y:\overline{m}|}$ can be written in the usual actuarial notation as

$$M_x / N_x = \frac{M_y - M_{y+m} + D_{y+m}}{N_y - N_{y+m}} \qquad (5.7.2)$$

and if y+m is equated with a constant, say L, then:

$$M_x / N_x = \frac{M_y - M_L + D_L}{N_y - N_L} \quad \text{for a unique value of } y. \qquad (5.7.3)$$

The solution to the problem then boiled down to choosing an appropriate value for the constant L - once this was done, the unique value of y could be obtained by computer simulation.

It has already been argued that L should be replaced by $x + e_x$, although the above analysis would indeed work for any other chosen value of L.

When $L = x + e_x$, Equation (5.7.3) produces a unique value of y that satisfies

$$P_x = P_{y:\overline{x+e_x-y}|} \tag{5.7.4}$$

In practice it was found that $x - 5 \leq y \leq x - 1$ for most rates of interest when calculation mortality rates were obtained from the A1949/52 Tables of Mortality (7). (This choice effectively gives the value for y in the year 1950/1: the value for y is, in fact, time specific since the expectation of life at each age changes through time).

Then for a policyholder of inception age x paying pure premium P_x, the proportion of pure premium that can be allocated to financial non-lifetime saving is given by

$$f = \frac{D_L}{M_y - M_L + D_L} \tag{5.7.5}$$

using the standard actuarial notation, where

$$L = x + e_x$$

e_x is the expectation of life of the average policyholder at inception age x ;

and y is the solution to

$$M_x / N_x = \frac{M_y - M_L + D_L}{N_y - N_L} \tag{5.7.3}$$

In an exactly similar manner to the endowment life insurance case, it is possible to determine that the proportion of financial non-lifetime saving in a level whole of life insurance office premium is given by

$$\emptyset_{\alpha,\beta} = \frac{(P_x - P'_{y:\overline{m}|}) + BL}{P_x + BL + EL} \tag{5.7.6}$$

where BL is the Bonus Loading and EL is the Expenses Loading.

Equation (5.7.6) may be rewritten as

$$\emptyset_{\alpha,\beta} = \frac{f + \alpha}{1 + \alpha + \beta} \tag{5.7.7}$$

where f is defined by Equation (5.7.5), $\alpha = \frac{BL}{P_x}$ and $\beta = \frac{EL}{P_x}$

In the next Section, the behaviour of the ratios f and $\emptyset_{\alpha,\beta}$ will be further examined while the comments made at the end of Section 5.5 apply in the whole of life as well as in the endowment case. However, it is necessary to emphasise again that the ratio f in the whole of life case is time specific in that the value of e_x changes from year to year. The ratio f in the endowment case is only time specific to the extent that the 'calculation' rates of interest and mortality change from year to year.

5.8 Whole of Life Insurance : Saving Flow Behaviour

Figure 5.8.1 shows how the financial saving ratio f varies as inception age x varies for different rates of interest. Figure 5.8.1 uses both calculation mortality and expectation of life data from the A1949/52 Table of Assured Lives (7) and therefore really refers to the situation in the year 1950/51. The situation corresponding to the year 1969 (with calculation mortality rates from the A1949/52 Table and average expectation of life from the A1967/70 Tables (8)) is virtually identical. The initial simplifying assumption of zero Bonus and Expense loadings will be dropped in due course.

The Shape of the Financial Saving Ratio

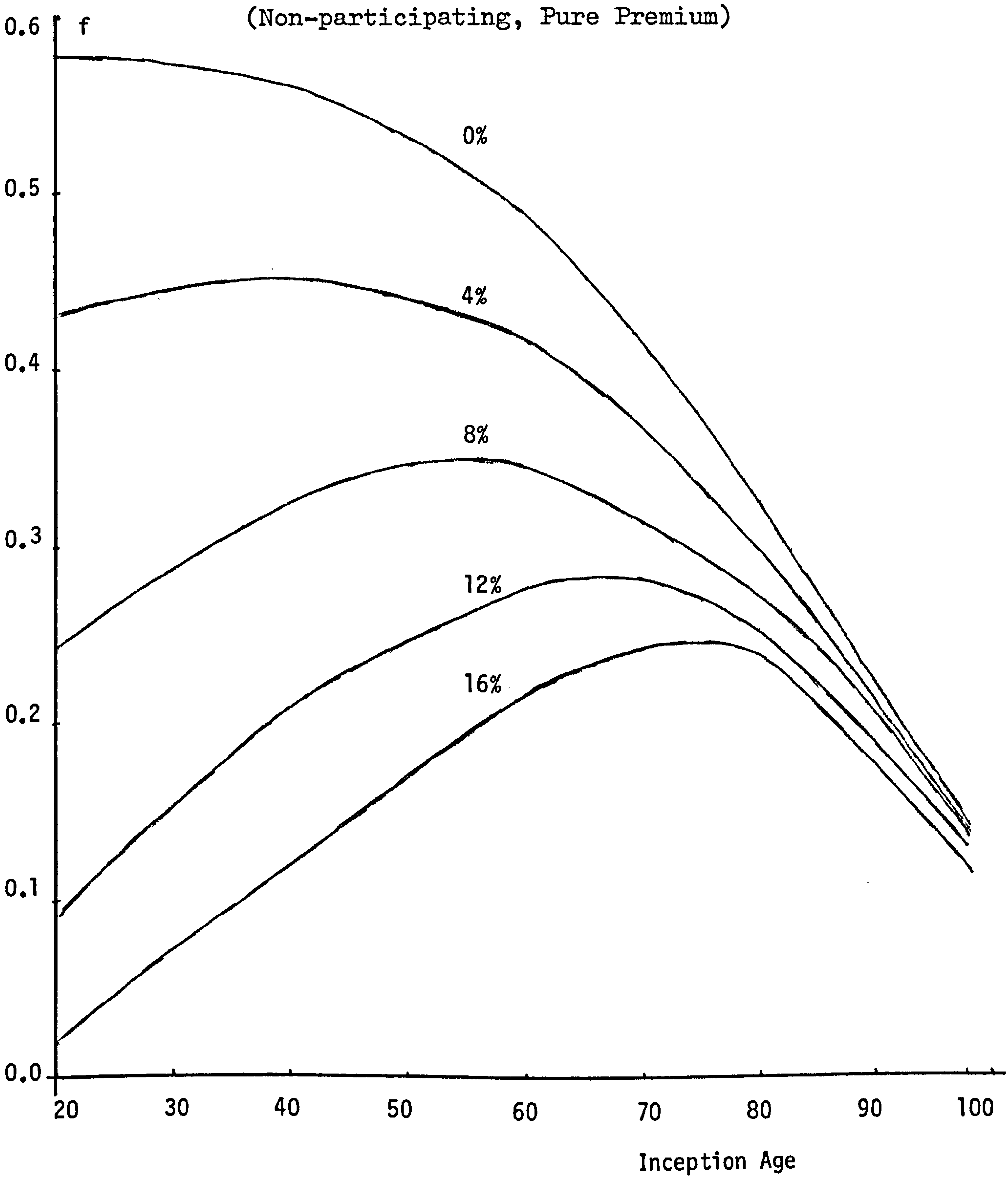
For all rates of interest greater than 2%, the Financial Saving Ratio peaks for an inception age greater than 20 in Figure 5.8.1. Thus at rate of interest equal to 5%, the ratio peaks at age 43, while for 10% the ratio is at a maximum at inception age 64.

Thus in terms of the Financial Saving Ratio, there appears in almost every case, to be an optimum inception age that maximises the proportion of whole life premium devoted to financial non-lifetime saving. However it should be noted that the amount of premium saved, ie. FS ratio multiplied by P_x rises continuously as inception age rises.

The locus of the peak points of the Financial Saving Ratio corresponding to the years 1950 and 1969 are given in Table 5.8.1.

Figure 5.8.1 Whole Life Saving Flow by Inception Age

(Non-participating, Pure Premium)



Source : Equation (5 7 5)

Table 5.8.1 Locus of Maximum Whole Life Financial Saving Ratio.

Rate of Interest (%)	Inception Age 1950	Inception Age 1969	Maximum of f 1950	Maximum of f 1969
3	34	38	0.480	0.420
4	39	41	0.450	0.391
5	43	45	0.421	0.362
6	46	52	0.392	0.337
7	52	58	0.367	0.315
8	58	61	0.345	0.297
9	61	64	0.327	0.282
10	64	66	0.312	0.268
11	66	68	0.299	0.257
12	68	70	0.287	0.246
13	69	71	0.277	0.236
14	70	72	0.267	0.228
15	72	74	0.259	0.220

Source : Equation (5 7 5)

One of the most interesting features of the shape of the Financial Saving Ratio f is that the ratio varies with inception age: up to some point, the policyholder can increase the proportion of saving in each pure premium by delaying the inception of the policy to a later age. The higher the rate of interest used in the premium calculation, the later the inception age corresponding to the ratio peak.

It is interesting to compare Figure 5.8.1 with the corresponding case for an endowment life insurance (Figure 5.6.1) since the latter does not have the 'humped' shape of the whole of life case. The explanation for this behaviour lies mainly in the offsetting effect of $(x + e_x - y)$ in Equation (5.7.4) at young inception ages. Since the difference between x and y is very slight, the quantity $(x + e_x - y)$ is governed by the behaviour of e_x, which, of course, decreases as x increases. It can be seen from Figure 5.6.2 that this, therefore, exerts an upward pressure on the f ratio which, in part, counteracts the downward pressure of an increasing surrogate inception age y.

The Response to the Rate of Interest

It is immediately obvious from Figure 5.8.1 that for any inception age, the proportion of pure premium contributing to Financial Saving decreases as the rate of interest assumed in premium calculations increases. Similarly, the amount of pure premium saved (ie. FS ratio multiplied by P_x) continues to fall as the rate of interest increases. Appendix 5.1 shows that the response of the Financial Saving Ratio to changes in the rate of interest is always negative, irrespective of inception age x.

A Note on Loadings

By assuming initially that the whole of life insurance is still on a non-participating basis, the office premium OP_x is commonly represented by the following expression:

$$OP_x = \frac{1}{1-k} \cdot (P_x + \frac{I}{\ddot{a}_x} + c)$$

where, as before, k allows for those expenses which vary in direct proportion to the premium: I represents the excess of initial expenses over the renewal expenses (I per unit sum insured) and c allows for those expenses which vary in direct proportion to the sum insured.

If the participating whole of life policy has bonus loadings with assumed rate of simple bonus b per unit sum insured, then the formula for the office premium becomes:

$$OP_x = \frac{1}{1-k} \cdot \left(P_x + \frac{I}{\ddot{a}_x} + c + \frac{b.(IA)_x}{\ddot{a}_x}\right)$$

where $(IA)_x$ is the value/single premium of an increasing whole life insurance where the sum insured during the t^{th} year is t.

If the participating whole of life policy has bonus loadings with assumed rate of compound bonus b, then the formula for the office premium is given by:

$$OP_x = \frac{1}{1-k} \cdot \left(P_x + \frac{(A_x^{(r)} - A_x)}{\ddot{a}_x} + \frac{I}{\ddot{a}_x} + c\right)$$

where $A_x^{(r)}$ is the value/single premium of a whole life policy calculated at interest rate r, i being the rate of interest used to calculate the non-participating policy premiums.(13)

If we consider first the case of a zero expenses

(13) See Hooker and Longley-Cook (6) Ch.10.
Note that if the rate of compound bonus is b per annum, then $r = \frac{i-b}{1+b}$.

loading, then the Financial Saving Ratio is given by:

$$\emptyset_{\alpha,0} = \frac{P_x - P'_{y:\overline{m|}} + \text{Bonus Loading}}{P_x + \text{Bonus Loading}}$$

Dividing both numerator and denominator by P_x:

$$\emptyset_{\alpha\,0} = \frac{\dfrac{P_x - P'_{y:\overline{m|}}}{P_x} + \dfrac{BL}{P_x}}{1 + \dfrac{BL}{P_x}} \qquad (5.8.1)$$

This can be expressed as $\emptyset_{\alpha,0} = \frac{f+\alpha}{1+\alpha}$ where $\alpha = \frac{BL}{P_x}$

and f is the Financial Saving Ratio based on <u>pure</u> premiums. In order to examine the characteristics of the ratio $\emptyset_{\alpha,0}$ differentiate Equation (5.7.1) with respect to x, the inception age: then

$$\frac{\partial\emptyset_{\alpha\,0}}{\partial x} \quad \frac{(1+\alpha).\frac{\partial}{\partial x}(f+\alpha) - (f+\alpha).\frac{\partial\alpha}{\partial x}}{(1+\alpha)^2}$$

$$= \frac{1}{(1+\alpha)^2} . \left(\frac{\partial f}{\partial x} .(1+\alpha) + \frac{\partial\alpha}{\partial x} .(1-f) \right) \qquad (5.8.2)$$

Now suppose that $\left.\frac{\partial f}{\partial x}\right|_{x = x_0} = 0$; then in order to see whether the addition of the bonus loading has altered the inception age corresponding to the maximum point of the f, we need to know the sign of $\left.\frac{\partial\emptyset_{\alpha\,0}}{\partial x}\right|_{x=x_0}$

Now $\left.\frac{\partial\emptyset_{\alpha 0}}{\partial x}\right|_{x=x_0} = \frac{1}{(1+\alpha)^2} \cdot \left.\frac{\partial\alpha}{\partial x}.(1-f)\right|_{x=x_0}$

and since $\frac{1}{(1+\alpha)^2} > 0$ and $f < 1$, the sign of $\left.\frac{\partial\emptyset_{\alpha\,0}}{\partial x}\right|_{x = x_0}$

is determined by $\left.\frac{\partial\alpha}{\delta x}\right|_{x=x_o}$

Now in the case of the simple reversionary bonus

$$\alpha = \frac{b.(IA)_x}{P_x \cdot \ddot{a}_x} = b \cdot \frac{R_x}{D_x} \cdot \frac{D_x}{N_x} \cdot \frac{N_x}{M_x}$$

$$= b \cdot \frac{R_x}{M_x} \tag{5.8.3}$$

Therefore if we assume that R_x and M_x are continuous variables:

$$\frac{\partial\alpha}{\delta x} = \frac{b}{M_x^2} \cdot \left(R_x \cdot D_x \cdot \mu_x - M_x \cdot \sum_{t=0}^{\infty} D_{x+t} \cdot \mu_{x+t} \right)$$

since $\frac{\partial M_x}{\delta x} = -D_x \cdot \mu_x$ and $R_x = \sum_{t=0}^{\infty} M_{x+t}$

It is immediately obvious that as $\frac{b}{M_x^2}$ is positive, the sign of $\frac{\partial\alpha}{\delta x}$ is determined by the expression

$$R_x \cdot D_x \cdot \mu_x - M_x \cdot \sum_{t=0}^{\infty} D_{x+t} \cdot \mu_{x+t} \tag{5.8.4}$$

which is independent of the rate of bonus b.

Unfortunately, it is not possible to determine mathematically the parity of $\left.\frac{\partial\alpha}{\delta x}\right|_{x=x_o}$ and consequently a computer simulation was carried out to determine the behaviour of the quantity

$$R_{x_o} \cdot D_{x_o} \cdot \mu_{x_o} - M_{x_o} \cdot \sum_{t=0}^{\infty} D_{x_o+t} \cdot \mu_{x_o+t}$$

for values of x_0 at different rates of interest. This quantity proved to be negative for all the combinations of interest and inception age given in Table 5.8.1. Thus it follows that the function $\emptyset_{\alpha,0}$ peaks at an earlier inception age than does the function f.

In the case of the compound reversionary bonus

$$\alpha = \frac{(A_x^{(r)} - A_x)}{P_x \cdot \ddot{a}_x}$$

$$= \frac{D_x}{M_x} \cdot A_x^{(r)} - 1$$

where $A_x^{(r)}$ denotes the single premium on a whole of life insurance calculated at the rate of interest r. If the compound rate of bonus is b, then $r = \frac{i-b}{1+b}$ where i is the rate of interest used in the pure premium calculations (this is often approximated by $r = i-b$). Therefore, assuming the variables are continuous

$$\frac{\delta\alpha}{\delta x} = \frac{\delta}{\delta x} \cdot \left(\frac{D_x}{M_x} \cdot A_x^{(r)}\right)$$

and since $\frac{\delta}{\delta x} \cdot A_x = A_x(\mu_x + \delta) - \mu_x$

we have $$\frac{\delta\alpha}{\delta x} = \frac{D_x}{M_x} \cdot \left(A_x^{(r)} \cdot (\mu_x + \delta^{(r)}) - \mu_x \right) - A_x^{(r)} \cdot \left(\frac{M_x \cdot D_x \cdot (\mu_x + \delta) + D_x^2 \cdot \mu_x}{M_x^2} \right)$$

where $$\delta = -\frac{\delta}{\delta x} \log_e v^x = -\frac{\delta}{\delta x} \log_e (1+i)^{-x}$$

and $$\delta^{(r)} = -\frac{\delta}{\delta x} \log_e (1+r)^{-x}$$

ie. $$\partial\alpha/\partial x = \frac{D_x}{M_x} . A_x^{(r)} . \left(\delta^{(r)} - \delta + D_x \mu_x / M_x \right) - D_x . \mu_x / M_x$$

Now if $r = \frac{i-b}{1+b}$, then $\delta^{(r)} - \delta = \frac{\partial}{\partial x} \log_e \left(\frac{1}{1+b}\right)^x$ which is independent of the rate of interest i. Consequently the expression $\left(\delta^{(r)} - \delta + \frac{D_x \mu_x}{M_x} \right)$ cannot equal zero for all rates of interest i and thus $\frac{\partial\alpha}{\partial x}$ is <u>not</u> independent of the rate of compound bonus b. Therefore $\left.\frac{\partial\alpha}{\partial x}\right|_{x=x_0}$ will only equal zero under special circumstances connected with the choice of b.

We are now in a position to consider the case of a non-zero expenses loading. In this case, the Financial Saving Ratio is given by

$$\phi_{\alpha,\beta} = \frac{f + \alpha}{1 + \alpha + \beta}$$

where f and α are defined as above and $\beta = \frac{\text{Expenses Loading}}{\text{Pure premium}}$. Now when $\beta = 0$, the effect of the bonus loading was to increase the Financial Saving Ratio, since $\frac{f+\alpha}{1+\alpha} > f$. The non-zero expenses loading has the effect of <u>reducing</u> the Financial Saving Ratio, since $\phi_{\alpha,0} > \phi_{\alpha,\beta}$.

The trace illustrating the behaviour of the ratios f, $\phi_{\alpha,0}$ and $\phi_{\alpha,\beta}$ as inception age changes is pictured in Figure 5.8.2 below. Both calculation mortality rates and expectation of life figures are obtained from the A1949/52 Tables.

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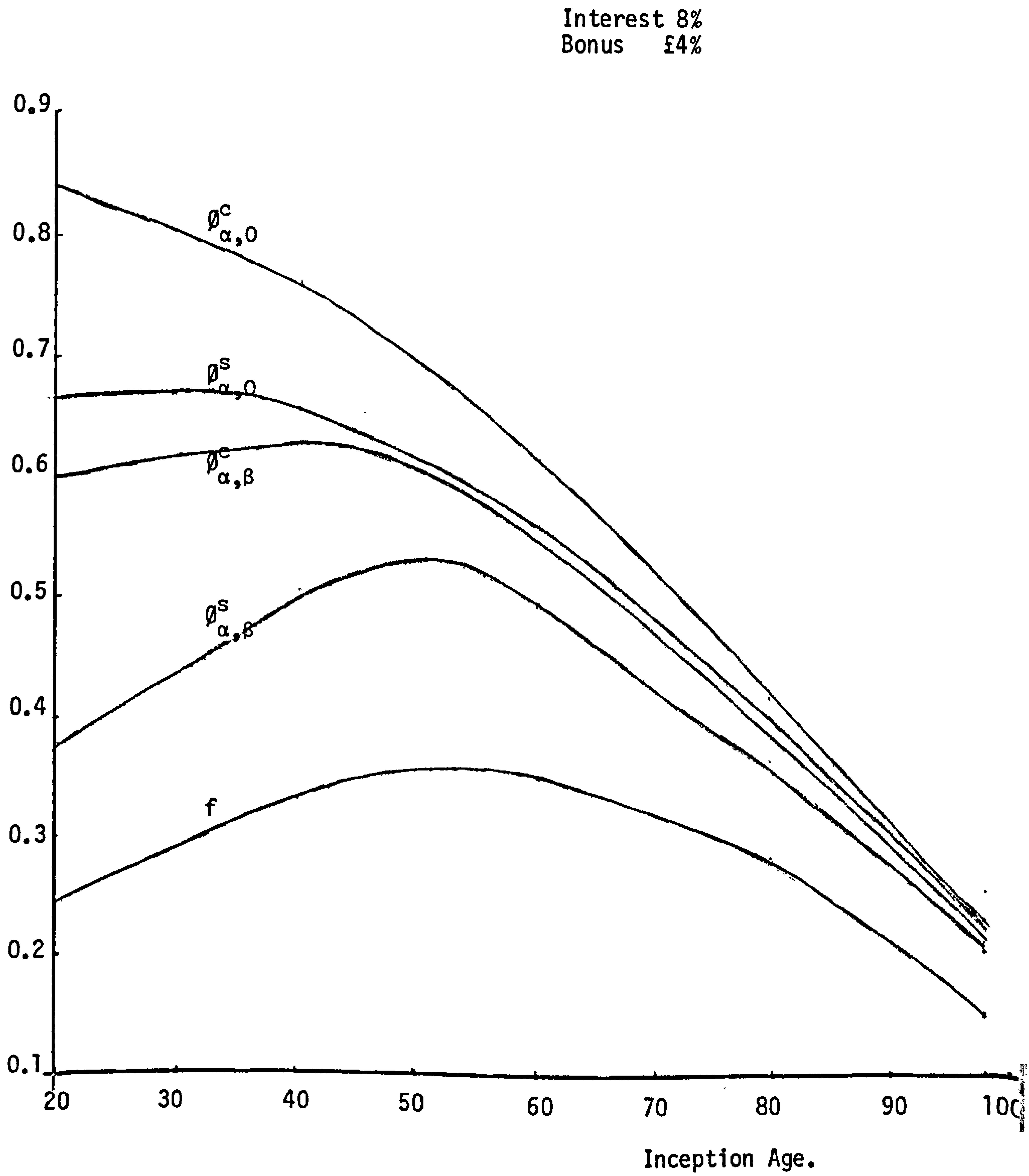
Figure 5.8.2 Whole Life Saving Flow by Inception Age
(Participating, Office Premium)



Source : Equation (5 8 1)

i=0.08 ; b=0.04 ; k=0.05 ; I=0.03 ; c=0.002

Superscripts : c - compound ; s - simple

We note from Figure 5.8.2 that first, the effect of the simple and compound bonus loadings is to shift the ratio f upwards and to the left thus reducing the inception age corresponding to the peak in the Financial Saving Ratio. Secondly, as before, the effect of the bonus loading is more pronounced in the case of the compound bonus: the Financial Saving Ratio is shifted higher and further to the left than in the simple bonus case. Thirdly, the addition of the expenses loading has shifted the ratio $\phi_{\alpha,0}$ in a south-easterly direction so that the curve $\phi_{\alpha,\beta}$ peaks at a higher inception age than the corresponding curve $\phi_{\alpha,0}$.

5.9 Policyholder Savings Stocks

Financial savings stocks at age x+t represent the accumulation of saving over the t time periods since saving began at age x; this accumulation of saving is normally held by some form of financial institution in trust for the small saver. This sum may not be the same as his readily realisable savings stocks: the difference depends on the degree of liquidity of the savings involved.

When we come to consider the financial savings stocks in a life insurance policy at age x+t, there are several important points of relevance. First, that unlike premium saving flows, financial savings stocks vary according to the time elapsed since inception and secondly, for all types of policy except the equity linked, the life office directly controls the amount of policyholder savings stocks. Thus there is a difference between the accumulated savings stocks held in trust by the life office and the amount available to the policyholder at any time. Thirdly, the financial savings stocks available to the policyholder are not fully realisable until the end of the maturity term or at death: early realisation (or voluntary termination) will involve some sort of penalty to the policyholder. Fourthly, the difference between the financial savings stocks available on death at age x+t and those held by the life office at age x+t when no death occurs must be highlighted.(14) Using standard mortality function notation, if l_x policyholders in a particular group start

(14) This difference is sometimes known as the 'actual death strain'.

life insurance policies at age x, then at age x+t, $l_x - l_{x+t}$ of those policyholders will have died: when we talk of policyholder savings stocks at age x+t, we are considering an average policyholder alive at age x+t. Those $l_x - l_{x+t}$ policyholders who died before attaining age x+t have already realised their financial savings stocks; however, it is still of interest to ascertain what proportion of the death benefits received by those policyholders constitutes return of savings stocks.

Following the argument in Section 5.4 for PIA savings stocks on a temporary life insurance, it is apparent that the financial savings stocks at time t for a non-participating n-year endowment with unit sum insured is given by

$$ {}^{T}_{t}F_{x:\overline{n}|} = {}^{T}_{t}V_{x:\overline{n}|} - {}^{T}_{t}V'_{x:\overline{N}|} \qquad (5.9.1)$$

where ${}^{T}_{t}V_{x:\overline{n}|}$ is the policy reserve value on the n-year endowment at age x+t and ${}^{T}_{t}V'_{x:\overline{n}|}$ is the policy reserve value on the corresponding temporary life insurance. The pre-superscript T denotes that allowance has been made for the 'true' rates of interest and mortality. ${}^{T}_{t}F_{x:\overline{n}|}$ represents the accumulation of financial saving flow $({}^{C}P_{x:\overline{n}|} - {}^{C}P'_{x:\overline{n}|})$ over t time periods where the pre-superscript C denotes the use of 'calculation' rates of interest and mortality. Thus the total financial saving flow from ${}^{T}l_{x+k}$ policyholders alive at age (x+k) is given by ${}^{T}l_{x+k} \cdot ({}^{C}P_{x:\overline{n}|} - {}^{C}P'_{x:\overline{n}|})$ per unit sum insured. Compounding this flow for t years using true rates of interest and mortality gives the accumulated financial savings stocks at age x+t, ie.

$$ {}_t^TF_{x:\overline{n}|} = \frac{1}{{}^Tl_{x+t}} \cdot \sum_{k=0}^{t-1} {}^Tl_{x+k} \cdot ({}^CP_{x:\overline{n}|} - {}^CP'_{x:\overline{n}|}) \cdot (1+{}^Ti)^{t-k} \tag{5.9.2} $$

Multiplying both the numerator and the denominator of Equation (5.9.2) by ${}^Tv^{x+t}$ gives

$$ {}_t^TF_{x:\overline{n}|} = \frac{1}{{}^Tv^{x+t} \cdot {}^Tl_{x+t}} \cdot \sum_{k=0}^{t-1} {}^Tv^{x+k} \cdot {}^Tl_{x+k} \cdot ({}^CP_{x:\overline{n}|} - {}^CP'_{x:\overline{n}|}) \tag{5.9.3} $$

$$ = \frac{{}^TN_x - {}^TN_{x+t}}{{}^TD_{x+t}} \cdot ({}^CP_{x:\overline{n}|} - {}^CP'_{x:\overline{n}|}) \tag{5.9.4} $$

$$ = {}_t^TV_{x:\overline{n}|} - {}_t^TV'_{x:\overline{n}|} $$ calculated retrospectively (non-participating case)

There are, unfortunately a number of difficulties involved in the use of Equation (5.9.1): both theoretical and practical.

First of all, Equation (5.9.1) has very little application to the aggregate data of Chapter Six because of its dependence on the time since inception (t): it would be impossible to obtain an 'average' time-since-inception for all endowments in force at any one time. Since Equation (5.9.1) has little foreseeable practical value, the discussion of it will be kept to a minimum.

Secondly, there are two alternative methods of calculating the policy reserve values ${}_tV_{x:\overline{n}|}$ and ${}_tV'_{x:\overline{n}|}$: the prospective and the retrospective. The prospective method calculates the policy reserve as the amount by which the present value of future net premiums falls short of the present value of future claims payments. The retrospective method calculates the policy reserve as the accumulated value of past premiums less the accumulated value of past claims. If the 'calculation' and 'true' rates of interest and mortality are equal, then the two methods of valuation will produce the same result. If, however, the 'calculation' rates are more conservative than the 'true' then the retrospective method will normally produce a greater reserve than the prospective.

At first sight, it would appear that the retrospective method is more suitable for the purpose of determining the Financial Savings Stocks. This is because the method is compatible with the concept that savings stocks at time x+t are derived by the compounding up of premium saving flow over the t years since inception (see Equation (5.9.2)). Furthermore, the method of calculation implies that when t=0, the accumulated savings stock is also zero (this is not the case with the prospective method). However, the retrospective method has some drawbacks in comparison with the (more widely used) prospective method:

a) it is quite possible that the policy reserve will be in excess of unity at the end of the policy term. This can cause difficulties when trying to allocate the savings content of the realised sum insured (plus bonuses) paid to the policyholder's estate.

b) this method cannot be used to determine the value of any lump sum bonuses accumulated. Bonuses can only be properly treated by assuming that they are at a guaranteed rate.

APPENDIX 5.1 : The Financial Saving Ratio

The financial saving ratio $f(v,x,n)$ is given by

$$f(v,x,n) = \frac{D_{x+n}}{M_x - M_{x+n} + D_{x+n}}$$

where $v = \frac{1}{1+i}$, i is the rate of interest;

x is the inception age; and n is the maturity term of the surrogate endowment policy.

Then, assuming that the variables are continuous,

$$\frac{\delta f}{\delta v} = \frac{(M_x - M_{x+n} + D_{x+n}) . \frac{\delta}{\delta v} D_{x+n} - D_{x+n} . \frac{\delta}{\delta v} (M_x - M_{x+n} + D_{x+n})}{(M_x - M_{x+n} + D_{x+n})^2}$$

Now $$\frac{\delta}{\delta v} D_{x+n} = \frac{\delta}{\delta v} v^{x+n} . l_{x+n} = \frac{(x+n)}{v} D_{x+n}$$

and $$\frac{\delta}{\delta v} M_{x+n} = \frac{\delta}{\delta v} \sum_{t=0}^{\infty} v^{x+n+t+1} . d_{x+n+t}$$

$$= \sum_{t=0}^{\infty} (x+n+t+1) v^{x+n+t} . d_{x+n+t}$$

Now remembering that

$$R_{x+n} = \sum_{t=0}^{\infty} M_{x+n+t}$$

$$= \sum_{t=0}^{\infty} (t+1) C_{x+n+t}$$

$$= \sum_{t=0}^{\infty} (t+1) . v^{x+n+t+1} . d_{x+n+t}$$

we have $\frac{\partial}{\partial v} M_{x+n} = \frac{(x+n)}{v} M_{x+n} + \frac{1}{v} R_{x+n}$

Thus $\frac{\partial}{\partial v} (M_x - M_{x+n} + D_{x+n})$

$$= 1/v \; (x.M_x + R_x - (x+n).M_{x+n} - R_{x+n} + (x+n).D_{x+n})$$

$$= 1/v \; (x.(M_x - M_{x+n} + D_{x+n}) - n.M_{x+n} + n.D_{x+n} + R_x - R_{x+n})$$

Consequently

$$\partial f/\partial v = \frac{\frac{1}{v} . D_{x+n} . (n.M_x - (R_x - R_{x+n}))}{(M_x - M_{x+n} + D_{x+n})^2}$$

$$= \frac{\frac{1}{v} . D_{x+n} . \sum_{t=0}^{n-1} (M_x - M_{x+t})}{(M_x - M_{x+n} + D_{x+n})^2}$$

and since $M_x > M_{x+t}$ for all $t > 0$, we have $\frac{\partial f}{\partial v} > 0$

(for $v > 0$).

Now $v = \frac{1}{1+i}$ therefore $\frac{\partial v}{\partial i} = \frac{-1}{(1+i)^2} < 0$

Thus $\frac{\partial f}{\partial i} = \frac{\partial f}{\partial v} . \frac{\partial v}{\partial i} < 0$

ie. the financial saving ratio decreases as the rate of interest increases.

Chapter Five : References

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CHAPTER SIX : CONSUMPTION AND FINANCIAL SAVING IN UNITED KINGDOM NON-GROUP ORDINARY LIFE INSURANCE 1946/1968.

6.1 Introduction

The objective of this Chapter is to utilise the theory of Chapter Five to derive data on consumption and financial saving from readily available information on UK non-group ordinary annual premium life insurance (over the period 1946/1968) - excluding annuities.

Regrettably, because of deficiencies in the information available, it was not possible to compile a complete series 1946/1968 without making a large number of subsidiary calculations and simplifying assumptions (which are explained in Appendices to this Chapter).

Discontinuities in the time series data proved to be a major constraint on the eventual choice of the period 1946/1968. The Insurance Companies (Accounts and Forms) Regulations 1968 substantially altered the reporting requirements so that the data on ordinary life insurance for 1969 and later years is not comparable with that for earlier years. In particular, much overseas business that had been included was excluded from 1969 onwards. Additionally, the post-1969 figures reported in the Annual

Abstract of Statistics (6) include capital redemption and long-term personal accident business (both of which were shown separately before 1969). Similarly, the Interest Income of Life Insurance Companies (Established in the UK) is shown Net of tax before 1968 and Gross afterwards. Data on mortality rates (from the Continuous Mortality Investigation (13)) also proved difficult to obtain beyond 1969.

The decision to restrict this study of the purchase of life insurance to ordinary non-group annual premium business was made for several reasons: first, because industrial life insurance and group life insurance exhibit markedly different characteristics to those of ordinary life insurance (see R.L. Carter (4) Section 2.2, S. Wynn Ch.2 (18) and B. Reitz (16)). Secondly, because group life and single premium business has proved to be much more volatile than ordinary life insurance (being heavily dependent on changes in tax legislation). Thirdly because, prior to the early 1970's, annual premium ordinary life insurance represented the dominant type of life insurance transacted.

As stated in Section 5.9, we will only be concerned with an analysis of saving and consumption flows rather than with savings stocks. This is principally because, in theory, the savings stocks associated with a life insurance contract depend on the length of time since the inception of that contract. Additionally it is not possible to separate PIA Savings stocks from Financial Savings Stocks unless that time is known. Consequently it is not sufficient simply to examine the reserves or funds of the life office in order to ascertain the level of savings stocks[(1)].

It should also be noted that we are only interested with new saving and new consumption of life insurance. Thus attention will be focussed entirely on the New Business Premium Income of ordinary life insurers conducting business within the UK. The reasons for this concentration have already been discussed in Chapter Four: namely that premium income from contracts already in force exhibits a marked degree of stability because of its contractual nature (and is correspondingly less interesting). Additionally, an examination of the policyholder's decision to purchase (or the demand for) life insurance is intended and therefore the new business premium figures are most appropriate.

Finally, although the majority of data used in this Chapter can be obtained from the Statutory returns to the DOT, a number of references are made to a 'survey' of fifteen of the largest individual life offices undertaken over the period 1975/6. This survey was undertaken by the author and a copy of the questions can be found in Appendix 6.3. Not all the fifteen respondents were able to answer all the questions for the entire period of study: in these cases the relevant number of respondent offices will be indicated. No indication can be given of individual replies although some of the data is available from the returns made to the Department of Trade.

[(1)] As has been suggested, for example, by J. Kindahl (14).

The rest of this Chapter will be devoted to a description and derivation of new business data on consumption and saving flows from UK (ordinary non-group, annual premium) life insurance statistics. The next Section re-examines the theoretical breakdown of Chapter Five and indicates how this may be applied to aggregate data. The following Sections investigate the principle determinants of consumption and saving (ie. inception age, mortality, interest, etc.) while the last Sections produce and comment upon the results.

6.2 The Application to Aggregate Data

The results of Chapter Five (when applied to an individual policyholder) indicate that Consumption Expenditure is made up of all temporary life insurance plus expenses loadings. Thus,

$$\text{Consumption Expenditure} = \begin{cases} \text{Temporary Life Insurance Office Premium Rate} \\ \\ P'_{x:\overline{n}|} + EL \quad \text{(for NP and WP Endowments)} \\ \\ P'_{y:\overline{m}|} + EL \quad \text{(for NP and WP Whole Life)} \end{cases}$$

where NP - Non-Profit, WP - With Profit and P' denotes the temporary life insurance part of the pure premium (see Equations (5.5.4) and (5.7.6)). However, if we wish to examine the expenditure on protection-based life insurance then only a proportion of the expenses loadings should be included (see Equations (5.5.6) and (5.5.7)):

$$\text{Expenditure on Protection-based Life Insurance} = \begin{cases} \text{Temporary Life Office Premium Rate} \\ \\ P'_{x:\overline{n}|} + \dfrac{P'_{x:\overline{n}|}}{OP} \cdot EL \quad \text{(for NP and WP Endowments)} \\ \\ P'_{y:\overline{m}|} + \dfrac{P'_{y:\overline{m}|}}{OP} \cdot EL \quad \text{(for NP and WP Whole Life)} \end{cases}$$

Similarly, the remainder of office premiums (after Consumption Expenditure) must constitute Financial Saving Flow, ie.:

$$\text{Financial Saving Flow} = \begin{cases} P_{x:\overline{n}|} - P'_{x:\overline{n}|} & \text{for NP Endowments} \\ P_{x:\overline{n}|} - P'_{x:\overline{n}|} + BL & \text{for WP Endowments} \\ P_x - P'_{y:\overline{m}|} & \text{for NP Whole Life} \\ P_x - P'_{y:\overline{m}|} + BL & \text{for WP Whole Life} \end{cases}$$

where BL denotes the Bonus Loading.

Likewise the remainder of office premiums (after 'Expenditure on Protection-based Life Insurance' has been removed) must constitute 'Expenditure on Savings-based Life Insurance'.

Now assume that a portfolio of N_t non-profit (NP) annual premium endowment policies were issued to UK policyholders in year t. These N_t policies have office premiums given by the vector $\overline{OP}_t = (OP_{1t}, \ldots, OP_{N_t t})$ with sums insured given by $\overline{S}_t = (S_{1t}, \ldots, S_{N_t t})$ and might appear in the official statistics in the following manner:

<u>New Endowment Non-Profit Policies</u> (year t)

Number : N_t

New Annual Premiums : $\Sigma\, OP_t = \sum_{k=1}^{N_t} OP_{kt}$

New Sums Insured : $\Sigma\, S_t = \sum_{k=1}^{N_t} S_{kt}$

Thus, the financial saving flow generated by the N_t policies is given by

$$\text{Financial Saving in year } t = \sum_{k=1}^{N_t} (OP_{kt} - P'_{kt} - EL_{kt}) \qquad (6.2.1)$$

where $P'_{kt} = S_{kt}.P'_{x_{kt}:\overline{n_{kt}}|}$ for the k^{th} policy and

EL_{kt} is the expenses loading (in the office premium) of the k^{th} policy.

Now Equation (6.2.1) can be rewritten as[2]:

$$\text{Financial Saving} = \sum_{k=1}^{N_t} (P_{kt} - P'_{kt}) \qquad (6.2.2)$$

where P_{kt} is the 'pure premium' associated with the k^{th} policy (ie. $P_{kt} = OP_{kt} - EL_{kt}$). In turn, Equation (6.2.2) can be represented as:

$$\text{Financial Saving} = \Sigma\, S_t \cdot (APR_t - APR'_t) \qquad (6.2.3)$$

where APR_t is the average pure premium rate per unit sum insured for the N_t policies and APR'_t is the corresponding average temporary life insurance pure premium

$$\text{ie. } \Sigma\, S_t \cdot APR_t = \sum_{k=1}^{N_t} P_{kt} = \sum_{k=1}^{N_t} S_{kt} \cdot P_{x_{kt}:\overline{n_{kt}}|} \qquad (6.2.4)$$

Since in practice, $\sum_{k=1}^{N_t} (P'_{kt} + EL_{kt})$ is unknown

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(2) where $OP_{kt} = P_{kt} + EL_{kt}$

$$= S_{kt} \cdot P_{x_{kt}:\overline{n_{kt}}|} + EL_{kt}$$

the problems of aggregation boil down to those involved in the choice of $(APR_t - APR_t')$. In actual fact, the problems centre on the appropriate choice of APR_t so that APR_t' can be deduced directly.

From Equation (6.2.4) it is apparent that APR_t' is derived from the formula:

$$APR_t = \frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \cdot P_{x_{kt}:\overline{n_{kt}|}} \tag{6.2.5}$$

which can be rewritten as:

$$APR_t = \frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \cdot \left[\frac{D_{x_{kt}}}{N_{x_{kt}} - N_{(x_{kt}+n_{kt})}} + v - 1 \right]$$

$$= \frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \left(\frac{D_{x_{kt}}}{N_{x_{kt}} - N_{(x_{kt}+n_{kt})}} \right) + v - 1 \tag{6.2.6}$$

where x_{kt} denotes the inception age of the k^{th} policy-holder, n_{kt} denotes the policy term and $v = (1 + i_c)^{-1}$ (i_c - the calculation rate of interest).

However, Equation (6.2.6) is of no great value because, typically, the individual values of S_{kt}, x_{kt} and n_{kt} will not be known. Consequently, an approximation of APR_t is required that can be computed from the overall average (or total) values of S (sum insured), x (inception age) and n (policy term) that are available.

Let the 'weighted average' values of x_{kt} and n_{kt} be represented by $\bar{x}_t$ and $\bar{n}_t$; then $\bar{x}_t$ and $\bar{n}_t$ are determined by the following relationship:

$$APR_t = P_{\overline{x}_t:\overline{n}_t|} \tag{6.2.7}$$

where $P_{\overline{x}_t:\overline{n}_t|}$ will be termed the 'average' pure premium rate. By utilising Equations(6.2.6), (6.2.7) can be rewritten as:

$$\frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \cdot \left(\frac{D_{x_{kt}}}{N_{x_{kt}} - N_{(x_{kt}+n_{kt})}}\right) = \frac{D_{\overline{x}_t}}{N_{\overline{x}_t} - N_{(\overline{x}_t + \overline{n}_t)}} \tag{6.2.8}$$

(assuming that the same 'calculation' rate of interest (i_c) is used for all N_t policies).

Once the appropriate values of $\overline{x}_t$ and $\overline{n}_t$ have been determined (this will be discussed in later Sections), the Financial Saving Flow arising from those N_t non-profit endowment policies can be expressed as:

$$\text{Financial Saving in year } t = \Sigma S_t \cdot \left(\frac{D_{(\overline{x}_t + \overline{n}_t)}}{N_{\overline{x}_t} - N_{(\overline{x}_t + \overline{n}_t)}}\right) \tag{6.2.9}$$

while the Consumption Expenditure is given by:

$$\text{Consumption Expenditure in year } t = \Sigma OP_t - \Sigma S_t \cdot \left(\frac{D_{\overline{x}_t + \overline{n}_t}}{N_{\overline{x}_t} - N_{(\overline{x}_t + \overline{n}_t)}}\right) \tag{6.2.10}$$

A similar analysis can be conducted if we assume a portfolio of N_t non-profit annual premium whole of life policies issued to UK policyholders in year t. The

Financial Saving Flow generated by the N_t policies is then given by

$$\text{Financial Saving in year } t = \sum_{k=1}^{N_t} (OP_{kt} - P'_{kt} - EL_{kt})$$

where

$$P'_{kt} = S_{kt} \cdot P'_{y_{kt}:\overline{m_{kt}}|} \qquad \text{(see Equation (5.7.6))}$$

In exactly the same way as before, the above equation can be rewritten as:

$$\text{Financial Saving in year } t = \Sigma S_t \, (APR_t - APR'_t)$$

where

$$APR_t = \frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \cdot P_{x_{kt}} = P_{\overline{x}_t} \tag{6.2.11}$$

The 'weighted average' inception age $\overline{x}_t$ is then determined by the following relationship:

$$\frac{1}{\Sigma S_t} \cdot \sum_{k=1}^{N_t} S_{kt} \left(\frac{D_{x_{kt}}}{N_{x_{kt}}}\right) = \frac{D_{\overline{x}_t}}{N_{\overline{x}_t}} \tag{6.2.12}$$

Once the appropriate value of $\overline{x}_t$ has been determined, the Financial Saving Flow arising from those N_t non-profit whole of life policies can be expressed as:

$$\text{Financial Saving in year } t = \Sigma S_t \cdot (P_{\overline{x}_t} - P'_{\overline{y}_t:\overline{\overline{m}_t}|}) \tag{6.2.13}$$

while the Consumption Expenditure is given by:

$$\text{Consumption Expenditure in year } t = \Sigma OP_t - \Sigma S_t.(P_{\bar{x}_t} - P'_{\bar{y}_t:\overline{m_t}|}) \qquad (6.2.14)$$

The change from non-profit policies to with profit policies is, in theory, easy to accomplish, simply by adding the total bonus loading $(BL_t = \sum_{k=1}^{N_t} BL_{kt})$ to the Financial Saving equations ((6.2.9) and (6.2.13)) and deducting BL_t from the Consumption Expenditure equations ((6.2.10) and (6.2.14)). However, in practice, BL_t is not observable for either endowment or whole of life contracts and an artificial construction is necessary in order to derive it.

When it comes to calculating the 'weighted average' inception age $\bar{x}_{kt}$ (endowment and whole life; non-profit and with profit) and the 'weighted average' maturity term $\bar{n}_t$ (non-profit and with profit endowment), little real assistance is given by Equation (6.2.8) because the vector $(n_{1t}, \ldots, n_{N_t t})$ is entirely unknown. However, for years up to 1958, the Continuous Mortality Investigation (13) published annual figures on the Numbers of In Force (Duration 0) at each new inception age for the NP and WP contracts. Consequently if we were willing to make the assumption that S_{kt} = Constant, for all k, then, since the inception age of every new policyholder is known, a value could be obtained for $\bar{x}_t$ (in (6.2.12)) that satisfied:

$$\sum_{k=1}^{N_t} \frac{D_{x_{kt}}}{N_{x_{kt}}} = \frac{D_{\bar{x}_t}}{N_{\bar{x}_t}} \qquad (6.2.15)$$

(whole of life contracts only). But the value for $\bar{x}_t$ so obtained from Equation (6.2.15) would have a number of serious defects:

a) $\bar{x}_t$ could be seriously underestimated if older policyholders purchased whole life insurance with larger sums insured than average;

b) the calculation is obviously dependent on the rate of interest used to calculate the commutation functions D and N. This rate of interest should theoretically correspond with the long-term yield expected to be earned on life office funds but obviously the industry-wide 'average long-term yield' is not known with certainty, thus undermining the accuracy of the solution;

c) the immediately preceding argument could similarly be applied to the mortality tables used to calculate D and N. Theoretically a mortality table should be used that epitomises the 'long-term average' trends in mortality. Obviously, great difficulty is experienced in even an approximation and this again undermines the accuracy of the solution.

In the light of the above arguments, it was felt that no feasible attempt to determine a meaningful value of $\bar{x}_t$ (by solving Equation (6.2.12)) could be undertaken. Furthermore, no solution of Equation (6.2.8) was possible because of the existence of two unknowns in the endowment case. Consequently it was decided that an attempt should be made to evaluate $\bar{x}_t$ (whole life and endowment, NP and WP) and $\bar{n}_t$ (endowment, NP and WP) directly (and independently of Equations (6.2.8) and (6.2.12)). The various methods employed to derive $\bar{x}_t$ and $\bar{n}_t$ (and therefore, by implication, $P_{\bar{x}_t}$ and $P_{\bar{x}_t:\overline{\bar{n}_t|}}$) are described in the following Sections.

6.3 The Application to UK Aggregate Data

The purpose of this Section is two-fold: first to describe the actual UK aggregate data that is available (with an explanation of its deficiencies) and second to explain how the methods outlined in the preceding Section can be applied to the UK data (specifically when it comes to dealing with the troublesome bonus loading BL_t on with profit policies).

The greater part of the available information on the ordinary life insurance business conducted by companies operating in the UK is published in the Annual Abstract of Statistics (6). This information is compiled from statutory returns that are submitted, by each individual life office, to the Department of Trade. There are minor year-to-year differences in the accounting periods of the individual companies but these have no appreciable effect on the comparability of the figures in successive years.

The most detailed information available covers 'Life assurances in force (ordinary business)' for companies established in the UK. The figures for each year show the total number and amount (sum insured) of the policies in force at the date of the last valuation of each company, which may have been at any time within the preceding five years (to 1969). In the case of with profits policies, the 'sum insured' includes reversionary bonuses in force at the date of valuation but excludes any further bonuses allotted as a result of that valuation. The figures are split between the major classes of life insurance policy.

Table 6.3.1 Description of Life Assurance In Force (Companies Established in UK)

Non-Profits:	Total
	Whole Life
	Endowment
	Joint life / Joint life endowments
	Other
With Profits:	Total
	Whole Life
	Endowment
	Joint life / Joint life endowments
	Other

Source: Annual Abstract of Statistics (6)

In the absence of further guidance, it has been assumed that 'Non-Profits Other' policies refer entirely to temporary life insurance while 'With Profits Other' is entirely composed of single premium policies (this simplifying assumption is not too drastic since 'WP Other' policies are almost of no consequence until the mid-1960's when unit-linked single premium policies were gathering strength).

Unfortunately, the new business statistics are much less detailed in that they do not differentiate between the different classes of ordinary life insurance - not even between non-profit and with profit. However 'New Long-Term assurances (ordinary business)' do split the information according to the domicile of the vendor and purchaser of the life insurance.

Table 6.3.2 Description of New Long-term Assurances

Companies established in UK

All business	(Number of Policies (Sum Assured (Single Premiums (Yearly Renewal Premiums
Business with UK	(Number of Policies (Sum Assured (Single Premiums (Yearly Renewal Premiums

Companies established out of UK

Business with UK	(Number of Policies (Sum Assured (Single Premiums (Yearly Renewal Premiums

Source: Annual Abstract of Statistics (6)

At this stage, it is sufficient to state that, if certain assumptions are made, then the information that we require (namely: Ordinary New Business Annual Premium Income and Sums Insured split by different classes of life insurance purchased by UK policyholders) can be obtained from the data described in Tables 6.3.1 and 6.3.2 in conjunction with the survey information (see Appendix 6.3). Table 6.3.3 below then illustrates the information that is available and the notation that will be used to describe it.

Table 6.3.3 Notation for New Life Insurance Business for Annual (Renewable) Policies within the UK.

Non-Profits	New Sums Insured	Numbers	New Premium Income
Whole Life	S_{WLNP}	N_{WLNP}	OP_{WLNP}
Endowment	S_{ENP}	N_{ENP}	OP_{ENP}
Other (Temporary)	S_T	N_T	OP_T
With Profits			
Whole Life	S_{WLWP}	N_{WLWP}	OP_{WLWP}
Endowment	S_{EWP}	N_{EWP}	OP_{EWP}
Overall Totals	S	N	OP

In this final part of Section 6.3 we jump ahead to assume that the 'average' pure premium rates of Equations (6.2.7) and (6.2.11) ($P_{\overline{x}_t:\overline{\overline{n}_t}|}$ and $P_{\overline{x}_t}$) have been calculated for both non-profit and with profit endowments and whole life policies (by making appropriate choices of $\overline{x}_t$ and $\overline{n}_t$). The notation for these rates is set out in Table 6.3.4 below.

Table 6.3.4 Notation for 'Average' Pure Premium Rates per unit sum insured (Business within UK)

Non-Profits		
Whole Life	P_{WLNP}	(= $P_{\overline{x}_t}$, NP)
Endowment	P_{ENP}	(= $P_{\overline{x}_t:\overline{\overline{n}_t}|}$, NP)
With Profits		
Whole Life	P_{WLWP}	
Endowment	P_{EWP}	

By multiplying the 'average' pure premium rates by the appropriate sums insured we can then obtain values for the Aggregate Pure Premiums. Deducting these Aggregate Pure Premiums from the Aggregate Office Premiums (OP) we obtain the Aggregate Expenses Loading (for non-profit policies) and the Aggregate Expenses Loading plus Bonus Loading for with profit policies, ie.,

$$EL_{NP} = (OP - P.S)_{NP} \qquad \text{(Non-Profits, Endowment and Whole Life)}$$

and

$$(EL + BL)_{WP} = (OP - P.S)_{WP} \qquad \text{(With Profits, Endowment and Whole Life)}$$

(6.3.1)

The remaining problem is then to strip out the Aggregate Bonus Loading from the With Profits New Business Office Premiums. This is possible, for example, if it is assumed that the quantity (Expenses as a proportion of Pure Premium) for non-profits policies is a fixed proportion, θ, of that for with profits policies. The Bonus Loadings can then be derived from the following formulae:-

$$BL_E = (EL + BL)_{EWP} - \frac{P_{EWP}}{\theta \cdot P_{ENP}} \cdot EL_{ENP}$$

$$= (OP_{EWP} - P_{EWP} \cdot S_{EWP}) - \frac{P_{EWP}}{\theta \cdot P_{ENP}} (OP_{ENP} - P_{ENP} \cdot S_{ENP})$$

and

$$BL_{WL} = (OP_{WLWP} - P_{WLWP} \cdot S_{WLWP})$$

$$- \frac{P_{WLWP}}{\theta \cdot P_{WLNP}} \cdot (OP_{WLNP} - P_{WLNP} \cdot S_{WLNP})$$

(6.3.2)

We would normally expect $\theta > 1$ since the life office is faced with greater risks in the non-profit case (as the claims payments are guaranteed) and this is reflected in the expenses loadings included in Non-Profits Office Premiums (see Section 5.4). Similarly there is no reason why life offices should include a similar loading to cover shareholders' profits in the non-profit and with profit cases.

The discussion on a suitable choice for θ is continued in Section 6.10. In fact, it was not possible to accurately estimate θ and the alternative was to estimate the Bonus Loading directly by making an assumption about the method of with profits premium calculation (and using information obtained from the Survey).

Finally, from Equations (6.2.9) and (6.2.13) the financial saving flow from UK ordinary renewable life insurance can be represented as:

Financial Saving = Financial Saving from Endowments (NP and WP) + Financial Saving from Whole Life (NP and WP)

$$= (S_{ENP} \cdot \alpha_{ENP} + S_{EWP} \cdot \alpha_{EWP} + BL_E)$$

$$+ (S_{WLNP} \cdot \alpha_{WLNP} + S_{WLWP} \cdot \alpha_{WLWP} + BL_{WL}) \qquad (6.3.3)$$

where

α_E is the ratio $\left(\frac{D_{(\bar{x}_t+\bar{n}_t)}}{N_{\bar{x}_t} - N_{(\bar{x}_t+\bar{n}_t)}}\right)$ ie. $P_{\bar{x}_t:\overline{\bar{n}_t}|} - P'_{\bar{x}_t:\overline{\bar{n}_t}|}$

and α_{WL} is the ratio $(P_{WL} - P'_{\bar{y}_t:\overline{\bar{m}_t}|})$

Consumption Expenditure comprises the remainder of the New Business Premium Income:

$$\text{ie. Consumption Expenditure} = OP_{ENP} + OP_{EWP} + OP_{WLNP} + OP_{WLWP} + OP_{T} - \text{Financial Saving}$$

Similarly, the aggregate expenditure on savings-based life insurance can be represented as:

$$\text{Expenditure on Savings-based Life Insurance} = \text{Financial Saving} + \sum_{\substack{\text{all} \\ \text{types}}} \frac{S,\alpha}{OP} \cdot EL \qquad (6.3.4)$$

6.4 The Derivation of Ordinary Life New Business Renewable Premium Income.

The objective is now to explain how figures for new renewal premium income were derived for the major classes of non-group life insurance (temporary, NP whole life, NP endowment, WP whole life and WP endowment). This explanation is necessary because none of the information described in Table 6.3.3 can be obtained direct from the Annual Abstract of Statistics, but has to be approximated from the information that is available (see Tables 6.3.1 and 6.3.2).

Essentially the new business figures are derived by examining the change in number of policies and sums insured in force and then making an allowance for policies that become Claims and Cancellations during the year. The figures for new numbers and sums insured can then be converted into new premium income by examining the ratios $\frac{\text{New Numbers}}{\text{New Premiums}}$ and $\frac{\text{New Sums Insured}}{\text{New Premium Income}}$ that were obtained from the survey data. The resultant figures were then finally 'scaled' in order to correspond to life insurance business purchased in the UK (rather than that conducted by UK-based companies).

The actual calculations involved in the derivation of the new-business premium income are explained in more detail in Appendix 6.1. However it is important, at this stage, to mention the specific assumptions that underpin the resultant figures:

a) that the 'With Profits Other' policies (In Force) relate entirely to single premium business;

b) secondly, any remaining single premium business is distributed among the different classes of policy in the same manner as the non-single premium business (in terms of sums insured in force);

c) that the numbers (or the sums insured) of policies which become claims or cancellations in year t are proportional to Numbers (or Sums Insured) In Force in year t;

d) that with profits policies are composed entirely of the compound reversionary bonus type. The annual rate of bonus declared is given by b_t - which in turn is obtained from the survey data (see Table A6.1.1);

e) that the data obtained from the survey of major UK life insurers is representative of the overall life insurance industry. An example of the survey questionnaire is given in Appendix 6.3 but unfortunately no specific details can be given in case individual life offices are identified. The market share of those companies providing the data used in this Section ranged between 10.3% and 15.5% of new business renewal premium income;

f) that the distribution of new premium income over the different classes of life insurance is the same for business purchased in the UK and for business (both home and overseas) undertaken by

UK-established life offices. This assumption is necessary because In Force figures relate to UK-established companies while we require business purchased by UK policyholders. Note that for new business after the 23rd June, 1972, exchange control regulations were imposed by the Bank of England which prohibited the remittance abroad of life and endowment premiums by UK policyholders (see Bank of England (2));

g) that the distribution of group life business (in terms of sums insured in force) among the different classes of policy is the same as for non-group business. Figures for Aggregate New Business Renewal Non-Group Office Premium Income are given in Table 6.4.1 : the group life insurance business has been largely excluded because, by definition, it constitutes single-premium business. Table 6.4.2 details figures on New Business Sums Insured: these figures must be interpreted carefully because the group life business cannot be stripped out separately.

Table 6.4.1 New Business Renewal Non-Group Office Premium Income (Business within the UK) £000

	Non-Profits			With Profits		
Year	Whole Life	Endowment	Other	Whole Life	Endowment	Grand Total
1946	580.56	3,367.39	691.78	724.40	10,517.77	15,881.90
47	869.52	3,717.95	790.48	1,232.15	14,160.52	20,770.62
48	805.16	3,771.94	862.74	1,242.59	13,292.57	19,975.00
49	1,158.33	5,050.57	1,438.80	1,194.41	11,267.84	20,109.95
1950	871.31	7,177.20	1,815.30	954.60	11,118.93	21,937.34
51	1,197.19	12,421.94	2,413.78	985.15	9,862.57	26,880.63
52	1,256.79	7,676.93	1,459.73	1,478.07	13,397.20	25,268.72
53	1,669.43	6,814.44	2,477.75	1,473.26	12,409.59	24,844.47
54	1,662.03	11,832.29	2,526.05	1,125.62	11,387.02	28,533.01
1955	2,194.15	10,858.85	2,597.94	1,698.87	15,522.36	32,872.17
56	2,041.67	11,241.26	3,925.49	1,906.50	14,357.95	33,472.87
57	1,396.67	10,340.22	4,102.35	1,870.11	15,206.32	32,915.67
58	2,169.38	9,341.82	4,052.36	2,923.25	20,338.04	38,824.85
59	1,899.87	13,158.92	7,564.08	2,903.81	18,519.31	44,045.99
1960	2,266.81	14,837.23	6,990.53	3,046.20	25,840.41	52,981.18
61	2,200.38	14,880.92	6,899.95	2,527.62	31,740.47	58,249.34
62	3,626.00	13,034.37	8,053.21	4,387.82	34,851.15	63,952.55
63	4,016.36	15,209.00	9,515.49	4,443.04	40,116.85	73,300.74
64	5,290.42	17,458.45	8,013.01	5,822.87	47,881.67	84,466.42
1965	4,902.07	9,797.16	9,731.13	6,760.11	56,299.23	87,489.70
66	5,082.06	20,365.54	9,400.10	6,737.62	51,862.71	93,448.03
67	5,201.26	12,437.69	9,290.73	8,852.09	63,721.99	99,503.76
68	5,484.43	11,926.61	15,663.84	9,263.61	69,634.58	111,973.07

Source: From calculations described in Appendix 6.1.

Table 6.4.2 New Business Sums Insured (Business within the UK) £m.

Year	Non-Profits			With Profits	
	Whole Life	Endowment	Other	Whole Life	Endowment
1946	27.3	67.1	69.0	26.5	192.4
47	39.7	76.0	80.8	42.7	265.6
48	36.8	78.4	90.9	42.2	250.9
49	44.3	94.5	116.9	50.4	196.8
1950	40.7	147.4	121.1	41.8	199.2
51	38.8	218.9	187.8	36.5	158.9
52	46.1	162.2	135.0	66.9	243.7
53	55.7	141.4	205.4	64.5	212.4
54	52.4	258.9	227.3	51.8	204.4
1955	77.1	241.5	235.6	78.3	281.8
56	78.4	253.0	317.3	86.3	262.6
57	60.0	245.0	390.7	85.6	284.5
58	109.1	251.2	390.1	144.4	423.0
59	79.5	301.4	670.3	120.6	333.9
1960	93.3	332.8	756.8	126.7	476.0
61	99.2	333.0	934.7	118.8	630.4
62	146.0	249.4	1,132.4	188.6	608.6
63	163.8	303.7	1,418.7	198.5	699.1
64	241.9	400.4	1,564.2	295.7	945.1
1965	210.0	215.0	2,080.3	323.4	1,024.8
66	237.8	495.9	2,482.1	331.3	955.2
67	333.0	354.8	2,780.2	471.0	1,237.9
68	333.7	277.8	3,968.3	404.7	1,131.1

Source: From calculations described in Appendix 6.1.

6.5 The Derivation of Average Inception Age $\bar{x}_t$

The objective of this Section is to describe a method for determining a value for the average inception age at time t ($\bar{x}_t$) for with and non-profit endowment and whole of life policies. A possible theoretical basis for the determination of $\bar{x}_t$ was given in Equations (6.2.8) and (6.2.12) but it was explained at the end of Section 6.2 that these equations provided little practical help.

Failing the theoretical approach it was felt that the next best alternative was to calculate the simple average inception age for all new policyholders (ie. at Duration Zero) based on <u>numbers</u> of new policyholders. Luckily, figures were available from the Continuous Mortality Investigation (13) giving the number of lives Exposed to Risk at each age. The figures up to 1958 were split by class of policy, but after 1958, only the overall average (all classes combined) was available. Consequently a least squares polynominal curve fitting procedure was used to estimate the missing ten years figures. (The methodology and computer programme for this is given in Appendix 6.2).

The average inception ages were obtained by the following formula:

$$\bar{x} = \frac{\sum_{x=11}^{75} x.\ E_{[x]}}{\sum_{x=11}^{75} E_{[x]}} \qquad (6.5.1)$$

where $E_{[x]}$ refers to the number of policies exposed to risk, at duration zero (medical and non-medical combined). The summation was started at age 11 and terminated at age 75 because data was only consistently available over those ages. It can be appreciated that the inception age range 11-75 encaptures the vast majority of new policyholders.

It is realised that Equation (6.5.1) represents only an approximation of $\bar{x}_t$ and that we might expect such values to be under-estimated since a calculation based on sums insured (rather than numbers) would attach greater weight to older new policyholders (who might be expected to have larger policies).

Table 6.5.1 Average Inception Ages (1946/1968)

Year	Non-Profit		With Profit		Average
	Whole Life	Endowment	Whole Life	Endowment	
1946	39.69	37.94	41.45	36.26	36.67
47	36.66	36.05	37.31	34.37	34.73
48	35.98	36.21	36.94	34.51	34.89
49	36.21	36.50	36.20	34.57	35.01
1950	36.09	36.78	33.34	34.90	35.22
51	35.45	36.84	32.76	35.01	35.25
52	35.98	37.12	32.66	35.22	35.47
53	36.11	37.05	31.96	35.11	35.30
54	36.08	36.76	31.09	35.25	35.30
1955	35.63	36.65	30.96	35.50	35.41
56	35.14	36.52	30.82	35.64	35.41
57	34.49	36.12	30.39	35.27	35.01
58	33.44	35.60	30.19	34.37	34.18
59	32.19	34.86	30.00	33.97	33.84
1960	30.80	34.62	30.20	33.69	33.71
61	29.33	34.03	30.03	32.99	33.22
62	27.87	33.61	22.94	32.42	32.90
63	26.56	33.03	29.53	31.69	32.42
64	25.53	32.66	29.06	31.22	32.17
1965	24.96	32.36	28.31	30.94	31.91
66	25.03	32.06	27.10	30.88	31.66
67	25.97	32.13	25.69	32.54	31.77
68	28.00	32.01	23.40	32.50	31.65

Source: Data from the Continuous Mortality Investigation (13). Calculations explained in Appendix 6.2.

All figures for 1959 onwards (except overall Average) are estimated from the overall Average by a least squares polynominal curve fitting procedure.

6.6. The Derivation of Annual Mortality Tables (1945/1968)

This and the following Section (on the calculation of 'true' interest rates) are a natural consequence of the discussion on pure premiums at the end of Section 5.4. where it was assumed that life offices use the best available forecast of long-term rates of interest and mortality at the time of the premium calculation. Thus the pure premium must be calculated using realistic assumptions about future long-term rates of interest and mortality.

This redefining of pure premiums produces two problems for the investigation: first, that 'calculation' mortality tables must be generated that are theoretically time specific (this involves the compilation of mortality tables reflecting mortality experience for each post-war year). Second, some decision must be taken on what constitutes a 'realistic' assumption about long-term future mortality.

Any attempt to derive a rigorous solution to the first problem would involve the researcher in an enormous task. This is because each time-specific mortality table would depend not only on the age of the (average) policyholder but also on the year of his birth. Thus the mortality rate for a policyholder aged 40 in 1960 would differ from that of a policyholder aged 40 in 1961. This variation is of course in addition to the variation in mortality rates caused by the length of time since inception (encaptured in Select Mortality Tables).

The second problem is equally difficult to resolve because in most cases it is impossible to predict long-term future mortality rates. Additionally, this prediction would only be sensible if time specific mortality tables were available (and used by life offices).

In practice, life offices use mortality tables that are not time-specific. So, for example, the A1967/70 Tables (11) illustrate the average mortality experience of policyholders alive (or dying) during the period 1967/70 (irrespective of their year of birth).

Because the details necessary to compile a time-specific mortality table are not available it was decided to solve both of the above problems in the way used by life offices ever since mortality tables were first compiled. Thus mortality tables will be derived that reflect the mortality experience of policyholders alive and dying in year t with no attempt made to make those tables time specific[(3)]. Furthermore, since mortality rates have consistently improved over the post-war period, it seems reasonable that a 'realistic' assumption about future mortality rates would involve year t pure premiums calculated on the basis of year (t-1) mortality tables.

The remainder of this Section will be taken up with

[(3)] In which case a separate mortality table would be needed for each age attained in year t. eg. for a 40 year old in year t, the mortality table would be of the form:

Year	Age	Mortality Rate
t	40	q_{40}^{t}
t+1	41	q_{41}^{t+1}
t+2	42	q_{42}^{t+2} etc.

an explanation of the method of determining annual ultimate mortality tables for the period 1945/1969.

An estimation of annual ultimate mortality tables over the post-war period is fortunately quite straightforward because of the availability of (albeit rough) ultimate central mortality rates compiled by the Continuous Mortality Investigation and Government Actuary's Department and published by the Institute of Actuaries ((9) and (8)). Additionally, full scale sophisticated ultimate mortality tables are available that cover the beginning (A1949/52 Tables (10)) and the end of the period (A1967/70 Tables (11)).

At this stage, it is instructive to investigate briefly the main features of the above mortality features in-so-far as they affect the subsequent calculations:

a) according to the Institute of Actuaries (12) p.136, the two mortality 'A' tables were compiled on slightly different bases with the emphasis of the former on 'smoothness' (through the process of graduation) and of the latter on 'fidelity'. In practice, this distinction revolves around the 'hump' in mortality rates for teenage males. From our point of view, the A1967/70 Tables provide the better basis because we are concerned with actual mortality experience;

b) all tables refer to male policyholders only (age last birthday) and combine medical and non-medically examined lives;

c) the tables were compiled from data on whole of life and endowment policyholders only: this will not affect the calculations as it is only endowment and whole life policies that need to be examined;

d) it has been assumed that the A1949/52 Tables yield mortality rates for the year 1950½/1951½ while the A1967/70 Tables apply to the year 1968½/1969½;

e) both the central mortality rates from the Continuous Mortality Investigation and the Death Rates from the Government Actuary's Department are only available for five-yearly age groups and refer to 'Ultimate' rather than 'Select' figures: the former for ages 30-79 and the latter for ages 10-30 and 80-84. Death Rates for age group 84-109 are also available;

f) of course, the Death Rates obtained from the Government Actuary's Department refer to the Great Britain population as a whole rather than ordinary life insurance policyholders. However an attempt has been made to minimise any distorting effects;

g) figures for the years 1967/1969 were only available from the Government Actuary.

The computer programme (written in FORTRAN) that produces the annual 'Ultimate' mortality tables for the years

1945/1969 is printed in full in Appendix 6.4. (see SUBROUTINE MORTALITYTABLE). Broadly speaking, SUBROUTINE MORTALITYTABLE can be divided into three main sections: the first changes the central mortality death rates (m_x) into mortality rates q_x, where

$$q_x = \frac{2m_x}{2 + m_x} \qquad (6.6.1)$$

and also calculates 'rough' l_x figures (RALX) on the basis of a linear interpolation of male 'Ultimate' mortality rates between the A1949/52 and the A1967/70 Tables.

The second main section calculates values for l_x in five-year age groupings corresponding to those groupings in (8) and (9) ie. ages 10, 15, 20, .., 85, 109. Essentially, the programme uses the rough values of l_x (RALX) to calculate five-year 'average' mortality rates[4] and then compares these with those rates obtained from the data (see Equation (6.6.1)). The final l_x are then obtained by adjusting RALX up or down depending on the results of the above comparison.

The third main section of MORTALITYTABLE is concerned with filling in between the five-yearly values of l_x. This was done on the basis of the rough mortality rates of the first section, weighted to agree with the

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[4] The five-year average mortality rates (comparable with the data in (8) and (9)) were assumed to be calculated in the following way,

(eg. for the average mortality rate for age group 30-34):

$$\text{'Average' } q = \frac{Rl_{30} - Rl_{35}}{\sum_{i=0}^{4} Rl_{30+i}}$$

where Rl_{30} denotes 'rough' values

five-year values of l_x.

The procedure was then repeated on an iterative basis using final figures as the rough RALX and repeating. Three iterations were used.

The ultimate mortality tables so calculated are given in full in an Appendix to this thesis; there are however, several features of interest:

a) it was not possible to completely remove the effects of the five-year grouping of mortality rates. Thus, for younger ages, the final mortality rates still move in five-year steps; this is especially true for the immediate post-war years;

b) the effects of the Second World War are particularly noticeable on younger mortality rates for the years 1945/1948;

c) the ultimate mortality tables have radix 999,999 at age 10, and are not tabulated beyond age 100;

d) the mortality tables represent the end of year mortality rather than some average of calendar years (ie. the rate for 1960 is an average from June 1960 to June 1961);

e) the presence of a 'hump' in the mortality rates is most noticeable for the majority of years;

f) in general mortality rates have been falling over time for most ages although the fall has not been continuous. As an example, Table 6.6.1 below illustrates how mortality rates have changed for selected ages.

Table 6.6.1 Selected Mortality Rates (from SUBROUTINE MORTALITYTABLE)

Year	Age			
	25	35	55	70
1950	0.00121	0.00132	0.01083	0.04450
1955	0.00089	0.00107	0.01023	0.04130
1960	0.00080	0.00103	0.00984	0.03931
1965	0.00077	0.00093	0.00938	0.03973
1969	0.00073	0.00081	0.00863	0.04136

Source: Calculated Ultimate Rates of Mortality (SUBROUTINE MORTALITYTABLE) (Males: Medical and Non-Medical combined).

6.7 The Derivation of Appropriate 'Calculation' Rates of Interest.

It was explained at the beginning of the previous Section that pure premiums must be calculated using realistic assumptions about future long-term rates of interest. Again, there are two specific problems: the first on how to calculate this rate of interest and the second to do with what constitutes a 'realistic' assumption about long-term rates of interest.

Following the argument of Section 5.4 we need to determine the long-term net of tax yield on invested life office funds. However, it is well known that an accurate calculation is virtually impossible in the United Kingdom because the market value of life office funds is unknown: only the book values are reported (over the period 1945/1968) - both in individual company accounts and in the submissions to the Department of Trade.

An alternative method is available (on the lines of N. Barr (3)) by compiling a theoretical life office portfolio and computing the yield and market value of the included assets. It was felt, however, that no attempt could be made to undertake a similar investigation (for the post-war period) with any accuracy. Consequently the annual yield on invested life and annuity funds (for companies established in the UK) has been computed on a book value basis using the following formula:

$$\text{Annual Net Yield} = \left(\left(\text{'Interest etc. (less tax)' plus 'Miscellaneous' Income}\right)\right) \div \tfrac{1}{2}\left(F_{t-1} + F_t\right)$$

where F_t denotes the end of year fund in book values (Government Securities in Nominal Values) and the data on 'Interest etc. (less tax)' and 'Miscellaneous' Income is obtained from the Annual Abstract of Statistics.

The resultant yields - which are probably exaggerated because market values were generally in excess of book values (over the period) are given in Table 6.7.1 below:

Table 6.7.1 Annual Yield on Life and Annuity Funds (companies established in the UK)

Year	Yield (%)
1945	3.8887
46	4.5660
47	3.5319
48	3.6415
49	3.4969
1950	3.5029
51	3.7871
52	3.8153
53	3.9355
54	4.2725
1955	4.4425
56	4.3576
57	4.5217
58	4.7026
59	5.1949
1960	5.2713
61	5.5488
62	5.3711
63	5.6182
64	5.9356
1965	6.1334
66	5.8761
67	7.2319
68	8.5906

Source: Data from the Annual Abstract of Statistics (Book Values)

It seems reasonable to assume that a 'realistic' assumption of future interest rates would be based on previous experience of those rates. Furthermore, future expectations ought to be adjusted in the light of current experience. These conditions can be approximated if the estimated yield in year t (r_t^*) is based on a weighted average of previous annual yields. The common procedure is to form a Koyck-type distributed-lag relationship:

$$r_t^* = br_{t-1} + b(1-b)r_{t-2} + b(1-b)^2 r_{t-3} + \ldots.$$

which can be re-arranged to give

$$r_t^* = br_{t-1} + (1-b)r_{t-1}^* \qquad (6.7.1)$$

The discussion then centres around a suitable choice of b (and a starting value for r_{t-k}^*). If the value of b should be small then expectations tail-back into the past (ie. with a long memory). On the other hand, if b is close to unity then recent experience is the most important determinant of long-term expectations. Instinctively, it would seem that in order to take a long-term view of future yields, we should incorporate a long-term view of past yields (so that b is fairly small). On the other hand it can be argued that competitive pressure ensures that offices adapt fairly quickly to rising interest rates (thereby decreasing pure premium rates).

The argument that competitive pressure forces life offices to use more 'realistic' calculation rates of interest is quite a persuasive one and applies particularly to with profits policies (where the combination of interest and bonus should closely approximate to future long-term yields). Consequently, an annual examination of with profits premium rates (at a specific age and maturity term) should give an indication of movement in calculation rates of interest (if an allowance can be made for expenses loadings and changes in mortality).

Survey data from seven of the larger life offices was collated showing with profit office premium rates (per unit sum insured) at inception age 45 and maturity term 10 years. This information is shown in Table 6.7.2 below:

Table 6.7.2 Average Office Premium Rates for With-Profit Endowment and Whole of Life Policies from Seven Life Offices. (x = 45, n = 10)

Year	Endowment	Whole Life
1946	0.1100	0.0382
47	0.1100	0.0386
48	0.1100	0.0386
49	0.1099	0.0383
1950	0.1089	0.0373
51	0.1084	0.0373
52	0.1091	0.0378
53	0.1088	0.0376
54	0.1088	0.0376
1955	0.1087	0.0376
56	0.1087	0.0374
57	0.1087	0.0374
58	0.1087	0.0374
59	0.1085	0.0373
1960	0.1084	0.0371
61	0.1084	0.0371
62	0.1086	0.0372
63	0.1086	0.0372
64	0.1096	0.0377
1965	0.1096	0.0374
66	0.1091	0.0376
67	0.1097	0.0379
68	0.1092	0.0376

Source: Survey Data.

It is immediately obvious that the premium rates of Table 6.7.2 do not exhibit the downward trend (over time) that we would expect if higher calculation rates of interest were being used. Furthermore, since we might also expect lighter mortality tables to have been used over time (thereby exerting a downward pressure on rates) the picture is even more difficult to interpret. There are only three conclusions to be drawn:

i) the seven life offices in the sample have not succumbed to competitive pressure on premium rates (or alternatively, that there is no competitive pressure). We note however that this does not preclude competition over the levels of declared bonus rates;

or ii) that a downtrend in pure premiums has been offset by increases in expenses loadings - however expenses loadings (expressed as a percentage of pure premiums) tend to remain unchanged for long periods;

or iii) roughly the same calculation rate of interest has been used by the sample life offices each year. By comparing with the A1949/52 Tables (10) and making reasonable assumptions about expenses loadings, it can be deduced that a calculation rate of interest of around 3% has been used. This ties in well with the situation up to and including the middle of the 1950's (Table 6.7.1) but not so well afterwards.

One can only conclude from a comparison of Tables 6.7.1 and 6.7.2 that, since the yield on invested funds has undoubtedly risen over time, competitive pressure does not force up the calculation rate of interest to any great extent for with profit policies. This seems a much more reasonable conclusion than the alternative - which is that the life offices (at least those in the sample) have assumed that the long-term rate of interest will remain constant (at around 3%) or may even fall over time.

Thus it turns out that Table 6.7.2 is not of great value in the problem of choosing a constant for Equation (6.7.1). All we can conclude is that, in the absence of competitive pressure, changes in the expected long-term interest rates are not necessarily reflected in office premium rates.

In the absence of any further evidence, arbitrary figures for b were chosen in Equation (6.7.1) ($b = 0.5$ for endowment and $b = 0.25$ for whole life) and the resultant figures for r^*_t are shown in Table 6.7.3 below. A starting value of $r^*_{1945} = 3.5\%$ was taken in both cases.

Table 6.7.3 Long-term Expected Yield on Invested Life Office Funds (r_t^*)

Year	(b = 0.5)	(b = 0.25)
1945	3.5000	3.5000
46	3.6943	3.5970
47	4.1301	3.8393
48	3.8310	3.7624
49	3.7362	3.7322
1950	3.6165	3.6733
51	3.5597	3.6307
52	3.6734	3.6698
53	3.7443	3.7062
54	3.8399	3.7635
1955	4.0562	3.8907
56	4.2493	4.0287
57	4.3034	4.1109
58	4.4125	4.2136
59	4.5575	4.3358
1960	4.8762	4.5506
61	5.0737	4.7307
62	5.3112	4.9352
63	5.3411	5.0442
64	5.4796	5.1877
1965	5.7076	5.3747
66	5.9205	5.5643
67	5.8907	5.6423
68	6.5613	6.0397
69	7.5759	6.6774

Source: Table 6.7.1 and Equation (6.7.1)

A smaller value for b was chosen in the whole of life case because typically, these policies are of longer duration than the corresponding endowment policies for each year. Thus the life office must take a longer term view of future yields and it seems reasonable that the expected long-term yield r_t^* would not be so responsive to short term changes in r_t.

6.8 The Derivation of Average Maturity Term

The objective of this Section is to describe a method for determining a value for the average maturity term at time t ($\bar{n}_t$) for with and non-profit endowment policies. A possible theoretical basis for the determination of $\bar{n}_t$ was given in Equation (6.2.8), but it was explained at the end of Section 6.2 that this provides little practical help. In particular, the individual information on maturity terms (n_{kt}) - on which Equation (6.2.8) relies - is simply not available.

Unfortunately, further problems are encountered in formulating an alternative method of calculating $\bar{n}_{kt}$ because no comprehensive information is available which could be used. There are however a small number of limited alternatives:

a) keep $\bar{n}_t$ fixed at, say, 20 or 25 years;

b) assume that the majority of endowments mature at retirement age;

c) use any survey data available (eg. Southern Television (17) indicate that $\bar{n}_{1975} = 23.5$);

d) attempt to relate $\bar{n}_t$ to the average Building Society mortgage repayment period (since a large proportion of endowment policies are used for house purchase purposes); or

e) attempt a rough approximation to $\bar{n}_t$: for example, by using $\bar{n}_t = \dfrac{\text{New Sums Insured}}{\text{New Office Premiums}}$ (for with profits policies).

It seems that alternative d) above is impracticable because of the paucity of detail published by the Building Societies Association. In any case, the practice of extending the mortgage repayment period instead of increasing interest repayments (in the event of a rise in the rate of interest received on mortgage advances) makes alternative d) undesirable.

Alternative e) was attempted by inverting the figures to be found in the final column of Table A6.1.2: the answers are reported in Table 6.8.1 below. The final column of Table 6.8.1 shows the estimated age at maturity obtained by adding the estimated maturity term to the inception ages of Table 6.5.1.

Table 6.8.1 Estimated Maturity Term and Age
(With Profit Endowments)

Year	New Sums Insured / New Office Premiums	Estimated Maturity Age
1946	18.34	54.60
47	18.34	52.71
48	18.36	52.87
49	18.42	52.99
1950	18.34	53.24
51	18.94	53.95
52	19.05	54.27
53	18.92	54.03
54	18.76	54.01
1955	18.85	54.35
56	18.96	54.60
57	19.62	54.89
58	19.84	54.21
59	20.30	54.27
1960	20.80	54.49
61	20.78	53.77
62	20.56	53.02
63	20.60	52.29
64	20.97	52.19
1965	20.79	51.73
66	20.54	51.42
67	19.77	51.31
68	20.41	52.91

Source: Table A6.1.2.

Now alternative e) is only a good approximation (for with profits endowments) if the other variables involved in the premium calculation remain constant over time. Table 6.7.2 shows that, as far as premium calculations are concerned, mortality, interest and expenses assumptions remain roughly constant. However Table 6.5.1 shows that average Inception Age generally declined over the post-war period for with profit endowments. Consequently, even if the average maturity term ($\bar{n}_t$) stayed constant over time, we would expect the ratio new sums insured over new premium income to rise. Thus it seems reasonable to conclude that $\bar{n}_t$ is overestimated for the period 1956/1968. Similarly, since Average Inception Age increased over the period 1946/1956, it would be reasonable to assume that the Ratio figures are underestimates of $\bar{n}_t$. Consequently, it would appear that any ratio figures under 20.0 (approximately) are underestimates while any over 20.0 are overestimates. Thus in the absence of further guidance, it will be assumed that $\bar{n}_t = 20.0$ for all t (1946/1968). In view of the above interpretation of Table 6.8.1, this does not seem an unreasonable simplification.

Further, it will be assumed that $\bar{n}_t = 20.0$ for all t in the case of non-profits endowment policies.

6.9 The Derivation of Annual Pure Premium Rates

The purpose of this Section is to describe how the results of the preceding Sections were combined to produce annual pure premium rates $P_{\overline{x}_t:\overline{n}_t|}$ and $P_{\overline{x}_t}$ and also the corresponding temporary life insurance elements $P^{1}_{\overline{x}_t:\overline{n}_t|}$ and $P^{1}_{\overline{y}_t:\overline{m}_t|}$. (see Section 6.3). The calculations involved are essentially very simple although complicated by the fact that for each year, a different mortality table, rate of interest, inception age (four types) and maturity term must be used (as described in the preceding Sections).

The actual calculations were performed by a computer programme - MASTER SAVINGS - which was written in FORTRAN and is reproduced in full in Appendix 6.4.

MASTER SAVINGS has four main sections: the MASTER section which reads the data and calls the other three Subroutines. One Subroutine calculates the annual mortality tables (SUBROUTINE MORTALITYTABLE); one calculates the commutation functions and final premium rates (SUBROUTINE COMMUTATION) and a third derives values for the expectation of life (FUNCTION EXPTN(X)).

The results of MASTER SAVINGS are set out in Tables 6.9.1 and 6.9.2 below showing true specific pure premium rates and the corresponding temporary premium element. Tables 6.9.1 and 6.9.2 incorporate mortality tables, the Appendix to this thesis, inception ages from Table 6.5.1, rates of interest from Table 6.7.3 and $\overline{n}_t = 20$ over the period 1946/68.

Table 6.9.1 Pure Premium Rates (New Business purchased in the UK 1946/68).

NON-PROFITS

Year	Endowment		Whole of Life	
	$P_{x_t:\overline{n_t}\rceil}$	$P'_{x_t:\overline{n_t}\rceil}$	P_{x_t}	$P'_{y_t:\overline{m_t}\rceil}$
1946	0.035771	0.005226	0.016610	0.009140
47	0.033704	0.004169	0.014008	0.007768
48	0.034695	0.004179	0.013783	0.007571
49	0.034939	0.003942	0.013465	0.007417
1950	0.035413	0.004108	0.013696	0.007473
51	0.035625	0.004164	0.013441	0.007310
52	0.035225	0.004288	0.013979	0.007577
53	0.034916	0.004070	0.013590	0.007438
54	0.034404	0.003805	0.013260	0.007220
1955	0.033565	0.003671	0.012715	0.007021
56	0.032844	0.003516	0.012014	0.006630
57	0.032545	0.003244	0.011496	0.006371
58	0.032071	0.003025	0.010641	0.005960
59	0.031414	0.002665	0.009812	0.005474
1960	0.030350	0.002604	0.008762	0.005023
61	0.029644	0.002398	0.007757	0.004445
62	0.028803	0.002210	0.006912	0.004045
63	0.028582	0.001980	0.006274	0.003690
64	0.028141	0.001979	0.005807	0.003486
1965	0.027447	0.001929	0.005312	0.003256
66	0.026744	0.001794	0.005100	0.003167
67	0.026822	0.001774	0.005173	0.003195
68	0.024820	0.001626	0.005143	0.003244

Source: MASTER SAVINGS

Table 6.9.2 Pure Premium Rates (New Business purchased in the UK 1946/68).

WITH PROFITS (without Bonus Loading)

	Endowment		Whole of Life	
Year	$P_{\overline{x}_t:\overline{n}_t\rceil}$	$P'_{\overline{x}_t:\overline{n}_t\rceil}$	$P_{\overline{x}_t}$	$P'_{\overline{y}_t:\overline{m}_t\rceil}$
1946	0.035536	0.004565	0.017989	0.009865
47	0.033492	0.003621	0.014425	0.007979
48	0.034426	0.003526	0.014400	0.007902
49	0.034657	0.003238	0.013459	0.007414
1950	0.035127	0.003388	0.012134	0.006632
51	0.035326	0.003416	0.011940	0.006504
52	0.034887	0.003471	0.012053	0.006544
53	0.034604	0.003295	0.011293	0.006189
54	0.034163	0.003195	0.010611	0.005793
1955	0.033383	0.003192	0.010308	0.005700
56	0.032703	0.003177	0.009843	0.005449
57	0.032424	0.002925	0.009505	0.005285
58	0.031910	0.002621	0.009135	0.005135
59	0.031314	0.002405	0.008846	0.004948
1960	0.030244	0.002344	0.008512	0.004883
61	0.029536	0.002141	0.008030	0.004595
62	0.028691	0.001951	0.007664	0.004465
63	0.028462	0.001709	0.007285	0.004250
64	0.028022	0.001687	0.006945	0.004125
1965	0.027343	0.001673	0.006301	0.003819
66	0.026657	0.001586	0.005675	0.003497
67	0.026768	0.001666	0.005097	0.003151
68	0.024849	0.001714	0.004014	0.002581

Source: MASTER SAVINGS

6.10 The Derivation of Aggregate Annual New Pure Premiums, Expenses Loadings and Bonus Loadings.

All is now ready to implement the theory developed in Section 6.3 so that aggregate figures can be obtained for financial saving flow from UK ordinary renewable life insurance.

The first step is to multiply the pure premium rates of Tables 6.9.1 and 6.9.2 by the new sums insured (Table 6.4.2) in order to obtain aggregate annual new pure premiums from renewable life insurance purchased in the UK. The resultant figures are given in Table 6.10.1 below. The corresponding aggregate temporary pure premiums are shown in Table 6.10.2.

The next step is to deduct the Aggregate Pure Premiums of Table 6.10.1 from the Aggregate Office Premiums of Table 6.4.1 (according to Equations (6.3.1)). This yields the Aggregate Expenses Loading in the case of non-profits policies and the Aggregate Expenses plus Bonus Loadings in the case of with profit policies. The figures are set out in Table 6.10.3 below. Details of Aggregate Expenses plus Bonus Loadings expressed as a proportion of Pure Premiums are given in Table 6.10.4.

An analysis of Table 6.10.4 confirms the suspicions voiced in Section 6.3 that the loading for Expenses (which includes expenses, contingencies and shareholder's profit) is indeed greater for non-profit than with profit policies in the whole life case, ie. $\theta > 1$ where

$$\theta = \frac{(EL/P)_{NP}}{(EL/P)_{WP}} \qquad (6.10.1)$$

Table 6.10.1 Aggregate Annual Pure Premiums (New Business purchased in the UK 1946/68). £000

	Non-Profits		With Profits	
Year	Whole Life	Endowment	Whole Life	Endowment
1946	453.5	2,400.2	476.7	6,837.1
47	556.1	2,561.5	615.9	8,895.5
48	507.2	2,720.1	607.7	8,637.5
49	596.5	3,301.7	678.3	6,820.5
1950	557.4	5,219.9	507.2	6,997.3
51	521.5	7,798.3	435.8	5,613.3
52	644.4	5,713.5	806.3	8,502.0
53	757.0	4,937.1	728.4	7,349.9
54	694.8	8,907.2	549.6	6,982.9
1955	980.3	8,105.9	807.1	9,407.3
56	941.9	8,309.5	849.5	8,587.8
57	689.8	7,973.5	813.6	9,224.6
58	1,160.9	8,056.2	1,319.1	13,497.9
59	780.1	9,468.2	1,066.8	10,455.7
1960	817.5	10,100.5	1,078.5	14,396.1
61	769.5	9,871.5	954.0	18,619.5
62	1,009.2	7,183.5	1,445.4	17,461.3
63	1,027.7	8,680.4	1,446.1	19,897.8
64	1,404.7	11,267.7	2,053.6	26,483.6
1965	1,115.5	5,901.1	2,037.7	28,021.1
66	1,212.8	13,262.3	1,880.1	25,462.8
67	1,722.6	9,516.4	2,400.7	33,136.1
68	1,716.2	6,895.0	1,624.5	28,106.7

Source: Tables 6.4.2 and 6.9.1

Table 6.10.2 Temporary (Protection) Element of Aggregate Annual Pure Premiums (New Business purchased in the UK 1946/68) £000

Year	Non-Life		With Profits		Grand Total
	Whole Life	Endowment	Whole Life	Endowment	
1946	249.5	350.7	261.4	878.3	1,739.9
47	308.4	316.8	340.7	961.7	1,927.6
48	278.6	327.6	333.5	884.7	1,824.4
49	328.6	372.5	373.7	637.2	1,712.0
1950	304.2	605.5	277.2	674.9	1,861.8
51	283.6	911.5	237.4	542.8	1,975.3
52	349.3	695.5	437.8	845.9	2,328.5
53	414.3	575.5	399.2	699.9	2,088.9
54	378.3	985.1	300.1	653.1	2,316.6
1955	541.3	886.5	446.3	899.5	2,773.6
56	519.8	889.5	470.2	834.3	2,713.8
57	382.3	794.8	452.4	832.2	2,461.7
58	650.2	759.9	741.5	1,108.7	3,260.3
59	435.2	803.2	596.7	803.0	2,638.1
1960	468.6	866.6	618.7	1,115.7	3,069.6
61	440.9	798.5	545.9	1,349.7	3,135.0
62	590.6	551.2	842.1	1,187.4	3,171.3
63	604.4	601.3	843.6	1,194.8	3,244.1
64	843.3	792.4	1,219.8	1,594.4	4,449.9
1965	683.8	414.7	1,235.1	1,714.5	4,048.1
66	753.1	889.6	1,158.6	1,514.9	4,316.2
67	1,063.9	629.4	1,484.1	2,062.3	5,239.7
68	1,082.5	451.7	1,044.5	1,938.7	4,517.4

Source: Tables 6.4.2 and 6.9.2

Table 6.10.3 Aggregate Expenses plus Bonus Loadings in Annual Office Premiums (New Business purchased in the UK 1946/68) £000

Year	Non-Profits		With Profits	
	Whole Life	Endowment	Whole Life	Endowment
1946	127.1	967.2	247.7	3,680.7
47	313.4	1,156.5	616.3	5,265.0
48	298.0	1,051.8	634.9	4,655.1
49	561.8	1,748.9	516.1	4,447.3
1950	313.9	1,957.3	447.4	4,121.6
51	675.7	4,623.6	549.4	4,249.3
52	612.4	1,963.4	671.8	4,895.2
53	912.4	1,877.3	744.9	5,059.7
54	967.2	2,925.1	576.0	4,404.1
1955	1,213.9	2,753.0	891.8	6,115.1
56	1,099.8	2,931.8	1,057.0	5,770.2
57	706.9	2,366.7	1,056.5	5,981.7
58	1,008.5	1,285.6	1,604.2	6,840.1
59	1,119.8	3,690.7	1,837.0	8,063.6
1960	1,449.3	4,736.7	1,967.7	11,444.3
61	1,430.9	5,009.4	1,573.6	13,121.0
62	2,616.8	5,850.9	2,942.4	17,389.9
63	2,988.7	6,528.6	2,996.9	20,219.1
64	3,885.7	6,190.8	3,769.3	21,398.1
1965	3,786.6	3,896.1	4,722.4	28,278.1
66	3,869.3	7,103.2	4,857.5	26,399.9
67	3,478.7	2,921.3	6,451.4	30,585.9
68	3,768.2	5,031.6	7,639.1	41,527.9

Source: Tables 6.4.1 and 6.10.1

Table 6.10.4 Aggregate Expenses plus Bonus Loadings expressed as a proportion of Pure Premiums (New Business purchased in the UK, 1946/68)

Year	Whole Life		Endowment	
	Non-Profit	With Profits	Non-Profit	With Profits
1946	0.2803	0.5196	0.4030	0.5383
47	0.5636	1.0006	0.4515	0.5919
48	0.5875	1.0448	0.3867	0.5389
49	0.9418	0.7609	0.5297	0.6520
1950	0.5632	0.8821	0.3750	0.5890
51	1.2957	1.2607	0.5929	0.7570
52	0.9503	0.8332	0.3436	0.5758
53	1.2053	1.0227	0.3802	0.6884
54	1.3921	1.0480	0.3284	0.6307
1955	1.2383	1.1049	0.3396	0.6500
56	1.1676	1.2443	0.3528	0.6719
57	1.0248	1.2985	0.2968	0.6485
58	0.8687	1.2161	0.1596	0.5068
59	1.4355	1.7220	0.3898	0.7712
1960	1.7728	1.8245	0.4690	0.7950
61	1.8595	1.6495	0.5075	0.7047
62	2.5929	2.0357	0.8145	0.9959
63	2.9081	2.0724	0.7521	1.0161
64	2.7662	1.8355	0.5494	0.8080
1965	3.3945	2.3175	0.6602	1.0092
66	3.1904	2.5836	0.5356	1.0368
67	2.0194	2.6873	0.3070	0.9230
68	2.1957	4.7024	0.7297	1.4775

Source: Tables 6.10.3 and 6.10.1

The final problem then revolves around a suitable choice for θ (assuming θ constant over time) before substitution into Equations (6.3.2).

Whether or not a value of $\theta = 1$ is appropriate for the endowment policies (as it clearly could be) is another issue that must be decided. An examination of the breakdown of the Expenses Loading should clarify the issue.

As has been previously stated, the size of any Expenses Loading (and hence the appropriate choice of θ) depends on a number of factors: the size of shareholders' profits, management expenses and an allowance for risk caused by unforeseen contingencies (this is often called a 'contingency' loading although it is not usually included specifically - see Chapter Five, note (8)). Management expenses are themselves often subdivided into three parts: initial expenses such as specific office expenses incurred at the commencement of the policy (including initial commission - expressed as a rate per cent of sum insured (up to October 1976)), renewal expenses such as renewal commission - expressed as a rate per cent of office premiums and thirdly, miscellaneous expenses to cover investment, claims, data services etc.

The following observations seem reasonable in the light of the preceding description:

a) we would expect the loading covering shareholders' profits to vary (over time and between policies) depending upon the competitive situation;

b) when expressed as a proportion of pure premiums we would not expect non-initial management expenses to vary between with and non-profit policies or between whole of life and endowment;

c) initial commission (expressed as a rate per cent of sums insured) as a proportion of pure premiums would cause variations between policies because of differing sums insured and inception ages (thus the greater the sum insured and/or inception age, the greater the Expenses Loading);

d) a larger contingency loading would be expected for non-profits policies (since sums insured are then guaranteed); and

e) a larger contingency loading for whole of life policies might be expected (in comparison with endowments) in view of the long time-horizon involved.

Thus a figure for θ in excess of unity can be caused by competitive pressure, initial commission or the contingency loading. However, it is only possible to investigate the effects of the initial commission (since commission rates, sums insured, inception ages and rates of interest are known).

In order to analyse the effects of initial commission, we need to know something about the level of the initial commission rates. These rates were fixed in 1954 (for Life Office Association members) at £2% of sums insured (whole of life and endowment) and it will be assumed that they remained unchanged over the period 1946/1968. Consequently the aggregate loading to cover initial expenses is given by

$$0.02 \frac{S}{P.\ddot{a}_x} \qquad (6.10.2)$$

where S represents the aggregate sum insured and P the aggregate pure premiums.(5)

Equation (6.10.2) can be rewritten as

$$0.02 \left(1 + \frac{(1-v)}{P_{\overline{x}}}\right) \qquad (6.10.3)$$

where $P_{\overline{x}}$ denotes the pure premium rates of Tables 6.9.1 and 6.9.2 and v uses the rates of interest (100r) in Table 6.7.3.

Aggregate Expenses Loadings to cover Initial Commission as a proportion of pure premiums are detailed in Table 6.10.5, and it is immediately obvious that these only have a very marginal influence on the figures of Table 6.10.4. Since it is unlikely that the addition of a contingency loading would solely account for the differences of Table 6.10.4. (especially for whole life policies) we must conclude that the major determinant of the size of $\theta > 1$ has been the effects of competitive pressure (or the lack of it) on the expenses loading to provide for shareholder's profit. Obviously the market for both with profits and non-profits endowment policies has been considerably more competitive than that for with profits

(5) Using what is known as 'Spragues' formula', eg. see P.D. Bacon and L.J. New p.178 (1).

Table 6.10.5 Aggregate Expenses Loadings to cover Initial Commission as a proportion of Pure Premiums (New Business purchased in the UK 1946/68).

	Whole Life		Endowment	
Year	Non-Profit	With Profit	Non-Profit	With Profit
1946	0.06180	0.05860	0.03992	0.04005
47	0.07279	0.07126	0.04354	0.04368
48	0.07262	0.07036	0.04127	0.04144
49	0.07344	0.07346	0.04062	0.04078
1950	0.07174	0.07840	0.03971	0.03987
51	0.07213	0.07870	0.03930	0.03946
52	0.07065	0.07874	0.04012	0.04031
53	0.07259	0.08329	0.04067	0.04086
54	0.07471	0.08836	0.04150	0.04165
1955	0.07891	0.09266	0.04323	0.04335
56	0.08447	0.09869	0.04482	0.04493
57	0.08869	0.10308	0.04535	0.04545
58	0.09599	0.10852	0.04635	0.04649
59	0.10470	0.11395	0.04775	0.04784
1960	0.11935	0.12227	0.05064	0.05075
61	0.13646	0.13250	0.05258	0.05270
62	0.15608	0.14273	0.05502	0.05516
63	0.17308	0.15183	0.05548	0.05563
64	0.18986	0.16203	0.05692	0.05708
1965	0.21204	0.18190	0.05934	0.05949
66	0.22671	0.20576	0.06180	0.06194
67	0.22692	0.22957	0.06148	0.06156
68	0.24149	0.30379	0.06962	0.06956

Source: Tables 6.9.1, 6.9.2, 6.7.3 and Equation (6.10.3) whole of life which, in turn, is more competitive than non-profit whole of life.

None of the above conclusions help in the allocation of with profit expenses loadings (although they do give an insight into the nature of the business). So, rather than make some kind of approximation which could not be anything but inaccurate it was felt that a better approach would be to calculate the bonus loading directly (hence revealing the with profits expenses loadings).

Bonus Loadings on with profit policies can be quite simply approximated in a direct fashion by utilising the Annual Average Rates of Compound Reversionary Bonus Declared (Table A6.1.1) obtained from the survey data. It seems reasonable to assume that life offices would compute a bonus loading which, if the yield on invested funds only equalled (rather than exceeded) the calculation rate of interest, would provide a future reversionary bonus almost as large as that which was currently declared. This seems sensible for four reasons: first, in view of the apparently large expenses loadings that have been allocated (so that bonus loadings have obviously not been forced down to low levels by competitive pressure). Secondly, to quote from P.F. Hooker and L.H. Longley-Cook, p.105 (7):

> "For ordinary assurances it is now customary to calculate with profits premiums from a formula which includes the cost of a bonus at a rate somewhat lower than that which it

is hoped to declare."

Thirdly, when with profits policies are sold, it is usual to quote an estimated benefit at maturity based on current bonus rates. Finally, where the comparatively recent 'bonus-reinforced' policies are used for house purchase purposes, they have been set up on the basis that the maturity value will be adequate to repay the loan in full if bonus declarations during the term of the policy average no more than 80% of the bonus rate in force when the policy was arranged.

The rate of compound reversionary bonus (b per annum) was therefore obtained by multiplying the rates of Table A6.1.1 by a factor of 0.8. The pure premiums plus bonus loading is then given by:

$$\frac{A_x^r}{\ddot{a}_x^i} \quad \text{in the whole life case}$$

or

$$\frac{A_{x:\overline{n|}}^r}{\ddot{a}_x^i} \quad \text{in the endowment case}$$

where i denotes the use of the calculation rate of interest and similarly $r = \frac{i-b}{1+b}$. It is recognised that the same problems as before arise in the use of compound reversionary bonus in that no account is taken of those simple bonus with profits policies.

Figures for the Bonus Loading per unit sums insured so obtained (ie. $(A_x^r - A_x^i) . \frac{1}{\ddot{a}_x^i}$) are given in Table 6.10.6 and the resultant aggregate Bonus and Expenses Loadings are separated and shown in Table 6.10.7. Aggregate Bonus and Expenses Loadings as a proportion of Pure Premiums are given in Table 6.10.8.

A comparison of Tables 6.10.4 and 6.10.8 confirm the conclusions about the influence of competitive pressure on Non-Profits loadings. Table 6.10.8 shows that the Expenses Loadings for With Profits policies have been fairly moderate in comparison with those for Non-Profits policies: this is especially true in the whole of life case.

We are finally in a position to obtain figures on financial saving flow by utilising Equation (6.3.3). Thus Financial Saving Flow (out of Office Premiums) is given by Pure Premiums (Table 6.10.1) less the Temporary Elements (Table 6.10.2) plus the Bonus Loadings (Whole Life and Endowment combined) (Table 6.10.7). These details are summarised in Table 6.10.9.

Similarly, it is possible to obtain aggregate figures on premium expenditure on savings-based life insurance by utilising Equation (6.3.4). Thus aggregate expenditure on savings-based life insurance is given by Financial Saving Flow plus a share of the Expenses Loadings of Tables 6.10.3 and 6.10.7 obtained by multiplying these loadings by a factor of $\left(\frac{\text{OP - Temporary Element}}{\text{OP}}\right)$ where the Temporary Elements are from Table 6.10.2 and the Office Premiums (OP) are from Table 6.4.1. The results are given in Table 6.10.10.

Table 6.10.6 Bonus Loadings per unit Sums Insured
(New Business purchased in the UK 1946/68)

Year	Whole Life	Endowment
1946	0.0067616	0.0083831
47	0.0060849	0.0079782
48	0.0067724	0.0089709
49	0.0066622	0.0090630
1950	0.0079443	0.0109155
51	0.0085837	0.0116998
52	0.0085671	0.0115527
53	0.0091254	0.0123861
54	0.0088636	0.0122529
1955	0.0087999	0.0122388
56	0.0091468	0.0128050
57	0.0089015	0.0127234
58	0.0102662	0.0145142
59	0.0099371	0.0142776
1960	0.0099415	0.0144475
61	0.0099523	0.0148008
62	0.0094060	0.0143962
63	0.0100810	0.0156101
64	0.0104724	0.0164438
1965	0.0102076	0.0166905
66	0.0095756	0.0164788
67	0.0093305	0.0169353
68	0.0093151	0.0179587

Source: MASTER SAVINGS

Table 6.10.7 Aggregate Bonus and Expenses Loadings (shown separately) in Annual New Business Office Premiums. £000

Year	Whole Life EL	With Profits BL	Endowment EL	With Profits BL
1946	68.5	179.2	2,067.8	1,612.9
47	356.5	259.8	3,146.0	2,119.0
48	349.1	285.8	2,404.3	2,250.8
49	180.3	335.8	2,663.7	1,783.6
1950	115.3	332.1	1,947.2	2,174.4
51	236.1	313.3	2,390.2	1,859.1
52	98.7	573.1	2,079.8	2,815.4
53	156.3	588.6	2,428.9	2,630.8
54	116.9	459.1	1,899.6	2,504.5
1955	202.8	689.0	2,666.2	3,448.9
56	267.6	789.4	2,407.6	3,362.6
57	294.5	762.0	2,361.9	3,619.8
58	121.8	1,482.4	700.6	6,139.5
59	638.6	1,198.4	3,296.3	4,767.3
1960	708.1	1,259.6	4,567.3	6,877.0
61	391.3	1,182.3	3,790.6	9,330.4
62	1,168.4	1,774.0	8,628.4	8,761.5
63	995.8	2,001.0	9,306.1	10,913.0
64	672.6	3,096.7	5,857.1	15,541.0
1965	1,421.3	3,301.1	11,173.7	17,104.4
66	1,685.1	3,172.4	10,659.4	15,740.5
67	2,056.7	4,394.7	9,621.7	20,964.2
68	3,869.3	3,769.8	21,214.8	20,313.1

Source: Tables 6.4.2, 6.10.3 and 6.10.6

Table 6.10.8. Aggregate Bonus and Expenses Loadings (shown separately) as a proportion of Pure Premiums.

Year	Whole Life EL	With Profits BL	Endowment EL	With Profits BL
1946	0.1437	0.3759	0.3024	0.2359
47	0.5788	0.4218	0.3537	0.2382
48	0.5745	0.4703	0.2784	0.2606
49	0.2658	0.4951	0.3905	0.2615
1950	0.2273	0.6548	0.2783	0.3107
51	0.5418	0.7189	0.4258	0.3312
52	0.1224	0.7108	0.2446	0.3311
53	0.2146	0.8081	0.3305	0.3579
54	0.2127	0.8353	0.2720	0.3587
1955	0.2513	0.8537	0.2834	0.3666
56	0.3150	0.9293	0.2804	0.3916
57	0.3620	0.9366	0.2560	0.3924
58	0.0923	1.1238	0.0519	0.4548
59	0.5986	1.1234	0.3153	0.4560
1960	0.6566	1.1679	0.3173	0.4777
61	0.4102	1.2393	0.2036	0.5011
62	0.8084	1.2273	0.4941	0.5018
63	0.6886	1.3838	0.4677	0.5485
64	0.3275	1.5079	0.2212	0.5868
1965	0.6975	1.6200	0.3988	0.6104
66	0.8963	1.6874	0.4186	0.6182
67	0.8567	1.8306	0.2904	0.6327
68	2.3818	2.3206	0.7548	0.7227

Source: Tables 6.4.1 and 6.10.7

Table 6.10.9 Financial Saving Flow in UK Ordinary Renewable Life Insurance New Business £000

Year	Saving	Consumption	Total
1946	10,219.7	5,662.3	15,882.0
47	13,080.2	7,690.5	20,770.7
48	13,184.7	6,790.3	19,975.0
49	11,804.4	8,305.5	20,109.9
1950	13,926.5	8,010.8	21,937.3
51	14,566.0	12,314.7	26,880.7
52	16,726.2	8,542.5	25,268.7
53	14,902.9	9,941.6	24,844.5
54	17,781.5	10,751.5	28,533.0
1955	20,664.9	12,207.4	32,872.3
56	20,126.9	13,346.1	33,473.0
57	20,621.6	12,294.1	32,915.7
58	28,395.7	10,429.2	38,824.9
59	25,098.4	18,947.6	44,046.0
1960	31,459.6	21,521.5	52,981.1
61	37,592.2	20,657.2	58,249.4
62	34,463.6	29,489.0	63,952.6
63	40,722.0	32,578.8	73,300.8
64	55,397.4	29,069.1	84,466.5
1965	53,432.8	34,056.9	87,489.7
66	56,414.7	37,033.3	93,448.0
67	66,895.0	32,608.8	99,503.8
68	57,907.9	54,065.1	111,973.0

Source: Tables 6.10.1, 6.10.2, 6.10.3 and 6.10.7

Table 6.10.10 New Premium Expenditure on Savings-based and Protection-based Renewable Life Insurance £000

Year	Savings-based	Protection-based
1946	13,097.6	2,784.4
47	17,530.6	3,240.1
48	16,839.7	3,135.3
49	16,463.7	3,646.2
1950	17,833.8	4,103.5
51	21,803.8	5,076.9
52	20,971.9	4,296.8
53	19,713.5	5,131.0
54	23,086.4	5,446.6
1955	26,772.9	6,099.4
56	26,115.8	7,357.2
57	25,775.7	7,140.0
58	31,036.2	7,788.7
59	33,087.9	10,958.1
1960	42,003.7	10,977.4
61	47,413.2	10,836.2
62	51,536.3	12,416.3
63	59,367.1	13,933.7
64	70,767.3	13,699.2
1965	72,417.4	15,072.3
66	78,246.8	15,201.2
67	83,457.8	16,046.0
68	89,830.5	22,142.5

Source: See Table 6.10.9

6.11 Conclusion

The objective of this Chapter has been to split new renewable ordinary life insurance premium income into components defined by their financial saving and protection contents. This has been done in two ways: first, by differentiating between financial saving and consumption (see Table 6.10.9) and second, by differentiating between savings-based premium expenditures and those which are protection-based (ie. those used to purchase life insurance for the purpose of protecting dependents). (see Table 6.10.10). The general method has been to compare Office Premiums derived from figures available in the Annual Abstract of Statistics with Aggregate Pure Premiums derived from the theory of Chapter Five.

Details of the importance of the Financial Savings content of Aggregate Office Premiums are given in Table 6.11.1. It can be seen that Financial Saving represents a fairly stable proportion of Office Premiums (with an average figure of 61.4%).

Now that separate data on the Financial Savings content of life insurance is available it will be possible to accurately analyse the importance of saving via the medium of those models of Section 4.5. Similarly, now that a differentiation of life insurance premium expenditures into saving-based and protection-based categories is possible, a demand model of life insurance can also be attempted without the disadvantages of those of Section 4.5.

Table 6.11.1 The Proportion of Aggregate Office Premiums Allocated to Financial Saving.

Year	Financial Saving as a Proportion of Office Premiums
1946	0.643
47	0.630
48	0.660
49	0.587
1950	0.635
51	0.542
52	0.662
53	0.600
54	0.623
1955	0.629
56	0.601
57	0.626
58	0.731
59	0.570
1960	0.594
61	0.645
62	0.539
63	0.556
64	0.656
1965	0.611
66	0.604
67	0.672
68	0.517

Source: Tables 6.4.1 and 6.10.9

APPENDIX 6.1.

The Calculation of New Business Renewal Premium Income

Let IFN_t denote the number of non-single premium policies in force (companies established in the UK) at the end of year t and IFS_t the sums insured plus accumulated reversionary bonuses in force at the end of year t (for non-single premium policies). Let the superscripts WLNP, ENP, T, WLWP, EWP and OWP denote the various classes of life insurance described in Table 6.3.1 (OWP refers to 'With Profit Other' policies which are assumed to be single premium policies).

The relationship between the renewal-premium in force in years t and (t-1) is then:

$$IFN_t = IFN_{t-1} + N_t - (NCl_t + NCa_t) \quad \text{(A6.1.1)}$$

$$\text{and } IFS_t = IFS_{t-1} + S_t + AB_t - (SCl_t + SCa_t) \quad \text{(A6.1.2)}$$

where N_t and S_t refer to new numbers and sums insured respectively; NCl_t and SCl_t refer to the numbers and sums insured (plus bonuses) deleted by claims and NCa_t and SCa_t refer to the numbers and sums insured (plus bonuses) deleted by cancellations. AB_t refers to the accumulated reversionary bonuses declared in year t.

Since, obviously, bonuses are only declared on with profit policies, we can rewrite Equation (A6.1.2) as

$$IFS_t = IFS^{NP}_{t-1} + IFS^{WP}_{t-1}(1+b_t) + S_t - (SCl_t + SCa_t) \tag{A6.1.3}$$

where b_t is the average rate of (compound) bonus declared in year t.

Equations (A6.1.1) and (A6.1.3) can be rewritten as

$$(NCl_t + NCa_t) = IFN_{t-1} + N_t - IFN_t \tag{A6.1.1a}$$

and

$$(SCl_t + SCa_t) = IFS^{NP}_{t-1} + IFS^{WP}_{t-1}(1+b_t) + S_t - IFS_t \tag{A6.1.3a}$$

where the right hand side of both equations can be obtained from the Annual Abstract of Statistics. The values of b_t obtained from the survey are given in a table at the end of this Appendix in Table A6.1.1.

The next step is to apportion the total values on the left hand side of the above Equations among the various classes of life insurance described in Table 6.3.3. This step uses the assumption that numbers (or sums insured) of policies which become claims or cancellations are proportional to IFN_t (or IFS_t). Essentially this means that the mortality and cancellation experience is the same (in proportion) for all classes of life insurance. In practice it turns out that this is not the case (with higher mortality and lower cancellations experienced by the protection-type policies. However, insufficient information is available to enable a more sophisticated assumption).

The claims and cancellations for any one class of life insurance (say EWP) is then given by:

$$(SCl_t + SCa_t)^{EWP} = \frac{IFS_t^{EWP}}{IFS_t} \cdot (SCl_t + SCa_t) \qquad \text{(A6.1.4)}$$

(similarly for numbers)

The only complication to Equation (A6.1.4) arises when $IFS_t^{EWP} < IFS_{t-1}^{EWP}$; the fall in in force must then be allocated solely to $(SCl_t + SCa_t)^{EWP}$.

Final values for new business numbers and sums insured (for business issued by UK-established companies) can then be obtained by re-arranging Equations (A6.1.1) and (A6.1.3) on an individual policy class basis, eg. for EWP we have:

$$S_{EWP} = IFS_t^{EWP} - IFS_{t-1}^{EWP}(1+b_t) + (SCl_t + SCa_t)^{EWP}$$

(similarly for numbers)

The final stage - to obtain new business renewal premium income (OP) - is to multiply each value of S_{EWP} by the ratio $\left(\frac{\text{New Premium Income}}{\text{New Sums Insured}}\right)^{EWP}$ (obtained from Survey non-group data and given in Tables A6.1.2 and A6.1.3 at the end of this Appendix). The resulting figures were then summed and scaled by the factor

$$\frac{\Sigma S_i}{S} \cdot \frac{OP}{\Sigma OP_i} \cdot Z$$

where Z is the ratio

$$\frac{\text{New Premium Income within UK}}{\text{New Premium Income : Companies established in UK}}$$

Obviously the new business renewal premium figures can be calculated by using either numbers or sums insured. It was found that figures computed on the basis of sums insured needed less scaling by the factor $\frac{\Sigma S_i}{S} \cdot \frac{OP}{\Sigma OP_i}$ and so the figures were calculated on this basis.

This method of deriving new premium income figures is heavily dependent on the appropriate use of the ratio $\left(\frac{\text{New Premium Income}}{\text{New Sums Insured}}\right)$. Naturally enough, we have to assume that figures obtained from the survey data are representative of ordinary branch new business in general. There are however three main points to watch out for:

i) that the average 'survey' inception age and the industry wide inception age are similar for each policy class;

ii) that the 'survey' premium rates are reasonably competitive, and

iii) whether the whole of life premiums cease at some maximum age. In fact the majority of 'survey' whole of life office premium rates were calculated to cease at age 60 and consequently must be revised downwards since theoretically we assume that no such restriction exists.

Unfortunately, not even an examination of the relevant Department of Trade returns can yield any information on the 'survey' inception ages so that no check is possible. A study of quinquennial 'survey' premium rates for selected policies reveals that, in a broad comparison with Table

6.7.2, they are much in line with general market rates (note: the 'survey' rates were not used in the compilation of Table 6.7.2). Although it would have been beneficial, not enough data was available to scale the 'survey' ratios of Tables A6.1.2 and A6.1.3 to bring them into line with the general market rates.

It was found necessary, however, to adjust the whole of life ratios of Table A6.1.2 to allow for cessation of premiums at age 60 (as reported in the relevant Department of Trade returns, Schedule 5). The whole of life pure premiums in this case is given by

$$\text{Premium at inception age} = \frac{M_x}{N_x - N_{60}} = \frac{P_x}{1 - \frac{N_{60}}{N_x}}$$

Thus to convert the whole of life ratios of Table A6.1.2 they were multiplied by the factor $(1 - \frac{N_{60}}{N_x})$ $(0 < (1 - \frac{N_{60}}{N_x}) < 1)$ where inception age was obtained from Table 6.5.1 and a 'calculation' rate of interest of 3% was used. The results are given in Table A6.1.3.

Table A6.1.1 'Average' Annual Compound Rate of Reversionary Bonus Declared b_t.

Year	b_t
1946	0.0140
47	0.0140
48	0.0151
49	0.0151
1950	0.0176
51	0.0186
52	0.0186
53	0.0199
54	0.0199
1955	0.0203
56	0.0215
57	0.0215
58	0.0243
59	0.0243
1960	0.0253
61	0.0263
62	0.0263
63	0.0282
64	0.0298
1965	0.0308
66	0.0311
67	0.0317
68	0.0353

Source: Survey Data

Table A6.1.2 The Ratio of New Non-Group Premium Income to New Non-Group Sums Insured

Year	Non-Profit			With Profit	
	Whole Life	Endowment	Other	Whole Life	Endowment
1946*	0.030000	0.050000	0.010000	0.040000	0.054500
47*	0.030000	0.050000	0.010000	0.040000	0.054500
48	0.029860	0.049426	0.009749	0.040703	0.054440
49	0.033000	0.050645	0.011665	0.029890	0.054270
1950	0.030554	0.047539	0.014634	0.028576	0.054516
51	0.034534	0.048245	0.010924	0.029204	0.052779
52	0.034506	0.045181	0.010322	0.026786	0.052495
53	0.036042	0.043586	0.010913	0.026034	0.052859
54	0.040317	0.043733	0.010636	0.025964	0.053313
1955	0.036191	0.043301	0.010620	0.026059	0.053052
56	0.032947	0.042870	0.011935	0.026526	0.052742
57	0.028842	0.040249	0.010013	0.025811	0.050963
58	0.026714	0.038984	0.010888	0.026247	0.050391
59	0.026825	0.038780	0.010022	0.026404	0.049263
1960	0.026787	0.039484	0.008181	0.026345	0.048076
61	0.026000	0.042698	0.007054	0.025110	0.048113
62	0.025530	0.044377	0.006039	0.024372	0.048627
63	0.024830	0.042371	0.005674	0.023272	0.048547
64	0.024446	0.041029	0.004821	0.022670	0.047680
1965	0.024168	0.039891	0.004095	0.022232	0.048094
66	0.022682	0.036827	0.003396	0.021932	0.048687
67	0.018295	0.034454	0.003284	0.021966	0.050587
68	0.015851	0.034173	0.003142	0.021316	0.049006

*Estimate
Source: Survey Data

Table A6.1.3 The Ratio of New Premium Income to New Sums Insured, adjusted by the factor $(1 - \frac{N_{60}}{N_x})$

Year	Whole Life	Whole Life
	NP	WP
1946*	0.021217	0.027241
47*	0.022379	0.029530
48	0.022504	0.030228
49	0.024786	0.022454
1950	0.022990	0.022317
51	0.026229	0.022967
52	0.026006	0.021090
53	0.027112	0.020662
54	0.030341	0.020802
1955	0.027417	0.020907
56	0.025133	0.021313
57	0.022197	0.020830
58	0.020838	0.021224
59	0.021235	0.021391
1960	0.021527	0.021301
61	0.021200	0.020336
62	0.021093	0.019756
63	0.020744	0.018940
64	0.020582	0.018531
1965	0.020433	0.018298
66	0.019167	0.018238
67	0.015351	0.018471
68	0.013081	0.018220

*Estimated
Source: Survey Data

APPENDIX 6.2

The Calculation of Inception Ages 1959/1968

Essentially, the method of estimating the unknown inception ages is a simple one involving a weighted least-squares polynominal curve fitting SUBROUTINE from the Nottingham Algorithms Group (NAG) Manual (15) (for use on an ICL 1906A computer). The subroutine uses Forsythe's Method (involving Orthogonal polynominals) (5) with weights inversely proportional to the square of the standard error of the dependent variable.

The SUBROUTINE requires the explanatory variable to be input in a monotomically increasing sequence.

For each class of policy, three different estimations were used, one using the overall Average Inception age as explanatory variable and the other two using time (to explain a) the inception ages and, b) the difference between a) and overall Average inception age).

A copy of the computer programme - written in FORTRAN - used to calculate average inception ages (MASTER INCPTNAGE) is given at the end of this Appendix.

```
LIBRARY NAME MAG
FORTRAN WITH (AIDS 0,LIST 0)
      MASTER 1.COTMAGE
      DIMENSION X(4,13),O(23),P(14),SI(14),IIT(14),SE(4,14),
     1W(4,13),XX(14),WW(14),OO(14),OZ(13),XXX(14),WWW(14),PP(14),
     2X68(4),SE68(4),ST(14),XO(14),SO(14),PPP(14)
      REAL IIT
      LOGICAL L
      L=.FALSE.
      M=14
      DO 1 J=1,13
1     READ(5,100)(X(I,J),I=1,4)
      DO 3 J=1,13
      READ(5,100)(SE(I,J),I=1,4)
      DO 2 I=1,4
2     W(I,J)=(SE(I,J)**(-2))*0.001
3     CONTINUE
      READ(5,101)(O(J),J=1,23)
      READ(5,100)(X68(I),I=1,4)
      READ(5,100 )(SE68(I),I=1,4)
      DO 14 I=1,4
      DO 4 J=1,13
4     OZ(J)=O(J)
      WRITE(6,200)
      Z=100.0
      OO(1)=31.65179
      XX(1)=X68(I)
      WW(1)=(SE68(I)**(-2))*0.001
      KK=1
5     DO 6 J=1,13
      IIT(J)=FLOAT(J)
      XXX(J)=X(I,J)
      WWW(J)=W(I,J)
6     Z=AMIN1(OZ(J),Z)
      DO 7 J=1,13
      R=O(J)-Z
7     IF(R.EQ.0.0)GO TO 8
8     OO(KK+1)=O(J)
      WW(KK+1)=W(I,J)
      XX(KK+1)=X(I,J)
      OZ(J)=OZ(J)+200.0
      Z=100.0
      KK=KK+1
      IF(KK.LE.13)GO TO 5
      OMIN=OO(1)
      OMAX=OO(4)
      IIT(14)=23.0
      XXX(14)=XX(1)
      WWW(14)=WW(1)
      DO 900 J=1,14
      IIT(J)=((IIT(J)-23)+(IIT(J)-1))/22
      OO(J)=((OO(J)-OMAX)+(OO(J)-OMIN))/(OMAX-OMIN)
      IF(J.EQ.14)GO TO 800
      XO(J)=XXX(J)-O(J)
      GO TO 900
800   XO(14)=XXX(14)-O(23)
900   CONTINUE
```

```
      DO 13 K1=2,13
      CALL E02ABF(M,OO,XX,WW,K1,N,ST,P,L)
      WRITE(6,202)K1
      DO 10 J=1,23
      IYEAR=1945+J
      OOO=O(J)
      OOO=((OOO-OMAX)+(OOO-OMIN))/(OMAX-OMIN)
      N1=N+1
      S=P(N1)
      DO 9 K=1,N
      I1=N1-K
9     S=S*OOO+P(I1)
10    WRITE(6,201)I,IYEAR,S
      WRITE(6,202)K1
      CALL E02ABF(M,ITT,XXX,WWW,K1,NN,ST,PP,L)
      CALL E02ABF(M,ITT,XO,WWW,K1,NNN,SO,PPP,L)
      DO 12 J=1,23
      IYEAR=1945+J
      RTT=FLOAT(J)
      RTT=((RTT-23)+(RTT-1))/22
      N1=NN+1
      NN1=NNN+1
      S=PP(N1)
      S1=PPP(NN1)
      DO 11 K=1,NN
      I1=N1-K
11    S=S*RTT+PP(I1)
      DO 111 K=1,NNN
      II1=NN1-K
111   S1=S1*RTT+PPP(II1)
      SS=S1+O(J)
12    WRITE(6,203)I,IYEAR,S,SS
13    CONTINUE
14    CONTINUE

100   FORMAT(4F0.0)
101   FORMAT(23F0.0)
200   FORMAT(1H ,///,20X,'PREDICTED INCEPTION AGES ',/,10X,
     1'FIRST MODEL USES OVERALL AVERAGE AGE, SECOND MODEL USES TIME'
     2,/,10X, '1 - WLWP',/,10X,'2 - WLNP',/,10X,'3 - FWP',/,10X,'4 - FNP
     3',//)
201   FORMAT(1H ,I1,5X,I4,10X,F8.5,'    AVERAGE')
202   FORMAT(1H ,//,60X,'K1 EQUALS ',I2,//)
203   FORMAT(1H ,I1,5X,I4,10X,F8.5,10X,F8.5,'    TIME')
      STOP
      END
```

APPENDIX 6.3 : The Survey Questionnaire

Department of Industrial Economics

University of Nottingham

INFLATION AND LIFE INSURANCE

The aim of this study is to build a time series demand model for life insurance and thus the data is required on an annual basis - hopefully for the past 25 years.

The data should only apply to the business transacted by your company in your own country, please do not include any foreign business you transact.

If your company has altered the basis on which any of these figures are calculated could you please indicate the fact and the date/s when the alteration occurred.

Only the figures for direct (ie. excluding reinsurance transactions) ordinary life insurance are required.

Types of ordinary life policy	Total	Whole life		Endowment		Term
		WP	NP	WP	NP	
Section A						
Per annum for the last 25 years.						
a) New (renewable) premiums ..						
b) New (single) premiums ..						
c) Total premium income.. ..						
d) Total premium income from Group-life policies.. ..						
Section B						
Per annum for the last 25 years.						
a) Number of surrenders, lapses, voids (ie. voluntary terminations)..						
b) Number of new policies ..						
c) New sums insured						

Types of ordinary life policy	Total	Whole life		Endowment		Term
		WP	NP	WP	NP	
Section C						
Per annum for the last 25 years.						
a) Management expenses						
b) Commission rates						
NB WP = with profit NP = non-profit						

Section D

Certain information is required on some specific policies issued by your company, with the aim of calculating the 'profitability' of these policies to the policyholder. Again, the data is required for the last 25 years (if possible); but if, say, valuations occurred every five years then there will only be five sets of figures.

Premium and Bonus Rates are required on the assumption that this will save time when filling in the questionnaire, leaving the questioner to work out the required data.

Non-Profit

	Age	Whole Life	Endowment Policy Term			Temporary (Term)			Date
			10	15	25	10	15	25	
Annual Premium Rates (% of SA)	25								1949 1950 : 1973 1974
	45								1949 1950 : 1973 1974
*Surrender Rates (values)	25								1949 1950 : 1973 1974
	45								1949 1950 : 1973 1974

*or for a specific policy.

With Profit

	Age	Whole Life	Endowment Policy Term			Temporary (Term)			Date
			10	15	25	10	15	25	
Annual Premium Rates (% if SA)	25								1949 1950 : 1973 1974
	45								1949 1950 : 1973 1974
Surrender Rates (values)	25								1949 1950 : 1973 1974
	45								1949 1950 : 1973 1974

Bonus Rates (Simple or Compound?)

	Whole Life	Endowment Policy Term			Temporary (Term)			Date
		10	15	25	10	15	25	
Reversionary (simple or compound)								1949 1950 : 1973 1974
Terminal or special								1949 1950 : 1973 1974

APPENDIX 6.4

MASTER SAVINGS

```
      COMMON//ALX(100,25),R(23),TERM(23),Q50(100),Q69(100),CM(16,25),
     1GM(16,25),X(4,23),B(23)
      READ(5,100)(Q50(I),I=1,100)
      READ(5,100)(Q69(I),I=1,100)
      DO 1000 J=1,25
1000  READ(5,101)(CM(K,J),K=1,16)
      DO 1001 J=1,25
1001  READ(5,101)(GM(K,J),K=1,16)
      DO 1002 J=1,23
1002  READ(5,102)(X(I,J),I=1,4)
      READ(5,103)(TERM(J),J=1,23)
      READ(5,103)(R(J),J=1,23)
      READ(5,103)(B(J),J=1,23)
      CALL MORTALITYTABLE
      CALL COMMUTATION

100   FORMAT(100F0.0)
101   FORMAT(16F0.0)
102   FORMAT(4F0.0)
103   FORMAT(23F0.0)
      STOP
      END
```

SUBROUTINE MORTALITYTABLE

```
      DIMENSION CQ(16,25),GQ(16,25),RQ66(16),RALX(100,25),EQ(100,25)
      COMMON//ALX(100,25),R(23),TERM(23),Q50(100),Q69(100),CM(16,25),
     1GM(16,25),X(4,23)
      DO 2 K=1,16
      DO 1 J=1,25
      CQ(K,J)= (2*CM(K,J))/(2+CM(K,J))
1     GQ(K,J)= (2*GM(K,J))/(2+GM(K,J))
2     CONTINUE
      DO 3 I=1,100
      EQ(I,1)=Q50(I)-11*(Q69(I)-Q50(I))/36
      DO 3 J=2,25
3     EQ(I,J)=EQ(I,1)+(J-1)*(Q69(I)-Q50(I))/18
      NN=1
30    DO 4 J=1,25
      RALX(1,J)=999999.0
      DO 4 I=2,100
      IF(NN.EQ.1)GO TO 31
      RALX(I,J)=ALX(I,J)
      GO TO 4
31    RALX(I,J)=RALX(I-1,J)*(1-EQ(I-1,J))
4     CONTINUE
```

```
      NN=NN+1
      DO 42 K=1,16
      KK=(K+1)*5-9
      IF(K.EQ.16)GO TO 40
      SUM=RALX(KK,22)+RALX(KK+1,22)+RALX(KK+2,22)+RALX(KK+3,22)+
     1RALX(KK+4,22)
      RQ66(K)=(RALX(KK,22)-RALX(KK+5,22))/SUM
      GO TO 42
40    SUM=0.0
      DO 41 JJ=0,23
41    SUM=SUM+RALX(KK+JJ,22)
      RQ66(16)=(RALX(KK,22)-RALX(100,22))/SUM
42    CONTINUE
      DO 10 K=1,16
      KK=(K+1)*5-9
      DO 9 J=1,25
      ALX(1,J)=999999.0
      RA=ALX(KK,J)/RALX(KK,J)
      IF((K.LE.4).OR.(K.GE.15).OR.(J.GE.23))GO TO 5
      FM=RQ66(K)*CQ(K,J)/CQ(K,22)
      IJK=5
      GO TO 8
5     IF(K.EQ.16)GO TO 6
      FM=RQ66(K)*GQ(K,J)/GQ(K,22)
      IJK=5
      GO TO 8
6     IJK=24
      ALX(100,J)=0.86+(ALX(76,J)-145388.3)*2.72/26331
      IF(ALX(100,J).GE.0.0)GO TO 80
      ALX(100,J)=0.0
      GO TO 80
8     SUM=RALX(KK+1,J)+RALX(KK+2,J)+RALX(KK+3,J)+RALX(KK+4,J)
      SUM=SUM*RA
      SUME=((ALX(KK,J)-RA*RALX(KK+IJK,J))/FM)-ALX(KK,J)
      SUMF=(SUM+SUME)/2
      ALX(KK+IJK,J)=ALX(KK,J)*(1-FM)-FM*SUMF
80    EL=ALX(KK,J)
      DO 81 JJ=0,(IJK-1)
81    EL=EL*(1-EQ(KK+JJ,J))
      F=ALOG(ALX(KK+IJK,J)/ALX(KK,J))/ALOG(EL/ALX(KK,J))
      DO 82 JJ=1,(IJK-1)
82    ALX(KK+JJ,J)=ALX(KK+JJ-1,J)*((1-EQ(KK+JJ-1,J))**F)
9     CONTINUE
10    CONTINUE
      IF(NN.NE.4)GO TO 30
      RETURN
      END
```

```
      SUBROUTINE COMMUTATION
      DIMENSION ADX(100,25),D(100,25),C(100,25),ANX(100,25),
     1AMX(100,25),AK(100),ANN(4,2)
      COMMON//ALX(100,25),R(23),TERM(23),Q50(100),Q69(100),CM(16,25),
     1GM(16,25),X(4,23),B(23)
      DO 6 J=1,23
      V=(1+R(J))**(-1)
      VB=(1+0.8*B(J))/(1+R(J))
      KKK=1
111   IJ=1945+J
      SUMD=ALX(100,J  )*(V**109)
      SUMC=ALX(100,J  )*(V**110)
      DO 1 I=1,99
      ADX(I,J  )=ALX(I,J  )-ALX(I+1,J  )
      D(I,J  )=ALX(I,J  )*(V**(I+9))
      C(I,J  )=ADX(I,J  )*(V**(I+10))
      SUMD=SUMD+D(I,J  )
1     SUMC=SUMC+C(I,J  )
      ANX(1,J  )=SUMD
      AMX(1,J  )=SUMC
      DO 2 I=2,100
      ANX(I,J  )=ANX(I-1,J  )-D(I-1,J  )
2     AMX(I,J  )=AMX(I-1,J  )-C(I-1,J  )

      DO 5 II=1,4
      IX=INT(X(II,J))
      RX=X(II,J)-IX
      AADX=RX*ADX(IX-9,J  )
      AALX=ALX(IX-9,J  )-AADX
      DX=AALX*(V**X(II,J))
      AANX=ANX(IX-9,J  )+RX*(ANX(IX-8,J  )-ANX(IX-9,J  ))
      AAMX=AMX(IX-9,J  )+RX*(AMX(IX-8,J  )-AMX(IX-9,J  ))

      IF(II.LE.2)GO TO 20
      XT=X(II,J)+TERM(J)
      IT=INT(XT)
      RT=X(II,J)+TERM(J)-IT
      AADT=RT*ADX(IT-9,J  )
      AALT=ALX(IT-9,J  )-AADT
      DT=AALT*(V**(X(II,J)+TERM(J)))
      AANT=ANX(IT-9,J  )+RT*(ANX(IT-8,J  )-ANX(IT-9,J  ))
      AAMT=AMX(IT-9,J  )+RT*(AMX(IT-8,J  )-AMX(IT-9,J  ))
      ANN(II,KKK)=(AANX-AANT)/DX
      IF(KKK.EQ.1)GO TO 200
      PEB=(AAMX-AAMT+DT)/(DX*ANN(II,1))
      WRITE(6,102)II,PEB
      GO TO 5
200   PE=(AAMX-AAMT+DT)/(AANX-AANT)
      PET=(AAMX-AAMT)/(AANX-AANT)
      WRITE(6,100)IJ,X(II,J),TERM(J),PE,PET,II
      GO TO 5

20    EX=EXPTN(X(II,J),J)
      XE=X(II,J)+EX
      IE=INT(XE)
      RE=X(II,J)+EX-IE
      AADE=RE*ADX(IE-9,J  )
      AALE=ALX(IE-9,J  )-AADE
      DE=AALE*(V**(X(II,J)+EX))
      AANE=ANX(IE-9,J  )+RE*(ANX(IE-8,J  )-ANX(IE-9,J  ))
      AAME=AMX(IE-9,J  )+RE*(AMX(IE-8,J  )-AMX(IE-9,J  ))
      CXL=(AAMX*AANE/AANX)-AAME+DE
      NN=IX-10
```

```
      DO 3 K=1,NN
      AK(IX-9-K)=(AAMX/AANX)*ANX(IX-9-K,J   )-AMX(IX-9-K,J   )
3     IF(AK(IX-9-K).GE.CXL)GO TO 4
4     AK1=ABS(AK(IX-9-K)-CXL)
      AK2=ABS(AK(IX-8-K)-CXL)
      BANX=(ANX(IX-9-K,J   )*AK2+ANX(IX-8-K,J   )*AK1)/(AK1+AK2)
      BAMX=(AAMX/AANX)*BANX-CXL
      ANN(II,KKK)=AANX/DX
      IF(KKK.EQ.1)GO TO 400
      PWLB=AAMX/(DX*ANN(II,1))
      WRITE(6,102)II,PWLB
      GO TO 5
400   PWL=AAMX/AANX
      PWLT=(BAMX-AAME)/(BANX-AANE)
      WRITE(6,101)IJ,X(II,J),EX,PWL,PWLT,II
5     CONTINUE
      V=VB
      KKK=KKK+1
      IF(KKK.EQ.2)GO TO 111
      WRITE(6,103)
6     CONTINUE

100   FORMAT(1H ,I4,5X,F9.5,5X,F6.3,5X,F13.9,5X,F13.9,4X,' ENDWMT ',I1)
101   FORMAT(1H ,I4,5X,F9.5,5X,F6.3,5X,F13.9,5X,F13.9,1X,'WHOLE LIFE ',I
     11)
102   FORMAT(1H ,77X,I1,5X,F13.9)
103   FORMAT(1H ,///)
      RETURN
      END
```

```
      FUNCTION EXPTN(T,J)
      COMMON//ALX(100,25),R(23),TERM(23),Q50(100),Q69(100),CM(16,25),
     1GM(16,25),X(4,23)
      SUM=0.0
      IX=INT(T)
      RX=T-IX
      IT=IX-8
      AL1=ALX(IX-9,J   )-RX*(ALX(IX-9,J   )-ALX(IX-8,J   ))
      DO 1 I=IT,100
1     SUM=SUM+ALX(I,J   )
      EXPTN=(SUM-RX*(ALX(IX-8,J   )-ALX(100,J   )))/AL1
      RETURN
      END
```

CHAPTER SIX : REFERENCES

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CHAPTER SEVEN : MODELS OF THE PURCHASE OF UK NON-GROUP LIFE INSURANCE : DESCRIPTION AND DATA.

7.1 Introduction

The objective of this Chapter is to build models of the purchase of UK non-group ordinary life insurance (excluding annuities) which will then be tested in Chapter Eight. Two main types of model will be examined (corresponding to those described in Section 4.5) ; the first tests Financial Saving (Table 6.10.9) as a proportion of Personal Disposable Income in order to investigate the importance of new financial saving via the medium of life insurance. The second involves a Demand Model of the purchase of life insurance with dependent variables from Table 6.10.10 (ie. the sales of savings-based and protection-based life insurance).

Both models are particularly concerned with the role of inflation expectations and these will be examined in greater detail in Section 7.5.

The starting hypothesis of the demand model is that life insurance is purchased for two main reasons: to provide protection of dependents against premature death and secondly, to provide a savings medium. Purchases of life insurance for these reasons will be influenced by several factors (such as income and inflation expectations) however, these factors vary in their importance and their effect depending on the reason for the purchase.

There is one theoretical complication to all demand models, known as the 'identification problem'. Briefly, no demand model can be examined in isolation because it is just one determinant of market equilibrium: supply must also be investigated. The statistical solution to this problem involves the construction of a set of simultaneous equations which must be estimated in their reduced forms (by the method of Two Stage Least Squares).

The financial savings model - using new financial saving as a proportion of Personable Disposable Income (PDI) as dependent variable - is analogous to those models described by the Bank of England (3). The objective is to test the responsiveness of semi-contractual financial saving to changes in certain economic variables (such as inflation expectations). Some attempt can be made to evaluate the various theories of saving in the context of new saving via life insurance.

The Financial Savings and Demand Models are, however, fairly similar in that the same explanatory variables are involved. These variables include inflation expectations, income (permanent and transitory), the 'optimum stock' of life insurance and wealth as well as several socio-economic variables such as births and marriages. The explanatory variables will be described in later Sections but first, the two models must be examined.

7.2 The Financial Saving Model

By the term 'Financial Saving Model' I mean the model of new financial saving via ordinary, non-group, renewable life insurance using as dependent variable: Financial Saving (Table 6.10.9) as a proportion of P.D.I. (see Table 7.2.1).

The figures of Table 7.2.1 show that (^{FS}/PDI) (to be called FSR) was fairly stable until the mid-1960's when there was a sharp rise. It must be remembered however, that we are only concerned with a very small part of the aggregate saving ratio: for example, in the 1960's saving represented an average of 8.1% of PDI[(1)] whereas <u>new</u> financial saving via life insurance averaged only 0.2% (though, of course, total saving via life insurance was much higher).

The theories of saving outlined in Section 4.3 were originally formulated to analyse the effects of inflation on aggregate saving rather than new saving via life insurance (which by definition is long-term and semi-contractual). Hence some of them are unable to explain the variation in the dependent variable (^{FS}/PDI) : for example, some of the theories only explain variations in one direction (Deaton (11) and Branson & Klevorick (5) cannot explain increases in (^{FS}/PDI) because of their emphasis on consumption as the complement of saving). Consequently, we are left with two

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(1) See Bank of England (3) p. 53. The aggregate saving ratio includes saving by unincorporated businesses as well as households.

Table 7.2.1 Financial Saving (via New Life Insurance Purchases) as a proportion of Personal Disposable Income.

$$(FSR = \frac{FS}{PDI})$$

Year	$(^{FS}/PDI) \times 1000$
1946	1.3433
47	1.5899
48	1.5199
49	1.2943
1950	1.4516
51	1.4048
52	1.5025
53	1.2508
54	1.4140
1955	1.5160
56	1.3779
57	1.3443
58	1.7790
59	1.4806
1960	1.7238
61	1.9227
62	1.6846
63	1.8716
64	2.3691
1965	2.1365
66	2.1301
67	2.4273
68	1.9761

Source: Table 6.10.9

main theories relating inflation to financial saving in a life insurance context:

i) the 'traditional' argument that inflation devalues the real value of maturity benefits paid to surviving policyholders (or death benefits paid to heirs in the financial saving whole of life case). An alternative viewpoint argues that inflation reduces the real rate of return on saving through life insurance (see M.J. Hamburger (24)). However it is not certain whether this will reduce or increase saving via life insurance;

ii) the Juster and Wachtel (27) hypothesis that when nominal incomes change rapidly, expectations of prices - and hence of the standard of living in real terms - becomes more uncertain. This results in precautionary saving.

Of course, as D. Barros (4) explains, the above hypotheses only attempt to explain marginal changes under special conditions of high inflation and do not attempt to replace the existing theories of the long-term determinants of personal saving. Thus we must still turn to traditional Keynesian theory, with its emphasis on the growth of real income, to provide the basis in the attempt to investigate the determinants of (^{FS}/PDI). Barros also points out that:

> "Two additional elements may have helped raise the personal saving rate: (1) the spread of owner-occupation of housing and the associated increase in building society deposits and (2) the promotion of private pension funds, especially after the introduction of state graduated

pensions and contracting-out in 1961." (4) p. 32 These two elements have a potential effect on saving through life insurance because first, endowment policies are used to purchase houses (in conjunction with a building society loan) and secondly, there exists an a priori relationship between saving through life insurance and the provision of pensions (endowment maturity benefits are used to provide the single premium on an annuity). In addition the provision of compulsory state pensions does not necessarily lead to a lower individual use of private provision. The study of A.M. El-Mokadem (16) indicates that, on the contrary, increased provision of state pensions leads to a greater awareness of the need to save for retirement and hence to an increase in the provision of private pensions.

Summarising then, the basic model used to explain the variations in the dependent variable (^{FS}/PDI) should have the following features:

i) real personal disposable income (per head of population) as an explanatory variable. Alternatively, following M. Friedman (19), a variable representing permanent income could be used;

ii) an explanatory variable to allow for the effects of inflation expectations;

iii) other explanatory variables that reflect the reasons for financial saving via the medium of life insurance (such as house purchase, pension rights etc.).

Expressed mathematically, we have

$$FSR_t = \left(\frac{FS}{PDI}\right)_t = F(\overline{PDI}_t, \dot{P}^e_t, X_{t1}, \ldots, X_{tk}) \qquad (7.2.1)$$

where $\overline{PDI}$ = real PDI per head of population

$\dot{P}^e$ = the expected rate of inflation

and $X_{t1}, \ldots, X_{tk}$ denote the other explanatory variables. F will commonly denote a linear function ie.,

$$FSR_t = a_0 + a_1 \overline{PDI}_t + a_2\dot{P}^e_t + \Sigma\, b_j X_{tj} + u_t \qquad (7.2.2)$$

where the term u_t denotes that residual (of the dependent variable) that cannot be explained by the linear relationship.

An alternative, and complicating version of Equation (7.2.2) splits $\overline{PDI}_t$ to differentiate between permanent (P) and transitory (T) income:

$$FSR_t = a_0 + a_1^P \overline{PDI}^P_t + a_1^T \overline{PDI}^T_t + a_2\dot{P}^e_t + \Sigma\, b_j X_{tj} + u_t \qquad (7.2.3)$$

A discussion of the determination of $\overline{PDI}^P_t$ and $\dot{P}^e_t$ will be deferred to Sections 7.6 and 7.5 respectively.

7.3 The Demand Model

By the term 'Demand Model' I mean the model of new purchases of ordinary, non-group renewable life insurance excluding annuities. The dependent variables used will be premium expenditures on 'protection-based' and 'savings-based' life insurance (as defined in Section 6.2) with data from Table 6.10.10 : these of course, are only a surrogate for the 'quantity demanded' but this is not a major difficulty.

The major explanatory variables of interest have been broadly outlined in the previous Section so that the potential demand model is of the form:

$$D_t^{Pr} = a_0^{Pr} + a_1^{Pr}\overline{PDI}_t + a_2^{Pr}\dot{P}_t^e + \Sigma\, b_j^{Pr} X_{tj} + u_t$$

$$D_t^{Sg} = a_0^{Sg} + a_1^{Sg}\overline{PDI}_t + a_2^{Sg}\dot{P}_t^e + \Sigma\, b_j^{Sg} X_{tj} + u_t \qquad (7.3.1)$$

where D_t^{Pr} denotes the real expenditure on 'protection-based' life insurance and D_t^{Sg} the real expenditure on 'savings-based' life insurance in year t.

Equation (7.3.1) may be subject to the same modifications as Equation (7.2.2) in order to include permanent and transitory income. Similarly, the determination of these variables, as well as inflation expectation $\dot{P}_t^e$, may alter the format of the model by introducing lagged variables. We note that if a stock adjustment model is required (eg. Houthakker and Taylor (26)) then one of the X_j will denote the amount of life insurance in force, It has also been hypothesised (in Section 4.6) that there is a direct relationship between D_t^{Sg} and D_t^{Pr} (S. Neumann (35) predicted a negative relationship while Lee and Whitaker (30) formulated the same kind of situation with different types of life insurance). Thus D_t^{Sg} and D_t^{Pr} (or their associated prices) should appear as explanatory variables in the model of Equation (7.3.1).

The other explanatory variables that could be included in Equation (7.3.1) include unemployment, the change in unemployment (see Section 4.3), tax, births, marriages, wealth, house purchases and pension provisions.

One further complication of Equation (7.3.1) is shared by all prospective demand models and that is the question of identification: the problem is that the time-series data represented in Table 6.10.10 represents a series of market equilibria so that the figures for D^{Pr} and D^{Sg} are synonymous with S^{Pr} and S^{Sg} - the 'supply' (in terms of premium expenditure) of 'protection-based' and 'savings-based' life insurance:

> "Market data register points of equilibrium of supply and demand at the price prevailing in the market at a certain point of time. A sample of time-series observations shows simultaneously the quantity demanded D, <u>and</u> the quantity supplied S, at the prevailing price, P. If we use these data for estimation, we actually measure the coefficients of a function of the form Q = f(P). This equation may be either the demand function or the supply function (or even a 'bogus' equation)."

A. Koutsoyiannis (28) p.33-37

The solution to the identification problem is to specify a supply equation - in addition to the demand equation - in such a way that both may be estimated simultaneously (see A.S. Goldberger (20) Section 7).

The complete model of the purchase of life insurance must then be of the following theoretical form:

$$\begin{aligned}
\sum_{m=1}^{M} \gamma_{m1} \cdot Y_{mt} &+ \sum_{k=1}^{K} \beta_{k1} \cdot X_{kt} + u_{1t} = 0 \\
\sum_{m=1}^{M} \gamma_{m2} \cdot Y_{mt} &+ \sum_{k=1}^{K} \beta_{k2} \cdot X_{kt} + u_{2t} = 0 \\
&\vdots \\
\sum_{m=1}^{M} \gamma_{mM} \cdot Y_{mt} &+ \sum_{k=1}^{K} \beta_{kM} \cdot X_{kt} + u_{Mt} = 0
\end{aligned}$$

for $t = 1, \ldots, T$

(7.3.2)

using the traditional notation (see Goldberger p.295) where there are T observations and:

"Y_{mt} is the t^{th} observation on the m^{th} jointly dependent (endogenous) variable ($m = 1, .., M$); X_{kt} is the t^{th} observation on the k^{th} predetermined (exogenous) variable ($k = 1, .., K$); γ and β are the coefficients of the endogenous and exogenous variables respectively; u_{mt} is the residual of the t^{th} observation in the m^{th} structural equation (having the usual econometric properties of zero mean and constant variance)." (20)

More specifically we have:

$$D_t^{Pr} = a^{(1)} IP_t^{Pr} + b^{(1)} IP_t^{Sg} + \sum_{k=1}^{K_1} c_j^{(1)} X_{tj} + u_t^{(1)}$$

$$D_t^{Sg} = a^{(2)} IP_t^{Pr} + b^{(2)} IP_t^{Sg} + \sum_{k=1}^{K_2} c_j^{(2)} X_{tj} + u_t^{(2)}$$

$$S_t^{Pr} = a^{(3)} IP_t^{Pr} + b^{(3)} IP_t^{Sg} + \sum_{k=1}^{K_3} c_j^{(3)} X_{tj} + u_t^{(3)}$$

$$S_t^{Sg} = a^{(4)} IP_t^{Pr} + b^{(4)} IP_t^{Sg} + \sum_{k=1}^{K_4} c_j^{(4)} X_{tj} + u_t^{(4)}$$

$$D_t^{Pr} = S_t^{Pr}$$

$$D_t^{Sg} = S_t^{Sg} \qquad (7.3.3)$$

where the X_{tj} represent the exogenous variables and D_t, S_t and IP_t are the only endogenous variables.

Equations (7.3.3) represent a complete system of simultaneous equations in that there are six equations in six endogenous variables: however, the system must still be checked to ascertain whether it is identified according to the rank and order conditions of identifiability (see Goldberger p. 316).

Finally the inclusion of the price variables IP_t^{Pr} and IP_t^{Sg} deserve some comment. The supply equations of Equation (7.3.3) are only necessary if it is hypothesised that the supply and demand equations have at least one common endogenous variable and, of course, economic theory indicates that market price is the most important of these. It is noted that we do not necessarily preclude the inclusion of a common exogenous variable in the demand and supply equations.

However, when it comes to quantifying, say, the market price of protection-based life insurance there are several difficulties because the Office Premium rate depends on many factors (such as the age, health, sex and occupation of the policyholder as well as the type of contract and the individual life office under consideration). There are additional problems in formulating a price for savings-based life insurance because the price is inextricably tied up with the benefits (bonuses) provided by such policies:

> "It is essential .... when discussing the demand for life insurance to recognise that the life offices deal in a range of contracts with fundamentally differing characteristics. At the one extreme is the pure insurance contract for which demand will tend to vary inversely to the price (usually expressed as a rate per cent of the sum insured). At the other extreme is the (principally) savings contract for which, ceteris paribus, demand will tend to vary directly with the expected rate of return on premiums payable; in other words, again demand will tend to vary inversely to the price, now defined as a premium rate expressed as a percentage of the expected maturity value."
>
> R.L. Carter (7) p. 93

Furthermore, some discussion is necessary on what actually constitutes the price of life insurance. The price of insurance is generally argued to be the office less the pure premium (ie. the expenses loading) on the basis that:

a) in the long-run, the potential buyer of insurance would pay losses equivalent to the pure premium if he did not buy insurance (assuming that premiums are rated on a 'merit' basis); and

b) again, in the long-run, the supplier of insurance would pay losses equivalent to the pure premiums collected.

Thus, when talking about the price of non-life insurance, R.L. Carter and N.A. Doherty (8) p. 7.1-04 report that:

> "If the pure premium is correctly calculated, in the very long run, the policyholder will pay for his own losses. Therefore the price he pays to obtain protection against the random fluctuation in annual losses is the premium loading ..."

This same argument can be used to determine the price of non-profit life insurance because the 'average' policyholder[(2)] will pay pure premiums equivalent to the claims benefits that he receives. Note, however, that this only applies if the pure premiums have been correctly calculated (as in Chapters Five and Six) and strictly is only feasible for a large number of policyholders aggregated (we have seen in Section 2.5 that - for an individual policyholder - the price is complicated by his own intuitive idea of the correct pure premium).

In the with profits case, some decision must be made about the bonus loading: one possible approach is to think in terms of an expected rate of return on the

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[(2)] Note that in this case, the equivalent meaning of the term 'long-run' (ie. over a large number of years) is the term 'average' (ie. over a larger number of policyholders).

bonus loading (assuming again that pure premiums - ie. those to provide protection plus a lump sum insured at maturity in the endowment case - are estimated correctly). This approach was suggested by R.L. Carter (7) and adopted by P. Fortune (18) and A.M. El-Mokadem (16) p. 88 (although the latter does not incorporate the possibility of early death (in the endowment case) so that his is not a strict expected rate of return).(3)

On the other hand, we must not lose sight of the fact that our concern is with life insurance in the marketplace and thus with the market price. The expected rate of return on the bonus loading is not part of a definition of market price but only affects it as a determinant of market demand. Consequently for our purposes, office premium less pure premium rates (including bonus loadings) form the correct definition of 'market' price. Table 7.3.1 illustrates the Office Premium rates: Table 7.3.2 - the Pure Premium rates and Table 7.3.3 - the Expenses Loading per £1,000 sums insured.

Of all the models of Section 4.5 only one (Mantis and Farmer (32)) used a price variable in the demand model (although Fortune (18) included a variable denoting the implicit nominal yield on saving through life insurance). The Mantis and Farmer price variable was a 'composed price index' obtained from M.H. Spencer and

(3) There is, in fact, a whole selection of literature on this subject, notably those articles by J.M. Belth and Stuart Schwarzchild.

L. Siegelman (37). It seems reasonable (indeed it is the only alternative) to compute the price indices IP_t^{Pr} and IP_t^{Sg} in a similar fashion using office premium rates obtained from the Survey Data and pure premium rates from MASTER SAVINGS. The price indices so obtained are exhibited in Table 7.3.4; however, certain qualifications must be made:

i) the indices were calculated according to the based-weighted Laspeyres index (for example, see E. Mansfield (31) Ch. 3) with quantities (ie. sums insured) from Table 6.4.2. (1963 = 100);

ii) it was not possible to split the saving and protection parts of the data (for this purpose) so the price index of protection-based policies was based on the purchases of temporary life insurances ('Other' in Table 6.4.2);

iii) the average office premium rates obtained from Survey Data refer to 45 year old policyholders with 10 year maturity terms (where necessary) which are assumed to be representative (these are shown in Table 7.3.1). The sums insured of Table 6.4.2 were applied to these rates;

iv) because office premium rates are altered infrequently, the rates of Table 7.3.1 show a certain 'stickiness';

v) because of the lack of more detailed information, no better way was found of combining the Survey Data (from the seven offices involved) than to take a straight-forward average. Of course, a basic assumption incorporated in Table 7.3.4 is that rate changes in the chosen policy types (ie. inception age 45, maturity term 10) are representative of the overall rate changes;

vi) the rates of Table 7.3.1 make allowance for any policy charge made by individual survey companies and, in cases where the rate depends on the size of the policy, premiums were determined according to a rough judgement on the UK average sums insured at that moment in time;

vii) the pure premium rates of Table 7.3.2 were obtained from MASTER SAVINGS - these rates correspond to inception age 45 and maturity term 10. The with profit rates include an allowance for the bonus loading.

The price indices of Table 7.3.4 are based on 1963 = 100, calculated using the formula:

$$\text{Index in Year } t = L_t = \frac{\Sigma_i \; SI_i^{63}.EL_i^t}{\Sigma_i \; SI_i^{63}.EL_i^{63}} \qquad (7.3.4)$$

where i runs over the various policy types (ie. Temporary for 'protection-based' and Whole Life (WP and NP) and Endowment (WP and NP) for 'savings-based' policies) and SI refers to the sums insured of Table 6.4.2.

The 'real-value' price indices of Table 7.3.4 have been adjusted by the Retail Price Index (1963 = 100).

Table 7.3.1 Average Office Premium Rates per £1,000 Sums Insured for selected policies.

Year	Non-Profits			With Profits	
	Whole Life 45	Endowment 45 : 10	Temporary 45 : 10	Whole Life 45	Endowment 45 : 10
1946	31.3	99.0	14.5	38.4	110.2
47	31.2	98.9	14.5	38.4	110.1
48	31.2	98.9	14.5	38.4	110.1
49	31.2	99.2	13.7	38.3	110.0
1950	31.0	99.0	13.7	38.1	109.7
51	30.6	98.2	12.8	38.2	109.2
52	29.9	97.8	12.2	37.9	109.2
53	29.3	97.0	11.9	37.8	108.8
54	28.8	96.3	10.8	37.8	108.7
1955	28.7	96.3	10.7	37.6	108.6
56	28.6	96.9	10.4	37.5	108.9
57	28.5	95.9	10.2	37.5	108.9
58	28.2	95.4	10.0	37.5	108.9
59	27.3	93.9	9.5	37.3	108.7
1960	26.8	93.6	9.1	37.3	108.6
61	26.6	93.5	8.8	37.2	108.6
62	26.4	93.2	8.7	37.3	108.7
63	26.3	92.7	8.5	37.3	108.7
64	26.0	92.4	8.3	37.6	109.4
1965	25.3	91.2	7.6	37.3	109.4
66	24.7	90.8	7.6	37.1	109.1
67	24.4	90.5	7.5	37.4	109.5
68	24.4	89.8	6.9	37.0	109.1

Source: Survey Data

Table 7.3.2 Pure Premium Rates per £1,000 Sums Insured for selected policies (including Bonus Loadings).

Year	Non-Profits			With Profits	
	Whole Life 45	Endowment 45 : 70	Temporary 45 : 70	Whole Life 45	Endowment 45 : 70
1946	21.3	84.1	6.1	28.3	93.7
47	20.8	82.2	6.1	27.6	91.5
48	21.2	83.5	6.1	28.7	93.8
49	20.4	83.7	5.6	27.9	94.1
1950	20.9	84.2	5.8	30.1	96.5
51	21.1	84.5	5.8	31.0	97.6
52	21.6	84.0	5.9	31.6	97.0
53	20.8	83.5	5.5	31.4	97.4
54	20.4	83.0	5.4	30.9	96.9
1955	20.2	82.0	5.3	30.7	96.0
56	19.6	81.2	5.2	30.6	95.9
57	19.5	80.8	4.9	30.4	95.5
58	19.1	80.4	5.0	31.6	97.0
59	19.0	79.7	4.9	31.2	96.2
1960	18.3	78.3	4.8	30.7	95.2
61	17.6	77.4	4.7	30.2	94.9
62	17.2	76.4	4.5	29.4	93.7
63	17.1	76.3	4.5	30.1	94.9
64	16.9	75.8	4.7	30.6	95.4
1965	16.0	74.9	4.6	29.7	94.9
66	15.7	73.9	4.6	29.2	94.0
67	15.3	74.0	4.4	28.9	94.5
68	14.2	71.3	4.1	28.8	93.6

Source: MASTER SAVINGS

Table 7.3.3 Average Premium Expenses Loading per £1,000 Sums Insured for selected policies.

Year	Non-Profits			With Profits	
	Whole Life 45	Endowment $45 : \overline{10}\rvert$	Temporary $45 : \overline{10}\rvert$	Whole Life 45	Endowment $45 : \overline{10}\rvert$
1946	10.0	14.9	8.4	10.1	16.5
47	10.4	16.7	8.4	10.8	18.6
48	10.0	15.4	8.4	9.7	16.3
49	10.8	15.5	8.1	10.4	15.9
1950	10.1	14.8	7.9	8.0	13.2
51	9.5	13.7	7.0	7.2	11.6
52	8.3	13.8	6.3	6.3	12.2
53	8.5	13.5	6.4	6.4	11.4
54	8.4	13.3	5.4	6.9	11.8
1955	8.5	14.3	5.4	6.9	12.6
56	9.0	15.0	5.2	6.9	13.0
57	9.0	15.1	5.3	7.1	13.4
58	9.1	15.0	5.0	5.9	11.9
59	8.3	14.2	4.6	6.1	12.5
1960	8.5	15.3	4.3	6.6	13.4
61	9.0	16.1	4.1	7.0	13.7
62	9.2	16.8	4.2	7.9	15.0
63	9.2	16.4	4.0	7.2	13.8
64	9.1	16.6	3.6	7.0	14.0
1965	9.3	16.3	3.0	7.6	14.5
66	9.0	16.9	3.0	7.9	15.1
67	9.1	16.5	3.1	8.5	15.0
68	10.2	18.5	2.8	8.2	15.5

Source: Tables 7.3.1 and 7.3.2

Table 7.3.4 Price Indices for Life Insurance 1963 = 100.

Year	Protection-based		Savings-based	
	L_t^{Pr}	RPI adjusted (IP_t^{Pr})	L_t^{Sg}	RPI adjusted (IP_t^{Sg})
1946	210.00	411.76	112.18	219.96
47	210.00	388.89	124.81	231.13
48	210.00	368.42	111.79	196.12
49	202.50	343.22	111.91	189.68
1950	197.50	323.77	96.59	158.34
51	175.00	261.19	86.85	129.63
52	157.50	215.75	87.28	119.56
53	160.00	213.33	83.88	111.84
54	135.00	177.63	85.59	112.62
1955	135.00	168.75	90.60	113.25
56	130.00	155.13	93.87	112.02
57	132.50	152.47	95.86	110.31
58	125.00	139.66	88.45	98.83
59	115.00	127.64	88.94	98.71
1960	107.50	118.13	95.18	104.59
61	102.50	108.93	98.67	104.86
62	105.00	107.14	106.26	108.43
63	100.00	100.00	100.00	100.00
64	90.00	87.21	100.82	97.69
1965	75.00	69.38	103.16	95.43
66	75.00	66.73	106.64	94.88
67	77.50	67.27	106.33	92.30
68	70.00	58.04	112.46	93.25

Source: Tables 6.4.2 and 7.3.3

7.4 The Supply of Life Insurance

The set of Equations numbered (7.3.3) included a supply function of the form:

$$S_t^{Pr} = a^{(3)} IP_t^{Pr} + b^{(3)} IP_t^{Sg} + \sum_{k=1}^{K_3} c_j^{(3)} X_{tj} + u_t^{(3)}$$

$$S_t^{Sg} = a^{(4)} IP_t^{Pr} + b^{(4)} IP_t^{Sg} + \sum_{k=1}^{K_4} c_j^{(4)} X_{tj} + u_t^{(4)}$$

where S_t^{Pr} and S_t^{Sg} denoted the supply of 'protection-based' and 'savings-based' life insurance respectively (in terms of new premium expenditure). These two equations were included in the demand model in order that the demand equations be properly specified and identified.

The main explanatory variables of demand model have already been mentioned - in this Section, those variables corresponding to supply side will be examined.

In addition to market price (which, of course, we would expect to have a positive effect) the following variables affect the supply of life insurance:

a) Surplus

b) Costs

c) Investment Performance (including net of tax yield)

d) Liability Structure

e) Expectations, and

f) Prices of Supply Alternatives.

These variables (or their surrogates) are summarised in an Appendix at the end of this Section.

a) Surplus

According to H.F. Fisher and J. Young (17) p. 21:

> "Surplus .... is the difference between the value placed upon the assets and the value of the liabilities, and it will vary according to the bases chosen for these valuations. It is derived in the main as a result of the actual experience in mortality, interest, expenses and asset values being more favourable than the experience assumed in the valuation."

Surplus has a three-fold effect on the supply of life insurance; first it gives a legal capacity to supply (ie. solvency), second, it acts as 'cushion' to protect the office against adverse claims fluctuations and third, it provides for the distribution to policyholders and shareholders (in a proprietary company) of bonuses and dividends. In all of these three cases, we would expect the existence and size of surplus to have a positive effect on supply.

Surplus has been imperfectly quantified in aggregate in the Annual Abstract of Statistics (22) under two items: 'Shareholders' surplus and transfers to profit and loss account' and 'Miscellaneous, including transfers to investment reserves etc.' (the larger item). Econometrically, we would expect these items to be lagged by (at least) one year in order to represent the correct explanatory relationship.

b) Costs

The costs associated with life insurance (and indeed with all types of insurance) fall under two headings: claims costs and management expenses. However, because of their broad statistical base, the life offices are able to estimate claims costs with a fair degree of accuracy. Moreover, large fluctuations in mortality experience occur infrequently (and are usually controlled by reinsurance) while the average mortality experience changes only very slowly over time. The most unstable element of claims costs - surrenders - is typically of no great concern (unless surrender values are guaranteed) because the amount so obtained by the policyholder is directly under the control of the life office. Consequently, although theoretically they influence the supply of life insurance, claims costs can be discounted as a significant determinant because of their invariability and predictivity.

'Management expenses' is a collective term and includes many separate items which can broadly be described under the sub-headings: initial, renewal and miscellaneous (see Section 6.10). Undoubtedly the initial costs (ie. costs incurred at the inception of the contract such as medical expenses, initial commission, underwriting and paperwork) impose the greatest strain (called the 'new business strain') on the viability of the (traditional) life office - so much so that a life office in financial difficulties needs only stop writing business in order to rectify the position. The other major items of management expenses are wages, salaries and pensions: life insurance is a very labour-intensive business and this is reflected in the size of the item 'Expenses of management and transfers to staff pension funds' in the Annual Abstract of Statistics - up to 80% of which is normally associated with labour costs. It is important to note, especially when examining individual companies, that labour costs differ depending on the structure of the sales force: thus to take an extreme example, an industrial life office will have proportionately larger labour costs because of the large number of employees necessary to service the business. Finally we note that labour costs are influenced to a great extent by inflation so that the life office must make an assessment of future wage and salary levels when attempting to fix premiums.

Summarising, the following variable can be used to simulate the unit costs of life insurance and as such is expected to have negative effects on supply:

- 'Commission'

plus 'Expenses of management and transfers to staff pension funds' (Annual Abstract of Statistics) expressed as a percentage of new sums insured (All Business by Companies established in the UK).

c) Investment Performance

The forces that shape the investment behaviour of life insurance companies have been described by many authors including G.M. Dickinson (13) and G. Clayton and W.T. Osborn (9). Essentially, the major concern of life office investment policy is to earn a net of tax yield that is sufficient to cover that rate of interest assumed in the premium calculations; this will enable the office to pay the sums insured to its non-profits and the sums insured plus at least the assumed rate of reversionary bonus to with profits policyholders. It should be noted however, that this concern with 'yield risk' (which is made up of 'income risk' and 'default risk') must be viewed in the context of the effects of competitive pressure on the level of the expenses loading; if large expenses loadings are possible then the yield risk is reduced because effectively a lower 'calculation' rate of interest can be assumed. Thus, to some extent, the major effects of investment performance on supply have been captured in the price variable: the higher the yield on invested funds relative to the 'calculation rate of interest' the greater the expenses loading and hence the price of life insurance.

d) Liability Structure

In order to avoid imposing an overt strain on the assets of the life office, it is usual practice to have a 'balanced' liability structure without putting great emphasis on one particular type of contract. Consequently the typical life office will not want to undertake a disproportionate amount of business which offers certain guarantees (such as guaranteed surrender values, equity linked policies) - the most common form of guarantee, of course, is the non-profits policy. Fisher and Young (17) p. 153, comment that the proportion that the with profits business bears to the total (known as the 'gearing' of the office) is of considerable importance:

> "An office with the major part of its liabilities in a non-participating (non-profits) form must have a greater problem in maintaining solvency than an office in which the business is predominantly on a participating basis. In the latter case, the bonus loadings reinforce the estate, at least to the extent that any surplus arising from the bonus loadings has not vested as declared bonus."

The obvious approach to the above problem would be to hypothesise a partial adjustment hypothesis (for example, see P.J. Dhrymes (12) p. 57) so that the life office is presumed to adjust its liability structure in order to achieve an 'optimum' gearing level. However, since this 'optimum' gearing level is unknown, this approach would involve the insertion of an additional equation in (7.3.3) in order to explain the level of this optimum.

Thus if G_t denotes the gearing level given by the proportion of with profits in force to the total in force and G_t^* the optimum gearing level, then the gist of Fisher and Young's explanation is that G_t^* depends primarily on the amount of surplus. So we have:

$$G_t^* = a.SP_{t-1} + v_t \tag{7.4.1}$$

and the partial adjustment hypothesis:

$$G_t - G_{t-1} = \lambda.(G_t^* - G_{t-1}) \quad (0 \leq \lambda \leq 1) \tag{7.4.2}$$

so that, by substituting from (7.4.1) to (7.4.2) we get

$$G_t = (1-\lambda)G_{t-1} + a\lambda\, SP_{t-1} + \lambda v_t \tag{7.4.3}$$

where v_t - the random error term - is assumed to possess the usual econometric properties of zero mean and constant variance.

The evaluation of λ from Equation (7.4.3) then yields the correct value of G_t^*, so that we can form the variable necessary for the supply equations $(G_{t-1} - G_t^*)$

which is an indicator of the adjustment necessary in year t:

$$(G_{t-1} - G_t^*) = \frac{1}{\lambda}(G_{t-1} - G_t) \qquad (7.4.4)$$

Equation (7.4.3) was then estimated using Ordinary Least Squares techniques with data on surplus and gearing from the Annual Abstract of Statistics (20). The results of the regression are illustrated in Equation (7.4.5) below yielding a value for λ of 0.02.

$$G_t = \underset{(136.9)}{0.98G_{t-1}} - \underset{(0.373)}{0.000074SP_{t-1}} \qquad (7.4.5)$$

(t-values are in parenthesis); $R^2 = 0.98$; $\bar{R}^2 = 0.98$; DW = 1.88; n = 23.

The resultant values for the optimum gearing level G_t^* are given in Table 7.4.1.

The negative values of G_t^* in Table 7.4.1, of course, are not feasible in practice. However, the variable $(G_{t-1} - G_t^*)$ gives an indication of the adjustment necessary in year t to bring the actual gearing level in line with the life offices' optimum gearing level.

Since $(G_{t-1} - G_t^*)$ represents an adjustment term, we would expect it to have a negative effect on the supply of with profits business (with a positive effect on non-profits). Thus if the actual liability structure contains too much with profits business at the end of year t-1 (ie. $(G_{t-1} - G_t^*)$ positive) then we would expect a reduction in the supply of with profits policies (and thus a negative coefficient of $(G_{t-1} - G_t^*)$).

Table 7.4.1 Optimum Gearing Level (G_t^*)

Year	G_t^*	$(G_{t-1} - G_t^*)$
1946	0.5890	0.0150
47	0.8637	-0.2600
48	0.6539	-0.0450
49	0.0548	0.5550
1950	-0.4063	1.0050
51	-1.2714	1.8500
52	0.4416	0.1000
53	-0.0754	0.6150
54	-0.7627	1.2900
1955	0.0665	0.4350
56	-0.2872	0.7800
57	-0.2278	0.7050
58	0.6631	-0.2000
59	-0.6829	1.1500
1960	-0.1059	0.5500
61	0.1431	0.2900
62	0.1423	0.2850
63	-0.0184	0.4400
64	0.0428	0.3700
1965	0.5054	-0.1000
66	-0.3126	0.7200
67	0.1680	0.2250
68	-0.4965	0.8850

Source: Equations (7.4.2) and (7.4.4)

e) Expectations

Expectations influence the supply of life insurance for the same reasons as they do demand; in particular the costs facing the life office (in terms of claims) cannot be known with certainty. Thus in order to determine supply, the life office must make forecasts about the future levels of costs and general economic and investment conditions. The major expectational variable included in the supply equation is concerned with predicting future wage inflation.

f) Price of Supply Alternatives

The elementary theory of supply predicts that as the market price of supply alternatives increases, supply will decrease. Thus, if for example, we consider the supply of protection-based life insurance then the price (index) of savings-based insurance (IP_t^{Sg}) should also be included as an explanatory variable.

APPENDIX TO SECTION 7.4

Explanatory Variables affecting the Supply of Life Insurance
(all variables in real terms)

SP_{t-1}	Real Surplus, lagged one time period: 'Shareholders' surplus and transfers to profit and loss account' plus 'Miscellaneous, including transfers to investment reserves etc.'
C_t	Costs in year t. 'Commission' plus 'Expenses of management and transfers to staff pension funds' as a % of new sums insured.
$(G_{t-1} - G_t^*)$	Adjustment necessary in Optimum Gearing level (with profits in force dividend by total in force).
IP_t^{Sg}	Real price of 'savings-based' Life Insurance.
IP_t^{Pr}	Real price of 'protection-based' Life Insurance.
YLD_t	Annual Yield on Life and Annuity Funds (companies established in the UK) based on Book Values.

7.5 Inflationary Expectations

The objective of this Section is to explore the formulation of those variables simulating price/inflation expectations. These variables have appeared several times in the model: in Equation (7.2.3), in Equation (7.3.3) (as undefined exogenous variables) and in the previous Section where expected wage levels were under discussion.

It has already been noted in Chapter Four that price/inflation expectations have played a significant role in the determination of saving and the purchase of life insurance. The theory of Chapter Three also indicates that inflation expectations are important in the purchase of 'protection-based' life insurance. However, the models of the preceding Chapters have indicated that inflation expectations, in the context of the purchase of life insurance, must have several distinguishing features.

First, expectations must be formed over the long-term because, obviously, most life insurance contracts are long-term ones (of typically over twenty years duration) (this point was raised by D.E.W. Laidler and J.M. Parkin (29)). A.E. Hofflander and R.M. Duvall (25) also recognised this fact and attempted to build an expected price (index) variable by including a trend element. The most common method of forming price expectations, however, is to use the adaptive expectations hypothesis (eg. see S. Neumann (35)) - this is typically only suitable for forecasting a short period into the future because of the assumption that expectations are revised in the light of past experience. Another method of obtaining price expectations is to rely on survey data (for example, J.A. Carlson and M. Parkin (6) and F.T. Juster and P. Wachtel (27)). However, survey data is more widely used for short-term expectations - Carlson and Parkin imposed a six months time horizon.

Secondly, not only anticipated (expected) inflation but also unanticipated inflation is of interest: this point was made with some force by Juster and Wachtel. But the exact effects of unanticipated inflation are confused once predictions are made for more than one time period: it seems reasonable to assume that the consumer places less reliance on the correctness of predictions made twenty years into the future than on one year hence.

Thirdly, although the adaptive expectations hypothesis is widely used, some doubt has been expressed on the validity of the approach on the basis that little evidence exists to show that price expectations are formed in this way (eg. see J.A. Trevithick and C. Mulvey (40)). Fortunately, (see Section 3.2) the work of Carlson and Parkin has restored confidence in the error learning models (albeit only for the short-term case).

The rest of this Section will be devoted to the formulation of a long-term adaptive expectations model which - as its name suggests - is established on the basis that the consumer modifies his price expectations in the light of recent experience.

To this end, define ${}_{n_t}\dot{P}^e_t$ as the 'average' long-term expected rate of inflation over the next n_t time periods (made at the start of time period t) where, for ease of exposition, it is assumed that compounding occurs continuously. Let P_t denote the level of the relevant price index (normally RPI - see Appendix 7.1) at the end of period t and $\dot{P}_t$ the rate of inflation that has occurred during time period t: again compounding is continuous so that we have(4):

$$P_t = P_{t-1} \cdot \exp(\dot{P}_t) \tag{7.5.1}$$

Now from our definition of ${}_{n_t}\dot{P}^e_t$ we get:

$$P^e_{t+n_t-1} = P_{t-1} \cdot \exp(n_t \cdot {}_{n_t}\dot{P}^e_t) \tag{7.5.2}$$

where $P^e_{t+n_t-1}$ denotes the expected price level at the end of time period $(t+n_t-1)$ and 'exp' is the exponential function (e^x).

Now the usual (short-term) adaptive expectations hypothesis would examine the course of inflation over the t^{th} time period and revise the $t+1^{th}$ expectation in the light of that examination. The difference in this 'long-term' adaptive expectations model is that consumers revise their $t+1^{th}$ (long-term) expectation in the light of the price level at the end of time period

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(4) $\dot{P}_t$ should more properly be called the 'force' of inflation (for example, see D.W.A. Donald (14) p.11). $\dot{P}_t$ based on changes in the RPI is given in Appendix 7.1.

$t+n_t-1$ [5] by assuming that the 'force' of inflation in time period t continues for a further $t+n_t-1$ periods. Thus if $\dot{P}_t$ exceeds ${}_{n_t}\dot{P}^e_t$ then the quantity $P_t.\exp((n_t-1).\dot{P}_t)$ will exceed $P^e_{t+n_t-1}$. The adaptive expectations approach then says that the $t+1^{th}$ forecast $({}_{n_{t+1}}\dot{P}^e{}_{t+1})$ will be 'adapted' by scaling ${}_{n_t}\dot{P}^e_t$ by the ratio $(P_t.\exp((n_t-1).\dot{P}_t) / P^e_{t+n_t-1})^{\lambda_t}$. It is then a simple matter to work backwards to obtain the $t+1$ long-term forecast.

So let the average long-term forecast (over the next n_{t+1} time periods) at time $t+1$ be given by ${}_{n_{t+1}}\dot{P}^e{}_{t+1}$. Then the expected price level at the end of period $\underline{n_t+t-1}$ is given by:

$$P_t.\exp((n_t-1).{}_{n_{t+1}}\dot{P}^e_{t+1})$$

which we can equate to

$$P^e_{t+n_t-1} \quad \text{scaled by} \quad \left[\frac{P_t.\exp((n_t-1).\dot{P}_t)}{P^e_{t+n_t-1}}\right]^{\lambda_t}$$

where the parameter λ_t denotes the degree of 'adaption' of $P^e_{t+n_t-1}$ according to the quantity $P_t.\exp((n_t-1)\dot{P}_t)$.

So we get

$$P_t.\exp((n_t-1).{}_{n_{t+1}}\dot{P}^e_{t+1}) = P^e_{t+n_t-1}.\left[\frac{P_t.\exp((n_t-1).\dot{P}_t)}{P^e_{t+n_t-1}}\right]^{\lambda_t} \qquad (7.5.3)$$

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(5) It seems reasonable to assume that an endowment policy-holder (for example) would make some (subjective) evaluation of the real value of his maturity benefits.

Taking natural logarithms of Equation (7.5.3) :

$$\ln(P_t) + (n_t-1)\cdot{}_{n_{t+1}}\dot{P}^e_{t+1} = \ln(P^e_{t+n_t-1}) +$$

$$\lambda_t\cdot(\ln(P_t) + (n_t-1)\cdot\dot{P}_t - \ln(P^e_{t+n_t-1})) \tag{7.5.4}$$

Expanding Equation (7.5.4) (using Equation (7.5.2)):

$$(n_t-1)\cdot{}_{n_{t+1}}\dot{P}^e_{t+1} = \ln(P_{t-1}) + n_t\cdot{}_{n_t}\dot{P}^e_t - \ln(P_t) +$$

$$\lambda_t\cdot(\ln(P_t) + (n_t-1)\dot{P}_t - \ln(P_{t-1}) - n_t\cdot{}_{n_t}\dot{P}^e_t) \tag{7.5.5}$$

Equation (7.5.5) can be rewritten (noting from (7.5.1) that $\ln(P_t) = \ln(P_{t-1}) + \dot{P}_t)$ to get

$${}_{n_{t+1}}\dot{P}^e_{t+1} = \frac{(1-\lambda_t)\cdot n_t}{(n_t-1)}\cdot{}_{n_t}\dot{P}^e_t + \frac{(\lambda_t n_t-1)}{(n_t-1)}\cdot\dot{P}_t \tag{7.5.6}$$

which can be abbreviated as:

$${}_{n_{t+1}}\dot{P}^e_t = (1-r)\cdot{}_{n_t}\dot{P}^e_t + r\cdot\dot{P}_t \tag{7.5.7}$$

where $r = \frac{\lambda_t n_t - 1}{n_t - 1}$

and $0 \leq r \leq 1$ in order to obtain convergence.

Finally, we note from Equation (7.3.3) that the variable ${}_{n_t}\dot{P}^e_t$ is included as an explanatory variable in the two demand functions. Thus we get (for example):

$$D_t^{Sg} = \text{CONST} + a^{(2)}.IP_t^{Pr} + b^{(2)}.IP_t^{Sg} + c^{(2)}.{}_{n_t}\dot{P}_t^e + \sum_{k=1}^{K_2} d_j.X_{tj} + u_t^{(2)} \quad (7.5.8)$$

which, by incorporating (7.5.7), can be written as

$$D_t^{Sg} = \text{CONST} + c^{(2)}(1-\Gamma)_{n_{t-1}}\dot{P}_{t-1}^e + c^{(2)}.\Gamma.\dot{P}_{t-1} + \Sigma_t^{(2)} + u_t^{(2)} \quad (7.5.9)$$

where $\Sigma_t^{(2)}$ denotes the other explanatory variables.

Lagging (7.5.8) by one time-period we get:

$$D_{t-1}^{Sg} = \text{CONST} + c^{(2)}.{}_{n_{t-1}}\dot{P}_{t-1}^e + \Sigma_{t-1}^{(2)} + u_{t-1}^{(2)} \quad (7.5.10)$$

Finally, substituting ${}_{n_{t-1}}\dot{P}_{t-1}^e$ from (7.5.10) into (7.5.9):

$$D_t^{Sg} = \Gamma.\text{CONST} + (1-\Gamma).(D_{t-1}^{Sg} - \Sigma_{t-1}^{(2)} - u_{t-1}^{(2)}) + c^{(2)}.\Gamma.\dot{P}_{t-1} + \Sigma_t^{(2)} + u_t^{(2)} \quad (7.5.11)$$

It is interesting to note from Equation (7.5.7) that $\Gamma \leq \lambda_t$ for all t (since $0 \leq \Gamma \leq 1$). Furthermore since

$$\lambda_t = \frac{(n_t-1)\Gamma + 1}{n_t}$$

the larger the value of n_t, the smaller (ie. nearer to $\underline{r}$) the value of λ_t. This latter characteristic is fully in line with what we would expect since it implies that the longer the time period involved the smaller the emphasis given to current experience.

I scarcely need to mention that n_t represents either the maturity term (in the endowment case) or the expectation of life in the case of whole of life insurance (these values have been used or calculated in MASTER SAVINGS).

7.6 Permanent Income

In the analyses of Chapter Four it was noted that there were good a priori reasons for the inclusion of permanent income as an explanatory variable in the demand and financial saving models. The difficulty lies in choosing an appropriate method of calculating the permanent income of potential purchasers of ordinary life insurance (and hence its complement - transitory income).

There are, in fact, several ways of determining permanent income:

1) the most common approach is to form permanent income out of a weighted average of all previous incomes. The geometric lag is the usual method[6] ie. $y_t^P = \sum_{k=0}^{\infty} \gamma(1-\gamma)^k . y_{t-k}$

$$(7.6.1)$$

where y_t^P denotes permanent income and y_{t-k} real disposable income in year t-k (RPI = 100 in 1963);

2) a similar approach to 1) above was adopted by the Bank of England (3) p. 68 but instead of a geometric lag structure, an Almon lag structure was used[7] utilising third degree polynomials;

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[6] For example, see R. Stone (38) p.127 or the Bank of England (3) p.10. The former obtains a value of γ = 0.7, the latter assumes the same value.

[7] See S. Almon (1)

3) there is the suggestion by Houthakker and Taylor (26) that the change in permanent income is proportional to the change in current income (see Equations (4.2.5) and (4.2.6));

4) the Bank of England (3) p.62 noted that, in studies of aggregate consumption and saving,

> "it is difficult in practice to decide whether the lagged dependent variable represents habit or persistence in spending decisions, a partial response to current income or the adjustment of current to permanent income". (Also see M.J.C. Surrey (39))

Thus it turns out that, in practice, aggregate consumption lagged by one year is also a good indicator (but this is rather a 'rule of thumb' solution).

Point 4) above also gives an insight into the factors to be taken into account in choosing the appropriate measure of permanent income. The demand model will quite probably already include the lagged dependent variable as an explanatory variable but in order to simulate inflation expectations and not permanent income. Thus to avoid confusion, the safest solution is either 1) (with predetermined γ), 2) or 4) above. It is also worth noting that, with only 23 time-series observations (1946-1968) there will be very little difference between the methods chosen. In addition the small number of observations cannot support a model that is too complicated.

The Almon lag structure solution was discarded because of the problems involved (eg. see P.J. Dhrymes (12) p.231) and because of the small number of observations. Data on permanent income from the two remaining solutions is represented in Table 7.6.1.[8] A value of $\gamma = 0.5$ was chosen because it was felt that Stone's figure of 0.7 was too large in the case of more long-term expectations (as was the figure of around 0.6 obtained by the Bank of England (3) p.64 for the purchase of non-consumer durables).

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[8] The income data is based on real Personal Disposable Income (1963 = 100) per head of UK population. Note that the formula for permanent income does not include any trend elements. Formulations of this type have been suggested by M.R. Darby (10) who uses quadratic trended estimates of the form:

$$INC_t^P = b\,INC_t + (1-b)(1+c+2dt)INC_{t-1}^P$$

where $b = 0.1$ and c and d are the estimates from the relationship $\log INC_t = e + c.t + dt^2 + u_t$.

Table 7.6.1 Permanent Income

Year	Real PDI per capita (RPI = 100 in 1963)	Permanent Income based on Equation (7.6.1) with $\gamma = 0.5$ (RPI = 100 in 1963)*	Permanent Income based on lagged consumer expenditure per capita (RPI = 100 in 1963)
1946	318.55	299.28	288.47
47	317.73	308.50	304.52
48	306.72	307.61	310.05
49	309.59	308.60	304.38
1950	313.43	310.19	304.46
51	307.74	308.96	309.08
52	302.39	305.67	303.17
53	314.03	309.85	292.44
54	325.90	317.88	302.43
1955	334.42	326.15	315.22
56	340.58	333.37	321.64
57	343.23	338.30	322.25
58	345.30	341.80	326.27
59	362.08	351.94	330.89
1960	382.95	367.44	344.26
61	393.45	380.45	355.31
62	391.59	386.02	358.79
63	405.63	395.82	361.96
64	419.51	407.67	374.48
1965	425.59	416.63	385.00
66	431.15	423.89	389.44
67	435.12	429.50	394.49
68	439.55	434.53	400.43

Source: Key Statistics 1900/1970 (23)

* Assumed starting value 280.00 in 1945

7.7 The Other Explanatory Variables

Those explanatory variables which exclusively affect the supply of life insurance have already been examined in Section 7.4 while the price of life insurance, price expectations and permanent income were discussed in Sections 7.3, 7.5, and 7.6. The objective of this Section is simply to summarise and evaluate those other explanatory variables in the demand model. Appendix 7.2 shows the raw data corresponding to these variables and Appendix 7.1 summarises the notation and sources of this data.

Following Neumann (35) Ch. VII, we can classify four pertinent groups of explanatory variables:

1) Factors related to the ability to purchase life insurance.
2) Factors related to the need for purchasing life insurance.
3) Factors related to the willingness to purchase life insurance, and
4) Factors related to the exposure of life insurance to potential purchasers.

(Note that the variables in monetary values are expressed in real terms by discounting by the Retail Price Index (1963 = 100)).

At this stage, I should point out that the explanatory variables described in the remainder of this Section are all deficient in one important respect: that, for the most part, they are aggregate variables relating to the total population of the UK, and not just to those households which represent potential purchasers of ordinary life insurance. Thus, for example, real permanent income per capita (INC_t^P) is used as a surrogate for that permanent income of purchasers of ordinary life insurance; yet recent surveys (eg. E.V. Morgan (33)) have indicated that, not unexpectedly, ordinary life insurance is purchased by households in the higher social classes (with correspondingly higher incomes).

Unfortunately, there is no easy solution to the aggregation problem (which arises in many areas where time-series data are used). Consequently, it may well be that the empirical analyses of Chapters Eight and Nine will suffer because the explanatory variables used are only surrogates and in many cases they may not be a close enough proxy to indicate the full significance of the underlying relationship.

1) The Ability of Purchase

Included under this heading are:

a) Real Personal Disposable Income per capita (PDI_t); Permanent (INC_t^P) and Transitory (INC_t^T);
b) The Price of Life Insurance (IP_t^{Sg} and IP_t^{Pr})
c) Real Wealth per capita, and
d) Real Holdings of Liquid Assets per capita (LIQ_t)

a) and b) have already been discussed: it suffices to say that we expect the demand for life insurance to be strongly influenced by income (in a positive sense).

Unfortunately, no direct measure of real per capita wealth is possible in a time-series context although figures are available for <u>shares and deposits in building societies</u> (BSW_t), holdings of liquid assets etc., as well as some indication of the level of indebtedness: <u>building society advances on mortgages</u> (BSA_t), <u>new hire</u> purchase commitments (HPC_t), etc.

The wealth effects on the purchase of life insurance are quite complicated since protection-based insurance may be needed to protect wealth; on the other hand it could be argued that the greater the wealth of the policyholder, the less need to protect dependents against premature death . Protection-based life insurance is often used to cover the indebtedness of the policyholder (so that he does not leave a potentially negative bequest).

It seems reasonable to hypothesise a negative wealth effect on the purchase of 'savings-based' life insurance on the basis that the presence of existing savings stocks will deter further saving. On the other hand, if the consumer has some idea of the 'desired' level of wealth/ assets (or, for example, of retirement income) then we cannot necessarily conclude that the existence of wealth will preclude saving (eg. see M.J. Hamburger (24)).

Again, <u>real holdings of liquid assets</u> (per capita) can, hypothetically, have two main effects: first, any of the wealth effects described above and secondly an effect depending on the desired level of cash for transactions or precautionary motives (see Morgan Grenfell (34)). Data on liquid assets can be obtained from the Annual Abstract of Statistics (22). The variable LIQ_t uses data on 'Notes and coin in circulation with the public' plus 'Total deposits of London Clearing banks'.

It is realised that these data will include liquid assets held by the non-personal sector but the corresponding figures for individuals are not available over the period 1946/1968.

2) The Need for Life Insurance

The need to obtain life insurance can be divided into two parts: to obtain protection of dependents against the financial losses associated with premature death and second, to effect some savings.

The first, and most obvious variables in this Section follows Hamburger (24) - the 'optimal stock' of life insurance already in existence. Thus it is of interest to know whether the purchase of life insurance is 'habit-forming' or alternatively whether - like the purchase of consumer durables - the existence of life insurance discourages further purchase. The 'stock' of protection-based life insurance will be represented by Temporary Sums Insured In Force (IF_t^{Pr}) and the stock of savings-based insurance by the complement (Total In Force minus Temporary Sums Insured In Force). (IF_t^{Sg})

So if we define IF_t^* to be the 'optimum stock' of life insurance (either savings- or protection-based) then a commonly used adjustment process of the following form can be hypothesised: where

$$IF_t - IF_{t-1} = k(IF_t^* - IF_{t-1}) \qquad (7.7.1)$$

where $0 \leq k \leq 1$.

The coefficient k measures the fraction of the adjustment made towards equating the actual and optimal 'stock' of life insurance in force. The larger is k the faster the adjustment. If k equals unity then the model implies that $IF_t^* = IF_t$ and adjustment is completed during the period of observation.

The usual subsequent procedure is then to provide a functional equation for IF_t^* and hence an equation to determine IF_t with $(1-k)IF_{t-1}$ appearing on the right-hand-side. However, obviously, this method does not give us any information about the determinants of the demand for (new) life insurance.

An alternative method which attempts to solve this problem is one which breaks down the term $IF_t - IF_{t-1}$ into its component parts. Thus in the demand model case we can write,

$$(IF_t - IF_{t-1}) = D_t - Cl_t - Ca_t \tag{7.7.2}$$

where D_t represents demand (new business), Cl_t - claims and Ca_t - cancellations in year t. Unfortunately this new method is again largely unsuccessful because it introduces further problems:

a) the method cannot cope with an analysis of the financial saving ratio FSR_t;

b) the term $Cl_t + Ca_t$ is not known separately for the protection and savings elements of life insurance. In fact, it is not even known separately for non-single premium policies;

c) furthermore, no entirely successful approximation of $Cl_t + Ca_t$ can be made without introducing some specification error into the model (for instance, by hypothesising $(Cl_t + Ca_t) = \alpha . IF_{t-1}$ where α is a constant).

Consequently the only remaining alternative is to include the term IF_t^* in the model explicitly: this can be done by reworking Equation (7.7.1) to obtain,

$$IF_t^* = \frac{1}{k}(IF_t - IF_{t-1}) + IF_{t-1}$$

$$= \frac{1}{k} IF_t + \frac{k-1}{k} IF_{t-1} \qquad (7.7.3)$$

where $0 < k \leq 1$ (in this case, we preclude a zero value of k). The model can then be run with different values of k so that the 'best' value of k can be chosen by some appropriate method (see S.M. Goldfeld & R.E. Quandt (21)).

The need for protection is also associated with the existence of dependents and thus we would expect births and marriages to be important variables (E.V. Morgan (33) reported a higher incidence of life insurance ownership among married persons (29.4% as opposed to 19.0% of unmarried in the sample) and also among two to four person households). We have already noted that life insurance protection is also needed to cover the risk of death with debts outstanding. Thus Morgan reports that,

> "savers who are buying their house on a mortgage loan were more likely to hold life assurance, presumably because of the links that often exist between the two transactions."

This implies that new mortgage advances would also be an important explanatory variable.

The need for savings is more difficult to simulate, but one would expect the following variables to be important:

new mortgage advances - since endowment policies are often used for house purchase purposes (in conjunction with a building society);

private pension 'wealth' - since endowment policies are often used to save for retirement, membership of a private pension scheme might discourage any further provision. Unfortunately insufficient data of suitable quality prohibited the inclusion of this important variable in any further analysis;

national insurance pension 'wealth' (see El-Mokadem (16)) - intuitively we might expect the existence of pension arrangements (whether private or state provided) to discourage saving for retirement. However, several authors have reported the opposite conclusion (eg. Morgan (33), Revell (34) and El-Mokadem (16)): presumably caused by what is known as the 'recognition' effect. Unfortunately, the series compiled by El-Mokadem was too short to be used in this study; however, figures are available for the total receipts of the National Insurance Fund (see Barros (4) Table 6) which includes employer and employee contributions, interest on government securities and government grants. Although the Fund provides other benefits as well as pensions, Barros reports that pensions constitute an average of nearly two-thirds of the benefits paid during the years 1960/1975.

3) The Willingness to Purchase

The consumer's willingness to purchase life insurance can be explained in the context of three areas: the economy, the savings 'industry' and the life insurance industry.

In the context of the economy, the following factors might be expected:

the long-term expected rate of inflation - we have seen from Chapter Three that expected inflation may have either negative or positive effects on sales of protection-based life insurance. Similarly, the effects of inflation on savings-based life insurance are uncertain although the authors listed in Chapter Four report that inflation discourages sales of this type of life insurance. However, the recent high levels of inflation have been proposed as the cause of the historically high saving ratios of the middle 1970's. In particular it has been argued that consumers increase their saving in order to maintain the real value of personal wealth (eg. see 'Personal Savings level likely to Remain High', Financial Times, 17th August 1978, p.6). The same arguments could be applied directly to the purchase of savings-based life insurance with increased saving in order to maintain the real value of retirement income;

the standard rate of income tax - by altering the rate of income tax, the government can effectively change the price of life insurance;

the level of unemployment - Juster and Wachtel (27) found evidence that this variable influenced aggregate saving: they reason that a high level of unemployment may reduce saving in order that consumption be maintained;

the change in unemployment - again, Juster and Wachtel argue that if unemployment is rising then fear of being unemployed is probably rising too. We would expect that this would lead to a decrease in the long-term commitments of the consumer.

In the context of the savings industry it is possible that the sales of savings-based life insurance would be influenced by the success and performance of the other savings institutions (for example: the real yield on building society shares).

In the context of the life insurance industry, it has already been explained that purchases of savings-based life insurance can influence those of protection-based insurance (and vice versa). Additionally, the 'yield' on life insurance savings could also be important; however, this can be measured in a variety of ways (for example: by the rate of declared reversionary bonus (Table A6.1.1) or by forming an index of the return on with profits policies similar to that of the Economist Intelligence Unit Ltd. (15)).

4) The Exposure to Life Insurance

Variables under this heading relate to the class and education of individual policyholders and are, therefore, not really suitable for a time-series analysis (see D.R. Anderson and J.R. Nevin (2)).

APPENDIX 7.1 Abbreviations and Sources

Abbreviation	Description	Source
B_t	Births in year t (000)	Annual Abstract (22)
BSA_t	Real Building Society Advances on Mortgages per capita (1963 £)	Annual Abstract (22)
BSW_t	Real wealth in Building Society shares and deposits per capita (1963 £)	Annual Abstract (22)
C_t	Real Life Insurance Supply Costs in year t (1963 £m)	see Appendix 7.4.1
$CONS_t$	Real Consumer Expenditure in year t	Key Statistics (23)
D_t^{Pr}	Real Demand for Protection -based Life Insurance per 1,000 head of population (1963 £)	Table 6.10.10
D_t^{Sg}	Real Demand for Savings-based Life Insurance per 1,000 head of population (1963 £)	Table 6.10.10
FS_t	Real Financial Saving via new Life Insurance Purchases (£000)	Table 6.10.9
FSR_t	Financial Saving via new Life Insurance divided by PDI (£000m)	Table 7.2.1
$(G_{t-1}-G_t^*)$	Adjustment necessary in Optimal Gearing Level	Table 7.4.1
HPC_t	Total Hire Purchase Credit Outstanding per capita (1963 £)	Key Statistics (23)
IF^{Pr}	Real Protection-based sums insured in force per capita	Annual Abstract (22)
IF_t^{Sg}	Real Savings-based sums insured in force per capita	Annual Abstract (22)
INC_t^P	Real Permanent Income in year t per capita (1963 £)	Table 7.6.1

Abbreviation	Description	Source
INC_t^T	Real Transitory Income per capita (1963 £)	
IP_t^{Pr}	Real Market price index of protection-based life insurance (1963 = 100)	Table 7.3.4
IP_t^{Sg}	Real market price index of savings-based Life Insurance (1963 = 100)	Table 7.3.4
LIQ_t	Real Liquid Assets per capita (1963 £)	Annual Abstract (22)
M_t	Marriages in year t (000)	Annual Abstract (22)
NIF_t	National Insurance Fund : Total Current Account Receipts per capita (1963 £)	Barros (4) (only after 1945)
$\dot{P}_t$	Actual Rate of Inflation of RPI over year t (Compounded Continuously)	Equation (7.5.1)
$\dot{P}_t^e$	Expected Rate of Inflation	
PDI_t	Real Personal Disposable Income per capita (1963 £)	Key Statistics (23)
POP_t	UK Population	Key Statistics (23)
r_t^{BS}	Real yield on Building Society shares	Annual Abstract (22)
RPI_t	Retail Price Index (1963 = 100)	Key Statistics (23)
S_t^{Pr}	Real Supply of Protection -based Life Insurance per 1,000 head of population (1963 £)	Table 6.10.10
S_t^{Sg}	Real Supply of Savings-based Life Insurance per 1,000 head of population (1965 £)	Table 6.10.10
SP_t	Real Surplus in year t (1963 £m)	see Appendix 7.4.1
T_t	Standard Rate of Income Tax (p in £)	Key Statistics (23)

Abbreviation	Description	Source
U_t	%Unemployment in UK in year t	Key Statistics (23)
ΔU_t	Change in %Unemployment	
WNI_t	National Insurance Pension Wealth	El-Mokadem (16) (1952/1966 only)
YLD_t	Annual Yield on Life and Annuity Funds (companies established in the UK) based on Book Values	Table 6.7.1

APPENDIX 7.2 Data

Endogenous Variables (Real Values)

Year	$D_t^{Pr}(= S_t^{Pr})$	$D_t^{Sg}(= S_t^{Sg})$	FSR_t	IP_t^{Pr}	IP_t^{Sg}
1946	116.58	548.40	1.3433	411.76	219.96
47	125.13	667.04	1.5899	388.89	231.13
48	110.85	595.39	1.5199	368.42	196.12
49	123.77	558.87	1.2943	343.22	189.68
1950	134.06	582.62	1.4516	323.77	158.34
51	150.68	647.11	1.4048	261.19	129.63
52	116.72	569.67	1.5025	215.75	119.56
53	135.23	519.56	1.2508	213.33	111.84
54	141.16	598.32	1.4140	177.63	112.62
1955	149.64	656.84	1.5160	168.75	113.25
56	171.54	608.92	1.3779	155.13	112.02
57	159.76	576.73	1.3443	152.47	110.31
58	168.49	671.39	1.7790	139.66	98.83
59	234.07	706.77	1.4806	127.64	98.71
1960	230.34	881.38	1.7238	118.13	104.59
61	218.06	954.10	1.9227	108.93	104.86
62	237.66	986.46	1.6846	107.14	108.43
63	259.76	1,106.77	1.8716	100.00	100.00
64	245.78	1,269.63	2.3691	87.21	97.69
1965	256.49	1,232.36	2.1365	69.38	95.43
66	247.47	1,273.83	2.1301	66.73	94.88
67	253.34	1,317.68	2.4273	67.27	92.30
68	332.13	1,347.44	1.9761	58.04	93.25

Exogenous Variables : Supply Equations (Real Values)

Year	SP_{t-1}	C_t	$(G_{t-1} - G^*_t)$	YLD_t
1946	8.05102	4.5826	0.0150	4.5660
47	4.73333	4.2642	-0.2600	3.5319
48	4.73148	4.7343	-0.0450	3.6415
49	8.17719	4.8548	0.5550	3.4969
1950	12.34748	4.6354	1.0050	3.5029
51	12.91148	4.5193	1.8500	3.7871
52	38.40746	4.7114	0.1000	3.8153
53	34.49315	4.7646	0.6150	3.9355
54	8.74133	4.4134	1.2900	4.2725
1955	10.11579	4.3165	0.4350	4.4425
56	16.76000	4.3397	0.7800	4.3576
57	41.17900	4.4975	0.7050	4.5217
58	32.45915	3.9337	-0.2000	4.7026
59	13.90279	3.6984	1.1500	5.1949
1960	16.09212	3.4906	0.5500	5.2713
61	13.70000	3.3238	0.2900	5.5488
62	11.98406	3.3184	0.2850	5.3711
63	27.67755	3.1223	0.4400	6.6182
64	16.87800	2.8472	0.3700	5.9356
1965	14.11047	2.8336	-0.1000	6.1334
66	20.10083	2.7402	0.7200	5.8761
67	19.09698	2.6328	0.2250	7.2319
68	26.78819	2.6050	0.8850	8.5906

Exogenous Variables : Demand Equations (Real Values)

Year	B_t	BSA_t	BSW_t	HPC_t	IF_t^{Pr}	IF_t^{Sg}	INC_t^P
1946	955	7.88	33.91	2.09*	19.28	95.78	299.28
47	1025	9.37	34.07	2.63	18.70	94.52	308.50
48	905	9.33	33.94	3.71	18.36	92.01	307.61
49	855	9.37	35.90	4.35	19.90	94.18	308.60
1950	818	8.81	38.15	5.46	22.11	101.06	310.19
51	797	7.94	37.55	6.17	24.75	102.10	308.96
52	793	7.23	37.58	6.55	24.15	99.67	305.67
53	804	7.89	40.62	7.27	26.61	101.13	309.85
54	795	9.67	45.54	9.95	30.41	108.96	317.88
1955	789	9.63	47.78	11.31	33.02	115.41	326.15
56	825	7.81	49.03	8.77	35.60	115.89	333.37
57	851	8.37	50.88	10.02	40.53	119.65	338.30
58	871	8.11	53.37	12.03	43.58	125.36	341.80
59	879	11.05	58.46	18.13	55.15	135.84	351.94
1960	918	11.75	61.75	19.62	67.79	150.20	367.44
61	944	10.98	63.32	18.81	73.89	147.80	380.45
62	976	11.73	67.13	16.98	85.97	152.94	386.02
63	990	15.84	74.64	17.88	102.24	162.23	395.82
64	1015	18.70	80.94	20.00	113.33	175.44	407.67
1965	997	16.26	87.53	20.35	133.68	180.16	416.63
66	980	20.26	95.96	17.97	153.38	188.49	423.89
67	962	23.09	110.31	16.70	177.26	204.89	429.50
68	947	23.85	116.82	16.33	225.29	224.89	434.53

*Estimated

Exogenous Variables : Demand Equations (Real Values) cont'd....

Year	LIQ_t	M_t	NIF_t	$\dot{P}_t$ (%)	PDI_t	POP_t (m)
1946	269.18	441.19	6.98	4.00	318.55	46.83
47	270.38	455.09	11.96	5.72	317.73	47.95
48	252.52	449.97	13.79	5.40	306.72	49.62
49	244.82	425.97	15.28	3.45	309.59	49.93
1950	237.11	408.03	14.93	3.33	313.43	50.18
51	221.19	411.40	14.13	9.38	307.74	50.29
52	202.45	399.76	14.83	8.58	302.39	50.43
53	203.41	395.32	15.84	2.70	314.03	50.59
54	208.53	392.86	15.66	1.32	325.90	50.77
1955	198.99	410.63	16.91	5.13	334.42	50.95
56	187.76	406.27	17.37	4.64	340.58	51.18
57	185.13	398.96	17.25	3.63	343.23	51.43
58	184.76	390.36	20.62	2.95	345.30	51.65
59	190.19	390.18	22.11	0.67	362.08	51.96
1960	195.10	393.60	21.82	0.99	382.95	52.37
61	192.09	397.10	23.65	3.35	393.45	52.81
62	187.05	397.82	24.23	4.06	391.59	53.31
63	189.80	401.14	27.50	2.02	405.63	53.64
64	195.23	410.16	28.03	3.15	419.51	54.01
1965	195.22	422.05	31.41	4.64	425.59	54.36
66	195.13	437.08	32.04	3.90	431.15	54.65
67	196.44	439.09	34.17	2.46	435.12	54.98
68	198.54	462.76	36.76	4.58	439.55	55.28

Exogenous Variables : Demand Equations (Real Values) cont'd....

Year	r_t^{BS}	RPI_t	T_t	U_t	ΔU_t
1946	-1.85	51.0	45	2.5	1.3
47	-3.57	54.0	45	3.1	0.6
48	-3.24	57.0	45	1.8	-1.3
49	-1.30	59.0	45	1.6	-0.2
1950	-1.11	61.0	45	1.5	-0.1
51	-7.16	67.0	47.5	1.2	-0.3
52	-6.20	73.0	47.5	2.1	0.9
53	-0.25	75.0	45	1.8	-0.3
54	1.13	76.0	45	1.5	-0.3
1955	-2.52	80.0	42.5	1.2	-0.3
56	-1.56	83.8	42.5	1.3	0.1
57	-0.18	86.9	42.5	1.6	0.3
58	0.53	89.5	42.5	2.2	0.6
59	2.76	90.1	38.75	2.3	0.1
1960	2.38	91.0	38.75	1.7	-0.6
61	0.19	94.1	38.75	1.6	-0.1
62	-0.36	98.0	38.75	2.1	0.5
63	1.54	100.0	38.75	2.6	0.5
64	0.35	103.2	38.75	1.7	-0.9
1965	-0.86	108.1	41.25	1.5	-0.2
66	0.11	112.4	41.25	1.6	0.1
67	1.74	115.2	41.25	2.5	0.9
68	-0.21	120.6	41.25	2.5	0

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CHAPTER EIGHT : MODELS OF THE PURCHASE OF UK NON-GROUP LIFE INSURANCE : AN ANALYSIS AND RESULTS.

8.1 Introduction

The objective of this Chapter is to test empirically the various models and hypotheses described in earlier Chapters. The main hypothesis is that inflation (represented mainly in the form of price expectations) has affected the purchase of new ordinary life insurance within the UK over the period 1946 to 1968. The influence of the other variables that affect the purchase of life insurance is also measured. Two separate models of the purchase of life insurance are of interest: the Demand and the Financial Saving Ratio (FSR) Models. These will be examined in the subsequent Sections. A study of the demand model, in particular, enables further insight into the differences between the demand for and the supply of savings-based and protection-based life insurance.

In Section 8.7 we will be concerned with an attempt to forecast the purchase of life insurance for the years 1969/1975 inclusive. However, the success of this forecast is marred by several difficulties:

a) the major discontinuity in the premium income data at 1969 means that post and pre-1969 data are not truely comparable (for example, annuity data is included in new business figures after 1969 but excluded before);

b) in some cases, data that is available before 1969 is not yet available for the later years;

c) the most obvious problem is that the period 1946/1968 was one of comparatively mild inflation in relation to the early 1970's. Consequently, as Carlson and Parkin (4) report, the price expectations variable generated by the 1946/1968 data may not be suitable (ie. not properly defined) for use on the later period.

As with all studies using time-series data, this Chapter is affected by the statistical problems generated when highly multicollinear data is used; these are apparent in two main cases: multicollinearity and misinterpretation of lag-structure.

The term multicollinearity is used to denote the presence of linear or near linear relationships among the explanatory variables. This condition is particularly prevalent in economic studies because of the general interdependence of economic phenomenon: thus, for example, income and positive wealth variables are often closely related as are income and negative wealth (ie. debt) variables (eg. building society advances).

The close collinearity of the more important explanatory variables is obvious in an examination of the zero order correlation matrix:

Table 8.1.1 Correlation Matrix

	INC^P	BSW	BSA	IF^{Sg}	RPI
INC^P	1.0	0.972	0.913	0.984	0.963
BSW		1.0	0.954	0.993	0.938
BSA			1.0	0.940	0.814
IF^{Sg}				1.0	0.945
RPI					1.0

A key to the abbreviations can be found in Appendix 7.1

The practical consequences of multicollinearity are uncertain but theoretically, a perfect linear relationship among explanatory variables produces estimates of the coefficients which are indeterminate and the standard errors of these estimates become infinitely large. This means that we run the danger of ignoring possible significant factors because their standard errors have been affected. In many of the FSR models, multicolinearity manifests itself by causing certain instability - more often than not in the appropriate value of Γ in the inflation expectations variable.

In an attempt to reduce the effects of multicollinearity in the FSR model, several different alternatives were tried; initially, first differences were used in the place of current values but this proved unsatisfactory because the first differencing largely removed the explanatory power of the model. Secondly, the simple solution of omitting highly correlated explanatory variables was tried. This produced reasonable results but has the obvious drawback of introducing specification error into the model.

The problem of possible misinterpretation of the lag-structure evolved from the use of the expected inflation variable (see Equation (7.5.7)). Equation (7.5.11) demonstrates that this variable involves the inclusion of the lagged dependent variable on the right-hand-side of the equation. Furthermore, the coefficient of this lagged variable $(1-r)$ should be between zero and unity. However, when this formulation was attempted in the financial saving ratio model, the coefficient $(1-r)$ turned out to be consistently negative (at approximately - 0.6) ; and a value of $r = 1.6$ implies that inflation expectations would diverge to infinity. The reasons for this behaviour were thought to lie in the stock adjustment model described in Section 7.7: in some way, the lagged dependent variable describes a stock-adjustment rather than an inflation-expectations model. The negative coefficient is then easily explained since the previous purchase of life insurance discourages current purchase. Various methods of eliminating the 'lag-structure' problem were attempted in the FSR model but the only successful solution was to include the inflation expectations variable 'manually' (and disperse with the lag-structure of Equation (7.5.11)). Luckily, this problem did not occur in any of the demand models.

Finally, it should be noted that the explanatory variables are to a large extent derived from aggregate data and are therefore only surrogates for the underlying variables which relate to the potential purchasers of ordinary life insurance.

8.2 The Financial Saving Model

The objective of the financial saving model is to provide a comparison with recent models explaining the behaviour of the aggregate saving ratio (eg. see Bank of England (1)). However, these models differ from the ones presented in some important respects:

a) quarterly rather than annual observations are often used thus providing a greater number of degrees of freedom: this enables the use of more sophisticated statistical techniques;

b) the financial saving ratio (FSR) is only a very small part of the aggregate saving ratio (averaging 0.2% as opposed to 8.1% (1946/68)). Furthermore, since the FSR represents new saving via ordinary life insurance (excluding annuities), it exhibits less fluctuation than the corresponding aggregate saving ratio. Consequently, considerably less success was achieved by the conventional explanatory variables in explaining the variation of the FSR;

c) one can only assume that other time-series studies were not adversely affected by multicollinearity problems: unfortunately, these problems limited the scope of this Section.

The models of this Section are of two main types, firstly, those based on Equation (7.2.3) (linear type) and secondly, a log-linear type similar to that used by Branson and Klevorick (3) to test the existence of money illusion. The difference between the models is that the former type includes an inflation expectations variable $(\dot{P}^e_t)$, based on Equation (7.5.7), whereas the latter uses the retail price index instead.

Linear Models

Different formulations of the linear model were used in order to minimise the disturbing effects of multicollinearity and misinterpretation of the lag-structure. These models can be summarised under the following headings:

Adaptive Expectations Type

$$FSR_t = a\Gamma + (1-\Gamma)FSR_{t-1} + b.\Gamma\dot{P}_{t-1} + \Sigma_t - (1-\Gamma).\Sigma_{t-1} + u_t - (1-\Gamma).u_{t-1} \quad (8.2.1)$$

where Σ_t denotes the other explanatory variables and u_t is the random error term.

Adaptive Expectations (First Difference) Type

$$\Delta FSR_t = (1-\Gamma)\Delta FSR_{t-1} + b\Gamma\Delta\dot{P}_{t-1} + \Delta\Sigma_t - (1-\Gamma)\Delta\Sigma_{t-1} + \Delta u_t - (1-\Gamma)\Delta u_{t-1} \quad (8.2.2)$$

where Δ denotes the first difference operator eg. $\Delta FSR_t = FSR_t - FSR_{t-1}$.

Adaptive Expectations (Stock Adjustment) Type

$$FSR_t = ar + (1-r)FSR_{t-1} + br\dot{P}_{t-1} + cIF^*_t$$
$$+ c(1-r)IF^*_{t-1} + \Sigma_t - (1-r)\Sigma_{t-1}$$
$$+ u_t - (1-r)u_{t-1} \quad (8.2.3)$$

where IF^*_t denotes the optimal stock of savings-based life insurance.

'Manual' Expectations Type

$$FSR_t = a + b\dot{P}^e_t + \Sigma_t + u_t \quad (8.2.4)$$

where $\dot{P}^e_t$ denotes the long-term expected rate of inflation.

Table 8.2.1 illustrates selected regressions from the Adaptive Expectations Type model using ordinary least squares. I have already commented on the outstanding features of this type of model, viz. the negative coefficient of FSR_{t-1}.

The results of Table 8.2.1 are, in the main, disappointing although not entirely unexpected given the disturbing presence of multicollinearity. There are, however, a few comments that can be salvaged:

a) although all the standard errors are large in relation to the corresponding coefficients, in most cases these coefficients have the expected sign. Thus Permanent Income (INC^P_t) has a positive coefficient and the 'stock'

Table 8.2.1 FSR : Adaptive - Expectations Type Results

Dependent Variable FSR_t
Sample Period 1946/1968
Ordinary Least Squares

Model No.

	I	II	III	IV	V	VI
CONST	0.693 (0.261)	-0.793 (0.428)	-0.970 (0.463)		-0.900 (1.053)	-0.928 (0.969)
INC_t^P		0.004 (0.008)	-0.026 (0.115)	0.007 (0.009)	0.002 (0.011)	0.005 (0.009)
INC_{t-1}^P		0.004 (0.008)	0.035 (0.115)	-0.002 (0.009)	0.007 (0.010)	0.004 (0.009)
INC_t^T			0.020 (0.117)			
INC_{t-1}^T			0.006 (0.005)			
BSW_t						0.010 (0.020)
BSW_{t-1}						-0.014 (0.022)
BSA_t				0.012 (0.022)		
BSA_{t-1}				0.033 (0.031)		
IF_t^{Sg}					-0.088 (0.009)	
IF_{t-1}^{Sg}					0.009 (0.013)	
FSR_{t-1}	0.693 (0.137)	-0.221 (0.260)	-0.296 (0.275)	-0.434 (0.285)	-0.282 (0.297)	-0.156 (0.290)
$\dot{P}_{t-1}$	-0.040 (0.023)	-0.003 (0.023)	-0.005 (0.024)	0.004 (0.023)	-0.010 (0.026)	-0.005 (0.025)
F	16.4	17.4	11.7	13.8	11.1	10.9
R^2	0.62	0.80	0.81	0.83	0.81	0.80
$\bar{R}^2$	0.58	0.75	0.75	0.77	0.73	0.73
DW	2.39	1.95	2.02	1.86	1.92	1.99

Standard Errors in ( )
DW - Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

of savings-based life insurance (IF_t^{Sg}) has a negative effect;

b) in the only case where all coefficients are significant (Model I), FSR_{t-1} has a positive coefficient (indicating a value of $\Gamma = 0.3$) and long-run inflation expectations have a negative effect on FSR_t;

c) the multicollinear situation is exacerbated by the presence of lagged explanatory variables since obviously, these will be highly correlated with their present values;

d) it is especially interesting to note that the coefficient of FSR_{t-1} is negative even in Model V - which includes IF_t^{Sg} in order to pick up any stock adjustment effects;

e) many of the explanatory variables suggested in Section 7.7 have been excluded because of the multicollinearity that their inclusion would entail. This is particularly so of the variables that are closely correlated with INC_t^P (eg. NIF (receipts by the National Insurance Fund), IF_t^{Pr}, LIQ_t and HPC_t etc.).

Table 8.2.2 illustrates the results of the second main model attempted - the <u>Adaptive Expectations (First Difference) Type</u>. A comparison of Equations (8.2.1) and (8.2.2) shows that taking first-differences does not spoil the interpretation of the lag coefficient Γ. The object of the first difference model was the cure of the multicollinearity problem: unfortunately, as can be seen, first differencing removes virtually all the explanatory power of the right-hand-side variables.

Again, the results of Table 8.2.2 are disappointing in that the value of $\bar{R}^2$ is consistently poor. Furthermore the only significant variable, FSR_{t-1}, had an incorrect sign, giving a value of r between 1.4 and 1.6.

Equation (8.2.3) gives the formulation of the third main linear model attempted - the Adaptive Expectations (Stock Adjustment) Type. A comparison of Equations (8.2.1) and (8.2.3) shows that the only difference is the inclusion of the term $c(1-r)IF^*_t$ in the latter Equation. IF^*_t denotes the 'optimum stock' of savings-based life insurance and is defined by:

$$IF_t - IF_{t-1} = g.(IF^*_t - IF_{t-1}) \qquad (7.7.1)$$

where $0 < g \leq 1$.

In Section 7.7 it was explained that the only sensible way to allow for adjustment to the 'optimum' stock of life insurance is to include IF^*_t 'manually'. IF^*_t is then obtained from Equation (7.7.1):

$$IF^*_t = \frac{1}{g} IF_t + \frac{g-1}{g} IF_{t-1} \qquad (7.7.3)$$

and the model run with different values of g.

The rationale for the inclusion of IF^*_t is that a stock adjustment model should sort out the problems caused by possible misinterpretation of the lag-structure in the models of Tables 8.2.1 and 8.2.2 where the

Table 8.2.2 FSR : Adaptive Expectations (First Difference) Type Results

Dependent Variable ΔFSR_t
Sample Period 1946/1968
Ordinary Least Squares

	MODEL NO.				
	VII	VIII	IX	X	XI
CONST	0.062 (0.050)	-0.020 (0.078)	-0.019 (0.082)	-0.001 (0.090)	-0.019 (0.094)
ΔINC^P_t		0.004 (0.014)	0.006 (0.109)	0.0004 (0.017)	0.004 (0.016)
ΔINC^P_{t-1}		0.007 (0.007)	0.006 (0.107)	0.008 (0.008)	0.007 (0.008)
ΔINC^T_t			-0.006 (0.106)		
ΔINC^T_{t-1}			0.004 (0.007)		
ΔBSW_t					0.009 (0.021)
ΔBSW_{t-1}					-0.012 (0.021)
ΔBSA_t					
ΔBSA_{t-1}					
ΔIF^{Sg}_t				-0.009 (0.011)	
ΔIF^{Sg}_{t-1}				0.012 (0.012)	
ΔFSR_{t-1}	-0.433 (0.229)	-0.509 (0.225)	-0.531 (0.246)	-0.614 (0.255)	-0.465 (0.249)
$\Delta \dot{P}_{t-1}$	-0.010 (0.021)	-0.017 (0.024)	-0.016 (0.026)	-0.026 (0.027)	-0.020 (0.026)
F	2.3	2.1	1.3	1.5	1.3
R^2	0.19	0.32	0.33	0.36	0.33
$\bar{R}^2$	0.10	0.17	0.08	0.12	0.08
DW	2.27	2.55	2.52	2.49	2.60

Standard Errors in ( )
DW - Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

coefficients of FSR_{t-1} and ΔFSR_{t-1} are significantly negative rather than positive.

Unfortunately, the model of Equation (8.2.3) proved unsuccessful because no value of g between zero and unity induced a positive coefficient for FSR_{t-1}. The results have not been tabulated.

One further possible explanation for the behaviour of FSR_{t-1} is that, theoretically, ordinary least squares is an inappropriate technique for estimating equations with an error term of the form $u_t - \rho . u_{t-1}$. This is because (i) the error terms are autocorrelated, (ii) the error term is not independent of FSR_{t-1}; thus OLS yields biased estimates in small samples like ours, (iii) the estimates are inconsistent (ie. biased even with very large samples) and (iv) the Durbin Watson statistic will not necessarily detect the inherent autocorrelation (for proofs see Koutsoyiannis (11)). J. Johnson (9) shows that, in the absence of autocorrelation the bias referred to above is negative (ie. the expected value of the estimate is less than the actual value of the coefficient) while if the residuals are autocorrelated then the bias may be positive.

Although econometric techniques exist for the estimation of distributed-lag models (for example, see Houthakker and Taylor (8) ('Three Pass Least Squares') and Koutsoyiannis (11) p.311) the simplest solution is to adopt Equation (8.2.4) instead - thus removing the troublesome FSR_{t-1} from the right-hand-side of the equation. Equation (8.2.4) is termed the 'Manual Expectations Type' because $\dot{P}_t^e$ is included 'manually' by computing it from Equation (7.5.7) for selected values of Γ. The 'best' value of Γ is then the one which yields the best statistical results (in terms of the highest values for F and $\bar{R}^2$). Of course, in order to calculate $\dot{P}^e_t$ in this way a suitable starting value must be chosen and in fact, a value of $\dot{P}^e_{1934}$ = 0.0 was used. This seemed appropriate since 1934 had been preceded by two years of zero inflation and several years of slump before that. In any case, for $\Gamma \neq 0.0$, the effects of a minor error in the choice of starting value should have disappeared by 1946.

One further option is then available once the application of Equation (8.2.4) has yielded the best value of Γ, say Γ^*. Equation (8.2.1) can be re-examined with the known value, Γ^*, substituted for the unknown Γ ie.,

$$FSR_t - (1-\Gamma^*)FSR_{t-1} = b\Gamma^*\dot{P}_{t-1} + \Sigma_t - (1-\Gamma^*)\Sigma_{t-1} + u_t - (1-\Gamma^*)u_{t-1}$$

which can be rewritten as,

$$\Delta^*FSR_t = b\Gamma^*\dot{P}_{t-1} + \Delta^*\Sigma_t + \Delta^*u_t \qquad (8.2.5)$$

where $\Delta^*FSR_t = FSR_t - (1-\Gamma^*)FSR_{t-1}$.

Equation (8.2.5) can then be estimated by Ordinary Least Squares without any further difficulty (see Koutsoyiannis (11) p.311).

Table 8.2.3 shows the results of the <u>'Manual' Expectations Type</u> of Equation (8.2.4). In each case, the model was attempted with a variety of different values for r and the one appearing in the Table is the 'best' in terms of the highest F -score, R^2 and $\bar{R}^2$ figures.

Table 8.2.3 FSR : 'Manual' Expectations Type Results

Dependent Variable FSR_t

Sample Period 1946/1968

Ordinary Least Squares

MODEL NO.

	XII	XIII	XIV	XV
CONST	-1.015 (0.322)	0.682 (0.742)	-0.429 (1.649)	-0.826 (0.755)
$\dot{P}^e_t$	-1.169 (0.635)	-0.115 (0.081)	-0.126 (0.085)	-0.225 (0.127)
INC^P_t	0.009 (0.002)	0.003 (0.002)	0.005 (0.003)	0.009 (0.003)
BSW_t				-0.002 (0.006)
BSA_t		0.020 (0.016)	0.010 (0.020)	
T_t			0.016 (0.022)	
r	0.005	0.145	0.135	0.035
F	45.3	29.5	21.6	28.8
R^2	0.82	0.82	0.83	0.82
$\bar{R}^2$	0.80	0.80	0.79	0.79
DW	2.58	2.52	2.55	2.58

continued .....

Table 8.2.3 FSR : 'Manual' Expectations Type Results continued

	MODEL NO.			
	XVI	XVII	XVIII	XIX
CONST	-1.060 (0.801)	-0.761 (1.475)	-0.500 (1.176)	-0.995 (2.089)
$\dot{P}^e_t$	-0.224 (0.127)	-0.041 (0.104)	-0.055 (0.060)	-0.067 (0.062)
INC^P_t	0.010 (0.004)	0.008 (0.004)	0.009 (0.005)	0.002 (0.004)
BSW_t		-0.011 (0.010)		
BSA_t		0.043 (0.026)	0.042 (0.019)	0.041 (0.030)
IF^{Sg}_t	-0.003 (0.005)		-0.009 (0.006)	
T_t				0.058 (0.031)
B_t				0.002 (0.001)
M_t				-0.006 (0.003)
U_t				-0.004 (0.135)
ΔU_t				-0.040 (0.086)
r	0.035	0.145	0.250	0.350
F	29.3	22.8	23.4	12.3
R^2	0.82	0.83	0.84	0.88
$\bar{R}^2$	0.77	0.80	0.80	0.80
DW	2.60	2.70	2.66	2.78

Standard Errors in ( )
DW - Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

The inclusion of the inflation expectations term 'manually' side-steps the problems of autocorrelation and inconsistency that are present when ordinary least squares is used on an Adaptive-Expectations Type model. However, the problem of multicollinearity is still present and consequently the models of Table 8.2.3 have been specifically formulated so that they do not include a large number of collinear variables. Other models with greater numbers of explanatory variables were attempted but proved to be very badly affected. The models of Table 8.2.3 show up the following points:

a) the Inflation Expectations variable $\dot{P}^e_t$ turns out to be largely significant (at just below the 95% level in most cases) and consistently negative in its effect on new saving through life insurance;

b) Permanent Income (INC^P) also turns out to be largely significant with a positive effect;

c) Building Society Advances (BSA) again appear to be positively associated with FSR;

d) the Stock of savings-related life insurance in force (IF^{Sg}) again appears to discourage new saving;

e) although neither of the coefficients of BSW is significant both indicate that Building Society Wealth has a negative effect (ie. as a substitute). However Models VI and XI indicate a positive effect;

f) the effects of the other possible explanatory variables are much in line with those that were expected: the standard rate of Income Tax (T) has a positive effect on FSR as does Births (B) while the coefficients of Marriages (M), Unemployment (U) and the Change in Unemployment (ΔU) are negative;

g) again, because of multicollinearity problems, certain variables (eg. LIQ_t, HPC_t, NIF_t) were omitted;

h) the 'best' value of r is of interest: when BSA is included as an explanatory variable r is of the order of 0.14; when BSA is excluded, r is much smaller (around 0.03). Now, since a smaller value of r could be interpreted to mean that the consumer takes a more long-term view of inflation, it looks as if a shorter-term view is taken when savings-based life insurance is purchased in conjunction with a building society advance for house purchase purposes. In fact, this result is not surprising because, as Revell (17) reports,

> "very few mortgages run to their full term: the average 'life' of a building society mortgage is less than ten years". (p.381)

Several of the models including BSA as an explanatory variable have been reworked using Equation (8.2.5) (by transforming the variables with Δ^*) with a value of $r^* = 0.14$: the results are illustrated in Table 8.2.4 below.

Table 8.2.4 FSR : Adaptive Expectations Type ($r^* = 0.14$)

Results — Dependent Variable Δ^*FSR_t
Sample Period 1946/1968
Ordinary Least Squares

MODEL NO.

	XX	XXI	XXII	XXIII	XXIV
CONST	0.439 (0.519)	0.351 (0.533)	0.329 (0.601)	0.247 (0.568)	0.240 (0.603)
$\dot{P}_{t-1}$	-0.035 (0.030)	-0.038 (0.031)	-0.032 (0.032)	-0.035 (0.030)	-0.045 (0.034)
Δ^*INC^P	-0.002 (0.009)	0.002 (0.010)	0.0005 (0.011)	-0.002 (0.009)	-0.002 (0.012)
Δ^*BSW			-0.005 (0.013)		
Δ^*BSA	0.034 (0.033)	0.043 (0.035)	0.038 (0.035)	0.043 (0.035)	0.051 (0.042)
Δ^*IF^{Sg}		-0.007 (0.008)			
Δ^*T				0.031 (0.036)	0.072 (0.043)
Δ^*B					0.002 (0.002)
Δ^*M					-0.008 (0.007)
Δ^*U					0.061 (0.210)
$\Delta^*\Delta U$					-0.062 (0.128)
F	1.2	1.0	0.9	1.1	0.94
R^2	0.16	0.19	0.16	0.19	0.35
$\bar{R}^2$	0.02	0.009	-0.022	-0.010	-0.023
DW	2.71	2.86	2.79	2.64	2.79

Standard Errors in ( )
DW - Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

Table 8.2.4 confirms the signs of most of the important explanatory variables although it will be noted that the semi-differencing has destroyed the explanatory power of the models. The only conclusive results from Table 8.2.4 are that the coefficient of $\dot{P}_{t-1}$ is consistently negative while that for BSA_t is positive: both these variables show some degree of significance.

Log-Linear Models

In addition to the linear models of Equations (8.2.1) - (8.2.4), a log-linear model along the lines suggested by Branson and Klevorick (3) was used:

$$\log FSR_t = a + b \log RPI_t + \Sigma c_i \log X_{it} + V_t \tag{8.2.6}$$

If the coefficient of $\log RPI_t$ is then insignificantly different from zero we can conclude that FSR_t only depends on the real value of INC^P, BSW, BSA, etc. On the other hand, if $b \neq 0$ then money illusion exists in the Patinkin sense. (see D. Patinkin (15)).

Unfortunately, the taking of (natural) logarithms does not spoil the collinearity of the explanatory variables so that, once again, strongly multicollinear models have been avoided. The results of the various log-linear models are given in Table 8.2.5.

Table 8.2.5 tells much the same story as the other models of the financial saving ratio but has, on the

Table 8.2.5 FSR : Log-Linear models

Dependent Variable $\log_e FSR_t$
Sample Period 1946/1968
Ordinary Least Squares

	MODEL NO.				
	XXV	XXVI	XXVII	XXVIII	XXIX
CONST	-9.515 (1.445)	-8.334 (3.243)	-11.040 (4.247)	-10.288 (3.790)	-12.330 (4.409)
$\log RPI_t$	-0.349 (0.190)	-0.296 (0.234)	-0.292 (0.244)	-0.330 (0.214)	-0.275 (0.489)
$\log INC_t^P$	1.972 (0.371)	1.704 (0.759)	2.285 (0.900)	2.187 (1.042)	2.157 (1.179)
$\log BSA_t$		0.065 (0.159)			
$\log BSW_t$			-0.139 (0.364)		
$\log IF_t^{Sg}$				-0.118 (0.532)	
$\log r_t^{BS}$					-0.062 (0.082)
$\log B_t$					0.491 (0.704)
$\log M_t$					-0.938 (0.763)
$\log T_t$					1.030 (0.784)
$\log U_t$					0.084 (0.124)
*$\log U_t$					-0.088 (0.072)
F	44.1	28.3	28.2	28.0	11.1
R^2	0.82	0.82	0.82	0.82	0.86
$\bar{R}^2$	0.80	0.79	0.79	0.79	0.79
DW	2.61	2.64	2.61	2.62	2.78

*Actually $\log(\Delta U_t + 2.0)$
Standard Errors in ( )
DW - Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

other hand, some additional facets: first, although the log-linear method has certain theoretical drawbacks (eg. see A. Deaton (5) Ch.2 and H.A.J. Green (7)), it has the advantage that the various elasticity coefficients can be observed directly (c_i in Equation (8.2.6)). Thus Models XXV to XXIX show that the (permanent) income elasticity settles around a value of 2.0 (and this variable is also consistently significant).

Secondly, the position regarding the existence of money illusion seems inconclusive: in Models XXV and XXVIII, the coefficients of log RPI seem almost significantly negative (at around the 90% level). However, in Models XXVI, XXVII and XXIX, the coefficients are not significant (although still negative). The corresponding elasticity coefficient averages out around -0.3 and thus a 1% increase in the Retail Price Index produces a 0.3% fall in FSR.

Finally, the remaining explanatory variables show a disappointing lack of significance although, as has been pointed out before, this insignificance is, in part, a consequence of the multicollinearity in the model. However, the signs of the coefficients are in agreement with those illustrated in the previous tables.

In order to summarise the models of Tables 8.2.1 to 8.2.5, the following points can be noted: first, both the retail price index (Table 8.2.5) and inflation expectations (Tables 8.2.1 - 8.2.4) have a negative effect on the new long-term financial saving ratio.

Furthermore, despite all the multicollinearity problems log RPI, $\dot{P}^e$ and $\dot{P}_{t-1}$ are often significant (ie. with small standard errors). It is, therefore, possible to conclude that the existence of inflation definitely discourages new long-term saving via life insurance, even though the inflationary experience of 1946/1968 was comparatively mild.

Secondly, the following variables have a negative effect on FSR_t:

- Building Society Wealth (BSW_t) ie. shares and deposits in building societies: thus, as expected, building societies act as a substitute alternative for personal saving. However, since no model shows a significantly negative effect, we might conclude, albeit tentatively, that BSW_t is not a strong substitute for new long-term savings;

- Savings-based Sums Insured In Force (IF_t^{Sg}): thus existing 'stocks' of life insurance discourage the purchase of further savings-based life insurance;

- holdings of Liquid Assets (LIQ_t): although the evidence is slight (and the results have not been reported), it does indicate that LIQ_t discourages FSR_t. However, it is not possible to determine whether this is because of a 'wealth effect' or the 'transactions effect' described by Morgan Grenfell (14);

- Marriages in year t (M_t): this study is not unique in the revelation of a negative coefficient for Marriages (eg. see Neumann (15) and Mantis & Farmer (13)). There seem to be two possible explanations for this result : first of all that a simple budget constraint prevents a newly-established household from purchasing savings-based life insurance in the first year of marriage. Secondly, it may be that short-term saving rather than its long-term counterpart is more important for newly-marrieds.

The third main conclusion relates to those variables which have a positive effect on FSR_t:

- the most important of these variables is Permanent Income (INC_t^P) which has a consistently positive and often significant effect. Furthermore, the elasticity figures of Table 8.2.5 show that FSR_t is fairly responsive to changes in INC_t^P;
- Building Society Advances (BSA_t); although Revell (17) notes that only 15% of building society mortgage advances are associated with endowment policies, BSA_t seems to have a reasonably important effect on new long-term saving via life insurance. In view of this result, it is anticipated that BSA_t should be an important determinant of the purchase of protection-based life insurance. Finally, we note again that when the value of r was determined (Table 8.2.3), a much larger figure was obtained when BSA_t was included in the model;

- the Standard Rate of Income Tax (T_t): the higher the rate of income tax, the greater is the tax relief on 'qualifying' policies: this effectively reduces the net of tax 'price' of life insurance (or alternatively improves the net yield obtained on savings);
- Births (B_t): not surprisingly the birth of a child encourages the purchase of savings-based life insurance.

The figures for the Durbin Watson Statistic in the extreme right-hand column of the above Tables consistently reject the hypothesis of positive autocorrelation of residuals. However, in some Models, the DW figure is quite high so that the hypothesis of negative autocorrelation of residuals cannot be rejected. Additionally, a high value of the DW Statistic is also to be found in Tables 8.2.2 and 8.2.4 when models using first-differenced and semi-first-differenced variables were attempted. This seems to imply that any autocorrelation that is present does not necessarily follow a first-order autoregressive scheme.

The traditional solution to the problem of autocorrelation involves first differencing (and may be subsequent differencing) of the variables in the model. However, the results of Tables 8.2.2 and 8.2.4 make it abundantly clear that no real value is to be gained by first-differencing because, in this case, the cure is more harmful than the disease. The results must, therefore, be interpreted in the light of possible negative autocorrelation which may have the following effects:

a) the variance of the parameter estimates are under-estimated and thus, the significance of these estimates is over-estimated. However, the standard errors of most of the variables are already so high that it is difficult to believe that autocorrelation has had any harmful effects;

b) any predictions based on ordinary least squares will be inefficient.

8.3 The Independent and Dependent Demand Models

The objective of the demand model is to analyse and explain the demand for protection-based and savings-based life insurance: thus the demand of these two elements of life insurance are analysed separately (although not independently in general). A notable feature of all the demand models is that they are part of a system of equations which describe the market equilibrium (assuming that such an equilibrium occurs) and so, as Section 7.3 explained, we have to include the corresponding supply equations in order to guarantee that the demand model is properly identified. The method of two stage least squares must then be utilised.

The factors that affect the supply side of the model were described in Section7.4. Unfortunately, it was not possible to separate those factors affecting the supply of life insurance into the savings-based and protection-based elements. We would expect the variable representing the adjustment to the optimum gearing level to have a negative effect on savings-based life insurance but a positive effect on protection-based policies. This is because _if_ the optimum gearing level (G_t^*) represents a proportion of with profits in force _greater than_ the actual level in the previous year (G_{t-1}) then we would expect the adjustment variable ($G_{t-1} - G_t^*$), which is negative,to induce first an increase in the supply of with profits policies (ie. ($G_{t-1} - G_t^*$) having a negative coefficient) and second a decrease in the supply of non-profits policies (ie. ($G_{t-1} - G_t^*$) having a positive coefficient).

The large number of different demand models that are subsequently analysed can be separated into two main groups:

first those with the protection-based models independent of the savings-based ones and secondly those models which are not independent. We can say that the protection-based and savings-based models are independent if no one protection-based equation contains a savings-based endogenous variable and vice versa. The independent models then form sets of pairs of demand and supply equations which can be estimated in isolation. The dependent models, on the other hand, form sets of four equations with each equation determining the behaviour of the other three.

In all of the subsequent demand models, demand and supply are measured in terms of premium expenditure (as explained in Chapter Six). This, of course, is not the usual demand relationship which plots price (ceteris paribus) against quantity (there is, however, no clear concept of the quantity demanded of life insurance). The use of expenditure causes no problems though: it can be readily demonstrated that the expenditure/own price elasticity is just $(1 + \eta)$ (where η (≤ 0) is the quantity/own price elasticity) while the other elasticity coefficients have their usual interpretation (ie. are not altered by using premium expenditure as a surrogate for demand). Similarly, instead of quantity supplied, the following supply relationships are formulated in terms of expenditure supplied.

The Independent Models

The simplest independent model is the case where demand depends solely on Own Price (IP_t) and Permanent Income (INC_t^P) in both the savings-based and protection-based cases. The supply equations include the normal supply exogenous variables $(SP_{t-1}, C_t, (G_{t-1} - G_t^*)$ and $YLD_t)$ as well as the endogenous

Own Price. The results from this model are illustrated in Table 8.3.1.

Table 8.3.1 shows that the model is by no means complete: in particular, Equations A, C and D indicate significantly low values for the Durbin Watson statistic (ie. positive autocorrelation of residuals) which could be caused by the omission of important explanatory variables from the model.

A more complex version of the independent model is described in Table 8.3.2 which is basically the same as that of Table 8.3.1 with the addition of the exogenous variables IF_t^{Sg}, BSA_t in the demand for savings-based equation and IF_t^{Pr}, BSA_t, in the demand for protection-based equation. The structure of the supply equation remains unaltered.

The improving qualities of Table 8.3.2 are mainly reflected in the supply equations which (in comparison with those of Table 8.3.1) indicate a better degree of explanation and better Durbin Watson statistics. The addition of Building Society Advances (BSA_t) to the demand for savings-based equation (A) seems to be beneficial as it has a significantly positive effect. It is interesting to note that the same variable is much less significant in the protection-based case (B).

The Dependent Models

Once we move from the use of the independent to the dependent models, the problems of identification become more acute because there are now <u>four</u> endogenous variables in the model (requiring all four equations) rather than two before (with two pairs of equations). Specifically, the 'order condition' of identification (see Koutsoyiannis (11) p.342)

Table 8.3.1 Independent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Pr})
CONST	-1777.815 (201.582)	-157.859 (99.602)
IP_t^{Sg}	1.315 (0.533)	-
IP_t^{Pr}	-	-0.115 (0.106)
INC_t^P	6.901 (0.413)	1.040 (0.231)
R^2	0.96	0.91
$\bar{R}^2$	0.96	0.90
F	267.5	98.6
DW	1.067	1.671

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium condition:

$$D_t^{Sg} = S_t^{Sg}$$

$$D_t^{Pr} = S_t^{Pr}$$

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.3.1 Independent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	1965.257 (867.883)	170.123 (80.431)
IP_t^{Sg}	-6.966 (3.231)	-
IP_t^{Pr}	-	-0.394 (0.125)
SP_{t-1}	-14.961 (7.089)	-1.278 (0.637)
C_t	-48.949 (69.821)	1.434 (8.156)
$(G_{t-1} - G_t^*)$	-218.580 (122.795)	-0.129 (11.234)
YLD_t	62.545 (70.058)	21.794 (9.025)
R^2	0.60	0.88
$\bar{R}^2$	0.48	0.84
F	5.06	24.61
DW	1.090	1.283

Sample Period 1946-1968
Two Stage Least Squares
Equilibrium condition : $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard errors in ( )

DW -Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.3.2 Independent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Sg})
CONST	-1442.102 (300.266)	-1.391 (155.181)
IP_t^{Sg}	0.633 (0.559)	-
IP_t^{Pr}	-	-0.161 (0.109)
INC_t^{P}	6.375 (1.390)	0.591 (0.467)
BSA_t	20.058 (9.170)	-1.572 (3.995)
IF_t^{Pr}	-	0.433 (0.379)
IF_t^{Sg}	-2.264 (2.347)	-
R^2	0.97	0.91
$\bar{R}^2$	0.97	0.89
F	166.2	46.1
DW	1.389	1.476

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition : $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson Statistic

A key to abbreviations can be found in Appendix 7.1.

Table 8.3.2 Independent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	874.313 (411.491)	129.356 (63.101)
IP_t^{Sg}	-2.080 (1.298)	-
IP_t^{Pr}	-	-0.306 (0.084)
SP_{t-1}	-6.524 (3.469)	-1.008 (0.519)
C_t	-72.438 (41.689)	0.079 (7.123)
$(G_{t-1} - G_t^*)$	-90.695 (63.722)	2.933 (9.617)
YLD_t	134.357 (36.560)	26.570 (6.994)
R^2	0.85	0.91
$\bar{R}^2$	0.81	0.88
F	19.7	32.5
DW	1.478	1.414

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson Statistic

A key to abbreviations can be found in Appendix 7.1.

means that we now need at least two more exogenous variables in the demand equations.

The solution to the problem of identification creates its own additional headaches: the model now requires a minimum of seven exogenous variables (four in the supply equations and three for demand). Thus although the structural equations may only have a moderate number of variables, the corresponding exogenous variables in the reduced form of the model quickly become much more numerous. Since, of course, the method of two stage least squares uses the reduced form to calculate initial estimates of the included endogenous variables this means that the model can quickly become 'strained' with too few degrees of freedom. With a sample of only twenty three observations this means that a very strict control must be kept on the number of exogenous variables. This problem is particularly noticeable when distributed-lag type structures are used for inflation expectations (because these structures can effectively double the number of exogenous variables in the model).

Table 8.3.3 illustrates virtually the simplest dependent model: this has more exogenous variables than is strictly necessary but these are important because of the difficulty involved in computing the 'second stage' equations accurately, when there are two included endogenous variables.

The supply equations of Table 8.3.3 are simply those of Tables 8.3.1 and 8.3.2 with the addition of the price of the supply alternative. The demand for savings-based equation is similar to Equation 8.3.2 A with the addition of IP_t^{Pr} . However the demand for protection-based life insurance includes both IP_t^{Sg} and Births (B_t) as additional variables.

Table 8.3.3 Dependent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Pr})
CONST	-1848.626 (951.885)	21.212 (391.003)
IP_t^{Sg}	-0.578 (3.794)	-0.175 (1.703)
IP_t^{Pr}	0.864 (2.236)	-0.147 (0.880)
INC_t^P	7.450 (3.222)	0.127 (1.434)
BSA_t	12.863 (14.292)	-2.640 (4.878)
B_t	-	0.174 (0.238)
IF_t^{Pr}	-	0.686 (0.438)
IF_t^{Sg}	-1.468 (2.348)	-
R^2	0.97	0.92
$\bar{R}^2$	0.97	0.89
F	129.8	30.8
DW	1.521	1.604

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition : $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.3.3 Dependent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	259.963 (465.415)	-2.506 (75.369)
IP_t^{Sg}	13.057 (5.949)	2.486 (0.963)
IP_t^{Pr}	-5.989 (2.457)	-1.222 (0.398)
SP_{t-1}	1.246 (4.482)	0.474 (0.726)
C_t	-53.304 (48.318)	2.191 (7.824)
$(G_{t-1} - G_t^*)$	97.763 (96.635)	36.207 (15.649)
YLD_t	32.864 (62.108)	13.533 (10.058)
R^2	0.82	0.90
$\bar{R}^2$	0.75	0.86
F	12.2	22.7
DW	1.388	1.846

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

The inclusion of non-own-prices should clarify the effects of the own price for a number of reasons, since it could be argued that savings-based and protection-based life insurance are so closely connected that they are bound to influence each other. In practice the most common forms of ordinary non-group life insurance include both savings-based and protection-based elements and the market price actually paid is a 'composite' price reflecting both.[1] Thus the two elements are almost complementary to the extent that they are almost always purchased in conjunction with one another (in practice, only pure endowments exclude the savings-based element). On the other hand, the two elements are demand substitutes if only to the extent that they represent alternative ways of spending disposable income. Additionally a life insurance policy with a high savings content (eg. a with-profits endowment) is a substitute for an alternative with little or no savings content (eg. a temporary life insurance policy).

The same broad description can also be applied to the supply of savings-based and protection-based life insurance: that although the 'characteristics' of savings and protection are complementary, the actual products in which they are combined are substitutes.

We can conclude therefore that there are strong theoretical conclusions to indicate that the non-own-price should not be omitted for either the demand or supply equations. It is by no means certain, however, what the sign of the coefficients

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(1) It should be remembered that the price indices IP_t^{Sg} and IP_t^{Pr} do not exactly correspond to the saving and protection elements in ordinary life insurance (see Section 7.3).

of these various variables should be.

The next point of interest is the low values of the Durbin Watson statistic (particularly in Tables 8.3.1 and 8.3.2): this phenomenon could be caused by the omission of the non-own-prices . A comparison of the supply of protection-based equations shows that the DW figure is substantially increased when IP_t^{Sg} is included. However it is only fair to say that the low values of the Durbin Watson statistic have not been completely removed in Table 8.3.3 (the dependent model): the null hypothesis of autocorrelation cannot therefore be completely rejected.

Next, we note that in some of the equations, there is a tendency for some of the standard errors to be large. This, of course, is a symptom of multicollinearity and it seems to be more prevalent in the demand equations: the sources of this multicollinearity will be examined in Section 8.4.

Finally, the explanatory variables denoting the stocks of life insurance (IF_t^{Sg} and IF_t^{Pr}) have interesting effects. The coefficient of IF_t^{Sg} is consistently negative in the demand for savings-based equations (Tables 8.3.2 and 8.3.3) although not significant at the 5% level. Thus existing stocks of savings-based life insurance may discourage the purchase of further similar policies. However the coefficient of IF_t^{Pr} is consistently and more significantly positive so that existing stocks of protection-based life insurance encourage a further purchase. Several reasons can be forwarded for this latter result.

a) the Houthakker & Taylor analysis (8) speaks of habit forming consumption and while it is doubtful if purchases of protection-based insurance can be so described, it may be that once consumer resistence has been initially broken down (by the first purchase) consumers are more receptive to further purchases;

b) a more currently popular argument might be that consumers are updating their protection cover to allow for the eroding effects of inflation. If this is the case then any inflationary expectations variable introduced at a later date should show a positive coefficient as well;

c) the effects of frequent changes in mortgage arrangements mean that owner occupiers are continually purchasing new mortgage protection cover. This seems, intuitively, to be the most acceptable reason but the models of Tables 8.3.1, 8.3.2 and 8.3.3 indicate that BSA_t is not a significant explanatory factor (BSA_t should have a positive effect on the purchase of protection-based life insurance): there may, however, be econometric reasons which explain this apparent anomaly.

Tables 8.3.4, 8.3.5 and 8.3.6 illustrate some results from a number of other dependent models and introduce the variables BSA, B, BSW, M, T, U, Δ U, and LIQ into the demand equations. The supply equations, of course, remain unaltered.

The four dependent models examined so far show up some important results about the nature of the demand and supply relationships. The most outstanding feature of Tables 8.3.3 to 8.3.6 is the insignificance of the price variables in the demand equations: none of the demand equations (for either savings-based or protection-based life insurance) attribute any significance to the coefficients of IP_t^{Pr} and IP_t^{Sg}. On the other hand, the price variables in the supply equations

Table 8.3.4 Dependent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Pr})
CONST	-2458.751 (874.471)	579.856 (972.925)
IP_t^{Sg}	-4.229 (4.451)	1.685 (3.703)
IP_t^{Pr}	2.624 (2.237)	-1.436 (2.251)
INC_t^P	8.513 (3.391)	-0.850 (2.544)
BSA_t	15.339 (17.083)	7.015 (14.478)
BSW_t	-7.501 (6.743)	-6.731 (7.813)
B_t	0.202 (0.705)	0.121 (0.354)
IF_t^{Pr}	-	2.438 (2.135)
IF_t^{Sg}	3.010 (3.899)	-
R^2	0.97	0.84
$\bar{R}^2$	0.96	0.77
F	77.3	11.5
DW	1.946	1.712

Sample Period 1946-1968
Two Stage Least Squares
Equilibrium Condition $D_t^{Sg} = S_t^{Sg}$
$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )
DW Durbin Watson Staistic
A key to abbreviations can be found in Appendix 7.1.

Table 8.3.4 Dependent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	357.304 (46.817)	14.806 (66.687)
IP_t^{Sg}	9.921 (5.078)	1.928 (0.812)
IP_t^{Pr}	-4.684 (2.096)	-0.990 (0.335)
SP_{t-1}	-0.165 (3.970)	0.223 (0.635)
C_t	-58.261 (43.553)	1.310 (6.968)
$(G_{t-1}-G_t^*)$	60.973 (84.815)	29.664 (13.570)
YLD_t	57.392 (54.389)	17.895 (8.702)
R^2	0.85	0.91
$\bar{R}^2$	0.80	0.89
F	15.5	29.2
DW	1.214	1.609

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.3.5 Dependent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Pr})
CONST	-1298.619 (1170.467)	308.342 (1039.367)
IP_t^{Sg}	3.169 (7.030)	-2.140 (3.519)
IP_t^{Pr}	-1.518 (3.142)	1.090 (1.721)
INC_t^P	0.909 (5.175)	1.066 (3.433)
BSA_t	23.444 (16.266)	-7.438 (7.706)
M_t	-2.543 (2.972)	0.142 (1.509)
T_t	27.549 (19.698)	-11.605 (8.030)
U_t	-46.470 (61.075)	13.586 (34.835)
ΔU_t	-25.117 (40.677)	-5.238 (16.628)
B_t	1.174 (0.889)	-0.004 (0.465)
IF_t^{Pr}	-	1.095 (1.167)
IF_t^{Sg}	2.351 (3.226)	-
R^2	0.98	0.93
$\bar{R}^2$	0.96	0.86
F	61.1	15.0
DW	1.718	2.906

Sample Period 1946-1968
Two Stage Least Squares
Equilibrium Condition $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )
DW - Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1

Table 8.3.5 Dependent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	459.679 (386.587)	23.286 (65.404)
IP_t^{Sg}	7.979 (4.422)	1.940 (0.748)
IP_t^{Pr}	-3.966 (1.853)	-1.014 (0.314)
SP_{t-1}	-1.318 (3.625)	0.170 (0.613)
C_t	-59.934 (41.630)	1.624 (7.043)
$(G_{t-1}-G_t^*)$	35.023 (76.801)	29.144 (12.993)
YLD_t	67.656 (50.669)	16.760 (8.572)
R^2	0.87	0.91
$\bar{R}^2$	0.82	0.88
F	17.2	28.5
DW	1.109	1.608

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.3.6 Dependent Demand Model

	A. Demand for Savings-based (D_t^{Sg})	B. Demand for Protection-based (D_t^{Pr})
CONST	-2339.710 (673.719)	-394.163 (436.639)
IP_t^{Sg}	-3.343 (2.968)	-1.881 (2.145)
IP_t^{Pr}	-0.661 (1.394)	-0.999 (0.910)
INC_t^P	5.413 (2.087)	-0.760 (1.480)
BSA_t	10.960 (9.591)	-8.954 (7.423)
LIQ_t	7.099 (4.858)	4.704 (3.201)
B_t	0.548 (0.498)	0.379 (0.389)
IF_t^{Pr}	-	0.809 (0.667)
IF_t^{Sg}	-2.319 (2.119)	-
R^2	0.98	0.83
$\bar{R}^2$	0.98	0.74
F	129.9	10.1
DW	1.750	1.311

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson statistic

A key to the abbreviations can be found in Appendix 7.1

Table 8.3.6 Dependent Demand Model (Cont.)

	C. Supply of Savings-based (S_t^{Sg})	D. Supply of Protection-based (S_t^{Pr})
CONST	291.946 (417.428)	1.202 (69.179)
IP_t^{Sg}	9.854 (4.913)	2.144 (0.814)
IP_t^{Pr}	-4.510 (1.982)	-1.050 (0.329)
SP_{t-1}	0.253 (3.972)	0.359 (0.658)
C_t	-60.611 (43.125)	1.344 (7.147)
$(G_{t-1}-G_t^*)$	65.244 (84.124)	32.437 (13.942)
YLD_t	65.826 (51.423)	17.355 (8.522)
R^2	0.86	0.91
$\bar{R}^2$	0.80	0.88
F	15.7	27.4
DW	1.251	1.691

Sample Period 1946-1968

Two Stage Least Squares

Equilibrium Condition: $D_t^{Sg} = S_t^{Sg}$

$D_t^{Pr} = S_t^{Pr}$

Standard Errors in ( )

DW - Durbin Watson Staistic

A key to the abbreviations can be found in Appendix 7.1.

are all largely significant (at the 5% level). The most immediate explanation of this behaviour lies in the multicollinearity present in the demand equations (as evidenced by the consistently large standard errors throughout). This is particularly so since we would not have expected the inclusion of the non-own-prices to have had any improving effect upon the equations if their coefficients were actually insignificantly different from zero. On the other hand, a comparison of the coefficients of IP_t^{Sg} and IP_t^{Pr} in the demand equations shows that there is considerable variability among Tables 8.3.3 - 8.3.6 whereas the coefficients of the other explanatory variables vary less. Furthermore an examination of the t-values (obtained by dividing the coefficients by their corresponding standard errors) shows that IP_t^{Sg} and IP_t^{Pr} are among the least significant of all the demand explanatory variables. Consequently we can very tentatively conclude that the two demand equations only seem to be marginally affected by changes in either their own price or the non-own-price so that all expenditure/price elasticities seem to be very small. An examination of the signs of the demand equation own prices shows that, with the exception of Table 8.3.5 , the coefficients have a negative sign which is consistent with a downward sloping (quantity) demand curve. The coefficients of the non-own prices do not exhibit any consistent pattern at all.

An examination of the protection-based demand equations of Tables 8.3.3 to 8.3.6 shows up many puzzling peculiarities (such as the large standard errors, the variations in the signs of coefficients and the low R^2 and $\bar{R}^2$ values). Consequently it seems as if this part of the model needs further examination so that comments on the protection-based demand equations will be deferred until Section 8.4.

The effect of Building Society Advances (BSA_t) on the savings-based demand equations is also of interest. BSA_t seems to have a reasonably strong consistently positive influence on D_t^{Sg}. Because of the multicollinearity in the equations an examination of the standard errors could be misleading but BSA_t is reasonably significant in at least one model (Table 8.3.5). Thus there is some evidence to suggest that Building Society Advances are complementary to D_t^{Sg} : this was only to be expected since savings-based life insurance is used for house purchase purposes. However, examination of the corresponding 'average' elasticity coefficient shows that a 1% rise in $\overline{BSA}$ produces only a 0.2% rise in D_t^{Sg}.

Similarly, Permanent Income (INC_t^P) has a consistently positive, strongly significant (above the 5% level with the exception of Table 8.3.5) effect on D_t^{Sg}. Not surprisingly therefore, Permanent Income turns out to be the major determinant of the demand for long-term savings-based life insurance. The size of the coefficient of INC_t^P is also relevant because we can then deduce the (permanent) income elasticity of demand: Table 8.3.7 below gives the 95% Confidence Intervals for the coefficient and the corresponding 'average' income elasticity.(2)

Table 8.3.7 shows that the mean (permanent) income

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(2) The 'average' Income Elasticity is approximated by the formula

$$\text{Elasticity} = \frac{dD_t^{Sg}}{dINC_t^P} \cdot \frac{\overline{INC}^P}{\bar{D}^{Sg}} \quad \text{where } \overline{INC}^P \text{ and } \bar{D}^{Sg}$$

the mean values, are £352.61m and £821.19m respectively.

Table 8.3.7 Savings-based Income Elasticity

Source	Interval for Coefficient			Interval for Elasticity		
	Lower	Mean	Upper	Lower	Mean	Upper
Table 8.3.3	0.652	7.450	14.248	0.280	3.199	6.118
Table 8.3.4	1.290	8.513	15.736	0.554	3.655	6.757
Table 8.3.6	0.968	5.413	9.858	0.416	2.324	4.233

elasticity of demand for savings-based life insurance is around 3.0, that is, quite highly elastic.

The other exogenous variables in the demand equation for savings-based life insurance exhibit few surprises: BSW_t has a negative coefficient (Table 8.3.4) so that Building Society Wealth (Shares and Deposits) represents a substitute (in the savings market) for long-term life insurance. However, the 'average' elasticity coefficient shows that a 1% rise in BSW produces only a 0.54% fall in D_t^{Sg} . It is interesting therefore to note that if the Building Societies received a 1% rise in BSW and consequently increased their lending by 1% then the net effect would be a 0.3% fall(approximately) in the demand for new long term savings-based life insurance. The explanatory variable B_t (Births) has a positive effect on D_t^{Sg} as does the Standard Rate of Income Tax (T_t). Both U_t and ΔU_t have a negative effect as also does the number of marriages (M_t) : this latter result only serves to confirm the comments made towards the end of Section 8.2.

We now turn to an examination of the supply equations which, although not producing as high R^2 and $\bar{R}^2$ values, seem much more stable than the corresponding demand equations. In terms of the Durbin Watson statistic, the savings-based supply equation yields a small DW figure (although not small enough to accept positive autocorrelation).

The savings-based supply equations all exhibit positive own price and negative non-own price coefficients with corresponding 'average' elasticities of around 1.5 and -1.00 (although by 1969, these had worsened to 0.7 and -0.2 respectively). Thus the corresponding expenditure supply curve is upward-sloping and protection-based life insurance is regarded as a supply substitute.

The protection-based supply equations indicate a rather peculiar relationship because the own price is consistently and significantly negative (with a coefficient very close to -1.00) while the non-own price has a positive coefficient (which is very near 2.0). Thus it would appear that the expenditure supply curve for protection-based life insurance is downward-sloping while savings-based life insurance has a complementary relationship with S_t^{Pr}. Now, of course, any re-examination of the demand for protection-based life insurance will also imply a closer look at the supply curve so that it is possible that the underlying instability in the demand behaviour is also adversely affecting supply. On the other hand the downward-sloping expenditure supply curve can be explained by stressing the complementary nature of the savings and protection-based elements. Thus it could be argued that as IP_t^{Pr} rises, although initially S_t^{Pr} might rise, once IP_t^{Pr} reaches a certain level the life office starts to switch its protection-based business from temporary policies to endowment and whole life policies (which of course have a small protection element): this involves an overall decrease in S_t^{Pr} but an increase in the supply of the more heavily savings-orientated policies.

Of the other supply variables YLD_t has the expected positive coefficient and is also significant at the 5% level in most cases. The coefficient of C_t is negative in the savings-based equations but highly insignificant in the protection-based case. The coefficient of SP_{t-1} is largely positive but not at all significant in any equation. Finally, the coefficient of $(G_{t-1}-G_t^*)$ is significantly positive in the protection-based case as expected but insignificantly different from zero in the savings-based case.

So it would appear that the major determinants of the supply of savings-based life insurance are IP_t^{Sg} , IP_t^{Pr} , C_t and YLD_t while those for protection-based contracts are IP_t^{Sg} , IP_t^{Pr} , $(G_{t-1}G_t^*)$ and YLD_t.

8.4 Using Ordinary Least Squares

The dependent models of the preceding Section show up some peculiar points that require some further attention. In particular, the following phenomena of the demand equations deserve attention:

- the large standard errors in the protection-based equations
- the insignificance of the own price and non-own price variables
- the instability endemic in the equations

The demand equations were retested without any price variables to see whether their exclusion appreciably harmed the explanatory power of the equations. Of course, since there are no included endogenous variables ordinary least squares can be used. The best results are summarised in Table 8.4.1.

A comparison of Table 8.4.1 with any of the dependent demand equations of the previous Section shows that the omission of the price variables has, if anything, improved the demand equations: especially in terms of the R^2, $\bar{R}^2$, F and DW statistics. (An examination of the residuals gives no indication of any autocorrelation).

It can be seen from the savings-based equation that the most significant explanatory variables are Permanent Income (INC_t^P), Building Society Advances(BSA_t) and Wealth (Shares and Deposits BSW_t) and the Standard Rate of Income Tax (T_t). A study of the corresponding 'average' elasticity coefficients indicates values of 3.48, 0.41, -0.58 and 0.98 respectively. Thus a 1% rise in Building Society Wealth and hence Mortgage Advances has a net effect of a 0.17%

Table 8.4.1 The Demand Equations

	Demand for Savings-based (D_t^{Sg})	Demand for Protection-based (D_t^{Pr})
CONST	-2803.026 (555.773)	82.569 (141.950)
INC_t^P	8.094 (1.511)	1.213 (0.154)
BSA_t	28.342 (7.145)	
BSW_t	-8.146 (2.830)	
B_t	0.294 (0.252)	-0.208 (0.095)
M_t	-0.379 (0.787)	0.091 (0.287)
T_t	18.918 (6.139)	-5.183 (2.988)
U_t		25.080 (12.480)
ΔU_t		-8.619 (8.737)
R^2	0.99	0.94
$\bar{R}^2$	0.98	0.92
F	216.6	42.9
DW	1.818	2.186

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW - Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

decrease in the demand for savings-based life insurance (the effects on protection-based contracts are negligible). Similarly a 1% rise in the Standard Rate of Income Tax produces a 0.98% rise in new savings-based business; however, a 1% rise in T_t also produces a fall in INC_t^P of about 0.7% so that the net result is a decrease in D_t^{Sg} by around 1.5%.

Similarly, it can be seen from the protection-based equation that the most significant explanatory variables are Permanent Income, Births (B_t), the Standard Rate of Income Tax and the percentage rate of Unemployment (U_t). Neither BSA_t nor BSW_t figured as significant variables in any of the attempted models. The elasticity coefficients corresponding to INC_t^P and U_t are 2.28 and 0.25 respectively. Both B_t anf T_t have significantly negative coefficients which indicate that these both discourage the purchase of new protection-based life insurance: this could be because of the effect on Disposable Income of the latter and in the case of B_t, because of arguments similar to those used to explain the negative coefficient of Marriages in Section 8.2.

The import of Table 8.4.1 is that the removal of the price variables IP_t^{Sg} and IP_t^{Pr} has no appreciable effect on the equations so that we can indeed confirm that the coefficients of these variables are likely to be insignificantly different from zero. We can therefore conclude that all expenditure/price elasticities are zero, ie. a change in price produces no corresponding change in premium expenditure. Consequently, the own demand curve (using quantity instead of expenditure) will have unit quantity/own price elasticity. Finally the problems caused by any large standard errors seem to have disappeared: it seems, therefore that the inherent multicollinearity in the demand model was primarily caused by the price variables themselves.

A number of other models were attempted including NIF_t (total current account receipts of the National Insurance Fund) and LIQ_t (Liquid Assets). The results showed that neither LIQ_t nor NIF_t demonstrated any significance. The insignificance of NIF_t is perhaps a little surprising since Barros (2) intimates that there might be a substitutory relationship between the demand for savings-based life insurance and state pensions provisions. In a recent lecture (3), Professor Feldstein summarises the possible relationships between private financial saving and social security and concludes that, on balance, the existence of social security seems to substantially depress private saving (in the USA). Although it is dangerous to apply such results across international frontiers, Feldstein does provide in the early part of his paper, a possible explanation for the insignificance of our NIF_t variable. Since the national insurance scheme formerly provided only a minimum level of benefits, the more affluent members of the community would have to supplement their pensions with private savings of their own: this would then offset the depressing effect of NIF_t because ordinary life insurance is generally purchased by higher income families in the UK.

Table 8.4.2 shows the results of including the In Force variables IF_t^{Sg} and IF_t^{Pr} with those significant variables of

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(3) 'The Effect of Social Security on Saving'. Presented to the Association Internationale pour l'Etude de l'Economie de l'Assurance, Paris, 16 Jan. 1980.

Table 8.4.2 The Demand Equations

Demand for Savings-based (D_t^{Sg})		Demand for Protection-based (D_t^{Pr})	
CONST	-3208.168 (397.032)	CONST	598.892 (252.616)
INC_t^P	8.094 (0.930)	INC_t^P	0.038 (0.522)
BSA_t	31.968 (4.388)	B_t	-0.087 (0.079)
BSW_t	-19.065 (2.967)	T_t	-9.493 (2.714)
T_t	23.062 (4.693)	U_t	3.737 (10.345)
IF_t^{Sg}	6.973 (2.039)	IF_t^{Pr}	0.744 (0.324)
R^2	0.99		0.95
$\bar{R}^2$	0.99		0.94
F	425.6		66.5
DW	2.230		2.106

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

Table 8.4.1. Both In Force variables are significantly positive thereby implying that (according to Houthakker and Taylor (8)) the new purchase of both types of ordinary life insurance is habit forming.

Since it has been conclusively established that the coefficients of IP_t^{Sg} and IP_t^{Pr} are insignificantly different from zero in the demand equations, the corresponding supply equations can also be estimated by ordinary least squares. The results of this estimation are given in Table 8.4.3 and the coefficients are of similar size and sign to those reported in the preceding Section. However both models indicate the presence of positive autocorrelation (with low values of the Durbin Watson statistic and a definite pattern in the residuals): no attempt has been made to try alternative formulations since we are primarily concerned with the demand equations.

Table 8.4.3 The Supply Equations

	Supply of Savings-based (S_t^{Sg})		Supply of Protection-based (S_t^{Pr})
CONST	427.397 (309.119)	CONST	29.430 (33.493)
IP_t^{Sg}	5.126 (2.335)	IP_t^{Sg}	1.091 (0.424)
IP_t^{Pr}	-2.526 (1.090)	IP_t^{Pr}	-0.623 (0.187)
C_t	-69.567 (37.889)	$G_{t-1} - G_t^*$	20.449 (9.225)
YLD_t	99.912 (39.651)	YLD_t	25.434 (5.945)
R^2	0.87		0.93
$\bar{R}^2$	0.84		0.91
F	30.7		59.9
DW	1.063		1.276

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

8.5 Inflationary Expectations

In this Section, we introduce the last in the selection of demand and supply models under discussion in Chapter Eight. These models incorporate a first-order distributed-lag type inflationary expectation variable in exactly the same manner as those of Section 8.2. In order to keep down the number of explanatory variables, the recommendations of the previous Sections will be incorporated so that no price variables will be included in the demand equations and SP_{t-1} excluded from the supply equations. Ordinary least squares is used to estimate both the demand and supply equations.

The models of this Section are of two main types: the first includes an inflationary expectations variable in the demand equations only, the second includes such expectations in both the demand and supply sides.

Tables 8.5.1 and 8.5.2 represent the best results from fitting the inflationary expectations model of Chapter Seven to the demand equations.

The demand for savings-based equation in Table 8.5.1 shows that r^{Sg} is approximately equal to unity. The coefficient of $\dot{P}_{t-1}$ also seems insignificant which indicates that inflationary expectations have no effect on the purchase of savings-based life insurance. Permanent Income, on the other hand, has a significantly positive effect (at the 95% level although multicollinearity acts to reduce the significance of the estimates). The coefficient of INC_t^P (10.861) reveals a slightly larger average income elasticity figure (at around 4.6) than

Table 8.5.1 Inflationary Expectations (Demand Equations Only)

	Demand for Savings-based (D_t^{Sg})		Demand for Protection-based (D_t^{Pr})
CONST	-3106.947 (1736.217)	D_{t-1}^{Pr}	-0.175 (0.317)
D_{t-1}^{Sg}	0.086 (0.349)	$\dot{P}_{t-1}$	-2.401 (2.801)
$\dot{P}_{t-1}$	-7.548 (7.948)	INC_t^P	0.416 (2.106)
INC_t^P	10.861 (4.152)	INC_{t-1}^P	1.131 (2.046)
INC_{t-1}^P	-1.932 (2.904)	B_t	0.038 (0.206)
BSA_t	28.485 (7.686)	B_{t-1}	-0.298 (0.185)
BSA_{t-1}	0.953 (13.515)	U_t	10.753 (12.694)
T_t	20.652 (8.938)	U_{t-1}	23.981 (11.637)
T_{t-1}	2.174 (15.597)	T_t	-1.226 (3.983)
BSW_t	-13.481 (7.025)	T_{t-1}	-2.107 (3.943)
BSW_{t-1}	3.635 (5.224)		
R^2	0.99		0.96
$\bar{R}^2$	0.98		0.93
F	132.6		32.4
DW	1.948		2.312

Dependent Model
Sample Period 1946-1968
Ordinary Least Squares
Standard Errors in ( )
DW Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 7.1

Table 8.5.2 Inflationary Expectations (Demand Equations Only)

	Demand for Savings-based (D_t^{Sg})		Demand for Protection-based (D_t^{Pr})
CONST	-3180.512 (474.079)	$\dot{P}_{t-1}$	-3.375 (2.133)
$\dot{P}_{t-1}$	-8.733 (4.517)	INC_t^P	1.252 (0.109)
INC_t^P	9.469 (1.023)	U_t	21.412 (9.234)
BSA_t	30.234 (5.204)	B_t	-0.181 (0.070)
BSW_t	-10.560 (1.904)	T_t	-2.776 (0.799)
T_t	22.535 (5.792)		
R^2	0.99		0.94
$\bar{R}^2$	0.99		0.93
F	306.7		74.0
DW	1.659		2.357

Dependent Model

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in (   )

DW   Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1.

those described in Tables 8.3.7 and 8.4.1. The effects of Building Society Advances (BSA_t) and the Standard Rate of Income Tax (T_t) are much in line with those of Table 8.4.1 (with almost identical coefficients). The present coefficient of BSW_t is smaller than those previously reported. The conclusion that inflation does not affect D_t^{Sg} is an interesting one which is difficult to accept on a priori grounds. The explanation may lie in the role played by the declared rates of reversionary bonus which, if they keep pace with inflation, preserve the real value of policyholders' savings stocks. This hypothesis is tested in Table 8.5.3.

The demand for protection-based equation shows first that $r^{Pr} = 1$ (since the coefficient of D_{t-1}^{Pr} is insignificant and of incorrect sign) ie. that, as in the savings-based case, future inflation expectations depend entirely on the immediately preceding rate of inflation ($\dot{P}_{t-1}$): the consumer has no long-term memory and does not make any allowance for past trends of inflation in his future expectations. Unfortunately no further analysis of D_t^{Pr} can be undertaken because of the strong multicollinearity present in the model; Table 8.5.1 was therefore reworked omitting all lagged variables since both D_{t-1}^{Sg} and D_{t-1}^{Pr} proved to be insignificant - the results are reported in Table 8.5.2.

Table 8.5.2 considerably clarifies the effects of inflation expectations: in both equations the coeffiecient of $\dot{P}_{t-1}$ is marginally significant (at the 90% level for D_t^{Sg} and 85% for D_t^{Pr}). This is a surprising result for the savings-based case and Table 8.5.3 tests the hypothesis that inflationary expectations are partially offset by expected increases in the rate of reversionary bonus declared. Otherwise Table 8.5.2 exhibits no surprises and confirms the signs and coefficients of the other variables.

Table 8.5.3 replaces $\dot{P}_{t-1}$ by the Net Rate of Inflation NRI_{t-1} in the savings-based demand equation, where $NRI_t = \frac{\dot{P}_t - b_t}{1 + \dot{P}_t}$ (4) and b_t, the rate of compound reversionary bonus declared, is obtained from Table A 6.1.1. If inflationary expectations are indeed modified by bonus anticipations then we would expect the coefficient of NRI_{t-1} to be more significantly negative than that of $\dot{P}_{t-1}$: this is because a positive value for NRI_t implies a fall in the net real value of life insurance savings and should therefore discourage such saving. On the other hand, a negative value of NRI_t implies a rise in the real value of savings since reversionary bonus rates are expected to be higher than the rate of inflation: life insurance savings should then be encouraged. Table 8.5.3 illustrates two models : the first is the full expectations model which shows an insignificant coefficient of D^{Sg}_{t-1} . The second model therefore excludes D^{Sg}_{t-1} and all the other lagged explanatory variables and provides more stable results.

A comparison of Tables 8.5.2 and 8.5.3 shows a remarkable similarity between the coefficients of the various explanatory variables. However the coefficient

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(4) Strictly we require NRI_t to be compounded continuously so that the formula given here is only an approximation.

Table 8.5.3 Net Inflationary Expectations (Savings-based Demand Equations)

CONST	-3018.000 (1724.002)	-3190.848 (469.645)
D^{Sg}_{t-1}	0.115 (0.347)	
NRI_{t-1}	-9.263 (8.432)	-9.856 (4.846)
INC^P_t	10.464 (4.152)	9.420 (1.011)
INC^P_{t-1}	-1.854 (2.851)	
BSA_t	28.864 (7.477)	30.831 (5.122)
BSA_{t-1}	0.009 (13.399)	
BSW_t	-13.294 (6.893)	-10.781 (1.889)
BSW_{t-1}	3.545 (5.150)	
T_t	20.772 (8.832)	22.892 (5.770)
T_{t-1}	1.752 (15.420)	
R^2	0.99	0.99
$\bar{R}^2$	0.98	0.99
F	135.7	312.6
DW	2.015	1.685

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 7.1

NRI_t - Net Rate of Inflation

(ie. $NRI_t = \dfrac{\dot{P}_t - b_t}{1 + \dot{P}_t}$ where

b_t is the rate of compound reversionary bonus from Table A 6.1.1.)

of NRI_{t-1} $\underset{(4.846)}{-9.856}$ is slightly more negative and significant than that of $\dot{P}_{t-1}$ $\underset{(4.517)}{-8.733}$ (with t-values 2.03 and 1.93 respectively) and moreover, the R^2, $\bar{R}^2$, F and DW statistics are all improved in the former case. In the absence of more rigorous tests of significance, we can only tentatively conclude that the incorporation of NRI_{t-1} gives more concrete results so that inflationary expectations are partially offset by anticipated increases in the rate of reversionary bonus declared.

Finally Table 8.5.4 illustrates the results of including inflationary expectations in the supply models. Although the results should be treated with caution because of the obvious presence of multicollinearity it seems that the coefficient of S_{t-1}^{Sg} is certainly less than unity indicating a strongly significant $\Gamma \approx 0.1$. Inflationary expectations have a significantly negative effect on S_t^{Sg} - due no doubt to the effects of inflation on future administration costs. It seems therefore that life offices are more concerned about inflation (with a longer 'memory') than their policyholders. The results in the protection-based case are not strong enough to draw any conclusions. The Durbin h statistic (see J. Durbin (6)) indicates the absence of positive autocorrelation in the S_t^{Sg} case (the Durbin Watson statistic cannot be used because of the presence of the lagged dependent variable on the right hand side of the equation).

Table 8.5.4 Inflationary Expectations (Supply Equations)

Supply of Savings-based (S_t^{Sg})		Supply of Protection-based (S_t^{Pr})	
S_{t-1}^{Sg}	0.894 (0.195)	S_{t-1}^{Pr}	0.332 (0.210)
$\dot{P}_{t-1}$	-22.617 (8.117)	$\dot{P}_{t-1}$	-2.663 (1.808)
IP_t^{Sg}	0.383 (2.275)	IP_t^{Sg}	-0.712 (0.621)
IP_{t-1}^{Sg}	-0.840 (2.114)	IP_{t-1}^{Sg}	0.006 (0.494)
IP_t^{Pr}	0.139 (2.059)	IP_t^{Pr}	0.063 (0.434)
IP_{t-1}^{Pr}	0.505 (1.087)	IP_{t-1}^{Pr}	0.153 (0.319)
C_t	-0.535 (19.096)	$(G_{t-1}-G_t^*)$	8.528 (9.103)
C_{t-1}	-4.903 (16.486)	$(G_{t-2}-G_{t-1}^*)$	-18.858 (8.920)
YLD_t	-23.279 (37.114)	YLD_t	8.836 (7.805)
YLD_{t-1}	79.885 (62.231)	YLD_{t-1}	31.034 (14.898)
R^2	0.97		0.97
$\bar{R}^2$	0.96		0.95
F	54.6		43.9
DW	2.056		1.990
h	-0.379		-

Sample Period 1946-1968
Ordinary Least Squares
Standard Errors in ( )
DW Durbin Watson Statistic
h Durbin h statistic
A key to the abbreviations can be found in Appendix 7.1

8.6 Conclusion

This Chapter has attempted to analyse and explain the purchase of UK non-group life insurance by examining the determinants of the Financial Saving Ratio and the Demand for Life Insurance. Although the Section on forecasting is still to come, it seems sensible to draw together the existing results of Chapter Eight at this stage.

Chapter Eight has not been without its econometric problems and both multicollinearity and autocorrelation have caused difficulty in places. However the former has been largely combatted in a variety of ways by reducing the number of collinear variables in the models. Autocorrelation is a far more difficult problem to solve and no real solution has been adopted so that some of the results should be interpreted in the light of unconfirmed positive autocorrelation of residuals.

The analysis of the financial saving ratio (FSR) given in Table 7.2.1 shows up several points, the first demonstrating that inflation expectations have a significantly negative effect on FSR. Expectations are generally fairly long-term and longer if BSA_t is omitted as an explanatory variable. The value of $\Gamma = 0.14$ (when BSA_t is included) indicates that the immediately preceding rate of inflation $\dot{P}_{t-1}$ only contributes 14% towards future inflationary expectations. Permanent Income has a positive effect on FSR with a constant income elasticity figure of around 2.0. Both BSA_t and T_t also have positive influences on FSR_t.

The results of the demand models were generally more conclusive than those of the financial saving ratio. The primary observation to note is that the demand expenditure for both savings-based and protection-based life insurance was not affected by either own price or non-own price. While this may be because of the construction of the price indices involved (which were based on expenses loadings rather than 'market place' prices (which in turn were fairly constant over the sample period)) it is felt that the demand expenditure for life insurance is genuinely unresponsive to price.

The demand for savings-based life insurance is primarily affected by Permanent Income, Building Society Advances and Wealth (Shares and Deposits), and the Standard Rate of Income Tax: all but BSW making a positive contribution. The average permanent income elasticity coefficient comes out at around 3.5 to 4.0. The inflationary expectations model for D_t^{Sg} showed that these have little effect in the savings-based case. No strong evidence was found for the hypothesis that inflationary expectations are partially offset by expected increases in the rate of reversionary bonus declared.

The demand for protection-based life insurance is determined mainly by Permanent Income (with an average elasticity coefficient of around 2.5), Births, Tax and the Unemployment Rate. B_t and T_t have negative effects while U_t has a strongly positive effect (thus Juster and Wachtel's results (10) carry over to the consumption of life insurance). The inflationary expectations model for D_t^{Pr}(and D_t^{Sg}) showed that these had a negative effect with an adjustment coefficient of $r = 1$: thus purchasers of ordinary life insurance have a very short memory concerning inflation.

The supply models both show that prices affect supply producing the conventional upward-sloping expenditure supply curve in the savings-based case (protection-based life insurance is observed to be a supply substitute). Administrative costs (C_t) and the yield on invested funds have a negative and positive effect on S_t^{Sg} respectively; including life office inflationary expectation shows that these have a strongly negative effect on S_t^{Sg}.

The supply of protection-based life insurance exhibits more unconventional characteristics : all the models attempted indicate a negative coefficient for IP_t^{Pr} (with a positive coefficient of IP_t^{Sg}). It is hypothesised that this indicates the case of a backward-bending supply curve. YLD_t again has a positive effect on S_t^{Pr} as does $(G_{t-1}-G_t^*)$ indicating that the life offices vary S_t^{Pr} in order to maintain an optional gearing ratio of with-profits to total business. Inflationary expectations do not appear to figure strongly in the supply of protection-based life cover.

The remainder of the explanatory variables mentioned in Section 7.7 that have not already been mentioned were either insignificant (such as U_t, r_t^{BS}), highly collinear with other more important variables (such as IF_t^{Pr} and IF_t^{Sg}) or not available for the full sample period.

8.7 Forecasts of Demand

In this Section, an attempt is made to use the demand models of the previous Section to predict the demand for new ordinary life insurance for the years 1969-1977. However a number of problems hinder this forecast, not the least of which is that 1969 saw a major discontinuity in the aggregate ordinary life new business figures. Consequently the prediction cannot be properly judged against figures comparable with the 1946-1968 series. Additionally, no split can be provided for new business figures by class of policy (whole life, endowment etc.). Consequently it is expedient to use a different series as the basis of comparison - with figures from the Life Offices Association and the Associated Scottish Life Offices (12). These figures - for New Ordinary Life Assurances in the UK (not including linked business) - are given in Table 8.7.1 below.

Table 8.7.1 New Yearly Premiums (1963 values per 1,000 head of population) (£)

1967	966.3
68	918.5
69	910.5
1970	1080.2
71	1105.7
72	1221.6
73	1363.9
74	1068.2
1975	1061.2
76	1071.0
77	1020.7

Sources: LOA / ASLO contributors (12).

Table 8.7.2 The Explanatory Variables (1967-1977)

	U_t	ΔU_t	M_t	B_t	T_t	POP_t	RPI_t	BSA_t	BSW_t	INC_t^P	$\dot{P}_t$
1967	2.3	0.7	439.09	962	41.25	54.80	115.2	23.17	110.67	432.25	2.46
68	2.5	0.2	462.76	947	41.25	55.05	120.6	23.95	117.31	440.20	4.58
69	2.5	0.0	451.63	920	41.25	55.26	127.2	22.17	124.10	445.80	5.33
1970	2.6	0.1	470.99	904	41.25	55.42	135.3	26.05	135.62	454.53	6.17
71	3.4	0.8	459.39	902	38.75	55.61	148.0	32.87	148.06	462.51	8.97
72	3.8	0.4	480.29	834	38.75	55.78	158.5	41.06	163.02	482.94	6.85
73	2.7	-0.9	453.67	780	30.00	55.91	173.1	36.29	171.70	506.51	8.81
74	2.6	-0.1	436.35	737	33.00	55.92	200.9	26.21	166.04	523.01	14.89
1975	4.2	1.6	430.68	698	35.00	55.90	249.5	35.19	164.16	528.09	21.66
76	5.7	1.5	406.02	676	35.00	55.89	290.7	38.06	163.78	529.20	15.28
77	6.2	0.5	403.91	657	34.00	55.95	336.8	35.86	171.89	523.26	14.72

Sources: Annual Abstract

Inland Revenue Statistics 1976

The figures in Table 8.7.1 cover business transacted by member offices within the UK and so do not include the UK business of non-member overseas based offices.

The figures which form the basis of the predictions are given in Table 8.7.2.

Predictions can then be made using two separate models: with and without the inflationary expectations model. These two models can then be evaluated by comparing the relative 'success' of the predictions using the standard techniques developed for this purpose (eg. see Koutsoyiannis (11) Ch.20).

The predictions and percentage changes from the two models (Tables 8.5.2 and 8.4.1 respectively) are shown in Table 8.7.3 below:

Table 8.7.3 Forecast Values for $(D_t^{Sg} + D_t^{Pr})$

Year	Actual Value (UK Co's. only)	%Δ	Predicted Value 1	%Δ	Predicted Value 2	%Δ
1968	966.3	-	1675.4	-	1691.6	-
69	918.5	-4.9	1624.3	-3.1	1605.3	-5.1
1970	1080.2	17.6	1716.4	5.7	1690.6	5.3
71	1105.7	2.4	1865.5	8.7	1808.8	7.0
72	1221.6	10.5	2167.5	16.2	2104.4	16.3
73	1363.9	11.6	2047.4	-5.5	1960.2	-6.9
74	1068.2	-21.7	1994.5	-2.6	1933.5	-1.4
1975	1061.2	-0.7	2362.8	18.5	2286.4	18.3
76	1071.0	0.9	2501.3	5.9	2343.1	2.5
77	1020.7	-4.7	2324.0	-7.1	2199.0	-6.1

Source: Tables 8.4.1 and 8.5.2

(Predicted Value 1 is from Table 8.4.1 ie. no inflation expectations)
(Predicted Value 2 is from Table 8.5.2)

A comparison of the predicted values is obviously limited because the comparable data for the actual values is not available. The only data available does not include the UK business transacted by overseas companies so that a straight comparison with this information is meaningless. However, if we make the assumption that the value of new business obtained by overseas companies (not contributing to the LOA/ASLO figures) is a constant proportion of the total then some analysis is possible using data on the percentage changes. We should note that the existing data on new business figures (for the years 1946-1968) shows that the above assumption is not really a very good one, moreover there is some evidence to show that the proportion of business obtained by overseas companies was increasing throughout the 1960's. If this trend continued into the early 1970's, then the forecasts would be better than they appear in Table 8.7.4.

Table 8.7.4 An Evaluation of the Forecasts

Method	Prediction 1	Prediction 2
MSE of % Δ	141.5	147.3
Correlation of % Δ	0.28	0.33
Thiel's Inequality Coefficient U	1.087	1.109

On the basis of Table 8.7.4 it seems as if Prediction 1 is the more accurate of the two : the evaluation methods of minimised mean squared errors (MSE), the correlation coefficient of actual and predicted percentage changes and

Thiel's Inequality Coefficient (19) pp.26-36 , all tend to favour Prediction 1. We note, however, that the period 1969-1977 was a very difficult one for forecasting purposes and even the more sophisticated forecasts of such economic indicators as consumer spending, price indices and unemployment proved to be unreliable (see Society for Business Economists (18)).

CHAPTER EIGHT : REFERENCES

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CHAPTER NINE : AN ANALYSIS OF SURRENDERS IN UK ORDINARY LIFE INSURANCE

9.1 Introduction

The emphasis on the purchase of new contracts in Chapter Eight does not properly cover the effects of the explanatory variables (and especially inflation) on the demand for contractual savings-based policies. The reason is that it is not until several payments have been made that these come to be regarded as contractual - obviously some discretion is involved at the inception of the contract.

Other authors (such as S.Neumann (11)) have tackled this problem by analysing total premium payments on contracts in force rather than just looking at new purchases. However, such an approach has several pitfalls and will not be adopted here because:

i) the contractual nature of business in force means that the situation changes very slowly over time. Additionally, declared reversionary bonuses are added to the sums insured in force and this further emphasises a trend element;

ii) an analysis of in force figures cannot differentiate between discretionary and contractual elements. In any case, an attempt to remove the effects of new business is only partially successful because claims (either on death or maturity) and surrenders must also be taken into account;

iii) United Kingdom in force figures cannot be obtained separately for non-group, non-single premium business (this is always a problem in any analysis of UK life insurance purchases).

As an alternative this Chapter will concentrate on the surrenders of ordinary life insurance and, as a result of the method of calculating the dependent variable, focusses particularly on the surrender of contractual savings-based insurance. A study of surrender rates is much more interesting than one of in force premium income because surrenders are more unpredictable (eg. see D.J.Ward and C.O.Kroncke (14)) and are not so dominated by trend elements.

In general, it would be better to talk about 'voluntary withdrawals' rather than surrenders because the former includes those withdrawals where no surrender value is paid:

> "The expression 'withdrawal' is used to denote a policy removed from the live file, due to premature termination of the contract by the policyholder, with or without payment of a surrender value. It does not include a policy which is converted to a paid-up amount or continued with a reduced premium and sum assured". (F.D.Patrick and A.Scobbie (12)).

However, the term 'surrenders' will continue to be used because no information is available on the number of withdrawals - only on surrender values paid out. Consequently no analysis can be conducted on contracts that do not pay a surrender value - which commonly include:

- all temporary life insurance policies, and
- all savings-based contracts of less than two years in force.

The next Section develops a theoretical framework for the study of voluntary withdrawals and . Section 9.5 attempts to apply a quantitative analysis of the dependent variable (whose calculation is described in Section 9.3).

9.2 A Theoretical Framework

Because of the important difference between contractual and discretionary saving it is necessary to distinguish between those withdrawals from policies that are still regarded as part of discretionary saving and those from policies that have been in force for long enough to be considered contractual.

Those policies whose premium payments are still regarded by the policyholder as discretionary will be called Not Yet Contractual (as opposed to Contractual) policies; the premiums paid out while the policy is still regarded as Not Yet Contractual will be called NYC premiums. The term 'not yet contractual' is preferred to 'discretionary' because it emphasises that eventually after the payment of an unspecified number of renewal premiums, the policy will be regarded by the purchaser as a contractual commitment.

As Neumann(11) pointed out, we would expect withdrawals from NYC policies to exhibit a different pattern to those from contractual policies. It would also be reasonable to assume that the reasons for withdrawal would have differing importance depending upon whether or not the exit came from a NYC policy. These two points underline the reasons why the NYC/Contractual split is a meaningful one: it provides part of a useful framework to analyse the determinants of voluntary exits. Some of these determinants will only apply to withdrawals from NYC policies and this should enhance the isolation and analysis of these factors.

We would expect that withdrawals from NYC policies would be more common than from contractual policies for several reasons, arising from the fact that the NYC policies have not been in force for so long:

- since the expenditure is not contractual (by definition) the decision to purchase life insurance is reviewed every time the renewal premium is paid. Once the expenditure is regarded as contractual, consumption decisions are made on the basis of disposable income net of contractual payments (eg. see P.L.Cheng (3));
- the policyholder may reconsider his decision to purchase life insurance in the first few days of the contract. Thus the Insurance Companies Act 1974 suggested a 'cooling off' period (Section6.5) of ten days when the policyholder could change his mind after the initial purchase (eg. see Post Magazine 14th April 1977, 'Cooling off life insurance').

The factors influencing the decision to terminate a life insurance policy can be categorised under four headings: the policyholder has no further need for the policy; the policy would still be useful but is terminated because a 'better buy' substitute exists elsewhere; a cash lump sum is required by the policyholder and to avoid the expenditure on premiums.

In the first case, the need for the services provided by the policy has disappeared. The main uses of life insurance are: family protection, mortgage repayment, tax avoidance, partnership protection, to cover school fees and for special savings and investment plans.(1) So, for example, if the mortgage arrangements of the policyholder change, then the associated policies (mortgage protection and/or house purchase policies) may be superfluous. The 'Scott' Committee (9) recognised, in 1973, that the policyholders may realise within the 'cooling off' period that they no longer need a life insurance policy if firstly they have been induced to take out policies on the basis of wrong, misleading or incomplete information or secondly where inducement, via pressure selling, results in the purchase of policies unsuited to their circumstances.

In the second case, the substitute may just be another policy offering better terms; however it is possible that in this case the policyholder may be a victim of heavy sales pressures by a salesman for commission purposes. On the other hand, the policy may be terminated in favour of another 'financial' savings institution; alternatively, in times of extreme uncertainty, the money could be held in cash or near-money balances.

Reason four is basically concerned with the relative expense to the policyholder of the premium; it can be further subdivided into two: first, where the premiums were beyond the means of the purchaser from the very start of the policy (this is likely to be a case of heavy 'sales pressure' by the broker or salesman) and second, where the economic circumstances of the policyholder change so that the

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(1) See R.L.Carter (2) part 2.1.3.

policy becomes too expensive (for example with sickness or unemployment).

A questionnaire was circulated to twelve of the largest ordinary life offices in the country in May 1975. In this, the companies were asked why some policyholders surrender or lapse their ordinary life insurance policies. Table 9.2.1. below represents a summary of the replies: the percentages add up to more than 100 because most offices mentioned more than one category.

Table 9.2.1 Reasons for Voluntary Exit

(the % of offices mentioning this category)

1 No further need	2 Substitution	3 Cash Required	4 To Avoid Expense
73%	55%	45%	73%

We are now in a position to combine the different elements into a framework which will aid the analysis of voluntary exit rates. This framework is pictured diagrammatically in Table 9.2.2.

Table 9.2.2. An Analysis of Voluntary Exits

	1. Need	2. Substitution	3. Cash	4. Expense
N,Y.C.			Not important	
Contractual				

The category NYC/Need as already been discussed since it includes those cases where a 'cooling off' period might apply. Similarly NYC/Expense comes under the aegis of the 'cooling off' argument since one would expect the policyholder to quickly discover if the premiums were too expensive. The two remaining NYC categories are not expected to be so important since in the case of NYC/Substitution, the policyholder will probably not discover that a better buy exists elsewhere until the premiums have come to be regarded as contractual. The case NYC/Cash is not an important one because of the common practice of not declaring a cash value in the early (normally two) years of the contract (assuming, of course, that the NYC period is less than two years).

The category Contractual/Expense is particularly interesting because it seems reasonable to assume that once the policy has been in force for some time, the policyholder will try to avoid discontinuing it. This is particularly pertinent because a penalty is paid by those policyholders who surrender their contract. The theory of Section 2.12 shows that a small surrender value will have an adverse effect on the decision to surrender. As the need for the policy continues the policyholder can either cut back on some other form of expenditure or convert the policy to 'paid-up' (see P.D.Bacon and L.J.New (1)):

> "For some, the chief reason for thinking of surrender is simply the increasing difficulty of meeting the regular premium..... For somebody in this position, getting his hands on the cash may not be so important as avoiding the regular payment of premiums. .... A compromise 'solution' to that problem could be to make the policy 'paid up'." (2)

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(2) J.Drummond. "Sums to do before cashing in a policy" The Times , 8th March 1975.

In fact, policies that are paid-up can only be analysed in the context of the Contractual/Expenses combination: this fact serves two useful purposes, first it makes surrenders in this category more unlikely and second it shows that some future analysis of paid-up policies may throw some light on the Contractual/Expense combination.

9.3 The Dependent Variable ($SURR_t$)

The basis for the dependent variable was derived from the figures 'surrenders' (from the ordinary life file of companies doing business in the United Kingdom) available under the heading 'Outgoings of life assurance companies' in the Annual Abstract of Statistics (7). The figures 'surrenders' relate to surrender values paid on policies that are withdrawn and therefore give no indication of the number of voluntary exits from policies that have not yet acquired a surrender value. Obviously, it would be incorrect to analyse the surrender data as it stands, without making any allowance for the volume of business from which the surrenders spring; therefore, it was necessary to make an approximation to the 'original population' from which the surrenders in year t could have come. This 'original population' was then used to form what will be known as the Surrender Rate.

Let E_t represent the 'volume' of new ordinary life policies issued in year t and let S_t represent the 'volume' of voluntary exits in year t. Let I_t represent some form of cost of living index for year t. We use the results from the paper by F.D.Patrick and A.Scobbie (12) which are represented in their first table.

Thus w_j represents the probability of withdrawal between the start of policy year j and the start of policy year (j + 1). The cumulative probability of withdrawal (W_k) is then given by

$$W_k = 1 - \prod_{j=0}^{k} (1 - w_j)$$ and represents the

probability of withdrawal after k curtate policy years.

Table 9.3.1 Crude Rates of Withdrawal

Curtate duration (i)	Rate (w_j)	Cumulative Rate (W_k)
0	0.067	0.067
1	0.056	0.119
2	0.053	0.166
3	0.042	0.201
4	0.039	0.232
5	0.033	0.258
6	0.030	
7	0.027	
8	0.023	
9	0.021	

So, if we ignore the possibility of withdrawal by death, the 'original population' in year T is given by

$$\sum_{k=1}^{N} E_{T-k}.(1-W_{k-1}) + E_T$$ where, theoretically

N should be very large. In effect, we are defining the 'original population' as the cumulative volume of new policies issued for up to N years preceding year T less those that are voluntarily withdrawn prior to year T. The voluntary exit rate in year T in real terms is then given by

$$\frac{100 * \qquad S_T/I_T}{\sum_{k=1}^{N} \frac{E_{T-k}}{I_{T-k}} (1-W_{k-1}) + \frac{E_T}{I_T}} \qquad (9.3.1)$$

For no better reason than that it suits the available data, N was chosen to equal 6: this limits the validity of the results so that the rates are only useful in comparison with each other. In addition, since the mortality experience has been ignored, the rates are underestimated; however, since mortality experience only changes very slowly through time, this underestimation should not affect the inter-rate comparisons. I have had to assume that the crude rates of withdrawal produced by Patrick and Scobbie for 1965 are time-invariant.

Since S_T must be represented by the volume of surrenders paid out in year T this means that lapses with no surrender values are ignored. In other words, the use of surrender values ignores those withdrawals in the early (normally first two) policy years; therefore, the 'original population' must be adjusted to allow for this omission. Similarly, it is not usual practice to allow a surrender value on temporary life insurance so that these must not be included in the 'original population'.

The eventual 'original population' is then given by:

$$\sum_{k=2}^{6} \frac{E_{T-k}}{I_{T-k}} \cdot (1-W_{k-1}) \qquad (9.3.2)$$

To be theoretically correct, the appropriate measure of E_T -the volume of new ordinary life policies issued in year T - should be the new sums insured issued in year T. However, aggregate sums insured data from the Annual Abstract of Statistics does not differentiate between single and renewable premium policies. Presupposing that most surrenders in the period 1946 to 1968 were from renewable premium policies,(3) it was decided to use new renewable office premiums (excluding temporary life business) to represent E_t (see Table 6.4.1). The resultant Surrender Rates are illustrated in Table 9.3.2 below.

Table 9.3.2 Surrender Rates (%) ($SURR_t$)

1945	24.8220
46	41.1060
47	36.1261
48	32.3579
49	28.5692
1950	26.4393
51	25.8361
52	25.9527
53	27.6890
54	29.0254
1955	30.4203
56	33.8196
57	36.7746
58	36.6655
59	36.8648
1960	40.3890
61	46.0081
62	46.8713
63	42.1100
64	43.0160
1965	41.4119
66	41.3658
67	42.6244
68	43.5737

Source: Equations (9.3.1) and (9.3.2)

An initial examination of the surrender rates of Table (9.3.2) shows up the following points:

a) from a high level immediately after the War, the rates dropped to a low of 25.8361 in 1951. Since then rates have been on an upward trend, peaking in 1962 at 46.8713.

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(3) This is obviously a simplification since single premium bonds may also be surrendered. However single premium bond business was most volatile in the early 1970's after the end of our period of interest.

b) the high level of 41.1060 in 1946 is understandable in view of the pent-up demand that had accumulated during the war years. However, it is interesting that surrender rates subsequently passed this level after 1961. Whether this is due to the operation of 'economic' factors or alternatively simply due to the method of calculating the rate must be investigated.

The problem associated with the construction of the surrender rates referred to above has to be overcome: it arises because surrender values paid have been used as the dependent variable instead of the number of surrenders. Thus, what could appear to be a rise in surrenders could just be an increase in the value paid per surrender in that year. It is impossible to offset this effect by juggling with the dependent variable and, consequently, an explanatory variable must be inserted to cover this possibility.

A change in the value paid per surrender could be caused by two different factors: first, an increase in the generosity of the individual life offices and second by a shift in the composition of life business to contracts that offer differing surrender values. An increase in the generosity of life offices in their treatment of surrenders can be caused both by a gradual change in life office attitudes towards surrendering policyholders and also by the increasing success of investment departments. An examination of the surrender values paid out on standard policies by the offices contributing to the Survey (see Appendix 6.3) showed, however, that there was very little change in the basis for calculating surrender values over the period 1946-1968. We must, therefore, come to the conclusion that any increase in 'generosity' has been caused not by a change in the method of calculating the surrender values but through an increase in the value of surrendered reversionary bonus. Consequently, an attempt can be made to 'pick-up' the effects of changes in generosity by including the Rate of Reversionary Bonus Declared (Table A 6.1.1) as an explanatory variable (RB_t).

It is hoped that the effects of a shift towards policies with different surrender values (either by a movement towards endowment (with profit) policies or alternatively by any increase in average inception age) will be offset by using new renewable office premium income (excluding temporary business) in the denominator of Equation (9.3.1). This offsetting effect arises because the policies with higher surrender values normally involve higher office premium payments.

9.4 The Explanatory Variables

Basically, the relevant explanatory variables fall into the four categories which are illustrated in Table 9.2.1: 'No further need', 'Substitution', 'Cash Required' and 'To Avoid Expense'.

No Further Need

If an endowment policy was used for house purchase purposes, then - in certain circumstances - a change of house might involve the surrender of the associated house-purchase policy. Thus we would expect an increase in housing turnover to cause surrenders to rise and since New Mortage Advances (BSA_t) are a surrogate for housing turnover (which itself is not available on an aggregate level), the coefficient of this variable should be positive.

Other variables which could reflect a change in the need for life insurance (both for savings and protection purposes) include:

- the number of divorces DIV_t ;
- the number of business partnerships dissolved;
- the mortality rate of dependents (say children under 14) DRD_t ;
- a change in the structure of taxation (eg. the admissibility of certain types of life insurance for tax-exemption).

Substitution

Surrenders in this category may either be in favour of another form of life insurance or alternatively (and more probably) in favour of the product of another savings institution. Unfortunately, the existing data is insufficient for the inclusion of a variable relating to previous business: this would have been of interest because the 1956 Finance Act and the 1959 National Insurance Act both substantially increased the membership of superannuation/pension schemes. However some idea of the increased importance of private pension schemes can be obtained from the pattern of new Considerations for Annuities purchased from life insurance companies established in the U.K. (ANN_t).

We might also expect surrenders to increase if consumers anticipated a decrease in the real value of their savings. Consequently an increase in inflation expectations should increase the surrender rate. The effects of inflationary anticipations can be incorporated by including a lag-structure similar to that described in Section 7.5.

Other variables which exhibit a substitute relationship to life insurance include:

- a change in Building Society Wealth (Shares and Deposits) (ΔBSW_t);
- a change in the Real Net of Tax Return from Unit Trusts (ΔUTY_t) (ideally net sales from Unit Trusts are required but figures are not available back to 1946). This information can be obtained from Economist Intelligence Unit Ltd. (5);
- a change in the holdings of liquid assets by the personal sector (ΔLIQ_t).

<u>Cash Required</u> (for consumption)

Surrenders in this category are used to finance the purchase of extra consumption goods and consequently the explanatory variables should reflect any changes in this propensity to consume. Suitable variables are thus:

- the change in the ratio of consumption to disposable income $(\Delta\frac{CONS}{PDI}\ t)$
- the change in the amount of Hire Purchase Credit outstanding (ΔHPC_t)

To Avoid Expense

We would expect any surrenders in this category to occur if the consumer's income decreased in relation to his level premium payments. Consequently suitable explanatory variables include:

- the change in personal disposable income (ΔPDI_t);
- the change in unemployment rate (ΔU_t) ;
- the change in the standard rate of income tax (ΔT_t).

Before we move onto a description of the actual analysis and results, it ought to be pointed out that, once again, problems of aggregation mean that the variables described above are only surrogates for the underlying explanatory variables. Unfortunately in some cases, the aggregate variable (pertaining to all UK residents) does not adequately reflect the situation of those individual holders of life insurance. So, for example, the variable ΔPDI is intended to pick up those surrenders by policy-holders who experienced a fall in disposable income (in relation to the constant premium payments). But an examination of the aggregate figures for Real PDI per capita shows that a year-to-year drop only occured in six of the post-war years. Thus in the majority of post-war years the aggregate figures do not adequately reflect those individuals (the minority) who experienced an overall drop in PDI.

The implications of the aggregation problem are that the explanatory variables involved may well prove to be insignificant because they are inadequate surrogates for the underlying variables. If no better surrogate can be found then the results must obviously be interpreted in the light of this defect.

9.5 A Quantitative Analysis

Before conducting a multiple regression analysis of the surrender rates derived in Section 9.3 , a potential problem first outlined in Sections 2.10 - 2.12 must be examined. The theory (although not applying directly to the surrenders of savings-based policies) indicates that for the individual consumer, the decision on surrender exhibits a certain 'stickiness'.(4) It is therefore a matter of importance whether, on aggregate, this situation can be adequately replaced by a formulation which implies that marginal changes in the explanatory variables produce marginal changes in the aggregate surrender rate.

For the i^{th} individual policyholder, one might hypothesise the following relationship (at time t):

$$\delta_{it} = \sum_{j=1}^{k} b_j \cdot X^*_{ijt} + u_{it} \qquad (9.5.1)$$

where $\delta_{it} = \begin{cases} 1 \text{ if surrender occurs} \\ 0 \text{ if no surrender occurs} \end{cases}$

X^*_{jt} is the j^{th} explanatory variable (at an individual level) and u_{it} is the random residual element.

Then, aggregating Equation (9.5.1) for N_t policyholder, we get

$$n_t = \sum_{i=1}^{N_t} \sum_{j=1}^{k} b_j . X^*_{ijt} + \sum_{i=1}^{N_t} u_{it} \qquad (9.5.2)$$

(4)In fact, it was hypothesised in Section 2.11. that no surrender would take place until $\bar{U} > U^*$ ie. until underlying circumstances changed to such an extent that the expected lifetime utility after surrender ($\bar{U}$) exceeded that expected lifetime utility obtained from continuing the old insurance arrangements (U^*_t).

where n_t denotes the number of surrendering policyholders.

Equation (9.5.2) can be rewritten as

$$\frac{n_t}{N_t} = \sum_{j=1}^{k} b_j . X_{jt} + v_t \qquad (9.5.3)$$

where $$X_{jt} = \frac{1}{N_t} . \sum_{i=1}^{N_t} X^*_{ijt}$$

and $$v_t = \frac{1}{N_t} . \sum_{i=1}^{N_t} u_{it}$$

The term $\frac{n_t}{N_t}$ then represents the probability of surrender at time t (which is essentially the dependent variable ($SURR_t$) that Section 9.3 attempts to approximate). The dependent variables X_{jt} correspond to per capita aggregate values of X^*_{jt} but it has already been pointed out that the pertinent characteristics of X_{jt} ($= \frac{1}{N_t} \sum_{i=1}^{N_t} X^*_{ijt}$) may be destroyed by approximating X_{jt} by the corresponding UK aggregate (which includes non-holders of ordinary life insurance) (5). Equation (9.5.3) then indicates that the process of aggregation has replaced the sticky 'switch-type' Kronecker delta with the continuous variable $\frac{n_t}{N_t}$. Although theoretically $\frac{n_t}{N_t}$ is bounded above by unity and below by zero this should

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(5) Thus, for example,

$$\sum_{i=1}^{N_t} \Delta\, PDI^*_{it} \neq \Delta \sum_{i=1}^{POP} PDI^*_{it}$$

where POP denotes the population of the UK.

Table 9.5.1 Linear Model of Surrender Rates

$$
\begin{aligned}
SURR_t = & - \underset{(26.069)}{5.098} + \underset{(920.232)}{1660.751}\, RB_t + \underset{(45.102)}{65.716}\, DRD_t \\
& - \underset{(0.245)}{0.134}\, DIV_t + \underset{(2.095)}{1.742}\, \Delta U_t - \underset{(0.934)}{1.379}\, BSA_t \\
& + \underset{(0.0032)}{0.0045}\, ANN_t + \underset{(78.243)}{4.928}\, \Delta \frac{CONS}{PDI}_t + \underset{(0.636)}{0.211}\, \Delta \dot{P}_t \\
& + \underset{(0.588)}{0.271}\, \Delta UTY_t - \underset{(0.660)}{1.041}\, \Delta HPC_t + \underset{(0.360)}{0.165}\, \Delta LIQ_t \\
& - \underset{(1.001)}{0.733}\, \Delta T_t + \underset{(0.918)}{0.546}\, \Delta BSW_t - \underset{(0.322)}{0.147}\, \Delta PDI_t
\end{aligned}
$$

$R^2 = 0.883$

$\bar{R}^2 = 0.678$

$F = 4.322$

$DW = 1.692$

Dependent Variable: $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbrevaitions can be found in Appendix 9.1.

not cause any difficulties (see Goldberger (6) p.251). Thus the potential problem referred to above has been resolved by the process of aggregation. This aggregation however has caused further problems. Initially the straightforward Ordinary Least Squares technique was applied to a linear function of the explanatory variables detailed in Section 9.4 but not including a lag structure of inflation expectations. The results of this first attempt are illustrated in Table 9.5.1 and a point of immediate interest is that the Durbin Watson statistic (D.W.) indicates a significant degree of autocorrelation of residuals. This autocorrelation has dangerous consequences for the interpretation of Table 9.5.1 (eg. see Koutsoyiannis (10)); in particular, the reliability of the estimates may be overstated with the result that explanatory variables may be accepted as significant when in fact they are not.

Table 9.5.1 indicates that the coefficient of the Rate of Reversionary Bonus (RB_t) is significantly positive. This can be interpreted in two ways:

i) the method of calculating surrender rates described in Section 9.3 is indeed deficient and does not take account of changes in generosity on the part of life offices in the surrender values paid by them, or

ii) that increases in the reversionary bonus rate declared (which obviously increase the surrender values paid) increase the probability of surrender. This, of course, was one of the major (but not surprising) conclusions of Sections 2.10 - 2.12.

Obviously, the results of Table 9.5.1 are disappointing and difficult to interpret (because of the inherent positive autocorrelation of residuals). Additionally the doubts that have been expressed about the suitability of ΔPDI have essentially been confirmed (by the insignificance and incorrect signs of this variable). Consequently, in future ΔPDI will be replaced with another surrogate for ΔPDI* which is a better indicator that the policyholder may be experiencing hard times: the number of bankruptcies (in England and Wales) per million of UK population (see Annual Abstract of Statistics (7)) $BUST_t$.

Further attempts were made to remove the positive autocorrelation inherent in the model of Table 9.5.1 - even to the extent of utilising Durbin's 'Two Step' method to estimate the degree of residual autocorrelation (see J.Durbin (4)).

However, no significant success was experienced using these methods and the results are not reported.

It seemed logical to conclude that the autocorrelation of Table 9.5.1 was therefore caused by some other factor such as the omission of an important explanatory variable. Consequently, more complicated models were attempted using a lag structure to incorporate inflationary expectations into the model. Although an ordinary least squares technique was used it should be pointed out again that there are certain drawbacks to this technique when the lagged dependent variable is included on the righthand side of the explanatory equation (see Section 8.2).

Two separate models incorporating a lag structure were used, the first utilising the simple first-order adaptive expectations lag of Section 7.5. The second model uses a more sophisticated error-learning mechanism taking account of both the recent rate of inflation and also its rate of change. As D.E.Rose (13) suggested, this behaviour can be captured by postulating that people adapt to the size of two previous errors in inflationary expectations. These two linear models can be represented by the following equations:

<u>First Order</u>

$$SURR_t = SURR_t\,(SURR_{t-1}, \dot{P}_{t-1}, X_{it}, X_{it-1}, u_t, u_{t-1}) \qquad (9.5.4)$$

and

<u>Second Order</u>

$$SURR_t = SURR_t\,(SURR_{t-1}, SURR_{t-2}, \dot{P}_{t-1}, \dot{P}_{t-2}, X_{it}, X_{it-1}, X_{it-2}, u_t, u_{t-1}, u_{t-2}) \qquad (9.5.5)$$

(see Appendix 9.2)

A notable disadvantage of both these models is that the lag-structure necessitates a large number of explanatory variables (especially for Equation (9.5.5)). Thus, as we saw in Chapter Eight, the only effective solution is to maintain a small number of right-hand-side variables: Tables 9.5.2 and 9.5.3 therefore concentrate on the variables: RB, DRD, ANN, Δ UTY and BUST.

An analysis of the F statistic shows that all the models of Tables 9.5.2 and 9.5.3 have something to contribute in

Table 9.5.2 First Order Inflationary Expectations Model

	A	B
CONST	8.711 (3.119)	11.053 (15.524)
$SURR_{t-1}$	0.220 (0.251)	0.271 (0.199)
$\dot{P}_{t-1}$	-0.470 (0.596)	-0.465 (0.555)
RB_t	-296.806 (1038.210)	-15.994 (908.782)
RB_{t-1}	1237.623 (1018.161)	1348.239 (889.663)
DRD_t	39.005 (35.559)	48.848 (29.200)
DRD_{t-1}	-22.234 (40.306)	-4.093 (29.482)
ANN_t	0.0055 (0.0052)	0.0023 (0.0027)
ANN_{t-1}	-0.0030 (0.0037)	-0.0039 (0.0029)
ΔUTY_t	-0.316 (0.459)	-0.303 (0.392)
ΔUTY_{t-1}	0.436 (0.399)	0.536 (0.352)
$BUST_t$	0.0873 (0.2378)	
$BUST_{t-1}$	-0.249 (0.351)	
R^2	0.87	0.86
$\bar{R}^2$	0.71	0.75
F	5.5	7.5
DW	1.853	1.860

Dependent Variable $SURR_t$
Sample Period 1946-1968
Ordinary Least Squares
Standard Errors in ( )
DW Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 9.1

Table 9.5.3 Second Order Inflationary Expectations Model

	A		
CONST	68.549 (28.586)	$BUST_t$	-0.133 (0.071)
$SURR_{t-1}$	0.354 (0.169)	$BUST_{t-1}$	0.178 (0.118)
$SURR_{t-2}$	0.098 (0.316)	$BUST_{t-2}$	-0.545 (0.150)
$\dot{P}_{t-1}$	-0.245 (0.133)	R^2	0.999
$\dot{P}_{t-2}$	0.011 (0.130)	$\bar{R}^2$	0.991
RB_t	-1383.750 (343.509)	F	127.1
RB_{t-1}	-417.836 (348.926)	DW	3.133
RB_{t-2}	1279.741 (417.541)		
DRD_t	-17.362 (44.205)		
DRD_{t-1}	-7.678 (12.322)		
DRD_{t-2}	-55.985 (5.507)		
ANN_t	0.0043 (0.0034)		
ANN_{t-1}	0.0077 (0.0017)		
ANN_{t-2}	0.0010 (0.0019)		
ΔUTY_t	-0.0107 (0.114)		
ΔUTY_{t-1}	0.022 (0.109)		
ΔUTY_{t-2}	0.687 (0.086)		

Dependent Variable $SURR_t$
Sample Period 1946-1968
Ordinary Least Squares
Standard Errors in ( )
DW Durbin Watson Statistic
A key to the abbreviations can be found in Appendix 9.1.

Table 9.5.3 Second Order Inflationary Expectations Model

(Cont.)

	B	C
CONST	-21.678 (9.928)	3.389 (5.465)
$SURR_{t-1}$	-0.125 (0.138)	0.151 (0.145)
$SURR_{t-2}$	0.788 (0.211)	0.320 (0.158)
$\dot{P}_{t-1}$	-0.480 (0.186)	-0.367 (0.247)
$\dot{P}_{t-2}$	0.219 (0.179)	0.259 (0.233)
RB_t	-441.751 (349.514)	-210.349 (421.224)
RB_{t-1}	445.147 (413.043)	191.705 (491.796)
RB_{t-2}	1598.301 (524.777)	628.724 (525.970)
DRD_t	110.677 (26.377)	57.015 (21.790)
DRD_{t-1}	-19.425 (18.099)	8.784 (19.256)
DRD_{t-2}	-46.705 (7.553)	-48.402 (8.875)
ANN_t	-0.0041 (0.0017)	
ANN_{t-1}	0.0021 (0.0013)	
ANN_{t-2}	-0.0047 (0.0015)	
ΔUTY_t	-0.208 (0.145)	-0.045 (0.169)
ΔUTY_{t-1}	-0.102 (0.168)	0.165 (0.193)
ΔUTY_{t-2}	0.789 (0.130)	0.835 (0.171)
R^2	0.993	0.982
$\bar{R}^2$	0.975	0.955
F	55.70	37.0
DW	3.009	2.142

explaining the variation of $SURR_t$. All models have reasonable values for $\bar{R}^2$ and the pattern of residuals reveals that none seem to be troubled by autocorrelation (the DW figure is overestimated in these cases). Moreover a comparison of Tables 9.5.2 and 9.5.3 indicates some interesting results:

i) the second order models seem to behave better than their fist order counterparts (in certain respects): they have better $\bar{R}^2$ and F figures. Furthermore, either the coefficient of $SURR_{t-1}$ or that of $SURR_{t-2}$ is always significant (at the 5% level) in Table 9.5.3, whereas Table 9.5.2 does not show a significant coefficient for $SURR_{t-1}$;

ii) the coefficients of $SURR_{t-1}$ are largely positive and those of $SURR_{t-2}$ (Table 9.5.3) also have a positive sign. Thus by reference to Equation (A 9.2.7) we see that

$$1 \geq \frac{n_{t-1} \cdot (1-\lambda_t^0)}{n_{t-1} - 1} \geq 0 \quad \text{so that approximately}$$

$$0 < \lambda_t^0 \leq 1$$

But also

$$1 \geq \frac{-\lambda_t^1 \cdot n_{t-2}}{n_{t-1} - 1} \geq 0 \quad \text{so that approximately}$$

$$0 \geq \lambda_t^1 \geq -1$$

We must therefore conclude that the addition of the second order adaptive expectations <u>reduces</u> the expected rate of inflation (although the first order will still have a positive effect).

Consequently, it would seem that, over the period 1946-1968 (when inflation rates were comparatively moderate), policyholders generally took an optimistic view of the course of future inflation so that the rate of change of inflation had a negative effect on expectations;

iii) the coefficient of $\dot{P}_{t-1}$ is consistently negative in both Tables 9.5.2 and 9.5.3 although only significantly so in the latter case (at the 5% level in Model B). Again, the coefficient of $\dot{P}_{t-2}$ in Table 9.5.3 is consistently positive. Both of these results imply a negative coefficient for 'a' in Equations (A 9.2.7) and (A 9.2.3) . Thus inflationary expectations discourage surrenders of life insurance. There may be several reasons for this result but first the possibility that $\dot{P}^e$ is consistently negative may be discounted since the coefficient of $SURR_{t-2}$ is generally smaller than that of $SURR_{t-1}$.
Of course, if surrenders were largely because there was 'No Further Need' for the life insurance arrangements then we would not expect inflation expectations to have any effect. But inflationary expectations may well prevent surrenders in the 'To Avoid Expense' category since the real value of premiums paid is reduced. Another factor of importance occurs in connection with with-profits policies (which form the major part of savings-

based life insurance). If positive inflation expectations are closely linked with expectations of an increase in the reversionary bonus rate then surrenders may well be discouraged because of the increased competitiveness of the life insurance policy. A similar point arises because of the relationship between inflation and the rate of interest: if interest rates are expected to rise then correspondingly capital values are expected to fall. This may reduce surrender rates since the surrender values paid are related to the market value of the underlying securities.

A final possible explanation lies in the protection element incorporated into most savings-based life insurance policies. It may be that if inflation has a positive effect on the demand for protection-based life insurance then this will discourage surrenders. Chapter Eight, however, gives no indication that inflationary expectations have this effect on protection-based policies;

iv) the coefficients of RB_t (which are negative) and RB_{t-1} and RB_{t-2} (which are positive) <u>all</u> indicate that, contrary to the results of Table 9.5.1, an increase in reversionary bonus <u>discourages</u> surrenders. We would not expect this result if surrenders were taking place to obtain cash (in fact none of the 'Cash' variables are important). On the other hand, if surrenders were occurring as a result of competitive

Table 9.5.4 Second Order Inflationary Expectations Model

	A
CONST	12.636 (6.731)
$SURR_{t-1}$	0.455 (0.227)
$SURR_{t-2}$	0.122 (0.262)
$\dot{P}_{t-1}$	-0.345 (0.306)
$\dot{P}_{t-2}$	-0.305 (0.308)
RB_t	-528.779 (657.207)
RB_{t-1}	247.242 (799.207)
RB_{t-2}	584.918 (814.142)
DRD_t	32.340 (35.563)
DRD_{t-1}	16.588 (32.705)
DRD_{t-2}	-46.033 (14.903)
R^2	0.93
$\bar{R}^2$	0.87
F	15.4
DW	2.027

Dependent Variable $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 9.1.

Table 9.5.4 Second Order Inflationary Expectations Model (Cont.)

	B
CONST	17.684 (23.803)
$SURR_{t-1}$	1.336 (0.447)
$SURR_{t-2}$	-1.044 (0.606)
$\dot{P}_{t-1}$	0.841 (0.614)
$\dot{P}_{t-2}$	0.060 (0.533)
RB_t	-436.232 (1018.840)
RB_{t-1}	476.942 (750.264)
RB_{t-2}	65.252 (875.440)
DRD_t	-61.140 (104.810)
DRD_{t-1}	83.304 (70.537)
DRD_{t-2}	67.977 (75.406)
BSA_t	1.554 (1.151)
BSA_{t-1}	1.290 (0.890)
BSA_{t-2}	0.355 (0.718)
DIV_t	-1.248 (0.509)
DIV_{t-1}	-0.262 (0.417)
DIV_{t-2}	-0.440 (0.350)
R^2	0.97
$\bar{R}^2$	0.91
F	14.4
DW	2.808

Dependent Variable $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 9.1.

Table 9.5.4 Second Order Inflationary Expectations Model (Cont.)

	C
CONST	1.424 (1.391)
$SURR_{t-1}$	0.203 (0.308)
$SURR_{t-2}$	0.447 (0.479)
$\dot{P}_{t-1}$	-0.746 (0.511)
$\dot{P}_{t-2}$	0.539 (0.853)
RB_t	-760.068 (1133.463)
RB_{t-1}	-14.366 (985.298)
RB_{t-2}	1385.390 (984.664)
DRD_t	56.174 (47.721)
DRD_{t-1}	16.042 (35.982)
DRD_{t-2}	-57.360 (36.231)
ΔBSW_t	-0.528 (0.412)
ΔBSW_{t-1}	-0.561 (0.667)
ΔBSW_{t-2}	0.801 (1.168)
R^2	0.94
$\bar{R}^2$	0.86
F	11.5
DW	1.870

Dependent Variable $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 9.1.

Table 9.5.5 Second Order Inflationary Expectations Model

	A
CONST	18.715 (16.274)
$SURR_{t-1}$	0.447 (0.306)
$SURR_{t-2}$	0.510 (0.456)
$\dot{P}_{t-1}$	-0.049 (0.486)
$\dot{P}_{t-2}$	0.468 (0.664)
RB_t	-243.017 (932.604)
RB_{t-1}	412.286 (1093.156)
RB_{t-2}	-507.204 (1262.755)
DRD_t	29.078 (43.982)
DRD_{t-1}	-12.325 (54.371)
DRD_{t-2}	-54.862 (23.749)
$\Delta(\frac{CONS}{PDI})_t$	122.694 (94.204)
$\Delta(\frac{CONS}{PDI})_{t-1}$	80.498 (101.539)
$\Delta(\frac{CONS}{PDI})_{t-2}$	4.672 (67.196)
ΔHPC_t	-0.800 (0.803)
ΔHPC_{t-1}	0.588 (0.680)
ΔHPC_{t-2}	1.181 (0.058)
R^2	0.96
$\bar{R}^2$	0.84
F	8.3
DW	2.348

Dependent Variable $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 9.1.

Table 9.5.5 Second Order Inflationary Expectations Model

(Cont.)

	B
CONST	5.987 (8.277)
$SURR_{t-1}$	0.619 (0.331)
$SURR_{t-2}$	0.110 (0.321)
$\dot{P}_{t-1}$	0.358 (0.496)
$\dot{P}_{t-2}$	-0.031 (0.413)
RB_t	353.835 (807.934)
RB_{t-1}	-184.682 (922.302)
RB_{t-2}	615.519 (959.251)
DRD_t	79.824 (46.644)
DRD_{t-1}	-8.610 (41.704)
DRD_{t-2}	-79.050 (23.135)
ΔU_t	-2.724 (2.103)
ΔU_{t-1}	-3.259 (2.045)
ΔU_{t-2}	0.280 (1.443)
$BUST_t$	-0.129 (0.244)
$BUST_{t-1}$	0.265 (0.314)
$BUST_{t-2}$	-0.372 (0.227)
R^2	0.96
$\bar{R}^2$	0.86
F	9.6
DW	2.623

Dependent Variable $SURR_t$

Sample Period 1946-1968

Ordinary Least Squares

Standard Errors in ( )

DW Durbin Watson Statistic

A key to the abbreviations can be found in Appendix 9.1.

pressure then this result is quite acceptable (note that this is the same type of conclusion as in iii) above)[6];

v) the variable DRD_t (Death Rates of Dependents) has a consistent significantly positive effect on $SURR_t$;

vi) again contrary to the indications of Table 9.5.1, the variable ΔUTY has a negative effect on surrenders. This result cannot really be explained in terms of the competitive effects of Unit Trusts. A possible interpretation may be that the variable ΔUTY reflects the change in the yield earned by the life office on its own investments. ΔUTY_t would then have a positive effect (ΔUTY_{t-1} and ΔUTY_{t-2} negative) for similar reasons to those of iii) above.

To finish this Section, a number of other explanatory variables were applied to the model of Table 9.5.3 (second order). ΔUTY was omitted because of its similarity to RB. ANN and BUST were largely omitted because of the insignificance exhibited in Tables 9.5.2 and 9.5.3. The results of these final models are explained in Tables 9.5.4 and 9.5.5.

The results from these final models show very few

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(6) Dryden Gilling-Smith notes that, before the tax regulations were changed in the late 1960's, a policyholder contemplating the surrender of bonus might have been better advised to obtain a loan against the policy (see Carter (2) p.2.1.3 -24)

surprises and by-and-large, they correspond to those from Table 9.5.3. Thus, for example, the coefficients of $SURR_{t-1}$ and $SURR_{t-2}$ are predominantly positive (with those of $SURR_{t-1}$ exhibiting a reasonable degree of significance - generally at around the 5% level).

The coefficients of $\dot{P}_{t-1}$ and $\dot{P}_{t-2}$, typically have the same signs as before although none of them exhibits any degree of significance. Similarly, the signs of RB_t, RB_{t-1} and RB_{t-2} correspond to those of Table 9.5.3 but again without the same level of significance.

In Table 9.5.4, Model B includes those variables in the 'No Further Need' category of Section 9.4. BSA_t does indeed show a positive coefficient but it is not particularly significant. The coefficient of DIV_t, on the other hand, is significant at the 5% level but has a negative sign (this same sign is born out in other models containing DIV but not reported). Thus it would seem that Divorce discourages surrender: a priori, this seems a difficult conclusion to accept. However, the results of Model B (Table 9.5.4) must be treated with suspicion because of the peculiar behaviour of $SURR_{t-1}$ and $SURR_{t-2}$.

In Table 9.5.4 , Model C includes ΔBSW in an attempt to examine surrenders in the 'Substitution' category. The negative coefficient of ΔBSW_t implies that as the shares and deposits held by building societies decrease, so surrenders of savings-based life insurance increase. Although this behaviour cannot be explained by the effects of competitive pressure, it may be that as the general level of prosperity falls, more money is withdrawn from the building societies and, at the same time, withdrawn from the life offices.

Table 9.5.5 , Model B attempts to characterise the situation where surrenders occur because the premiums are too expensive. None of the coefficients of or BUST are significant at the 5% level and the only indication that these variables might positively explain surrenders is given by ΔU_{t-1} and $BUST_{t-2}$ (significant at the 20% level).

Finally, Model A (table 9.5.5) includes those variables in the 'Cash Required' category of the previous Section. Again, the results are disappointing with none of the coefficients of $\Delta(\frac{CONS}{PDI})$ or ΔHPC significant at even the 10% level. However the signs of $\Delta(\frac{CONS}{PDI})_t$ and ΔHPC_t (positive and negative respectively) show some indication that the desire for cash (for spending) cannot be ignored.

9.6 Conclusion

Section 9.2 was concerned with describing a theoretical framework for the analysis of voluntary exits from ordinary life insurance contracts. Four main reasons for termination were explained: 'No Further Need', 'Substitution', 'Cash Required' and 'Expense'. The framework indicated that terminations for the purpose of obtaining cash would not normally occur until the policyholder regarded the premium payments as contractual. The framework also indicated that terminations because of the expense of the (contractual) premiums would most likely be in the form of policies converted to 'paid up'.

Section 9.3 was devoted to a description of how the dependent variable $SURR_t$ was derived. Since the only data available related to surrender values paid, the surrender rate so produced ($SURR_t$) applies only to the surrender of savings-based life insurance whose premiums are more then likely to be regarded by the policyholder as contractual.

Section 9.5 attempted a variety of models of analysis on $SURR_t$ and most success was met with the use of the second order inflationary expectations model (described in Appendix 9.2).

Most of the explanatory variables described in Section 9.4 were tested to see if they contributed anything to the explanation of $SURR_t$. The results of Section 9.5 , although unfortunately not very conclusive, show that as suspected, the 'Expense' motive is not important as an explanation of surrenders. The 'No

Further Need' motive appeared to be moderately important with the variables DRD_t and DIV_t showing some significance. The 'Cash Required' motive did not show up as very important but the variables reflecting the 'Substitution' motive did, on the other hand, show themselves to contribute to the explanation of $SURR_t$. Furthermore, the behaviour of the variables RB_t and $\dot{P}_{t-1}$ also indicated that surrenders have taken place because of the effects of competitive pressure.

On the whole, inflationary expectations do not seem to encourage surrenders, and in some models (Table 9.5.3) they seem to have the opposite effect. This behaviour may be for the reasons described in Section 9.5 or may be caused by the fact that if prices are expected to increase then, as wages too are expected to rise, the policyholder will be better able to afford to pay the premiums. Another explanation may lie in the fact that consumers generally seem to take an optimistic view of the rate of increase in future inflation - expecting increases in inflation to 'tail-off'.

Appendix 9.1 Abbreviations and Sources

Abbreviation	Description	Source
ANN_t	Surrender Rates (as a percentage of 'original population')	Table 9.3.2
BSA_t	Real Building Society Advances on Mortgages per capita (1963 £)	Annual Abstract of Statistics (7)
BSW_t	Real Building Society Shares and Deposits per capita (1963 £)	Annual Abstract (7)
$BUST_t$	The Number of Bankruptcies Declared in England and Wales per million UK population	Annual Abstract (7)
$CONS_t$	Real Consumers Expenditure per capita (1963 £)	Key Statistics (8)
DIV_t	The Number of Divorces in England and Wales (Decrees Absolute Granted) 000's	Annual Abstract (7)
DRD_t	Death Rate of Dependents (Deaths as a % of UK population under 15)	Annual Abstract (7)
HPC_t	Total Real Hire Purchase Credit Outstanding per Capita (1963 £)	Annual Abstract (7)
LIQ_t	Real Liquid Assets per capita (1963 £)	Annual Abstract (7)
$\dot{P}_t$	Actual Rate of Inflation of RPI (compounded continuously)	Equation (7.5.1)
PDI_t	Real Personal Disposable Income per capita (1963 £)	Key Statistics (8)
RB_t	The Rate of Compound Reversionary Bonus Declared	Table A 6.1.1
$SURR_t$	Surrender Rates (as a percentage of 'original population')	Table 9.3.2
T_t	Standard Rate of Income Tax (p in £)	Key Statistics (8)
U_t	% UK Unemployment	Key Statistics (8)
UTY_t	Real Net of Tax Yield on Unit Trusts	Economist Intelligence Unit (5)

Appendix 9.2 The Derivation of the Second Order Inflation Expectations Process

Using the notation developed in Section 7.5 we now assume that the consumer's adaptation to inflationary expectations has two components: first according to the error in their previous expectations of $P^e_{t+n_t-1}$ (as before) and secondly according to the error involved in the expectations of the preceding future price level $P^e_{t-1+n_{t-1}-1}$.

So instead of Equation (7.5.3) we now have

$$P_t \cdot \exp(\,(n_t-1)\cdot {}_{n_{t+1}}\dot{P}^e_{t+1}\,) =$$

$$P^e_{t+n_t-1} \cdot \left[\frac{P_t \cdot \exp(\,(n_t-1).\dot{P}_t\,)}{P^e_{t+n_t-1}}\right]^{\lambda^0_t} \cdot \left[\frac{P_{t-1} \cdot \exp(\,(n_{t-1}-1).\dot{P}_{t-1}\,)}{P^e_{t+n_{t-1}-2}}\right]^{\lambda^1_t}$$

(A 9.2.1)

Substitution for $P^e_{t+n_{t-1}}$ using Equation (7.5.2) and P_t using Equation (7.5.1) yields the following relationship:

$${}_{n_{t+1}}\dot{P}^e_{t+1} = \frac{1}{n_t-1} \cdot (\; {}_{n_t}\dot{P}^e_t \cdot (\,n_t - \lambda^0_t . n_t\,) - \lambda^1_t . n_{t-1} \cdot {}_{n_{t-1}}\dot{P}^e_{t-1}$$

$$+ \dot{P}_t \cdot (\,\lambda^0_t . n_t - 1\,) + \dot{P}_{t-1} \cdot \lambda^1_t \cdot n_{t-1}\;)$$

(A 9.2.2)

Equation (A 9.2.2) can then be incorporated into the demand function $D_t = D_t(\,{}_{n_t}\dot{P}^e_t\,, X_{it}, u_t)$ in the following way:

Let the demand function be

$$D_t = CONST + a . {}_{n_t}\dot{P}^{e}_{t} + \Sigma\, b_i . X_{it} + u_t \qquad (A\ 9.2.3)$$

then from Equation (A 9.2.2), this can be written as

$$D_t = CONST + \frac{a}{n_{t-1} - 1} . \Big({}_{n_{t-1}}\dot{P}^{e}_{t-1} . (n_{t-1} - \lambda^{0}_{t} . n_{t-1}) - \lambda^{1}_{t} . n_{t-2} . {}_{n_{t-2}}\dot{P}^{e}_{t-2} + \dot{P}_{t-1} . (\lambda^{0}_{t} . n_{t-1} - 1) + \dot{P}_{t-2} . \lambda^{1}_{t} . n_{t-2} \Big) + \Sigma\, b_i . X_{it} + u_t \qquad (A\ 9.2.4)$$

Lagging (A 9.2.3) by one and two time periods respectively we get

$$D_{t-1} = CONST + a . {}_{n_{t-1}}\dot{P}^{e}_{t-1} + \Sigma\, b_i . X_{it-1} + u_{t-1} \qquad (A\ 9.2.5)$$

and

$$D_{t-2} = CONST + a . {}_{n_{t-2}}\dot{P}^{e}_{t-2} + \Sigma\, b_i . X_{it-2} + u_{t-2} \qquad (A\ 9.2.6)$$

Finally, substituting for ${}_{n_{t-1}}\dot{P}^{e}_{t-1}$ and ${}_{n_{t-2}}\dot{P}^{e}_{t-2}$ in Equation (A 9.2.4) (from (A 9.2.5) and (A 9.2.6)):

$$\begin{aligned} D_t = CONST &+ \frac{n_{t-1} . (1-\lambda^{0}_{t})}{n_{t-1} - 1} . (D_{t-1} - CONST - \Sigma\, b_i . X_{it-1} - u_{t-1}) \\ &- \frac{\lambda^{1}_{t} . n_{t-2}}{n_{t-1} - 1} . (D_{t-2} - CONST - \Sigma\, b_i . X_{it-2} - u_{t-2}) \\ &+ \frac{a}{n_{t-1} - 1} . ((\lambda^{0}_{t} . n_{t-1} - 1) . \dot{P}_{t-1} + \lambda^{1}_{t} . n_{t-2} . \dot{P}_{t-2}) \\ &+ \Sigma\, b_i . X_{it} + u_t \end{aligned} \qquad (A\ 9.2.7)$$

Chapter Nine : References

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CHAPTER TEN : SUMMARY AND CONCLUSIONS

This study has attempted to analyse the demand for life insurance and the factors which influence that demand. The analysis has been both theoretical and practical; the latter aspect using data on UK ordinary life insurance over the years 1946 to 1968. The demand for life insurance has been examined on the basis of new business premium income and also business in force (by looking at the surrenders of existing life insurance contracts). Some special attention has been devoted to the effects of inflation (mainly in the form of inflationary expectations) on the various aspects of demand.

Chapter Two examines the consumer's motives for the purchase of protection-based life insurance and begins by breaking income down into it's component parts: current consumption, lifetime saving and bequesting flow. The bequest made at the end of the consumer's lifetime can in turn be split into two parts: to provide protection and to provide non-lifetime savings stocks: consequently the corresponding bequesting flows count as consumption and non-lifetime saving respectively. The argument is extended to show that bequests for protection purposes yield satisfaction at every instant in the consumer's lifetime and are not conditional upon his death: this is a major departure from the standard Yaari 'life cycle' models.

The model restricted to lifetime saving only, with any bequest solely via life insurance (for protection purposes), shows that the bequest plan can be 'humped' shaped. However the greater the inception age, the more likely a downward sloping bequest plan becomes. It is contended that this is more realistic than the corresponding 'life cycle' result (with bequests independent of inception age) because protection is most needed (and desired) when uncertainty about death is greatest (ie. at younger ages) and when premature death would do most harm to dependents.

The models which describe non-lifetime saving do so in two ways, first by treating it as precautionary saving and secondly by dropping the restriction of 'lifetime' saving only (although a bequest for protection purposes is still provided by the purchase of life insurance). The former type does not alter the basic pattern of consumption or bequest although each bequest is smaller than it would have been in the absence of precautionary saving. The latter type alters the pattern of consumption and concentrates it more towards the younger ages.

Finally, Chapter Two extends the existing theory to allow the consumer to experience unanticipated changes in his circumstances which might lead him to surrender his protection-based life insurance contract. The results show first that there will be a certain stickiness in the consumers' reactions to these changes and secondly that the higher is the surrender value the more likely that surrender becomes.

Chapter Three extends the theory of Chapter Two to describe the effects of anticipated and unanticipated inflation on the purchase and surrender of protection-based life insurance. The major conclusions are that both anticipated and unanticipated inflation may effect the bequest by bringing forward the bequest plan to younger ages. Whether or not this reduces the overall purchase of life insurance depends on the extent of the offsetting 'income' effect but it seems that if $\dot{P} > \delta$ then that new purchase may well be increased by older policyholders.

An examination of the effects of unanticipated changes in $\dot{P}$ at time τ shows that surrender is more likely if $\dot{P}$ is revised in an upward direction. Furthermore, surrender becomes less likely with a decrease in the size of the surrender value paid and also if consumers are not highly 'impatient' for early consumption and/or bequest.

Chapter Four is mainly devoted to a review of the literature concerning saving and the purchase of savings-based life insurance. The major points of interest concern the criticisms of the alternative models: that little consideration has been given to an appropriate distinction between new and in force business and also between saving and protection-based policies.

The objective of Chapter Five is to split the savings and protection elements of the new business premiums on standard life insurance contracts. Temporary insurance premiums are allocated entirely to protection as are the temporary elements in endowment and whole of life contracts. The results show that the financial saving ratio (financial saving as a proportion of premium) is a declining function of inception age as well as the calculation rate of interest. The addition of the bonus and expenses loadings has the effect of 'swivelling' the endowment ratio in an anticlockwise direction: the effect on the whole of life ratio is more simply limited to movements up (dependent on the size of the bonus loading) and down (depending on the size of the expenses loading). Finally a discussion of policyholder savings stocks shows how these depend on the time since inception so that a more rigorous analysis cannot be conducted.

Chapter Six is concerned with the application of Chapter Five's theory to differentiate between protection-based and savings-based new business premium income for UK ordinary life insurance over the period 1946 to 1968. This is done by calculating, for each year, an appropriate pure premium rate (based on realistic assumptions about interest and mortality) and then using this rate to derive aggregate pure premiums and hence the desired information. The data is then utilised to provide the dependent variables for the analysis of Chapter Eight.

Chapter Seven is basically devoted to a description of the sources and derivation of the many explanatory variables to be used in Chapter Eight. A series is derived for a measure of permanant income and a theory of long-term inflation expectations is developed which is similar to the basic adaptive-expectations hypothesis.

Chapter Eight is the first of two chapters applying econometric techniques to analyse the influences on the demand for ordinary UK life insurance. For a number of reasons, the results of Chapters Eight and Nine are not as clear-cut as perhaps one would like: there are problems with data that are common to most time-series econometric studies (eg problems of aggregation, surrogation and availability) as well as those particular to this work (mainly caused by the only approximate methodology described in Chapter Six. A similar study conducted in a few years time would have less trouble because of the greater availability of data (both reported to the Department of Trade and retained by the individual life offices)). Additionally, the usual econometric problems of autocorrelation and multicollinearity have been encountered and sometimes their appropriate solutions have limited applicability.

The first part of Chapter Eight is devoted to an analysis of the Financial Saving Ratio (that is, new financial saving via ordinary life insurance expressed as a proportion of personal disposable income). The thinking behind this study developed along two lines: reflecting first the similar work conducted on the aggregate Saving Ratio (reported in Chapter Four) and secondly examining specifically the new semi-contractual saving via life insurance (rather than the demand for this characteristic in life insurance contracts). No real success was obtained with any form of adaptive expectations model allowing automatically for inflation and the best results were found using a 'manual expectations' formulation (by choosing that model yielding the best R^2, $\bar{R}^2$ and F coefficients). The major results indicated that inflation expectations have a negative effect on new contractual saving (via life insurance) whereas permanent income has a positive effect. Building Society Advances were positively associated with FSR and, additionally, the inclusion of this variable seems to shorten the saver's time horizon for inflation expectations (an alternative view could be that mortgagees are less worried about the effects of inflation because of the fixed-price nature of the mortgage). Models that were linear in the logarithms were also attempted and these indicated much the same results giving a (constant) permanent income elasticity figure of around 2.0.

The second part of Chapter Eight is concerned with testing a demand model for the protection-based and savings-based elements of new life insurance (new premium expenditures were used as a surrogate for these quantities). Because the demand equation is just one part of the Demand and Supply model the method of Two Stage Least Squares was employed to avoid simultaneous equation bias and identification problems.

Initially models were attempted that had only Own-Price as the included endogenous variable but these proved too simplistic to be of any use. Consequently, all equations subsequently included both prices as endogenous variables; however one of the major results is that the coefficients of these prices are insignificantly different from zero in both Demand equations (ie the expenditure/price elasticities are zero).

The most important demand explanatory variable turned out to be Permanent Income (with 'average' elasticity figures of around 3.5 and 2.5 for the savings-based and protection-based cases respectively). Building Society Advances, Building Society Wealth (a negative effect) and the Standard Rate of Income Tax were the most important savings-based (demand) variables. Inflationary expectations were bound to have a marginal negative effect on demand (D^{Sg}). The demand for protection-based life insurance is negatively influenced by Births and Tax whereas Unemployment has a positive effect. Inflationary expectations again had a negative effect and policyholders' memories of inflation in both savings and protection-based cases are very short.

The Supply models both show that Prices affect Supply producing the conventional upward-sloping Expenditure Supply curve in the savings-based case: administrative costs, the price of protection-based insurance and inflationary expectations have a negative effect while the coefficient of yield on invested funds is positive. The supply of protection-based life insurance is more unconventional because there is strong evidence that the supply (expenditure) curve is backward bending; the price coefficient for savings-based insurance is now positive. Inflationary expectations do not appear to have any effect and the major explanatory variables are yield on invested funds and the adjustment to the optimum gearing level.

Chapter Nine represents the results of an econometric analysis into the demand for in force savings-based life insurance: this is undertaken via study of surrender rates. Inflationary expectations are incorporated by the use of a second-order adaptive expectations approach which indicates that policyholders generally took an optimistic view of the trends in future inflation. The major conclusions are that inflation and the reversionary bonus declared have a negative effect on surrender rates while divorces and the deaths of dependents have a positive effect. Consequently, the main reasons for surrender appear to be in the categories of 'No Further Need' and competitive pressure.

The main results of this study can hence be summarised as follows:

a) life insurance is purchased for three main motives -- lifetime saving, non-lifetime saving and the provision of protection. These three functions are primarily undertaken by endowment, whole of life and temporary life insurance contracts respectively. Non-lifetime saving and the provision of protection for dependents are provided by making a bequest;

b) a life-cycle model explaining consumption and protection-based bequest plans, shows these to be determined by the force of interest, the rate of impatience and (in the latter case) by the force of mortality, which depends on the age at inception of the insurance contract;

c) the theory predicts that the decision to surrender a protection-based life insurance tends to be 'sticky'. Any consumer who wishes to revise his insurance arrangements may well put up with a sub-optimal situation because of the penalty involved in the termination of a contract;

d) inflation, both anticipated and unanticipated, can affect the purchase of life insurance by bringing forward consumption and bequests in time. The actual effect of inflation on premium expenditures is clouded by the reduction in the cost of the bequest plans;

e) because most life insurance contracts include elements of both saving (whether lifetime or non-lifetime) and protection (for dependents), an analysis of aggregate premium expenditures cannot properly be conducted without separating these two elements. An indication has been given of how this may be achieved and the method is implemented on UK data over the period 1946 - 1968;

f) an analysis of data has indicated that new financial saving via life insurance was adversely affected by inflation expectations over the period. However, this saving represents only a very small part of Personal Disposable Income (0.2% average, 1960 - 1968);

g) the premium expenditure on new renewable life insurance demanded does not seem to depend on price but is mainly affected by income. The (permanent) income elasticity of demand averages around 3.5. Inflationary expectations have a marginally negative effect;

h) surrenders of existing contractual savings-based life insurance seem to have been caused mainly by either a lack of need or by competitive pressure. Second-order inflationary expectations play a negative role in discouraging surrenders.

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APPENDIX

Estimated Mortality Tables 1945/1969
Source : SUBROUTINE MORTALITYTABLE.
See Section 6.6
Ultimate ; Medical and Non-medical Combined ;
Males.

	x	l_x	q_x
1945	10	999999.00000001	0.00099672
1945	11	999002.27795410	0.00099672
1945	12	998006.54950220	0.00099672
1945	13	997011.81324006	0.00098988
1945	14	996024.80755509	0.00097393
1945	15	995054.83383160	0.00138106
1945	16	993680.00201364	0.00131423
1945	17	992374.67745210	0.00123223
1945	18	991151.64006501	0.00125181
1945	19	989911.11067056	0.00127032
1945	20	988053.60538120	0.01114354
1945	21	977310.25006907	0.01161010
1945	22	965957.71042304	0.01174027
1945	23	954611.32152614	0.01199940
1945	24	943106.10541535	0.01208110
1945	25	931712.19151825	0.00810917
1945	26	924150.47093201	0.00829457
1945	27	916480.47355007	0.00833178
1945	28	908844.50934847	0.00843457
1945	29	901178.80029296	0.00862150
1945	30	893409.40090571	0.00484652
1945	31	889079.68030458	0.00502479
1945	32	884612.33482361	0.00509154
1945	33	880108.91574895	0.00519281
1945	34	875538.68057251	0.00532933
1945	35	870872.66406111	0.00273379
1945	36	868491.85950573	0.00285641
1945	37	866009.35405732	0.00299794
1945	38	863413.11126235	0.00320126
1945	39	860649.96279407	0.00343958
1945	40	857689.34030151	0.00231715
1945	41	855701.94053211	0.00255383
1945	42	853516.62973786	0.00282636
1945	43	851104.28440961	0.00316389
1945	44	848411.48319240	0.00356546
1945	45	845386.50672040	0.00377965
1945	46	842191.24495677	0.00425929
1945	47	838604.10650265	0.00480983
1945	48	834570.56046296	0.00542999
1945	49	830038.85360821	0.00611834
1945	50	824960.39727784	0.00656175

1945	51	819596.70746613	0.00727915
1945	52	813630.74330243	0.00812978
1945	53	807016.10050202	0.00906513
1945	54	799700.39448548	0.01006978
1945	55	791647.59043122	0.01114336
1945	56	782825.97510158	0.01234236
1945	57	773164.05732727	0.01365633
1945	58	762605.47322083	0.01509572
1945	59	751093.39533234	0.01667070
1945	60	738572.13935090	0.01622031
1945	61	726587.84000397	0.01789495
1945	62	713585.58890533	0.01973284
1945	63	699504.52206420	0.02174807
1945	64	684291.64996338	0.02396022
1945	65	667895.86845398	0.02675500
1945	66	650026.31607056	0.02945384
1945	67	630880.54409790	0.03243372
1945	68	610418.74183655	0.03570133
1945	69	588625.98163605	0.03926090
1945	70	565500.68974304	0.04165434
1945	71	541945.13069153	0.04581290
1945	72	517117.05441665	0.05036472
1945	73	491072.59661866	0.05533810
1945	74	463897.57175826	0.06077266
1945	75	435705.28110695	0.07042974
1945	76	405018.07317199	0.07727693
1945	77	373740.32316878	0.08461946
1945	78	342114.62053299	0.09263594
1945	79	310422.51260478	0.10131720
1945	80	278971.35509108	0.09562926
1945	81	252237.73663901	0.10464257
1945	82	225842.93189048	0.11411648
1945	83	200070.07982063	0.12428703
1945	84	175203.96300696	0.13516872
1945	85	151521.86696149	0.16913980
1945	86	125893.48894118	0.18313016
1945	87	102338.59411906	0.19788615
1945	88	82488.26062678	0.21346038
1945	89	64884.73935031	0.22964169
1945	90	49984.49798107	0.24656943
1945	91	37659.84883117	0.26411933
1945	92	27713.15494942	0.28222507
1945	93	19891.86339449	0.30081446
1945	94	13908.10326447	0.31980455
1945	95	9460.22855175	0.33906148
1945	96	6252.44021255	0.35854788
1945	97	4010.64104146	0.37809575
1945	98	2494.23468891	0.39760630
1945	99	1502.51126511	0.41697479
1945	100	876.00194989	0.43609782
1945	101	493.97941082	0.45483487
1945	102	269.30034941	0.47312211
1945	103	141.88839999	0.49084518
1945	104	72.24316307	0.50794057
1945	105	35.54792957	0.52431874
1945	106	16.96948400	0.53995540
1945	107	7.77915056	0.55476576
1945	108	3.46354422	0.56876564
1945	109	1.49359926	1.00000000

	x	l_x	q_x
1946	10	999999,00000001	0,00094937
1946	11	999049.62689973	0,00094937
1946	12	998101.15510560	0,00094937
1946	13	997153,58377076	0,00094387
1946	14	996212,39562989	0,00093164
1946	15	995284.87880708	0,00133140
1946	16	993959.75915240	0,00127750
1946	17	992689.97838593	0.00121137
1946	18	991487.46684267	0.00122715
1946	19	990270.76156616	0.00124268
1946	20	989040.76155855	0.00429817
1946	21	984789.69621277	0,00434249
1946	22	980513.25929262	0.00438295
1946	23	976215.71404268	0.00446525
1946	24	971856.66842652	0.00449667
1946	25	967486.35180006	0.00384037
1946	26	963763.10904693	0,00390541
1946	27	959999.22240840	0,00391009
1946	28	956237.85820008	0,00391426
1946	29	952447.08190918	0.00400419
1946	30	948633.29999543	0,00254829
1946	31	946215.91104890	0,00256920
1946	32	943765.97343445	0,00262492
1946	33	941288.66555786	0,00267969
1946	34	938766.30365754	0,00275207
1946	35	936181.91119385	0.00185105
1946	36	934448.98963107	0.00193776
1946	37	932638.24946595	0.00203523
1946	38	930740.11484527	0.00217516
1946	39	928715.60168458	0.00234089
1946	40	926541.57656199	0.00231190
1946	41	924399.50318910	0.00254973
1946	42	922042.53819275	0.00282350
1946	43	919439.14990235	0.00316155
1946	44	916532.29922485	0,00356306
1946	45	913266.63732911	0.00384363
1946	46	909756.37767793	0.00433160
1946	47	905815.67916871	0,00489126
1946	48	901385.09497853	0,00552154
1946	49	896408.05951691	0.00622124
1946	50	890831.29421234	0.00649215
1946	51	885047.88363648	0.00726907
1946	52	878614.40859222	0,00811959
1946	53	871480.42074585	0.00905504
1946	54	863589.12924195	0,01006083
1946	55	854900.70584870	0.01090386
1946	56	845578,99092865	0.01208014
1946	57	835364.27551270	0.01336954
1946	58	824195.83598328	0,01478233
1946	59	812012.29698836	0.01632858
1946	60	798753,29310608	0,01641590
1946	61	785641.04018403	0.01810755
1946	62	771415.00193025	0,01997053
1946	63	756009,43475342	0.02201325
1946	64	739367.20800018	0,02425540
1946	65	721433.56107331	0,02752352
1946	66	701577.17157740	0.03030169
1946	67	680318.19440460	0.03336782

1946	68	657017.46046737	0.03672725
1946	69	633463.67658996	0.04041665
1946	70	607861.19449616	0.04169719
1946	71	582515.09158324	0.04585899
1946	72	555801.53974914	0.05041347
1946	73	527781.65680693	0.05538890
1946	74	498548.41120529	0.06082456
1946	75	468224.42259217	0.07311036
1946	76	433992.36824417	0.08014669
1946	77	399208.44975280	0.08779974
1946	78	364158.05079269	0.09609274
1946	79	329165.10549927	0.10506904
1946	80	294580.04483794	0.10213801
1946	81	264492.22397232	0.11148516
1946	82	235005.26009802	0.12152776
1946	83	206445.60194587	0.13229001
1946	84	179133.67209433	0.14381004
1946	85	153372.45220957	0.16711521
1946	86	127733.91433620	0.18100345
1946	87	104013.63479899	0.19560530
1946	88	84150.65349483	0.21097040
1946	89	66397.35617160	0.22705265
1946	90	51321.66021066	0.24383102
1946	91	38807.84730060	0.26123878
1946	92	28069.73249387	0.27921079
1946	93	20064.83370168	0.29766360
1946	94	14513.25158869	0.31657170
1946	95	9918.76683850	0.33576730
1946	96	6588.36926413	0.35517608
1946	97	4248.33810973	0.37469314
1946	98	2656.51495162	0.39420286
1946	99	1609.30916607	0.41360263
1946	100	943.69465615	0.43279076
1946	101	535.27232939	0.45162969
1946	102	293.52745315	0.47005483
1946	103	155.55345514	0.48795305
1946	104	79.65067261	0.50525958
1946	105	39.40640723	0.52188463
1946	106	18.84080898	0.53775882
1946	107	8.70824412	0.55292253
1946	108	3.89325973	0.56727101
1946	109	1.68476527	1.00000000

	x	l_x	q_x
1947	10	999999.00000001	0.00090206
1947	11	999096.94413759	0.00090206
1947	12	998195.70190060	0.00090206
1947	13	997295.27279663	0.00089786
1947	14	996399.84177400	0.00088807
1947	15	995514.97310639	0.00126025
1947	16	994254.40319825	0.00122546
1947	17	993035.98431397	0.00117542
1947	18	991868.75440217	0.00118736
1947	19	990691.04845429	0.00119866
1947	20	989503.54541780	0.00170658
1947	21	987814.88048555	0.00172043
1947	22	986115.41671755	0.00173307
1947	23	984406.40577699	0.00176208
1947	24	982671.80162648	0.00177197
1947	25	980930.54121400	0.00152872
1947	26	979430.97515870	0.00154953
1947	27	977913.31534577	0.00155353
1947	28	976394.09474946	0.00157092
1947	29	974860.25804138	0.00158634
1947	30	973313.79892732	0.00155493
1947	31	971800.36811067	0.00158028
1947	32	970264.65601349	0.00160311
1947	33	968709.21787264	0.00163793
1947	34	967122.53772737	0.00168449
1947	35	965493.43285371	0.00155324
1947	36	963993.79089356	0.00162800
1947	37	962424.40505220	0.00171237
1947	38	960776.37999727	0.00183230
1947	39	959015.94829560	0.00197428
1947	40	957122.58502199	0.00217763
1947	41	955038.32714845	0.00240327
1947	42	952743.11060438	0.00266297
1947	43	950205.98577119	0.00298265
1947	44	947371.85471345	0.00336172
1947	45	944187.05609895	0.00377082
1947	46	940626.69845583	0.00424979
1947	47	936629.23684694	0.00479871
1947	48	932134.62615968	0.00541675
1947	49	927085.49028016	0.00610297
1947	50	921427.51207734	0.00662404
1947	51	915323.94150648	0.00741738
1947	52	908534.03360226	0.00828628
1947	53	901006.26131440	0.00924218
1947	54	892679.00234224	0.01027099
1947	55	883510.30314636	0.01125664
1947	56	873564.94352722	0.01247380
1947	57	862668.26660920	0.01380825
1947	58	850756.33037568	0.01527061
1947	59	837764.75970852	0.01687137
1947	60	823630.52059938	0.01688807
1947	61	809720.99016572	0.01863180
1947	62	794634.43357087	0.02055178
1947	63	778303.28207397	0.02265691
1947	64	760669.33098603	0.02496729
1947	65	741677.48014837	0.02836471
1947	66	720640.01242829	0.03122959
1947	67	698134.71733856	0.03438998

1947	68	674125.87789154	0.03785390
1947	69	648607.58180236	0.04165262
1947	70	621591.37953186	0.04323301
1947	71	594718.11192322	0.04754330
1947	72	566443.25134277	0.05225655
1947	73	536841.74806213	0.05740779
1947	74	506022.85174942	0.06303099
1947	75	474127.72887039	0.07137428
1947	76	440287.20395279	0.07824648
1947	77	405836.28018951	0.08571704
1947	78	371049.19643020	0.09381508
1947	79	336239.18489456	0.10258124
1947	80	301747.35300826	0.10557061
1947	81	269889.68941573	0.11520764
1947	82	238796.51323890	0.12555109
1947	83	208815.35075758	0.13663716
1947	84	180283.41407774	0.14846585
1947	85	153513.87852668	0.16652956
1947	86	127949.27935408	0.18031402
1947	87	104378.22986220	0.19486325
1947	88	84441.31666852	0.21017755
1947	89	66693.64803599	0.22621310
1947	90	51606.67144917	0.24294991
1947	91	39068.83503388	0.26037394
1947	92	28898.28191995	0.27827234
1947	93	20856.68950056	0.29673358
1947	94	14067.80937456	0.31562467
1947	95	10038.28694534	0.33484093
1947	96	6677.05763441	0.35429022
1947	97	4311.44143480	0.37386996
1947	98	2699.52301592	0.39346692
1947	99	1637.35001813	0.41298006
1947	100	961.15711205	0.43230068
1947	101	545.64035325	0.45131835
1947	102	299.38285145	0.46994301
1947	103	158.68997317	0.48807081
1947	104	81.23802929	0.50563567
1947	105	40.16116732	0.52254831
1947	106	19.17501719	0.53877004
1947	107	8.84397744	0.55423897
1947	108	3.94230059	0.56893833
1947	109	1.69937464	1.00000000

	x	l_x	q_x
1948	10	999999.00000001	0,00085477
1948	11	999144.22640992	0,00085477
1948	12	998290.18345643	0,00085477
1948	13	997436.87052155	0,00085184
1948	14	996587.21666718	0,00084498
1948	15	995745.11672211	0,00120196
1948	16	994548.27343751	0,00117365
1948	17	993381.02206422	0,00113892
1948	18	992249.64146424	0.00114721
1948	19	991111.32376100	0,00115505
1948	20	989960.53902436	0.00127414
1948	21	988705,18434907	0,00128161
1948	22	987438.05449677	0.00128842
1948	23	986165,81606293	0,00130725
1948	24	984876,64703370	0.00131258
1948	25	983583,91500856	0,00129802
1948	26	982307,20405290	0,00131404
1948	27	981016,41052247	0,00131652
1948	28	979724,88481904	0.00133042
1948	29	978421,43875597	0,00134311
1948	30	977107,31501771	0,00122118
1948	31	975914,09412305	0,00124140
1948	32	974702,59780226	0,00126017
1948	33	973474,30573273	0,00128866
1948	34	972219.82357790	0,00132073
1948	35	970929.95430386	0,00147388
1948	36	969498.92423249	0.00154679
1948	37	967999.31610975	0.00152937
1948	38	966422.09033967	0,00174563
1948	39	964735.07580671	0.00188519
1948	40	962918,29269411	0,00191480
1948	41	961074.49465654	0.00211470
1948	42	959042.11431880	0.00234471
1948	43	956793.43810377	0.00262698
1948	44	954279,96193696	0.00296113
1948	45	951454.21717073	0,00356123
1948	46	948065.87280274	0,00401584
1948	47	944260,48558808	0,00453217
1948	48	939980.93844605	0,00511563
1948	49	935172.34286500	0.00576360
1948	50	929782.37889100	0,00583636
1948	51	924355,83251192	0,00653631
1948	52	918313,95492555	0,00730337
1948	53	911607,17161560	0.00814754
1948	54	904179,81645702	0.00905718
1948	55	895990,49993897	0,01049927
1948	56	886583.25122071	0,01163793
1948	57	876265.25447846	0.01288670
1948	58	864973.08663940	0,01425558
1948	59	852642.39062500	0,01575448
1948	60	839209,45199585	0,01622914
1948	61	825589,80206588	0,01790925
1948	62	810804.11120236	0,01975908
1948	63	794783.36571503	0,02178750
1948	64	777467,02319338	0.02401375
1948	65	758797.12818910	0,02689241
1948	66	738391.24797059	0.02961392
1948	67	716524.58875276	0,03261555

1948	68	693154.74479675	0.03590561
1948	69	668266.60134887	0.03951367
1948	70	641360.93746475	0.03936872
1948	71	616591.69117737	0.04330165
1948	72	589892.25548553	0.04760509
1948	73	561810.37909699	0.05230606
1948	74	532424.28902434	0.05744134
1948	75	501841.12350081	0.06438096
1948	76	469532.10846617	0.07060010
1948	77	436383.09302907	0.07736481
1948	78	402622.39811325	0.08470308
1948	79	368519.04270172	0.09265316
1948	80	334374.58710861	0.09230234
1948	81	303511.03047179	0.10078283
1948	82	272922.33043289	0.10990412
1948	83	242927.04190635	0.11969604
1948	84	213849.63640022	0.13018016
1948	85	186010.65582656	0.15398748
1948	86	157367.34419249	0.16683440
1948	87	131113.05823134	0.18041626
1948	88	107458.13106061	0.19473730
1948	89	86532.02491187	0.20976219
1948	90	68380.87811279	0.22547087
1948	91	52962.57180554	0.24182744
1948	92	40154.76880656	0.25876070
1948	93	29764.29260754	0.27622424
1948	94	21542.67354154	0.29414561
1948	95	15205.99060773	0.31243199
1948	96	10455.15272974	0.33100098
1948	97	6994.48688936	0.34975976
1948	98	4548.09679985	0.36860466
1948	99	2871.64711910	0.38744196
1948	100	1759.05054213	0.40617725
1948	101	1044.56422444	0.42468307
1948	102	600.95548430	0.44289476
1948	103	334.79545151	0.46070353
1948	104	180.55400625	0.47804221
1948	105	94.24156968	0.49482042
1948	106	47.60891695	0.51100076
1948	107	23.28072425	0.52650049
1948	108	11.02341152	0.54131248
1948	109	5.05630124	1.00000000

	X	l_x	q_x
1949	10	999999.00000001	0.00080753
1949	11	999191.47001649	0.00080753
1949	12	998384.59214021	0.00080753
1949	13	997578.36583711	0.00080581
1949	14	996774.50970564	0.00080179
1949	15	995975.30964662	0.00113855
1949	16	994841.34034730	0.00112207
1949	17	993725.05410005	0.00110186
1949	18	992630.10915376	0.00110668
1949	19	991531.58097841	0.00111125
1949	20	990429.74224855	0.00129841
1949	21	989143.76309969	0.00130302
1949	22	987854.88896943	0.00130723
1949	23	986563.53142545	0.00132347
1949	24	985257.03973695	0.00132677
1949	25	983950.63113405	0.00123513
1949	26	982737.29640199	0.00124071
1949	27	981512.10343172	0.00124615
1949	28	980287.02471926	0.00126051
1949	29	979051.36310081	0.00127216
1949	30	977805.85560609	0.00129641
1949	31	976536.26204683	0.00132025
1949	32	975246.98945618	0.00134115
1949	33	973939.03957367	0.00137271
1949	34	972602.10490419	0.00141484
1949	35	971226.03254700	0.00146692
1949	36	969801.32617188	0.00154151
1949	37	968306.36769105	0.00162031
1949	38	966731.60484316	0.00174455
1949	39	965045.09034730	0.00188440
1949	40	963226.56140900	0.00178120
1949	41	961510.85839844	0.00196855
1949	42	959618.07280733	0.00218469
1949	43	957522.18467713	0.00244775
1949	44	955178.41052247	0.00275934
1949	45	952542.75299836	0.00348842
1949	46	949219.88330841	0.00393202
1949	47	945487.53317261	0.00443961
1949	48	941289.93770600	0.00501085
1949	49	936573.27229310	0.00564538
1949	50	931285.96460065	0.00625097
1949	51	925464.52474977	0.00700116
1949	52	918985.20038600	0.00782363
1949	53	911795.39736940	0.00872899
1949	54	903836.34641266	0.00970554
1949	55	895064.12752533	0.01048192
1949	56	885682.14131166	0.01162167
1949	57	875389.03630458	0.01287192
1949	58	864121.09760388	0.01424271
1949	59	851813.66744232	0.01574403
1949	60	838402.69078828	0.01726007
1949	61	823926.77105423	0.01905658
1949	62	808225.54349518	0.02102750
1949	63	791230.57092719	0.02318645
1949	64	772883.16392517	0.02555968
1949	65	753128.51513672	0.02657088
1949	66	733112.71065522	0.02927002
1949	67	711654.48722078	0.03223937

1949	68	688711.19555117	0.03549372
1949	69	664266.27578736	0.03906206
1949	70	638318.66778564	0.04168349
1949	71	611711.32083129	0.04584115
1949	72	583669.76886749	0.05038844
1949	73	554259.56188965	0.05535351
1949	74	523579.34748839	0.06077445
1949	75	491759.09816361	0.06797536
1949	76	458331.59754562	0.07452170
1949	77	424175.94632721	0.08163810
1949	78	389547.02693939	0.08935345
1949	79	354739.65781784	0.09770686
1949	80	320079.15941619	0.10149580
1949	81	287592.46834182	0.11075835
1949	82	255739.20128059	0.12071046
1949	83	224868.80491828	0.13138231
1949	84	195325.02155303	0.14279504
1949	85	167433.57773398	0.15845512
1949	86	140902.87036993	0.17162913
1949	87	116719.83311939	0.18555291
1949	88	95062.12794779	0.20022997
1949	89	76027.84080894	0.21562465
1949	90	59634.36422156	0.23172188
1949	91	45815.77700387	0.24846722
1949	92	34432.05851316	0.26580664
1949	93	25279.78876280	0.28368659
1949	94	18108.25160392	0.30203370
1949	95	12638.94934856	0.32075448
1949	96	8584.94966784	0.33976550
1949	97	5668.07997954	0.35897300
1949	98	3633.39229506	0.37827265
1949	99	2258.97936296	0.39756978
1949	100	1360.87743650	0.41676910
1949	101	793.70577797	0.43574338
1949	102	447.85374209	0.45447645
1949	103	244.33715728	0.47270960
1949	104	128.83063645	0.49052463
1949	105	65.63909309	0.50778118
1949	106	32.30879699	0.52444029
1949	107	15.36476200	0.54041967
1949	108	7.06134241	0.55571003
1949	109	3.13728364	1.00000000

	x	l_x	q_x
1950	10	999999.00000001	0.00072936
1950	11	999269.63896180	0.00072936
1950	12	998540.80989075	0.00072936
1950	13	997812.51239777	0.00072882
1950	14	997085.28263856	0.00072757
1950	15	996359.83394624	0.00094565
1950	16	995417.62298585	0.00094098
1950	17	994480.95655535	0.00093524
1950	18	993550.87547303	0.00093661
1950	19	992620.30404663	0.00093791
1950	20	991689.31859590	0.00116190
1950	21	990537.07120618	0.00116330
1950	22	989384.78414919	0.00116457
1950	23	988232.58010102	0.00117640
1950	24	987070.01930341	0.00117740
1950	25	985907.84585572	0.00120659
1950	26	984718.26055910	0.00121922
1950	27	983518.65677170	0.00121870
1950	28	982320.04288483	0.00122992
1950	29	981111.86728669	0.00124091
1950	30	979894.40005494	0.00112955
1950	31	978787.56070813	0.00114886
1950	32	977663.06791687	0.00116790
1950	33	976521.25868226	0.00119651
1950	34	975352.84035497	0.00123468
1950	35	974148.59467318	0.00131665
1950	36	972866.56683350	0.00138487
1950	37	971519.27540589	0.00146538
1950	38	970097.57501984	0.00157183
1950	39	968572.74194357	0.00170964
1950	40	966926.12844850	0.00184060
1950	41	965146.40440371	0.00203566
1950	42	963181.69220076	0.00226001
1950	43	961004.88977052	0.00253358
1950	44	958570.10737610	0.00285629
1950	45	955832.15172577	0.00348400
1950	46	952502.03545380	0.00392726
1950	47	948761.31653595	0.00443404
1950	48	944554.47297669	0.00500423
1950	49	939827.70925904	0.00563769
1950	50	934529.25234223	0.00631196
1950	51	928630.54325104	0.00707016
1950	52	922064.98075867	0.00790182
1950	53	914778.98634230	0.00881754
1950	54	906712.89094545	0.00980633
1950	55	897821.36098480	0.01083389
1950	56	888094.46685792	0.01201481
1950	57	877424.18389130	0.01331046
1950	58	865745.26431274	0.01473126
1950	59	852991.74302674	0.01628760
1950	60	839098.55171204	0.01710142
1950	61	824748.77182770	0.01837917
1950	62	809178.20341493	0.02083582
1950	63	792318.31127930	0.02298120
1950	64	774109.88578797	0.02533526
1950	65	754497.61122894	0.02591045
1950	66	734955.78199005	0.02852911
1950	67	713988.14715576	0.03142650

1950	68	691549.99732971	0,03460180
1950	69	667621.12403107	0.03808509
1950	70	642196.05160152	0,04450214
1950	71	613616.95365906	0.04893271
1950	72	583591.01450451	0,05377596
1950	73	552207.84605407	0,05906147
1950	74	519593,64159010	0,06432882
1950	75	485909,00094986	0.06631002
1950	76	453445,41083908	0,07324268
1950	77	420235.85568619	0.08023507
1950	78	386516.36201477	0,08781005
1950	79	352574.02179718	0,09602413
1950	80	318718.40684509	0,10378283
1950	81	285640.90937041	0,11323048
1950	82	253297,65134237	0,12337880
1950	83	222046,09029579	0,13425800
1950	84	192234,62640102	0,14588929
1950	85	164189,65419197	0,15916569
1950	86	138056,29460715	0,17238612
1950	87	114257,30630779	0,18636104
1950	88	92964.19555187	0.20109424
1950	89	74269,63086604	0.21655169
1950	90	58186.41075043	0.23271080
1950	91	44645.34388112	0,24954293
1950	92	33504.41403054	0.26697155
1950	93	24559,68854856	0.28495222
1950	94	17561,35066368	0,30341316
1950	95	12233,00569188	0,32226299
1950	96	8290.76063409	0.34141961
1950	97	5460,13237458	0,36079071
1950	98	3490.16733810	0.38027354
1950	99	2162.94904885	0.39977432
1950	100	1298.25750861	0.41919840
1950	101	754,03007288	0.43842047
1950	102	423,44785205	0,45737333
1950	103	229,77409892	0.47594909
1950	104	120.41332562	0.49407844
1950	105	60.91969709	0,51167101
1950	106	29,74885414	0,52868544
1950	107	14.02106819	0.54503923
1950	108	6.37903593	0,56071961
1950	109	2,80218539	1,00000000

	x	l_x	q_x
1951	10	999999.00000001	0.00071318
1951	11	999285.82368470	0.00071318
1951	12	998573.15599062	0.00071318
1951	13	997860.99655915	0.00071372
1951	14	997148.80056001	0.00071500
1951	15	996435.84339143	0.00083291
1951	16	995605.90555573	0.00083713
1951	17	994772.45545197	0.00084231
1951	18	993934.55005647	0.00084107
1951	19	993098.57937622	0.00083990
1951	20	992264.47346498	0.00110169
1951	21	991171.30463410	0.00110036
1951	22	990080.66265871	0.00109914
1951	23	988992.42710876	0.00110775
1951	24	987896.87119294	0.00110680
1951	25	986803.46966554	0.00112217
1951	26	985696.10053087	0.00113138
1951	27	984580.91419220	0.00113092
1951	28	983467.43193054	0.00114051
1951	29	982345.77389527	0.00115033
1951	30	981215.75094606	0.00120655
1951	31	980031.86409761	0.00122753
1951	32	978828.84540560	0.00124882
1951	33	977606.46842958	0.00128067
1951	34	976354.47320950	0.00132314
1951	35	975062.62294007	0.00123762
1951	36	973855.86945818	0.00130418
1951	37	972585.78303529	0.00135039
1951	38	971243.23291780	0.00148470
1951	39	969801.23024751	0.00160794
1951	40	968241.84893799	0.00177146
1951	41	966526.64295197	0.00196066
1951	42	964631.01190797	0.00217521
1951	43	962530.43859864	0.00244263
1951	44	960179.33251191	0.00275399
1951	45	957535.01105009	0.00341119
1951	46	954268.67869500	0.00384542
1951	47	950599.11007692	0.00434148
1951	48	946472.10739137	0.00489945
1951	49	941834.91537476	0.00551947
1951	50	936636.49024965	0.00653492
1951	51	930468.81841279	0.00737654
1951	52	923605.17595673	0.00824529
1951	53	915989.78256989	0.00920207
1951	54	907560.78136445	0.01023631
1951	55	898270.70537567	0.01105773
1951	56	888355.83957673	0.01224398
1951	57	877478.82830811	0.01356771
1951	58	865573.45335390	0.01501956
1951	59	852572.92090606	0.01661021
1951	60	838411.50207521	0.01799647
1951	61	823323.05219269	0.01987075
1951	62	806963.00724794	0.02193318
1951	63	789263.74212646	0.02419432
1951	64	770168.04183959	0.02667495
1951	65	749623.84693910	0.02919606
1951	66	727736.28561401	0.03215933
1951	67	704332.77491761	0.03542090

1951	68	679384.67188264	0.03899335
1951	69	652893.18544007	0.04290753
1951	70	624679.15127564	0.04689977
1951	71	595572.46565246	0.05156149
1951	72	564863.86454773	0.05665501
1951	73	532861.49490357	0.06221104
1951	74	499711.62913132	0.06827029
1951	75	465596.17264938	0.07312666
1951	76	431548.67794417	0.08013444
1951	77	396966.76630401	0.08774513
1951	78	362134.86720761	0.09598692
1951	79	327373.93302154	0.10490582
1951	80	293030.50126648	0.11417289
1951	81	259574.36174010	0.12448782
1951	82	227260.51463699	0.13555468
1951	83	196454.28826522	0.14740345
1951	84	167496.24771308	0.16005426
1951	85	140687.76035308	0.18286712
1951	86	114960.59418486	0.19780754
1951	87	92220.54529284	0.21356118
1951	88	72525.81653403	0.23012560
1951	89	55835.76948164	0.24745824
1951	90	42018.85972165	0.26552890
1951	91	30861.63804221	0.28427828
1951	92	22088.34456515	0.30363897
1951	93	15381.46247803	0.32354541
1951	94	10404.86087727	0.34391176
1951	95	6826.50657671	0.36463265
1951	96	4337.34360840	0.38561073
1951	97	2664.81740820	0.40674249
1951	98	1580.92295768	0.42791247
1951	99	904.42629156	0.44901648
1951	100	498.32398031	0.46995097
1951	101	264.13614323	0.49058329
1951	102	134.55536583	0.51084150
1951	103	65.81890080	0.53061471
1951	104	30.69442386	0.54983264
1951	105	13.90766132	0.56840011
1951	106	6.00246170	0.58620610
1951	107	2.48324182	0.60342501
1951	108	0.98479159	0.61978465
1951	109	0.37445288	1.00000000

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1952	10	999999.00000001	0.00064549
1952	11	999353.51315783	0.00064549
1952	12	998708.44292451	0.00064549
1952	13	998063.78910065	0.00064703
1952	14	997418.01474000	0.00065062
1952	15	996769.07501984	0.00081220
1952	16	995959.49874115	0.00082487
1952	17	995137.95871736	0.00084042
1952	18	994301.62401581	0.00083671
1952	19	993469.68220521	0.00083320
1952	20	992641.92449952	0.00108345
1952	21	991566.44635485	0.00107947
1952	22	990496.08064271	0.00107583
1952	23	989430.47095202	0.00108168
1952	24	988360.22840883	0.00107683
1952	25	987293.95158387	0.00097951
1952	26	986326.88509370	0.00098606
1952	27	985354.29240521	0.00098486
1952	28	984383.85958100	0.00099246
1952	29	983406.89415742	0.00100067
1952	30	982422.82835864	0.00103612
1952	31	981402.95713045	0.00105649
1952	32	980366.11795045	0.00107567
1952	33	979311.57130433	0.00110425
1952	34	978230.16724397	0.00114232
1952	35	977112.71424867	0.00130259
1952	36	975840.12979126	0.00137446
1952	37	974498.87331390	0.00145726
1952	38	973078.77771761	0.00156954
1952	39	971551.48870088	0.00170215
1952	40	969897.76271058	0.00183061
1952	41	968122.25565339	0.00202766
1952	42	966159.23130799	0.00225419
1952	43	963981.32855226	0.00252860
1952	44	961543.60810547	0.00285112
1952	45	958802.33051300	0.00320163
1952	46	955732.59877778	0.00360945
1952	47	952282.92650604	0.00407494
1952	48	948402.43144227	0.00459841
1952	49	944041.28823853	0.00518022
1952	50	939150.94937898	0.00608047
1952	51	933440.47149659	0.00681238
1952	52	927081.51904297	0.00761607
1952	53	920020.80079652	0.00850151
1952	54	912199.23300936	0.00945980
1952	55	903570.00945283	0.01057680
1952	56	894013.12849426	0.01173617
1952	57	883520.84094239	0.01300881
1952	58	872027.28824615	0.01440501
1952	59	859465.72520447	0.01593513
1952	60	845770.02480317	0.01627815
1952	61	832002.45028687	0.01797920
1952	62	817043.70817567	0.01985114
1952	63	800824.45775605	0.02190590
1952	64	783283.27715303	0.02415032
1952	65	764362.03868865	0.02808600
1952	66	742894.16421508	0.03093972
1952	67	719909.22782897	0.03408202

1952	68	695373.26797485	0.03752380
1952	69	669280.22145843	0.04129460
1952	70	641042.56114197	0.04380509
1952	71	613535.34986115	0.04816607
1952	72	583983.76523589	0.05293166
1952	73	553072.53403472	0.05813104
1952	74	520921.85025404	0.06380247
1952	75	487685.74812316	0.06761660
1952	76	454713.99781035	0.07410288
1952	77	421018.37951659	0.08115891
1952	78	386848.98556137	0.08880607
1952	79	352494.44783019	0.09708260
1952	80	318273.37035751	0.10258359
1952	81	285623.74642943	0.11190875
1952	82	253659.95036315	0.12192677
1952	83	222732.01133536	0.13266790
1952	84	193182.62367248	0.14415417
1952	85	165334.54279898	0.15762492
1952	86	139273.69944190	0.17071848
1952	87	115497.10525986	0.18456834
1952	88	94179.99664115	0.19917962
1952	89	75421.26060198	0.21452307
1952	90	59241.65990208	0.23058631
1952	91	45581.34400899	0.24732219
1952	92	34308.06612968	0.26466297
1952	93	25227.30536651	0.28262019
1952	94	18097.55951118	0.30106728
1952	95	12648.97645222	0.31993924
1952	96	8602.07254970	0.33915850
1952	97	5684.60051696	0.35863749
1952	98	3645.69352718	0.37627840
1952	99	2266.73077115	0.39799064
1952	100	1364.59314799	0.41768207
1952	101	794.62705097	0.43723246
1952	102	447.19031084	0.45657294
1952	103	243.01531685	0.47559782
1952	104	127.43770163	0.49423557
1952	105	64.45348654	0.51239565
1952	106	31.42780024	0.53005130
1952	107	14.77008238	0.54705913
1952	108	6.68997399	0.56345829
1952	109	2.92045271	1.00000000

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1953	11	999390.24257660	0.00060876
1953	12	998781.85573578	0.00060876
1953	13	998173.83925631	0.00061128
1953	14	997563.67871094	0.00061715
1953	15	996948.02824403	0.00076207
1953	16	996188.28009035	0.00078242
1953	17	995408.84188844	0.00080738
1953	18	994605.16471864	0.00080143
1953	19	993808.06284332	0.00079579
1953	20	993017.20195772	0.00097992
1953	21	992044.12601473	0.00097385
1953	22	991078.02442171	0.00096831
1953	23	990118.35622407	0.00097116
1953	24	989156.79312898	0.00097683
1953	25	988200.44838716	0.00095258
1953	26	987259.11119844	0.00095747
1953	27	986313.84439850	0.00095545
1953	28	985371.47406770	0.00096207
1953	29	984423.48261262	0.00096968
1953	30	983468.91075135	0.00103328
1953	31	982452.71274213	0.00105189
1953	32	981419.27660370	0.00107189
1953	33	980367.30549623	0.00110156
1953	34	979287.37021638	0.00114106
1953	35	978169.94310761	0.00115298
1953	36	977042.12812806	0.00121666
1953	37	975851.44938661	0.00129435
1953	38	974588.35170088	0.00139610
1953	39	973227.73188020	0.00151620
1953	40	971752.12676239	0.00163355
1953	41	970164.72458649	0.00181081
1953	42	968407.93577575	0.00201455
1953	43	966457.02530011	0.00225054
1953	44	964272.30706314	0.00254913
1953	45	961814.25463868	0.00306046
1953	46	958670.65705110	0.00345054
1953	47	955562.03156282	0.00389539
1953	48	951839.74110413	0.00439553
1953	49	947655.89910126	0.00495153
1953	50	942963.55532075	0.00614130
1953	51	937172.53186036	0.00688126
1953	52	930723.60678101	0.00769421
1953	53	923562.42794801	0.00859010
1953	54	915628.93309023	0.00956081
1953	55	906874.77470399	0.01070647
1953	56	897165.35101320	0.01188325
1953	57	886504.11119081	0.01317530
1953	58	874824.15222932	0.01459308
1953	59	862057.76937104	0.01614718
1953	60	848137.97162630	0.01611374
1953	61	834471.29727174	0.01780189
1953	62	819616.13098907	0.01965942
1953	63	803502.95133209	0.02169646
1953	64	786069.78165457	0.02393154
1953	65	767257.92411805	0.02596428
1953	66	747336.62693023	0.02860883
1953	67	725956.20360566	0.03152037

1953	68	703073.79675293	0.03470976
1953	69	678670.27205657	0.03820433
1953	70	652742.12773896	0.04199679
1953	71	625329.05390929	0.04618118
1953	72	596450.61909485	0.05075378
1953	73	566178.49614715	0.05574282
1953	74	534618.11257935	0.06118491
1953	75	501907.55058669	0.06807977
1953	76	467737.79845810	0.07461127
1953	77	432839.28645706	0.08170597
1953	78	397473.73316481	0.08939354
1953	79	361942.14828871	0.09771212
1953	80	326576.01435442	0.10371767
1953	81	292704.31274413	0.11312694
1953	82	259590.98481749	0.12323818
1953	83	227599.46419142	0.13407567
1953	84	197083.91324805	0.14566571
1953	85	168375.93938254	0.15595794
1953	86	142114.69095229	0.16893222
1953	87	118106.94052219	0.18265130
1953	88	96534.55436038	0.19713123
1953	89	77504.57918166	0.21234556
1953	90	61046.82595967	0.22828336
1953	91	47110.65114049	0.24490078
1953	92	35573.36699628	0.26215297
1953	93	26247.70307445	0.27999403
1953	94	18698.50305315	0.29836054
1953	95	13259.93538570	0.31717136
1953	96	9054.26370835	0.33635174
1953	97	6008.84635037	0.35581705
1953	98	3070.7963704[illegible]	0.37547251
1953	99	2417.41872364	0.39522960
1953	100	1461.98327950	0.41499785
1953	101	855.26330183	0.43465996
1953	102	483.51462572	0.45414044
1953	103	263.92817779	0.47335295
1953	104	138.99699705	0.49220689
1953	105	70.58171804	0.51061768
1953	106	34.54144479	0.52853577
1953	107	16.28505579	0.54587720
1953	108	7.39541513	0.56261686
1953	109	3.23462987	1.00000000

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1954	11	999437.19836427	0.00056180
1954	12	998875.71234896	0.00056180
1954	13	998314.54177858	0.00056520
1954	14	997750.29936982	0.00057511
1954	15	997178.47708893	0.00072731
1954	16	996453.22166443	0.00075524
1954	17	995700.66131593	0.00078951
1954	18	994914.54711915	0.00078133
1954	19	994137.19184113	0.00077359
1954	20	993368.13710023	0.00094687
1954	21	992427.55156708	0.00093856
1954	22	991496.09822848	0.00093098
1954	23	990573.03787995	0.00093133
1954	24	989650.48674013	0.00092540
1954	25	988734.66031648	0.00089973
1954	26	987845.07101441	0.00090287
1954	27	986953.17832948	0.00090014
1954	28	986064.78593447	0.00090562
1954	29	985171.78827668	0.00091244
1954	30	984272.87664796	0.00110984
1954	31	983180.49464417	0.00113020
1954	32	982069.30107882	0.00115270
1954	33	980937.26989746	0.00118596
1954	34	979773.92120363	0.00123020
1954	35	978568.60750682	0.00114033
1954	36	977446.84716798	0.00121355
1954	37	976260.66505907	0.00129129
1954	38	975000.02526383	0.00139486
1954	39	973640.03511649	0.00151706
1954	40	972162.96631623	0.00156477
1954	41	970641.75260267	0.00173597
1954	42	968956.74372865	0.00193269
1954	43	967084.04772188	0.00216940
1954	44	964986.05413819	0.00244657
1954	45	962625.15174103	0.00305603
1954	46	959683.34310914	0.00344576
1954	47	956376.50730525	0.00388980
1954	48	952656.38979340	0.00438890
1954	49	948475.27529908	0.00494385
1954	50	943786.15772248	0.00591969
1954	51	938199.23585510	0.00663577
1954	52	931975.43811035	0.00741874
1954	53	925061.35491181	0.00828409
1954	54	917398.06716158	0.00922283
1954	55	908937.05941019	0.01120465
1954	56	898752.73811341	0.01243956
1954	57	887572.82643891	0.01379527
1954	58	875528.51927949	0.01528335
1954	59	861950.56387330	0.01691476
1954	60	847370.87780763	0.01594933
1954	61	833855.88405610	0.01762457
1954	62	819159.53560742	0.01946768
1954	63	803212.39682008	0.02148899
1954	64	785952.17539216	0.02370071
1954	65	767319.83143616	0.02644246
1954	66	747030.00777437	0.02913332
1954	67	725262.80539703	0.03210504

1954	68	701978.21482066	0.05535420
1954	69	677160.33791352	0.05891527
1954	70	650809.81254577	0.04374901
1954	71	622337.52912902	0.04810258
1954	72	592401.48916626	0.05285811
1954	73	561088.26552582	0.05804470
1954	74	528520.06816390	0.06369967
1954	75	494853.51597213	0.06591572
1954	76	462234.09199448	0.07224168
1954	77	428842.26856612	0.07911541
1954	78	394915.00337997	0.08650034
1954	79	360731.10781859	0.09461970
1954	80	326598.83730327	0.10427423
1954	81	292542.99605941	0.11372277
1954	82	259274.34300803	0.12387040
1954	83	227157.92757225	0.13474692
1954	84	196548.04160450	0.14638047
1954	85	167777.75853157	0.15562033
1954	86	141668.12910281	0.16855294
1954	87	117789.54952620	0.18224254
1954	88	96323.28233337	0.19665578
1954	89	77376.89958666	0.21188641
1954	90	60981.63151072	0.22781945
1954	91	47089.37830257	0.24442058
1954	92	35579.76524972	0.26107011
1954	93	26269.39086126	0.27953287
1954	94	18926.23260402	0.29792980
1954	95	13287.54384612	0.31678861
1954	96	9078.20129418	0.33603655
1954	97	6027.59386456	0.35559103
1954	98	3884.23557659	0.37535945
1954	99	2426.25105715	0.39525459
1954	100	1467.26418047	0.41510099
1954	101	858.07517748	0.43504162
1954	102	484.77670347	0.45474799
1954	103	264.32550615	0.47420265
1954	104	138.93164979	0.49333182
1954	105	70.41757976	0.51204475
1954	106	34.36062805	0.53028915
1954	107	16.13955965	0.54798067
1954	108	7.29539291	0.56509024
1954	109	3.17283761	1.00000000

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1955	11	999412.36637117	0,00058663
1955	12	998826.07680142	0.00058663
1955	13	998240.13132477	0.00059139
1955	14	997649.78057094	0,00060249
1955	15	997048.70462036	0,00075001
1955	16	996300.90920260	0.00078809
1955	17	995515.73926545	0.00083480
1955	18	994684.68125917	0,00082365
1955	19	993865.40858461	0,00081310
1955	20	993057.29519655	0,00098533
1955	21	992078.80841065	0,00097409
1955	22	991112.43636611	0,00096382
1955	23	990157.17799379	0,00095163
1955	24	989205.00925156	0,00095361
1955	25	988261.69110501	0,00088565
1955	26	987386.43721010	0,00088722
1955	27	986510.40552522	0,00088369
1955	28	985638.63980103	0,00088829
1955	29	984763.10815431	0,00089463
1955	30	983882.10972596	0,00094207
1955	31	982955.22500617	0,00095969
1955	32	982011.88940823	0,00097971
1955	33	981049.80635072	0.00100918
1955	34	980059.75290681	0,00104036
1955	35	979032.29935455	0,00106886
1955	36	977985.85425569	0,00113342
1955	37	976877.38810732	0,00120832
1955	38	975697.00529480	0,00130724
1955	39	974421.53424837	0.00142390
1955	40	973034.05600848	0.00162377
1955	41	971454.07294465	0.00180290
1955	42	969702.63525103	0,00200868
1955	43	967754.81110384	0,00225544
1955	44	965572.10199739	0,00254379
1955	45	963115.88500215	0.00305159
1955	46	960176.84915637	0,00344098
1955	47	956872.90442658	0.00388422
1955	48	953156.20220578	0,00438227
1955	49	948979.21842195	0,00493616
1955	50	944294.90463598	0.00583931
1955	51	938780.87350564	0.00654447
1955	52	932637.05077363	0,00732007
1955	53	925810.07932282	0,00817536
1955	54	918241.24600221	0.00910432
1955	55	909881.28378296	0.01022813
1955	56	900574.89868165	0.01135914
1955	57	890345.14440919	0.01260161
1955	58	879125.36245728	0.01396568
1955	59	866847.78199005	0.01540167
1955	60	853444.86392975	0.01627812
1955	61	839552.38390454	0.01799191
1955	62	824447.25408509	0.01987711
1955	63	808059.60522461	0,02194452
1955	64	790327.12738801	0.02421258
1955	65	771191.26568604	0,02598227
1955	66	751153.96881103	0.02863525
1955	67	729644.48530209	0,03155377

1955	68	706621.45158387	0.03474984
1955	69	682066.46678162	0.03825024
1955	70	655977.25814056	0.04129936
1955	71	628885.81481934	0.04541407
1955	72	600325.54957581	0.04990895
1955	73	570363.93083953	0.05481184
1955	74	539101.23655700	0.06015602
1955	75	506669.97222900	0.06702717
1955	76	472709.31958770	0.07344931
1955	77	437989.14802933	0.08042325
1955	78	402764.63525102	0.08797886
1955	79	367329.85251617	0.09615350
1955	80	332009.80167769	0.10656677
1955	81	296627.92714309	0.11620018
1955	82	262159.71001815	0.12654264
1955	83	228985.32735251	0.13762652
1955	84	197470.87377929	0.14947458
1955	85	167953.99698257	0.15499790
1955	86	141921.43056319	0.16787050
1955	87	118095.91501236	0.18151759
1955	88	96659.42945575	0.19597245
1955	89	77721.67097429	0.21107123
1955	90	61310.65670923	0.22695590
1955	91	47400.69697523	0.24353512
1955	92	35856.96230612	0.26077125
1955	93	26506.49758385	0.27862113
1955	94	19121.22724151	0.29702021
1955	95	13441.72167623	0.31591126
1955	96	9195.33043122	0.33520570
1955	97	6113.00325078	0.35482916
1955	98	3943.93143627	0.37469138
1955	99	2466.17433310	0.39470058
1955	100	1492.75910409	0.41476639
1955	101	873.58294798	0.43481617
1955	102	493.73321153	0.45475047
1955	103	269.21767852	0.47442080
1955	104	141.49521113	0.49381416
1955	105	71.62287252	0.51281963
1955	106	34.69325754	0.53138216
1955	107	16.35160287	0.54941091
1955	108	7.36775580	0.56684076
1955	109	3.19104316	1.00000000

	x	l_x	q_x
1956	10	999999.00000001	0.00053970
1956	11	999459.29614258	0.00053970
1956	12	998919.88356019	0.00053970
1956	13	998380.76210022	0.00054530
1956	14	997836.34250641	0.00055837
1956	15	997279.18560792	0.00068719
1956	16	996593.86239625	0.00073107
1956	17	995865.28141023	0.00078491
1956	18	995083.62049867	0.00077206
1956	19	994315.36001587	0.00075990
1956	20	993559.78030396	0.00089450
1956	21	992671.03516513	0.00088188
1956	22	991795.61670686	0.00087036
1956	23	990932.39490613	0.00086600
1956	24	990074.25075532	0.00085699
1956	25	989225.76574707	0.00085860
1956	26	988370.41893770	0.00085858
1956	27	987527.82043459	0.00085429
1956	28	986684.18727112	0.00085794
1956	29	985837.66729737	0.00086371
1956	30	984986 18970489	0 00093715
1956	31	984063.10652161	0.00095504
1956	32	983123.28892518	0.00097591
1956	33	982163.85129548	0.00100653
1956	34	981175.27642061	0.00104721
1956	35	980147.77944185	0.00113264
1956	36	979037.62642671	0.00120313
1956	37	977859.72074129	0.00128517
1956	38	976603.00799562	0.00139258
1956	39	975243.01276398	0.00151919
1956	40	973701.43321250	0.00149134
1956	41	972309.21911622	0.00165728
1956	42	970697.83061982	0.00184785
1956	43	968904.12499237	0.00207558
1956	44	966893 09086900	0 00234117
1956	45	964629.43022919	0.00291042
1956	46	961821.94975283	0.00328204
1956	47	958665.21309663	0.00370466
1956	48	955113.68458558	0.00417941
1956	49	951121.87377931	0.00470751
1956	50	946644.45395662	0.00554739
1956	51	941393.05233002	0.00621313
1956	52	935539.35192109	0.00695632
1956	53	929031.44126893	0.00777064
1956	54	921812.27135468	0.00865622
1956	55	913832.86231994	0.01079916
1956	56	903964.23296357	0.01199647
1956	57	893119.85303498	0.01331202
1956	58	881230.62538259	0.01475654
1956	59	868226.71060573	0.01634101
1956	60	854039.01055146	0.01674755
1956	61	839735.95040131	0.01851483
1956	62	824188.38498688	0.02045850
1956	63	807326.72559356	0.02258994
1956	64	789089.26041412	0.02492796
1956	65	769418.87247468	0.02646019
1956	66	749059.90599000	0.02916471
1956	67	727213.79460143	0.03213855

1956	68	703842.19454956	0.03539454
1956	69	678930.02655792	0.03895955
1956	70	652479.21824645	0.04077265
1956	71	625875.90895081	0.04483520
1956	72	597814.63496398	0.04927240
1956	73	568358.87136841	0.05411179
1956	74	537603.95575713	0.05938788
1956	75	505676.79440688	0.06942510
1956	76	470570.13013859	0.07606099
1956	77	434778.10220718	0.08526365
1956	78	398576.39040478	0.09106370
1956	79	362281.00342178	0.09949885
1956	80	326234.45875930	0.10553869
1956	81	291853.03714751	0.11490976
1956	82	258316.27575492	0.12513516
1956	83	225991.82728767	0.13609530
1956	84	195235.40164756	0.14781550
1956	85	166376.97409247	0.15499077
1956	86	140590.07900531	0.16786538
1956	87	116989.87139510	0.18150182
1956	88	95755.99695872	0.19590789
1956	89	76996.64185713	0.21106385
1956	90	60745.43451833	0.22696100
1956	91	46958.55370997	0.24356488
1956	92	35521.09930657	0.26083533
1956	93	26255.94152784	0.27873229
1956	94	18937.56294393	0.29719927
1956	95	13309.53314597	0.31616371
1956	96	9101.40499997	0.33555074
1956	97	6047.36717194	0.35529980
1956	98	3898.73883256	0.37530451
1956	99	2435.52457514	0.39548614
1956	100	1472.30830420	0.41575705
1956	101	860.18578308	0.43600660
1956	102	485.13910246	0.45616212
1956	103	263.83702247	0.47612182
1956	104	138.21845972	0.49580929
1956	105	69.68840362	0.51513332
1956	106	33.78961408	0.53403606
1956	107	15.74474185	0.55243215
1956	108	7.04684029	0.57028455
1956	109	3.02813617	1.00000000

	x	l_x	q_x
1957	10	999999.00000001	0.00056428
1957	11	999434.71762085	0.00056428
1957	12	998870.75365448	0.00056428
1957	13	998307.10792542	0.00057154
1957	14	997736.53804018	0.00058846
1957	15	997149.40534211	0.00073604
1957	16	996415.45995332	0.00079324
1957	17	995625.05979920	0.00086342
1957	18	994765.41377259	0.00084667
1957	19	993925.17372132	0.00083082
1957	20	993097.39805605	0.00095507
1957	21	992148.91783907	0.00093896
1957	22	991217.32785798	0.00092425
1957	23	990301.19505022	0.00091699
1957	24	989593.09810742	0.00093549
1957	25	988497.21023560	0.00085108
1957	26	987655.92435930	0.00084945
1957	27	986815.95871735	0.00084430
1957	28	985983.78658294	0.00084709
1957	29	985148.57039643	0.00085241
1957	30	984308.82273107	0.00085113
1957	31	983471.05107117	0.00086771
1957	32	982617.68490601	0.00088759
1957	33	981745.52651978	0.00091665
1957	34	980845.61179354	0.00095524
1957	35	979908.07137147	0.00105544
1957	36	978874.43788911	0.00112315
1957	37	977775.01855995	0.00120270
1957	38	976599.54028321	0.00130482
1957	39	975325.25617219	0.00142573
1957	40	973934.70736695	0.00148652
1957	41	972486.93701955	0.00165335
1957	42	970879.07069067	0.00184490
1957	43	969087.90016938	0.00207299
1957	44	967078.99461365	0.00233046
1957	45	964017.52235413	0.00297435
1957	46	961947.81780243	0.00335434
1957	47	958721.12082673	0.00378606
1957	48	955091.34877778	0.00427087
1957	49	951012.27310945	0.00481029
1957	50	946437.62805176	0.00560008
1957	51	941129.93283845	0.00628688
1957	52	935213.15938569	0.00703438
1957	53	928634.51395417	0.00785972
1957	54	921330.17147828	0.00875734
1957	55	913267.71752930	0.01026544
1957	56	903892.62139150	0.01140756
1957	57	893581.59338380	0.01266245
1957	58	882266.66082765	0.01404099
1957	59	869878.75971985	0.01555358
1957	60	856349.03446487	0.01651210
1957	61	842208.91214754	0.01825932
1957	62	826830.74611664	0.02018073
1957	63	810144.69765473	0.02228785
1957	64	792088.31391144	0.02459912
1957	65	772603.63955401	0.02621630
1957	66	752348.83311462	0.02889970
1957	67	730606.13211059	0.03184935

1957	68	707330.01550228	0.03507633
1957	69	682524.62041473	0.03861315
1957	70	656170.19617462	0.04109971
1957	71	629201.79342651	0.04519314
1957	72	600766.18622589	0.04966293
1957	73	570930.37705230	0.05455675
1957	74	539793.68849945	0.05984891
1957	75	507487.62619399	0.06739955
1957	76	473283.19030103	0.07384321
1957	77	438334.43844604	0.08083740
1957	78	402900.62375640	0.08841249
1957	79	367279.17775726	0.09660538
1957	80	331798.03175734	0.10015253
1957	81	298567.61844253	0.10921974
1957	82	265958.14052200	0.11896349
1957	83	234318.83245849	0.12941415
1957	84	203994.65917395	0.14059590
1957	85	175313.84563327	0.15103217
1957	86	148730.62719344	0.16425056
1957	87	124301.53837013	0.17762435
1957	88	102222.56088732	0.19176263
1957	89	82020.09372424	0.20664924
1957	90	65546.71399592	0.22227016
1957	91	50977.11072825	0.23861733
1957	92	38013.08859443	0.25563226
1957	93	26091.21112775	0.27326577
1957	94	20995.65421843	0.29157569
1957	95	14874.88155540	0.31028450
1957	96	10259.43038804	0.32949728
1957	97	6878.97991445	0.34905957
1957	98	4477.59982787	0.36897716
1957	99	2925.46774107	0.38907844
1957	100	1720.13915106	0.40930839
1957	101	1019.61591817	0.42956029
1957	102	581.62941358	0.44976147
1957	103	320.03491555	0.46981214
1957	104	169.67862771	0.48963524
1957	105	86.59799186	0.50913985
1957	106	42.50750302	0.52826538
1957	107	20.05220090	0.54692594
1957	108	9.08515928	0.56507998
1957	109	3.95131762	1.00000000

	x	l_x	q_x
1958	10	999999.00000001	0.00050719
1958	11	999491.80590822	0.00050719
1958	12	998984.86906434	0.00050719
1958	13	998478.18933106	0.00051510
1958	14	997963.86899568	0.00053356
1958	15	997431.39524842	0.00068770
1958	16	996745.46371460	0.00075120
1958	17	995996.64529421	0.00082925
1958	18	995170.71146394	0.00081064
1958	19	994363.98830517	0.00079303
1958	20	993575.45124391	0.00090708
1958	21	992674.17856125	0.00088921
1958	22	991791.48062890	0.00087290
1958	23	990925.74990082	0.00086347
1958	24	990070.11150464	0.00085072
1958	25	989227.83768463	0.00081747
1958	26	988419.17777254	0.00081429
1958	27	987614.32139589	0.00080844
1958	28	986815.89715288	0.00081027
1958	29	986016.31079102	0.00081498
1958	30	985212.73065168	0.00084621
1958	31	984379.03521730	0.00086305
1958	32	983529.46630964	0.00088379
1958	33	982660.23648072	0.00091400
1958	34	981762.08845233	0.00095409
1958	35	980825.39908601	0.00090880
1958	36	979934.02323915	0.00096893
1958	37	978984.53785707	0.00103934
1958	38	977967.03842927	0.00112999
1958	39	976861.94757844	0.00123674
1958	40	975653.82070264	0.00148164
1958	41	974208.25130856	0.00164940
1958	42	972601.39490509	0.00184196
1958	43	970809.90195466	0.00207042
1958	44	968799.91069406	0.00233578
1958	45	966537.01630794	0.00290154
1958	46	963732.56094030	0.00327247
1958	47	960578.78189851	0.00369348
1958	48	957030.90560914	0.00416613
1958	49	953043.79267120	0.00469213
1958	50	948571.99150850	0.00552775
1958	51	943328.52548218	0.00619762
1958	52	937482.13602448	0.00693572
1958	53	930980.02069093	0.00775047
1958	54	923764.49100124	0.00863873
1958	55	915784.33980562	0.01024707
1958	56	906400.23251344	0.01139047
1958	57	896075.90680036	0.01264752
1958	58	884742.76847841	0.01402854
1958	59	872331.11775208	0.01554421
1958	60	858771.41841889	0.01634712
1958	61	844732.97991944	0.01808165
1958	62	829458.81521607	0.01998887
1958	63	812878.87493134	0.02208050
1958	64	794930.00917450	0.02437470
1958	65	775553.91797639	0.02626098
1958	66	755187.11079406	0.02895252
1958	67	733322.53945923	0.03190773

1958	68	709922.41589355	0.03514657
1958	69	684971.07984924	0.03868916
1958	70	658470.12455461	0.04263609
1958	71	630394.21501160	0.04688001
1958	72	600841.32897949	0.05151010
1958	73	569891.93397521	0.05655681
1958	74	537650.66566467	0.06205496
1958	75	504296.15690994	0.07459753
1958	76	466676.91041183	0.08169160
1958	77	428553.32909012	0.08938395
1958	78	390247.53800964	0.09776671
1958	79	352117.73375701	0.10669831
1958	80	314547.36511612	0.10592486
1958	81	281228.98031615	0.11546945
1958	82	248755.62499046	0.12571971
1958	83	217482.13875960	0.13676617
1958	84	187750.98846626	0.14845294
1958	85	159878.80136299	0.15655733
1958	86	134848.60379600	0.16953799
1958	87	111986.64185098	0.18329207
1958	88	91460.37897777	0.19782803
1958	89	73366.95254325	0.21312982
1958	90	57730.26685093	0.22919045
1958	91	44499.04110860	0.24597763
1958	92	33553.27235603	0.26345616
1958	93	24713.45598435	0.28158759
1958	94	17754.45345711	0.30031663
1958	95	12422.46025240	0.31958090
1958	96	8452.47920235	0.33931782
1958	97	5584.48693174	0.35942326
1958	98	3577.29245776	0.37984143
1958	99	2218.48857695	0.40047854
1958	100	1330.03150177	0.42124729
1958	101	769.75933018	0.44203997
1958	102	429.49494028	0.46278051
1958	103	230.73305066	0.48336772
1958	104	119.20414316	0.50372170
1958	105	59.15842985	0.52375011
1958	106	28.17419567	0.54336915
1958	107	12.86464358	0.56255229
1958	108	5.62760890	0.58119459
1958	109	2.35687303	1.00000000

	x	l_x	q_x
1959	10	999999.00000001	0,00050102
1959	11	999497.97785950	0,00050102
1959	12	998997.20674898	0,00050102
1959	13	998496.68653108	0,00051036
1959	14	997987.09747315	0,00053213
1959	15	997456.03363801	0,00072003
1959	16	996737.83119906	0,00079786
1959	17	995942.57632446	0,00089334
1959	18	995052.86003876	0,00087055
1959	19	994186.61701204	0,00084899
1959	20	993342.56421663	0,00095398
1959	21	992394.93924714	0,00093242
1959	22	991469.61058045	0.00091274
1959	23	990564.66088867	0,00090010
1959	24	989673.04962921	0,00088472
1959	25	988797.46654512	0,00079690
1959	26	988009.49561311	0,00079214
1959	27	987226.85433961	0,00078551
1959	28	986451.37670136	0,00078643
1959	29	985675.60094453	0,00079061
1959	30	984896.31652832	0,00084121
1959	31	984067.81279756	0,00085833
1959	32	983223.16650062	0,00087907
1959	33	982357.95735169	0,00091139
1959	34	981462.64866928	0,00095307
1959	35	980527.25076294	0,00104148
1959	36	979506.04771425	0.00111257
1959	37	978416.28142540	0.00119606
1959	38	977246.03524781	0.00130266
1959	39	975973.01546478	0.00142814
1959	40	974579.18611146	0,00154017
1959	41	973078.16845415	0.00171612
1959	42	971408.25021364	0.00191803
1959	43	969545.05767823	0.00215671
1959	44	967454.03115846	0,00243534
1959	45	965099.88870241	0,00282674
1959	46	962369.87390901	0.00319060
1959	47	959299.34130202	0,00360090
1959	48	955845.00389864	0,00406139
1959	49	951962.94922639	0.00457397
1959	50	947608.69856263	0.00551786
1959	51	942379.92786306	0.00618731
1959	52	936549.13084412	0,00692540
1959	53	930063.15239716	0.00774040
1959	54	922864.09100343	0,00863009
1959	55	914899.69340516	0,01008160
1959	56	905676.03627778	0,01121018
1959	57	895523.24305725	0.01245129
1959	58	884372.62527101	0.01381515
1959	59	872155.08490754	0,01531238
1959	60	858800.31137848	0,01660434
1959	61	844540.50089264	0,01837067
1959	62	829025.72492982	0.02031253
1959	63	812186.11765289	0,02244212
1959	64	793958.93583679	0.02477772
1959	65	774286.44612885	0,02681075
1959	66	755527.24885559	0.02956149
1959	67	731251.86390686	0.03258228

1959	68	707426.07193236	0.05505797
1959	69	682037.97910004	0.03950484
1959	70	655094.13240815	0.04189092
1959	71	627647.70797730	0.04600594
1959	72	598734.52622224	0.05061581
1959	73	568429.09259033	0.05557459
1959	74	536838.88156787	0.06097616
1959	75	504104.50672148	0.06663053
1959	76	470515.75562667	0.07299076
1959	77	436172.45354080	0.07989208
1959	78	401525.72809218	0.08736518
1959	79	366263.83340835	0.09544622
1959	80	331505.33419799	0.10300208
1959	81	297180.19675063	0.11228950
1959	82	263809.98076247	0.12226698
1959	83	231554.73139190	0.13296507
1959	84	200766.04076575	0.14440059
1959	85	171773.70024299	0.15147207
1959	86	145754.78166580	0.16406849
1959	87	121841.01535888	0.17742089
1959	88	100223.14265918	0.19155756
1959	89	81024.64245986	0.20644064
1959	90	64297.21506690	0.22209591
1959	91	50017.06669902	0.23847220
1959	92	38089.38695764	0.25554713
1959	93	28355.75349068	0.27328077
1959	94	20606.50112724	0.29164327
1959	95	14596.75375055	0.31055449
1959	96	10063.66635903	0.32995904
1959	97	6743.06864208	0.34978607
1959	98	4584.43714458	0.35995521
1959	99	2762.39178979	0.39038687
1959	100	1683.99029795	0.41099744
1959	101	991.87459249	0.43106416
1959	102	563.09804193	0.45237175
1959	103	308.69697437	0.47296169
1959	104	162.69513120	0.49357598
1959	105	82.42558690	0.51351092
1959	106	40.09865312	0.53532413
1959	107	10.71307591	0.55270856
1959	108	8.37019733	0.57162967
1959	109	3.58561956	1.00000000

	x	l_x	q_x
1960	10	999909.00000001	0,00048461
1960	11	999514.38849640	0,00048461
1960	12	999030.01184846	0.00048461
1960	13	998545.86993410	0,00049528
1960	14	998051.31160840	0,00052017
1960	15	997532,15816500	0,00068516
1960	16	996848.68098120	0,00077065
1960	17	996080.46917725	0,00087553
1960	18	995208.57140759	0.00085049
1960	19	994301.95221711	0.00082081
1960	20	993539.80330762	0.00098680
1960	21	992559.37819673	0.00096157
1960	22	991604.96074678	0.00093854
1960	23	990674.30450832	0.00092258
1960	24	989760.32476807	0.00090458
1960	25	988865.00881958	0,00086265
1960	26	988071.29945375	0,00079609
1960	27	987284.70162964	0,00078844
1960	28	986506.29091646	0.00078844
1960	29	985728.48581696	0,00079222
1960	30	984947.57454083	0,00083612
1960	31	984124.63862000	0,00085353
1960	32	983284.05631256	0,00087612
1960	33	982422.57671357	0.00090882
1960	34	981529.72754671	0.00095217
1960	35	980595.14041902	0.00105429
1960	36	979580.92547608	0.00110715
1960	37	978496.38169861	0.00119299
1960	38	977329.04552461	0.00130169
1960	39	976056.86442567	0.00142959
1960	40	974061.50160981	0.00159846
1960	41	973103.54901124	0.00178274
1960	42	971368.76135255	0.00190415
1960	43	969431.70220947	0,00224314
1960	44	967257.13542939	0.00253108
1960	45	964808.93167115	0,00282430
1960	46	962084.02500154	0,00314582
1960	47	959019.00247194	0.00359531
1960	48	955571.03543092	0.00405474
1960	49	951696.44687653	0.00456626
1960	50	947350.75189210	0.00529671
1960	51	942332.90620424	0.00594018
1960	52	936735.27999116	0.00665008
1960	53	930505.91609193	0,00743424
1960	54	923588.31295488	0.00829141
1960	55	915930.46049500	0,00984280
1960	56	906915.14472962	0,01094832
1960	57	896985.94850159	0.01217443
1960	58	886074.62168886	0.01350120
1960	59	874111.55475328	0.01496910
1960	60	861026.88971710	0,01615777
1960	61	847114.61370088	0,01788177
1960	62	831966.70777131	0,01977690
1960	63	815512.98325349	0,02105542
1960	64	797689,60511017	0.02413498
1960	65	778437.38088228	0.02404538
1960	66	759721.11135576	0.02651748
1960	67	739575.22137452	0.02923416

1960	68	717954.36282349	0,03220759
1960	69	694830.78204345	0,03546141
1960	70	670191.10457610	0,03930529
1960	71	645849.04848479	0,04322140
1960	72	616020.99024200	0,04749551
1960	73	586762.76051330	0,05215430
1960	74	556160.55845353	0,05722962
1960	75	524331.69877625	0,07210961
1960	76	486522.34222794	0.07890334
1960	77	448104.91100311	0,08639404
1960	78	409391.31793976	0,09443385
1960	79	370730.91785430	0.10312011
1960	80	332501.10401534	0.10007815
1960	81	299225.00799179	0,10910781
1960	82	266577.22179412	0,11881168
1960	83	234904.73323059	0,12922032
1960	84	204550.26833914	0,14035917
1960	85	175839.76197813	0.14975538
1960	86	149506.81123733	0,16221039
1960	87	125254.05761623	0,17544180
1960	88	103279.26037882	0,18943683
1960	89	83714.36504840	0.20419448
1960	90	66620.35377787	0.21971200
1960	91	51983.06264161	0,23596554
1960	92	39716.85130452	0,25292767
1960	93	29671.36063718	0,27056702
1960	94	21643.26901125	0,28885095
1960	95	15391.85001127	0,30768517
1960	96	10656.00597059	0.32704718
1960	97	7170.98929268	0,34685706
1960	98	4683.68105206	0,36703748
1960	99	2964.59450077	0,38751095
1960	100	1815.78170748	0,40819552
1960	101	1074.58775404	0,42899139
1960	102	613.59885897	0,44982268
1960	103	337.58817804	0,47059220
1960	104	178.72161315	0,49121894
1960	105	90.93027388	0.51161118
1960	106	44 40932904	0 53169977
1960	107	20.79689908	0,55139719
1960	108	9.32954741	0.57064960
1960	109	4.00564493	1.00000000

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1961	12	999042.78276063	0.00047622
1961	13	998565.01707459	0.00049056
1961	14	998075.15884401	0.00051935
1961	15	997556.80574800	0.00068268
1961	16	996875.79365540	0.00078001
1961	17	996098.21593476	0.00089944
1961	18	995202.28824616	0.00087093
1961	19	994335.53495789	0.00084396
1961	20	993496.35105133	0.00094598
1961	21	992556.52731678	0.00091891
1961	22	991644.45623780	0.00089420
1961	23	990757.72984315	0.00087607
1961	24	989889.75609589	0.00085676
1961	25	989041.66284181	0.00080857
1961	26	988241.95690919	0.00080010
1961	27	987451.26747132	0.00079134
1961	28	986669.85840712	0.00079037
1961	29	985890.02165223	0.00079371
1961	30	985107.50925447	0.00083094
1961	31	984288.94652559	0.00084366
1961	32	983453.61862946	0.00087226
1961	33	982595.70119874	0.00090631
1961	34	981705.25444794	0.00095143
1961	35	980771.23211670	0.00102693
1961	36	979764.05107881	0.00110164
1961	37	978684.70075989	0.00118991
1961	38	977520.14921572	0.00130080
1961	39	976248.58917238	0.00143121
1961	40	974851.36901094	0.00152997
1961	41	973359.87697601	0.00170602
1961	42	971697.56078645	0.00191223
1961	43	969839.25052644	0.00215182
1961	44	967752.32019715	0.00242830
1961	45	965402.33342744	0.00281986
1961	46	962680.03707150	0.00318104
1961	47	959617.71179963	0.00358972
1961	48	956172.95407106	0.00404808
1961	49	952302.28656006	0.00455854
1961	50	947961.17562104	0.00500537
1961	51	943216.27842714	0.00561428
1961	52	937920.79495240	0.00628651
1961	53	932074.54244996	0.00702935
1961	54	925473.01573945	0.00784248
1961	55	918215.01434327	0.00967753
1961	56	909328.95746614	0.01076815
1961	57	899537.16613007	0.01196822
1961	58	888771.30749511	0.01328770
1961	59	876961.58522797	0.01473702
1961	60	864037.78377534	0.01592241
1961	61	850280.22424316	0.01762027
1961	62	835292.95430859	0.01949906
1961	63	819005.52713013	0.02155517
1961	64	801353.36407471	0.02380588
1961	65	782276.43849183	0.02603363
1961	66	761910.94039017	0.02871324
1961	67	740034.00577545	0.03165366

1961	68	716009.22371778	0.03487066
1961	69	691020.54764648	0.03838925
1961	70	665069.79082490	0.04146324
1961	71	637480.54322816	0.04561021
1961	72	608404.92293549	0.05011231
1961	73	577916.34567260	0.05501736
1961	74	546120.91581725	0.06035007
1961	75	513158.11310195	0.06972018
1961	76	477380.65937759	0.07634940
1961	77	440932.91596727	0.08353705
1961	78	404098.67955398	0.09131505
1961	79	367198.38821029	0.09971986
1961	80	330581.41787338	0.10469682
1961	81	295970.59389114	0.11410517
1961	82	262198.81880187	0.12423101
1961	83	229030.83864402	0.13504507
1961	84	198620.32660968	0.14603296
1961	85	169496.04056357	0.15098517
1961	86	143904.65150554	0.16353575
1961	87	120371.09659098	0.17685435
1961	88	99082.94470595	0.19095246
1961	89	80162.81237028	0.20582289
1961	90	63063.47064781	0.22146330
1961	91	49564.34815165	0.23785173
1961	92	37775.38235664	0.25496213
1961	93	28144.09032058	0.27276404
1961	94	20467.39459657	0.29121414
1961	95	14506.99992871	0.31025595
1961	96	10006.11092428	0.32983175
1961	97	6705.78183460	0.34967448
1961	98	4359.59990126	0.37031768
1961	99	2745.20656258	0.39105388
1961	100	1671.68287373	0.41203084
1961	101	982.89796950	0.43313977
1961	102	557.16577204	0.45430260
1961	103	304.04391113	0.47542202
1961	104	159.49474221	0.49641401
1961	105	80.31918983	0.51718835
1961	106	38.77904096	0.53767014
1961	107	17.92870859	0.55777184
1961	108	7.92857966	0.57743546
1961	109	3.35033665	1.00000000

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1962	12	998955.35800935	0.00052196
1962	13	998433.94553376	0.00053766
1962	14	997897.12917329	0.00057429
1962	15	997324.04349519	0.00066086
1962	16	996658.97174836	0.00077465
1962	17	995886.91167451	0.00090690
1962	18	994983.74073793	0.00087533
1962	19	994112.79841615	0.00084547
1962	20	993272.30718995	0.00094212
1962	21	992336.52815721	0.00091272
1962	22	991431.29537965	0.00088493
1962	23	990553.95201875	0.00086398
1962	24	989698.13564302	0.00084264
1962	25	988864.17380523	0.00075506
1962	26	988117.52255249	0.00074531
1962	27	987381.07045459	0.00073610
1962	28	986654.25750362	0.00073424
1962	29	985929.81885240	0.00073690
1962	30	985203.28691101	0.00074550
1962	31	984468.81826019	0.00076180
1962	32	983718.84597015	0.00078408
1962	33	982947.53584291	0.00081611
1962	34	982145.33845521	0.00085855
1962	35	981302.12197114	0.00088218
1962	36	980430.44154358	0.00094850
1962	37	979506.49670411	0.00102708
1962	38	978500.46211245	0.00112502
1962	39	977399.63060102	0.00124015
1962	40	976187.50985719	0.00139848
1962	41	974822.32698823	0.00156280
1962	42	973298.87054445	0.00172122
1962	43	971594.40781403	0.00197144
1962	44	969678.96557619	0.00222499
1962	45	967521.43844605	0.00281542
1962	46	964797.46337891	0.00317628
1962	47	961733.00090028	0.00358413
1962	48	958286.02098085	0.00404143
1962	49	954413.17971040	0.00455081
1962	50	950069.87232666	0.00506585
1962	51	945256.91292574	0.00568287
1962	52	939885.14445497	0.00630448
1962	53	933905.26293946	0.00711794
1962	54	927255.79090883	0.00794382
1962	55	919889.83356478	0.00943906
1962	56	911206.93871307	0.01050649
1962	57	901633.34991456	0.01168141
1962	58	891101.00016317	0.01297360
1962	59	879540.21561437	0.01439536
1962	60	866880.67591096	0.01645996
1962	61	852611.85777284	0.01822583
1962	62	837072.29912567	0.02016637
1962	63	820191.59130098	0.02229468
1962	64	801905.68213653	0.02462845
1962	65	782155.99150850	0.02672714
1962	66	761251.19755553	0.02946096
1962	67	738808.77870941	0.03250125

1962	68	714796.57012176	0.03580481
1962	69	689203.41175081	0.03941690
1962	70	662037.15087128	0.03995855
1962	71	635583.10511017	0.04393644
1962	72	607657.84764099	0.04827562
1962	73	578322.79077149	0.05300509
1962	74	547669.89633179	0.05815020
1962	75	515822.77973173	0.07291695
1962	76	478209.52181244	0.07983181
1962	77	440033.18860245	0.08732270
1962	78	401608.30063247	0.09542451
1962	79	363285.02437591	0.10417420
1962	80	325440.09711837	0.10467578
1962	81	291575.05315017	0.11406909
1962	82	258136.16534233	0.12416118
1962	83	226087.42615699	0.13498085
1962	84	195569.95332335	0.14655421
1962	85	166908.35417938	0.15124473
1962	86	141564.34563445	0.16381026
1962	87	118458.27180289	0.17714856
1962	88	97473.55948351	0.19127163
1962	89	78829.63330172	0.20617444
1962	90	62576.97765492	0.22185548
1962	91	48693.93244409	0.23829514
1962	92	37090.40491151	0.25546935
1962	93	27614.94344138	0.27334905
1962	94	20066.42477106	0.29189243
1962	95	14209.17734789	0.31104695
1962	96	9789 45605671	0 33075468
1962	97	6551.54567933	0.35095113
1962	98	4252.27332896	0.37156080
1962	99	2672.29520734	0.39250703
1962	100	1623.40060064	0.41371784
1962	101	951.78704063	0.43506001
1962	102	537.69684744	0.45650170
1962	103	292.23732169	0.47791778
1962	104	152.57190973	0.49922914
1962	105	76.40356632	0.52034237
1962	106	36.64755369	0.54118172
1962	107	16.81456754	0.56165798
1962	108	7.37053148	0.58170888
1962	109	3.08302789	1.00000000

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1963	11	999524.03332520	0.00047497
1963	12	999049.29224396	0.00047497
1963	13	998574.77664947	0.00049157
1963	14	998083.90954591	0.00053030
1963	15	997554.62310028	0.00063239
1963	16	996923.78205109	0.00074754
1963	17	996178.53826904	0.00088883
1963	18	995293.10299683	0.00085511
1963	19	994442.02060041	0.00082520
1963	20	993623.39370729	0.00095339
1963	21	992676.08658601	0.00092008
1963	22	991762.74124910	0.00088967
1963	23	990880.39603425	0.00086547
1963	24	990022.81075611	0.00084170
1963	25	989189.51432040	0.00077445
1963	26	988423.43775177	0.00076245
1963	27	987669.81465623	0.00075189
1963	28	986927.19380190	0.00074894
1963	29	986188.04682924	0.00075117
1963	30	985447.24750624	0.00074031
1963	31	984717.71494294	0.00075692
1963	32	983972.35819246	0.00078021
1963	33	983204.65551759	0.00081360
1963	34	982404.71723175	0.00085781
1963	35	981562.00027755	0.00094343
1963	36	980635.96817017	0.00101676
1963	37	979638.89797975	0.00110588
1963	38	978557.49182130	0.00121162
1963	39	977371.85655213	0.00133820
1963	40	976063.95331909	0.00145049
1963	41	974642.30463411	0.00162930
1963	42	973054.32347108	0.00182739
1963	43	971276.17605590	0.00205801
1963	44	969277.27851105	0.00232292
1963	45	967025.72312166	0.00294774
1963	46	964175.18692017	0.00332579
1963	47	960968.54680634	0.00375259
1963	48	957362.43054963	0.00423094
1963	49	953311.89213563	0.00476390
1963	50	948770.40602112	0.00533716
1963	51	943706.66835023	0.00598795
1963	52	938055.79631806	0.00670733
1963	53	931763.94232942	0.00750280
1963	54	924773.10523226	0.00837589
1963	55	917027.30548097	0.00986000
1963	56	907985.41370496	0.01097857
1963	57	898017.03398896	0.01221005
1963	58	887052.19815827	0.01356472
1963	59	875019.58531952	0.01505343
1963	60	861847.54152681	0.01608378
1963	61	847985.77455140	0.01781467
1963	62	832879.18880092	0.01971656
1963	63	816457.67583466	0.02180263
1963	64	798656.75205994	0.02409004
1963	65	779417.08046724	0.02612194
1963	66	759057.19243621	0.02881024
1963	67	737182.50163268	0.03177432

1963	68	713759.03015899	0.03500736
1963	69	688772.21060181	0,03854179
1963	70	662225.69693756	0,04185342
1963	71	634509.28701020	0.04601443
1963	72	605312.70359038	0.05055132
1963	73	574713.34539795	0.05549207
1963	74	542821.31503295	0,06086065
1963	75	509780.51573943	0,06687605
1963	76	475688.40594101	0.07323220
1963	77	440852.69778824	0,08012293
1963	78	405530.28620911	0.08759001
1963	79	370013.94003296	0,09563863
1963	80	334626.31523894	0,10639499
1963	81	299023.75052643	0,11592322
1963	82	264359.95419693	0.12615637
1963	83	231009.26146222	0.13712536
1963	84	199332.03307151	0.14885067
1963	85	169660.12974107	0.14971375
1963	86	144259.67005399	0.16216035
1963	87	120866.47621440	0,17537893
1963	88	99069.04290389	0.18938242
1963	89	80793.47882556	0,20415862
1963	90	64297.98571490	0,21973745
1963	91	50169.31005096	0.23607259
1963	92	38325.71091508	0.25315283
1963	93	28623.44659790	0.27095161
1963	94	20667.87919974	0,28943068
1963	95	14028.07464349	0.30853979
1963	96	10253.02354824	0,32822540
1963	97	6687.72070398	0.34842471
1963	98	4487.86865133	0,36900569
1963	99	2831.55032897	0.39007335
1963	100	1727.03800246	0.41136720
1963	101	1016.59120996	0,43285276
1963	102	576.55689441	0.45444957
1963	103	314.54097748	0.47606103
1963	104	164.80027698	0,49760141
1963	105	82.79542721	0.51897691
1963	106	39.82651213	0,54010087
1963	107	18.31585986	0.56090742
1963	108	8.04235809	0.58130600
1963	109	3.36728711	1.00000000

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1964	11	999521.06220349	0.00047794
1964	12	999043.35295107	0.00047794
1964	13	998565.87195588	0.00049731
1964	14	998069.27030184	0.00054253
1964	15	997527.79051209	0.00066999
1964	16	996859.46131135	0.00080672
1964	17	996055.27220919	0.00097449
1964	18	995084.62841798	0.00093444
1964	19	994154.77797700	0.00089056
1964	20	993263.45868684	0.00094228
1964	21	992327.53079225	0.00090027
1964	22	991428.21714021	0.00087339
1964	23	990562.31798554	0.00084642
1964	24	989723.88899994	0.00082072
1964	25	988911.60468294	0.00077413
1964	26	988146.05760193	0.00076002
1964	27	987395.04444885	0.00074829
1964	28	986656.18795014	0.00074423
1964	29	985921.88640596	0.00074595
1964	30	985186.44266511	0.00081471
1964	31	984383.79825594	0.00083351
1964	32	983563.30930432	0.00086650
1964	33	982716.94936372	0.00089912
1964	34	981833.37030793	0.00095021
1964	35	980900.42182160	0.00100380
1964	36	979915.79345705	0.00108452
1964	37	978853.05997468	0.00118067
1964	38	977697.35656730	0.00128066
1964	39	976427.66367341	0.00143723
1964	40	975024.51350051	0.00151423
1964	41	973547.90547180	0.00169567
1964	42	971897.08767700	0.00190361
1964	43	970046.97705079	0.00214473
1964	44	967966.48341371	0.00242105
1964	45	965622.98582459	0.00294330
1964	46	962780.86521149	0.00332105
1964	47	959583.42127728	0.00374702
1964	48	955987.84600068	0.00422426
1964	49	951949.50312806	0.00475614
1964	50	947421.90174103	0.00504595
1964	51	942641.25872449	0.00560215
1964	52	937303.88601684	0.00634377
1964	53	931357.84806825	0.00709779
1964	54	924747.26837922	0.00792061
1964	55	917417.16165925	0.00925540
1964	56	908926.09767152	0.01030938
1964	57	899555.62984468	0.01147018
1964	58	889237.56093598	0.01274753
1964	59	877901.98172760	0.01415176
1964	60	865478.12422944	0.01479000
1964	61	852672.50627899	0.01639417
1964	62	838693.65113069	0.01815022
1964	63	823471.17797851	0.02007668
1964	64	806938.61020661	0.02218952
1964	65	789033.19439697	0.02453044
1964	66	769638.41370393	0.02712359
1964	67	748763.05549622	0.02991100

1964	68	726366.80065629	0,03295964
1964	69	702426.00901032	0,03629238
1964	70	676933.29693604	0,04089771
1964	71	649248.27429199	0,04496503
1964	72	620054.80875396	0,04939915
1964	73	589424.62035040	0,05422755
1964	74	557461.57043458	0,05948119
1964	75	524303.09121703	0,06406180
1964	76	490715.29360962	0.07015409
1964	77	456289.60752105	0.07675936
1964	78	421265.10728073	0,08390628
1964	79	385917.22533035	0,09103679
1964	80	350553.00840377	0,09475035
1964	81	317337.98846435	0.10328681
1964	82	284561.15672192	0,11246687
1964	83	252557.45512198	0.12232114
1964	84	221064.33883475	0,13287705
1964	85	192210.23513412	0,14399046
1964	86	164533.79562376	0,15609222
1964	87	138866,15845298	0,16877131
1964	88	115429.53515338	0,18231243
1964	89	94585.29605006	0.19662752
1964	90	75826.54911427	0.21171933
1964	91	59772,60266256	0,22757666
1964	92	46169,75321054	0,24410527
1964	93	34895,87207507	0,26151734
1964	94	25769,99644946	0,27934037
1964	95	18566,08743595	0,29822671
1964	96	13029.18424141	0.31751052
1964	97	8092.28120124	0,33734091
1964	98	5892.55074451	0,35765197
1964	99	3785.06852022	0.37837377
1964	100	2352,89786795	0,39943045
1964	101	1413,07832276	0,42073340
1964	102	818,54930322	0,44220388
1964	103	456.58366269	0,46374964
1964	104	244.84315294	0.48526536
1964	105	126.02435600	0.50671914
1964	106	62,10540258	0,52797079
1964	107	29,34388611	0,54895088
1964	108	13.23553414	0.56957399
1964	109	5,69671959	1,00000000

	x	l_x	q_x
1965	10	999999.00000001	0.00049044
1965	11	999508.56291962	0.00049044
1965	12	999018.36037117	0.00049044
1965	13	998528.41023256	0.00051350
1965	14	998015.66423035	0.00056732
1965	15	997449.47198487	0.00066389
1965	16	996787.27994538	0.00081513
1965	17	995974.77079774	0.00100069
1965	18	994978.10926057	0.00095640
1965	19	994026.51473999	0.00091450
1965	20	993117.48233796	0.00091595
1965	21	992207.83574678	0.00087786
1965	22	991336.81523896	0.00084508
1965	23	990501.03970338	0.00081582
1965	24	989694.94562531	0.00076064
1965	25	988916.41171265	0.00077399
1965	26	988150.99578095	0.00075765
1965	27	987402.32707216	0.00074466
1965	28	986667.04542547	0.00073944
1965	29	985937.46725464	0.00074059
1965	30	985207.28701762	0.00080904
1965	31	984410.21270503	0.00082624
1965	32	983594.88240443	0.00085652
1965	33	982752.41202546	0.00089686
1965	34	981871.02290737	0.00095020
1965	35	980938.04666902	0.00092817
1965	36	980027.57070923	0.00100543
1965	37	979042.22121431	0.00109772
1965	38	977967.50721741	0.00121010
1965	39	976784.06760407	0.00134203
1965	40	975473.18997193	0.00144603
1965	41	974062.62710675	0.00162108
1965	42	972483.59527588	0.00182163
1965	43	970712.09051514	0.00205326
1965	44	968718.96301270	0.00231806
1965	45	966473.41150665	0.00280210
1965	46	963765.25670728	0.00316201
1965	47	960717.81771090	0.00356740
1965	48	957290.55578614	0.00402142
1965	49	953440.88562777	0.00452757
1965	50	949124.11849976	0.00524066
1965	51	944144.38230897	0.00588816
1965	52	938585.10771180	0.00659874
1965	53	932392.09387208	0.00738402
1965	54	925507.29231263	0.00824697
1965	55	917872.81209565	0.00938289
1965	56	909260.51099397	0.01045510
1965	57	899754.09814454	0.01163631
1965	58	889284.28200635	0.01293042
1965	59	877780.12644196	0.01436604
1965	60	865169.89808654	0.01610581
1965	61	851237.36965942	0.01784717
1965	62	836045.19384766	0.01976218
1965	63	819523.11865235	0.02186272
1965	64	801606.11410524	0.02416567
1965	65	782234.76155091	0.02469638
1965	66	762916.39736939	0.02725520
1965	67	742122.96139526	0.03005846

1965	68	719815.88965606	0.03312587
1965	69	695972.80432891	0.03647406
1965	70	670587.84762573	0.03972856
1965	71	643946.35587311	0.04308137
1965	72	615817.89700420	0.04799016
1965	73	586264.69825745	0.05268177
1965	74	555379.23635101	0.05776602
1965	75	523286.08021163	0.06139325
1965	76	491159.84810637	0.06723463
1965	77	458136.89782333	0.07356828
1965	78	424432.55400847	0.08042496
1965	79	390297.58463287	0.08783778
1965	80	356014.71065139	0.09704472
1965	81	321465.36344528	0.10576498
1965	82	287465.58632660	0.11514060
1965	83	254366.62565040	0.12520243
1965	84	222519.30490683	0.13597837
1965	85	192261.49227141	0.14337427
1965	86	164096.14037039	0.15533564
1965	87	139112.96061134	0.16805034
1965	88	115734.14501380	0.18155182
1965	89	94722.40067005	0.19582548
1965	90	76173.24645997	0.21088419
1965	91	60109.51324986	0.22671654
1965	92	46481.69205520	0.24331970
1965	93	35172.24557351	0.26064379
1965	94	26004.81814121	0.27860883
1965	95	18757.56574010	0.29741426
1965	96	13178.98582374	0.31674431
1965	97	9004.61702049	0.33665425
1965	98	5973.17443132	0.35707035
1965	99	3840.33090299	0.37792396
1965	100	2388.97766856	0.39914009
1965	101	1435.44101520	0.42063233
1965	102	851.64811409	0.44232064
1965	103	463.79298413	0.46411352
1965	104	248.54059190	0.48592408
1965	105	127.76806995	0.50766004
1965	106	62.90560249	0.52923014
1965	107	29.01355873	0.55054828
1965	108	13.30927278	0.57157581
1965	109	5.70201447	1.00000000

	X	l_x	q_x
1966	10	999999.00000001	0.00047278
1966	11	999526.22268678	0.00047278
1966	12	999053.66889191	0.00047278
1966	13	998581.33850861	0.00049861
1966	14	998083.43470105	0.00055889
1966	15	997525.61705018	0.00067944
1966	16	996847.85380556	0.00085167
1966	17	995998.87171174	0.00106297
1966	18	994940.15865326	0.00101253
1966	19	993932.75106049	0.00096482
1966	20	992973.78820801	0.00092010
1966	21	992060.15383913	0.00087865
1966	22	991188.47031837	0.00084081
1966	23	990355.07705690	0.00080829
1966	24	989554.58652497	0.00077871
1966	25	988784.01324463	0.00075386
1966	26	988038.60440827	0.00073562
1966	27	987311.78372193	0.00072168
1966	28	986599.26219941	0.00071538
1966	29	985893.47127534	0.00071593
1966	30	985187.63948061	0.00072460
1966	31	984474.36143494	0.00074170
1966	32	983744.17373735	0.00076843
1966	33	982988.23687744	0.00080645
1966	34	982195.51065064	0.00085670
1966	35	981354.06269075	0.00092025
1966	36	980450.97515870	0.00099960
1966	37	979470.91825868	0.00109463
1966	38	978398.75700378	0.00120949
1966	39	977215.39375365	0.00134427
1966	40	975901.75352479	0.00150541
1966	41	974434.57279206	0.00168730
1966	42	972790.41041566	0.00189792
1966	43	970944.13062721	0.00214018
1966	44	968866.14005281	0.00241045
1966	45	966524.92537690	0.00272928
1966	46	963887.00680171	0.00308012
1966	47	960918.12218476	0.00347480
1966	48	957579.12570294	0.00391067
1966	49	953828.60000715	0.00440941
1966	50	949622.77445985	0.00495563
1966	51	944916.79218293	0.00556248
1966	52	939660.70827485	0.00623472
1966	53	933802.18894959	0.00697693
1966	54	927285.25195313	0.00779937
1966	55	920053.01117707	0.00870638
1966	56	912042.68003846	0.00971527
1966	57	903191.06211092	0.01086611
1966	58	893451.07863618	0.01201821
1966	59	882693.63879395	0.01335155
1966	60	870908.31471253	0.01481684
1966	61	858004.20507050	0.01642695
1966	62	843909.80907441	0.01819559
1966	63	828554.37031555	0.02013594
1966	64	811870.64922333	0.02226350
1966	65	793795.56477357	0.02459615
1966	66	774271.25197603	0.02714863
1966	67	753250.85210418	0.02994367

1966	68	730695.75527955	0.03299961
1966	69	706583.08154297	0.03633867
1966	70	680906.78937531	0.03998428
1966	71	653681.22051238	0.04396103
1966	72	624944.71913147	0.04829472
1966	73	594763.19168091	0.05301229
1966	74	563233.43222045	0.05814324
1966	75	530485.21278381	0.06371813
1966	76	496683.68659210	0.06976571
1966	77	462032.19420624	0.07631976
1966	78	426770.00584792	0.08341199
1966	79	391172.27216721	0.09107594
1966	80	355545.89097213	0.09934264
1966	81	320225.02342223	0.10824584
1966	82	285561.99819564	0.11781604
1966	83	251918.21576526	0.12808430
1966	84	219651.44609642	0.13907901
1966	85	189102.54140853	0.14329589
1966	86	162004.92375564	0.15524768
1966	87	136854.03452491	0.16796276
1966	88	113867.65207944	0.18145718
1966	89	93205.55903355	0.19573770
1966	90	74961.70971297	0.21080915
1966	91	59159.09533547	0.22660572
1966	92	45749.75624704	0.24329572
1966	93	34619.03653096	0.26068094
1966	94	25594.51366280	0.27879359
1966	95	18458.92720341	0.29759585
1966	96	12965.62701332	0.31704365
1966	97	8854.95733344	0.33708394
1966	98	5870.09341234	0.35765477
1966	99	3770.62648594	0.37861830
1966	100	2342.73435479	0.40011994
1966	101	1405.38305581	0.42183499
1966	102	812.54331368	0.44373170
1966	103	451.95140148	0.46585884
1966	104	241.40587847	0.48797764
1966	105	123.60520844	0.51004501
1966	106	60.56098907	0.53197438
1966	107	28.34409474	0.55367472
1966	108	12.65065590	0.57506700
1966	109	5.37569392	1.00000000

	x	l_x	q_x
1967	10	999999.00000001	0.00047435
1967	11	999524.64676668	0.00047435
1967	12	999050.51854707	0.00047435
1967	13	998576.61523459	0.00050457
1967	14	998072.76097108	0.00057508
1967	15	997498.78672791	0.00061008
1967	16	996890.23107148	0.00076172
1967	17	996110.94165041	0.00099231
1967	18	995122.49403382	0.00094204
1967	19	994185.04743958	0.00089449
1967	20	993295.76071168	0.00086269
1967	21	992438.85717011	0.00082076
1967	22	991624.30125429	0.00078248
1967	23	990848.37740326	0.00074897
1967	24	990106.26530562	0.00071904
1967	25	989394.33477784	0.00073377
1967	26	988668.35160829	0.00071360
1967	27	987962.83609010	0.00069869
1967	28	987272.55693054	0.00069130
1967	29	986590.05390562	0.00069125
1967	30	985908.07822419	0.00067019
1967	31	985241.41670228	0.00069325
1967	32	984558.40150452	0.00071964
1967	33	983849.87609102	0.00075708
1967	34	983105.02063752	0.00080656
1967	35	982312.08712769	0.00085555
1967	36	981471.66621400	0.00093203
1967	37	980556.90210828	0.00102386
1967	38	979552.94430172	0.00113403
1967	39	978442.10279846	0.00126325
1967	40	977206.08983613	0.00144737
1967	41	975791.71240998	0.00162630
1967	42	974204.78569031	0.00183117
1967	43	972420.84779360	0.00209586
1967	44	970411.95857239	0.00233282
1967	45	968148.16442109	0.00265279
1967	46	965579.86691176	0.00299407
1967	47	962088.85562398	0.00337752
1967	48	959437.35186006	0.00380667
1967	49	955785.09394074	0.00428535
1967	50	951689.22200116	0.00469512
1967	51	947220.92646027	0.00527097
1967	52	942228.15390119	0.00596935
1967	53	936060.19985190	0.00661638
1967	54	930462.89992524	0.00759705
1967	55	923580.22175598	0.00844111
1967	56	915784.17941285	0.00941329
1967	57	907165.63874055	0.01048506
1967	58	897651.97627259	0.01165550
1967	59	887180.41694647	0.01296443
1967	60	875678.62585450	0.01414920
1967	61	863288.47531892	0.01569213
1967	62	849741.63872524	0.01733684
1967	63	834967.31463624	0.01924629
1967	64	818897.29360592	0.02128519
1967	65	801466.90870297	0.02568871
1967	66	782481.18880567	0.02615212
1967	67	762017.64687347	0.02884664

1967	68	740034.47540283	0.03179066
1967	69	716503.84790802	0.03501741
1967	70	691413.73745728	0.03828197
1967	71	664945.05502319	0.04209228
1967	72	636956.00034333	0.04624431
1967	73	607500.40802766	0.05076411
1967	74	576661.18994141	0.05567973
1967	75	544552.84885405	0.05980531
1967	76	511985.69950103	0.06548831
1967	77	478456.62224960	0.07164845
1967	78	444175.94681549	0.07831651
1967	79	409389.63540649	0.08552465
1967	80	374570.73120498	0.09234015
1967	81	339806.73413406	0.10064118
1967	82	305608.18413543	0.10957127
1967	83	272122.30635451	0.11910115
1967	84	239095.80953422	0.12943960
1967	85	208669.74261284	0.13988466
1967	86	179480.04665657	0.15157443
1967	87	152275.46156310	0.16401987
1967	88	127299.25956243	0.17723828
1967	89	104736.95779036	0.19125958
1967	90	84707.09567259	0.20003087
1967	91	67254.81942081	0.22101000
1967	92	52350.47917365	0.23796907
1967	93	39892.68410158	0.25509334
1967	94	29716.32617711	0.27295921
1967	95	21004.98120053	0.29155383
1967	96	15306.30841207	0.31077719
1967	97	10549.51087369	0.33064653
1967	98	7061.42041230	0.35104627
1967	99	4582.39387828	0.37199008
1967	100	2877.78833472	0.39334046
1967	101	1745.83004089	0.41503712
1967	102	1021.25044593	0.43699907
1967	103	574.96494646	0.45913774
1967	104	310.97683873	0.48136456
1967	105	161.28364060	0.50355752
1967	106	80.06319739	0.52571823
1967	107	37.97251512	0.54706595
1967	108	17.17620168	0.56934000
1967	109	7.39699153	1.00000000

	x	l_x	q_x
1968	10	999999.00000001	0.00043640
1968	11	999562.60019685	0.00043640
1968	12	999126.39985804	0.00043640
1968	13	998690.37184144	0.00046099
1968	14	998221.99626923	0.00054503
1968	15	997677.93370923	0.00055918
1968	16	997120.05252840	0.00073546
1968	17	996388.70093536	0.00094730
1968	18	995444.82385728	0.00089626
1968	19	994552.64913942	0.00084797
1968	20	993709.29770766	0.00084351
1968	21	992871.09806826	0.00079943
1968	22	992077.36370192	0.00075919
1968	23	991324.18997956	0.00072339
1968	24	990607.07580568	0.00069194
1968	25	989921.63897706	0.00070690
1968	26	989221.86009213	0.00068501
1968	27	988544.23750306	0.00066926
1968	28	987882.64527131	0.00066085
1968	29	987229.80140027	0.00066019
1968	30	986578.04615784	0.00064425
1968	31	985942.44249727	0.00065104
1968	32	985290.69235993	0.00065767
1968	33	984613.14207460	0.00072534
1968	34	983898.95951344	0.00077511
1968	35	983136.33107758	0.00083026
1968	36	982320.06831361	0.00090726
1968	37	981428.84590150	0.00099995
1968	38	980447.46311168	0.00111033
1968	39	979358.83847809	0.00123975
1968	40	978144.67894745	0.00148811
1968	41	976689.09503937	0.00167407
1968	42	975054.04580030	0.00188695
1968	43	973214.17179109	0.00212977
1968	44	971141.45125292	0.00240525
1968	45	968805.60847763	0.00268438
1968	46	966204.96881867	0.00302998
1968	47	963277.38464356	0.00341780
1968	48	959985.09648696	0.00385165
1968	49	956287.57198335	0.00433572
1968	50	952141.38092042	0.00483075
1968	51	947494.21686555	0.00548018
1968	52	942301.78247835	0.00614517
1968	53	936511.17840969	0.00688195
1968	54	930066.15085603	0.00769670
1968	55	922907.70726776	0.00858983
1968	56	914980.08328248	0.00958279
1968	57	906212.01761628	0.01067779
1968	58	896535.67588807	0.01188412
1968	59	885881.13475037	0.01321189
1968	60	874176.96825864	0.01442241
1968	61	861569.22774506	0.01599989
1968	62	847784.21891023	0.01773220
1968	63	832751.13663484	0.01963289
1968	64	816401.82649995	0.02171678
1968	65	798672.20520783	0.02458820
1968	66	779034.29631044	0.02714789
1968	67	757885.16143036	0.02994818

1968	68	735187.88379609	0.05303076
1968	69	710920.24053191	0.05635123
1968	70	685077.41660309	0.04050116
1968	71	657330.99520659	0.04452027
1968	72	628062.48875428	0.04691015
1968	73	597343.85569763	0.05368002
1968	74	565278.42264557	0.05880467
1968	75	532003.49709319	0.06480213
1968	76	497485.97883987	0.07102213
1968	77	462153.46714402	0.07707236
1968	78	426256.91012244	0.08486545
1968	79	390082.43080624	0.09263476
1968	80	353947.23724364	0.10045372
1968	81	318391.92086792	0.10942798
1968	82	283550.93740462	0.11907410
1968	83	249787.36559787	0.12942344
1968	84	217459.02312659	0.14050542
1968	85	186904.85128020	0.14243542
1968	86	160282.98007064	0.15431534
1968	87	135548.85767744	0.10696433
1968	88	112917.03296279	0.18039786
1968	89	92540.81574535	0.19463372
1968	90	74534.08512972	0.20907285
1968	91	58906.31127977	0.22551704
1968	92	45021.93430709	0.24215928
1968	93	34574.15951442	0.25958510
1968	94	25509.22292113	0.27777156
1968	95	18488.48095203	0.29668677
1968	96	13003.19740815	0.31624052
1968	97	8690.40930818	0.33653367
1968	98	5898.48730928	0.35735025
1968	99	3790.61422139	0.37869845
1968	100	2355.11450371	0.40048092
1968	101	1411.93008467	0.42262478
1968	102	815.21691256	0.44504511
1968	103	452.40861173	0.46765152
1968	104	240.83903879	0.49035157
1968	105	122.74323806	0.51305076
1968	106	59.76972706	0.53565587
1968	107	27.75372177	0.55807406
1968	108	12.26506965	0.58021735
1968	109	5.14867189	1.00000000

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1969	10	999999.00000001	0.00041746
1969	11	999581.54045106	0.00041746
1969	12	999164.25517274	0.00041746
1969	13	998747.14409639	0.00045429
1969	14	998293.41995240	0.00054024
1969	15	997754.10501862	0.00055647
1969	16	997198.88346864	0.00074836
1969	17	996452.61660766	0.00098380
1969	18	995472.30780031	0.00092760
1969	19	994548.90570069	0.00087444
1969	20	993679.23492432	0.00083990
1969	21	992844.64046040	0.00079286
1969	22	992057.46120453	0.00074991
1969	23	991313.50075202	0.00071116
1969	24	990608.52145386	0.00067759
1969	25	989937.29042054	0.00072785
1969	26	989216.76031494	0.00070259
1969	27	988521.74994659	0.00068485
1969	28	987844.75650788	0.00067478
1969	29	987178.18085481	0.00067342
1969	30	986513.39601899	0.00066977
1969	31	985852.65811153	0.00068784
1969	32	985174.54928590	0.00071719
1969	33	984467.99417115	0.00075662
1969	34	983721.15597535	0.00081332
1969	35	982921.07221986	0.00081557
1969	36	982121.39850618	0.00089191
1969	37	981245.43502045	0.00098645
1969	38	980277.48455049	0.00109622
1969	39	979200.92050335	0.00122921
1969	40	977997.27345277	0.00148245
1969	41	976547.44086457	0.00166978
1969	42	974916.82404329	0.00188414
1969	43	973079.94075777	0.00212762
1969	44	971009.59173585	0.00240313
1969	45	968676.13100706	0.00287360
1969	46	965892.54721070	0.00324382
1969	47	962759.36432648	0.00365871
1969	48	959236.91130067	0.00412263
1969	49	955282.33236694	0.00464039
1969	50	950849.44809723	0.00490239
1969	51	946188.01290894	0.00550537
1969	52	940978.90062715	0.00617481
1969	53	935168.53530121	0.00691083
1969	54	928700.13740644	0.00773065
1969	55	921513.25479126	0.00862665
1969	56	913563.68079377	0.00962769
1969	57	904768.16859437	0.01073195
1969	58	895058.24034552	0.01194881
1969	59	884365.36062293	0.01329055
1969	60	872611.46421815	0.01480682
1969	61	859690.86099244	0.01643122
1969	62	845565.08943936	0.01821471
1969	63	830163.37017822	0.02017148
1969	64	813417.74840650	0.02231659
1969	65	795265.03531647	0.02519445
1969	66	775228.76779939	0.02782059
1969	67	753061.44855490	0.03069197

1969	68	730530.09282683	0.03362946
1969	69	705816.65232086	0.03725474
1969	70	679521.63361358	0.04136164
1969	71	651415.50399017	0.04546928
1969	72	621796.11328888	0.04994145
1969	73	590742.71678161	0.05480580
1969	74	558366.59160614	0.06009105
1969	75	524813.75531004	0.06351976
1969	76	491477.71188354	0.06952880
1969	77	457305.85705184	0.07603661
1969	78	422533.87189865	0.08307569
1969	79	387431.57985016	0.09067879
1969	80	352299.75483703	0.09693521
1969	81	318150.20848003	0.10561083
1969	82	284553.28092194	0.11492130
1969	83	251852.03181075	0.12497006
1969	84	220389.14884947	0.13564495
1969	85	190494.47383498	0.14116344
1969	86	163603.01891173	0.15294390
1969	87	135581.44315528	0.16549349
1969	88	115047.11651897	0.17883055
1969	89	94765.88141821	0.19294955
1969	90	76640.35754600	0.20791087
1969	91	60705.38112026	0.22365140
1969	92	47126.71073467	0.24025287
1969	93	35804.38772487	0.25762114
1969	94	26580.42046522	0.27576021
1969	95	19250.43854188	0.29465975
1969	96	13576.10912501	0.31426406
1969	97	9310.99744952	0.33453259
1969	98	6196.10537803	0.35541003
1969	99	3993.98602870	0.37683227
1969	100	2488.92321423	0.39877222
1969	101	1496.52176808	0.42101059
1969	102	866.46127830	0.44361447
1969	103	482.08651917	0.46643145
1969	104	257.22620454	0.48937379
1969	105	131.34644251	0.51234670
1969	106	64.05152604	0.53525400
1969	107	29.76769036	0.55800219
1969	108	13.15725384	0.58049901
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